Specifying models – fixed and random effects

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Aims for the class

1. Understand the use of *Information Criteria* and *Likelihood Ratio Tests* for evaluating models
2. Practise running linear mixed-effects models with varying fixed or random effects structures
3. Practise evaluating models Practise reporting models

Linear mixed-effects models – Model comparison approach

We focus on building a series of models from the simplest to the most complex model supported by the data

- A minimal model – data observed can be predicted only by the average of observations – the intercept
- Observed values are then random deviations from the average
- Will our our capacity to predict observations be improved by adding other terms?
- Factors we manipulate or predict should influence outcomes

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Evaluating models

Trade-off between too much and too little simplicity in model selection – variable selection

- Models with too few parameters – included variables, effects – have bias
- Bias – the estimate of the effect coefficient will not on average equal the true value of the coefficient in the population

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We can use Information Criteria statistics like AIC or BIC to evaluate models

Understood within an approach: Information-theoretic methods – for linear models we can compare the F, R^2 and information criteria indices

- Information-theoretic methods are grounded in the insight that researchers have reality and have approximating models
- The distance between a model and reality corresponds to the 'information lost' when you use a model to approximate reality
- Information criteria – AIC or BIC – estimates of information loss
- The process of model selection aims to minimise information loss

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Akaike Information Criteria: AIC

Akaike showed you could estimate information loss in terms of the likelihood of the model given the data

$$AIC = -2ln(l) + 2k \quad (1)$$

- $-2ln(l)$ -2 times the log of the likelihood of the model given the data
- (l) – likelihood
  - Is proportional to the probability of observed data conditional on some hypothesis being true

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- You can link models that are more likely – closer to reality – with models with smaller residuals
- Linear models with smaller residuals would have larger R^2

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Bayesian Information Criteria: BIC

Schwartz proposed an alternative estimate

$$BIC = -2ln(l) + kln(N) \tag{3}$$

- $-2ln(l)$ – -2 times the log of the likelihood of the model given the data
- $+kln(N)$ – is the number of parameters in the model times the log of the sample size
- Crudely, the penalty for greater complexity is heavier in BIC
- Models with more parameters may fit the data better but some of those effects may be spurious

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Model selection and judgment: *Information-theoretic methods* are grounded in the insight that you have reality and you have approximating models

- Compare models varying in fixed effects: model 1, just main effects; model 2, main effects plus interactions
- Compare models varying in random effects: model 1, just random effect of subjects on intercepts; model 2, random effects of subjects and items on intercepts
- If the more complex model better approximates reality then it will be more likely given the data
  - BIC or AIC will be closer to negative infinity: $-2ln(l)$ will be larger
  - e.g. 10 is better than 1000, -1000 better than -10

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Reporting standards

Using AIC and BIC

- Report briefly the model comparisons: "Compared a simpler model: model 1, just main effects; model 2, main effects plus interactions"
- Report the AIC or BIC for the different models
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Likelihood ratio test comparison

- The test statistic is the comparison of the likelihood of the simpler model with the more complex model
- Comparison by division $2log\frac{likelihood-complex}{likelihood-simple}$
- The likelihood ratio is compared to the χ^2 distribution for a significance test
- Assuming the null hypothesis that the simpler model is adequate
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Model comparisons among mixed-effects models – use *anova()* function

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anova(full.lmer0, full.lmer1)
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- `anova(...,....)` – compare pairs of models
- `full.lmer0` – a simpler model – more limited assumptions about sources of variance
- `full.lmer1` – a more complex model – more predictors – includes simpler model as a special case

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Running the *anova*(,) comparison will deliver AIC, BIC, and likelihood comparisons for varying models

```
> anova(full.lmer1, full.lmer2)
Data: subjects.behaviour.items.nomissing
Models:
full.lmer1: logrt ~ cAge + cTOWRE_wordacc + cTOWRE_nonwordacc + cART_HRminusFR +
full.lmer1:     (1 | subjectID) + (1 | item_name)
full.lmer2: logrt ~ cAge + cTOWRE_wordacc + cTOWRE_nonwordacc + cART_HRminusFR +
full.lmer2:     item_type + cLength + cOrtho_N + cBG_Mean + (1 | subjectID) +
full.lmer2:     (1 | item_name)
           Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
full.lmer1  8 -17983 -17925 8999.4   -17999
full.lmer2 12 -18319 -18232 9171.3   -18343 343.81      4  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> 
```

Figure : Comparison of model with subject attribute predictors and model also with item effects

Model comparison

- AIC, BIC and LRT comparisons should be consistent in their indications – which model to prefer
- Can be tricky where dealing with complex sets of predictors – indicators may diverge
- Remember that BIC may penalise complexity more heavily – especially if conducting exploratory research
- Remember that may be obliged to include all effects built-in by design – if conducting confirmatory study

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A more realistic repeated measures model of word frequency and reading ability effects

Taking into account random effect of subjects and items

$$RT = \beta_0 + \beta_{ability} + \beta_{frequency} + \beta_{ability*frequency} + \beta_{subject} + \beta_{item} + \beta_{subject*frequency} + \beta_{item*ability} + \epsilon \quad (4)$$

- What about effect of random variation between participants?
  - Allow intercept to vary – random effect of subject – some have slower average some have faster average than average overall
  - Allow effect of frequency to vary – random effect of subject – subjects can be affected by word frequency in different ways

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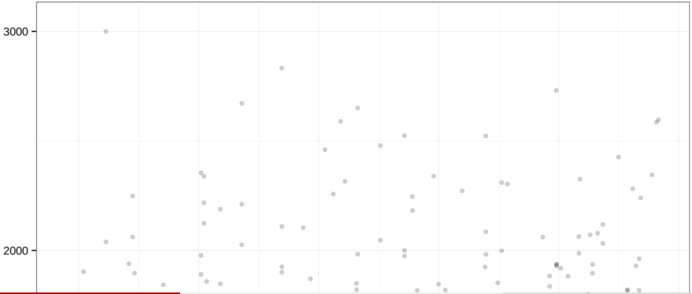
Random effect of subjects on intercepts

Differences in individuals' average response speed compared to the group average

$$Y_{ij} = \beta_0 + \beta_1 X_i + U_{0,j} + \epsilon_{ij} \tag{6}$$

- β_0 common intercept, average outcome given the other effects
- $U_{0,j}$ adjustments to the intercept required to explain differences between common average and average for each j individual



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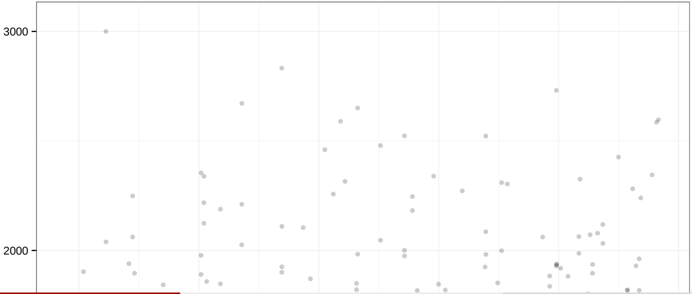

Random effect of subjects on slopes of fixed effects

Accounting for variation in *within-subjects* frequency effect on RTs

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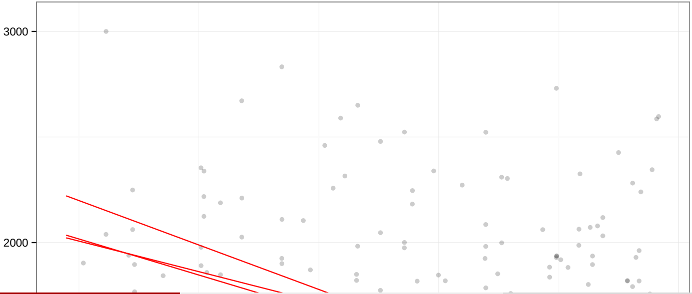

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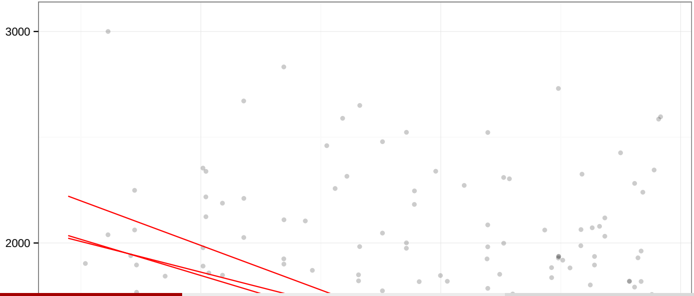

Can allow random effects of both subjects and items

Solving the 'language-as-fixed-effect' problem in one model

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Mixed effects models – both fixed effects and random effects

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- $\beta_0 + \beta_1 X_i$ fixed effect of predictors – fixed because replicable by manipulation or selection
- $U_{0,j} + U_{1,j} + +W_{0,i} + W_{1,i} + \epsilon_{ij}$ random effects of group on intercepts and slopes – random because differences due to sampling

Mixed effects models – both fixed effects and random effects

$$Y_{ij} = \beta_0 + \beta_1 X_i + U_{0,j} + U_{1,j} + W_{0,i} + W_{1,i} + \epsilon_{ij} \tag{9}$$

- $\beta_0 + \beta_1 X_i$ fixed effect of predictors – fixed because replicable by manipulation or selection
- $U_{0,j} + U_{1,j} + +W_{0,i} + W_{1,i} + \epsilon_{ij}$ random effects of group on intercepts and slopes – random because differences due to sampling

We could model random differences between participants items as fixed effects but we do not

We incorporate random effects terms in models to capture the spread, *the variance*, associated with random differences in intercepts or slopes

$$Y_{ij} = \beta_0 + \beta_1 X_i + U_{0,j} + U_{1,j} + W_{0,i} + W_{1,i} + \epsilon_{ij} \tag{10}$$

- $\sigma^2_{U_{0,j}} + \sigma^2_{U_{1,j}} + \sigma^2_{U_{0,j}U_{1,j}}$ variances and covariance of random effects
- $\sigma^2_{W_{0,j}} + \sigma^2_{W_{1,j}} + \sigma^2_{W_{0,j}W_{1,j}}$ variances and covariance of random effects
- $\sigma^2_{\epsilon_{ij}}$ residuals – error variance left over
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Estimation methods

An intuitive account of estimation in mixed-effects models

- If we knew the random effects, we could find the fixed effects estimates by minimising differences between predicted and observed outcomes – like linear modelling
- If we knew the fixed effects – the regression coefficients – we could work out the residuals and the random effects

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- At the start, we know neither, but we can move between partial estimation of fixed and random effect in an iterative approach
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Restricted maximum likelihood ($REML = TRUE$) and maximum likelihood ($REML = FALSE$) methods

- Restricted maximum likelihood ($REML = TRUE$)
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- REML method recommended for comparing the likelihood of models with *the same fixed effects* but *different random effects*
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Model comparisons among mixed-effects models fitted using maximum likelihood (*REML* = *FALSE*) method

```
full.lmer.0 <- lmer(logrt ~

(1|subjectID) + (1|item_name),

data = ML.all.correct, REML = F)
```

- Maximum likelihood (*REML* = *FALSE*)
  - ML estimation methods can be used to fit models with varying fixed effects but the same random effects

Model comparisons among mixed-effects models fitted using maximum likelihood ($REML = FALSE$) method

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full.lmer.0 <- lmer(logrt ~

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```

- Maximum likelihood ($REML = FALSE$)
  - ML estimation methods can be used to fit models with varying fixed effects but the same random effects

Examine the fixed effects then the random effects

Start by examining models varying in the *fixed effects* but constant in the *random effects*

- Compare maximum likelihood ($REML = FALSE$) models varying in fixed effects
  - Think about simpler models as simplifications or subsets of more complex models
- Add effects of interest in blocks or sets of predictors

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Compare maximum likelihood models with varying fixed effects but the same random effects

```
full.lmer.1 <- lmer(logrt ~
  zAge + zTOWRE_wordacc + zTOWRE_nonwordacc +
  (1|subjectID) + (1|item_name),
  data = ML.all.correct, REML = FALSE)
summary(full.lmer.1)
```

- `zAge + zTOWRE_wordacc + zTOWRE_nonwordacc +` add fixed effects
- `(1|subjectID) + (1|item_name)` to random effects of subjects and items on intercepts

Compare maximum likelihood models with varying fixed effects but the same random effects

```
full.lmer.1 <- lmer(logrt ~
  zAge + zTOWRE_wordacc + zTOWRE_nonwordacc +
  (1|subjectID) + (1|item_name),
  data = ML.all.correct, REML = FALSE)
summary(full.lmer.1)
```

- `zAge + zTOWRE_wordacc + zTOWRE_nonwordacc +` add fixed effects
- `(1|subjectID) + (1|item_name)` to random effects of subjects and items on intercepts

Compare the model 1 with just subject effects to the model 2 also with item effects

```
full.lmer.2 <- lmer(logrt ~

zAge + zTOWRE_wordacc + zTOWRE_nonwordacc +

item_type + zLength + zOrtho_N +

(1|subjectID) + (1|item_name),

data = ML.all.correct, REML = F)

summary(full.lmer.2)
```

- `item_type + zLength + zOrtho_N` add item effects
- Everything else stays the same

Compare the model 1 with just subject effects to the model 2 also with item effects

```
full.lmer.2 <- lmer(logrt ~

zAge + zTOWRE_wordacc + zTOWRE_nonwordacc +

item_type + zLength + zOrtho_N +

(1|subjectID) + (1|item_name),

data = ML.all.correct, REML = F)

summary(full.lmer.2)
```

- `item_type + zLength + zOrtho_N` add item effects
- Everything else stays the same

Compare model 2 with subject and item effects to model 3 specifying interactions between subject and item effects

```
full.lmer.3  <- lmer(logrt ~

    (zAge + zTOWRE_wordacc + zTOWRE_nonwordacc)*

    (item_type + zLength + zOrtho_N) +

    (1|subjectID) + (1|item_name),

    data = ML.all.correct, REML = FALSE)
summary(full.lmer.3)
```

- Notice that the `(something)*(something)` get you interactions and main effects for all possible pairs of variables in the first set `(zAge + zTOWRE_wordacc + zTOWRE_nonwordacc)` and the second set `(item_type + zLength + zOrtho_N)` of predictors

How do we know if increasing *model complexity* by adding predictors actually helps us to account for variation in outcome values?

We can use the *anova()* function to do the comparison

```
anova(full.lmer.0, full.lmer.1, full.lmer.2, full.lmer.3)
```

- `anova(full.lmer.0, full.lmer.1, full.lmer.2, full.lmer.3)` compare the named models in pairs

See that *anova()* results present information criteria statistics for each model plus *likelihood ratio test* comparisons

Each step increase in complexity appears warranted by improved model fit to data

```
> anova(full.lmer.0, full.lmer.1, full.lmer.2, full.lmer.3)
Data: ML.all.correct
Models:
full.lmer.0: logrt ~ (1 | subjectID) + (1 | item_name)
full.lmer.1: logrt ~ zAge + zTOWRE_wordacc + zTOWRE_nonwordacc + (1 | subjectID) +
full.lmer.1:     (1 | item_name)
full.lmer.2: logrt ~ zAge + zTOWRE_wordacc + zTOWRE_nonwordacc + item_type +
full.lmer.2:     zLength + zOrtho_N + (1 | subjectID) + (1 | item_name)
full.lmer.3: logrt ~ (zAge + zTOWRE_wordacc + zTOWRE_nonwordacc) * (item_type +
full.lmer.3:     zLength + zOrtho_N) + (1 | subjectID) + (1 | item_name)
            Df    AIC    BIC logLik deviance    Chisq Chi Df Pr(>Chisq)
full.lmer.0  4 -17981 -17952 8994.4   -17989
full.lmer.1  7 -17982 -17931 8998.0   -17996   7.1782      3    0.06643 .
full.lmer.2 10 -18320 -18247 9169.7   -18340 343.5400      3    < 2e-16 ***
full.lmer.3 19 -18416 -18278 9227.0   -18454 114.5129      9    < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
```

Figure : Model comparisons

The distinction between exploratory and confirmatory studies may be useful here

Cumming (2014): between *pre specified* and *exploratory* or *question answering* and *question formulating* studies

- Confirmatory – pre specified – models test the effects of the variables or conditions you manipulated
- Exploratory – question formulating – models may be developed by building up complexity, with initially no clear idea about predictions

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Examine the random effects

When the goal of a confirmatory analysis is to test hypotheses about one or more critical fixed effects, what random-effects structure should one use?

- Current recommendations (Barr et al., 2013; JML): Maximal random effects structure
- If you are testing effects manipulated according to a prespecified – confirmatory study – design
  - Test random intercepts – subjects and items
  - Test random slopes for all within-subjects or within-items fixed effects

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Current recommendations (Barr et al., 2013; JML): Maximal random effects structure – what does this involve?

Conceptually, we are working within a framework where we consider a range of potential random effects

$$Y_{ij} = \beta_0 + \beta_1 X_i + U_{0,j} + U_{1,j} + W_{0,i} + W_{1,i} + \epsilon_{ij} \qquad (11)$$

- $\sigma^2_{U_{0,j}} + \sigma^2_{U_{1,j}} + \sigma^2_{U_{0,j}U_{1,j}}$ variances and covariance of random effects
- $\sigma^2_{W_{0,j}} + \sigma^2_{W_{1,j}} + \sigma^2_{W_{0,j}W_{1,j}}$ variances and covariance of random effects
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Examine the utility of random effects by comparing REML models with the same fixed effects but varying random effects

You can begin with random effects of subjects and items on intercepts

```
full.lmer.3  <- lmer(logrt ~

    (zAge + zTOWRE_wordacc + zTOWRE_nonwordacc)*

    (item_type + zLength + zOrtho_N) +

    (1|subjectID) + (1|item_name),

    data = ML.all.correct, REML = FALSE)
summary(full.lmer.3)
```

- Notice that *REML* = *TRUE* – we are focused on accurate comparisons of the random effects component

Compare a model with both random effects of subjects and items on intercepts to a model with just the random effect of items on intercepts

```
full.lmer.3.i  <- lmer(logrt ~

    (zAge + zTOWRE_wordacc + zTOWRE_nonwordacc)*

    (item_type + zLength + zOrtho_N) +

    (1|item_name),

    data = ML.all.correct, REML = FALSE)
summary(full.lmer.3.i)
```

- `(1|item_name)` a comparison with the model including both random intercepts will tell us if the inclusion of the random effect of subjects on intercepts helps the model to fit the data better
- Repeat the comparison but for a second model including a random effect of subjects but not of items on intercepts to examine if the random effect of items on

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```
full.lmer.3.i  <- lmer(logrt ~

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    (1|item_name),

    data = ML.all.correct, REML = FALSE)
summary(full.lmer.3.i)
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- Repeat the comparison but for a second model including a random effect of subjects but not of items on intercepts to examine if the random effect of items on

Examine the utility of random effects of subjects on slopes of fixed effects are required

```
full.lmer.3.slopes  <- lmer(logrt ~

    (zAge + zTOWRE_wordacc + zTOWRE_nonwordacc)*

    (item_type + zLength + zOrtho_N) +

    (item_type + zLength + zOrtho_N + 1|subjectID) + (1|item_name),

    data = ML.all.correct, REML = FALSE)
summary(full.lmer.3.slopes)
```

- `(item_type + zLength + zOrtho_N + 1|subjectID)` we specify a random effect of subjects on intercepts and on the slopes of the item type, length and neighbourhood effects

How do we know if increasing *model complexity* helps us to account for variation in outcome values?

We can use the *anova()* function to compare models with or without the random effect of subjects on the slopes of the within-subjects fixed effects

```
anova(full.lmer.3, full.lmer.3.slopes)
```

```
> anova(full.lmer.3, full.lmer.3.slopes)
Data: ML.all.correct
Models:
full.lmer.3: logrt ~ (zAge + zTOWRE_wordacc + zTOWRE_nonwordacc) * (item_type +
full.lmer.3:     zLength + zOrtho_N) + (1 | subjectID) + (1 | item_name)
full.lmer.3.slopes: logrt ~ (zAge + zTOWRE_wordacc + zTOWRE_nonwordacc) * (item_type +
full.lmer.3.slopes:     zLength + zOrtho_N) + (item_type + zLength + zOrtho_N + 1 |
full.lmer.3.slopes:     subjectID) + (1 | item_name)
                   Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
full.lmer.3        19 -18416 -18278 9227.0   -18454
full.lmer.3.slopes 28 -18740 -18538 9398.1   -18796 342.25      9  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure : Comparison of models with vs. without random effects of subjects on slopes of fixed effects

Estimation procedures may run into convergence problems where there is too much model complexity and not enough data

Reporting standards

Model comparisons

- Report briefly the model comparisons: "Compared a simpler model: model 1, just main effects; model 2, main effects plus interactions"
- Report the AIC or BIC for the different models, or LRT for pair-wise comparisons
  - Report and explain the model selection choice, based on the aims of the study and the information criteria comparisons results

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Reporting standards in psychology

Likelihood Ratio Test comparisons are favoured

- Recommendations (Bates et al., 2015; glmm.wikidot) – to compare models of varying complexity
- Use Likelihood Ratio Test comparisons between models varying in fixed or random effects

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```
confint(full.lmer3, method = "Wald")
```

- `confint(full.lmer3)` – ask for confidence intervals for effect estimates – if do not include 0 then significant
- `method = "Wald"` – can use different methods – "Wald" is faster but alternatives
- Can ask for differing levels of confidence

Reporting the model

- Summary of fixed effects – just like in linear models – with confidence intervals
- Report random effects variance and covariance (if applicable)
- In text, report likelihood comparisons

Fixed effects	Estimated coefficient	SE	Wald confidence intervals 2.50%	97.50%	z	Pr(>|z|)
(Intercept)	-0.2669	0.1544	-0.5482	0.0143	-1.7300	0.0840
Experimental condition (noun vs. noun-verb)	-0.0193	0.1797	-0.3539	0.3149	-0.1100	0.9150
Experimental condition (noun vs. verb)	0.2661	0.2028	-0.0948	0.6266	1.3100	0.1900
Block effect	0.1871	0.0421	0.1049	0.2694	4.4500	< 0.0001
Experimental condition (noun-verb):block interaction	0.0117	0.0594	-0.1047	0.1282	0.2000	0.8440
Experimental condition (verb):block interaction	-0.1287	0.0596	-0.2447	-0.0126	-2.1600	0.0310
Random effects						
	Name	Variance	Std.Dev.	Corr		
Subject effect on intercepts	(Intercept)	0.0447	0.2115			
Random effect of subjects on slopes of block effects		0.0258	0.1606	-0.6800		
Item effect (action) on intercepts	block	< 0.0001	0.0023			
Random effect of items (actions) on slopes of block effects	block	0.0001	0.0096			
Item effect (objects) on intercepts	(Intercept)	0.0153	0.1238			
Random effect of items (objects) on slopes of block effects	(Intercept)	0.0084	0.0919			
	AIC	BIC	logLik	deviance		
	16496.736	16594.681	-8235.368	16470.736		

Reporting the model

- Summary of fixed effects – just like in linear models – with confidence intervals
- Report random effects variance and covariance (if applicable)
- In text, report likelihood comparisons

Fixed effects	Estimated coefficient	SE	Wald confidence intervals 2.50%	97.50%	z	Pr(>|z|)
(Intercept)	-0.2669	0.1544	-0.5482	0.0143	-1.7300	0.0840
Experimental condition (noun vs. noun-verb)	-0.0193	0.1797	-0.3539	0.3149	-0.1100	0.9150
Experimental condition (noun vs. verb)	0.2661	0.2028	-0.0948	0.6266	1.3100	0.1900
Block effect	0.1871	0.0421	0.1049	0.2694	4.4500	< 0.0001
Experimental condition (noun-verb):block interaction	0.0117	0.0594	-0.1047	0.1282	0.2000	0.8440
Experimental condition (verb):block interaction	-0.1287	0.0596	-0.2447	-0.0126	-2.1600	0.0310
Random effects						
	Name	Variance	Std.Dev.	Corr		
Subject effect on intercepts	(Intercept)	0.0447	0.2115			
Random effect of subjects on slopes of block effects		0.0258	0.1606	-0.6800		
Item effect (action) on intercepts	block	< 0.0001	0.0023			
Random effect of items (actions) on slopes of block effects	block	0.0001	0.0096			
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