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Lectures in Image Processing

Chapter 6
Interpolation

6. Interpolation

6. Interpolation, 6.1 Introduction

Problem :

Geometrical adjustment of images to different imaging conditions.
- Mutual adjustment of two images taken in different situations :
  - seasons (summer / winter),
  - positions (airplane – satellite)
  - orientations (lines of sight)
  - different technical sensors (visible wave lengths – IR / CT - NMR),
- Correction of single image (according to deficiencies of image sensors)

Note:
- Only two-dimensional problems considered.
- No perspective correction of three-dimensional objects, e.g. buildings etc.

6. Interpolation, 6.1 Introduction

Extension and reduction of image size : $F(g(x, y)) \rightarrow F'(g'(x', y'))$

with spatial transformation equation

$x' = f_1(x,y)$ and $y' = f_2(x,y)$ (no statement about grey value)

If constant aspect ratio is required : $x' = f_1(x,y) = f_1(x) = a * x$
$y' = f_2(x,y) = f_2(y) = a * y$

The image expands from upper left down to the right

6. Interpolation, 6.1 Introduction

In case of 512² - images and for expansion out of image center :

image center M = (255.5 / 255.5)

$$x' = a * x + (1-a) * 255.5 \qquad (Gl. 1a)$$
$$y' = a * y + (1-a) * 255.5$$

Other frequently used image sizes :
768 x 576
640 x 480

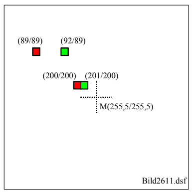
Where is the image center ?

6. Interpolation, 6.1 Introduction

Example for a = + 3 :
Consider two adjacent points :

p(200,200) → p(89, 89)
p(201,200) → p(92, 89),

The points in between p(90,89) and p(91,89) are not assigned.



Consequences of image extension

6. Interpolation, 6.1 Introduction

Resampling : Calculate positions of target pixels back into source image

$x = f_1^{-1}(x',y')$ = int ((x'-255.5)/a + 255.5 + 0.5) (1)
$y = f_2^{-1}(x',y')$ = int ((y'-255.5)/a + 255.5 + 0.5)

x'	88	89	90	91	92	93	94
xfloat	199.7	200	200.3	200.7	201	201.3	201.7
xfloat +0.5	200.2	200.5	200.8	201.2	201.5	201.8	202.2
xint (2)	200	200	200	201	201	201	202

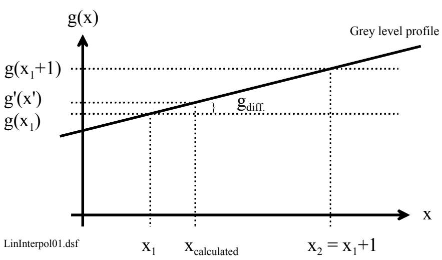
Note 1 : Correct rounding of values
Note 2 : If assigning g'(x',y') the value g(x,y), the new image shows a rough pixel structure
Note 3 : Grey values g'∈ { G* }; G* : values of source image

6. Interpolation, 6.1 Introduction, Image Expansion

Original Image

Expanded Image by Factor a = 5

6. Interpolation, 6.2 Linear Interpolation



Linear Interpolation of grey values

6. Interpolation, 6.2 Linear Interpolation

$$\frac{g(x+1) - g(x)}{1} = \frac{g_{diff.}}{x_{calculated} - x}$$

g'(x') = g(x) + $g_{diff.}$

g'(x') = g(x) + (g(x+1) - g(x))*($x_{calculated}$ - x) (Eq. 2)

g'(x') = int [(g(x) + (g(x+1) - g(x))*($x_{calculated}$ - x)) + 0.5]

For the above mentioned example follows :

g'(90) = int [(g(200) + (g(201) - g(200)) * 0.33)+0.5]

6. *Interpolation, 6.2 Linear Interpolation*

If we denote in Eq. 2

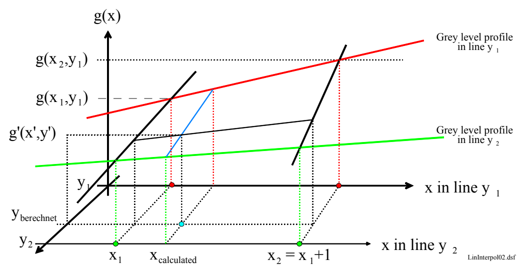
Eq.2 : $g'(x') = g(x) + (g(x+1) - g(x))*(x_{calculated} - x)$

$(x_{calculated} – x)$ by x_d, the distance of the destination pixel to the target pixel, we get :

$$g'(x') = (1 – x_d) * g(x) + x_d * g(x+1)$$

for 1-dimensional interpolation.

6. *Interpolation, 6.3 Bilinear Interpolation*



Bilinear interpolation of grey values

6. *Interpolation, 6.3 Bilinear Interpolation*

In line y :

$g'(x',y) = g(x,y) + (g(x+1,y) - g(x,y)) *(x_{calculated} - x)$

In line y+1 :

$g'(x',y+1) = g(x,y+1) + (g(x+1,y+1) - g(x,y+1)) *(x_{calculated} - x)$

$g'(x',y') = \textbf{int} [(g'(x',y)+(g'(x',y+1) - g'(x',y)) * (y_{calculated} -y))+0.5]$ (Eq.3)

6. *Interpolation, 6.3 Bilinear Interpolation*

Expanded Image without Interpolation

Expanded Image with Interpolation

6. *Interpolation, 6.3 Bilinear Interpolation*

If we denote in Eq.3
$(y_{calculated} – y)$ by y_d and
$(x_{calculated} – x)$ by x_d
the vertical resp. horizontal distance of the destination pixel to the target pixel, we get :

$$
\begin{aligned}
g'(x', y') = &(1 – x_d) * (1 - y_d) * g(x,y) \\
&+ x_d * (1 - y_d) * g(x+1,y) \\
&+ (1 – x_d) * y_d * g(x,y+1) \\
&+ x_d * y_d * g(x+1,y+1)
\end{aligned}
$$

for 2-dimensional interpolation.

6. *Interpolation, 6.4 Trilinear Interpolation*

For 3-dimensional data (volume data, voxels) e.g. DICOM data format, use tri-linear interpolation accordingly :

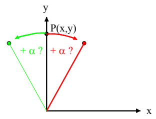
$$
\begin{aligned}
g'(x', y', z') = &(1 – x_d) * (1 - y_d) * (1 – z_d) * g(x,y,z) \\
&+ x_d * (1 - y_d) * (1 – z_d) * g(x+1,y,z) \\
&+ (1 – x_d) * y_d * (1 – z_d) * g(x,y+1,z) \\
&+ x_d * y_d * (1 – z_d) * g(x+1,y+1,z) \\
&+ (1 – x_d) * (1 - y_d) * z_d * g(x,y,z+1) \\
&+ x_d * (1 - y_d) * z_d * g(x+1,y,z+1) \\
&+ (1 – x_d) * y_d * z_d * g(x,y+1,z+1) \\
&+ x_d * y_d * z_d * g(x+1,y+1,z+1)
\end{aligned}
$$

6. Interpolation, 6.5 Rotation of Images

Rotation of a vector by rotational matrix R: $\begin{pmatrix} x' \\ y' \end{pmatrix} = R_1 * \begin{pmatrix} x \\ y \end{pmatrix}$

$R_1 = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

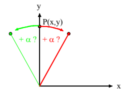
$R_2 = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$



Which matrix rotates the pixel into the red and which one into the green direction?

6. Interpolation, 6.5 Rotation of Images

Rotation of images :



Apply rotational Matrix $R = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$

R performs a rotation around the origin (0, 0).

For rotation around an arbitrary point, e.g. image center (x_m=383.5 / y_m=287.5 for 768x576 or x_m = 255.5/ y_m= 255.5) for 512² shift the co-ordinate system:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R \cdot \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_m \\ y_m \end{pmatrix} \right) + \begin{pmatrix} x_m \\ y_m \end{pmatrix}$$

6. Interpolation, Homogeneous Coordinates

Translation of a Vector Using Vector Addition	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$
Result of Vector Addition	x' = x + a y' = y + b
Addition of a Vector by a Matrix T using Homogeneous Coordinates	$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

6. Interpolation, Homogeneous Coordinates

Rotation of a Vector using Rotation Matrix R	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} * \begin{pmatrix} x \\ y \end{pmatrix}$
Rotation of a Vector by Matrix R using Homogeneous Coordinates	$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$
Rotation and Translation of a Vector using Homogeneous Coordinates	$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & a \\ -\sin\alpha & \cos\alpha & b \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

6. Interpolation, Homogeneous Coordinates

General 2-dimensional Form	$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$
General n-dimensional Form	$\begin{pmatrix} x'_1 \\ \bullet \\ x'_n \\ 1 \end{pmatrix} = \begin{pmatrix} r_{11} & \bullet & r_{1n} & t_1 \\ \bullet & \bullet & \bullet & \bullet \\ r_{1n} & \bullet & r_{nn} & t_n \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x_1 \\ \bullet \\ x_n \\ 1 \end{pmatrix}$

6. Interpolation, 6.5 Rotation of Images

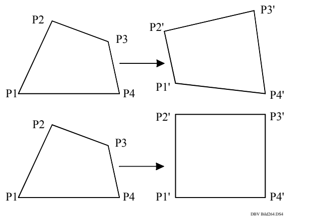
Use homogeneous coordinates and apply :

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = T^{-1} * R * T * \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & -x_m\cos\alpha - y_m\sin\alpha + x_m \\ -\sin\alpha & \cos\alpha & +x_m\sin\alpha - y_m\cos\alpha + y_m \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

6. Interpolation, 6.6 Projective Collineation

Wanted: Transformation of a tetragon onto another tetragon.



In general : Transformation of a tetragon onto tetragon

Special case : Transformation of a tetragon onto rectangle

Brasov, RO
Pizzeria Venezia
Str. Appolonia Hirscher

6. Interpolation, 6.6 Projective Collineation

Points P_i und P_i' (i = 1,...,4) in homogeneous coordinates

$$P_1 = \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \ldots P_4 = \begin{pmatrix} x_4 \\ y_4 \\ 1 \end{pmatrix} \quad \text{und} \quad P_1' = \begin{pmatrix} x_1' \\ y_1' \\ 1 \end{pmatrix} \ldots P_4' = \begin{pmatrix} x_4' \\ y_4' \\ 1 \end{pmatrix}$$

Assumed, the tetragons are not degenerated
↔ neither three of the P_i nor P_i' lie onto a line,
↔ every three of the P_i or P_i' are linear independent vectors,

↔ it exists $a_i \neq 0$ and $a_i' \neq 0$ so that

$$P_4 = a_1 * P_1 + a_2 * P_2 + a_3 * P_3 \qquad \text{(Eq. 1)}$$
$$P_4' = a_1' * P_1' + a_2' * P_2' + a_3' * P_3'$$

6. Interpolation, 6.6 Projective Collineation

or that $\quad P_4 = \mathbf{P} * \mathbf{a} \qquad P_4' = \mathbf{P'} * \mathbf{a'}$, (Eq. 1a)

with matrices $\quad \mathrm{P} = (P_1 \quad P_2 \quad P_3) \qquad \mathrm{P}' = (P_1' \quad P_2' \quad P_3')$

and vectors : $\quad a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad a' = \begin{pmatrix} a_1' \\ a_2' \\ a_3' \end{pmatrix}$

Multiply (Eq. 1) a from left by the inverse matrices $\mathbf{P^{-1}}$ resp. $\mathbf{P'^{-1}}$ and calculate the unknown vectors **a** resp. **a'** :

$$a = P^{-1} * P_4 \qquad a' = P'^{-1} * P_4' \qquad \text{(Eq. 1b)}$$

6. Interpolation, 6.6 Projective Collineation

Wanted : 3 X 3 - Matrix **A**, which maps the vectors P_i onto P_i' except for an unknown scaling factor.

$\mathbf{A} * \mathbf{P_i} = \lambda_i * \mathbf{P_i}'$ in which $\lambda_i \neq 0$ for i = 1, …, 4 (Eq. 2)

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} * \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} = \begin{pmatrix} a_{11}x_i + a_{12}y_i + a_{13} \\ a_{21}x_i + a_{22}y_i + a_{23} \\ a_{31}x_i + a_{32}y_i + a_{33} \end{pmatrix} = \lambda_i \begin{pmatrix} x_i' \\ y_i' \\ 1 \end{pmatrix} = \lambda_i * P_i'$$

6. Interpolation, 6.6 Projective Collineation

Elimination of the scaling factors $\lambda_1, \lambda_2, \lambda_3$:

$$A * P_4 \underset{(Gl.1)}{=} \sum_{i=1}^{3} A * (P_i * a_i) = \sum_{i=1}^{3} (A * P_i) * a_i \underset{(Gl.2)}{=} \sum \lambda_i * P_i' * a_i \qquad \text{(Eq. 3)}$$

$$A * P_4 \underset{(Gl.2)}{=} \lambda_4 * P_4' \underset{(Gl.1)}{=} \lambda_4 * \sum P_i' * a_i' = \sum \lambda_4 * P_i' * a_i' \qquad \text{(Eq. 4)}$$

Comparison of coefficients in the right side of Eq. 3 and 4 :

$a_i * \lambda_i = a_i' * \lambda_4$ for i = 1, 2, 3 (Eq.5)

Thus

$\lambda_i = \lambda_4 * a_i' / a_i$ for i = 1, 2, 3 with $a_i \neq 0$

6. Interpolation, 6.6 Projective Collineation

Simple procedure :

Known are the points P_i und P_i' for i = 1, 2, 3 and 4

Compose matrix **P** with i = 1, 2, 3

$$A * P = A * \begin{pmatrix} P_1 & P_2 & P_3 \end{pmatrix} \underset{(Eq.2)}{=} \begin{pmatrix} \lambda_1 P_1' & \lambda_2 P_2' & \lambda_3 P_3' \end{pmatrix} \underset{(Eq.5)}{=} \lambda_4 * \begin{pmatrix} \frac{a_1'}{a_1} P_1' & \frac{a_2'}{a_2} P_2' & \frac{a_3'}{a_3} P_3' \end{pmatrix}$$

Calculate $\mathbf{P^{-1}}$ and multiply from the right by $\mathbf{P^{-1}}$:

$$A = A * P * P^{-1} = \lambda_4 * \begin{pmatrix} \frac{a_1'}{a_1} P_1' & \frac{a_2'}{a_2} P_2' & \frac{a_3'}{a_3} P_3' \end{pmatrix} * P^{-1}$$

In which the a_i and a_i' are known from (Eq. 1b).

6. Interpolation, 6.6 Projective Collineation

The inverse matrix P^{-1} exists, because P_1, P_2 and P_3 are linear independent.

$\lambda_4 \neq 0$ is arbitrary, because the matrix A is defined except for an unknown factor.

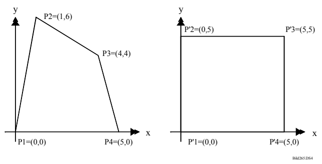
Use A * P_i = P_i' to calculate the new positions in the target image.

Note 1 : These equations were derived without resampling !
Use resampling to avoid undefined grey values in the target image.

Note 2 : Only two-dimensional problems are considered.

6. Interpolation, 6.6 Projective Collineation, Example

Example for projective collineation :
Transformation of the left tetragon onto the right square :



The vertices are known. i.e.

$$P = \begin{pmatrix} 0 & 1 & 4 \\ 0 & 6 & 4 \\ 1 & 1 & 1 \end{pmatrix} \quad P_4 = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} \quad P' = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 5 & 5 \\ 1 & 1 & 1 \end{pmatrix} \quad P_4' = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$$

6. Interpolation, 6.6 Projective Collineation, Example

Known are the vertices

$$P = \begin{pmatrix} 0 & 1 & 4 \\ 0 & 6 & 4 \\ 1 & 1 & 1 \end{pmatrix} \quad P_4 = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} \quad P' = \begin{pmatrix} 0 & 0 & 5 \\ 0 & 5 & 5 \\ 1 & 1 & 1 \end{pmatrix} \quad P_4' = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$$

Calculate P^{-1} , a, a'

$$P^{-1} = \begin{pmatrix} -0,1 & -0,15 & 1 \\ -0,2 & 0,2 & 0 \\ 0,3 & -0,05 & 0 \end{pmatrix} \quad a = P^{-1} P_4 = \begin{pmatrix} 0,5 \\ -1 \\ 1,5 \end{pmatrix} \quad P'^{-1} = \begin{pmatrix} 0 & -0,2 & 1 \\ -0,2 & 0,2 & 0 \\ 0,2 & 0 & 0 \end{pmatrix} \quad a' = P'^{-1} P'_4 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

thus A :

$$A = \lambda_4 \begin{pmatrix} \frac{a_1'}{a_1} P_1' & \frac{a_2'}{a_2} P_2' & \frac{a_3'}{a_3} P_3' \end{pmatrix} * P^{-1}$$

6. Interpolation, 6.6 Projective Collineation, Example

Choose λ_4 = 1 and get :

$$\lambda_1 = \frac{a_1'}{a_1} * \lambda_4 = 2 \qquad \lambda_2 = \frac{a_2'}{a_2} * \lambda_4 = 1 \qquad \lambda_3 = \frac{a_3'}{a_3} * \lambda_4 = 0,666...$$

Therefore get for A :

$$A = \begin{pmatrix} 0 & 0 & 3,333.. \\ 0 & 5 & 3,33.. \\ 2 & 1 & 0,66.. \end{pmatrix} * \begin{pmatrix} -0,1 & -0,15 & 1 \\ -0,2 & 0,2 & 0 \\ 0,3 & -0,05 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -0,16665 & 0 \\ 0 & 0,83335 & 0 \\ -0,2 & -0,13335 & 2 \end{pmatrix}$$

A = 1 * { } * P^{-1}

6. Interpolation, 6.6 Projective Collineation, Example

Prove for the known vertices :

$$P_1' = A * \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \xrightarrow[\text{div. by } \lambda_1]{} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad P_2' = A * \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix}$$

$$P_3' = A * \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3,33.. \\ 3,33.. \\ 0,66.. \end{pmatrix} \xrightarrow[\text{div. by } \lambda_3]{} \begin{pmatrix} 5 \\ 5 \\ 1 \end{pmatrix} \qquad P_4' = A * \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$$

6. *Interpolation, 6.6 Projective Collineation, Example*

Original Image

Adjusted Image without Resampling

6. *Interpolation, 6.6 Projective Collineation, Example*

Original Image

Adjusted Image using Resampling

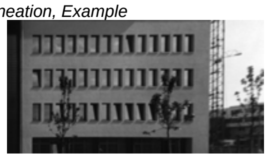
6. *Interpolation, 6.6 Projective Collineation, Example*

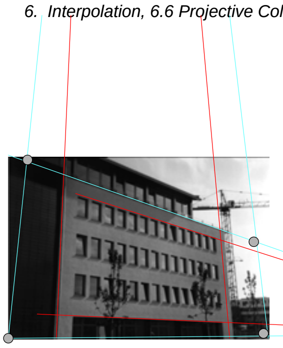
Example of rectification : Photo shot in Brasov Pizzeria, 8 Bit grey level image

6. *Interpolation, 6.6 Projective Collineation, Example*

6. *Interpolation, 6.6 Projective Collineation, Example*

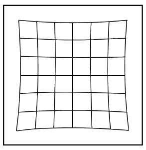
6. *Interpolation, 6.6 Projective Collineation, Example*



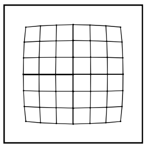


- Analyze image by Hough transform for lines.
- Calculate vanishing points (red).
- Connect these points with opposite corners of image (cyan lines).
- Calculate intersection points (grey) to define a tetragon for the projective collineation.

6. Interpolation, 6.7 Lens Distortion



DBV Bild4223.DS4

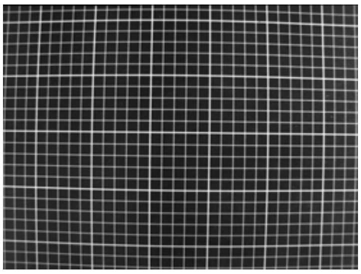


DBV Bild4225.DS4

Distortion by lens aberration :
- Pillow like distortion (above)
- Barrel like distortion (below)
- Mathematical modeling necessary

- Model :
  - „ideal" pixel image resulting from pin hole camera (geometrical optical laws !),
  - pixels with square aspect ratio (linear interpolation !)
  - radial symmetric distortion with origin in image center

6. Interpolation, 6.7 Lens Distortion



Conventional objective

Image „little_focal_length.bmp" in the folder

6. Interpolation, 6.7 Lens Distortion, Conventional Digital Camera

6. Interpolation, 6.7 Lens Distortion

$$\vec{x}_d = (1 + k_1 * r^2 + k_2 * r^4 + ...) * \vec{x}_1$$

$$\vec{x}_1 = (1 + k_1 * r^2 + k_2 * r^4)^{-1} * \vec{x}_d$$

Parameters k_1 and k_2 are unknown and specific for the lens and have to be defined by a calibrating process.

$k_1 > 0$ results in a pillow like distortion

$k_1 < 0$ results in a barrel like distortion

Radius : $r^2 = x^2 + y^2$

$\vec{x}_d = \begin{pmatrix} x_d \\ y_d \end{pmatrix}$: distorted coordinates, analog for pinhole coordinates $\vec{x}_1$

6. Interpolation, 6.7 Lens Distortion

- Image of calibration pattern
- Linear Interpolation to adjust the pixel aspect ratio
- High-pass filtering to get edges
- Hough-Transformation (Chap. 7) to find linear equations of edges
- Calculation of approximate intersection points
- Calculation in the vincinity of intersecting points suppixel precise calibation points

6. Interpolation, 6.7 Lens Distortion

- Assume image center to be optical center (according to model)
- Calculation of parameters k_1 and k_2 by overdefined equation system
- Solution by Householder-Transformation
- Result : k_1 und k_2

Application :
- Transform image data by resampling

6. Interpolation, 6.8 Correction of Distorted Images

Projective Collineation
- Gray value overflow and underflow suppressed
- Resampling necessary
- 2D interpolation

Radial symmetric distortion
- Origin of coordinate system shifted to image center, i. e.

$$\vec{x}_{target} = \left(1 + k * r^2\right) * \left(\vec{x}_{source} - \vec{x}_m\right) + \vec{x}_m$$

- Empirical choice of k
- Position data restricted to image frame
- 2D interpolation

6. Interpolation, 6.8 Correction of Distorted Images

6. Interpolation, 6.8 Correction of Distorted Images

Original Image

Corrected Color Image

Summary of Chapter 6

- Importance of Resampling
- Linear interpolation
- Bilinear interpolation
- Tri-linear interpolation
- Projective Collineation
  (Mathematical fundamentals: homogeneous coordinates, matrix inversion)
- Treatment of radial symmetric distortions
- Modeling of optical lenses
- Parameter calibration of optical lenses
- Application on images