Convolutional Neural Network (CNN)

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Convolutional Neural Network

Convolutional Neural Network (**CNN**) or **CovNet**

Closest model on how human vision works

Uses an operation called **Convolution**

- Con – Latin word for *together*
- Volvere – Latin word for *roll up*

Convolution operator

$$\boldsymbol{y} = \boldsymbol{x} * \boldsymbol{k}$$

$\boldsymbol{x} \in \mathbb{R}^{w \times h \times d}$ is input: d feature maps with dimensions $w \times h$

$\boldsymbol{k} \in \mathbb{R}^{k \times k \times f}$ is kernel: f filters or kernels with dimensions $k \times k$

$\boldsymbol{y} \in \mathbb{R}^{w \times h \times f}$ is output: f feature maps with dimensions $w \times h$ assuming sufficient padding is applied

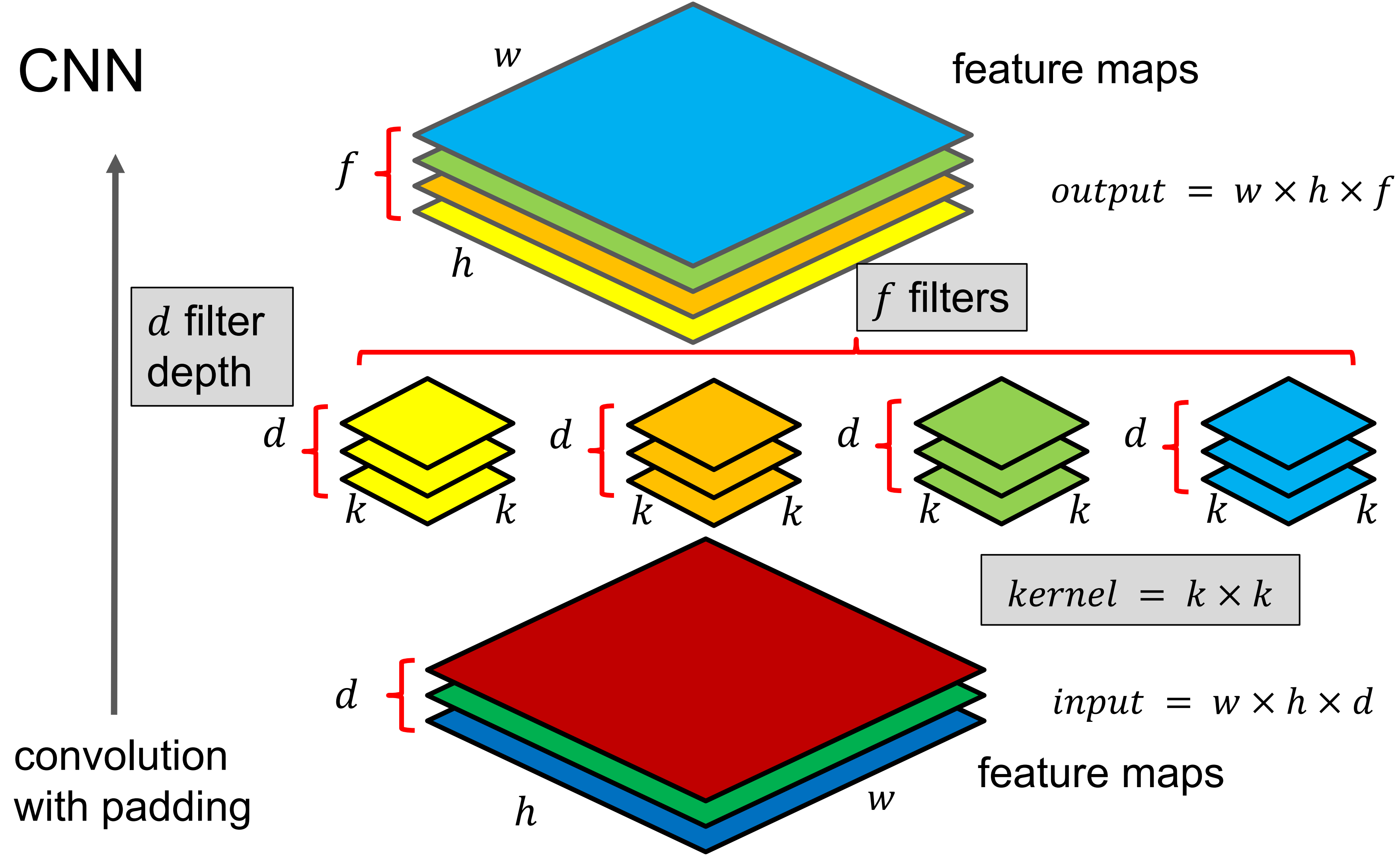
Dimensions are based on 2D inputs and square kernels

CNN Parameters: Kernel or Filter + Bias

In MLP, we learn weights and biases

In CNN, we learn the weights and bias of a kernel

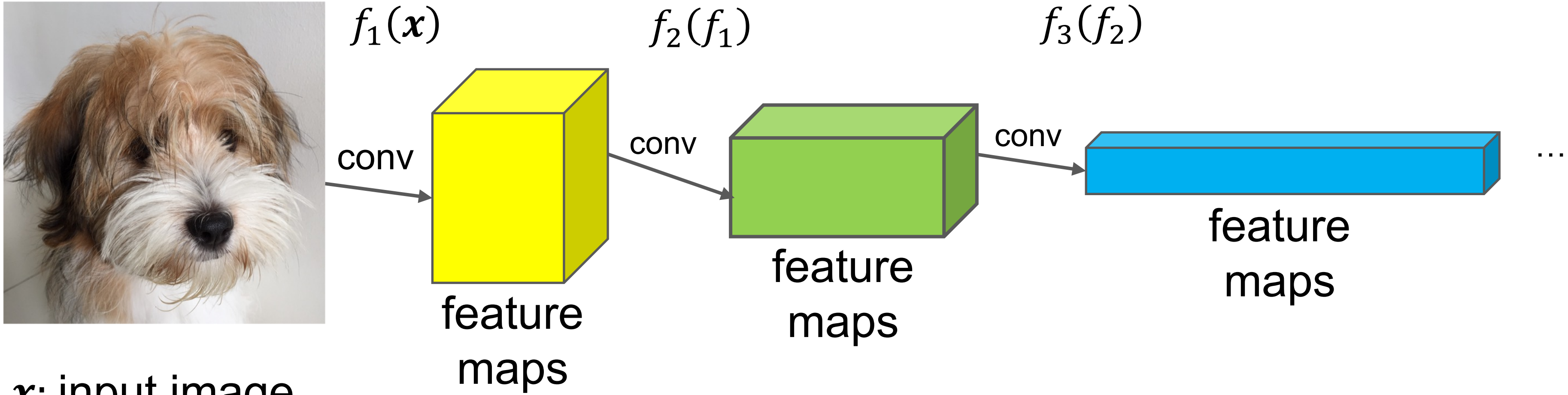
CNN



w
feature maps
f
$output = w \times h \times f$
h
f filters
d filter depth
d
k
k
d
k
k
d
k
k
d
k
k
$kernel = k \times k$
d
$input = w \times h \times d$
convolution with padding
feature maps
h
w

A CNN is made of multiple convolutional layers

The deeper the network, the more representations the network learns



$\boldsymbol{x}$: input image or input feature

$$\boldsymbol{y} = f(\boldsymbol{x}) \approx f_n \circ f_{n-1} \circ f_{n-2} \circ \cdots \circ f_1(\boldsymbol{x})$$

CNN models can also be used as function approximators

Reasons why CNN models are effective and efficient

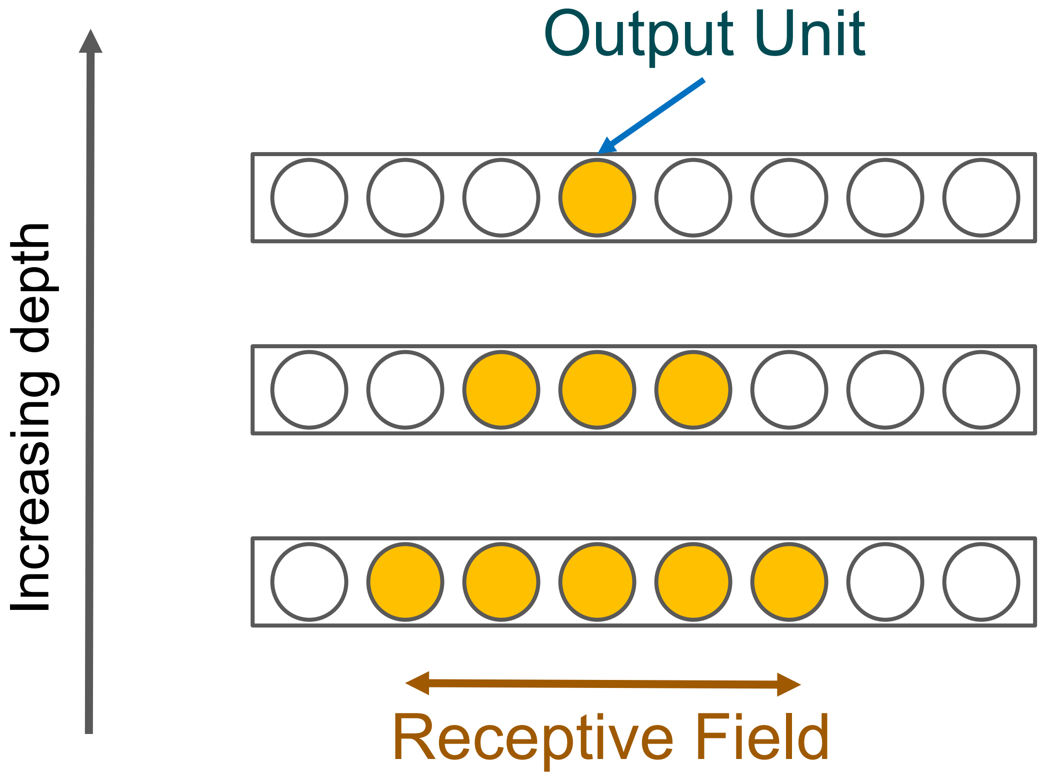
Sparse Interaction - kernel as a feature detector is small compared to the input image thus requires few interaction only

Parameter Sharing - same set of parameters used for more than 1 function in the model

Receptive Field - the input units that affect the output unit

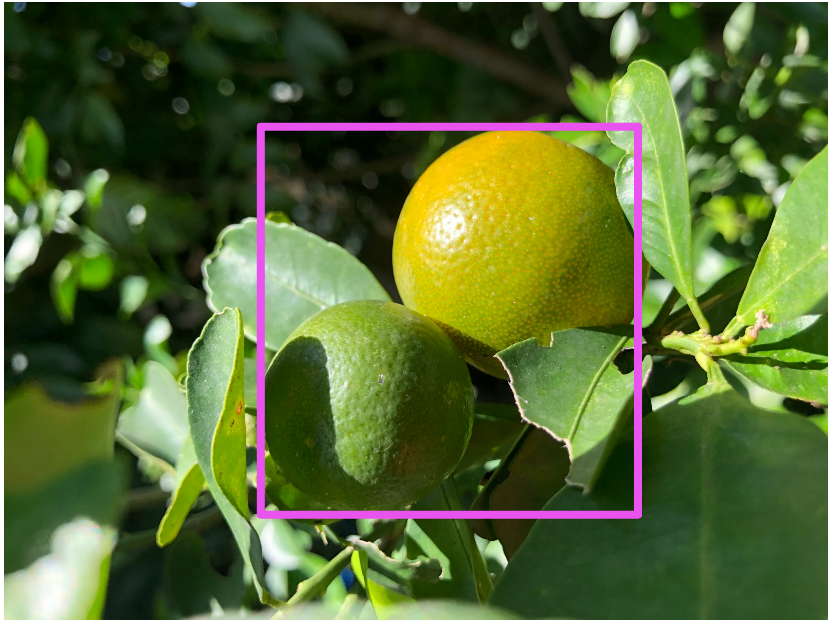
The deeper the network, the larger is the receptive field

Receptive Field



Assume:
Side view
3×3 kernel

CNN models are scale invariant



Due to max pooling, CNN models detect the same object regardless of scale

CNN models are translation equivariant

Due to kernel convolution, CNN models detect the same object even if it moves in the image

CNN models are not rotation invariant

No object detected

No built-in operators to make CNN models rotation invariant.
Solution: Train the model with rotated objects.

CNN Ops

Convolution

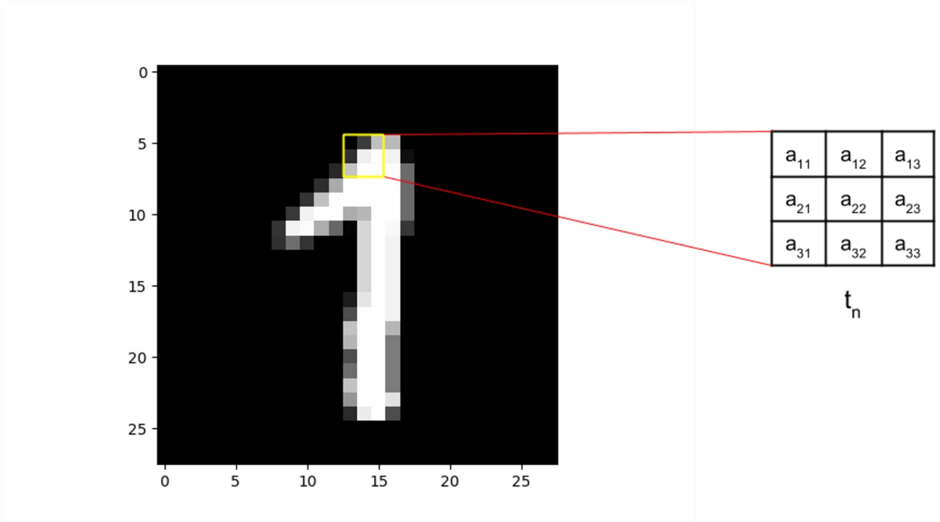
Activation

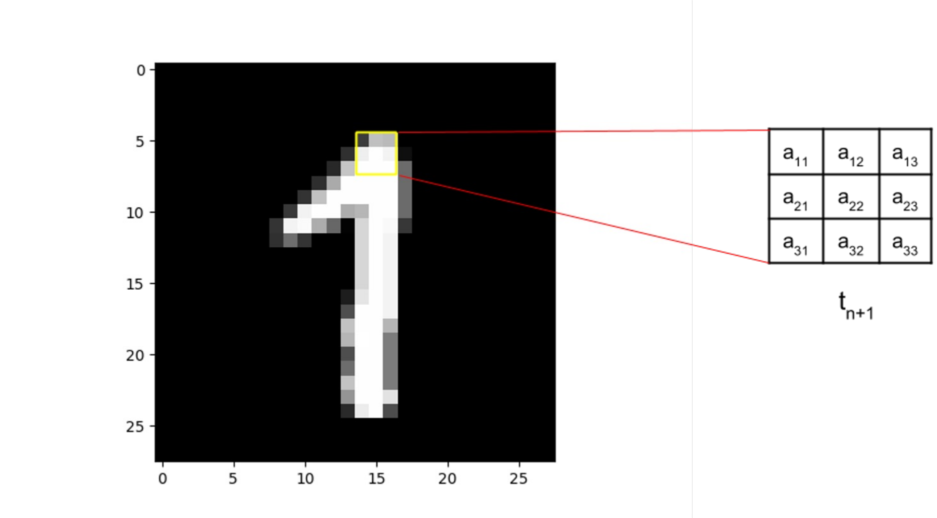
Padding

Pooling

Strides

Convolution





CNN: Convolution No Padding (Valid)

$$X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} * K = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = y = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

$$y_{11} = aw + bx + ey + fz$$

Convolution No Padding (Valid)

$$\underset{X}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} * \underset{K}{\begin{bmatrix} w & x \\ y & z \end{bmatrix}} = \underset{y}{\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}}$$

$y_{12} = bw + cx + fy + gz$

Convolution No Padding (Valid)

$$\underset{X}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} * \underset{K}{\begin{bmatrix} w & x \\ y & z \end{bmatrix}} = \underset{y}{\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}}$$

$$y_{13} = cw + dx + gy + hz$$

Convolution No Padding (Valid)

$$\underset{X}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} * \underset{K}{\begin{bmatrix} w & x \\ y & z \end{bmatrix}} = \underset{y}{\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}}$$

$y_{21} = ew + fx + iy + jz$

Convolution No Padding (Valid)

$$\underset{X}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} * \underset{K}{\begin{bmatrix} w & x \\ y & z \end{bmatrix}} = \underset{y}{\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}}$$

$$y_{22} = fw + gx + jy + kz$$

Convolution No Padding (Valid)

$$\overset{X}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} * \overset{K}{\begin{bmatrix} w & x \\ y & z \end{bmatrix}} = \overset{y}{\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}}$$

$$y_{23} = gw + hx + ky + lz$$

Convolution No Padding (Valid)

$$\underset{X}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} * \underset{K}{\begin{bmatrix} w & x \\ y & z \end{bmatrix}} = \underset{y}{\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}}$$

$$y_{31} = iw + jx + my + nz$$

Convolution No Padding (Valid)

$$
\underset{X}{\begin{array}{|c|c|c|c|}\hline a & b & c & d \\ \hline e & f & g & h \\ \hline i & j & k & l \\ \hline m & n & o & p \\ \hline \end{array}}
\; * \;
\underset{K}{\begin{array}{|c|c|}\hline w & x \\ \hline y & z \\ \hline \end{array}}
\; = \;
\underset{y}{\begin{array}{|c|c|c|}\hline y_{11} & y_{12} & y_{13} \\ \hline y_{21} & y_{22} & y_{23} \\ \hline y_{31} & y_{32} & y_{33} \\ \hline \end{array}}
$$

$$y_{32} = jw + kx + ny + oz$$

Convolution No Padding (Valid)

$$\underset{X}{\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}} * \underset{K}{\begin{bmatrix} w & x \\ y & z \end{bmatrix}} = \underset{y}{\begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}}$$

$$y_{33} = kw + lx + oy + pz$$

Activation Function - ReLU

$$\overset{X}{\begin{array}{|c|c|c|c|}\hline a & b & c & d \\ \hline e & f & g & h \\ \hline i & j & k & l \\ \hline m & n & o & p \\ \hline\end{array}} * \overset{K}{\begin{array}{|c|c|}\hline w & x \\ \hline y & z \\ \hline\end{array}} = \overset{y}{\begin{array}{|c|c|c|}\hline y_{11} & y_{12} & y_{13} \\ \hline y_{21} & y_{22} & y_{23} \\ \hline y_{31} & y_{32} & y_{33} \\ \hline\end{array}} \xrightarrow{f(\cdot)} \overset{R}{\begin{array}{|c|c|c|}\hline r_{11} & r_{12} & r_{13} \\ \hline r_{21} & r_{22} & r_{23} \\ \hline r_{31} & r_{32} & r_{33} \\ \hline\end{array}}$$

$R = \mathrm{ReLU}(y)$

$r_{ij} = \mathrm{ReLU}(y_{ij})$

Element-wise ReLU

Downsampling - Pooling (eg MaxPooling)

R

r_{11}	r_{12}	r_{13}
r_{21}	r_{22}	r_{23}
r_{31}	r_{32}	r_{33}

=

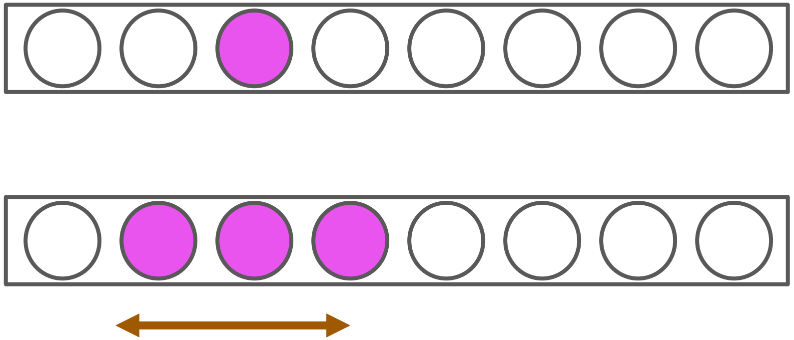
P

p_{11}

$p_{11} = \max(r_{11}, r_{12}, r_{21}, r_{22})$

Why MaxPool? To Increase the Receptive Field

Before MaxPool

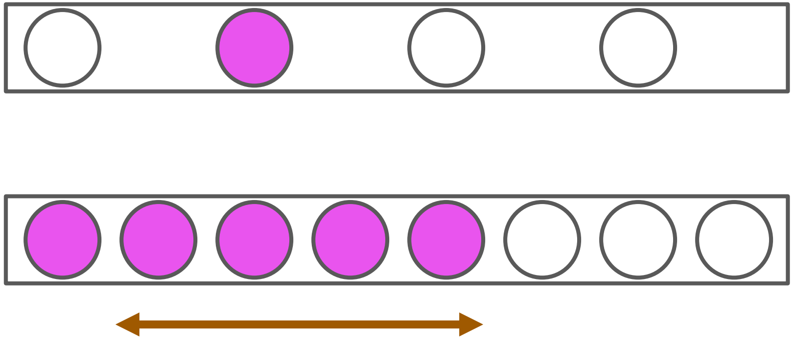


Receptive Field is 3

Assume:
Side view
3×3 kernel

Why MaxPool? To Increase the Receptive Field

After 2x2 MaxPool



Receptive Field is now 5

Assume:
Side view
3×3 kernel

Downsampling using MaxPooling (MP)

$\mathrm{MP}(X) = y$

X

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

y

y_{11}	y_{12}
y_{21}	y_{22}

$y_{11} = \max(a, b, e, f)$

$\mathrm{MP}(X) = y$

X

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

y

y_{11}	y_{12}
y_{21}	y_{22}

$y_{12} = \max(c, d, g, h)$

$\mathrm{MP}(X) = y$

X

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

y

y_{11}	y_{12}
y_{21}	y_{22}

$y_{21} = \max(i, j, m, n)$

$\mathrm{MP}(X) = y$

X

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

y

y_{11}	y_{12}
y_{21}	y_{22}

$y_{22} = \max(k, l, o, p)$

Downsampling using Stride > 1, (e.g. 2)

$$X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} * K = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$y_{11} = aw + bx + ey + fz$

$$X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} * K = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$y_{12} = cw + dx + gy + hz$

$$X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} * K = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$y_{21} = iw + jx + my + nz$

$$X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} * K = \begin{bmatrix} w & x \\ y & z \end{bmatrix} = y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$y_{22} = kw + lx + oy + pz$

Zero Padding

X

0	0	0	0	0	0
0	a	b	c	d	0
0	e	f	g	h	0
0	i	j	k	l	0
0	m	n	o	p	0
0	0	0	0	0	0

*

K

r	s	t
u	v	w
x	y	z

=

y

y_{11}	y_{12}	y_{13}	y_{14}
y_{21}	y_{22}	y_{23}	y_{24}
y_{31}	y_{32}	y_{33}	y_{34}
y_{41}	y_{42}	y_{43}	y_{44}

$y_{11} = av + bw + ey + fz$

$y_{12} = au + bv + cw + ex + fy + gz$

$y_{13} = bu + cv + dw + fx + gy + hz$

$y_{14} = cu + dv + gx + hy$

etc

K kernels/filters

$$X = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} * K: \begin{bmatrix} s & t \\ u & v \end{bmatrix}, \begin{bmatrix} w & x \\ y & z \end{bmatrix} = y: \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}, \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$$

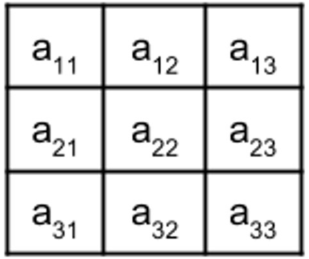
K = 2

$y_{11} = as + bt + eu + fv$
etc.

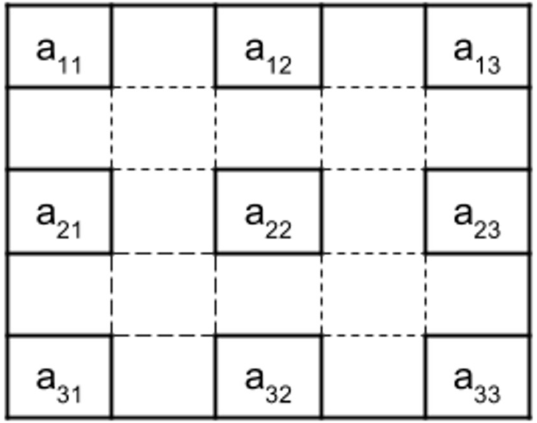
$t_{11} = aw + bx + ey + fz$
etc.

Dilated Convolution

Dilation rate > 1 increases kernel coverage w/o increasing computation time



dilation_rate=1



dilation_rate=2

Dilated Convolution No Padding (Valid)

X

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

*

K

w	x
y	z

=

y

y_{11}	y_{12}	y_{13}
y_{21}	y_{22}	y_{23}
y_{31}	y_{32}	y_{33}

dilation_rate=1

$$y_{11} = aw + bx + ey + fz$$

X

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

*

K

w	x
y	z

=

y

y_{11}	y_{12}
y_{21}	y_{22}

dilation_rate=2

$$y_{11} = aw + cx + iy + kz$$

UpSampling

UP(

a	b
c	d

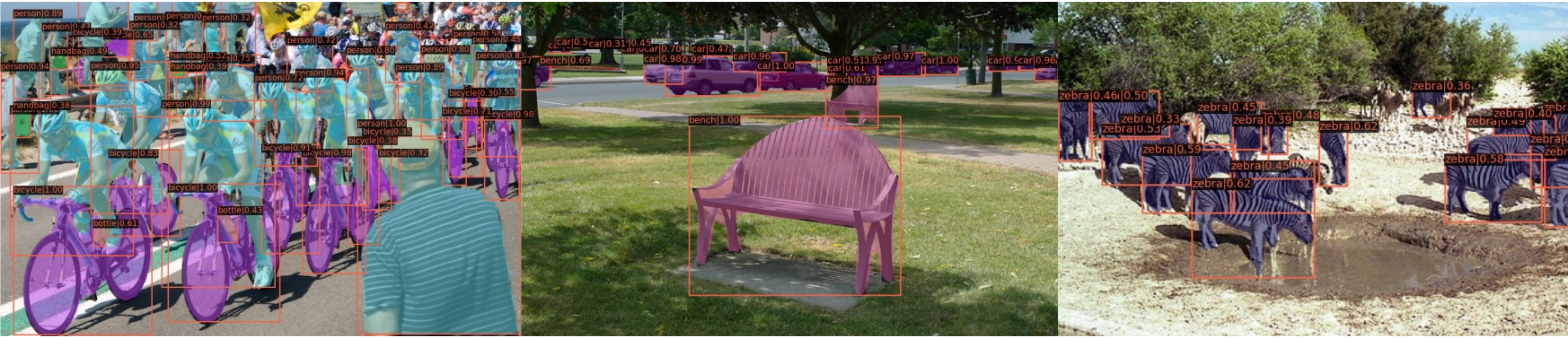
) =

a	a	b	b
a	a	b	b
c	c	d	d
c	c	d	d

Interpolation: same data repeated n times
Other interpolation algorithms: Bilinear

Transposed Convolution for Dense Prediction

Convolution + UpSampling (if strides>2)



Segmentation Masks as Dense Prediction

https://github.com/open-mmlab/mmdetection

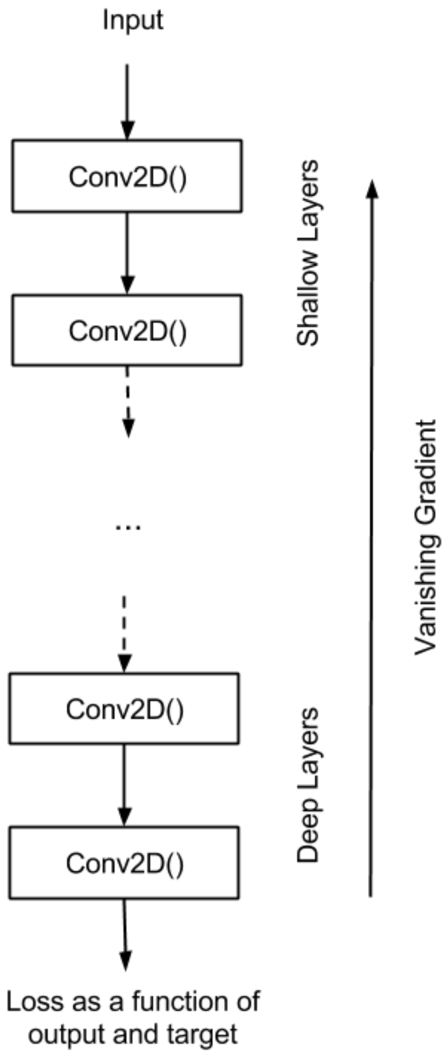
Issues with Deep Neural Networks

Vanishing Gradients

Exploding Gradients

Unstable Training

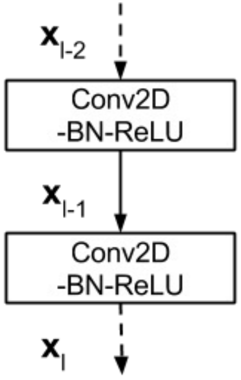
Vanishing Gradients



ResNet

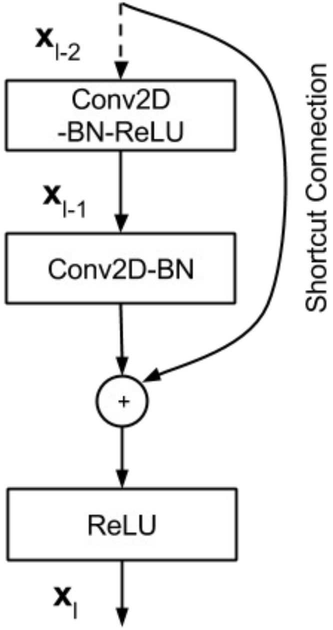
$$\boldsymbol{x}_{l-1} = H(\boldsymbol{x}_{l-2})$$

By introducing a skip connection, ResNet avoids the problem of vanishing gradients



CNN Block

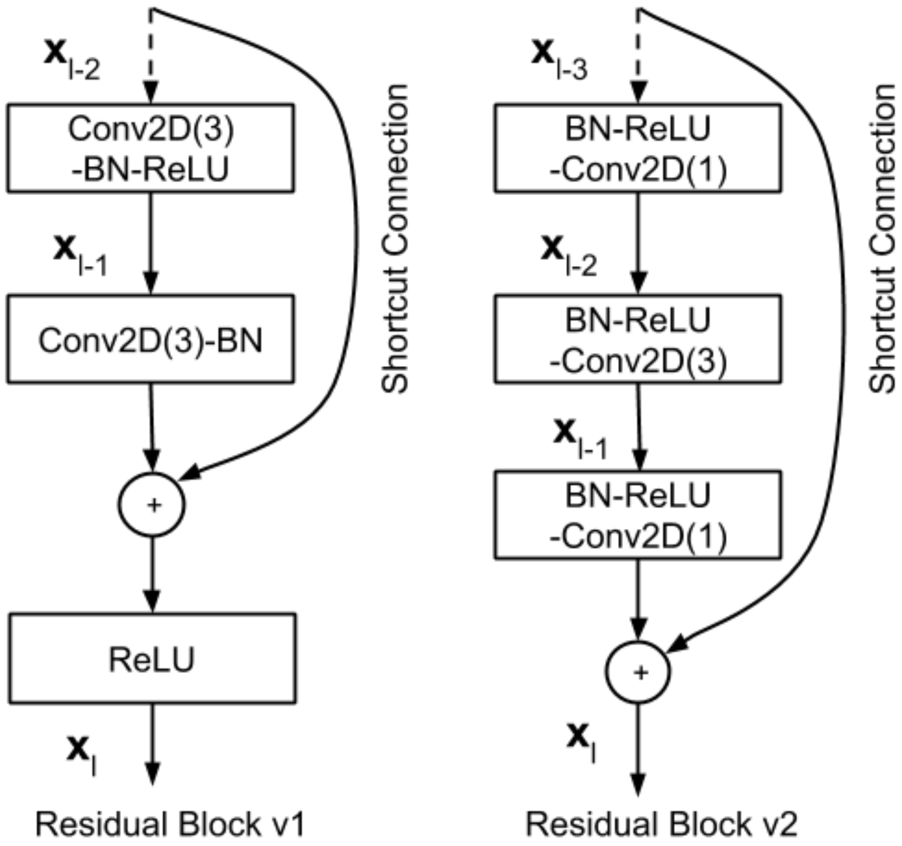
$$\boldsymbol{x}_{l} = H(\boldsymbol{x}_{l-1})$$



Residual Block

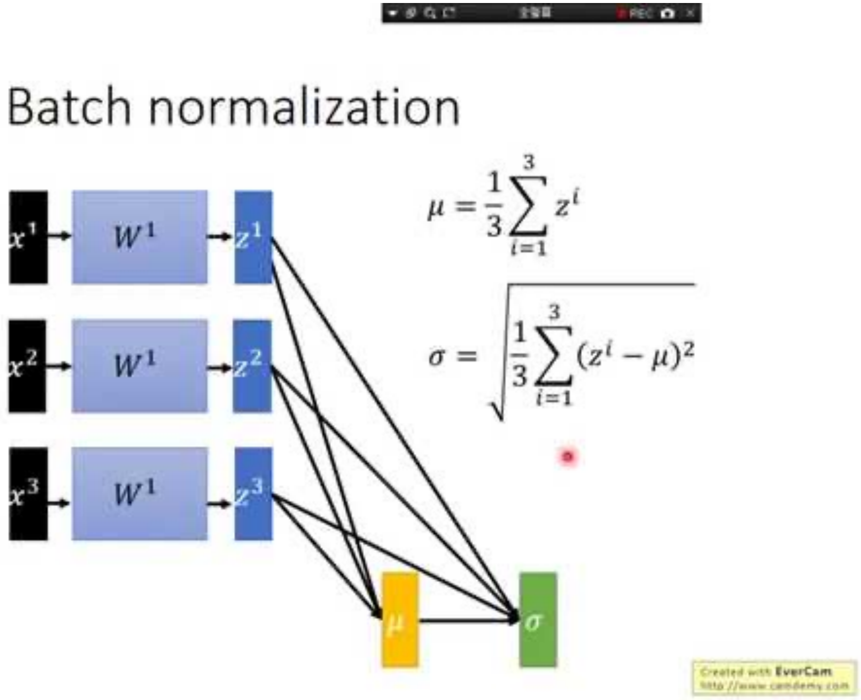
$$\boldsymbol{x}_{l} = ReLU(F(\boldsymbol{x}_{l-1}) + \boldsymbol{x}_{l-2})$$

Improved ResNet



Batch Normalization

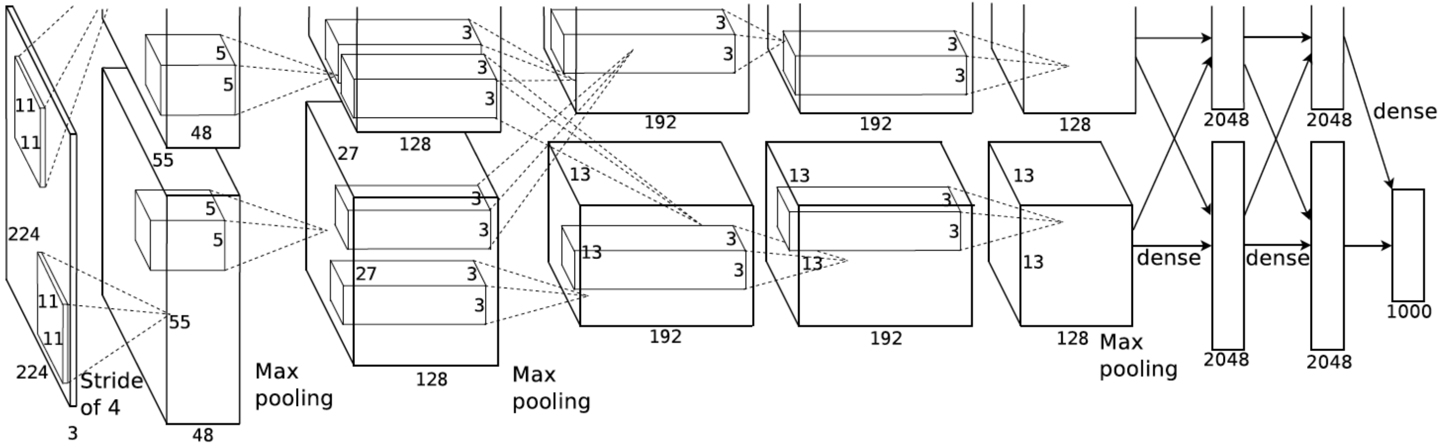
Applied layer-wise to maintain zero mean and variance of 1 for activation outputs
Allows use of larger learning rate in deep models w/o causing instability



In Summary

CNN is parameter efficient, parallelizable, translation equivariant, and scale invariant model

Deep CNN exhibits state-of-the-art (SOTA) performances not only in vision tasks



AlexNet [Krizhevsky et al (2012)]

Code demo is next