

## Exercise 2.3.1

Show the following formulas by induction on  $n$  starting at  $n = 1$ .

$$\text{a) } \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

Basis case: LHS when  $n = 1$  yields 1.

RHS when  $n = 1$  is 1

LHS = RHS therefore  $s(n)$  is true when  $n$  is 1.

Inductive step: Assume that  $n \geq 1$  and that  $s(n)$  is true as proved above.

I must prove that  $s(n + 1)$  is true which is:

$$\sum_{i=1}^{n+1} i^3 = \frac{(n+1)^2(n+2)^2}{4} \quad (2)$$

Replacing the LHS of the above equation with the following:

$$\sum_{i=1}^{n+1} i^3 = \left(\sum_{i=1}^n i^3\right) + (n+1)^3$$

which can be rewritten as:

$$\begin{aligned} \sum_{i=1}^{n+1} i^3 &= \frac{n^2(n+1)^2}{4} + (n+1)^3 \\ &= \frac{n^2(n+1)^2 + 4(n+1)^3}{4} \\ &= \frac{(n+1)^2(n^2 + 4(n+1))}{4} \\ &= \frac{(n+1)^2(n^2 + 4n + 4)}{4} \\ &= \frac{(n+1)^2(n+2)^2}{4} \end{aligned}$$

The final equation the right is equal to (2) proving that  $s(n + 1)$  is also true. Therefore  $s(n)$  hold for all  $n \geq 1$