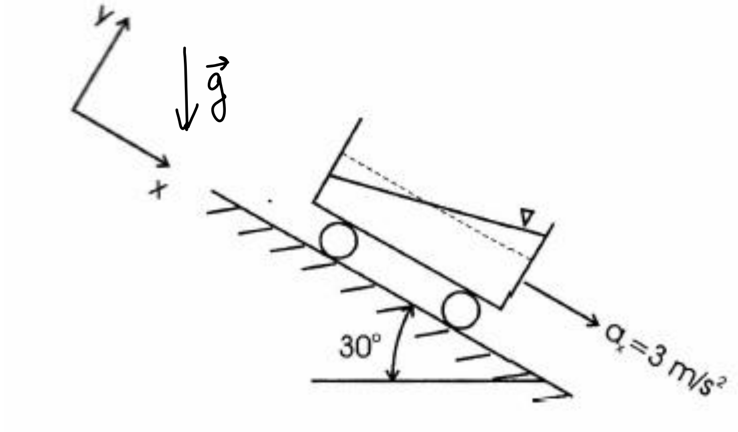
A rectangular container of water undergoes acceleration down an incline as shown in the diagram. Determine the slop of the free surface using coordinate system shown.

Note: $\sin(30^o) = 0.5$, $\cos(30^o) = \sqrt{3}/2$



$$\vec{g} = g_x \vec{i} + g_y \vec{j}$$

$$\boxed{\vec{g} = g\sin 30\,\vec{i} - g\cos 30\,\vec{j}}$$

$$\vec{a} = a_x \vec{i} \Rightarrow \boxed{a = 3\vec{i}}$$

$$-\vec{\nabla} p + \rho\vec{g} = \rho\vec{a}$$

$$-\left(\frac{\partial p}{\partial x}\vec{i} + \frac{\partial p}{\partial y}\vec{j} + \frac{\partial p}{\partial z}\vec{k}\right) + \rho\left(g\sin 30\,\vec{i} - g\cos 30\,\vec{j}\right) = \rho a_x \vec{i}$$

$$\vec{i}: \quad -\frac{\partial p}{\partial x} + \rho g \sin 30 = \rho a_x \Rightarrow \frac{\partial p}{\partial x} = \rho\left(g\sin 30 - a_x\right)$$

$$\vec{j}: \quad -\frac{\partial p}{\partial y} - \rho g \cos 30 = 0 \Rightarrow \frac{\partial p}{\partial y} = -\rho g \cos 30$$

$$\vec{k}: \quad -\frac{\partial p}{\partial z} = 0 \Rightarrow p \neq f(z)$$

$$p = p(x,y) \xRightarrow{\text{total differential}} dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$$

$$dp = \rho\left(g\sin 30 - a_x\right)dx - \rho g\cos 30\, dy$$

integrating

$$p = \rho\left(g\sin 30 - a_x\right)x - \rho g\cos 30\, y + C$$

$$p = \rho\left(10 \cdot 0.5 - 3\right)x - \rho\left(10 \cdot \frac{\sqrt{3}}{2}\right)y + C$$

At the free surface we have atmospheric pressure:

at $y = y_s$ $\quad P = P_{atm}$

$$P_{atm} = 2\rho x - 5\sqrt{3}\rho y_s + C$$

$$\Rightarrow \quad y_s = \underbrace{\frac{C - P_{atm}}{5\sqrt{3}\rho}}_{\text{constants}} + \underbrace{\frac{2}{5\sqrt{3}}}_{\text{slope}} x$$

$\therefore$ The free surface has a slope of $\dfrac{2}{5\sqrt{3}}$.