Fairness - Homework

October 28, 2021

1 Simpson's paradox

Table 2. Admissions data for the graduate programs in the six largest majors at University of California, Berkeley.

	Men		Women	
Major	*Number of applicants*	*Percent admitted*	*Number of applicants*	*Percent admitted*
A	825	62	108	82
B	560	63	25	68
C	325	37	593	34
D	417	33	375	35
E	191	28	393	24
F	373	6	341	7

Note: University policy does not allow these majors to be identified by name.
Source: The Graduate Division, University of California, Berkeley.

1. What is the percentages of women/men admitted ? Does it seem biased?
2. Compare this result with the detailed percentages by department. Does it seem biased? How can you explain this result?

Size	Treatment A	Treatment B
Small	(96%) 84/87	(87%) 234/270
Large	(73%) 192/263	(68%) 55/80

3. Two treatments are experimented to deal with kidney stones, the results are given according to the size of the stones. Compute the recovering percentages for the two treatments, which treatment seems to be the best?
4. Compute the detailed recovering percentages by size of stone. Which treatment seems to be the best in the two cases? How can you explain this result?

5. Defining the event C ("The patient is recovered"), the random variable T (equal to 1 for A and 0 for B) and the random variable S (size), describe in terms of conditional probabilities the observations of the two previous questions.

2 Some fairness criteria

In the binary classification setting, let Y be the target variable, A the sensitive attribute and R the classifier. Recall that the triple (R, A, Y) satisfies the *separation criteria* if $R \perp A \mid Y$ and the *sufficiency criteria* if $Y \perp A \mid R$.

1. Assume that R is a binary classifier and that there are only two groups. What does the separation criteria mean in terms of false positive rate and false negative rate for the two groups, a and b?

2. If we observe the two group-level ROC curves, graphically, which point corresponds to the classifier satisfying the separation criteria?

3. We say that R satisfies *calibration by group* if:

$$P(Y = 1 \mid R = r, A = a) = P(Y = 1 \mid R = r, A = b) = r$$

   for all r. Show that: if R satisfies sufficiency, there exists a function ℓ such that $\ell(R)$ satisfies calibration by group.

4. Show that: if A is not independent of Y and R is not independent of Y, then independence and separation cannot both hold.