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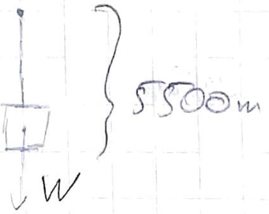
Assignment 2

Problem Set 2

1)



a) • Maximum stress on the wire occurs at top of cable (Surface level)

$W = W_{lift} + W_{people} + W_{cable}$

$W = 16 + 20(0.85) + 77 \cdot 5500 \cdot \left(\pi \left(\frac{0.006}{2}\right)^2\right) \cdot n$

where n is the number of strands

$W = 33 + 11.9742n$; in kN

$\frac{\sigma_{required}}{n} = \sigma_{allowable}$

$\frac{(33 + 11.9742n)\frac{1000N}{kN}}{\pi \cdot 3^2 \cdot n} = 650$

$33 + 11.9742n = 5.85\pi n$

$33 = 18.378n - 11.9742n$

$n = \frac{33}{6.4038}$

$n = 5.15$

∴ Since we can only use whole numbers of strands, $\boxed{6 \text{ strands}}$ are required

b) $\sigma_{ult} = \sigma_{total}$

$1300 = \frac{W}{A_{strand} \cdot n}$

$1300 = \frac{16000 + 20 \cdot 850 + 77000\pi\left(\frac{0.006}{2}\right)^2 \cdot 6 \cdot d}{\pi \cdot \left(\frac{6}{2}\right)^2 \cdot 6}$, where d is the depth of the lift

$77000\pi \cdot 54 \cdot 10^{-6} \cdot d = 70200\pi - 16000 - 20 \cdot 850$

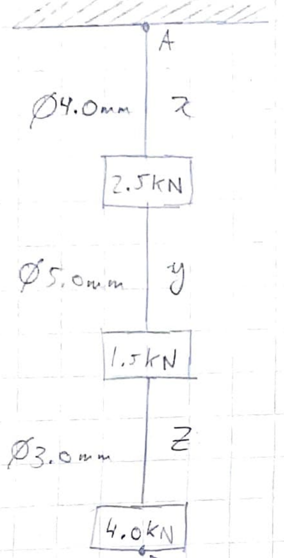
$d = \frac{187540}{13.063}$

$\boxed{d = 14.36 \text{ km}}$

∴ the maximum depth to which the lift can be lowered is 14.36 km

Problem Set 2

2)



x, y, and z are used to distinguish the indicated cables, Δl_x, Δl_y, Δl_z are the respective deformation lengths, and similarly for other quantities

$$\sigma_z = \frac{W_z}{A_z} = \frac{4000}{\pi\left(\frac{3}{2}\right)^2} = 565.88\,MPa$$

$$\sigma_y = \frac{W_z + W_y}{A_y} = \frac{4000+1500}{\pi\left(\frac{5}{2}\right)^2} = 280.11\,MPa$$

$$\sigma_x = \frac{W_z + W_y + W_z}{A_x} = \frac{4000+1500+2500}{\pi\left(\frac{4}{2}\right)^2} = 636.62\,MPa$$

$$\sigma = E\varepsilon$$

$$\varepsilon = \frac{\sigma}{E}$$

$$\frac{\Delta l}{L_o} = \frac{\sigma}{E}$$

$$\Delta l = \frac{\sigma L_o}{E}$$

$$W = \int \sigma \, d\varepsilon \cdot V_o$$

$$W = \frac{1}{2}\left(\varepsilon_z \sigma_z V_{o_z} + \varepsilon_y \sigma_y V_{o_y} + \varepsilon_z \sigma_z V_{o_z}\right)$$

$$= \frac{1}{2}\left(\frac{\Delta l_z}{L_{oz}} \sigma_z \pi\left(\frac{d_z}{2}\right)^2 L_{oz} + \frac{\Delta l_y}{L_{oy}} \sigma_y \pi\left(\frac{d_y}{2}\right)^2 L_{o_y} + \frac{\Delta l_x}{L_{o_x}} \sigma_x \pi\left(\frac{d_x}{2}\right)^2 L_{o_x}\right)$$

$$= 44.2\,J$$

$$\Delta l_z = \frac{\sigma_z L_{oz}}{E} = 7.07\,mm$$

$$\Delta l_y = \frac{\sigma_y L_{oy}}{E} = 2.80$$

$$\Delta l_x = \frac{\sigma_x L_{ox}}{E} = 5.57\,mm$$

∴ the total energy stored in all 3 cables is $\boxed{44.2\,J}$

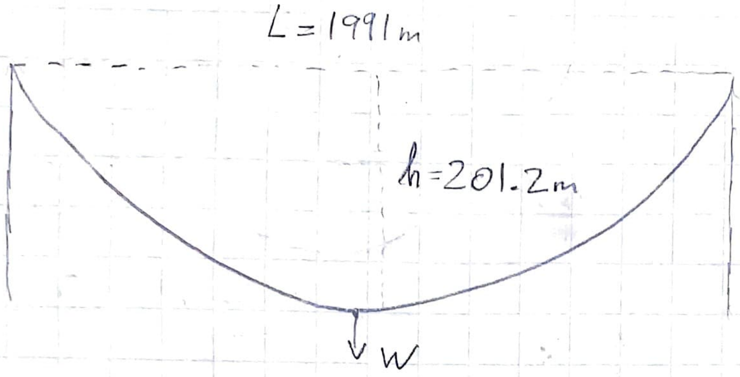
$$L_{AB} = 1750 + 2000 + 2500 + 3(300) + 7.07 + 2.80 + 5.57$$

$$L_{AB} = \boxed{7.17\,m}$$

$\boxed{\text{∴ the deformed length of AB is } 7.17\,m}$

Problem Set 2

3)



W is the distributed force per unit length accross the deck, equivelent to a point load at the center of the main cables

$$W = \frac{27.5\,kN}{m^2} \cdot 36m$$
$$= 0.990\,MN/m$$

$$T_{max} = \sqrt{\left(\frac{WL^2}{8h}\right)^2 + \left(\frac{WL}{2}\right)^2}$$
$$= 2629.8\,MN$$
$$= 2.6298 \cdot 10^9\,N$$

$$\frac{\sigma_{required}}{2n} = \sigma_{allowable}$$, where n is number of wires per cable

$$n = \frac{T_{max}}{2 A_{wires}\, \sigma_{allowable}}$$

$$n = \frac{2.6298 \cdot 10^9}{2\pi\left(\frac{5.23}{2}\right)^2 \cdot 805}$$

$$\boxed{n = 76100}$$

$\therefore$ 76100 wire are needed for the main cables

Problem Set 2

4a) i) $\sigma_P = \frac{P}{A}$

$\boxed{\sigma_P = 2.29 MPa}$

$\varepsilon_\sigma = \frac{\sigma}{E}$

$E_{concrete} = 30{,}000 \text{ MPa}$

$\boxed{\varepsilon_\sigma = 7.62 \cdot 10^{-5} \frac{mm}{mm}}$

ii) $\varepsilon_{th} = \alpha \Delta T$

$\alpha_{concrete} = 9 \cdot 10^{-6} \ /°C$

$\boxed{\varepsilon_{th} = 1.350 \cdot 10^{-4} \frac{mm}{mm}}$

No, as the temperature increases, the block is free to expand, meaning no stress will develop as a result

iii) $\varepsilon_{total} = \varepsilon_\sigma + \varepsilon_{th}$

$\frac{\Delta l}{L_o} = \varepsilon_\sigma + \varepsilon_{th}$

$\Delta l = (\varepsilon_\sigma + \varepsilon_{th}) L_o$

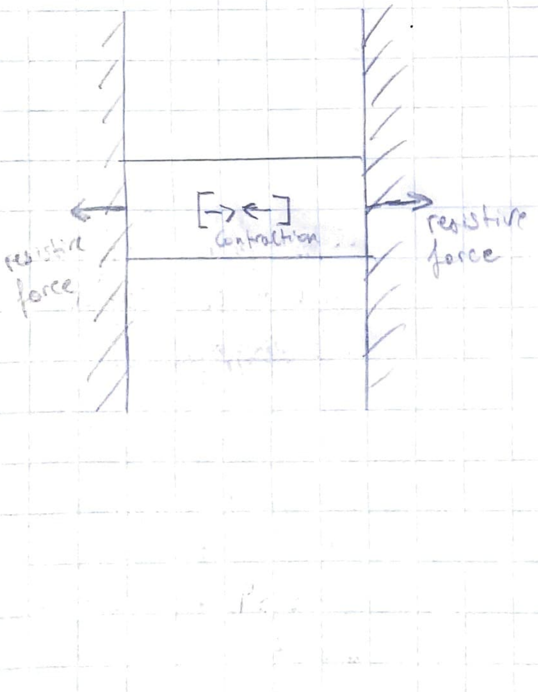
$\boxed{\Delta l = 0.211 \text{ mm}}$

$\varepsilon_{total} = \varepsilon_\sigma + \varepsilon_{th}$

$\boxed{\varepsilon_{total} = 2.11 \cdot 10^{-4} \frac{mm}{mm}}$

∴ the total change in length due to thermal and mechanical strain is 0.211 mm

4b) as the block is cooled, it will begin to contract. Since the block is rigidly attatched to the walls (and the walls are presumably rigid), the walls will resist the contraction due to temperature, causing a tensile stress to form in the block along the axis of the block.



$\sigma_{ult} = 3 \text{ MPa}$

$\sigma_{ult} = E \varepsilon_{th}$

$\sigma_{ult} = E(\alpha \Delta T)$

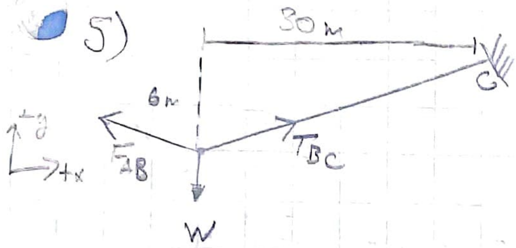
$\Delta T = \frac{\sigma_{ult}}{E\alpha}$

$\boxed{\Delta T = -11.11 °C}$

∴ the temperature must be lowered by 11.1°C for the block to fail.

Problem Set 2

5)



$T_{AB} = T_{BC}$; Symmetrical

$\sigma_{AB} = \sigma_{BC}$

a) $L_{AB_0} = L_{BC_0} = \sqrt{6^2+30^2}$

$= 30.594\,m$

$\Sigma F_y = 0$

$0 = T_{AB_y} + T_{BC_y} - W$

$W = T_{AB}\left(\frac{6}{30.594}\right) + T_{BC}\left(\frac{6}{30.594}\right), T_{AB} = T_{BC}$

$2000 = T_{BC} \cdot 2\left(\frac{6}{30.594}\right)$

$\boxed{T_{AB} = T_{BC} = 5.10 \cdot 10^3 N}$

$\sigma_{AB} = \sigma_{BC} = \frac{T_{BC}}{\pi\left(\frac{3}{2}\right)^2}$

$\boxed{\sigma_{AB} = \sigma_{BC} = 721\ MPa}$

$\sigma = E\epsilon$, $E_{steel} = 200{,}000\ MPa$

$\epsilon = \frac{\sigma_{BC}}{E_{steel}}$

$\Delta l = \frac{\sigma_{BC}}{E_{steel}} \cdot L_{BC_0}$

$\boxed{\Delta l = 110.3\ mm}$

$L'_{BC_y} = \sqrt{(L_{BC_0} + \Delta l)^2 - 30^2}$

$L_{BC_y} = 6.5392\,m$

$\Delta l_{BC_y} = L_{BC_y} - L_{BC_{y_0}}$

$\boxed{\Delta l_{BC_y} = 0.539\ m}$

$fos = \frac{\sigma_{capacity}}{\sigma_{demand}}$

$= \frac{1500}{721}$

$\boxed{fos = 2.08}$

∴ the forces on AB and BC are both $5.10 \cdot 10^3$ N, the stresses on AB and BC are both 721 MPa, wires AB and BC both become 110.3 mm longer, the 2000 N block moves 0.539 m downward and the fos for the system is 2.08

Problem Set 2

5) b) $L_{AB_1} = L_{BC_1} = 30.704\,m$

$$\sum F_y = 0$$

$$0 = T_{BC_y} + T_{AB_y} - W$$

$$W = T_{BC}\left(\frac{6.5392}{30.704}\right) + T_{AB}\left(\frac{6.5392}{30.704}\right), \quad T_{BC} = T_{AB}$$

$$T_{BC} = \frac{2000 \cdot 30.704}{2 \cdot 6.5392}$$

$$\boxed{T_{AB} = T_{BC} = 4.70 \cdot 10^3\,N}$$

$$\sigma_{BC} = \frac{T_{BC}}{\pi\left(\frac{3}{2}\right)^2}$$

$$\boxed{\sigma_{BC} = \sigma_{AB} = 664\,MPa}$$

$$fos = \frac{\sigma_{capacity}}{\sigma_{demand}} = \frac{1500}{664}$$

$$\boxed{fos = 2.26}$$

∴ after deformation, the forces in AB and BC are both $4.70 \cdot 10^3\,N$, the stresses in both AB and BC are 664 MPa and the factor of safety has increased to 2.26

c) $\sigma = E\varepsilon$

$$\frac{\Delta l}{L_0} = \frac{\sigma}{E}$$

$$\Delta l_{BC} = \frac{\sigma_{BC}}{E_{steel}} \cdot L_{BC_0}$$

$$\Delta l_{BC} = 101.6\,mm$$

$$L_{BC} = L_{BC_0} + \Delta l_{BC}$$

$$\boxed{L_{BC} = 30.7\,m}$$

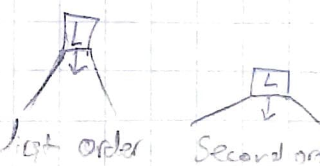
∴ this is very close, but slightly less than what was found in (a)

$$l_{BC_y} = \sqrt{(L_{BC_0} + \Delta l_{BC})^2 - 30^2} = 6.4982$$

$$\Delta l_{BC_y} = l_{BC_y} - 6 = \boxed{0.498\,m}$$

∴ the 2000 N load has moved between 0.498 m and 0.539 m down

d) the value calculated in c is about 9 mm longer than the one obtained in a. for this system, the second order analysis showed that the system stabilizes at some point in the range of 0.498 m and 0.539 m bellow where it started, because the value obtained through second order analysis is less then the first. This because the stress on the wire decreases as stress increases, because the tension decreases. Second order effects could be dangerous if we were dealing with compressive loads that increased as the system deformed (pictured). If the system's deformation caused the stress on the members to increase, this would create a positive feedback loop that may result in failure



1st order Second order