ITCS 532:
W1 Homework Solutions

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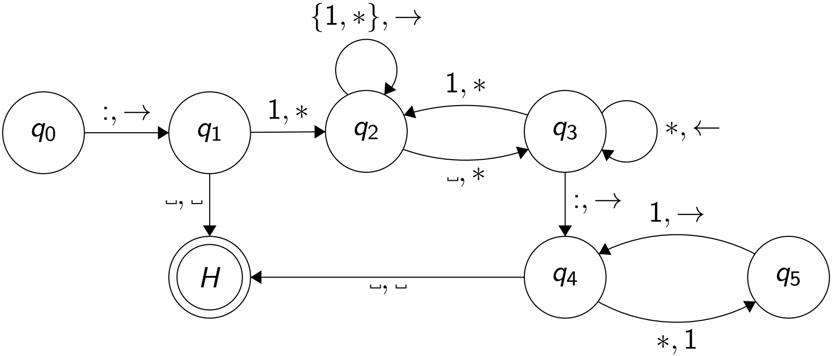
Q1

Design a suitable encoding system to encode quadratics $ax^2 + bx + c$.

- Use $\Sigma = \{1, -, *\}$.
- Store a, b and c in unary, separated by $*$, using $-$ to denote negatives.
- E.g. we express $2x^2 - 3x + 1$ as $11 * -111 * 1$.

Q2

Let $\Sigma = \{1, *\}$. Design a Turing machine that accepts as input a unary number (i.e. a finite string containing only 1s), and outputs that number multiplied by 2.



Q3

Can a formal language L exist that is recursive but not r.e.?

- No.
- We know from the notes that decidable implies semidecidable, and this is just the formal language version of that.
- I.e. if a TM exists that decides a language we can change the reject state into an infinite loop to get a machine the semidecides the language.
- Or use results from the class:

$$L \text{ recursive} \iff D_L \text{ decidable} \implies D_L \text{ semidecidable} \iff L \text{ r.e.}$$

Q4

Suppose we define a class $\mathcal{C}$ of abstract computational devices similar to Turing machines but without the $\leftarrow$ command (so the tape head may never move backwards).

(a) Give an informal argument for why this model of computation is strictly weaker than that of Turing machines.

(b) Would it make any difference if we replaced the $\leftarrow$ command with a $-$ command that keeps the tape head in the same place?

(c) (Hard) Give a rigorous proof for part (a).

- (a) No memory.
- (b) No. Still no memory.

Q4 part c

- Consider the problem of checking to see if two strings are the same length.
- Input is a string of 1's followed by a $*$, followed by another string of 1's.
- Suppose we have a $-$ machine M that solves this problem.
- Then it has a finite number of states, n say.
- Consider the set X that contains all strings of k ones followed by a $*$, for $k \in \{1, \ldots, n+1\}$.
- As $|X| = n+1$, by the pigeon hole principle there must be (at least) two strings in X such that M is in the same state when it gets to $*$.
- Call these strings $s*$ and $t*$.
- Then M should accept $s * s$, but then it must also accept $t * s$, which is incorrect.