

THIN SHELL STRUCTURE DESIGN TOOL

By

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ABSTRACT

Thin-shell structures are becoming increasingly useful in construction and design of buildings. They allow the usage of less material to enclose larger spaces, are structurally efficient, and have a natural aesthetic beauty. However, they can be difficult to design, as the exact shape required for structural stability depends on the material used, the size of the shell, and other features. Fortunately, it is possible to simulate these structures quickly and accurately, allowing architects to concentrate more on their design and less on ensuring that their building is stable. The tool described in this thesis simulates thin-shell structures and aids architects in their design and optimization.

1. INTRODUCTION

1.1 Project Goals

The goal of this project was to create a tool that could be used by architects to construct buildings more cheaply. There are many factors that can make construction of a building expensive. Labor, materials, and planning are all expensive elements which can be optimized. This project focuses primarily on the third of these, reducing design time by giving the architect a tool that allows them to quickly design a structurally efficient building.

Structural efficiency is a very important element of construction. With traditional construction methods, this tends not to be an issue, since the tried-and-true construction conventions will keep a building standing. Houses, for example, have been built using the same structural conventions for years and do not require any advanced structural analysis. Walls are constructed with studs every 16 inches, and the house stands up. Even in non-residential structures, the studded or cinder-block walls convention tends to be followed. However, when creating buildings that fall outside the norm of studded walls, cinder block construction, and other such traditional methods, more complex analysis tools are necessary.

1.2 Thin Shell Structures

1.2.1 Overview

A thin shell structure is a structure which has a small thickness compared to its other dimensions. While this may seem to be an obvious definition, the design and construction of these structures can be complicated. In traditional construction, load-bearing members are flat, carrying forces straight through themselves. A simplified 2D representation of the load-bearing elements of a house can be seen in Figure 1.1. Larger buildings which are constructed using traditional techniques use very similar techniques as those used in Figure 1.1, employing vertical members and cross-pieces to support the weight of the building. One consideration for larger buildings is that large rooms will have large unsupported expanses of floor. Beams

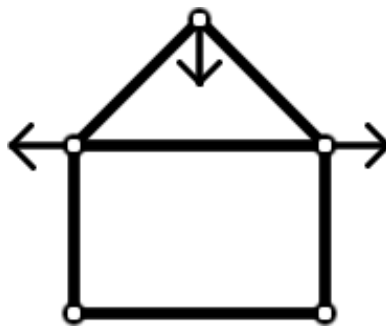


Figure 1.1: This simplified representation of the load-bearing elements of a house show how traditional construction techniques require additional material to be stable. Were it not for the horizontal piece which forms the ceiling, the walls would be forced outward by the forces caused by the weight of the roof.

which are loaded transversely in that manner are subjected to bending according to the Euler-Bernoulli equation:

$$EI \frac{d^4 u}{dx^4} = w(x)$$

If too much force is applied, the beam will buckle, perhaps causing catastrophic failure. Therefore, beams and other flat structural elements must have a high second moment of area (have a large dimension parallel to the applied force) in order to ensure that it will not buckle under load. Alternately, columns can be installed to effectively shorten the span which the beam is crossing, but depending on the application, this may not be desirable.

Unlike normal beam and plate structures, thin shell structures are curved, which allows the force to travel through the thinner structural elements. Structures such as those in figure 1.3 can cover a large span with a minimal amount of material, saving the construction company money. Since the structure completely supports itself, no internal columns are necessary, allowing an unobstructed interior. These structures are very stable because of their unique shape, called a catenary shell. Catenaries are covered in more detail in section 1.2.3 Some prominent thin shell structures include the TWA Flight Center Building at the JFK International Airport in New York, New York (Fig. 1.4), the Kresge Auditorium on the MIT campus in Cambridge, Massachusetts (Fig. 1.5), and the Montreal Biosphere in Montreal,

¹"Bending", Daniel De Leon Martinez, <http://en.wikipedia.org/wiki/File:Bending.png>

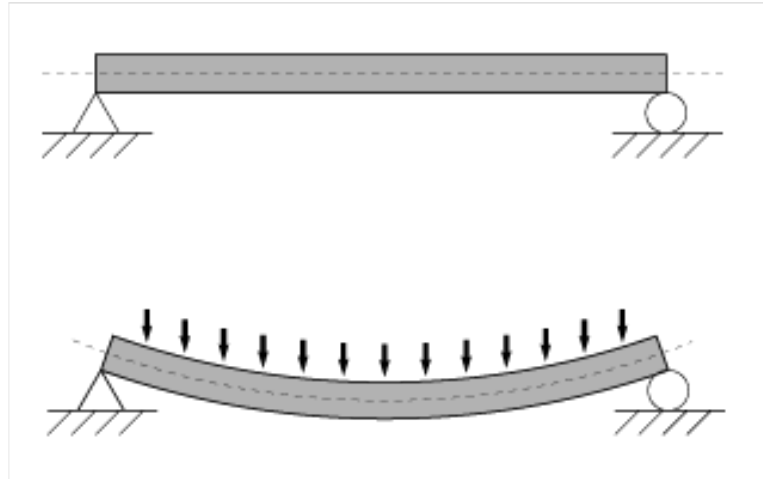


Figure 1.2: This is an image of a beam which is deforming under a uniform load.¹



Figure 1.3: This dome, designed by Swiss civil engineer Heinz Isler, gracefully arches over a service area, protecting it from the elements with a minimal amount of material.²

Canada (Fig 1.6).

²"Service Area", Heinz Isler, http://commons.wikimedia.org/wiki/File:Deitingen_Sued_Raststaette,_Schalendach_04_09.jpg

³"TWA Flight Center", Eero Saarinen, <http://en.wikipedia.org/wiki/File:08terminal5.jpg>

⁴"Kresge Auditorium", Eero Saarinen, http://en.wikipedia.org/wiki/File:Kresge_Auditorium,_MIT_\\%28view_with_Green_Building\\%29.JPG

⁵"Montreal Biosphere", Richard Buckminster Fuller, http://en.wikipedia.org/wiki/File:Biosphere_montreal.JPG FIGURE OUT HOW TO GET THE BACKSLASH TO GO AWAY FROM IN FRONT OF THE '%'



Figure 1.4: The TWA Flight Center at JFK International Airport in New York, New York is an excellent modern example of thin-shell structures providing a much-needed unobstructed internal space.³



Figure 1.5: The Kresge Auditorium on the MIT campus in Cambridge, Massachusetts is an example of the problem thin-shell structures can cause when not designed properly. Since the roof is octanisperical rather than catenarian, the forces do not travel as intended and the building has been plagued with structural problems since its construction.⁴



Figure 1.6: The Montreal Biosphere is an example of a lattice-based thin-shell structure, which relies on a lattice of struts to support the huge expanse of the dome.⁵

1.2.2 Structural Stability

The core concept for structural stability for masonry buildings is the concept of lines of thrust. Lines of thrust are lines that can be drawn in the direction of the forces neighboring elements of the structure impart on one another. If all the discrete forces are connected together into a generalized curve, the traditional lines of thrust are obtained. In order for a building to stand up, these lines of thrust must pass through structural elements. As can be seen in Figure 1.7, traditional arches must be rather thick to contain the lines of thrust produced by their weight. However, a catenary arch can be built much thinner for the same stability, as it contains the line of thrust exactly. For example, the Gateway Arch in St. Louis, Missouri (Figure 1.8 is constructed in the shape of a catenary arch. This allows it to be thin and elegant while remaining very stable. To extrapolate the concept of lines of thrust to entire buildings, traditional construction methods require very thick elements such as walls and columns to be used in order to keep the lines of thrust within a building's structural elements. This is especially true of large masonry structures such as cathedrals. However, if the shape of the building is instead matched to the shape of the lines of thrust, the structural elements can be

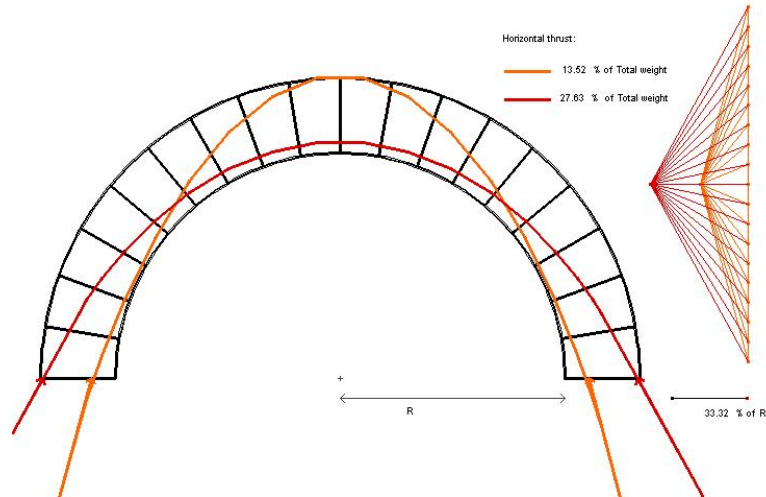


Figure 1.7: This figure shows the lines of thrust within a standard masonry arch. As can be seen from the minimum and maximum lines, the arch must be rather thick in order to contain the lines of thrust, thereby wasting material.⁶

much thinner, since they only need to support the direct compressive force.

1.2.3 Catenary

The term for the shape that lines of thrust tend to take is called a catenary. A catenary is a curve described by the function

$$y = a \cosh\left(\frac{x}{a}\right)$$

where cosh is the hyperbolic cosine function. Several examples of catenaries can be seen in Figure 1.9a. In addition to being an interesting mathematical figure, the catenary is the shape taken by a cable, rope, or chain suspended at both ends, as seen in Figure 1.9b. Since this is the shape formed by a freely hanging object under pure tension, it is not surprising that if inverted, it is similarly stable under pure compression. For this reason, catenary arches and catenary shells are the primary building blocks of thin-shell structures. One very important thing to note is that a catenary is only the optimal shape when the chain or arch is evenly loaded. If there

⁶Screenshot of "Interactive Thrust", John Ochsendorf, <http://web.mit.edu/masonry/interactiveThrust/applets/applet01.html>

⁷"Gateway Arch", Eero Saarinen, http://en.wikipedia.org/wiki/File:Gateway_Arch.jpg



Figure 1.8: The Gateway Arch in St. Louis, Missouri is an example of a catenary arch. Since the lines of thrust travel directly through the structure of the arch, it can be built very thin.⁷

is an uneven load, for example if the arch has a decorative mass at some point or if a secondary arch rests on another arch, the catenary is not the optimal shape, as seen in Figure 1.10.

The shape of the catenary has been used by many architects. One example mentioned earlier is the Gateway Arch in St. Louis, Missouri, designed by the Finnish-American architect Eero Saarinen, seen in Figure 1.8. However, the shape has also been used as an integral design principle for much larger and more complex structures. Hanging chains have been used by a number of architects to design structures for stability and aesthetics. One famous user of hanging chains is Antoni Gaudí, whose catenary-rich projects include such Barcelona landmarks as the Casa Milà (Figure 1.11), Park Guell (Figure 1.12), and Sagrada Família (Figure 1.13). Figure 1.14 shows one of the models Gaudí used in creating these graceful structures.

⁹”Catenary Curves”, Geek3, <http://en.wikipedia.org/wiki/File:Catenary-pm.svg>

⁹”Hanging Chain”, Kamell15, http://en.wikipedia.org/wiki/File:Kette_Kettenkurve_Catenary_2008_PD.JPG

¹⁰”Casa Milà”, Antoni Gaudí, http://en.wikipedia.org/wiki/File:Casa_Mil%C3%A0_-_Barcelona,_Spain_-_Jan_2007.jpg, <http://en.wikipedia.org/wiki/File:>

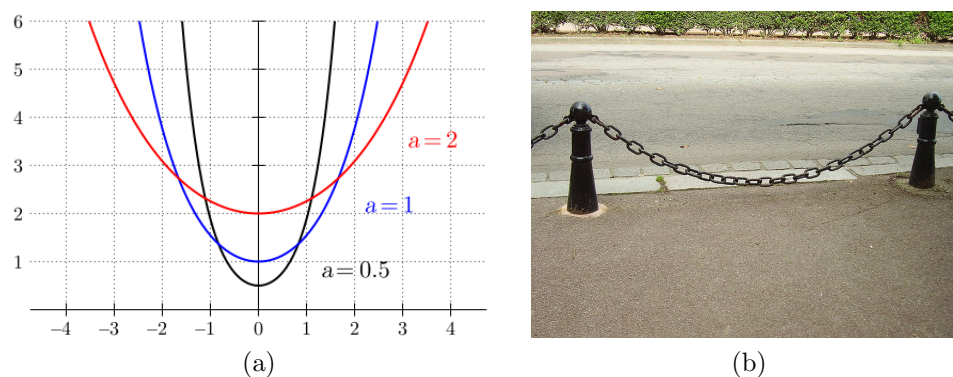


Figure 1.9: Image (a) shows a few catenary curves for various values of a .⁸ Image (b) shows a natural catenary formed by a freely hanging chain.⁹

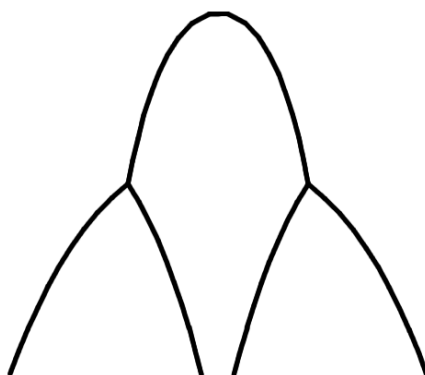


Figure 1.10: This image shows the necessary deformation of supporting catenary arches when a third arch is placed on top of them.

Another architect who is famous for his use of catenary shells in his thin-shell structures is Heinz Isler. A civil engineer from Switzerland, Isler designed some very beautiful and elegant structures using the simple tools of cloth and water. Since a sheet of cloth will behave as an interconnected set of hanging strings, it can be used to create catenary shell structures. What Isler did was to take the shape a sheet of cloth formed when suspended and freeze it by soaking the cloth evenly

LaPedreraParabola.jpg

¹¹"Parc Guell", Antoni Gaudí, <http://en.wikipedia.org/wiki/File:Parcguell.jpg>, http://en.wikipedia.org/wiki/File:Parc_Guell_10.jpg

¹²"Sagrada Família", Antoni Gaudí, <http://en.wikipedia.org/wiki/File:Sagradafamilia-overview.jpg>, <http://en.wikipedia.org/wiki/File:SagradaFamiliaRoof.jpg>

¹³"Hanging model", Antoni Gaudí, http://2.bp.blogspot.com/_PZ0VPTsrTJ0/SR7z2F_h93I/AAAAAAAAAIs/hAv1--bslzQ/s1600-h/Gaudimodel.jpg



Figure 1.11: Image (a) shows an exterior view of Casa Milà, one of Antoni Gaudí's stunning buildings in Barcelona, Spain. Image (b) is of the catenary arches under the terrace of Casa Milà.¹⁰



Figure 1.12: Image (a) shows the entrance to Parc Guell in Barcelona. Image (b) shows the columns supporting the roadway that runs past the park. These columns form an offset catenary, which is asymmetric because the loading of the arch is asymmetric.¹¹

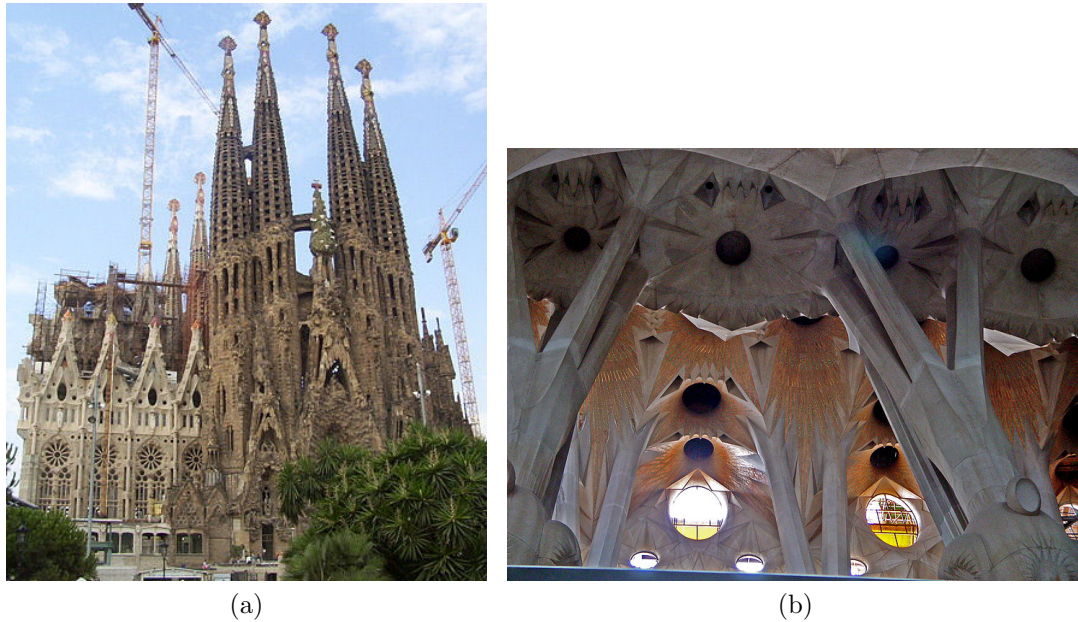


Figure 1.13: Image (a) shows the nativity façade of Antoni Gaudí’s masterpiece, Sagrada Família, slated to be completed some time after 2026. Image (b) shows the structural columns that are possible when designing with catenaries in mind. Rather than the monolithic columns found in most gothic cathedrals, Gaudí has whittled away the nonessential stone to reveal the core load-bearing elements. This results in a gracefully arcing column that supports the huge structure as well as a monolithic column would have.¹²

with water. The resulting frozen structure was then inverted and measured very accurately with a device he created. Once these measurements had been taken, he built forms and poured the shell using standard concrete construction techniques. The resulting buildings, such as those in Figure 1.3, are elegant, graceful structures with an exquisite simplicity of form and conservation of material.

One drawback to the thin-shell structure work done by Gaudí and Isler is that the design process is very time-consuming. The amount of time it takes to create a hanging model from strings and lead shot or freeze a cloth shell is prohibitive to the fast-paced, quick turnaround time of the modern architecture world. Fortunately, both hanging chains and cloth are rather easy to simulate, and therefore software can be created to allow these designs to be rapidly prototyped, tweaked, and refined on the computer.



Figure 1.14: This is a photo of one of the hanging models used by Antoni Gaudí to understand the forces in the buildings he constructed. The bags are full of small lead weights which are proportional to various structural elements and ornaments in the planned building. The strings holding them together are the necessary columns, arches, and other core structural elements that will make up the building.¹³

1.3 Related Works

1.3.1 Procedural Modeling

In "Procedural Modeling of Structurally-Sound Masonry Buildings"[1], Whiting, Ochsendorf, and Durand explore the possibilities of creating existing or novel structures procedurally. They began by creating a grammar which can be used to construct masonry buildings. Arches, buttresses, domes, and vaults are some of the structural elements which are then combined in their software. These grammar elements are assembled into a structure through a procedural algorithm which cuts windows in walls and assembles all the various masonry elements of the building. Once the initial configuration is generated, the software runs static analysis on the building. If it is feasibly stable, the program is done. If not, the program determines a measure of infeasibility, which is a measure of how far away from stable a structure is. The static analysis only allows for compressive forces, as the tensile strength of masonry elements is close to zero. Friction is also modeled, allowing for some shear. Once the measure of infeasibility is calculated, a parameter search is conducted iteratively, searching the parameter space for a stable configuration. Depending on

the application, this stable configuration will take into account a factor of safety. The more likely a structure is to have changing loads, the higher a factor of safety is needed. For example, a bridge needs a higher factor of safety than a cathedral. In the event that there is no feasible configuration for a structure, the least infeasible structure is returned and the user is required to add new structural elements.

In "Creating Models of Truss Structures with Optimization" [2], Smith, Hodgins, Oppenheim, and Witkin propose a method of creating trusses procedurally. This work allows the user to define several anchor points and loads for a truss, then have the software automatically generate a truss. In this work, the risk of pieces falling apart is not an issue as it was in the previous paper. The primary failure method in this case is buckling, since all forces are axial. Therefore, the core of the algorithm is a multivariable optimization with constraints. The algorithm attempts iteratively to minimize weight while ensuring that none of the members will fail, either in tension or compression.

My work had initially intended to go in this direction, using static analysis of structures within Google Sketchup. However, Sketchup proved to be a poor environment for the program I wanted to write, so the project was moved to a standalone application and the focus shifted to thin-shell structures.

2. BACKEND / SIMULATION ALGORITHM (BETTER TITLE GOES HERE)

The primary data structure is the cloth object. This object is a collection of points which are connected to each other through springs. These springs exert forces on one another based on constants determined when the cloth is loaded.

At each step of the simulation, the system iterates over all the vertices in the shell, and for each vertex calculates the forces acting upon it. The forces that act upon any given point are gravity and the spring force exerted by all connected points. Once the forces have been calculated, the algorithm divides the forces by the masses of the points, yielding an acceleration. The acceleration can be multiplied by the time step to obtain a velocity, which is again multiplied by the time step to get the new position of the point. However, this single-step explicit integration is very imprecise. If the timestep or forces involved are very large, the result can be very imprecise or cause oscillations. To combat this problem, a second step is taken which calculates the midpoint of the original integration and takes an additional half-step from that point. This second-order integration is much more stable, so much larger timesteps can be taken. A fourth-order system was created, but despite its stability, each step took too long for it to be useful in an interactive simulation.

This simulation will find equilibrium when the forces exerted by the springs balance the force of gravity. Depending on the number of points in the shell and the amount the shape was changed, this equilibrium could take anywhere from a few seconds to a minute to reach.

3. THIN SHELL DESIGN TOOL

3.1 Layout

As can be seen in figure 3.1, there are 3 main panels in the design tool. On the left is the viewing panel, where the structure is shown in 3D. This view can be rotated, zoomed, and panned to give the architect the view they need. There is no actual interaction with the structure in this panel; all of the interaction is done via the two right-hand panels. The top-right panel is the floorplan panel. This panel contains a point for each point of the structure that is in contact with the ground. These points can be moved around in order to modify the shape of the structure to fit the architect's vision. The bottom-right panel is the grid panel. This panel shows all the vertexes of the structure in 2D. This panel can be used to disable points, creating voids in the structure. It can also be used to associate different points in the shell with certain points in the floor plan. This is most useful when adding new points to the floor plan, allowing the architect to define a new point of support on the fly. Together, these windows allow the architect broad control over the design of their structure, while the tool works in the background to keep the shape of the structure optimal.

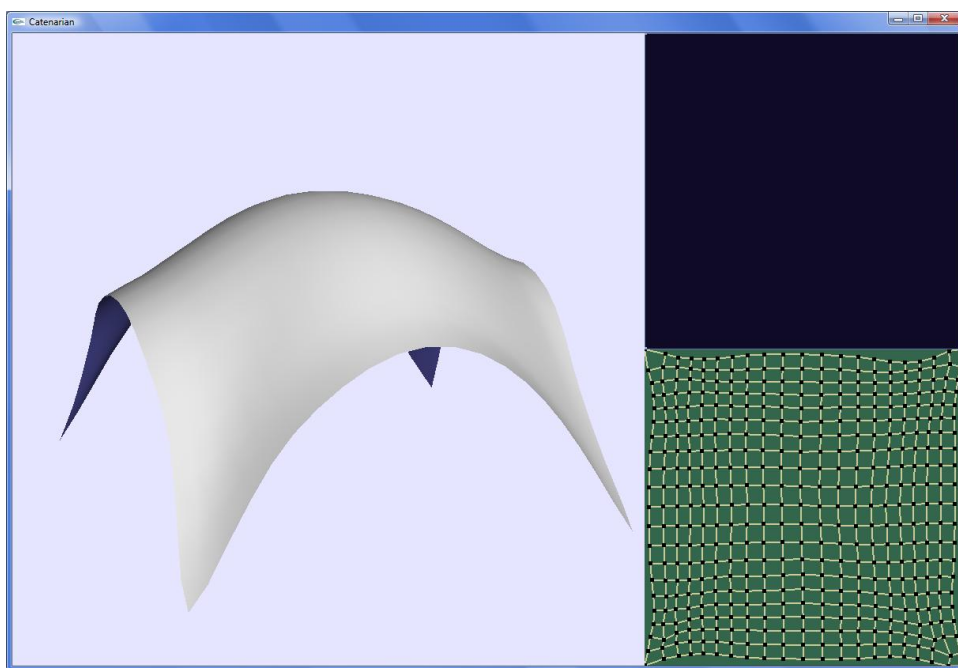


Figure 3.1: A screenshot of the tool

4. USER STUDY

4.1 John

The first user I had in to use the software was John. John is an architecture student, and was very excited about the software. The first thing I noticed when he started using the software was that he tried to zoom with the mouse wheel. That's a feature that should be added. Mouse wheel scrolling is a common functionality that shouldn't be too hard to implement.

As he continued to use the software and get more comfortable with it, he noted that having left-click have an immediate action was not what he was used to. Most architectural software allows the user to select points with the left mouse button, then right-click to bring up the interaction menu. A standardization would be a good idea. This is especially true in the floorplan window, where left-clicking places a new point, which is not what the user wants when they miss selecting a point by a few pixels. On a related note, an undo feature would be very welcome. While it would be less of an issue if accidental left-clicks did not create points, an undo feature would still help the user to revert changes, whether this reversion stems from error or indecision.

Something that became very clear to me as the John continued to use the software is that the current implementation of the grid interface is a bit clunky. Firstly, new points should automatically associate with the closest point in the grid. That's almost always what the user does next, and it saves them from having to guess which point is the closest, which is especially useful in a high-density grid. However, the real problem with grid association is the snapping. Since the floorplan point does not move when it is associated with a new grid point, the mesh is forced to warp, sometimes drastically, to acquiesce to the user's request. I am not sure what the best approach to this would be. Moving the floorplan point to match the grid does not make much sense, since that's not usually what the user is trying to do. Moving them both to the average also does not make a lot of sense. I suspect, however, that the main problem with the snapping is less that it snaps and more

that it surprises the user when it happens. I think having a less immediate interface would help with this. For example, if rather than left-click associating the points it selected them, the user could see where on the mesh he was about to associate with a floorplan point. He could then associate the two via a right-click menu, which is a more natural workflow. It also gives the architect finer control, allowing them to have more information before making a change to the model.

Some creative features that were requested include changing the height of points and length of springs. Point height changing would be a fairly simple feature, and allow for more complex catenary shapes that incorporate traditional structural parts. Changing the length of springs would be a very useful feature, allowing the user to modify the shape of the structure in much finer detail than is currently possible. By manually tightening and relaxing springs, the structure can be allowed to slip into any shape the architect wants.

Several visualization options were discussed during the trial, foremost of which was the option to highlight the selected floorplan and/or grid point on the model. This would make it much easier to orient the model to the floorplan and to keep one's bearings when looking at the model from various angles. Another visualization option that was requested was the ability to pause and rewind the simulation of the model. While rewinding is probably not feasible, pausing certainly is, though I'm not certain that it is necessarily a good idea since a model is not stable until it reaches equilibrium.

Several suggestions were also made with regards to audience and distribution. John suggested that this tool would work very well in a web-based medium, perhaps with the ability for users to save and share their designs. This could be a fun tool with potential for collaboration. In addition, it would be a great market to get a number of testers. The downside to this, of course, is that this would require a complete re-write of the code, as c++ is not well-supported on the internet.

A workflow pattern that I noticed is that he tended to start with an idea in mind of what he wanted to create. Once that was made, he would look at it from a few angles, then change it. If the simulation started to get tangled up or if he made a mis-click such that the model did something he was not expecting, he would

continue to push it in that direction, ususally ending up in an inescapable oscillation which would either require the mesh to be reset or crash the program. I'm not sure what was so intriguing about the failures, but he seemed much more interested in them than in the successful structures.

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