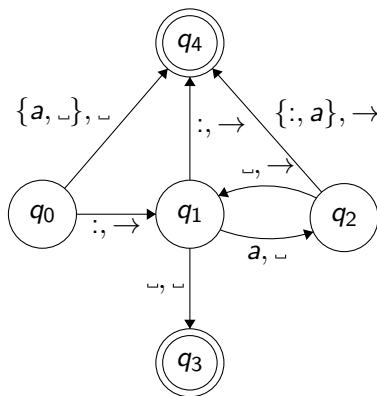


ITCS 532: W3 Homework Solutions

Rob Egrot

Q1

What does this machine do?



Erases its input.

Q2

Write out the transition function δ for T from Q1 in terms of tuples (i.e. (q, σ, q', σ')). Since $\sigma_0, \sigma_1, \sigma_2$, and σ_3 are taken by $:, \sqcup, \leftarrow$ and \rightarrow respectively we let $a = \sigma_4$.

- ▶ $(q_0, :, q_1, \rightarrow) = (q_0, \sigma_0, q_1, \sigma_3)$.
- ▶ $(q_0, \sqcup, q_4, \sqcup) = (q_0, \sigma_1, q_4, \sigma_1)$.
- ▶ $(q_0, a, q_4, \sqcup) = (q_0, \sigma_4, q_4, \sigma_1)$.
- ▶ $(q_1, :, q_4, \rightarrow) = (q_1, \sigma_0, q_4, \sigma_3)$.
- ▶ $(q_1, \sqcup, q_3, \sqcup) = (q_1, \sigma_1, q_3, \sigma_1)$.
- ▶ $(q_1, a, q_2, \sqcup) = (q_1, \sigma_4, q_2, \sigma_1)$.
- ▶ $(q_2, :, q_4, \rightarrow) = (q_2, \sigma_0, q_4, \sigma_3)$.
- ▶ $(q_2, \sqcup, q_1, \rightarrow) = (q_2, \sigma_1, q_1, \sigma_3)$.
- ▶ $(q_2, a, q_4, \rightarrow) = (q_2, \sigma_4, q_4, \sigma_3)$.

Q3

Using the system from the notes and your transition function from Q2 write down **code**(T).

$Code(q_4)$

10110111011110111110010110111011110111110011110111110

$Code(Q)$ $Code(\Sigma)$ $Code(q_3)$

101011011110101101111101101011111011101101011111011110

110110111101101101111101101110110110111101110111110111110

$Code(\delta)$

101101110111101111100101101110111101111100111101111101010110111010110

111110110101111101111101101101011111011110110110111101101111101111011101

101110110110111101111011111011111011110

$Code(T)$

Q4

Suppose T is a Turing machine over the alphabet $\{0, 1, *\}$ that takes as input two binary numbers separated by $*$ and outputs their sum (in binary). Describe a way we could use T to construct a Turing machine that takes as input two positive binary numbers separated by $*$ and outputs their product.

1. Copy first number to tape 2.
2. Copy second number to tape 3.
3. Trim tape 1 so it just contains first number.
4. If tape 3 is empty then erase tape 1 and halt.
5. If the number on tape 3 is one then halt.
6. Add number on tape 2 to the number on tape 1, then decrease number on tape 3 by one (we use T in this step).
7. Go to step 5).