Tail Risk and Asset Prices

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We propose a new measure of time-varying tail risk that is directly estimable from the cross-section of returns. We exploit firm-level price crashes every month to identify common fluctuations in tail risk among individual stocks. Our tail measure is significantly correlated with tail risk measures extracted from S&P 500 index options and negatively predicts real economic activity. We show that tail risk has strong predictive power for aggregate market returns. Cross-sectionally, stocks with high loadings on past tail risk earn an annual three-factor alpha 5.4% higher than stocks with low tail risk loadings. We explore potential mechanisms giving rise to these asset pricing facts. (*JEL* G11, G12, G17)

The goal of this paper is to investigate the effects of time-varying extreme event risk in asset markets. The chief obstacle to this investigation is a viable measure of tail risk over time. Ideally, one would directly construct a measure of aggregate tail risk dynamics from the time series of, say, market returns or GDP growth rates, much like dynamic volatility estimated from a GARCH model. But dynamic tail risk estimates are infeasible in a univariate time series model because of the infrequent nature of extreme events.

To overcome this problem, we devise a panel estimation approach that captures common variation in the tail risks of individual firms. If firm-level tail distributions possess similar dynamics, then the cross-section of crash events for individual firms can be used to identify the common component of their tail risk at each point in time.

Our empirical framework centers on a reduced-form description for the tail distribution of returns. The time t lower tail distribution is defined as the set

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of return events falling below some extreme negative threshold u_t. We assume that the lower tail of asset return i behaves according to

$$P(R_{i,t+1} < r \mid R_{i,t+1} < u_t \text{ and } \mathcal{F}_t) = \left(\frac{r}{u_t}\right)^{-a_i/\lambda_t}, \tag{1}$$

where $r < u_t < 0$. Equation (1) states that extreme return events obey a power law. The key parameter of the model, a_i/λ_t, determines the shape of the tail and is referred to as the tail exponent. Because $r < u_t < 0$, $r/u_t > 1$. Therefore $a_i/\lambda_t > 0$ to ensure that the probability $(r/u_t)^{-a_i/\lambda_t}$ always lies between zero and one. High values of λ_t correspond to "fat" tails and high probabilities of extreme returns.

In contrast to past power law research, Equation (1) is a model of the conditional return tail. The $1/\lambda_t$ term in the exponent may vary with the conditioning information set $\mathcal{F}_t$. Although different assets can have different levels of tail risk (determined by the constant a_i), dynamics are the same for all assets because they are driven by the common process λ_t. Thus, we refer to λ_t as "tail risk" at time t, and we refer to the tail structure in (1) as a "dynamic power law."

We build a tail risk measure from the dynamic power law structure (1). The identifying assumption is that tail risks of individual assets share similar dynamics. Therefore, in a sufficiently large cross section, enough stocks will experience individual tail events each period to provide accurate information about the prevailing level of tail risk. Applying Hill's (1975) power law estimator to the time t cross-section recovers an estimate of λ_t.[1]

We find that the time-varying tail exponent is highly persistent. We estimate λ_t separately each month, so there is no mechanical persistence in this series, yet we find a monthly AR(1) coefficient of 0.927. Thus, λ_t has strong predictive power for future extreme returns of individual stocks, offering a first indication that λ_t is a potentially important determinant of asset prices. We also find a high degree of commonality in time-varying tail exponents across firms, supporting our assumption of common firm-level tail dynamics. For example, when we estimate separate tail risk series for each industry, we find time series correlations in their tail risks ranging from 57% to 87%.

Our primary contribution is an empirical analysis of the impact of tail risk on asset markets. First, we test the hypothesis that tail risk forecasts aggregate stock market returns. Predictive regressions show that a one-standard-deviation increase in tail risk forecasts an increase in annualized excess market returns of 4.5%, 4.0%, 3.7%, and 3.2% at the one-month, one-year, three-year, and five-year horizons, respectively. These are all statistically significant with t-statistics of 2.1, 2.0, 2.4, and 2.7. These results are robust out-of-sample, producing a

[1] This allows us to capture common fluctuations in individual firms' tails over time. This procedure avoids having to accumulate years of tail observations from the aggregate series in order to estimate tail risk, and therefore avoids using stale observations that carry little information about current tail risk.

4.5% R^2 at the annual frequency, compared to 6.1% in-sample. The forecasting power of tail risk is also robust to controlling for a broad set of alternative predictors, outperforming the dividend-price ratio and other common predictors surveyed by Goyal and Welch (2008).

The tail exponent also has substantial predictive power for the cross-section of average returns. We run predictive regressions for each stock, then sort stocks based on their predictive tail risk exposures. Stocks in the highest quintile earn annual value-weighted three-factor alphas 5.4% higher than stocks in the lowest quintile over the subsequent year. This tail risk premium is robust to controlling for other priced factors and characteristics, including momentum (Carhart 1997), liquidity (Pastor and Stambaugh 2003), idiosyncratic stock volatility (Ang, Hodrick, Xing and Zhang 2006), downside beta (Ang, Chen, and Xing 2006), and coskewness (Harvey and Siddique 2000). We also find a strong association between our tail risk measure and the crash insurance premium on deep out-of-the-money equity put options.

We explore two channels through which our tail risk measure may correlate with state variables driving the stochastic discount factor. Each channel provides a potential explanation for why the common component of firm-level tail risk is related to equity premia.

First, aggregate tail risks, which we expect to have important pricing implications, are mathematically linked to common dynamics in firm-level tails. In particular, power law distributions are stable under aggregation: a sum of power law returns inherits the tail behavior of the individual returns.[2] This implies that firm-level tail distributions are informative about the likelihood of market-wide extremes.

A second link between individual firm risks and aggregate effects may arise from the impact of uncertainty shocks on real outcomes. Bloom (2009) argues that, because of capital and labor adjustment costs, an increase in uncertainty raises the value of a firm's "real options," such as the option to postpone investment decisions. In his framework, firm-level uncertainty for all firms fluctuates in concert through time. A common rise in uncertainty depresses aggregate economic activity by inducing all firms to simultaneously reduce investment and hiring. Although Bloom focuses on uncertainty in the form of volatility, his rationale also implies that common changes in firm-level tail risk can have important aggregate real effects.[3] Because we find common fluctuations in tail risk across firms, firm-level tail uncertainty may adversely affect aggregate real outcomes, representing a second potential channel through which tail risk impacts equity premia.

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[2] Gabaix (2009) provides a summary of aggregation properties for variables with power law tails. In particular, power law tails are conserved under addition, so that portfolio return tails behave like the tails of stocks in the portfolio. Further details and derivations are found in Jessen and Mikosch (2006).

[3] Gourio (2012) presents a theoretical model showing that shocks to aggregate tail risk induce qualitatively similar business fluctuations as the volatility uncertainty studied by Bloom (2009).

We explore both of these mechanisms empirically. Over the 1963 to 2010 sample, an increase in our tail risk measure significantly forecasts higher market return kurtosis and lower (more negative) market return skewness (after controlling for own lags of market skewness and kurtosis). Options data, though only available for the last twenty years, provide a second opportunity to investigate whether our tail estimator is correlated with tail risk of the market portfolio. In particular, we compare our measure to option-implied risk measures for the S&P 500 index. We find that our tail measure has a significant 33% correlation with option-implied kurtosis and −30% correlation with option-implied skewness, suggesting that our measure is closely associated with tail risks perceived by option market participants. In summary, tests based on higher moments of the market return distribution, estimated either from market returns data or S&P 500 options data, corroborate the power law aggregation property that firm-level tail distributions contain information about the likelihood of aggregate extreme events.

Motivated by the uncertainty shocks argument of Bloom (2009), we investigate whether there is evidence of time-varying tail risk in firms' fundamentals. We apply our estimation approach to the panel of firm-level sales growth and show that dynamics in stock return tails share a significant correlation of 31% with fluctuations in the tail distribution of cash flows (p-value of 0.008). Furthermore, we find that economic activity is highly sensitive to tail risk shocks. Aggregate investment, output, and employment drop significantly following an increase in tail risk. These facts provide a bridge between empirical studies of fat-tailed stock return behavior and theoretical models of tail risk in the real economy.

Our research question draws on several strands of literatures. Recently, researchers have hypothesized that heavy-tailed shocks to economic fundamentals help explain certain asset pricing behavior that has proved otherwise difficult to reconcile with traditional macrofinance theory. Examples include the Rietz (1988) and Barro (2006) rare disaster hypothesis and its extensions to dynamic settings by Gabaix (2012), Gourio (2012), and Wachter (2013), as well as extensions of Bansal and Yaron's (2004) long-run risks model that incorporate fat-tailed endowment shocks (Eraker and Shaliastovich 2008; Bansal and Shaliastovich 2010, 2011; Drechsler and Yaron 2011).[4] Model calibrations show that this class of models matches a number of key asset pricing moments. Ours is the first paper to directly document time-varying tail risk in fundamentals. We also provide direct estimates of the association between tail risk and risk premia (as opposed to model calibrations). There are two key equity premium implications from these models, and we find that tail risk significantly relates to return data in the manner predicted. First, tail risk positively forecasts excess market returns. Because investors are tail risk averse, increases in tail

[4] These long-run risk extensions build on a large body of literature that models extreme events with jump processes, most notably the widely used affine class of Duffie, Pan, and Singleton (2000).

risk raise the return required by investors to hold the market, thereby inducing a positive predictive relationship between tail risk and future returns. The second implication applies to the cross-section of expected returns. High tail risk is associated with bad states of the world and high marginal utility. Hence, assets that hedge tail risk are more valuable (have lower expected returns) than those that are adversely exposed to tail risk.

Since at least Mandelbrot (1963) and Fama (1963), a separate thread of literature has developed arguing that unconditional return distributions are heavy tailed and aptly described by a power law. More recent empirical work suggests that the return tail distribution varies over time.[5] We show that empirical studies of fat-tailed stock return behavior and theoretical models of tail risk in the real economy are closely linked.

There are two current approaches to measuring tail risk dynamics for stock returns: one based on option price data and another on high frequency return data. Examples of the option-based approach include Bakshi, Kapadia, and Madan (2003), who study risk-neutral skewness and kurtosis, Bollerslev, Tauchen, and Zhou (2009), who examine how the variance risk premium relates to the equity premium, and Backus, Chernov, and Martin (2011) and Gao and Song (2013), who infer disaster risk premia from options. Tail estimation from high-frequency data is exemplified by Bollerslev and Todorov (2011). These approaches are powerful but subject to data limitations (sample horizons are at most 20 years for returns, and the method cannot be applied to low frequency cash flow data). Our tail risk series is estimated using returns and sales growth data since 1963, and may be used in any setting in which a large cross-section is available.[6]

1. Empirical Methodology

1.1 The tail distribution of returns

We posit that returns obey the dynamic power law structure in Equation (1). An extensive literature in finance, statistics, and physics has thoroughly documented power law tail behavior of equity returns.[7] Evidence suggests that the key parameter of this power law may vary over time (Quintos, Fan, and Phillips 2001). We propose a novel specification for equity returns in which the tail distribution obeys a potentially time-varying power law. Modeling dynamic tail risk is challenging because observations that are informative about tails

[5] A seminal paper documenting variation in the power law tail of returns is that of Quintos, Fan, and Phillips (2001), with additional evidence presented by Galbraith and Zernov (2004), Werner and Upper (2004), and Wagner (2003).

[6] The cross-section procedure that we propose has subsequently been adopted as a measure of systemic banking sector risk by Allen, Bali, and Tang (2012) and liquidity risk by Wu (2013).

[7] See, for example, Mandelbrot (1963), Fama (1963, 1965), Officer (1972), Blattberg and Gonedes (1974), Akgiray and Booth (1988), Hols and De Vries (1991), Jansen and De Vries (1991), Kearns and Pagan (1997), Gopikrishnan et al. (1999), and Gabaix et al. (2006).

occur rarely by definition. To overcome this challenge, our approach relies on commonality in the tail risks of individual assets, in turn exploiting the comparatively rich information about tail risk in the cross-section of returns. We allow for a different level of firm-specific tail risk across assets, but assume that tail risk fluctuations for all assets are governed by a single process. This structure implies that firms have different unconditional tail risks, but their tail risk dynamics are similar (we provide evidence below that supports this assumption).

Conditional upon exceeding some extreme lower "tail threshold," u_t, and given information $\mathcal{F}_t$, we assume that an asset's return obeys the tail probability distribution in Equation (1).[8] We estimate the common time-varying component of return tails, λ_t, month-by-month by applying Hill's (1975) power law estimator to the set of daily return observations for all stocks in month t.[9] Applied to the pooled cross-section each month, it takes the form

$$\lambda_t^{Hill} = \frac{1}{K_t} \sum_{k=1}^{K_t} \ln \frac{R_{k,t}}{u_t},$$

where $R_{k,t}$ is the kth daily return that falls below an extreme value threshold u_t during month t, and K_t is the total number of such exceedences within month t.[10]

The threshold parameter u_t is chosen by the econometrician and defines where the center of the distribution ends and the tail begins. It represents a suitably extreme quantile such that any returns below this cutoff are assumed to obey the specified tail distribution. We define u_t as the fifth percentile of the cross-section each period.[11]

The extreme value approach constructs Hill's measure using only those observations that exceed the tail threshold (observations such that $R_{i,t}/u_t > 1$, referred to as "u-exceedences") and discards nonexceedences. To understand

8 This specification is motivated by the Pickands-Balkema-de Haan limit theorem, which states that for a wide class of heavy-tailed distributions for $R_{i,t+1}$, $P(R_{i,t+1} < r \mid R_{i,t+1} < u_t)$ will converge to a generalized power law distribution as u_t approaches the support boundary of $R_{i,t+1}$. This limit result is operationalized by treating the power law as an exact relationship for threshold exceedences (Embrechts et al. 1997).

9 Gabaix and Ibragimov (2011) propose an alternative "rank-1/2" regression approach to estimating λ. Our λ estimates and asset pricing test results are nearly identical when we use their estimator in place of λ_t^{Hill}.

10 For simplicity, the Hill formula is written as though the cross-sectional u-exceedences are the first K_t elements of R_t. This is without loss of generality because the elements of R_t are exchangeable from the perspective of the estimator.

11 An inappropriately mild threshold will contaminate tail exponent estimates by using data from the center of the distribution, whose behavior can vary markedly from tail data. A very extreme threshold can result in noisy estimates resulting from too few data points. Sophisticated methods for threshold selection have been developed (Dupuis 1999 and Matthys and Beirlant 2000, among others), though these typically require estimation of additional parameters and can be unstable. In light of this fact, Gabaix et al. (2006) advocate a simple rule that fixes the u-exceedence probability at 5% for unconditional power law estimation. We follow these authors by applying a similar simple rule in the dynamic setting. We find very similar empirical results if we use thresholds ranging from the first to fifth percentile.

why this is a sensible estimate of the exponent, first note that nonexceedences are part of the nontail domain and thus they need not obey a power law and are appropriately omitted from tail estimates. Second, because u-exceedences obey a power law with exponent a_i/λ_t, *log* exceedences are exponentially distributed with scale parameter a_i/λ_t. By the properties of an exponential random variable, $E_{t-1}[\ln(R_{i,t}/u_t)] = \lambda_t/a_i$. When all stocks have the same ex ante probability of experiencing a threshold exceedence, the expected value of λ_t^{Hill} becomes the cross-sectional harmonic average tail exponent:

$$E_{t-1}\left[\frac{1}{K_t}\sum_{k=1}^{K_t}\ln\frac{R_{k,t}}{u_t} \mid \lambda_t, R_{k,t} < u_t\right] = \lambda_t\frac{1}{\bar{a}}, \quad \text{where} \quad \frac{1}{\bar{a}} \equiv \frac{1}{n}\sum_{i=1}^{n}\frac{1}{a_i}. \qquad (2)$$

Equation (2) states that, in expectation, the Hill estimator is equal to the true common tail risk component λ_t times a constant multiplicative bias term. Thus, expected value of period-by-period Hill estimates is perfectly correlated with λ_t.

1.2 Other empirical considerations

A potential empirical concern is bias in tail estimates arising from dependence among returns. This can be mitigated by first removing common return factors and then estimating the tail process from return residuals. We implement this strategy by removing common return factors with Fama and French's (1993) three-factor model regressions and then estimating tail risk from the residuals.[12]

Next, because the tail threshold varies over time, common time-variation in volatility is largely taken into account in the construction of our tail estimates. The threshold expands and contracts with volatility so that a fixed fraction of the most extreme observations is used for estimation each period. This mitigates the potential contamination of the tail risk time series because of volatility dynamics.

A third potential concern is the influence that cross-sectional volatility heterogeneity may have on tail estimates. One way to address this heterogeneity is to variance-standardize returns in a preliminary estimation step. Substantial measurement-related problems arise with this approach, as the behavior of standardized returns can be highly sensitive to estimation error in the divisor. Furthermore, variance heterogeneity has little effect on the performance of the Hill estimator–Monte Carlo evidence in Appendix A demonstrates that tail risk can be accurately estimated from raw returns in the presence of cross-sectional volatility heterogeneity. Therefore, our results focus on tail estimates of nonstandardized returns. As a robustness check, we estimate tail risk using volatility-standardized returns.[13] This series is 90% correlated with our main

[12] These results are very similar to tail estimates based on raw returns.

[13] If sampling error produces too low an estimate of volatility, volatility scaling excessively inflates returns, making them appear as tail observations when they are not. On the other hand, if a stock experiences a tail event,

tail risk series that relies on unstandardized data, and produces qualitatively similar results in our asset pricing tests.

Appendix A discusses additional potential confounding issues that can arise when estimating tail risk. We show via simulation that Hill estimates appear consistent amid common forms of dependence and heterogeneity known to exist in return data. The simulations corroborate theoretical results from the extreme value literature (see Hill 2010).

1.3 Hypotheses

Our hypothesis is that investors' marginal utility (and hence the stochastic discount factor) is increasing in tail risk and that tail risk is persistent. These hypotheses have two testable asset pricing implications. The first applies to the equity premium time series. Because investors are averse to tail risk, a positive tail risk shock increases the return required by investors to hold any tail risky portfolio, including the market portfolio. Tail risk persistence is a necessary condition for time series effects because investors will only dynamically adjust their discount rates in response to shocks that are informative about future levels of tail risk. Empirically, we test whether tail risk positively forecasts market returns. Second, assets that better hedge tail risk will command a relatively high price and earn low expected returns. This implication may be tested in the cross-section by comparing average returns of assets to their estimated tail risk sensitivities.

2. Empirical Results

2.1 Tail risk estimates

We estimate the dynamic power law exponent using daily CRSP data from January 1963 to December 2010 for NYSE/AMEX/NASDAQ stocks with share codes 10 and 11. Large data sets are crucial to the accuracy of extreme value estimates because only a small fraction of data is informative about the tail distribution. Because our approach to estimating the dynamic power law exponent relies on the cross-section of returns, we require a large panel of stocks in order to gather sufficient information about the tail at each point in time. The number of stocks in CRSP varies dramatically over time.[14] We focus on the 1963 to 2010 sample because of the cross-section expansion of CRSP beginning in August 1962. To further increase the sample size and reduce sampling noise, we estimate the tail exponent monthly, pooling all daily observations within the month.

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this mechanically raises its measured volatility, in which case standardization can overshrink precisely those observations that are most informative about the tail distribution. We address this empirically by winsorizing 10% of the highest and lowest estimated volatilities to dampen the influence of the noisiest volatility estimates.

[14] The period-wise Hill approach to the dynamic power law in Section 1 naturally accommodates changes in cross-section size over time.

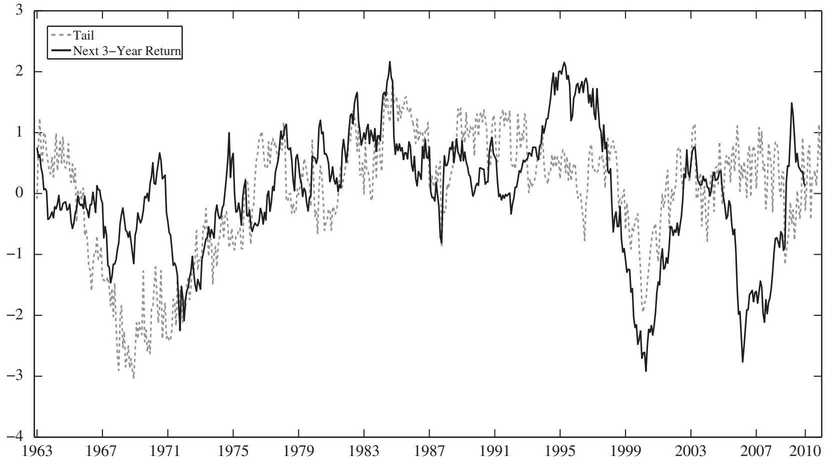

Figure 1
Tail exponent estimates and subsequent market returns
Plotted is the monthly estimated tail risk time series. Tail estimates are calculated each month by pooling daily returns of NYSE/AMEX/NASDAQ stocks. Also plotted in each month t is the realized market return over the three years following month t. To emphasize comparison, both series have been scaled to have mean zero and variance one.

Figure 1 plots the estimated tail risk series alongside the market return over the subsequent three-year period (the series are scaled for comparison). Tail risk appears countercyclical. Our sample begins just after a 28% drop in the aggregate U.S. stock market during the first half of 1962. This major market decline was the first in the postwar era. Estimated tail risk is high at this starting point, but begins to decline steadily until December 1968, when it reaches its lowest level in the sample. This tail risk minimum corresponds to a late 1960's bull market peak, the level of which is not reached again until the mid-1970s. Tail risk rises throughout the 1970s, accelerating its ascent during the oil crisis. It fluctuates above its mean for several years. Tail risk recedes in the four bull market years leading to 1987, rising quickly in the months following the October crash. During the technology boom, tail risk retreats sharply but briefly, rising to its highest post-2000 level amid the early 2003 market trough. At this time the value-weighted index was down 49% from its 2000 high and NASDAQ was 78% off its peak. During the last half of the decade, tail risk hovers close to its mean, and is roughly flat through the 2007–2009 financial crisis and recession.

The absence of an increase in measured tail risk during the recent financial crisis may be surprising prima facie, but is potentially consistent with the account of the recent financial crisis by Brownlees, Engle, and Kelly (2011). They argue that the financial crisis was characterized by soaring volatility, but that this volatility was predictable over short horizons using standard volatility forecasting models and that volatility-adjusted residuals do not appear extreme compared with their historical distribution. This argument is also consistent

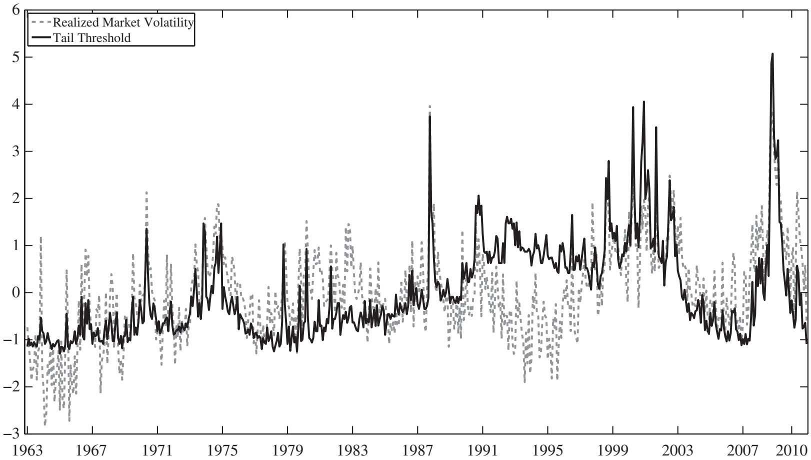

Figure 2
Tail threshold and aggregate market volatility
Plotted is the monthly tail threshold series. The threshold is the absolute value of the fifth percentile of monthly pooled daily returns of NYSE/AMEX/NASDAQ stocks. Also plotted is the monthly realized volatility of the CRSP value-weighted index. To emphasize comparison, both series have been scaled to have the mean zero and variance one.

with Figure 2, which plots the cross-section tail threshold series $\hat{u}_t$ (in absolute value) alongside monthly realized volatility of the CRSP value-weighted index. The lower tail threshold has a 60% correlation with market volatility. During the crisis period, the threshold, which measures the dispersion of the cross-section distribution, spikes drastically along with market volatility. A fixed percentile is used to define the tail region for exactly this reason. If volatility rises dramatically but the shape of return tails is unchanged, then a widening of the threshold will absorb the effect of volatility changes and leave estimates of the tail exponent unaffected.[15]

The tail series is highly persistent, possessing a monthly AR(1) coefficient of 0.927. Because the Hill measure is estimated month-by-month with nonoverlapping data, this autocorrelation is strong evidence that the severity of extreme returns is highly predictable. The estimated persistence in tail risk is on par with that of equity volatility. Because tail shocks are persistent, they have the potential to weigh significantly on equilibrium prices.

2.2 Predicting stock market returns

We test the hypothesis that tail risk forecasts returns of the aggregate market portfolio with a series of predictive regressions. All regressions are conducted

[15] The incidence of tail events at the firm-level is fairly evenly distributed across firms. In a large homogenous cross-section we would expect each firm to show up in the tail 5% of the time. Over 99% of all firms experience at least 1 tail event, 86% of firms show up in the tail more than 1% of the time; and only 8% of firms show up in the tail more than 15% of the time. In the average month, 44.8% of available firms appear in the cross-sectional lower tail on at least one day.

at the monthly frequency, meaning that observations are overlapping for the one-, three-, and five-year analyses. We conduct inference using the Hodrick's (1992) standard error correction for overlapping data.[16]

The dependent variable is the return on the CRSP value-weighted index at frequencies of one month, one year, three years, and five years. To illustrate economic magnitudes, all reported predictive coefficients are scaled to be interpreted as the effect of a one-standard-deviation increase in the regressor on future annualized returns. Table 1 shows that tail risk has large, significant forecasting power over all horizons. A one-standard-deviation increase in lower tail risk predicts an increase in future excess returns of 4.5%, 4.0%, 3.7%, and 3.2% per annum, based on data for one-month, one-year, three-year, and five-year horizons, respectively. The corresponding Hodrick t-statistics are 2.1, 2.0, 2.4, and 2.7.[17]

Table 1 compares the forecasting power of tail risk with a large set of alternative forecasting variables studied in a survey by Goyal and Welch (2008).[18] The aggregate dividend-price ratio is the only other predictor with performance comparable to tail risk. The long-term bond return strongly predicts one-month returns, but its effect dies out at longer horizons. The long-term yield is successful at long horizons, but has weak short horizon predictability.

We next run bivariate regressions using lower tail risk alongside each Goyal and Welch variable to assess the robustness of tail risk's return forecasts after controlling for alternative predictors. Table 2 presents these results. Conclusions regarding the predictive ability of tail risk are unaffected by including alternative regressors. For one-month forecasts, the tail risk predictive coefficient remains above 4% when combined with each of the Goyal and Welch variables, with a t-statistic above 1.8 in all cases. At longer horizons, the performance of tail risk relative to alternatives becomes stronger. At the five-year horizon, the t-statistic is always above 2.2, except when included with the long-term yield, in which case the t-statistic is 1.7. Tail risk, when combined with the dividend-price ratio, achieves impressive levels of predictability, reaching R^2 values of 38% at three years and 54% at five years.[19]

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16 Richardson and Smith (1991), Hodrick's (1992), and Boudoukh and Richardson (1993) (among others) have noted the inferential problems concomitant with overlapping horizon predictive regressions. Ang and Bekaert (2007) demonstrate in a Monte Carlo study that the standard error correction of Hodrick's (1992) provides the most conservative test statistics relative to other commonly employed procedures, maintaining appropriate test size over horizons as long as five years. We also find in our tests that Hodrick's correction produces more conservative results than do Newey-West (1987) and other methods.

17 We find that Goyal and Welch (2008) bootstrap standard errors, which are valid under the Stambaugh (1999) specification, produce even stronger statistical results than those based on the Hodrick correction.

18 We thank Amit Goyal for providing the data of Goyal and Welch (2008), updated through 2010.

19 Our estimates for upper tail risk have a correlation of 81% with lower tail risk, suggesting that tail risk dynamics are fairly symmetric. However, lower tail risk dominates upper tail risk in bivariate predictive regressions of the market return.

Table 1
Market return predictability: univariate predictor performance

	One-month horizon			One-year horizon			Three-year horizon			Five-year horizon		
	Coeff.	*t-stat.*	R^2	Coeff.	*t-stat.*	R^2	Coeff.	*t-stat.*	R^2	Coeff.	*t-stat.*	R^2
Tail	4.54	*2.08*	0.7	4.02	*2.04*	6.1	3.65	*2.40*	16.6	3.16	*2.65*	20.9
Book-to-market	2.49	*1.14*	0.2	3.12	*1.34*	3.7	2.26	*1.12*	6.3	2.76	*1.82*	15.5
Default return spread	2.96	*1.36*	0.3	0.43	*0.57*	0.1	0.28	*1.22*	0.1	0.02	*0.18*	0.0
Default yield spread	2.82	*1.29*	0.3	2.93	*1.63*	3.2	1.90	*1.19*	4.5	3.04	*2.80*	14.8
Dividend payout ratio	0.79	*0.36*	0.0	1.55	*0.90*	0.9	1.90	*1.38*	4.4	3.55	*3.68*	10.4
Dividend price ratio	4.24	*1.94*	0.7	4.75	*2.07*	8.5	4.34	*2.56*	23.1	4.19	*3.60*	36.5
Earnings price ratio	3.23	*1.48*	0.4	3.16	*1.48*	3.8	2.54	*1.65*	8.0	3.75	*2.90*	21.5
Inflation	−5.07	*−2.33*	0.9	−1.67	*−1.09*	1.1	0.40	*0.40*	0.2	0.81	*0.71*	1.2
Long-term return	5.40	*2.48*	1.1	1.83	*3.04*	1.3	0.56	*2.16*	0.4	0.68	*2.69*	0.9
Long-term yield	1.95	*0.89*	0.1	3.72	*1.70*	5.2	4.26	*3.48*	21.9	4.59	*5.17*	40.5
Net equity expansion	−0.71	*−0.33*	0.0	−0.03	*−0.01*	0.0	0.44	*0.27*	0.2	−0.14	*−0.13*	0.0
Stock volatility	−6.24	*−2.87*	1.4	0.61	*0.50*	0.1	0.07	*0.13*	0.0	0.01	*0.03*	0.0
Term spread	2.28	*1.04*	0.2	2.57	*1.35*	2.5	2.53	*1.69*	7.7	2.18	*1.72*	9.1
Treasury-bill rate	0.44	*0.20*	0.0	1.78	*0.80*	1.2	2.33	*1.48*	6.3	3.00	*2.56*	15.6
Var. risk prem.*	11.22	*3.45*	4.5	3.32	*2.18*	3.5	0.63	*0.33*	0.3	−1.24	*−1.90*	1.1
R.N. skewness*	−0.74	*−0.17*	0.0	−1.13	*−0.34*	0.3	−0.35	*−0.18*	0.1	−0.32	*−0.37*	0.4
R.N. kurtosis*	−1.82	*−0.43*	0.1	1.56	*0.53*	0.6	1.16	*0.48*	1.0	0.56	*0.59*	1.1

The table reports results from monthly predictive regressions of CRSP value-weighted market index returns over one-month, one-year, three-year, and five-year horizons. The first row reports forecasting results based on our estimated tail risk time series. Next are the results from predictors studied in the survey by Goyal and Welch (2008) (data from Amit Goyal's Web site), as well as the variance risk premium (Bollerslev, Tauchen, and Zhou 2009, data from Hao Zhou's Web site) and risk-neutral skewness and kurtosis based on S&P 500 index options. (*) denotes that a variable is available for a truncated sample: the variance risk premium is only available beginning in 1990, and risk-neutral moments are only available beginning in 1996. Because overlapping monthly observations are used, test statistics are calculated using Hodrick's (1992) standard error correction for overlapping data with lag length equal to the number of months in each horizon. For comparison, reported predictive coefficients are scaled to be interpreted as the percentage change in annualized expected market returns resulting from a one-standard-deviation increase in each predictor variable.

Table 2
Market return predictability: bivariate predictor performance

	One-month horizon					One-year horizon				
	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2
Book-to-market	4.68	*2.15*	2.73	*1.25*	1.0	4.19	*2.27*	3.34	*1.49*	10.3
Default return spread	4.57	*2.10*	3.01	*1.38*	1.1	4.02	*2.05*	0.46	*0.61*	6.2
Default yield spread	4.32	*1.97*	2.42	*1.10*	1.0	3.78	*2.01*	2.58	*1.51*	8.6
Dividend payout ratio	4.67	*2.13*	1.26	*0.58*	0.8	4.22	*2.19*	1.98	*1.14*	7.6
Dividend price ratio	4.32	*1.98*	4.00	*1.84*	1.3	3.77	*2.07*	4.54	*2.03*	13.8
Earnings price ratio	4.22	*1.92*	2.73	*1.25*	1.0	3.70	*1.92*	2.72	*1.31*	8.9
Inflation	4.12	*1.89*	−4.70	*−2.16*	1.5	3.90	*2.01*	−1.32	*-0.88*	6.7
Long-term return	4.04	*1.85*	5.00	*2.29*	1.6	3.88	*1.99*	1.44	*2.82*	6.9
Long-term yield	4.34	*1.91*	0.71	*0.31*	0.8	3.22	*1.54*	2.80	*1.23*	8.8
Net equity expansion	4.86	*2.10*	0.93	*0.40*	0.8	4.53	*2.31*	1.51	*0.53*	6.9
Stock volatility	4.04	*1.86*	−5.90	*−2.71*	2.0	4.10	*2.07*	0.95	*0.77*	6.4
Term spread	4.27	*1.83*	0.80	*0.34*	0.8	3.56	*1.74*	1.33	*0.67*	6.7
Treasury-bill rate	4.53	*2.07*	0.18	*0.08*	0.8	3.93	*1.99*	1.55	*0.74*	7.0

	Three-year horizon					Five-year horizon				
	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2	Tail Coeff.	*Tail t-stat.*	Coeff.	*t-stat.*	R^2
Book-to-market	3.77	*2.56*	2.44	*1.40*	24.0	3.31	*2.55*	2.92	*2.31*	38.4
Default return spread	3.66	*2.40*	0.34	*1.61*	16.7	3.16	*2.65*	0.04	*0.39*	20.9
Default yield spread	3.51	*2.25*	1.57	*1.14*	19.6	2.83	*2.28*	2.57	*2.79*	31.2
Dividend payout ratio	3.86	*2.57*	2.27	*1.72*	22.8	3.36	*2.80*	3.97	*4.55*	33.9
Dividend price ratio	3.40	*2.47*	4.13	*2.65*	37.5	2.88	*2.47*	3.99	*3.66*	53.8
Earnings price ratio	3.40	*2.28*	2.14	*1.54*	22.2	2.84	*2.21*	3.38	*2.77*	38.2
Inflation	3.72	*2.47*	0.73	*0.83*	17.2	3.30	*2.61*	1.25	*1.26*	23.6
Long-term return	3.63	*2.38*	0.18	*0.85*	16.6	3.13	*2.63*	0.28	*1.74*	21.1
Long-term yield	2.63	*1.72*	3.46	*2.53*	29.7	1.99	*1.74*	3.94	*4.68*	48.0
Net equity expansion	4.31	*2.68*	1.91	*1.14*	20.6	3.73	*2.71*	1.62	*1.49*	24.4
Stock volatility	3.69	*2.41*	0.38	*0.67*	16.8	3.17	*2.66*	0.24	*0.66*	21.0
Term spread	3.17	*2.03*	1.41	*1.06*	18.7	2.77	*2.46*	1.11	*0.99*	23.0
Treasury-bill rate	3.51	*2.43*	2.07	*1.45*	21.6	2.98	*2.28*	2.77	*2.75*	34.2

The table reports results from monthly predictive regressions of CRSP value-weighted market index returns over one-month, one-year, three-year, and five-year horizons. The table repeats the analysis of Table 1 but instead reports bivariate regressions that include each alternative predictor alongside the estimated tail risk process. For each horizon, the first two columns are the coefficient estimate and t-statistic for the tail risk process, whereas the third and fourth columns are the coefficient and t-statistic for the alternative predictor. Because overlapping monthly observations are used, test statistics are calculated using Hodrick's (1992) standard error correction for overlapping data with lag length equal to the number of months in each horizon. For comparison, reported predictive coefficients are scaled to be interpreted as the percentage change in annualized expected market returns resulting from a one-standard-deviation increase in each predictor variable.

We also investigate the out-of-sample predictive ability of tail risk. Using data only through month t (beginning at $t = 120$ to allow for a sufficiently large initial estimation period), we run univariate predictive regressions of market returns on tail risk. This coefficient is used to forecast the $t+1$ return. The estimation window is then extended by one month to obtain a new predictive coefficient, and an out-of-sample forecast of the following month's return is constructed. This procedure is repeated until the full sample has been exhausted. Because coefficients are based only on data through t, this procedure mimics the information set an investor would work with in real time. Using the forecast errors from this approach, we calculate the out-of-sample R^2 as $1 - \sum_t (r_{m,t+1} - \hat{r}_{m,t+1|t})^2 / \sum_t (r_{m,t+1} - \bar{r}_{m,t})^2$, where $\hat{r}_{m,t+1|t}$ is the out-of-sample forecast of the

Table 3
Market return predictability: out-of-sample R^2 (%)

	One month	One year	Three years	Five years
Tail	0.3	4.5*	15.7*	20.1*
Book-to-market	−1.6	−9.9	−14.5	−34.0
Default return spread	−0.9	−0.6	−0.4	−0.3
Default yield spread	−0.6	−0.5	−9.2	3.0
Dividend payout ratio	−1.7	−23.6	−13.7	−46.4
Dividend price ratio	−1.3	−6.5	−4.4	15.8*
Earnings price ratio	−1.8	−15.6	−9.3	1.3
Inflation	−0.4	−4.0	−3.1	−11.4
Long-term return	−0.1	0.9	−0.2	0.8
Long-term yield	−0.9	−4.6	20.6*	10.2*
Net equity expansion	−0.8	−10.6	−6.6	−9.0
Stock volatility	−2.5	−25.0	−28.1	−22.1
Term spread	−0.6	−2.9	−7.1	7.0
Treasury-bill rate	−1.3	−9.2	5.8	−14.4

The table reports the out-of-sample forecasting R^2 in percent from predictive regressions of CRSP value-weighted market index returns over one-month, one-year, three-year, and five-year horizons. In each month t (beginning at $t=120$ to allow for a sufficiently large initial estimation period), we estimate rolling univariate forecasting regressions of monthly market returns on the estimated tail risk series and alternative predictors. Predictive coefficient estimates only use data through date t, which are then used to forecast returns at $t+1$. The out-of-sample R^2 is calculated as $1-\sum_t(r_{m,t+1}-\hat{r}_{m,t+1|t})^2/\sum_t(r_{m,t+1}-\bar{r}_{m,t})^2$, where $\hat{r}_{m,t+1|t}$ is the out-of-sample forecast of the $t+1$ return based only on data through t, and $\bar{r}_{m,t}$ is the historical average market return through t. A negative R^2 implies that the predictor performs worse than setting forecasts equal to the sample mean. Because of the short time series for the variance risk premium, out-of-sample forecasts are infeasible. An asterisk (*) beside an estimate denotes that it is statistically significant at the 5% level or better based on the Clark and McCracken (2001) ENC-NEW test of out-of-sample predictability.

$t+1$ return based only on data through t, and $\bar{r}_{m,t}$ is the historical average market return through t. A negative R^2 implies that the predictor performs worse than setting forecasts equal to the historical mean. This recursive out-of-sample forecast approach is also performed using each of the alternative predictors from the preceding tables.[20] The results from this analysis are reported in Table 3. Tail risk forecasts demonstrate similar predictive success out-of-sample. At the one-month, one-year, three-year, and five-year horizons, the tail risk out-of-sample R^2 is 0.3%, 4.5%, 15.7%, and 20.1%, versus 0.7%, 6.1%, 16.6%, and 20.9% in-sample. We conduct tests of out-of-sample predictive power based on Clark and McCracken's (2001) method, which is the benchmark out-of-sample predictive test in the forecasting literature. According to this test, only tail risk and the long-term yield demonstrate statistically significant out-of-sample performance at multiple horizons (at the 5% significance level or better).

In summary, predictive regressions suggest that tail risk is positively and significantly related to market discount rates.

2.3 Tail risk and the cross-section of expected stock returns

We next test the hypothesis that tail risk helps explain differences in expected returns across stocks, consistent with the priced tail risk hypothesis. If investors

20 Due to the short time series for the variance risk premium, out-of-sample forecasts based on options measures are infeasible and thus omitted.

are averse to tail risk, stocks with high predictive loadings on tail risk will be discounted more steeply and thus have higher expected returns going forward. On the other hand, stocks with low or negative tail risk loadings serve as effective hedges and therefore will have comparatively higher prices and lower expected returns.

In line with the aggregate predictive analysis above, we estimate tail risk sensitivities of individual stocks with regressions of the form $E_t[r_{i,t+1}] = \mu_i + \beta_i \lambda_t$. Consistent with the intuition from aggregate tail risk predictive regressions, stocks with high values of β_i are those that are most sensitive to tail risk, and thus are deeply discounted when tail risk is high and have high expected returns going forward. On the other hand, stocks with low or negative β_i are good tail risk hedges because, when tail risk rises, their prices rise contemporaneously, and their expected future returns fall.[21]

Each month, we estimate the tail loading for each stock in regressions that use the most recent 120 months of data.[22] Stocks are then sorted into quintile portfolios based on their estimated tail risk loadings. We track average monthly value- and equal-weighted quintile portfolio returns in a twelve-month postformation window, reported in panel A of Table 4. Portfolio returns are out-of-sample–there is no overlap between data used for estimating betas and data used in the postformation performance period.

Stocks in the highest tail risk loading quintile earn value-weighted average annual returns 4.2% higher than stocks in the lowest quintile, with a t-statistic of 2.2 based on Newey-West standard errors using twelve lags. The equal-weighted high minus low tail risk portfolio average return is 4.0% per annum (t=2.5). Average portfolio returns demonstrate a monotonic pattern that is increasing in tail risk.

Next, we test if the high average return for the long/short tail risk portfolio is robust to considering alternative priced factors. We report alphas from regressions of portfolio returns on the three Fama-French factors, alphas with respect to the Fama-French-Carhart four-factor momentum model, and alphas with respect to the Fama-French-Carhart model plus the Pastor and Stambaugh (2003) traded liquidity factor as a fifth control. Alphas of the value-weighted high minus low tail risk portfolio are large and statistically significant for each of these models. For the three-factor model, the alpha is 5.4% per annum (t=3.0). On an equal-weighted basis, the high minus low tail risk portfolio alpha is 4.0% for the three-factor model (t=2.9). Portfolio alphas retain the same monotonicity that was observed for average portfolio returns.

Panel B reports average returns in a one-month postformation window. These results show that short horizon portfolio returns have the same qualitative

[21] Estimating tail risk sensitivity using predictive regressions on levels of λ, as opposed to contemporaneous regressions on λ's shocks, helps reduce the influence of λ's estimation error on the estimates of tail sensitivities.

[22] This analysis uses all NYSE/AMEX/NASDAQ stocks with CRSP share codes 10 and 11 and at least 36 months out of 120 with nonmissing returns. Portfolios are reconstituted each month.

Table 4
Tail beta-sorted portfolio returns

	Low	2	3	4	High	High-low	t-stat.
Panel A: Twelve-month returns							
			Equal-weighted				
Average return	1.14	1.24	1.31	1.39	1.48	0.33	*2.48*
CAPM alpha	0.10	0.31	0.4	0.47	0.49	0.38	*2.63*
FF alpha	−0.13	0.01	0.09	0.17	0.20	0.33	*2.85*
FF + Mom alpha	0.04	0.10	0.15	0.22	0.26	0.21	*2.15*
FF + Mom + Liq alpha	0.02	0.10	0.17	0.24	0.27	0.25	*2.54*
			Value-weighted				
Average return	0.86	0.99	1.03	1.14	1.21	0.35	*2.15*
CAPM alpha	−0.17	0.05	0.12	0.20	0.18	0.35	*2.10*
FF alpha	−0.20	−0.02	0.08	0.19	0.25	0.45	*3.00*
FF + Mom alpha	−0.02	0.07	0.11	0.18	0.32	0.34	*2.24*
FF + Mom + Liq alpha	−0.08	0.08	0.14	0.21	0.35	0.43	*2.93*
Panel B: One-month returns							
			Equal-weighted				
Average return	1.14	1.24	1.28	1.4	1.45	0.31	*2.12*
CAPM alpha	0.10	0.31	0.37	0.48	0.47	0.37	*2.52*
FF alpha	−0.11	0.02	0.09	0.19	0.19	0.30	*2.22*
FF + Mom alpha	−0.06	0.06	0.11	0.22	0.24	0.29	*2.14*
FF + Mom + Liq alpha	−0.08	0.06	0.12	0.24	0.26	0.34	*2.50*
			Value-weighted				
Average return	0.84	0.96	0.98	1.18	1.20	0.36	*2.00*
CAPM alpha	−0.19	0.03	0.08	0.25	0.18	0.37	*2.08*
FF alpha	−0.19	−0.04	0.05	0.22	0.27	0.46	*2.58*
FF + Mom alpha	−0.16	−0.03	0.03	0.18	0.30	0.45	*2.22*
FF + Mom + Liq alpha	−0.21	−0.03	0.05	0.21	0.35	0.55	*2.78*

The table reports monthly return statistics for portfolios formed on the basis of tail risk beta. Each month stocks are sorted into quintile portfolios based on predictive tail loadings that are estimated from monthly data over the previous ten years. Portfolios are based on NYSE/AMEX/NASDAQ stocks with CRSP share codes 10 and 11. Panel A reports equal- and value-weighted average out-of-sample twelve-month holding period portfolio returns and panel B reports out-of-sample one-month holding period portfolio returns. The table also reports portfolio alphas from regressions of portfolio returns using the Fama-French three-factor model as well as extended four- and five-factor models controlling for momentum and liquidity (Pastor and Stambaugh 2003) factors. The right-most columns report results for the high minus low zero net investment portfolio that is long quintile portfolio five and short quintile one and associated t-statistics. For twelve-month returns, t-statistics use Newey-West (1987) standard errors based on twelve lags. Stocks with prices below $5 at the portfolio formation date are excluded.

behavior as those over longer horizons. The value-weighted three-factor alpha for the high minus low tail risk portfolio is 5.5% annualized (t=2.6), whereas the equal-weighted three-factor alpha is 3.6% annualized (t=2.2).

We also examine the robustness of tail risk's cross-section return explanatory power to controlling for other individual stock characteristics that are potentially associated with return tails. We test whether the return spread between high and low tail risk portfolios is robust to controlling for four alternative firm characteristics. The first characteristic we examine is firm size, measured as equity market value at the time of portfolio formation, which may be an important driver of tail risk if smaller firms are particularly susceptible to tail risk shocks. Next, because our tail measure is derived from tail events among individual firms, we explore its association with the idiosyncratic volatility effect of Ang, Chen, and Xing (2006). We measure firm volatility as the standard deviation of daily residuals from the Fama-French three-factor model in the month prior to portfolio formation. The results are qualitatively unchanged if we use raw returns rather than factor model residuals or different window lengths to calculate firms' volatility. Because our tail risk measure captures an asymmetric downside risk, we investigate how tail risk interacts with the downside beta of Ang, Chen, and Xing (2006) and coskewness of Harvey and Siddique (2000). Downside beta is estimated as the regression coefficient of firm returns on market returns based only on months in which the market return was negative, using the most recent 120 months of data prior to portfolio formation, whereas coskewness is estimated from a regression of firm returns on squared market returns.

Results from independent two-way portfolio sorts are reported in Table 5. We report monthly four-factor postformation alphas. Within each alternative characteristic quartile, we calculate the average returns on the high minus low tail risk portfolio and the corresponding Newey-West t-statistic with twelve lags. Results are broadly consistent with findings reported thus far. Value-weighted spreads within size quartiles are above 3.6% per annum for all but the smallest stocks, are between 2.2% and 5.4% within volatility quartiles, between 2.4% and 4.0% within downside beta quartiles, and between 2.9% and 6.8% for coskewness quartiles.

2.4 Crash insurance

The preceding analysis shows that stocks with low tail risk exposure have low average returns, consistent with the view that investors value the ability of such stocks to hedge against fluctuations in tail risk. We next examine the relative values of contracts explicitly designed to hedge against tail risk. We form portfolios of individual equity put options on the basis of option moneyness following the approach of Frazzini and Pedersen (2012).[23] Moneyness is defined as the absolute value of the Black-Scholes delta of an option, and the five portfolios are deep out-of-the-money (DOTM, $|\Delta| < 0.20$), out-of-the-money (OTM, $0.20 \leq |\Delta| < 0.40$), at-the-money (ATM, $0.40 \leq |\Delta| < 0.60$), in-the-money (ITM, $0.60 \leq |\Delta| < 0.80$), and deep-in-the-money (DITM, $0.80 \leq |\Delta|$). Portfolios are rebalanced corresponding to the monthly expiration schedule for exchange-listed options (the Saturday immediately following the third Friday of the month). Our option sample covers 1996 to 2010.

We compute the return of selling a put with one month to maturity on the first trading day following each expiration date and holding it to the next month's expiration. Each put position is delta hedged daily. We use the standard put return calculation, incorporating the change in option value, the profit or loss

[23] We use data from OptionMetrics and apply data filters that include dropping all observations for which the bid-ask spread is smaller than the minimum tick size, the bid is zero, open interest is zero, embedded leverage is in the top or bottom 1% of the distribution, or time value is below 5%. The time value filter controls for the American exercise feature as discussed in Frazzini and Pedersen (2012).

Table 5
Double-sorted portfolio returns

	Low	2	3	High	High-low	t-stat.
Panel A: Firm size and tail risk beta						
			Equal-weighted			
Small	0.10	0.16	0.26	0.15	0.05	*0.37*
2	−0.07	0.09	0.24	0.23	0.30	*2.54*
3	−0.07	0.11	0.15	0.24	0.31	*2.11*
Large	−0.08	0.03	0.08	0.27	0.35	*1.94*
			Value-weighted			
Small	0.08	0.10	0.19	0.10	0.01	*0.10*
2	−0.07	0.06	0.23	0.24	0.31	*2.57*
3	−0.07	0.10	0.16	0.23	0.30	*2.03*
Large	−0.14	0.01	0.09	0.20	0.34	*1.69*
Panel B: Idiosyncratic volatility and tail risk beta						
			Equal-weighted			
Low IV	0.15	0.16	0.22	0.30	0.15	*1.52*
2	0.10	0.17	0.25	0.33	0.22	*2.33*
3	0.11	0.13	0.24	0.30	0.19	*2.02*
High IV	−0.10	−0.01	0.08	0.11	0.21	*2.02*
			Value-weighted			
Low IV	0.12	0.11	0.14	0.30	0.18	*1.53*
2	−0.01	0.09	0.16	0.30	0.30	*2.25*
3	−0.05	0.07	0.18	0.30	0.36	*2.32*
High IV	−0.17	−0.09	0.11	0.28	0.45	*2.61*
Panel C: Downside beta and tail risk beta						
			Equal-weighted			
Low down beta	0.20	0.18	0.25	0.28	0.08	*0.76*
2	0.08	0.12	0.20	0.29	0.21	*1.92*
3	0.06	0.13	0.21	0.29	0.24	*2.43*
High down beta	−0.04	0.06	0.08	0.17	0.21	*1.97*
			Value-weighted			
Low down beta	0.07	0.11	0.14	0.40	0.33	*2.15*
2	0.11	0.14	0.19	0.40	0.29	*1.72*
3	−0.15	0.01	0.03	0.16	0.31	*2.26*
High down beta	−0.04	−0.04	0.25	0.16	0.20	*0.94*
Panel D: Coskewness and tail risk beta						
			Equal-weighted			
Low coskew	0.00	0.33	0.40	0.40	0.40	*2.91*
2	0.10	0.17	0.28	0.29	0.19	*1.45*
3	0.01	0.06	0.12	0.18	0.17	*1.24*
High coskew	−0.30	−0.23	−0.09	0.03	0.34	*2.30*
			Value-weighted			
Low coskew	−0.17	0.30	0.39	0.41	0.57	*2.43*
2	−0.19	0.12	0.22	0.36	0.54	*2.47*
3	0.04	−0.04	0.01	0.28	0.24	*1.13*
High coskew	−0.30	−0.22	−0.04	−0.03	0.28	*1.30*

The table reports monthly holding period four-factor (Fama-French and momentum) alphas for double-sorted portfolios that are formed on the basis of tail risk loadings and size (panel A), idiosyncratic volatility (panel B), downside beta (panel C) or coskewness (panel D). Each month stocks are independently sorted into four quartile portfolios based each of these characteristics (rows), and four quartiles based on predictive tail loadings (columns). Loading are estimated from monthly data over the previous ten years. Portfolios are based on NYSE/AMEX/NASDAQ stocks with CRSP share codes 10 and 11. The right-most columns report results for the high minus low zero net investment portfolio that is long quartile portfolio four and short quartile one. The portfolios are held for one year t-statistics use the Newey-West (1987) standard errors based on twelve lags. Stocks with prices below \$5 at the portfolio formation date are excluded.

from the delta hedge, and interest on the margin account. We recalculate our monthly tail risk measure to correspond to the expiration schedule, so there is no timing overlap between tail risk in month t and option portfolio returns in $t+1$. We then estimate a predictive regression of each portfolio's return on lagged tail risk. Because of the relatively short sample for options data, we estimate a single in-sample predictive coefficient for each portfolio.

Panel A of Table 6 reports predictive tail betas and average monthly returns on delta-hedged put option portfolios. An investor that is willing to sell crash protection in the form of DOTM puts earns a large insurance premium–the difference between DOTM and DITM short put returns is 16.7% per month (t=3.6). The exposure of option portfolios to tail risk is also monotonically decreasing in moneyness. The difference in tail risk coefficients for the DOTM portfolio versus DITM is 7.2 (t=2.4), meaning that a one-standard-deviation increase in tail risk predicts an increase in the expected return spread (DOTM−DITM) of over 7% in the next month.[24] The far right column reports the correlation between tail beta and portfolio alpha across the five portfolios. There is a 94% correlation between exposures and average portfolio returns.

Frazzini and Pedersen (2012) argue that much of the spread in panel A is due to a premium that financially constrained investors are willing to pay to hold implicitly levered options positions. To account for differences in "embedded leverage," we modify weights in our portfolio construction to equalize the embedded leverage of each portfolio.[25] Panel B reports leverage-adjusted average returns and tail exposures for short put portfolios, again showing that tail risk exposures decrease monotonically with moneyness. Our beta estimate imply that a one-standard-deviation increase in tail risk corresponds to a predicted increase in the DOTM-DITM return spread of 0.8% (t=2.4) in the following month.

3. Potential Mechanisms

In this section we investigate how our estimated tail risk series, which describes tail distributions for individual firms, may be tied to equity premia. A variety of models can potentially generate the hypothesized association between tail risk and risk premia. Rather than specifying a detailed model of preferences and fundamentals, we discuss two general mechanisms that give rise to asset pricing effects of firm-level tail risk.

[24] In all regressions, tail risk is first standardized to have unit variance for ease of interpreting the estimated coefficients.

[25] Equity positions can be levered as much as twenty times using out-of-the-money options. To adjust for leverage, we follow Karakaya (2013) and scale option positions by the elasticity of an option's price with respect to the underlying price. Because embedded leverage also magnifies risk exposure and expected returns, deleveraged return magnitudes are more easily compared to our earlier equity portfolio results.

Table 6
Moneyness-sorted short put portfolios

	DOTM	OTM	ATM	ITM	DITM	DOTM–DITM	t-stat.	Corr.
Panel A: Delta-hedged								
Tail risk beta	7.17	4.20	2.89	1.07	0.01	7.17	*2.39*	–
Average return	19.46	16.64	10.21	5.31	2.80	16.66	*3.55*	94.10
CAPM alpha	18.50	15.88	9.74	5.01	2.53	15.96	*3.43*	93.91
FF alpha	18.05	15.28	9.52	4.90	2.51	15.54	*3.31*	94.54
FF + Mom alpha	17.87	15.21	9.51	4.96	2.62	15.25	*3.18*	94.36
FF + Mom + Liq alpha	15.83	14.17	9.17	4.94	2.68	13.15	*2.82*	92.27
Panel B: Delta-hedged and Leverage-adjusted								
Tail risk beta	0.79	0.46	0.25	0.08	0.00	0.79	*2.43*	–
Average return	1.63	1.57	1.28	0.99	0.56	1.07	*2.43*	79.15
CAPM alpha	1.55	1.49	1.22	0.94	0.50	1.05	*2.29*	77.79
FF alpha	1.55	1.44	1.20	0.91	0.50	1.04	*2.28*	80.79
FF + Mom alpha	1.54	1.43	1.22	0.93	0.54	1.00	*2.07*	80.51
FF + Mom + Liq alpha	1.29	1.32	1.16	0.94	0.56	0.73	*1.54*	65.97

We form portfolios of individual equity put options on the basis of option moneyness following the approach of Frazzini and Pedersen (2012) over 1996 to 2010. Moneyness is defined as absolute value of the Black-Scholes delta of an option, and the five portfolios are deep out-of-the-money (DOTM, $|\Delta| < 0.20$), out-of-the-money (OTM, $0.20 \le |\Delta| < 0.40$), at-the-money (ATM, $0.40 \le |\Delta| < 0.60$), in-the-money (ITM, $0.60 \le |\Delta| < 0.80$) and deep-in-the-money (DITM, $0.80 \le |\Delta|$). We also report results from the difference between the DOTM and DITM portfolios. Portfolio rebalancing dates and one-month holding periods correspond to the monthly expiration schedule. We compute the return of selling an put with one month to maturity on the first trading day following the an expiration date and holding it to the next month's expiration. Each put position is delta-hedged daily. Tail betas are estimated from a predictive regression of each portfolio's return on lagged tail risk (which is recalculated to correspond to the option portfolio formation schedule and variance standardized). Because of the relatively short sample for options data, we estimate a single in-sample predictive coefficient for each portfolio. The far right column reports the correlation between tail beta and portfolio alpha across the five portfolios. Panel A reports results for monthly returns on delta-hedged short put option portfolios. Panel B reports results for leverage-adjusted versions of these portfolios following the procedure of Karakaya (2013).

3.1 Power law aggregation

Power law distributions are stable under aggregation. In particular, when random variables with power law tails are summed, the tail of the sum is dominated by the variable with the heaviest individual tail.[26] Therefore, if individual stock return tails obey a power law, the tail risk of the market portfolio will "inherit" the tail risk of the individual stocks. This property offers a means of inferring aggregate tail risk from the common tail risk of individual stocks.

3.1.1 Tail risk and realized skewness and kurtosis. Assessing the link between our firm-level tail risk and tail risk of the market portfolio is a challenge because time variation in the market's tail exponent is difficult (if not infeasible) to estimate from the time series of market returns alone. This is the original motivation for our panel-based estimator. We examine the third and fourth moments for the market return distribution. Third and fourth moments are only coarse proxies for the market's tail exponent, but they remain useful benchmarks because they are sensitive to movements in lower tail risk, while remaining calculable at the monthly frequency.

26 This property is employed in economic settings by Gabaix et al. (2006) and Gabaix (2009) among others. Jessen and Mikosch (2006) provide a detailed analysis of power law aggregation properties.

We construct monthly realized skewness and kurtosis for the value-weighted market portfolio using daily returns within each month from 1963 to 2010. To test the association between our tail risk measure and realized higher moments of the market portfolio, we run monthly regressions of the form

$$\text{Moment}_{t+k} = \text{constant} + b_1 \text{Moment}_t + b_2 \text{Tail}_t + e_{t+k}, \tag{3}$$

where "Moment" is either realized skewness or kurtosis and k ranges from -24 to 24 months (we drop the Moment_t regressor when $k=0$). This regression measures the common dynamics among Moment_{t+k} and tail risk after controlling for own lags/leads of the realized moments.[27]

Panel A of Figure 3 reports results for realized skewness in regression 3. Bars show estimated b_2 coefficients for different values of k, and the line plot shows corresponding Newey-West t-statistics (using twelve lags). The results illustrate that although there is little association between tail risk and past market skewness, rises in tail risk predict more negatively skewed market returns in the future. This predictive coefficient is negative for all $k > 2$ and is significant after one year. Panel B reports regression results for realized market kurtosis. Tail risk and market kurtosis tend to significantly predict each other. The estimated b_2 coefficients are positive for all k indicating that relatively high levels of tail risk tend to be preceded and followed by higher kurtosis in the market portfolio. The estimated coefficients are larger and more significant for $k > 0$. We find, however, that neither market skewness nor kurtosis demonstrate predictive power for returns in time series or cross-section tests.

3.1.2 Tail risk and option implied risk measures. S&P 500 index options present an alternative means of measuring aggregate market tail risk (under the risk neutral rather than physical measure). In Table 7, we compare our tail risk estimates to various options-based measures of tail risk during the 15-year subsample in which options are available. First, we compare against risk-neutral skewness and kurtosis estimated from S&P 500 index options, following Bakshi, Kapadia, and Madan (2003). We find correlations of -30% and 33%, respectively, indicating that when tail risk rises the risk-neutral market return distribution also becomes more negatively skewed and more leptokurtic. Next, we compare our tail risk time series to the slope of the implied volatility smirk for out-of-the-money S&P 500 put options.[28] Our tail risk measure

[27] We also analyze the association between our tail risk measure and realized moments of individual stock returns. Tail risk has a correlation with the average monthly skewness for all CRSP stocks of -23% ($t=-4.7$) and a correlation with average stock-level kurtosis of 25% ($t=2.5$).

[28] We only use options with positive open interest when calculating risk neutral skewness and kurtosis and the smirk slope. Each of these measures is estimated separately for two sets of options with maturities closest to 30 days (one set for the maturity just greater than 30 days, and one set for the maturity just less than 30 days), then the estimates are linearly interpolated to arrive at a measure with constant 30-day maturity. We estimate the smirk slope in a regression of OTM put-implied volatility on option moneyness (strike over spot) using options with Black-Scholes delta greater than -0.5 and one month to maturity. A more negative slope of the smirk means that OTM puts are especially expensive relative to ATM puts.

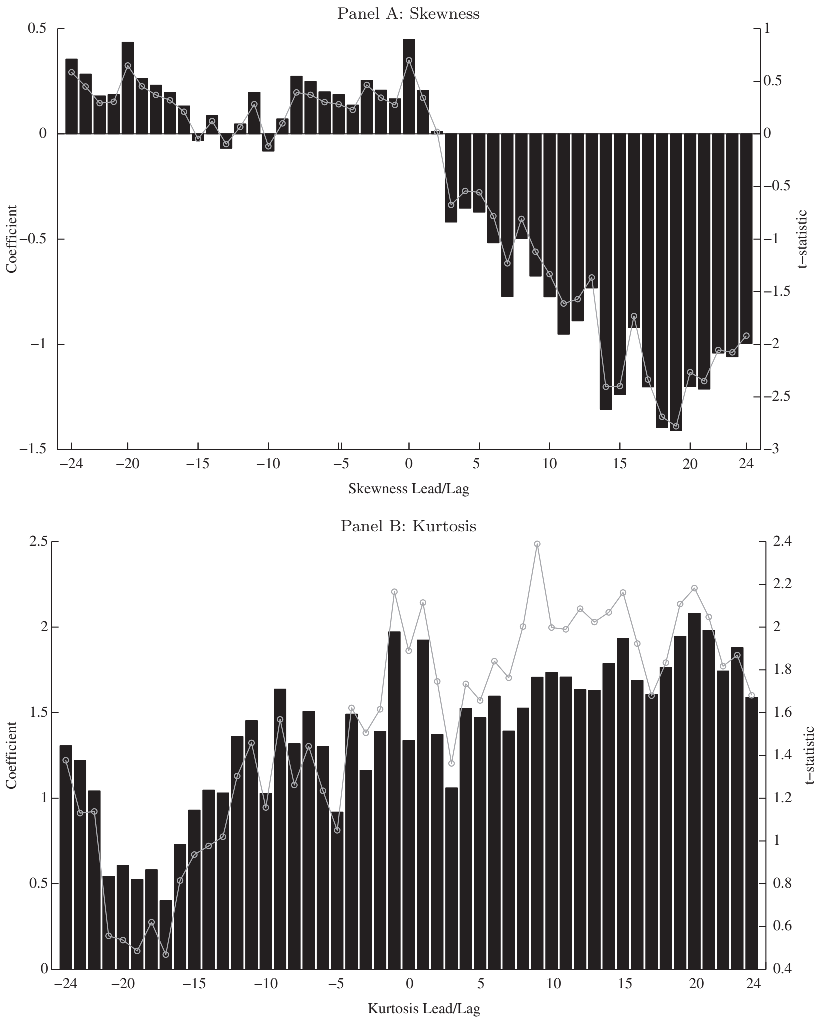

Figure 3
Tail risk versus skewness and kurtosis of the market portfolio
The figure reports estimates of the monthly regression $\text{Moment}_{t+k} = \text{constant} + b_1 \text{Moment}_t + b_2 \text{Tail}_t + e_{t+k}$, where "Moment" is either realized skewness (panel A) or kurtosis (panel B) of aggregate stock market returns, and k ranges from -24 to +24 months (we drop the Moment_t regressor when $k=0$). Realized moments are estimated from daily returns within each month. We report estimated b_2 coefficients for each k (bars corresponding to left axis) and Newey-West t-statistics using twelve months of lags (line plot corresponding to right axis).

has a correlation of -17% with the smirk slope, indicating that OTM puts become especially expensive when tail risk is high. We then calculate the slope of the OTM put-option implied volatility smirk for all individual equities in OptionMetrics and calculate a monthly average across stocks. Our tail risk measure has a correlation of -53% with the average smirk slope of individual

Table 7
Correlations with risk measures based on S&P 500 index options

		(1)	(2)	(3)	(4)	(5)	(6)	(7)
Tail	(1)	1.00						
R.N. skewness	(2)	−0.30	1.00					
		0.02	–					
R.N. kurtosis	(3)	0.33	−0.92	1.00				
		0.01	*<0.01*	–				
OTM put IV slope	(4)	−0.17	0.22	−0.25	1.00			
(S&P 500)		*0.15*	*0.06*	*0.05*	–			
OTM put IV slope	(5)	−0.53	0.49	−0.58	0.26	1.00		
(individual stocks)		*<0.01*	*<0.01*	*<0.01*	*0.13*	–		
Put/call ratio	(6)	0.42	−0.49	0.38	<0.01	−0.65	1.00	
		0.01	*<0.01*	*0.01*	*0.97*	*<0.01*	–	
Variance risk premium	(7)	0.04	0.10	−0.17	−0.10	0.22	−0.22	1.00
		0.67	*0.13*	*0.02*	*0.41*	*0.02*	*0.03*	–

The table reports monthly correlations between tail risk estimated from the cross-section of returns on NYSE/AMEX/NASDAQ stocks, and various options-based risk measures derived from prices of S&P 500 index options from 1996 to 2010. Below each correlation estimate we report its p-value in italics (based on Newey-West (1987) standard errors with twelve lags). Risk-neutral skewness and kurtosis are estimated following Bakshi, Kapadia, and Madan (2003). The slope of the implied volatility smirk for out-of-the-money S&P 500 put options is estimated from a regression of put-implied volatility on option moneyness (strike over spot) using options with Black-Scholes delta greater than −0.5 and one month to maturity. We similarly calculate the implied volatility smirk for all individual stocks in OptionMetrics and construct an equal-weighted average across stocks. The put/call ratio measures the number of new put contracts purchased by non-market-makers relative to new calls purchases and comes from the CBOE. The variance risk premium is the difference between the squared VIX and realized variance of the S&P 500 index and comes from Hao Zhou's Web site. Risk-neutral moments and the IV slope only use options with positive open interest. These three measures are estimated separately for two sets of options with maturities closest to 30 days (one set for the maturity just greater than 30 days, and one set for the maturity just less than 30 days), then the estimates are linearly interpolated to arrive at a daily measure with constant 30-day maturity. Finally, all daily measures are averaged within the month to arrive at a monthly time series.

equity options. Finally, we find a correlation of 42% between tail risk and the CBOE put/call ratio (Pan and Poteshman 2006).[29] With the exception of the variance risk premium, we find that none of the options variables in Table 7 demonstrates predictive power for market returns.

3.1.3 Tail risk dynamics within industry and size groups.

The power law aggregation mechanism relating firm tails to the market tail builds from our specification that tail risks of all assets share a common factor as in Equation (1). To demonstrate empirical support for this specification, we split the sample of CRSP stocks into nonoverlapping subsets and apply our cross-sectional tail risk estimator to each subset. We then show that dynamic tail risk estimates are highly correlated across subgroups.

Because our estimation approach requires a large cross section, we split stocks into moderately large subsets. First, we group stocks into five industries

[29] This ratio measures the number of new put contracts purchased by non-market-makers relative to new calls purchases, depending in part on crash risk perceived by investors. We compute monthly averages of daily put/call ratios from 1996 to 2010 for all option contracts traded on the CBOE.

Table 8
Correlation of dynamic tail risk estimates among subgroups

		(1)	(2)	(3)	(4)	(5)
Panel A: By industry						
Consumer	(1)	1.00				
Manufacturing	(2)	0.85	1.00			
Technology	(3)	0.81	0.75	1.00		
Healthcare	(4)	0.67	0.58	0.69	1.00	
Other	(5)	0.86	0.87	0.77	0.57	1.00
Panel B: By size						
Small	(1)	1.00				
	(2)	0.71	1.00			
	(3)	0.59	0.77	1.00		
	(4)	0.47	0.66	0.76	1.00	
Big	(5)	0.38	0.56	0.67	0.86	1.00

The table reports time series correlation between monthly tail risk series estimated from the cross-section of stocks in each of the five Fama-French industry SIC classifications (panel A) and in each size quintile (panel B) for 1963 to 2010.

according to the SIC code classification of Fama and French. Within each industry, we calculate the cross-section lower tail estimate by pooling daily observations within a month, as in our main tail series construction above. Panel A of Table 8 shows that industry-level tail risks are highly correlated over time, ranging between 57% and 87%. Panel B conducts the same test, but instead groups stocks into equally spaced size (market equity) quintiles each month. Time series correlations for size quintiles range between 38% and 86%. All correlation estimates in Table 8 are highly statistically significant ($p < 0.001$).

3.2 Tail risk shocks and the real economy

The real business-cycle literature suggests a second channel by which shifts in firm-level risk impact investors' marginal utility and therefore asset prices. Bloom (2009) argues that an increase in uncertainty raises the value of a firm's "real options." Because firms face capital and labor adjustment costs, higher uncertainty makes the option to postpone investment more valuable. This can produce aggregate effects if uncertainty at the firm-level tends to rise and fall in unison across firms. Bloom (2009) focuses on uncertainty in the form of volatility, but the economic predictions of his framework are qualitatively identical when uncertainty is measured by tail risk.

3.2.1 Evidence supporting the uncertainty shocks channel. If asset pricing effects arise through a tail uncertainty shocks channel, tail risk should manifest itself not only in returns but also in firms' fundamental growth rate shocks. We check this implication directly by testing for comovement between the tail risk of firm-level sales growth rates and tail risk measured from stock returns. We estimate sales growth tail risk by applying our cross-section tail estimator to the panel of quarterly sales growth data from Compustat. To ensure a sufficiently

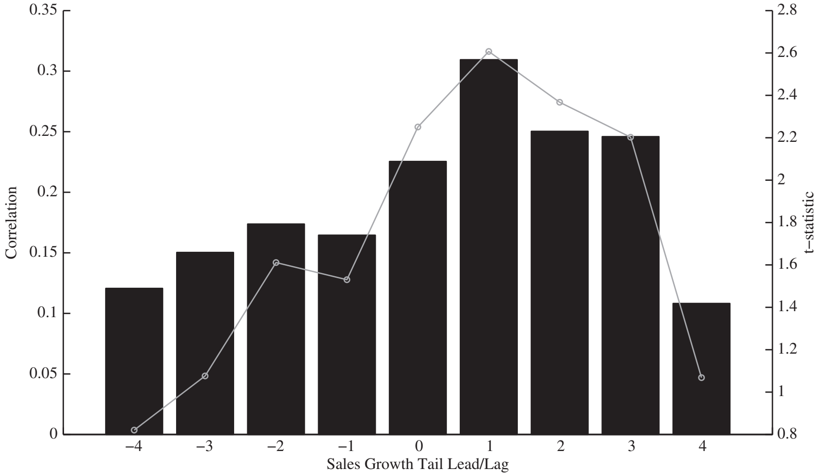

Figure 4
Correlogram: Sales growth tails and stock return tails
The figure shows the percentage correlation (bars corresponding to left axis) between the estimated return tail series in quarter t with the sales growth tail series in quarter $t+j$ for $j=-4,...,4$ and Newey-West t-statistics using twelve months of lags (line plot corresponding to right axis).

large cross section, we pool all reported sales data that occur within the same calendar quarter and use data beginning in 1975.[30]

Figure 4 reports cross-correlations between stock return tail risk in quarter t, and sales growth tail risk in quarters $t-4$ to $t+4$. Despite the coarseness of quarterly sales data, we still find that fundamental cash flow tail risk shares a significant contemporaneous correlation of 23% with the stock return tails (Newey-West p-value of 0.024). Return tails are most strongly correlated with sales growth tails one quarter ahead (31%, $p=0.008$), and remain significantly correlated up to three quarters ahead. The notion that return tail risk leads tail risk measured from sales growth is perhaps unsurprising given the comparatively rapid response of market prices to news and the infrequent reporting of accounting data.

To have pricing effects via the uncertainty shocks channel, tail risk measured from the cross-section ultimately must be associated with aggregate real economic outcomes. Bloom (2009) provides a useful framework to gauge the influence of uncertainty on economic activity and shows that the evolution of uncertainty (measured by stock market volatility) has a large influence on industrial production and employment.

30 Due to the quarterly nature of sales data, we are forced to use a substantially smaller number of observations to estimate the tail risk series. We therefore define the sales growth tail threshold as the 7.5th percentile of the cross-section distribution each year. Quarterly stock return tails are calculated as an average of the monthly tail risk series within each calendar quarter.

We examine the impact of time-varying tail risk on macroeconomic aggregates in a monthly vector autoregression (VAR) that extends Bloom (2009) econometric model to include tail risk. In our VAR ordering, stock market volatility is first, followed by tail risk, the Federal Funds Rate, log average hourly earnings, the log consumer price index, hours, log employment, and log industrial production (the resulting impulse responses are robust to alternative orderings). Because our sample period coincides largely with Bloom (2009), we estimate the VAR using monthly data from July 1963 to June 2008 (as available from Bloom) so that we can quantify the incremental impact of tail risk relative to volatility.

The left-hand plot in panel A of Figure 5 shows the response of industrial production to a one-standard-deviation shock to tail risk.[31] It indicates that industrial production displays an immediate decline of 0.6% within one year of the shock, with a subsequent recovery that peaks at two years. For comparison, the right plot in panel A shows that a volatility shock produces a decline in industrial production of 1.4% with a similar pattern to that of tail risk.[32] These are distinct effects, however, as tail risk and volatility are weakly negatively correlated and included side-by-side in the VAR. Panel B estimates the impulse response for employment. These plots indicate that a shock to tail risk produces the same effect that it does for production, declining in the first year by just over 0.6% and then rebounding at around two years. Panels C and D indicate that, following a shock to tail risk, investment displays an immediate drop of 2.5% to 4% in the subsequent year, followed by a recovery by year three.[33]

In summary, after controlling for the impact of the volatility shocks as emphasized in the previous literature, we find that a positive shock to tail risk precedes an immediate and prolonged contraction in economic activity in the subsequent year. These effects on the real economy, coupled with the effects of tail risk on expected stock returns, suggest that tail risk plays an important role in the marginal utility of investors and in determining equilibrium asset prices.

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[31] Because industrial production and employment are only calculated for the manufacturing sector, the VARs in panels A and B use tail risk estimated from the cross-section of manufacturing firms. The results are nearly identical when tails are estimated including nonmanufacturing firms.

[32] Volatility produces a comparatively large effect because of our use of the volatility indicator constructed by Bloom (2009). It equals one when the peak of HP detrended volatility is more than 1.65 standard deviations above the mean. A "shock" is defined as a movement of this variable from zero to one and thus represents an extreme shift in volatility. If instead we use raw stock market volatility in the VAR (to be more closely comparable to the tail risk measure that we use), the effect of a one-standard-deviation volatility shock is qualitatively similar, but quantitatively much smaller, producing a decline in IP growth of 0.4% after one year, whereas the effect of a tail risk shock is effectively identical to that reported in Figure 5.

[33] Investment is available quarterly (and thus was omitted from Bloom's monthly analysis). We estimate a quarterly trivariate VAR that includes stock market volatility, tail risk, and aggregate investment. Investment is measured as either quarterly gross private domestic or private nonresidential fixed investment (as in Cochrane 1991, 1996)

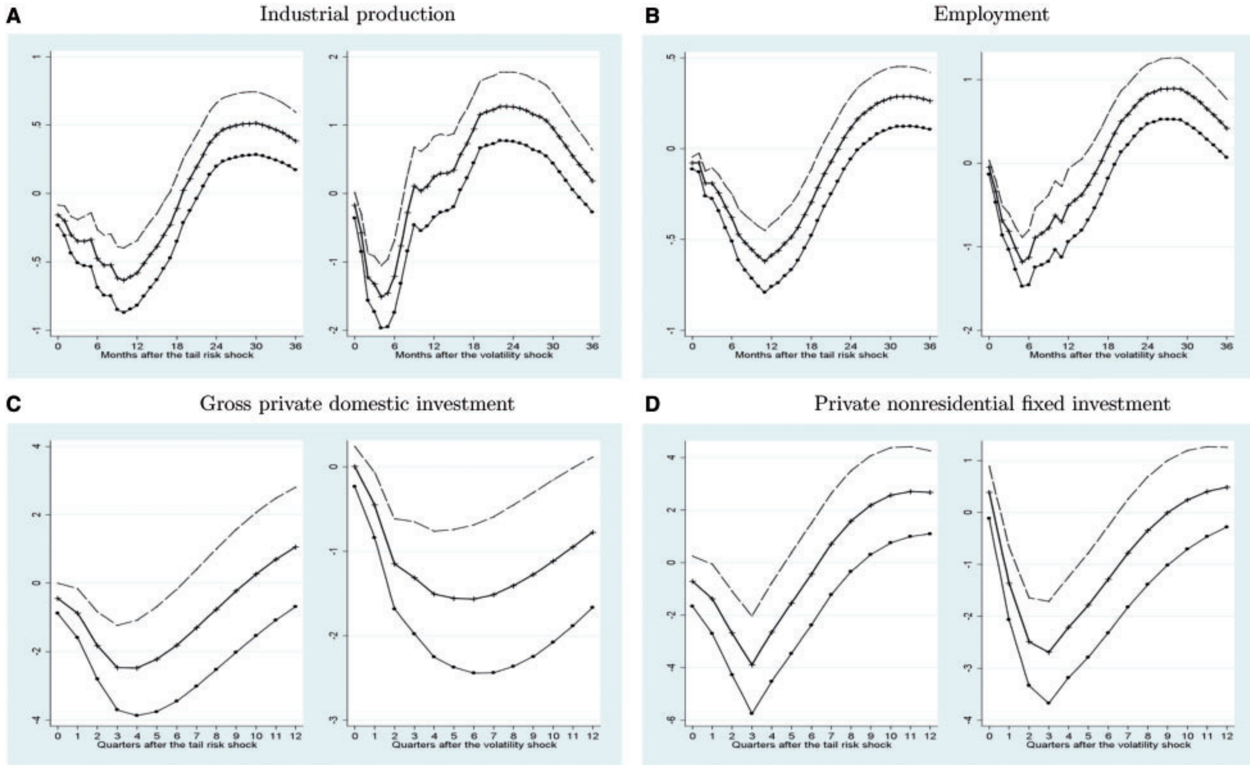

Figure 5
Tail risk impulse response functions
The figure plots the estimated impact of uncertainty shocks on industrial production (panel A), employment (panel B), gross private domestic investments (panel C) and private nonresidential fixed investments (panel D). Within each panel, the impulse response for a one-standard-deviation shock to tail risk on the left and for a one-standard-deviation shock to volatility on the right. For industrial production and employment we estimate a monthly VAR that includes stock market volatility, tail risk, Federal Funds Rate, log average hourly earnings, the log consumer price index, hours, log employment, and log industrial production over the period July 1963 to June 2008. Because investment is only available quarterly, panels C and D are for quarterly trivariate VARs that includes stock market volatility, tail risk, and aggregate investment over the period 1963 Q3 to 2008 Q2. Because industrial production and employment are only calculated for the manufacturing sector, the VARs in panels A and B use tail risk estimated from the cross-section of manufacturing firms. Dashed lines are one standard-error bands following Bloom (2009). Vertical axis is in percent.

4. Conclusion

A measure of extreme event risk is crucial for evaluating modern theoretical asset pricing paradigms. Estimates based on the univariate time series of aggregate market returns are incapable of accurately tracking conditional tail risk. We present a new dynamic tail risk measure that overcomes this difficulty. It uses the cross-section of individual stock returns to estimate conditional tail risk at each point in time.

We provide evidence that tail risk has large predictive power for aggregate stock market returns over horizons of one month to five years, performing as well as the most successful alternative predictors considered in the literature. Furthermore, tail risk has substantial explanatory power for the cross-section of stock and put option returns. Stocks that are effective tail risk hedges earn annual three-factor alphas that are 5.4% lower than their high tail risk counterparts.

These results can be understood from the perspective of structural models with heavy-tailed firm-level shock distributions that are preserved under aggregation. In this case, common fluctuations in tail risk across firms can lead them to simultaneously disinvest, which impairs aggregate economic activity. Power law aggregation and the real effects of uncertainty shocks represent potential channels through which firm-level tail risk can influence asset prices.

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