Regularization

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Generalization

After training, the model should be validated using data never seen before: $\mathcal{D}_{test} = \{(\boldsymbol{x}_m, y_m)\}, \quad n = 1 \ldots M$

The test scores called generalization performance are the ones reported

Issue: Overfitting and Memorization

It is possible that for a given training set, a model with sufficient complexity is able to memorize the input-output data

This leads to a problem called overfitting

Regularization

To prevent overfitting, a regularizer is used

Examples: weight penalty, noise injection, inductive bias on dataset, novel model architecture, dropout, data augmentation

Weight Penalty: Scaled Dot Product of Parameters

$$R_{emp}(f, \boldsymbol{X}, \boldsymbol{y}) = \frac{1}{N}\sum_{i=1}^{n}\left(y_n - f(\boldsymbol{x}_n, \boldsymbol{\theta})\right)^2 + \lambda \boldsymbol{\theta}^{\boldsymbol{T}}\boldsymbol{\theta}$$

Cross-Validation: Use when there is a small dataset (100s to few 1000s)

For Example, 4-fold Validation

Probabilistic Parameter Estimation

Maximum Likelihood Estimation (MLE)

Function of parameters that explain the data well:

$$\mathcal{L}_{\mathcal{D}}(\boldsymbol{\theta}) = -\log p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta})$$

The likelihood of parameters $\boldsymbol{\theta}$ given that we observed data $\mathcal{D}$

Maximum because the loss function is minimized when $p(\mathcal{D}|\boldsymbol{\theta}) \rightarrow 1.0$

Log does not change the location of minima

Supervised Learning using Gaussian Distribution

$$\mathcal{D}_{train} = \{(\boldsymbol{x}_n, y_n)\}, \quad n = 1 \ldots N$$

Target model: $p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta}) \rightarrow p(\hat{y}_n|\boldsymbol{x}_n, \boldsymbol{\theta})$

Assume a Gaussian distribution with mean as a linear function $\mu = \boldsymbol{x}_n^T\boldsymbol{\theta}$ and with the parameter perturbed by a Gaussian noise with zero mean and variance σ^2:

$$p(\hat{y}_n|\boldsymbol{x}_n, \boldsymbol{\theta}) = \mathcal{N}(\hat{y}_n|\boldsymbol{x}_n^T\boldsymbol{\theta}, \sigma^2)$$

Independent Identically Distributed (IID) Assumption

Independent : probabilities can be multiplied $p(a, b) = p(a)p(b)$

Identically Distributed : can share parameters

We can factorize the data distribution:

$$p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta}) = p(\mathcal{Y}|\mathcal{X}, \boldsymbol{\theta}) = \prod_{n=1}^{N} p(\hat{y}_n|\boldsymbol{x}_n, \boldsymbol{\theta})$$

MLE

$$\mathcal{L}_{\mathcal{D}}(\boldsymbol{\theta}) = -\log p(\mathcal{Y}|\mathcal{X}, \boldsymbol{\theta}) = -\sum_{n=1}^{N} \log p(\hat{y}_n|\boldsymbol{x}_n, \boldsymbol{\theta})$$

Since $\log ab = \log a + \log b$

If $p(\hat{y}_n|\boldsymbol{x}_n, \boldsymbol{\theta})$ is a Gaussian

$$p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta}) = p(\mathcal{Y}|\mathcal{X}, \boldsymbol{\theta}) = \prod_{n=1}^{N} \mathcal{N}(\hat{y}_n|\boldsymbol{x}_n^T\boldsymbol{\theta}, \sigma^2)$$

$$\mathcal{L}_{\mathcal{D}}(\boldsymbol{\theta}) = -\log p(\mathcal{Y}|\mathcal{X}, \boldsymbol{\theta}) = -\sum_{n=1}^{N} \log \mathcal{N}(\hat{y}_n|\boldsymbol{x}_n^T\boldsymbol{\theta}, \sigma^2)$$

If $p(\hat{y}_n|\boldsymbol{x}_n, \boldsymbol{\theta})$ is a Gaussian

$$\mathcal{L}_{\mathcal{D}}(\boldsymbol{\theta}) = -\sum_{n=1}^{N} \log \frac{1}{\sqrt{2\pi\sigma^2}} \exp -\left(\frac{(\hat{y}_n - \boldsymbol{x}_n^T\boldsymbol{\theta})^2}{2\sigma^2}\right)$$

$$\mathcal{L}_{\mathcal{D}}(\boldsymbol{\theta}) = \sum_{n=1}^{N} \left(\frac{(\hat{y}_n - \boldsymbol{x}_n^T\boldsymbol{\theta})^2}{2\sigma^2}\right) - \sum_{n=1}^{N} \log \frac{1}{\sqrt{2\pi\sigma^2}}$$

Squared Losses

Constant

Maximum A Posteriori (MAP) Principle

$$p(\boldsymbol{\theta}|\boldsymbol{\mathcal{D}}) = \frac{p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\boldsymbol{\mathcal{D}})} \propto p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

Since $p(\boldsymbol{\mathcal{D}})$ is not a function of $\boldsymbol{\theta}$, it does not affect the optimization

$$\log p(\boldsymbol{\theta}|\boldsymbol{\mathcal{D}}) = \log p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta})$$

$$\underset{\boldsymbol{\theta}}{\operatorname{argmax}} \log p(\boldsymbol{\theta}|\boldsymbol{\mathcal{D}}) = \underset{\boldsymbol{\theta}}{\operatorname{argmax}} (\log p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta}) + \log p(\boldsymbol{\theta}))$$

Maximum A Posteriori (MAP) Principle

$$\underset{\boldsymbol{\theta}}{\operatorname{argmax}} \log p(\boldsymbol{\theta}|\boldsymbol{\mathcal{D}}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} (\underbrace{-\log p(\boldsymbol{\mathcal{D}}|\boldsymbol{\theta})}_{\text{MLE}} \underbrace{-\log p(\boldsymbol{\theta})}_{\text{Regularizer}})$$

If we assume $p(\boldsymbol{\theta}) = \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$, then:

$$-\log p(\boldsymbol{\theta}) = \lambda \boldsymbol{\theta}^T \boldsymbol{\theta}$$

Capacity

Capacity - ability to fit a wide variety of functions

↓ Capacity → Underfitting: ↑ MSE(train) , ↑ MSE(test)

↑ Capacity → Overfitting: ↓ MSE(train) , ↑ MSE(test)

✓ Capacity → Optimal Fit: ↓ MSE(train) , ↓ MSE(test)

Sample Data

Input: x is a random number from -3 to +3

Output: $f(x) = y = x^2 + x + 1 + \lambda \sin \beta x$

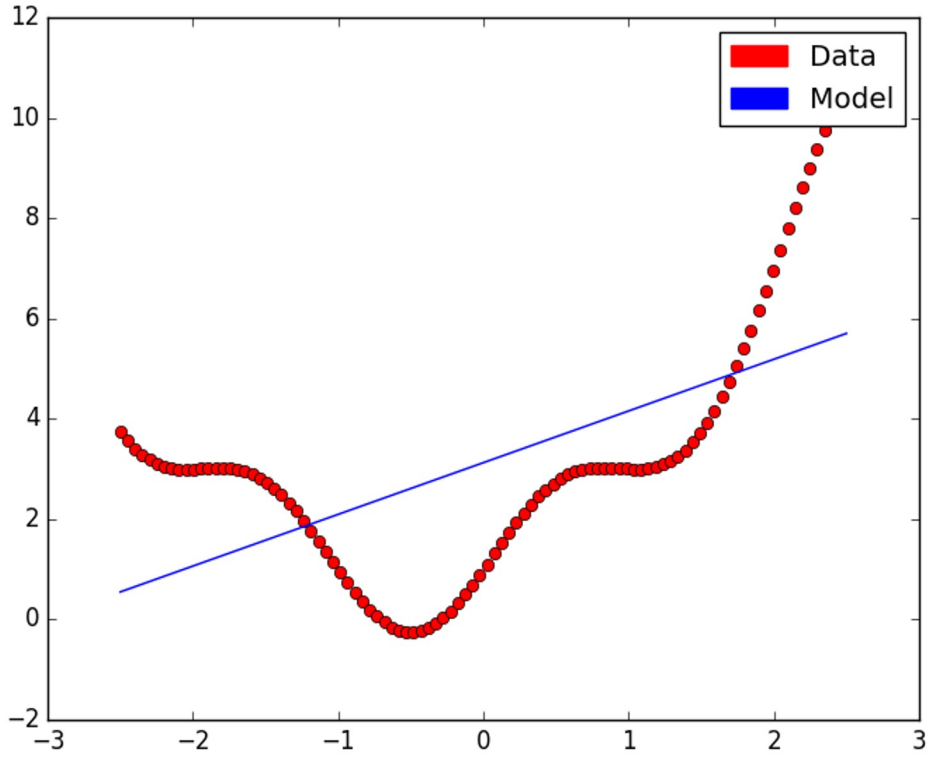
Noise

Model: Our model should be a 2nd degree polynomial

Suppose we can only see the data generated by $f(x)$

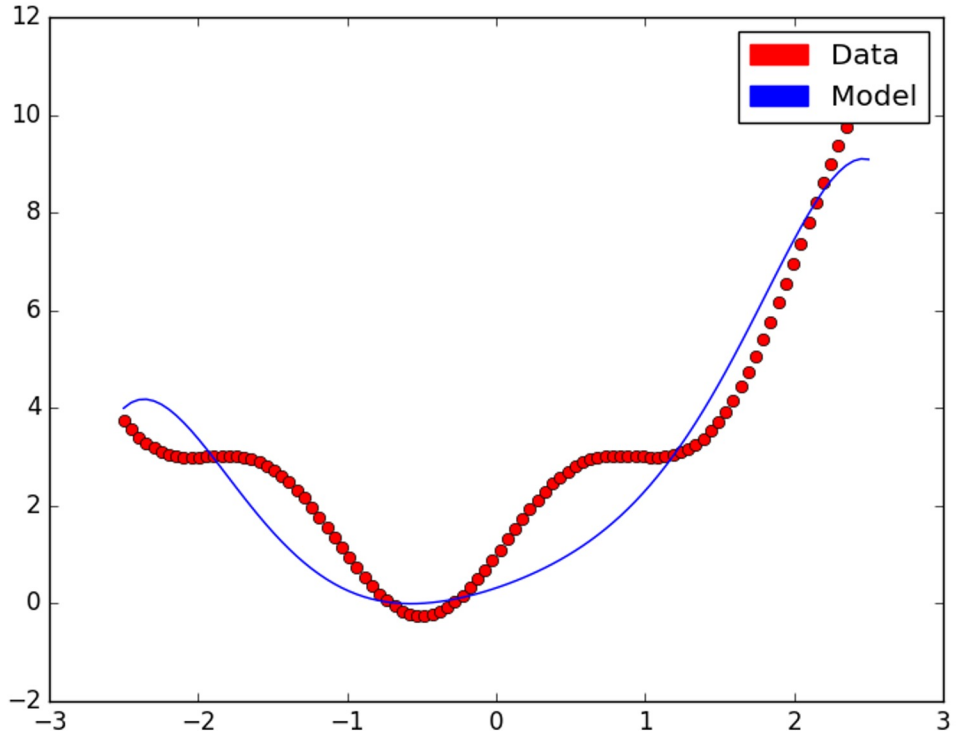
Model: $f(x) = y = \theta_1 x + \theta_0$

Underfitting: the model has both big train and test error



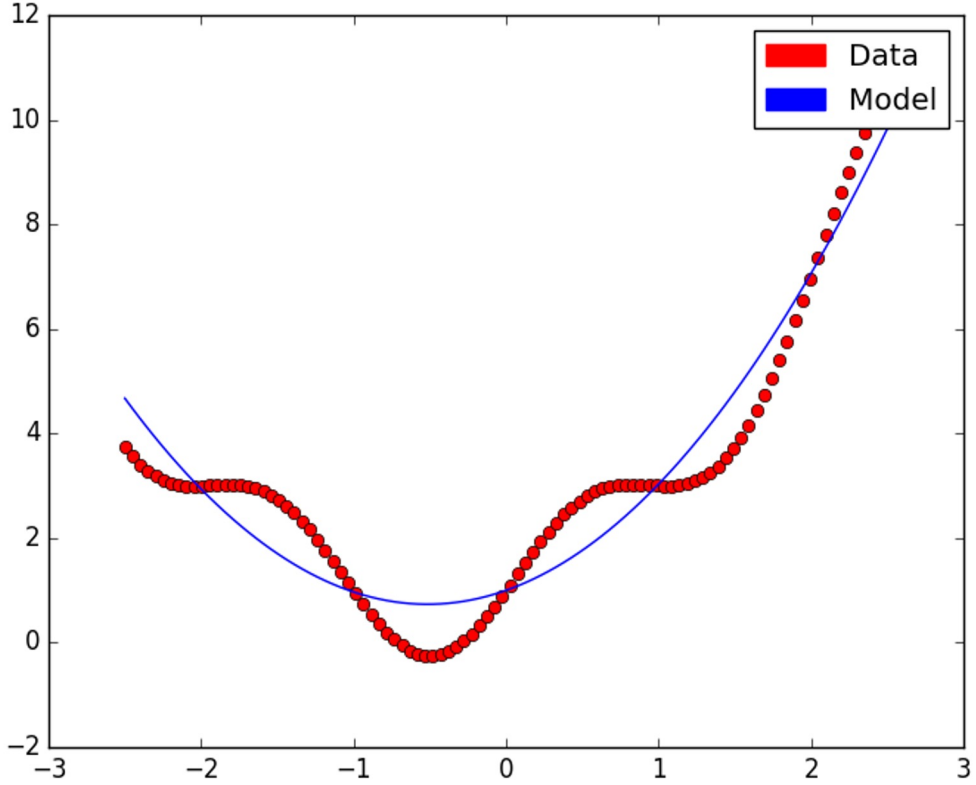
Model: $f(x) = y = \theta_6 x^6 + \cdots + \theta_1 x + \theta_0$

Overfitting: the model has a small train error but has a big test error



Model: $f(x) = y = \theta_2 x^2 + \theta_1 x + \theta_0$

Optimal fit: the model has a small train and test errors



Dataset Augmentation

Increasing the amount of training data by creating fake data

Effective in computer vision (eg image of a dog rotated, resized, translated, etc) and speech

Original

Flipped

Rotated

Dataset Augmentation

Some image data are not amenable to geometric transformation as the meaning is altered

For example,

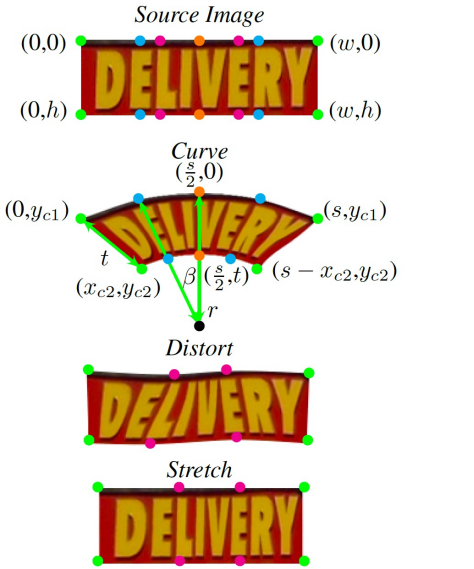
Orig Rot/Flipped Mirrored

Data Augmentation

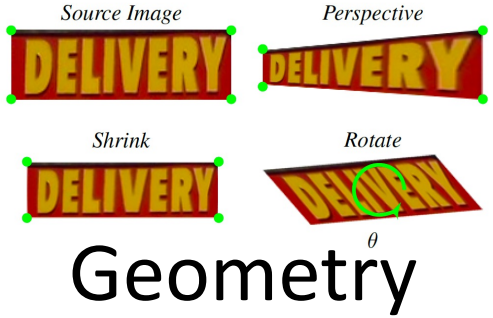
Other operations

1. Scaling
2. Translating
3. Minor distortion
4. Normalization
5. ZCA Whitening
6. etc

STRAug: Data Augmentation for STR



Warp



Geometry

Figure 6. Examples of images affected by *Noise* data augmenta tion.

Figure 7. Example images affected by *Blur* data augmentation.

Noise and Blur

Figure 8. Example images affected by *Weather* data augmentation.

Figure 9. Example images affected by *Camera* data augmentation.

Figure 10. Example images affected by *Process* data augmenta- tion.

Weather, Camera, Process

Atienza, Rowel. "Data Augmentation for Scene Text Recognition." *Proceedings of the IEEE/CVF International Conference on Computer Vision*. 2021.

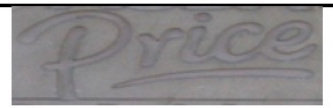
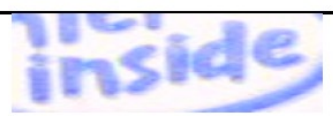
Model	Input Text Image	Baseline Prediction	+STRAug Prediction	%Acc Gain
CRNN [33]		Pyice	Price	1.30
R2AM [22]		OLDTOW	OLDTOWN	1.48
GCRNN [36]		TiMmES	Times	0.89
Rosetta [5]		eizu	eBizu	2.10
RARE [34]		Washii1	Washing	1.35
TRBA [3]		insiid	inside	1.06

Figure 1. STRAug data augmentation significantly improves the overall accuracy of STR models especially on challenging input text images. We follow the evaluation protocol used in most STR models of case sensitive training and case insensitive validation.

Data Augmentation – Key Concepts

x_1:

y_1: bird

x_2:

y_2: cat

CutOut

x_1:

y_1: bird

Terrance DeVries and Graham W Taylor. Improved regularization of convolutional neural networks with cutout. arXiv:1708.04552, 2017

MixUp

$$\tilde{x} = \lambda x_1 + (1 - \lambda) x_2$$

Mix of bird and cat

$$\tilde{y} = \lambda y_1 + (1 - \lambda) y_2$$

Hongyi Zhang, Moustapha Cisse, Yann N Dauphin, and David Lopez-Paz. MixUp: Beyond empirical risk minimization. ICLR 2018.

CutMix

$$\tilde{x} = \mathbf{M} \odot x_1 + (\mathbf{1} - \mathbf{M})x_2$$

Mix of bird and cat

$$\tilde{y} = \lambda y_1 + (1 - \lambda)y_2$$

Sangdoo Yun, Dongyoon Han, Seong Joon Oh, Sanghyuk Chun, Junsuk Choe, and Youngjoon Yoo. Cutmix: Regularization strategy to train strong classifiers with localizable features. CVPR 2019.

RandAugment

Policy for combining multiple data augmentations

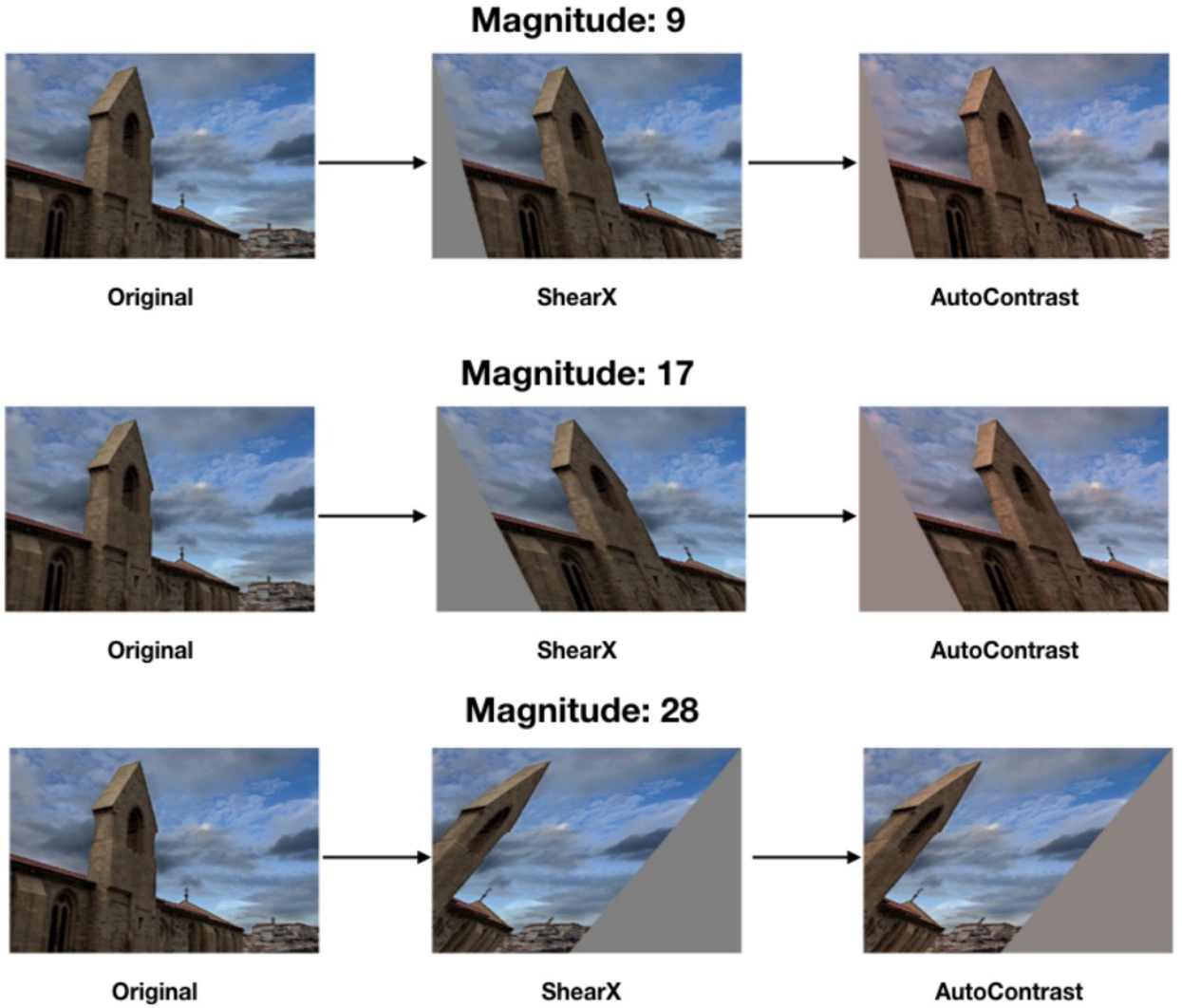


Figure 1. **Example images augmented by RandAugment.** In these examples N=2 and three magnitudes are shown corresponding to the optimal distortion magnitudes for ResNet-50, EfficientNet-B5 and EfficientNet-B7, respectively. As the distortion magnitude increases, the strength of the augmentation increases.

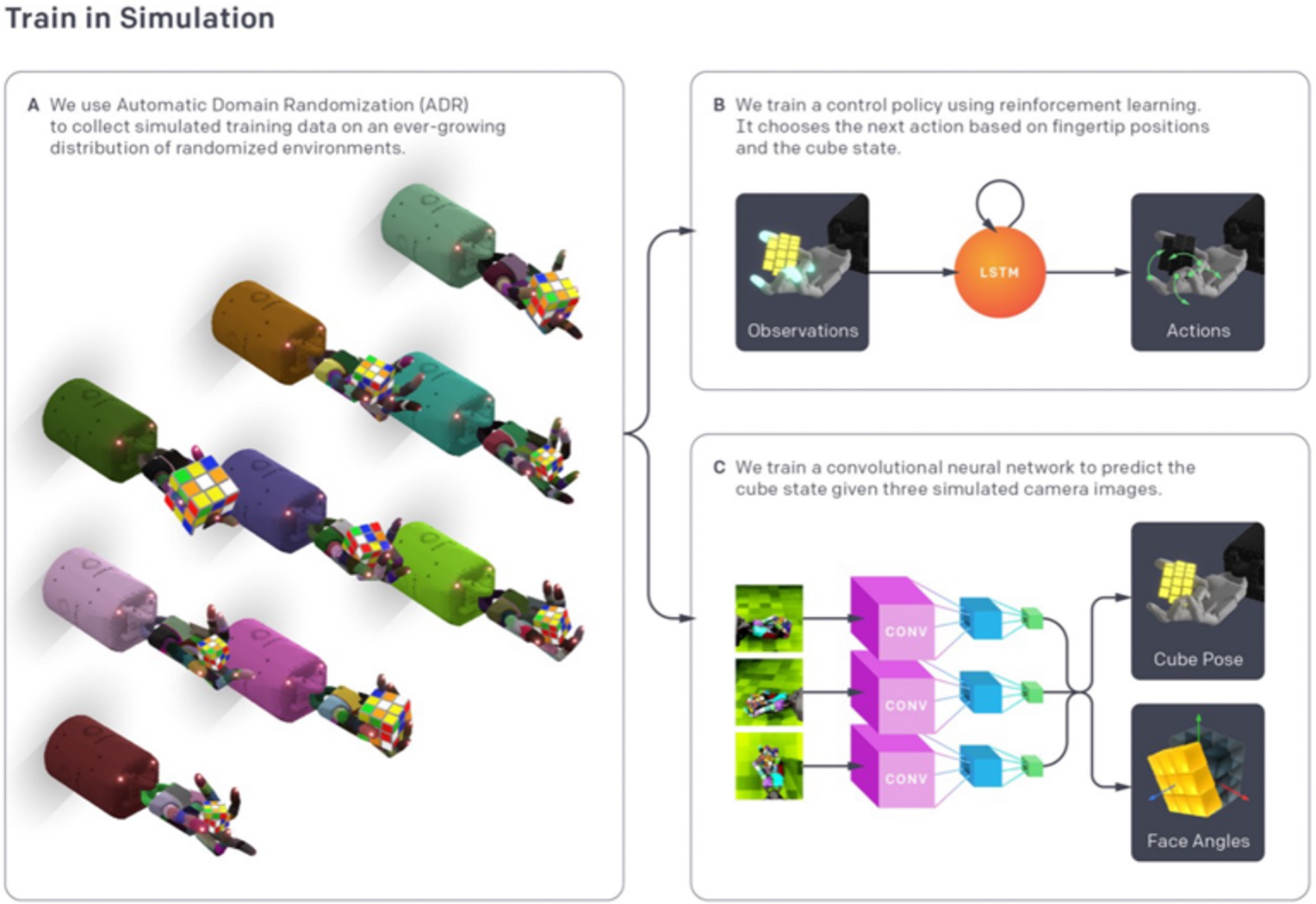
Cubuk, Ekin D., et al. "Randaugment: Practical automated data augmentation with a reduced search space." *Proceedings of the IEEE/CVF conference on computer vision and pattern recognition workshops*. 2020.

Data Augmentation

Other methods of Dataset Augmentation: Adding noise to input

- Neural Networks are inherently not insensitive to noise
- Training a NN with a small amount of noise added to the data can make the network more robust

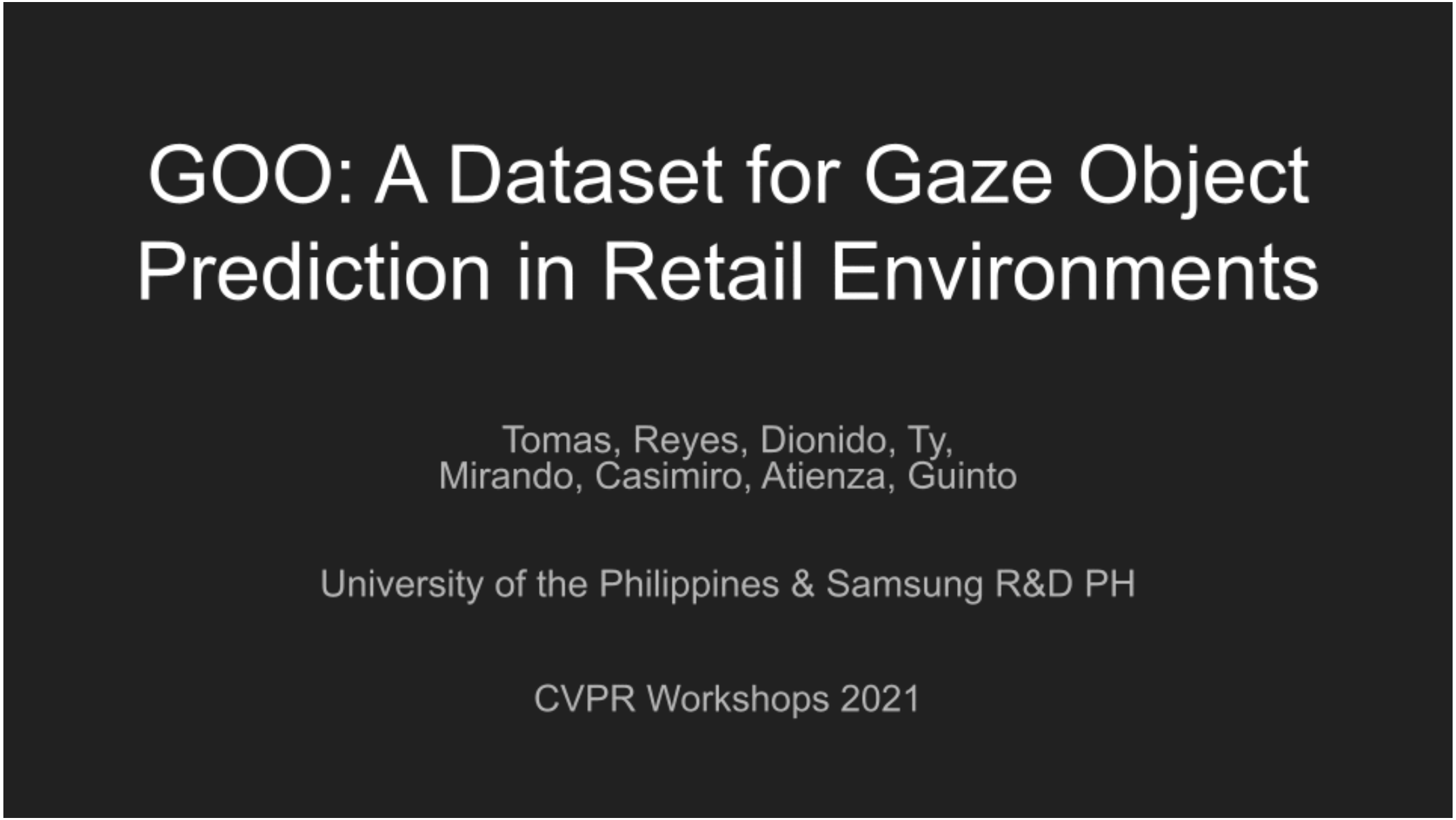
Automatic Domain Randomization (ADR)



Tobin, Josh, et al. "Domain randomization and generative models for robotic grasping." *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018.

https://youtu.be/FVl8OkNXiDk


TWO MINUTE
PAPERS
WITH KÁROLY ZSOLNAI FEHÉR (KZF)
TRANSFERRING AI
TO THE REAL WORLD



Tomas, Henri, et al. "Goo: A dataset for gaze object prediction in retail environments." *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition*. 2021.

Noise as Weights Regularizer

Noise can also be added to weights

In Bayesian Approach, model weights are stochastic

Adding noise is one way to model stochasticity

For example, MSE: $L = (\mathbf{Xw}-\mathbf{y})^T(\mathbf{Xw}-\mathbf{y})$

Suppose the weights are corrupted

$\mathbf{w}$ + noise = $(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$

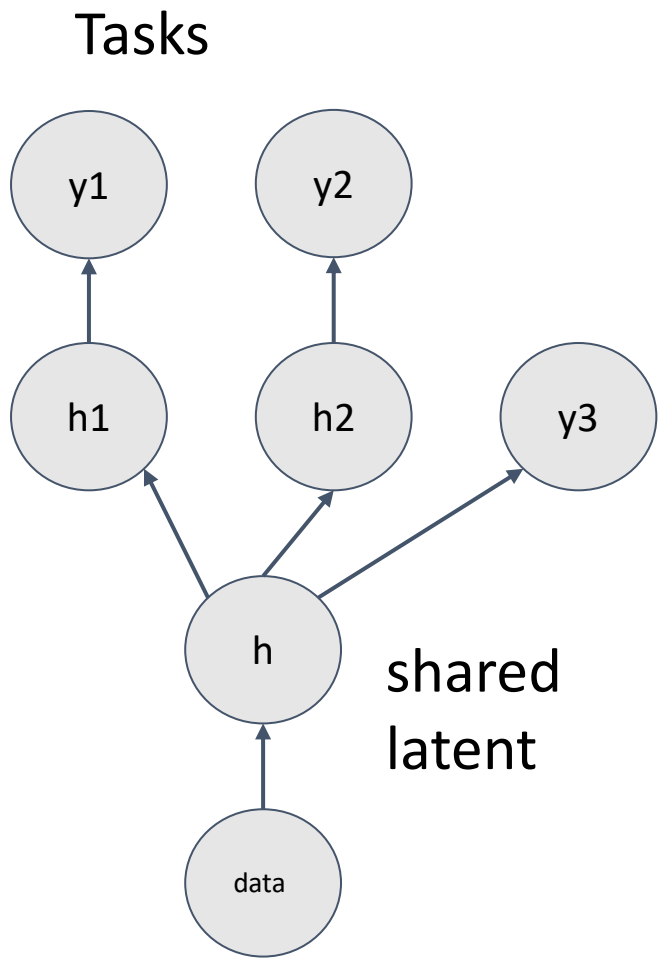
$\mathbf{w} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y}$ - noise

The constraints drive the values of $\mathbf{w}$ to a flat region such that a small perturbation will not alter the prediction

Multi-task Learning

Same data is used to learn multiple tasks

- This is a form of regularization since the network is subjected to soft constraints
- Shared parameters across multiple tasks
- In the factors that explain variations of observed data across multiple tasks, some are shared among multiple tasks
- y1 may be classification, y2 may be bounding box detection, y3 may segmentation
- Example: Mask RCNN



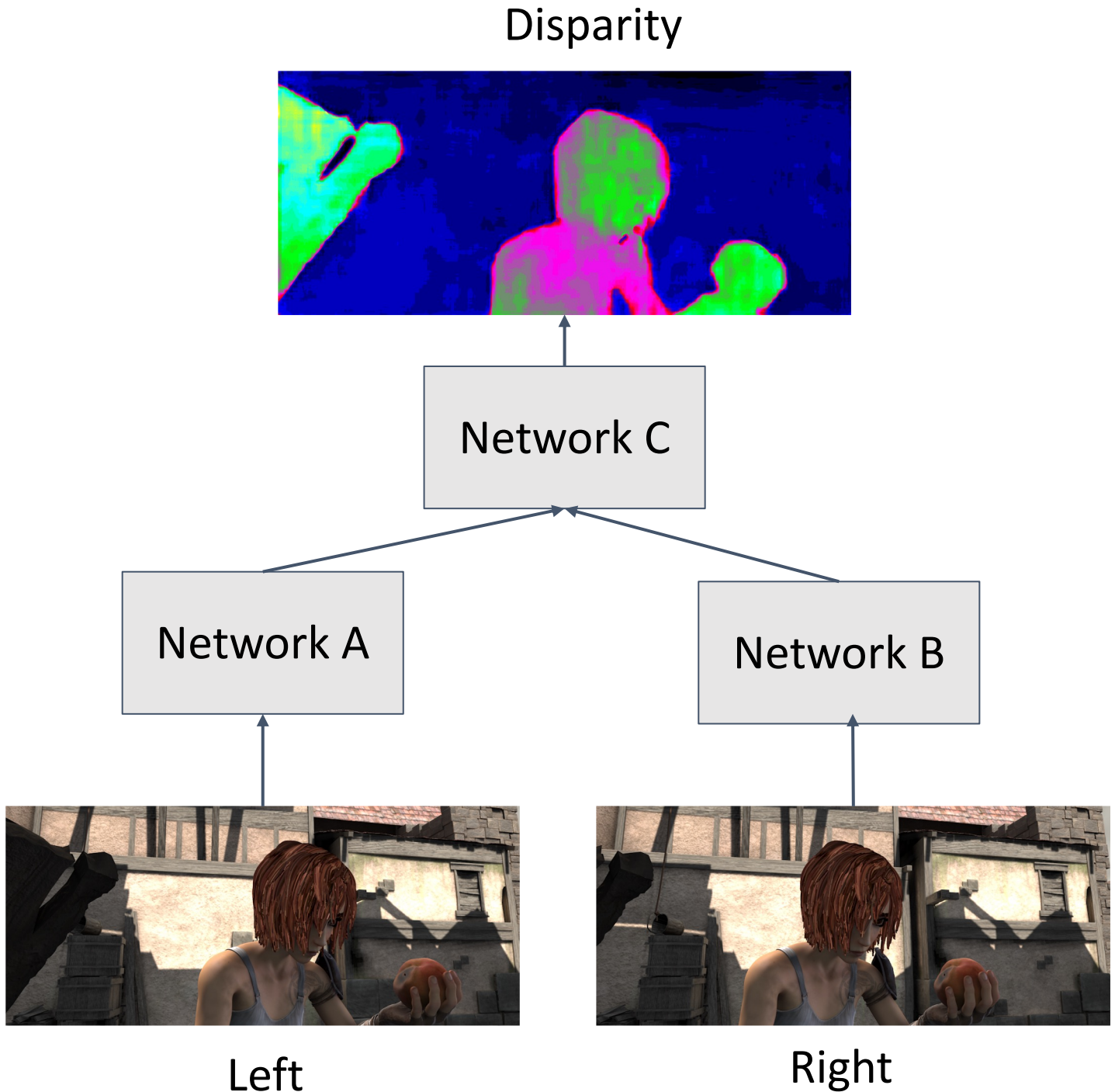
Parameter Sharing

Two different networks doing similar tasks can share parameters

Very common in Siamese Networks

Example, Networks A and B share the same set of parameters in a Siamese Network to perform disparity estimation

Forces Networks A and B to be more robust by forcing them to work on different images from the same distribution



Atienza, Rowel. "Fast disparity estimation using dense networks." *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018.

Early Stopping

Initially, validation error decreases as training error decreases

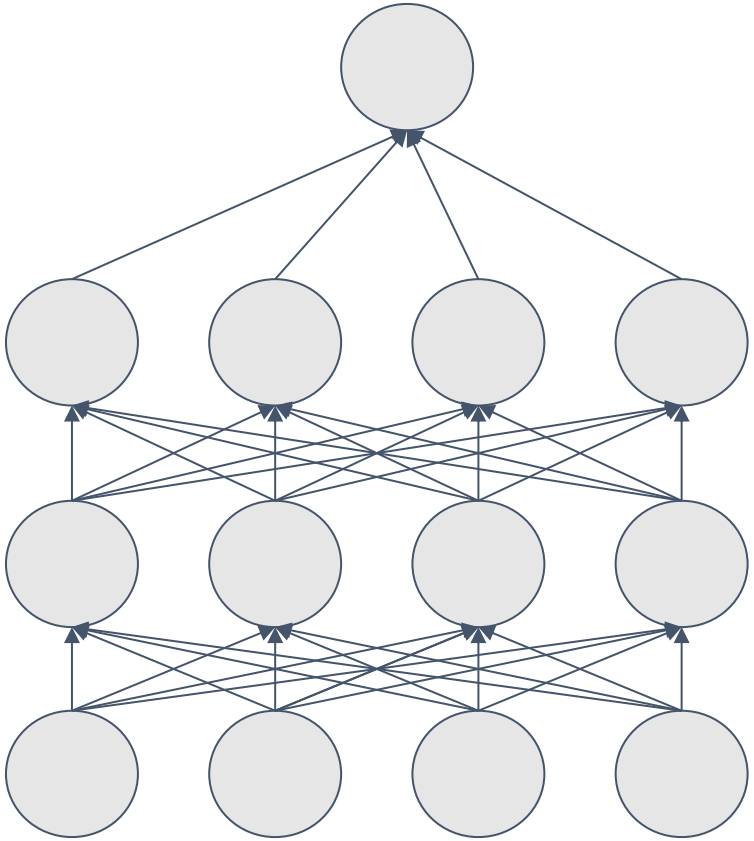
As training continues, the training error decreases while the validation error increases; network is overfitting

Early Stopping stops the training and saves the parameters when the validation error starts to increase

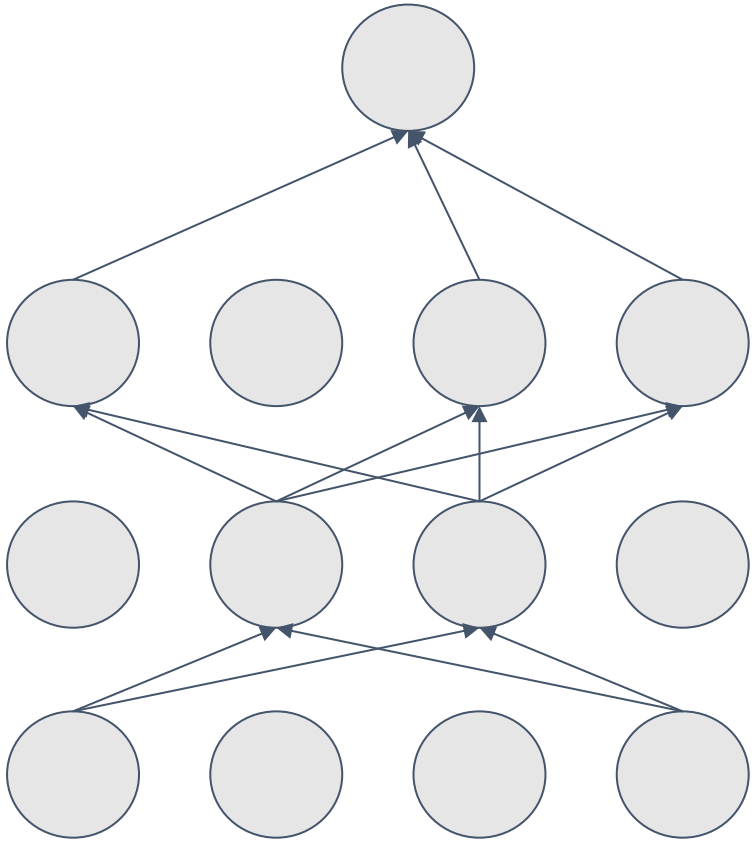
Early Stopping is a form of regularization since it limits the parameter space

Dropout

Strongly inspired by biological processes

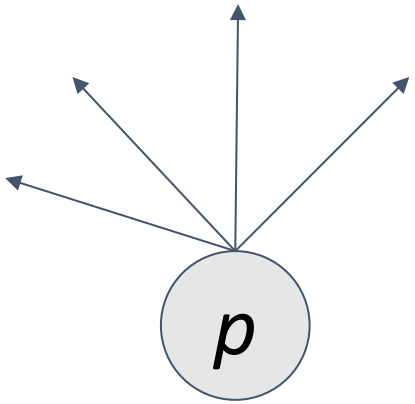


Standard Deep Neural Network

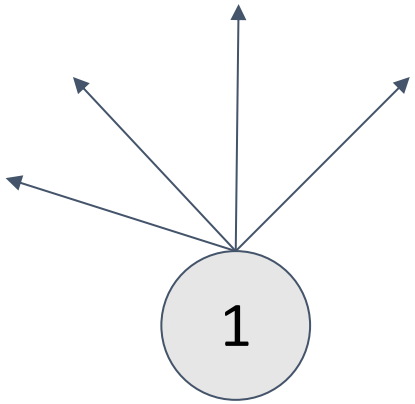


Standard Deep Neural Network With Dropout

Dropout



During training, a neuron will be included in the network with probability p



During test, a neuron will always be included in the network

Reference

Deep Learning, Ian Goodfellow and Yoshua Bengio and Aaron Courville, MIT Press, 2016, http://www.deeplearningbook.org

Dropout: A Simple Way to Prevent Neural Networks from Overfitting, Srivastava, et. al. Journal of Machine Learning, 2014

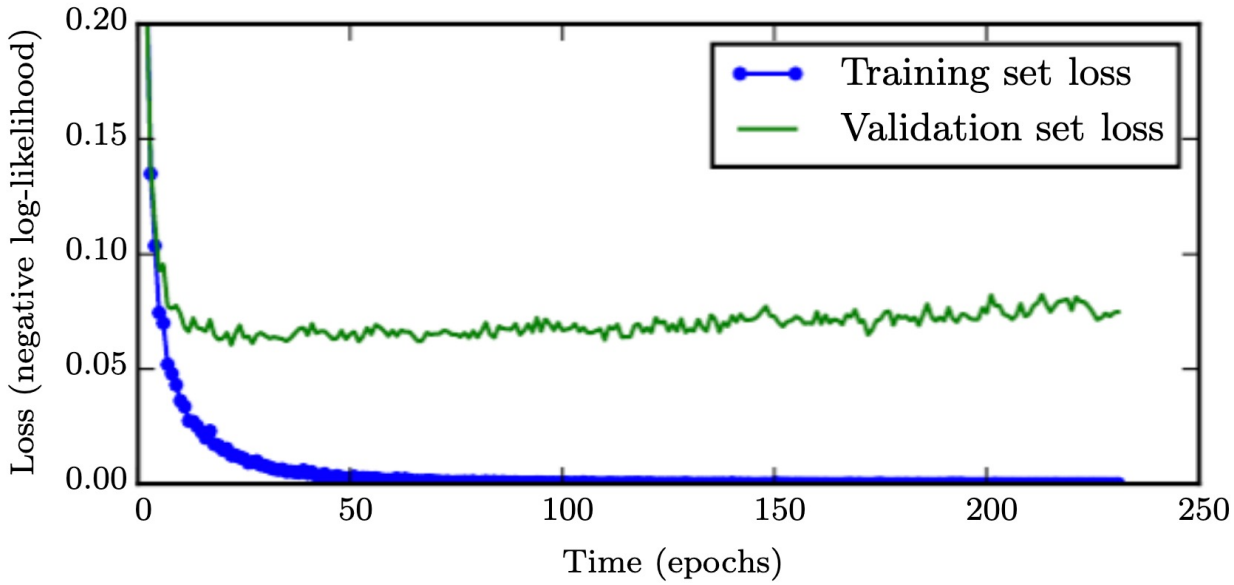
Marc Peter Deisenroth, A. Aldo Faisal, and Cheng Soon Ong , "Mathematics for Machine Learning". Copyright 2020 by. Published by Cambridge University Press.

In Summary

Regularization prevents a model from overfitting

Regularization algorithms include L2/L1, data augmentation, noise injection, parameter sharing, dropout

Overfitting



Deep Learning Book, Goodfellow et al 2016