Problem Set 3

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Assignment 3

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1)

platform

a) $2\cdot\sigma_{max} = \frac{F_{max}}{A}$

$F_{Max} = 2\cdot\sigma_{max}\cdot A$

$\sigma_{max} = \frac{\Delta l_{max}}{L_o}\cdot E$

$\sigma_{max} = \frac{15}{3000}\cdot 10000$

$\sigma_{max} = 50\ MPa$

$F_{max} = 50\cdot\pi\left(\frac{10}{2}\right)^2$

$= 3.927\ kN$

$\sum F = 0$

$0 = 2F_T - F_I - F_g$

$0 = 2F_T - 200(a) - 200g$

$a = \frac{2F_T - 200g}{200}$

$a = 29.46\ m/s^2$

Person on platform

$\sum F = 0$

$0 = F_N - F_I - F_g$

$F_N = mg + ma$

$\boxed{F_N = 3.14\ kN}$

c) $\omega_n = \sqrt{\frac{k}{m}}$, $k = \frac{AE}{L_o}$

$\omega_n = \sqrt{\frac{2\cdot AE}{mL_o}}$

$= \sqrt{\frac{2\cdot 5^2\pi\cdot 10000}{(80+120)\cdot 3000}}$

$\boxed{\omega_n = 51.2\ \frac{rad}{s}}$

$= 8.14\ Hz$

b) $W_{rope} = \frac{1}{2}\sigma\varepsilon V_o$

$W = \frac{1}{2}\sigma\frac{\Delta l}{L_o}\cdot L_o A$

$W = \frac{1}{2}\sigma\Delta l A$

$\boxed{W = 58.9\ J}$

$fos = \frac{\sigma_{ult}}{\sigma_{max}}$

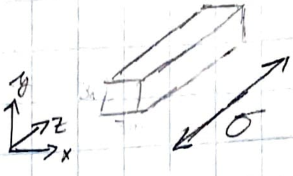
$= \frac{60}{50}$

$\boxed{fos = 1.200}$

Problem Set 3 cont'd

2) $E = 200{,}000$ MPa

$\sigma_{yield} = 1650$ MPa



$$\sigma_{yield} = E\varepsilon$$

$$\varepsilon = \frac{\sigma_{yield}}{E}$$

$$\varepsilon = 8.25 \cdot 10^{-3} \ \frac{mm}{mm}$$

$$\sigma_{yield} = \frac{F_{yield}}{A}$$

$$F_{yield} = \sigma_{yield} \cdot A$$

$$F_{yield} = 1650 \cdot (3 \cdot 3)$$

$$\boxed{= 14.85 \text{ KN}}$$

b) $\mu = -\frac{\varepsilon_y}{\varepsilon_z}$

$$\varepsilon_y = -\mu\varepsilon_z$$

$$\boxed{\varepsilon_y = -2.48 \cdot 10^{-3} \ \frac{mm}{mm}}$$

$$\mu = -\frac{\varepsilon_x}{\varepsilon_z}$$

$$\varepsilon_z = -\mu\varepsilon_z$$

$$\boxed{\varepsilon_z = -2.48 \cdot 10^{-3} \ \frac{mm}{mm}}$$

c) $V_f = V_o$

$$V_o = \left(L_{oz} + \frac{\sigma_{yield}}{E} \cdot L_{oz}\right)\left(L_{oy} - \mu\varepsilon_z L_{oy}\right)\left(L_{ox} - \mu\varepsilon_z L_{ox}\right)$$

$$L_{oz} = L_{oy}$$

$$\frac{V_o}{L_{oz} + \frac{\sigma_{yield}}{E} \cdot L_{oz}} = \left(L_{oy} - \mu\varepsilon_z L_{oy}\right)^2$$

$$L_{oy} - \mu\varepsilon_z L_{oy} = \sqrt{\frac{V_o}{L_{oz} + \frac{\sigma_{yield}}{E} \cdot L_{oz}}}$$

$$\mu = \frac{L_{oy} - \sqrt{\frac{V_o}{L_{oz} + \frac{\sigma_{yield}}{E} \cdot L_{oz}}}}{\varepsilon_z L_{oy}}$$

$$\boxed{\mu = 0.497}$$

$$V_f = \left((L_{oz} + \Delta l_z) \cdot (L_{oy} + \Delta l_{oy}) \cdot (L_{ox} + \Delta l_x)\right)$$

$$\varepsilon_y = -\mu\varepsilon_z \qquad \varepsilon_x = -\mu\varepsilon_z$$

$$\frac{\Delta l_y}{L_{oy}} = -\mu\varepsilon_z \qquad \frac{\Delta l_x}{L_{ox}} = -\mu\varepsilon_z$$

$$\Delta l_y = -\mu\varepsilon_z L_{oy} \qquad \Delta l_x = -\mu\varepsilon_z L_{ox}$$

$$V_f = \left(L_{oz} + \frac{\sigma_{yield}}{E} \cdot L_{oz}\right)\left(L_{oy} - \mu\varepsilon_z L_{oz}\right)\left(L_{ox} - \mu\varepsilon_z L_{ox}\right) - V_o$$

$$= 45146.9 \text{ mm}^3$$

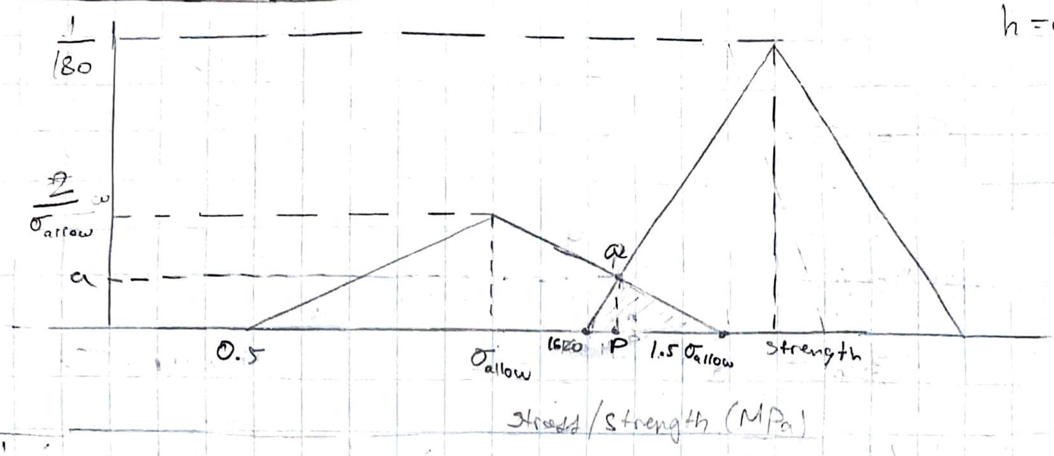
$$\Delta V = V_f - V_o$$

$$\Delta V = 45146.9 - 45000$$

$$\boxed{= 146.9 \text{ mm}^3 \text{ increase}}$$

Problem Set 3 con'd

$h = 0.02$



By similar triangles,

$$\frac{\frac{2}{\sigma_{allow}}}{a} = \frac{1.5\sigma_{allow} - \sigma_{allow}}{1.5\sigma_{allow} - P}$$

$$3\sigma_{allow} - 2P = 0.5\, a\, \sigma_{allow}^2$$

$$P = \frac{3\sigma_{allow} - 0.5a\sigma_{allow}^2}{2}$$

$$\frac{\frac{1}{180}}{a} = \frac{1800-1620}{P-1620}$$

$$P - 1620 = 180^2 a$$

$$P = 180^2 a + 1620$$

$$\frac{3\sigma_{allow} - 0.5a\sigma_{allow}^2}{2} = 180^2 a + 1620$$

$$0.02 = \frac{1}{2}(1.5\sigma_{allow} - 1620)(a)$$

$$a = \frac{0.04}{1.5\sigma_{allow} - 1620}$$

$$3\sigma_{allow} - \frac{0.04 \cdot 0.5\,\sigma_{allow}^2}{1.5\sigma_{allow} - 1620} = \frac{0.04 \cdot 2 \cdot 180^2}{1.5\sigma_{allow} - 1620} + 1620 \cdot 2$$

$$4.5\sigma_{allow}^2 - 4860\sigma_{allow} - 0.02\sigma_{allow}^2 = 2592 + 4860\sigma_{allow} - 2 \cdot 1620^2$$

$$4.48\sigma_{allow}^2 - 9720\sigma_{allow} + 5246208 = 0$$

$\sigma_{allow} = 1161$ MPa or 1009 MPa

$1.5 \cdot 1161 = 1742$

$1742 > 1620$

$1.5 \cdot 1009 = 1514$

$1514 < 1620$, $\therefore$ not accepted

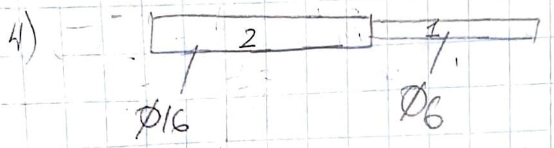
$$\boxed{\therefore \sigma_{allow} = 1161 \text{ MPa}}$$

$$fos = \frac{\sigma_{max}}{\sigma_{allow}} = \frac{1800}{1161}$$

$$\boxed{fos = 1.550}$$

Problem Set 3 Con't

4)



a) Before yielding, we can treat both wires like a pair of springs in series

$F = K\Delta l$

$K_1 \Delta l_1 = K_2 \Delta l_2$, $K = \frac{AE}{L_0}$, $\Delta l = \varepsilon L_0$

$\frac{A_1 E}{L_{0_1}} \cdot \varepsilon_1 L_{0_1} = \frac{A_2 E}{L_{0_2}} L_{0_2} \varepsilon_2$

$A_1 \varepsilon_1 = A_2 \varepsilon_2$

$\varepsilon_{yield} = \frac{\sigma_{yield}}{E}$

$\varepsilon_{yield} = 2.1 \cdot 10^{-3} \frac{mm}{mm}$

Since $A_1 < A_2$, ε_1 will reach ε_{yield} first

$\varepsilon_2 = \frac{A_1 \varepsilon_1}{A_2}$, $\varepsilon_1 = \varepsilon_{yield}$

$\varepsilon_1 = 2.95 \cdot 10^{-4} \frac{mm}{mm}$; confirms ε_1 yields first

when $\sigma_1 = \sigma_{yield}$,

$\frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_2}{\varepsilon_2}$

$\sigma_2 = \frac{\sigma_1 \cdot \varepsilon_2}{\varepsilon_1}$, $\sigma_1 = \sigma_{yield}$

$\sigma_2 = 59.1\ MPa$

$W_{yield} = \frac{1}{2}\sigma_1 \varepsilon_1 V_{0_1} + \frac{1}{2}\sigma_2 \varepsilon_2 V_{0_2}$

$= \frac{1}{2}\sigma_1 \varepsilon_1 A_1 L_{0_1} + \frac{1}{2}\sigma_2 \varepsilon_2 A_2 L_{0_2}$

$= 14.22\ J$

$mgh = W_{yield}$

$h = \frac{14.22 J}{4 \cdot 9.81}$

$\boxed{h = 0.363 m}$

b) the bottom wire will continue to yield as the height of the drop increases. Since the strain increases, but the stress is constant, the stress in the larger wire doesn't increase. Since there is no strain hardening, the smaller wire will eventually fail in this state

c) the 2m Ø6mm wire would withstand more drops. Using $W = \frac{1}{2}\sigma\varepsilon A_0 L_0$, we find that a 2m long 6mm cable could absorb 24.9 J before breaking, which is greater than the 14.22 J the first cable absorbs, ∴ the smaller wire absorbs more drops