Exercise 2 – Odometry, Dead Reckoning & Error Predictions

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Task 1:

Calculate the changes in the forward direction (Δd) and heading (ΔΘ) of the robot, i.e., the changes expressed in the robot coordinate system caused by the latest movement. Also, calculate the variances (co-variances) of Δd and Δθ.

What uncertainties do you assume?

Uncertainties:

- Odometry: limitations of wheel encoders, gear slippage, non-linearities, and drift due to integration of velocity measurements over time.
- Dead Reckoning: inaccuracies in estimating position based on velocity and heading, non-linear motion, sensor noise, and heading errors.
- Error Predictions: inherent limitations in predicting the future state based on past measurements and system dynamics, environmental changes, and measurement noise.

For calculate the variance (covariance) of Δd and $\Delta\theta$ using the equation 1 and 2 and also 3.

$$1\Big)\ \Delta d = \frac{(\Delta r + \Delta l)}{2},\ 2\Big)\ \Delta\theta = \frac{(\Delta r - \Delta l)}{\text{WheelBase}},$$

$$3\Bigg)\ \Sigma_{\Delta d \Delta\theta} = \begin{bmatrix} \sigma^2_{\Delta d} & 0 \\ 0 & \sigma^2_{\Delta\theta} \end{bmatrix},\ \sigma^2_{\Delta d} = \frac{\sigma_r^2 + \sigma_l^2}{4},\ \sigma^2_{\Delta\theta} = \frac{\sigma_r^2 + \sigma_l^2}{\text{WheelBase}^2}$$

Task 2: Calculate the new state variables of the robot under the assumption that the robot moves according to a circular trajectory (at this point, you could skip the compensation term given in Wang 1988 [1])? At the same time, calculate the covariance matrix of these new positions? You can assume that the robot always starts in origin and with a heading of 90º, i.e. (x0, y0, θ0) = (0, 0, 90*π/180).

No there is no point of having an compensation term according to the paper Wang 1988 [1]. *(See equation 6 for the compensation term.)* Because of if the encoder values are low example 1-10, it will still be 1 and wont make any huge different/changes.

$$4)\ X_k = \begin{bmatrix} X_{k-1} + \Delta\text{dcos}\left(\theta_k + \frac{\Delta\theta}{2}\right) \\ Y_{k-1} + \Delta\text{dsin}\left(\theta_k + \frac{\Delta\theta}{2}\right) \\ \theta_{k-1} + \Delta\theta \end{bmatrix}, J_{\Delta d \Delta\theta} = \begin{bmatrix} \text{Cos}\left[\frac{\Delta\theta}{2} + \theta_k\right] & -\frac{1}{2}\ \Delta\text{d Sin}\left[\frac{\Delta\theta}{2} + \theta_k\right] \\ \text{Sin}\left[\frac{\Delta\theta}{2} + \theta_k\right] & \frac{1}{2}\ \Delta\text{d Cos}\left[\frac{\Delta\theta}{2} + \theta_k\right] \\ 0 & 1 \end{bmatrix}, J^T_{\Delta d \Delta\theta} = \begin{bmatrix} \text{Cos}\left[\frac{\Delta\theta}{2} + \theta_k\right] & \text{Sin}\left[\cdots \right. \\ -\frac{1}{2}\ \Delta\text{d Sin}\left[\frac{\Delta\theta}{2} + \theta_k\right] & \frac{1}{2}\ \Delta\text{d Co}\cdots \end{bmatrix}$$

$$5)\ \Sigma_{X_k} = J_{X_{k-1}} \Sigma_{X_{k-1}} J^T_{X_{k-1}} + J_{\Delta d \Delta\theta} \Sigma_{\Delta d \Delta\theta} J^T_{\Delta d \Delta\theta}$$

$$6)\ \text{Compensation Term} = \frac{\text{Sin}\left(\frac{\Delta\theta}{2}\right)}{\frac{\Delta\theta}{2}}$$

Task 3: Run the entire sequence of encoder but also incorporate the compensation term given in the paper Wang [1]?

After the compensation term have been incorporated. This is the function: $\frac{\text{Sin}\left(\frac{\Delta\theta}{2}\right)}{\frac{\Delta\theta}{2}} \Delta\text{dcos}\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right)$

$$7)\ \Delta x = \Delta\text{dcos}\left(\theta_{k-1} + \frac{\Delta\theta}{2}\right)$$

Are the estimated state variables and covariance matrices the same?

Answer: There is a small difference between incoperating the term versus not, we save the difference in a matrix called "ct" in ex2_khepera.m. When we ran 1:1 iterations, ct was equal to 1 but with fewer iterations it will approximate accordingly under 1. When zooming into a thousands of a millimeter you'll see a difference (see figure 1).

If you read the encoder values less often, i.e. if you read them 2, 5 or maybe ten times as seldom, what will happen to the state variables? When do the state variables start to differ – and why? (Hint: Check, e.g., the Δd and Δθ values).

Answer: If the encoder values are read less often the varibles will become less accurate. They already start changing after changing the read to 2, but it's a very small difference. At 5 you can start to see a little bigger difference since some of the "ct" values will stop being aproximated as 1, instead slightly bellow. At 10 almost all the the "ct" values will have changed and the change in the variables is obvious.

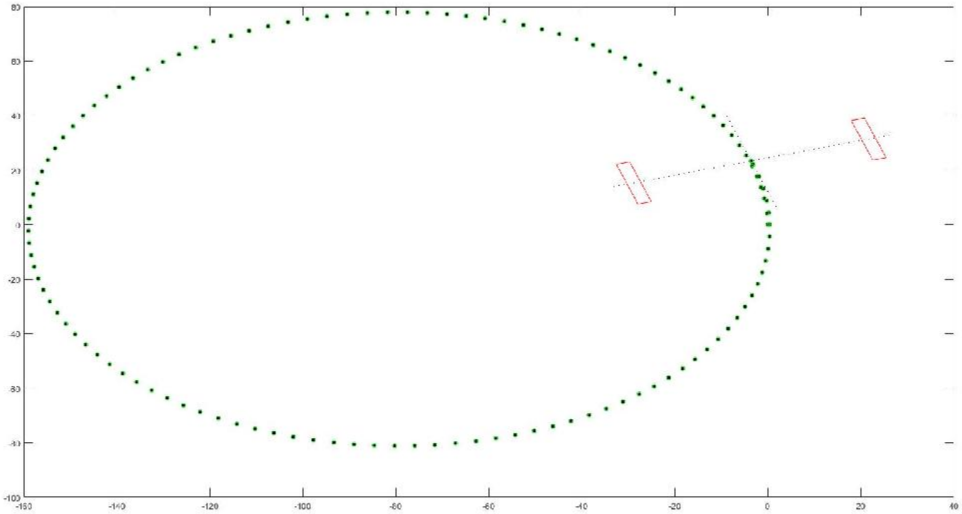

Figure 1. Here you can see the plot of both the khepera with and without compensation term, the black plot is the one without and the one with is the green one.

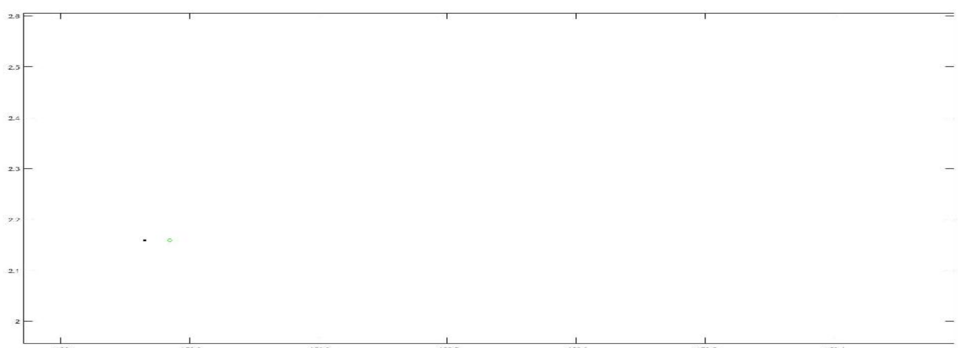

Figure 2. This figure shows the difference between having a compensation term and dont. The encoder values are 1:10. The scale of figure is 1:100.

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Task 4: How are the covariance matrices evolving during the run (plot the entire run together with the calculated covariance matrices)? You don't have the true values of the state variables (a common problem in the mobile robot community) – but still, is the uncertainties realistic?

Answer: Since the uncertanty in the new position is based on the previous state the error will grow after each step if there is no correction of the position. It is realistic that the values are somewhat uncertain since when working with the real world you never know the actual value of anything, everything is an approximation. The better the approximation the closer the values are to the actual value but it will never be 100% accurate. Well since the uncertainties are growing in north and south direction, I am not really sure what that depends on even if it far or close to the starting position. But yes they could be realistic.

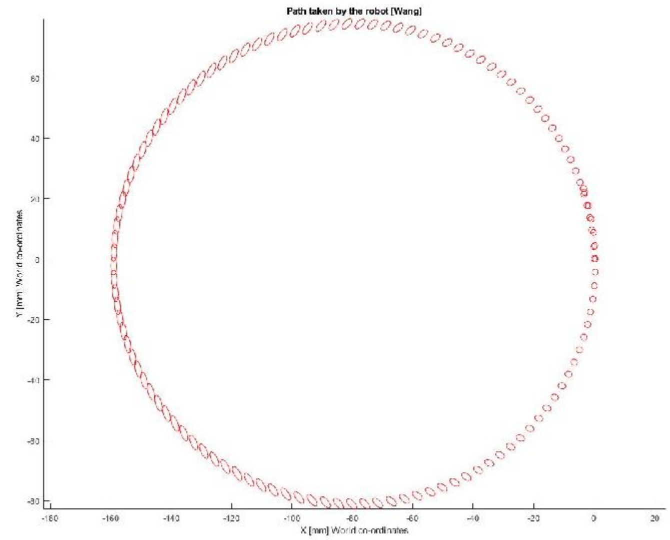


Figure 3. This is a plot of the uncertianty of khepera with encoder values of 1:10

Task 5: Once more, do the same run but use worse known values of the wheel diameter and the wheelbase, e.g., D = 14mm and WB = 45mm? How does it affect the estimated state variables? Plot the trajectories in the same plot as the other trajectories! **What assumptions on these errors would you make?**

According the code we used the diameter of the wheel equals 14 mm and wheel base 45 mm.

$\mathrm{cB} = 0.1$ (The measurement error/estimation error of the wheelbase itself.), $k = 0.01^2$ (kr + kl)(Velocity accelerometer misestin

Answer: The new, smaller values of the wheel diameter and wheelbase will affect the accuracy of the measurements obtained from the IMU(Inertial Measurement Unit). The smaller size of the wheels and wheelbase will result in errors in the variables, despite the fact that the same encoder is being used for the same robot. This is because the encoder was designed for a larger set of wheel and wheelbase dimensions, and its measurements may not be precisely applicable to the new, smaller dimensions.

Answer: Errors in the wheel diameter and wheelbase measurements can greatly impact the accuracy of state estimation. Smaller values result in underestimated position and velocity due to the vehicle traveling a shorter distance per wheel revolution. These measurements are crucial inputs in the estimation process and any errors will affect the outcome. To improve accuracy, it's recommended to use manufacturer specifications and incorporate additional sensors like encoders or an IMU.

$$8)\ X_k = \begin{bmatrix} X_{k-1} + \frac{\Delta r + \Delta l}{2}\cos\left(\theta_{k-1} + \frac{\Delta r + \Delta l}{2b}\right) \\ Y_{k-1} + \frac{\Delta r + \Delta l}{2}\sin\left(\theta_{k-1} + \frac{\Delta r + \Delta l}{2b}\right) \\ \theta_{k-1} + \frac{\Delta r + \Delta l}{b} \end{bmatrix},$$

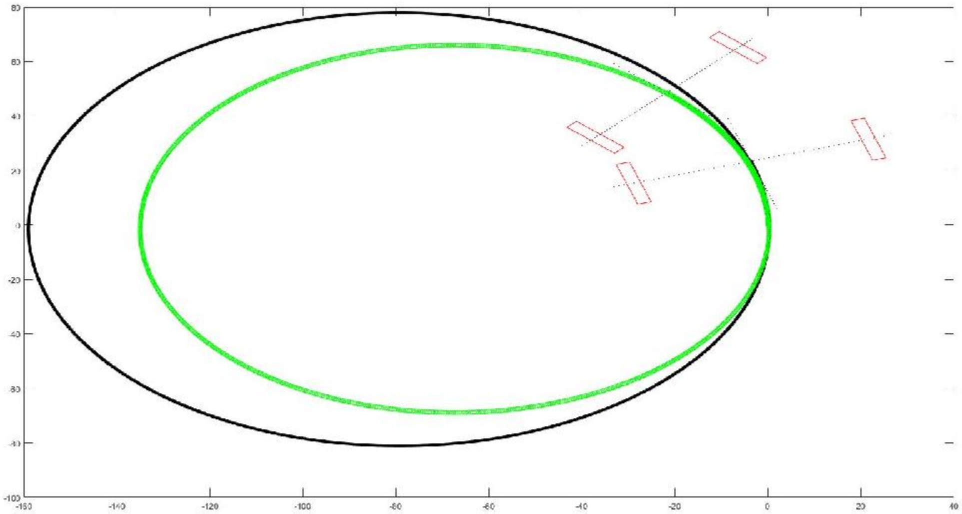
$$J_{\Delta r\Delta l} = \begin{bmatrix} \frac{1}{2}\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right] - \frac{(\Delta L + \Delta R)\,\mathrm{Sin}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b} & \frac{1}{2}\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right] + \frac{(\Delta L + \Delta R)\,\mathrm{Sin}\left[\frac{-\Delta L + \Delta \cdots}{2\,b} \cdots\right.}{4\,b} \cdots \\ \frac{(\Delta L + \Delta R)\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b} + \frac{1}{2}\,\mathrm{Sin}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right] & -\frac{(\Delta L + \Delta R)\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b} + \frac{1}{2}\,\mathrm{Sin}\left[\frac{-\Delta L + \cdots}{2\,b} \cdots\right. \\ \frac{1}{b} & -\frac{1}{b} \end{bmatrix}$$

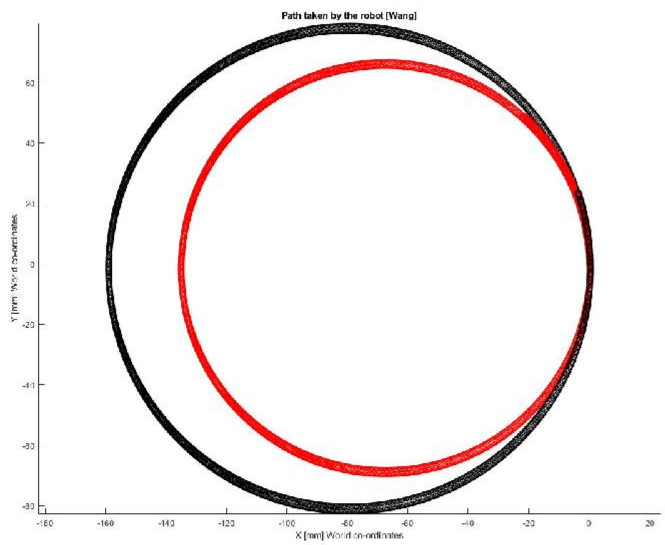
$$J^T_{\Delta r\Delta l} = \begin{bmatrix} \frac{1}{2}\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right] - \frac{(\Delta L + \Delta R)\,\mathrm{Sin}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b} & \frac{(\Delta L + \Delta R)\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b} + \frac{1}{2}\,\mathrm{Sin}\left[\frac{-\Delta L + \cdots}{2\,b} \cdots\right. \\ \frac{1}{2}\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right] + \frac{(\Delta L + \Delta R)\,\mathrm{Sin}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b} & -\frac{(\Delta L + \Delta R)\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b} + \frac{1}{2}\,\mathrm{Sin}\left[\frac{-\Delta L + \cdots}{2\,b} \cdots\right. \end{bmatrix}$$

$$J_b = \begin{bmatrix} \frac{(-\Delta L + \Delta R)\,(\Delta L + \Delta R)\,\mathrm{Sin}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b^2} \\ -\frac{(-\Delta L + \Delta R)\,(\Delta L + \Delta R)\,\mathrm{Cos}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b^2} \\ -\frac{-\Delta L + \Delta R}{b^2} \end{bmatrix}, J_b^T = \left[\frac{(-\Delta L + \Delta R)\,(\Delta L + \Delta R)\,\mathrm{Sin}\left[\frac{-\Delta L + \Delta R}{2\,b} + \theta_k\right]}{4\,b^2} \quad -\frac{(-\Delta L + \Delta \cdots}{\cdots} \cdots\right.$$

$$9)\ \Sigma_{X_k} = J_{X_{k-1}}\Sigma_{X_{k-1}}J^T_{X_{k-1}} + J_{\Delta r\Delta l}\Sigma_{\Delta r\Delta l}J^T_{\Delta r\Delta l} + J_b\Sigma_b J_b^T$$

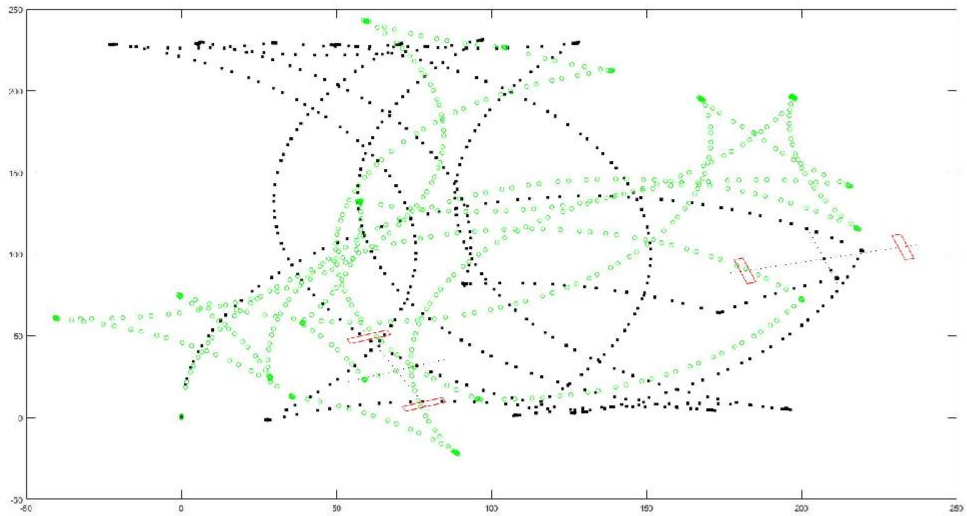
$$10)\ \Sigma_b = 0.1, \Sigma_{\Delta r\Delta l} = \begin{bmatrix} k\ \mathrm{abs}(\Delta r) & 0 \\ 0 & k\ \mathrm{abs}(\Delta l) \end{bmatrix}$$

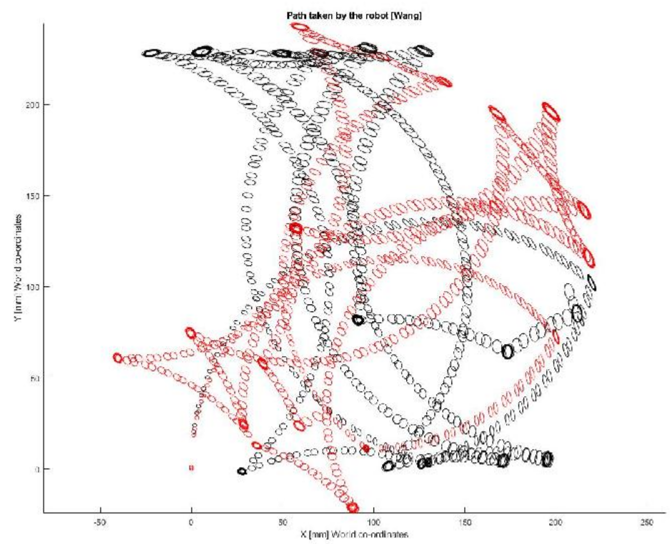




Task 6: Repeat the experiment in task 5 but use the data file 'khepera.txt'? Plot the trajectories in the same plot as the other trajectories, i.e. different wheelbase and wheel diameter!

Answer:





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Task 7: Calculate the new state variables of the robot? Also, calculate the covariance matrix of these new positions? (To do this, you have to derive the Jacobian matrices with respect to the uncertain parameters. Follow the example given in the Wang paper [1] or chapter 5 in the textbook [2].) The robot always starts in the first ground truth position and with the first ground truth heading, i.e. (x0, y0, θ0) = (TrueX(0), TrueY(0), Trueθ(0)).

Answer:

For task 7, 8 and 9 we use the Σ_{vaT} where k = 0.01^2. Because of an 1% wrong error should be realistic.

$$10)\; X_k = \begin{bmatrix} X_{k-1} + v\cos(a)\,T\cos\left(\theta_{k-1} + \frac{v\sin(a)\,T}{2L}\right) \\ Y_{k-1} + v\cos(a)\,T\cos\left(\theta_{k-1} + \frac{v\sin(a)\,T}{2L}\right) \\ \theta_{k-1} + \frac{v\sin(a)\,T}{L} \end{bmatrix},$$

$$J_{\mathrm{vaT}} = \begin{bmatrix} T\,\mathrm{Cos}\left[\alpha\right]\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right] - \frac{T^2\,v\,\mathrm{Cos}\left[\alpha\right]\mathrm{Sin}\left[\alpha\right]\mathrm{Sin}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]}{2\,L} & -T\,v\,\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]\mathrm{Sin}\left[\alpha\right] - \frac{T^2}{} \\ \frac{T^2\,v\,\mathrm{Cos}\left[\alpha\right]\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]\mathrm{Sin}\left[\alpha\right]}{2\,L} + T\,\mathrm{Cos}\left[\alpha\right]\mathrm{Sin}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right] & \frac{T^2\,v^2\,\mathrm{Cos}[\alpha]^2\,\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]}{2\,L} - T\,v \\ \frac{T\,\mathrm{Sin}[\alpha]}{L} & \frac{T\,v\,\mathrm{Cos}[\alpha]}{L} \end{bmatrix}$$

$$J^T_{\mathrm{vaT}} = \begin{bmatrix} T\,\mathrm{Cos}\left[\alpha\right]\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right] - \frac{T^2\,v\,\mathrm{Cos}\left[\alpha\right]\mathrm{Sin}\left[\alpha\right]\mathrm{Sin}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]}{2\,L} & \frac{T^2\,v\,\mathrm{Cos}\left[\alpha\right]\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]\mathrm{Sin}\left[\alpha\right]}{2\,L} \\ -T\,v\,\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]\mathrm{Sin}\left[\alpha\right] - \frac{T^2\,v^2\,\mathrm{Cos}[\alpha]^2\,\mathrm{Sin}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]}{2\,L} & \frac{T^2\,v^2\,\mathrm{Cos}[\alpha]^2\,\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]}{2\,L} - \\ v\,\mathrm{Cos}\left[\alpha\right]\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right] - \frac{T\,v^2\,\mathrm{Cos}\left[\alpha\right]\mathrm{Sin}\left[\alpha\right]\mathrm{Sin}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]}{2\,L} & \frac{T\,v^2\,\mathrm{Cos}\left[\alpha\right]\mathrm{Cos}\left[\frac{T\,v\,\mathrm{Sin}[\alpha]}{2\,L} + \theta_k\right]\mathrm{Sin}\left[\alpha\right]}{2\,L} \end{bmatrix}$$

$$11)\; \Sigma_{X_k} = J_{X_{k-1}}\Sigma_{X_{k-1}}J^T_{X_{k-1}} + J_{\mathrm{vaT}}\Sigma_{\mathrm{vaT}}J^T_{\mathrm{vaT}}$$

$$12)\; \Sigma_{\mathrm{vaT}} = \begin{bmatrix} k\,\mathrm{abs}(v) & 0 & 0 \\ 0 & k\,a & 0 \\ 0 & 0 & k\,T \end{bmatrix}$$

Task 8: Run the entire sequence of speeds and steering angles and compare the estimated state variables to the ground truth values. Also, compare the error in the state variable estimates, i.e., the difference between the estimated state variable and the ground truth state variables, to the standard deviations of the estimated variances (square root of the diagonal elements in the covariance matrix). What errors do you assume in the steering angle and the speed? Make sure the estimated standard deviations (the uncertainty of the estimated state variables) stay close to the error of the state variables.

Answer: In a steer-drive robot, the accuracy of the steering angle and speed measurement can be affected by various sources of error, including **sensor error** (e.g. due to inherent error in readings from encoders), **mechanical error** (e.g. due to slippage, wear and tear, or mechanical misalignment of the steering mechanism, drive train, or wheels), **electromagnetic interference** (e.g. from motors or other electronic devices), **software** error (e.g. due to bugs or other issues in the software used to control the robot and process the sensor readings), and **environmental conditions** (e.g. temperature, humidity, or vibrations).

Task 9: Don't forget to plot the error ellipses along the path taken by the robot? How long (distance and time) is the path?

Answer: For calculate the time, we take

$$\text{Time} = \text{samplings time} * \text{number of sampling}$$
$$\text{Time} = 0.050 * 4050 = 202.5 \text{ sec}$$

The average of V = adding all the different V and divide those with number of samplings.

$$S = V(\text{Average}) * T = ((723140/1000)/4050)x202.5 = 36.2m$$

An alternative way for calculate the robots distance is with pythagoras with dx and dy and then adding those two lines between the samplings witch equals 3.18 x 10^4 mm etc.,3.18 x 10^1 = 31.8m