Exercise 2.3.1

Show the following formulas by induction on n starting at n = 1.

a) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Basis case: LHS when n = 1 yields 1.

RHS when n = 1 is 1(1 + 1)(2 + 1)/6 = 6/6 = 1
LHS = RHS therefore s(n) is true when n is 1.
Inductive step: Assume that n $\geq$ 1 and that s(n) is true as proved above.
I must prove that s(n + 1) is true which is:

$$\sum_{i=1}^{n+1} i^2 = \frac{(n+1)(n+2)(2n+3)}{6} \qquad (2)$$

Replacing the LHS of the above equation with the following:

$$\sum_{i=1}^{n+1} i^2 = \left(\sum_{i=1}^{n} i^2\right) + (n+1)^2$$

which can be rewritten as:

$$\sum_{i=1}^{n+1} i^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{n(n+1)(2n+1)+6(n+1)^2}{6}$$

$$= \frac{(n+1)(2n^2+n+6n+6)}{6}$$

$$= \frac{(n+1)(2n^2+7n+6)}{6}$$

$$= \frac{(n+1)(n+2)(2n+3)}{6}$$

The final equation the right is equal to (2) proving that s(n + 1) is also true. Therefore s(n) hold for all n $\geq$ 1