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ISIP 2019
International School on Image Processing
July 15th - 26th, 2019

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University of Applied Sciences
Department of
Design, Computer Science, Media
Study Courses AI und MI
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Lectures in Image Processing

Chapter 2
Image Analysis Tools

Chapter 2

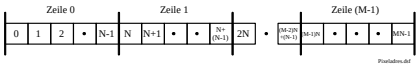
2. Image Analysis Tools

All these operations are (more or less) pixel operations

2.0 Image Analysis Tools, Addressing Scheme

$$\text{Byte-Matrix}_{(N,M)}(x,y) = \begin{pmatrix} g(0,0) & g(1,0) & \bullet & g(N-1,0) \\ g(0,1) & g(1,1) & \bullet & g(N-1,1) \\ \bullet & \bullet & \bullet & \bullet \\ g(0,M-1) & g(1,M-1) & \bullet & g(N-1,M-1) \end{pmatrix}$$

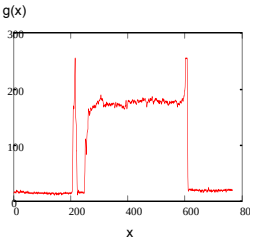
Byte-Array [i]



Calculation of index i from coordinates (x, y) :
i = y * N + x

2.1 Image Analysis Tools, Grey Level Profile

Image Size 576 x 768



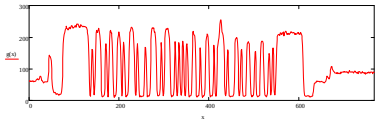
Example of grey level profile

2.1 Image Analysis Tools, Grey Level Profile, Applications

EAN-Code in arbitrary position and orientation

Standard directions for analysis

2.1 Image Analysis Tools, Grey Level Profile, Applications

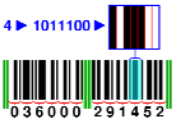


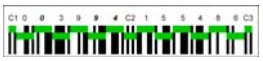
Known are
- the number of dark and light lines within EAN-Code,
- the ratios of line thicknesses.

Analysis
- In which direction is a complete EAN-Code represented?
- Numerical output of the code (and country of origin).

2.1 Image Analysis Tools, Grey Level Profile, Applications

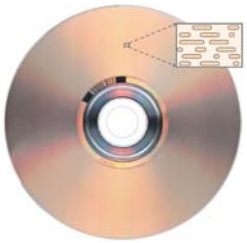
- EAN-13-Code: 12 numbers + 1 automatically generated checksum number.
- Each number consists of 2 black and 2 white bars.
- Each number is coded with width of 7 units.
- Additional start bars (010), center bars (01010), end bars (010).
- Sum of widths: 12*7 + 2*3 + 1*5 units = 95 units.
- Left side: L-code, right side: R-code and G-code.
- Depending on image resolution, no solution in binary image possible.



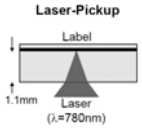


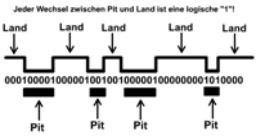
[Source of graphics: Wikipedia]

2.1 Image Analysis Tools, Grey Level Profile & Scaling, Applications



Track pitch (Spurabstand) 1500 nm
Track width (Spurbreite) 600 nm
Pit size (Vertiefungen) 833 – 3050 nm
CLV (constant linear velocity) 1.2 m/s
Diameter 12 cm
Eight-to-fourteen-modulation EFM (8-14 Kodierung)



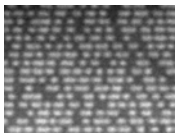


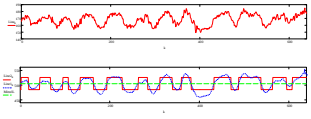
Eight-to-fourteen-modulation EFM (8-14 Kodierung)

14 bits are transferred by look-up-table to 8 bit

Land / Pit or Pit / Land change: logical 1

2.1 Image Analysis Tools, Grey Level Profile & Scaling, Applications

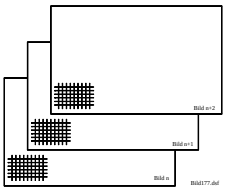




2.1 Image Analysis Tools, Grey Level Profile & Scaling, Applications

- Adjust the tracks into horizontal direction
- Search center of track width
- Read grey level profile
- Scale to maximum contrast
- Search for Land/Pit or Pit/Land changes
- Correct Land/Pit or Pit/Land changes with respect to EFM
- Reconstruct any faults on CD

2.2 Image Analysis Tools, Noise



Taking an image of a homogeneous (without any structure) surface with homogeneous illumination :

Noise of quantization

Ideal grey value : $g = g_o \pm 1$

Grey levels inhere errors in measurements.

Mean grey level : $g_m = \frac{1}{N}\sum_{i=1}^{i=N} g_i$

Heterologue pixels : pixels with similar coordinates in the same frame

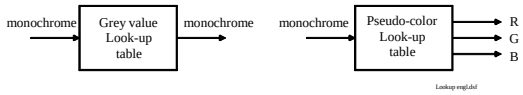
Homologue pixels : pixels with same coordinates in different frames.

2.2 Image Analysis Tools, Noise

Variance σ^2 of grey levels :

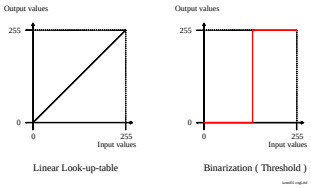
- Mean value of discrete random variable : $g_m = \frac{1}{N}\sum_{i=1}^{i=N} g_i$
- Variance : $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(g(i)-g_m)^2$
- Index i ($1 \le i \le N$) denotes the whole domain of grey levels or a defined subset.

2.3 Image Analysis Tools, Look-up-Tables



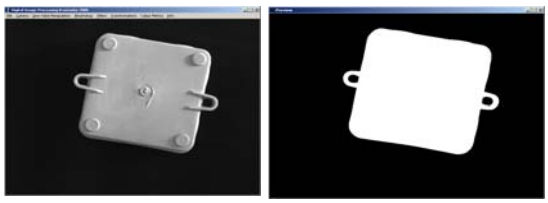
- 1 input / 1 output : grey level Look-up-table
- 1 input / 3 outputs : pseudo color Look-up-table
- 7 inputs / 3 outputs : satellite images
- Typically an array with 256 elements
- Type of data : byte (input data / output data)
- Content of array : „grey level transformation function“

2.3 Image Analysis Tools, Look-up-Tables



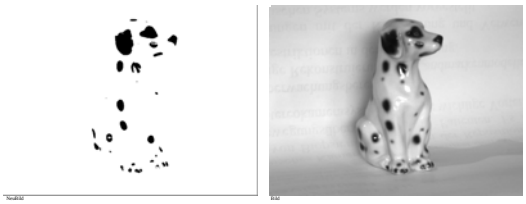
- Linear function : output = input
- Binarization $g_{new} \in \{ 0, 1 \}$ resp. $\{ 0, 255 \}$
- Definition of binarization threshold T $\quad g_{new} = \begin{cases} 0, \text{if } 0 \le g_{old} < T \\ 255, \text{if } T \le g_{old} \le 255 \end{cases}$

2.3 Image Analysis Tools, Look-up-Tables

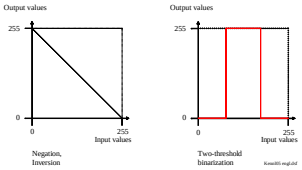


Example of useful binarization

2.3 Image Analysis Tools, Look-up-Tables



2.3 Image Analysis Tools, Look-up-Tables



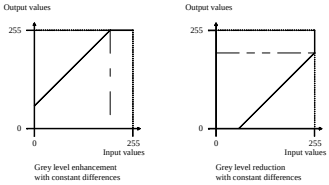
- Negation or Inversion (Preparation for paperprints)
- Two-threshold binarization
- Use this tool to find similar grey values within range of $T_1 \leq g \leq T_2$

2.3 Image Analysis Tools, Look-up-Tables

Inversion

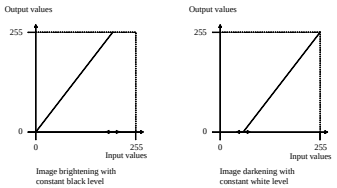
2-Binarisation (thres.:165/225)

2.3 Image Analysis Tools, Look-up-Tables



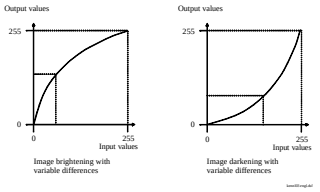
- Image brightening with linear function, constant grey level differences, loss of information by not used and by multiple used grey levels
- Image darkening with linear function
- Definition of threshold necessary

2.3 Image Analysis Tools, Look-up-Tables



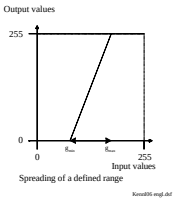
- Brightening and darkening with constant black level / white level.
- Definition of threshold necessary (Used for restoration of neg. and pos. movie images)
- Constant but modified grey level differences.

2.3 Image Analysis Tools, Look-up-Tables



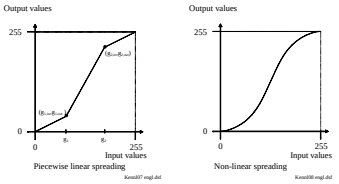
- Brightening with variable grey level distances
- Darkening with variable grey level distances
- All output grey levels may be occupied
- No interaction (no thresholds) necessary

2.3 Image Analysis Tools, Look-up-Tables



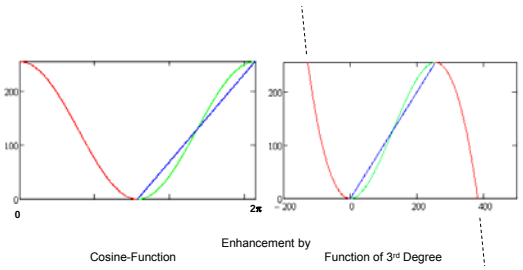
- Linear spreading of input grey range onto the whole output range

2.3 Image Analysis Tools, Look-up-Tables

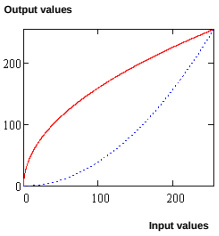


- Analytic spreading of grey levels to the whole output dynamic range by defined mathematical functions
  (piecewise linear and nonlinear)

2.3 Image Analysis Tools, Enhancement Functions



2.3 Image Analysis Tools, Look-up-Tables



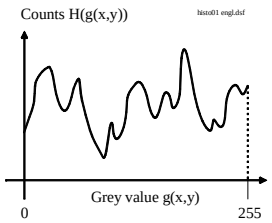
Gamma-Look-up-table

$$g' = c * g^{\gamma}$$

According to the chosen value of Gamma brightening and darkening of images are possible.

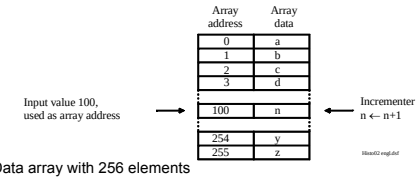
The Gamma-Look-up-table allows inverse operations (except rounding effects).

2.4 Image Analysis Tools, Grey Value Distribution, Histogram



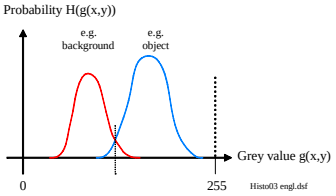
- Measurement of grey value distribution H(g(x,y))
- Realization by HW or SW

2.4 Image Analysis Tools, Grey Value Distribution, Histogram



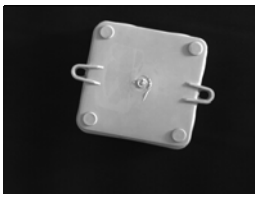
- Data array with 256 elements
- Data types Bytes / Integers
- Unimodal / bimodal / multimodal distributions (Histograms)
- Necessary for checking the illumination of objects and the lens aperture
- Decision for choosing thresholds for binarization
- Definition of parameter values for spreading grey levels to achieve $g_{min} \rightarrow 0$ and $g_{max} \rightarrow 255$

2.4 Image Analysis Tools, Grey Value Distribution, Histogram

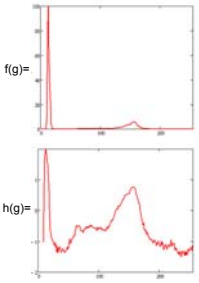


- Bimodal histogram
- Problems : possibly occurring overlapping (ideal / non ideal distribution)
- Tri-modal probability (two-threshold binarization)

2.4 Image Analysis Tools, Grey Value Distribution, Histogram

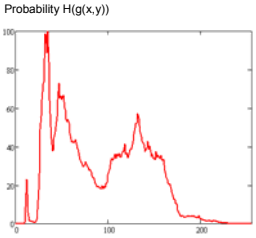


Example of bimodal histogram



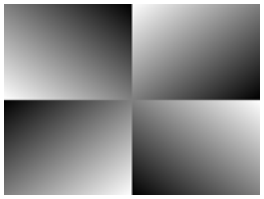
f(g): histogram, normalized linear scale (right, top)
h(g): histogram, logarithmic scale (right, bottom)

2.4 Image Analysis Tools, Grey Value Distribution, Histogram

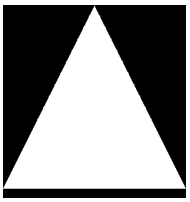


Example of multimodal histogam

2.4 Image Analysis Tools, Grey Value Distribution, Histogram

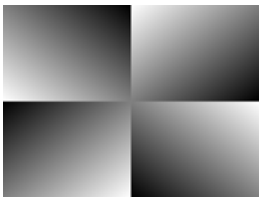


Synthetic Image

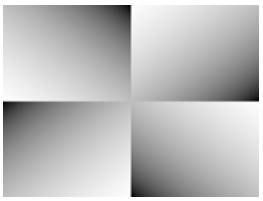


Histogram

2.4 Image Analysis Tools, Grey Value Distribution, Histogram

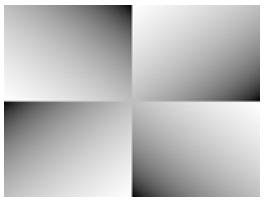


Synthetic image

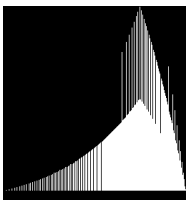


Increased grey levels

2.4 Image Analysis Tools, Grey Value Distribution, Histogram



Brightened Image



Histogram

2.4 Image Analysis Tools, Grey Value Distribution, Histogram

Options for automatic thresholds, derived from histogram:

Correct thresholds are related to the content of an image. Thus there is no general rule to find an optimal threshold T for the grey values g(i) within the grey value intervall I, e.g. $0 \le g(i) \le 255$

1. Median of the provided grey values: T = med{0, …, 255} = 127.5
2. Median of the occurring grey values: T = med{g(i)}
3. Mean of the occurring grey values: T = mean{g(i)} = μ_I
4. Mid-range value,
   min(I): minimum value of I with $g(i) \neq 0$
   max(I): maximum value of I with $g(i) \neq 255$ T = round{½(min(I)+max(I)}

2.4 Image Analysis Tools, Grey Value Distribution, Histogram

Options for thresholds, derived from histogram:

5. Quantile threshold, applied for bimodal histograms (e.g. newspapers or similar):

$$T = \min\left\{ j \;\middle|\; \sum_{i=0}^{j} h(i) \ge N * b \right\}$$

   h(i) : Histogram, *N: total number of pixels, 0<b<1 estimated fraction of white background pixels*
   For b = 0.5 ⇒ Median

6. Otsu's method: see later on this chapter
7. Iterative procedures

2.4 Image Analysis Tools, Quantiles (Option 5, b = 0.75)

Sie brachte der Postkutsche das Aus

JUBILÄUM Vor 175 Jahren fuhr die erste Taunusbahn Strecke Kastel – Wiesbaden zunächst im Pendelverkehr

Von Helmut Wirth

KASTEL. Sang- und klanglos ging ein Jubiläum über die Bühne der Zeitgeschichte, das eine ganze Region hätte feiern können und nicht nur die beteiligten Großstädte Wiesbaden und Frankfurt. 175 Jahre Taunusbahn. Sie löste die Postkutsche ab und brachte schon nach wenigen Betriebsjahren einen ungeahnten Aufschwung in das Land zwischen Untermain und Taunus. Als eine der fünf ersten Eisenbahnen auf deutschem Boden ist die Taunusbahn in die Geschichte eingegangen. Und Geschichten über sie gibt es jede Menge.

Angefangen mit dem Anti-Rauchabzug. Unbekümmert um die schwebenden Grundstücksverhandlungen gab das Taunusbahnkomitee den Startschuss für den Baubeginn an beiden Endpunkten der insgesamt rund 43 Kilometer langen Strecke, in Wiesbaden und in Frankfurt. Besonders zeitraubend war der Hickhack mit den Hochheimer Winzern. Wegen der Hochwassergefahr mussten die Gleise auf der sogenannten Hochterrasse verlegt werden. Hier durchschnitten sie die Weinberge. Und jeden Weinstock wollten die Winzer vergoldet haben, berichtete noch 20 Jahre danach der Reiseschriftsteller Karl Baedeker in einem seiner Reisehandbücher.

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2.4 Image Analysis Tools, Grey Value Distribution, Histogram

7. Options for iterative threshold, derived from histogram:

- Chose any arbitrary threshold according to one of the previous options (option 1, 2, 3, 4).
- This threshold separates the grey values into two classes.
- Calculate the mean values μ_0 and μ_1 of the two classes.
- Replace the threshold by the average of the means, i.e. $½(\mu_0 + \mu_1)$.
- Iterate the process until the threshold doesn't change.

2.5 Image Analysis Tools, Relative Cumulated Probability

Probability p(g) of occurrence of grey level g :

$$p(g) = \frac{\text{number n of pixels with grey value g}}{\text{total number N of all pixels}} = \frac{n(g)}{N} \qquad 0 \le g \le 255$$

Definition : Cumulated Probability P(g) of occurrence of grey values between 0 and g :

$$P(g) = \sum_{j=0}^{j=g} \frac{n(j)}{N} \qquad \text{with } 0 \le P(g) \le 1$$

Calculate a **Relative** Cumulated Probability S_{rH} :

$$S_{rH} = 255 * P(g) = 255 * \sum_{j=0}^{j=g} \frac{n(j)}{N}$$

2.5 Image Analysis Tools, Relative Cumulated Probability

S_{rH} is a monotonous increasing function with $0 \le S_{rH} \le 255$

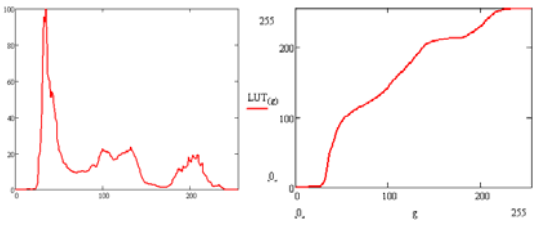
$S_{rH}(g+1) = S_{rH}(g) + 255 * p(g+1)$

Define a Look-up-table INT{ $S_{rH}(g)$} for g = 0 to g = 255

This look-up-table produces an equidistant distribution of gray values.

Input data type : Byte
Calculation data type : Floating point
Output data type : Byte

2.5 Image Analysis Tools, Relative Cumulated Probability

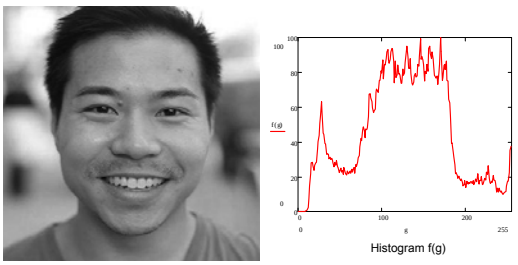


Histogram of Building02 (left), corresponding Look-up-table (right)

2.5 Image Analysis Tools, Relative Cumulated Probability

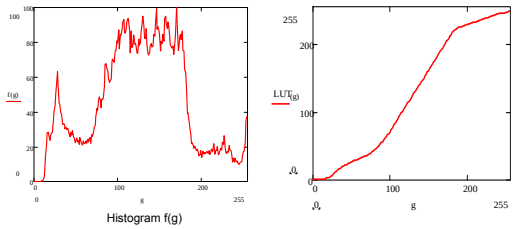
Application of cumulated probability Look-up-Table :
original image (left), manipulated image (right)

2.5 Image Analysis Tools, Relative Cumulated Probability



Histogram f(g)

2.5 Image Analysis Tools, Relative Cumulated Probability



Histogram f(g)

2.5 Image Analysis Tools, Relative Cumulated Probability

Original Processed

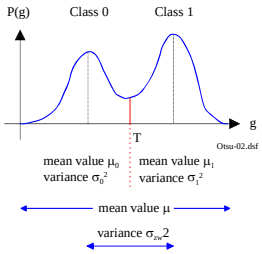
2.6 Image Analysis Tools, Numerical Pixel Magnifier

Example of numerical pixel magnifier

2.7 Image Analysis Tools, Otsu – Binarization (optional)

- Automatic binarization according procedure of N. Otsu
  (IEEE Trans System Man and Cybernetics, Vol. 9, 1979)
- Suppose to have a bimodal histogram (Chapter 2.4)
- Look for optimal threshold according to a performance criterion. The threshold separates the histogram into two exactly not overlapping classes
- Performance criterion :
  - Variance of grey values within both of the two classes ⇒ minimum
  - Variance of mean values of the two classes ⇒ maximum

2.7 Image Analysis Tools, Otsu – Binarization (optional)



- Separation of grey level histogram into two classes by threshold T
- Mean values of grey values within the classes.
- Calculation of variance σ_{in}^2 within the classes with intention $\sigma_{in}^2 \Rightarrow$ small
- Calculation of variance σ_{zw}^2 between the classes with intention $\sigma_{zw}^2 \Rightarrow$ large
- Look for T ∈ {1, .. 254} for which $\sigma_{zw}^2 / (\sum \sigma_{in}^2) \Rightarrow$ maximum

2.7 Image Analysis Tools, Otsu – Binarization (optional)

- Probability for g to appear in one of the classes :

$$P_0 = \sum_{g=0}^{T} p(g) \qquad P_1 = \sum_{g=T+1}^{G-1} p(g)$$

- Mean grey value μ of the image :

$$\mu = \mu_0 P_0 + \mu_1 P_1$$

  μ_0 and μ_1 are the mean values of the classes

- The variances σ² of the classes are :

$$\sigma_0^2 = \sum_{g=0}^{T} p(g) * (g - \mu_0)^2 \qquad \sigma_1^2 = \sum_{g=T+1}^{G-1} p(g) * (g - \mu_1)^2$$

- The variance between the classes :

$$\sigma_{zw}^2 = P_0 (\mu_0 - \mu)^2 + P_1 (\mu_1 - \mu)^2$$

2.7 Image Analysis Tools, Otsu – Binarization (optional)

- We define the variances σ_{in}^2 within the classes :

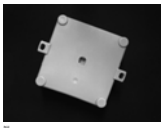
$$\sigma_{in}^2 = P_0 \sigma_0^2 + P_1 \sigma_1^2$$

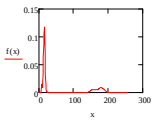
- Performance criterion :

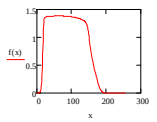
$$\frac{\sigma_{zw}^2}{\sigma_{in}^2} \Rightarrow \max$$

- The procedure is also applicable for 2-threshold binarization with T_1 and T_2 (see Chap. 2). Calculate P_0, P_1, P_2, μ, σ_{zw}^2 and σ_{in}^2.
- Problem : For 8 bit grey level resolution 256 * 256 = 64K combinations for T_1 und T_2 have to be calculated.

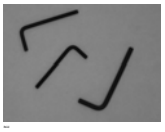
2.7 Image Analysis Tools, Otsu – Binarization (optional)

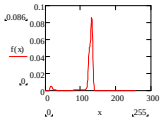


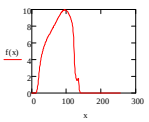




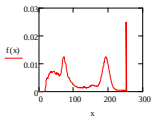
2.7 Image Analysis Tools, Otsu – Binarization (optional)

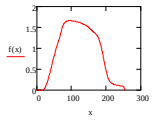






2.7 Image Analysis Tools, Otsu – Binarization (optional)





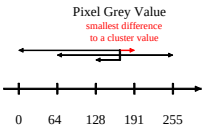
2.8 Image Analysis Tools, k-Means Clustering

- Grouping of similar grey values into clusters
- Problem:
  - Definition of number k of clusters
    - e.g. interactive input
    - e.g. extracting number of local maxima from histogram
  - Definition of grey values of cluster centers
    - Chose equidistant grey values according to k
    - Iteration of clustering after updating the cluster centers
    - Terminating the iteration

2.8 Image Analysis Tools, k-Means Clustering

- Application on grey value images
- Grey value space { 0, …, 255 }
- Definition of e.g. k = 5 clusters
- Automatically define equidistant grey values { 0, 64, 128, 191, 255 }
- Calculate for each pixel the smallest difference to the defined grey values and assign each pixel the correspondent cluster value
- Calculate the average grey value of the pixels for each cluster value
- Substitute the cluster values by the averages and iterate the procedure until no changes occur.

2.8 Image Analysis Tools, k-Means Clustering



Calculation of mean grey value of all pixels,
which have smallest difference to cluste value 191.
Substitute cluster value 191
by the corresponding mean value.
Iterate procedure.

k-means 01-engl.dsf

2.8 Image Analysis Tools, k-Means Clustering

G ∈ {0, 64, 128, 191, 255}

G_{neu} ∈ {24, 76, 128, 180, 244} after 1st step

Review of Chapter 2

- Tools to judge digital images
  - Grey level profile to control correct lens aperture
  - Noise in homologous and heterologous pixels
  - Variance as measure of noise
  - Look-up-tables to modify the grey levels
  - Histograms to judge the dynamic range and to get correct thresholds
  - Ideal grey value distribution by applying Look-up-table of cumulative probability distribution
  - Numeric magnifier to see small details in the image
  - Optional: Otsu Binarization
  - k-means clustering with iteration