Exercise 2.3.1

Show the following formulas by induction on n starting at n = 1.

c) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Basis case: LHS when n = 1 yields 1.

RHS when n = 1 is 1
LHS = RHS therefore s(n) is true when n is 1.
Inductive step: Assume that n $\geq$ 1 and that s(n) is true as proved above.
I must prove that s(n + 1) is true which is:

$$\sum_{i=1}^{n+1} i^3 = \frac{(n+1)^2(n+2)^2}{4} \tag{2}$$

Replacing the LHS of the above equation with the following:

$$\sum_{i=1}^{n+1} i^3 = (\sum_{i=1}^{n} i^3) + (n+1)^3$$

which can be rewritten as:

$$\sum_{i=1}^{n+1} i^3 = \frac{n^2(n+1)^2}{4} + (n+1)^3$$

$$= \frac{n^2(n+1)^2+4(n+1)^3}{4}$$

$$= \frac{(n+1)^2(n^2+4(n+1))}{4}$$

$$= \frac{(n+1)^2(n^2+4n+4))}{4}$$

$$= \frac{(n+1)^2(n+2)^2}{4}$$

The final equation the right is equal to (2) proving that s(n + 1) is also true. Therefore s(n) hold for all n $\geq$ 1