Aim of the exercise

The idea is that we get a joint understanding of where you stand in terms of programming skills as well as your insurance knowledge, and whether you are versatile in filling gaps. Please don't use tools like ChatGPT. However, feel encouraged to use the old-style research: Google, Wikipedia, Stack Overflow, and so on are great.

At Solveva, we have a long-term perspective on our staff. This implies that we respect the work-life balance of our employees – and also our candidates. Thus, the exercise should not require an excessive amount of your time. Let us know in your answer how much time you needed. Feel free to adjust the scope to the time you want to invest.

The "Basic challenge" is kind of a minimum. The sections following on it you can add depending on your level of seniority (and willingness to invest time, of course).

Reinsurance

Reinsurance is about transferring peak risks. Usually, the client is an insurance company, seeking protection for some losses of high severity and low frequency. Let us give you a simple example.

Assume your reinsurer is offering an „XL" cover. It's not related to dress size but short for „excess of loss". ☺

An excess of loss means that you will not pay any single loss occurring lower than the defined **deductible** and not pay more for any loss than the **limit** in excess of the deductible.

Example: The reinsurer will not pay any single loss smaller than USD 5m (the deductible) and you would also not pay more than USD 10m for a single loss (the limit).

In reinsurance language you would write USD 10m xs USD 5m ("xs" slang for excess).

The contract would start January 1 next year and end December 31 next year. This defines the "cover period".

Note that this does not limit your annual exposure due to that contract yet, because you could have several losses in the cover period and so far we have not said how many times we would pay.

The basic challenge

Let's assume you have done a "loss modelling" for the risks which are covered by your contract.

You came to the conclusion that the loss severity (size of a single loss) is distributed according to a so-called Pareto function (not related to the famous "Pareto principle"):

$$F(x) = 1 - \left(\frac{\theta}{x}\right)^{\alpha}$$

with parameters α=1.2, θ=2m.

For the frequency of such a loss occurring you concluded that it is best modelled by a Poisson distribution with λ=1, with the definition as given in https://en.wikipedia.org/wiki/Poisson_distribution.

Could you develop a program (in the language of your preference) to run a Monte Carlo simulation which produces the losses according to the frequency and severity distributions given above? We call those “ground-up losses”.

Could you calculate the average loss for that XL and, in the best case, even VaR 99% for that XL?

Introducing AAD and AAL

This is probably the easiest case we can imagine. In real life, reinsurance contracts can be far more complex.

If you want, you can extend the example and tackle more complex examples.

Let us give you some ideas.

You can introduce an annual aggregated deductible (AAD), which means you would not pay the first losses qualifying for the layer upon this AAD is reached.
Example: 10m xs 5m, AAD 2m.
You have losses to the contract of 8m and 9m (before applying the deductible, we call this “from ground up”).
Thus, without the AAD you would pay 3m for the first and 4m for the second loss. With an AAD of 2m you would only pay after the AAD is used up. So in the example, you would pay only 1m for the first loss, but the full 4m for the second loss (as AAD had been exhausted).

The other side of the medal is an annual aggregate limit (AAL). An AAL is an upper limit of losses which are paid by a reinsurer in a certain layer.
Example: 10m xs 5m, AAL 12m. You have losses of 8m, 9m, 16m (before applying the deductible, we call this “from ground up”).
Then you would pay for the first loss 3m, for the second 4m, for the third 5m only, as the total amount of losses you would pay in this year is 12m max.

Applying a simple pricing model

Let’s take one step closer into a real reinsurer’s life – let’s introduce a simple reinsurance pricing model.

Of course, you would not like to charge less than the average loss to the layer, right? However, a reinsurer would be required to able to pay losses far higher than that, too – it means the reinsurer has to reserve capital for that! Let us assume the reinsurer uses an internal model where they would strive to be able to pay an average loss out of the 1% worst losses possible. This is called tail value at risk (TVAR). Let us further assume that the reinsurer applies internal cost of capital at 8%. Then you would get this simple formula for a reinsurance "price" (=reinsurance premium):
Premium= Average loss + (TVaR(99%) loss - Average loss)*8%

Could you calculate such a premium for the four cases

1. 10m xs 5m
2. 10m xs 5m, AAD 2m
3. 10m xs 5m, AAL 12m.
4. 10m xs 5m, AAD 2m, AAL 12m

Patterns

Losses are not paid immediately. The "original insured" (the client of the insurance company which in turn reinsured itself) will need time to make the claim, the insurance company will need time to process it. Some claims would end in court. Payments can also occur in several instalments. Thus, even the distribution of payments over time is stochastic, not only the total amount.
We distinguish these two types of randomness by modelling loss amounts on the one hand and patterns describing the share of the total amount paid in a certain period on the other hand.
Example: Let's assume a loss to the original insured (= from ground up loss) of 10m and a payment pattern (0.3,0.6,0.1). Then 3m of the loss are paid in the first year, 6m in the second year, and 1m in the third year.
Note that reinsurance often shows a **delayed** pattern! If the loss of 10m as above goes to the 10m xs 5m layer we had discussed before, the payments for the reinsurer are as follows. First year payment is 3m by the insurer, that is below the 5m deductible, therefore there is no payment in the first year. In the second year, in total 9m are paid already by the insurer, thus the reinsurer pays 4m (9m – 5m deductible). And in the third year, another 1m is paid. Therefore, the pattern of the reinsurer is (0,0.8,0.2).
You can also see it depends on the loss size…
Could you calculate for the cases

1. 10m xs 5m
2. 10m xs 5m, AAD 2m
3. 10m xs 5m, AAL 12m.
4. 10m xs 5m, AAD 2m, AAL 12m

an average payment pattern of the reinsurer, given a f.g.u. (from ground up) payment pattern of (0.3,0.6,0.1)? For this average payment pattern we just add all payment patterns (regardless of loss size) and divide each entry of the vector by the number of losses.

Freestyle

Do you want to show us even more of your skills? Here are some ideas, but feel free to come up with your own ones:

- You can add another line of business with a loss model you invent for it.
- You can then apply a measure like it is used in Solvency II or the Swiss Solvency Test and calculate the underwriting risk involved.

How your answer should look like

Please share with us not only the results but also your code (and the time you needed). Thanks!!
You can use the language of your preference, C++, Java, Python, Typescript… Note that our backend is in TypeScript, which you most likely don't know yet, but this is fine! If you want to give it a try in TS, feel free, though. ☺

The idea of the exercise is of course that we get an impression of your coding style. Therefore, please use only absolutely basic math libraries, not ones which are closer to actuarial needs providing all you need out of the box. For example, if you use Python, please only use the math library, same for C++, Java, TS.

Hints

1. If a ground-up loss is lower than USD 5m, it is not paid by the XL. If it is larger than USD 15m, only USD 10m (15m-5m) will be paid. If the ground-up loss is USD 7m, the XL would pay USD 2m.
2. Using the inverse of the cumulative distribution function, you can generate Pareto-distributed random numbers out of equally distributed random numbers (which you usually get from your random number generator) with this
$$r_{Pareto} = \theta/(1 - r_{equally\ distributed})^{1/\alpha}$$

Good luck!