ITCS 532:
Bonus question

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Is HAI r.e.?

- No.

- Consider the complement of the halting problem $\bar{H}$.
  - Yes instance is pair (T, I) such that $T(I)$ does not halt.
  - No instance is (T, I) such that $T(I)$ halts.

- $\bar{H}$ is not r.e., as otherwise H would be recursive (as H is r.e.).

- Given an instance (T, I) of $\bar{H}$, define an instance T_I of HAI.
- $T_I(J)$ erases its input then simulates $T(I)$ for $|J|$ steps.
- If $T(I)$ halts within $|J|$ steps then $T_I(J)$ loops.
- If $T(I)$ does not halt within $|J|$ steps then $T_I(J)$ halts.

$$\begin{aligned}(T, I) \text{ is yes of } \bar{H} &\implies T(I) \text{ does not halt} \\ &\implies T_I(J) \text{ halts for all } J \\ &\implies T_I \text{ is yes of } HAI.\end{aligned}$$

$$\begin{aligned}(T, I) \text{ is no of } \bar{H} &\implies T(I) \text{ halts (in } n \text{ steps say)} \\ &\implies T_I(J) \text{ does not halt when } |J| > n \\ &\implies T_I \text{ is no of } HAI.\end{aligned}$$

- So $\bar{H} \leq HAI$, and if HAI were r.e. $\bar{H}$ would be too.