Initialisation

Draw N samples $\mathbf{x}_0^{(i)}$ from the initial state distribution:

$$\left\{\left(\mathbf{x}_0^{(i)}, N^{-1}\right)\right\}_{i=1}^{N}, \quad \mathbf{x}_0^{(i)} \sim p(\mathbf{x}_0),$$

$$\bar{\mathbf{x}}_0^{(i)} = \mathbb{E}\left[\mathbf{x}_0^{(i)}\right], \quad \bar{\mathbf{x}}_0^{(i)a} = \left[\bar{\mathbf{x}}_0^{(i)\mathsf{T}}, \mathbb{E}\left[\mathbf{w}_0{}^{\mathsf{T}}\right], \mathbb{E}\left[\mathbf{v}_0{}^{\mathsf{T}}\right]\right]^{\mathsf{T}},$$

$$\mathbf{P}_0^{(i)} = \mathbb{E}\left[\left(\mathbf{x}_0^{(i)} - \bar{\mathbf{x}}_0^{(i)}\right)\left(\mathbf{x}_0^{(i)} - \bar{\mathbf{x}}_0^{(i)}\right)^{\mathsf{T}}\right],$$

$$\mathbf{P}_0^{(i)a} = \mathbb{E}\left[\left(\mathbf{x}_0^{(i)a} - \bar{\mathbf{x}}_0^{(i)a}\right)\left(\mathbf{x}_0^{(i)a} - \bar{\mathbf{x}}_0^{(i)a}\right)^{\mathsf{T}}\right] = \begin{bmatrix} \mathbf{P}_0^{(i)} & 0 & 0 \\ 0 & \mathbf{Q}_0 & 0 \\ 0 & 0 & \mathbf{R}_0 \end{bmatrix}$$

Calculation of sigma points

$$\mathcal{X}_{k-1}^{(i)a} = \left\{\bar{\mathbf{x}}_{k-1}^{(i)a}, \quad \bar{\mathbf{x}}_{k-1}^{(i)a} + \gamma_a\sqrt{\mathbf{P}_{k-1}^{(i)a}}, \quad \bar{\mathbf{x}}_{k-1}^{(i)a} - \gamma_a\sqrt{\mathbf{P}_{k-1}^{(i)a}}\right\}$$

Time update

Propagate sigma points:

$$\mathcal{X}_{k|k-1}^{(i)x} = \boldsymbol{\phi}_{k-1}\left(\mathcal{X}_{k-1}^{(i)x}, \mathbf{u}_{k-1}, \mathcal{X}_{k-1}^{(i)w}\right)$$

Compute a priori state estimate:

$$\bar{\mathbf{x}}_{k|k-1}^{(i)} = \sum_{j=0}^{2l} W_j^{(m)} \mathcal{X}_{j,k|k-1}^{(i)x}$$

Compute a priori error covariance:

$$\mathbf{P}_{k|k-1}^{(i)} = \sum_{j=0}^{2l} W_j^{(c)} \left(\mathcal{X}_{j,k|k-1}^{(i)x} - \bar{\mathbf{x}}_{k|k-1}^{(i)}\right)\left(\mathcal{X}_{j,k|k-1}^{(i)x} - \bar{\mathbf{x}}_{k|k-1}^{(i)}\right)^{\mathsf{T}}$$

Predict measurement:

$$\mathcal{Z}_{k|k-1}^{(i)} = \mathbf{h}_k\left(\mathcal{X}_{k|k-1}^{(i)x}, \mathcal{X}_{k|k-1}^{(i)v}\right)$$

$$\bar{\mathbf{z}}_{k|k-1}^{(i)} = \sum_{j=0}^{2l} W_j^{(m)} \mathcal{Z}_{j,k|k-1}^{(i)}$$

Measurement update

Compute innovation and cross covariance matrices:

$$\mathbf{P}_{\tilde{\mathbf{z}}_k\tilde{\mathbf{z}}_k}^{(i)} = \sum_{j=0}^{2l} W_j^{(c)} \left(\mathcal{Z}_{j,k|k-1}^{(i)} - \bar{\mathbf{z}}_{k|k-1}^{(i)}\right)\left(\mathcal{Z}_{j,k|k-1}^{(i)} - \bar{\mathbf{z}}_{k|k-1}^{(i)}\right)^{\mathsf{T}}$$

$$\mathbf{P}_{\tilde{\mathbf{x}}_k\tilde{\mathbf{z}}_k}^{(i)} = \sum_{j=0}^{2l} W_j^{(c)} \left(\mathcal{X}_{j,k|k-1}^{(i)x} - \bar{\mathbf{x}}_{k|k-1}^{(i)}\right)\left(\mathcal{Z}_{j,k|k-1}^{(i)} - \bar{\mathbf{z}}_{k|k-1}^{(i)}\right)^{\mathsf{T}}$$

Compute Kalman gain:

$$\mathcal{K}_k^{(i)} = \mathbf{P}_{\tilde{\mathbf{x}}_k\tilde{\mathbf{z}}_k}^{(i)} \mathbf{P}_{\mathbf{z}_k\mathbf{z}_k}^{(i)\;-1}$$

Compute a posteriori mean:

$$\bar{\mathbf{x}}_k^{(i)} = \bar{\mathbf{x}}_{k|k-1}^{(i)} + \mathcal{K}_k^{(i)}\left(\mathbf{z}_k - \bar{\mathbf{z}}_{k|k-1}^{(i)}\right)$$

Update error covariance:

$$\mathbf{P}_k^{(i)} = \mathbf{P}_{k|k-1}^{(i)} - \mathcal{K}_k^{(i)} \mathbf{P}_{\tilde{\mathbf{z}}_k\tilde{\mathbf{z}}_k}^{(i)} \mathcal{K}_k^{(i)\mathsf{T}}$$

$\vdots$ $\vdots$