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Lectures on Image Processing

Chapter 5
Texture Analysis, Classification

5. Statistical Analysis, Textures

5. Statistical Analysis, Textures, Definitions

- What are **textures** ?
- Texture **recognition**
  Automation of visual inspection of surfaces,
  recognition of surface faults
- Texture **classification**
  To which different classes of textures belongs an individual texture ?

5. Statistical Analysis, Textures, Definitions

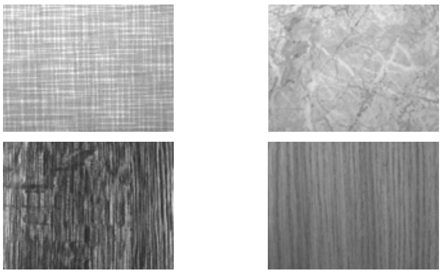
Exact periodical structure

Periodical structure with perturbation

Apparent periodical structure

Aperiodic structure

5. Statistical Analysis, Textures, Examples from Folder



5. Statistical Analysis, Measures from Grey Value Histogram

- Analysis of distribution of grey levels (histograms)
- Grey levels are very dependent of incident light,
- Histograms are a function (probability p(g) in dependence of grey level g)
- Consider
  - Images with 512 * 512 pixels
  - Grey level resolution 8 Bit
- We have : $0 \le p(g) \le 1$; $0 \le g \le 255 = N$; $\Sigma\, p(g) = 1$ für g = 0 .. N

5. Statistical Analysis, Measures from Grey Value Histogram

- Average grey level value M1 (expected value)

$$M1 = \sum_{g=0}^{g=255} g * p(g)$$

- Calculated from histogram
- M1 defines the average luminance in the image
- M1 is strongly dependent on lighting
- Using p(g) and M1 calculate the following values M2 to M6. Accept the calculation rules or try to understand their meanings.

5. Statistical Analysis, Measures from Grey Value Histogram

- Average squared deviation M2 (variance)

$$M2 = \sum_{g=0}^{g=255} (M1 - g)^2 * p(g) \ge 0$$

- M2 describes the average deviation of all individual grey values from M1
- An area, containing the same amount of only black and white pixels, results in M1 = 127,5 and M2 = 127,5² = 16.256,25
- Pixels of an area containing only pixels with (assumed) grey values g = 127,5 ,result in M1 = 127,5 and M2 = 0

5. Statistical Analysis, Measures from Grey Value Histogram

Example for M2 :

Look at 6 pixels	M1	M2
90, 90, 90, 110, 110, 110	100	100
90, 100, 100, 100, 100, 110	100	33

5. Statistical Analysis, Measures from Grey Value Histogram

- Asymmetry or Skewness M3

M3 = Σ (M1 – g)³ * p(g)

M3 describes the asymmetric behavior of grey values from the average value M1.

M3 = 0, if no asymmetry exists,
M3 ≠ 0, if an asymmetry exists.

5. Statistical Analysis, Measures from Grey Value Histogram

Example for M3 :

Pixels	M1	M2	M3
90, 90, 90, 110, 110, 110	100	100	0
90, 100, 100, 100, 100, 110	100	33	0
95, 95, 95, 95, 110, 110	100	50	-250
90, 90, 105, 105, 105, 105	100	50	+250

5. Statistical Analysis, Measures from Grey Value Histogram

- Excess M4 :

  $M4 = \Sigma (M1 – g)^4 * p(g) \geq 0$

  M4 is small, if many small deviations of g contribute to M2,

  M4 is large, if few large deviations of g contribute to M2.

5. Statistical Analysis, Measures from Grey Value Histogram

Example for M4 :

Pixels	M1	M2	M3	M4
70, 70, 70, 130, 130, 130	100	901	0	810.000
48, 100, 100, 100, 100, 152	100	900	0	2.437.250

5. Statistical Analysis, Measures from Grey Value Histogram

- Squared probability sum M5 :

  $M5 = \Sigma (p(g))^2 > 0$

  (a) Homogeneous area : $p(g_0\text{=const}) = 1$ $p(g \neq g_0) = 0$
  $M5 = \Sigma (p(g))^2 = 1$

  (b) All grey values are uniformly distributed : $p(g) = {}^{1}/_{256}$
  $M5 = \Sigma ({}^{1}/_{256})^2 = 256 * ({}^{1}/_{256})^2 = {}^{1}/_{256} \approx 0{,}004$

  Thus : $1 \geq M5 \geq 0{,}004$

5. Statistical Analysis, Measures from Grey Value Histogram

- Alternative to squared probability sum M5 is the entropy M6 :

  $M6 = - \Sigma\ p(g) * \log[\ p(g)\ ]$

  (a) Homogeneous area : $p(g_0\text{=const}) = 1$ $p(g \neq g_0) = 0$
  $M6 = - \Sigma\ p(g) * \log[\ p(g)\ ] = -1 * \log[\ 1\ ] = 0$

  (b) All grey values are uniformly distributed : $p(g) = {}^{1}/_{256}$
  $M6 = - \Sigma\ p(g) * \log[\ p(g)\ ] = - 256 * {}^{1}/_{256} * \log[{}^{1}/_{256}] \approx +2{,}4$

  Thus : $0 \leq M6 \leq \approx +2{,}4$

5. Statistical Analysis, Measures from Grey Value Histogram

Additional notice : So far we considered

- Pixel oriented operations
  (Binarization, Look-up-Tables, Histograms)

- Mask oriented operations (Low-pass / high-pass filtering, morphologic operations, skeletonizing)

New: *Region oriented operations* (texture analysis)

5. Statistical Analysis, Textures, Run Length Matrix

- Suppose we have a two-dimensional matrix with indices [length], [grey value].
- Calculate the amount of subsequent pixels with the same grey values (pixel chain or run length).
- Increment the respective matrix element for each single run length.

5. Statistical Analysis, Textures, Run Length Matrix

- Because of image noise it is unlikely to find two adjacent pixels with identical grey values. ⇒
- Calculate chains of pixels within a defined interval of grey values.
- Because of the large number of pixels within one line, the matrix must be very large. To reduce matrix size ⇒
- Calculate the run lengths of defined grey intervalls.
- Calculate the run lengths for defined intervals of run lengths.
- Define according size of matrix.

5. Statistical Analysis, Textures, Run Length Matrix

- An element of the matrix [length], [grey value] indicates, how often a pixel chain within a given run length interval j and within a given grey value interval i is contained within the respective image.
- Within a 8-neighborhood-metrics run lengths can be defined for the directions of 0°, 45°, 90° and 135°.
- Simple systems prefer calculation of run lengths within the image lines, i.e. 0°.

5. Statistical Analysis, Textures, Run Length Matrix

	x			
y	0	1	2	3
	0	2	3	3
	2	1	1	1
	3	0	3	0

Small image of size 4 * 4
2 bit grey level resolution

Mini-Bild 4*4
2 Bit Grauwertauflösung

RL01.dsf

5. Statistical Analysis, Textures, Run Length Matrix

	x			
y	0	1	2	3
	0	2	3	3
	2	1	1	1
	3	0	3	0

Mini-Bild 4*4
2 Bit Grauwertauflösung
RL01.dsf

Angle — Run Lenghts — Grey values

Winkel 0°		Lauflängen			
		j = 1	j = 2	j = 3	j = 4
Grauwert	i = 1	4	0	0	0
	i = 2	1	0	1	0
	i = 3	3	0	0	0
	i = 4	3	1	0	0

RL02.dsf

Winkel 90°		Lauflängen			
		j = 1	j = 2	j = 3	j = 4
Grauwert	i = 1	2	1	0	0
	i = 2	4	0	0	0
	i = 3	3	0	0	0
	i = 4	3	1	0	0

RL03.dsf

5. Statistical Analysis, Textures, Run Length Matrix

N_g	Total number of grey values or grey value intervals	$1 \le N_g \le 256$
N_r	Total number of run lengths or run length intervals	$1 \le N_r \le 768$ (512, 640)
i	Index for grey values or grey value interval	$1 \le i \le N_g$
j	Index for run lengths or run length intervals	$1 \le j \le N_r$
p(i,j)	Element of run length matrix	
P	Total sum of elements	$P = \sum_{i=1}^{N_g} \sum_{j=1}^{N_r} p(i,j)$
W	Window, total number of pixels concerned (ROI : region of interest)	$W = \Delta x * \Delta y$

5. Statistical Analysis, Textures, Run Length Matrix, Interpretation

Parameter RL1 : Short Runs Emphasis

$$RL1 = \frac{1}{P}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r}\frac{p(i,\ j)}{j^2}$$

Division of p(i,j) by j^2 suppresses contribution of elements with long run lengths.

Factor 1/P for normalization.

RL1 gets $\left\{\begin{matrix} \text{large, if mainly the left columns contain large elements} \\ \text{small, if mainly the right columns contain large elements} \end{matrix}\right\}$

5. Statistical Analysis, Textures, Run Length Matrix, Interpretation

Parameter RL2 : Long Runs Emphasis

$$RL2 = \frac{1}{P}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r} p(i,\ j) * j^2$$

Multiplication of p(i,j) by j^2 suppresses contribution of elements with small run lengths

Factor 1/P for normalization

RL2 gets $\left\{\begin{matrix} \text{large, if mainly the right columns contain large elements} \\ \text{small, if mainly the left columns contain large elements} \end{matrix}\right\}$

5. Statistical Analysis, Textures, Run Length Matrix, Interpretation

Parameter RL3 : Grey Level Non-uniformity

$$RL3 = \frac{1}{P}\sum_{i=1}^{N_g}\left(\sum_{j=1}^{N_r} p(i,\ j)\right)^2$$

Elements p(i,j) are added within the lines, independent of the run length j. The squared sum gets large, if the grey levels are non-uniformly distributed.

Factor 1/P for normalization.

RL3 gets $\left\{\begin{matrix} \text{large, if the run lengths are distributed in only a few grey value intervals.} \\ \text{small, if the run lengths are distributed uniformly within the grey value intervals.} \end{matrix}\right\}$

5. Statistical Analysis, Textures, Run Length Matrix, Interpretation

Parameter RL4 : Run Lengths Non-uniformity

$$RL4 = \frac{1}{P}\sum_{j=1}^{N_r}\left(\sum_{i=1}^{N_g} p(i,\ j)\right)^2$$

Elements p(i,j) are added within the columns independent of the grey levels. The squared sum gets large, if the run lengths are non-uniformly distributed.

Factor 1/P for normalization.

RL4 gets $\left\{\begin{matrix} \text{large, if the single run lengths are non - uniformly distribute d.} \\ \text{small, if all the run lengths are uniformly distribute d.} \end{matrix}\right\}$

5. Statistical Analysis, Textures, Run Length Matrix, Interpretation

Parameter RL5 : Run Lengths Percentage

$$RL5 = \frac{1}{W}\sum_{j=1}^{N_r}\sum_{i=1}^{N_g} p(i,\ j) = \frac{P}{W}$$

All matrix elements are added and the sum divided by the pixels considered.

RL5=1, if the ROI contains only run lengths of 1 (P = W).

(I.e. in the ROI exists high frequent noise)

RL5 = $^1/_m$, if the ROI (n lines / m columns) is a homogeneous area.

(I.e. the maximal run length m appears in every of the n lines.

5. Statistical Analysis, Textures, Run Length Matrix, Interpretation

Parameter RL6 : *small grey level emphasis* and RL7 : *large grey level emphasis* : Illumination dependent parameters.

$$RL6 = \frac{1}{P}\sum_{j=1}^{N_r}\sum_{i=1}^{N_g} p(i,\ j) * i^2 \qquad RL7 = \frac{1}{P}\sum_{j=1}^{N_r}\sum_{i=1}^{N_g}\frac{p(i,\ j)}{i^2}$$

Parameters RL1 to RL5 are independent of illumination, because the index i doesn't appear within the formulas.

Parameters RL6 and RL7 are applicable only with constant (arbitrary) illumination of objects.

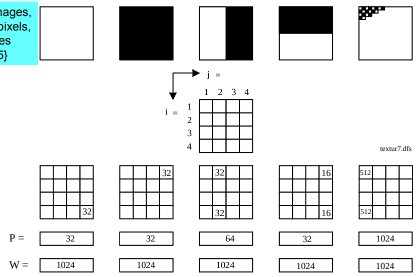
RL6 gets large for bright images and small for dark images.

RL7 gets small for bright images and large for dark images

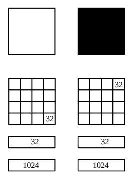
5. Statistical Analysis, Textures, Run Length Matrix, Examples

Testbilder 32 x 32 Pixel, Grauwerte G aus (0, 255)

5 synthetic images, size 32 x 32 pixels, grey values G:{0, 255}



5. Statistical Analysis, Textures, Run Length Matrix, Examples



$$RL1 = \frac{1}{P}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r}\frac{p(i,j)}{j^2}$$

$$RL2 = \frac{1}{P}\sum_{i=1}^{N_g}\sum_{j=1}^{N_r}p(i,j) * j^2$$

$$RL3 = \frac{1}{P}\sum_{i=1}^{N_g}\left(\sum_{j=1}^{N_r}p(i,j)\right)^2$$

$$RL4 = \frac{1}{P}\sum_{j=1}^{N_r}\left(\sum_{i=1}^{N_g}p(i,j)\right)^2$$

$$RL5 = \frac{1}{W}\sum_{j=1}^{N_r}\sum_{i=1}^{N_g}p(i,j) = \frac{P}{W}$$

Table left : 2 homogeneous areas a and b with different grey values

	Pattern Group 1 : Both patterns are homogeneous areas differing in grey values	
	Pattern a	Pattern b
RL1	$^1/_{16}$	$^1/_{16}$
RL2	16	16
RL3	32	32
RL4	32	32
RL5	$^1/_{32}$	$^1/_{32}$
RL6	16	1
RL7	$^1/_{16}$	1

5. Statistical Analysis, Textures, Run Length Matrix, Examples

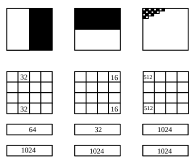


Table left : Pattern c, d and e with different grey value patterns

	Pattern Group 2 : These three patterns consist of the same number of pixels in different configurations		
	Pattern c	Pattern d	Pattern e
RL1	$^4/_{16}$	$^1/_{16}$	1
RL2	4	16	1
RL3	32	16	512
RL4	64	32	1024
RL5	$^1/_{16}$	$^1/_{32}$	1
RL6	$^{17}/_2$	$^{17}/_2$	$^{17}/_2$
RL7	$^{17}/_{32}$	$^{17}/_{32}$	$^{17}/_{32}$

5. Statistical Analysis, Textures, Run Length Matrix, Examples

- Examples a and b (homogeneous areas) : Values of RL1 bis RL5 are independent of illumination and therefore for both patterns equal. Values of RL6 und RL7 are dependent of lighting and show opposite values.
- Examples c, d and e (average grey levels are equal) : Values of RL6 and RL7 are equal.
- RL1 (Short Runs Emphasis) : smallest for a, b, und d, largest for e.
- RL2 (Long Runs Emphasis) : largest for a, b und d, smallest for e.
- RL3 (Grey Level Non-uniformity) : largest for e.
- RL4 (Run Lengths Non-uniformity) : largest for e.
- RL5 (Run Lengths Percentage) : $^1/_m$ for a, b und d; 1 for e

5. Statistical Analysis, Textures, Classification

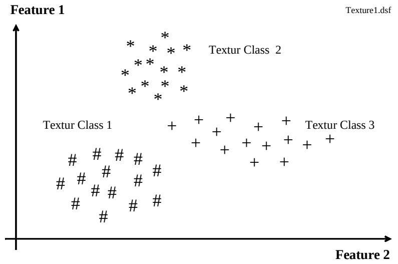
- Classification : Assignment of textures to one of N different known texture classes
- N+1 classes required (N known classes + 1 class for undefined or unknown textures)
- N = 1 : Texture belongs to a specified class, e.g. product quality control, surface inspection for correctness, fault recognition. (production is correct or not correct)
- N ≥ 2 : Classifying into different classes (texture classification)

5. Statistical Analysis, Textures, Classification

Class and classifier are defined by numerical values.

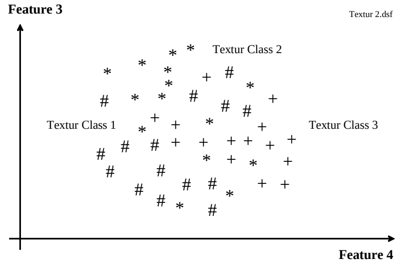
- Texture is defined by a limited feature set (set of size n) of suitable parameters (e.g. M2 to M5 and RL1 to RL5) or subset thereof.
- The number n of parameters is considered to be a n-component vector within a n-dimensional vector space.
- Each vector defines a point in the space.
- Each individual texture corresponds to exactly one point.

5. Statistical Analysis, Textures, Classification



Two-dimensional (n = 2) example using a suitable feature set :
each of the 3 different textures form an isolated point cluster.

5. Statistical Analysis, Textures, Classification



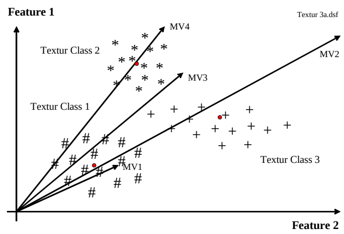
Two-dimensional example using a not suitable feature set :
3 different textures form only one common point cluster.

5. Statistical Analysis, Textures, Classification

Numerical Classifier

- Classifiers assign a new individual vector to one of the N defined texture classes or to the remaining class N+1.
- Fundamental idea : In vector space close-by lying vectors belong to the same texture.

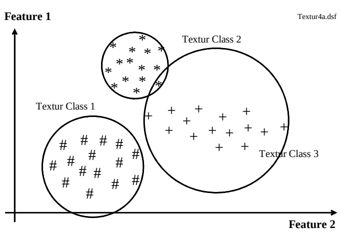
5. Statistical Analysis, Textures, Classification



Texture clusters are represented by their centers of mass.

- Vector MV1 has shortest distance to cluster 1.
- Vector MV2 has shortest distance to cluster 3, correct classification ?
- Vectors MV3 and MV4 have shortest distance to cluster 2, correct classification ?

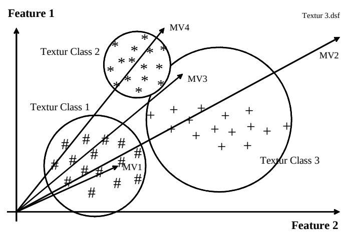
5. Statistical Analysis, Textures, Sphere Classifier



Define clusters by circumscribed circles resp. N-dimensional spheres.

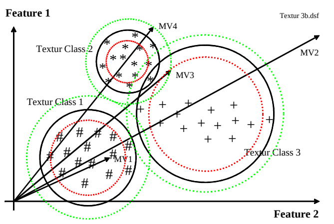
- The spheres may result in not desired overlapping (ambiguity)
- To avoid this, the feature set may be changed

5. Statistical Analysis, Textures, Sphere Classifier



- Vector MV1 still is classified to cluster 1
- Vector MV2 is classified to cluster N+1 (unknown texture)
- Vector MV3 changed from cluster 2 into cluster 3
- Vector MV4 changed into cluster N+1 (unknown texture)

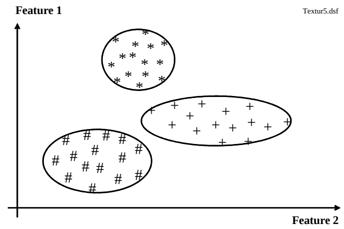
5. Statistical Analysis, Textures, security margin



Security margin ± ΔR
ΔR positive (green) :
Use, if the clusters are separate.
Overlapping increases.
Error rate (faulty objects declared as correct) increases.

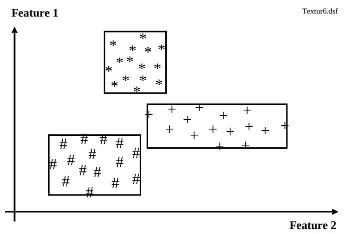
- ΔR negative (red) : Prevents overlapping. Outliers of correct objects might be declared faulty. Error rate (correct objects declared as faulty) increases.
- ΔR might be defined as absolute or as relative size.

5. Statistical Analysis, Textures, Ellipsoid Classifier



- Texture clusters are unlikely spheres
- Define clusters by circumscribed ellipses resp. N-dimensional ellipsoids
- Better prevention of overlapping (uniqueness)
- Time consuming algorithms

5. Statistical Analysis, Textures, Box Classifier



- Compromise : Define clusters by circumscribed boxes resp. N-dim. boxes
- Classifying vectors components by operators "≥" resp. "≤".
- Only small probability of overlapping
- Fast algorithms

5. Statistical Analysis, Textures, Dynamic Cluster Definition

- Dynamic cluster definition :
- Use small number of samples during teach-in. Use vectors during work phase as new additional samples. ⇒ floating center of cluster.
- Gain confidence in classification :
  - Vector has to lie with a defined security margin within the box.
  - Vector has to lie within smaller box containing 90 % of samples.
    ⇒ floating rejection
- Dynamic cluster definition takes a risk of continuous shifting the cluster boxes.

Summary of Chapter 5

- Definition, recognition and classification of textures
- Generation of grey level characteristics out of histograms
- Definition and generation of run length matrices
- Generation of run lengths characteristics out of the matrix
- Numerical classification
- Numerical classifiers
- Selection of qualified characteristics for classification