A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u,v) = (0.5 + 0.8x)\,\hat{\imath} + (1.5 - 0.8y)\,\hat{\jmath}$$

where the x- and y- coordinates are in meters and the magnitude of velocity is in m/s.

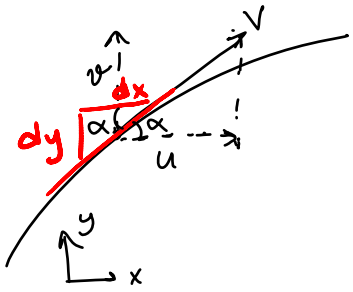
(a) Determine if there are any stagnation points in this flow field, and if so, where? (**2 points**)
(b) For this velocity field, generate an analytical expression for the flow streamlines (**6 points**)
(c) and draw a couple of streamlines in the right half of the flow ($x > 0$) (**2 points**).

(a) $0.5 + 0.8x = 0 \Rightarrow x = \frac{-0.5}{0.8} = -0.625\,m$

$1.5 - 0.8y = 0 \Rightarrow y = \frac{-1.5}{-0.8} = 1.875\,m$

$\Rightarrow$ Yes. There is one stagnation point. It is located at $x = -0.625$, $y = 1.875$.

(b)



$$\tan\alpha = \frac{dy}{dx} = \frac{v}{u}$$

$$\frac{dy}{dx} = \frac{1.5 - 0.8y}{0.5 + 0.8x}$$

$$\int \frac{dy}{1.5 - 0.8y} = \int \frac{dx}{0.5 + 0.8x}$$

$$A = \int \frac{dy}{1.5 - 0.8y} = \int \frac{d\eta}{(-0.8)\,\eta} = -\frac{1}{0.8}\ln\eta + C_1 = -\frac{1}{0.8}\ln(1.5 - 0.8y) + C_1$$

$1.5 - 0.8y = \eta$

$-0.8\,dy = d\eta$

$dy = \frac{d\eta}{-0.8}$

$$B = \int \frac{dx}{0.5 + 0.8x} = \int \frac{d\xi}{(0.8)\,\xi} = \frac{1}{0.8}\ln(0.5 + 0.8x) + C_2$$

$0.5 + 0.8x = \xi$

$0.8\,dx = d\xi$

$dx = \frac{1}{0.8}\,d\xi$

$$\underbrace{\int \frac{dy}{1.5-0.8y}}_{A} = \underbrace{\int \frac{dx}{0.5+0.8x}}_{B} \Rightarrow$$

$$-\frac{1}{0.8}\ln(1.5-0.8y) + C_1 = \frac{1}{0.8}\ln(0.5+0.8x) + C_2$$

$$-\frac{1}{0.8}\ln(1.5-0.8y) = \frac{1}{0.8}\ln(0.5+0.8x) + \ln C$$

$$\ln(1.5-0.8y) = -\ln(0.5+0.8x) + \ln C'$$

$$1.5-0.8y = e^{-\ln(0.5+0.8x)+\ln C'}$$

$$1.5-0.8y = \frac{C'}{0.5+0.8x} \Rightarrow \boxed{y = \frac{C}{0.8(0.5+0.8x)} + 1.875}$$

where C is a constant of integration that can be set to various values in order to plot the streamlines. Several streamlines are given below.