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Intangible factor and idiosyncratic volatility puzzles

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ABSTRACT

In this paper, we explore whether intangible capital (IC) can help explain idiosyncratic volatility puzzles. The underlying assumption is that firms produce and accumulate IC as part of their normal operations. Investments in IC can either raise a company's future ability to produce or lower its cost of production. The applied model finds empirical support for the hypothesis that IC can help explain idiosyncratic volatility puzzles, especially for firms with higher IC-to-total asset ratios. This paper contributes to existing literature on idiosyncratic volatility puzzles from an IC investment perspective and provides implications for IC on stock markets.

1. Introduction

Contradictory to the classical return-risk relationship, previous research has been able to demonstrate a significant negative correlation between idiosyncratic risks and the expected return on stocks. This is usually referred to as the “idiosyncratic volatility puzzle”.[1] The phenomenon of the idiosyncratic volatility puzzle appears to prevail in all stock markets. Most studies claim that the idiosyncratic volatility puzzle occurs due to short-selling restrictions and heterogeneous beliefs [2]. This claim is, amongst others, supported by Hong and Stein (2007), who found a significant negative correlation between the expected return rate and heterogeneous beliefs. Another explanation for this relationship was provided by Long et al. (2018). In a study conducted on the Chinese market, the scholars found that a negative correlation between the idiosyncratic tail risks and expected stock returns could be explained by a firm's turnover. Contradicting results, however, were obtained by Wang et al. (2016). They demonstrated, that after introducing new control variables, the relationship between idiosyncratic volatility and cross-sectional returns is no longer significant. This indicates that further research is needed to better understand the idiosyncratic volatility puzzle.

Academic literature has already recognized the importance of IC with regard to a firm's tangible capital in asset pricing from a theoretical point of view (Hall, 2000; McGrattan & Prescott, 2010; Gunn & Johri, 2011). However, while the idiosyncratic volatility puzzle has received increasing attention in research, to the best of our knowledge, no prior research has examined the relationship between idiosyncratic volatility and IC. In this article, we therefore investigate the explanatory power of IC on the idiosyncratic volatility puzzle by applying a model that incorporates independent variables representative for IC. Thereby, this article contributes

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[1] Ang et al. (2009) document the so-called “idiosyncratic volatility puzzle,” in each G7 country.

[2] Zaremba et al. (2018) provide both theoretical and numerical evidence that this risk-return relationship might be driven purely by mathematical properties of return distributions.

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to the idiosyncratic volatility puzzle literature from an IC investment perspective and provides information on the impact of IC on market values of listed firms.

Our work builds on the recently published studies of Hou and Johri (2018) as well as Gunn and Johri (2011), who incorporated various variables representative of intangible or knowledge capital into dynamic stochastic general equilibrium models (DSGE). The results of their investigation show that these variables can play an important role in explaining second-order moments of macro variables. In our analysis, we propose a simple modification of the Hou and Johri (2018) model, which is capable of explaining the negative correlation between expected returns and volatility in stock returns. The applied IC model thereby captures the variation of the price of equity shares that are attributable to IC investments. Investments in IC are generally defined as any corporate expenditures (not included in physical capital investment) that are either used to improve future production abilities or to reduce production costs for a given level of technology and conventional inputs of physical capital and labor. IC comprises expenses for new business processes for old and new machinery, expenses for marketing and sales (including advertising) as well as expenses for the development of new products. As firms accumulate IC, a rise in the price of equity shares is accompanied by a decrease in the volatility of equity share returns.

We apply the IC model to the U.S. economy, using three of the major indexes. The results of this research show that IC is positively correlated with a firm's expected returns and negatively correlated with the volatility of a firm's stock returns.

To validate our theoretical model, we use the Fama–French three-factor model to extract idiosyncratic volatility. Furthermore, we apply the Fama–Macbeth two-step regression to determine the quantitative relationship between idiosyncratic volatility and expected returns in the U.S. stock market. Using the ratio of intangible assets to total assets, we divide firms into different categories. Afterwards, we examine the relationship between the ratio of intangible assets to total assets and the idiosyncratic volatility puzzle. The results of the examination indicate a negative correlation between idiosyncratic volatility and the ratio of intangible assets and are therefore consistent with the prediction of the IC model. We further create different subsamples by dividing firms based on their idiosyncratic volatility and their firm-level ratio of intangible assets to total assets, respectively. Derived from the results of this analysis, we can confirm that the idiosyncratic volatility puzzle could be explained by a firm's intangible assets. The explanatory power is even higher for firms with low idiosyncratic volatility and for firms with high intangible asset ratios.

Next to the contribution this paper makes towards better understanding the idiosyncratic volatility puzzle, it also contributes to finance literature on intangible capital. Related topics in this field of research were for example addressed by Eisfeldt and Papanikolaou (2013) who found that firms with more organizational capital exhibit higher average stock returns, suggesting this type of IC makes these firms riskier to shareholders. Li and Hou (2019) demonstrated that in the Chinese stock market, firms with investments in knowledge capital will have a higher valuation growth than firms without investments in knowledge capital. Qiu et al. (2019) found that a firm's patent-to-market ratio is a pricing factor distinct from known factors in the cross-section of stock returns. The patent-to-market ratio is furthermore positively associated with future profitability.

2. The theoretical model: firm value and investment in intangible capital

In this section, we explore the idea that firms may improve future productivity at the cost of diverting resources to engage in the creation of IC. IC is included in the research model as the third production input variable next to labor and physical capital. We embed this feature into an otherwise standard dynamic general equilibrium model with imperfect competition. If firm-level investments in IC are pro-cyclical, then the extra productivity generated by these investments can result in profits rising more than output in future periods of high activity and consequently can increase the value of firms.[3]

As discussed in the introduction, a key feature of the IC model is its ability to explain the negative correlation between expected returns and idiosyncratic volatility. While IC is positively correlated with expected returns, it is negatively correlated with return volatility. The firm's dividend or profit for each period is calculated using the following equation:

$$d_t(i) = \vartheta_t(i)y_t(i) - w_tN_t(i) - r_t{}^kK_t(i) \tag{1}$$

where a good-producing firm i pays the dividend d_t for each unit of firm equity, whereby cost factors occur in form of labor $N_t(i)$ and physical capital $K_t(i)$ taking as given the wage rate w_t and the rental rate of capital, $r_t{}^k$.

A firm's investments into IC is defined as $I_t{}^z$ and is calculated as follows:

$$I_t{}^z = w_tN_t(1 - u_t{}^n) + r_t{}^kK_t(1 - u_t{}^k) \tag{2}$$

However, not the entire amount of labor and capital employed by the firm is used for the production of the final good, $y_t(i)$. The variables $u_t^n(i) \in [0, 1]$, and $u_t^k(i) \in [0, 1]$ denote the fraction of labor and physical capital that a firm allocates to each input factor for the production output. As shown in the appendix, we can express the relationship between firm's dividend, output and investment in IC as:

$$d_t = \left(1 - \frac{1-\varepsilon}{\eta}\right)y_t - I_t^z \tag{3}$$

Eq. (3) presents the trade-off a firm encounters. Investments in IC will improve a firm's future efficiency, thus raising expected dividends and consequently, increasing returns of equity shares. However, this development is accompanied by the reduction of

[3] The model economy is presented in the appendix due to space constraints.

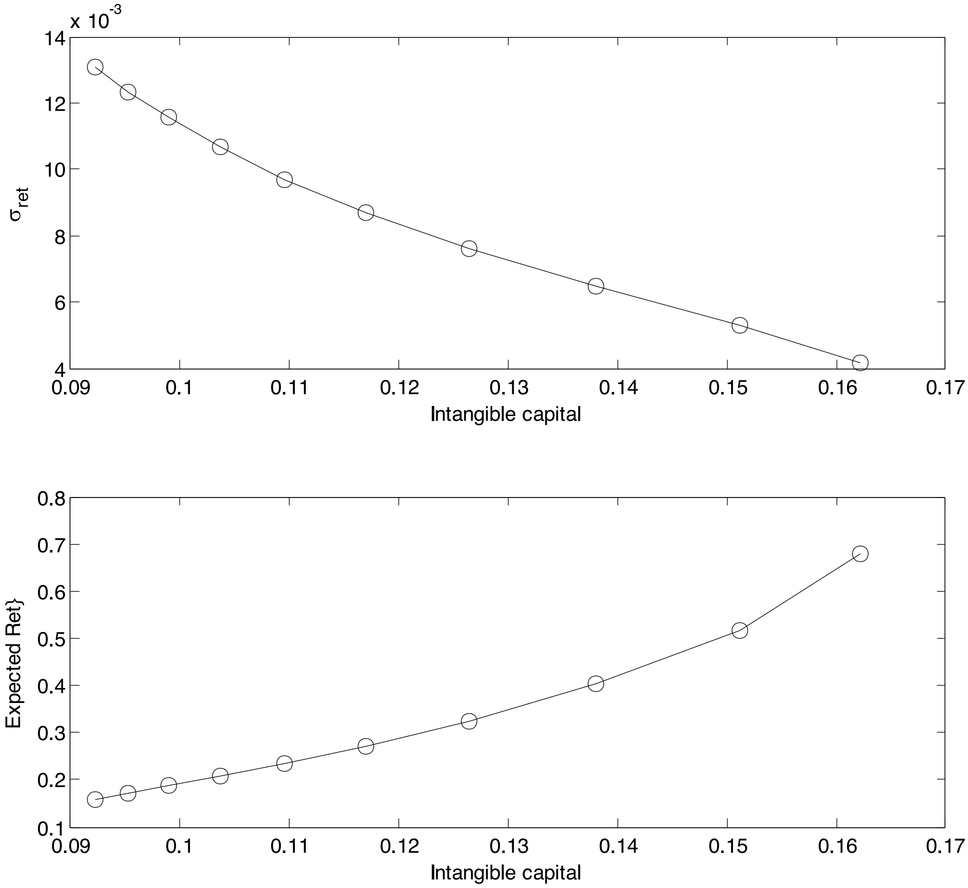

Fig. 1. Intangible capital, expected return and volatility.

current period profits due to reallocation of resources away from the production of the final good.

The results of the variations in firm values will be presented later in this research. Following the approach of Gunn and Johri (2011), the value of a firm, measured at the end of period *t*, and therefore the price of equity can be defined as (see appendix for further explanation):

$$V_t = \frac{\lambda_t^z}{\lambda_t^y} Z_{t+1} \tag{4}$$

where $\frac{\lambda_t^z}{\lambda_t^y}$ refers to the relative marginal price of new IC in terms of consumption. Eq. (4) shows that the value of a firm is determined by the total value of its existing stock of knowledge, obtained as a product of the marginal value of firm-specific IC, $\frac{\lambda_t^z}{\lambda_t^y}$, and the stock of firm-specific IC. In addition, the contrary movements in u_t^n and u_t^k counteract the cyclical variations in the price of equity shares, V_t. Therefore, IC raises the firm-specific expected returns but moderates the volatility of the returns of equity shares. The results of the IC model can be seen in Fig. 1. While IC is positively correlated with the expected firm-specific returns, it exhibits a negative correlation with the volatility of returns.

As shown in Fig. 2, a firm with IC investment will have a higher valuation growth than a firm without IC investment. This prediction from our IC model is largely consistent with the stylized facts in the US stock market.

3. Empirical results

3.1. Data descriptions

Our sample consists of firms listed on the NYSE, AMEX, and NASDAQ. We obtain accounting data from Compustat and stock returns data from the Center for Research in Security Prices (CRSP). Only domestic common shares traded companies from these indices are included for which accounting and return data is available. Firms operating in the financial sector are excluded. Furthermore, this analysis follows the approach of Fama and French (1993) and therefore excludes closed-end funds, trusts, American depository receipts, real estate investment trusts, units of beneficial interest, and firms with negative book values of equity from our

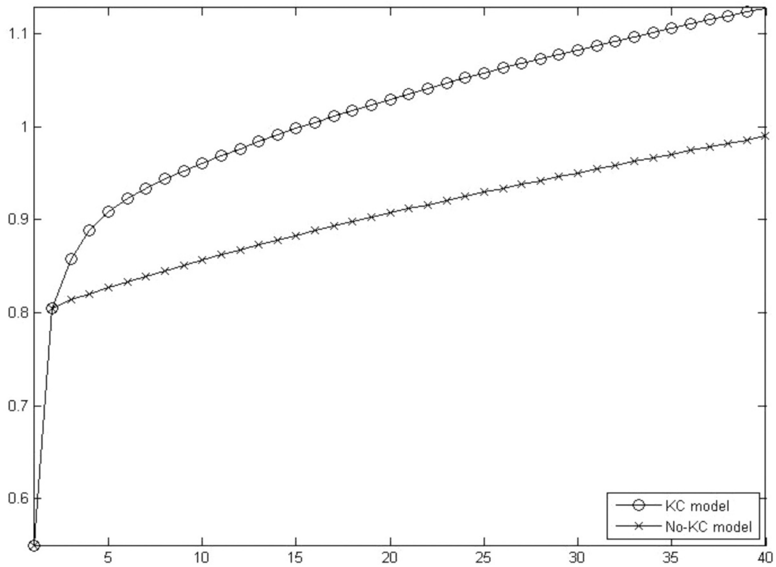

Fig. 2. Simulated values of firms with/without IC.

Table 1
Descriptive statistics of idiosyncratic volatility.

Num	EIV mean(%)	Std.(%)	Skewness	Kurtosis
9837	4.435	2.772	2.261	10.444

investigation. To mitigate backfilling bias, only firms are considered in the sample which were listed on Compustat for at least two years. The sample consists in total of 9,837 different firms, covering a period of 21 years from 1982 to 2012.

3.2. *Idiosyncratic volatility analysis*

We perform a three-factor regression analysis on the eligible stocks to obtain the residuals for each stock. Afterwards, we execute a basic descriptive statistics analysis. The results of this analysis can be seen in Table 1. In addition, due to the large number of stocks, an arithmetic mean analysis of all the stocks is applied.

3.3. *Fama–Macbeth cross-sectional regression estimates*

To examine the conditional relationship between firm-level idiosyncratic volatility and intangible assets, we estimate a series of Fama-MacBeth monthly cross-sectional regressions. The dependent variable in this analysis is the firm-level idiosyncratic volatility in month *t*. The explanatory variables included in the applied regressions are various firm characteristics. These include stock prices and firm size measured at the end of the previous month, lagged idiosyncratic volatility, intangible assets divided by total assets, the past 12-month stock returns with a one month gap as momentum, and absolute monthly stock returns divided by monthly dollar trading volume as a measure of stock illiquidity.[4] As we can tell from Table 2, the intangible asset measure exhibits a statistically significant negative correlation with idiosyncratic volatility. This finding is consistent with the theoretical prediction of the correlation between IC and firm-specific volatility in the IC model.

We also calculate the empirical distribution of intangible assets for firms with high idiosyncratic volatility and firms with low idiosyncratic volatility, respectively. Fig. 3 shows that the distribution of intangible assets of firms with low idiosyncratic volatility exhibit a fatter tail than the distribution of intangible assets of firms with high idiosyncratic volatility. This result suggests that firms with high intangible assets are more likely to have a lower idiosyncratic volatility.

In Table 3, we examine if the idiosyncratic volatility puzzle still exists when we control for the firm-level intangible assets to total assets ratio. From the results depicted in Table 3, we can conclude that the idiosyncratic puzzle still exists after adding the intangible assets to total assets ratio into the regression, although both the coefficient and the *t*-statistics of the IVOL factor decreased. This indicates that the idiosyncratic puzzle can at least partially be explained by a firm's intangible assets. Consequently, the estimates are largely in line with the IC model's predictions.

[4] All independent variables are normalized to zero mean and one standard deviation after winsorization at the 1% and 99% levels.

Table 2
Determinants of idiosyncratic volatility: Fama–MacBeth cross-sectional regression estimates.

Explanatory variable	(1)	(2)	(3)
Log(Size)	−0.37%	−0.37%	−0.38%
	(−38.34)	(−38.51)	(−38.99)
Log(Price)	−0.19%	−0.19%	−0.18%
	(−21.35)	(−21.43)	(−23.29)
Log(Lagged Idiosyncratic Volatility)	1.74%	1.74%	1.72%
	(−31.73)	(−31.72)	(31.36)
Intangible assets to total assets		−0.02%	−0.02%
		(−7.93)	(−7.02)
Log(Book to Market)			−0.09%
			(−15.53)
Momentum			−0.03%
			(−2.38)
Illiquidity			0.09%
			(12.57)
Industry Dummy	Yes	Yes	Yes
Adjusted R square	50.02%	57.04%	57.94%

Note: All models control for industry effects based on the Fama–French 48 industries. *R*-square (in percentage) is the time-series average of the *R*-square from the monthly cross-sectional regressions for each model.

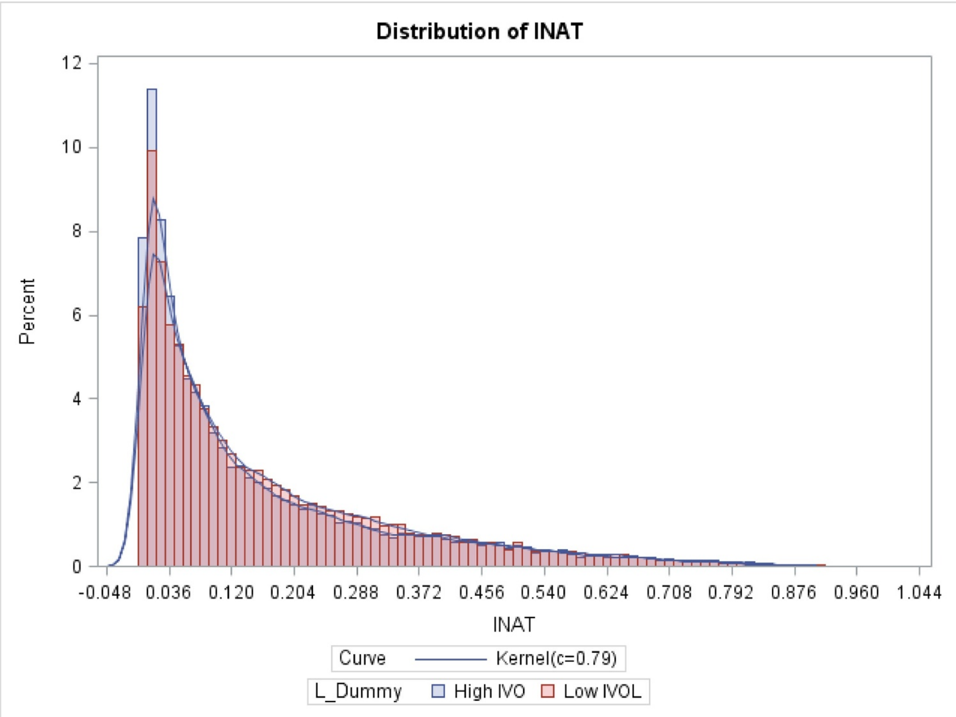

Fig. 3. Empirical distribution of intangible assets.

To gauge the relationship between firm-level intangible assets and the idiosyncratic volatility, we divide the sample, into different subsamples, based on idiosyncratic volatility and the intangible assets to total assets ratio. We conduct subsample Fama-MacBeth cross-sectional regressions to test the explanatory power of intangible assets in relation to the idiosyncratic puzzle. The results can be seen in Table 4. We first divide firms independently into two idiosyncratic volatility groups (high and low idiosyncratic volatility). This categorization is based on the 30th and 70th percentiles of idiosyncratic volatility of the previous month. We then conduct Fama-MacBeth cross-sectional regressions for the high and low idiosyncratic volatility groups. As we can tell from Table 4, the idiosyncratic puzzle exhibits an insignificant value after adding intangible assets into the group with low idiosyncratic volatility. However, it can be noticed that the factor of intangible assets has a significant positive impact of 0.6% on the expected annual returns in the low idiosyncratic volatility group. These empirical findings further support the theoretical predictions of the relationship between IC and idiosyncratic volatility.

We also examine subsamples that are categorized based on firm-level ratios of intangible assets to total assets. Table 5 depicts the

Table 3
Idiosyncratic volatility and intangible assets: Fama–MacBeth cross-sectional regression estimates.

Explanatory variable	(1)	(2)	(3)	(4)
Market beta	−0.14%	−0.14%	0.05%	0.03%
	(−1.30)	(−1.30)	(0.48)	(0.30)
SMB beta	−0.29%	−0.29%	−0.26%	−0.25%
	(−4.39)	(−4.40)	(−4.20)	(−4.11)
HML beta	0.10%	0.10%	−0.01%	−0.02%
	(0.99)	(1.03)	(−0.11)	(−0.26)
Idiosyncratic volatility	−0.31%	−0.26%	−0.23%	−0.22%
	(−3.77)	(−2.81)	(−2.12)	(−2.03)
Intangible Assets to Total Assets		−0.13%	−0.13%	−0.13%
		(−4.34)	(−4.68)	(−4.75)
Log(Size)			0.24%	0.25%
			(2.97)	(3.12)
Log(Price)			−1.14%	−1.13%
			(−12.98)	(−14.41)
Log(Book to market)			0.31%	0.32%
			(5.33)	(5.84)
Momentum				0.05%
				(0.78)
Illiquidity				0.16%
				(3.87)
Industry dummy	Yes	Yes	Yes	Yes
Adjusted *R* square	5.36%	6.11%	6.88%	7.36%

Note: The dependent variable is monthly stock return. All models control for industry effects based on the Fama–French 48 industries. *R*-square (in percentage) is the time-series average of the *R*-square from the monthly cross-sectional regressions for each model.

Table 4
Idiosyncratic volatility and intangible assets: subsample Fama–MacBeth cross-sectional regression estimates.

Explanatory variable	High	Low
Market beta	0.16%	0.14%
	(1.15)	(2.08)
SMB beta	−0.33%	−0.02%
	(−3.48)	(−0.56)
HML beta	−0.11%	−0.06%
	(−0.87)	(−1.18)
Idiosyncratic volatility	−0.34%	−0.02%
	(−3.52)	(−0.50)
Intangible assets to total assets	−0.20%	0.05%
	(−1.90)	(2.03)
Log(Size)	−0.66%	0.15%
	(−4.65)	(2.39)
Log(Price)	−1.24%	−0.53%
	(−11.47)	(−10.82)
Log(Book to Market)	0.40%	0.10%
	(4.51)	(2.58)
Momentum	0.02%	0.01%
	(0.21)	(0.20)
Illiquidity	0.27%	−0.07%
	(3.60)	(−1.70)
Industry Dummy	Yes	Yes
Adjusted R square	9.09%	13.92%

Note: All models control for industry effects based on the Fama–French 48 industries. The sample periods are from 1982 to 2012. *R*-square (in percentage) is the time-series average of the *R*-square from the monthly cross-sectional regressions for each model.

estimates of the subsample Fama–MacBeth cross-sectional regressions, in which the dependent variable is a firm's monthly stock return. We divide firms independently into two intangible assets groups (high and low intangible assets). The categorization is again based on the 30th and 70th percentiles of intangible assets. As shown in Table 5, in the “high” category, the intangible asset factor has a statistically significant impact of 0.36% on the expected annual returns at the 10% level. However, the idiosyncratic puzzle exhibits insignificant results after controlling for intangible assets to the “high” group of intangible assets. This finding also validates the implications of the IC model.

Table 5
Idiosyncratic volatility and intangible assets: subsample Fama–MacBeth cross-sectional regression estimates sorted on intangible assets.

Explanatory variable	High	Low
Market beta	0.07%	−0.01%
	(0.56)	(−0.06)
SMB beta	−0.33%	−0.21%
	(−3.88)	(−2.63)
HML Beta	−0.01%	−0.14%
	(−0.11)	(−1.18)
Idiosyncratic volatility	−0.26%	−0.37%
	(−1.69)	(−3.70)
Intangible assets to total assets	0.03%	−0.15%
	(1.86)	(−4.17)
Log(Size)	0.30%	0.13%
	(3.26)	(1.41)
Log(Price)	−1.10%	−1.11%
	(−11.50)	(−13.11)
Log(Book to Market)	0.22%	0.39%
	(3.65)	(6.28)
Momentum	0.09%	0.07%
	(1.12)	(0.94)
Illiquidity	0.26%	0.12%
	(3.40)	(2.18)
Industry dummy	Yes	Yes
Adjusted R square	10.77%	10.04%

Note: All models control for industry effects based on the Fama–French 48 industries. *R*-square (in percentage) is the time-series average of the *R*-square from the monthly cross-sectional regressions for each model.

4. Conclusion

In this paper, we explored whether intangible capital (IC) can help explain idiosyncratic volatility puzzles. The underlying assumption was that firms produce and accumulate IC as part of their normal operations. The IC model captures the variation of the price of equity shares that are attributable to IC investments. As firms accumulate IC, a rise in the price of equity shares is accompanied by a decrease in the volatility of returns of equity shares. Empirical support has been found for this phenomenon. We find that idiosyncratic volatility is negatively correlated to the ratio of intangible assets to total assets, which is consistent with the theoretical predictions of the IC model. We further construct different subsamples by dividing the dataset based on the factors of idiosyncratic volatility and the firm-level ratio of intangible assets to total assets, respectively. In addition, we demonstrated the explanatory power of intangible assets with regard to the idiosyncratic volatility puzzle for firms with idiosyncratic volatility and firms with high intangible asset ratios.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.frl.2019.101403.

Appendix A. Intangible factor and idiosycratic volatility puzzles

1. The model economy

In the appendix, we incorporate intangible capital into the production technology of firms in a standard DSGE model with imperfect competition. For convenience, we refer to this model as the IC model.

1.1. The household's problem

The representative household maximizes its expected discounted utility over an infinite time horizon:

$$Max\, E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, B_t),$$
$$where\ U(C_t, N_t, B_t) = \frac{(C_t^{\theta}(1-N_t)^{1-\theta})^{1-\sigma}}{1-\sigma} \tag{A1}$$

Here, β is the discount factor, σ > 0 determines the household's coefficient of relative risk aversion and intertemporal elasticity of substitution. The parameter θ governs the fraction of time spent on working. The utility function in period t depends positively on contemporaneous consumption, C_t, and labor supply, N_t. The variable B_t represents a shock to preferences and follows a first-order autoregressive process with an *i.i.d.* error term:

$$lnB_t = \rho_b lnB_{t-1} + \varepsilon_{bt} \tag{A2}$$

In each period, the representative household supplies labor and physical capital to good-producing firm, taking as given the wage rate w_t and the rental rate on capital, r_t^k. In addition, as the owner of the firm, the household receives the dividend income d_t for each unit of its outstanding holdings of firm equity a_t. v_t is the price of equity. For convenience, we normalize the firm's outstanding number of shares to unity. The household allocates its earnings between consumption, investment in physical capital and equity shares. The sequence of budget constraints is given by:

$$C_t + I_t + v_t a_{t+1} = w_t N_t + r_t^k K_t + (d_t + v_t)a_t \tag{A3}$$

Investment augments the physical capital stock over time according to:

$$K_{t+1} = I_t + (1-\delta)K_t \tag{A4}$$

where δ ∈ (0, 1) is a constant depreciation rate for physical capital.

Given the initial values, the household chooses $\{C_t, N_t, I_t, K_{t+1}, a_{t+1}\}$, $t = 0, 1, 2, \ldots$, to maximize the objective function (1) subject to the budget constraint (3) and the capital accumulation Eq. (4). The first-order conditions associated with this problem are:

$$w_t = \Xi_t \frac{U_{n,t}}{U_{c,t}} \tag{A5}$$

$$1 = \beta E_t \left[\frac{U_{n,t}}{U_{c,t}} (r_{t+1}^k + 1 - \delta) \right] \tag{A6}$$

$$v_t(i) = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} [v_{t+1} + d_{t+1}] \right\} = \sum_{s=1}^{\infty} E_t \left\{ \beta^s \frac{\lambda_{t+s}}{\lambda_t} d_{t+s} \right\} \tag{A7}$$

where $U_{c,t}$ and $U_{n,t}$ are, respectively, the marginal utility of consumption and marginal utility of leisure. Ξ_t is the Lagrange multiplier of the constraint maximization of the household's problem. It is also clear from (7) that the price of the firm's equity will equal the stochastically-discounted lifetime stream of the firm's profits beginning in period $t+1$.

1.2. The final good producers

There are large number of final good producers of measure 1 who behave competitively and use $y_t(i)$ units of a continuum of intermediate good $i \in [0, 1]$, to produce Y_t units of the final good. Assuming that all intermediate goods are imperfect substitutes with a constant elasticity of substitution, $\frac{\eta}{\eta-1}$, the corresponding Dixit-Stiglitz aggregator can be defined as:

$$Y_t = \left[\int y_t(i)^{\eta} di \right]^{\frac{1}{\eta}}, \quad \eta > 1 \tag{A8}$$

Given the relative price vector, $\vartheta_t(i)$, the final-good producer chooses the quantity of intermediate good $y_t(i)$ that maximizes its profits,

$$\max_{y_t(i)} Y_t - \int \vartheta_i(i) y_t(i) di$$

Subject to the constraint imposed by (2.6). Note that $\vartheta_t(i)$ is the relative price charged by the ith intermediate good producer. The first order conditions give us the input demand functions:

$$y_t(i) = \vartheta_t(i)^{-\frac{\eta}{\eta-1}} Y_t \tag{A9}$$

where $\frac{\eta}{\eta-1}$ measures the price elasticity of demand for intermediate good i.

1.3. Intermediate good producers

The economy is populated by a continuum of intermediate good producers indexed by $i \in [0, 1]$. Firm I produces the differentiated good I according to the following constant returns to scale technology which uses labor $N_t(i)$, physical capital $K_t(i)$ and intangible capital $Z_t(i)$ as inputs:

$$y_t(i) = (A_t u_t^n(i) N_t(i))^{\alpha} (u_t^k(i) K_t(i))^{1-\alpha-\varepsilon} Z_t(i)^{\varepsilon} \tag{A10}$$

The presence of a third input, knowledge capital, $Z_t(i)$, in addition to the usual labor, $N_t(i)$, and physical capital, $K_t(i)$, is what distinguishes the model from the standard structure. Not all of the labor and capital hired by the firm goes to the production of the final good, $y_t(i)$. The variables, $u_t^n(i) \in [0, 1]$, and $u_t^k(i) \in [0, 1]$ denote, respectively, the fraction of labor and physical capital that the firm allocates to output production. The remainder of labor and capital are used to produce new IC. The technology shock, A_t, is assumed to follow a random walk with drift process:

$$lnA_t = \gamma_a + lnA_{t-1} + \varepsilon_{at}, \quad \varepsilon_{at} \sim iid(0, \sigma_{at}) \tag{A11}$$

The stock of IC for the next period requires labor, physical capital, and IC. The IC technology is given by:

$$Z_{t+1}(i) = \left[(A_t(1 - u_t^n(i)) N_t(i))^{\alpha_1} ((1 - u_t^k(i)) K_t(i))^{1-\alpha_1} \right]^{1-\gamma} Z_t^{\gamma}(i) \tag{A12}$$

where $\alpha_1(1 - \gamma)$ and $(1 - \alpha_1)(1 - \gamma)$ represents, respectively, the elasticity of hours and capital spent on creating IC in the current period with respect to IC in the next period. When $\alpha_1 = 0$, the firm allocates all of its labour to the production of the final good, and when $\alpha_1 = 1$, physical capital is no longer used in IC creation. The other parameter, $\gamma \in (0, 1)$, indicates that the contribution of past IC decays further back in time from when it was created. This captures the idea that the relevance of knowledge falls with time as the economic environment undergoes changes. This is consistent with the notion of organizational forgetting and the depreciation of organizational capital discussed in the learning literature.[5] We would expect the performance of the IC model to approach that of the no-IC model for values of γ close to unity. To see this, note that $\gamma = 1$ implies that IC is constant over time. Note also that $\gamma = 0$ implies that IC available to the firm at present makes no contribution to future IC levels. The productivity shock appears in Eq. (A9) to ensure a balanced growth path. It can imply that increases in the productivity of labour over time apply to both activities of the firm production. This appears to be a reasonable assumption.

In each period, producer i choose contingency plans to for $\{\vartheta_t(i), u_t^n(i), u_t^k(i), N_t(i), K_{t+1}(i), Z_{t+1}(i)\}_{t=0}^{\infty}$ to maximize the current and expected future lifetime dividends:

$$V_t(i) = d_t(i) + E_t \sum_{s=1}^{\infty} \Xi_s \{d_{t+s}(i)\} = d_t(i) + \Xi_t E_t[V_{t+1}(i)] \tag{A13}$$

subject to (7) and (9), where the variable $\Xi_t = \beta \frac{U_{c,t+1}}{U_{c,t}}$ is the appropriate endogenous discount factor for the firm, and where we have defined

$$d_t(i) = \vartheta_t(i) y_t(i) - w_t N_t(t) - r_t^k K_t(i) \tag{A14}$$

and $E_t[V_{t+1}(i)] = E_t\{d_{t+1} + \sum_{s=1}^{\infty} (\Xi_{t+1+s} d_{t+1+s}(i))\}$ as the discounted value of the firm's future lifetime stream of dividends at period t +1. The first order conditions are then:

$$w_t = \alpha \lambda_t^y(i) \frac{y_t(i)}{N_t(i)} + \alpha_1(1 - \gamma) \lambda_t^Z(i) \frac{Z_{t+1}(i)}{N_t(i)} \tag{A15}$$

$$r_t^k = (1 - \alpha - \varepsilon) \lambda_t^y(i) \frac{y_t(i)}{K_t(i)} + (1 - \alpha_1)(1 - \gamma) \lambda_t^Z(i) \frac{Z_{t+1}(i)}{K_t(i)} \tag{A16}$$

$$\alpha \lambda_t^y(i) \frac{y_t(i)}{u_t^n(i)} = \alpha_1(1 - \gamma) \lambda_t^Z(i) \frac{Z_{t+1}(i)}{1 - u_t^n(i)} \tag{A17}$$

$$(1 - \alpha - \varepsilon) \lambda_t^y(i) \frac{y_t(i)}{u_t^k(i)} = (1 - \alpha_1)(1 - \gamma) \lambda_t^Z(i) \frac{Z_{t+1}(i)}{1 - u_t^k(i)} \tag{A18}$$

$$\lambda_t^Z(i) = E_t \left[\frac{\Xi_{t+1}}{\Xi_t} \left(\varepsilon \lambda_{t+1}^y(i) \frac{y_{t+1}(i)}{Z_{t+1}(i)} + \gamma \lambda_{t+1}^Z(i) \frac{Z_{t+2}(i)}{Z_{t+1}(i)} \right) \right] \tag{A19}$$

$$\vartheta_t(i) = \eta \lambda_t^y(i) y_t(i) \tag{A20}$$

where λ_t^y and λ_t^Z is the Lagrange multiplier associated with Eq. (A9). Eqs. (A10) and (A11) differ from the typical conditions in that the firm will not equate the marginal product of labor and physical capital, respectively, to their factor prices. Rather, the prices will be higher than the marginal products. This occurs because only a part of the labour and capital is used in production; the rest is used to produce IC, which in turn raises production and hence profits in the future. This dynamic consideration facing the firm when it

[5] Note that the IC technology may be viewed as a log-linear accumulation equation for IC, with γ governing the depreciation rate of IC.

decides how much labour and capital to hire shows up in the additional term involving Z_{t+1}, which appears on the right-hand side of both conditions.

Eqs. (A12) and (A13) state that the firm should allocate inputs from the production of final goods to IC in such a way that the marginal decrease in output is exactly equal to the value of the marginal increase in IC made available to the firm as a result of the switch. Substituting (12) and (13) in (10) and (11) respectively yields (15) and (16) below. Since u_t^n and u_t^k are positive fractions, factor prices exceed their marginal products in the final output production. Note also that the u_t^i's act as time-varying wedges between factor prices and marginal products.[6]

$$w_t = \frac{\alpha}{\eta}\frac{1}{u_t^n(i)}\frac{y_t(i)}{N_t(i)} \tag{A21}$$

$$r_t^k = \frac{(1-\alpha-\varepsilon)}{\eta}\frac{1}{u_t^k(i)}\frac{y_t(i)}{K_t(i)} \tag{A22}$$

Eq. (A14) establishes the marginal value of an extra unit of IC to the firm. The benefit comes not only from the extra production of the final good made possible but also from the additional IC that can be produced in the future.

1.3. Equilibrium

Factor market clearing requires that the total demand for hours worked from firms and the total demand for physical capital from firms is equal to the amount of each supplied by the representative household:

$$N_t^D = \int_0^1 N_t(i)di = N_t \; K_t^D = \int_0^1 K_t(i)di = K_t \tag{A23}$$

The resource constraint for this economy is:

$$Y_t = C_t + I_t \tag{A24}$$

A competitive equilibrium consists of sequences of allocations of the endogenous variables together with prices $\{w_t, r_t^k, \vartheta_t(i)\}_{t=0}^{\infty}\ \forall\ i \in [0, 1]$ that satisfy the conditions below:

- $\{C_t, N_t, I_t, K_{t+1}, a_{t+1}\}_{t=0}^{\infty}$ solves the household problem, taking, as given, K_0, Z_0, a_0 and exogenous processes $\{A_t, B_t\}_{t=0}^{\infty}$;
- $\{Y_t, y_t(i)\}_{t=0}^{\infty} \forall i \in [0, 1]$ solves the final good producer problem taking prices as given;
- $\{u_t^n(i), u_t^k(i), Z_{t+1}(i), N_t(i), K_t(i)\}_{t=0}^{\infty}\ \forall\ i \in [0, 1]$ solves the intermediate producer i's problem where all prices but its own are taken as given and its own price is chosen to maximize the expected discounted firm value;
- Market clearing conditions for goods, labor, and physical capital are satisfied, and the resource constraint holds.

We assume that all intermediate good producers face identical sequences of aggregate shocks and start out with identical quantities of tangible capital, K_0, and intangible capital, Z_0. Since all producers have the same technology and face the same sequence of factor prices, it is natural to focus our attention on a symmetric equilibrium where all producers in the intermediate good sector charge the same relative price and produce the same quantity of intermediate output. In this case:

$$Y_t = \left[\int_0^1 y_t(i)^{\eta} di\right]^{\frac{1}{\eta}} = y_t \tag{A25}$$

and the relative price $\vartheta_t = 1$ for all firms.

Finally, we note that (x) and (x) imply that $v_t = V_t$.

1.4. Calibration and simulation results

The calibrated parameter values are largely in line with the literature. The quarterly depreciation rate of capital δ is assumed to 0.025, which implies an annual depreciation rate of 0.1. α is calibrated at 0.55, which implies a labor share of 0.6. Regarding the labor supply elasticity, we assume $\phi = 2$. We calibrate the discount factor β equal to 0.99, which implies a steady-state annual real interest rate of 4%. The parameter η is set to 1.06 which matches the average value of gross dividend over the period of 1963-2014. We use $\sigma = 2$ which is quite common in the literature.

For the IC production process, we set $\varepsilon = 0.3$ and $\gamma = 0.55$, which are based on the estimates obtained by Hou and Johri (2018). Instead of calibrating α_1, we use the capital output ratio, $\frac{\bar{k}}{\bar{y}}$, to pin down its value[7]:

[6] The wedge between marginal product of labour and wages in the data is often interpreted as evidence of monopoly power. In a steady state, our model calibration would imply a markup of 1/0.8 = 1.25, even though the firm behaves competitively.

[7] $\bar{x}$ denotes the steady-state value.

$$\alpha_1 = 1 - \frac{\eta \frac{\bar{k}}{\bar{y}} \frac{\bar{r}^k}{e^{\gamma_a}}}{\beta \varepsilon \frac{1-\gamma}{1-\beta\gamma}} \tag{A26}$$

here we set the capital output ratio to 10 which is within the commonly used range of values in the literature. For example, Johri (2009) uses 9.6 while Chari et al. (2007) use 10.6.

The equilibrium system of the IC model can be linearly approximated around the steady-state and then using the singular linear difference system reduction method of King and Watson (2002). Note also that some variables are growing in the steady-state because there is a stochastic trend in the model. It comes from the unit root in the technology process given in Eq. (A11). [8]

References

Ang, A., Hodrick, R.J., Xing, Y., et al., 2009. High idiosyncratic volatility and low returns: international and further U.S. evidence. J. Financ. Econ. 91 (1), 1–23.

Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2007. Business cycle accounting. Econometrica 75 (3), 781–836.

Fama, E.E.F., French, K.K.R., 1993. Common risk factors in the returns on stocks and bonds. J. Financ. Econ. 33 (1), 3–56.

Gunn, C.M., Johri, A., 2011. News and knowledge capital. Rev. Econ. Dyn. 14, 92–101.

Hall, B.B.H., 2000. Innovation and market value. Product. Innovat. Econ. Perform. 177–198.

Harrison, H., Stein, J.C., 2007. Disagreement and the stock market. J. Econ. Perspect. 21 (2), 109–128.

Hou, K., Johri, A., 2018. Intangible capital, the labor wedge and the volatility of corporate profits. Rev. Econ. Dyn. 29, 216–234.

Li, X., Hou, K., 2019. R&D based knowledge capital and future firm growth: evidence from China's growth enterprise market firms. Econ. Modell forthcoming.

Long, H., Jiang, Y., Zhu, Y., 2018. Idiosyncratic tail risk and expected stock returns: evidence from the Chinese stock markets. Finance Research Letters 24. Elsevier, pp. 129–136.

McGrattan, E.R., Prescott, E.C., 2010. Technology capital and the US current account. Am. Econ. Rev. 100 (4), 1493–1522.

Qiu, J, Tseng, K., Zhang, C., 2019. Patent-to-market premium. McMaster University working paper.

Wang, Li.-Hsun., Lin, Chu.-Hsiung., Kang, Jui.-Heng., Fung, Hung.-Gay., 2016. Idiosyncratic volatility and excess return: evidence from the Greater China region. Finance Research Letters 19. Elsevier, pp. 126–129. https://ideas.repec.org/a/eee/finlet/v19y2016icp126-129.html.

Zaremba, A., Czapkiewicz, A., Będowska-Sójka, B., 2018. Idiosyncratic volatility, returns, and mispricing: no real anomaly in sight. Finance Research Letters 24. Elsevier, pp. 163–167. https://ideas.repec.org/a/eee/finlet/v24y2018icp163-167.html.

[8] Thus solving the model involves using stationary transformations of variables with unit roots – they are rendered stationary according to the following formula: $\widetilde{X_t} = \frac{X_t}{A_{t-1}}$.