Exercise 2.3.1

Show the following formulas by induction on n starting at n = 1.

d) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{(n+1)}$

Basis case: LHS when n = 1 yields 1/2.

RHS when n = 1 is 1/2
LHS = RHS therefore s(n) is true when n is 1.
Inductive step: Assume that n $\geq$ 1 and that s(n) is true as proved above.
I must prove that s(n + 1) is true which is:

$$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \frac{n+1}{(n+2)} \tag{2}$$

Replacing the LHS of the above equation with the following:

$$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \left(\sum_{i=1}^{n} \frac{1}{i(i+1)}\right) + \frac{1}{(n+1)(n+2)}$$

which can be rewritten as:

$$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \frac{n}{(n+1)} + \frac{1}{(n+1)(n+2)}$$

$$= \frac{n(n+2)+1}{(n+1)(n+2)}$$

$$= \frac{n^2+2n+1}{(n+1)(n+2)}$$

$$= \frac{(n+1)^2}{(n+1)(n+2)}$$

$$= \frac{(n+1)}{(n+2)}$$

The final equation the right is equal to (2) proving that s(n + 1) is also true. Therefore s(n) holds for all n $\geq$ 1