Exercise 2.3.1

Show the following formulas by induction on n starting at n = 1.

a) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Basis case: LHS when n = 1 yields 1.

RHS when n = 1 then 1(1 + 1)/2 = 1(2)/2 = 1
LHS = RHS therefore s(n) is true when n is 1.
Inductive step: Assume that n $\geq$ 1 and that s(n) is true as proved above.
I must prove that s(n + 1) is true which is:

$$\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2} \tag{2}$$

Replacing the LHS of the above equation with the following:

$\sum_{i=1}^{n+1} i = (\sum_{i=1}^{n} i) + (n + 1)$

which can be rewritten as:

$\sum_{i=1}^{n+1} i = \frac{n(n+1)}{2} + (n + 1) = \frac{n+(n+1)+2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$

The final equation the right is equal to (2) proving that s(n + 1) is also true. Therefore s(n) hold for all n $\geq$ 1