Pairwise Sequence Alignment ILP Model

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parameter
$$a_{ij} = \begin{cases} 1 & \text{if } S_1[i] = S_2[j] \\ 0 & \text{otherwise} \end{cases}$$

minimize
$$z = \sum_{j=1}^{m} \sum_{i=1}^{n} x[i,j] \cdot r + \sum_{i=1}^{n} y[i] \cdot gp + \sum_{j=1}^{m} z[j] \cdot gp +$$
$$mp \cdot \sum_{i=1}^{n} \left(1 - \sum_{j=1}^{m} x[i,j] + y[i]\right) + v \cdot gp;$$

s.t.
$$\sum_{i=1}^{m} x_{ij} + y_i \le 1$$
 $i = 1, \dots, n$ (1)

$$\sum_{i=1}^{n} x_{ij} + z_j \le 1 j = 1, \dots, m (2)$$

$$x_{ij} + x_{kh} \le 1$$
 $i = 1, \dots, n, \ j = 1, \dots, m, \ k < i, \ h > j$ (3)

$$x_{ij} + x_{kh} \le 1$$
 $i = 1, \dots, n, \ j = 1, \dots, m, \ k > i, \ h < j$ (4)

$$x_{ij} \le a_{ij}$$
 $i = 1, \dots, n, \ j = 1, \dots, m$ (5)

$$x_{ij} \cdot \left(\sum_{k=1}^{i} y[k] - \sum_{h=1}^{j} z[h]\right) = x_{ij} \cdot (i-j) \qquad i = 1, \dots, n, \ j = 1, \dots, m$$
 (6)

$$-v \le \left(n + \sum_{j=1}^{m} z[j]\right) - \left(m + \sum_{i=1}^{n} y[i]\right) \le v \qquad i = 1, \dots, n, \ j = 1, \dots, m$$
 (7)

$$x_{ij} \in \{0, 1\}$$
 $i = 1, \dots, n, \ j = 1, \dots, m$ (8)

$$y_{ij} \in \{0, 1\}$$
 $i = 1, \dots, n, \ j = 1, \dots, m$ (9)

$$z_{ij} \in \{0, 1\}$$
 $i = 1, \dots, n, \ j = 1, \dots, m$ (10)

$$v \ge 0 \tag{11}$$