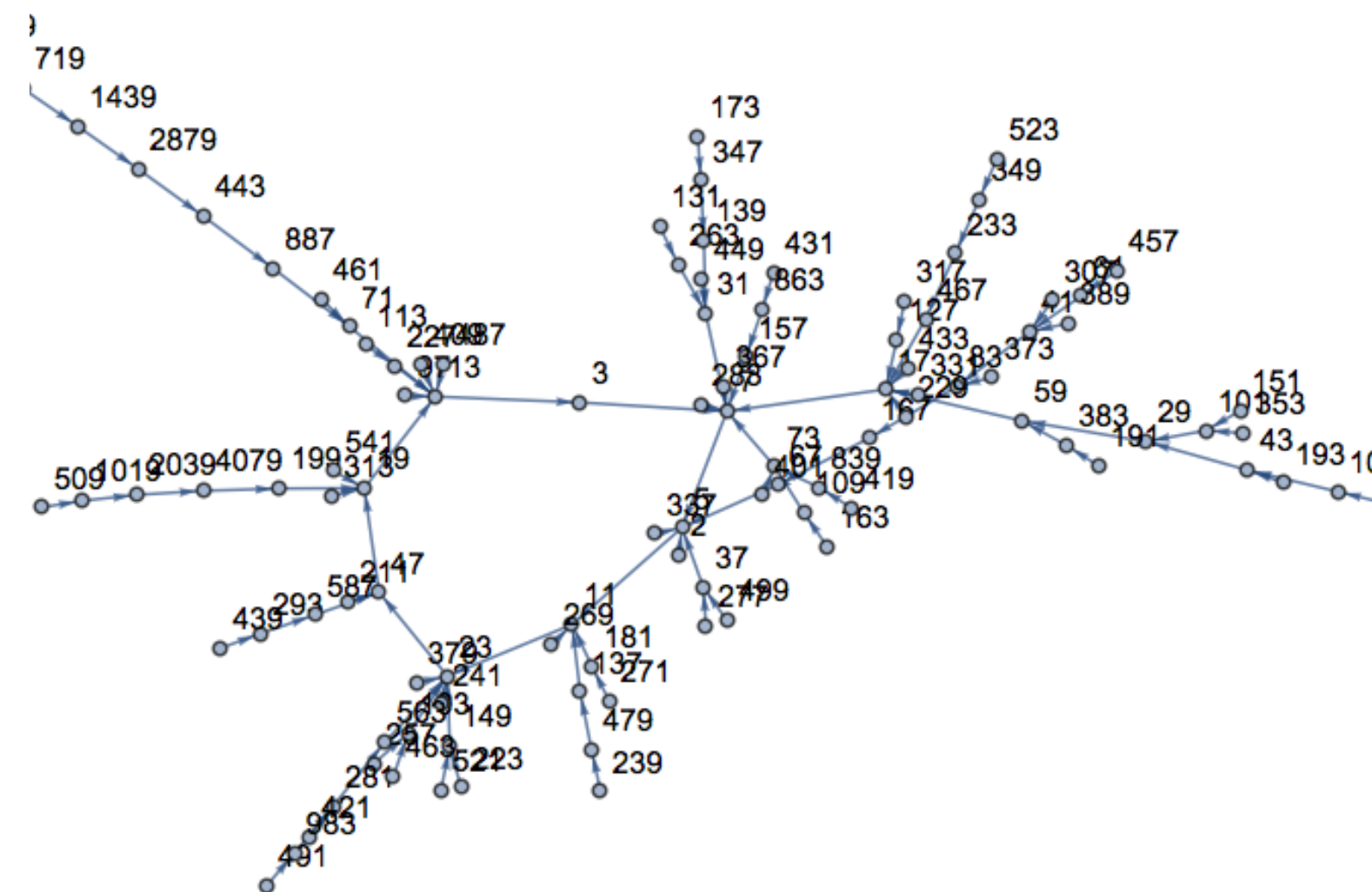
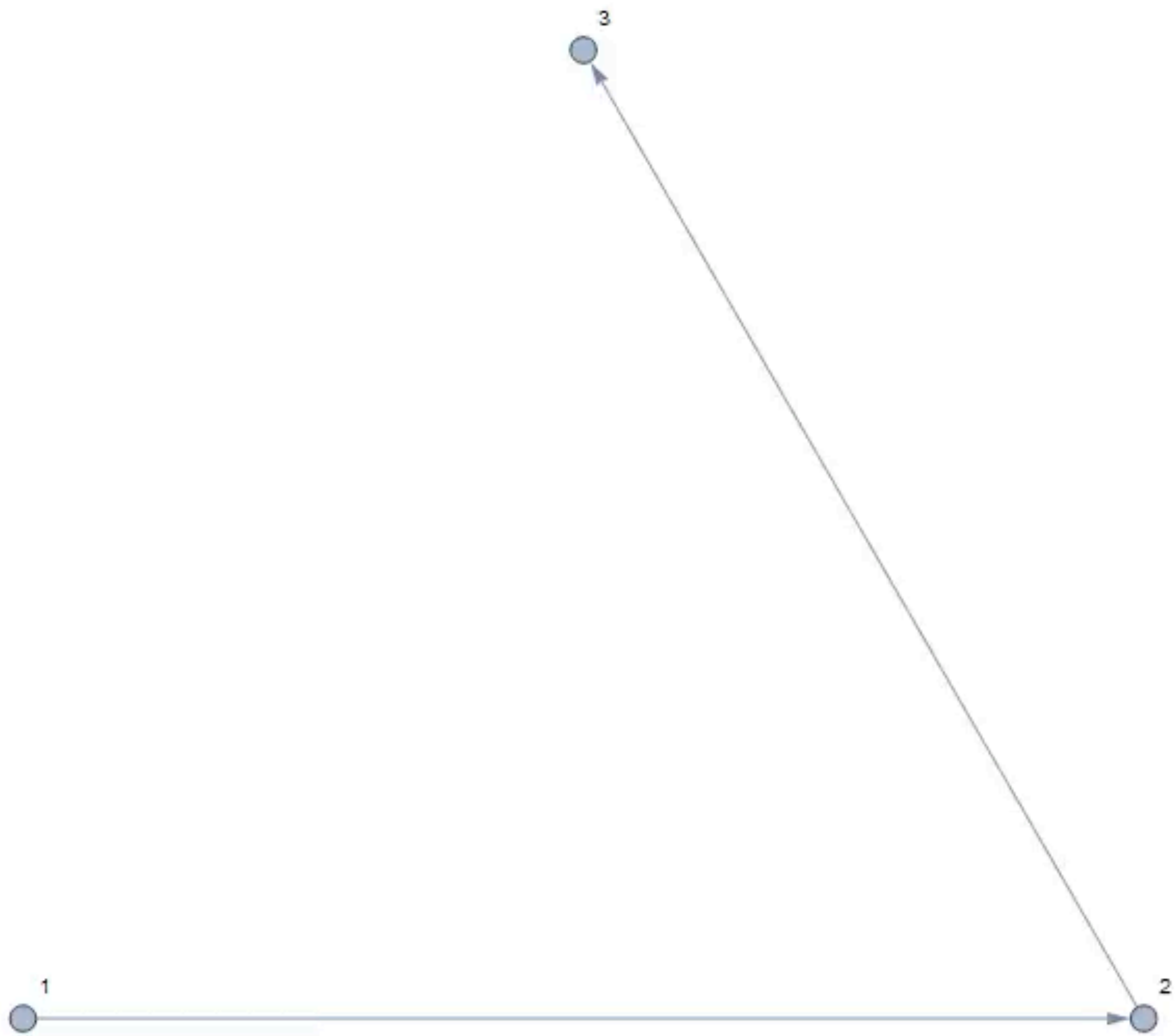


# Extracting { structure }

Robbie VanDerzee, Friday July 30th



$$\sum_{n=1}^k 1 = \frac{k}{1}$$

$$\sum_{n=1}^k n = \frac{k}{1} \cdot \frac{k+1}{2}$$

$$\sum_{n=1}^k n^2 = \frac{k}{1} \cdot \frac{k+1}{2} \cdot \frac{2k+1}{3}$$

$$\sum_{n=1}^k n^3 = \frac{k}{1} \cdot \frac{k+1}{2} \cdot \frac{2k+1}{3} \cdot \frac{3k+1}{4}$$



What ways do we think about numbers?



$$v = \sum_{n=0}^k a_n b^n = a_0 b^0 + a_1 b^1 + \dots + a_n b^n + \dots + a_k b^k$$
$$b \geq 2, \quad 0 \leq a_n < b$$



What other ways could we think  
about numbers?





$$v = \mathit{integer}\langle k \rangle = \{ p_0, p_1, p_2, \dots, p_k \}$$



$\mathbb{Z}^+(v)$	$\mathbb{Z}_{new}^+(v)$
<b>0</b>	$\emptyset$
<b>1</b>	$\{0\}$
<b>2</b>	$\{1\}$
<b>3</b>	$\{0,1\}$
<b>4</b>	$\{2\}$
<b>45</b>	$\{0,2,1\}$



$$v = \pm \textit{integer}\langle k \rangle = \{0/1 \in p_{-1}, p_0, p_1, p_2, \dots, p_k \}$$



$$v = \frac{p}{q} \ni (p, q) \in \mathbb{Z} \wedge \gcd(p, q) \equiv 1$$

$$v = \textit{rational}\langle k \rangle = \left\{ \pm p_{-1}, \pm p_0, \pm p_1, \pm p_2, \dots, \pm p_k \right\}$$



$\mathbb{R}^{\pm}(v)$	$\mathbb{R}_{new}^{\pm}(v)$
<b>1/2</b>	$\{0, -1\}$
<b>1/3</b>	$\{0,0, -1\}$
<b>1/4</b>	$\{0, -2,0\}$
<b>1/5</b>	$\{0,0,0, -1\}$
<b>-5/7</b>	$\{1,0,0,1, -1\}$
<b>11/5</b>	$\{0,0,0, -1,0,1\}$



$$v = \textit{complex}\langle k \rangle = \{ r_{-1}, r_0, r_1, r_2, \dots, r_k \}$$



$\mathbb{C}(v)$	$\mathbb{C}_{new}(v)$
$\sqrt{2}$	$\{\{0\}, \{0, -1\}\}$
$\sqrt{6}$	$\{\{0\}, \{0, -1\}, \{0,0, -1\}\}$
$\sqrt[3]{7.5}$	$\{\{0\}, \{1,0, -1\}, \{0,0, -1\} \{0,0, -1\}\}$
$i$	$\{\{1\}\}$
$\sqrt{-6}$	$\{\{1\}, \{0, -1\}, \{0,0, -1\}\}$
$\sqrt[{\sqrt{2}}]{2}$	$\{\{0\}, \{0,\{0, -1\}\}\}$



**We can keep going, but there is  
an issue!**



How can we decouple it?



How could we represent it  
better?



$$0 := \circ$$

$$1 = (-1)^0 * (2)^0:$$

$$\begin{array}{c} \circ \\ | \\ \hline \hline \end{array} \begin{array}{c} \circ \\ \circ \end{array}$$

$$2 = (-1)^0 * (2^{(-1)^0}) :=$$

$$\begin{array}{c} \circ \\ | \\ \hline \hline \end{array} \begin{array}{c} \circ \\ \circ \\ | \\ \hline \end{array} \circ$$

$$30 = (-1)^0 * 2^{(-1)^0} * 3^{(-1)^0} * 5^{(-1)^0} :=$$

$$\begin{array}{c} \circ \\ | \\ \hline \hline \hline \hline \hline \end{array} \begin{array}{c} \circ \\ \circ \\ | \\ \hline \end{array} \begin{array}{c} \circ \\ \circ \\ | \\ \hline \end{array} \begin{array}{c} \circ \\ \circ \\ | \\ \hline \end{array} \circ$$





How could we compute it?



```
class Atomic {
public:

    Atomic() : atoms({}) {}

    Atomic(const std::initializer_list<Atomic> & Γ) : atoms {Γ} {}

    Atomic(const Atomic & ζ) = default;

    friend Atomic operator*(const Atomic & α, const Atomic & β) {
        if (!α.atoms.size()) return β;
        if (!β.atoms.size()) return α;
        if (!α.atoms.size() && !β.atoms.size()) return {};

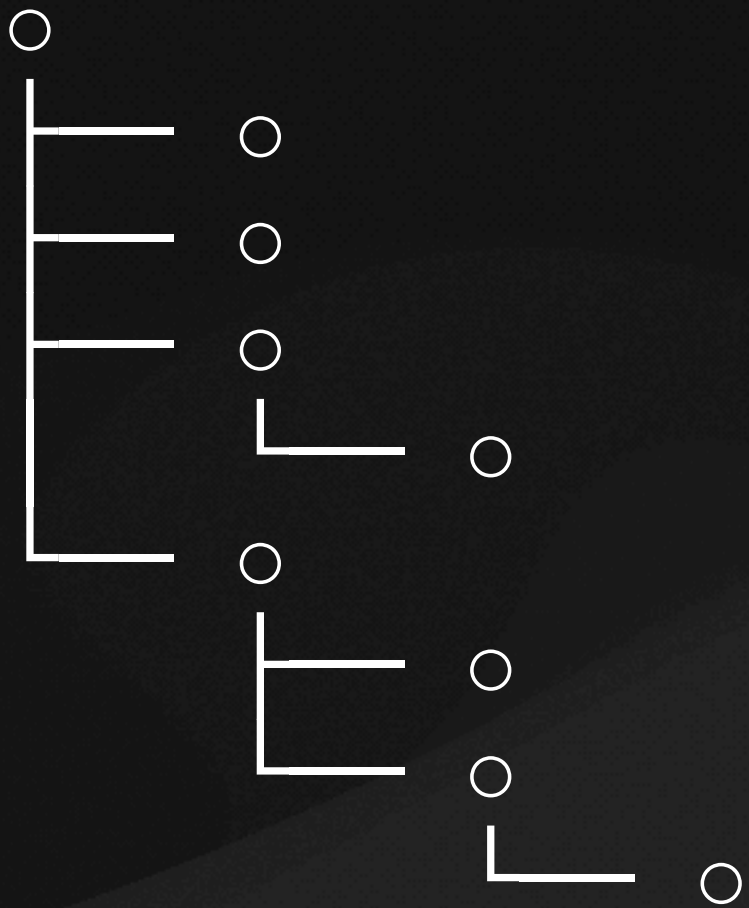
        Atomic μ;
        μ.atoms.resize(std::max(α.atoms.size(), β.atoms.size()));

        for (int element = 0; element < μ.atoms.size(); element++) {
            μ.atoms[element] = (element < α.atoms.size() ? α.atoms[element] : Atomic {{{}}) *
                               (element < β.atoms.size() ? β.atoms[element] : Atomic {{{}});
        }
        return μ;
    }

private:
    std::vector<Atomic> atoms;
};
```



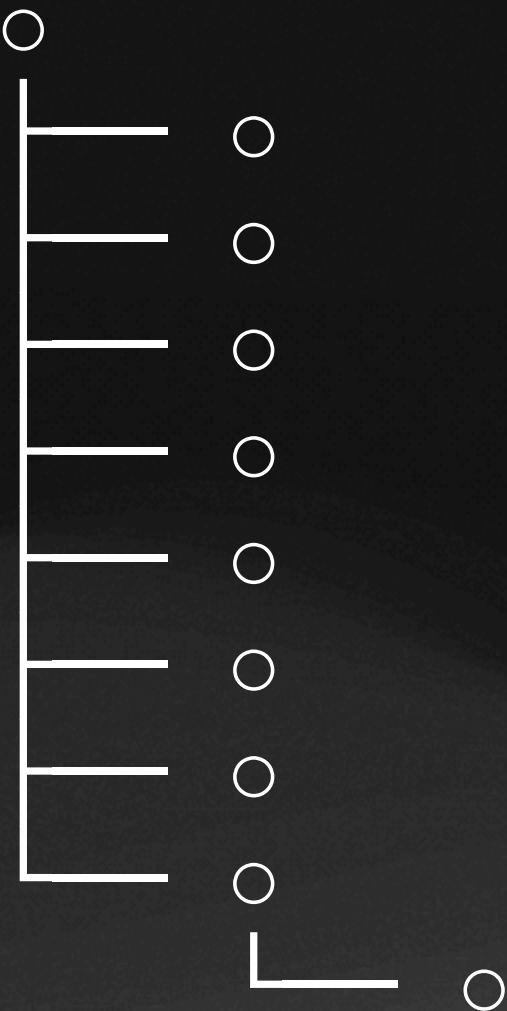
75 :=



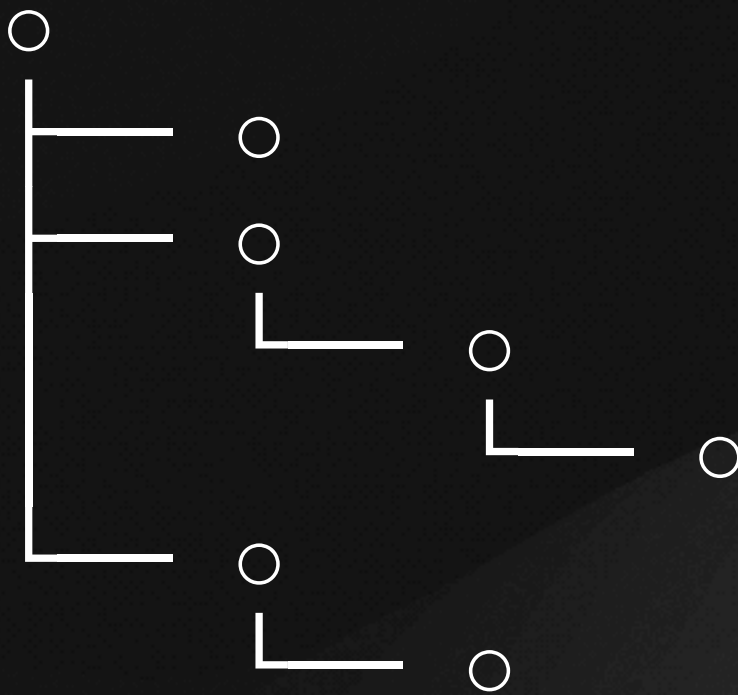
3/2 :=



17 :=



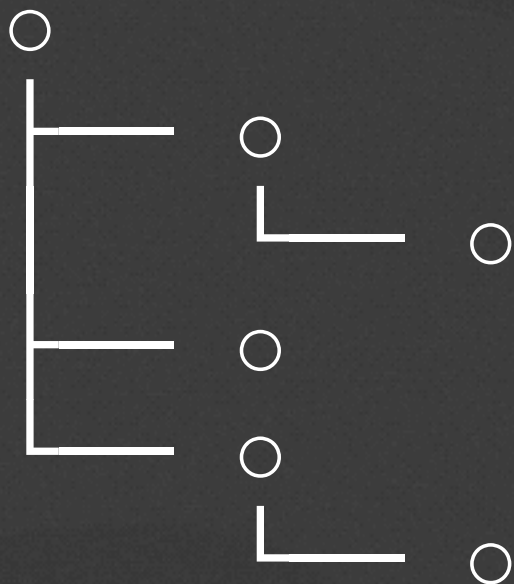
3/2 :=



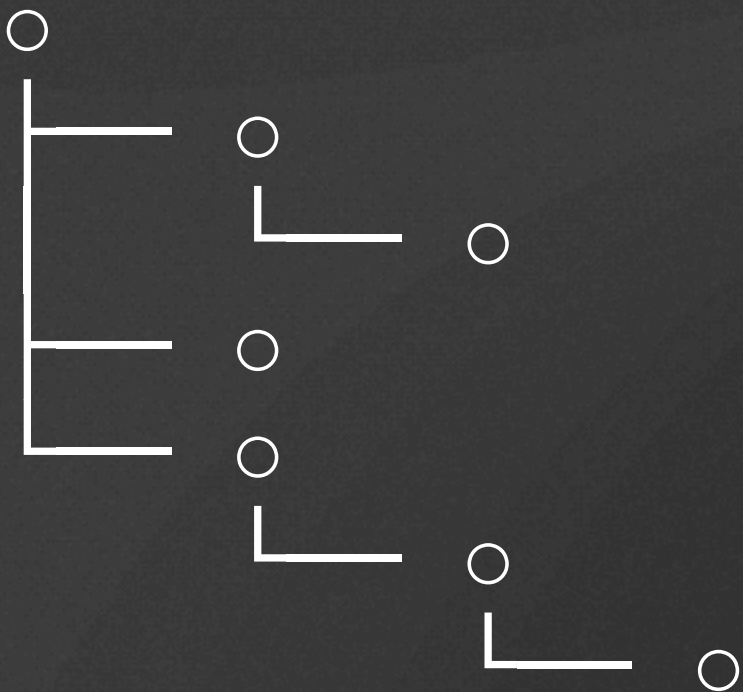
i^4 :=



3i :=



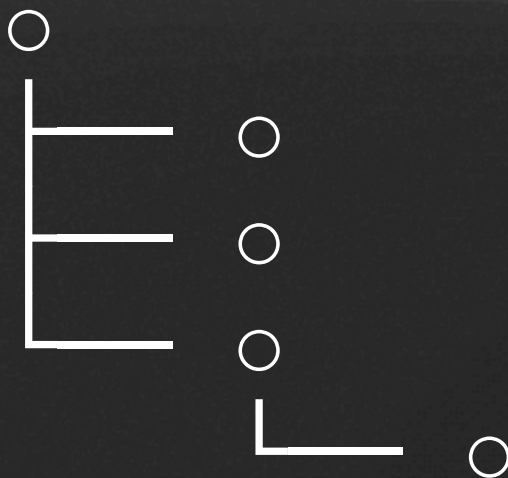
3i\_{2} :=



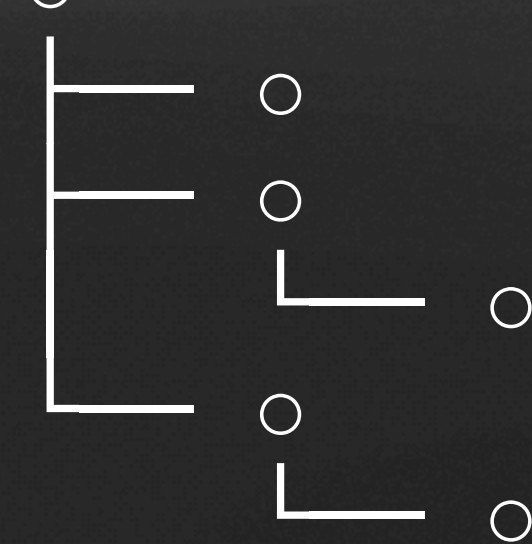
2\*3 :=



\*



:=





How about leveraging temporary defeat?









**We're further than we started!**



The background of the slide is a dark, monochromatic illustration of a landscape. It features several layers of rolling hills and mountains. The hills in the foreground are a dark charcoal grey, while the mountains in the distance are a slightly lighter, dark grey, creating a sense of depth. The sky above is a solid, deep black. The overall mood is mysterious and contemplative.

Where else can we go?



