

1. Assumption: Incredible detail about the game rules isn't necessary

Monopoly

State descriptions:

Initial State:

The beginning of the game, each player has his/her token on the go tile. Each player has \$1500.00 dollars in the following denominations: 2 each of (\$500.00, \$100.00, \$50.00), 5 each of (\$10.00, \$5.00, \$1.00), and 6 \$20.00. There are two stacks of cards: Chance and Community Chest, the board is otherwise clear. No property is yet owned.

Player start State:

Current player rolls two six-sided dice, the player must move the number of tiles indicated by the sum of these dice from their current tile. If a player rolls doubles they proceed to the move state and may roll again, three sequential double rolls causes the player to move directly to the jail tile. If they player is currently in jail on the beginning of their turn they may attempt to roll doubles , pay a \$50.00 fine, or redeem a currently held get out of jail free card to be freed from jail.

Player move State:

Once the dice are rolled and the player has moved they must obey stipulations regarding the tile they landed on. If the go tile is landed on or passed during movement, the player may collect \$200.00 from the bank. If player lands on an unowned property they may purchase it. If the player lands on owned property they must pay rent to the player that owns it. If the player lands on chance or community chest they must take a card from the respective pile and obey what it says to do. If they player lands on luxury tax they must pay the amount specified. If the player lands on the go to jail tile they must go to jail

Player end state:

Once the player has moved and taken care of any directives indicated by cards or tiles the player may simply end their turn. If the player owns all of the properties of a color group (excluding railroads) they may now buy houses or hotels to put on their property. The player may also mortgage currently owned property to the bank or obtain a loan from the bank before ending their turn. The player may also trade properties with other players.

Goal State:

One player bankrupts all other players meaning they do not have any money or properties.

Move generators

- i. the outcome of the dice roll. The player is to move the number of spaces indicated: a value from 1 to 12. The player must also move to jail if three sets of doubles are thrown back to back.

- ii. When a chance or community chest card directs the player to move to a specific tile they must do so
- iii. When the player lands on the go to jail card they must move to jail

Terminal tests

If there are still at least two players who are not bankrupt then continue playing, otherwise the last remaining player wins.

Utility functions

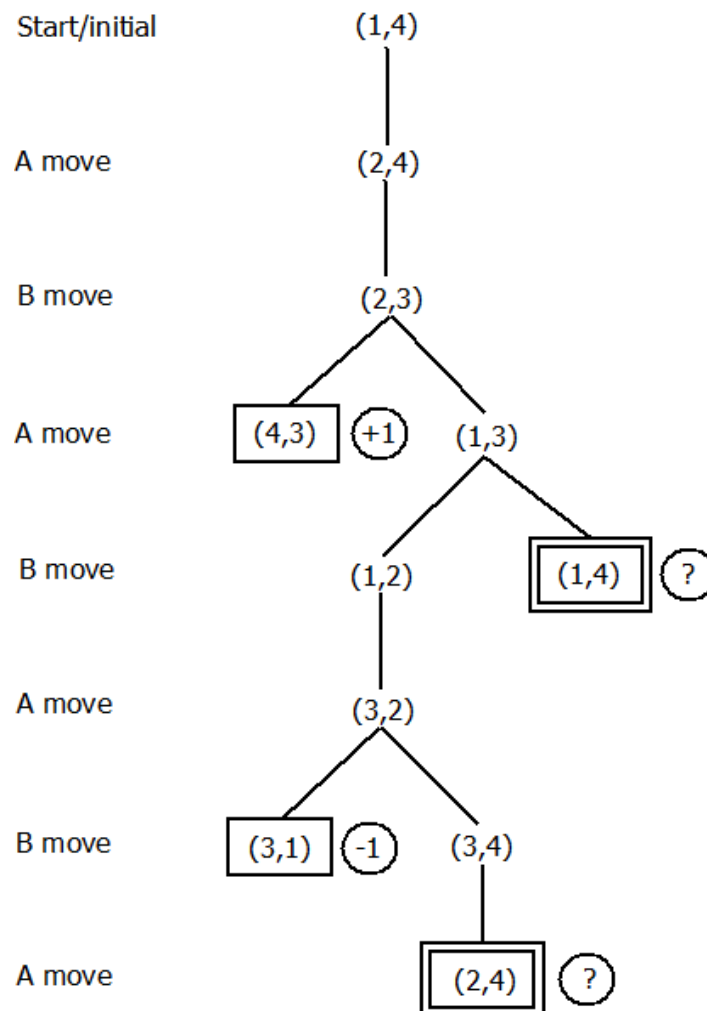
As the way players generate money is by owning property there are two general approaches to winning the game: Prioritize properties with high payouts and/or prioritize owning as many developed properties as possible.

Evaluation functions

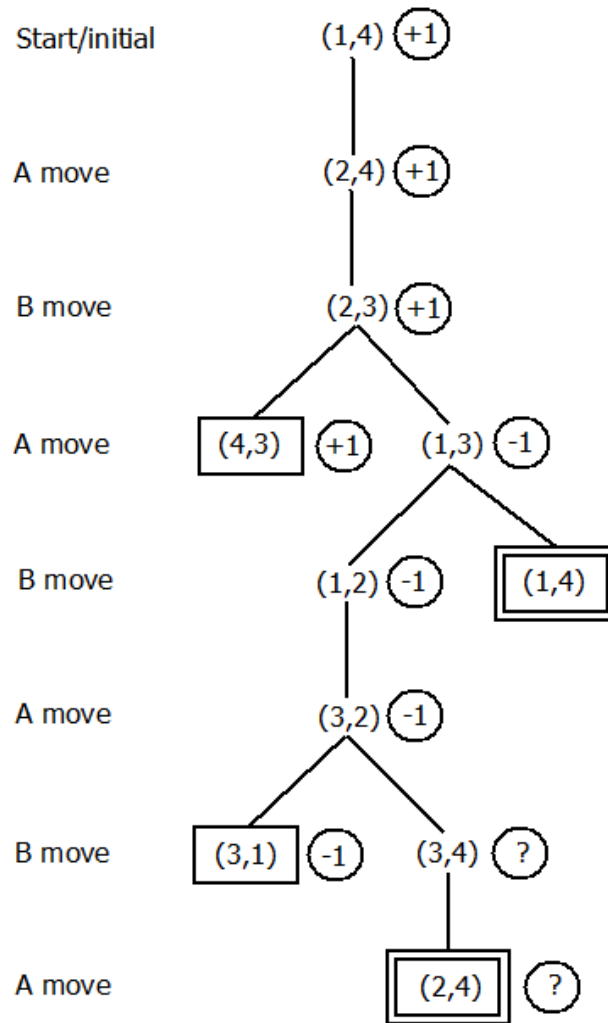
The decision to purchase a property can be influenced by several strategies: If a player owns all but one property a player may choose to buy the remaining property if they land on it to prevent the owner of the remainder of the property from increasing their rent. A player will also put more value in buying a property among a color they already own one or more properties in order to do the same thing.

2.

a.



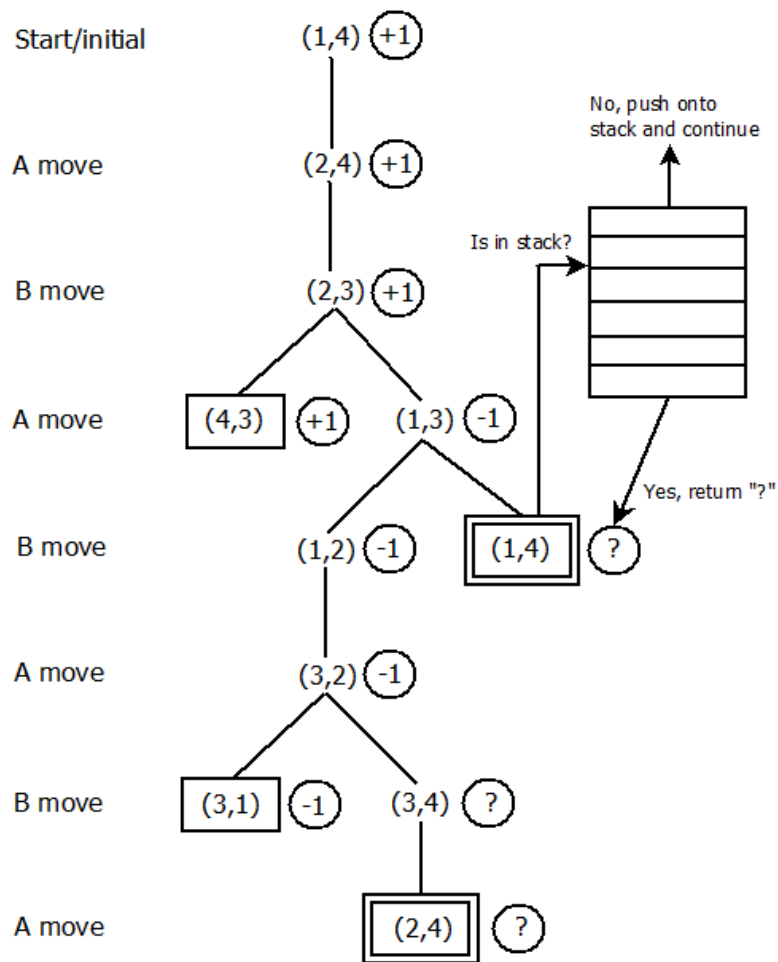
b.



The “?” only becomes the back-up minimax value if there is no other option. This is because if the player has the option of winning or not they will choose to win. So only if all successors are “?” the back-up value becomes “?”.

c.

- i. The standard minimax algorithm is a depth-first search and so it goes into an infinite loop, not terminating. This can be fixed by comparing the current state to the stack that hold the states we have already been to, if the state is already in the stack we just return the “?” which is backed up according to part b.
- ii. Assuming by all games with loops you mean any game that has a possible loop state and not just this particular game, I would have to say it isn't clear whether or not it guarantees optimal decisions. This game is very straight forward and it works here but other games may have conditions such as not simply winning or losing that could affect whether we want to avoid loops or not in certain situations.



- d. If we Assume that no character will move back towards their home spot we can see that the first player that has an opportunity to jump the other character will win as they are skipping a tile while the other player must travel the full length. Since A goes first any board with an even number of tiles will always result in A jumping B and for any board with an odd number of tiles B will jump A.

To put it in terms of n , if we assume n is always some even number then we can say that whoever moves first in a game of n squares will win. A game with an odd number of tiles would actually begin at $n + 1$. When the number of tiles is even, A moves first in the game of n tiles. However, if the number of tiles is odd, A moving first brings the play area from $n + 1$ to n and now B is the first to move in the n tile game and so is the winner. We can generalize this to whichever player moves with n number of tiles between each player will win the game.

3. Smoke: S, Fire: F, Heat: H, Big: B, Dumb: D

a. Valid

S	(S → S)
F	T
T	T

b. Neither

S	F	$(S \rightarrow F)$
F	F	T
F	T	T
T	F	F
T	T	T

c. Neither

S	F	$\neg S$	$\neg F$	$S \rightarrow F$	$\neg S \rightarrow \neg F$	$(S \rightarrow F) \rightarrow (\neg S \rightarrow \neg F)$
F	F	T	T	T	T	T
F	T	T	F	T	F	F
T	F	F	T	F	T	T
T	T	F	F	T	T	T

d. Valid

S	F	$\neg F$	$S \vee F \vee \neg F$
F	F	T	T
F	T	F	T
T	F	T	T
T	T	F	T

e. Valid

S	H	F	$S \wedge H$	$(S \wedge H) \rightarrow F$	$S \rightarrow F$	$H \rightarrow F$	$(S \rightarrow F) \vee (H \rightarrow F)$	$((S \wedge H) \rightarrow F) \leftrightarrow (S \rightarrow F) \vee (H \rightarrow F)$
F	F	F	F	T	T	T	T	T
F	F	T	F	T	T	T	T	T
F	T	F	F	T	T	F	T	T
F	T	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
T	F	T	F	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	T	T	T	T	T	T	T	T

f. Valid

S	H	F	$S \rightarrow F$	$S \wedge H$	$(S \wedge H) \rightarrow F$	$(S \rightarrow F) \rightarrow ((S \wedge H) \rightarrow F)$
F	F	F	T	F	T	T
F	F	T	T	F	T	T
F	T	F	T	F	T	T
F	T	T	T	F	T	T
T	F	F	F	F	T	T
T	F	T	T	F	T	T
T	T	F	F	T	F	T
T	T	T	T	T	T	T

g. Valid

B	D	$B \rightarrow D$	$B \vee D \vee (B \rightarrow D)$
F	F	T	T
F	T	T	T
T	F	F	T
T	T	T	T

4.

a.

- Not correct; It is not necessary for a conservative to be radical
- Correct; A radical is only electable if they are also a conservative
- Not correct; After implication elimination this becomes $E \vee \neg E$ which is a tautology

b. All can be written in Horn form

- $((R \wedge E) \rightarrow C) \wedge (C \rightarrow (R \wedge E)) \equiv ((R \wedge E) \rightarrow C) \wedge (C \rightarrow R) \wedge (C \rightarrow E)$
- $R \rightarrow ((E \rightarrow C) \wedge (C \rightarrow E)) \equiv \neg R \vee ((\neg E \vee C) \wedge (\neg C \vee E))$
 $\equiv (\neg R \vee \neg E \vee C) \wedge (\neg R \vee \neg C \vee E)$
- True \rightarrow true

5.

a. I assume we are just doing the truth table portion of enumeration.

The sentence is valid

Food: F, Party: P, Drinks: D

F	P	D	$F \rightarrow P$	$D \rightarrow P$	$(F \rightarrow P) \vee (D \rightarrow P)$	$F \wedge D$	$(F \wedge D) \rightarrow P$	$((F \rightarrow P) \vee (D \rightarrow P)) \rightarrow ((F \wedge D) \rightarrow P)$
F	F	F	T	T	T	F	T	T
F	F	T	T	F	T	F	T	T
F	T	F	T	T	T	F	T	T
F	T	T	T	T	T	F	T	T
T	F	F	F	T	T	F	T	T
T	F	T	F	F	F	T	F	T
T	T	F	T	T	T	F	T	T
T	T	T	T	T	T	T	T	T

b. LHS:

- Implication elimination: rewrite $a \rightarrow b$ as $\neg a \vee b$
 $(\neg F \vee P) \vee (\neg D \vee P)$
- Or expansion
 $\neg F \vee P \vee \neg D \vee P$
- Eliminate superfluous terms
 $\neg F \vee P \vee \neg D$

RHS:

- i. Implication elimination: rewrite $a \rightarrow b$ as $\neg a \vee b$
 $\neg(F \wedge D) \vee P$
- ii. DeMorgan's
 $\neg F \vee \neg D \vee P$

We can rewrite the RHS as: $\neg F \vee P \vee \neg D$ and see that the LHS = RHS.

Let's call $\neg F \vee P \vee \neg D$, X. Now we have $X \rightarrow X$ so whether X is True or False the expression will be True so we have a tautology which confirms what we got for the truth table in part a.

- c. First we can take our LHS and RHS from part b and convert it completely to CNF:
 - i. From part b:
 $(\neg F \vee P \vee \neg D) \rightarrow (\neg F \vee \neg D \vee P)$
 - ii. Implication elimination:
 $\neg(\neg F \vee P \vee \neg D) \vee (\neg F \vee \neg D \vee P)$
 - iii. DeMorgan's:
 $(F \wedge \neg P \wedge D) \vee (\neg F \vee \neg D \vee P)$
 - iv. Expand or
 $(F \wedge \neg P \wedge D) \vee \neg F \vee \neg D \vee P \leftarrow \text{Complete CNF}$

This sentence is true no matter what because while the ANDed terms on the left have to be a specific value to be true, every term that is ORed is the negation of all these terms. So if F is 0 then $\neg F$ will be 1, the same is true for all of the other terms.

We can continue our resolution proof by negating our CNF sentence:

- i. Negate CNF:
 $\neg((F \wedge \neg P \wedge D) \vee \neg F \vee \neg D \vee P)$
- ii. DeMorgan's:
 $\neg(F \wedge \neg P \wedge D) \wedge F \wedge D \wedge \neg P$
- iii. More DeMorgan's:
 $(\neg F \vee P \vee \neg D) \wedge F \wedge D \wedge \neg P$

This sentence is false no matter what and so we have the empty clause, our proof is complete.

6.

- a. This translation is not good, it says nothing about two people not having the same social security number it simply states they have one, they could be the same given this sentence. Also, implication should not be used with the existential quantifier, conjunction should be used instead.

Correct sentence:

$$\neg \exists x, y, n \text{ Person}(x) \wedge \text{Person}(y) \wedge (x \neq y) \wedge \text{HasSS\#}(x, n) \wedge \text{HasSS\#}(y, n)$$

- b. This translation is good
- c. This translation is not good. This insinuates that every person has a social security card. We should clarify that the person has a social security number.

Correct sentence:

$$\forall x, n (\text{Person}(x) \wedge \text{HasSS\#}(x, n)) \rightarrow \text{Digits}(n, 9)$$

- d. Assumption: we no longer need to clarify n as the function should do so
- i. $\neg \exists x, y \text{ Person}(x) \wedge \text{Person}(y) \Rightarrow \text{SS \#}(x) = \text{SS \#}(y)$
 - ii. $\text{SS \#}(\text{John}) = \text{SS \#}(\text{Mary})$
 - iii. $\forall x \text{ Person}(x) \Rightarrow \text{Digits}(\text{SS \#}(x), 9)$

7.

- a. Goal: $7 \leq 3 + 9$

Steps:

- i. Axiom 8
 $x = 7 + 0, y = 7, z = 3 + 9$
 $7 + 0 \leq 7 \wedge 7 \leq 3 + 9 \rightarrow 7 + 0 \leq 3 + 9$
- ii. Axiom 6
 $x = 3, y = 9$
 $3 + 9 \leq 9 + 3$
- iii. Axiom 7
 $w = 7, x = 0, y = 9, z = 3$
 $7 \leq 9 \wedge 0 \leq 3 \rightarrow 7 + 0 \leq 9 + 3$
- iv. Axiom 1
 $0 \leq 3$
- v. Axiom 2
 $7 \leq 9$

- b. Goal: $7 \leq 3 + 9$

Steps:

- i. Axiom 1
 $0 \leq 3$
- ii. Axiom 2
 $7 \leq 9$
- iii. Axiom 7
 $w = 7, x = 0, y = 9, z = 3$
 $7 \leq 9 \wedge 0 \leq 3 \rightarrow 7 + 0 \leq 9 + 3$
- iv. Axiom 4
 $x = 7$
 $7 \leq 7 + 0$
- v. Axiom 6
 $x = 3, y = 9$
 $3 + 9 \leq 9 + 3$
- vi. Axiom 8
 $x = 7 + 0, y = 7, z = 3 + 9$
 $7 + 0 \leq 7 \wedge 7 \leq 3 + 9 \rightarrow 7 + 0 \leq 3 + 9$