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Assignment 1

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Problem Set 1

1) a) Force: N

$1MPa = \frac{1MN}{m^2}$

$\frac{MN}{m^2} = \frac{P}{m^2}$

$P = MN$

∴ P is a force with units MN

b) $MPa = \frac{M \cdot mm}{mm^4}$

$\frac{MN}{m^2} = \frac{M \cdot mm}{mm^4}$

$10^6 \frac{N}{m^2} = \frac{M}{mm^3}$

$10^6 \frac{N}{m^2} = \frac{M}{(10^{-3}m)^3}$

$M = 10^{-3} Nm$

$M = Nmm$

∴ M has units Nmm

c) $\frac{1}{mm} = \frac{Nmm}{E\,mm^4}$

$E = \frac{N}{mm^2} = \frac{MN}{m^2} = MPa$

∴ E has units Nmm^{-2}

Note: rad is a dimensionless unit, so we can exclude it from our unit analysis

d) $J = MPa \cdot \frac{mm}{mm} \cdot mm^2 \cdot L$

$N \cdot m = \frac{MN}{m^2} \cdot mm^2 \cdot L$

$N \cdot m = \frac{10^6 N}{(10^3 mm)^2} \cdot mm^2 \cdot L$

$N \cdot m = N \cdot L$

$L = m$

∴ L is a length with units m

Note: we ignore $\frac{1}{2}$ because it is a dimensionless constant.

e) $N = \frac{mm^2 f_y \cdot mm}{mm}$

$N = f_y \cdot mm^2$

$f_y = \frac{N}{mm^2} = \frac{MN}{m^2} = MPa$

∴ f_y is a stress with units Nmm^{-2}

We exclude ϕ_s and θ from our analysis because they are dimensionless

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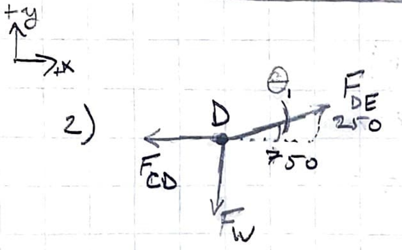
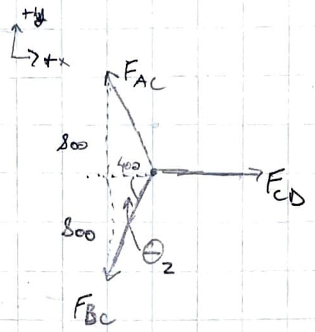
2)





$\tan \theta_1 = \frac{250}{750}$

$\theta_1 = \tan^{-1} \frac{250}{750}$

$\theta_1 = 18.4°$

$F_W = mg$

$\boxed{F_W = 715\ N}$

$\sum F_y = 0$

$0 = F_{DEy} - F_W$

$F_W = F_{DE} \sin \theta_1$

$F_{DE} = \frac{F_W}{\sin \theta}$

$\boxed{F_{DE} = 2.26 \cdot 10^3 N}$

$\sum F_x = 0$

$0 = F_{DEx} - F_{CD}$

$F_{CD} = F_{DE} \cos \theta_1$

$\boxed{F_{CD} = 2.15 \cdot 10^3 N}$

$\theta_2 = \tan^{-1}\left(\frac{800}{400}\right)$

$= 63.4°$

F_{AC} and F_{BC} are symmetrical about F_{CD}

$\therefore F_{AC} = F_{BC}$

$\sum F_x = 0$

$0 = F_{CD} - F_{AC} - F_{BCx}$

$F_{CD} = 2(F_{AC} \cos \theta_2)$

$F_{AC} = \frac{F_{CD}}{2 \cos \theta_2}$

$\boxed{F_{AC} = 2.40 \cdot 10^3 N}$

$F_{BC} = F_{AC}$

$\boxed{F_{BC} = 2.40 \cdot 10^3 N}$

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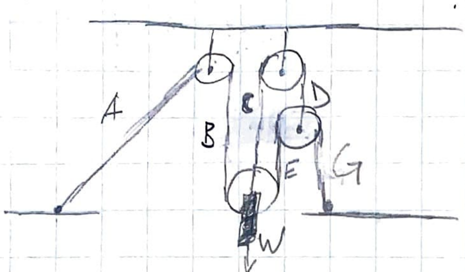
3)

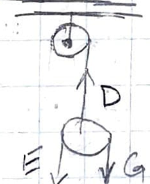


$A = B = E = G$
$C = D$

$\uparrow +y$

$\sum F_y = 0$
$0 = B + E + C - W$
$W = B + E + C, \quad B = E$
$\boxed{W = 2B + C}$ (1)

$\uparrow +y$



$\sum F_y = 0$
$0 = D - (E + G), \quad E = G = B$
$D = 2B, \quad D = C$
$\boxed{C = 2B}$ (2)

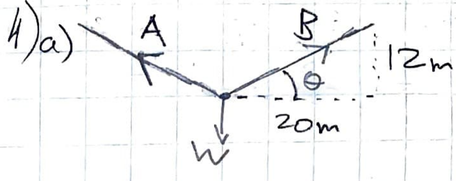
Sub (2) into (1)

$W = 2B + 2B$
$B = \frac{W}{4}$
$C = 2B$
$C = \frac{W}{2}$

Segments A, B, E, G are all part of the same rope, all with Tension T_1, and segments C and D are part of the same rope, with tension T_2.

$\boxed{T_1 = \frac{W}{4} \qquad T_2 = \frac{W}{4}}$

4) a)



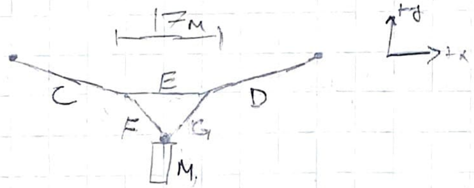
Since A and B are symmetrical, $A = B$

$\theta = \tan^{-1} \frac{12}{20}$
$= 31.0°$

$\sum F_y = 0$
$0 = A_y + B_y - W$
$W = A\sin\theta + B\sin\theta; \quad A = B$
$W = 2A\sin\theta$
$A = \frac{Mg}{2\sin\theta}$
$A = 1.335 \text{ kN}$
$A = B$
$\boxed{A, B = 1.335 \text{ kN}}$

Problem Set 1 Con'd

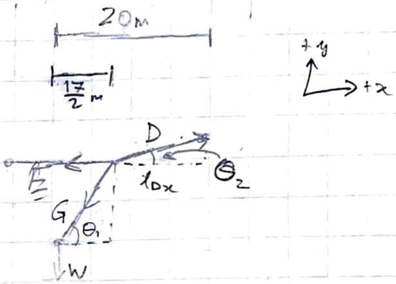
4) b)



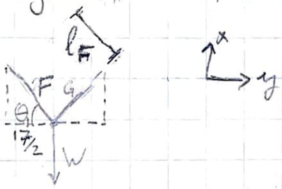
$l_C = l_D = l_F = l_G = \frac{1}{2} l_A$

$l_C, l_D, l_F, l_G = \frac{1}{2}\sqrt{20^2+12^2}$

$l_C, l_D, l_F, l_G = 11.66\text{ m}$



Due to symmetry, A = B, C = D



$\theta_1 = \cos^{-1}\left(\frac{17}{2l_F}\right)$
$= 43.2°$

$l_{Dx} = 20\text{m} - \frac{17}{2}\text{m}$
$= 11.5\text{m}$

$\sum F_y = 0$
$0 = F_y + G_y - W$
$W = F\sin\theta_1 + G\sin\theta_1,\ F = G$
$Mg = 2F\sin\theta_1$
$F = \frac{Mg}{2\sin\theta_1}$
$\boxed{F, G = 1.003\text{ kN}}$

$\theta_2 = \cos^{-1}\left(\frac{l_{Dx}}{l_D}\right)$
$= \cos^{-1}\left(\frac{11.5}{11.66}\right)$
$= 9.56°$

$\sum F_y = 0$
$0 = D_y - G_y$
$D\sin\theta_2 = G\sin\theta_1$
$D = \frac{G\sin\theta_1}{\sin\theta_2}$
$D = 4.16\text{ kN}$
$D = C$
$\boxed{C, D = 4.16\text{ kN}}$

$\sum F_x = 0$
$0 = D_x - E - G_x$
$D\cos\theta_2 = E + G\cos\theta_1$
$E = D\cos\theta_2 - G\cos\theta_1$
$\boxed{E = 3.37\text{ kN}}$

* This is <u>not</u> feasible; the tension in wires C and D are greater than 4 kN

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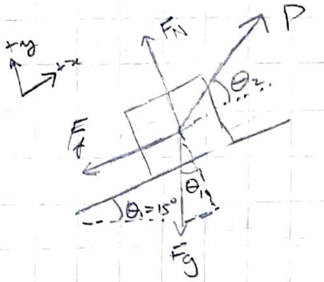
5) a)



• find equilibrium:

$$\sum F_y = 0$$
$$0 = F_N + P_y - F_{g_y}$$
$$F_N = mg\cos\theta_1 - P\sin\theta_2$$

$$\sum F_x = 0$$
$$0 = P_x - F_f - F_{g_x}$$
$$P\cos\theta_2 = \mu_s F_N + mg\sin\theta_1$$
$$P\cos\theta_2 = \mu_s(mg\cos\theta_1 - P\sin\theta_2) + mg\sin\theta$$
$$P\cos\theta_2 + \mu_s P\sin\theta_2 = mg\sin\theta_1 + \mu_s mg\cos\theta_1$$
$$\boxed{P = \frac{mg(\sin\theta_1 + \mu_s\cos\theta_1)}{\cos\theta_2 + \mu_s\sin\theta_2}} \quad (1)$$
$$P = 779\,N$$

for equilibrium, 779N are required.

∴ Any force greater than 779 N will result in the box moving

b) $$P = \frac{mg(\sin\theta_1 + \mu_s\cos\theta_1)}{\cos\theta_2 + \mu_s\sin\theta_2}$$

$$\frac{d}{d\theta_2}(P) = \frac{d}{d\theta}\left(\frac{mg\sin\theta_1 + \mu_s\cos\theta_1}{\cos\theta_2 + \mu_s\sin\theta_2}\right)$$

$$\frac{dP}{d\theta_2} = (mg\sin\theta_1 + \mu_s\cos\theta_1)\cdot(-1)\frac{\frac{d}{d\theta_2}(\cos\theta_2 + \mu\sin\theta_2)}{(\cos\theta_2 + \mu_s\sin\theta_2)^2}$$

local maxima and minima occur when $\frac{dP}{d\theta_2} = 0$

$$0 = (mg\sin\theta_1 + \mu\cos\theta_1)\cdot(-1)\frac{\frac{d}{d\theta_2}(\cos\theta_2 + \mu_s\sin\theta_2)}{(\cos\theta_2 + \mu_s\sin\theta_2)^2}$$

$$0 = -\sin\theta_2 + \mu_s\cos\theta_2$$

$$\sin\theta_2 = \mu_s\cos\theta_2, \quad 0 \le \theta \le 90°$$
$$\frac{\sin\theta_2}{\cos\theta_2} = \mu_s$$
$$\tan\theta_2 = \mu_s$$
$$\theta_2 = \tan^{-1}(\mu_s)$$
$$\theta_2 = 36.9°$$

Using (1),
$$P = 772\,N$$

∴ the minimum force which makes the box move is 772N at an angle of 36.9°