Pairwise Sequence Alignment ILP Model

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$$\text{parameter } a_{ij} = \begin{cases} 1 & \text{if } S_1[i] = S_2[j] \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \text{minimize } z = & \sum_{j=1}^{m}\sum_{i=1}^{n} x[i,j]\cdot r + \sum_{i=1}^{n} y[i]\cdot gp + \sum_{j=1}^{m} z[j]\cdot gp + \\ & mp\cdot\sum_{i=1}^{n}\left(1 - \sum_{j=1}^{m} x[i,j] - y[i]\right) + v\cdot gp; \end{aligned}$$

$$s.t. \quad \sum_{j=1}^{m} x_{ij} + y_i \leq 1 \qquad i = 1,\dots,n \qquad (1)$$

$$\sum_{i=1}^{n} x_{ij} + z_j \leq 1 \qquad j = 1,\dots,m \qquad (2)$$

$$x_{ij} + x_{kh} \leq 1 \qquad i = 1,\dots,n,\ j = 1,\dots,m,\ k < i,\ h > j \qquad (3)$$

$$x_{ij} + x_{kh} \leq 1 \qquad i = 1,\dots,n,\ j = 1,\dots,m,\ k > i,\ h < j \qquad (4)$$

$$x_{ij} \leq a_{ij} \qquad i = 1,\dots,n,\ j = 1,\dots,m \qquad (5)$$

$$x_{ij}\cdot\left(\sum_{k=1}^{i} y[k] - \sum_{h=1}^{j} z[h]\right) = x_{ij}\cdot(i-j) \qquad i = 1,\dots,n,\ j = 1,\dots,m \qquad (6)$$

$$-v \leq \left(n + \sum_{j=1}^{m} z[j]\right) - \left(m + \sum_{i=1}^{n} y[i]\right) \leq v \qquad i = 1,\dots,n,\ j = 1,\dots,m \qquad (7)$$

$$x_{ij} \in \{0,1\} \qquad i = 1,\dots,n,\ j = 1,\dots,m \qquad (8)$$

$$y_i \in \{0,1\} \qquad i = 1,\dots,n \qquad (9)$$

$$z_j \in \{0,1\} \qquad j = 1,\dots,m \qquad (10)$$

$$v \geq 0 \qquad (11)$$