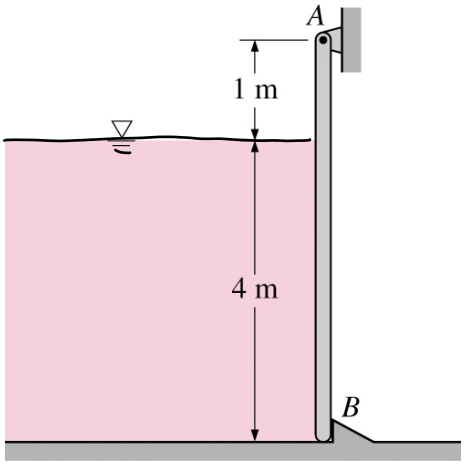
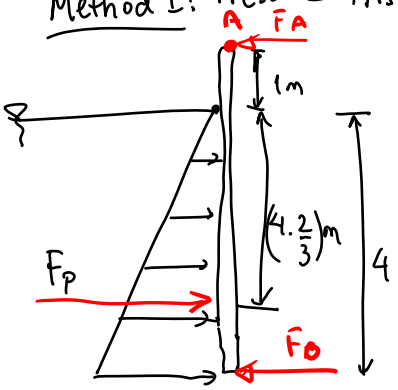
A 4-m-high, 5-m-wide rectangular plate blocks the end of a 4-m-deep freshwater channel, as shown in Figure below. The plate is hinged about a horizontal axis along its upper edge through a point A and is restrained from opening by a fixed ridge at point B. Determine the force exerted on the plate by the ridge. (10 points)



Method I: Pressure Prism: In equilibrium: $\Sigma M_A = 0$ (Moment about an axis passing through A is zero)



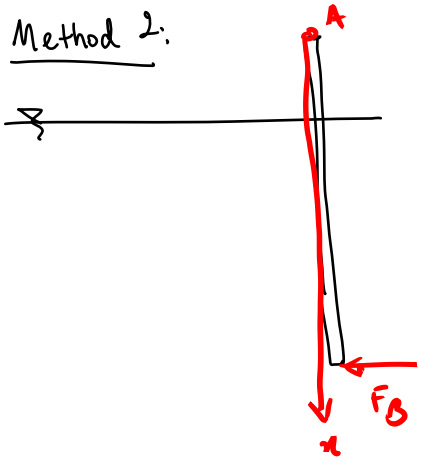
$$F_P = \frac{1}{2}\rho g 4 \underbrace{(4\times 5)}_{\text{Area}} = \frac{1}{2}\times 1000 \times 10 \times 4 \times \underbrace{20}_{10} = 400{,}000\ N$$

$$F_P\left(1 + 4\times\frac{2}{3}\right) = F_B \times 5$$

$$F_B = F_P \frac{11}{3}\cdot\frac{1}{5} = 400{,}000 \times \frac{11}{3}\times\frac{1}{5}$$

$$\boxed{F_B \cong 293{,}333\ N}$$

Method 2:



$$\text{Opening moment} = \int_A x\,p\,dA$$

$$= \int_{x=1}^{x=5} x\,\rho g (x-1)\,\underbrace{5\,dx}_{dA}$$

$$= 5\rho g \int_{x=1}^{x=5} (x^2 - x)\,dx$$

$$\text{Opening moment} = 5\rho g \left[\frac{x^3}{3} - \frac{x^2}{2}\right]\Bigg|_{x=1}^{x=5}$$

$$= 5 \times 1000 \times 10 \times \left(\frac{5^3}{3} - \frac{5^2}{2} - \frac{1^3}{3} + \frac{1^2}{2}\right)$$

$$= 1{,}466666.7 \text{ Nm}$$

$$\text{Closing moment} = 5F_B$$

$$\text{Closing moment} = \text{Opening moment}$$

$$5F_B = 1466666.7$$

$$\boxed{F_B \simeq 293333 \text{ N}}$$