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Lectures on Image Processing

Chapter 4
Morphology

4. Morphology

4.1 Morphology, Definitions

- Let us consider **binary images**, G ∈ {0, 1} resp. G ∈ {0, 255}
- Produced from grey level images
  - by using binarization Look-up-Tables with binarization threshold at g(x, y) = T
  - by edge detection filters
- Definition : g = 255 : pixel is part of the object / edge
  g = 0 : pixel is not part of the object / edge

  i.e. white objects on black background or white edges on black background
- Use the following notation : $H = h(u,v) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

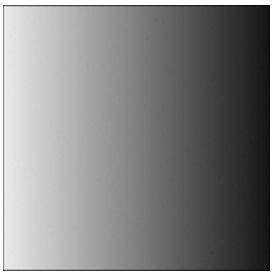
4.2 Morphology, Dilation or Expansion

- Dilation or Expansion :
- Set g'(x,y) = g'(e) = g(a) v g(b) v …. v g(i); v : OR
- Effect : $$g'(x,y) = \begin{cases} 255, \text{if } g(x,y) = 255 \\ 255, \text{if within H is at least one element } g(k) = 255,\ k = a, ..., i \\ 0, \text{otherwise} \end{cases}$$

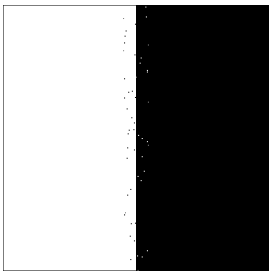
  i.e. the white object is extended at its edge by one pixel,
  black holes within the object smaller than mask H disappear

4.2 Morphology, Dilation or Expansion

Test image: Linear grey slope (g_{min}=20 / g_{max}=235) with superimposed noise

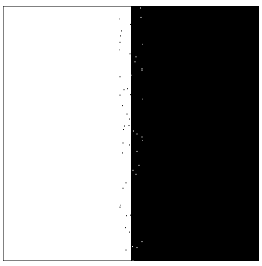


4.2 Morphology, Dilation or Expansion

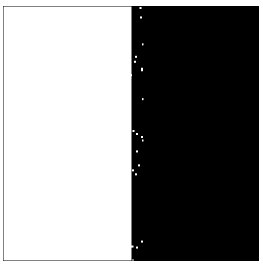


4.2 Morphology, Dilation or Expansion

Binary test image with threshold th=128



Dilation applied on binary test image



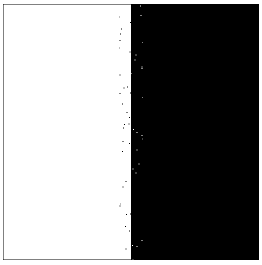
4.3 Morphology, Erosion or Contraction

- Erosion or Contraction
- Set g'(x, y) = g'(e) = g(a) ∧ g(b) ∧ …. ∧ g(i); ∧ : AND
- Effect : $$g'(x,y)=\begin{cases}0, \text{if } g(x,y)=0\\ 0, \text{if within H is at least one element } g(k)=0,\ k=a,...,i\\ 255, \text{otherwise}\end{cases}$$

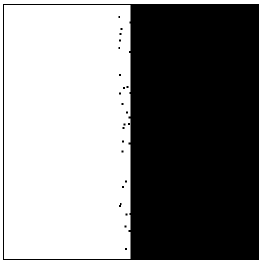
i.e. the white object is eroded at its edge by one pixel, white holes in the background smaller than H disappear

4.3 Morphology, Erosion or Contraction

Binary test image with threshold th=128



Erosion applied on binary test image



4.4 Morphology, Edges

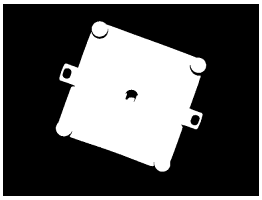
- Generation of edges (high-pass filtering of binary images)
- The operation $$\begin{Bmatrix}\text{expanded image F' XOR source image F}\\ \text{source image F XOR eroded image F'}\end{Bmatrix}$$

results in an edge image of a binary object. Edges are one pixel wide.

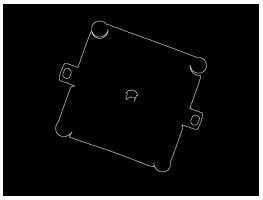
- What is the difference between the two operations ?

4.4 Morphology, Edges

Original binary image

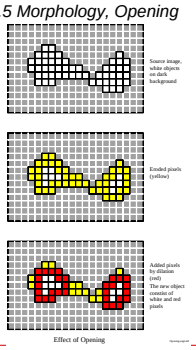


XOR-ed image



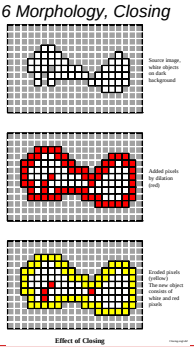
Question: dilated XOR original or eroded XOR original ?

4.5 Morphology, Opening



- The operations of a dilation and of an erosion are not inverse operations
- Subsequent application of erosion and dilation → Opening
- Isolating adjacent areas

4.6 Morphology, Closing



- Subsequent application of dilation and erosion → Closing
- Unifying of closely positioned areas
- Closing of small holes

4.6 Morphology, Closing

Original grey level Image (Diskette01.bmp)

Binary image, threshold = 64

Twofold dilated image by 3x3 mask

Twofold eroded image, original size

4.7 Grey Level Dilation and Erosion

Define any structuring element, e.g. $H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Define as grey level dilation $(I \oplus H)(x,y) = \max_{(i,j)\in H}\{I(x+i, y+j) + H(i,j)\}$

Example:

$$\begin{bmatrix} 105 & 90 & 100 \\ 100 & 90 & 100 \\ 103 & 90 & 100 \end{bmatrix} \oplus \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 105 & 91 & 100 \\ 101 & 92 & 101 \\ 103 & 91 & 100 \end{bmatrix} \Rightarrow (I \oplus H)(x,y) = 105$$

4.7 Grey Level Dilation and Erosion

Example: Grey level dilation

4.7 Grey Level Dilation and Erosion

3rd Iteration of Dilation

4.7 Grey Level Dilation and Erosion

Define any structuring element, e.g. $H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

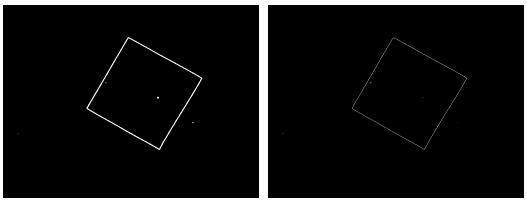
Define as grey level erosion $(I \ominus H)(x,y) = \min_{(i,j)\in H} \{I(x+i, y+j) - H(i,j)\}$

4.7 Grey Level Dilation and Erosion

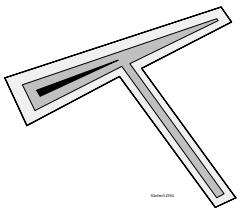
3rd Iteration of Erosion

4.8 Morphology, Skeletonizing, Thinning

General Problem: Creating One Pixel Wide Edges



4.8 Morphology, Skeletonizing, Thinning



- Skeletonizing or Thinning
- Eroding a connected pixel area I does not produce a skeleton of the area
- The discrete skeleton S(I) of a connected pixel area I is not unambiguously defined.

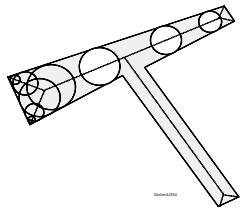
4.8 Morphology, Skeletonizing, Thinning

The following boundary conditions are met :

- The skeleton S(I) is composed by lines of one pixel width.
- Connected areas I are described by a connected skeleton S(I).
- The skeleton S(I) is positioned in the middle of the area I
- Endpoints of a skeleton may not be deleted.
- Additional connections may not be created.

Complexity of problem needs time consuming algorithms.
Alternatively, simple and empirical solutions are requested.

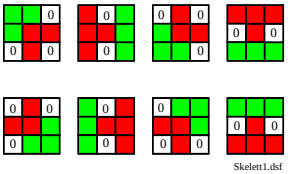
4.8 Morphology, Skeletonizing, Thinning



- Skeleton
- An attempt of a potential definition :

  Connection of the centers of all inscribed circles

4.8 Morphology, Skeletonizing, Thinning



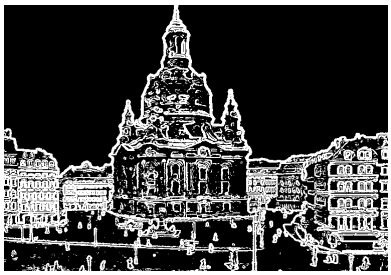
Copy source image into target image.
Target pixel is registered for deleting, if precisely all red elements are part and all green elements of a mask are not part of the object.
Zero-elements are ignored.
Deletion is done after processing the whole image with **one** of the masks.
Algorithm terminates, if $S_{i+1}(I) = S_i(I)$ for **all** masks.

4.8 Morphology, Skeletonizing, Thinning

Original image, historical Frauenkirche of Dresden

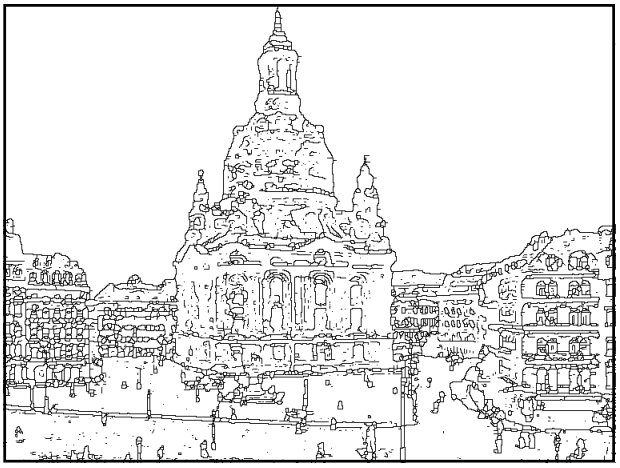
4.8 Morphology, Skeletonizing, Thinning

Sobel filtering using low threshold



4.8 Morphology, Skeletonizing, Thinning

Skeletonizing



4.9 Morphology, Skeletonizing, Method of Zhang-Suen

- Object pixel white (g = 255)
- Background pixel black (g = 0)
- Source pixel P_1.
- Apply 3 x 3 mask, i.e. all pixels have 8 neighbors.
- Define as $A(P_1)$ = number of all transitions 255 → 0 in the sequence $P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_2$. (Note P_2 also at the end)
- Define as $B(P_1)$ as the sum of all pixels with g = 0 in neighboring pixels $P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9$

P_5	P_4	P_3
P_6	P_1	P_2
P_7	P_8	P_9

Zhang-Suen-Mask.dsf

4.9 Morphology, Skeletonizing, Method of Zhang-Suen

Step 1:

Scan source image from upper left to lower right and test the following conditions:

1. $P_1 = 255$
2. $2 \le B(P_1) \le 6$ (sum of black pixels)
3. $A(P_1) = 1$ (transitions)
4. At least for one of the pixels $P_4, P_2, P_8 : g = 0$
5. At least for one of the pixels $P_2, P_8, P_6 : g = 0$

Note all pixels for which all the conditions are met.
After the scan set the noted pixels to 0.

Zhang-Suen-Mask.dsf

4.9 Morphology, Skeletonizing, Method of Zhang-Suen

Step 2:

Scan source image from upper left to lower right and test the following conditions:

1. $P_1 = 255$
2. $2 \le B(P_1) \le 6$ (sum of black pixels)
3. $A(P_1) = 1$ (transitions)
4. At least for one of the pixels $P_4, P_2, P_8 : g = 0$
5. At least for one of the pixels $P_4, P_8, P_6 : g = 0$

Note all pixels for which all the conditions are met.
After the scan set the noted pixels to 0.

Zhang-Suen-Mask.dsf

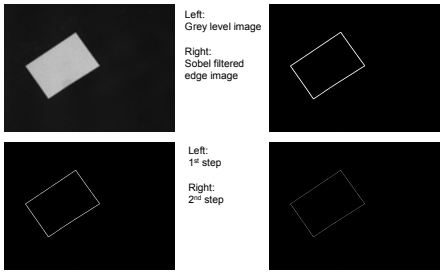
4.9 Morphology, Skeletonizing, Method of Zhang-Suen

Step 3 and further:

Iterate steps 1 and 2, as long as any pixel during step 1 and 2 was changed from 255 to 0.

Terminate the process, if during step 1 and 2 no more pixel is changed.

4.9 Morphology, Skeletonizing, Method of Zhang-Suen



Left:
Grey level image

Right:
Sobel filtered edge image

Left:
1st step

Right:
2nd step

Summary of Chapter 4

- Information content of binary images
- Removal of white pixels in black areas
- Removal of black pixels in white areas
- How to connect distinct binary areas
- How to separate connected binary areas
- How to produce edges in binary images
- Skeletons of binary objects
  - by empirical masks
  - by Method of Zhang-Suen
- How to dilate or erode grey level images