

THIN SHELL STRUCTURE DESIGN TOOL

By

R. Allan Pendergrast

A Thesis Submitted to the Graduate
Faculty of Rensselaer Polytechnic Institute
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE

Major Subject: COMPUTER SCIENCE

Approved:

Barbara M. Cutler, Thesis Adviser

Rensselaer Polytechnic Institute
Troy, New York

May 2010
(For Graduation May 2010)

CONTENTS

LIST OF TABLES	ii
LIST OF FIGURES	iii
ACKNOWLEDGMENT	iv
ABSTRACT	v
1. INTRODUCTION	1
1.1 Project Goals	1
1.2 Thin Shell Structures	1
1.2.1 Overview	1
1.2.2 Structural Stability	3
1.2.3 Catenary	5
1.3 Related Works	10
1.3.1 Procedural Modeling	10
2. THIN SHELL DESIGN TOOL	12
2.1 Layout	12
2.2 Simulator	12
LITERATURE CITED	12

LIST OF TABLES

LIST OF FIGURES

1.1	Simplified representation of a house	2
1.2	A beam under uniform loading	3
1.3	A thin-shell dome over a service area in Switzerland	3
1.4	The TWA Flight Center	4
1.5	The Kresge Auditorium	4
1.6	The Montreal Biosphere	5
1.7	Lines of thrust	6
1.8	The Gateway Arch	6
1.9	Example catenary curves	7
1.10	A composite catenary structure	7
1.11	Casa Milà, Barcelona, Spain	8
1.12	Parc Guell, Barcelona, Spain	9
1.13	Sagrada Família, Barcelona, Spain	9
1.14	A hanging model used by Gaudí	10
2.1	A screenshot of the tool	13

ACKNOWLEDGMENT

Thanks to Barbara Cutler for all her help in creating, testing, and writing about this tool.

ABSTRACT

Thin-shell structures are becoming increasingly useful in construction and design of buildings. They allow the usage of less material to enclose larger spaces, are structurally efficient, and have a natural aesthetic beauty. However, they can be difficult to design, as the exact shape required for structural stability depends on the material used, the size of the shell, and other features. Fortunately, it is possible to simulate these structures quickly and accurately, allowing architects to concentrate more on their design and less on ensuring that their building is stable. The tool described in this thesis simulates thin-shell structures and aids architects in their design and optimization.

1. INTRODUCTION

1.1 Project Goals

The goal of this project was to create a tool that could be used by architects to construct buildings more cheaply. There are many factors that can make construction of a building expensive. Labor, materials, and planning are all expensive elements which can be optimized. This project focuses primarily on the third of these, reducing design time by giving the architect a tool that allows them to quickly design a structurally efficient building.

Structural efficiency is a very important element of construction. With traditional construction methods, this tends not to be an issue, since the tried-and-true construction conventions will keep a building standing. Houses, for example, have been built using the same structural conventions for years and do not require any advanced structural analysis. Walls are constructed with studs every 16 inches, and the house stands up. Even in non-residential structures, the studded or cinder-block walls convention tends to be followed. However, when creating buildings that fall outside the norm of studded walls, cinder block construction, and other such traditional methods, more complex analysis tools are necessary.

1.2 Thin Shell Structures

1.2.1 Overview

A thin shell structure is a structure which has a small thickness compared to its other dimensions. While this may seem to be an obvious definition, the design and construction of these structures can be complicated. In traditional construction, load-bearing members are flat, carrying forces straight through themselves. A simplified 2D representation of the load-bearing elements of a house can be seen in Figure 1.1. Larger buildings which are constructed using traditional techniques use very similar techniques as those used in Figure 1.1, employing vertical members and cross-pieces to support the weight of the building. One consideration for larger buildings is that large rooms will have large unsupported expanses of floor. Beams

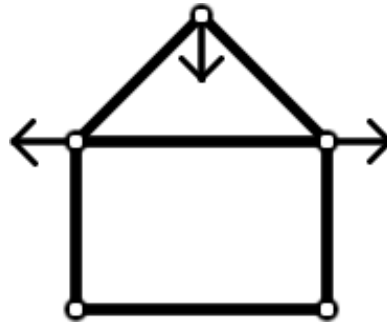


Figure 1.1: This simplified representation of the load-bearing elements of a house show how traditional construction techniques require additional material to be stable. Were it not for the horizontal piece which forms the ceiling, the walls would be forced outward by the forces caused by the weight of the roof.

which are loaded transversely in that manner are subjected to bending according to the Euler-Bernoulli equation:

$$EI \frac{d^4 u}{dx^4} = w(x)$$

If too much force is applied, the beam will buckle, perhaps causing catastrophic failure. Therefore, beams and other flat structural elements must have a high second moment of area (have a large dimension parallel to the applied force) in order to ensure that it will not buckle under load. Alternately, columns can be installed to effectively shorten the span which the beam is crossing, but depending on the application, this may not be desirable.

Unlike normal beam and plate structures, thin shell structures are curved, which allows the force to travel through the thinner structural elements. Structures such as those in figure 1.3 can cover a large span with a minimal amount of material, saving the construction company money. Since the structure completely supports itself, no internal columns are necessary, allowing an unobstructed interior. These structures are very stable because of their unique shape, called a catenary shell. Catenaries are covered in more detail in section 1.2.3 Some prominent thin shell structures include the TWA Flight Center Building at the JFK International Airport in New York, New York (Fig. 1.4), the Kresge Auditorium on the MIT campus in Cambridge, Massachusetts (Fig. 1.5), and the Montreal Biosphere in Montreal, Canada (Fig 1.6).

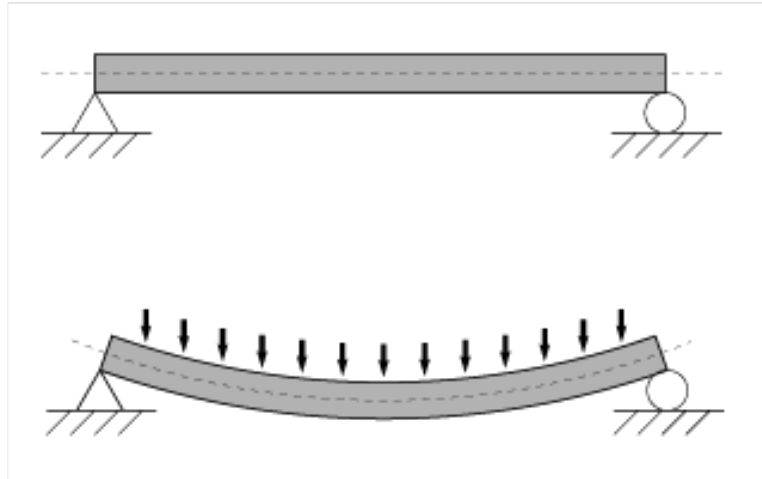


Figure 1.2: This is an image of a beam which is deforming under a uniform load. [1]



Figure 1.3: This dome, designed by Swiss civil engineer Heinz Isler, gracefully arches over a service area, protecting it from the elements with a minimal amount of material. [2]

1.2.2 Structural Stability

The core concept for structural stability for masonry buildings is the concept of lines of thrust. Lines of thrust are lines that can be drawn in the direction of the forces neighboring elements of the structure impart on one another. If all the discrete forces are connected together into a generalized curve, the traditional lines of thrust are obtained. In order for a building to stand up, these lines of thrust must pass through structural elements. As can be seen in Figure 1.7, traditional



Figure 1.4: The TWA Flight Center at JFK International Airport in New York, New York is an excellent modern example of thin-shell structures providing a much-needed unobstructed internal space. [3]



Figure 1.5: The Kresge Auditorium on the MIT campus in Cambridge, Massachusetts is an example of the problem thin-shell structures can cause when not designed properly. Since the roof is octanisperical rather than catenarian, the forces do not travel as intended and the building has been plagued with structural problems since its construction. [4]



Figure 1.6: The Montreal Biosphere is an example of a lattice-based thin-shell structure, which relies on a lattice of struts to support the huge expanse of the dome. [5]

arches must be rather thick to contain the lines of thrust produced by their weight. However, a catenary arch can be built much thinner for the same stability, as it contains the line of thrust exactly. For example, the Gateway Arch in St. Louis, Missouri (Figure 1.8 is constructed in the shape of a catenary arch. This allows it to be thin and elegant while remaining very stable. To extrapolate the concept of lines of thrust to entire buildings, traditional construction methods require very thick elements such as walls and columns to be used in order to keep the lines of thrust within a building's structural elements. This is especially true of large masonry structures such as cathedrals. However, if the shape of the building is instead matched to the shape of the lines of thrust, the structural elements can be much thinner, since they only need to support the direct compressive force.

1.2.3 Catenary

The term for the shape that lines of thrust tend to take is called a catenary. A catenary is a curve described by the function

$$y = a \cosh\left(\frac{x}{a}\right)$$

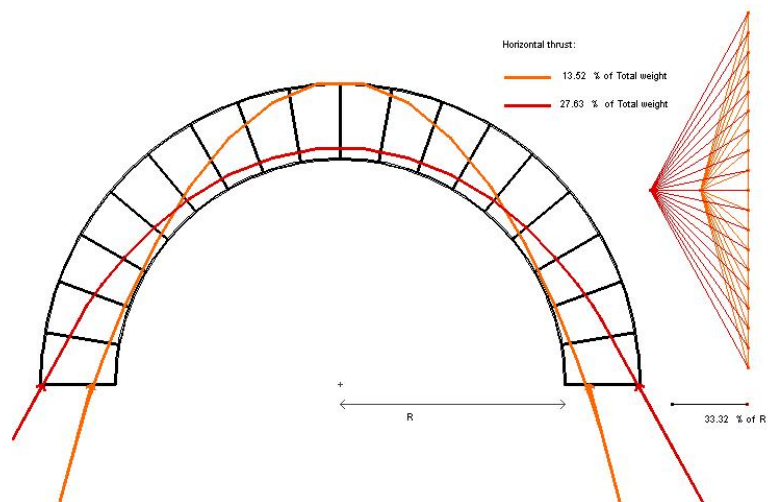


Figure 1.7: This figure shows the lines of thrust within a standard masonry arch. As can be seen from the minimum and maximum lines, the arch must be rather thick in order to contain the lines of thrust, thereby wasting material. [6]



Figure 1.8: The Gateway Arch in St. Louis, Missouri is an example of a catenary arch. Since the lines of thrust travel directly through the structure of the arch, it can be built very thin. [7]

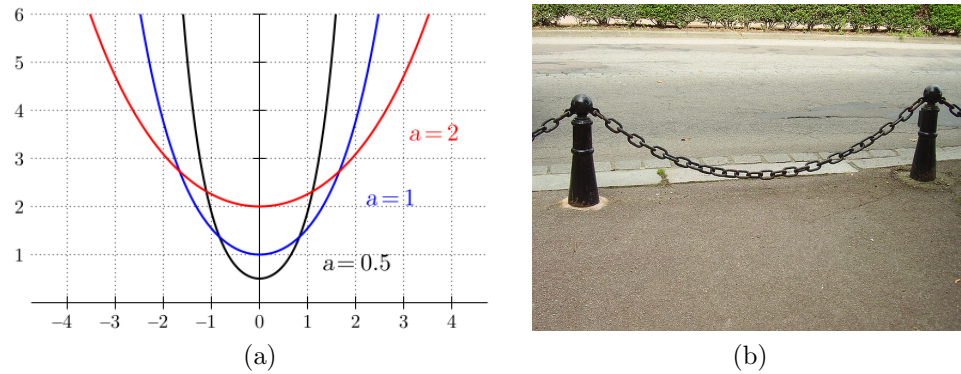


Figure 1.9: Image (a) shows a few catenary curves for various values of a .
 [8] Image (b) shows a natural catenary formed by a freely hanging chain.
 [9]

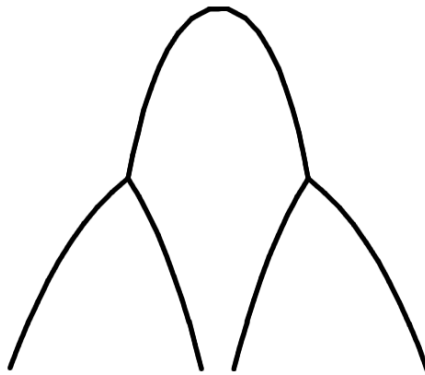


Figure 1.10: This image shows the necessary deformation of supporting catenary arches when a third arch is placed on top of them.

where \cosh is the hyperbolic cosine function. Several examples of catenaries can be seen in Figure 1.9a. In addition to being an interesting mathematical figure, the catenary is the shape taken by a cable, rope, or chain suspended at both ends, as seen in Figure 1.9b. Since this is the shape formed by a freely hanging object under pure tension, it is not surprising that if inverted, it is similarly stable under pure compression. For this reason, catenary arches and catenary shells are the primary building blocks of thin-shell structures. One very important thing to note is that a catenary is only the optimal shape when the chain or arch is evenly loaded. If there is an uneven load, for example if the arch has a decorative mass at some point or if a secondary arch rests on another arch, the catenary is not the optimal shape, as seen in Figure 1.10.



Figure 1.11: Image (a) shows an exterior view of Casa Milà, one of Antoni Gaudí's stunning buildings in Barcelona, Spain. [10] Image (b) is of the catenary arches under the terrace of Casa Milà. [11]

The shape of the catenary has been used by many architects. One example mentioned earlier is the Gateway Arch in St. Louis, Missouri, designed by the Finnish-American architect Eero Saarinen, seen in Figure 1.8. However, the shape has also been used as an integral design principle for much larger and more complex structures. Hanging chains have been used by a number of architects to design structures for stability and aesthetics. One famous user of hanging chains is Antoni Gaudí, whose catenary-rich projects include such Barcelona landmarks as the Casa Milà (Figure 1.11), Park Guell (Figure 1.12), and Sagrada Família (Figure 1.13). Figure 1.14 shows one of the models Gaudí used in creating these graceful structures.

Another architect who is famous for his use of catenary shells in his thin-shell structures is Heinz Isler. A civil engineer from Switzerland, Isler designed some very beautiful and elegant structures using the simple tools of cloth and water. Since a sheet of cloth will behave as an interconnected set of hanging strings, it can be used to create catenary shell structures. What Isler did was to take the shape a sheet of cloth formed in when suspended and freeze it by soaking the cloth evenly with water. The resulting frozen structure was then inverted and measured very



Figure 1.12: Image (a) shows the entrance to Parc Guell in Barcelona. [12] Image (b) shows the columns supporting the roadway that runs past the park. These columns form an offset catenary, which is asymmetric because the loading of the arch is asymmetric. [13]

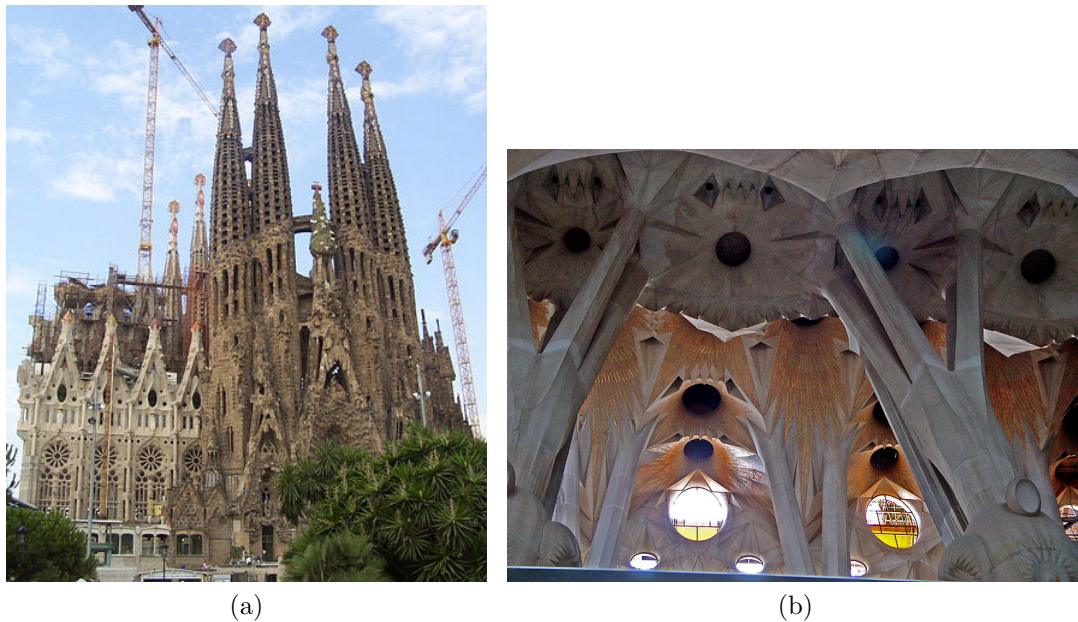


Figure 1.13: Image (a) shows the nativity façade of Antoni Gaudí's masterpiece, Sagrada Família, slated to be completed some time after 2026. [14] Image (b) shows the structural columns that are possible when designing with catenaries in mind. Rather than the monolithic columns found in most gothic cathedrals, Gaudí has whittled away the nonessential stone to reveal the core load-bearing elements. This results in a gracefully arching column that supports the huge structure as well as a monolithic column would have. [15]



Figure 1.14: This is a photo of one of the hanging models used by Antoni Gaudí to understand the forces in the buildings he constructed. The bags are full of small lead weights which are proportional to various structural elements and ornaments in the planned building. The strings holding them together are the necessary columns, arches, and other core structural elements that will make up the building. [16]

accurately with a device he created. Once these measurements had been taken, he built forms and poured the shell using standard concrete construction techniques. The resulting buildings, such as those in Figure 1.3, are elegant, graceful structures with an exquisite simplicity of form and conservation of material.

One drawback to the thin-shell structure work done by Gaudí and Isler is that the design process is very time-consuming. The amount of time it takes to create a hanging model from strings and lead shot or freeze a cloth shell is prohibitive to the fast-paced, quick turnaround time of the modern architecture world. Fortunately, both hanging chains and cloth are rather easy to simulate, and therefore software can be created to allow these designs to be rapidly prototyped, tweaked, and refined on the computer.

1.3 Related Works

1.3.1 Procedural Modeling

In "Procedural Modeling of Structurally-Sound Masonry Buildings" [17], Whiting, Ochsendorf, and Durand explore the possibilities of creating existing or novel

structures procedurally. They began by creating a grammar which can be used to construct masonry buildings. Arches, buttresses, domes, and vaults are some of the structural elements which are then combined in their software. These grammar elements are assembled into a structure through a procedural algorithm which cuts windows in walls and assembles all the various masonry elements of the building. Once the initial configuration is generated, the software runs static analysis on the building. If it is feasibly stable, the program is done. If not, the program determines a measure of infeasibility, which is a measure of how far away from stable a structure is. The static analysis only allows for compressive forces, as the tensile strength of masonry elements is close to zero. Friction is also modeled, allowing for some shear. Once the measure of infeasibility is calculated, a parameter search is conducted iteratively, searching the parameter space for a stable configuration. Depending on the application, this stable configuration will take into account a factor of safety. The more likely a structure is to have changing loads, the higher a factor of safety is needed. For example, a bridge needs a higher factor of safety than a cathedral. In the event that there is no feasible configuration for a structure, the least infeasible structure is returned and the user is required to add new structural elements.

In "Creating Models of Truss Structures with Optimization" [18], Smith, Hodgins, Oppenheim, and Witkin propose a method of creating trusses procedurally. This work allows the user to define several anchor points and loads for a truss, then have the software automatically generate a truss. In this work, the risk of pieces falling apart is not an issue as it was in the previous paper. The primary failure method in this case is buckling, since all forces are axial. Therefore, the core of the algorithm is a multivariable optimization with constraints. The algorithm attempts iteratively to minimize weight while ensuring that none of the members will fail, either in tension or compression.

My work had initially intended to go in this direction, using static analysis of structures within Google Sketchup. However, Sketchup proved to be a poor environment for the program I wanted to write, so the project was moved to a standalone application and the focus shifted to thin-shell structures.

2. THIN SHELL DESIGN TOOL

2.1 Layout

As can be seen in figure 2.1, there are 3 main panels in the design tool. On the left is the viewing panel, where the structure is shown in 3D. This view can be rotated, zoomed, and panned to give the architect the view they need. There is no actual interaction with the structure in this panel; all of the interaction is done via the two right-hand panels. The top-right panel is the floorplan panel. This panel contains a point for each point of the structure that is in contact with the ground. These points can be moved around in order to modify the shape of the structure to fit the architect's vision. The bottom-right panel is the grid panel. This panel shows all the vertexes of the structure in 2D. This panel can be used to disable points, creating voids in the structure. It can also be used to associate different points in the shell with certain points in the floor plan. This is most useful when adding new points to the floor plan, allowing the architect to define a new point of support on the fly. Together, these windows allow the architect broad control over the design of their structure, while the tool works in the background to keep the shape of the structure optimal.

2.2 Simulator

The heart of the tool is the simulator. This simulator uses a second-order explicit integral solver to simulate the forces on the structure. These forces cause the structure to form catenaries, shapes that will support themselves. The floorplan, grid, and viewing window can all be interacted with in real time while the simulation is going on, allowing the architect to make changes on the fly. Depending on the magnitude of the changes made, the simulator will take anywhere from a second to a couple minutes to reach equilibrium. It should be noted that until the simulation does reach equilibrium there are no guarantees of the stability of the structure.

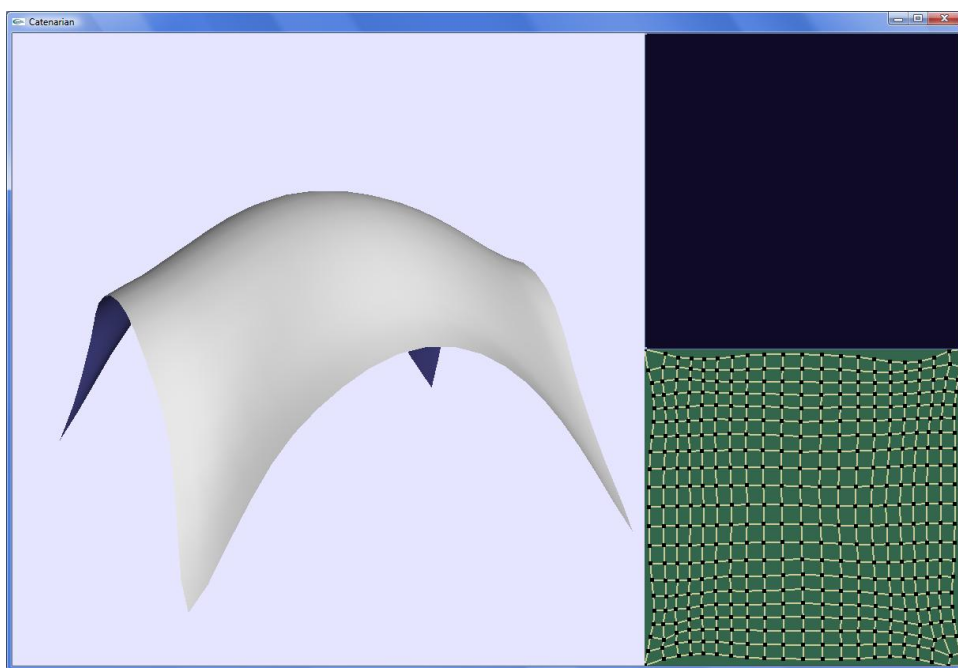


Figure 2.1: A screenshot of the tool

LITERATURE CITED

- [1] Daniel De Leon Martinez. Bending. Online Image, 2006.
<http://en.wikipedia.org/wiki/File:Bending.png>.
- [2] Chriusha. Deitingen sued raststaette, schalendach 04 09. Online Image, 2009.
http://commons.wikimedia.org/wiki/File:Deitingen_Sued_Raststaette,_Schalendach_04_09.jpg.
- [3] Marc N. Weissman. 08terminal5. Online Image, 2004.
<http://en.wikipedia.org/wiki/File:08terminal5.jpg>.
- [4] Ibn Battuta. Kresge auditorium, mit (view with green building). Online Image, 2007. http://en.wikipedia.org/wiki/File:Kresge_Auditorium,_MIT_%28view_with_Green_Building%29.JPG.
- [5] Philipp Hienstorfer. Biosphere montreal. Online Image, 2007.
http://en.wikipedia.org/wiki/File:Biosphere_montreal.JPG.
- [6] John Ochsendorf. Interactive thrust screenshot. Screenshot of Online Applet, 2010. <http://web.mit.edu/masonry/interactiveThrust/applets/applet01.html>.
- [7] David K. Staub. Gateway arch. Online Image, 2006.
http://en.wikipedia.org/wiki/File:Gateway_Arch.jpg.
- [8] Geek3. Catenary-pm. Online Image, 2009.
<http://en.wikipedia.org/wiki/File:Catenary-pm.svg>.
- [9] Kamell15. Kette kettenkurve catenary 2008 pd. Online Image, 2008.
http://en.wikipedia.org/wiki/File:Kette_Kettenkurve_Catenary_2008_PD.JPG.
- [10] Diliff. Casa mil - barcelona, spain - jan 2007. Online Image, 2007.
http://en.wikipedia.org/wiki/File:Casa_Mil%C3%A0_-_Barcelona,_Spain_-_Jan_2007.jpg.
- [11] Error. Lapedreraparabola. Online Image, 2005.
<http://en.wikipedia.org/wiki/File:LaPedreraParabola.jpg>.
- [12] Maksim. Parcuell. Online Image, 2006.
<http://en.wikipedia.org/wiki/File:Parcguell.jpg>.
- [13] Dodo. Parc guell 10. Online Image, 2005.
http://en.wikipedia.org/wiki/File:Parc_Guell_10.jpg.

- [14] Montrealais. Sagradafamilia-overview. Online Image, 2002.
<http://en.wikipedia.org/wiki/File:Sagradafamilia-overview.jpg>.
- [15] Etan J. Tal. Sagradafamiliaroof. Online Image, 2009.
<http://en.wikipedia.org/wiki/File:SagradaFamiliaRoof.jpg>.
- [16] Pamela Angus. Gaudi model. Online Image, 2008.
http://2.bp.blogspot.com/_PZ0VPTsrTJ0/SR7z2F_h93I/AAAAAAAAA1s/hAv1--bslzQ/s1600-h/Gaudimodel.jpg+.
- [17] Emily Whiting, John Ochsendorf, and Frédo Durand. Procedural modeling of structurally-sound masonry buildings. *ACM Transactions on Graphics*, 28(5):112, 2009.
- [18] Jeffrey Smith Carnegie and Jeffrey Smith. Creating models of truss structures with optimization. In *Contemporary Mathematics*, pages 101–136. American Mathematical Society, 2002.