ESC103F Engineering Mathematics and Computation: Tutorial #3

Question 1: Show, using a vector algebra approach, that the plane whose intercepts with the coordinate axes are $x = a$, $y = b$ and $z = c$ where a, b, c are nonzero has the scalar equation:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Question 2: Find the vector equation for the line through (-2,5,0) that is parallel to the planes $2x + y - 4z = 0$ and $-x + 2y + 3z + 1 = 0$.

Question 3: Let $A, B,$ and $C(2,-1,1)$ be the vertices of a triangle where $\overrightarrow{AB}$ is parallel to $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\overrightarrow{AC}$ is parallel to $\begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$, and angle $C = 90^0$. Find the equation of the line through B and C.

Question 4: Find all unit vectors parallel to the *yz*-plane that are perpendicular to the vector $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$.

Question 5: The volume of a tetrahedron is given by:

1/3(area of base)(height)

Use this result to show that the volume of a tetrahedron with sides defined by the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is given by $\frac{1}{6}|\vec{a} \cdot (\vec{b} \times \vec{c})|$, taking the base to be defined by vectors $\vec{b}$ and $\vec{c}$.

(Note, $\vec{a} \cdot (\vec{b} \times \vec{c})$ is referred to as the scalar triple product.)

Question 6: Find the distance between the point P(-3,1,3) and the plane 5x+z=3y-4.

Question 7: Show that the planes 3x-y+6z=7 and -6x+2y-12z=1 are parallel and find the distance between the planes.

Question 8: Find the vector equation for the line in R^3 that contains the point P(-1,6,0) and is orthogonal to the plane 4x-z=5.

Question 9: Find the scalar equation for the plane that is represented by the vector equation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 5 \\ 6 \end{bmatrix} + t\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} + s\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

Question 10: The equation ax+by=0 represents a line through the origin in R^2 if a and b are not both zero. What does this equation represent in R^3 if you think of it as ax+by+0z=0? Explain.

In both Questions 11 and 12, the solution refers to the values of x, y, z that satisfy the equations.

Question 11: Consider the following planes in R^3:

$$1x + 0y + 0z = 2$$
$$0x + 1y + 0z = 3$$
$$0x + 0y + 1z = 4$$

Construct a mental image of the row picture. Now express these 3 equations as a single vector equation using appropriately defined column vectors and construct a mental image of the column picture. Convince yourself that both pictures have the same solution. What does the solution represent in R^3?

Question 12: Consider the following planes in R^3:

$$x + y + z = 2$$
$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5$$

Express these 3 equations as a single vector equation using appropriately defined column vectors. Find two linear combinations of the columns on the left-hand side that give the vector on the right-hand side. Show that this is only possible for a right-hand side vector equal to $[4\ 6\ c]^T$ if c is equal to what value? What does the solution of the original 3 equations represent in R^3?