Recurrent Neural Networks (RNN)

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Recurrent Neural Network (RNN)

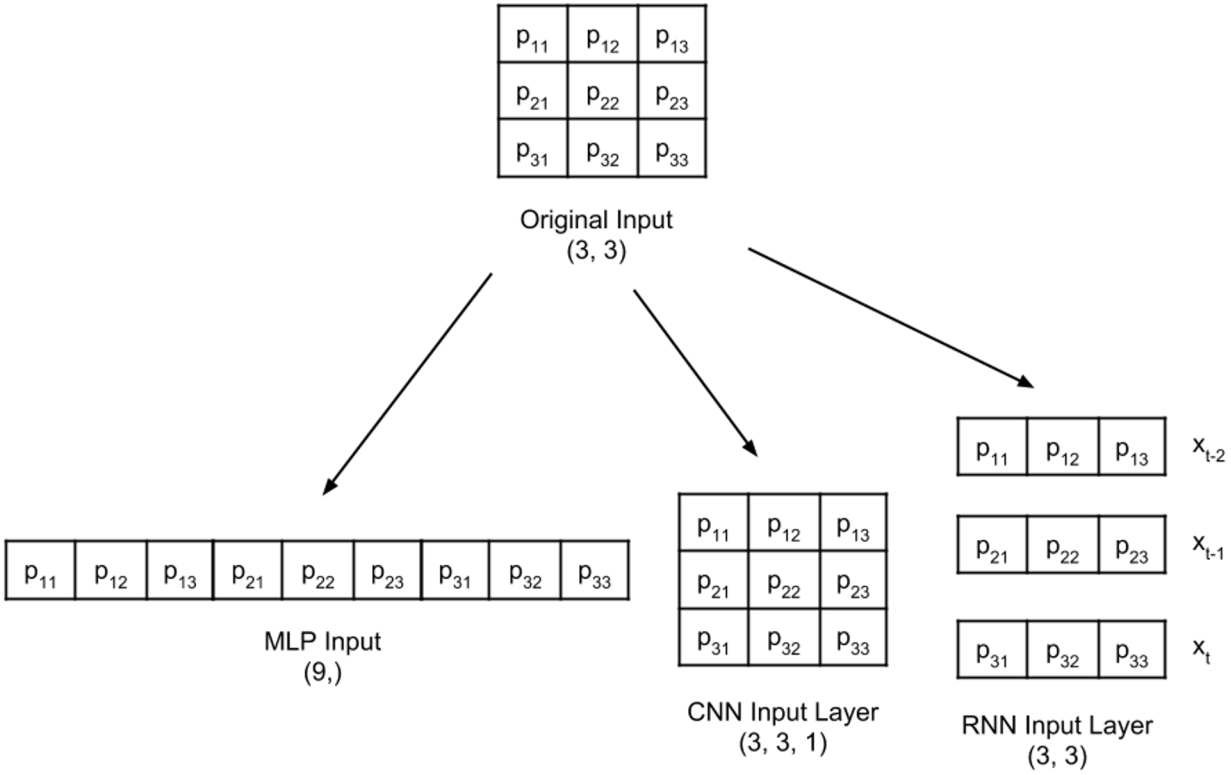
Specializes in processing sequential data: $\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots \boldsymbol{x}_t$

Sequence of words : the quick brown fox jumps over the lazy dog (NLP)

Sequence of signals : series of quantized audio signal for speech to text (ASR)

Sequence of pixels : 2D image can be converted into 1D array of pixels

An image can be interpreted in 3 different ways

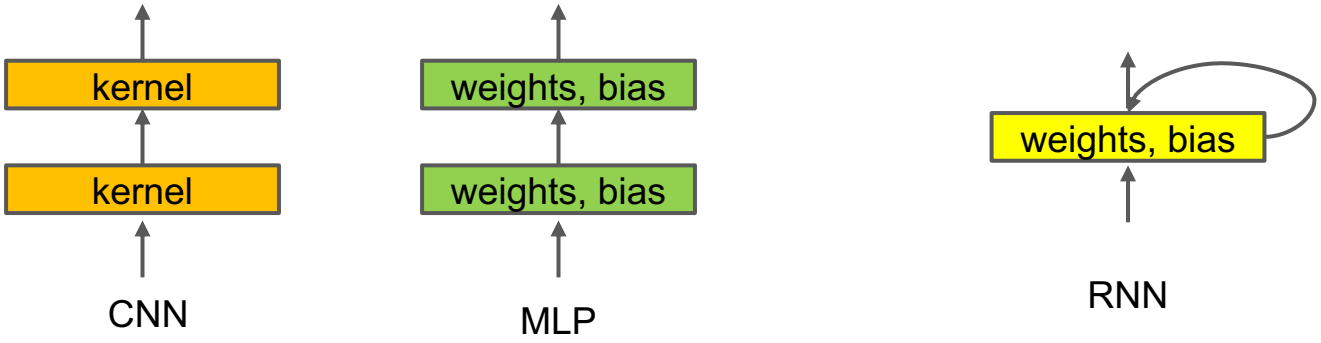


Recurrent Neural Network

One difference between RNN and CNN and MLP is parameters to be learned are shared among inputs

$x_0, x_1, .. x_t$ share the same set of parameters over the sequence

CNN and MLP try to learn a set parameters per feature



Recurrent Neural Network

Specializes in processing sequential data: $\boldsymbol{x}_0, \boldsymbol{x}_1, .. \boldsymbol{x}_t$

In MLP and CNN, $\boldsymbol{x}_0, \boldsymbol{x}_1, .. \boldsymbol{x}_t$ are treated as independent inputs whose features needed to be learned

In RNN, sequential inputs are dependent on one another, $\boldsymbol{x}_0, \boldsymbol{x}_1, .. \boldsymbol{x}_t$

Recurrent Neural Network - Unfolding

Consider a dynamical system with states: $\boldsymbol{s}_0, \boldsymbol{s}_1, \ldots, \boldsymbol{s}_t$

$$\boldsymbol{s}_t = f(\boldsymbol{s}_{t-1}; \boldsymbol{\theta}_{t-1})$$

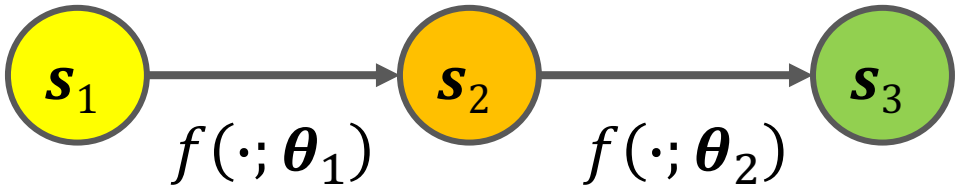
$\boldsymbol{\theta}_{t-1}$ are the parameters at time $t-1$

For example:

$$\boldsymbol{s}_3 = f(\boldsymbol{s}_2;\ \boldsymbol{\theta}_2) = f(f(\boldsymbol{s}_1; \boldsymbol{\theta}_1); \boldsymbol{\theta}_2)$$

This process is called **unfolding**

Recurrent Neural Network - Unfolding in Graphical Model



Recurrent Neural Network - with external signal

The dynamical system can also have external input: $\boldsymbol{x}_t$

$$\boldsymbol{s}_t = f(\boldsymbol{s}_{t-1}, \boldsymbol{x}_{t-1}; \boldsymbol{\theta}_{t-1})$$

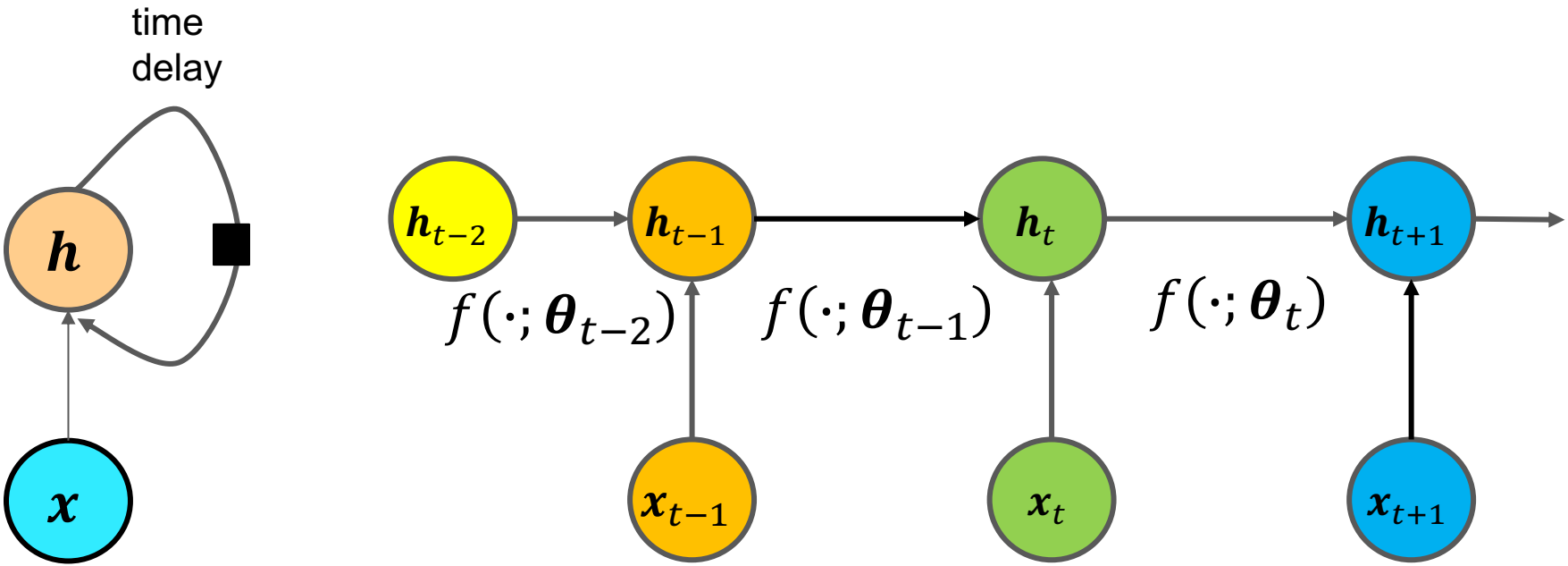
If we use a neural network to store the state, a hidden layer can be modeled as:

$$\boldsymbol{h}_t = f(\boldsymbol{h}_{t-1}, \boldsymbol{x}_{t-1}; \boldsymbol{\theta}_{t-1})$$

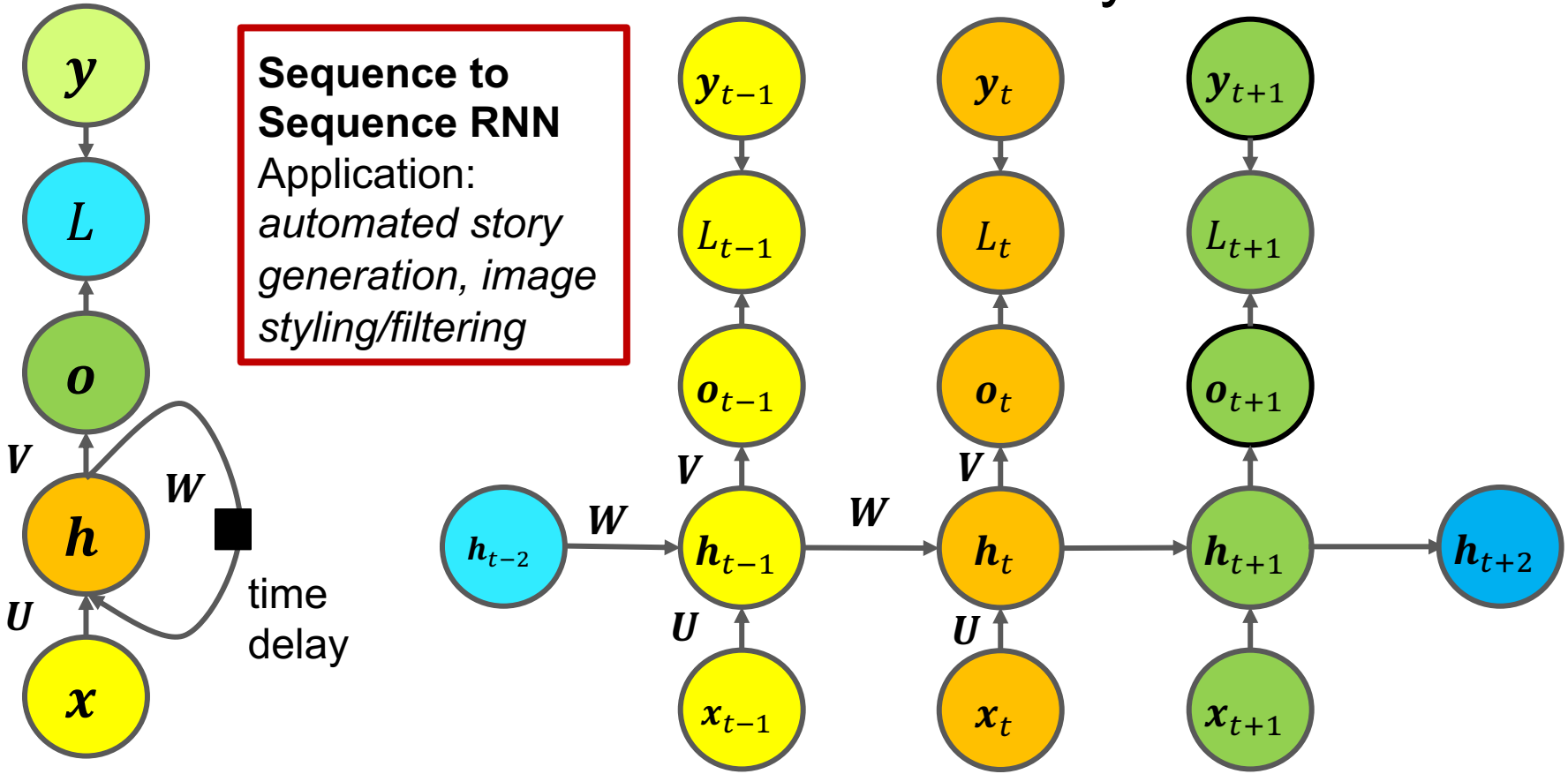
The output at time t is a function of previous output of hidden layer and the current input

$\boldsymbol{h}_t$ encodes (lossy) all previous inputs $\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_{t-1}$

Unfolding of RNN (Graphical Model)



RNN - Recurrence between hidden layers



RNN - Recurrence between hidden layers

$$\boldsymbol{a}_t = \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}_{t-1} + \boldsymbol{U}\boldsymbol{x}_t$$
$$\boldsymbol{h}_t = \tanh(\boldsymbol{a}_t)$$
$$\boldsymbol{o}_t = \boldsymbol{c} + \boldsymbol{V}\boldsymbol{h}_t$$
$$L_t = -\log p(\boldsymbol{y}_t | \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_t\})$$
$$\boldsymbol{y}_t = softmax(\boldsymbol{o}_t)$$

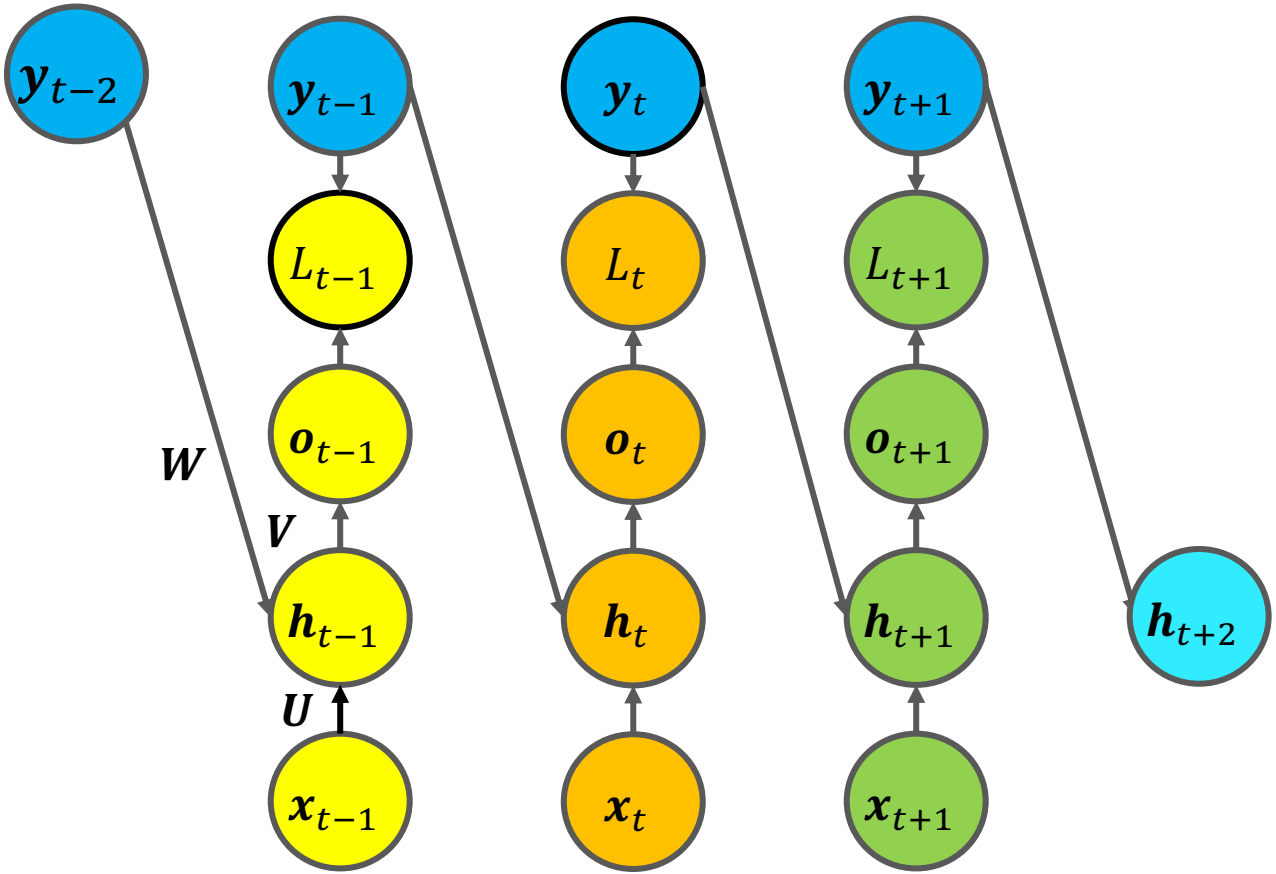
$\boldsymbol{W}$, $\boldsymbol{U}$ and $\boldsymbol{V}$ are parameter matrices

$\boldsymbol{b}$ and $\boldsymbol{c}$ are biases

RNN maximizes the conditional probability to minimize L

$$L = -\log p(\boldsymbol{y}_t | \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_t\})$$

RNN - Teacher forcing (training)

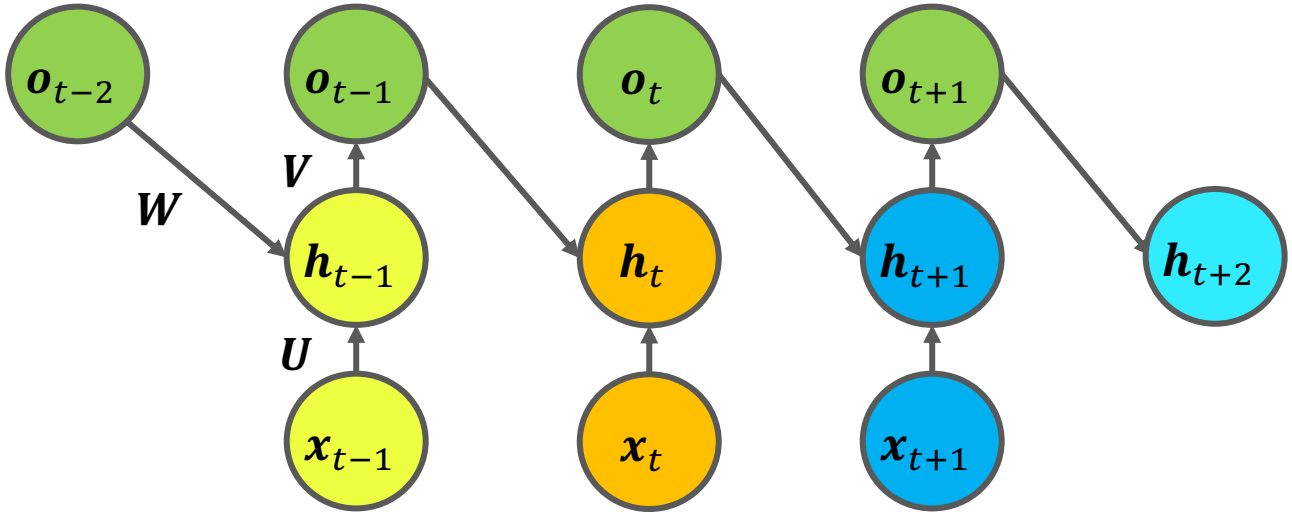


Teacher forcing maximizes the conditional probability to minimize L

$$L = -\log p(\boldsymbol{y}_t, \boldsymbol{y}_{t-1} | \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_t\})$$

$$L = -\log p(\boldsymbol{y}_t | \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_t, \boldsymbol{y}_{t-1}\}) - \log p(\boldsymbol{y}_{t-1} | \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_t\})$$

RNN - Teacher forcing (testing)



Since we have no access to training outputs, we use our predicted outputs

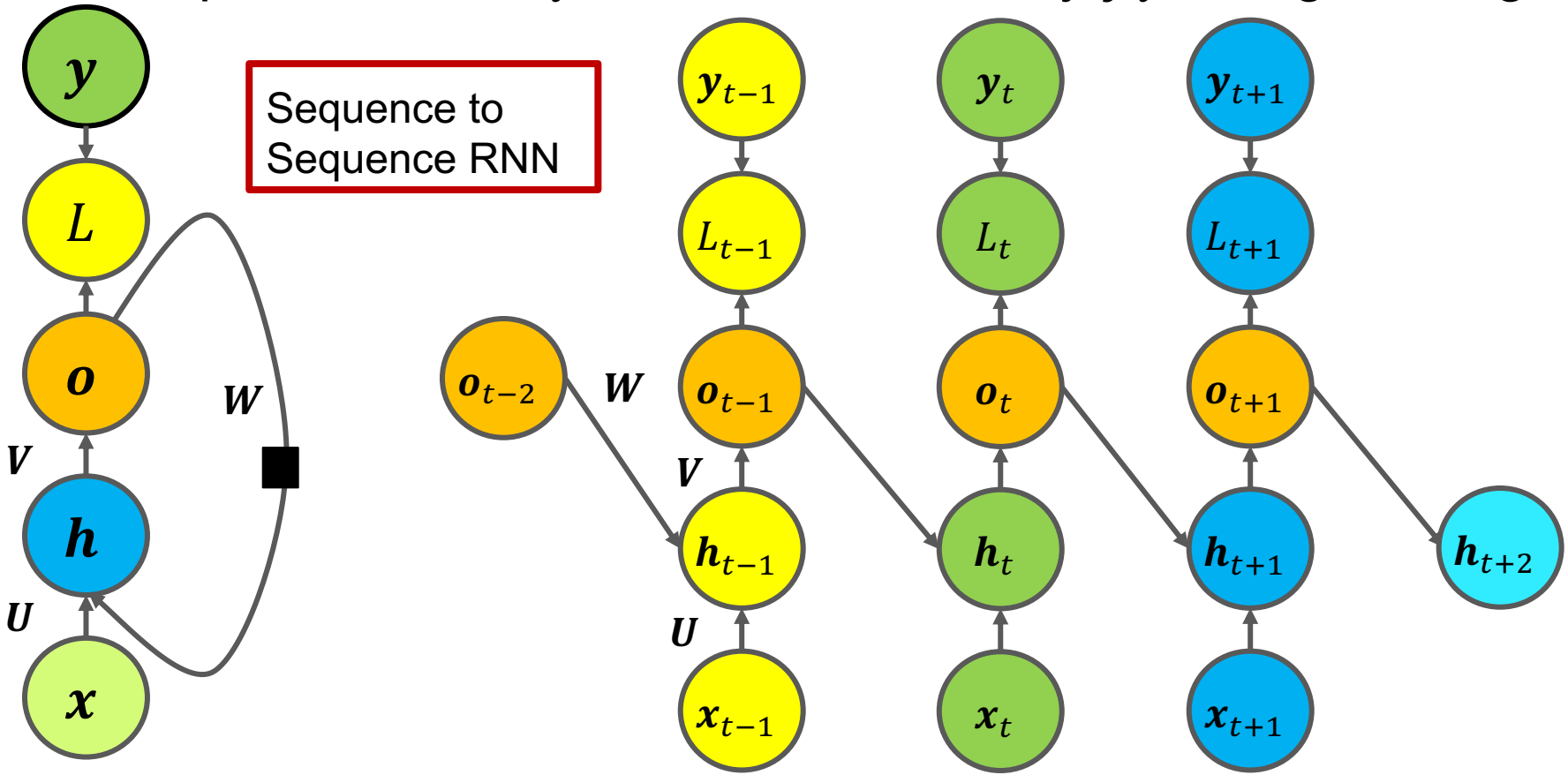
RNN on GPU

RNN is not suitable for parallelization

Before computing $\boldsymbol{y}_t$, we have to wait for $\boldsymbol{y}_{t-1}$

RNN - Recurrence between output & hidden layers

Can be parallelized! $\boldsymbol{o}_t$ can be estimated by $\boldsymbol{y}_t$ during training



RNN - Recurrence between output & hidden layers

$$\boldsymbol{a}_t = \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}_t + \boldsymbol{U}\boldsymbol{x}_t$$
$$\boldsymbol{h}_t = \tanh(\boldsymbol{a}_t)$$
$$\boldsymbol{o}_t = \boldsymbol{c} + \boldsymbol{V}\boldsymbol{h}_t$$
$$L = -\log p(\boldsymbol{y}_t | \{\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_t\})$$
$$\boldsymbol{y}_t = softmax(\boldsymbol{o}_t)$$

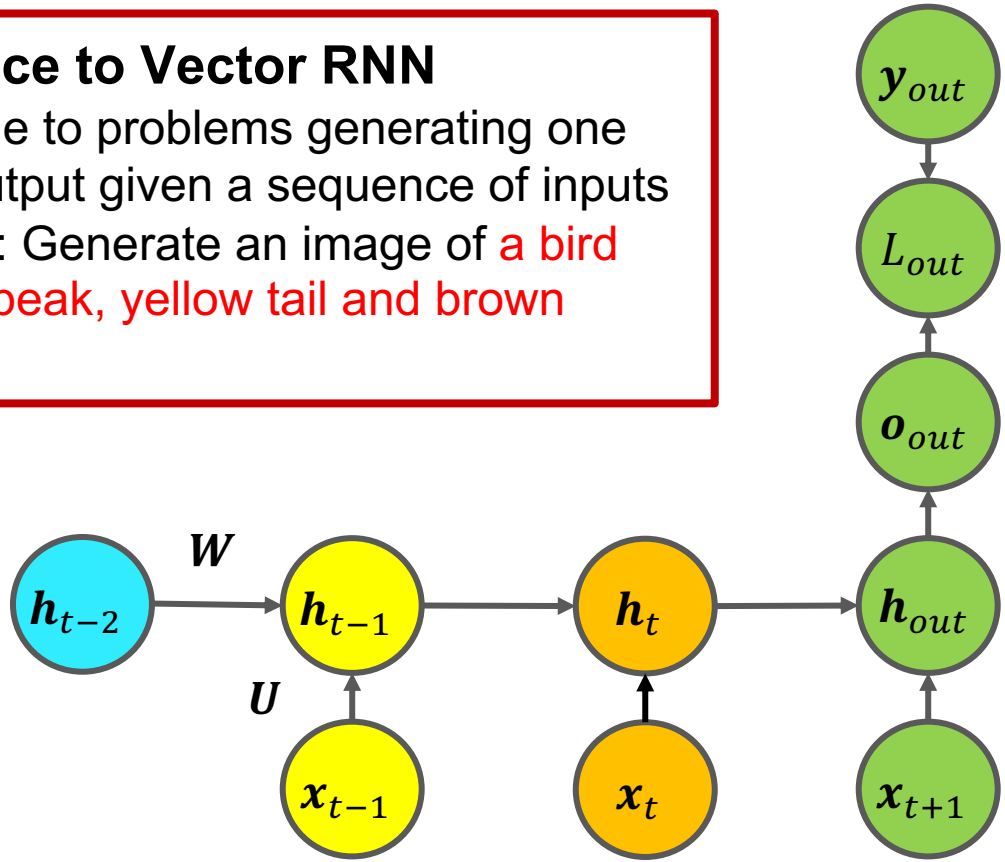
$\boldsymbol{W}$, $\boldsymbol{U}$ and $\boldsymbol{V}$ are parameter matrices

$\boldsymbol{b}$ and $\boldsymbol{c}$ are biases

Exercise: Modify the equation for the preceding figure

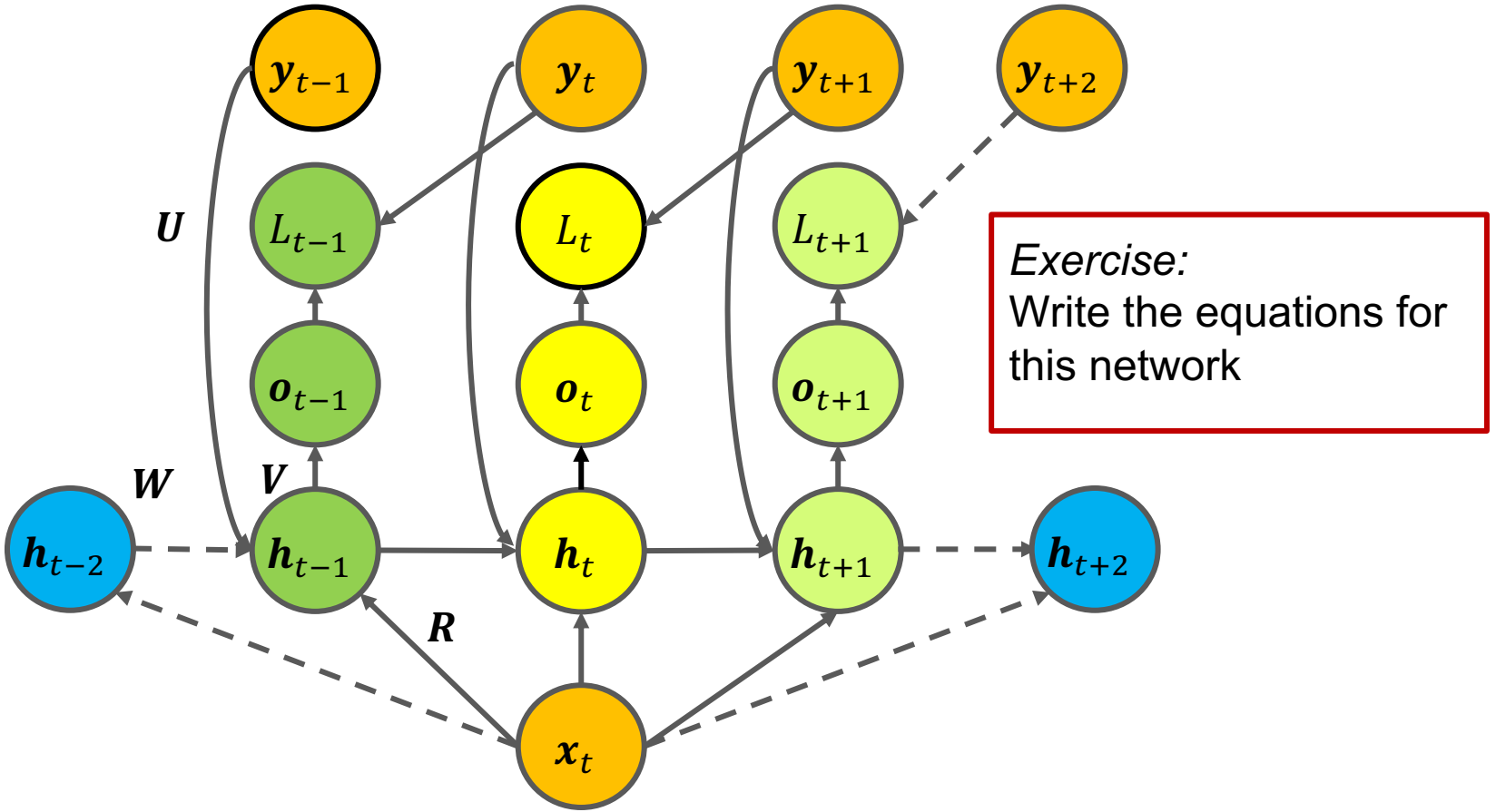
RNN - Recurrence between hidden layers; 1 output

Sequence to Vector RNN
Applicable to problems generating one vector output given a sequence of inputs
Example: Generate an image of a bird with red beak, yellow tail and brown feathers



Exercise:
Write the equations for this network

Vector to Sequence RNN



Vector to Sequence

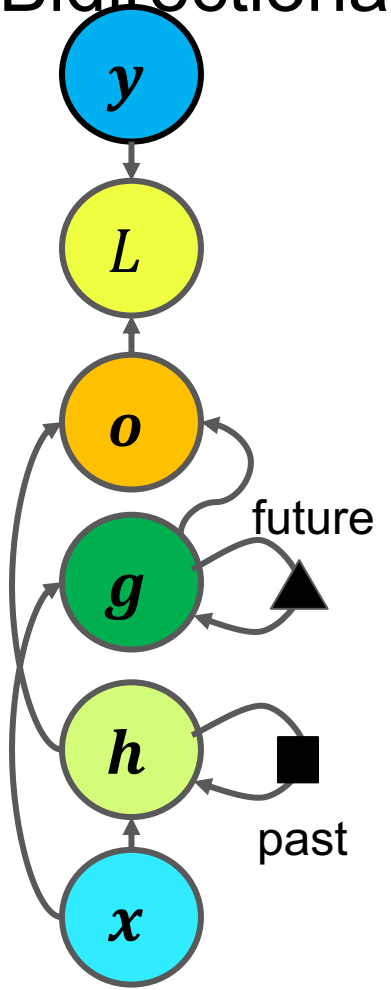
Applicable to problems converting fixed data to a variable length sequence

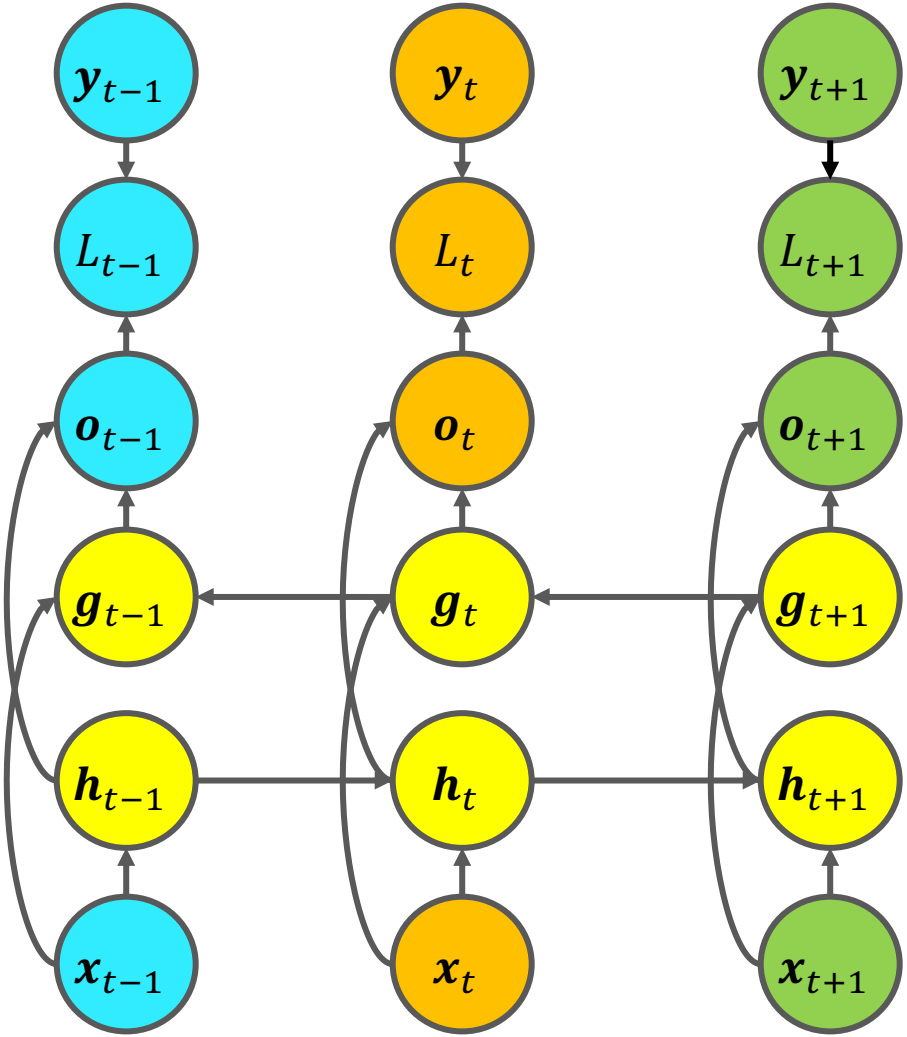
Image (fixed) → Caption (variable)

See http://visualqa.org

for good problems to solve.

Bidirectional RNN





Bidirectional RNN

Prediction is not only dependent on the past but also on the future

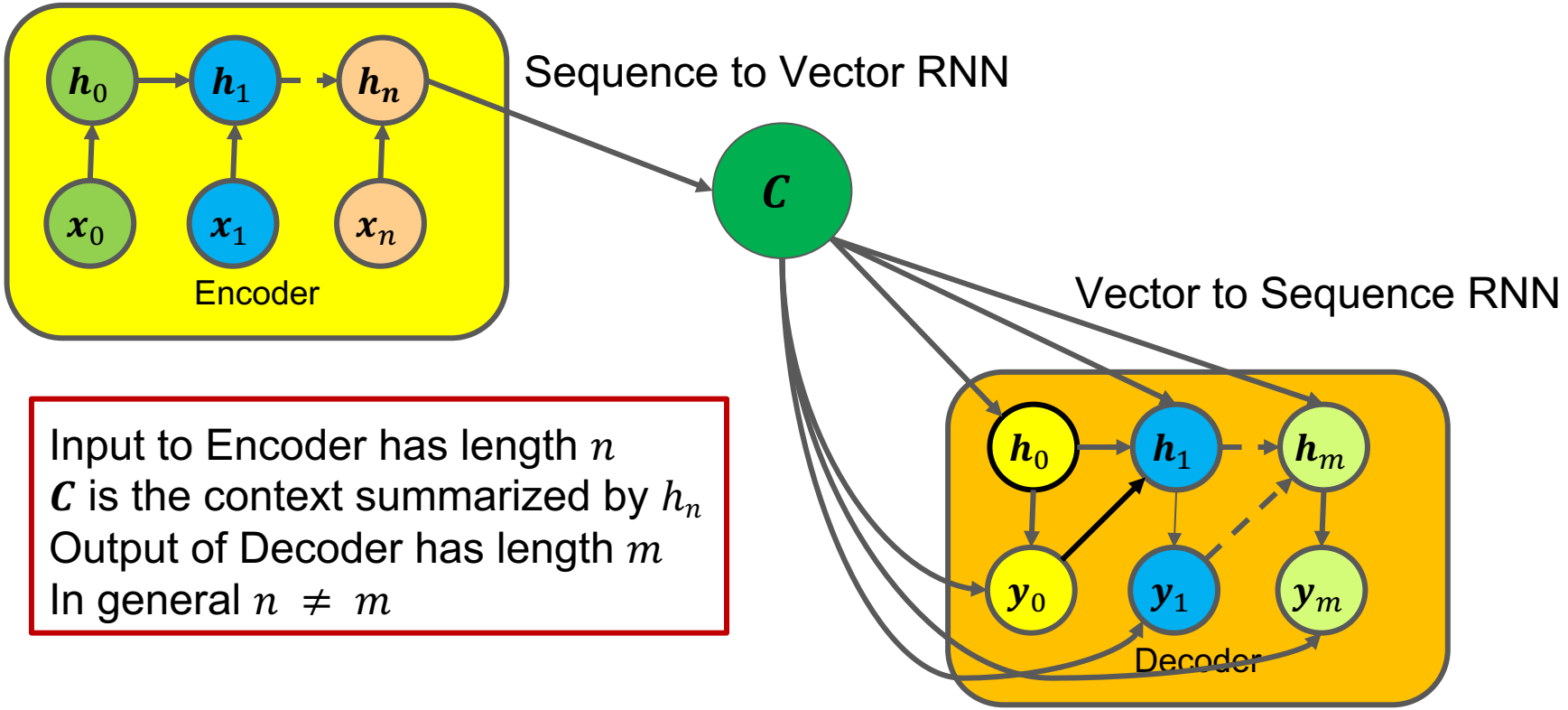
RNN prediction: *bring a golden* → ring

Even if RNN sees the next word is chicken, there is no way to correct ring

Bi RNN prediction: *bring a golden* → spring ← chicken

Applicable to problems where correction/disambiguation on the prediction is needed: Speech to text, Video understanding, NLU

Encoder-Decoder Sequence-to-Sequence



Input to Encoder has length n
C is the context summarized by h_n
Output of Decoder has length m
In general $n \neq m$

Encoder-Decoder Sequence-to-Sequence

A vector, $\boldsymbol{x}$, is encoded into context, $\boldsymbol{C}$. The information in $\boldsymbol{C}$ is used to decode output vector, $\boldsymbol{y}$.

Applicable in problems generating a sequence from a given sequence:

Language translation, speech to text, question-answering, etc

Sequence to Sequence for Machine Translation

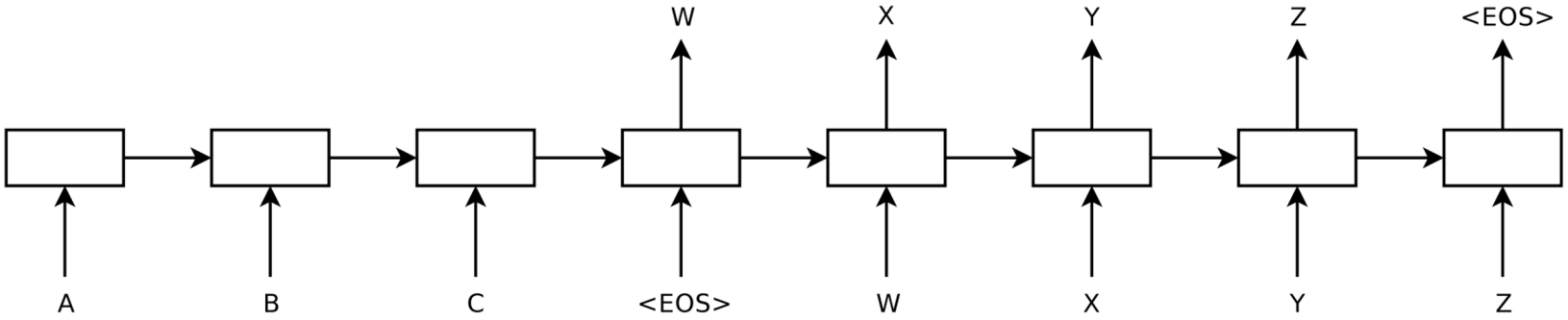
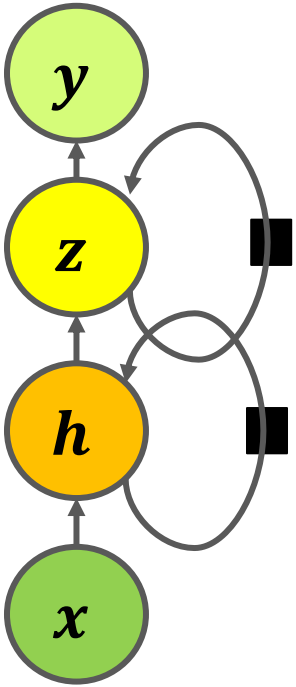


Figure 1: Our model reads an input sentence "ABC" and produces "WXYZ" as the output sentence. The model stops making predictions after outputting the end-of-sentence token. Note that the LSTM reads the input sentence in reverse, because doing so introduces many short term dependencies in the data that make the optimization problem much easier.

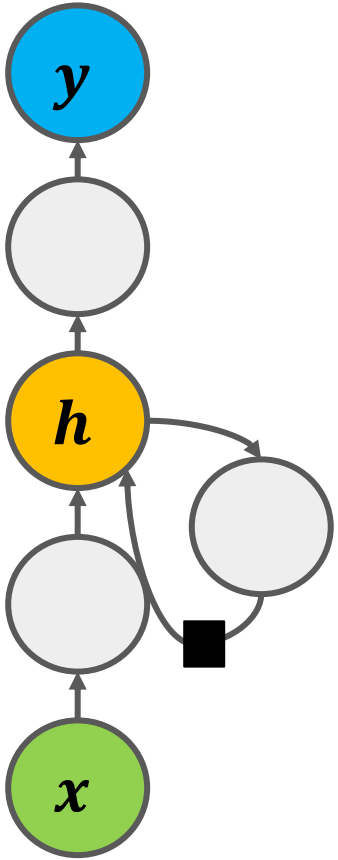
[2015 Sutskever et al Sequence to Sequence Learning with Neural Networks]

Deep RNN

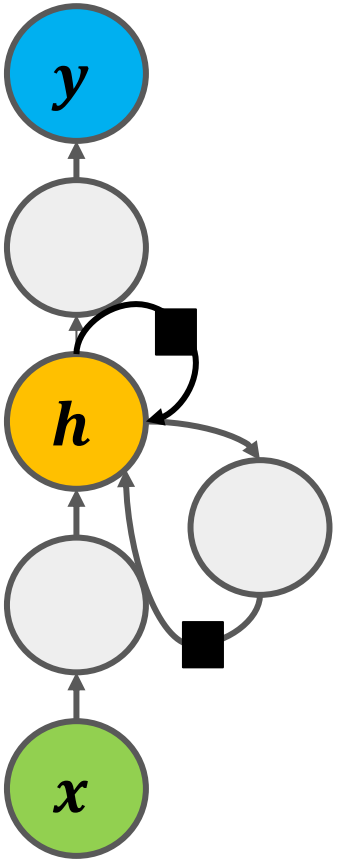
Deep Hidden Layer



MLP in input, hidden and output layers



Skip connection



Problem with Long-Term Dependency

Consider: $\boldsymbol{h}_t = \boldsymbol{W}\boldsymbol{h}_{t-1} = \boldsymbol{W}^t\boldsymbol{h}_0 = \boldsymbol{Q}^T\boldsymbol{\Lambda}^t\boldsymbol{Q}\boldsymbol{h}_0$

As $t \to \infty$, eigenvalues < 1.0 will decay to zero while those > 1.0 will explode in time

Problem with Long-Term Dependency

How to deal with long-term dependency problems

Time-delay e.g. $t \rightarrow t/2$

Skip connections/Remove connections

Leaky Unit: $u_t = \alpha u_{t-1} + (1-\alpha)v_t$ where $\alpha \in [0.0, 1.0]$, u is running average

Gradient clipping

Use LSTM

Gated Recurrent Units (GRU)

Leaky Units and GRU - Values must not vanish nor explode in time

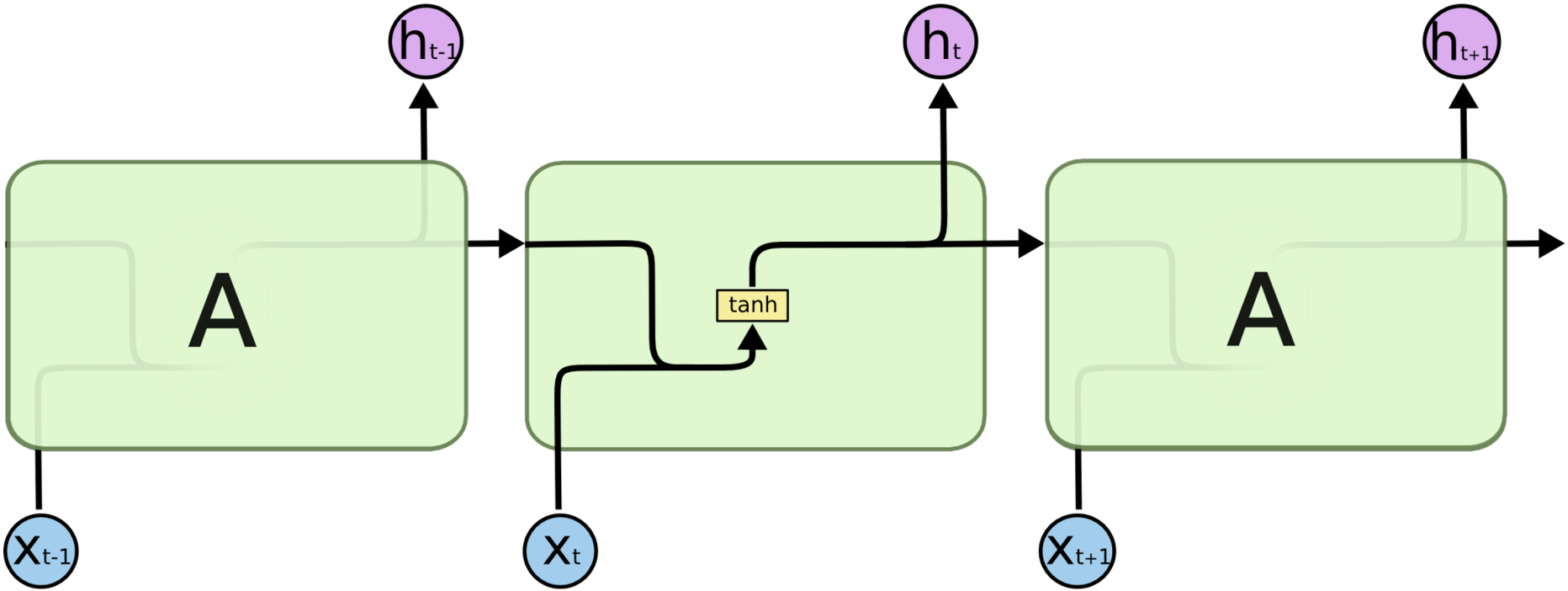
Leaky units - past sequence of states affect the current state; manual setting of weights

GRU - a sequence of states is divided into subsequences. Each new subsequence forgets about the previous subsequence state by using a learned gating mechanism

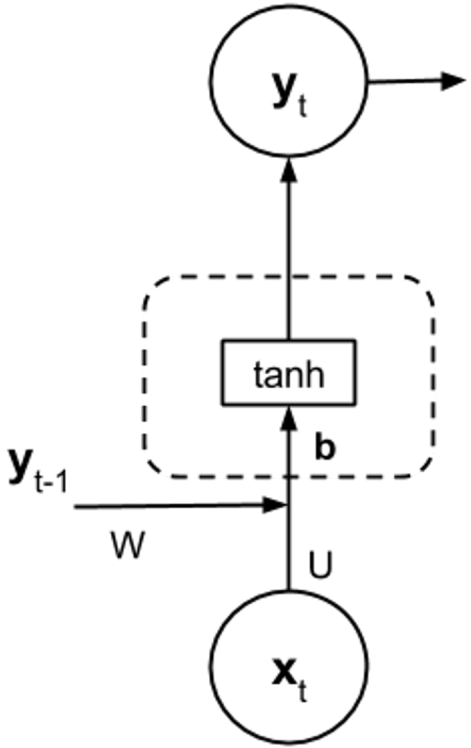
Long Short-Term Memory (LSTM)

RNN Cell [by Chris Olah]

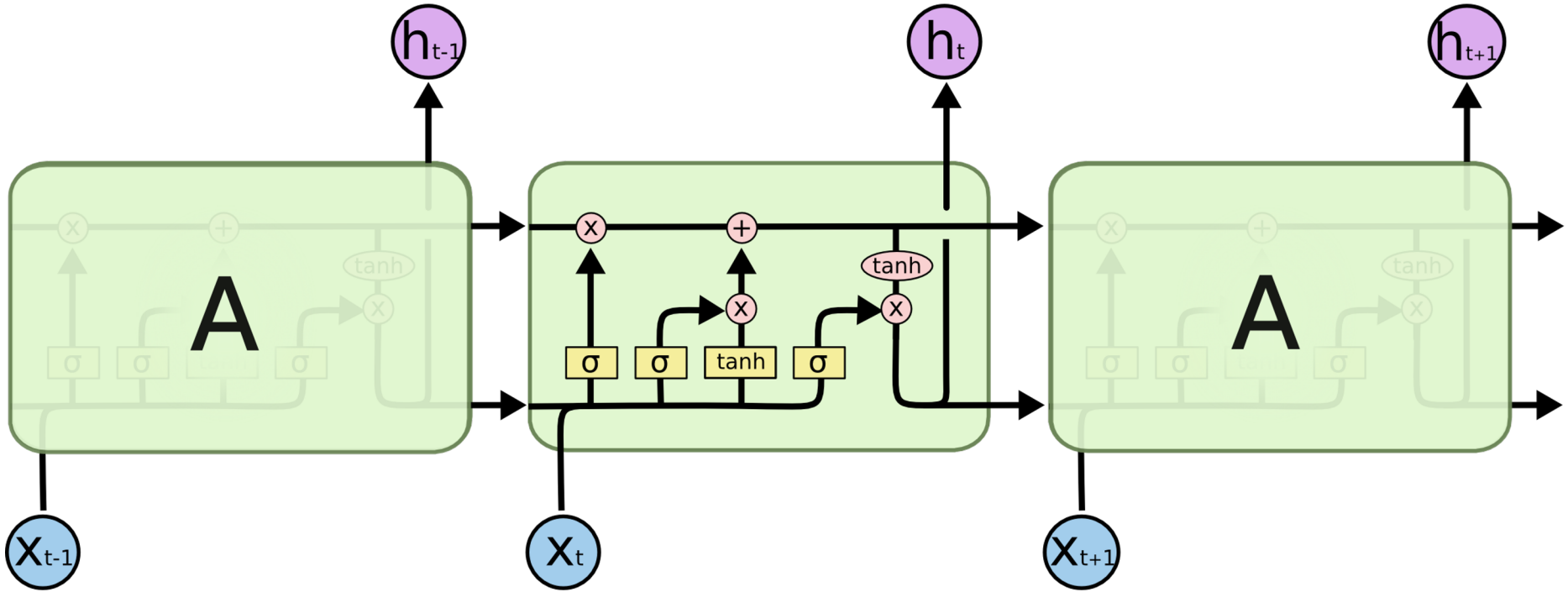
$$\boldsymbol{a}_t = \boldsymbol{b} + \boldsymbol{W}\boldsymbol{h}_{t-1} + \boldsymbol{U}\boldsymbol{x}_t$$
$$\boldsymbol{h}_t = \tanh(\boldsymbol{a}_t)$$



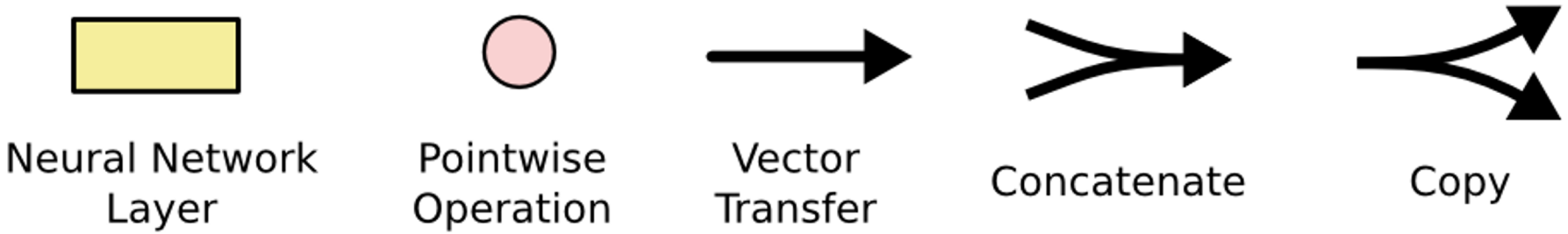
SimpleRNN in Keras



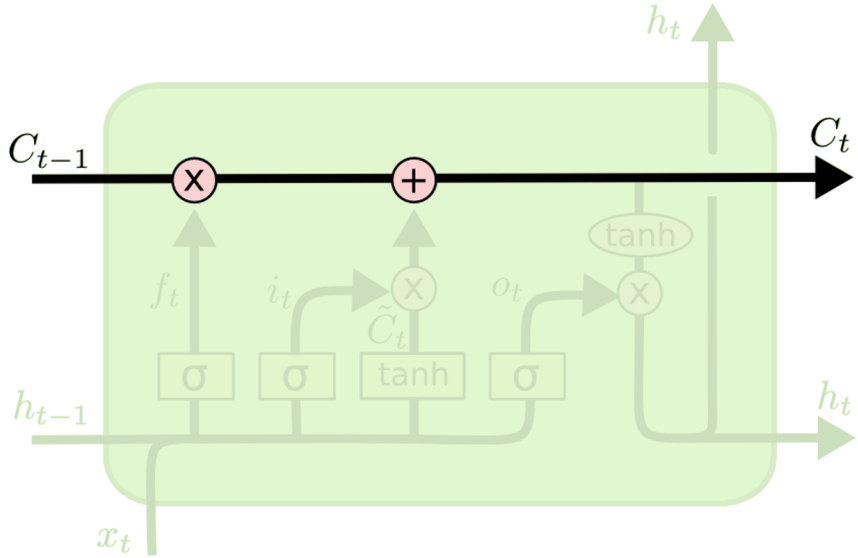
LSTM Cell [by Chris Olah]



LSTM Cell Operators [by Chris Olah]

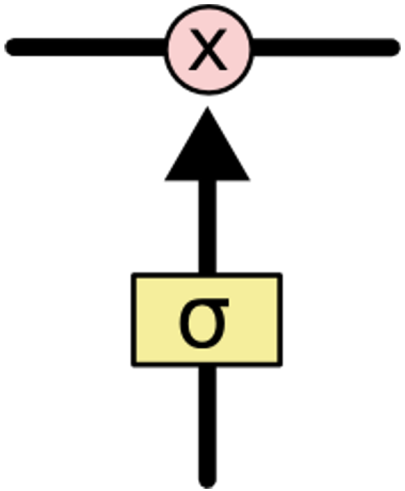


LSTM Cell State Sequence [by Chris Olah]



Cell states are like items in a conveyor belt
Cells states represent the summary of what the network knows so far. It is like the "context" of what we are talking about.
Example: History: "Maria likes summer." Context: "A person's opinion about season"

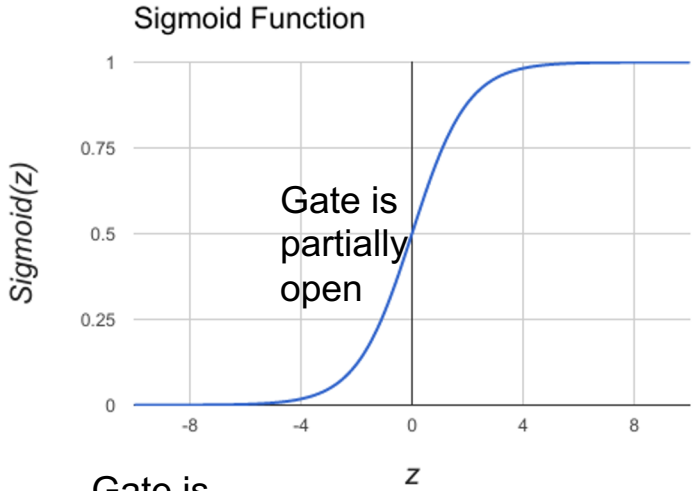
LSTM Cell Sigmoid as Gate [by Chris Olah]



Gate controls the amount of information to pass

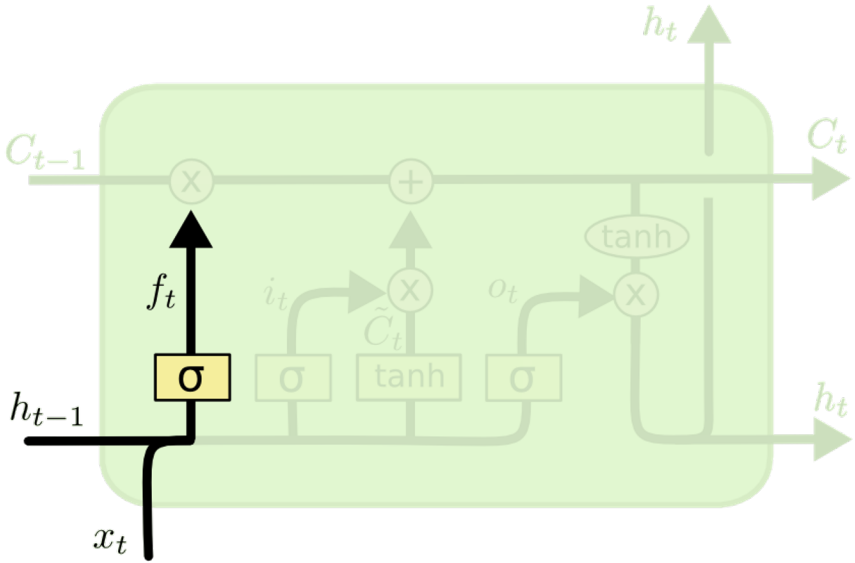
$$\sigma(z) = 1/(1+e^{-z})$$

Gate is open



Gate is closed

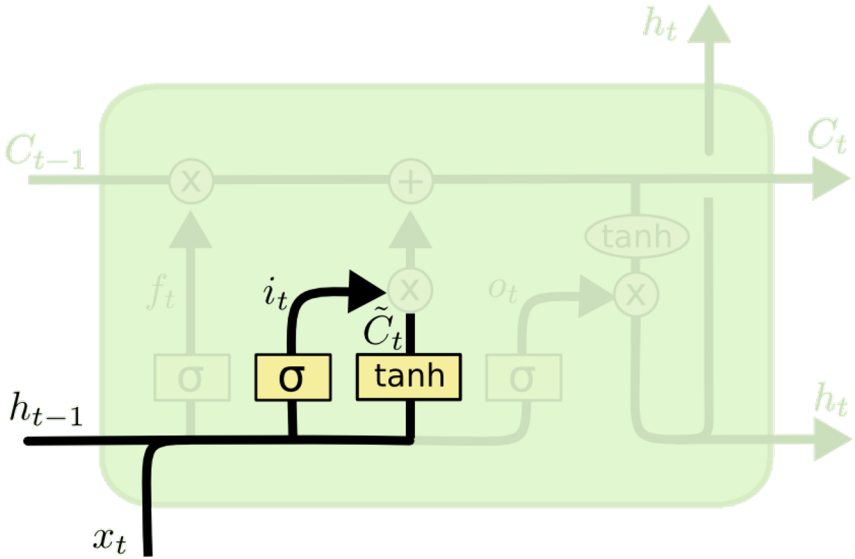
LSTM Cell Gated State [by Chris Olah]



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

Information from previous cell state is filtered by current input and previous hidden layer output (cell output) using forget gate f_t.
Example: “Maria likes summer. She enjoys the beach. Jim likes tennis.”
Meaning: Let’s change our topic from Maria.

LSTM Cell Candidate State Update [by Chris Olah]

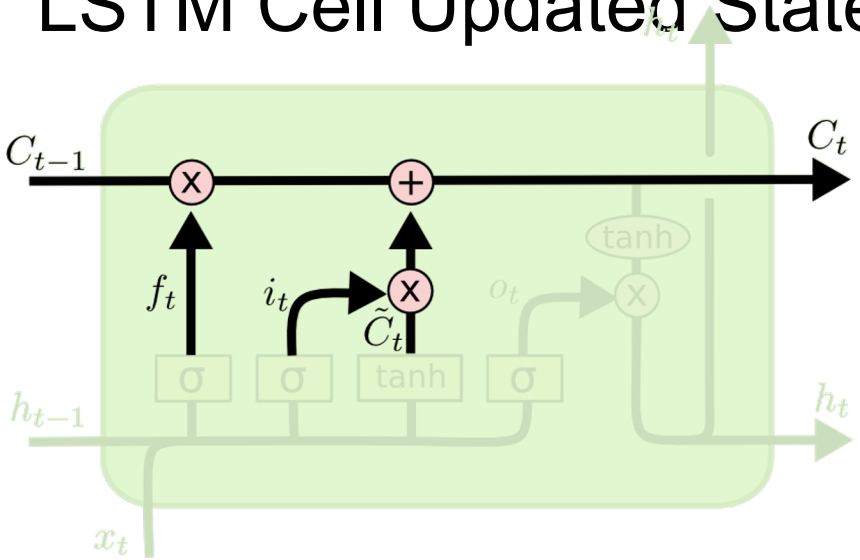


$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \; + \; b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)$$

$\tilde{C}_t$ is the proposed update to the previous state. i_t is another forget gate that filters the proposed update. Both are controlled by present input and previous cell output
Example: “Maria likes summer. She enjoys the beach. Jim likes tennis.”
Meaning: Let’s talk about Jim now.

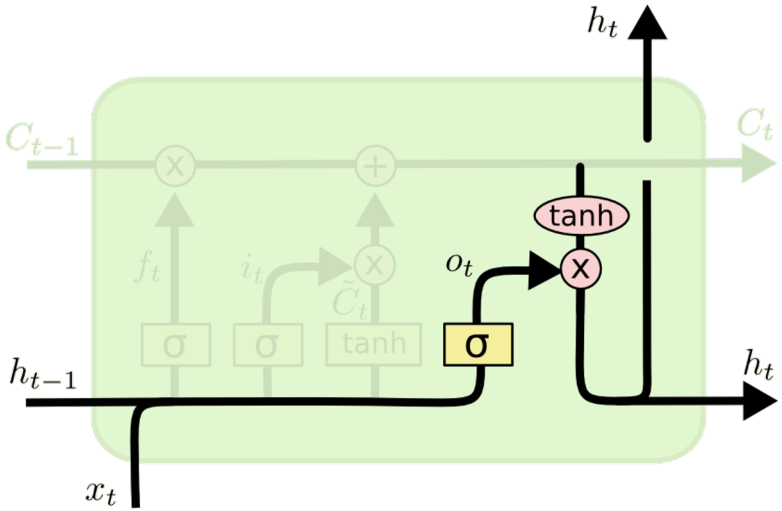
LSTM Cell Updated State [by Chris Olah]



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

The new state, C_t, is the old state with some old information forgotten plus the new proposed state with filtered information.
Example: “Maria likes summer. She enjoys the beach. Jim ...”
Meaning: Context: A person’s opinion on sports.. History is still intact. We have not forgotten what Maria likes.

LSTM Cell Output [by Chris Olah]



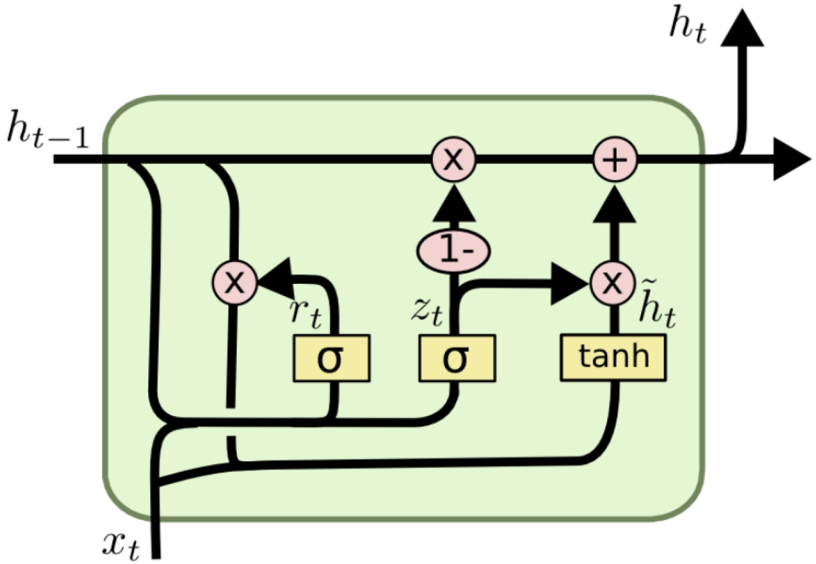
$$o_t = \sigma\left(W_o\ [h_{t-1}, x_t]\ +\ b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

The final cell output is a squashed version of the new state (by applying non-linearity) filtered by another forget gate controlled by past cell output and new input
Example: “Maria likes summer. She enjoys the beach. Jim likes tennis.”
Meaning: If the network generates an image - A girl on a beach, it will start drawing an image of a man playing tennis near the beach.

Variations of LSTM

Simpler LSTM is GRU



$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

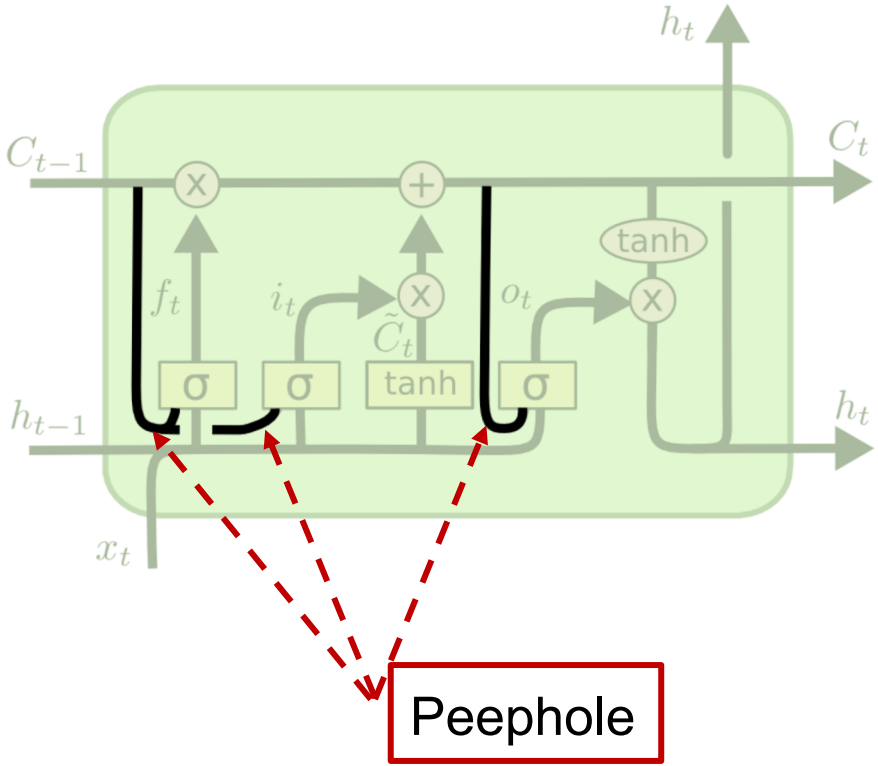
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

Cell and output states are merged. Forget and input gates perform a single update.

LSTM with Peephole

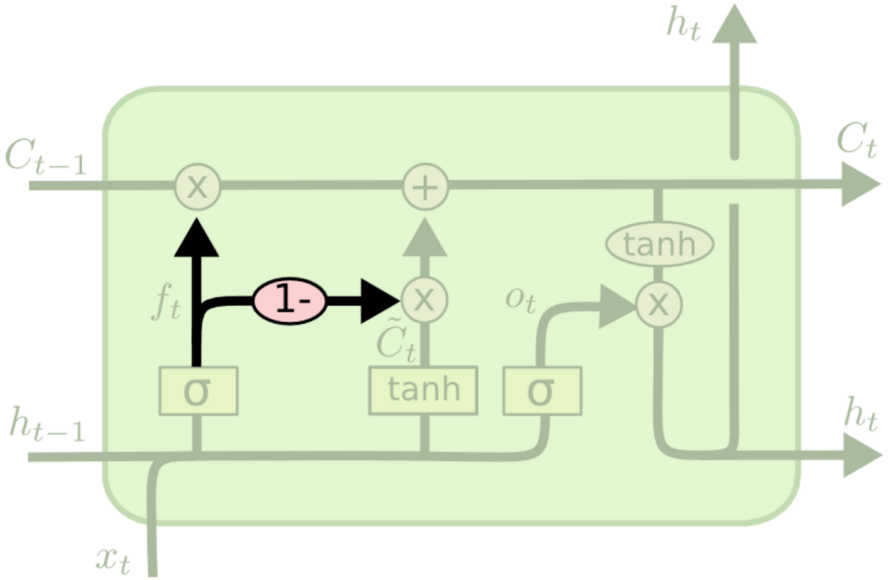


$$f_t = \sigma\left(W_f \cdot [\boldsymbol{C_{t-1}}, h_{t-1}, x_t] + b\right.$$

$$i_t = \sigma\left(W_i \cdot [\boldsymbol{C_{t-1}}, h_{t-1}, x_t] + b\right.$$

$$o_t = \sigma\left(W_o \cdot [\boldsymbol{C_t}, h_{t-1}, x_t] + b_o\right)$$

LSTM with combined forget and input gates



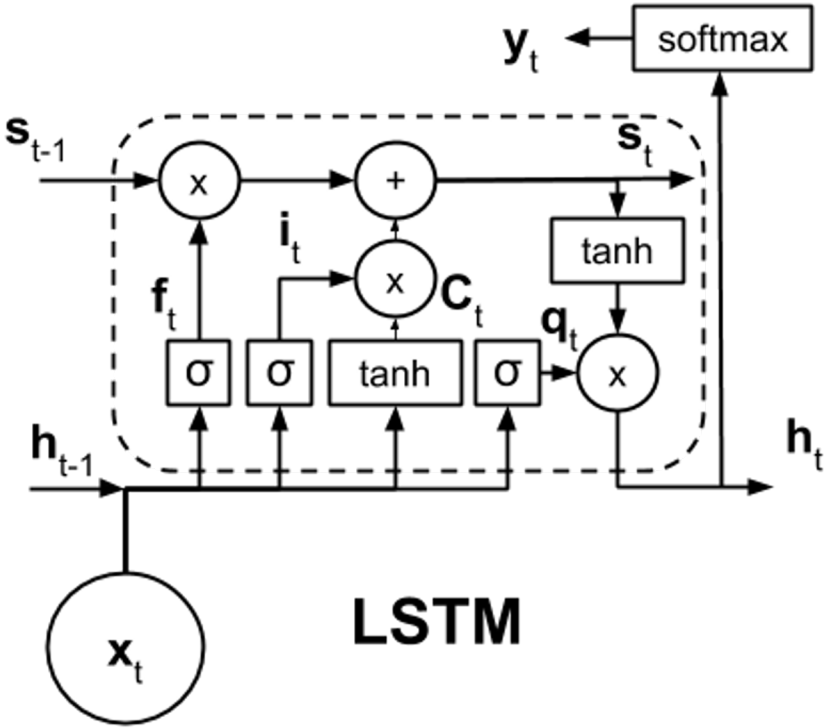
$$C_t = f_t * C_{t-1} + (1 - \boldsymbol{f_t}) * \tilde{C}_t$$

In short, the percentage of information to forget is inversely proportional to the new information to introduce

RNN in Keras

`SimpleRNN` - simplified implementation of RNN
`GRU` - GRU based on Cho's paper https://arxiv.org/pdf/1406.1078v3.pdf
`LSTM` - Long Short-Term Memory layer - Hochreiter 1997.
`ConvLSTM2D` - Convolutional LSTM. It is similar to an LSTM layer, but the input transformations and recurrent transformations are both convolutional.

LSTM Cell



Reference

Deep Learning, Ian Goodfellow and Yoshua Bengio and Aaron Courville, MIT Press, 2016, http://www.deeplearningbook.org

Kaparthy, A. The Unreasonable Effectiveness of RNN, http://karpathy.github.io/2015/05/21/rnn-effectiveness/

Olah, C. Understanding LSTM Networks, http://colah.github.io/posts/2015-08-Understanding-LSTMs/

In Summary

RNN family excels in learning patterns in sequential data

RNN layers are inherently slow and tricks are needed to parallelize

Transformer network has emerged as a superior replacement of RNN

seq2seq

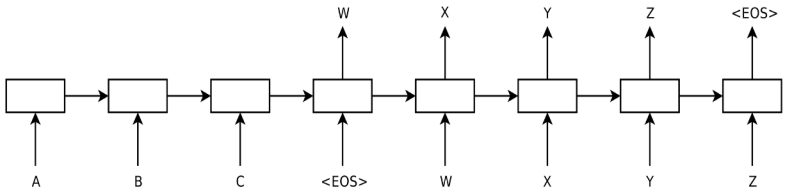


Figure 1: Our model reads an input sentence "ABC" and produces "WXYZ" as the output sentence. The model stops making predictions after outputting the end-of-sentence token. Note that the LSTM reads the input sentence in reverse, because doing so introduces many short term dependencies in the data that make the optimization problem much easier.

[2015 Sutskever et al Sequence to Sequence Learning with Neural Networks]