

Initialisation

Draw N samples $\mathbf{x}_0^{(i)}$ from the initial state distribution:

$$\left\{ \left(\mathbf{x}_0^{(i)}, N^{-1} \right) \right\}_{i=1}^N, \quad \mathbf{x}_0^{(i)} \sim p(\mathbf{x}_0)$$



Importance sampling step

Draw N samples $\hat{\mathbf{x}}_k^{(i)}$ from the proposal distribution:

$$\left\{ \left(\hat{\mathbf{x}}_k^{(i)}, N^{-1} \right) \right\}_{i=1}^N, \quad \hat{\mathbf{x}}_k^{(i)} \sim \pi(\mathbf{x}_k | \mathbf{X}_{k-1}^{(i)}, \mathbf{Z}_k, \mathbf{U}_{k-1})$$

Evaluate importance weights:

$$w_k^{(i)} = w_{k-1}^{(i)} \frac{p(\mathbf{z}_k | \hat{\mathbf{x}}_k^{(i)}) p(\hat{\mathbf{x}}_k^{(i)} | \mathbf{x}_{k-1}^{(i)}, \mathbf{u}_{k-1})}{\pi(\hat{\mathbf{x}}_k^{(i)} | \mathbf{X}_{k-1}^{(i)}, \mathbf{Z}_k, \mathbf{U}_{k-1})}, \quad i \in \{1, \dots, N\}$$

Normalise importance weights:

$$\tilde{w}_k^{(i)} = w_k^{(i)} \left[\sum_{j=1}^N w_k^{(j)} \right]^{-1}, \quad i \in \{1, \dots, N\}$$



Resampling step

Draw N samples $\mathbf{x}_k^{(i)}$ from the set $\left\{ \left(\hat{\mathbf{x}}_k^{(j)}, \tilde{w}_k^{(j)} \right) \right\}_{j=1}^N$:

$$\left\{ \left(\mathbf{x}_k^{(i)}, N^{-1} \right) \right\}_{i=1}^N, \quad \Pr(\mathbf{x}_k^{(i)} = \hat{\mathbf{x}}_k^{(j)}) = \tilde{w}_k^{(j)}, \quad i, j \in \{1, \dots, N\}$$



Recombine particles

Compute conditional mean:

$$\hat{\mathbf{x}}_k = \tilde{\mathbb{E}}_{p(\mathbf{x}_k | \mathbf{Z}_k, \mathbf{U}_{k-1})}[\mathbf{x}_k] = \frac{1}{N} \sum_{i=1}^N \mathbf{x}_k^{(i)}$$

Compute covariance:

$$\mathbf{P}_k = \sum_{i=1}^N (\mathbf{x}_k^{(i)} - \hat{\mathbf{x}}_k)(\mathbf{x}_k^{(i)} - \hat{\mathbf{x}}_k)^T$$



Output