CIV102F Assignment # 3 – September 30, 2020

Due Wednesday October 7, 2020 at 23:59 Toronto time

General Instructions

- There are four questions on this assignment. All questions must be attempted; however, only one question will be graded.
- Submissions which are incomplete and do not contain a serious attempt to solve each question will receive a grade of 0.
- Intermediate steps must be provided to explain how you arrived at your final answer. Receiving full marks requires both the correct process and answer.
- All final answers must be reported using slide-rule precision (ie, four significant figures if the first digit is a "1", three otherwise), and engineering notation for very large or very small quantities.
- Submissions must be prepared neatly and be formatted using the requirements discussed in the course syllabus. Marks will be deducted for poor presentation of work.

Assignment-Specific Instructions

- Question 1 can be done using the provided material properties and assuming that the materials are always linear elastic. Make sure to draw plenty of free body diagrams!
- When solving question 2, you may need to keep many significant digits during your intermediate process because the effects are very small. Use 5 or 6 sig figs before rounding your final answer to slide rule precision.
- When solving Question 3, you may end up with two values. Show why one of them makes sense and why the other should be discarded.
- Parts b and c in Question 4 can be solved qualitatively, although it may be helpful to perform supporting calculations to understand what is happening.

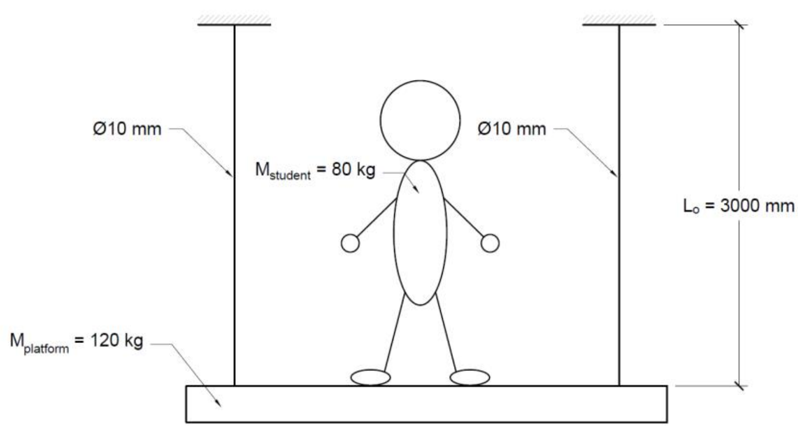
Free Vibration

1. D'Alembert's principle permits us to analyze a dynamic system (i.e. a system with accelerating bodies) by converting it to an equivalent static system where the accelerating body has a fictitious inertial force equal to $F_{\text{inertial}} = ma$, acting in the direction opposite to the acceleration.

Dynamic system, not in equilibrium. $\sum F \neq 0$ *Equivalent static system in equilibrium.* $\sum F = 0$

A student weighing 80 kg jumps directly onto the centre of a platform which is supported by two ropes. Each rope is 10 mm in diameter, has an undeformed length of 3000 mm, and is made from a material with E = 10,000 MPa and σ_{ult} = 60 MPa. When the student lands onto the platform, the maximum length of the rope is measured to be 3015 mm.

a) Using a series of free body diagrams and D'Alembert's principle, calculate the maximum force that the platform exerts onto the student.
b) Calculate the amount of energy absorbed by the ropes. What is the factor of safety?
c) Calculate the natural frequency of the system containing the student standing on top of the platform.



Poisson's Ratio

2. Many students this year have been curious about how the cross-section of a member changes when it is loaded in tension. A commonly used parameter which is useful for global behaviour (i.e. not local behaviour like necking) is the **Poisson's ratio, μ**. For a member which is being stretched or compressed in one direction, the ratio of the change in lengths in the directions orthogonal to the applied load to the change in length parallel to the load is defined as:

$$\mu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

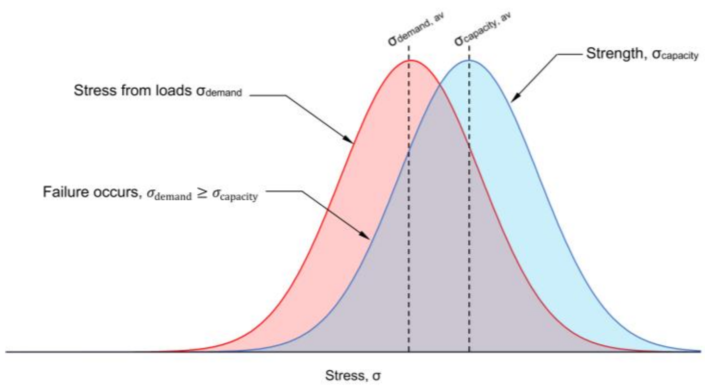
For example, if a prismatic member was stretched along its longitudinal (x direction) axis with a strain of ε_x, it will contract in the y direction with a strain having magnitude $\varepsilon_y = \mu\varepsilon_x$ and also contract in the z direction with a strain having magnitude $\varepsilon_z = \mu\varepsilon_x$. The negative sign in the definition of μ means that when a member is stretched, its cross section will shrink, and if a member is compressed, its cross section will expand.

Consider a wire which is made from high tensile steel, has an undeformed length of 5000 mm, and has a square cross section which is 3 mm x 3mm. The Poisson's ratio of steel can be taken as 0.3.

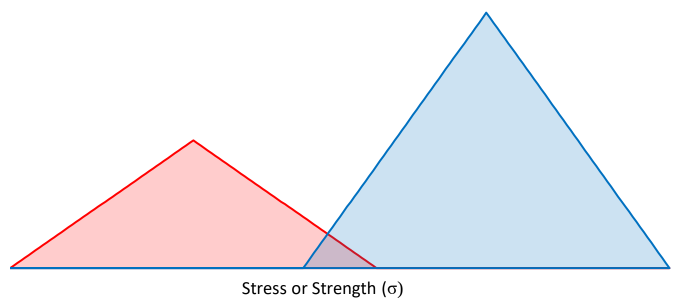
a) Calculate the force and associated longitudinal strain which causes the wire to start yielding.
b) Using the longitudinal strain calculated in part a, calculate the transverse strains and hence calculate the change in volume of the wire after it has been stretched.
c) What value of μ is needed so that the volume stays the same when the material stretches?

Factors of Safety and Structural Reliability

3. The probability distributions discussed in the lectures and course notes are often assumed to be Gaussian distributions – in this question, we will simplify these distributions to just be triangular distributions as shown below:

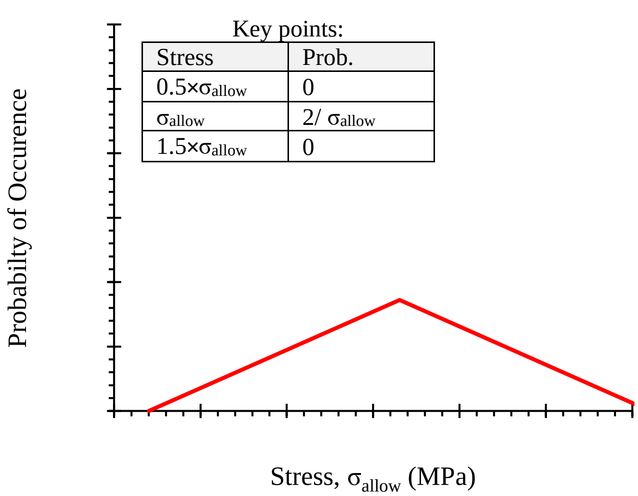


Gaussian Distribution Representation

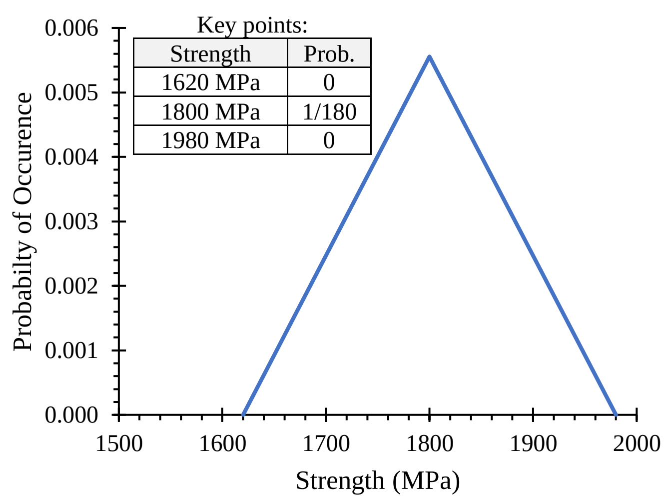


Simplified Distributions

Consider a wire made of steel having an average ultimate strength of 1800 MPa which varies between 1620 MPa and 1980 MPa according to the distribution shown in the figure below on the right. The maximum allowable stress, σ_{allow}, varies between $0.5\times\sigma_{allow}$ and $1.5\times\sigma_{allow}$ as shown in the figure below on the left. The height of each distribution, the probability, is provided in each figure and the area under each distribution is equal to 1 (i.e. there is a 100% probability that the actual capacity is within 1620 MPa and 1980 MPa and the actual demand is within $0.5\times\sigma_{allow}$ and $1.5\times\sigma_{allow}$). Recall that the overlapping area between the two curves is the probability of failure.



Capacity Distribution Curve



Demand Distribution Curve

Calculate the maximum allowable stress, σ_{allow}, which corresponds to a 2% probability of failure (i.e. the overlapping area is equal to 0.02). What is the factor of safety?

Nonlinear Material Behaviour

4. A 2 m long wire made of low alloy steel is attached to a ceiling and has a rigid catch-plate securely attached to the other end. The diameter of the wire is 16 mm for the top 1 m, and 6 mm for the bottom 1 m. Ignore the self-weight of the wire and the flange and neglect strain hardening for this question.

A 4 kg weight is dropped onto the flange from a height h:

a) What is the maximum value of h which does not cause any permanent deformations in the wire?
b) What will happen to the top and bottom segments of the wire if the weight is dropped from a higher height than what you calculated in part a?
c) If the weight is dropped repeatedly from a height of 2 m, the wire will eventually break. Suppose we repeated the process with a wire which was identical to the one shown below but had a 6 mm diameter over the entire 2 m length. Would this new wire be able to withstand more drops than the nonuniform rod considered in this question? Why?