Development Proposal:

Paper Folding as Problem Solving

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1 INTRODUCTION

Origami is the art of folding paper into 3D models without cuts or gluing, and for readers familiar with the practice, it likely has a certain natural draw. You see a square piece of paper, a magician figuratively passes their wand over it, and suddenly it's a dove, a boat, a heart - before your very eyes it has transformed. This is certainly a popular perspective on the value of the art, perhaps because it fits the historical context: although it can be traced to paper folding practices for various ceremonial, religious, and artistic uses dating back to the Muromachi Period in Japan (1333-1573) and the 16th century in Europe (Hatori, 2011), modern origami instruction was defined in part by the work of magician Robert Harbin. In 1956 he published his first book of instructions, titled *Paper Magic: The art of paper folding,* which was quite popular and led to him teaming up with enthusiasts Samuel Randlett and Lillian Oppenheimer in an effort to standardize the young field. Together they expanded and popularized worldwide the diagraming system developed a few years earlier by origami pioneer Akira Yoshizawa, which provided a consistent list of arrows, lines, and other shapes for easy and clear communication of individual steps (Hull, 2005).

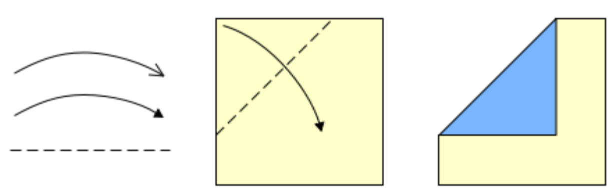

Figure 1 - Standardized symbol example for a "valley" fold (Grigory, 2008)

The resulting system (called the Yoshizawa-Randlett system, presumably to Harbin & Oppenheimer's chagrin) allowed for rapid growth in the development of and demand for "mass-marketed origami instruction books" (Hull, 2005). On a most basic level, this defined origami instruction as a series of discrete categorizable steps attached to periodic written captions, and that design has stuck. Many of the specifics have, too; the closest thing to a standard for modern origami instruction, the work *Origami Diagramming Conventions* by researcher and origami celebrity Robert J. Lang, still bases all its advice on this notation, while summing up the reasoning well: "[Yoshizawa-Randlett] is clear, it has stood the test of time, and therefore it should be the diagrammer's first source" (Lang, 2011).

From this point, other authors have continued the account of the history of origami by describing the progressive development of new, more complex models, such as paper grasshoppers, spiders, and even people (Hull, 2005). This narrative choice meshes well with the magic-like view of origami by asserting that the basic value proposition of the practice is to create a (preferably impressive) artwork, and this is exactly why this choice is dismissed. Instead, we pause here, and examine the effects of the shocking idea implied above: origami instruction is still built around a system that hasn't been fundamentally modified in 60 years.

This paper is an honest attempt to reexamine origami instructional methods in a modern technical, economic, & social context, and propose some changes. To do this, the analysis is based on the opposing assertion that origami's modern value proposition is not in the *final product*, but rather in the *activity itself*. From this starting point, the priorities for instructional content change, and a new medium begins to take conceptual shape.

2 RELATED WORK

2.1 Paper Folding

2.1.1 In Person Instruction

The first medium, in person instruction, is the source of almost all academic work on the practice in general, though mostly in a scholastic context using 2D diagrams as support, not in hobbyist classes taught by an origami expert. Still, there's quite a large body of experimental work on the use of paper folding in the classroom to improve results on spatial and geometric reasoning tests (Yuzawa & Bart, 2002; Boakes, 2009; Taylor & Hutton, 2013; Cakmak et al., 2014; Arici & Aslan-Tutak, 2015; Akayuure et al., 2016), and multiple examples of lesson plans for teaching geometry and mathematics using origami (Chen, 2006; Hull, 2013; Higginson & Colgan, 2001; Golan & Jackson, 2009). Beyond these recorded benefits on achievement tests, there's also evidence on the intuitive notion that origami is enjoyable and engaging for learners. One study recorded that 32 out of 38 tested grade school students (84%) described the classes as "entertaining and enjoyable" (Cakmak et al., 2014), and Hull (2013) notes that origami is a "logical choice" for "discovery-based" teaching methods, a constructivist approach centered on student agency that is "designed to engage students in inquiry" (Hammer, 1997).

This depth of academic backing is promising yet frustrating, since this particular medium likely makes up a very small percentage of all origami learning experiences. For example, the largest such program found in the literature is an ongoing Israeli program used in 70 schools to teach grade school "curriculum geometry" called *Origametria* (Golan & Jackson, 2009), while just one of multiple large origami YouTube channels, *Origami with Jo Nakashima*, has over 349 million views. This drives home the intuitive modern truth that online instruction is rapidly gaining ground over in person methods where the fit is natural, and this non-online aspect is why in person instruction will not be directly contrasted with the proposed novel medium. Still, these results are important, since they provide reassurance that this content is worth delivering in the first place for reasons intrinsic to the task itself.

2.1.2 Diagrams

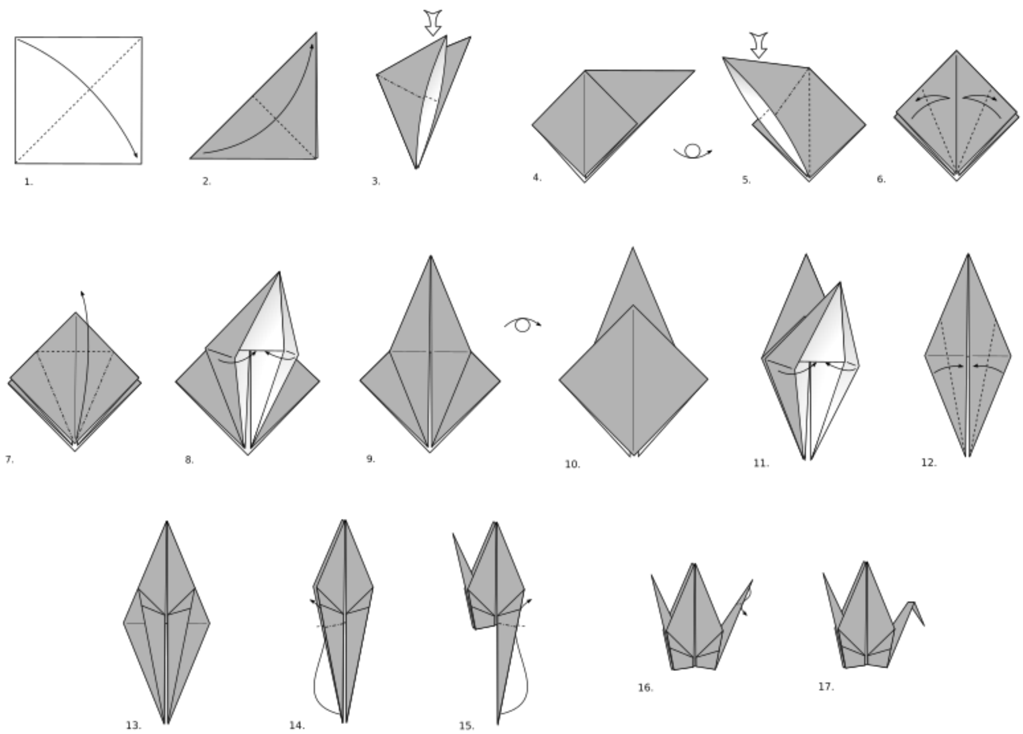

Figure 2 **-** Set of captionless diagrams for an origami crane (Hudson, 2011)

As discussed in the introduction, modern Origami was defined by this medium, and it's still the primary form of instructional materials available for many copyrighted models. It's hard to say which medium is more popular between this and videos today, especially since 2D diagrams are published in a multitude of venues from printed books to magazines to web pages, but it's objectively clear that these two mediums make up the vast majority of modern origami education.

Much has been written on detailed techniques for drawing 2D diagrams to best represent a 3D object, but since the proposed medium doesn't face that challenge (see 3.2: Instructional Methods), those are not explored here. Instead, the focus is put on the underlying principles at play, and the goals they imply. Lang's *Origami Diagramming Conventions* (2011) explicitly lays out 10 such principles, which are discussed in Appendix 5.2, but for brevity's sake attention is focused on the following quote, added by Lang in an August, 2000 epilogue to the aforementioned work:

> "As the juggernaut of computer technology continues to roll… there will undoubtedly be new means to convey origami instruction... Whatever the medium, however, the same basic principles of diagram apply: keep it simple, keep it clear, and keep it consistent!" (Lang, 2011)

These final three concepts are the most important part, and will be used in the manner clearly intended by Lang: during the evaluation of the proposed medium.

2.1.3 Videos

Figure 3 **-** Screenshot of the third most popular origami video on YouTube (Nakashima, 2013)

The video medium is notable in that it is clearly an exception to the inflammatory remark in the introduction on origami instruction being 60 years old. The basic idea at play here is that someone will film themselves folding an origami model, then that footage - either alone or attached to narration/captions - serves as instructional content. Video takes advantage of modern social, economic, and technological trends by being widely shareable and cheap to produce, with some even taken on smartphones. Also, it is perhaps a more natural fit for origami than 2D diagrams, since it is a) temporally continuous so the results of every procedure are always shown, b) is in 3D, which allows the learner to see many different angles of a model at any given stage, and c) maps better to what the learner is seeing in real life while following along. Content examples vary dramatically in quality and style, but Figure 3 depicts a particularly high quality video by Jo Nakashima, in which it seems clear that Lang's "basic principles of diagram" are all satisfied comprehensively. For example, the video is cropped so that only the model and the instructor's hands are visible with no overlays or sounds (simplicity), time is taken to show different angles of the results of complex folds, the hands rarely obscure the model, and the camera is aimed at a soft-yet-contrasting backdrop (clarity), and almost every step has a Yoshizawa-Randlett diagram shown alongside it and is presented in similar ways across the channel (consistency).

Why then, if this medium can result in content that fits the stated goals well, should we seek out another? Besides the easy dismissal that many origami videos do not perform as well on these metrics (especially those made by amateurs), all videos perform badly on the important metrics of spatial and temporal *user control*. The spatial limitation is clear: the content creator chooses when to rotate the model and to what places, and since they can't completely rotate the model after every step, this means users might want to see an angle or portion that is impossible to ever see in the video. The temporal controls limitation is less clear, since to an outsider, it seems that existing pacing controls for a site like YouTube (namely jump back/forward 15s, pause/play, and playback speed controls) would perform fine for this use case. However, one can intuitively deduce at least one issue: for a medium depicting a series of steps of various temporal durations, any subset of which might be repeated over-and-over by the learner, tying the step-selection controls to arbitrary values in seconds can lead to over/undershooting of target timestamps. This, when repeated perhaps dozens of times for difficult steps, clearly contains the potential for learner frustration. For a more basic argument of why origami might be so complex that videos can't properly convey it, see 2.2: Problem Solving.

2.1.4 *Crease Patterns*

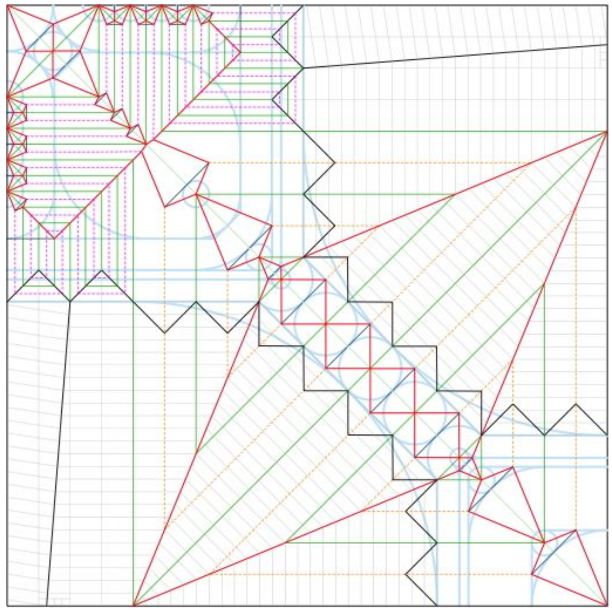

Figure 4 - Crease pattern for a 3D model titled *Turkey Vulture "Richard"* (Lang, 2009)

Instead of providing step-by-step instructions, these describe the final model in one diagram, where different line colors indicate the type of fold to perform along that line. If you folded a piece of paper into a model following instructions in another medium, then unfolded it to be completely flat, the creases still visible in the paper would (with some complex exceptions) match the crease pattern diagram for that model. Lang (2015) gives an informal list of benefits of crease diagrams over sequential instructions, which will be helpful when evaluating the proposed medium: a) they're less time-consuming to create (partially because they're often produced incidentally as part of the design process), b) they can give folders useful insight to the overall model design, and c) some very difficult folds can only be represented in this format (i.e. there is no "intermediary state").

Beyond that, the details of crease patterns aren't too relevant here, since they're intended only for expert folders. For non-experts they are too complex, and "[sequential] origami diagrams are still generally required" (Tsuruta et al., 2011).

2.2 Problem Solving

The critical reader may ask, reasonably, what is even being taught by all these mediums? The term "instruction" is typically associated with how-tos, manuals, furniture assembly booklets, and other simple materials that are advertised to be follow-able by anyone, regardless of prior experience. If this were the case, then origami instruction would fundamentally be a process of description, not education; unsurprisingly, that view is not taken here. This opposition is based in Tenbrink and Taylor's *Conceptual Transformation and Cognitive Processes in Origami Paper Folding* (2015), which draws a line between instructions that "leave few or no problems to be solved" and those that do. They undertake to show that origami falls in the second category with a think aloud study recording 24 undergraduate participant's efforts to follow instructions for folding an origami tulip, categorizing user utterances along a few axes. The results are complex, but the final answer to this question is stated clearly:

> "Apparently, specifying the main solution steps in a complete set of step-by-step instructions does not eliminate the need for problem solving. Instead, the main solution steps provide a coarse level of problem solving, but leave room for more fine-grained challenges. Our verbal data highlight the cognitive complexity and creativity involved in this process, going well beyond the immediate and automatic interpretation of clearly laid out instructions." (Tenbrink & Taylor, 2015)

Their analysis is explicitly based on a very influential work by Herbert Simon and Allen Newell on human problem solving (1971), which will here be used to provide a path from this basic assertion - that origami involves complex and creative problem solving - to pedagogical prescriptions. The theory is that human problem solving involves examining the "task environment" and constructing a corresponding mental "problem space", which can be thought of as connected "nodes'', each of which contains information from the current state of the environment (i.e. shape of a paper model) and actively recalled concepts from memory (i.e. "I recognise this fold!"). They argue that humans typically only cognitively "occupy" one of these nodes at a time, then iteratively loop through an algorithm to look at their current node and apply some "operator" to change to another node, such as by folding and unfolding a paper flap, or examining the corresponding diagram again. In other words, they summarize the theory as such: "The problem solver's search for a solution is an odyssey through the problem space, from one knowledge state to another, until… [they] know the answer." Tenbrink and Taylor seem to think this applies to Origami, and this supplies direct support to several aspects of the proposed work.

3 PROPOSED WORK

3.1 Audience

The proposed work is aimed at those that practice Origami as a self-motivated "arts and craft" activity, and not an educational tool in the classroom. This group is composed of two non-exclusive subgroups, contrasted by their differing motivations: the *casual* audience, who seek to complete a particular model as accurately and quickly as possible, and the *hobbyist* audience, who seek to learn to fold familiar models from memory and attempt progressively more "difficult" models over time.

3.2 Thesis

A new web application for teaching origami should be developed that uses animated 3D simulations with temporal & spatial controls, hierarchical annotations of folds, and hobbyist augmentations to improve learner performance on audience specific self-assessments.

3.3 Animated 3D Simulations

This is the part of the proposed work that can be described as definitely part of the "proposed medium", with the others being categorized more as methods that augment this medium. In light of the classification of origami as a problem solving process, a goal of any medium should be to construct a "task environment" that is built explicitly to aid users in problem space construction and traversal. In other words, a key task is to convey information about what the whole model should look like at each step in the instructions. Once it's put in these words, it seems uncontroversial that an animated 3D simulation would be an intuitively good fit for these goals, since it allows the users to "directly" see a 3D model that maps to the actual paper they have in front of them. This seems to be backed up by the literature as well, with support for the assertion that materials that incorporated 3D animations increased student's spatial reasoning skills (Hoyek et al., 2014; Cohen & Hegarty, 2008).

To discuss this medium comparatively with a) sequential 2D diagrams and b) videos, it will be evaluated in terms of Lang's aforementioned three principles for origami instructions in any medium: simplicity, clarity, and consistency (see 2.1.2: Diagrams).

Simplicity is the easiest to see advantages for, at least for the learner; 3D animations do not require special symbols or terminology like 2D diagrams do, and since the animation is computer generated, none of the incidental distractions present in videos (such as busy backgrounds, hands, lighting problems, and miscellaneous objects in frame) are possible. Also, the simplicity-damaging problem caused by action-intensive temporal video controls is avoided (see 2.1.3: Videos).

Animated 3D simulations also lead to increased clarity, since they don't need to imply the shape of a 3D object using 2D drawings, and the lack of the aforementioned distractions present in some videos also serves to avoid some common impediments to clarity, such as the folder's hands obscuring the model.

Finally, in terms of consistency, this medium improves on both others by being far easier to modify and perfect later. This fact is derived from the nature of the instructions, which will be a set of data structures that define the behavior of a hierarchical set of axial/translational springs over time (see 3.8: Technical Details). If new conventions are defined for animation style, notation, terminology, colors, or anything else, new instructions for old data could easily be made by just modifying the interface component, and reusing the same structures. When compared to redrawing diagrams or reshooting videos, not to mention potentially reprinting materials, this is clearly an attractive proposition.

3.4 Temporal & Spatial Controls

This aspect is central to the value proposition of the medium, and what's mentioned above likely wouldn't work well without it. Generally, cognitive load theorists Moreno and Mayer (2007) tell us that making multimodal instruction interactive can lead to greatly increased outcomes when used properly, and these two controls are the best way to do that in this context. These concepts don't really apply to 2D diagrams by definition, so comparison here will solely be between these capabilities and those of video instructions.

Temporal controls, which in this case mean controls to easily pick which instructional steps to play animations for and when to do it, have been shown to increase learner outcomes on experimental tests (Akpinar, 2014; Bétrancourt et al., 2003; Hasler et al., 2007; Izmirli & Kurt, 2016). They also tie directly into the criticism already leveled at the video medium, namely that jumping around the video in arbitrary seconds-based intervals doesn't map well to a task where users want to rewatch arbitrary sets of steps. Tying these controls to those steps directly allows the user to accomplish the same task with less interaction.

Spatial controls are very important, and represent something that animation can probably do better than any other medium. In this application the user should be allowed to rotate the model and to zoom in and out during any step, but not allowed to "pan" or transform the origin coordinates of the model in any way. This is both to generally improve simplicity, and to cater to the technical limitations of 3D interaction on a multi-touch smart phone. Such controls also have strong experimental backing in a study conducted by Ha and Fang (2018), which showed that allowing students to freely rotate simulated 3D models of real-life objects lead to improved outcomes on spatial skills tests. There are some key differences between this application and the tested one, namely that Ha and Fang's application used orientation sensors hidden in real-life objects as the spatial controls rather than mouse-based interaction, but the importance of this shockingly specific experimental backing is hard to overstate in justifying these controls.

3.5 Hierarchical Annotations of Folds

This method is more incidental; although there's evidence to indicate that it might lead to improved learning outcomes, it's fairly separable from the others. The motivation here is derived not from comparison with other mediums, but rather the underlying ideas in 2.2: Problem Solving, specifically Simon and Newell's "problem space" theory (1971). Work within this field has shown that humans often organize sequential steps into larger super-steps based on spatial (Klippel et al., 2002) and algorithmic (Esponda-Argüero, 2008) relationships, so if we're trying to structure the task environment in order to ease the processes of problem space creation and translation, it follows that we should present the steps in a similar hierarchical structure. Additionally, other research in a subfield called "concept learning" describes how humans tend to categorize information in general (Rosch et al., 1976; Medin, 1989; Wisniewski & Medin, 1994), the most relevant of which discusses the characteristics of "ad-hoc" categories, which "violate the correlational structure of the environment, and are not well established in memory," which leads to them being frequently re-constructed

(Barsalou, 1983). All of this is applied to origami in Tenbrink and Taylor (2015), who's whole approach is centered on "additional ideas about a step" that, when verbalized, indicate that learners are engaged in understanding and categorizing this step using metrics not explicitly shown in typical sequential diagrams. Some examples of such ideas are "across-step" comparisons and relations, describing the step in terms of paper orientation or alignment, and assigning teleological roles to individual steps.

3.6 Hobbyist Augmentations

Some special care should be taken to cater to the particular needs of the second audience subgroup, origami hobbyists. On a fundamental level these learners are engaged in self-guided "generative learning" that builds understanding through hands-on interaction and repetition (Wittrock, 1992), which in this context has been described as "skilled practice" (Torrey et al., 2009). This is a highly time-constrained activity in today's online world where there's too much information to go through in full, and Torrey et al. go on to detail some expectations of this audience for online applications: representation of information via "images, video, and diagrams" for ease of skimming, focus on a smooth "keyword search" functionality, and filter-able information based on things like skill level or time required for the task. At the moment, these three goals are the only ones to fall specifically within this point, but the hobbyist-specific self-assessments detailed below are also clearly expanded in order to better serve this audience.

3.7 Audience Specific Self-Assessments

The casual audience is generally concerned about time by the very definition of their motivations, so any self-assessments that involve user interaction are not considered for them. Instead, functionality will be added to allow for a quick comparison between their final model and some example pictures of what it should look like. This is a natural choice already adopted by most good origami instructional materials, and is also the first of three assessments for origami folding success devised by Tenbrink and Taylor (2015), the other two being 1) time required to complete the instructions, and 2) the implied-but-important final assessment, which is whether the learner is able to complete the task at all without personal guidance. To facilitate timing assessments, the option will be given to display a pausable timer that tracks the learner's folding speed. Binary completion assessments could easily be performed automatically (i.e. log if the learner closes the page half-way through), but won't be acknowledged explicitly by the interface in order to avoid a pejorative tone.

For the hobbyist audiences, assessment comes in two forms corresponding to the two goals of this audience, which in turn correspond to the well-supported scholarly distinction between retention learning and transfer learning (Perkins & Salomon, 1992; Bétrancourt et al., 2003; Piksööt & Sarapuu, 2014). First, to help learners remember how to fold familiar models (retention learning), functionality should be added to the app to support optional self-assessments given at spaced intervals (as inspired by Kang, 2016). The exact intervals have yet to be determined, but will take one of two forms: either they will be simple arbitrary ranges based on a rough power law, or if time permits, they will be based on the MEMORIZE stochastic optimization algorithm (Tabibian et al., 2019) because of its positive experimental backing and the availability of its python source code.

Second, to help learners attempt more difficult models (transfer learning), the above two assessments (similarity to example, and time taken to fold) should be applied over long time ranges and tracked in a user profile, which includes a list of which models they have successfully completed so far. Similarity would be entered on a 5-Point Likert Scale (Joshi et al., 2015), and timing would be automatically recorded for these users.

3.8 Technical Details

The proposed application is a React.js based CRA single page web application that uses react-three-fiber (a 3D WebGL canvas renderer for Three.js) to build animated 3D models. This is very new technology, with this specific library only coming out in 2018, but it's a good fit for the task at hand primarily because it allows us to deliver high-quality 3D simulations in an accessible manner (i.e. in-browser at low computational cost). The project will also use Material-UI for layout, react-spring for animated transitions, and (likely) Auth0 for authentication and basic user management.

The proposed task is apparently novel, so it necessarily doesn't contain too many confidently asserted technical details. A basic sketch of the underlying engine is clear, however:

- A basic sheet will be displayed, where the shape of the sheet is defined by the position of the vertices and the ordering that connects them sequentially.
- A "foldState" structure will be defined, which contains a list of edges between vertices, each of which is tied to a rotational value. This structure can be translated into the vertex positions needed for the above step.
- A basic "step" data structure will be defined, which contains both a) breakpoints in existing edges that create new vertices, and b) the foldState at this step, either for the whole model or as a diff from the previous foldState
- An "instructions" data structure will be defined that holds a series of these steps, within a hierarchical labelled structure (See 3.5: Hiercharchial Annotations of Folds).
- A system will be implemented to step through, undo, and redo instructions dynamically, preferably with support for animated transitions.

It should also here be mentioned that there already exists a research project on the use of Three.js to display origami by Ghassaei et al. (2018), for which there is source code available, and a very impressive demo. This was certainly an exciting find, and will no doubt be used for comparison and planning purposes, but it's likely that no code can actually be adapted; not only is this code in native Three.js rather than react-three-fiber (a non-trivial distinction), the whole thing aims to simulate the folding of an origami model from crease patterns, which presents entirely different challenges than our task (folding a model from a set of steps) does.

4 REFERENCES

1. Akayuure, P., Asiedu-Addo, S. K., & Alebna, V. (2016). Investigating the effect of origami instruction on preservice teachers' spatial ability and geometric knowledge for teaching. *International Journal of Education in Mathematics, Science and Technology*, *4*(3), 198-209.

2. Akpinar, E. (2014). The use of interactive computer animations based on POE as a presentation tool in primary science teaching. *Journal of Science Education and Technology*, *23*(4), 527-537.

3. Arici, S., & Aslan-Tutak, F. (2015). The effect of origami-based instruction on spatial visualization, geometry achievement, and geometric reasoning. *International Journal of Science and Mathematics Education*, *13*(1), 179-200.

4. Bétrancourt, M., Dillenbourg, P., & Montarnal, C. (2003). Computer technologies in powerful learning environments: the case of using animated and interactive graphics for teaching financial concepts. *Unravelling basic components and dimensions of powerful learning environments*, 143-157.

5. Barsalou, L. W. (1983). Ad hoc categories. *Memory & cognition*, *11*(3), 211-227.

6. Boakes, N. J. (2009). Origami instruction in the middle school mathematics classroom: Its impact on spatial visualization and geometry knowledge of students. *RMLE Online*, *32*(7), 1-12.

7. Cakmak, S., Isiksal, M., & Koc, Y. (2014). Investigating effect of origami-based instruction on elementary students' spatial skills and perceptions. *The Journal of Educational Research*, *107*(1), 59-68.

8. Chen, K. (2006). Math in motion: Origami math for students who are deaf and hard of hearing. *Journal of deaf studies and deaf education*, *11*(2), 262-266.

9. Cohen, C. A., & Hegarty, M. (2008, June). Spatial visualization training using interactive animation. In *Conference on Research and Training in Spatial Intelligence. Sponsored by National Science Foundation, Evanston, IL*.

10. Esponda-Argüero, M. (2008). Algorithmic animation in education—review of academic experience. *Journal of Educational Computing Research*, *39*(1), 1-15.

11. Ghassaei, A., Demaine, E. D., & Gershenfeld, N. (2018). Fast, interactive origami simulation using GPU computation. *Origami*, *7*, 1151-1166.

12. Golan, M., & Jackson, P. (2009). Origametria: A program to teach geometry and to develop learning skills using the art of origami. *Origami*, *4*, 459-469.

13. Grigory, S. (2008). *Origami symbol valley fold* [Diagram]. Wikipedia. https://commons.wikimedia.org/wiki/File:Origami_symbol_valley_fold.svg

14. Ha, O., & Fang, N. (2018). Interactive virtual and physical manipulatives for improving students' spatial skills. *Journal of Educational Computing Research*, *55*(8), 1088-1110.

15. Hammer, D. (1997). Discovery learning and discovery teaching. *Cognition and instruction*, *15*(4), 485-529.

16. Hasler, B. S., Kersten, B., & Sweller, J. (2007). Learner control, cognitive load and instructional animation. *Applied Cognitive Psychology: The Official Journal of the Society for Applied Research in Memory and Cognition*, *21*(6), 713-729.

17. Hatori, K. (2011, June). History of Origami in the East and the West before Interfusion. In *Origami 5: Fifth International Meeting of Origami Science, Mathematics and Education* (pp. 1-13).

18. Higginson, W., & Colgan, L. (2001). Algebraic thinking through origami. *Mathematics Teaching in the Middle School*, *6*(6), 343-349.

19. Hoyek, N., Collet, C., Di Rienzo, F., De Almeida, M., & Guillot, A. (2014). Effectiveness of three-dimensional digital animation in teaching human anatomy in an authentic classroom context. *Anatomical sciences education*, *7*(6), 430-437.

20. Hudson, A. (2011). *Tsuru Wiki* [Diagram]. Wikipedia. https://commons.wikimedia.org/wiki/File:Tsuru_wiki.svg

21. Hull, T. (2005). Origami design secrets: mathematical methods for an ancient art. *The Mathematical Intelligencer*, *27*(2), 92-95.

22. Hull, T. (2012). *Project origami: activities for exploring mathematics*. CRC Press.

23. Izmirli, S., & Kurt, A. A. (2016). Effects of modality and pace on achievement, mental effort, and positive affect in multimedia learning environments. *Journal of educational computing research*, *54*(3), 299-325.

24. Joshi, A., Kale, S., Chandel, S., & Pal, D. K. (2015). Likert scale: Explored and explained. *Current Journal of Applied Science and Technology*, 396-403.

25. Kang, S. H. (2016). Spaced repetition promotes efficient and effective learning: Policy implications for instruction. *Policy Insights from the Behavioral and Brain Sciences*, *3*(1), 12-19.

26. Klippel, A., Tappe, H., & Habel, C. (2002, May). Pictorial representations of routes: Chunking route segments during comprehension. In *International Conference on Spatial Cognition* (pp. 11-33). Springer, Berlin, Heidelberg.

27. Lang, R. J. (2009). *Turkey Vulture "Richard"* [Illustration]. Robert J. Lang Origami. https://langorigami.com/crease-pattern/turkey-vulture-richard/

28. Lang, R. J. (2011). Origami diagramming conventions. *article available at https://www.langorigami.com/info/diagramming_series.pdf. (last access: June 9th, 2010).*

29. Lang, R. J. (2015). *Crease Patterns for Folders*. Robert J. Lang Origami. https://langorigami.com/article/crease-patterns-for-folders/

30. Medin, D. L. (1989). Concepts and conceptual structure. *American psychologist*, *44*(12), 1469.

31. Moreno, R., & Mayer, R. (2007). Interactive multimodal learning environments. *Educational psychology review*, *19*(3), 309-326.

32. *Origami Rose (Jo Nakashima)*. (2013, February 14). [Video]. YouTube. https://www.youtube.com/watch?v=wSesWQ-51ZM

33. Nielsen, J. (2005). Ten usability heuristics.

34. Norman, D. (2013). *The design of everyday things: Revised and expanded edition*. Basic books.

35. Perkins, D. N., & Salomon, G. (1992). Transfer of learning. *International encyclopedia of education*, *2*, 6452-6457.

36. Piksööt, J., & Sarapuu, T. (2014). Supporting Students' Knowledge Transfer in Modeling Activities. *Journal of Educational Computing Research*, *50*(2), 213-229.

37. Rosch, E., Mervis, C. B., Gray, W. D., Johnson, D. M., & Boyes-Braem, P. (1976). Basic objects in natural categories. *Cognitive psychology*, *8*(3), 382-439.

38. Simon, H. A., & Newell, A. (1971). Human problem solving: The state of the theory in 1970. *American Psychologist*, *26*(2), 145.

39. Tabibian, B., Upadhyay, U., De, A., Zarezade, A., Schölkopf, B., & Gomez-Rodriguez, M. (2019). Enhancing human learning via spaced repetition optimization. *Proceedings of the National Academy of Sciences*, *116*(10), 3988-3993.

40. Taylor, H. A., & Hutton, A. (2013). Think3d!: Training spatial thinking fundamental to STEM education. *Cognition and Instruction*, *31*(4), 434-455.

41. Tenbrink, T., & Taylor, H. A. (2015). Conceptual transformation and cognitive processes in Origami paper folding. *The Journal of Problem Solving*, *8*(1), 1.

42. Torrey, C., Churchill, E. F., & McDonald, D. W. (2009, April). Learning how: the search for craft knowledge on the internet. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems* (pp. 1371-1380).

43. Tsuruta, N., Mitani, J., Kanamori, Y., & Fukui, Y. (2011). A CAD system for diagramming origami with reduction of folding processes. *LRJ, editor, Origami*, 335-345.

44. Wisniewski, E. J., & Medin, D. L. (1994). On the interaction of theory and data in concept learning. *Cognitive Science*, *18*(2), 221-281.

45. Wittrock, M. C. (1992). Generative learning processes of the brain. *Educational Psychologist*, *27*(4), 531-541.

46. Yuzawa, M., & Bart, W. M. (2002). Young children's learning of size comparison strategies: Effect of origami exercises. *The Journal of genetic psychology*, *163*(4), 459-478.