CS7646 Machine Learning for Trading — Assignment 1
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I. INTRODUCTION

The goal of this assignment is to explore a betting strategy by implementing the following pseudocode:

```
episode winnings = $0
while episode winnings < $80:
     won = False
     bet amount = $1
     while not won
          wager bet amount on black
          won = result of roulette wheel spin
          if won == True:
               episode winnings = episode winnings + bet amount
          else:
               episode winnings = episode winnings - bet amount
               bet amount = bet amount * 2
```

The algorithm was revised to simulate 1000 successive bets on spins of the roulette wheel using the betting scheme outlined above. Winnings (results from a spin) were tracked by storing them in a numpy array, where the initial value is set to 0 (just before the first spin). Winnings were capped at $80, that is, for an episode, if 80$ in winnings is reached, betting is stopped.

II. Experiment 1: Explore the Strategy and Make Some Charts

Experiment 1 utilized an unlimited bankroll and was executed based on the following requirements:

Figure 1: Run your simple simulator 10 times and track the winnings, starting from 0 each time. Plot all 10 runs on one chart using matplotlib functions. The horizontal (X) axis should range from 0 to 300, the vertical (Y) axis should range from -256 to +100. Note that we will not be surprised if some of the plot lines are not visible because they exceed the vertical or horizontal scales.

Figure 2: Run your simple simulator 1000 times. Plot the mean value of winnings for each spin using the same axis bounds as Figure 1. Add an additional line above and below the mean at mean + standard deviation and mean - standard deviation of the winnings at each point.

Figure 3: Use the same data you used for Figure 2 but plot the median instead of the mean. Be sure to include the standard deviation lines above and below the median as well.

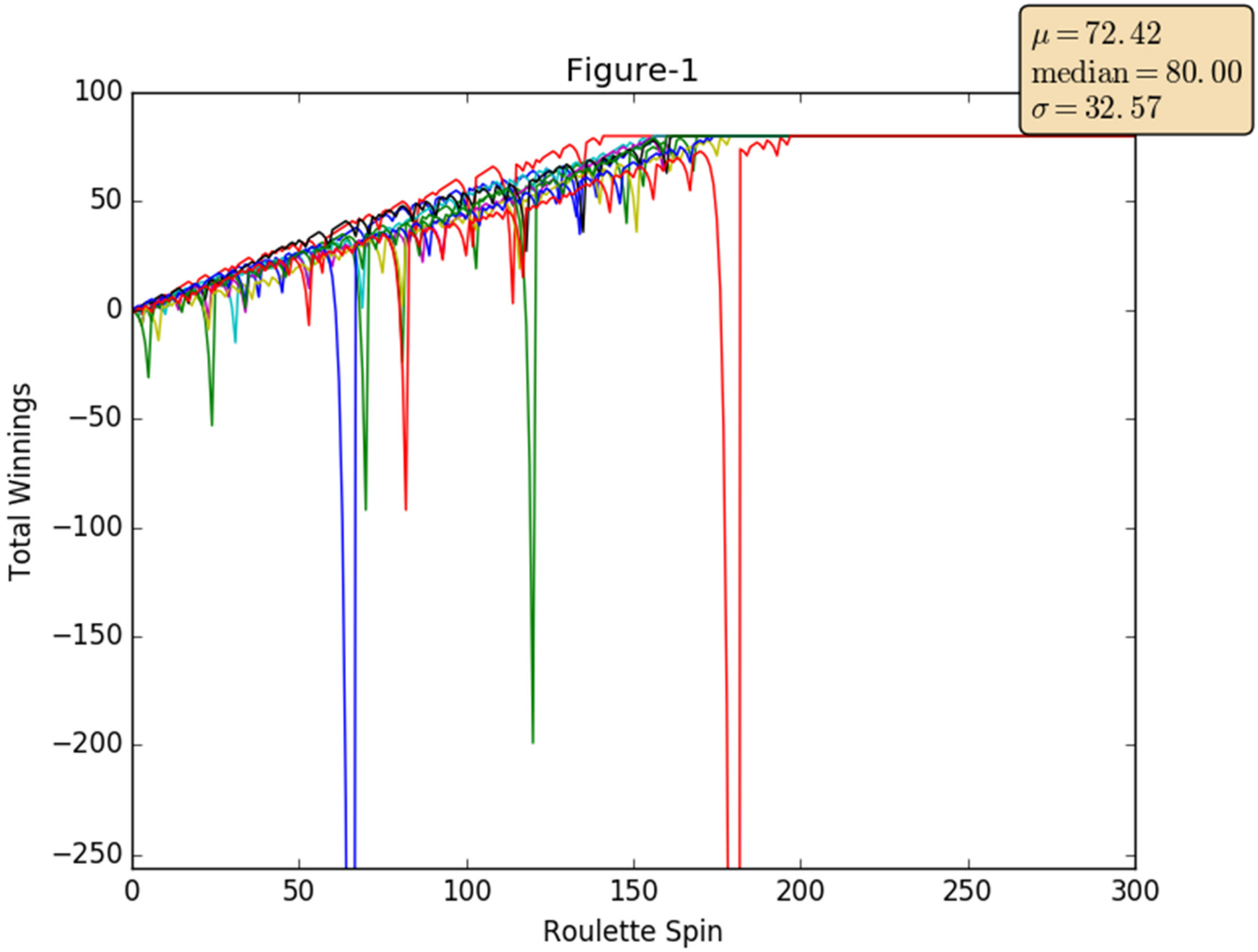


Figure-1
$\mu = 72.42$
$\mathrm{median} = 80.00$
$\sigma = 32.57$
Total Winnings
Roulette Spin

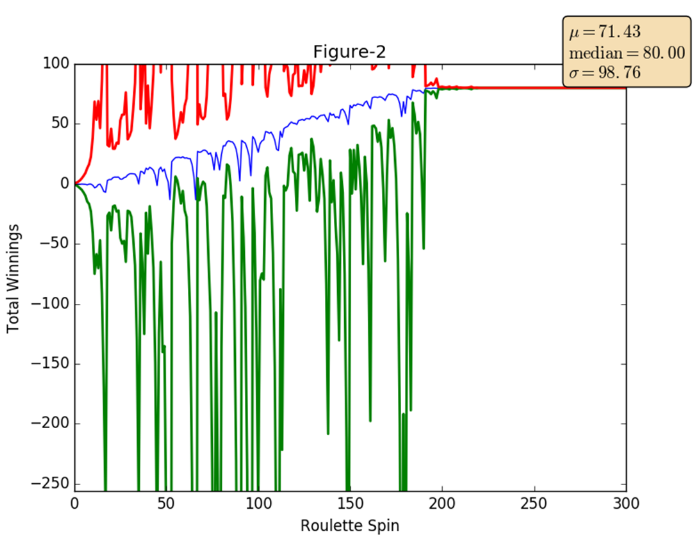


Figure-2
$\mu = 71.43$
$\mathrm{median} = 80.00$
$\sigma = 98.76$
Total Winnings
Roulette Spin

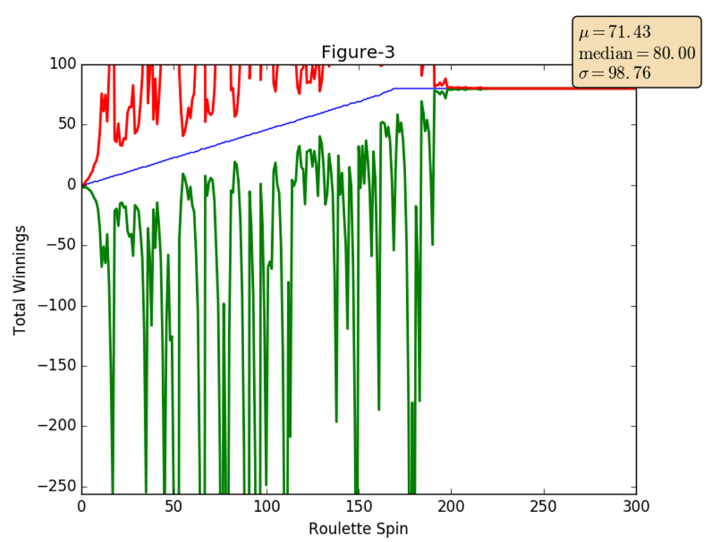


Figure-3
$\mu = 71.43$
$\mathrm{median} = 80.00$
$\sigma = 98.76$
Total Winnings
Roulette Spin

1. In Experiment 1, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning.

   Assume normal distribution, $N(\mu, \sigma^2)$, then $Z = \frac{(X - \mu)}{\sigma}$.

   $\mu = 71.43$

   $\sigma = 98.76$

   $\dot{X} = 80$

   $P(X \geq 80) = 1 - P(X \leq 80)$

   $P(Z \leq [(80 - 71.43) / 98.76]) = 0.087$

   Complementary Cumulative Mean Table:
   0.46414, or ~**46% probability of winning $80 within 1000 sequential bets**.

2. In Experiment 1, what is the expected value of our winnings after 1000 sequential bets? Explain your reasoning. Go here to learn about expected value: https://en.wikipedia.org/wiki/Expected_value

   In probability theory, the expected value of a random variable, intuitively, is the long-run average value of repetitions of the experiment it represents[1]. Given $\mu = \sum_{i=1}^{N} Xi$, the average, or mean value and $E[X] = \sum_{i=1}^{N} Xi\, Pi$ the expected value. The from ***Figures 2 and 3***, $\mu = 71.43$. This was calculated by taking the mean of all spins 1000*1000 (1000 sequential bets by 1000 simulations). Thus, the expected value of winnings is **71.43$** after 1000 sequential bets.

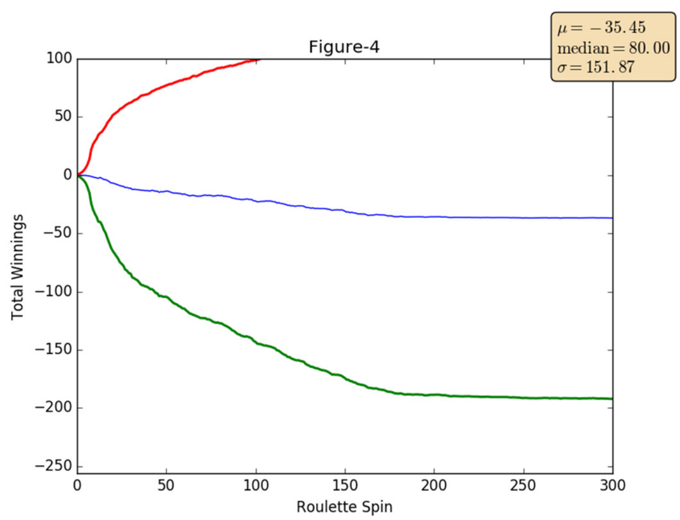
3. In Experiment 1, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

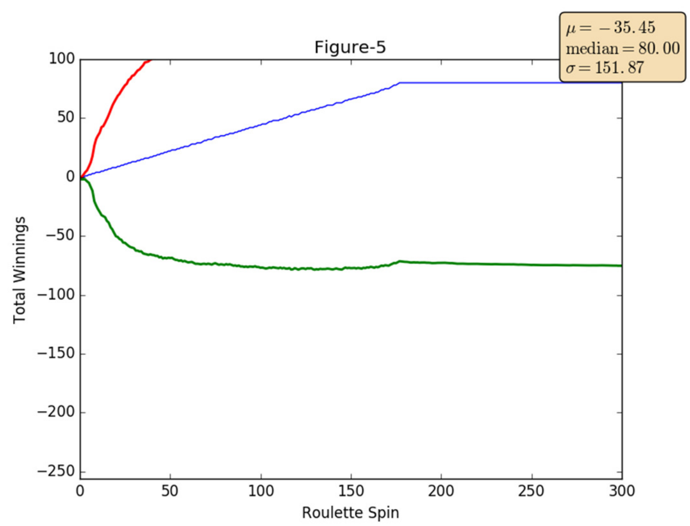
   $\sigma = \sqrt{\sum_{i=1}^{N} (xi - \mu i)^2}$. The underlying uncertainty in generating the data is high, ~50% (18/38 for the algorithm's bet on black approach). This will cause a large difference in the Xi. This large uncertainty in combination with an unlimited bankroll causes a large variance in the $(xi - \mu i)^2$. Thus, until the max value of 80$ is achieved, σ never stabilizes, or converges. Once the 80$ threshold is reached, σ stabilizes and converges to zero.

III. Experiment 2: A More Realistic Gambling Simulator

Experiment 2 provides a more realistic simulation by giving the gambler a $256 bank roll. If the bankroll is exceeded. The game is over. Experiment 2 repeats experiment 1, but with the bankroll. Note that once the player has lost all of their money (i.e., episode winnings = -256), betting is stopped and (-256) is filled forward. A necessary corner case that was addressed handles the situation where the next bet should be $N, but the bankroll only has $M (where M<N). Thus, the bet must be $M. The following charts were created:

> **<u>Figure 4</u>**: Run your realistic simulator 1000 times. Plot the mean value of winnings for each spin using the same axis bounds as Figure 1. Add an additional line above and below the mean at mean + standard deviation and mean - standard deviation of the winnings at each point.
>
> **<u>Figure 5</u>**: Repeat the same experiment as in Figure 4 but use the median instead of the mean. Be sure to include the standard deviation lines above and below the median as well.





4. In Experiment 2, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning.

Assume normal distribution, $N(\mu, \sigma^2)$, then $Z = \frac{(X - \mu)}{\sigma}$.

μ = -35.45

σ = 151.87

$\dot{X} = 80$

P(x≥80) = 1 - P(x≤80)

P(Z ≤ [(80 - -35.45) / 151.87]) = 0.76

From a Complementary Cumulative Mean Table:
0.22363, or ~**22% probability of winning $80 within 1000 sequential bets**.

5. In Experiment 2, what is the expected value of our winnings after 1000 sequential bets? Explain your reasoning.

   In probability theory, the expected value of a random variable, intuitively, is the long-run average value of repetitions of the experiment it represents[2]. Given $\mu = \sum_{i=1}^{N} Xi$, the average, or mean value and $E[X] = \sum_{i=1}^{N} Xi\, Pi$ the expected value. The from ***Figures 4 and 5***, $\mu = -35.45$. This was calculated by taking the mean of all spins 1000*1000 (1000 sequential bets by 1000 simulations). Thus, the expected value of winnings is -**35.45$** (loss) after 1000 sequential bets.

6. In Experiment 2, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

   $\sigma = \sqrt{\sum_{i=1}^{N}(xi - \mu i)}^2$. The underlying uncertainty in generating the data is high, ~50% (18/38 for the algorithm's bet on black approach). This will cause a large difference in the Xi. However, unlike the data from the first Experiment, this data was generated under conditions that the gambler had a finite bankroll to spend. So, notice from the figures that σ is still large, it is much more stable. This stability id due to the limit placed on how much the gambler could lose, thus limiting the variance in the $(xi - \mu i)^2$. From Figure 4, that as the mean of the winnings stabilizes at ~ -35.00, σ seems to slowly converge to a stable value.

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[1] https://en.wikipedia.org/wiki/Standard_normal_table