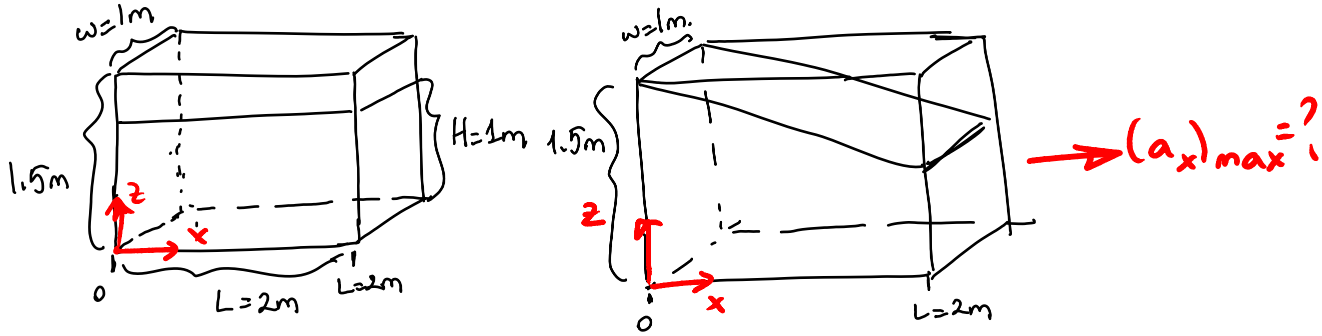
An open rectangular tank 1 m wide and 2 m long contains gasoline to a depth of 1m. If the height of the tank sides is 1.5 m, what is the maximum horizontal acceleration (along the long axis of the tank) that can develop before the gasoline would begin to spill?

Solution:



$$-\vec{\nabla}p - \rho g\hat{k} = \rho\vec{a}$$

$$-\left(\frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}\right) - \rho g\hat{k} = \rho a_x\hat{i}$$

$\hat{i}$: $\frac{\partial p}{\partial x} = -\rho a_x$

$\hat{j}$: $\frac{\partial p}{\partial y} = 0$

$\hat{k}$: $\frac{\partial p}{\partial z} = -\rho g$

$p = p(x,z)$

Total differential of $p=p(x,z)$:

$$dp = \underbrace{\frac{\partial p}{\partial x}}_{-\rho a_x} dx + \underbrace{\frac{\partial p}{\partial z}}_{-\rho g} dz$$

$$\int dp = -\int \rho a_x dx - \int \rho g\, dz$$

$$\boxed{p = -\rho a_x x - \rho g z + C}$$

$P = P_{atm}$ @ $z = z_s$ $\Rightarrow$ $P_{atm} = -\rho a_x x - \rho g z_s + C$

$$z_s = \underbrace{\frac{C - P_{atm}}{\rho g}}_{C_1 \text{ (new constant)}} - \frac{a_x}{g}x$$

$$\boxed{z_s = C_1 - \frac{a_x}{g}x}$$ Equation of the surface

@ $x=0$ $z_s = 1.5m$ $\Rightarrow$ $1.5 = C_1 - \cancel{\frac{a_x}{g}(0)}$ $\Rightarrow$ $C_1 = 1.5$ then $\boxed{z_s = 1.5 - \frac{a_x}{g}x}$

Volume of the gasoline inside the tank in motion and at rest should be the same before the gasoline would begin to spill...

$$\forall_{initial} = \forall_{in\ motion}$$

$$1 \times 2 \times 1 = \int_0^{L=2} z_s \, w \, dx$$

$w = 1$

$$z_s = C_1 - \frac{a_x}{g} x$$

$$2 = \left(C_1 x - \frac{a_x}{2g} x^2 \right) \Big|_0^2$$

$$\cancel{2} = \cancel{2} C_1 - \cancel{4} \frac{a_x}{\cancel{2} g}$$

$C_1 = 1.5$

$$1 = C_1 - \frac{a_x}{g}$$

$g = 10$

$$a_x = (1.5 - 1) \times 10 = 0.5 \times 10 = 5 \, m/s^2$$