# The Rabin-Karp algorithm : A different approach to exact matching

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# Eliminating spurious comparisons through "fingerprinting"

Rabin-Karp is a form of semi-numerical string matching:

Instead of focusing on comparing characters, think of string as a **sequence of bits or numbers** and use arithmetic operations to search for patterns.

Tends to work best for short patterns, and when there are relatively few occurrences of the pattern in the text.

# Characters as digits

- Assume $\sum = \{0,\ldots,9\}$
- Then a string can be thought of as the decimal representation of a number:

**427328**

- In general, if $|\sum| = d$, a string represents a number in base $d$.
- Let $p$ = the number represented by query $P$.
- Let $t_s$ = the number represented by the $|P|$ digits of $T$ that start at position $s$.

$P$ occurs at position $s$ of $T \Leftrightarrow p = t_s$.

# If the pattern is "small", comparison can be fast (O(1))

- Imagine $\log_2(|\sum|) * |P| <= 64$ (typical word size)
- Then, both $p$ and $t_s$ can fit in a machine word, and comparison can be done in constant time.
- 2 problems:
  - How do we *encode* the string into a word in constant time?
  - What do we do when $\log_2(|\sum|) * |P| > 64$ ?

# Computing p and $t_s$

- Consider representing P via the following polynomial:

$$p = P[m] + P[m\text{-}1]10^1 + P[m\text{-}2]10^2 + \ldots + P[1]10^{m-1}$$

- Use Horner's rule to compute O(|$P$|=$m$):

$$p = P[m] + 10(P[m\text{-}1] + 10(P[m\text{-}2] + \ldots + 10(P[2] + 10P[1])\ldots)$$

- Example: $427328 = (8+10(2+10(3+10(7+10(2 + 10 \times 4)))))$

- $t_0$ can be computed the same way in time O(|$P$|=$m$).

- $t_s$ can be computed from $t_{s-1}$ **in O(1) time**:

$$t_s = \underset{\text{shift left by 1 digit}}{10}(\underbrace{t_{s-1} - 10^{m-1}T[s-1]}_{\text{remove high-order digit}}) + \underbrace{T[s+m-1]}_{\text{add next digit of T as the low-order digit}}$$

# Rabin-Karp

Compute $p$.

Iteratively compute $t_s$.

Output $s$ when $t_s = p$.

Problem: $p$ and $t_s$ might be huge numbers.

Solution: compute everything modulo some large prime number $q$.

- If $10q$ is $\leq$ word size, then $p$ mod $q$ and $t_s$ mod $q$ can be computed in a single word.
- If $p$ occurs at $t_s$, then $p \equiv t_s \pmod{q}$

New problem: If $p \equiv t_s \pmod{q}$, it doesn't necessarily mean there is a match at $s$.

New solution: if $p \equiv t_s \pmod{q}$, check match explicitly.

Worst-case runtime = $O(mn)$, if every position is a match or false positive.

# Rabin-Karp Notes

- If your pattern is very small, don't need to use the (mod $q$) trick, and you can avoid false positive matches.
- You can also pick several different primes $q_1, q_2, \ldots, q_k$ and then require that:

$$p \equiv t_s \pmod{q_1}$$

$$p \equiv t_s \pmod{q_2}$$

$$\vdots$$

$$p \equiv t_s \pmod{q_k}$$

# Rabin-Karp Notes

- Think about this with respect to DNA / RNA; how long of a pattern can we search for, without using the mod trick, if we choose the right encoding (assume machine word = 64-bits)?
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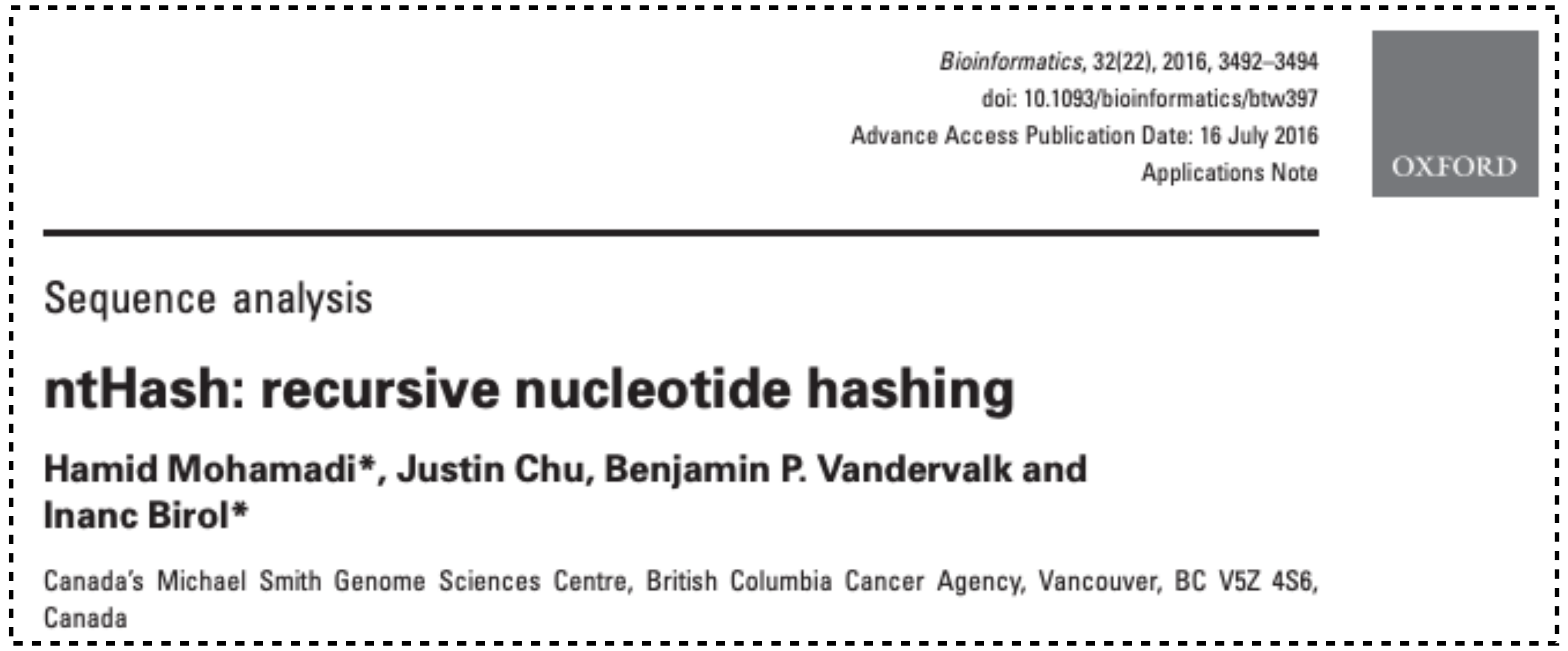
  - We can search for a pattern of length <= 32. Consider encoding each nucleotide in 2-bits e.g. A = 00, C = 01, G = 10, T = 11. Then a string of up to 32 nucleotides fits in a single machine word.

- For a good rolling hash for nucleotides, see the ntHash paper (https://academic.oup.com/bioinformatics/article/32/22/3492/2525588)



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Applications Note

OXFORD

Sequence analysis

**ntHash: recursive nucleotide hashing**

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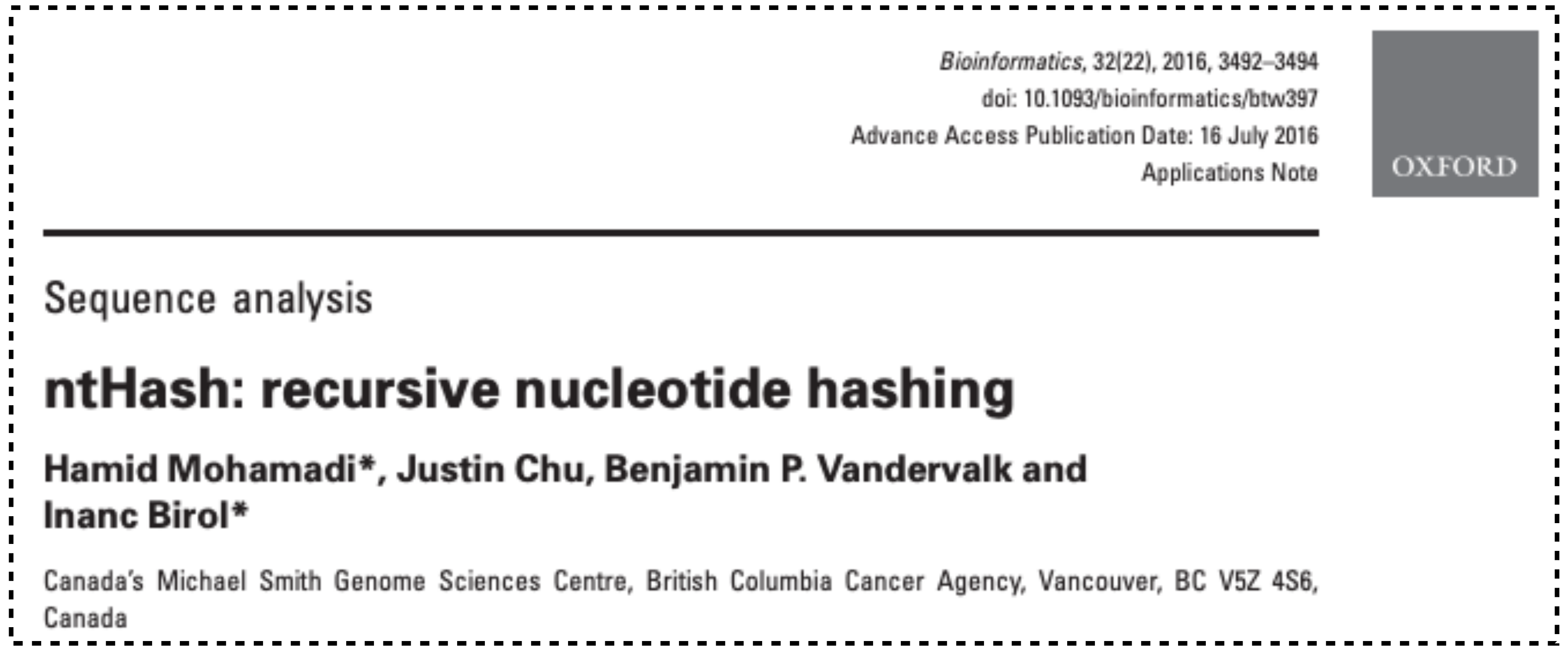

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## Basic implementation of Rabin-Karp following implementation in CLRS (code from https://www.geeksforgeeks.org/rabin-karp-algorithm-for-pattern-searching/)

```
/* Following program is a C++ implementation of Rabin Karp
Algorithm given in the CLRS book */
#include <bits/stdc++.h>
using namespace std;

// d is the number of characters in the input alphabet
#define d 256

/* pat -> pattern
        txt -> text
        q -> A prime number
*/
```

```
void search(char pat[], char txt[], int q)
{
        int M = strlen(pat);
        int N = strlen(txt);
        int i, j;
        int p = 0; // hash value for pattern
        int t = 0; // hash value for txt
        int h = 1;
        // The value of h would be "pow(d, M-1)%q"
        for (i = 0; i < M - 1; i++)
                h = (h * d) % q;
        // Calculate the hash value of pattern and first
        // window of text
        for (i = 0; i < M; i++)
        {
                p = (d * p + pat[i]) % q;
                t = (d * t + txt[i]) % q;
        }
        // Slide the pattern over text one by one
        for (i = 0; i <= N - M; i++)
        {
                // Check the hash values of current window of text
                // and pattern. If the hash values match then only
                // check for characters one by one
                if ( p == t )
                {
                        bool flag = true;
                        /* Check for characters one by one */
                        for (j = 0; j < M; j++)
                        {
                                if (txt[i+j] != pat[j])
                                {
                                flag = false;
                                break;
                                }
                                if(flag)
                                cout<<i<<" ";

                        }
                        // if p == t and pat[0...M-1] = txt[i, i+1, ...i+M-1]
                        if (j == M)
                                cout<<"Pattern found at index "<< i<<endl;
                }
                // Calculate hash value for next window of text: Remove
                // leading digit, add trailing digit
                if ( i < N-M )
                {
                        t = (d*(t - txt[i]*h) + txt[i+M])%q;
                        // We might get negative value of t, converting it
                        // to positive
                        if (t < 0)
                        t = (t + q);
                }
        }
}
```

```
/* Driver code */
int main()
{
        char txt[] = "GEEKS FOR GEEKS";
        char pat[] = "GEEK";

        // A prime number
        int q = 101;

        // Function Call
        search(pat, txt, q);
        return 0;
}

// This is code is contributed by rathbhupendra
```