# Can the Premium for Idiosyncratic Tail Risk be Explained by Exposures to its Common Factor?

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**Abstract**

Stocks in the highest idiosyncratic tail risk decile earn 8% higher average annualized returns than in the lowest. I propose a risk-based explanation for this premium, in which shocks to intermediary funding cause idiosyncratic tail risk to follow a strong factor structure, and the factor, common idiosyncratic tail risk (CITR), comoves with intermediary funding. Consequently, firms with high idiosyncratic tail risk have high exposure to CITR shocks, and command a risk premium due to their low returns when intermediary constraints tighten. To test my explanation, I create a novel measure of idiosyncratic tail risk that is estimated using high-frequency returns, and theoretically establish its time-aggregation properties. Consistent with my explanation, CITR shocks are procyclical, are correlated to intermediary factors, are priced in assets, and explain the idiosyncratic tail risk premium. Furthermore, volume tail risk also earns a premium, follows a strong factor structure, and its common factor is priced. This duality of idiosyncratic tail risk and volume tail risk provides evidence for my risk-based explanation, and further supports the hypothesis that intermediaries' large trades cause idiosyncratic tail risk and volume tail risk from Gabaix, Gopikrishnan, Plerou, and Stanley (2006).

*JEL Classification:* G12, C14, C58

*Keywords:* Idiosyncratic tail risk, Volume tail risk, Common idiosyncratic tail risk factor, Power law, High-Frequency factor model, Tail risk premia, Intermediary asset pricing

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## Introduction

A stock's idiosyncratic tail risk (ITR), denoted as $\xi$ in this paper, measures the density of the left tail of idiosyncratic returns, the residual returns after removing the effect of systematic factors. Empirically, stocks with higher idiosyncratic tail risk have higher average excess returns. This idiosyncratic tail risk premium is recently documented in Savor (2012), Jiang and Zhu (2017), Bégin, Dorion, and Gauthier (2019) and Kapadia and Zekhnini (2019), and it contradicts classical asset pricing theory where only systematic risk earns a risk premium. These papers speculate that the premium is caused by either the inability for investors to diversify due to frictions or market under-reaction to firm-specific news.[1] However, none of them find that the premium can be explained in the conventional manner by exposures to systematic risk.

In this paper, I propose a risk-based explanation for the idiosyncratic tail risk premium that combines two ideas. First, I hypothesize that idiosyncratic tail risk is caused by large intermediaries trading on news or propriety analysis, which is motivated by Gabaix, Gopikrishnan, Plerou, and Stanley (2006).[2] Since intermediaries require funding to trade, shocks to intermediary funding cause idiosyncratic tail risks of different firms to comove over time and share a common factor, which is denoted as the common idiosyncratic tail risk (CITR). This explanation is consistent with the recent finding of commonality in idiosyncratic tail risk (Qin and Todorov (2019) and Bégin, Dorion, and Gauthier (2019)). Since CITR decreases when intermediary constraints tighten, it is correlated to the intermediary marginal utility of wealth and is *procyclical*.

Second, the recent intermediary asset pricing literature shows that intermediaries are the marginal investor in many financial assets. Subsequently, shocks to their funding, correlated with shocks to CITR, are an important source of undiversifiable risk and may explain cross-sectional differences in average returns (see e.g. Brunnermeier and Pedersen (2009)). Since intermediaries play a central role in explaining asset prices, they may explain the idiosyncratic tail risk premium. I hypothesize that firms with higher idiosyncratic tail risk have higher exposure to shocks to CITR, thus commanding a risk premium. Differences in exposure to CITR shocks provide a risk-based explanation for the idiosyncratic tail risk premium.

[1] Examples of frictions include short-sale constraints, narrow framing, or market incompleteness due to non-traded assets (e.g. human capital or private businesses).

[2] Examples of intermediaries include hedge funds, mutual funds, or banks' propriety trading desks.

To test my explanation, I first introduce a new measure of *idiosyncratic* tail risk, that is the shape parameter of the power-law distribution of idiosyncratic returns, denoted as $\xi$. A large $\xi$ corresponds to high density in the left tail of idiosyncratic returns. Therefore $\xi$, which is inversely proportional the slope of the tails, is an intuitive measure of idiosyncratic tail risk. To measure $\xi$, I use a high-frequency factor model, then use Extreme Value Theory to prove that my idiosyncratic tail risk measure is inherited under time-aggregation, meaning that within each month, the idiosyncratic tail risk calculated on high-frequency returns can be extrapolated to longer time horizons (i.e. daily, weekly, or monthly), even with a finite number of return observations. Moreover, the measure is robust to certain microstructure noise processes.

Testing my explanation requires a large sample of medium and small stocks, since their idiosyncratic tail risks are most affected by large intermediary trades. Existing options-based measures of idiosyncratic tail risk require a large cross-sections of options at different strike prices (i.e. Bégin, Dorion, and Gauthier (2019), Kapadia and Zekhnini (2019), and Qin and Todorov (2019)), which are available only for a limited number of large stocks. Instead, I use the richness of high-frequency data to study a much larger sample of stocks. Additionally, existing options-based measures only extract the market factor, while my high-frequency model can remove multiple factors from returns.

I begin my empirical analysis by establishing that the idiosyncratic tail risk premium is a significant and prevalent phenomenon. I estimate the factor model with respect to the Fama and French (2015) five factor model and estimate the idiosyncratic tail risk as the Hill estimate of idiosyncratic returns. In each month, I sort stocks into deciles based on their idiosyncratic tail risk and hold the portfolios for a month. Stocks in the decile with the highest idiosyncratic tail risk earn 0.66% (t-stat of 3.16), approximately 8% annually, higher value-weighted returns than stocks in the lowest decile. This idiosyncratic tail risk premium cannot be explained by the market, size, value, profitability, investment, or momentum factors. The premium exists even when stocks are conditionally sorted on other firm characteristics and is robust to different factor model specifications and tail risk estimation methods. Additionally, Gabaix, Gopikrishnan, Plerou, and Stanley (2006) theorize that large intermediaries cause both idiosyncratic tail risk and trading volume tail risk, predicting that their tail distributions are proportional. Under my explanation, volume tail risk (VTR) as a proxy for idiosyncratic tail risk should also earn a premium. To test my explanation, I estimate volume tail risk as the Hill estimate of the right tail of changes in trading volume. Stocks in the decile with the highest volume tail risk earn 0.67% (t-stat of 3.70) higher value-weighted returns than stocks in the lowest decile. This volume tail risk premium is highly correlated to the idiosyncratic tail risk premium, indicating that these premia are driven by the same factor.

The idiosyncratic tail risk and volume tail risk premia are highly persistent. In each month, I sort stocks into deciles based on their idiosyncratic tail risk and hold the portfolios for up to two years. For the two-year holding period, stocks in the decile with the highest idiosyncratic tail risk earn 16.60% (t-stat of 3.44) higher value-weighted returns than stocks in the lowest decile. Likewise, stocks in the decile with the highest volume tail risk earn 22.46% (t-stat of 3.71) higher value-weighted returns than stocks in the lowest decile for the two-year holding period. This finding is consistent with the intuition that firm exposure to CITR shocks does not frequently change. If the idiosyncratic tail risk premium is compensation for exposure to CITR shocks, and stocks' risk exposures do not frequently change, then the risk premium should be persistent, which is exactly what I document.

Next, I test the large intermediary hypothesis of idiosyncratic tail risk by conducting firm-level regressions. A cross-sectional regression shows that idiosyncratic tail risk is correlated to intermediary trading volume as a percentage of total trading volume, where a one percent increase in intermediary volume increases idiosyncratic tail risk by 0.06. Furthermore, idiosyncratic tail risk is highly correlated to volume tail risk, providing empirical evidence that intermediary trades cause both idiosyncratic tail risk and volume tail risk (Gabaix, Gopikrishnan, Plerou, and Stanley (2006)). Finally, idiosyncratic tail risk is highly autocorrelated, which provides evidence against the explanation that the measure is only driven by fundamental news-shocks. This finding is consistent with previous studies that show large intermediaries frequently trade the same stocks (Sias (2004)).

Comovement in idiosyncratic tail risks is pervasive. Commonality of idiosyncratic tail risk exists for firms with different sizes, values, and industries. Their common factor, CITR estimated as the average cross-sectional idiosyncratic tail risk, explains 42.6% of the time variation in firm-level tail risk. This synchronization of idiosyncratic tail risks is robust across various specifications and does not arise from omitted factors, since the factor model residuals are virtually uncorrelated. Consistent with my explanation, the CITR factor, defined as shocks to CITR, is procyclical as evidenced by its positive correlation to log changes in the price-to-earnings ratio, market return, gross domestic product, investment, and consumption. In addition, the CITR factor is highly correlated to existing intermediary factors, demonstrated by its strong positive correlation to the intermediary capital factor (He, Kelly, and Manela (2017)) and the broker-dealer leverage factor (Adrian, Etula, and Muir (2014)), and its negative correlation to the Leverage Constraint Tightness of mutual funds (Boguth and Simutin (2018)). Furthermore, the CITR factor is uncorrelated with shocks to market volatility, shocks to the VIX, and the common idiosyncratic volatility factor of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), showing that it's not driven by volatility.

The CITR factor is a systematic risk factor, since CITR is correlated to the intermediary marginal utility of wealth. Consequently, exposure to this risk should be priced in equilibrium. To test that, I examine whether the CITR factor is a priced risk factor and find support for this hypothesis in stock returns. First, I sort stocks into portfolios based on their CITR-betas, which are estimated by regressing individual stock excess returns on the CITR factor, then hold the portfolios for a month. Excess returns and alphas monotonically increase in CITR-betas. Stocks in the highest CITR-beta quintile earns a 0.62% (t-stat of 2.01) higher value-weighted returns than the lowest quintile, reflecting their compensation for higher exposures to shocks to CITR.

The main question of this paper is whether high idiosyncratic tail risk firms have high exposure to the CITR factor and if that can explain their cross-sectional differences in average returns, which are confirmed by my following findings. My analysis shows that betas to the CITR factor are increasing in the idiosyncratic tail risk deciles and the relationship is nearly monotonic. Furthermore, the CITR factor alone explains 73%, up to 86% along with the market factor, of the cross-sectional variation of average excess returns for the portfolios sorted by idiosyncratic tail risk. These findings confirm my hypothesis that exposure to the CITR factor explain most of the idiosyncratic tail risk premium. The CITR factor alone also explains nearly 32% of its cross-sectional variation in the volume tail risk deciles and adding the market factor increases the cross-sectional $R^2$ to 68%.

My explanation for the idiosyncratic tail risk premium is supported by a battery of robustness checks. First, the CITR factor also helps to explain cross-sectional differences in average returns of portfolios conditionally double-sorted on size and then idiosyncratic tail risk or volume tail risk. The CITR factor is also priced in anomaly portfolios independently double-sorted on size and the following characteristics: operating profitability, investment, momentum, or idiosyncratic volatility. Second, the CITR factor risk price is positive for all asset classes used in He, Kelly, and Manela

(2017) and is statistically significant for the sophisticated options, CDS, commodities, and foreign exchange portfolios. Third, some intermediary models predict that the intermediary factor negatively forecasts market returns (He, Kelly, and Manela (2017)). I test this hypothesis by regressing equity market returns on CITR. A one-standard-deviation increase in CITR forecasts a decrease in annualized excess market returns of -9.56%, -7.09%, -5.31%, and -3.65% at the one-month, six-month, one-year, and two-year horizons, respectively. The results are statistic significant with Hodrick t-statistics of -2.02, -2.31, -2.36, and -2.05 and produce an $R^2$ of 9.83% at the annual frequency. Finally, I use the idiosyncratic tail risk long-short portfolio as a factor-mimicking portfolio for the non-traded CITR factor, and find that this traded CITR factor is also priced in equities and sophisticated asset classes.

Finally, in my explanation volume tail risk is a substitute for idiosyncratic tail risk, since they are both caused by large intermediary trades (Gabaix, Gopikrishnan, Plerou, and Stanley (2006)). Hence, a litmus test is to evaluate whether volume tail risk exhibits commonality that is correlated to the intermediary marginal utility of wealth and helps explain average returns across multiple asset classes. Consistent with my explanation, volume tail risk also exhibits a strong factor structure and common volume tail risk (CVTR), estimated as the average cross-sectional volume tail risk, explains 46.3% of the time variation in volume tail risks. The common volume tail risk factor, defined as 3-month shocks to CVTR is procyclical, correlated to intermediary factors, and helps explain cross-sectional differences in average returns for anomalies and sophisticated assets. The CITR and CVTR factors are highly correlated and have similar prices of risk, supporting the theory that they are driven by shocks to intermediary funding. This paper is the first to show duality in the *tail distributions* of idiosyncratic returns and trading volume in empirical asset pricing, providing strong support for the large intermediary hypothesis of tail risk and my explanation of their risk premia.

My paper is related to several strands of literature. My power-law measure of idiosyncratic tail risk is closest to the seminal work of Bollerslev and Todorov (2011a), which measures the total tail risk (tail risk of returns) from high-frequency returns. My measure complements their work by measuring idiosyncratic tail risk after removing common factors. Danielsson and De Vries (1997) use power-law to estimate the tail distribution of high-frequency foreign exchange data. Kelly and Jiang (2014) use power-law to measure market tail risk. Bollerslev and Todorov (2011b), Bollerslev, Todorov, and Li (2013), and Bollerslev and Todorov (2014) use power-law to measure tail risk implied from options prices. Qin and Todorov (2019) study the asymmetry of power-law measures of idiosyncratic tail risk using high-frequency returns and options, but focusing on the latter. Van Oordt and Zhou (2016) uses power-law theory to estimate exposures to systematic tail risk. My paper complements these studies by measuring the realized idiosyncratic tail risk estimated using high-frequency equity returns. My model allows removal of multiple factors and provides a much larger sample of stocks. Additionally, I theoretically demonstrate the time-aggregation properties of my idiosyncratic tail risk measure, which justifies extrapolating the high-frequency idiosyncratic tail risk measure to longer time horizons.

Next, my research on the idiosyncratic tail risk premium is motivated by the recent literature on *idiosyncratic* tail risk and returns. Bégin, Dorion, and Gauthier (2019) shows that idiosyncratic jump risk explains 28% of the variation in risk premium on a stock. Kapadia and Zekhnini (2019) show ex-ante jump probabilities predict cross-sectional average returns. Pederzoli (2018) shows that idiosyncratic skewness is priced in individual stocks. Kelly, Lustig, and Van Nieuwerburgh (2016) show that for firms in the financial sector, idiosyncratic risk had a higher price than sector risk during the financial crisis. Long, Jiang, and Zhu (2018) study idiosyncratic tail risk in Chinese stock markets and finds that it negatively predicts stock returns. My paper complement this literature by demonstrating that idiosyncratic tail risk estimated using high-frequency returns predicts cross-sectional average returns. Furthermore, I demonstrate that volume tail risk measured from high-frequency trading volume also predicts cross-sectional average returns. Finally, I show both the idiosyncratic tail risk and volume tail risk premia are highly persistent and can be explained by exposures to a common factor.

In addition, my paper is related to the literature on *total* tail risk and returns. Savor (2012) finds momentum after large absolute returns with information and reversals in the absence of information. Jiang and Zhu (2017) find that markets under-react to jumps interpreted as information shocks and Jiang and Yao (2013) argue that jumps are due to new information and not systematic shocks. My paper complements these studies by measuring *idiosyncratic* tail risk with systematic factors removed. Additionally, I demonstrate that idiosyncratic tail risk has a strong factor structure, which provides evidence against their interpretation that tail risk is only caused by firm-specific news and that the premium is due to market under-reaction.

Furthermore, my research on the commonality of idiosyncratic tail risk is connected to the literature on cross-sectional studies of firm-level risk. Campbell, Lettau, Malkiel, and Xu (2001) show between 1962 to 1997 that firm-level volatility increased relative to market volatility. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) find that idiosyncratic volatility follows a strong factor structure, where their common idiosyncratic volatility factor is priced and related to labor income risk. Dew-Becker and Giglio (2020) study the cross-section of implied volatility and find a strong factor structure. Bégin, Dorion, and Gauthier (2019) find a high degree of comovement in idiosyncratic tail risk measured from options and daily returns. Lin and Todorov (2019) study the asymmetry between positive and negative idiosyncratic tail risk from options and find their asymmetry measure predicts the market risk premium. My paper compliments this literature in several ways. First, I study the *realized* idiosyncratic tail risk measured from high-frequency returns and show that idiosyncratic tail risk follows a strong factor structure. Second, I show that CITR is correlated to the intermediary marginal utility wealth and is procyclical. Finally, my paper studies the cross-sectional asset pricing implications of the CITR factor, showing exposures to this factor earn higher average returns and that the factor explains the idiosyncratic tail risk premium.

Finally, my study is similar in spirit to the recent work on intermediary asset pricing including Adrian and Shin (2010), Adrian, Etula, and Muir (2014), Brunnermeier and Pedersen (2009), Brunnermeier and Sannikov (2014), Boguth and Simutin (2018), Fontaine and Garcia (2012), Gromb and Vayanos (2002), He and Krishnamurthy (2013), He, Kelly, and Manela (2017). My paper complements this literature by showing that intermediaries also drive the tail distribution of returns and trading volume and that common idiosyncratic tail risk and common volume tail risk are both correlated to the intermediary marginal utility of wealth. I also demonstrate that the common idiosyncratic tail risk and common volume tail risk factors are priced in many sophisticated asset classes, which further supports the hypothesis that intermediaries are the marginal investor in these assets.

The paper is organized as follows: Section 1 provides my explanation for the idiosyncratic tail risk premium. Section 2 presents the econometric model, idiosyncratic tail risk measure, time-aggregation properties, robustness to microstructure noise, and estimation of idiosyncratic tail risk. Section 3 describes the data and factors, then documents the idiosyncratic tail risk and volume tail risk premia, persistence of the tail risk premia, and firm-level evidence that idiosyncratic tail risk is driven by large intermediary trades. Section 4 explores the factor structure in idiosyncratic tail risk, defines the CITR factor and shows that it's priced and explains the idiosyncratic tail risk premium. Section 5 demonstrates robustness of the results by showing that the CITR factor is priced in anomaly portfolios and in sophisticated assets, the common volume tail risk factor has similar pricing abilities, the traded idiosyncratic tail risk factor has similar pricing abilities, and CITR predicts market returns. Section 6 concludes.

# 1 Explanation for the Idiosyncratic Tail Risk Premium

In this section, I describe my explanation for the idiosyncratic tail risk premium using a highly stylized model. Motivated by Gabaix, Gopikrishnan, Plerou, and Stanley (2006), I hypothesize that intermediaries' large trades, either due to fundamental news or proprietary analysis (absent news), causes idiosyncratic tail risk. Intermediaries require funding to trade on investment opportunities. Intuitively, as funding constraints tighten, intermediaries cannot trade on investment opportunities and idiosyncratic tail risk *decreases* on average. Hence, the common component of each stock's idiosyncratic tail risk is a decreasing function of the shadow cost of capital in Brunnermeier and Pedersen (2009), which is a measure of the tightness of intermediary funding constraints. A higher shadow cost of capital means low intermediary funding, indicating that investment opportunities are higher and therefore idiosyncratic tail risks are lower across stocks due to fewer large intermediary trades. My explanation is consistent with intermediaries deleveraging when their funding constraints tighten as demonstrated by Adrian, Etula, and Muir (2014), impairing their ability to conduct large trades. Consequently, the common factor of the idiosyncratic tail risks is correlated to their leverage factor.

## 1.1 Stylized Model of Idiosyncratic Tail Risk

Denote the idiosyncratic tail risk of firm $l = 1...L$, as $\xi_{l,1}$. Each firm's time-one idiosyncratic tail risk is driven by intermediary funding and firm-specific news $N_{l,1}$, and equals

$$\xi_{l,1} = b - c\phi_1 + N_{l,1}, \tag{1}$$

where $N_{l,1}$ is i.i.d., with mean $\mu_N$, variance $\sigma_N^2$, independent of $\phi_1$, $b, c > 0$ are constants, and $\phi_1$ is the time-one intermediary shadow cost of capital from Brunnermeier and Pedersen (2009). Taking an average over a large number of firms yields approximation

$$CITR_1 \equiv \frac{1}{L}\sum_{l=1}^{L}\xi_{l,1} \approx b + \mu_N - c\phi_1, \tag{2}$$

where $CITR_1$ is the common idiosyncratic tail risk. $CITR_1$ is a negative function of the intermediary shadow cost of capital, since

$$\phi_1 \approx \frac{b + \mu_N - CITR_1}{c}, \tag{3}$$

such that a lower $CITR_1$ corresponds to tighter funding constraints. Also, each firm's idiosyncratic tail risk comoves with $CITR_1$, since substituting (3) into (1) yields approximate factor structure

$$\xi_{l,1} \approx -\mu_N + CITR_1 + N_{l,1}, \tag{4}$$

implying that $Cov_0(CITR_1, \xi_{l,1}) > 0$ for all firms. Brunnermeier and Pedersen (2009) demonstrate how funding liquidity enters the pricing kernel even when investors are risk-neutral. Let $W_1$ be time-one wealth such that the risk-neutral investor maximizes $E_0[\phi_1 W_1]$. Then, the stochastic discount factor is $\phi_1/E_0[\phi_1]$ and asset $k$'s time-zero expected excess return $R_{k,1}^e$ is

$$E_0[R_{k,1}^e] = -\frac{Cov_0(\phi_1, R_{k,1}^e)}{E_0[\phi_1]}. \tag{5}$$

Equation (5) states that stocks with a negative covariance term have higher expected excess returns, since the stock has a low payoff during future funding liquidity crises when $\phi_1$ is high. Substituting Equation (3) into Equation (5) yields approximation

$$E_0[R_{k,1}^e] \approx \lambda_{CITR} Cov_0(CITR_1, R_{k,1}^e), \tag{6}$$

where price of risk $\lambda_{CITR} > 0$. Thus, CITR is a priced risk factor and assets that covary with CITR are risky and earn a higher risk premium. A risk-based explanation of the idiosyncratic tail risk premium requires a further assumption that exposure to CITR is increasing in idiosyncratic tail risk, that is if $\xi_{n,1} > \xi_{m,1}$, then

$$Cov_0(CITR_1, R^e_{n,1}) > Cov_0(CITR_1, R^e_{m,1}), \tag{7}$$

and consequently $E_0[R^e_{n,1}] > E_0[R^e_{m,1}]$ by (6). One possible economic mechanism for Assumption (7) is flight-to-quality. When funding is tight, intermediaries decrease ownership in small and medium stocks (Næs, Skjeltorp, and Ødegaard (2011), Papaioannou, Park, Pihlman, and Van der Hoorn (2013)) due to their higher risk and increased margins on leverage. Since small and medium stocks have high idiosyncratic tail risks, their exposure to intermediary funding is high, increasing their riskiness.

My explanation relies on two central assumptions. First, the common component of idiosyncratic tail risk is correlated to the intermediary marginal utility of wealth, which implies CITR is a priced risk factor. Second, exposure to CITR is increasing in idiosyncratic tail risk. Large exposures to CITR for firms with high idiosyncratic tail risk can explain their premium. These simple assumptions have several testable implications, listed below, that allow me to distinguish my theory from existing theories of what causes idiosyncratic tail risk and its premium.

a) CITR is correlated to the intermediary marginal utility of wealth and can be estimated as the cross-sectional mean of idiosyncratic tail risks. Since intermediary funding is procyclical, then CITR should be as well. This is the opposite prediction of several existing explanations of what drives the commonality of idiosyncratic tail risk like fire-sales and labour tail risk, which predict counter-cyclical comovement. Also, if idiosyncratic tail risk were only caused by firm-specific news, there should be no commonality.
b) CITR is a risk factor with a positive price. Stocks with higher exposures to CITR should earn higher average returns. The prediction that CITR has a positive price of risk distinguishes the model from existing possibilities like fire-sales and labour tail risk, which predict a negative price of risk, and firm-specific news which should have no price of risk. Additionally, CITR's correlation to the intermediary marginal utility of wealth means it should negatively forecast market returns as predicted by some intermediary models (i.e. He, Kelly, and Manela (2017)).
c) The idiosyncratic tail risk premium is explained by exposures to CITR. Firms with high idiosyncratic tail risk also have high betas to CITR and are compensated with high returns.

This is the main test that distinguishes my explanation from the inability of investors to diversify or under-reaction to news, which should be firm-specific.

d) Firms with higher idiosyncratic tail risk should continue to enjoy higher future expected returns. If the idiosyncratic tail risk is driven by intermediary trading, then it should be persistent, since intermediaries can frequently trade these stocks according to their proprietary models. If the idiosyncratic tail risk premium is a systematic risk premium, then firms with higher idiosyncratic tail risks are riskier due to their higher exposures to the CITR factor. If the relationship between idiosyncratic tail risk and CITR do not change rapidly, then this risk premium should be persistent. This test allows me to distinguish my explanation from firm-specific explanations like under-reaction to fundamental news-shocks, which should be short-lived.

A litmus test for my explanation uses the relationship between the tail distribution of returns and trading volume predicted by Gabaix, Gopikrishnan, Plerou, and Stanley (2006). Under their model, the tail distribution of returns and volume are proportional, meaning volume tail risk is a proxy for idiosyncratic tail risk. This implies that trading volume should follow the same predictions as above, where volume tail risk earns a premium, exhibits commonality, its common factor is driven by intermediary funding, is a priced risk factor, and explains the volume tail risk premium.

This paper uses a power-law measure of idiosyncratic tail risk, which is motivated by the power-law economic model in Gabaix, Gopikrishnan, Plerou, and Stanley (2006). Power-law conveniently summarizes the tail distribution using a single parameter corresponding to the idiosyncratic tail risk $\xi_l$. Additionally, tail risk under power-law is preserved under time aggregation, which justifies measuring tail risk using high-frequency returns resulting in a time-varying measure of idiosyncratic tail risk. Section A presents empirical evidence that idiosyncratic returns and trading volume have power-law distributed tails.

# 2 Econometric Framework

## 2.1 Idiosyncratic Tail Risk Measure

In each month $t$, I observe a large series of high-frequency observations $i = 1, .., N$, where $N$ is the number of observations. For example, in a month with 21 trading days and 5-minute time intervals, $N = 1659$.[3] The high-frequency log return during month $t$ for firm $l = 1, .., L$ is

$$r_{l,t,i} = p_{l,t-1+\frac{i}{N}} - p_{l,t-1+\frac{i-1}{N}}, \tag{8}$$

where $p$ is the natural log of price. I assume return dynamics follow a standard factor model

$$r_{l,t,i} = \boldsymbol{\beta}_{l,t}^{\intercal} \mathbf{f}_{t,i} + x_{l,t,i}, \tag{9}$$

where the $\boldsymbol{\beta}_{l,t}$ are the factor loadings of firm $l$ on systematic factors $\mathbf{f}_{t,i}$, and $x_{l,t,i}$ are unobservable idiosyncratic returns. The factors are assumed to be observable and log returns of equity portfolios, for example in the CAPM $\mathbf{f}_{t,i}$ is the high-frequency return of the market portfolio. $\boldsymbol{\beta}_{l,t}$ is assumed to be constant throughout each month t, but can vary month-to-month. The constant $\boldsymbol{\beta}_{l,t}$ assumption is common in the literature (e.g., Todorov and Bollerslev (2010), Aït-Sahalia and Xiu (2017), Dai, Lu, and Xiu (2019)); Reiß, Todorov, and Tauchen (2015) find evidence supportive of this assumption using high-frequency data. Furthermore, I assume that the tails of idiosyncratic returns are power-law distributed.

**Assumption 1.** *(a) For each firm l, the idiosyncratic returns $x_{l,t,i}$, $i = 1, ..., N$, are independent and identically distributed, and are regularly varying with tail risk parameter $\xi_{l,t} > 0$. That is, $P(|x_{l,t,i}| > y) = y^{-\frac{1}{\xi_{l,t}}} L(y)$, where $L$ is a slowly varying function.*[4]

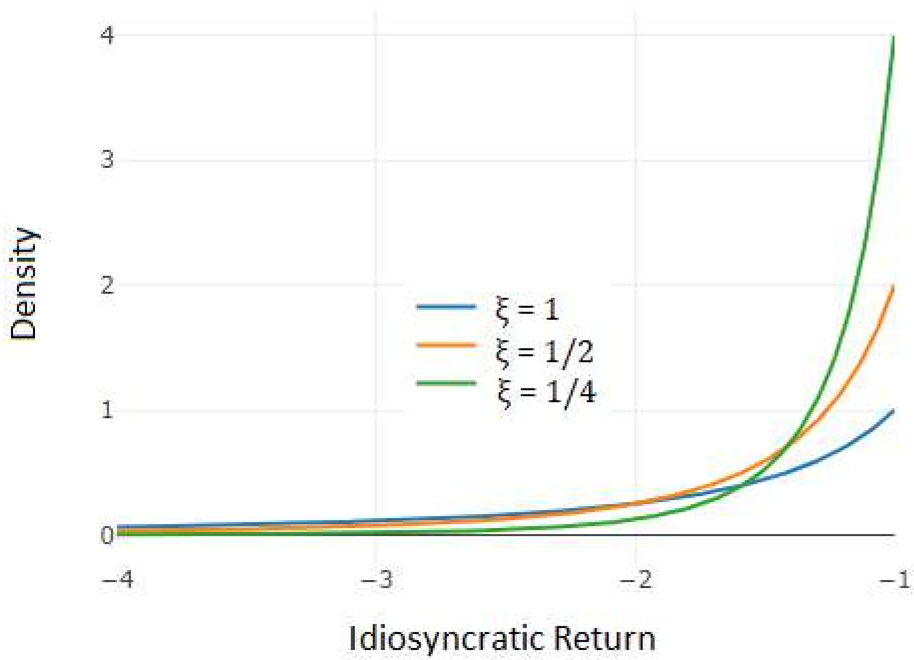
*(b)* $\lim_{y \to \infty} \frac{P(x_{l,t,i} \leq -y)}{P(|x_{l,t,i}| > y)} = \theta \in (0, 1]$

Assumption 1(a) postulates that the tails of idiosyncratic returns are power-law distributed with tail risk parameter $\xi_{l,t}$, which is supported by empirical evidence presented in Figure 8 in Section A. Tail risk parameter $\xi_{l,t}$ is firm-specific, dynamic, and able to capture monthly changes

[3] In a trading day, there are 78 intraday returns when sampled in 5-minute intervals and 1 overnight return.

[4] Function L is slowly varying if $L$ is strictly positive and $\lim_{x \to \infty} L(tx)/L(x) = 1$ for all $t > 0$. Prototypical examples include $L(x) = log(x)$ and $L(x) = c$ for a $c > 0$.

**Figure 1: Probability Density of Power-Law Distributed Tails**




The figure plots the probability density of Pareto distributed tails with various tail risk parameters $\xi$ and L(y)=1. Pareto tails become heavier as the tail risk parameter $\xi$ increases. A larger $\xi$ has a smaller slope, which increases the severity of extreme negative returns.

in idiosyncratic tail risk. The tail risk parameter $\xi_{l,t}$ is inversely proportional to the slope of the tails. Figure 2.1 illustrates the relationship between tail risk parameter $\xi_{l,t}$ and the slope of the left tail. A large tail risk parameter corresponds to a small slope and a high probability of extreme observations. Hence, $\xi_{l,t}$ is an intuitive measure of idiosyncratic tail risk. Assumption 1(b) is a standard regularity condition that ensures the right tail of idiosyncratic returns does not dominate the left tail. Assumption 1(b) can be economically interpreted as the probability of bankruptcy is non-zero for any firm. For simplicity, I assume that idiosyncratic returns are independent and identically distributed within each month. The i.i.d. assumption is stronger than necessary for the theoretical results and estimation methods in my paper, which can still hold for idiosyncratic returns that are dependent and heterogeneous.[5]

The strict positivity of $\xi_{l,t}$ rules out light tails that understate the probability of rare events, such as normally distributed tails. Most parametric models of financial returns are power-law distributed.

[5]See Hill (2010) for tail risk estimation using dependent and heterogeneous data.

For example, a Student's t distribution with d degrees of freedom satisfies Assumption 1 with tail risk $\xi_{l,t} = 1/d$. Other prototypical examples of power-law distributions are the Pareto, Levy-Stable, and Cauchy distributions (see Embrechts, Klüppelberg, and Mikosch (1997) for more examples). Assumption 1 is semi-parametric and agnostic on the body (region of the distribution not belonging to the tail) of idiosyncratic returns, allowing for skewness and excess kurtosis commonly exhibited by returns.[6]

Factor models of equity returns are commonly used in financial economics due to their connection to investor preferences. In the absence of arbitrage, expected stock returns can be expressed as a function of the stochastic discount factor (SDF), where in equilibrium the stochastic discount factor is the investor's marginal utility. As a result, systematic factors correlated with the stochastic discount factor determine expected returns.[7] For example, in the Capital Asset Pricing model (CAPM) by Sharpe (1964) and Lintner (1965), the market portfolio is the single factor determining expected returns. Models with multiple factors include the Fama and French (1993) and Carhart (1997) four factor model, the Fama and French (2015) five factor model, and statistical factor models. However, most factor models of returns assume the idiosyncratic returns are light-tailed. Light-tailed idiosyncratic returns understate the probability of rare events and fitting a light-tailed model on power-law distributed data causes severe econometric issues (Balkema and Embrechts (2018)).

The model assumes *high-frequency* idiosyncratic returns have power-law distributed tails. High-frequency returns have more observations than daily, which should improve the precision of tail risk estimates. However, this gain in precision assumes that the idiosyncratic tail risk is the same at longer time-intervals (e.g. daily, monthly). Assumption 1 describes the tail behaviour of high frequency idiosyncratic returns. This is different from many factor models and risk measures that are measured at longer time horizons. The next section demonstrates how the high-frequency idiosyncratic tail risk $\xi_{l,t}$ maps to longer time horizons.

[6] While I assume the left and right tails have the same tail risk parameter $\xi$, the body of the distribution can still exhibit skewness. For example, a Levy-Stable distribution has a tail parameter and a separate skewness parameter.

[7] For a further explanation, see Chapter 6 of Cochrane (2009).

## 2.2 Temporal Aggregation of Idiosyncratic Tail Risk

In this section, I theoretically demonstrate that within each month, longer-horizon (i.e. daily, weekly, or monthly) idiosyncratic returns inherit the tail risk of the underlying high-frequency returns, which I refer to as the inheritance property under time-aggregation. Heuristically, a property of power-law distributed random variables is that their sum inherits the tail risk of the underlying random variables. For example, the sum of two t-distributed random variables with tail risk $\xi$ will also have tail risk $\xi$, despite not being t-distributed. This section uses extreme value theory to formalize these heuristic arguments. I show that the inheritance property under time-aggregation is theoretically guaranteed even with a finite number of observed returns. Additionally, the measure is robust to certain microstructure noise. To simplify the notation in this section, I drop the firm $l$ superscript with the understanding that all idiosyncratic returns $x_{t,i}$ and idiosyncratic tail risk parameters $\xi_t$ are firm-specific.

I prove that idiosyncratic tail risk is inherited under time-aggregation using progressively weaker assumptions. First, I show that the monthly idiosyncratic returns weakly converge to a Levy distribution with the same tail risk as the high-frequency returns, when returns are continuously observed in a month. Second, I show that when the number of returns is finite and the distribution of the monthly idiosyncratic return is unknown, the inheritance property under time-aggregation still holds. Finally, I show the high-frequency idiosyncratic tail risk can also be measured when certain microstructure noise exists, which justifies using my measure of idiosyncratic tail risk even when returns are contaminated with light-tailed noise such as the bid-ask bounce.

### 2.2.1 Monthly Idiosyncratic Tail Risk

The literature overwhelmingly specifies factor models at the monthly time horizon, so this section uses only the definition of log returns to show the connection between high-frequency and monthly idiosyncratic returns. To do so, I first construct a monthly factor model defined analogously to the high-frequency model. By definition, monthly log returns are $R_t = p_t - p_{t-1}$, where $p_t$ is the log price at month $t$. I then assume the monthly factor model of each firm follows

$$R_t = \boldsymbol{\beta}_t^{\intercal} \mathbf{F}_t + X_t, \tag{10}$$

where returns $R_t$, factors $\mathbf{F}_t$, and idiosyncratic returns $X_t$ are *monthly* log returns. By definition, the observable returns and factors can be written as the sum of their underlying high-frequency returns by using the aggregation properties of log returns, such that $R_t = \sum_{i=1}^{N} r_{t,i}$ and $F_t = \sum_{i=1}^{N} f_{t,i}$. Since $\boldsymbol{\beta}_t$ are constant for each month $t$, $\boldsymbol{\beta}_t$ is assumed to be the same in the monthly and high-frequency models. Furthermore, I impose no direct assumptions on the distribution of the unobservable monthly idiosyncratic returns $X_t$. Using only the definition of the models and log returns, the following Lemma shows the monthly idiosyncratic return can be expressed as the sum of the underlying high-frequency idiosyncratic returns.

**Lemma 1.** *The monthly idiosyncratic return is equal to the sum of high-frequency idiosyncratic returns, that's* $X_t = \sum_{i=1}^{N} x_{t,i}$.

*Proof.* See Section B.1 in the appendix. □

Lemma 1 shows the identity that the monthly idiosyncratic return is equal to the sum of high-frequency idiosyncratic returns. However, the distribution of the monthly idiosyncratic return is unknown. In general, the distribution of $x_{t,i}$ will be different from the distribution of $X_t = \sum_{i=1}^{N} x_{t,i}$. For example, the sum of two t-distributed random variables is generally not t-distributed. The monthly idiosyncratic return $X_{l,t} = \sum_{i=1}^{N} x_{t,i}$ has the same distribution as $x_{t,i}$ if and only if $x_{t,i}$ is a stable distribution.[8] However, the monthly idiosyncratic tail risk inherits the high-frequency idiosyncratic tail risk $\xi_t$, which is proved in the next section.

### 2.2.2 Idiosyncratic Tail Risk with Continuously Observed Returns

I first assume that high-frequency returns are continuously observed, such that $N \to \infty$. As the number of returns approach infinity, the limiting distribution of the sum of returns can be derived using Levy's Theorem. Levy's Theorem in Section B.2 is the generalization of the Central Limit Theorem for heavy-tailed random variables. The theorem states that the sum of i.i.d. power-law distributed random variables converge to a Levy distribution with the same tail risk. I use Levy's Theorem to derive the limiting distribution and tail risk of the monthly idiosyncratic returns in the following theorem.

[8]A distribution is stable if a sum two i.i.d. random variables with this distribution has the same distribution. The normal distribution is the only stable distribution to be light-tailed, while all other stable distributions are heavy-tailed.

**Theorem 1.** *Suppose the high-frequency idiosyncratic returns $x_{t,i}$, $i = 1, .., N$, satisfy Assumption 1 and $\xi_t > \frac{1}{2}$. Then there exist $a_N$ and $b_N$ such that the monthly idiosyncratic return $X_t$ satisfies*

$$(\sum_{i=1}^{N} x_{t,i} - b_N)/a_N \xrightarrow{d} u_t \text{ as } N \to \infty, \tag{11}$$

*where $u_t$ is a Levy-distributed random variable with tail risk $\xi_t$, and $a_N = inf\{y : P(|x_{t,1}| > y) \leq N^{-1}\}$, and $b_N = NE(x_{t,1}\mathbb{1}_{(x_{t,1} \leq a_N)})$.*

*Proof.* See Section B.3 in the appendix. □

If the high-frequency returns are continuously observed, then the distribution and tail risk of the monthly idiosyncratic returns is derived in Theorem 1. In practice, microstructure effects prevent returns from being sampled continuously, since microstructure noise dominates the returns process at ultra high frequencies. Therefore, returns must be sampled discretely, where the number of returns N is finite. When N is finite, monthly idiosyncratic returns are no longer Levy distributed and in general the distribution cannot be found. However, the next section shows that the monthly idiosyncratic *tail risk* can still be measured.

#### 2.2.3 Idiosyncratic Tail Risk with a Finite Number of Returns

The following theorem is the main theoretical result for the tail risk of the monthly idiosyncratic return when the number of returns observed in a month is finite.

**Theorem 2.** *Suppose the high-frequency idiosyncratic returns $x_{t,i}$, $i = 1, .., N$, satisfy Assumption 1 and $N \geq 2$. Then,*

$$P(X_t \leq -y) \sim NP(x_{t,i} \leq -y), \tag{12}$$

*where $f(y) \sim g(y)$ means $\lim_{y\to\infty} f(y)/g(y) \to 1$. This implies the left tail of the monthly idiosyncratic return $X_t$ is power-law distributed with tail risk parameter $\xi_t$.*

*Proof.* See Section B.5 in the appendix. □

The above implies that when the number of returns $N$ is finite and the distribution of the monthly idiosyncratic return $X_t$ is unknown, the left tail of $X_t$ still inherits the tail risk of the high-frequency idiosyncratic returns $\xi_t$ under very general power-law assumptions.

In practice, microstructure effects introduce an unobservable noise process to returns. Separating the noise from the idiosyncratic returns is difficult, since both processes are unobservable. However, the next section shows that the high-frequency idiosyncratic tail risk can still be measured under certain microstructure noise processes.

#### 2.2.4 Idiosyncratic Tail Risk with Microstructure Noise

Suppose high-frequency returns are observed with microstructure noise process $\eta_{t,i}$. While papers often assume microstructure noise follows a normal or light-tailed distribution, I allow the microstructure noise to be power-law distributed in the following assumption.

**Assumption 2.** *(a) The microstructure noise $\eta_{t,i}$ are independent and identically distributed, and are regularly varying with tail risk $\xi_t > \gamma_t > 0$. That's $P(|\eta_{t,i}| > y) = y^{-\frac{1}{\gamma_t}} L_\eta(y)$, where $L_\eta$ is a slowly varying function.*

*(b) $\lim_{y\to\infty} \frac{P(\eta_{t,i} \leq -y)}{P(|\eta_{t,i}| > y)} = p \in [0,1]$*

*(c) $\eta_{t,i}$ is independent of idiosyncratic returns $x_{t,i}$ and systematic factors $\boldsymbol{f}_{t,i}$.*

The key condition is that $\xi_t > \gamma_t$, meaning the tails of the idiosyncratic returns are heavier than the noise. If the condition is violated, the idiosyncratic tail risk cannot be measured. Next, I assume high-frequency returns are contaminated with additive microstructure noise, such that the returns contaminated with noise follow

$$\begin{aligned} r^*_{t,i} &= r_{t,i} + x^*_{t,i}, \\ x^*_{t,i} &= x_{t,i} + \eta_{t,i}, \end{aligned} \tag{13}$$

where the $r_{t,i}$ are the returns in Equation (9), and $x^*_{t,i}$ are unobservable noisy idiosyncratic returns. Additive noise is by far the most common assumption in the microstructure literature.[9] In general, the distribution of the idiosyncratic returns contaminated with noise depend on the distribution of the microstructure noise. For example the volatility of $x^*_{t,i}$ is a function of the volatility of $x_{t,i}$ and $\eta_{t,i}$. However, if the tails of the microstructure noise $\eta_{t,i}$ are lighter than the tails of the idiosyncratic returns $x_{t,i}$, i.e. when $\xi_t > \gamma_t$, then the idiosyncratic returns contaminated with

[9]The theory also holds for other noise processes, such as multiplicative microstructure noise.

noise $x^*_{t,i}$ only inherit the tail risk of the idiosyncratic returns $x_{t,i}$ without the noise, which is demonstrated in the following theorem.

**Theorem 3.** *Under Assumption 1 for idiosyncratic returns $x_{t,i}$, Assumption 2 for noise $\eta_{t,i}$, and additive microstructure noise* (13)*, then $\forall i$,*

$$P(x^*_{t,i} \leq -y) \sim P(x_{t,i} \leq -y), \tag{14}$$

*where the $x^*_{t,i}$ are idiosyncratic returns contaminated with noise defined in Equation* (13)*.*

*Proof.* See Section B.6 in the appendix. □

Since microstructure noise such as the bid-ask bounce (Roll (1984)) have lighter tails than returns, Theorem (3) indicates that the bid-ask bounce will not affect idiosyncratic tail risk measurement. However, $\gamma_t$ is larger than $xi_t$ during rare microstructure events such as the 2013 Flash Crash, meaning the idiosyncratic tail risk cannot be measured during those events.

### 2.3 Volume Tail Risk Measure

In Gabaix, Gopikrishnan, Plerou, and Stanley (2006), tail returns are caused by the price impact of large intermediaries trading an abnormally large volume of shares, which implies the tails of idiosyncratic returns and trading volume are power-law distributed and proportional. In this section, I define a power-law measure of the right tail of trading volume, which is supported by the empirical evidence in Figure 9 of Section A. Let $s_{t,i}$ denote the total trading volume between time interval $i$ and $i-1$. Since trading volume has intraday seasonality, I use daily differences and measuring changes in trading volume $v_{t,i} = s_{t,i} - s_{t,i-d}$, where d denotes the number of intraday observations in a day. Next, I assume that changes in trading volume are power-law distributed with tail risk parameter $\nu_t$.

**Assumption 3.** *(a) Changes in trading volume $v_{t,i}$, $i = 1, ..., N$, are independent and identically distributed, and regularly varying with volume tail risk parameter $\nu_t > 0$. That is, $P(v_{t,i} > y) = y^{-\frac{1}{\nu_t}} L_{\nu_t}(y)$, where $L_\nu$ is a slowly varying function.*

*(b)* $\lim\limits_{y\to\infty} \frac{P(v_{t,i} \leq -y)}{P(|v_{t,i}| > y)} = p \in [0,1]$

I define volume tail risk (VTR) as the tail risk parameter $\nu_t$ of changes in trading volume $v_{t,i}$. Gabaix, Gopikrishnan, Plerou, and Stanley (2006) predict that large intermediary trades cause idiosyncratic tail risk and volume tail risk to be related according to $\xi_t \sim \rho\nu_t$ for some price impact measure $0 \leq \rho \leq 1$. This theoretical relationship can be tested by substituting volume tail risk in the place of idiosyncratic tail risk in the empirical asset pricing tests. If ITR and VTR are proportional, they should produce similar asset pricing results.

### 2.4 Estimation of Idiosyncratic Tail Risk and Volume Tail Risk

The monthly idiosyncratic tail risk $\xi_t$ for each firm is the main parameter of interest. Section 2.2 demonstrates that the monthly idiosyncratic tail risk inherits the tail risk of high-frequency idiosyncratic returns. However, idiosyncratic returns $x_{t,i}$ are unobservable and must be estimated. The factor model's filtered residuals are frequently used as the estimator of idiosyncratic returns.[10] Filtered residuals depend on the estimates of the factor betas, hence this section also discusses estimation of betas in heavy-tailed regressions. The asset pricing literature overwhelmingly uses ordinary least squares (OLS) to estimate factor model betas. When $\xi_t \leq 1/2$, the idiosyncratic returns have finite variance, and the Gauss-Markov Theorem proves the OLS estimate has the minimum variance of all linear unbiased estimators. However, when $\xi_t > 1/2$, the idiosyncratic returns have infinite variance and OLS is no longer efficient.[11]

Least Absolute Deviations (LAD) is a robust alternative to OLS when the errors have heavy tails, in the sense that LAD is generally more efficient than OLS when the regression errors have infinite variance.[12] Blattberg and Sargent (1971) show that the LAD estimator is the maximum likelihood estimate when the i.i.d. regression errors have infinite variance with a Laplace distribution (two-tailed exponential distribution). It is well known that the LAD estimator is consistent and asymptomatically normal when the errors have a distribution function that is differentiable at 0

[10]For example, see Ang, Hodrick, Xing, and Zhang (2006), Kapadia and Zekhnini (2019).

[11]Davis and Wu (1997) show that when $\xi_t > 1/2$, the least square estimate converges weakly to a ratio of stable distributions with heavy tails. Mikosch and de Vries (2013) derive explicit finite sample expressions for the tails of the OLS estimate. They show the OLS estimate is heavy-tailed in finite samples. OLS estimates that are heavy-tailed can have large errors. Balkema and Embrechts (2018) perform a simulation study and demonstrate the shortcomings of OLS and other estimators when the errors are heavy-tailed.

[12]This may not be true if the regressors have infinite variance, as discussed in Balkema and Embrechts (2018). While I find the factors are heavy-tailed, there is no evidence to suggest they have infinite variance. Additionally, the results are quantitatively similar when the factors are estimated using the weighted-LAD from Ling (2005), which is robust to infinite variance regressors.

with the derivative positive (See Koenker and Bassett (1978)). Knight (1998) shows that the LAD estimate is asymptotically normal under more general regularity conditions. To find the limiting distribution of the LAD-estimated betas $\hat{\boldsymbol{\beta}}_t$, requires the following assumption.

**Assumption 4.** *(a) The returns and factors $(r_{t,i}, \boldsymbol{f}_{t,i}^{\intercal})^{\intercal}$ are independently and identically distributed across i.*

*(b) The factors have bounded second moment, i.e., $E[||\boldsymbol{f}_{t,i}||^2] < \infty$.*

*(c) Idiosyncratic returns $x_{t,i}$ are continuously distributed given systematic factors $\boldsymbol{f}_{t,i}$, with conditional density $g(x_{t,i}|\boldsymbol{f}_{t,i})$, and with median zero conditional on the systematic factors, i.e., $\int_{-\infty}^{0} g(\lambda|\boldsymbol{f}_{t,i})d\lambda = \frac{1}{2}$.*

*(d) The systematic factors and idiosyncratic return density satisfy a "local identification" condition, meaning the matrix $C = E[g(0|\boldsymbol{f}_{t,i})\boldsymbol{f}_{t,i}\boldsymbol{f}_{t,i}^{\intercal}]$ is positive definite.*

From Assumption 4, the LAD estimates of the betas $\hat{\boldsymbol{\beta}}_t$ converge to in distribution to a multivariate normal distribution with covariance matrix $\frac{1}{4}C^{-1}E[\mathbf{f}_{t,i}\mathbf{f}_{t,i}^{\intercal}]C^{-1}$ (Powell (1991)).[13] Estimates of idiosyncratic returns are then the LAD filtered residuals

$$\hat{x}_{t,i} = r_{t,i} - \hat{\boldsymbol{\beta}}_t\mathbf{f}_{t,i}. \tag{15}$$

Following power-law literature, I use the Hill (1975) method to estimate the idiosyncratic tail risk $\xi_t$. When a distribution's tail is exactly Pareto (i.e. when L(x) is a constant) the Hill estimate is the maximum likelihood estimate of $\xi_t$. If the idiosyncratic returns are observable, the Hill estimate is

$$\hat{\xi}_t^{Hill} = \frac{1}{K_t}\sum_{k=1}^{K_t} log\frac{x_{t,(k)}}{x_{t,(K_t+1)}}, \tag{16}$$

where $x_{t,(k)}$ is the k-th order statistic of high-frequency idiosyncratic returns ($x_{t,(1)} \leq x_{t,(2)} \leq ... \leq x_{t,(N)}$) and $K_t$ is the total number of returns below threshold $x_{t,(K_t+1)}$ in month t. The Hill estimator only uses idiosyncratic returns less than negative threshold $x_{t,(K_t+1)}$, disregarding returns that do not belong to the left tail. The threshold is chosen to be sufficiently negative so that the estimator

[13] LAD estimation is only used in the creation of the idiosyncratic tail risk estimates and not in the asset pricing sections. All the regressions in the asset pricing sections follow the standard procedure by estimating betas with OLS.

only uses returns belonging to the left tail. Panel (a) of Figure 8 shows idiosyncratic returns greater than 2 standard deviations (approximately the 0.95 quantile of absolute idiosyncratic returns) are power-law distributed. Hence, a natural value for threshold $x_{t,(K_t+1)}$ is the 0.05 quantile of the stock's idiosyncratic returns in month t.[14] When $K_t \to \infty$ and $K_t/N \to 0$, the Hill estimate is consistent and asymptotically normal with variance $\xi_t^2$.

The Hill estimate in Equation (16) is infeasible, because the idiosyncratic returns are unobservable. Hill (2015) extends the Hill estimator to residuals filtered from a regression. According to Lemma 2 in Hill (2015), the Hill estimator applied to LAD filtered residuals is asymptotically normal with variance $\xi_t^2$. Hence, the firm's idiosyncratic tail risk estimate is

$$\hat{\xi}_t^{Hill-Res} = \frac{1}{K_t} \sum_{k=1}^{K_t} log \frac{\hat{x}_{t,(k)}(\hat{\boldsymbol{\beta}}_t)}{\hat{x}_{t,(K_t+1)}(\hat{\boldsymbol{\beta}}_t)}, \tag{17}$$

where $\hat{x}_{t,(K_t+1)}(\hat{\boldsymbol{\beta}}_t)$ is the k-th order statistic of the estimated idiosyncratic returns in Equation (15).

To estimate the volume tail risk $\nu_t$, I use the Hill (1975) estimator

$$\hat{\nu}_t^{Hill} = \frac{1}{M_t} \sum_{m=M_t+1}^{N} log \frac{v_{t,(m)}}{v_{t,(M_t+1)}}, \tag{18}$$

where $v_{t,(k)}$ is the m-th order statistic of changes in trading volume ($v_{t,(1)} \leq v_{t,(2)} \leq ... \leq v_{t,(N)}$) and $M_t$ is the total number of $v_{t,i}$ above threshold $v_{t,(M_t+1)}$ in month t. Figure 9 shows trading volume greater than 3 absolute deviations (approximately the 0.9 quantile of trading volume) are power-law distributed. Hence, a natural value for threshold $v_{t,(K_t+1)}$ is the 0.9 quantile of the stock's trading volume in month t. When $M_t \to \infty$ and $M_t/N \to 0$, the Hill estimate is consistent and asymptotically normal with variance $\nu_t^2$. This section's procedures result in a monthly idiosyncratic tail risk and volume tail risk estimate for each firm.

[14] Kelly and Jiang (2014) also use the 0.05 quantile for their Hill estimator. As a robustness check, I show that my empirical results are similar for the 0.025 and 0.1 quantiles. Liu and Stentoft (2020) conducts a large simulation study to show that the 0.05 quantile is accurate for market tail risk estimation.

# 3 Empirical Firm-Level Tail Risk

## 3.1 Data

I conduct a large empirical study of high-frequency equity returns, merging the Trade and Quotes (TAQ), the Center for Research in Security Prices (CRSP), and Compustat databases. The TAQ is the primary dataset, containing intraday transactions data for all stocks on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), NASDAQ, and other U.S. regional exchanges. I use the Monthly TAQ database up to 2003 then use the Daily TAQ database from 2004 to 2016.[15] I analyze all common stocks (Share Code 10 or 11) and price above $5. I clean the TAQ according to the procedures in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008), and extract second-by-second price, trade size, bid, and ask data between 9:30am - 4:00pm. The data is then aggregated into 5-minute intervals and merged with the monthly CRSP by the TAQ CUSIP key. During each month, I keep stocks with more than 150 negative returns (roughly 10% of 5-minute returns in a month) to ensure enough left tail observations and liquidity. Section C.1 in Appendix C provides further details on my TAQ data cleaning procedures. The empirical analysis includes 6,213 unique securities and 414,336 firm-month observations during the post decimalization sample period from January 2001 to December 2016,[16] and the least number of firms in a month is 2,616.[17]

In addition to intraday returns, I also include the overnight return of each stock in estimation procedures, which is consistent with the daily estimation procedures in Savor (2012), Jiang and Yao (2013), and Jiang and Zhu (2017). Overnight returns are calculated using daily CRSP data according to the formula in Lou, Polk, and Skouras (2019), which accounts for dividend adjustments, share splits, and other corporate events. I take the natural logarithm of their simple return to obtain log overnight return,

$$R_{overnight} = log(\frac{1 + R_{close-to-close}}{1 + R_{intraday}}), \tag{19}$$

[15]The monthly database reports data in second time-intervals, while the daily database reports data in milliseconds from January 2004 to July 27, 2015, microseconds to October 2016, and from November 2016 onwards.

[16]While the TAQ data is available from January 1993, I find a structural break in tail risk measures in 1997 across all stocks, caused by microstructure changes (tick size changes from 1/8 to 1/16). To avoid influence from tick size changes, the asset pricing analysis focuses on the post decimalization period.

[17]Factor betas are estimated using a 36 month rolling window, so idiosyncratic tail risk data is used from January 1998 to December 2001 to estimate the factor betas.

where $R_{close-to-close}$ is the standard return from the daily CRSP, and $R_{intraday} = \frac{P_{close}}{P_{open}} - 1$. Overnight returns are merged with intraday returns for idiosyncratic tail risk estimation.

The CRSP monthly database is used to obtain each stock's price, shares outstanding, and returns. Delisted stocks are adjusted using the delisting return from CRSP to avoid survivorship bias. Daily and monthly portfolio returns for the Fama and French and momentum factors are downloaded from Kenneth French's website. Anomaly portfolios and industry classifications are also downloaded from French's website.

The Compustat annual database is used to create the Fama and French characteristics from firm fundamentals. Compustat is merged with CRSP to replicate the Fama and French portfolio sorts and determine daily constituents for each portfolio. The daily constituents are merged with the TAQ database to create the high-frequency Fama and French factors. My exact procedures to create the high-frequency factors is described in Section C.2 of Appendix C.

I replicate an extensive set of control variables that may be correlated with idiosyncratic tail risk, and have demonstrated predictive power for cross-sectional stock returns. I include characteristics from Fama and French (2015), which are market beta, size, book-to-market, operating profitability, and investment. Momentum, short-term reversal, and illiquidity are included, since they can lead to tail risk. Finally, idiosyncratic volatility, coskewness, downside beta, and extreme positive returns are included, because they are related to the distribution of returns. The control variables are created using the CRSP and Compustat databases. Section C.3 in Appendix C describes the variables and procedures to create them.

### 3.2 High-Frequency Systematic Factors

The high-frequency factor literature generally uses two types of systematic factors: observable or latent. Observable factors are often created using portfolios that are sorted on characteristics known to explain the cross-sectional variation of expected returns (i.e. Fama and French (1993), Carhart (1997), Fama and French (2015)). In this paper, I adopt the approach of Aït-Sahalia, Kalnina, and Xiu (2020) to create high-frequency Fama and French (2015) factors using portfolios sorted on characteristics (see Section C.2 of Appendix C for further details). The baseline model in this paper is the well known Fama and French (2015) five factor model. I also use the market and Fama-French-

Cahart four factor models as robustness checks and find similar results.[18] Furthermore, I create a five factor model that uses high-frequency cross-sectional moments and liquidity variables. Specifically, I calculate the high-frequency cross-sectional mean absolute deviation around the median as a robust measure of dispersion, Hinkley (1975)'s robust coefficient of asymmetry skewness at the 0.95 quantile, the power-law tail risk measure of Kelly and Jiang (2014) using the Hill estimate at the 0.05 quantile, the mean percentage NBBO spread, and the mean dollar trading volume.

As a further robustness check, I consider an alternative latent factor model. Latent factors use statistical methods to create systematic factors. Pelger (2020) shows that the high-frequency statistical factors can be proxied by the equal-weighted market, oil, finance, and electricity portfolios. Motivated by their findings, I use their methods to construct the above market and industry factors for my sample of stocks, to create the industry statistical model.

### 3.3 The Idiosyncratic Tail Risk Premium

In this section, I verify the idiosyncratic tail risk premium by documenting that stocks with higher idiosyncratic tail risk have higher average excess returns. For each month from January 2001 until December 2016, each firm's idiosyncratic tail risk estimate is the Hill estimate of LAD residuals from the Fama and French (2015) five factor model. Stocks are sorted into decile groupings based on their idiosyncratic tail risk in that month. Decile 1 (low) contains stocks with the lowest idiosyncratic tail risk and decile 10 (high) contains stocks with the highest idiosyncratic tail risk. I form an equally-weighted portfolio and a value-weighted portfolio in each decile, and hold each portfolio for 1 month.

Panel A of Table 1 reports the average idiosyncratic tail risk in each decile. The high values of idiosyncratic tail risk illustrates that the returns of some stocks have infinite moments. A random variable's moment is infinite if its tail risk parameter exceeds the reciprocal of the moment. The skewness is infinite for deciles 2-10, since $\xi > 1/3$ in these deciles. The variance is infinite for deciles 9 and 10, because $\xi > 1/2$ in these deciles. All deciles have $\xi > 1/4$ indicating infinite kurtosis.

[18]Using high-frequency equity factors may have been controversial pre-decimalization due to high trading costs. However, in the post-decimalization sample used in this paper, trading costs have significantly decreased so that they are no longer a barrier to forming these traded factors. Note that the observable factors used in this paper are also openly traded as Exchange Traded Funds (ETFs), which accurately track the factors due to tracking-error constraints.Examples of ETFs that track factors include the SPY/VTI market, VTV value, VB size, VFQY quality (investment and profitability), VFMO momentum, and VFMF multi-factor ETFs).

Table 1: Excess Returns and Alphas of Portfolios Sorted on Idiosyncratic Tail Risk

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Panel A: Average Portfolio Idiosyncratic Tail Risk | | | | | | | | | | |
| Idiosyncratic Tail Risk | 0.32 | 0.36 | 0.38 | 0.39 | 0.41 | 0.43 | 0.45 | 0.47 | 0.51 | 0.65 | |
| | Panel B: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | |
| Excess Return | 0.47 | 0.51 | 0.67 | 0.63 | 0.62 | 0.75 | 0.71 | 0.73 | 0.87 | 1.03 | 0.57 |
| t-stat | (1.04) | (1.16) | (1.52) | (1.40) | (1.44) | (1.76) | (1.61) | (1.76) | (2.11) | (2.76) | (2.77) |
| FFC4 alpha | 0.51 | 0.57 | 0.72 | 0.69 | 0.66 | 0.80 | 0.74 | 0.78 | 0.89 | 1.05 | 0.54 |
| FFC4 t-stat | (1.23) | (1.38) | (1.75) | (1.70) | (1.67) | (1.98) | (1.83) | (2.07) | (2.30) | (2.96) | (2.69) |
| FF5 alpha | 0.68 | 0.70 | 0.85 | 0.84 | 0.78 | 0.94 | 0.85 | 0.87 | 0.96 | 1.11 | 0.43 |
| FF5 t-stat | (1.56) | (1.59) | (1.94) | (1.96) | (1.88) | (2.24) | (2.01) | (2.19) | (2.39) | (3.04) | (2.07) |
| | Panel C: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | |
| Excess Return | 0.23 | 0.39 | 0.50 | 0.52 | 0.41 | 0.42 | 0.53 | 0.48 | 0.77 | 0.89 | 0.66 |
| t-stat | (0.61) | (1.15) | (1.46) | (1.41) | (1.20) | (1.23) | (1.34) | (1.26) | (2.05) | (2.23) | (3.16) |
| FFC4 alpha | 0.27 | 0.45 | 0.55 | 0.56 | 0.45 | 0.46 | 0.55 | 0.56 | 0.80 | 0.88 | 0.61 |
| FFC4 t-stat | (0.77) | (1.41) | (1.72) | (1.77) | (1.42) | (1.50) | (1.61) | (1.65) | (2.21) | (2.40) | (2.90) |
| FF5 alpha | 0.43 | 0.58 | 0.59 | 0.69 | 0.50 | 0.55 | 0.67 | 0.68 | 0.84 | 1.00 | 0.57 |
| FF5 t-stat | (1.15) | (1.82) | (1.78) | (2.19) | (1.57) | (1.80) | (1.90) | (1.93) | (2.24) | (2.57) | (2.61) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to December 2016. Panel A reports the average idiosyncratic tail risk for portfolios sorted on idiosyncratic tail risk. Panel B reports equally-weighted excess returns and alphas for portfolios sorted on idiosyncratic tail risk. Panel C reports value-weighted excess returns and alphas for portfolios sorted on idiosyncratic tail risk.

These findings highlight the usefulness of power-law approximations that can model returns with infinite moments.

Panel B of Table 1 reports the average equal-weighted excess return of each decile portfolio and a portfolio that goes long the high idiosyncratic tail risk decile and shorts the low idiosyncratic tail risk decile (10-1). Average excess returns increase as the portfolio's average idiosyncratic tail risk increases. The long-short portfolio has an average monthly return of 0.57% with a t-statistic of 2.77.[19] The next 4 rows report returns and t-statistics relative to the Fama and French (1993) and Carhart (1997) four factor model (FFC4) and the Fama and French (2015) five factor model (FF5). Alphas are increasing in idiosyncratic tail risk. The long-short portfolio has a four factor alpha of 0.54% (t-stat of 2.69) and a five factor alpha of 0.43% (t-stat of 2.07).

Panel C of Table 1 reports the average value-weighted excess return of each decile portfolio and the long-short portfolio. Average value-weighted excess returns and alphas increase as the portfolio's average idiosyncratic tail risk increases. The long-short portfolio has an average return of 0.66% with a t-statistic of 3.16. The long-short portfolio has a four factor alpha of 0.61% (2.90)

[19] All t-statistics are calculated using Newey and West (1987) standard errors with one lag.

and a five factor alpha of 0.57% (2.61). In summary, stocks with higher idiosyncratic tail risk have statistically and economically higher average returns than stocks with lower idiosyncratic tail risk.

Appendix D verifies the robustness of these results. Appendix D.1 shows the cross-sectional relationship remains when estimating idiosyncratic tail risk using residuals from the CAPM, the Fama-French-Carhart four factor model, the high-frequency cross-sectional variable model, and the industry statistical model. The premium is also robust to using the 0.025 and 0.1 Extreme Value Theory thresholds $x_{t,(K_t+1)}$. In addition, results are similar for idiosyncratic tail risk estimated using midquotes instead of trades. Appendix D.2 reports double sorts on other firm characteristics that have been shown to predict returns. The double sorts demonstrate that conditional on other variables, average returns are still increasing in idiosyncratic tail risk.

I conclude that the idiosyncratic tail risk premium is a large economic phenomenon, highly statistically significant, and robust to different high-frequency factor models, tail risk thresholds, control variables, and midquotes. This section's cross-sectional findings are remarkably similar in magnitude to the cross-sectional results using jump risk implied by equity options. Kapadia and Zekhnini (2019) and Bégin, Dorion, and Gauthier (2019) also find a positive cross-sectional relationship between implied jump risk and average returns and argue the premium is caused by the inability to diversity jump risk and not due to a systematic factor. Option implied jump risk can be considered as the price of insurance against jumps. It's not obvious that the realized idiosyncratic tail risk estimated from historical returns will also earn a premium, since it is an realized measure of jumps. This section demonstrates that the premium is a pervasive phenomenon that also exists for realized measures of idiosyncratic tail risk.

### 3.4 The Volume Tail Risk Premium

The large intermediary hypothesis of tail risk predicts that there is a strong firm-level relationship between idiosyncratic tail risk and volume tail risk. As a litmus test, I evaluate whether stocks with higher Volume Tail Risk earn higher average returns. Volume tail risk is distinct from idiosyncratic tail risk, since it measured using trading volume data. For each month from January 2001 to December 2016, I estimate each firm's volume tail risk as the Hill estimate of the firm's changes in trading volume $v_{t,i}$. I sort stocks into decile groupings based on their VTR in that month. Decile 1 (low) holds stocks with the lowest VTR and decile 10 (high) holds with the highest VTR. I form an

Table 2: Excess Returns and Alphas of Portfolios Sorted on Volume Tail Risk

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Panel A: Average Portfolio Volume Tail Risk | | | | | | |
| Volume Tail Risk | 0.45 | 0.52 | 0.57 | 0.61 | 0.65 | 0.69 | 0.73 | 0.79 | 0.87 | 1.07 | |
| | | | | | Panel B: Univariate Sort on Volume Tail Risk (Equal-Weighted) | | | | | | |
| Excess Return | 0.30 | 0.58 | 0.61 | 0.67 | 0.65 | 0.63 | 0.79 | 0.73 | 0.84 | 1.17 | 0.88 |
| t-stat | (0.66) | (1.38) | (1.44) | (1.57) | (1.53) | (1.43) | (1.82) | (1.75) | (1.99) | (2.83) | (3.88) |
| FFC4 alpha | 0.34 | 0.62 | 0.66 | 0.73 | 0.70 | 0.69 | 0.85 | 0.80 | 0.88 | 1.16 | 0.83 |
| FFC4 t-stat | (0.82) | (1.64) | (1.66) | (1.83) | (1.73) | (1.66) | (2.04) | (1.98) | (2.21) | (3.14) | (3.80) |
| FF5 alpha | 0.48 | 0.77 | 0.78 | 0.86 | 0.80 | 0.82 | 0.96 | 0.89 | 0.97 | 1.25 | 0.77 |
| FF5 t-stat | (1.16) | (1.95) | (1.88) | (2.02) | (1.88) | (1.89) | (2.22) | (2.10) | (2.32) | (3.21) | (3.85) |
| | | | | | Panel C: Univariate Sort on Volume Tail Risk (Value-Weighted) | | | | | | |
| Excess Return | 0.31 | 0.50 | 0.69 | 0.77 | 0.77 | 0.63 | 0.68 | 0.78 | 0.79 | 0.98 | 0.67 |
| t-stat | (0.89) | (1.41) | (1.93) | (2.11) | (2.12) | (1.65) | (1.64) | (2.03) | (2.01) | (2.47) | (3.70) |
| FFC4 alpha | 0.35 | 0.54 | 0.72 | 0.83 | 0.80 | 0.68 | 0.72 | 0.82 | 0.85 | 0.97 | 0.61 |
| FFC4 t-stat | (1.09) | (1.68) | (2.32) | (2.59) | (2.38) | (1.96) | (1.97) | (2.29) | (2.48) | (2.84) | (3.22) |
| FF5 alpha | 0.46 | 0.65 | 0.82 | 0.95 | 0.87 | 0.80 | 0.79 | 0.90 | 0.96 | 1.03 | 0.58 |
| FF5 t-stat | (1.42) | (1.98) | (2.55) | (2.91) | (2.49) | (2.22) | (2.16) | (2.45) | (2.63) | (2.93) | (3.17) |

The table reports monthly average volume tail risk, excess returns, and alphas for portfolios sorted on Volume Tail Risk between January 2001 to December 2016. Panel A reports the average volume tail risk for portfolios sorted on volume tail risk. Panel B reports equally-weighted excess returns and alphas sorted on volume tail risk. Panel C reports value-weighted excess returns and alphas for portfolios sorted on volume tail risk.

equally-weighted and a value-weighted portfolio in each decile, and hold the portfolio for 1 month.

Panel A of Table 2 reports the average volume tail risk in each decile. VTR has very heavy tails. In deciles 2-10 trading volume has an infinite variance as indicated by $\xi > 1/2$. In decile 10 trading volume has an infinite expectation as indicated by $\xi > 1$. Power-law is particularly suitable for modeling trading volume, since moment-based measures become intractable with such heavy tails.

Panel B of Table 2 reports the average equal-weighted excess return of each decile portfolio and a portfolio that goes long the highest VTR decile and shorts the lowest VTR decile. The results are remarkably similar to the idiosyncratic tail risk anomaly in Section 3.3. Average excess returns and alphas increase as portfolio VTR increases. The long-short portfolio has an average return of 0.88% with a t-stat of 3.88. The long-short portfolio has a four-factor alpha of 0.83% (t-stat of 3.80) and five-factor alpha of 0.77% (t-stat of 3.85).

Panel C of Table 2 reports the average value-weighted excess returns of each decile portfolio and a long-short portfolio. The value-weighted returns and alphas of the long-short portfolio are nearly identical to the average returns and alphas of the idiosyncratic tail risk long-short portfolio

Table 3: Persistence in Excess Returns of Portfolios Sorted on Tail Risk

| | Idiosyncratic Tail Risk | | | | Volume Tail Risk | | | |
|---|---|---|---|---|---|---|---|---|
| Hold Period (Months) | 3 | 6 | 12 | 24 | 3 | 6 | 12 | 24 |
| Panel A: Univariate Sort on Idiosyncratic Tail Risk and Volume Tail Risk (Equal-Weighted) | | | | | | | | |
| (10-1) Return | 2.97 | 4.39 | 6.16 | 7.98 | 3.84 | 7.23 | 13.42 | 25.01 |
| t-stat | (5.08) | (5.56) | (5.00) | (3.13) | (4.40) | (3.95) | (3.77) | (3.51) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk and Volume Tail Risk (Value-Weighted) | | | | | | | | |
| (10-1) Return | 2.85 | 5.14 | 9.35 | 16.60 | 2.96 | 5.97 | 11.18 | 22.46 |
| t-stat | (4.39) | (3.78) | (4.04) | (3.44) | (3.85) | (4.13) | (3.37) | (3.71) |

The table reports long-horizon excess returns for portfolios sorted on idiosyncratic tail risk and volume tail risk between January 2001 to December 2016. Panel A reports equally-weighted excess returns for the idiosyncratic tail risk and volume tail risk long-short portfolios. Panel B reports value-weighted excess returns for the idiosyncratic tail risk and volume tail risk long-short portfolios.

in Section 3.3. These results confirm that VTR has predictive ability for the cross-section of average stock returns. The returns of the long-short portfolio for ITR and VTR are highly correlated. The equal-weighted long-short portfolios have a correlation of 0.71 (t-stat of 13.88) while the value-weighted long-short portfolios have a correlation of 0.43 (t-stat of 6.57). These results suggest that the ITR and VTR premiums may be driven by the same risk factor.

### 3.5 Persistence in Idiosyncratic Tail Risk and Volume Tail Risk Premia

Existing theories argue that the idiosyncratic tail risk premium is caused by an inability to fully diversity or an under-reaction to firm-specific news. In both theories, the premium should be short-lived as markets will quickly adjust to the firm-specific information. My risk-based theory predicts the premium is highly persistent as long as firms' tail risks and intermediary betas are fairly stable. If risk exposures do not change rapidly over time, then lagged idiosyncratic tail risk is correlated to current idiosyncratic tail risk and the risk premium should be persistent across stocks, that is those with high (low) past idiosyncratic tail risks should continue to experience high (low) future average returns.

To evaluate this hypothesis, in each month I sort stocks into decile groupings based on their ITR in that month, form an equally-weighted and a value-weighted portfolio in each decile, and hold the portfolio for 3, 6, 12, or 24 months. I perform an analogous portfolio sort for volume tail risk.

Table 3 reports the average excess returns of portfolios that buys the highest tail risk decile and shorts the lowest tail risk decile for 3, 6, 12, or 24 months. The left side of the table shows results for idiosyncratic tail risk long-short portfolios and the right side shows results for volume tail risk long-short portfolios. Panel A shows that the equal-weighted idiosyncratic tail risk and volume tail risk premia are highly persistent. Excess returns of the idiosyncratic tail risk long-short portfolio range from 2.97% (t-stat of 5.08) at the three-month horizon to 7.98% (t-stat of 3.13) at the two-year horizon. Similarly, excess returns of the volume tail risk long-short portfolio range from 3.84% (t-stat of 4.40) at the three-month horizon to 25.01% (t-stat of 3.51) at the two-year horizon.

Panel B shows that the value-weighted idiosyncratic tail risk and volume tail risk premia are also highly persistent and have higher magnitudes than the equal-weighted portfolios. Excess returns of the idiosyncratic tail risk long-short portfolio range from 2.85% (t-stat of 4.39) at the three-month horizon to 16.60% (t-stat of 3.44) at the two-year horizon. Similarly, excess returns of the volume tail risk long-short portfolio range from 2.96% (t-stat of 3.85) at the three-month horizon to 22.46% (t-stat of 3.71) at the two-year horizon.

The persistence of the idiosyncratic tail risk and volume tail risk long-short portfolio returns is consistent with the risk-based explanation proposed in this paper. This finding also provides evidence against existing theories of the premium, which should dissipate as the holding period increases. Instead, I find the returns increase as the holding period increases. Additionally, transaction costs should not affect the premium, since turnover is likely to be low when returns are so highly persistent.

### 3.6 Idiosyncratic Tail Risk Covariates

My explanation for the idiosyncratic tail risk premium relies on Gabaix, Gopikrishnan, Plerou, and Stanley (2006)'s hypothesis that large intermediaries cause tail risk through large trades. This section provides empirical evidence for their large intermediary hypothesis. In their economic model, the price impact of a trade scales with the trading volume, and the tail distribution of idiosyncratic returns is proportional to the tail distribution of trading volume. Hence, their hypothesis can be tested by analyzing the firm-level relationship between idiosyncratic tail risk and volume tail risk. Additionally, I use the Lee and Radhakrishna (2000) algorithm to identify intermediary trades and provide direct evidence on whether a higher percentage of intermediary trades is associated with

Table 4: Idiosyncratic Tail Risk Covariates

| | Panel A: VTR | | Panel B: Intermediary Dollar Volume | | | | | | Panel C: Lagged ITR | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) |
| Intercept | 0.32 | 0.31 | 0.39 | 0.37 | 0.39 | 0.38 | 0.40 | 0.38 | 0.28 | 0.28 | 0.32 | 0.32 | 0.35 | 0.35 |
| t-stat | 52.17 | 60.62 | 174.66 | 125.99 | 163.22 | 131.64 | 234.91 | 141.00 | 47.08 | 49.80 | 68.90 | 52.61 | 106.43 | 73.52 |
| VTR | 0.14 | 0.12 | | | | | | | | | | | | |
| t-stat | 14.01 | 16.21 | | | | | | | | | | | | |
| %Intermediary Total | | | 0.06 | 0.06 | | | | | | | | | | |
| t-stat | | | 17.98 | 13.60 | | | | | | | | | | |
| %Intermediary Sell | | | | | 0.12 | 0.10 | | | | | | | | |
| t-stat | | | | | 15.64 | 13.31 | | | | | | | | |
| %Intermediary Buy | | | | | | | 0.09 | 0.09 | | | | | | |
| t-stat | | | | | | | 14.86 | 14.21 | | | | | | |
| ITR t-1 | | | | | | | | | 0.32 | 0.27 | | | | |
| t-stat | | | | | | | | | 23.90 | 17.02 | | | | |
| ITR t-12 | | | | | | | | | | | 0.22 | 0.17 | | |
| t-stat | | | | | | | | | | | 18.36 | 12.29 | | |
| ITR t-24 | | | | | | | | | | | | | 0.14 | 0.10 |
| t-stat | | | | | | | | | | | | | 13.68 | 9.05 |
| Size | | 0.00 | | 0.00 | | 0.00 | | 0.00 | | 0.00 | | 0.00 | | 0.00 |
| t-stat | | 1.59 | | -1.93 | | -1.73 | | -1.87 | | -0.36 | | -0.40 | | -0.05 |
| ILLIQ | | 0.12 | | 0.13 | | 0.12 | | 0.13 | | 0.10 | | 0.11 | | 0.11 |
| t-stat | | 3.96 | | 4.58 | | 4.56 | | 4.61 | | 4.54 | | 5.55 | | 5.93 |
| IVOL | | 0.01 | | 0.01 | | 0.01 | | 0.01 | | 0.01 | | 0.01 | | 0.01 |
| t-stat | | 17.71 | | 14.53 | | 14.18 | | 14.99 | | 17.99 | | 16.73 | | 16.88 |
| % Adj. $R^2$ | 10.19 | 16.75 | 1.96 | 13.35 | 2.24 | 13.18 | 1.25 | 12.84 | 10.87 | 18.63 | 5.20 | 13.87 | 2.43 | 11.32 |

Monthly firm-level regression of idiosyncratic tail risk on volume tail risk, percent intermediary dollar volume, and lagged idiosyncratic tail risk, controlling for size, illiquidity, and idiosyncratic volatility. The table reports point estimates, Newey-West t-statistics with one lag, and adjusted $R^2$.

a higher idiosyncratic tail risk.[20] I also classify the direction of intermediary trades using the Lee and Ready (1991) algorithm to test whether a higher percentage of intermediary selling (buying) as a percentage of total trades is associated with a higher idiosyncratic tail risk. Finally, I verify the persistence of idiosyncratic tail risk. In each month from January 2001 to December 2016, I run Fama-MacBeth cross-sectional regressions on nested versions of

$$ITR_{l,t} = \pi_{0,t} + \pi_{1,t} VTR_{l,t} + \pi_{2,t} \% IntermediaryVolume_{l,t} + \pi_{3,t} ITR_{l,t-k} + \pi_{4,t} Controls_{l,t} + \epsilon_{l,t}, \quad (20)$$

where for each firm $l$ and month t, $ITR_{l,t}$ is the idiosyncratic tail risk, $VTR_{l,t}$ is the volume tail risk, $\%IntermediaryVolume_{l,t}$ is the dollar volume of intermediaries' large trades as a percentage of the stock's total dollar volume, $ITR_{l,t-k}$ is idiosyncratic tail risk with lag $k$, and $Controls_{l,t}$ include size, illiquidity, and idiosyncratic volatility.[21]

Table 4 reports point estimates, Newey-West t-statistics with one lag, and the adjusted $R^2$ for regression (20). Column (1) of Panel A regresses idiosyncratic tail risk on just volume tail risk. The intercept term of 0.32 can be interpreted as the base level of ITR. Volume tail risk contributes

[20]Large trades are classified as intermediary driven if their dollar volume exceeds \$100,000 for the largest, \$50,000 for the medium, and \$20,000 for the smallest tripartite of stocks (Lee and Radhakrishna (2000)).

[21]The results are robust to controlling for firm-level and month fixed effects, available upon request.

$0.14 \times VTR$ to the firm's monthly idiosyncratic tail risk. VTR also has a high degree of explanatory power for the cross-section of ITR, with an adjusted $R^2$ of 10%. In column (2), VTR coefficient does not substantially change after controlling for size, idiosyncratic volatility, illiquidity. This finding supports the theoretical link between idiosyncratic tail risk and volume tail risk.

Column (3) in Panel B reports results for a regression of idiosyncratic tail risk on percent intermediary dollar volume. There is a strong relationship between ITR and percent intermediary dollar volume. Stocks with 1% higher intermediary dollar volume have 0.06 high ITR. Since the average ITR is roughly 0.44 with a standard deviation of 0.1, this increase is highly economically significant. One potential cause of this relationship may be that stocks that are small, illiquid or volatile may be associated with higher intermediary dollar volume. However, after controlling for these characteristics in column (4), the coefficient is unchanged. Column (5) and (8) report results for a regression of ITR on percent intermediary selling and buying volume respectively. Since ITR is a measure of the left tail, it should be more affected by intermediary selling volume, which is what I show. Stocks with 1% higher intermediary selling volume have 0.12 high ITR. ITR is also associated with higher buying volume, which is consistent with large trades absent of information being quickly reversed by arbitragers. These results provide strong empirical support for the large intermediary theory of idiosyncratic tail risk.

In contrast to existing theories, my explanation of predicts idiosyncratic tail risk is persistent. Columns (9), (11), and (13) in Panel C report results for a regression of idiosyncratic tail risk on itself with a 1-month, 1-year, and 2-year lag. Idiosyncratic tail risk is highly persistent at the 1-month horizon with an adjusted $R^2$ of nearly 11%. The persistence decreases at the 1-year and 2-year horizons, but their coefficients remain statistically significant. Since, idiosyncratic tail risk is estimated with non-overlapping data, this persistence is not mechanical. This finding is consistent with investor herding, where intermediaries frequently purchase the same stocks resulting in autocorrelated stock trades as documented in Sias (2004). The persistence of idiosyncratic tail risk is also consistent with the persistence of its premium, which requires the risk measure to be fairly stable. This finding provides evidence against the existing theory that idiosyncratic tail risk is only caused by inability to diversify or firm-specific news, which should have no persistence.

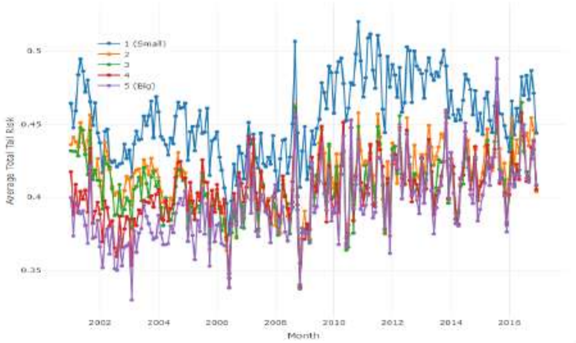
# 4 Commonality in Idiosyncratic Tail Risk and Pricing Implications

A key hypothesis in my explanation of the ITR premium is that idiosyncratic tail risk follows a strong factor structure and that common idiosyncratic tail risk is correlated to the intermediary marginal utility of wealth. In this section, I verify that idiosyncratic tail risk follows a strong factor structure and that a single factor explains a high degree of the time variation in firm-specific tail risk. The common idiosyncratic tail risk factor is highly correlated to existing intermediary factors and is procyclical. Stocks with higher exposures to this common idiosyncratic tail risk factor earn higher average returns, suggesting it's a priced risk factor. Asset pricing tests show that the common idiosyncratic tail risk factor explains the idiosyncratic tail risk premium.

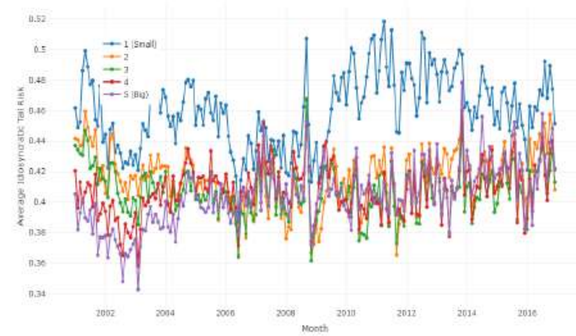
## 4.1 Factor Structure in Idiosyncratic Tail Risk

Firms with different characteristics share common idiosyncratic tail risk dynamics over time. Figure 2 plots average total tail risk (tail risk of returns) and idiosyncratic tail risk for portfolios formed on size, book-to-market, and industry. Panels (a) and (b) plot average total tail risk and idiosyncratic tail risk of size quintiles. As expected, levels of tail risk decreases as firm size increases. However, the idiosyncratic tail risk of different size quintiles are also strongly correlated through time. The largest firms are several orders of magnitude larger than the smallest firms, but their idiosyncratic tail risk has a time-series correlation of 44.6%. Panels (c) and (d) plot average total tail risk and idiosyncratic tail risk of firms sorted by book-to-market. The idiosyncratic tail risk of quintiles with the highest and lowest book-to-market have a correlation of 60.5%. Panels (e) and (f) plots average idiosyncratic tail risk for firms sorted into the five-industry SIC code categories from Kenneth French's website. The idiosyncratic tail risk of the industries have an average pairwise correlation of 68.7% with a minimum correlation of 58.4% between the health and manufacturing sectors. Firms with different size, book-to-market and in different industries all share common idiosyncratic tail risk dynamics. Additionally, the total tail risk time-series dynamics in Panels (a), (c), and (e) are nearly identical to the idiosyncratic tail risk dynamics in Panels (b), (d), and (f), indicating that most of the time-series variation in total tail risk is coming from the idiosyncratic component, and not the systematic component.

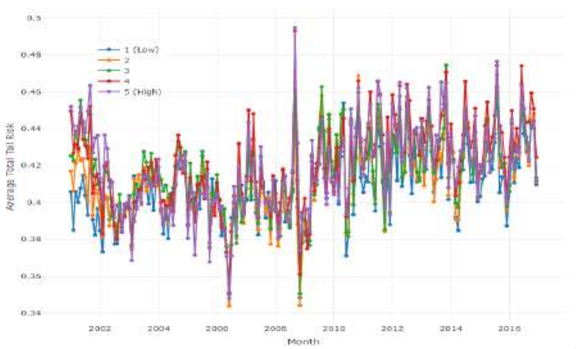
**Figure 2: Average Idiosyncratic Tail Risk of Portfolios Formed on Firm Characteristics**



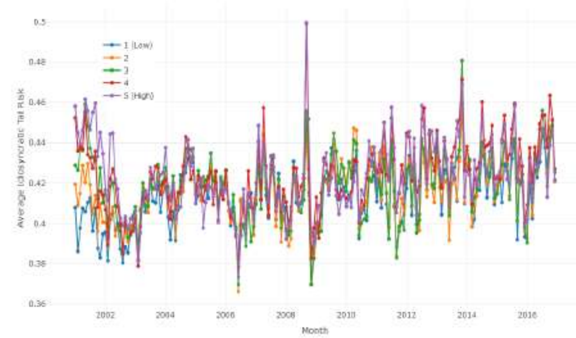

(a) Average Total Tail Risk by Size



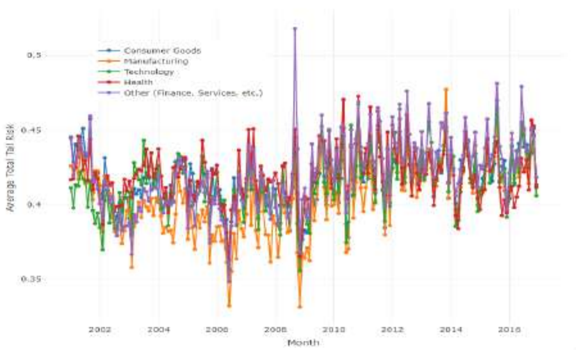

(b) Average Idiosyncratic Tail Risk by Size



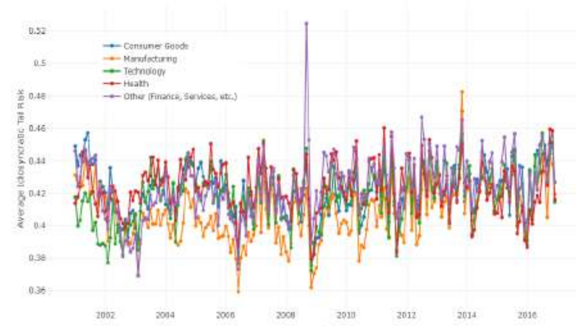

(c) Average Total Tail Risk by Book-to-Market




(d) Average Idiosyncratic Tail Risk by Book-to-Market




(e) Average Total Tail Risk by Industry




(f) Average Idiosyncratic Tail Risk by Industry

The figure plots monthly total tail risk and idiosyncratic tail risk averaged within size quintiles, book-to-market quintiles, and industries from January 2001 to December 2016. Each month, total tail risk for each stock is the Hill estimator of log returns, and idiosyncratic tail risk for each stock is calculated as the Hill estimator of residuals from the Fama and French (2015) five factor model. Panel (a) and (b) shows total tail risk and idiosyncratic tail risk averaged within market capitalization quintiles. Panel (c) and (d) shows total tail risk and idiosyncratic tail risk averaged within book-to-market quintiles. Panel (e) and (f) shows total tail risk and idiosyncratic tail risk averaged within the five-industry SIC categorizes on Kenneth French's website.

Tail risk of returns is expected to be correlated by Equation (1), since the systematic factors are heavy-tailed.[22] Returns inherit the tail behaviour of systematic factors, and time variation in factor tail risk causes the tail risk of returns to be correlated. For example in the CAPM, returns inherits the tail risk of the market factor. However, the high correlation of idiosyncratic tail risks is unexpected, since idiosyncratic returns are uncorrelated to systematic factors. If idiosyncratic tail risk is only caused by firm-specific shocks as in Merton (1976), then there should be no correlation of firm-specific tail risks. The observed commonality suggests firm-level tail risk is driven by a common factor.

The high correlation of idiosyncratic tail risks suggests modeling the dynamics using a single factor model. Consistent with my explanation in Equation (2), I define common idiosyncratic tail risk (CITR) as the mean cross-sectional idiosyncratic tail risk in each month. For each firm, I run monthly and annual time-series regressions of idiosyncratic tail risk on CITR,[23]

$$\xi_{l,t} = \kappa_{l,0} + \kappa_{l,1} CITR_t + e_{l,t}, \tag{21}$$

which is analogous to the hypothesized factor structure in Equation (4) with $\kappa_{l,0} = -\mu_N$ and $\kappa_{l,1} = 1$. This single factor model for idiosyncratic tail risk has an average $R^2$ of 13.6% for the monthly regression and an average $R^2$ of 42.6% for the annual regression. In comparison, Bégin, Dorion, and Gauthier (2019) document an $R^2$ of 56.4%. However, their sample of 260 S&P stocks is much smaller and more homogeneous than the 6,213 equities sampled in this paper. Considering that my sample contains small and medium stocks, the 42.6% average $R^2$ is economically significant.

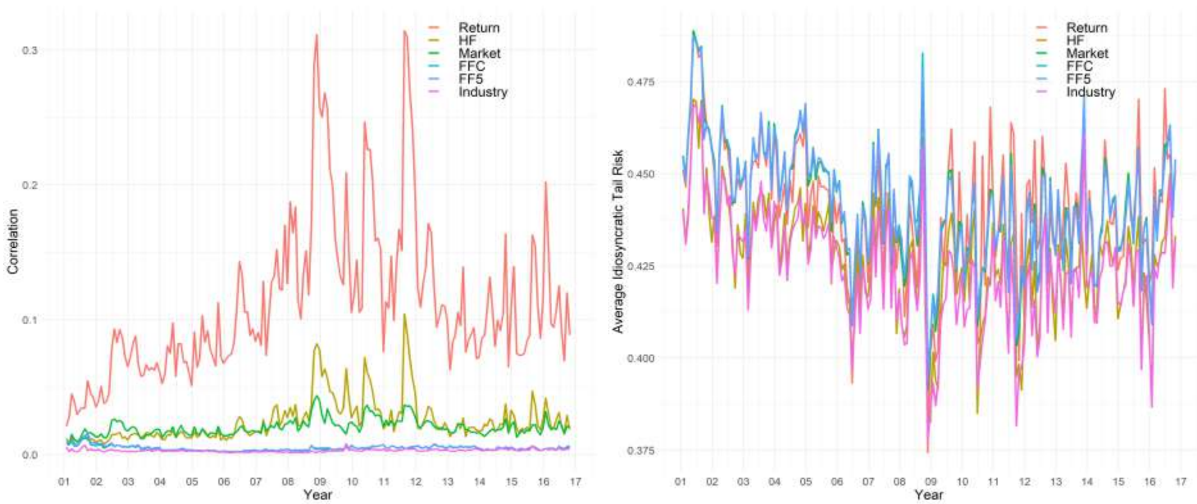
Another natural comparison can be done with common idiosyncratic volatility (CIV) from Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016).[24] They document that the annual regression of idiosyncratic volatility on CIV has an average $R^2$ of 35%. However, they examine a larger sample at a longer time-span, so it is again difficult to make direct comparisons. However, it is evident that idiosyncratic tail risk follows a strong factor structure, where CITR explains most of the variation in idiosyncratic tail risk.

[22] Since the systematic factors are portfolios of heavy-tailed returns, they will also be heavy-tailed. For example, it is well documented that the market return is heavy-tailed (Kelly and Jiang (2014))

[23] Annual idiosyncratic tail risk and CITR is calculated as the average of their monthly values within each year.

[24] The CITR factor only has a pairwise correlation of 0.08 (t-stat of 0.97) with the CIV factor in Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) and the opposite price of risk. Additionally, while the volatility factor of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) is countercyclical, I affirm that the idiosyncratic tail risk factor is strongly procyclical indicating they are separate factors.

**Figure 3: Average Pairwise Correlations and Average Idiosyncratic Tail Risks**




(a) Average pairwise correlations (b) Average idiosyncratic tail risks

The figure plots the average correlation and idiosyncratic tail risks of returns and factor model residuals from 2001 to 2016. Idiosyncratic tail risk is the Hill estimate of residuals from the market model (Market), high-frequency cross-section variable model (HF), industry statistical model (Industry), Fama-French-Cahart four factor model (FFC), or Fama and French five factor model (FF5). Panel A shows the average pairwise Spearman correlation for returns and residuals for each month. The figures for Pearson and Kendall correlations look nearly identical. Panel B shows the average idiosyncratic tail risk across firms for each month.

## 4.2 Robustness to Omitted Variables

The commonality of the idiosyncratic tail risks cannot be explained by omitted common factors if any. Panel A of Figure 3 shows that high-frequency returns exhibit substantial common variation, with an average monthly pairwise correlation of 11% during 2001 to 2016, and a monthly pairwise correlation exceeding 30% during the Subprime Mortgage and Euro Debt crises. Even after removing the market factor, the residuals continue to exhibit some common variation, with an average and maximum pairwise correlation of 2% and 4.4% respectively. The Fama and French five factor model captures nearly all the common variation of high-frequency returns, as the average pairwise correlation among its residuals is only 0.05% and the maximum is 1.5% during the 9/11 terrorist attack (after 2001, the average pairwise correlation never exceeds 0.9%).[25] The Fama-French-Cahart four factor and industry statistical models also remove most of the covariation from returns.

[25] Conversely, the high-frequency cross-sectional variable model does not remove as much of the covariation as the Fama and French models. The failure of the high-frequency cross-sectional variable model is consistent with the finding from Huddleston, Liu, and Stentoft (2020) that price trends and liquidity variables do not help forecast high-frequency market returns.

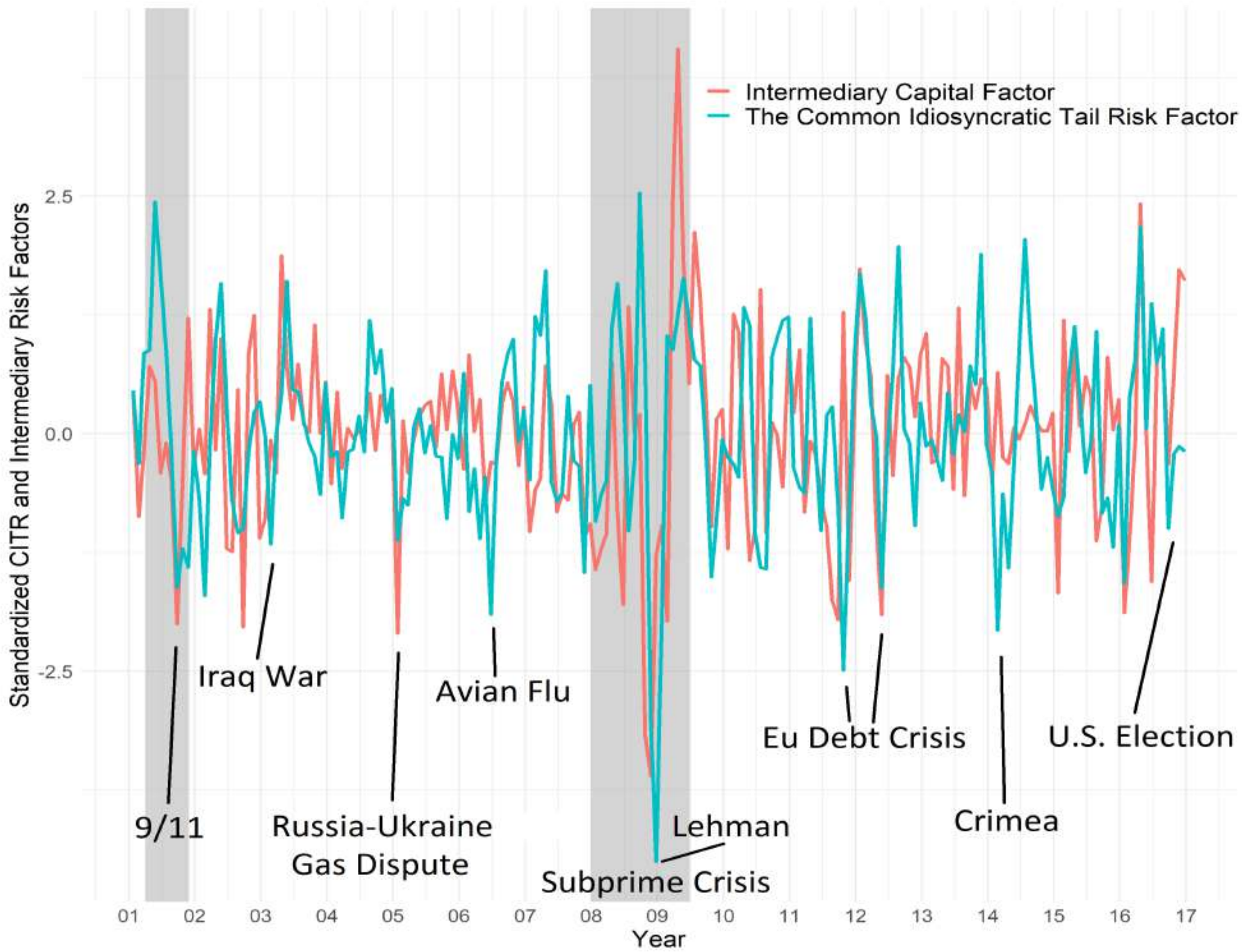
Panel B of Figure 3 shows that the average total tail risk of returns and the idiosyncratic tail risk from the various factor models are nearly identical, despite the fact that the Fama and French models and industry model saturate nearly all of the covariation in returns. This makes omitted factors an unlikely explanation for the strong commonality in idiosyncratic tail risks. Furthermore, Section D.1 shows that the idiosyncratic tail risk premium is robust to these alternative factor models, verifying that omitted factors also cannot explain the premium.

### 4.3 Link Between Common Idiosyncratic Tail Risk and Intermediary Factors

A key assumption in my explanation is that CITR, the factor driving idiosyncratic tail risks, is correlated to the intermediary marginal utility of wealth. When intermediary funding is high, they trade on firm-specific signals, and the average idiosyncratic tail risk increases. When constraints tighten, investment opportunities increase as intermediaries are unable to fund many firm-specific trades, and the average idiosyncratic tail risk falls. A testable implication is that CITR is highly correlated to intermediary factors in the literature and negative economic news should be associated with *decreases* to CITR.

As empirical evidence, I plot the CITR factor and the intermediary capital factor from He, Kelly, and Manela (2017), downloaded from Asaf Manela's website. Since their intermediary factor is defined as quarterly shocks to intermediary capital levels, I define the CITR factor $\triangle CITR$ as 3-month differences in CITR levels, that is $\triangle CITR_t = CITR_t - CITR_{t-3}$. The intermediary capital factor is the change in the equity capital ratio of large primary dealers, a prominent example of sophisticated intermediaries. Figure 4 shows that the CITR factor and intermediary capital factor are remarkably correlated, despite the fact that CITR is measured using high-frequency equity returns and intermediary capital is measured using quarterly accounting data. The CITR factor and the intermediary capital factor have a monthly correlation of 0.32 (t-stat of 4.58) and a quarterly correlation of 0.54 (t-stat of 4.30). Additionally, the CITR factor and the broker-dealer leverage factor from Adrian, Etula, and Muir (2014) have a quarterly correlation of 0.31 (t-stat of 2.15). The CITR factor and the (negative) Leverage Constraint Tightness factor (the average market beta of actively managed mutual funds is correlated to intermediary funding liquidity tightness) from Boguth and Simutin (2018) has a correlation of 0.20 (t-stat of 2.67). The empirical correlation between CITR and existing intermediary factors provide strong support for my hypothesis that

**Figure 4: The Common Idiosyncratic Tail Risk and Intermediary Capital Factors**




The common idiosyncratic tail risk factor and the intermediary capital factor. Both time-series are standardized to zero mean and unit variance. The monthly sample is from January 2001 to December 2016. CITR is the mean monthly idiosyncratic tail risk.

CITR is correlated to the intermediary marginal utility of wealth.

Figure 4 also illustrates that there is strong evidence that CITR decreases during negative news shocks. Significant negative news like the 9/11 terrorist attack, Iraq war, Russia-Ukraine gas dispute, Lehman Brothers bankruptcy, EU debt crisis, and 2016 election are associated with large decreases in both the CITR and intermediary factors. The largest decrease to CITR occurred in October 2008 after the bankruptcy of Lehman Brothers when intermediary funding was at its lowest. Interestingly, the 2004 Avian flu outbreaks and 2014 Crimea Annexation resulted in much larger decreases in CITR relative to the intermediary capital factor, illustrating that CITR contains

Table 5: Correlations between the CITR factor and Economic Variables

| Monthly Variables | P/E Growth | Market Volatility Growth | Market Return |
|---|---|---|---|
| Pairwise Correlation | 0.22 | -0.07 | 0.15 |
| t-stat | (3.16) | (-0.93) | (2.14) |

| Quarterly Variables | GDP Growth | Investment Growth | Consumption Growth |
|---|---|---|---|
| Pairwise Correlation | 0.27 | 0.25 | 0.18 |
| t-stat | (2.23) | (1.98) | (1.42) |

Time-series correlations between the common idiosyncratic tail risk factor and economic variables from January 2001 to December 2016. The common idiosyncratic tail risk factor each month is the mean firm-level tail risk. Monthly growth (log changes) factors are Price-to-Earnings of the S&P 500, market volatility, and market prices. Quarterly growth (log changes) factors are GDP, investment, and consumption.

additional information. These two events show that large intermediaries decreased large trades as possible risks increased, but quickly reversed course when the risk failed to materialize.

Consistent with my hypothesis, CITR is correlated to existing intermediary factors. Negative economic news is associated with decreases to CITR, which contradicts the fire-sale and labor risk theories of idiosyncratic tail risk. The strong empirical connection between shocks to CITR and negative economic news rules out the theory that idiosyncratic tail risk is only caused by firm-specific news shocks.

### 4.4 Cyclicality of Common Idiosyncratic Tail Risk

In my explanation, idiosyncratic tail risk comoves with intermediary funding, which is a negative function of the shadow cost of capital $\phi_1$. A testable implication is that the common idiosyncratic tail risk factor should be procyclical, since $\phi_1$ is counter-cyclical. In this section, I examine the empirical correlation of CITR to innovations in financial and real economic variables.

Table 5 reports correlations of the common idiosyncratic tail risk factor and growth (log changes) of aggregate macro variables. Monthly economic variables include Robert Shiller's (Shiller (2015)) Cyclically Adjusted Price-to-Earnings Ratio of the S&P 500, market volatility, and market prices. Seasonally adjusted quarterly economic variables include real GDP, gross private domestic investment, and real personal consumption expenditures. All correlations support the procyclicality of idiosyncratic tail risk. Shocks to common idiosyncratic tail risk are positively correlated with positive economic growth, measured as increases to Price-to-Earnings, market prices, GDP, investment, and consumption. CITR and market volatility have nearly zero correlation, further illustrating that volatility does not drive tail risk.

The commonality and procyclicality of idiosyncratic tail risk is difficult to reconcile with the theories positing that idiosyncratic tail risk is caused by firm-specific shocks (Merton (1976)), investor fire-sales, or labor risk. If idiosyncratic tail risk is only caused by firm-specific news, then there would be no commonality in idiosyncratic tail risk. Alternatively, if idiosyncratic tail risk is primarily caused by fire-selling during a liquidity crisis or labor risk during an economic crisis, then there may be common variation in idiosyncratic tail risk, but it would be countercyclical. The procyclical common idiosyncratic tail risk factor documented in this section strongly supports my hypothesis that idiosyncratic tail risk is driven by shocks to intermediary funding.

### 4.5 CITR Exposure and Average Returns

In my explanation for the idiosyncratic tail risk premium, CITR is a priced risk factor. This section documents that stocks' exposure to the common idiosyncratic tail risk factor helps to explain differences in the cross-section of expected returns. For each month from January 2001 to December 2016, I estimate each stock's CITR-beta by regressing the stock's monthly excess return on the CITR factor using a 36-month trailing window.[26] This procedure results in a CITR-beta for each stock in each month. A stock's CITR-beta is a measure of its exposure to the common idiosyncratic tail risk factor.

For the analysis, I sort stocks in each month into quintiles based on their CITR-betas. Stocks in quintile 1 have the least exposure to CITR and stocks in quintile 5 have the most CITR exposure. I form an equal-weighted portfolio and a value-weighted portfolio in each quintile and hold the portfolios for 1 month. Panel A of Table 6 reports the average CITR-beta in each quintile. There is a large range of CITR-betas from -0.97 in the lowest quintile to 2.28 in the highest quintile. Stocks in quintile 1 hedge states of low CITR, while stocks in quintile 5 lose value when CITR is low.

Panel B reports excess returns for the equal-weighted portfolios, and a portfolio that goes long the highest CITR-beta quintile and shorts the lowest CITR-beta quintile. Excess returns and four

[26] A stock is included only if it has no missing return in the 36-month estimation window. Additional returns from 1998-2001 are included in estimating betas.

Table 6: Excess Returns and Alphas of Portfolios Sorted on CITR-betas

| | 1 (Low) | 2 | 3 | 4 | 5 (High) | (5-1) |
|---|---|---|---|---|---|---|
| Panel A: Average CITR-beta | | | | | | |
| Average CITR-beta | -0.97 | 0.05 | 0.57 | 1.13 | 2.28 | |
| Panel B: Univariate Sort on CITR-beta (Equal-Weighted) | | | | | | |
| Excess Return | 0.50 | 0.70 | 0.70 | 0.89 | 0.90 | 0.40 |
| t-stat | (1.11) | (1.82) | (1.79) | (2.07) | (1.72) | (2.06) |
| FFC4 alpha | 0.55 | 0.74 | 0.74 | 0.94 | 0.95 | 0.40 |
| FFC4 t-stat | (1.30) | (2.08) | (2.00) | (2.28) | (1.93) | (2.21) |
| FF5 alpha | 0.73 | 0.86 | 0.83 | 0.99 | 1.01 | 0.28 |
| FF5 t-stat | (1.72) | (2.31) | (2.10) | (2.28) | (1.97) | (1.77) |
| Panel C: Univariate Sort on CITR-beta (Value-Weighted) | | | | | | |
| Excess Return | 0.14 | 0.37 | 0.54 | 0.63 | 0.76 | 0.62 |
| t-stat | (0.37) | (1.20) | (1.67) | (1.71) | (1.42) | (2.01) |
| FFC4 alpha | 0.15 | 0.42 | 0.60 | 0.70 | 0.80 | 0.65 |
| FFC4 t-stat | (0.42) | (1.44) | (1.98) | (1.96) | (1.65) | (2.21) |
| FF5 alpha | 0.29 | 0.50 | 0.70 | 0.78 | 0.84 | 0.55 |
| FF5 t-stat | (0.86) | (1.72) | (2.25) | (2.08) | (1.73) | (2.08) |

The table reports monthly average CITR-beta, excess returns, and alphas for portfolios sorted on CITR-betas between January 2001 to December 2016. Panel A reports the average CITR-beta for portfolios sorted on CITR-beta. Panel B reports equally-weighted excess returns and alphas sorted on CITR-beta. Panel C reports value-weighted excess returns and alphas sorted on CITR-beta.

factor alphas are monotonically increasing in CITR-betas. The long-short portfolio has an average return of 0.40% (t-stat of 2.06). The next 4 rows report excess returns relative to the Fama-French-Cahart four factor model and Fama and French five factor model. The long-short portfolio has a four factor alpha of 0.40% (t-stat of 2.21) and five-factor alpha of 0.28% (t-stat of 1.77). Panel C reports excess returns and alphas for value-weighted portfolios. Value-weighted excess returns and alphas are monotonically increasing in CITR-betas. The long-short portfolio has an average monthly return of 0.62% (t-stat of 2.01), four factor alpha of 0.65% (t-stat of 2.11), and five factor alpha of 0.55% (t-stat of 2.08).

In summary, stocks with high CITR-betas have economically and statistically higher returns than stocks with low CITR-betas. These results support the hypothesis that stocks with low or negative CITR exposure provide hedges for states of low CITR, and stocks with high CITR exposure are compensated for the additional risk. These results support my hypothesis that CITR is a priced risk factor. Additionally, the positive price of risk for CITR contradicts the existing fire-sale or labour risk theories of tail risk that imply a negative price of risk, and also contradicts the news explanation that implies no price of risk.

### 4.6 Pricing Idiosyncratic Tail Risk and Volume Tail Risk Portfolios

This section tests the risk-based explanation for the idiosyncratic tail risk premium using formal asset pricing procedures. To do so, I demonstrate that the CITR factor is priced in portfolios sorted on idiosyncratic tail risk and volume tail risk. I also show that differences in CITR exposure of idiosyncratic tail risk and volume tail risk portfolios can account for most of the differences in average returns. I conduct a two-stage Fama and MacBeth (1973) estimation procedure from January 2001 to December 2016. In the first stage, I estimate factor betas for each test asset k from time-series regression,

$$R_{k,t+1} - R_{f,t} = a^k + \beta_{k,CITR} \triangle CITR_{t+1} + \beta_{k,M}(R_{M,t+1} - R_{f,t}) + e_{k,t+1}, \tag{22}$$

where $R_{k,t+1} - R_{f,t}$ are monthly excess returns for test asset k, $\triangle CITR_{t+1}$ is the common idiosyncratic tail risk factor in percentage terms (that is, three-month changes in CITR times 100), and $R_{M,t+1} - R_{f,t}$ is the excess return on the market portfolio. In the second stage, I use the estimated betas to run cross-sectional regression,

$$E[R_{k,t+1} - R_{f,t}] = \alpha + \beta_{k,CITR}\lambda_{CITR} + \beta_{k,M}\lambda_M + \epsilon_k, \tag{23}$$

where $E[R_{k,t+1} - R_{f,t}]$ is the average excess return of test asset k, $\lambda_{CITR}$ is the risk price for the common idiosyncratic risk factor, and $\lambda_M$ is the risk price of the market, reported in percentage terms. In addition to point estimates of the risk prices, I adjust the Fama and MacBeth standard errors for time-series correlation by reporting Newey-West t-statistics with one lag and Shanken t-statistics (Shanken (1992)). To evaluate model fit, I report the cross-sectional $R^2$ and the mean absolute pricing error (MAPE), both in percentage terms.

Panel A in Table 7 reports results for using the decile portfolios sorted on idiosyncratic tail risk from Section 3.3 as test assets, to determine whether the CITR factor explains the idiosyncratic tail risk premium. Column (1) reports results for the pricing model using only the CITR factor. Consistent with the risk-based explanation, CITR is priced in the idiosyncratic tail risk deciles

Table 7: Cross-sectional Asset Pricing Tests on ITR and VTR Deciles

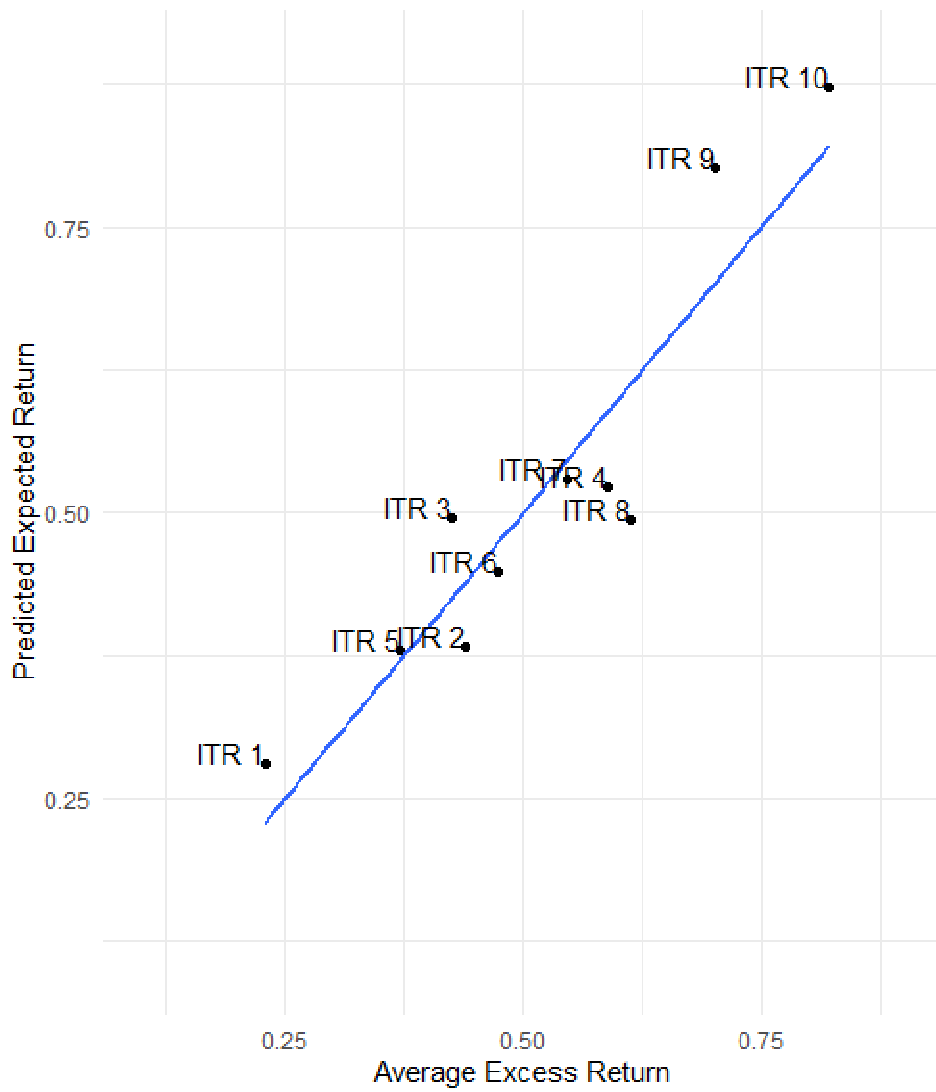
| | Panel A: Idiosyncratic Tail Risk Deciles | | Panel B: Volume Tail Risk Deciles | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Intercept | 0.00 | 1.49 | 0.20 | 3.33 |
| NW t-stat | (0.01) | (1.73) | (0.55) | (4.15) |
| Shanken t-stat | [0.00] | [1.28] | [0.47] | [2.99] |
| $R^M - R^f$ | | -1.04 | | -2.84 |
| NW t-stat | | (-1.13) | | (-3.31) |
| Shanken t-stat | | [-0.84] | | [-2.43] |
| CITR | 1.21 | 0.99 | 1.08 | 0.73 |
| NW t-stat | (2.69) | (2.27) | (2.90) | (2.03) |
| Shanken t-stat | [2.10] | [1.83] | [2.19] | [1.42] |
| % Adj. $R^2$ | 73.45 | 86.05 | 31.60 | 68.05 |
| % MAE | 0.08 | 0.06 | 0.11 | 0.08 |
| Months | 192 | 192 | 192 | 192 |

In Panel A, the test assets are decile portfolios sorted on idiosyncratic tail risk. In Panel B, the test assets are decile portfolios sorted on volume tail risk. The Fama MacBeth analysis is from January 2001 to December 2016. The model in columns (1) and (3) uses innovations in CITR as the factor. The model in columns (2) and (4) uses both the market portfolio and innovations in CITR as the factors. The table reports the risk price estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

with a statistically significant risk price $\lambda_{CITR}$ of 1.21%. CITR explains nearly all the differences in average returns of the idiosyncratic tail risk decile portfolios, with an adjusted $R^2$ of 74% and a small pricing error of 0.08%. Column (2) shows that adding the market factor increases the adjusted $R^2$ to 86% and reduces the pricing error to 0.06%. In the two-factor model, CITR has positive and statistically significant risk price of 0.99%. Figure 5 plots the expected excess returns predicted in the two-factor model against actual returns for the idiosyncratic tail risk deciles. Test assets line up closely on the 45 degree line, indicating that the model does a good job of pricing these assets. Panel B reports results using the decile portfolios sorted on volume tail risk from Section 3.4 as test assets. Column (3) shows that the CITR single factor model has some ability to explain the VTR premium with an adjusted $R^2$ of 32% and pricing error of 0.11%. CITR has a statistically significant risk price of 1.08%, showing the factor is priced in these portfolios. Column (4) shows that adding the market factor increases the $R^2$ to 68% and reduces the pricing error to 0.08%. The CITR factor remains statistically significant with a risk price of 0.73%.

To confirm that exposures to CITR explains the idiosyncratic tail risk premium, I examine the $\beta_{k,CITR}$ in each of the idiosyncratic tail risk deciles. Table 8 reports the $\beta_{k,M}$ and $\beta_{k,CITR}$

**Figure 5: Realized Versus Predicted Mean Excess Returns of ITR Deciles**




Actual average percent excess returns on all anomaly portfolios versus predicted expected returns using exposures to CITR and the market portfolio. The test assets are decile portfolios sorted on idiosyncratic tail risk. Distance from the 45-degree line represents pricing errors (alphas). The monthly sample is from January 2001 to December 2016.

of each decile and the difference between the highest and lowest decile. Panel A shows that in the single factor model, $\beta_{k,CITR}$ is nearly monotonically increasing in idiosyncratic tail risk. The highest idiosyncratic tail risk decile has 0.35 higher $\beta_{k,CITR}$ than the lowest decile. Since the CITR risk price is 1.21%, the factor explains 0.42% ($0.35 \times 1.21\% = 0.42\%$) or roughly two-thirds of the 0.66% idiosyncratic tail risk premium. Hence, the idiosyncratic tail risk can be mostly explained by exposures to CITR. In Panel B, I add the market factor and find that $\beta_{k,CITR}$ is still increasing

Table 8: CITR Betas for Portfolios Sorted on Idiosyncratic Tail Risk

| Panel A: One Factor Model: CITR | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 (Low ITR) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High ITR) | (10-1) Beta |
| $\beta_{CITR}$ | 0.29 | 0.35 | 0.37 | 0.45 | 0.23 | 0.34 | 0.50 | 0.55 | 0.55 | 0.64 | 0.35 |
| **Panel B: Two Factor Model: CITR and Market** | | | | | | | | | | | |
| | 1 (Low ITR) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High ITR) | (10-1) Beta |
| $\beta_{M}$ | 1.07 | 0.97 | 0.99 | 0.93 | 0.93 | 0.94 | 0.99 | 0.98 | 0.92 | 0.90 | -0.17 |
| $\beta_{CITR}$ | -0.15 | -0.05 | -0.04 | 0.06 | -0.15 | -0.05 | 0.09 | 0.14 | 0.17 | 0.26 | 0.42 |

The table presents factor betas $\beta_{k,CITR}$ and $\beta_{k,M}$ for each idiosyncratic tail risk decile and the difference in factor betas between the highest and lowest decile. CITR beta $\beta_{k,CITR}$ and market beta $\beta_{k,M}$ are estimated in stage one of the Fama and MacBeth regression in (22) using the full sample from January 2001 to December 2016.

in idiosyncratic tail risk. The highest idiosyncratic tail risk decile has 0.42 higher $\beta_{k,CITR}$ than the lowest decile. Since the CITR risk price is 0.99%, the factor explains 0.41% of the premium and the two factors combined explain 0.59% $(0.18 + 0.41)$ of the 0.66% premium.

In summary, exposures to CITR helps to explain the abnormal returns in the idiosyncratic tail risk and volume tail risk portfolios. The CITR factor provides a risk-based explanation for the idiosyncratic tail risk premium. Portfolios with high idiosyncratic tail risk earn high average returns due to their high exposure to the CITR factor. Portfolios with low idiosyncratic tail risk earn low average returns, since they have less CITR exposure and hedge against states of low CITR.

# 5 Robustness

This section examines a battery of robustness tests for my explanation of the idiosyncratic tail risk premium. In my explanation, CITR is correlated to the intermediary marginal utility of wealth and its shocks should be positively priced in anomaly portfolios and asset classes traded by intermediaries, and should negatively forecast market returns. Additionally, Gabaix, Gopikrishnan, Plerou, and Stanley (2006)'s economic model theorizes that volume tail risk is a proxy for idiosyncratic tail risk, hence I create a common volume tail risk factor and test whether it is a risk factor that prices the test assets above. Finally, if the idiosyncratic tail risk premium is caused by different exposures to the CITR factor, then the long-short idiosyncratic tail risk portfolio is factor-mimicking portfolio for CITR and can be seen as a traded CITR factor. I test whether this traded CITR factor is priced in the test assets above. I find that the results of these tests are consistent with my explanation.

## 5.1 Pricing Anomaly Portfolios

In my explanation of the ITR premium, CITR shocks are a risk factor and should be priced in portfolios traded by intermediaries. In this section, I examine if the CITR factor can price test assets with anomalous returns. Recent work by Lewellen, Nagel, and Shanken (2010) advocates expanding the set of test assets beyond book-to-market. Motivated by their recommendations, I focus my analysis on less known anomalies in which intermediaries are likely to be the marginal investor. My test assets include portfolios independently double-sorted on size and another asset characteristic, including operating profitability, investment, momentum, reversal, and idiosyncratic volatility from January 2001 to December 2016.[27] Anomaly portfolios are sorted by size to have more granular test assets for cross-sectional tests. All anomaly portfolios are downloaded from Kenneth French's website. Additionally, Lewellen, Nagel, and Shanken (2010) advocates including portfolios sorted by exposures to the CITR factor in tests. Since Section 4.6 shows that average returns for ITR and VTR sorted portfolios are largely driven by their exposures to the CITR factor, I include portfolios conditionally double-sorted on size then idiosyncratic tail risk and size then volume tail risk.[28]

Table 9 reports results for the CITR and market two-factor model. In each column, the CITR risk price $\lambda_{CITR}$ is positive and statistically significant at the 10% confidence level. The magnitude of $\lambda_{CITR}$s are remarkably similar, ranging from 0.70% to 1.05%, and in line with the tail risk decile long-short return of 0.66% in Table 7. The two-factor model explains a large degree of the variation in average returns as indicated by the high adjusted $R^2$s ranging from 25% for the portfolios sorted by size and operating profitability up to 67% for the portfolios sorted by size and ITR. Likewise, pricing errors are relatively low, ranging from 0.11% for the portfolios sorted by size and ITR to 0.2% for the portfolios sorted by size and idiosyncratic volatility. In particular, the high $R^2$s and low pricing errors for the tail risk portfolios in Panel A confirm that the CITR factor explains most of the idiosyncratic tail risk and volume tail risk premia. Column (7) reports the results for the all-in portfolio, which includes all 150 test assets from Columns (1) to (6). The CITR

[27] In each month, stocks are sorted into 25 (5x5) groups based on their size and characteristic simultaneously, as the intersection of 5 quintiles sorted on size and 5 quintiles sorted on the characteristic. Value-weighted portfolios are formed in each grouping, and held for one month. See Kenneth French's website for more details.

[28] In each month, stocks are sorted into quintile groupings based on their size. Then, within each size quintile, stocks are sorted into idiosyncratic tail risk or volume tail risk quintiles, value-weighted portfolios are formed in each grouping, and held for one month.

Table 9: Asset Pricing Tests on Characteristic Portfolios

| | Panel A: Size and Tail Risk Portfolios | | Panel B: Size and Characteristic Portfolios | | | | |
|---|---|---|---|---|---|---|---|
| | 25 ITR | 25 VTR | 25 OP | 25 INV | 25 MOM | 25 IVOL | ALL |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| Intercept | 0.03 | 0.44 | 0.89 | 1.62 | 1.35 | 1.52 | 1.15 |
| NW t-stat | (0.08) | (1.24) | (2.34) | (3.51) | (2.88) | (4.29) | (3.54) |
| Shanken t-stat | [0.05] | [0.86] | [2.10] | [3.31] | [2.26] | [3.63] | [2.95] |
| $R^M - R^f$ | 0.56 | 0.21 | -0.20 | -0.91 | -0.61 | -0.77 | -0.43 |
| NW t-stat | (1.19) | (0.47) | (-0.37) | (-1.56) | (-1.03) | (-1.54) | (-0.90) |
| Shanken t-stat | [0.92] | [0.36] | [-0.38] | [-1.63] | [-0.97] | [-1.56] | [-0.90] |
| CITR | 1.05 | 1.05 | 0.70 | 0.97 | 1.01 | 0.89 | 0.84 |
| NW t-stat | (2.66) | (2.18) | (1.76) | (2.27) | (2.53) | (2.23) | (2.34) |
| Shanken t-stat | [2.25] | [1.81] | [1.61] | [1.83] | [1.95] | [1.80] | [2.03] |
| % Adj. $R^2$ | 67.49 | 50.16 | 25.08 | 45.07 | 43.92 | 35.17 | 26.89 |
| % MAE | 0.11 | 0.12 | 0.17 | 0.18 | 0.16 | 0.20 | 0.18 |
| Months | 192 | 192 | 192 | 192 | 192 | 192 | 192 |

This table presents asset pricing tests on double-sorted portfolios using the CITR and market two-factor model from 2001 to 2016. In Panel A, the test assets are 25 portfolios conditionally sorted on size and idiosyncratic tail risk or volume tail risk. Stocks are first grouped into size quintiles, then within each size quintile, stocks are grouped by their ITR or VTR. In Panel B, test assets are 25 portfolios independently sorted by size and the characteristic. Stocks are grouped by the intersection of 5 quintiles sorted on size and 5 quintiles sorted on the characteristic. These anomaly portfolios are downloaded from from Kenneth French's website and include operating profitability, investment, momentum, reversal, and idiosyncratic volatility. The table reports the risk premia estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

risk price remains statistically and economically significant. The cross-section standard deviation of CITR-betas across all 150 test assets is 0.19, hence a one standard deviation increase in an asset's CITR-beta corresponds to a 2% ($0.19 \times 0.84 \times 12 = 2\%$) increase in its annual risk premia. Consistent with my risk-based explanation, CITR shocks are a risk factor that prices anomaly portfolios traded by intermediaries.

## 5.2 Pricing Sophisticated Asset Classes

In my explanation, the common idiosyncratic tail risk factor is correlated to the intermediary marginal utility of wealth. A natural test of my explanation is to evaluate whether the CITR factor can price the sophisticated asset classes in which large intermediaries are the marginal investor. I use the well-known asset returns from He, Kelly, and Manela (2017) as test assets. The test assets are downloaded from Asaf Manela's website, and include quarterly returns for equities, US government and corporate bonds, sovereign bonds, options, credit default swaps, commodities, and foreign exchange up to December 2012.

Table 10: Asset Pricing Tests on Sophisticated Assets, Quarterly

| | FF25 | | Bond | | Sov | | Options | | CDS | | Conmod | | FX | | All | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) |
| Intercept | 1.27 | 1.09 | 0.82 | 0.82 | 1.24 | 1.20 | -3.16 | -5.49 | -0.28 | -0.21 | -0.09 | 0.34 | -0.57 | -1.30 | 0.12 | 0.28 |
| NW t-stat | 0.96 | 1.04 | 4.02 | 4.05 | 2.92 | 3.04 | -4.08 | -5.08 | -5.09 | -4.07 | -0.07 | 0.37 | -1.05 | -2.25 | 0.30 | 0.97 |
| Shanken t-stat | 1.06 | 0.77 | 4.03 | 4.08 | 2.80 | 2.81 | -1.86 | -3.06 | -3.67 | -1.72 | -0.07 | 0.30 | -0.84 | -1.47 | 0.35 | 0.78 |
| $R^M - R^f$ | | 0.71 | | 1.35 | | 1.70 | | 5.55 | | 10.09 | | 0.99 | | 9.35 | | 0.95 |
| NW t-stat | | 0.52 | | 0.68 | | 0.64 | | 2.63 | | 3.93 | | 0.41 | | 3.03 | | 0.67 |
| Shanken t-stat | | 0.37 | | 0.89 | | 0.62 | | 2.22 | | 2.86 | | 0.42 | | 2.59 | | 0.67 |
| CITR | 0.19 | 0.18 | 0.56 | 0.61 | 0.49 | 0.44 | 2.00 | -1.07 | 0.93 | -1.16 | 0.75 | 0.79 | 1.24 | 0.86 | 0.51 | 0.75 |
| NW t-stat | 0.74 | 0.65 | 1.00 | 1.51 | 1.33 | 1.11 | 2.50 | -1.96 | 2.21 | -2.77 | 2.04 | 2.18 | 2.96 | 1.86 | 1.42 | 2.87 |
| Shanken t-stat | 0.65 | 0.59 | 1.39 | 1.30 | 1.52 | 1.17 | 1.45 | -1.34 | 2.27 | -1.74 | 1.57 | 1.63 | 2.77 | 1.75 | 1.35 | 2.48 |
| % $R^2$ | 5.21 | 5.28 | 63.37 | 64.03 | 72.90 | 72.97 | 88.61 | 95.07 | 42.83 | 73.64 | 26.53 | 29.88 | 20.08 | 37.26 | 29.91 | 34.55 |
| % MAE | 0.67 | 0.68 | 0.30 | 0.30 | 0.46 | 0.47 | 0.36 | 0.27 | 0.19 | 0.18 | 1.42 | 1.41 | 0.71 | 0.65 | 0.92 | 0.90 |
| Quarters | 48 | 48 | 48 | 48 | 41 | 41 | 44 | 44 | 47 | 47 | 48 | 48 | 36 | 36 | 48 | 48 |

This table presents asset pricing tests using the CITR single-factor model and the CITR and market two-factor model at a quarterly frequency from January 2001 to December 2012. Quarterly CITR is defined as the mean of monthly CITRs within the quarter, and quarterly CITR shocks are defined as quarterly changes in CITR. Test assets are the portfolios in He, Kelly, and Manela (2017) downloaded from Asaf Manela's website. Assets include equities, US government and corporate bonds, sovereign bonds, options, credit default swaps, commodities, and foreign exchange. The table reports the risk premium estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

To be consistent with their main analysis, I conduct pricing tests at the quarterly frequency. I construct quarterly CITR levels as the average monthly CITR within each quarter and define the quarterly CITR factor as first differences in quarterly CITR levels. I then conduct a two-stage Fama and MacBeth (1973) estimation procedure from January 2001 to December 2012 to estimate the CITR risk premium for each asset class. I analyze both the CITR single-factor model and the CITR and market two-factor model.

Table 10 reports the pricing ability of the CITR factor for the asset classes investigated in He, Kelly, and Manela (2017) from January 2001 to December 2012. Due to its correlation with the intermediary marginal utility of wealth, the CITR factor should have a positive risk premium, which is what I find in every asset class for the single-factor model. In odd columns, $\lambda_{CITR}$ is positive and economically large. $\lambda_{CITR}$ is also statistically significant for the options, CDS, commodities, and foreign exchange test assets in columns (7), (9), (11), and (13), which are highly sophisticated assets and are most likely to be traded by large intermediaries. However, for the two-factor model, $\lambda_{CITR}$ is negative for options and CDS in columns (8) and (10), suggesting that some of the CITR factor's ability to price these assets may be due to its correlation with the market factor. Columns (15) and (16) report results for the one- and two-factor models using all test assets, where $\lambda_{CITR}$ is positive in both columns and statistically significant in the two-factor model with a risk price of 0.75% (t-stat of 2.87). Furthermore, the two-factor model achieves an $R^2$ of 34.55% when pricing all assets in Column (16). $\lambda_{CITR}$ is not statistically significant for equities, bonds, and sovereign bonds in columns (1) to (6), which is likely because these assets are frequently traded by less sophisticated investors, hence intermediaries may not be the marginal investor. In summary, there is some evidence that the CITR factor is priced in highly sophisticated asset classes, further supporting the factor's correlation to the intermediary marginal utility of wealth.

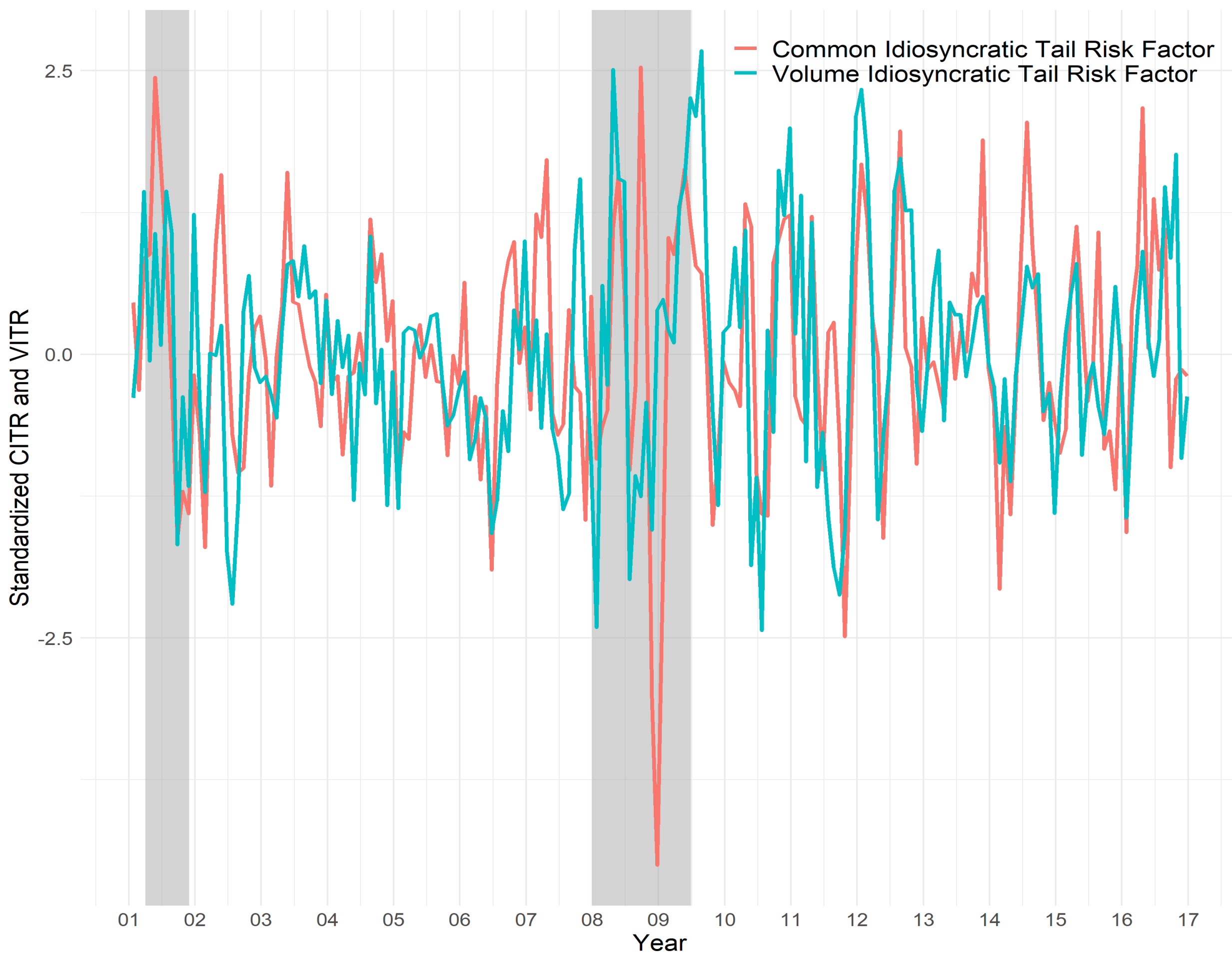
### 5.3 Duality of the Common Idiosyncratic Tail Risk and Common Volume Tail Risk Factors

In my explanation, volume tail risk is a proxy for idiosyncratic tail risk. Section 3 presents new evidence of duality in idiosyncratic tail risk and volume tail risk, showing that stocks with higher volume tail risks earn higher average returns, this premium is highly persistent, and that volume tail risk is cross-sectionally correlated to idiosyncratic tail risk. A further test of my explanation is if volume tail risk has a factor structure and if the common volume tail risk factor has similar empirical asset pricing results as the CITR factor, which is exactly what I show in this section, illustrating the duality of the common idiosyncratic tail risk and common volume tail risk factors.

Analogous to Section 4 and Equation (21), I test the commonality of volume tail risk by regressing each firm's VTR on the mean VTR, denoted as the common volume tail risk (CVTR). Volume tail risk has a stronger factor structure than idiosyncratic tail risk, where CVTR explains 25.18% of the monthly variation in firm-level volume tail risk and 46.27% of the annual variation in firm-level volume tail risk.

In my explanation, volume tail risk is driven by large intermediary trades, and innovations in CVTR should be driven by innovations in intermediary funding. I define the CVTR factor $\triangle CVTR$ as 3-month differences in CVTR levels, that is $\triangle CVTR_t = CVTR_t - CVTR_{t-3}$. Figure 6 plots the CVTR and CITR factors, showing that they are highly correlated and procyclical. The common volume tail risk factor has a monthly pair-wise correlation of 0.46 (t-stat of 7.18) with the common idiosyncratic tail risk factor. While idiosyncratic tail risk and volume tail risk may be cross-sectionally correlated due to other characteristics like size or idiosyncratic volatility, the high correlation between CITR and CVTR is not automatic and there is no mechanical reason why the two factors should be correlated over time. Furthermore, the CVTR factor has a monthly pair-wise correlation of 0.35 (t-stat of 5.19) with the intermediary capital factor of He, Kelly, and Manela (2017). These empirical findings strongly support Gabaix, Gopikrishnan, Plerou, and

**Figure 6: Common Idiosyncratic Tail Risk and Volume Tail Risk Factors**




The common idiosyncratic tail risk factor and the common volume tail risk factor. Both time-series are standardized to zero mean and unit variance. The monthly sample is from January 2001 to December 2016. CITR is the mean monthly idiosyncratic tail risk and common volume tail risk is the mean monthly volume tail risk.

Stanley (2006)'s hypothesis that both idiosyncratic tail risk and volume tail risk are caused by large intermediary trades and my explanation that the CITR and CVTR factors are dually driven by shocks to intermediary funding.

Next, I test whether shocks to CVTR is a priced risk factor by analyzing portfolios with different exposures to CVTR using the same methodology as in Section 4.5. For each month from January 2001 to December 2016, I estimate each stock's CVTR-beta by regressing the stocks monthly excess return on the CVTR factor using a 36-month trailing window. I sort stocks in each month into quintiles based on their CVTR-betas. Stocks in quintile 1 have the least exposure to CVTR and

Table 11: Excess Returns and Alphas of Portfolios Sorted on CVTR-betas

| | 1 (Low) | 2 | 3 | 4 | 5 (High) | (5-1) |
|---|---|---|---|---|---|---|
| Panel A: Average CVTR-beta | | | | | | |
| Average CVTR-beta | -0.54 | 0.11 | 0.47 | 0.85 | 1.67 | |
| Panel B: Univariate Sort on CVTR-beta (Equal-Weighted) | | | | | | |
| Excess Return | 0.52 | 0.71 | 0.78 | 0.84 | 0.84 | 0.32 |
| t-stat | (1.24) | (1.82) | (2.04) | (1.93) | (1.46) | (1.07) |
| FFC4 alpha | 0.57 | 0.75 | 0.83 | 0.89 | 0.88 | 0.30 |
| FFC4 t-stat | (1.40) | (2.05) | (2.26) | (2.18) | (1.71) | (1.10) |
| FF5 alpha | 0.78 | 0.86 | 0.92 | 0.95 | 0.92 | 0.14 |
| FF5 t-stat | (1.94) | (2.22) | (2.35) | (2.23) | (1.75) | (0.56) |
| Panel C: Univariate Sort on CVTR-beta (Value-Weighted) | | | | | | |
| Excess Return | 0.07 | 0.38 | 0.60 | 0.52 | 0.85 | 0.77 |
| t-stat | (0.17) | (1.21) | (1.83) | (1.43) | (1.66) | (2.25) |
| FFC4 alpha | 0.10 | 0.41 | 0.66 | 0.56 | 0.91 | 0.80 |
| FFC4 t-stat | (0.27) | (1.40) | (2.10) | (1.65) | (1.95) | (2.21) |
| FF5 alpha | 0.27 | 0.51 | 0.76 | 0.65 | 0.98 | 0.72 |
| FF5 t-stat | (0.74) | (1.71) | (2.34) | (1.92) | (2.09) | (2.09) |

The table reports monthly average CVTR-beta, excess returns, and alphas for portfolios sorted on CVTR-betas between January 2001 to December 2016. Panel A reports the average CVTR-beta for portfolios sorted on CVTR-beta. Panel B reports equally-weighted excess returns and alphas sorted on CVTR-beta. Panel C reports value-weighted excess returns and alphas sorted on CVTR-beta.

stocks in quintile 5 have the most CVTR exposure.

Table 11 reports results for the univariate sort. Panel A shows that CVTR-betas range from -0.54 to 1.67, which is slightly less disperse than CITR-betas. Panel B shows that for the equal-weighted results, the long-short portfolio return is not statistically significant, but still has a positive average return of 0.32%. In comparison, the CITR-beta long-short portfolio has a similar equal-weighted return of 0.40%. Panel C shows that for the value-weighted results, the long-short portfolio return is 0.77% (t-stat of 2.25), which is higher than the 0.62% for CITR-beta. Both equal-weighted and value-weighted returns and alphas are generally increasing in CVTR-betas. These results are consistent with my explanation that shocks to CVTR are a priced risk factor, are driven by shocks in intermediary funding, and has a pricing duality with CITR.

Next, Table 12 reports the ability of the CVTR and market two-factor model to price the test assets in Sections 4.6 and 5.1. CVTR explains most of the VTR deciles with an $R^2$ of 81%, and has some ability to price the idiosyncratic tail risk deciles with an $R^2$ of 48%. The VTR risk price

Table 12: Asset Pricing Tests on Anomaly Portfolios using the CVTR factor

| | Panel A: Tail Risk Deciles | | Panel B: Size & Tail Risk | | Panel C: Size & Characteristic | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | ITR Dec. | VTR Dec. | 25 ITR | 25 VTR | 25 OP | 25 INV | 25 MOM | 25 IVOL | ALL-DS |
| Intercept | 1.05 | 1.58 | -0.47 | 0.03 | 0.79 | 1.61 | 0.94 | 1.38 | 0.86 |
| NW t-stat | (1.02) | (1.90) | (-1.16) | (0.08) | (2.08) | (3.52) | (2.19) | (4.18) | (2.92) |
| Shanken t-stat | [0.71] | [1.23] | [-0.86] | [0.06] | [1.70] | [3.23] | [1.72] | [2.84] | [2.41] |
| $R^M - R^f$ | -0.59 | -1.08 | 1.05 | 0.64 | -0.07 | -0.88 | -0.21 | -0.72 | -0.13 |
| NW t-stat | (-0.53) | (-1.15) | (1.97) | (1.33) | (-0.15) | (-1.55) | (-0.35) | (-1.44) | (-0.27) |
| Shanken t-stat | [-0.38] | [-0.79] | [1.62] | [1.09] | [-0.13] | [-1.56] | [-0.34] | [-1.29] | [-0.27] |
| CVTR | 1.39 | 2.02 | 1.15 | 1.08 | 0.88 | 1.50 | 1.48 | 2.26 | 0.86 |
| NW t-stat | (1.45) | (2.51) | (2.19) | (2.26) | (1.20) | (2.21) | (2.07) | (2.97) | (1.75) |
| Shanken t-stat | [1.11] | [1.64] | [2.14] | [1.97] | [1.30] | [2.02] | [1.67] | [2.36] | [1.83] |
| % $R^2$ | 47.75 | 80.80 | 50.77 | 29.82 | 7.04 | 30.00 | 28.62 | 40.85 | 8.47 |
| % MAE | 0.11 | 0.06 | 0.14 | 0.14 | 0.19 | 0.20 | 0.18 | 0.20 | 0.21 |
| Months | 192 | 192 | 192 | 192 | 192 | 192 | 192 | 192 | 192 |

This table presents asset pricing tests on double-sorted portfolios using the CVTR and market two-factor model from 2001 to 2016. In Panel A, the test assets are the decile portfolios sorted on idiosyncratic tail risk or volume tail risk examined in Table 7. In Panel B, the test assets are 25 portfolios conditionally sorted on size and idiosyncratic tail risk or volume tail risk. Stocks are first grouped into size quintiles, then within each size quintile, stocks are grouped by their ITR or VTR. In Panel C, test assets are 25 portfolios independently sorted by size and the characteristic. Stocks are grouped by the intersection of 5 quintiles sorted on size and 5 quintiles sorted on the characteristic. These anomaly portfolios are downloaded from from Kenneth French's website and include operating profitability, investment, momentum, reversal, and idiosyncratic volatility. The table reports the risk premia estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

$\lambda_{CVTR}$ is positive for all test assets and statistically significant for many. Column (9) reports results using all double-sorted portfolios as test assets, where the CVTR factor has a risk price of 0.86% (t-stat of 1.75). The magnitude of the CVTR risk prices $\lambda_{CVTR}$ are in line with the magnitude of the CITR risk prices $\lambda_{CITR}$, further supporting the duality between risk factors.

Finally, Table 13 reports the pricing ability of the CVTR single-factor and the CVTR and market two-factor models on the sophisticated asset classes investigated in He, Kelly, and Manela (2017) from January 2001 to December 2012 at the quarterly frequency. In each asset class, the CVTR risk price is positive for both the single-factor and two-factor models. Analogous to the CITR results, the CVTR risk price is statistically significant for options, CDS, commodities, and foreign exchange while not being statistically significant for equities, bonds, and sovereign bonds. Using all assets in column (16), CVTR has a risk price of 1.05 (t-stat of 2.62). Furthermore, for each asset class the $R^2$ and pricing errors are similar in magnitude for the CITR and CVTR results. The duality of CITR and CVTR in pricing sophisticated assets further links these factors to intermediaries.

Table 13: Asset Pricing Tests on Sophisticated Assets using the CVTR factor, Quarterly

| | FF25 | | Bond | | Sov | | Options | | CDS | | Conmod | | FX | | All | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) |
| Intercept | 0.60 | 1.11 | 0.77 | 0.83 | 0.94 | 0.91 | -2.71 | 8.06 | -0.32 | -0.20 | -0.52 | -0.01 | -1.78 | -1.77 | 0.03 | 0.00 |
| NW t-stat | 0.50 | 0.94 | 4.11 | 4.88 | 2.72 | 2.46 | -3.58 | 2.08 | -5.31 | -5.63 | -0.50 | -0.01 | -2.89 | -2.91 | 0.12 | 0.44 |
| Shanken t-stat | 0.57 | 0.67 | 3.99 | 4.83 | 2.14 | 1.81 | -2.57 | 0.54 | -3.89 | -2.22 | -0.55 | -0.01 | -1.34 | -1.44 | 0.15 | 0.41 |
| $R^M - R^f$ | | 0.56 | | 1.55 | | 1.50 | | -9.38 | | -0.75 | | -0.17 | | 7.92 | | 0.86 |
| NW t-stat | | 0.39 | | 0.81 | | 0.60 | | -1.66 | | -0.28 | | -0.07 | | 2.53 | | 0.62 |
| Shanken t-stat | | 0.26 | | 0.72 | | 0.52 | | -0.55 | | -0.24 | | -0.08 | | 1.52 | | 0.60 |
| VITR | 0.48 | 0.61 | 0.72 | 1.06 | 0.99 | 1.22 | 1.41 | 6.35 | 1.56 | 3.04 | 0.77 | 0.86 | 3.52 | 3.27 | 0.62 | 1.05 |
| NW t-stat | 1.00 | 1.30 | 0.99 | 1.47 | 1.32 | 1.00 | 2.57 | 3.20 | 2.64 | 2.97 | 1.85 | 2.25 | 4.00 | 4.00 | 1.23 | 2.62 |
| Shanken t-stat | 0.96 | 1.22 | 1.47 | 1.27 | 1.44 | 0.95 | 2.45 | 0.79 | 2.84 | 2.33 | 1.67 | 1.86 | 2.29 | 1.82 | 1.23 | 2.07 |
| % $R^2$ | 9.16 | 10.23 | 65.14 | 61.98 | 72.31 | 72.42 | 95.21 | 98.34 | 67.11 | 71.81 | 23.27 | 34.05 | 45.56 | 45.74 | 27.39 | 37.49 |
| % MAE | 0.66 | 0.65 | 0.29 | 0.30 | 0.50 | 0.49 | 0.25 | 0.15 | 0.19 | 0.16 | 1.48 | 1.38 | 0.66 | 0.65 | 0.93 | 0.85 |
| Months | 48 | 48 | 48 | 44 | 41 | 41 | 44 | 44 | 47 | 47 | 48 | 48 | 36 | 36 | 48 | 48 |

This table presents asset pricing tests using the CVTR single-factor model and the CVTR and market two-factor model at a quarterly frequency from January 2001 to December 2012. Quarterly CVTR is defined as the mean of monthly CVTRs within the quarter, and quarterly CVTR shocks are defined as quarterly changes in CVTR. Test assets are the portfolios in He, Kelly, and Manela (2017) downloaded from Asaf Manela's website. Assets include equities, US government and corporate bonds, sovereign bonds, options, credit default swaps, commodities, and foreign exchange. The table reports the risk premium estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

This section presents new results on the duality of the tail distributions of idiosyncratic returns and trading volume, estimated on two distinct sources of data. Like idiosyncratic tail risk, volume tail risk follows a factor structure and shocks to common volume tail risk is a priced risk factor. These results are not automatic and strongly support the hypothesis in this paper that shocks to intermediary funding drive the time-series variation in common idiosyncratic tail risk and common volume tail risk.

## 5.4 Pricing Assets using a Traded CITR Factor

Section 4.6 demonstrates that the idiosyncratic tail risk premium is explained by exposures to the CITR factor. If the idiosyncratic tail risk deciles are primarily driven by the CITR factor, then the ITR long-short portfolio itself can be used as a traded factor that mimics the non-traded CITR factor. I define the traded CITR factor as the value-weighted return on a portfolio that goes long the highest idiosyncratic tail risk decile and shorts the lowest idiosyncratic tail risk decile.[29] The traded CITR factor should price assets better than the non-traded CITR factor, due to the measurement error in the latter.[30]

[29] Motivated by to Fama and French (1993), I use a characteristic-managed portfolio to create the traded CITR factor. The finance literature on characteristic-managed portfolios include Feng, Giglio, and Xiu (2019), Giglio and Xiu (2018), Gu, Kelly, and Xiu (2019), Kelly, Pruitt, and Su (2019), and Kozak, Nagel, and Santosh (2020).

[30] Economically motivated macro variables, such as CITR, will always have measurement error. Due to measurement error, the factor-mimicking portfolio will always price assets better than an estimate of the underlying factor that uses measured macroeconomic variables Cochrane (2009). Additionally, the traded CITR factor can be a factor-mimicking portfolio for CITR, since they both are both driven by intermediary funding. Using the idiosyncratic

Table 14: Asset Pricing Tests on Anomaly Portfolios using the Traded CITR factor

| | Panel A: Tail Risk Deciles | | Panel B: Size & Tail Risk | | Panel C: Size & Characteristic | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | ITR Dec. | VTR Dec. | 25 ITR | 25 VTR | 25 OP | 25 INV | 25 MOM | 25 IVOL | ALL-DS |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept | 0.35 | 2.81 | -0.03 | 0.54 | 0.46 | 0.73 | 0.44 | 0.58 | -0.28 |
| NW t-stat | (0.35) | (3.71) | (-0.10) | (1.80) | (1.39) | (2.07) | (1.02) | (2.11) | (-0.72) |
| Shanken t-stat | [0.30] | [2.92] | [-0.55] | [-0.07] | [1.06] | [1.70] | [0.82] | [1.62] | [1.27] |
| $R^M - R^f$ | 0.12 | -2.39 | 0.55 | -0.03 | 0.06 | -0.21 | 0.02 | -0.13 | 0.78 |
| NW t-stat | (0.12) | (-2.81) | (1.19) | (-0.05) | (0.11) | (-0.40) | (0.03) | (-0.25) | (1.57) |
| Shanken t-stat | [0.10] | [-2.31] | [1.32] | [0.98] | [0.11] | [-0.39] | [0.03] | [-0.26] | [0.08] |
| Traded CITR | 0.63 | 1.44 | 1.05 | 1.50 | 1.37 | 1.48 | 1.79 | 1.80 | 0.76 |
| NW t-stat | (2.74) | (2.90) | (2.38) | (3.46) | (2.65) | (3.09) | (3.16) | (3.82) | (2.50) |
| Shanken t-stat | [2.50] | [2.46] | [2.17] | [2.15] | [2.67] | [2.96] | [2.46] | [3.31] | [3.06] |
| % Adj. $R^2$ | 80.50 | 77.03 | 73.43 | 71.94 | 75.54 | 82.16 | 66.87 | 88.75 | 71.74 |
| % MAE | 0.06 | 0.08 | 0.09 | 0.11 | 0.09 | 0.10 | 0.12 | 0.08 | 0.09 |
| Months | 192 | 192 | 192 | 192 | 192 | 192 | 192 | 192 | 192 |

This table presents asset pricing tests on double-sorted portfolios using the traded CITR and market two-factor model from 2001 to 2016. In Panel A, the test assets are the decile portfolios sorted on idiosyncratic tail risk or volume tail risk examined in Table 7. In Panel B, the test assets are 25 portfolios conditionally sorted on size and idiosyncratic tail risk or volume tail risk. Stocks are first grouped into size quintiles, then within each size quintile, stocks are grouped by their ITR or VTR. In Panel C, test assets are 25 portfolios independently sorted by size and the characteristic. Stocks are grouped by the intersection of 5 quintiles sorted on size and 5 quintiles sorted on the characteristic. These anomaly portfolios are downloaded from from Kenneth French's website and include operating profitability, investment, momentum, reversal, and idiosyncratic volatility. The table reports the risk premia estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

This section investigates the ability of the traded CITR factor to price assets. According to my risk-based explanation, the traded CITR factor is correlated to the intermediary marginal utility of wealth, and should price assets in which intermediaries are the marginal investor. I test this hypothesis by evaluating the pricing of the traded CITR factor on the previous anomalies and the sophisticated assets classes in He, Kelly, and Manela (2017).

Table 14 reports the ability of the traded CITR and the market two-factor model to price the same portfolios in Sections 4.6 and 5.1. The traded CITR factor prices the test assets similarly to the non-traded CITR factor, validating that it is an appropriate factor-mimicking portfolio. In each column, the traded CITR factor risk price is statistically significant and economically large. Since the traded CITR factor is a traded portfolio, its risk price can be interpreted as a monthly excess return. Estimates of the traded CITR factor risk premium is economically large, ranging from 0.63% for the idiosyncratic tail risk deciles to 1.80% for the portfolios sorted on idiosyncratic

tail risk premium may be statistically preferred to the projection of non-traded CITR on stocks, since it avoids the errors-in-variables bias and variance issues associated with estimating factor loadings plagued by many other factor-mimicking approaches.

Table 15: Tests on HKM Portfolios using CITR Factor-Mimicking Portfolio, Monthly

| | FF25 | | Bond | | Sov | | Options | | CDS | | Conmod | | FX | | All | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) |
| Intercept | 0.61 | 0.16 | 0.24 | 0.22 | 0.50 | 0.29 | 5.93 | 0.67 | 0.00 | -0.10 | 0.38 | 0.27 | 0.06 | -0.66 | 0.35 | 0.15 |
| NW t-stat | 1.09 | 0.35 | 3.63 | 3.45 | 2.00 | 1.63 | 1.82 | 1.09 | -0.02 | -5.08 | 0.90 | 0.98 | 0.27 | -2.97 | 1.43 | 1.48 |
| Shanken t-stat | 1.26 | 0.28 | 2.88 | 3.17 | 1.65 | 1.40 | 0.36 | 0.42 | -0.01 | -2.53 | 0.95 | 0.87 | 0.31 | -2.11 | 1.68 | 1.46 |
| $R^M - R^f$ | | 0.14 | | 0.25 | | 1.53 | | -0.46 | | -0.03 | | -0.09 | | 2.73 | | 0.03 |
| NW t-stat | | 0.21 | | 0.33 | | 1.64 | | -0.56 | | -0.03 | | -0.11 | | 2.28 | | 0.06 |
| Shanken t-stat | | 0.20 | | 0.57 | | 1.65 | | -0.28 | | -0.02 | | -0.12 | | 1.88 | | 0.07 |
| Tr. CITR | 1.19 | 1.15 | 4.11 | 2.19 | -5.11 | -2.26 | 27.41 | 9.44 | 12.94 | 7.54 | 1.60 | 1.66 | 2.61 | 3.01 | 1.53 | 1.80 |
| NW t-stat | 1.72 | 1.64 | 2.43 | 1.77 | -2.06 | -1.73 | 2.03 | 2.93 | 4.73 | 4.65 | 1.87 | 1.87 | 1.95 | 2.34 | 2.69 | 3.44 |
| Shanken t-stat | 2.09 | 2.00 | 2.33 | 2.19 | -1.32 | -1.21 | 0.41 | 1.30 | 1.46 | 2.36 | 1.47 | 1.49 | 1.89 | 1.90 | 2.36 | 3.01 |
| % $R^2$ | 45.01 | 48.49 | 26.05 | 69.79 | 64.00 | 74.69 | 88.67 | 95.63 | 59.82 | 81.71 | 12.25 | 13.12 | 10.51 | 34.93 | 16.36 | 29.74 |
| % MAE | 0.17 | 0.16 | 0.14 | 0.08 | 0.15 | 0.14 | 0.14 | 0.09 | 0.09 | 0.06 | 0.56 | 0.55 | 0.25 | 0.23 | 0.35 | 0.29 |
| Months | 144 | 144 | 144 | 144 | 124 | 124 | 133 | 133 | 143 | 143 | 144 | 144 | 109 | 109 | 144 | 144 |

This table presents asset pricing tests using the traded CITR single-factor model and the traded CITR and market two-factor model at a monthly frequency from January 2001 to December 2012. Traded CITR is defined as the value-weighted return on a portfolio that goes long the highest idiosyncratic tail risk decile and shorts the lowest idiosyncratic tail risk decile. Test assets are the portfolios in He, Kelly, and Manela (2017) downloaded from Asaf Manela's website. Assets include equities, US government and corporate bonds, sovereign bonds, options, credit default swaps, commodities, and foreign exchange. The table reports the risk premium estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

volatility. Column (9) reports results using all double-sorted portfolios as test assets, where traded CITR factor has a risk price of 0.76% (t-stat of 2.5), $R^2$ of 71.74%, and a pricing error of 0.09. This traded CITR factor risk price is in line with the long-short return of 0.62% for quintile portfolios sorted on CITR-betas in Table 6.

Table 15 reports the pricing ability of the traded CITR factor for the asset classes investigated in He, Kelly, and Manela (2017) from January 2001 to December 2012 at the monthly frequency. In each asset class except for sovereign bonds, the traded CITR factor risk price is positive, economically large, and statistically significant at the 10% level. This is strong evidence that supports the link between the traded CITR factor and the intermediary marginal utility of wealth. Column (16) reports the two-factor model results using all test assets. The traded CITR factor risk premium across all the asset classes is 1.80% and statistically significant at the 1% level. The two-factor model provides the best fit for options with an $R^2$ of 95% and an $R^2$ of 29.74% in Column (16) using all asset classes.

The results in this section show that the traded CITR factor, defined as the long-short ITR portfolio, has similar pricing results as the non-traded CITR factor, supporting my explanation that the idiosyncratic tail risk premium is driven by the CITR factor. Furthermore, if the ITR and VTR premia are both driven by shocks to the intermediary marginal utility of wealth, then the traded VTR factor, defined as the VTR long-short portfolio, should have similar pricing abilities as the traded CITR factor, which is what I find in the data. Section D.4 investigates the pricing ability of traded VTR to price anomalies and sophisticated assets. Tables 26 and 27 show that traded VTR has nearly the same pricing ability as the traded CITR factor, providing further support for the empirical duality between ITR and VTR and their risk premia.

### 5.5 Forecasting the Equity Market Premium

A common prediction of intermediary asset pricing models is that expected returns are a function of lagged state variables that capture financial sector distress. In my explanation, CITR is negatively related to the shadow cost of capital for intermediaries, implying it should negatively forecast the equity market risk premium. I test this hypothesis using a monthly time-series regression of market returns on CITR. Denote $r_{[t,t+h]}$ as the CRSP value-weighted return from time t to $t+h$, where time t is measured in months. The return regression is

$$r_{[t,t+h]} = a_h + b_h CITR_t + c_h X_t + \epsilon_{t,t+h}, \quad t = 1, 2, ..., T-h, \tag{24}$$

where $CITR_t$ is standardized to have zero mean and unit variance, $X_t$ refers to other explanatory variables, and the horizon ranges from $h = 1$ (one month) to $h = 24$ (two years). I report t-statistics calculated using Hodrick (1992) standard errors to account for the overlapping returns when $h > 1$.[31]

Panel A in Table 16 presents the regression results with no control variables. A one standard deviation increase in CITR results in a decrease in future returns of 9.6%, 5.3%, 3.7% at the 1 month, 1 year, and 2 year horizons. The respective t-statistics are all significant at the 95% confidence level. Panel B in Table 16 presents the results for the regression controlling for Fama-French-Cahart factors. The coefficients remain economically and statistically significant at the 10% confidence level. The regression $R^2$s are also relatively high, increasing from 3% at the 1-month horizon up to 11% at the 16-month horizon. These results further support the hypothesis that CITR is a priced risk factor and is driven by intermediary funding. The results are inconsistent with the fire-sale or labour income risk explanation of idiosyncratic tail risk, in which a high average

[31] Ang and Bekaert (2007) demonstrate through simulation that Hodrick (1992)'s standard error correction provide the most conservative test statistics relative to other commonly used procedures. I also find that Hodrick's correction produces more conservative t-statistics than those calculated using Newey and West (1987) standard errors lag length equal to $2 \times h$.

Table 16: Equity Market Premium Time-Series Predictability

| Horizon | 1 | 4 | 6 | 8 | 12 | 16 | 20 | 24 |
|---|---|---|---|---|---|---|---|---|
| | | | | Panel A: Univariate Regression | | | | |
| CITR | -9.56 | -7.24 | -7.09 | -5.79 | -5.31 | -4.81 | -4.09 | -3.65 |
| Hodrick t-stat | (-2.02) | (-2.05) | (-2.31) | (-2.20) | (-2.36) | (-2.01) | (-1.85) | (-2.05) |
| Intercept | 4.28 | 4.87 | 4.99 | 5.12 | 5.40 | 5.68 | 6.05 | 6.47 |
| Hodrick t-stat | (1.17) | (1.35) | (1.37) | (1.40) | (1.49) | (1.57) | (1.69) | (1.87) |
| Adj. $R^2$ | 3.03 | 6.78 | 9.02 | 7.75 | 9.83 | 10.86 | 9.57 | 8.72 |
| | | | | Panel B: Fama French Cahart Regression | | | | |
| CITR | -8.48 | -6.57 | -5.97 | -5.17 | -5.12 | -4.79 | -4.01 | -3.39 |
| Hodrick t-stat | (-1.85) | (-1.93) | (-2.03) | (-1.99) | (-2.26) | (-2.03) | (-1.84) | (-1.96) |
| Intercept | 4.32 | 5.14 | 5.56 | 5.47 | 5.62 | 5.69 | 6.08 | 6.55 |
| Hodrick t-stat | (1.09) | (1.37) | (1.49) | (1.49) | (1.53) | (1.55) | (1.67) | (1.86) |
| Market | 1.88 | 0.62 | 0.32 | 0.25 | 0.03 | 0.17 | 0.13 | 0.05 |
| Hodrick t-stat | (1.29) | (0.72) | (0.44) | (0.40) | (0.06) | (0.40) | (0.34) | (0.15) |
| SMB | -2.08 | -1.18 | -1.30 | -0.82 | -0.32 | -0.16 | -0.23 | -0.09 |
| Hodrick t-stat | (-1.27) | (-1.41) | (-1.95) | (-1.37) | (-0.73) | (-0.47) | (-0.76) | (-0.34) |
| HML | -0.38 | -0.25 | -0.81 | -0.40 | -0.06 | 0.08 | 0.07 | -0.22 |
| Hodrick t-stat | (-0.19) | (-0.28) | (-1.26) | (-0.88) | (-0.17) | (0.28) | (0.27) | (-0.90) |
| UMD | -0.15 | -0.67 | -0.74 | -0.70 | -0.56 | -0.26 | -0.27 | -0.24 |
| Hodrick t-stat | (-0.14) | (-1.54) | (-2.10) | (-2.28) | (-2.32) | (-1.21) | (-1.29) | (-1.34) |
| % Adj. $R^2$ | 3.97 | 9.08 | 13.42 | 11.10 | 11.27 | 10.49 | 9.31 | 8.18 |

This table shows the results of regression (14) for a horizon of 1 - 24 months. The top panel is a regression of returns on only CITR and the bottom panel is a regression of returns on CITR and the Fama French Cahart factors. The first row in each panel shows the regression coefficient for a one standard deviation increase in CITR. The second row shows the t-statistic calculated using Hodrick standard errors. The third row shows the adjusted-$R^2$.

ITR should predict positive average returns.

## 6 Conclusion

The idiosyncratic tail risk premium is a significant recent discovery in the asset pricing literature. Savor (2012) and Jiang and Zhu (2017) interpret the idiosyncratic tail risk as news shocks, and argues its premium is driven by under-reaction to firm-specific news. Bégin, Dorion, and Gauthier (2019) and Kapadia and Zekhnini (2019) argue that the premium is caused by the inability to diversify and persists due to limits to arbitrage.

My paper offers a risk-based explanation for the idiosyncratic tail risk premium. I show that idiosyncratic tail risk is driven by intermediary funding and the common idiosyncratic tail risk factor is correlated to the intermediary marginal utility of wealth. Stocks with high idiosyncratic tail risk also have high exposures to the common idiosyncratic tail risk (CITR) factor, earning a risk premium due to their low returns when intermediary constraints tighten.

I test my explanation using a new measure of idiosyncratic tail risk. First, I show that stocks with high idiosyncratic tail risk earn higher average returns that persists over years. Second, idiosyncratic tail risk has a strong firm-level correlation to volume tail risk and intermediary trading volume as a percentage of total trading volume. Third, idiosyncratic tail risk follows a strong factor structure, and the common idiosyncratic tail risk factor is procyclical and correlated to existing intermediary factors. Fourth, the common idiosyncratic tail risk factor explains cross-sectional differences in average returns, including the idiosyncratic tail risk premium. I confirm that high (low) idiosyncratic tail risk portfolios have high (low) exposures to the common idiosyncratic tail risk factor, and that the difference can explain most of the premium. Fifth, the common idiosyncratic tail risk factor is priced in other anomalies and sophisticated assets, and forecasts the equity market premium. Sixth, the idiosyncratic tail risk long-short portfolio is a factor-mimicking portfolio for the CITR factor and is priced in anomalies and sophisticated assets. Finally, I document that these asset pricing results are similar when using volume tail risk in the place of idiosyncratic tail risk. Volume tail risk earns a persistent premium and exhibits commonality that's correlated to intermediary factors. The common volume tail risk factor is also priced in anomaly portfolios and sophisticated assets. This duality between idiosyncratic tail risk and volume tail risk in asset pricing provides new empirical evidence for the intermediary hypothesis of idiosyncratic tail risk from Gabaix, Gopikrishnan, Plerou, and Stanley (2006).

Some of these results allow me to distinguish my hypothesis from leading alternative explanations of the idiosyncratic tail risk premium. Savor (2012) and Jiang and Zhu (2017) propose behavioral explanations for the premium based on short-term under-reaction to news shocks caused by limited investor inattention. Their short-term explanation is inconsistent with my finding that the idiosyncratic tail risk premium is persistent. Additionally, if idiosyncratic tail risk were only caused by news shocks, then it would not exhibit such a strong factor structure. Bégin, Dorion, and Gauthier (2019) and Kapadia and Zekhnini (2019) propose that the premium is caused by the inability to hedge idiosyncratic tail risk in a diversified portfolio, and that the premium persists due to limits to arbitrage. This explanation is inconsistent with my finding that the premium can be explained by differences in exposure to the common idiosyncratic tail risk factor.

Finally, my explanation and empirical findings support the large intermediary hypothesis of idiosyncratic tail risk and volume tail risk by Gabaix, Gopikrishnan, Plerou, and Stanley (2006). Their economic model assumes intermediary funding is constant, while my explanation allows intermediary funding to change, which provides a stylized framework linking intermediaries, asset prices, and the tail distributions of returns and trading volume.

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# A Empirical Evidence of Power-Law Distributed Tails

In this section, I provide empirical evidence for the power-law definition of idiosyncratic tail risk $\xi$ used in this paper. Since Mandelbrot (1963), numerous studies have documented that the tails of *daily* and *monthly* equity returns are power-law distributed.[32] Recently, researchers have been using high-frequency equity data to show the tails of *high-frequency* returns are power-law distributed.[33] Gabaix, Gopikrishnan, Plerou, and Stanley (2006) shows the tail distribution of 15-minute US equity returns for the 1000 largest stocks from 1994-1995 has power-law distributed tails according to

$$P(|r| > x) \sim x^{-1/\xi} L(x) \tag{25}$$

where $r$ is the log return (log denotes natural logarithm), $L(x)$ is a slowly varying function, and $\sim$ denotes asymptotic equivalence.[34]

Following similar procedures as in Gabaix, Gopikrishnan, Plerou, and Stanley (2006), Figure 7 illustrates the empirical complementary cumulative distribution of a sample of 5-minute normalized absolute returns for the 1000 largest stocks from 2001 to 2016 using a natural log scale for the horizontal and vertical axis. The standard normal distribution is plotted in dashed lines in both panels for comparison. According to Figure 7, the natural log of probability that returns are greater than 2 standard deviations is approximately linear in log of absolute returns following,

$$logP(|r| > x) \approx -\frac{1}{\xi} logx + c, \tag{26}$$

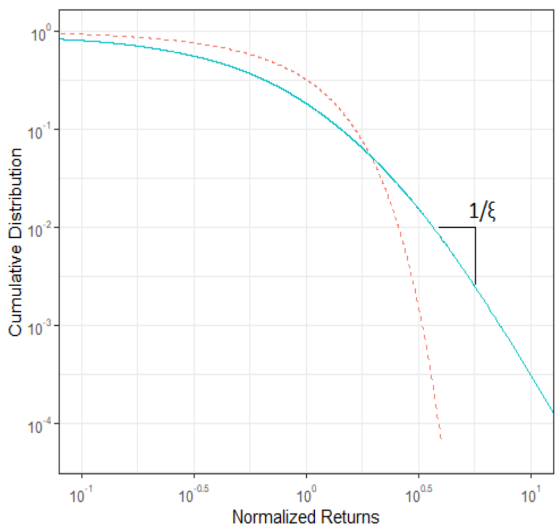
and confirming return tails are power-law distributed. Panels (a) and (b) of Figure 7 show that return tails are much heavier than normal tails. Furthermore, in the data empirical returns greater than 3 standard deviations constitute 1.76% of the sample – approximately 6 times the probability implied by a normal distribution.

[32] Power law tail behaviour of equity returns has been thoroughly documented in, for example, Mandelbrot (1963), Fama (1963), Jansen and De Vries (1991), Kearns and Pagan (1997), Samanta and LeBaron (2005), and Kelly and Jiang (2014). For an overview, refer to Gabaix (2009).

[33] For examples, see Danielsson and De Vries (1997), Gabaix, Gopikrishnan, Plerou, and Stanley (2006), Stanley, Plerou, and Gabaix (2008), and Bollerslev and Todorov (2011a)

[34] Function L is slowly varying if $L$ is strictly positive and $\lim_{x\to\infty} L(tx)/L(x) = 1$ for all $t > 0$. Prototypical examples include $L(x) = log(x)$ and $L(x) = c$ for a $c > 0$. $f(x) \sim g(x)$ means $\lim_{x\to\infty} f(x)/g(x) \to 1$.

**Figure 7: Complementary Cumulative Distribution of High-Frequency Returns**



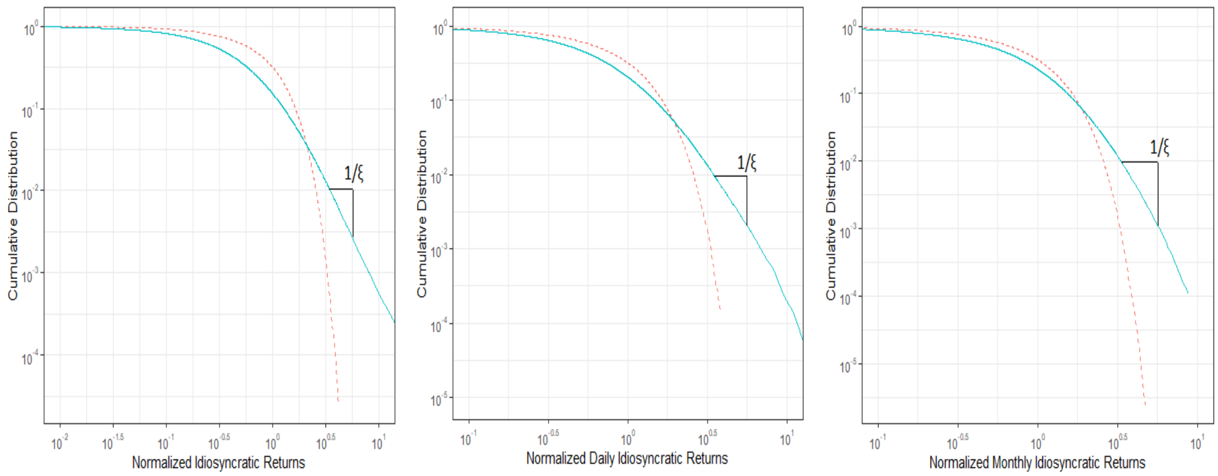

This figure plots the empirical complementary cumulative distribution function $P(|r| > x)$ of absolute values of a sample of 5-minute log returns for the 1000 largest U.S stocks from 2001 to 2016. 10% of the returns are sampled resulting in approximately 30 million observations. Returns are normalized by stock to have mean 0 and standard deviation 1. $log$x is plotted on the horizontal axis and $logP(|r| > x)$ on the vertical axis.

While empirical evidence shows that return tails obey a power-law, this does not guarantee that idiosyncratic return tails are power-law distributed. In factor models, returns are a function of systematic factors and idiosyncratic returns. Returns could be inheriting the power-law behaviour only from the tails of systematic factors and not the idiosyncratic returns. I present new evidence that the tails of idiosyncratic returns are also power-law distributed. Figure 8 plots the complementary cumulative distribution of residuals from Fama and French (2015) five factor model.[35] Panel (a) plots high-frequency residuals, panel (b) plots daily residuals, and panel (c) plots monthly residuals. The distribution is linear in logs, confirming that idiosyncratic returns are power-law distributed. Returns and residuals have similar slopes $1/\xi$, suggesting that returns are inheriting their power-law behaviour mostly from idiosyncratic returns.[36] Additionally, slopes $1/\xi$ appear related for high-frequency, daily, and monthly idiosyncratic returns. Section 2.2 proves that that idiosyncratic tail risk $\xi$ is preserved under time aggregation and that $\xi$ is theoretically

[35]For each firm, high-frequency, daily, or monthly residuals are created by regressing returns on high-frequency, daily, or monthly Fama and French (2015) five factors, where the factor betas are estimated using Least Absolute Deviation. See Section 2.4 for more details on estimation.

[36]$\xi$ here is pooled across firms and time, while $\xi_{l,1}$ in Equation (1) is for a single firm-month.

**Figure 8: Complementary Cumulative Distribution of High-Frequency, Daily and Monthly Idiosyncratic Returns**




(a) High-Frequency Complementary Cumulative Distribution

(b) Daily Complementary Cumulative Distribution

(c) Monthly Complementary Cumulative Distribution

Panel (a) plots the empirical complementary cumulative distribution function of absolute values of a sample of normalized 5-minute residuals for the 1000 largest U.S stocks from 2001 to 2016. Residuals are estimated every month from regressing log returns on the high-frequency Fama and French (2015) five factors. 10% of the residuals are sampled resulting in approximately 30 million observations. Panel (b) plots the empirical cumulative distribution function of absolute values of daily residuals for the 1000 largest U.S stocks from 2001 to 2016. Panel (c) plots the empirical cumulative distribution function of absolute values of normalized monthly residuals for all U.S common stocks from 1963 to 2016. Daily and Monthly residuals are estimated for each firm by regressing log excess returns on the log of Fama and French (2015) five factors over the entire period. For further details on estimation see Section 2.4. High-frequency, daily, and monthly residuals are normalized by stock to have mean 0 and standard deviation 1. $log$x is plotted on the horizontal axis and $logP(|r| > x)$ on the vertical axis.

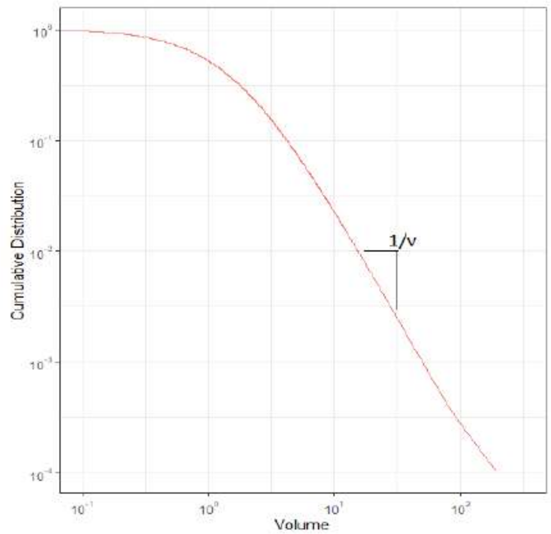
equivalent for high-frequency, daily, and monthly idiosyncratic returns.

Gabaix, Gopikrishnan, Plerou, and Stanley (2006) shows that high-frequency and daily trading volume is power-law distributed. Following the procedures in their paper, Figure 9 plots the cumulative distribution function of 5-minute and daily trading volume for the 1000 largest stocks from January 2001 to December 2016. Each stock's volume v is normalized by its mean absolute deviation.

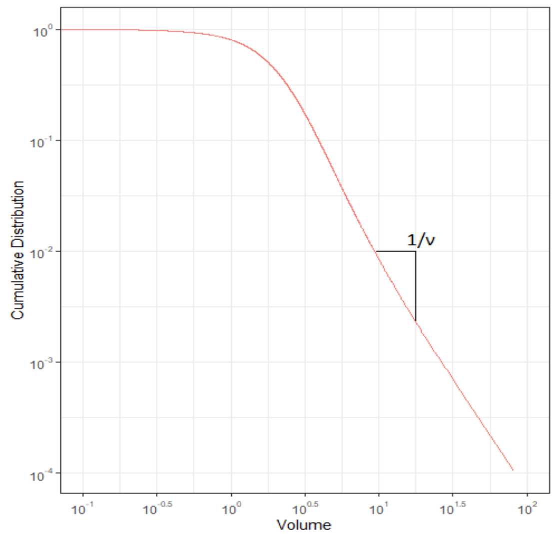
Panel (a) of Figure 9 shows that high-frequency volume greater than 1 absolute deviation follows a power law with volume tail risk parameter $\nu$. Trading volume has much heavier tails than returns. The 10% largest 5-minute volumes represents 65.82% of the total volume traded and the 1% largest volumes represents 27.81% of the total volume traded.[37] Panel (b) of Figure

[37]The 0.1% largest volumes represents 9.52% of the total volume traded.

**Figure 9: Cumulative Distribution of High-Frequency and Daily Trading Volume**




(a) High-Frequency Volume Cumulative Distribution




(b) Daily Volume Cumulative Distribution

Panel (a) plots the empirical cumulative distribution function $P(|r| > x)$ of high-frequency trading volume for the 1000 largest U.S stocks from 2001 to 2016. 10% of the volume observations are sampled resulting in approximately 30 million observations. Panel (b) plots the empirical cumulative distribution function $P(|v| > x)$ of daily trading volumes for all U.S common stocks from 2001 to 2016. For each stock, high-frequency and daily trading volumes are normalized by the stock's mean absolute deviation $\frac{v}{\frac{1}{N}\sum_{i=1}^{N}|v-\bar{v}|}$. $log$x is plotted on the horizontal axis and $logP(|v| > x)$ on the vertical axis.

9 shows that the tail of daily volume is also power-law distributed. This is unsurprising, since daily volume is the sum of high-frequency volumes with power-law distributed tails and power-law random variables are preserved under time aggregation. Additionally, high-frequency and daily trading volume have similar slopes $1/\nu$, suggesting that daily trading volume also inherits the tail distribution of high-frequency trading volume.

In summary, high-frequency returns are power-law distributed. Their availability provides a new means to effectively capture time-varying idiosyncratic tail risk. In addition, the empirical evidence shows that idiosyncratic returns are also power-law distributed, as such this is used as one of the key assumptions in this paper. Furthermore, daily and monthly idiosyncratic returns inherit the power-law distribution and tail risk parameter $\xi$ from high-frequency idiosyncratic returns. Trading volume is also power-law distributed and its tail risk can be summarized by slope parameter $\nu$. The next section introduces a new power-law factor model and idiosyncratic tail risk measure.

# B Proofs

## B.1 Proof for Lemma 1

*Proof.* Since $\boldsymbol{\beta}_t$ are constant for each month $t$, $\boldsymbol{\beta}_t$ is assumed to be the same in the monthly and high-frequency models. Take the sum of both sides of Equation (9) for periods $i = 1, .., N$, to obtain

$$\sum_{i=1}^{N} r_{t,i} = \boldsymbol{\beta}_t \sum_{i=1}^{N} \mathbf{f}_{t,i} + \sum_{i=1}^{N} x_{t,i}. \tag{27}$$

Then by the aggregation property of log returns, (27) can be written as

$$R_t = \boldsymbol{\beta}_t \mathbf{F}_t + \sum_{i=1}^{N} x_{t,i}, \tag{28}$$

which combined with (10) implies $X_t = \sum_{i=1}^{N} x_{t,i}$. □

## B.2 Levy's theorem (Auxiliary)

**Theorem 4.** *Durrett (2019), Page 186. Suppose $Z_1, Z_2, ...$ are i.i.d. with a distribution that satisfies*

*(i)* $\lim\limits_{z\to\infty} \frac{P(Z_1 > z)}{P(|Z_1| > z)} = \theta \in [0, 1]$

*(ii)* $P(|Z_1| > z) = z^{-\alpha} L(z)$,

*where $\alpha < 2$ and $L$ is a slowly varying function. Let $S_n = Z_1 + ... + Z_n$,*

$$a_n = inf\{z : P(|Z_1| > z) \leq n^{-1}\} \text{ and } b_n = nE(Z_1 \mathbb{1}_{(Z_1 \leq a_n)})$$

*As $n \to \infty$, $(S_n - b_n)/a_n \Rightarrow Y$ where $Y$ has a nondegenerate distribution. $Y$ follows a Levy distribution with tail shape $\alpha$.*

## B.3 Proof for Theorem 1

*Proof.* Let $\alpha_t = \frac{1}{\xi_t}$ in Levy's Theorem 4 in Section B.2. Parameter $\alpha$ is often called the tail exponent. Assumption 1 (a) and (b) satisfy conditions (i) and (ii) of Levy's Theorem. Hence, by Levy's Theorem,

$$(\sum_{i=1}^{N} x_{t,i} - b_N)/a_N \xrightarrow{d} u_t \text{ as } N \to \infty, \tag{29}$$

where u is a Levy-distributed random variable with tail risk $\xi_t$, $a_N = inf\{y : P(|x_{t,1}| > y) \leq N^{-1}\}$, and $b_N = NE(x_{t,1}\mathbb{1}_{(x_{t,1} \leq a_N)})$. By Lemma 1, the monthly idiosyncratic return is equal to the sum of high-frequency returns, that is $X_t = \sum_{i=1}^{N} x_{t,i}$. Substituting $X_t = \sum_{i=1}^{N} x_{t,i}$ into (29) gives the result in (11).

□

### B.4 Jessen and Mikosch Lemma (Auxiliary)

**Lemma 2** (Jessen and Mikosch (2006), Lemma 3.1)**.** *Assume $|Z_1|$ is regularly varying with shape $\alpha \geq 0$. Assume $Z_1, ..., Z_n$ are random variables satisfying*

$$\lim_{z\to\infty} \frac{P(Z_i > z)}{P(|Z_1| > z)} = c_i^+ \text{ and } \lim_{z\to\infty} \frac{P(Z_i \leq -z)}{P(|Z_1| > z)} = c_i^-, \; i = 1,..,n, \tag{30}$$

*for some non-negative numbers $c_i^\pm$ and*

$$\lim_{z\to\infty} \frac{P(Z_i > z, Z_j > z)}{P(|Z_1| > z)} = \lim_{z\to\infty} \frac{P(Z_i \leq -z, Z_j > z)}{P(|Z_1| > z)} = \lim_{z\to\infty} \frac{P(Z_i \leq -z, Z_j \leq -z)}{P(|Z_1| > z)} = 0, \; i \neq j, \tag{31}$$

*then*

$$\lim_{z\to\infty} \frac{P(Z_1 + ... + Z_n > z)}{P(|Z_1| > z)} = c_1^+ + ... + c_n^+ \text{ and } \lim_{z\to\infty} \frac{P(Z_1 + ... + Z_n \leq -z)}{P(|Z_1| > z)} = c_1^- + ... + c_n^-. \tag{32}$$

*In particular if the $Z_i$'s are independent non-negative regularly varying random variables then*

$$P(Z_1 + ... + Z_n > z) \sim P(Z_1 > z) + ... + P(Z_n > z). \tag{33}$$

## B.5 Proof for Theorem 2

*Proof.* The result is an application of Lemma 3.2 (a) in Mikosch and de Vries (2013), and I follow their proof closely with additional details. The proof uses the result from auxiliary Lemma 2 (Jessen and Mikosch (2006)) in Section B.4 for the sum of regularly varying random variables. I show the high-frequency idiosyncratic returns, $x_{t,1}, .., x_{t,N}$, satisfy the conditions of Lemma 2.[38] Condition (30) is satisfied by the tail balance condition in Assumption 1(b),

$$\lim_{y\to\infty} \frac{P(x_{t,1} > y)}{P(|x_{t,1}| > y)} = (1-\theta) \text{ and } \lim_{y\to\infty} \frac{P(x_{t,1} \leq -y)}{P(|x_{t,1}| > y)} = \theta, \tag{34}$$

where $\theta \in (0, 1]$. Since the idiosyncratic returns are i.i.d., then

$$\lim_{y\to\infty} \frac{P(x_{t,i} > y)}{P(|x_{t,1}| > y)} = (1-\theta) \text{ and } \lim_{y\to\infty} \frac{P(x_{t,i} \leq -y)}{P(|x_{t,1}| > y)} = \theta, \text{ i = 1,..,N.} \tag{35}$$

To show condition (31), note that

$$\begin{aligned}\lim_{y\to\infty} \frac{P(x_{t,i} \leq -y, x_{t,j} \leq -y)}{P(|x_{t,1}| > y)} &= \lim_{y\to\infty} \frac{P(x_{t,i} \leq -y)P(x_{t,j} \leq -y)}{P(|x_{t,1}| > y)} \\ &= \lim_{y\to\infty} \frac{P(x_{t,i} \leq -y)}{P(|x_{t,1}| > y)} \frac{P(x_{t,j} \leq -y)}{P(|x_{t,1}| > y)} P(|x_{t,1}| > y),\end{aligned} \tag{36}$$

where the first equality uses the independence of $x_{t,i}$ and the second equality is from multiplying the denominator and numerator by $P(|x_{t,1}| > y)$. It has been proved that for any $\rho > 0$ and slowly varying function $L_\rho(y)$, $\lim_{y\to\infty} y^{-\frac{1}{\rho}} L_\rho(y) = 0$ (Karamata (1962)), hence $\lim_{y\to\infty} P(|x_{t,1}| > y) = \lim_{y\to\infty} y^{-\frac{1}{\xi_t}} L(y) = 0$. Applying the product rule of limits to the last equality in Equation (36) and using Assumption 1(b) gives,

$$\lim_{y\to\infty} \frac{P(x_{t,i} \leq -y, x_{t,j} \leq -y)}{P(|x_{t,1}| > y)} = \lim_{y\to\infty} \frac{P(x_{t,i} \leq -y)}{P(|x_{t,1}| > y)} \lim_{y\to\infty} \frac{P(x_{t,j} \leq -y)}{P(|x_{t,1}| > y)} \lim_{y\to\infty} P(|x_{t,1}| > y) = \theta^2 \lim_{y\to\infty} y^{-\frac{1}{\xi_t}} L(y) = 0. \tag{37}$$

The other 2 conditions in (31) are proved analogously. Since the conditions in Lemma 2 are

[38] In the notation of Lemma 2, $Z_1 = x_{t,1}, Z_2 = x_{t,2}, ..., Z_N = x_{t,N}$.

satisfied, then

$$\lim_{y\to\infty} \frac{P(x_{t,1} + ... + x_{t,N} \le -y)}{P(|x_{t,1}| > y)} = N\theta = N \lim_{y\to\infty} \frac{P(x_{t,1} \le -y)}{P(|x_{t,1}| > y)}, \tag{38}$$

where the first equality is by Lemma 2 and the second equality is by Assumption 1(b). Hence,

$$P(x_{t,1} + ... + x_{t,N} \le -y) \sim NP(x_{t,1} \le -y), \tag{39}$$

and with the identity in Lemma 1 implies (12).

□

### B.6 Proof for Theorem 3

*Proof.* The result an application of Lemma 3.2 (c) in Mikosch and de Vries (2013), and I follow their proof closely with additional details. Similar to the previous theorem, this proof uses the result from auxiliary Lemma 2 (Jessen and Mikosch (2006)) in Appendix B.4 for the sum of regularly varying random variables. However, this theorem requires showing the conditions of Lemma 2 are satisfied for any i by the pair $x_{t,i}$ and $\eta_{t,i}$.[39] and I provide.

Fix any $i \in [1, N]$, then $|x_{t,i}|$ is regularly varying with shape $\frac{1}{\xi_t} > 0$. As in Theorem (2), $x_{t,i}$ satisfies condition (30) by Assumption 1(b). To show $\eta_{t,i}$ satisfies condition (30), note that

$$\lim_{y\to\infty} \frac{P(\eta_{t,i} \le -y)}{P(|x_{t,i}| > y)} = \lim_{y\to\infty} \frac{P(\eta_{t,i} \le -y)}{P(|\eta_{t,i}| > y)} \frac{P(|\eta_{t,i}| > y)}{P(|x_{t,i}| > y)}. \tag{40}$$

By Assumption 2(b), $\lim\limits_{y\to\infty} \frac{P(\eta_{t,i} \le -y)}{P(|\eta_{t,i}|>y)} = p$. Also,

$$\lim_{y\to\infty} \frac{P(|\eta_{t,i}| > y)}{P(|x_{t,i}| > y)} = \lim_{y\to\infty} \frac{y^{-\frac{1}{\gamma_t}} L_\eta(y)}{y^{-\frac{1}{\xi_t}} L(y)} = \lim_{y\to\infty} y^{-(\frac{1}{\gamma_t} - \frac{1}{\xi_t})} \frac{L_\eta(y)}{L(y)} = 0, \tag{41}$$

where the last equality is because $\frac{1}{\gamma_t} - \frac{1}{\xi_t} > 0$, $\frac{L_\eta(y)}{L(y)}$ is a slowly varying function, and for any $\rho > 0$ and slowly varying function $L_\rho(y)$, $\lim\limits_{y\to\infty} y^{-\frac{1}{\rho}} L_\rho(y) = 0$ (see Karamata (1962)). Furthermore, by the product rule of limits,

[39] In the notation of Lemma 2, $Z_1 = x_{t,i}$ and $Z_2 = \eta_{t,i}$

$$\lim_{y\to\infty}\frac{P(\eta_{t,i}\le -y)}{P(|x_{t,i}|>y)}=\lim_{y\to\infty}\frac{P(\eta_{t,i}\le -y)}{P(|\eta_{t,i}|>y)}\lim_{y\to\infty}\frac{P(|\eta_{t,i}|>y)}{P(|x_{t,i}|>y)}=p0=0. \tag{42}$$

The other case in condition (30) is proved analogously. To show condition (31), note that

$$\begin{aligned}\lim_{y\to\infty}\frac{P(x_{t,i}\le -y,\eta_{t,i}\le -y)}{P(|x_{t,1}|>y)}&=\lim_{y\to\infty}\frac{P(x_{t,i}\le -y)P(\eta_{t,i}\le -y)}{P(|x_{t,1}|>y)}\\&=\lim_{y\to\infty}\frac{P(x_{t,i}\le -y)}{P(|x_{t,1}|>y)}\frac{P(\eta_{t,i}\le -y)}{P(|\eta_{t,1}|>y)}P(|\eta_{t,1}|>y),\end{aligned} \tag{43}$$

where the first equality uses the independence of $x_{t,i}$ and $\eta_{t,i}$ and the second equality is from multiplying the denominator and numerator by $P(|\eta_{t,1}|>y)$. Applying the product rule of limits to the last equality in Equation (43) gives,

$$\lim_{y\to\infty}\frac{P(x_{t,i}\le -y,x_{t,j}\le -y)}{P(|x_{t,1}|>y)}=\lim_{y\to\infty}\frac{P(x_{t,i}\le -y)}{P(|x_{t,1}|>y)}\lim_{y\to\infty}\frac{P(\eta_{t,i}\le -y)}{P(|\eta_{t,1}|>y)}\lim_{y\to\infty}P(|\eta_{t,1}|>y)=\theta p\lim_{y\to\infty}y^{-\frac{1}{\eta_t}}L_\eta(y)=0. \tag{44}$$

The other 2 conditions in (31) are proved analogously. Since the assumptions of Lemma 2 are satisfied for $x_{t,i}$ and $\eta_{t,i}$, then

$$\begin{aligned}\lim_{y\to\infty}\frac{P(x^*_{t,i}\le -y)}{P(|x_{t,i}|>y)}&=\lim_{y\to\infty}\frac{P(x_{t,i}+\eta_{t,i}\le -y)}{P(|x_{t,i}|>y)}=\lim_{y\to\infty}\frac{P(x_{t,i}\le -y)+P(\eta_{t,i}\le -y)}{P(|x_{t,i}|>y)}=\\&\lim_{y\to\infty}\frac{P(x_{t,i}\le -y)}{P(|x_{t,i}|>y)}+\lim_{y\to\infty}\frac{P(\eta_{t,i}\le -y)}{P(|x_{t,i}|>y)}=\lim_{y\to\infty}\frac{P(x_{t,i}\le -y)}{P(|x_{t,i}|>y)},\end{aligned} \tag{45}$$

which implies that

$$P(x^*_{t,i}\le -y)\sim P(x_{t,i}\le -y). \tag{46}$$

□

# C Data

## C.1 TAQ Cleaning

The TAQ requires substantial cleaning due to contamination from market microstructure noise. As demonstrated in Section 2.2.4, large outliers from microstructure effects must be filtered for tail risk measurement. I filter noisy observations following the procedures in Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008). My econometric analysis primarily uses prices and trade size from the trades database. Bid and ask observations from the quotes database are also used for cleaning trades and robustness analysis. For both trades and quotes, entries with time stamps outside of the trading day (9:30am to 4:00pm) are removed. Entries with a zero or negative bid, ask, or price are also removed. For each stock, I keep only entries from the exchange with the highest volume in the month, and delete entries from other exchanges.

For only the quotes database, entries with a negative spread (ask minus bid) are deleted. Multiple quotes with the same second timestamp are replaced with a single entry with the median bid and median ask. Entries with a spread more than 50 times the median spread on that day are removed. Finally, entries are deleted when the midquote (midpoint of bid and ask) deviates by more than 10 mean absolute deviations (excluding the considered observation) from a rolling centered mean of 50 observations (25 observations before and 25 after).

For only the trades data, entries with corrected trades (CORR $\neq 0$) are deleted. Entries with an abnormal sale condition are removed, only keeping entries with COND equal to E, F, @, *, @E, @F, *E, and *F. Multiple trades with the same second timestamp are replaced with an entry with the median price. Finally, quotes are used to discipline the trade prices. Entries are removed if their price is above the ask plus spread or below the bid minus spread.

Merging the TAQ data with the CRSP data is challenging, because CRSP tickers often differ from TAQ tickers. Additionally, tickers change over time due to mergers, acquisitions, and other corporate events. I instead merge the TAQ and CRSP databases using CUSIPs. Each stock's CUSIP identifier is obtained by merging trades with the TAQ master files. Finally, each stock is indexed by CRSP PERMINOs, which are unique and do not change over time.

## C.2 Constructing High-Frequency Factors

I construct the high-frequency equity factors in the heavy-tailed factor model in Equation (9) following the procedures in Aït-Sahalia, Kalnina, and Xiu (2020). The models considered are the one-factor CAPM model (Sharpe (1964);Lintner (1965)), the Fama and French (1993) and Carhart (1997) four-factor model, and the Fama and French (2015) five-factor model. Factors in the models are reconstructed at 5-minute time intervals. Following Aït-Sahalia, Kalnina, and Xiu (2020), the construction combines the TAQ, CRSP, and COMPUSTAT databases in a multiple step procedure.

Since the constituents for the Fama and French factors are not publicly available, the first step is to replicate their factor construction. Compustat is used to create annual book equity (BE), operating profit (OP), and investment (INV) variables for each firm in North America. Only firms on Compustat for more than two years are included, to avoid survival bias. The firm-year variables are then merged with the CRSP monthly dataset. CRSP data is filtered according to Fama and French (1993), keeping only ordinary common equity (Share Code 10 or 11) from the NYSE, AMEX, and NASDAQ exchanges. Market equity (ME) for each month is the product of a firm's price and shares outstanding. Book-to-market (BE/ME) for June of year y is the ratio of book equity for y-1 fiscal year end divided by market equity for December of year y-1. Operating profitability and investment are also calculated during June of year y according to the formulas in Section C.3.

NYSE breakpoints are calculated during June of year y for ME, BE/ME, OP, and INV. Breakpoints determine portfolio groupings for each variable. Size groupings divide stocks with ME smaller (S) and bigger (B) than the median NYSE ME. Book-to-market groupings are denoted growth (L), neutral (N), and value (H), based on breakpoints for the lowest 30%, neutral 40%, and highest 30% of ranked BE/ME values for NYSE stocks. Six value-weighted portfolios are formed from the intersection of the two ME and three BE/ME groupings. The value factor (HML) is the return of the high value portfolios minus low value (growth) portfolios, given by

$$HML = \frac{1}{2}(Small\,Value + Big\,Value) - \frac{1}{2}(Small\,Growth + Big\,Growth). \tag{47}$$

The size factor ($SML_{B/M}$) is the return of the small portfolios minus big portfolios, given by

$$SMB_{B/M} = \frac{1}{3}(Small\ Value + Small\ Neutral + Small\ Growth) - \frac{1}{3}(Big\ Value + Big\ Neutral + Big\ Growth). \tag{48}$$

In Fama and French (2015), the operating profitability groupings are denoted robust (R), neutral (N), and weak (W). The investment groupings are conservative (C), neutral (N), and aggressive (A). The operating profitability and investment breakpoints are the 30th and 70th NYSE percentiles of OP and INV. The profitability factor (RMW) is

$$RMW = \frac{1}{2}(Small\ Robust + Big\ Robust) - \frac{1}{2}(Small\ Weak + Big\ Weak),$$

where the value-weighted portfolios are formed by the intersection of OP and ME groupings. The investment factor (CMA) is

$$CMA = \frac{1}{2}(Small\ Conservative + Big\ Conservative) - \frac{1}{2}(Small\ Aggressive + Big\ Aggressive),$$

where the value-weighted portfolios are formed by the intersection of INV and ME groupings. Additionally, the size factor (SMB) in Fama and French (2015) is adjusted to

$$SMB = \frac{1}{3}(SMB_{B/M} + SMB_{OP} + SMB_{INV}), \tag{49}$$

where

$$SMB_{OP} = \frac{1}{3}(Small\ Robust + Small\ Neutral + Small\ Weak) - \frac{1}{3}(Big\ Robust + Big\ Neutral + Big\ Weak), \tag{50}$$

$$\begin{aligned} SMB_{INV} = \frac{1}{3}(&Small\ Conservative + Small\ Neutral + Small\ Aggressive) - \\ &\frac{1}{3}(Big\ Conservative + Big\ Neutral + Big\ Aggressive), \end{aligned} \tag{51}$$

and $SMB_{B/M}$ is given in Equation (48).

A stock's momentum is its return in the previous 12 months excluding the most recent month. Momentum portfolios are denoted up (U), neutral (N), and down (D) based on breakpoints at the 30th and 70th NYSE percentiles. Following Ken French's website, six value-weighted portfolios are reformed every month by intersecting MOM and ME groupings. The momentum factor (UMD) is

$$UMD = \frac{1}{2}(Small\ Up + Big\ Up) - \frac{1}{2}(Small\ Down + Big\ Down).$$

In each model, the market factor (MKT) is the value-weighted portfolio of all stocks considered in Fama and French (1993).

The replication procedure provides daily portfolio constituents and weights for each equity factor. The daily constituents are merged with the TAQ to create intraday factors at 5-minute intervals. The constituents are also merged with the CRSP daily database to create overnight factors. Portfolios are weighted using the previous day's market capitalization. The high-frequency factors are transformed from simple returns to log returns using

$$f_{t,i} = log(1 + f_{t,i}^{Simple}),$$

where $f_{t,i}^{Simple}$ is the simple return of the high-frequency factor created from the procedures in this section.

## C.3 Additional Firm Characteristics

**Market Beta:** Market beta is based on single factor model

$$R_{l,d} - r_d^f = \alpha_l + \beta_l(R_d^m - r_d^f) + \epsilon_{l,d}, \tag{52}$$

where $R_{l,d}$ is the return of stock l on day d, $R_{m,d}$ is the market return, and $r_d^f$ is the risk-free rate. Equation (52) is estimated monthly using a rolling window of the previous 252 days. BETA for stock l is the least squares estimate $\hat{\beta}_l$.

**Size:** Following Fama and French (1993), a firm's SIZE is the natural logarithm of the market value of equity. Market equity (ME) is the product of price and number of shares outstanding in millions of dollars. The size factor (SMB) includes all common stocks in the NYSE, AMEX, and NASDAQ exchanges with non-missing market equity for June of year y.

**Book-to-Market:** Following Fama and French (1993), the BM ratio in June of year y is the firm's ratio of book value of common equity in fiscal year y-1 and market value of equity in December of year y-1. Book value of common equity is defined as book value of shareholders' equity, plus balance sheet deferred taxes and investment tax credit (if available), minus book value of preferred stock. The book value of common equity for firm l in fiscal year y-1 is

$$BE_{l,y-1} = SEQ_{l,y-1} + TXDB_{l,y-1} + ITCB_{l,y-1} - BVPS_{l,y-1},$$

where $SEQ_{l,y-1}$ is book value of shareholder's equity, $TXDB_{l,y-1}$ is deferred taxes, $ITCB_{l,y-1}$ is investment tax credit, and $BVPS_{l,y-1}$ is book value of preferred stock. Depending on availability, book value of preferred stock is estimated using the redemption ($PSTKRV_{l,y-1}$), liquidation ($PSTKL_{l,y-1}$), or par ($PSTK_{l,y-1}$) value in that order. The book-to-market factor (HML) includes all common stocks in the NYSE, AMEX, and NASDAQ exchanges with non-missing market equity for December of year y-1 and June of year y, and non-missing book equity for fiscal year y-1.

**Operating Profitability:** Following Fama and French (2015), the OP of firm l for June of year y is annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity for fiscal year y-1, that is,

$$OP_{l,y} = \frac{REVT_{l,y-1} - COGS_{l,y-1} - XINT_{l,y-1} - XSGA_{l,y-1}}{BE_{l,y-1}}$$

where $REVT_{l,y-1}$ is annual revenues, $COGS_{l,y-1}$ is cost of goods sold, $XINT_{l,y-1}$ is interest and related expenses, $XSGA_{l,y-1}$ is selling, general, and administrative expenses, and $BE_{l,y-1}$ is book equity for fiscal year y-1. The profitability factor (RMW) includes all common stocks on the NYSE, AMEX, and NASDAQ with market equity data for June of year y, positive $BE_{l,y-1}$, non-missing $REVT_{l,y-1}$, and non-missing data for at least one of $COGS_{l,y-1}$, $XINT_{l,y-1}$, or $XSGA_{l,y-1}$.

**Investment:** Following FFama and French (2015), the INV of firm l for June of year y is the change of total assets from fiscal year y-2 to fiscal year y-1, divided by the total assets in fiscal year y-2, that is,

$$INV_{l,y} = \frac{AT_{l,y-1} - AT_{l,y-2}}{AT_{l,y-2}},$$

where $AT_{l,y-1}$ is the firm's total assets in fiscal year y-1. The investment factor (CMA) includes all common stocks on the NYSE, AMEX, and NASDAQ exchanges with market equity data for June of year y and total assets data for fiscal years y-1 and y-2.

**Momentum:** Following Jegadeesh and Titman (1993), the MOM of stock l in month t is the cumulative return for the 11 months from month t-12 to t-2, skipping the most recent month.

**Short-term reversals:** Following Jegadeesh (1990) and Lehmann (1990), the REV of stock l in month t is the return of the stock in the previous month, that is the return in month t-1.

**Illiquidity:** Following Amihud (2002), the ILLIQ of stock l in month t is the average daily ratio of the absolute stock return to the dollar volume, averaged in month t, that is,

$$ILLIQ_l = \frac{1}{D}\sum_{d=1}^{D}\frac{|R_{l,d}|}{VOLD_{l,d}},$$

where $D$ is the number of trading days in the month, $R_d$ is the return on day d, $VOLD_d$ is the respective daily volume in millions of dollars.

**Maximum Daily Return:** Following Bali, Cakici, and Whitelaw (2011), the MAX is defined as the maximum daily return within month t.

**Idiosyncratic Volatility:** Following Ang, Hodrick, Xing, and Zhang (2006), idiosyncratic volatility is based on the Fama and French (1993) three factor model

$$R_{l,d} - r_d^f = \alpha_l + \beta_l(R_d^m - r_d^f) + \gamma_l SMB_d + \phi_l HML_d + \epsilon_{l,d},$$

where $SMB_d$ and $HML_d$ are daily returns on the size and book-to-market factors defined in Section C.2. The IVOL of stock l in month t is the standard deviation of daily residuals in month t, that is, $IVOL_l = \sqrt{var(\epsilon_{l,d})}$.

**Coskewness:** Following Harvey and Siddique (2000), the COSKEW for stock l in month t is

$$COSKEW_l = \frac{E[(R_{l,d} - \bar{r}_{l,t})(R_d^m - \bar{r}_t^m)^2]}{\sqrt{var(R_{l,d})}var(R_d^m)},$$

where $R_{l,d}$ is the return of stock l on day d and $R_d^m$ is the market return on day d over the past year. Variables $\bar{r}_{l,t}$ and $\bar{r}_t^m$ are respectively the average daily returns of stock l and the market portfolio over the past year.

**Downside Risk:** Following Ang, Chen, and Xing (2006), the $\beta_{l,down}$ for stock l in month t is calculated over the past year using excess daily stock returns and market returns, conditional on the excess market return moving below its average value, that is,

$$\beta_{l,down} = \frac{cov(R_{l,d} - r_d^f, R_d^m - r_d^f | R_d^m - r_d^f < \mu^m)}{var(R_d^m - r_d^f | R_d^m - r_d^f < \mu^m)},$$

where $R_{l,d}$ is the return of stock l on day d, $R_d^m$ is the market return on day d, and $\mu^m$ is the average daily excess market return over the past year.

# D Robustness for Idiosyncratic Tail Risk and Expected Returns

## D.1 Additional Univariate Sorts

Table 17: Portfolios Sorted on Idiosyncratic Tail Risk estimated using the Market model residuals

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | |
| Excess Return | 0.50 | 0.51 | 0.67 | 0.62 | 0.66 | 0.66 | 0.76 | 0.67 | 0.90 | 1.02 | 0.52 |
| t-stat | (1.12) | (1.15) | (1.50) | (1.40) | (1.56) | (1.57) | (1.73) | (1.55) | (2.25) | (2.72) | (2.61) |
| FFC4 alpha | 0.55 | 0.58 | 0.71 | 0.67 | 0.73 | 0.70 | 0.81 | 0.70 | 0.93 | 1.03 | 0.48 |
| FFC4 t-stat | (1.32) | (1.39) | (1.73) | (1.66) | (1.84) | (1.79) | (1.99) | (1.78) | (2.45) | (2.91) | (2.40) |
| FF5 alpha | 0.73 | 0.70 | 0.86 | 0.79 | 0.86 | 0.80 | 0.93 | 0.81 | 1.00 | 1.09 | 0.36 |
| FF5 t-stat | (1.65) | (1.61) | (1.97) | (1.85) | (2.11) | (1.97) | (2.20) | (1.92) | (2.51) | (3.00) | (1.67) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | |
| Excess Return | 0.28 | 0.39 | 0.47 | 0.41 | 0.40 | 0.49 | 0.45 | 0.48 | 0.73 | 1.02 | 0.74 |
| t-stat | (0.73) | (1.12) | (1.27) | (1.20) | (1.21) | (1.45) | (1.22) | (1.14) | (1.97) | (2.84) | (3.75) |
| FFC4 alpha | 0.33 | 0.45 | 0.50 | 0.43 | 0.44 | 0.52 | 0.53 | 0.49 | 0.78 | 1.05 | 0.71 |
| FFC4 t-stat | (0.89) | (1.37) | (1.53) | (1.47) | (1.40) | (1.70) | (1.67) | (1.30) | (2.29) | (3.17) | (3.48) |
| FF5 alpha | 0.49 | 0.55 | 0.57 | 0.51 | 0.54 | 0.60 | 0.63 | 0.63 | 0.80 | 1.18 | 0.68 |
| FF5 t-stat | (1.28) | (1.66) | (1.70) | (1.72) | (1.81) | (1.88) | (1.95) | (1.61) | (2.23) | (3.55) | (3.18) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to December 2016. Idiosyncratic tail risk is estimated from the high-frequency the market model residuals. Panel A reports equally-weighted excess returns and alphas sorted on idiosyncratic tail risk. Panel B reports value-weighted excess returns and alphas sorted on idiosyncratic tail risk.

Table 18: Portfolios Sorted on Idiosyncratic Tail Risk estimated using the FFC residuals

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | |
| Excess Return | 0.51 | 0.56 | 0.68 | 0.66 | 0.56 | 0.76 | 0.72 | 0.62 | 0.88 | 1.01 | 0.50 |
| t-stat | (1.16) | (1.26) | (1.61) | (1.51) | (1.27) | (1.77) | (1.66) | (1.45) | (2.13) | (2.70) | (2.45) |
| FFC4 alpha | 0.57 | 0.60 | 0.76 | 0.73 | 0.60 | 0.81 | 0.76 | 0.65 | 0.90 | 1.03 | 0.46 |
| FFC4 t-stat | (1.34) | (1.47) | (1.88) | (1.83) | (1.47) | (2.02) | (1.92) | (1.69) | (2.32) | (2.90) | (2.25) |
| FF5 alpha | 0.74 | 0.72 | 0.87 | 0.88 | 0.74 | 0.93 | 0.88 | 0.76 | 0.97 | 1.09 | 0.35 |
| FF5 t-stat | (1.63) | (1.68) | (2.04) | (2.10) | (1.72) | (2.19) | (2.16) | (1.89) | (2.39) | (2.99) | (1.60) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | |
| Excess Return | 0.34 | 0.43 | 0.36 | 0.61 | 0.29 | 0.57 | 0.42 | 0.52 | 0.60 | 0.91 | 0.57 |
| t-stat | (0.93) | (1.25) | (1.07) | (1.77) | (0.82) | (1.69) | (1.10) | (1.31) | (1.40) | (2.37) | (2.65) |
| FFC4 alpha | 0.40 | 0.46 | 0.42 | 0.66 | 0.30 | 0.63 | 0.45 | 0.57 | 0.62 | 0.91 | 0.51 |
| FFC4 t-stat | (1.09) | (1.46) | (1.33) | (2.10) | (0.96) | (2.13) | (1.37) | (1.62) | (1.55) | (2.64) | (2.39) |
| FF5 alpha | 0.54 | 0.55 | 0.52 | 0.74 | 0.37 | 0.75 | 0.58 | 0.65 | 0.66 | 1.02 | 0.48 |
| FF5 t-stat | (1.42) | (1.75) | (1.64) | (2.33) | (1.11) | (2.51) | (1.75) | (1.80) | (1.58) | (2.95) | (2.24) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to December 2016. Idiosyncratic tail risk is estimated from the high-frequency Fama-French-Cahart model residuals. Panel A reports equally-weighted excess returns and alphas sorted on idiosyncratic tail risk. Panel B reports value-weighted excess returns and alphas sorted on idiosyncratic tail risk.

Table 19: Portfolios Sorted on Idiosyncratic Tail Risk estimated using the Industry statistical model residuals

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | |
| Excess Return | 0.45 | 0.57 | 0.64 | 0.61 | 0.62 | 0.73 | 0.73 | 0.71 | 0.81 | 0.98 | 0.53 |
| t-stat | (1.00) | (1.20) | (1.36) | (1.28) | (1.31) | (1.51) | (1.53) | (1.50) | (1.77) | (2.24) | (2.95) |
| FFC4 alpha | 0.51 | 0.62 | 0.70 | 0.67 | 0.66 | 0.78 | 0.78 | 0.75 | 0.84 | 0.98 | 0.47 |
| FFC4 t-stat | (1.21) | (1.43) | (1.64) | (1.52) | (1.52) | (1.74) | (1.76) | (1.73) | (1.96) | (2.46) | (2.69) |
| FF5 alpha | 0.68 | 0.75 | 0.85 | 0.83 | 0.77 | 0.90 | 0.90 | 0.85 | 0.94 | 1.07 | 0.39 |
| FF5 t-stat | (1.49) | (1.66) | (1.89) | (1.79) | (1.68) | (1.91) | (1.96) | (1.87) | (2.07) | (2.57) | (2.25) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | |
| Excess Return | 0.21 | 0.42 | 0.48 | 0.59 | 0.50 | 0.57 | 0.72 | 0.60 | 0.67 | 0.91 | 0.70 |
| t-stat | (0.63) | (1.11) | (1.35) | (1.55) | (1.25) | (1.49) | (1.83) | (1.45) | (1.53) | (2.29) | (3.53) |
| FFC4 alpha | 0.27 | 0.45 | 0.54 | 0.63 | 0.55 | 0.60 | 0.76 | 0.64 | 0.68 | 0.94 | 0.67 |
| FFC4 t-stat | (0.81) | (1.34) | (1.67) | (1.95) | (1.57) | (1.78) | (2.19) | (1.70) | (1.74) | (2.67) | (3.39) |
| FF5 alpha | 0.37 | 0.53 | 0.64 | 0.74 | 0.65 | 0.72 | 0.89 | 0.72 | 0.84 | 1.02 | 0.65 |
| FF5 t-stat | (1.06) | (1.56) | (1.88) | (2.23) | (1.82) | (2.07) | (2.61) | (1.94) | (2.11) | (2.92) | (3.14) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to December 2016. Idiosyncratic tail risk is estimated from the high-frequency industry statistical model residuals. Panel A reports equally-weighted excess returns and alphas sorted on idiosyncratic tail risk. Panel B reports value-weighted excess returns and alphas sorted on idiosyncratic tail risk.

Table 20: Portfolios Sorted on Idiosyncratic Tail Risk estimated using the High-Frequency Cross-Sectional Variable model residuals

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | |
| Excess Return | 0.46 | 0.53 | 0.61 | 0.62 | 0.58 | 0.69 | 0.76 | 0.72 | 0.86 | 0.96 | 0.50 |
| t-stat | (1.00) | (1.12) | (1.32) | (1.27) | (1.21) | (1.41) | (1.57) | (1.50) | (1.87) | (2.15) | (2.80) |
| FFC4 alpha | 0.51 | 0.59 | 0.68 | 0.68 | 0.62 | 0.73 | 0.80 | 0.75 | 0.89 | 0.96 | 0.45 |
| FFC4 t-stat | (1.20) | (1.35) | (1.59) | (1.52) | (1.43) | (1.66) | (1.79) | (1.70) | (2.06) | (2.37) | (2.50) |
| FF5 alpha | 0.65 | 0.74 | 0.85 | 0.80 | 0.73 | 0.87 | 0.95 | 0.85 | 0.98 | 1.05 | 0.40 |
| FF5 t-stat | (1.43) | (1.64) | (1.89) | (1.70) | (1.59) | (1.92) | (2.03) | (1.80) | (2.13) | (2.50) | (2.15) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | |
| Excess Return | 0.30 | 0.31 | 0.50 | 0.55 | 0.32 | 0.69 | 0.66 | 0.76 | 0.62 | 0.72 | 0.42 |
| t-stat | (0.85) | (0.86) | (1.23) | (1.57) | (0.80) | (1.85) | (1.70) | (1.74) | (1.34) | (1.66) | (1.88) |
| FFC4 alpha | 0.33 | 0.37 | 0.50 | 0.58 | 0.38 | 0.75 | 0.73 | 0.78 | 0.62 | 0.72 | 0.40 |
| FFC4 t-stat | (1.02) | (1.13) | (1.45) | (1.79) | (1.06) | (2.21) | (2.05) | (2.20) | (1.49) | (1.88) | (1.74) |
| FF5 alpha | 0.40 | 0.46 | 0.65 | 0.62 | 0.49 | 0.88 | 0.88 | 0.89 | 0.73 | 0.87 | 0.47 |
| FF5 t-stat | (1.18) | (1.42) | (1.81) | (1.90) | (1.32) | (2.62) | (2.56) | (2.49) | (1.73) | (2.27) | (1.97) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to October 2016. Idiosyncratic tail risk is estimated from the high-frequency cross-Sectional variable model residuals. Panel A reports equally-weighted excess returns and alphas sorted on idiosyncratic tail risk. Panel B reports value-weighted excess returns and alphas sorted on idiosyncratic tail risk.

Table 21: Portfolios Sorted on Idiosyncratic Tail Risk estimated using Midquotes

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | |
| Excess Return | 0.52 | 0.48 | 0.59 | 0.37 | 0.58 | 0.56 | 0.61 | 0.79 | 0.83 | 0.96 | 0.44 |
| t-stat | (1.13) | (1.09) | (1.37) | (0.80) | (1.31) | (1.29) | (1.38) | (1.82) | (1.88) | (2.35) | (3.05) |
| FFC4 alpha | 0.53 | 0.54 | 0.67 | 0.39 | 0.63 | 0.61 | 0.64 | 0.83 | 0.83 | 0.97 | 0.43 |
| FFC4 t-stat | (1.32) | (1.26) | (1.71) | (0.93) | (1.51) | (1.49) | (1.60) | (2.07) | (2.06) | (2.57) | (3.13) |
| FF5 alpha | 0.67 | 0.71 | 0.84 | 0.52 | 0.77 | 0.77 | 0.74 | 0.91 | 0.88 | 1.03 | 0.36 |
| FF5 t-stat | (1.60) | (1.54) | (2.11) | (1.17) | (1.75) | (1.85) | (1.72) | (2.18) | (2.10) | (2.62) | (2.67) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | |
| Excess Return | 0.25 | 0.13 | 0.50 | 0.15 | 0.28 | 0.54 | 0.32 | 0.61 | 0.59 | 0.97 | 0.72 |
| t-stat | (0.61) | (0.34) | (1.49) | (0.38) | (0.75) | (1.41) | (0.76) | (1.52) | (1.46) | (2.65) | (3.10) |
| FFC4 alpha | 0.31 | 0.14 | 0.58 | 0.18 | 0.32 | 0.57 | 0.30 | 0.63 | 0.60 | 1.00 | 0.68 |
| FFC4 t-stat | (0.79) | (0.39) | (1.81) | (0.48) | (0.96) | (1.57) | (0.89) | (1.76) | (1.70) | (2.86) | (2.81) |
| FF5 alpha | 0.49 | 0.27 | 0.66 | 0.23 | 0.42 | 0.72 | 0.47 | 0.67 | 0.71 | 1.11 | 0.62 |
| FF5 t-stat | (1.21) | (0.74) | (1.95) | (0.62) | (1.18) | (1.90) | (1.34) | (1.85) | (1.96) | (3.13) | (2.75) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to October 2016. Idiosyncratic tail risk is estimated using midquote data and the high-frequency Fama-French five factor model residuals. Panel A reports equally-weighted excess returns and alphas sorted on idiosyncratic tail risk. Panel B reports value-weighted excess returns and alphas sorted on idiosyncratic tail risk.

Table 22: Portfolios Sorted on Idiosyncratic Tail Risk estimated using 0.025 quantile

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | |
| Excess Return | 0.46 | 0.62 | 0.58 | 0.52 | 0.68 | 0.75 | 0.80 | 0.80 | 0.85 | 0.90 | 0.45 |
| t-stat | (1.02) | (1.40) | (1.31) | (1.15) | (1.59) | (1.75) | (1.94) | (1.94) | (2.08) | (2.31) | (2.77) |
| FFC4 alpha | 0.50 | 0.66 | 0.63 | 0.56 | 0.72 | 0.80 | 0.84 | 0.85 | 0.90 | 0.93 | 0.43 |
| FFC4 t-stat | (1.23) | (1.61) | (1.53) | (1.37) | (1.82) | (1.98) | (2.17) | (2.22) | (2.39) | (2.52) | (2.66) |
| FF5 alpha | 0.66 | 0.79 | 0.76 | 0.69 | 0.85 | 0.91 | 0.97 | 0.96 | 1.00 | 0.99 | 0.33 |
| FF5 t-stat | (1.52) | (1.81) | (1.75) | (1.61) | (2.04) | (2.12) | (2.38) | (2.42) | (2.58) | (2.57) | (1.99) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | |
| Excess Return | 0.23 | 0.32 | 0.49 | 0.54 | 0.39 | 0.49 | 0.54 | 0.56 | 0.56 | 0.72 | 0.49 |
| t-stat | (0.62) | (0.94) | (1.31) | (1.55) | (1.04) | (1.33) | (1.65) | (1.54) | (1.38) | (1.92) | (2.71) |
| FFC4 alpha | 0.26 | 0.36 | 0.54 | 0.57 | 0.40 | 0.53 | 0.60 | 0.60 | 0.60 | 0.80 | 0.54 |
| FFC4 t-stat | (0.77) | (1.10) | (1.55) | (1.84) | (1.25) | (1.60) | (2.00) | (1.83) | (1.67) | (2.37) | (2.72) |
| FF5 alpha | 0.36 | 0.44 | 0.65 | 0.68 | 0.46 | 0.61 | 0.68 | 0.75 | 0.73 | 0.83 | 0.46 |
| FF5 t-stat | (1.00) | (1.29) | (1.80) | (2.12) | (1.45) | (1.80) | (2.23) | (2.31) | (2.03) | (2.36) | (2.32) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to December 2016. Idiosyncratic tail risk is estimated using a Hill threshold of 0.025 and the high-frequency market model residuals. Panel A reports equally-weighted excess returns and alphas sorted on idiosyncratic tail risk. Panel B reports value-weighted excess returns and alphas sorted on idiosyncratic tail risk.

Table 23: Portfolios Sorted on Idiosyncratic Tail Risk using 0.01 quantile

| | 1 (Low) | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 (High) | (10-1) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Univariate Sort on Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | |
| Excess Return | 0.45 | 0.55 | 0.63 | 0.63 | 0.70 | 0.62 | 0.68 | 0.79 | 0.89 | 1.01 | 0.56 |
| t-stat | (1.05) | (1.26) | (1.43) | (1.42) | (1.58) | (1.45) | (1.54) | (1.85) | (2.08) | (2.73) | (2.78) |
| FFC4 alpha | 0.51 | 0.60 | 0.67 | 0.68 | 0.76 | 0.67 | 0.73 | 0.84 | 0.91 | 1.02 | 0.51 |
| FFC4 t-stat | (1.29) | (1.50) | (1.62) | (1.70) | (1.87) | (1.68) | (1.78) | (2.05) | (2.28) | (2.89) | (2.58) |
| FF5 alpha | 0.68 | 0.74 | 0.81 | 0.83 | 0.87 | 0.81 | 0.83 | 0.93 | 1.00 | 1.08 | 0.40 |
| FF5 t-stat | (1.61) | (1.77) | (1.85) | (1.94) | (2.05) | (1.95) | (1.93) | (2.15) | (2.38) | (2.97) | (1.95) |
| Panel B: Univariate Sort on Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | |
| Excess Return | 0.18 | 0.59 | 0.47 | 0.60 | 0.49 | 0.71 | 0.70 | 0.74 | 0.72 | 0.86 | 0.68 |
| t-stat | (0.55) | (1.74) | (1.27) | (1.62) | (1.31) | (1.84) | (1.86) | (1.83) | (1.66) | (2.54) | (3.22) |
| FFC4 alpha | 0.23 | 0.64 | 0.47 | 0.65 | 0.53 | 0.75 | 0.74 | 0.79 | 0.72 | 0.86 | 0.63 |
| FFC4 t-stat | (0.76) | (2.01) | (1.43) | (1.99) | (1.67) | (2.26) | (2.16) | (2.09) | (1.82) | (2.72) | (2.88) |
| FF5 alpha | 0.33 | 0.74 | 0.58 | 0.76 | 0.63 | 0.88 | 0.87 | 0.89 | 0.80 | 0.93 | 0.61 |
| FF5 t-stat | (1.06) | (2.31) | (1.69) | (2.27) | (1.92) | (2.64) | (2.48) | (2.24) | (2.05) | (2.88) | (2.79) |

The table reports monthly average idiosyncratic tail risk, excess returns, and alphas for portfolios sorted on idiosyncratic tail risk between January 2001 to December 2016. Idiosyncratic tail risk is estimated using a Hill threshold of 0.1 and the high-frequency market model residuals. Panel A reports equally-weighted excess returns and alphas sorted on idiosyncratic tail risk. Panel B reports value-weighted excess returns and alphas sorted on idiosyncratic tail risk.

## D.2 Conditional Portfolio Sorts on Additional Firm Characteristics

Table 24: Double-Sorted Portfolios on Firm Characteristic then Idiosyncratic Tail Risk

| Firm Characteristic | BETA | SIZE | BM | OP | INV | MOM | COSKEW | IVOL | ILLIQ | MAX | $\beta^{Downside}$ | REV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Double-Sort on Firm Characteristic then Idiosyncratic Tail Risk (Equal-Weighted) | | | | | | | | | | | | |
| 1 (Low ITR) | 0.53 | 0.54 | 0.53 | 0.63 | 0.54 | 0.53 | 0.52 | 0.49 | 0.60 | 0.50 | 0.52 | 0.50 |
| 2 | 0.70 | 0.71 | 0.71 | 0.70 | 0.68 | 0.68 | 0.67 | 0.65 | 0.68 | 0.67 | 0.68 | 0.68 |
| 3 | 0.72 | 0.65 | 0.72 | 0.76 | 0.71 | 0.66 | 0.70 | 0.72 | 0.66 | 0.67 | 0.70 | 0.67 |
| 4 | 0.79 | 0.83 | 0.79 | 0.81 | 0.80 | 0.76 | 0.79 | 0.76 | 0.81 | 0.79 | 0.81 | 0.78 |
| 5 (High ITR) | 0.85 | 0.85 | 0.87 | 1.06 | 0.91 | 0.94 | 0.92 | 0.97 | 0.84 | 0.96 | 0.89 | 0.94 |
| Excess Return | 0.32 | 0.31 | 0.34 | 0.44 | 0.38 | 0.41 | 0.40 | 0.48 | 0.24 | 0.45 | 0.37 | 0.44 |
| t-stat | (2.68) | (2.25) | (2.38) | (2.92) | (2.82) | (2.77) | (2.75) | (2.73) | (2.16) | (2.91) | (3.03) | (2.97) |
| Panel B: Double-Sort on Firm Characteristic then Idiosyncratic Tail Risk (Value-Weighted) | | | | | | | | | | | | |
| | BETA | SIZE | BM | OP | INV | MOM | COSKEW | IVOL | ILLIQ | MAX | $\beta^{Downside}$ | REV |
| 1 (Low ITR) | 0.36 | 0.54 | 0.45 | 0.38 | 0.33 | 0.35 | 0.40 | 0.29 | 0.56 | 0.29 | 0.39 | 0.36 |
| 2 | 0.44 | 0.65 | 0.58 | 0.48 | 0.50 | 0.50 | 0.44 | 0.36 | 0.64 | 0.42 | 0.45 | 0.45 |
| 3 | 0.45 | 0.65 | 0.57 | 0.50 | 0.47 | 0.42 | 0.50 | 0.38 | 0.63 | 0.32 | 0.50 | 0.48 |
| 4 | 0.58 | 0.77 | 0.68 | 0.61 | 0.57 | 0.48 | 0.56 | 0.57 | 0.72 | 0.51 | 0.59 | 0.46 |
| 5 (High ITR) | 0.67 | 0.82 | 0.74 | 0.92 | 0.75 | 0.69 | 0.79 | 0.74 | 0.75 | 0.69 | 0.71 | 0.76 |
| Excess Return | 0.30 | 0.28 | 0.29 | 0.53 | 0.42 | 0.34 | 0.39 | 0.45 | 0.18 | 0.40 | 0.32 | 0.39 |
| t-stat | (2.59) | (2.03) | (2.06) | (3.24) | (3.13) | (2.33) | (2.68) | (2.25) | (1.86) | (2.51) | (2.40) | (2.42) |

Double-Sorted equal-weighted and value-weighted portfolios are formed every month from January 2001 to December 2016. Portfolios are created by sorting by idiosyncratic tail risk after first controlling for beta, size, book-to-market, operating profitability, investment, momentum, coskewness, idiosyncratic volatility, max, downside beta, or reversal. In each case, I first sort stocks in quintiles using the control variable, then in each quintile, I sort the stocks into quintiles based on idiosyncratic tail risk. The table presents average returns in percentage terms across the 5 control quintiles to produce quintile portfolios with different idiosyncratic tail risks but similar levels of the control variable. The table also reports the average return and Newey-West t-statistic with 1 lag of a portfolio that goes long the highest idiosyncratic tail risk and shorts the lowest idiosyncratic tail risk for the returns averaged across the 5 control quintiles.

## D.3 Descriptive Statistics

Table 25: Descriptive Statistics of Portfolios Sorted by Idiosyncratic Tail Risk and Unconditional Correlations

| | ITR | BETA | SIZE | BM | OP | INV | MOM | IVOL | COSKEW | ILLIQ | MAX | $\beta^{Downside}$ | REV |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Unconditional Correlations | | | | | | | | | | | | | |
| ITR | 1 | -0.18 | -0.09 | 0.08 | 0.00 | 0.01 | 0.02 | 0.14 | 0.01 | 0.25 | 0.07 | -0.16 | 0.04 |
| BETA | | 1 | -0.07 | -0.03 | -0.03 | -0.01 | 0.01 | 0.19 | 0.17 | -0.04 | 0.21 | 0.80 | 0.00 |
| SIZE | | | 1 | -0.06 | 0.01 | 0.00 | -0.03 | -0.14 | 0.02 | -0.05 | -0.10 | -0.05 | -0.01 |
| BM | | | | 1 | -0.06 | 0.00 | 0.15 | -0.05 | -0.03 | 0.04 | -0.02 | -0.01 | 0.05 |
| OP | | | | | 1 | -0.01 | 0.00 | -0.02 | 0.00 | -0.01 | -0.02 | -0.02 | 0.00 |
| INV | | | | | | 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.01 | 0.00 |
| MOM | | | | | | | 1 | 0.08 | -0.05 | 0.00 | 0.03 | 0.06 | 0.00 |
| IVOL | | | | | | | | 1 | -0.04 | 0.09 | 0.83 | 0.18 | -0.01 |
| COSKEW | | | | | | | | | 1 | 0.01 | 0.01 | -0.23 | -0.02 |
| ILLIQ | | | | | | | | | | 1 | 0.07 | -0.05 | 0.00 |
| MAX | | | | | | | | | | | 1 | 0.18 | -0.04 |
| $\beta^{Downside}$ | | | | | | | | | | | | 1 | 0.01 |
| REV | | | | | | | | | | | | | 1 |
| Panel B: Descriptive Statistics for Stocks Sorted by Idiosyncratic Tail Risk | | | | | | | | | | | | | |
| 1 (Low) | 0.32 | 1.22 | 9.14 | 0.58 | 0.32 | 0.20 | 0.18 | 1.44 | 0.00 | 0.06 | 0.05 | 1.18 | 0.01 |
| 2 | 0.36 | 1.19 | 9.07 | 0.57 | 0.33 | 0.18 | 0.19 | 1.50 | 0.00 | 0.06 | 0.05 | 1.17 | 0.02 |
| 3 | 0.38 | 1.18 | 9.06 | 0.57 | 0.37 | 0.18 | 0.19 | 1.55 | 0.00 | 0.07 | 0.05 | 1.15 | 0.02 |
| 4 | 0.39 | 1.16 | 9.01 | 0.57 | 0.38 | 0.17 | 0.20 | 1.59 | 0.00 | 0.07 | 0.05 | 1.14 | 0.02 |
| 5 | 0.41 | 1.14 | 8.96 | 0.56 | 0.35 | 0.17 | 0.20 | 1.64 | 0.00 | 0.08 | 0.05 | 1.13 | 0.02 |
| 6 | 0.43 | 1.13 | 8.95 | 0.56 | 0.33 | 0.17 | 0.21 | 1.68 | 0.00 | 0.09 | 0.05 | 1.12 | 0.02 |
| 7 | 0.44 | 1.11 | 8.90 | 0.57 | 0.35 | 0.16 | 0.22 | 1.74 | 0.00 | 0.10 | 0.05 | 1.10 | 0.02 |
| 8 | 0.47 | 1.09 | 8.77 | 0.57 | 0.35 | 0.16 | 0.22 | 1.83 | 0.00 | 0.14 | 0.05 | 1.07 | 0.02 |
| 9 | 0.51 | 1.05 | 8.48 | 0.60 | 0.37 | 0.98 | 0.22 | 1.95 | 0.00 | 0.23 | 0.05 | 1.03 | 0.02 |
| 10 (High) | 0.65 | 0.95 | 7.35 | 0.70 | 0.30 | 1.79 | 0.21 | 1.98 | 0.01 | 0.65 | 0.06 | 0.91 | 0.02 |

Panel A reports the unconditional correlations between all the variables from January 2001 to December 2016. Panel B reports the average value of each control variable for decile portfolios sorted on idiosyncratic tail risk. Control variables include beta, size, book-to-market, operating profitability, investment, momentum, coskewness, idiosyncratic volatility, max, downside beta, and reversal.

Table 26: Asset Pricing Tests on Anomaly Portfolios using Traded VTR

| | Panel A: Tail Risk Deciles | | Panel B: Tail Risk ME Portfolios | | Panel C: Characteristic ME Portfolios | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | ITR Dec. | VTR Dec. | 25 ITR | 25 VTR | 25 OP | 25 INV | 25 MOM | 25 IVOL | ALL |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Intercept | 1.28 | 1.81 | 0.58 | 0.25 | 1.20 | 1.55 | 0.64 | 0.97 | 0.79 |
| NW t-stat | (1.50) | (2.10) | (1.73) | (0.73) | (2.94) | (3.52) | (1.64) | (3.38) | (2.71) |
| Shanken t-stat | [1.25] | [1.84] | [1.24] | [0.57] | [2.85] | [3.62] | [1.44] | [2.85] | [2.26] |
| $R^M - R^f$ | -0.87 | -1.38 | -0.12 | 0.24 | -0.68 | -1.05 | -0.08 | -0.49 | -0.28 |
| NW t-stat | (-0.96) | (-1.42) | (-0.25) | (0.54) | (-1.14) | (-1.75) | (-0.14) | (-0.97) | (-0.56) |
| Shanken t-stat | [-0.80] | [-1.30] | [-0.21] | [0.45] | [-1.29] | [-1.99] | [-0.15] | [-1.06] | [-0.59] |
| Traded VTR | 0.89 | 0.69 | 0.89 | 0.62 | 0.86 | 1.00 | 0.70 | 0.96 | 0.79 |
| NW t-stat | (2.56) | (3.82) | (3.30) | (2.44) | (2.68) | (3.27) | (2.55) | (3.60) | (3.17) |
| Shanken t-stat | [2.31] | [3.55] | [2.79] | [2.30] | [2.60] | [3.18] | [2.32] | [3.34] | [2.95] |
| % $R^2$ | 76.53 | 93.05 | 84.71 | 79.85 | 64.48 | 89.33 | 71.19 | 80.91 | 67.85 |
| % MAE | 0.07 | 0.04 | 0.07 | 0.07 | 0.11 | 0.08 | 0.11 | 0.09 | 0.11 |
| Months | 192 | 192 | 192 | 192 | 192 | 192 | 192 | 192 | 192 |

This table presents asset pricing tests on double-sorted portfolios using the traded VTR and market two-factor model from 2001 to 2016. In Panel A, the test assets are the decile portfolios sorted on idiosyncratic tail risk or volume tail risk examined in Table 7. In Panel B, the test assets are 25 portfolios conditionally sorted on size and idiosyncratic tail risk or volume tail risk. Stocks are first grouped into size quintiles, then within each size quintile, stocks are grouped by their ITR or VTR. In Panel C, test assets are 25 portfolios independently sorted by size and the characteristic. Stocks are grouped by the intersection of 5 quintiles sorted on size and 5 quintiles sorted on the characteristic. These anomaly portfolios are downloaded from from Kenneth French's website and include operating profitability, investment, momentum, reversal, and idiosyncratic volatility. The table reports the risk premia estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

Table 27: Tests on HKM Portfolios using Traded VTR

| | FF | | Bond | | Sov | | Options | | CDS | | Conmod | | FX | | All | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) | (16) |
| Intercept | 0.63 | 1.25 | 0.26 | 0.28 | 0.84 | 0.34 | 0.90 | 0.78 | 0.02 | -0.11 | 0.24 | 0.10 | 0.22 | -0.34 | 0.34 | 0.14 |
| NW t-stat | 1.12 | 1.92 | 4.16 | 4.91 | 2.46 | 1.76 | 1.07 | 0.94 | 0.41 | -4.99 | 0.59 | 0.34 | 0.95 | -1.29 | 1.34 | 1.36 |
| Shanken t-stat | 1.27 | 1.83 | 2.72 | 5.20 | 1.94 | 1.54 | 1.01 | 0.74 | 0.27 | -4.79 | 0.62 | 0.30 | 0.73 | -0.78 | 1.59 | 1.37 |
| $R^M - R^f$ | | -1.01 | | 1.17 | | 2.04 | | -0.89 | | 2.51 | | -0.07 | | 0.98 | | -0.08 |
| NW t-stat | | -1.20 | | 0.99 | | 2.07 | | -0.86 | | 3.45 | | -0.08 | | 0.72 | | -0.17 |
| Shanken t-stat | | -1.30 | | 1.16 | | 1.85 | | -0.76 | | 3.57 | | -0.09 | | 0.49 | | -0.18 |
| CITR | 1.02 | 1.27 | 3.75 | -0.17 | -3.21 | -1.57 | 2.70 | 2.59 | 5.10 | -0.77 | 0.94 | 0.95 | 4.51 | 4.26 | 1.12 | 1.21 |
| NW t-stat | 2.49 | 2.51 | 1.22 | -0.10 | -2.06 | -1.29 | 2.00 | 2.18 | 2.57 | -0.99 | 1.63 | 1.62 | 3.64 | 3.35 | 3.05 | 3.25 |
| Shanken t-stat | 2.28 | 2.65 | 1.13 | -0.12 | -1.37 | -1.11 | 2.15 | 1.99 | 1.48 | -0.78 | 1.34 | 1.34 | 2.82 | 2.53 | 2.64 | 2.92 |
| % Adj. $R^2$ | 58.06 | 63.23 | 34.96 | 58.98 | 30.18 | 75.30 | 97.19 | 97.19 | 14.91 | 66.95 | 19.70 | 20.96 | 14.91 | 33.22 | 28.93 | 43.37 |
| % MAE | 0.12 | 0.12 | 0.11 | 0.10 | 0.26 | 0.14 | 0.07 | 0.07 | 0.09 | 0.07 | 0.50 | 0.49 | 0.26 | 0.23 | 0.31 | 0.24 |
| Months | 144 | 144 | 132 | 132 | 124 | 124 | 133 | 133 | 143 | 143 | 144 | 144 | 109 | 109 | 144 | 144 |

Test assets are the portfolios in He, Kelly, and Manela (2017) downloaded from Asaf Manela's website. Assets include equities, US government and corporate bonds, sovereign bonds, options, credit default swaps, commodities, and foreign exchange. The Fama MacBeth analysis is from January 2001 to December 2012. The model uses the market portfolio and traded CITR as the factors. Traded ITR is the value-weighted return on a portfolio that goes long the highest ITR decile and shorts the lowest ITR decile. The table reports the risk premia estimates, $R^2$, and mean absolute pricing errors in percentage terms, Newey-West t-statistics with one lag, and Shanken t-statistics.

## D.4 Traded Volume Tail Risk Pricing