

# THE DYNKIN DIAGRAMS PACKAGE

BEN MCKAY

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## 1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)

```
\usepackage{dynkin-diagrams}
```

Invoke it

The Dynkin diagram of  $(B_3)$  is  $\text{dynkin}\{B\}\{3\}$ .

The Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet \rightarrow \bullet$ .

Use the long form inside a `tikz` statement ...

```
\tikz \dynkin{B}{3};
```



...or a TikZ environment

```
\begin{tikzpicture}
\dynkin{B}{3}
\end{tikzpicture}
```



Indefinite rank Dynkin diagrams

```
\dynkin{B}{}
```

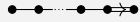
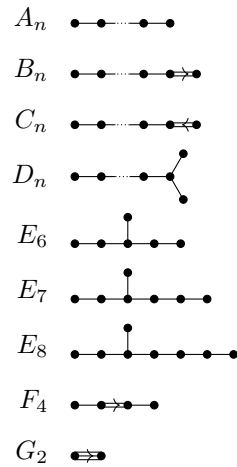


Table 1: The Dynkin diagrams of the reduced simple root systems  
[2] pp. 265–290, plates I–IX



## 2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,edgeLength=.5cm,foldradius=.5cm}
```

...or pass to the package

```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  mark=o,
  radius=.06cm]
{dynkin-diagrams}
```

### 3. COXETER DIAGRAMS

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```

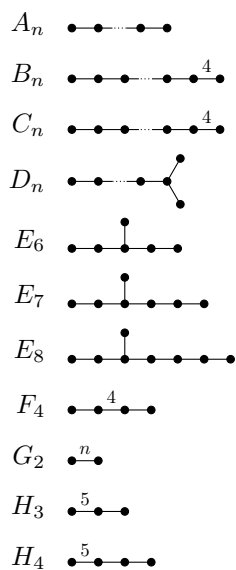


gonality option for  $G_2$  and  $I_n$  Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \
\ (I_n=\dynkin[Coxeter,gonality=n]{I}{n}\)
```

$G_2 = \bullet \overset{n}{\bullet} \bullet$ ,  $I_n = \bullet \overset{n}{\bullet}$

Table 2: The Coxeter diagrams of the simple reflection groups



continued below ...

Table 2: (continued)

$$I_n \quad \bullet \xrightarrow{n} \bullet$$

## 4. SATAKE DIAGRAMS

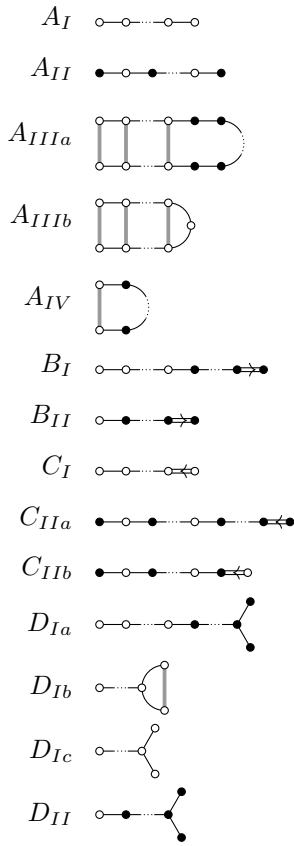
Satake diagrams use the standard name instead of a rank

$\backslash(A_{IIIb})=\backslash\text{dynkin}\{A\}\{IIIb\}\backslash$

$$A_{IIIb} = \begin{array}{c} \circ \text{---} \circ \text{---} \circ \\ | \quad | \quad | \\ \circ \text{---} \circ \text{---} \circ \end{array}$$

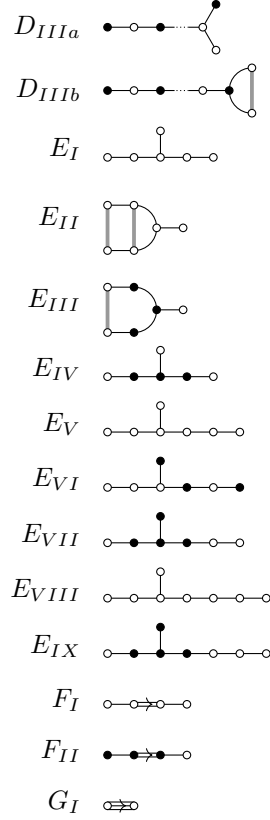
We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [9] p. 532–534



continued below ...

Table 3: (continued)

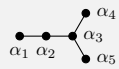


## 5. LABELS FOR THE ROOTS

Label the roots by root number

`\dynkin[label]{B}{3}`

Make a macro to assign labels to roots

`\dynkin[label,labelMacro/.code={\alpha_{#1}}]{D}{5}`

## Make a label for a single root

```
\begin{tikzpicture}
\dynkin{B}{3}
\dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



## Use a text style

```
\begin{tikzpicture}
\dynkin[text/.style={scale=1.2}]{B}{3};
\dynkinLabelRoot{2}{\alpha_2}
\end{tikzpicture}
```



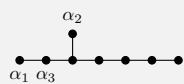
## Access root labels via TikZ

```
\begin{tikzpicture}
\dynkin{B}{3};
\node[below] at (root 2) {\alpha_2};
\end{tikzpicture}
```



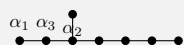
## The labels have default locations

```
\begin{tikzpicture}
\dynkin{E}{8};
\dynkinLabelRoot{1}{\alpha_1}
\dynkinLabelRoot{2}{\alpha_2}
\dynkinLabelRoot{3}{\alpha_3}
\end{tikzpicture}
```



The starred form flips labels to alternate locations

```
\begin{tikzpicture}
\dynkin{E}{8};
\dynkinLabelRoot*{1}{\alpha_1}
\dynkinLabelRoot*{2}{\alpha_2}
\dynkinLabelRoot*{3}{\alpha_3}
\end{tikzpicture}
```



## 6. STYLE

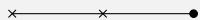
Colours

```
\dynkin[edge/.style={blue!50,thick},*/.style=blue!50!red]{F}{4}
```



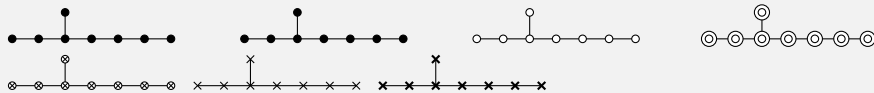
Edge lengths

```
\dynkin[edgeLength=1.2,parabolic=3]{A}{3}
```



Root marks

```
\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=0]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}
```



At the moment, you can only use:

- \* solid dot
- o hollow circle
- 0 double hollow circle
- t tensor root
- x crossed root
- X thickly crossed root

| Mark styles   |
|---|
| <code>\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}</code>             |
|  |
| Sizes of root marks   |
| <code>\dynkin[radius=.08cm,parabolic=3]{A}{3}</code>                              |
|  |

Table 4: Classical Lie superalgebras [7]. We need a slightly larger radius parameter to distinguish the tensor product symbols from the solid dots.

$$\begin{array}{ll}
 A_{mn} & \circ - \circ - \circ - \circ - \otimes - \circ - \circ - \circ \\
 B_{mn} & \circ - \circ - \circ - \circ - \otimes - \circ - \circ - \circ - \times \\
 B_{0n} & \circ - \circ - \circ - \circ - \circ - \circ - \circ - \times - \bullet \\
 C_n & \otimes - \circ - \circ - \circ - \circ - \otimes - \circ - \circ - \times \\
 D_{mn} & \circ - \circ - \circ - \circ - \circ - \otimes - \circ - \circ - \circ - \circ \\
 D_{21\alpha} & \circ - \otimes - \circ \\
 F_4 & \circ - \circ - \times - \circ - \otimes \\
 G_3 & \otimes - \bullet - \bullet - \bullet
 \end{array}$$

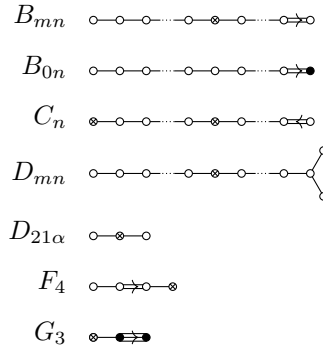
Table 5: Classical Lie superalgebras [7]. Here we see the problem with using the default radius parameter, which is too small for tensor product symbols.

$$A_{mn} \quad \circ - \circ - \circ - \circ - \otimes - \circ - \circ - \circ$$

continued below ...



Table 5: (continued)



## 7. SUPPRESS OR REVERSE ARROWS

Some diagrams have double or triple edges

```
\dynkin{F}{4}
\dynkin{G}{2}
```



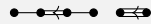
Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```



Reverse arrows

```
\dynkin[reverseArrows]{F}{4}
\dynkin[reverseArrows]{G}{2}
```



## 8. DRAWING ON TOP OF A DYNKIN DIAGRAM

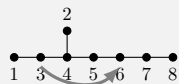
TikZ can access the roots themselves

```
\begin{tikzpicture}
\dynkin{A}{4};
\fill[white,draw=black] (root 2) circle (.15cm);
\fill[white,draw=black] (root 2) circle (.1cm);
\draw[black] (root 2) circle (.05cm);
\end{tikzpicture}
```



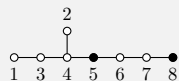
Draw curves between the roots

```
\begin{tikzpicture}
\dynkin[label]{E}{8}
\draw[very thick, black!50,-latex]
  (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{tikzpicture}
```



Change marks

```
\begin{tikzpicture}
\dynkin[mark=o,label]{E}{8};
\dynkinRootMark{*}{5}
\dynkinRootMark{*}{8}
\end{tikzpicture}
```

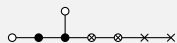


## 9. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



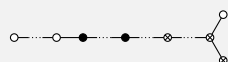
The mark list `oo**ttxx` has one mark for each root: `o`, `o`,  $\dots$ , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

## 10. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots,  $\bullet \cdots \bullet$  indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

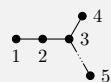
```
\dynkin{D}{o.o*.*.t.to.t}
```



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

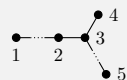
Indefinite edge option

```
\dynkin[makeIndefiniteEdge={3-5},label]{D}{5}
```



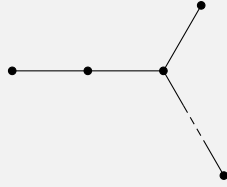
Give a list of edges to become indefinite

```
\dynkin[makeIndefiniteEdge/.list={1-2,3-5},label]{D}{5}
```



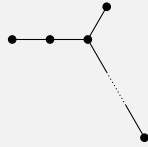
## Indefinite edge style

```
\dynkin[indefiniteEdge/.style={draw=black,fill=white,thin,densely
dashed},%
edgeLength=1cm,%
makeIndefiniteEdge={3-5}]
{D}{5}
```



## The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edgeLength = .5cm,%
indefiniteEdgeRatio=3,%
makeIndefiniteEdge={3-5}]
{D}{5}
```



## 11. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram  $\backslash\text{dynkin}[\text{parabolic}=3]\{A\}\{3\}$ .

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram  $\times - \times - \bullet$ .

Table 6: The Hermitian symmetric spaces

$A_n$  Grassmannian of  $k$ -planes in  $\mathbb{C}^{n+1}$

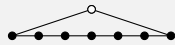
continued below ...

Table 6: (continued)

|       |  |   |
|-------|--|---|
| $B_n$ |  | $(2n - 1)$ -dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$ |
| $C_n$ |  | space of Lagrangian $n$ -planes in $\mathbb{C}^{2n}$  |
| $D_n$ |  | $(2n - 2)$ -dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$   |
| $D_n$ |  | one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$       |
| $D_n$ |  | the other component   |
| $E_6$ |  | complexified octave projective plane  |
| $E_6$ |  | its dual plane  |
| $E_7$ |  | the space of null octave 3-planes in octave 6-space   |

## 12. EXTENDED DYNKIN DIAGRAMS

## Extended Dynkin diagrams

`\dynkin[extended]{A}{7}`

The extended Dynkin diagrams are also described in the notation of Kac [11] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

## Extended Dynkin diagrams

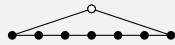
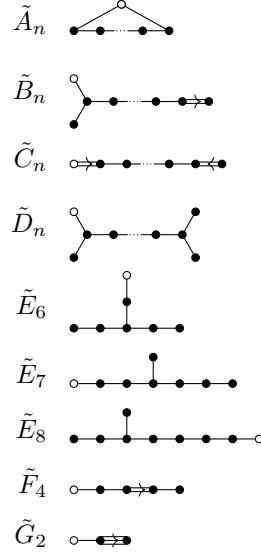
`\dynkin{A}[1]{7}`

Table 7: The Dynkin diagrams of the extended simple root systems

$$\tilde{A}_1 \Leftrightarrow \bullet$$

continued below ...

Table 7: (continued)



## 13. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

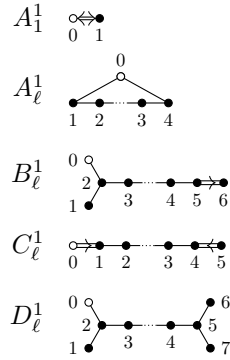
The affine Dynkin diagrams are described in the notation of Kac [11] p. 55:

Affine Dynkin diagrams

$\backslash(A^{(1)}_7 = \text{dynkin}\{A\}[1]\{7\}, \backslash$   
 $E^{(2)}_6 = \text{dynkin}\{E\}[2]\{6\}, \backslash$   
 $D^{(3)}_4 = \text{dynkin}\{D\}[3]\{4\}\backslash$

$$A_7^{(1)} = \text{Dynkin diagram for } A_7^{(1)}, E_6^{(2)} = \text{Dynkin diagram for } E_6^{(2)}, D_4^{(3)} = \text{Dynkin diagram for } D_4^{(3)}$$

Table 8: The affine Dynkin diagrams



continued below ...

Table 8: (continued)

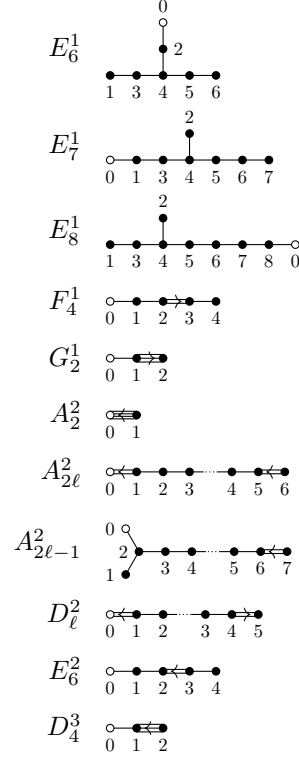
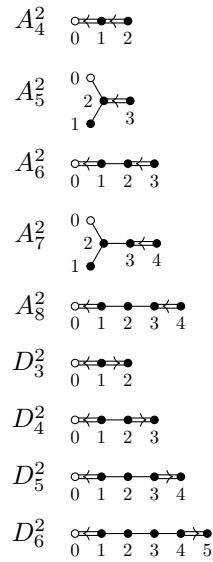
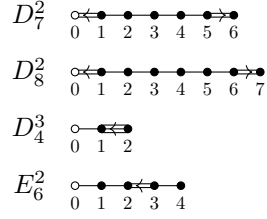


Table 9: Some more affine Dynkin diagrams



continued below ...

Table 9: (continued)



## 14. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

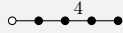
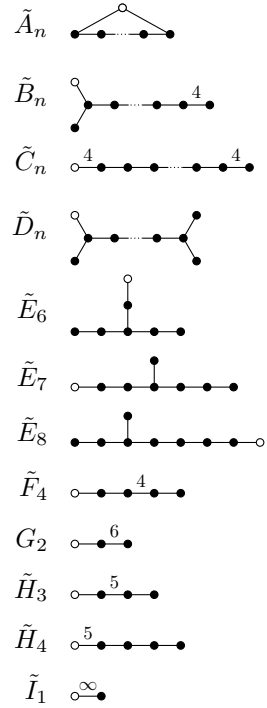
`\dynkin[extended,Coxeter]{F}{4}`

Table 10: The extended (affine) Coxeter diagrams



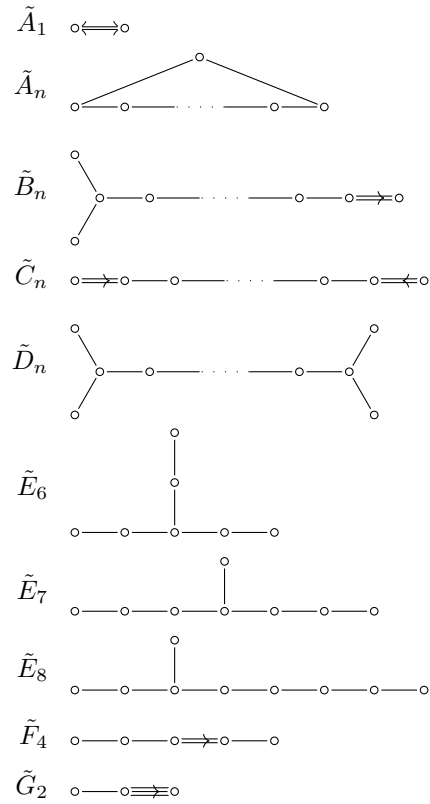
## 15. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [11].



| Kac style   |
|---|
| <code>\dynkin[Kac]{F}{4}</code>   |
|  |

Table 11: The Dynkin diagrams of the extended simple root systems in Kac style



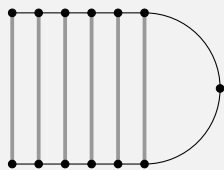
## 16. FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

| Folding   |
|---|
| <code>\dynkin[fold]{A}{13}</code>   |
|  |

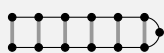
## Big fold radius

```
\dynkin[fold,foldradius=1cm]{A}{13}
```



## Small fold radius

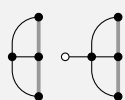
```
\dynkin[fold,foldradius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

## 3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



## 4-ply

```
\dynkin[ply=4]{D}[1]{4}
```



The  $D_\ell^{(1)}$  diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{ } \
\dynkin[foldleft]{D}[1]{ } \
\dynkin[foldright]{D}[1]{ } \
\dynkin[fold]{D}[1]{ }
```



We have to be careful about the 4-ply foldings of  $D_{2\ell}^{(1)}$ , for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default  $D_{2\ell}^{(1)}$  and the two ways to finish it

```
\begin{tikzpicture}
\dynkin[ply=4]{D}[1]{****.*****.*****}%
\end{tikzpicture} \
\begin{tikzpicture}
\dynkin[ply=4]{D}[1]{****.*****.*****}%
\dynkinFold[bend right=65]{1}{13}%
\dynkinFold[bend right=65]{0}{14}%
\end{tikzpicture} \
\begin{tikzpicture}
\dynkin[ply=4]{D}[1]{****.*****.*****}%
\dynkinFold{0}{1}%
\dynkinFold{1}{13}%
\dynkinFold{13}{14}%
\end{tikzpicture}
```

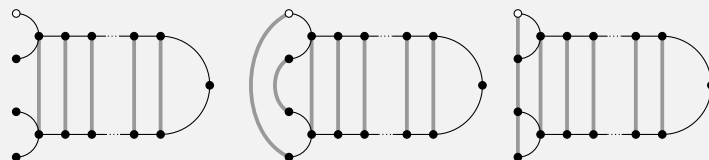
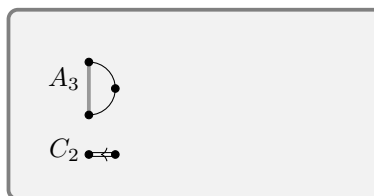
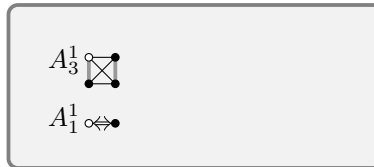
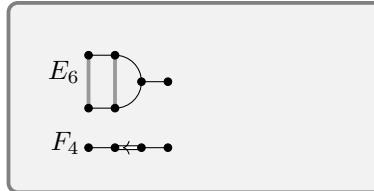
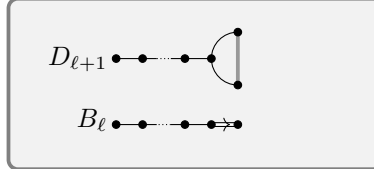
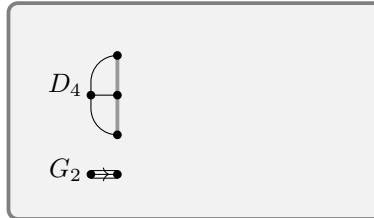
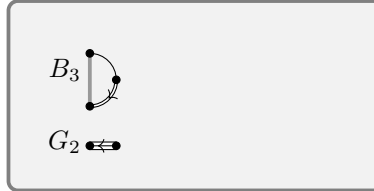
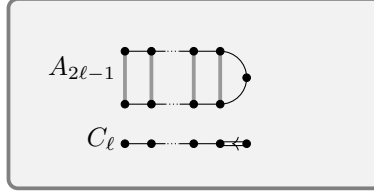


Table 12: Some foldings of Dynkin diagrams



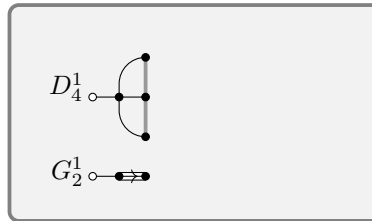
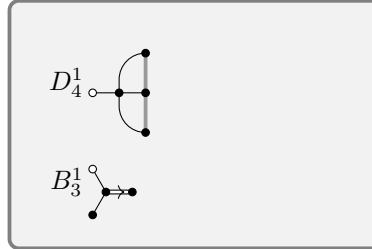
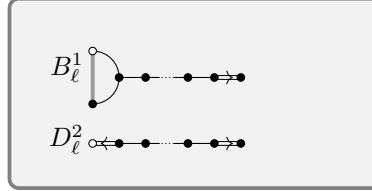
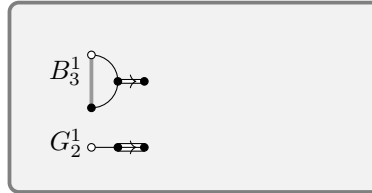
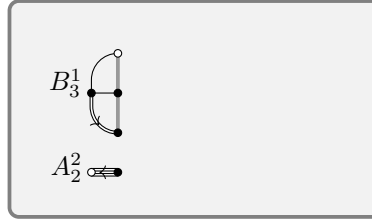
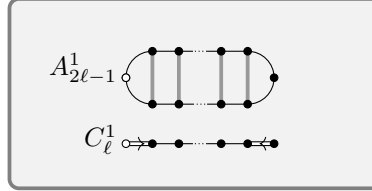
continued below ...

Table 12: (continued)



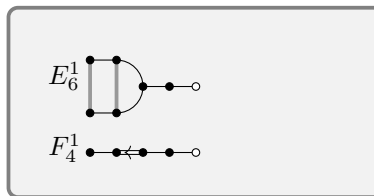
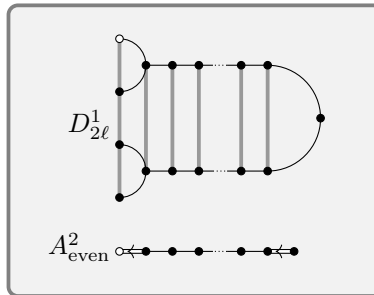
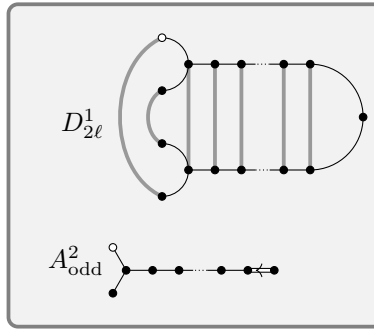
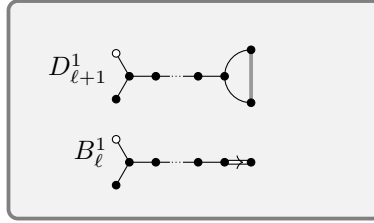
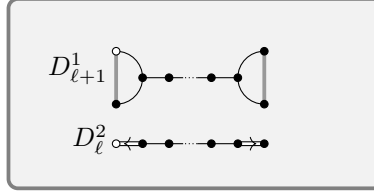
continued below ...

Table 12: (continued)



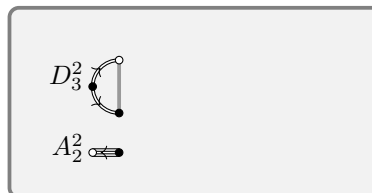
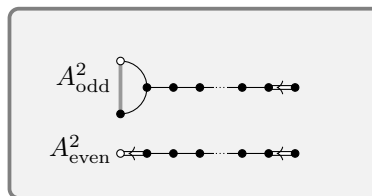
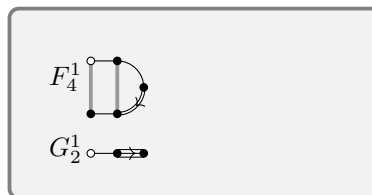
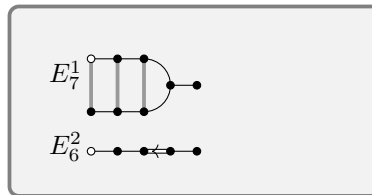
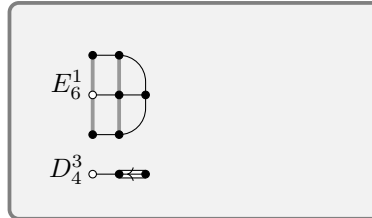
continued below ...

Table 12: (continued)



continued below ...

Table 12: (continued)



## 17. ROOT ORDERING

Root ordering

```
\dynkin[ordering=Adams]{E}{6}
\dynkin[ordering=Bourbaki]{E}{6}
\dynkin[ordering=Carter]{E}{6}
\dynkin[ordering=Dynkin]{E}{6}
\dynkin[ordering=Kac]{E}{6}
```



Default is Bourbaki.

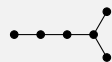
|       | Adams | Bourbaki | Carter | Dynkin | Kac |
|-------|-------|----------|--------|--------|-----|
| $E_6$ |       |          |        |        |     |
| $E_7$ |       |          |        |        |     |
| $E_8$ |       |          |        |        |     |
| $F_4$ |       |          |        |        |     |
| $G_2$ |       |          |        |        |     |

## 18. NAMING DYNKIN DIAGRAMS AND CONNECTING DIFFERENT ONES

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:



## Connect diagrams

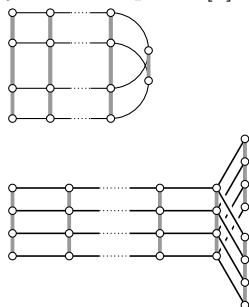
```

\begin{tikzpicture}
\dynkin[name=upper]{A}{3}
\dynkin[name=lower]{A}{3}
\foreach \i in {1,...,3}%
{%
    \draw[/Dynkin diagram/foldStyle] ($(\text{upper root } \i)$) --
        ($(\text{lower root } \i)$);%
}%
\end{tikzpicture}

```

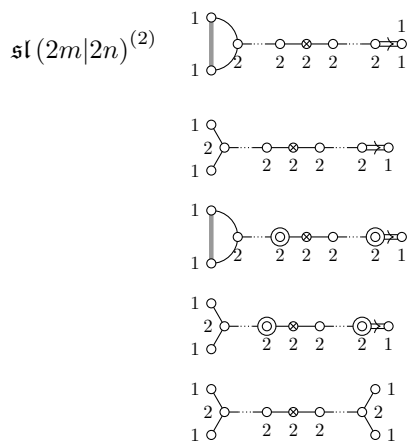


The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [1].



## 19. OTHER EXAMPLES

Table 14: The Vogan diagrams of some affine Lie superalgebras [16, 15]



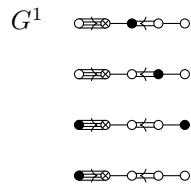
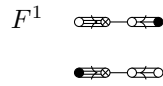
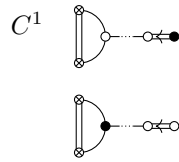
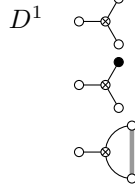
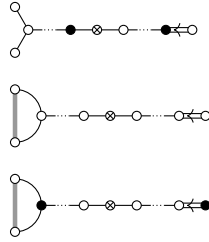
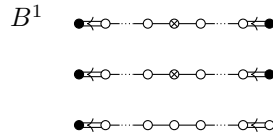
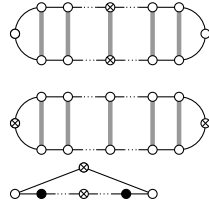
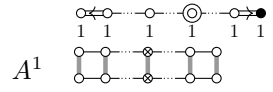
continued below ...

Table 14: (continued)

|                               |  |
|-------------------------------|--|
|                               |  |
|                               |  |
| $\mathfrak{sl}(2m+1 2n)^2$    |  |
|                               |  |
| $\mathfrak{sl}(2m+1 2n+1)^2$  |  |
|                               |  |
| $\mathfrak{sl}(2 2n+1)^{(2)}$ |  |
|                               |  |
|                               |  |
| $\mathfrak{sl}(2 2n)^{(2)}$   |  |
|                               |  |
| $\mathfrak{osp}(2m 2n)^{(2)}$ |  |
|                               |  |
| $\mathfrak{osp}(2 2n)^{(2)}$  |  |
|                               |  |
| $\mathfrak{sl}(1 2n+1)^4$     |  |

continued below ...

Table 14: (continued)



## 20. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is *A, B, C, D, E, F* or *G*, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type <sup>(1)</sup>
- 2 affine twisted root system of type <sup>(2)</sup>
- 3 affine twisted root system of type <sup>(3)</sup>

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

## 21. OPTIONS

`text/.style = <TikZ style data>`,

default : `scale=.7`

Style for any labels on the roots.

`name = <string>`,

default : `anonymous`

A name for the Dynkin diagram, with `anonymous` treated as a blank; see section 18.

`parabolic = <integer>`,

default : 0

A parabolic subgroup with specified integer, where the integer is computed as  $n = \sum 2^{i-1} a_i$ ,  $a_i = 0$  or 1, to say that root  $i$  is crossed, i.e. a noncompact root.

`radius = <number>cm`,

default : `.05cm`

size of the dots and of the crosses in the Dynkin diagram

`edgeLength = <number>cm`,

default : `.35cm`

distance between nodes in the Dynkin diagram

```

edge/.style = TikZ style data,
default : thin
           style of edges in the Dynkin diagram
mark = (o,0,t,x,X,*),
default : *
           default root mark
affineMark = o,0,t,x,X,*,
default : *
           default root mark for root zero in an affine Dynkin diagram
label = true or false,
default : false
           whether to label the roots according to the current labelling
           scheme.
labelMacro = (1-parameter TeX macro),
default : #1
           the current labelling scheme.
makeIndefiniteEdge = (edge pair  $i$ - $j$  or list of such),
default : {}
           edge pair or list of edge pairs to treat as having indefinitely many
           roots on them.
indefiniteEdgeRatio = (float),
default : 1.6
           ratio of indefinite edge lengths to other edge lengths.
indefiniteEdge/.style = (TikZ style data),
default : draw=black,fill=white,thin,densely dotted
           style of the dotted or dashed middle third of each indefinite edge.
arrows = (true or false),
default : true
           whether to draw the arrows that arise along the edges.
reverseArrows = (true or false),
default : true
           whether to reverse the direction of the arrows that arise along the
           edges.

```

```

fold = ⟨true or false⟩,
default : true
        whether, when drawing Dynkin diagrams, to draw them 2-ply.
ply = ⟨0,1,2,3,4⟩,
default : 0
        how many roots get folded together, at most.
foldleft = ⟨true or false⟩,
default : true
        whether to fold the roots on the left side of a Dynkin diagram.
foldright = ⟨true or false⟩,
default : true
        whether to fold the roots on the right side of a Dynkin diagram.
foldradius = ⟨length⟩,
default : .3cm
        the radius of circular arcs used in curved edges of folded Dynkin
        diagrams.
foldStyle = ⟨TikZ style data⟩,
default : draw=black!40,fill=none,line width=radius
        when drawing folded diagrams, style for the fold indicators.
*/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
        style for roots like •
o/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
        style for roots like ○
O/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
        style for roots like ⊙
t/.style = ⟨TikZ style data⟩,
default : draw=black,fill=black
        style for roots like ⊗
x/.style = ⟨TikZ style data⟩,

```

```

default : draw=black
           style for roots like  $\times$ 
X/.style =  $\langle$ TikZ style data $\rangle$ ,
default : draw=black,thick
           style for roots like  $\times$ 
leftFold/.style =  $\langle$ TikZ style data $\rangle$ ,
default :
           style to override the fold style when folding roots together on the
           left half of a Dynkin diagram
rightFold/.style =  $\langle$ TikZ style data $\rangle$ ,
default :
           style to override the fold style when folding roots together on the
           right half of a Dynkin diagram
doubleEdges =  $\langle$  $\rangle$ ,
default : not set
           set to override the fold style when folding roots together in a
           Dynkin diagram, so that the foldings are indicated with double
           edges (like those of an  $F_4$  Dynkin diagram without arrows).
doubleFold =  $\langle$  $\rangle$ ,
default : not set
           set to override the fold style when folding roots together in a
           Dynkin diagram, so that the foldings are indicated with double
           edges (like those of an  $F_4$  Dynkin diagram without arrows), but
           filled in solidly.
doubleLeft =  $\langle$  $\rangle$ ,
default : not set
           set to override the fold style when folding roots together at the
           left side of a Dynkin diagram, so that the foldings are indicated
           with double edges (like those of an  $F_4$  Dynkin diagram without
           arrows).
doubleFoldLeft =  $\langle$  $\rangle$ ,
default : not set
           set to override the fold style when folding roots together at the
           left side of a Dynkin diagram, so that the foldings are indicated
           with double edges (like those of an  $F_4$  Dynkin diagram without
           arrows), but filled in solidly.

```

`doubleRight` =  $\langle \rangle$ ,  
`default` : `not set`  
 set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows).

`doubleFoldRight` =  $\langle \rangle$ ,  
`default` : `not set`  
 set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows), but filled in solidly.

`Coxeter` =  $\langle \text{true or false} \rangle$ ,  
`default` : `false`  
 whether to draw a Coxeter diagram, rather than a Dynkin diagram.

`ordering` =  $\langle \text{Adams, Bourbaki, Carter, Dynkin, Kac} \rangle$ ,  
`default` : `Bourbaki`  
 which ordering of the roots to use in exceptional root systems as in section 17.

All other options are passed to TikZ.

## REFERENCES

1. Kurando Baba, *Satake diagrams and restricted root systems of semisimple pseudo-Riemannian symmetric spaces*, Tokyo J. Math. **32** (2009), no. 1, 127–158. MR 2541161
2. Nicolas Bourbaki, *Lie groups and Lie algebras. Chapters 4–6*, Elements of Mathematics (Berlin), Springer-Verlag, Berlin, 2002, Translated from the 1968 French original by Andrew Pressley. MR 1890629
3. R. W. Carter, *Lie algebras of finite and affine type*, Cambridge Studies in Advanced Mathematics, vol. 96, Cambridge University Press, Cambridge, 2005. MR 2188930
4. Meng-Kiat Chuah, *Cartan automorphisms and Vogan superdiagrams*, Math. Z. **273** (2013), no. 3–4, 793–800. MR 3030677
5. E. B. Dynkin, *Semisimple subalgebras of semisimple Lie algebras*, Mat. Sbornik N.S. **30(72)** (1952), 349–462 (3 plates), Reprinted in English translation in [6]. MR 0047629
6. ———, *Selected papers of E. B. Dynkin with commentary*, American Mathematical Society, Providence, RI; International Press, Cambridge, MA, 2000, Edited by A. A. Yushkevich, G. M. Seitz and A. L. Onishchik. MR 1757976
7. L. Frappat, A. Sciarrino, and P. Sorba, *Structure of basic Lie superalgebras and of their affine extensions*, Comm. Math. Phys. **121** (1989), no. 3, 457–500. MR 990776
8. L. C. Grove and C. T. Benson, *Finite reflection groups*, second ed., Graduate Texts in Mathematics, vol. 99, Springer-Verlag, New York, 1985. MR 777684
9. Sigurdur Helgason, *Differential geometry, Lie groups, and symmetric spaces*, Graduate Studies in Mathematics, vol. 34, American Mathematical Society, Providence, RI, 2001, Corrected reprint of the 1978 original. MR 1834454



10. James E. Humphreys, *Reflection groups and Coxeter groups*, Cambridge Studies in Advanced Mathematics, vol. 29, Cambridge University Press, Cambridge, 1990. MR 1066460
11. Victor G. Kac, *Infinite-dimensional Lie algebras*, third ed., Cambridge University Press, Cambridge, 1990. MR 1104219
12. S. Pratik Khastgir and Ryu Sasaki, *Non-canonical folding of Dynkin diagrams and reduction of affine Toda theories*, Progr. Theoret. Phys. **95** (1996), no. 3, 503–518. MR 1388245
13. A. L. Onishchik and È. B. Vinberg, *Lie groups and algebraic groups*, Springer Series in Soviet Mathematics, Springer-Verlag, Berlin, 1990, Translated from the Russian and with a preface by D. A. Leites. MR 91g:22001
14. A. L. Onishchik and È. B. Vinberg, *Lie groups and algebraic groups*, Springer Series in Soviet Mathematics, Springer-Verlag, Berlin, 1990, Translated from the Russian and with a preface by D. A. Leites. MR 1064110
15. B. Ransingh, *Vogan diagrams of affine twisted Lie superalgebras*, ArXiv e-prints (2013).
16. Biswajit Ransingh, *Vogan diagrams of untwisted affine Kac-Moody superalgebras*, Asian-Eur. J. Math. **6** (2013), no. 4, 1350062, 10. MR 3149279
17. V. Regelskis and B. Vlaar, *Reflection matrices, coideal subalgebras and generalized Satake diagrams of affine type*, ArXiv e-prints (2016).
18. Ichirô Satake, *Algebraic structures of symmetric domains*, Kanô Memorial Lectures, vol. 4, Iwanami Shoten, Tokyo; Princeton University Press, Princeton, N.J., 1980. MR 591460
19. È. B. Vinberg (ed.), *Lie groups and Lie algebras, III*, Encyclopaedia of Mathematical Sciences, vol. 41, Springer-Verlag, Berlin, 1994, Structure of Lie groups and Lie algebras, A translation of it Current problems in mathematics. Fundamental directions. Vol. 41 (Russian), Akad. Nauk SSSR, Vsesoyuz. Inst. Nauchn. i Tekhn. Inform., Moscow, 1990 [ MR1056485 (91b:22001)], Translation by V. Minachin [V. V. Minakhin], Translation edited by A. L. Onishchik and È. B. Vinberg. MR 1349140
20. Jean-Bernard Zuber, *Generalized Dynkin diagrams and root systems and their folding*, Topological field theory, primitive forms and related topics (Kyoto, 1996), Progr. Math., vol. 160, Birkhäuser Boston, Boston, MA, 1998, pp. 453–493. MR 1653035