Initialisation

Draw N samples $\mathbf{x}_0^{(i)}$ from the initial state distribution:

$$\left\{\left(\mathbf{x}_0^{(i)}, N^{-1}\right)\right\}_{i=1}^{N}, \quad \mathbf{x}_0^{(i)} \sim p(\mathbf{x}_0)$$

Importance sampling step

Draw N samples $\hat{\mathbf{x}}_k^{(i)}$ from the proposal distribution:

$$\left\{\left(\hat{\mathbf{x}}_k^{(i)}, N^{-1}\right)\right\}_{i=1}^{N}, \quad \hat{\mathbf{x}}_k^{(i)} \sim p(\mathbf{x}_k \,|\, \mathbf{x}_{k-1}^{(i)}, \mathbf{u}_{k-1}))$$

Evaluate importance weights:

$$w_k^{(i)} = p(\mathbf{z}_k \,|\, \hat{\mathbf{x}}_k^{(i)}), \quad i \in \{1, \ldots, N\}$$

Normalise importance weights:

$$\tilde{w}_k^{(i)} = w_k^{(i)} \left[\sum_{j=1}^{N} w_k^{(j)}\right]^{-1}, \quad i \in \{1, \ldots, N\}$$

Resampling step

Draw N samples $\mathbf{x}_k^{(i)}$ from the set $\left\{\left(\hat{\mathbf{x}}_k^{(j)}, \tilde{w}_k^{(j)}\right)\right\}_{j=1}^{N}$:

$$\left\{\left(\mathbf{x}_k^{(i)}, N^{-1}\right)\right\}_{i=1}^{N}, \quad \Pr\left(\mathbf{x}_k^{(i)} = \hat{\mathbf{x}}_k^{(j)}\right) = \tilde{w}_k^{(j)}, \quad i, j \in \{1, \ldots, N\}$$

Recombine particles

Compute conditional mean:

$$\hat{\mathbf{x}}_k = \tilde{\mathbb{E}}_{p(\mathbf{x}_k \,|\, \mathbf{z}_k, \mathbf{u}_{k-1})}\left[\mathbf{x}_k\right] = \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_k^{(i)}$$

Compute covariance:

$$\mathbf{P}_k = \sum_{i=1}^{N} \left(\mathbf{x}_k^{(i)} - \hat{\mathbf{x}}_k\right)\left(\mathbf{x}_k^{(i)} - \hat{\mathbf{x}}_k\right)^T$$

Output