Econ 573: Project 2

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1. Introduction

This empirical analysis aims to examine the dynamics of Apple Inc.'s (**AAPL**) stock price, focusing on its returns, volatility, and Value-at-Risk (**VaR**) measure. Additionally, I test for cointegration between Apple's stock price and that of Broadcom (**AVGO**), a key supplier, to determine if a long-term equilibrium relationship exists between the two. The CSV file *AAPL_AVGO.csv* was provided to me through Professor Yong Bao via Brightspace. My analysis will be conducted using *EViews 14*, unless otherwise stated. All figures mentioned are saved in the appendix at the end of the report.

2. Description of Data and Empirical Method

The data spans 1,005 observations, ranging from January 1st, 2021 through December 31st, 2024. For purposes of this exercise, the in-sample period is defined as January 5th, 2021 through December 29th, 2023.The out-of-sample period is January 2nd, 2024 through December 29, 2024.

Furthermore, calculations and tests rely on specific modeling assumptions. For the *ARCH/GARCH* models, the mean equation is specified as an *ARMA(0,0)*, while the volatility follows an *ARCH/GARCH(1,1)* process. Stock returns are computed as *100×log(price)*. Since volatility is not directly observable, it is proxied using the squared return residuals, defined as: ${u^2}_t = (y_t \ - u_{t|t-1})^2$.

One of the most fundamental assumptions when understanding time series modeling is stationarity. A time series is said to be stationary if its statistical properties do not change over time, meaning it has 1) Constant mean 2) Constant Variance & 3) Constant autocovariance. To test for stationarity within both sets of time series, I conducted the *Augmented Dickey-Fuller* **(ADF)** *Test* on Apple Inc.'s daily closing price series from the total sample period.. The test returned a p-value of *0.899*, indicating a failure to reject the null hypothesis of a unit root at the 10% significant level. This result suggests that Apple's stock price is *non-stationary* and consistent with a *random walk process*. A random walk process suggests that price changes are unpredictable, which is why analyzing the *returns* allows us to gain further insight into meaningful action. The results of the **ADF** test are documented in *Figure 1*.

3. Empirical Methodology

To analyze the behavior of Apple's stock returns and volatility, I apply a series of time series econometric techniques. These include unit root tests for stationarity, *ARCH/GARCH*-type models to capture and forecast volatility, and Value-at-Risk (**VaR**) calculations for risk assessment. Finally, I conduct a cointegration analysis to examine the long-run relationship between Apple and Broadcom's stock prices.

Modeling Volatility Using Returns

As mentioned above, the data for **AAPL** stock prices contains a unit root, indicating non-stationarity. Although returns are historically difficult to predict, volatility often shows clear, persistent patterns over time. In fact, while we typically assume that the conditional mean follows a simple *ARMA*(0,0) process, incorporating a volatility term enables us to model potential returns based on both past and expected future volatility.

This motivates the use of *ARCH/GARCH*-type models, which enable us to explicitly model and forecast volatility based on past squared errors and past volatility. To explore this further, we will fit the returns data of **AAPL** using the in-sample-period, estimating four *GARCH* models.

1. **ARMA(0,0)-GARCH(1,1)**

   Y *= 0.114154778571*

   GARCH *= 0.0329876993864 + 0.0486883078464*RESID(-1)^2 + 0.938666390087*GARCH(-1)*

This model is the standard *GARCH* model, designed to model conditional heteroskedasticity or the variance of the error terms over time. Essentially, the model comprises 3 parts 1) Mean Equation (**Y**): This represents the long-run average variance until a point in time. 2) *ARCH* Term: Models the influence of the previous period's squared shock to current volatility 3) *GARCH* term: Explains the magnitude of the previous period's volatility on the current periods

2. **ARMA(0,0)-EGARCH(1,1) - Asymmetric Order of 1**

   Y = 0.0757443606234

   LOG(GARCH) = -0.0627906901632 + 0.111961060314*ABS(RESID(-1)/@SQRT(GARCH(-1))) - 0.0749503352237*RESID(-1)/@SQRT(GARCH(-1)) + 0.973343072031*LOG(GARCH(-1))

   The exponential *GARCH* model is used when one believes that the past shocks of volatility are asymmetric. Typically, negative shocks might have a larger effect on volatility than positive shocks of the same magnitude. To achieve this, the model captures the log of the variance rather than the variance itself, ensuring positive variance.

3. **ARMA(0,0)-TGARCH(1,1)**

   Y = 0.086584416468

   GARCH = 0.0569810830368 + 0.0134288187811*RESID(-1)^2 + 0.0763079520625*RESID(-1)^2*(RESID(-1)<0) + 0.926656554013*GARCH(-1)

   Similar to an *EGARCH* model, *TGARCH* aims to model the larger impact of negative shocks on volatility compared to positive shocks of the same size. To do this, the model uses an additional term that activates only when the shock is negative, in this case a coefficient of .076.

4. **GARCH-in-Mean (GARCH-M)**

   Y = -0.144985926132*@SQRT(GARCH) + 0.334563840241

   GARCH = 0.0343796011463 + 0.0497964363097*RESID(-1)^2 + 0.937120151804*GARCH(-1)

GARCH-M focuses on modeling how volatility directly affects returns, incorporating the conditional variance in the mean equation of the model, which is modelled by -*0.145*@SQRT(GARCH)*. This suggests increases in volatility garner lower than expected returns.

EViews provides the Schwarz Information Criterion (**SIC**) for model selection, in this case *EGARCH* (3.86) is preferred.

Model	*GARCH*	*EGARCH*	*TGARCH*	*GARCH-M*
SIC Value	3.871	3.867	3.870	3.878

Residual Diagnostics for Volatility Models

Once the volatility of Apple returns has been modeled using *GARCH*-type models, it is essential to assess the adequacy and fit of each model. A well-specified volatility model should produce standardized residuals that behave like white noise. Specifically, we expect these standardized residuals to resemble a normal distribution, exhibit no autocorrelation, and show no remaining *ARCH* effects. If these properties do not hold, the model may be mis-specified or failing to capture key dynamics in the return series. To evaluate this, I examine the standardized residuals using the Jarque-Bera test, correlogram function, and tests for heteroskedasticity to detect remaining *ARCH* effects. *An explanation of each test and it results are summarized below:*

A) Jargue-Bera Diagnosis.

Model	*GARCH*	*EGARCH*	*TGARCH*	*GARCH-M*
Statistic	48.881	22.931	33.886	50.529
Probability	0.000	0.000	0.000	0.000

B) Correlogram Diagnosis

Model	*GARCH*	*EGARCH*	*TGARCH*	*GARCH-M*
Significant	No	No	No	No

Autocorrelation				

C) Heteroskedasticity Diagnosis

Model	*GARCH*	*EGARCH*	*TGARCH*	*GARCH-M*
F-Stat	0.243	0.374	0.179	0.225
P-Value	0.621	0.540	0.671	0.634
Conclusion	No ARCH effects present	No ARCH effects present	No ARCH effects present	No ARCH effects present

When specifying the volatility, all models were deemed sufficient based on the conditional requirements of their residuals. Although the residuals are not normally distributed across all models, this is common in financial data and considered acceptable. Therefore, based on the lack of significant autocorrelation and no remaining *ARCH* effects, we conclude that the models adequately capture the time varying volatility structure.

Out-of-Sample Volatility Forecasting and MSE Analysis

To evaluate how well each model predicts volatility, I use the in-sample-period to generate one-day-ahead forecasts, based on a rolling scheme. To efficiently compare the models I used the *mean squared error* (**MSE**).:

Model	*GARCH*	*EGARCH*	*TGARCH*	*GARCH-M*
MSE	21.889	21.941	22.070	21.938

Based on the out-of-sample **MSE** calculations, the standard *GARCH* model exhibited the highest forecasting accuracy among the four models. While all models produced similar **MSE** values - averaging around 22 - the *GARCH* model achieved the lowest at 21.889, indicating a marginally better performance in predicting return volatility. Taking the square root of the mean squared error, the average deviation in forecasting volatility is approximately 4.7 percentage points for each model.

Forecast Accuracy and Risk Assessment Using VaR

To better understand the risk profile of Apple's stock returns, I compute one-day 10% **VaR** forecasts for the out-of-sample period. These estimates, generated using the same rolling scheme described earlier, indicate the return threshold that is expected to be breached with 10% probability on any given day. To evaluate the accuracy and reliability of each model's risk forecasts, I assess the **VaR** predictions using three key performance metrics: **1)** Crossover Count **2)** Out-of-Sample Check Function Value **3)** Empirical Coverage Probability.

VaR Diagnostic	*GARCH*	*EGARCH*	*TGARCH*	*GARCH-M*
Crossover(s)	21	23	23	21
Check Function Value	26.5%	26.9%	27.0%	26.6%
Empirical Probability Coverage	8.03%	9.13%	9.13%	8.33%

Among the four models, *EGARCH* and *TGARCH* deliver empirical coverage probabilities closest to the target 10% level, suggesting accurate risk estimation. However, *GARCH* has the lowest check function value, indicating the best overall tradeoff between coverage and loss when violations do occur. Therefore, while *EGARCH* and *TGARCH* may be slightly better calibrated for **VaR** coverage, *GARCH* provides the most efficient risk forecast overall. To visualize each model's projected **VaR** against actual returns, see *Figure 2*.

Testing for Cointegration with Broadcom

Though our volatility models focus on Apple's returns, we can also explore whether a long-term relationship exists between Apple and Broadcom by testing for cointegration. Broadcom is a key supplier to Apple, so it's reasonable to suspect that the stock prices of both companies - **AVGO** and **AAPL** - might move together over time. Cointegration requires that both time series are non-stationary and integrated of the same order. Since I have already established that Apple's stock price contains a unit root, I conducted the same **ADF** test on the price of **AVGO**. The results confirmed that **AVGO** also has a unit root. With both series containing a unit root, we can now proceed with testing for cointegration. See *Figure 3*.

Using the cointegration function in EViews, I ran the *Engle-Granger Cointegration* test on both series of data, which tests whether a linear combination of two non-stationary series is itself stationary - indicating a long-term equilibrium relationship between them. At the 10% significance level, I was able to reject the null hypothesis that the series are not cointegrated, with p-values of 0.037 for **AVGO** dependent on **AAPL** and 0.083 for the reverse relationship, see *Figure 4*. This indicates that there is evidence of a long-run equilibrium relationship between the series, introducing the need for a *Vector Error Correction* (**VEC**) model to explain the dynamics between the two stock prices.

When specifying the **VEC** model, I used 2 lags and no exogenous variables, in the order of **AAPL** followed by **AVGO**. The model aims to explain short-run changes in each stock price as a function of their own past changes, the past changes of the other stock price, and the long-run disequilibrium from the previous period. The direction and speed of adjustment toward equilibrium are determined by the sign and magnitude of the error correction term's coefficient in the cointegration equation. The formal equations, along with its coefficient values, for both models are as follows (rounded to 3 decimal places).

D(AAPL) *= - 0.013*(AAPL(-1) - 0.587*AVGO(-1) - 119.840) + 0.0166*D(AAPL(-1)) - 0.049*D(AAPL(-2)) + 0.032*D(AVGO(-1)) + 0.005*D(AVGO(-2)) + 0.120*

D(AVGO) *= 0.013*(AAPL(-1) - 0.587*AVGO(-1) - 119.840) - 0.008*D(AAPL(-1)) + 0.053*D(AAPL(-2)) + 0.129*D(AVGO(-1)) - 0.049*D(AVGO(-2)) + 0.168*

The first coefficient, *-0.131*, indicates how quickly **AAPL** responds to the long-term disequilibrium between the two stocks. For example, if **AAPL** price is below its long-term equilibrium relative to **AVGO** price, it corrects upwards. In contrast **AVGO** price, with a coefficient of *0.0122*, adjusts to the long-run imbalance more weakly than **AAPL** price. Both cointegration equations were found to be significant at the 10%(*t-stat of 1.645*) level according to the model output. An interesting observation is that **AAPL** stock price tends to move in close alignment with **AVGO** price, at about 58.67%, along with a constant difference of $119.84. Although this constant may initially appear high, **AVGO** stock price surged from around $45 in 2021 to nearly $241 at the end of 2024. With 2025 data, the model's coefficients could significantly shift. See *Figure 5*.

When analyzing the short-run dynamics, many of the relationships (lags) were not statistically significant. The only notable interactions, beyond the cointegration equations, were between **AVGO** stock price and its first lag, as well as its constant. This suggests that, in the short run, **AVGO** stock price exhibits a consistent upward drift (*.168*), independent of its lagged values. A summary of the significant variables is available in *Figure 6*.

4. Empirical Results and Main Conclusions

The insights gained from this exercise shift the focus from predicting **AAPL** stock price to understanding its returns and price movements in relation to **AVGO**. I began by testing for stationarity in **AAPL** stock prices, concluding that price movements are not predictable based on stock prices alone. I estimated four *GARCH* models, all of which passed key residual diagnostic tests except for normality. Next, I used several metrics to evaluate each model and compare their performance. Based on the **SIC**, *EGARCH* was identified as the optimal model. However, when forecasting volatility and comparing it to a proxy using MSE, the standard *GARCH* model had the smallest error. Similarly, when generating one-day-ahead 10% VaR forecasts, the standard *GARCH* model again offered the best overall trade-off, as indicated by the lowest check function value. Lastly, I modeled the long-term dynamics between **AAPL** and **AVGO** stock prices, confirming statistically significant cointegration between the two.

With these insights in mind, the standard *GARCH* model appears to be the optimal choice for forecasting one-day-ahead volatility and 10% **VaR**. Furthermore, the cointegration analysis between **AAPL** and **AVGO** suggests that when both prices decline, **AAPL** tends to stabilize quicker by reverting to its long-term equilibrium.

Appendix

Figure 1.

Null Hypothesis: AAPL has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=21)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.443185	0.8992
Test critical values:	1% level	-3.436644	
	5% level	-2.864207	
	10% level	-2.568242	

*MacKinnon (1996) one-sided p-values.

Figure 2.

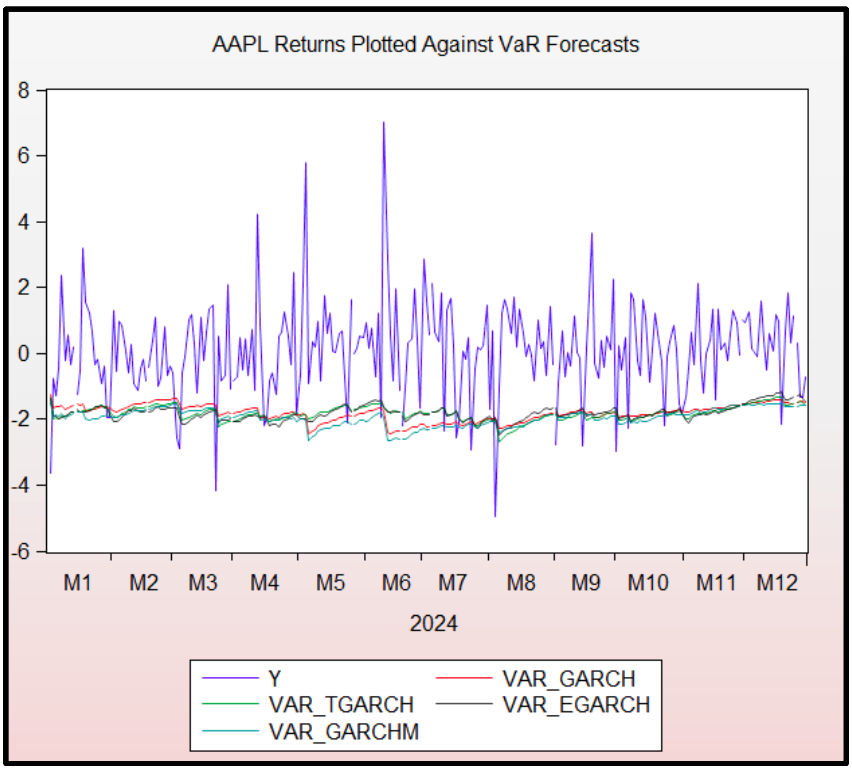

Figure 3.

Null Hypothesis: AVGO has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=19)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.879362	0.9953
Test critical values:	1% level	-3.438819	
	5% level	-2.865168	
	10% level	-2.568757	

*MacKinnon (1996) one-sided p-values.

Figure 4.

Date: 05/04/25 Time: 15:14
Series: AAPL AVGO
Sample: 1/04/2021 12/31/2024
Included observations: 1005
Null hypothesis: Series are not cointegrated
Cointegrating equation deterministics: C
Automatic lags specification based on Schwarz criterion (maxlag=21)

Dependent	tau-statistic	Prob.*	z-statistic	Prob.*
AAPL	-3.342188	0.0500	-21.93586	0.0370
AVGO	-2.835687	0.1556	-17.98428	0.0830

*MacKinnon (1996) p-values.

Figure 5.

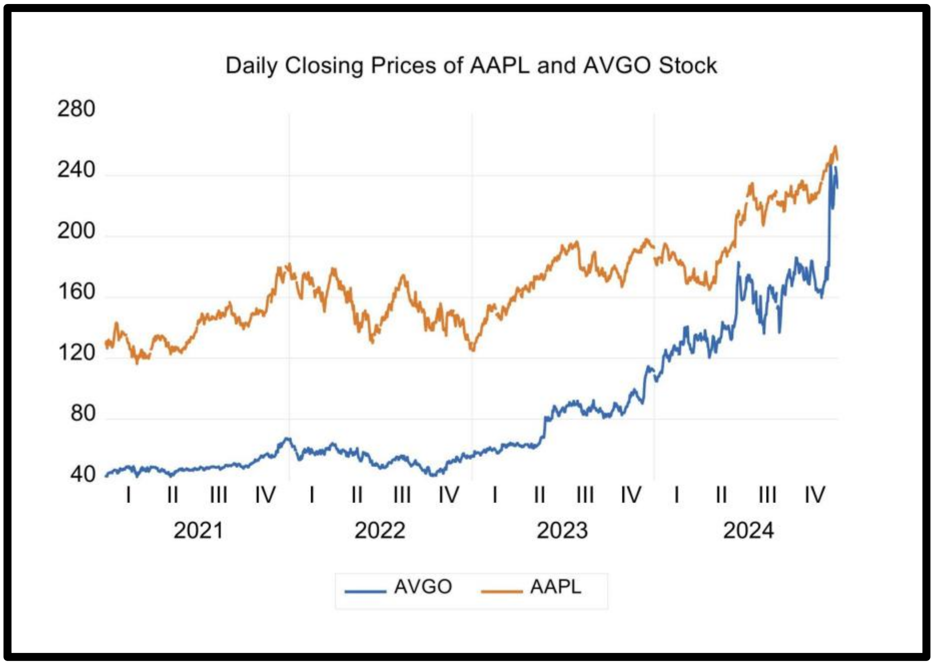

Figure 6.

Equation	Variable	Coefficient	t-statistic	Significance (10%)
Cointegrating Equation	AAPL(-1)	1	-	Significant
	AVGO(-1)	-0.5867	-6.33	Significant
Error Correction (D(AAPL))	COINTEQ1	-0.0131	-2.01	Significant
Error Correction (D(AVGO))	COINTEQ1	0.0123	1.70	Significant
	D(AVGO(-1))	0.1289	3.84	Significant
Drift Effect	Constant (C)	0.1681	1.76	Significant

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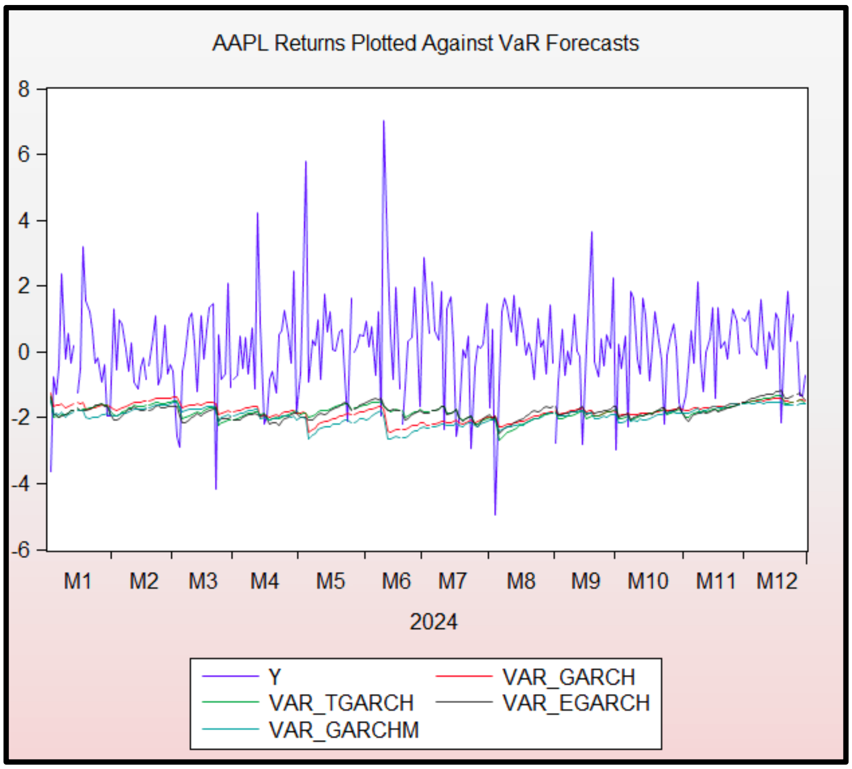

Figure 3.

Null Hypothesis: AVGO has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=19)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.879362	0.9953
Test critical values:	1% level	-3.438819	
	5% level	-2.865168	
	10% level	-2.568757	

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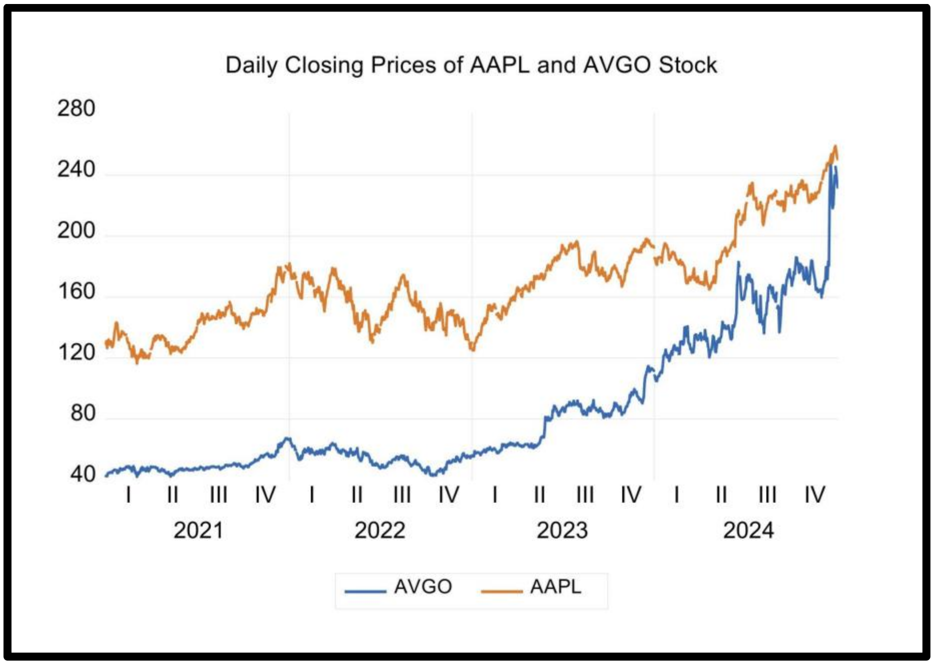
Daily Closing Prices of AAPL and AVGO Stock
280
240
200
160
120
80
40
I II III IV I II III IV I II III IV I II III IV
2021
2022
2023
2024
AVGO
AAPL