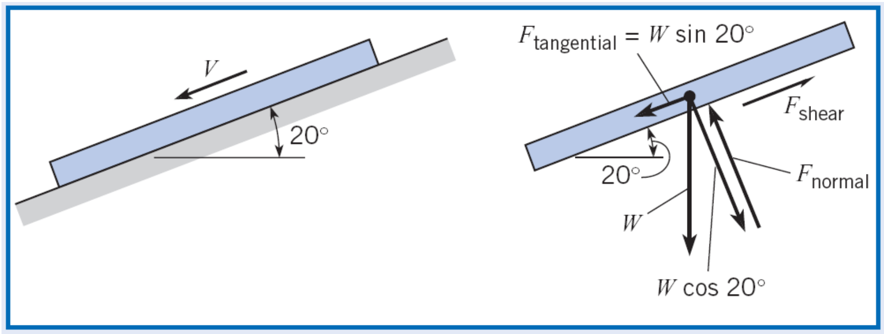
Problem 1: A board 1m by 1m that weighs 25 N slides down an inclined ramp (slope = 20°) with a velocity of 2.0 cm/s. The board is separated from the ramp by a thin film of oil with a viscosity of 0.05 N.s/m^2. Neglecting edge effects, calculate the spacing between the board and the ramp.



Solution: The board and ramp (left) and a free body diagram of the board (right) are shown below. For a constant sliding velocity, the resisting shear force is equal to the component of weight parallel to the inclined ramp. Therefore,

$$F_{tangential} = F_{shear}$$

$$W \sin 20^\circ = \tau A$$

$$W \sin 20^\circ = \mu \frac{dV}{dy} A$$

In this case we can assume a linear velocity distribution in the oil, so dV/dy can be expressed as $\Delta V/\Delta y$, where ΔV is the velocity of the board and Δy is the spacing between the board and the ramp. We then have

$$W \sin 20^\circ = \mu \frac{\Delta V}{\Delta y} A \Rightarrow \Delta y = \frac{\mu \, \Delta V \, A}{W \sin 20^\circ}$$

$$= \frac{0.05 \, N.s/m^2 \times 0.020 \, m/s \times 1 \, m^2}{25N \times \sin 20^\circ}$$

$$= 0.000117 m$$

$$\boxed{\Delta y = 0.117 mm}$$

Problem 2: The velocity distribution for water (20°C) near a wall is given by u = $a(y/b)^{1/6}$, where a = 10 m/s, b = 2 mm, and y is the distance from the wall in mm. Determine the shear stress in the water at y = 1mm. (For water at 20°C $\mu = 1.00\times10^{-3} N.s/m^2$)

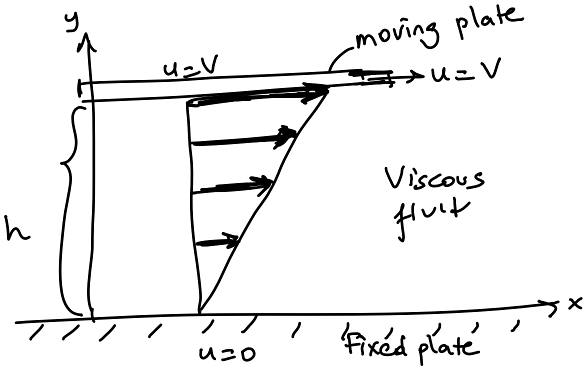
Solution:

Rate of strain:
$$\frac{du}{dy} = \frac{d}{dy}\left[a\left(\frac{y}{b}\right)^{1/6}\right]$$
$$= \frac{a}{b^{1/6}}\frac{1}{6\,y^{5/6}}$$
$$= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6}$$

Rate of strain at y=1mm ⇒
$$\left.\frac{du}{dy}\right|_{y=1mm} = \left.\frac{a}{6b}\left(\frac{b}{y}\right)^{5/6}\right|_{y=1mm}$$
$$= \frac{10\ m/s}{6\times 0.002m}\left(\frac{2mm}{1mm}\right)^{5/6}$$
$$= 1485\ s^{-1}$$

Shear stress
$$\tau|_{y=1mm} = \mu\frac{du}{dy}$$
$$= \left(1.00\times10^{-3}\ \frac{N.s}{m^2}\right)\left(1485\ s^{-1}\right)$$
$$= 1.485\ Pa$$
$$\boxed{\tau(y=1mm) \cong 1.49\ Pa}$$

Problem 3: Suppose that the fluid being sheared in below is SAE 30 oil at 20°C. Compute the shear stress in the oil if V = 3 m/s and h = 2 cm. ($\mu = 0.29$ kg/m-s for SAE30 oil @ 20°C)



Solution:

Assumptions: Linear velocity profile, laminar newtonian fluid, no slip at either plate surface.

$$\tau = \mu \frac{du}{dy}$$

$u = a + by$ (Since the velocity distribution is linear)

The constants a and b can be evaluated from the no-slip condition at the upper and lower walls:

$u = 0$ @ $y = 0 \Rightarrow \quad 0 = a + b(0) \Rightarrow \boxed{a = 0}$

$u = V$ @ $y = h \Rightarrow \quad V = \cancel{a} + bh \Rightarrow \boxed{b = \frac{V}{h}}$

Then, the velocity profile between the plates is given by

$$u = \frac{V}{h} y$$

(Please not that turbulent flow would not have this linear shape)

$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{h} = \left(0.29 \frac{kg}{m\,s}\right) \frac{(3\,m/s)}{(0.02\,m)} = 43.5 \frac{kg.m/s^2}{m^2}$$

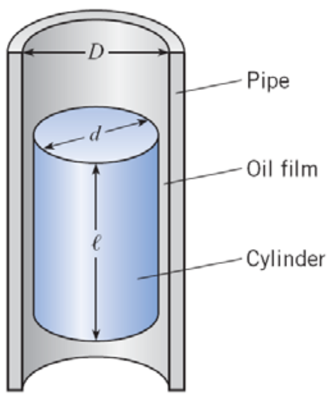
$$= 43.5 \frac{N}{m^2}$$

$$= 43.5\ Pa$$

Problem 4: Consider the ratio μ_{100}/μ_{50} where μ is the viscosity of oxygen and the subscripts 100 and 50 are temperatures of the oxygen in degrees Fahrenheit. Does this ratio have a value (a) less than 1, (b) equal to 1, or (c) greater than 1?

Solution: Because the viscosity of gases increases with temperature $\mu_{100}/\mu_{50}>1$. Correct choice is (c).

Problem 5: A solid circular cylinder of diameter d and length l slides inside a vertical smooth pipe that has an inside diameter D. The small space between the cylinder and the pipe is lubricated with an oil film that has a viscosity μ. Derive a formula for the steady rate of descent of the cylinder in the vertical pipe. Assume that the cylinder has a weight W and is concentric with the pipe as it falls. Se the formula to find the rate of descent of a cylinder 100 mm in diameter that slides inside a 100.5 mm pipe. The cylinder is 200 mm long and weighs 20 N. The lubricant is SAE 20W oil at 10°C with $\mu(10°C)=0.35$ N.s/m^2.



Solution:

$$\tau = \mu \frac{dV}{dy}$$

If we assume linear velocity distribution

$\frac{dV}{dy} = \frac{\Delta V}{\Delta y}$, where ΔV is the velocity of the falling cylinder and Δy is the spacing between the two cylinders.

$$\Downarrow$$

$$\left\{ \begin{array}{l} \Delta V = V_{fall} \\ \Delta y = (D-d)/2 \end{array} \right\}$$

$\frac{F}{A} = \frac{W}{\pi d l}$

$$\tau = \mu \frac{\Delta V}{\Delta y}$$

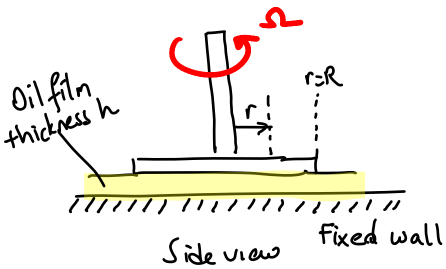
$$\frac{W}{\pi d l} = \mu \frac{V_{fall}}{(D-d)/2} \Rightarrow V_{fall} = \frac{W(D-d)}{2\pi d l \mu}$$

wetted area

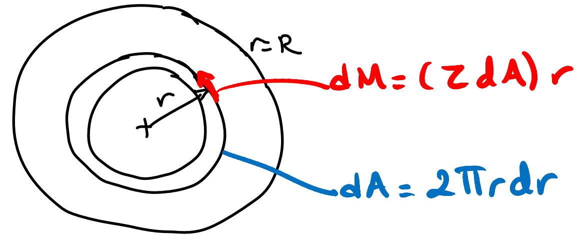
$$= \frac{20(0.5\times10^{-3})}{2\pi \times 0.1 \times 0.2 \times 3.5 \times 10^{-1}}$$

$$= 0.23 \, m/s$$

Problem 6: An oil film of viscosity μ and thickness h≪R lies between a solid wall and a circular disk as shown below. The disk is rotated at angular velocity Ω. Noting that both velocity and shear stress vary with radius r, derive a formula for torque M required to rotate the disk.



Side view
(Figure a)



Top view
(Figure b)

Solution:

Assumptions: Linear velocity profile, laminar flow, no-slip

Approach: Estimate the shear stress on a circular strip of width dr and area dA = 2πrdr (in Figure b above), then find the moment dM about the origin caused by this shear stress. Integrate over the entire disk to find the total moment M.

Solution steps: At radius r, the velocity in the oil is tangential, varying from zero at the fixed wall (no-slip) to u = Ωr at the disk surface (also no-slip). The shear stress at this position is thus:

$$\tau = \mu \frac{du}{dy} = \mu \frac{\Omega r}{h}$$

This shear stress is everywhere perpendicular to the radius from the origin. Then the total moment about the disk origin, caused by shearing this circular strip, can be found and integrated:

$$dM = \overbrace{(\tau)(dA)}^{F}\, r = \left(\mu \frac{\Omega r}{h}\right)(2\pi r\, dr)\, r$$

$$M = \int dM = \frac{2\pi \mu \Omega}{h} \int_0^R r^3 dr = \boxed{\frac{\pi \mu \Omega R^4}{2h}} \quad *$$

Comments: This is a simplified engineering analysis, which neglects possible edge effects, air drag on the top of the disk, and the turbulence that might ensue if the disk rotates too fast.

Problem 7: A pressure of $2\times10^6 N/m^2$ is applied to a mass of water initially filled a 1000-cm volume. Estimate its volume after the pressure is applied. (Modulus of elasticity $E_{\forall} = 2.2\times10^9$ Pa)

Solution:

Modulus of elasticity $E_V = -\Delta p \dfrac{\forall}{\Delta\forall}$ → ($\forall$ shows Volume)

$$\Downarrow$$

$$\Delta\forall = -\frac{\Delta p}{E}\forall$$

$$= -\left[\frac{(2\times10^6 \text{ Pa})}{(2.2\times10^9 \text{ Pa})}\right](1000 \text{ cm}^3)$$

$$= -0.9091 \text{ cm}^3$$

Final volume: $\forall_{final} = \forall + \Delta\forall$

$$= (1000 - 0.9091) \text{ cm}^3$$

$$= 999.1 \text{ cm}^3$$

$$\boxed{\forall_{final} \approx 999 \text{ cm}^3}$$