Statistical Methods for Bioinformatics

Exercise 1

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Show that the following function is a cubic regression spline

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + \beta_4 (x - \xi)_+^3$$

a. Find a cubic polynomial $f_1(x) = a_1 + b_1 x + c_1 x^2 + d_1 x^3$ that matched f(x) for $x \leq \xi$

b. Find a cubic polynomial $f_1(x) = a_2 + b_2 x + c_2 x^2 + d_2 x^3$ that matched f(x) for $x > \xi$

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We write out $\beta_4(x-\xi)^3 = \beta_4(x^3 - 3\xi x^2 + 3\xi^2 x - \xi^3)$ *, then rewriting gives the following coefficients:* $a_2 = \beta_0 - \xi^3 \beta_4$, $b_2 = \beta_1 + 3\xi^2 \beta_4$, $c_2 = \beta_2 - 3\xi\beta_4$, $d_2 = \beta_3 + \beta_4$

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c. Show that f1(ξ) $=$ f2(ξ).
d. Show that $f_1^{'}(\xi) = f_2^{'}(\xi)$.
e. Show that $f_1^{''}(\xi) = f_2^{''}(\xi)$.

c. Show that f1(ξ) = f2(ξ).

With $x = \xi$ *we have*

$f_2(\xi) = \beta_0 - \xi^3\beta_4 + \xi\beta_1 + 3\xi^3\beta_4 + \xi^2\beta_2 - 3\xi^3\beta_4 + \xi^3\beta_3 + \xi^3\beta_4$

$f_2(\xi) = \beta_0 + \xi\beta_1 + \xi^2\beta_2 + \xi^3\beta_3 = f_1(\xi)$

d. Show that $f_1^{'}(\xi) = f_2^{'}(\xi)$.

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d. Show that $f_1^{'}(\xi) = f_2^{'}(\xi)$.

$f_2^{'}(x) = \beta_1 + 3\xi^2\beta_4 + 2x(\beta_2 - 3\xi\beta_4) + 3x^2(\beta_3 + \beta_4)$

$f_2^{'}(\xi) = \beta_1 + 3\xi^2\beta_4 + 2\xi\beta_2 - 6\xi^2\beta_4 + 3\xi^2\beta_3 + 3\xi^2\beta_4$

$f_2^{'}(\xi) = \beta_1 + 2\xi\beta_2 + 3\xi^2\beta = f_1^{'}(\xi)$

e. Show that $f_1^{''}(\xi) = f_2^{''}(\xi)$.

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c. Show that f1(ξ) = f2(ξ).

With $x = \xi$ *we have*

$f_2(\xi) = \beta_0 - \xi^3\beta_4 + \xi\beta_1 + 3\xi^3\beta_4 + \xi^2\beta_2 - 3\xi^3\beta_4 + \xi^3\beta_3 + \xi^3\beta_4$

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d. Show that $f_1^{'}(\xi) = f_2^{'}(\xi)$.

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$f_2^{'}(\xi) = \beta_1 + 2\xi\beta_2 + 3\xi^2\beta = f_1^{'}(\xi)$

e. Show that $f_1^{''}(\xi) = f_2^{''}(\xi)$.

$f_2^{''}(x) = 2\beta_2 - 6\xi\beta_4 + 6x(\beta_3 + \beta_4)$

$f_2^{''}(\xi) = 2\beta_2 + 6\xi\beta_3 + 6\xi\beta_4 - 6\xi\beta_4 = f_1^{''}(\xi)$