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3. Global Operations in Spatial Domain, Filters

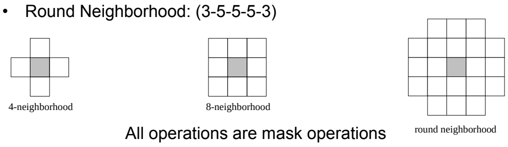
3. Global Operations in Spatial Domain, Filters

Following notation is used :

- Pixel position : p(x, y)
- Grey level at position p(x,y) : g(x, y)
- Modified grey level by an operation : g‘(x, y)
- Derivative of grey level at position p : g_x(x, y), g_y(x, y)
- Source pixel / Target pixel g_{old} / g_{new}

3. Global Operations in Spatial Domain, Filters

- 4-Neighborhood : (adjacent pixels by edges)
- 8-Neighborhood : (adjacent pixels by edges and corners)
- Round Neighborhood: (3-5-5-5-3)



All operations are mask operations
(most commonly used: 8-Neighborhood)

Note : In contrast to the mathematical coordinate system, the positive y-axis directs downwards. This fact originates from the time dependency of the pixel source in video.

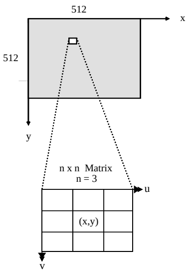
3.1 Global Operations in Spatial Domain, Low-pass Filters

H = h(x,y) =

x-1,y-1	x,y-1	x+1,y-1
x-1,y	x,y	x+1,y
x-1,y+1	x,y+1	x+1,y+1

- n*n filter
- n = 2*k+1, k=1,2,…
- p(x, y) = p(x-u, y-v)
- u and v ∈ {-(n-1)/2,..,0, ..+(n-1)/2}
- In the following :
  n = 3 unless stated otherwise

3.1.1 Global Operations in Spatial Domain, Low-pass Filters



- Mean Filter

$$g'(x,y) = \frac{1}{n^2} \sum_{u=-\frac{n-1}{2}}^{u=+\frac{n-1}{2}} \sum_{v=-\frac{n-1}{2}}^{v=+\frac{n-1}{2}} g(x-u, y-v)$$

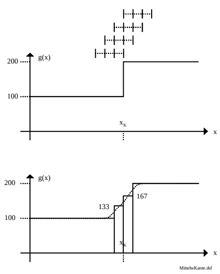
$$g'(x,y) = \frac{1}{n^2} \sum_{u=-\frac{n-1}{2}}^{u=+\frac{n-1}{2}} \sum_{v=-\frac{n-1}{2}}^{v=+\frac{n-1}{2}} g(x-u, y-v) * h(u,v)$$

$$H = h(u,v) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

3.1.1 Global Operations in Spatial Domain, Low-pass Filters

- Problem : margin area (upper and lower line / left and right column)
  - Use unchanged data, in most cases optimal procedure
- Result of operation :
  - No effect on areas with constant grey values
  - No effect on average grey value of image
  - Significant on image noise (desired)
  - Smoothing of edges (undesired)

3.1.1 Global Operations in Spatial Domain, Low-pass Filters



- Example :

  Filter mask 3 x 1 on ideal edge

  Weights of mask elements are 1

  Generation of new grey values

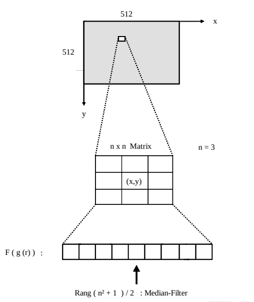
3.1.1 Global Operations in Spatial Domain, Low-pass Filters

- Empirical modification of mask weights (*edge sensitive mask*) :

$$H = \begin{vmatrix} 0,9 & 0,9 & 0,9 \\ 0,9 & 1,8 & 0,9 \\ 0,9 & 0,9 & 0,9 \end{vmatrix}$$

- Average value of mask elements : $h_{average}(u,v) = 1$
- Normalizing requires : $\Sigma\, h(u,v) = n^2$
  - Average grey level remains constant
  - Algorithm requires floating point values
  - Correct rounding and data conversion for output data necessary
  - $g'(x, y) \in \{0, \ldots, 255\}$

3.1.2 Global Operations in Spatial Domain, Low-pass Filters



- Rank order filter
  - Mean filter : $g' \notin \{G^*\}$
  - Rank ordering filter : $g' \in \{G^*\}$
  - G^* : Set of initial grey values
  - F(g(r)) increasing (or decreasing) sequence of grey values
  - $r = \{1, \ldots, n^2\}$; $n > 1$ and odd
  - Def. : $med\,\{F(g(r))\} = g((n^2+1)/2)$

    → Median filter
  - “median value of sorted sequence” instead of Mean value

3.1.3 Global Operations in Spatial Domain, Low-pass Filters

- Gauss filter
- Edge smoothing filter
- One dimensional Gauss function $\qquad h(u) = \frac{1}{\sigma\sqrt{2\pi}} * e^{-\frac{u^2}{2\sigma^2}}$

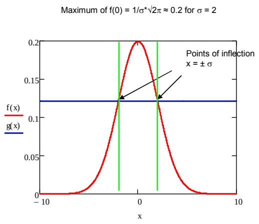
Maximum at h(u=0) $= \frac{1}{\sigma\sqrt{2\pi}}$

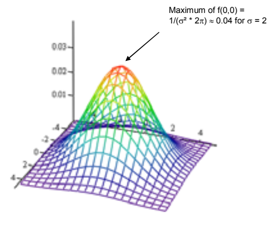
Minimum at h(u→±∞) $= 0$

Point of inflection at h(u=±σ) $= \frac{1}{\sigma\sqrt{2\pi}} * \frac{1}{\sqrt{e}}$

- Two dimensional Gauss function $\qquad h(u,v) = \frac{1}{\sigma^2 * 2\pi} * e^{-\frac{u^2+v^2}{2\sigma^2}}$

3.1.3 One-dimensional and two-dimensional Gauss function





3.1.3 Global Operations in Spatial Domain, Low-pass Filters

Let be σ = 1 and pre-factor = 4, one gets for n = 3 or n = 5 :

H = 1/16 *

oder

H = 1/20 *

0	0	1	0	0
0	1	2	1	0
1	2	4	2	1
0	1	2	1	0
0	0	1	0	0

Integer values

H = 1/21,6 *

0	0	0,5	0	0
0	1,5	2,4	1,5	0
0,5	2,4	4	2,4	0,5
0	1,5	2,4	1,5	0
0	0	0,5	0	0

Float values

Gauss00.DSF

σ defines the range of the filter
The pre-factor is arbitrary and is compensated by the normalizing factor

3.1.4 Global Operations in Spatial Domain, Low-pass Filters

- n-Binomial filter, n even
- Normalized binomial coefficients
  [e.g. n = 4 : $(a+b)^4 \rightarrow {}^1/_{16}$ * { 1 4 6 4 1 }]
- Multiplication column vector * row vector → Binomial mask elements

$$H = h(u, v) = \frac{1}{256} * \begin{pmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{pmatrix}$$

Question about Low-pass Filters

What are the image processing differences between the algorithms of

- Mean filter
- Median filter

- Gauss filter
- Binomial filter

?

3.1.5 Global Operations in Spatial Domain, Unsharp Masking

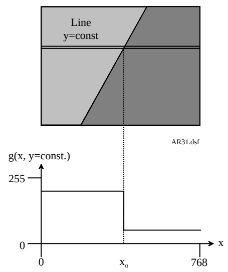
Original image: f(x,y)
Low-pass filtered image: $f_L(x,y)$
Unsharp mask: $g_{mask}(x,y) = f(x,y) – f_L(x,y) \approx 0$
Enhanced image: $f_{enhanced}(x,y) = f(x,y) + k * g_{mask}(x,y)$

If k = 1: unsharp masking
If k ≥ 1: highboost filtering

UnsharpMasking.xmcd

High-pass Filters

3.2 Global Operations in Spatial Domain, High-pass Filters



Edges : Significant change of grey values

Suppose to have a continuous step-function g(x)

How can we find changes of g(x) ?

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3.2 Global Operations in Spatial Domain, High-pass Filters

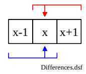
$$\frac{d}{dx}g(x) = \lim_{\Delta x \to 0} \frac{g(x+\Delta x) - g(x)}{\Delta x} = g_x(x)$$

In the discrete domain limit $\Delta x \to 0$ not realizable.
Which limit is realizable ?

$\Delta x \to ?$

We get

$g_x(x) = g(x+1) - g(x)$ forward differences
$g_x(x) = g(x) - g(x-1)$ backward differences



Problem : There exists no position between x and x+1 respectively between x-1 and x.

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3.2 Global Operations in Spatial Domain, High-pass Filters

Let be g(x,y) the central pixel of mask H

$$H = \begin{vmatrix} g(x-1,y-1) & g(x,y-1) & g(x+1,y-1) \\ g(x-1,y) & g(x,y) & g(x+1,y) \\ g(x-1,y+1) & g(x,y+1) & g(x+1,y+1) \end{vmatrix}$$

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3.2 Global Operations in Spatial Domain, High-pass Filters

The differential in x direction defines the edge in y direction

$$H_y = \begin{vmatrix} 0 & 0 & 0 \\ 0 & -1 & +1 \\ 0 & 0 & 0 \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} 0 & 0 & 0 \\ -1 & +1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

forward backward
differences

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3.2 Global Operations in Spatial Domain, High-pass Filters

Accordingly we get for edge in x direction :

$$H_x = \begin{vmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & +1 & 0 \end{vmatrix} \quad \text{or} \quad \begin{vmatrix} 0 & -1 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

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3.2 Global Operations in Spatial Domain, High-pass Filters

These masks are noise sensitive.

Edges are supposed to have certain lengths.

With distinct probability they continue into adjacent rows or columns.

Use $\Delta x \to 2$ instead of $\Delta x \to 1$ to find a pixel position for the result

We get an empirical extension of the filter mask.

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3.2.1 Global Operations in Spatial Domain, High-pass Filters

We define as **Prewitt-Operator** :

$$g_x = H_y = \begin{vmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{vmatrix} \text{ und } g_y = H_x = \begin{vmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{vmatrix}$$

<u>Note:</u>

- H_y finds edges in y-direction, derivative in x direction,
- H_x finds edges in x-direction, derivative in y direction,

All lines / columns are treated with the same weights

3.2.2 Global Operations in Spatial Domain, High-pass Filters

We define as **Sobel-Operator** :

$$H_y = \begin{vmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{vmatrix} \text{ und } H_x = \begin{vmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{vmatrix}$$

The source line / column are weighted double

3.2.2 Global Operations in Spatial Domain, High-pass Filters

The grey value function is a two dimensional function.
We define the **Nabla-Operator** :

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}$$

The Nabla-Operator is a vector operator, g(x, y) is a scalar function.
We define :

$$\nabla g(x, y) = \text{grad } g(x, y) = \begin{pmatrix} g_x(x, y) \\ g_y(x, y) \end{pmatrix}$$

grad g(x, y) is a vector.

3.2.2 Global Operations in Spatial Domain, High-pass Filters

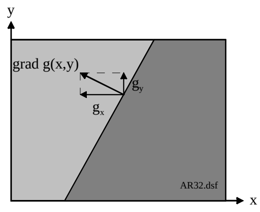
Magnitude of grad g(x,y) : $\left|\text{grad } g(x, y)\right| = \sqrt{g_x^2 + g_y^2}$

Here we lose the information of the direction.

Direction of grad g(x,y) : $\tan \alpha = \frac{g_y}{g_x}$

3.2.2 Global Operations in Spatial Domain, High-pass Filters

grad g(x,y) shows into direction of largest change of luminance.
Representation in mathematical coordinate system :



Mind the use of a mathematical coordinate system

3.2.2 Global Operations in Spatial Domain, High-pass Filters

Visualization as binary image (target image) :

$$g'(x, y) = \begin{cases} 0, & \text{if } \left|\text{grad } g(x, y)\right| \le T \\ 255, & \text{otherwise} \end{cases}$$

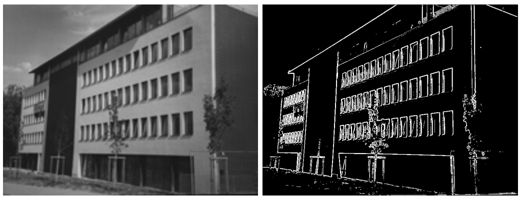
Visualization in grey level image (source image) :

$$g'(x, y) = \begin{cases} g(x, y), & \text{if } \left|\text{grad } g(x, y)\right| \le T \\ \text{const.} & \text{otherwise} \end{cases}$$

Visualization of vector direction (target image) :

$$g'(x, y) = \begin{cases} 0, & \text{if } \left|\text{grad } g(x, y)\right| \le T \\ \text{angle coded as grey value,} & \text{otherwise} \end{cases}$$

3.2.2 Global Operations in Spatial Domain, High-pass Filters



Example of direction-independent Sobel filtering as binary image.
Threshold = 150

HochpassFilter-Sobel.xmcd

3.2.2 Global Operations in Spatial Domain, High-pass Filters

Example of direction-independent Sobel filtering as grey value image

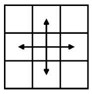
3.2.2 Global Operations in Spatial Domain, High-pass Filters

Example of direction-dependent Sobel filtering

atan2()-function atan()-function

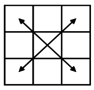
HochpassFilter-Sobel-RA

3.2.3 Global Operations in Spatial Domain, High-pass Filters



Sobelfilter

Roberts.dsf



Robertsfilter

Roberts.dsf

- Roberts filter
- The basis of gradient components are the diagonals

$g_+ = g(x+1,y-1) - g(x-1,y+1)$
$g_- = g(x+1,y+1) – g(x-1,y-1)$

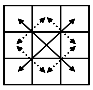
$|\text{grad } g(x,y)| = \text{sqrt}(g_+^2 + g_-^2)$

Difference to Sobel filter ?

3.2.3 Global Operations in Spatial Domain, High-pass Filters

$$H- = \begin{bmatrix} -2 & -1 & 0 \\ -1 & 0 & +1 \\ 0 & +1 & +2 \end{bmatrix} \qquad H+ = \begin{bmatrix} 0 & +1 & +2 \\ -1 & 0 & +1 \\ -2 & -1 & 0 \end{bmatrix}$$

Roberts4.dsf



Robertsfilter

Roberts.dsf

Filter masks, when using main and adjacent diagonals :

$g_+ = 2*[g(x+1,y-1)–g(x-1,y+1)] + g(x,y-1) – g(x-1,y) + g(x+1,y) –g(x,y+1)$

$g_- = 2*[g(x+1,y+1)–g(x-1,y-1)] + g(x,y+1) – g(x-1,y) + g(x+1,y) –g(x,y-1)$

$|\text{grad } g(x, y)| = \text{sqrt}(g_+^2 + g_-^2)$

3.2 Global Operations in Spatial Domain, High-pass Filters

The filters need interactions by thresholding, therefore we use :

Global adaptive threshold

- Calculate entire gradient image : $g'(x,y) = \text{grad } g(x,y)$
- Find maximum of all gradients : $\text{grad}_{max,glob} = \max\{ \text{grad } g(x,y) \}$
- Define adaptive threshold : $T_{adaptiv,glob} = \text{Factor}_{glob} * \text{grad}_{max,glob}$
- Empirical : approx. $0.2 \le \text{Factor}_{glob} \le$ approx. 0.7

3.2 Global Operations in Spatial Domain, High-pass Filters

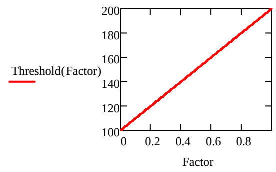
Alternative option : Local adaptive threshold

- Calculate entire gradient image : g‘(x,y) = grad g(x,y)
- Find within n x n – mask
  maximum of gradients : $grad_{max,local}$ = max{ grad g(x,y) }$_{nXn}$
  minimum of gradients : $grad_{min,local}$ = min{ grad g(x,y) }$_{nXn}$
- Define adaptive local threshold
  $T_{adaptiv,local} = Factor_{local} * grad_{max,local} + (1 - Factor_{local}) * grad_{min,local}$
- Empirical : approx. 0,5 ≤ $Factor_{local}$ ≤ approx. 0,7
- Empirical : approx. 11 ≤ n ≤ 25
  according to image content and structure

3.2 Global Operations in Spatial Domain, High-pass Filters

Example for $grad_{max,local}$ = 200 and $grad_{min,local}$ = 100

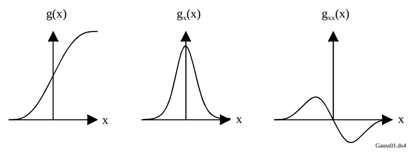
$T_{adaptiv,local} = Factor_{local} * grad_{max,local} + (1 - Factor_{local}) * grad_{min,local}$



3.2 Global Operations in Spatial Domain, High-pass Filters

- This procedure may produce very small threshold values in cases when
  $grad_{max,local} \approx grad_{min,local}$
- Define a minimal threshold, which may not be surpassed.

3.2.4 Global Operations in Spatial Domain, High-pass Filters



Laplace-Filter

The edge is defined by Zero-crossing of 2nd derivative of grey level function.

- Calculate 2nd derivative by applying Nabla operator onto 1st derivative of grey level function.
- Vector operator applied on vector function results in scalar value.

3.2.4 Global Operations in Spatial Domain, High-pass Filters

$$\Delta g(x,y) = \nabla(\nabla g(x,y)) = \nabla \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} g(x,y) =$$

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix} g(x,y) = \frac{\partial^2 g(x,y)}{\partial x^2} + \frac{\partial^2 g(x,y)}{\partial y^2} =$$

$$\frac{\partial}{\partial x}(g(x+1,y) - g(x,y)) + \frac{\partial}{\partial y}(g(x,y+1) - g(x,y)) =$$

$$g(x+1,y) - g(x,y) - g(x,y) + g(x-1,y) + g(x,y+1) - g(x,y) - g(x,y) + g(x,y-1)$$

Note : *backward* und *forward differences* are applied alternately !

3.2.4 Global Operations in Spatial Domain, High-pass Filters

$$H_{math} = const * \begin{vmatrix} 0 & -1 & 0 \\ -1 & +4 & -1 \\ 0 & -1 & 0 \end{vmatrix} \text{ oder } H_{emp1} = const * \begin{vmatrix} -1 & -1 & -1 \\ -1 & +8 & -1 \\ -1 & -1 & -1 \end{vmatrix} \text{ oder } H_{emp2} = const * \begin{vmatrix} -1 & -2 & -1 \\ -2 & +12 & -2 \\ -1 & -2 & -1 \end{vmatrix}$$

- H_{math} is deduced from Math, H_{emp1} und H_{emp2} are empirical deduced masks.
- Note : Σ h(u,v) = 0, i.e. image areas with constant grey level result in g‘ = 0.
- The masks produce symmetrical positive and negative grey values, negative grey values cannot be visualized.
- For visualization use according Look-up-Tables
- Find Zero-crossings (or crossing at 127.5 if scaled)

3.2.4 Global Operations in Spatial Domain, High-pass Filters

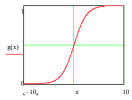
Zero crossing detection :

if ((Ai ≥ Zero point +T AND Bi ≤ Zero point -T) OR-ed for i = 1,2,3,4) = 1
OR
(Ai ≤ Zero point -T AND Bi ≥ Zero point +T)) OR-ed for i = 1,2,3,4) = 1
then g' = 255
else g' = 0

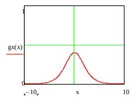
A1	A2	A3
B4	g'	A4
B3	B2	B1

Zero point = 0 or 127,5 if scaled;
Threshold T : {0 ≤ T < ≈10}

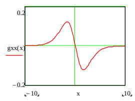
3.2.4 Global Operations in Spatial Domain, Enhancement by Laplace



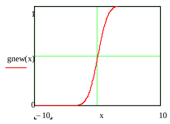
greylevel g(x)



1st derivative gx(x)



2nd derivative gxx(x)



gnew(x) = g(x)-gxx(x)

3.2.4 Global Operations in Spatial Domain, Enhancement by Laplace

Original Image

Enhanced Image by Laplace

3.2.5 Global Operations in Spatial Domain, High-pass Filters

- Laplace filter is extremely sensitive for noise.
- Apply low-pass filter before application of Laplace filter
- Ideal low-pass filter : Gauss filter
- Result : Laplacian of Gaussian (LoG)

3.2.5 Global Operations in Spatial Domain, LoG Filter

- Gauss-Filter : G(u, v, σ) (two-dimensional)

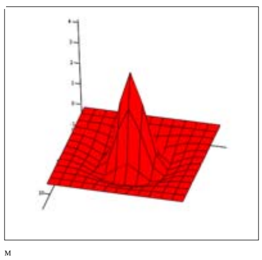
  g'(x,y) = Δ {G(u, v, σ) * g(x,y) } = { Δ G(u, v, σ) } * g(x,y)
  (stated without mathematical proof)

$$\Delta G(u,v,\sigma) = h(u,v) = \frac{1}{2*\pi*\sigma^4} * \left(+\frac{u^2+v^2}{\sigma^2} - 2\right) * e^{\frac{-(u^2+v^2)}{2\sigma^2}}$$

(Mexican hat shaped function)

g'(x,y) = h(u,v) * g(x,y)

3.2.5 Global Operations in Spatial Domain, LoG Filter



M

$M_{i,j}$ = h(u,v), u = 0 … 12, v = 0 … 12

using prefactor -2 to get center element $M_{6,6}$ = 4

$$M_{i,j} := -2 \cdot \left[\frac{\left[(i-6)^2 + (j-6)^2\right]}{\sigma^2} - 2 \right] \cdot e^{\frac{-\left[(i-6)^2+(j-6)^2\right]}{2\cdot\sigma^2}}$$

3.2.5 Global Operations in Spatial Domain, LoG Filter

M=

	0	1	2	3	4	5	6	7
0	-0	-0	-0	-0	-0	-0.01	-0.01	-0.01
1	-0	-0	-0	-0.01	-0.03	-0.06	-0.07	-0.06
2	-0	-0	-0.02	-0.07	-0.16	-0.25	-0.29	-0.25
3	-0	-0.01	-0.07	-0.22	-0.42	-0.53	-0.54	-0.53
4	-0	-0.03	-0.16	-0.42	-0.53	-0.15	0.18	-0.15
5	-0.01	-0.06	-0.25	-0.53	-0.15	1.42	2.49	1.42
6	-0.01	-0.07	-0.29	-0.54	0.18	2.49	4	2.49
7	-0.01	-0.06	-0.25	-0.53	-0.15	1.42	2.49	...

- 2D-example with σ=1,5
- Range : 6 elements resp. 13 x 13 mask
- Cut-out of mask shown, missing elements are symmetrical to h(6,6)
- Element values rounded to 2 decimals
- Mask may be cut-off, if absolute values of h(u,v) / h(6,6) are less then 0.1 ⇒ 9 x 9 mask

3.2.5 Global Operations in Spatial Domain, LoG Filter

For r → ∞ we obtain h(u,v) → 0

Cut-off for filter range assumed as h(u,v) / h(0,0) > -0.1

Range of filter : h(u,v) / h(0,0) =

σ = 1 : h(u,v) / h(0,0) > -0.1 for u ≈ 2,5; 7 x 7 mask
σ = 1,5 h(u,v) / h(0,0) > -0.1 for u ≈ 3,8 9 x 9 mask
σ = 2 : h(u,v) / h(0,0) > -0.1 for u ≈ 5,0; 11 x 11 mask
σ = 3 : h(u,v) / h(0,0) > -0.1 for u ≈ 7,5; 17 x 17 mask
σ = 4 : h(u,v) / h(0,0) > -0.1 for u ≈ 10,0; 21 x 21 mask

3.2.5 Global Operations in Spatial Domain, LoG Filter

cut-off

Let σ = 2 and get for h(u,v)/h(0,0) the following linear mask :

-0,039

-0,093	-0,135	-0,041	0,303	0,772	1.0	0,772	0,303	-0,041	-0,135	-0,093

-0,039

Sum of mask elements / 11 elements = 0.237

Subtract 0.237 from all mask elements to achieve Σ of elements = 0 :

-0,330	-0,372	-0,278	0,066	0,535	0,763	0,535	0,066	-0,278	-0,372	-0,330

Apply mask elements on grey level step from g = 100 to g = 200 :

0	-33	-70	-98	-91	-38	+38	+91	+98	+70	+33	0

Zero-crossing defines position of edge

LoG01 engl.dsf

3.2.5 Global Operations in Spatial Domain, LoG Filter

- Scaling to G ∈ { 0, …, 255 }
- g'(g) = m * g +n

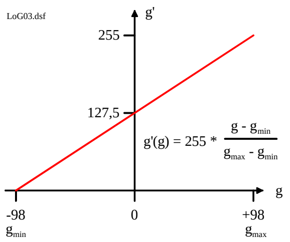
with boundary condition $g(g_{min} = -98) = 0$ und $g(g_{max} = +98) = 255$

$$g'(g) = \frac{255}{g_{max} - g_{min}} * (g - g_{min})$$

- In general in images g_{max} and g_{min} are not symmetrical to zero

3.2.5 Global Operations in Spatial Domain, LoG Filter

LoG03.dsf



⇒ g'(g=-98) = 0 g'(g=0) = 127,5 g'(g=+98) = 255

- Zero-crossing is shifted in this case to 127,5 !

3.2.5 Global Operations in Spatial Domain, LoG Filter

- Define appropriate Mask size
- Calculate the sum of all mask elements, which is ≠ 0
- Divide sum by number of all mask elements n x n
- Subtract result from all mask elements
- Check sum of all mask elements to be 0
- Apply mask on image
- Calculate crossings of average grey value of laplace filtered image.

3.2.5 Global Operations in Spatial Domain, LoG Filter, Results

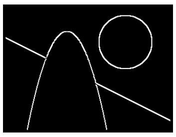


Image:
Synthetic image ParabolaTestImage04 without noise

Average grey value of Laplace filtered image 134.8

Target Image threshold T = 15

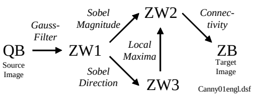
3.2.6 High Pass Filters, Canny Filter

Canny filter is a multi-step operator and generates one-pixel-wide edges

- Gauss Filter (σ = 1 or σ = 1.4)
- Direction dependent and independent Sobel Filter
- Thinning of edges
- Closing of gaps

3.2.6 High Pass Filters, Canny Filter

(www.pages.drexel.edu/~weg22/can_tut.html)



Canny filter is a multi-step operator and generates one-pixel-wide edges

5 x 5 Gauss filter

0.5	1.1	1.4	1.1	0.5
1.1	2.4	3.1	2.4	1.1
1.4	3.1	4.0	3.1	1.4
1.1	2.4	3.1	2.4	1.1
0.5	1.1	1.4	1.1	0.5

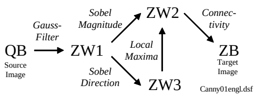
3 x 3 Gauss filter

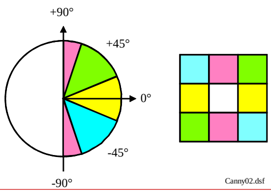
1	2	1
2	4	2
1	2	1

- Apply *low-pass filter on source image (QB)* for noise reduction, e.g. 5*5-Gauss filter with σ = 1 or σ = 1.4, (Action: QB → ZW1)

3.2.6 High Pass Filters, Canny Filter

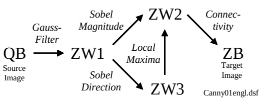
(www.pages.drexel.edu/~weg22/can_tut.html)

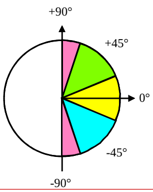




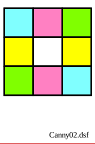
- Apply Sobel *highpass filter on ZW1* for edge extraction.
- Store direction independent magnitude (Action: ZW1 → ZW2)
- Scale ZW2 to values 0, … 255
- Store 4-direction dependent value in ZW3 (e.g. 0, 45, 90, 135 or 0, 1, 2, 3) for the discrete direction (Action: ZW1 → ZW3).

3.2.6 High Pass Filters, Canny Filter





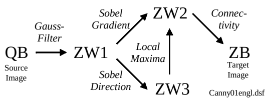
Mask in ZW2



Thinning of edges :

- Scan ZW2 :
- Set actual magnitude value (white element) in ZW2 to 0, if it is **not** the maximum within the direction of the respective mask defined in ZW3 (e.g. white and yellow elements, white and green elements etc).
- Action : (ZW2 → ZW2)

3.2.6 Global Operations in Spatial Domain, Canny Filter

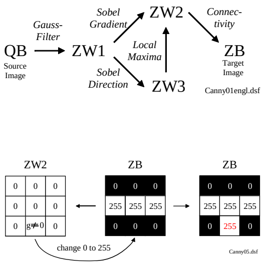


Closing of gaps (Connectivity)

- Define two thresholds $0 < T_1 < T_2 < 255$
- Scan ZW2 :
  - if $g > T_2$: set ZW2 = 0 and ZB = 255
  - if $g < T_1$: set ZW2 = 0
  - if $T_1 < g < T_2$: leave ZW2 = g unchanged

Action : (ZW2 → ZW2 and ZW2 → ZB)

3.2.6 Global Operations in Spatial Domain, Canny Filter



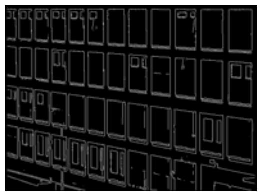
Completion of target image (ZB):

- Scan ZB :
  if in ZB the
  actual pixel = 255 AND
  within the mask in ZW2
  any value g ≠ 0, i.e. $T_1 < g < T_2$,
  → set the corresponding pixel
  in ZB to g = 255.

Action: (ZB → ZB)

3.2.6 Global Operations in Spatial Domain, Canny Filter


Bild


NeuBild

3.3 High Pass Filters, Harris Filter / Corner Filter

- Corners: significant structures in an image
- Applications:
  - Tracking of structures in image sequences
  - Search for corresponding points in stereo vision
  - Reference points for automatic measurements
  - Calibration of cameras
- Widely independent of illumination
- Requirements:
  - Differing between significant and non significant points
  - Independency of noise
  - Real time performance for video-tracking

3.3 High Pass Filters, Harris Filter / Corner Filter (1)

- Edges: significant magnitude of gradients in only one direction
- Corners: significant magnitude of gradients in more than one direction
- Calculate the gradients
  by using Prewitt or Sobel: $\mathrm{Ix}(x,y) = \frac{\partial}{\partial x}\mathrm{I}(x,y) \qquad \mathrm{Iy}(x,y) = \frac{\partial}{\partial y}\mathrm{I}(x,y)$
- Calculate matrices:. $\mathrm{A}(x,y) = \mathrm{Ix}^2 \quad \mathrm{B}(x,y) = \mathrm{Iy}^2 \quad \mathrm{C}(x,y) = \mathrm{Ix} * \mathrm{Iy}$
- Construct matrix M(x,y): $\mathrm{M} = \begin{pmatrix} \mathrm{Ix}^2 & \mathrm{Ix}*\mathrm{Iy} \\ \mathrm{Ix}*\mathrm{Iy} & \mathrm{Iy}^2 \end{pmatrix} = \begin{pmatrix} \mathrm{A} & \mathrm{C} \\ \mathrm{C} & \mathrm{B} \end{pmatrix}$
- Filter the matrices A, B and C by using e.g. 5 x 5 low pass filter:
  etc $\mathrm{A} \rightarrow \overline{\mathrm{A}}$

3.3 High Pass Filters, Harris Filter / Corner Filter (3)

Set $\overline{\mathrm{M}} = \begin{pmatrix} \overline{\mathrm{A}} & \overline{\mathrm{C}} \\ \overline{\mathrm{C}} & \overline{\mathrm{B}} \end{pmatrix}$

Define a „corner response function" Q(x,y):

$$\mathrm{Q}(x,y) = \det \overline{\mathrm{M}} \quad - \mathrm{thr1}\,(\mathrm{trace}\,\overline{\mathrm{M}}\,)^2$$
$$= \mathrm{thr2}\,\left\{(\overline{\mathrm{A}}\,\overline{\mathrm{B}} - \overline{\mathrm{C}}^2) - \mathrm{thr1}\,(\overline{\mathrm{A}} + \overline{\mathrm{B}})^2\right\} \rightarrow \text{large}$$

Default value for thr1⇒ 0.04 … 0.06, max. value 0.25

Check Q(x,y) > thr2 with thr2 ≈ 10^4 … 10^8

3.3 High Pass Filters, Harris Filter (Mathematical Background)

- Let be $\overline{\mathrm{M}} = \begin{pmatrix} \overline{\mathrm{A}} & \overline{\mathrm{C}} \\ \overline{\mathrm{C}} & \overline{\mathrm{B}} \end{pmatrix}$
- The matrices M resp. $\overline{\mathrm{M}}$ are symmetrical and can be diagonalized:

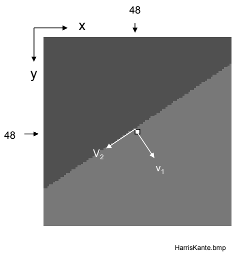
  $\overline{\mathrm{M}}' = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ in which $\lambda_1 > 0$ and $\lambda_2 = 0$

  λ_i are called eigenvalues of $\overline{\mathrm{M}}'$

  $$\lambda_{1,2} = \frac{\mathrm{trace}\,\overline{\mathrm{M}}}{2} \pm \sqrt{\left(\frac{\mathrm{trace}\,\overline{\mathrm{M}}}{2}\right)^2 - \det\left(\overline{\mathrm{M}}\right)}$$
  $$= \frac{1}{2}\left(\overline{\mathrm{A}} + \overline{\mathrm{B}} \pm \sqrt{\overline{\mathrm{A}}^2 - 2\overline{\mathrm{A}}\,\overline{\mathrm{B}} + \overline{\mathrm{B}}^2 + 4\overline{\mathrm{C}}^2}\right)$$

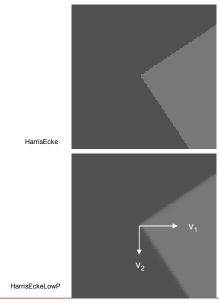
- The eigenvalues define the strength of the edge.
- The eigenvectors v_1 and v_2 define the direction of the corresponding edge.
- Calculation of eigenvectors: http://www.youtube.com/watch?v=i0ggTPAe1GU

3.3 High Pass Filters, Harris Filter (Mathematical Background)



Synthetic Image
if $y < -^2/_3\, x + 80 \rightarrow g = 80$
g = 120 else
g(48,48) = 120
Ix(x,y) = 44,44..
Iy(x,y) = 71,11..
Ix² = 1975,31..
Iy² = 5056,79..
IxIy = 3160,13..
http://www.arndt-bruenner.de/mathe/scripts/eigenwert2.htm
$\lambda_1 = 7031{,}77..$
$\lambda_2 = 0{,}3279..$
v_1 = eigenvector(λ_1) = {0,6249.., 1}
v_2 = eigenvector(λ_2) = {-1,600.., 1}

3.3 High Pass Filters, Harris Filter (Mathematical Background)



Synthetic Image
if $y < -^2/_3\, x + 80 \rightarrow g = 80$
If $y > +^3/_2\, x - 22 \rightarrow g = 80$
g = 120 else
g(48,48) = 120

filtered image by 3x3 Mean filter
g(49,48) = 106
$\overline{A} = 3919 \quad \overline{B} = 879,.. \quad \overline{C} = 36,..$
$\lambda_1 = 4203{,}9.. \quad v_1 = (84{,}45...,\ 1)$
$\lambda_2 = 752{,}0.. \quad v_2 = (-0{,}011..,1)$

3.3 High Pass Filters, Harris Filter (Mathematical Background)

- Both eigenvalues λ_1 and λ_2 should be significant in corner positions or

$$\lambda_1 - \lambda_2 = 2 * \sqrt{\left(\frac{\text{trace}\,\overline{M}}{2}\right)^2 - \det \overline{M}}$$

should be small. Note: $(0.5 * \text{trace}\ \overline{M})^2 > \det\ \overline{M}$

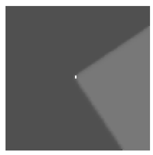
Define a „corner response function“ Q(x,y):

$$Q(x,y) = \det \overline{M} \quad - \text{thr}\ (\text{trace}\,\overline{M}\,)^2$$
$$= (\overline{A}\,\overline{B} - \overline{C}^2) - \text{thr}\ (\overline{A} + \overline{B})^2 \quad \rightarrow \quad \text{large}$$

Default value for thr 0.04 … 0.06, max. value 0.25

Check Q(x,y) > thr2 with thr2 ≈ 10^4 … 10^8

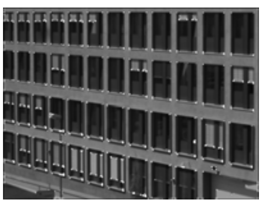
3.3 High Pass Filters, Harris Filter / Corner Filter



Result with th2 = $2*10^6$

3.3 High Pass Filters, Harris Filter / Corner Filter

Source image

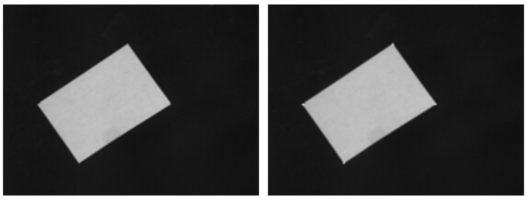


Harris filtered image,
for better visibility the grey intensity is set to 60%

Modellbildung:

Lineare Bewegung ohne Rotation
Rotation bei konstantem Schwerpunkt
Lineare Bewegung des Schwerpunktes mit Rotation
Nichtlineare Bewegung des Schwerpunktes mit Rotation
Zoom-in / zoom-out Bewegung

3.3 High Pass Filters, Harris Filter / Corner Filter



Orginalbild und Harris-gefiltertes Bild zur Bewegungsanalyse

Review of Chapter 3

- Low pass filters (Mean / Median / Gauss / Binomial)
- High pass filters (Prewitt / Sobel / Roberts / Laplace / LoG optional: Canny filter)
- Filtering as mask operation (no pixel operation)
- Filtering with masks of fixed and variable size
- Gradient of grey levels defined as vector
- Threshold operations (fixed / adaptive)
- Direction dependent and direction independent filtering
- Local and global filters
- Corner filter