100

ROBBIE CULKIN
COEN 140

HOMEWORK 1

1/16/18

Q1. $\text{POISSON}(x|\lambda) = e^{-\lambda}\dfrac{\lambda^x}{x!}$ FOR $x_1, x_2, \ldots, x_6 \Rightarrow n=6$

1. $\max L(\lambda) = \prod_{i=1}^{6} e^{-\lambda}\dfrac{\lambda^{x_i}}{x_i}$

$$\log L(\lambda) = \sum_{i=1}^{6}\left(-\lambda + \log(\lambda^{x_i}) - \log(x_i)\right)$$

$$\frac{\partial \log L(\lambda)}{\partial L} = \sum_{i=1}^{6} -1 + \sum_{i=1}^{6}\frac{x_i}{\lambda} = 0$$

$$= -6 + \frac{1}{\lambda}\sum_{i=1}^{6} x_i = 0$$

$$6\lambda = \sum_{i=1}^{6} x_i$$

$$\lambda = \frac{\sum_{i=1}^{6} x_i}{6}$$

* CHECK SECOND ORDER DERIVATIVE! FOR CONCAVITY

2. $\lambda = \dfrac{(2+4+3+0+3+6)}{6} = \dfrac{18}{6} = \boxed{3 \text{ TDS/GAME}}$

Q2. MULTINOMIAL $\left\{\begin{array}{l} n_{CLEAR} = 41 \\ n_{CLOUDY} = 23 \\ n_{RAINY} = 36 \end{array}\right\}$ $N = 100$.

— FOR DERIVATION, WILL GENERALIZE TO $n_{CONDITION}$, n_c FOR SHORT.

SIMILARLY, $p_{CONDITION} \Rightarrow p_c$ "

$$L(p_c) = p_c^{n_c} \cdot \underbrace{(1-p_c)}_{\text{PROB. WEATHER WILL NOT MATCH DESIRED CONDITION}}{}^{N-n_c}$$

$$\log L(p_c) = \log(p_c^{n_c}) + \log\left((1-p_c)^{N-n_c}\right)$$

$$\frac{\partial \log L(p_c)}{\partial p_c} = \frac{n_c}{p_c} - \frac{N-n_c}{1-p_c} = 0$$

NEG B/C p_c IS NEG IN LOG FCN

$$\frac{n_c}{p_c} = \frac{N-n_c}{1-p_c}$$

$$n_c(1-p_c) = (N-n_c)p_c$$

$$n_c - \cancel{n_c p_c} = Np_c - \cancel{n_c p_c}$$

$$n_c = Np_c$$

$$\boxed{p_c = \frac{n_c}{N}}$$

SO: $p_{CLEAR} = \dfrac{n_{CLEAR}}{N} = \dfrac{41}{100} = 0.41$

$p_{CLOUDY} = \dfrac{n_{CLOUDY}}{N} = \dfrac{23}{100} = 0.23$

$p_{RAINY} = \dfrac{n_{RAINY}}{N} \; \dfrac{36}{100} = 0.36$

ALTERNATE SOLUTION ON REVERSE. (LAGRANGE)

ALTERNATIVE SOLUTION TO Q2: LAGRANGE MULTS?

$f(p_i)$: MAX $P_{CLEAR}^{n_{CLEAR}} \cdot P_{CLOUDY}^{n_{CLOUDY}} \cdot P_{RAINY}^{n_{RAINY}}$

$g(p_i)$: s.t. $P_{CLEAR} + P_{CLOUDY} + P_{RAINY} = 1$

$\log f(p_i) = \log(P_{CLEAR}^{n_{CLEAR}}) + \log(P_{CLOUDY}^{n_{CLOUDY}}) + \log(P_{RAIN}^{n_{RAIN}})$

MAX $L = \log(P_{CLEAR}^{n_{CLEAR}}) + \log(P_{CLOUD}^{n_{CLOUDY}}) + \log(P_{RAIN}^{n_{RAIN}}) + \lambda(1 - P_{CLEAR} - P_{CLOUDY} - P_{RAINY})$

$$\frac{\partial \log L}{\partial P_{CLEAR}} = \frac{n_{CLEAR}}{P_{CLEAR}} - \lambda = 0 \qquad P_{CLEAR} = \frac{n_{CLEAR}}{\lambda}$$

$$\frac{\partial \log L}{\partial P_{CLOUDY}} = \frac{n_{CLOUDY}}{P_{CLOUDY}} - \lambda = 0 \qquad P_{CLOUDY} = \frac{n_{CLOUDY}}{\lambda}$$

$$\frac{\partial \log L}{\partial P_{RAIN}} = \frac{n_{RAIN}}{P_{RAIN}} - \lambda = 0 \qquad P_{RAINY} = \frac{n_{RAINY}}{\lambda}$$

$$P_{CLEAR} + P_{CLOUDY} + P_{RAINY} = 1$$

$$\frac{n_{CLEAR}}{\lambda} + \frac{n_{CLOUDY}}{\lambda} + \frac{n_{RAINY}}{\lambda} = 1$$

$$\underbrace{n_{CLEAR} + n_{CLOUDY} + n_{RAINY}}_{N=100} = \lambda$$

SO
$$\boxed{\begin{aligned} P_{CLEAR} &= \frac{n_{CLEAR}}{N} \\ P_{CLOUDY} &= \frac{n_{CLOUDY}}{N} \\ P_{RAINY} &= \frac{n_{RAINY}}{N} \end{aligned}} \quad \begin{aligned} &= 0.41 \\ &= 0.23 \\ &= 0.36 \end{aligned}$$

*CHECK SECOND ORDER DERIVATIVE FOR CONCAVITY