FOUNDATIONS

1

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{-2} m **2.** 10^2 m **3.** 10^2 m **4.** 10^1 m^2 **5.** 10^3 **6.** 10^3 kg **7.** 10^4 kg **8.** 10^{13} kg **9.** 10^5 kg **10.** 10^{11} **11.** 10^9 **12.** 10^5

Guided Problems

1.2 Solar oxygen

1. Getting Started Much of the plan used in Worked Problem 1.1 can be used here. We wish to use the volume of the sun and the percent of the Sun that is made up of oxygen to calculate the mass density and number density of oxygen atoms. As before we assume the Sun is a perfect sphere.

2. Devise Plan As before, we use the mass density $\rho = m/V$ and the number density $n = N/V$. We will use $V = \frac{4}{3}\pi R^3$ for the volume of the Sun. We will use the total mass of the Sun and the fraction of that mass that is made up of oxygen to calculate the mass of oxygen in the Sun. We will divide by volume to get the mass density. Then, one need only divide by the mass of a single oxygen atom to convert from mass density to number density.

3. Execute Plan First, the mass density is given by

$$\rho = \frac{m}{V} = \frac{m_{\text{oxygen}}}{\frac{4}{3}\pi R_{\text{Sun}}^3} = \frac{(0.00970)M_{\text{Sun}}}{\frac{4}{3}\pi R_{\text{Sun}}^3}$$

$$\rho = \frac{(0.00970)(1.99\times10^{30}\ \text{kg})}{\frac{4}{3}\pi(6.96\times10^8\ \text{m})^3} = 13.7\ \text{kg/m}^3$$

We convert from mass density to number density using

$$n = \frac{\rho}{m_{\text{atom}}} = \frac{13.7\ \text{kg/m}^3}{2.66\times10^{-26}\ \text{kg/atom}} = 5.14\times10^{26}\ \text{atoms/m}^3$$

4. Evaluate Result The calculated mass density of oxygen is approximately two orders of magnitude smaller than the mass density of hydrogen calculated in Worked Problem 1.1. This is what we expect, because the oxygen accounts for only about 1% of the mass of the Sun and hydrogen makes up approximately 70%. Since the oxygen accounts for two orders of magnitude less mass, it is perfectly sensible that the mass density of oxygen would also be two orders of magnitude smaller. Similarly, the number density of oxygen is about three orders of magnitude smaller than the number density of hydrogen. If the only issue were the relative percentages of the Sun's mass made up by oxygen and hydrogen, we would expect a difference of only two orders of magnitude. However, oxygen is also much more massive than hydrogen (around one order of magnitude more massive). Hence, mass densities that differ by two orders of magnitude mean number densities that differ by three orders of magnitude. Our results are consistent with those of Worked Problem 1.1.

1.4 Box volume

1. Getting Started This problem is similar to Worked Problem 1.3 in that we are given a mixture of units which we must convert to SI units. However, the expressions for volume here is completely different from that of Worked Problem 1.3.

2. Devise Plan We convert from feet to inches using the conversion factor $1 \text{ ft} = 12 \text{ in},$ and convert from inches to meters using $1 \text{ in} = 0.0254 \text{ m}.$ We can convert millimeters and centimeters to meters by simply dividing by the appropriate factor of ten. When all quantities are in units of meters, we proceed to find the volume of the box using $V_{\text{box}} = \ell wh,$ where $\ell,$ $w,$ and h are the length, width, and height of the box, respectively.

3. Execute Plan We first convert the length, width and height to units of meters:

$$\ell = 1420 \text{ mm} \times \frac{1 \text{ m}}{10^3 \text{ mm}} = 1.420 \text{ m}$$

$$w = 2.75 \text{ ft} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{0.0254 \text{ m}}{1 \text{ in}} = 0.838 \text{ m}$$

$$h = 87.8 \text{ cm} \times \frac{1 \text{ m}}{10^2 \text{ cm}} = 0.878 \text{ m}$$

Finally the volume is given by $V_{\text{box}} = \ell wh = (1.420 \text{ m})(0.838 \text{ m})(0.878 \text{ m}) = 1.05 \text{ m}^3.$

4. Evaluate Result A volume of 1.05 m^3 is reasonable. The box had a size that was on the order of 1.0 m in each dimension. One dimension had a length greater than 1.0 m and the other two were smaller than 1.0 m. This is the approximate size we would expect.

1.6 Digits on your own

1. Getting Started The methods used in Worked Problem 1.5 are essentially the same that will be used here. The expressions involve quantities with varying numbers of significant digits. The number of significant digits in the answer will depend on how many significant digits are in the given in the problem statement, and on the operation carried out between quantities (subtraction, multiplication, etc).

2. Devise Plan As in Worked Problem 1.5, the number of significant digits in a product or quotient is the same as the number of significant digits in the input quantity that has the *fewest* significant digits. The number of decimal places in a sum or difference is the same as the number of decimal places in the input quantity that has the *fewest* decimal places. To express our answers as orders of magnitude, we write each in scientific notation, round the coefficient either down to 1 (for coefficients $\leq 3)$ or up to 10 (for coefficients $> 3).$ We then write the answer as a power of ten without the coefficient.

3. Execute Plan (*a*) In $(205)(0.0041)(489.62),$ the middle quantity (0.0041) has the fewest significant digits, with only two. Hence the answer must be given to only two significant digits: $4.1 \times 10^2.$ Here the coefficient is greater than 3, so we round it to 10 and obtain for the order of magnitude $10 \times 10^2 = 10^3.$ (*b*) Here, the first factor is given to four significant digits, which is the fewest of all factors (since π is known to many digits). Hence the answer must be given to five significant digits: $2.475 \times 10^1.$ Here the prefactor is less than 3, so we round it down to 1. This yields an order of magnitude of $1 \times 10 = 10^1.$ (*c*) The first term is known to the thousandths place, whereas the second term is known only to the tenths place. Hence the answer can be given only out to the tenths place: $6.9802 \times 10^3.$ The prefactor is greater than 3, so we round it to 10. This yields an order of magnitude of $10 \times 10^3 = 10^4.$

4. Evaluate Result We could check our answers by getting order of magnitude estimates for each number. (*a*) Using orders of magnitude of $(205)(0.0041)(489.62)$ yields $(10^2)(10^{-2})(10^3) = 10^3$, which is consistent with our answer. (*b*) Similarly, $(190.8)(0.407\ 500)/\pi$ becomes $(10^2)(10^0)/(10^1) = 10^1$, which is again consistent with our answer. (*c*) Finally, to nearest order $(6980.035) + (0.2)$ yields $(10^4) - (10^{-1}) = 10^4$, which is also consistent.

1.8 Roof area

1. Getting Started We will approximate the United States as a rectangle 5,000 km wide and 3,000 km high. A very small percentage of this area is occupied by buildings. Only in cities can we approximate the surface as being covered by structures.

2. Devise Plan The approximate area of the United States is $(10^4\ \text{km})(10^3\ \text{km}) = 10^7\ \text{km}^2$. Depending on one's definition of a "city" the percentage of this surface area that is occupied by cities could be anywhere between 0.1% and 1%. But since even cities are not completely filled with structures, we will use the smaller of these two percentages for our estimate (0.1%). These two numbers can be used to find an order of magnitude estimate of the total roof area in the United States.

3. Execute Plan The roof area is given by $A_{\text{roof}} = (0.1\%)A_{\text{US}} = (0.001)(10^7\ \text{km}^2) = 10^4\ \text{km}^2$.

4. Evaluate Result We could check our result by estimating the roof surface area in a completely different way. Let us start with the population of the United States (3×10^8 people to one significant digit), and assume that there exists a structure that houses every set of three or four people, and assume further that these structures have a footprint of approximately $30\ \text{m}^2$. Of course, many people live alone and many people live in apartment buildings that have many floors (meaning less of a footprint per person). But these cases might cancel each other out, making our estimate plausible. This yields a surface area of $2.3\times10^9\ \text{m}^2$. Let us double this number to account for the places of business that employ many of these people. This gives a total surface area of order $10^{10}\ \text{m}^2$. Using known conversion factors, one can see that this is equivalent to $10^4\ \text{km}^2$. This agrees with our previous estimate.

Questions and Problems

1.1. The word "undetectable" prevents this from being a valid scientific hypothesis. A hypothesis must be experimentally verifiable.

1.2. You assume that the competing product contains non-zero fat, and that the serving sizes of the two are equal. Say the two foods contain the same amount of fat per ounce. The maker of the product being advertised could print his label showing a *recommended serving size* 50 percent smaller than the recommended serving size of the competing product. This makes the claim of 50 percent less fat *per serving* true but of course misleading.

1.3. You assume the sequence is linear, meaning each entry is larger than the previous one by a constant amount. As an alternative, the sequence could be formed by starting with 1, 2 and then setting the n^{th} term c_n equal to the sum of the previous two terms: $c_n = c_{n-1} + c_{n-2}$. This would work for $c_3 = c_2 + c_1 = 2 + 1 = 3$, and would yield 5 as the next number in the sequence.

1.4. If you assume that the coins are currently circulated U.S. currency, you would not be able to find a solution. If, however, you consider that the word "cents" is also used to refer to hundredths of other currencies, then all that would be required is that increments of 10 and 20 centers exist in some currency. As an example, "cents" may refer to hundredths of a Euro. You would say the coins must be worth 10 and 20 Euro cents, respectively (which do exist).

1.5. There are 12 ways:

4	3	2	1
1	2	3	4
3	1	4	2
2	4	1	3

4	3	2	1
1	2	3	4
2	1	4	3
3	4	1	2

4	3	2	1
1	2	3	4
2	4	1	3
3	1	4	2

4	3	2	1
1	2	3	4
3	4	1	2
2	1	4	3

4	3	1	2
1	2	3	4
2	1	4	3
3	4	2	1

4	3	1	2
1	2	3	4
3	4	1	2
2	1	4	3

4	3	1	2
1	2	4	3
2	1	3	4
3	4	2	1

4	3	1	2
1	2	4	3
2	4	3	1
3	1	2	4

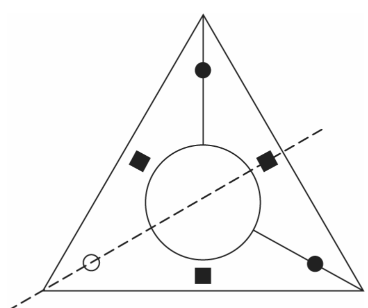
4	3	1	2
1	2	4	3
3	1	2	4
2	4	3	1

4	3	1	2
1	2	4	3
3	4	2	1
2	1	3	4

4	3	2	1
1	2	4	3
2	1	3	4
3	4	1	2

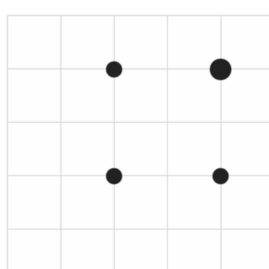
4	3	2	1
1	2	4	3
3	4	1	2
2	1	3	4

1.6. There is one axis of reflection symmetry. It is marked by the dashed line:



1.7. One. An axis of rotational symmetry is an axis about which the object can be rotated (through some angle other than a multiple of 360 degrees), that results in an indistinguishable appearance compared to the original orientation of the object. For a cone the axis passing through the center of the circular face and through the vertex (point) of the cone is the only axis of rotational symmetry.

1.8. One unit down and left from the upper right corner. This way the coins form a square:

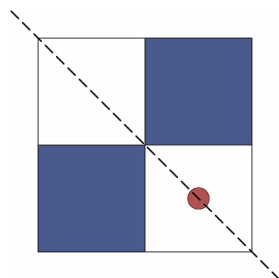


1.9. "T" and "A" are reflection symmetric across a vertical line passing through the center of the letter. "E" and "B" are reflection symmetric across a horizontal line passing through the center of the letter. "L" and "S" have no reflection symmetry.

1.10. Reflection and rotation symmetry. A sphere is reflection symmetric across any plane that passes through its center, and rotationally symmetric around any axis passing through its center.

1.11. A cube has 9 planes of reflection symmetry and 13 axes of rotational symmetry. 3 planes each bisect four sides of the cube. The remaining six planes pass through edges that are diagonally opposite each other. 3 of the axes of rotational symmetry pass orthogonally through the centers of two square faces. 4 of the axes pass through corners that are opposite each other along the body diagonal. 6 axes pass through the centers of edges that are opposite each other along the body diagonal.

1.12. There is one axis of reflection symmetry:



1.13. The maximum number of axes of reflection symmetry is two. There are two sides about which we have no information. Let us assume the side opposite the visible blue side is also blue, and the side opposite the visible red side is also red. In that case, the object would be reflection symmetric about a vertical plane that bisects both blue sides, and a about a vertical plane that bisects both red sides.

1.14. We use $v_x = \Delta x/\Delta t$ for constant speed in the x direction to write:

$$\Delta x = v_x \Delta t = \frac{299{,}792{,}458 \text{ m}}{1\text{s}} \times \frac{3600 \text{ s}}{1 \text{ h}} \times \frac{24 \text{ h}}{1 \text{ day}} \times \frac{365.25 \text{ days}}{1 \text{ yr}} \times 78 \text{ yrs} = 7.4 \times 10^{17} \text{ m}$$

1.15. (*a*) We convert units using known conversion factors:

$$9.3 \times 10^7 \text{ miles} \times \frac{1609 \text{ m}}{1 \text{ mi}} \times \frac{10^3 \text{ mm}}{1 \text{ m}} = 1.5 \times 10^{14} \text{ mm}$$

(*b*) If we divide the distance by the width of Earth, that will tell us how many Earths can fit in that distance.

$$\text{Number of Earths} = \frac{\text{Distance to Sun}}{2R_{\text{Earth}}} = \frac{9.3 \times 10^7 \text{ mi} \times \dfrac{1.609 \text{ km}}{1 \text{ mi}}}{2(6.38 \times 10^3 \text{ km})} = 1.2 \times 10^4 \text{ Earths}$$

1.16. The blue whale and a human have densities that are the same orders of magnitude, and contain similar concentrations of different atom types. Hence it is reasonable to say that the ratio of the numbers of atoms in these two species should be roughly equal to the ratio of volumes of these two species: $\dfrac{V_{\text{bw}}}{V_{\text{human}}} = \dfrac{10^{32}}{10^{29}} = 10^3$. Since whales and people are three dimensional, this corresponds to blue whales being roughly one order of magnitude larger in each of three dimensions. Hence, the blue whale is approximately 10 times longer than a human is tall.

1.17. Because the gastrotrich lifetime is given as three days, this should technically be treated as being on the order of one day. However, this is an approximate value of a lifetime, and if it were slightly higher, it would be treated as being on the order of ten days. Either of these is defensible in order of magnitude treatments. One tortoise lifetime can be related to gastrotrich lifetimes using the following order of magnitude conversions:

$$10^2 \text{ yr} \times \frac{10^3 \text{ days}}{1 \text{ yr}} \times \frac{1 \text{ gastrotrich lifetime}}{1 \text{ day}} = 10^5 \text{ gastrotrich lifetimes}$$

If we had used 10 days as the order of magnitude for a gastrotrich lifetime we would have obtained 10^4 gastrotrich lifetimes. Hence there are 10,000 to 100,000 gastrotrich lifetimes in one tortoise lifetime. Either of these is acceptable.

1.18. Given the diameter of a droplet of water, we can estimate the volume of a droplet using $V_{\text{drop}} = \frac{4}{3}\pi r^3 \approx 1 \times 10 \times (10^{-3} \text{ m})^3 = 10^{-8} \text{ m}^3$. We can estimate the volume of a human body to be on the order of $V_{\text{body}} = 10^{-1} \text{ m}^3$. Hence an order of magnitude estimate of the number of droplets in the human body is given by $\dfrac{V_{\text{body}}}{V_{\text{drop}}} = 10^7$.

1.19. Answers may vary by an order of magnitude since some textbooks may be somewhat thicker than 3.0 cm, and others may be thinner. My textbook has a thickness that is on the order of 10^{-1} m. The distance to the moon is 3.84×10^8 m, meaning the distance is of order 10^9 m. Dividing the distance by the thickness of one textbook yields $\text{Number of books} = \dfrac{\text{Distance to Moon}}{\text{Thickness of book}} = \dfrac{10^9}{10^{-1}} = 10^{10}$. Hence 10^{10} copies of my physics textbook could fit between Earth and the Moon.

1.20. We proceed by finding the total mass of water in the pool, and dividing this by the mass of a single molecule of water:
$N = \dfrac{m_{\text{pool}}}{m_{\text{molecule}}} = \dfrac{\rho_{\text{water}} V_{\text{pool}}}{M / N_{\text{A}}}$ where $\rho_{\text{water}} = 1000 \text{ kg/m}^3$ is the density of water, $V_{\text{pool}} = (15 \times 8.5 \times 1.5) \text{ m}^3$ is the volume of the swimming pool, $M = 0.018 \text{kg}$ is the molar mass of water, and $N_{\text{A}} = 6.02 \times 10^{23} \text{ mol}^{-1}$ is Avogadro's number. Using this information we find

$$N = \frac{(1.0 \times 10^3 \text{ kg/m}^3)(191 \text{ m}^3)}{(0.018 \text{ kg/mol})}(6.02 \times 10^{23} \text{ mol}^{-1}) = 6.4 \times 10^{30} \text{ molecules in the pool.}$$

1.21. (*a*) Since numbers are not given, it might be natural to cancel lengths and be left with a factor of 2^3. In that case the volume of the cube would increase by one order of magnitude. But if one were asked to use an order of magnitude estimate to first express ℓ_2 in terms of ℓ_1, one might find that they have the same order of magnitude and that the volume therefore does not increase. Either of these answers (one order of magnitude, or zero orders of magnitude) is acceptable. (*b*) Yes, because of the rules of rounding numerical values. For example, if $V_1 = 3.5 \text{ m}^3$, that value would round to an order of magnitude of 10 m^3. Then $V_2 = 8V_1 = 28 \text{ m}^3$, which also rounds to an order of magnitude of 10 m^3.

1.22. The speed of light is of order 10^8 m/s. The length of Earth's trip around the Sun is of order 10^{12} m. Hence the order of magnitude of the time light would need to make the same trip around the sun is $\text{time} = \dfrac{\text{distance}}{\text{speed}} = \dfrac{10^{12} \text{ m}}{10^8 \text{ m/s}} = 10^4 \text{ s}$. Hence light would take approximately 10^4 s to complete this trip.

1.23. The surface area of the roughly spherical distribution of leaves is $4\pi r^2$, where r is the radius of the tree's sphere of leaves. The surface area of an individual leaf is just its length ℓ times its width w. We can find the number of leaves by dividing the total surface area by the area of one leaf: $N = \dfrac{4\pi r^2}{\ell w} = \dfrac{10(10 \text{ m})^2}{(0.1\text{m})(0.1 \text{ m})} = 10^5$ leaves.

1.24. To find the number of generations since the universe began, we divide the age of the universe by the time required for one human generation. We simply have to get both times in the same units. Let us express one human generation in seconds: $30\ \text{yr}\times\frac{365.25\ \text{days}}{1\,\text{yr}}\times\frac{24\ \text{h}}{1\ \text{day}}\times\frac{3600\ \text{s}}{1\text{h}}=9.5\times10^{8}\ \text{s}.$ Hence the number of human generations N is given by $N=\frac{t_{\text{universe}}}{t_{\text{generation}}}=\frac{10^{17}\ \text{s}}{9.5\times10^{8}\ \text{s}}=10^{8}$ generations.

1.25. Not reasonable. Because light travels much faster than sound, any thunder peal is delayed compared to the light signal caused by the lightning bolt event. From the principle of causality, the lightning you see after you hear the peal cannot have caused the peal. The peal you heard must have come from a previous lightning strike.

1.26. Call the period of time required for one such oscillation T. Then the second is defined such that $1.0\ \text{s}=(9.19\times10^{9})T$ or $T=\frac{1.0\ \text{s}}{(9.19\times10^{9})}=1.09\times10^{-10}\ \text{s}.$

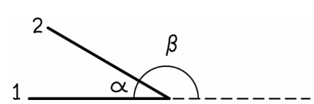
1.27. That the barrier lowers time after time 30 s before a train passes is consistent with a causal relationship between the two events. The single negative result, however, tells you that the lowering of the barrier cannot be the *direct* cause of the passing of the train. More likely, the lowering is triggered when the train passes a sensor quite a distance up the tracks from the barrier and the sensor sends an electrical signal to the lowering mechanism. A malfunction in either the sensor, the electrical connections, or the lowering mechanism would account for the one negative result you observed.

1.28. The period calculated in Problem 26 was $T=1.09\times10^{-10}\ \text{s}.$ The speed of light is $c=3.00\times10^{8}\ \text{m/s}.$ Here the distance travelled is just the speed times the time: $d=c\Delta t=(3.00\times10^{8}\ \text{m/s})(1.09\times10^{-10}\ \text{s})=0.0326\ \text{m}$.

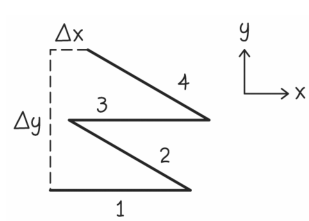
1.29. $E=mc^2$ Here, E is the type of energy described, m is the mass of the object, and c is the speed of light.

1.30. Rectangle, parallelogram, and two equilateral triangles meeting at a vertex forming an hourglass shape.

1.31. The problem states that two adjacent sides must make an angle of $30°$. This most likely means the interior angle between actual sides. This angle is labeled α in the figure below. But one might also describe the exterior angle in this way. This angle is labeled β below. If one accepts this interpretation, some of the $30°$ angles could be interior and some could be exterior.

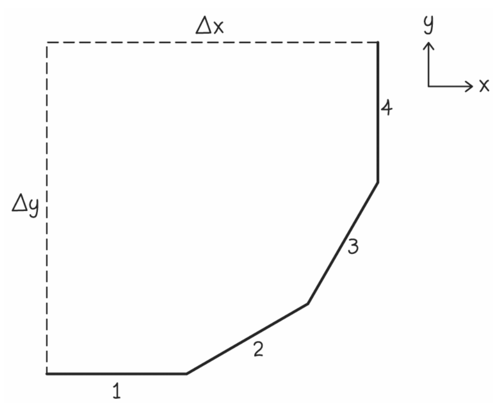


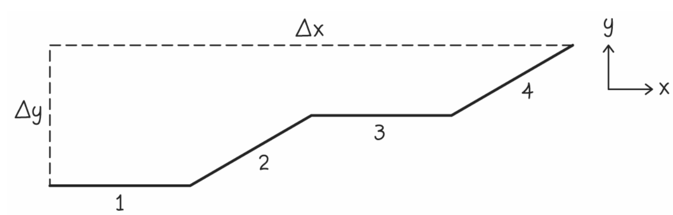
Let us first address the case in which the $30°$ angle refers to the interior angle. Remembering that the segments cannot cross, there is only one possible arrangement of segments that fits the description:



Clearly $\Delta y = 2\ell\sin(\theta)$ and $\Delta x = 2\ell(1-\cos(\theta))$. The Pythagorean Theorem tells us that the distance between the two unconnected points must be $d = \sqrt{\Delta x^2 + \Delta y^2} = 2\ell\left((1-\cos(30°))^2 + \sin^2(30°)\right)^{1/2} = 1.0\ell$.
So the distance between unconnected ends is ℓ.

If the $30°$ angle refers to the exterior angle, we can obtain several possible shapes. The two shapes that give the shortest and longest distances are shown below:

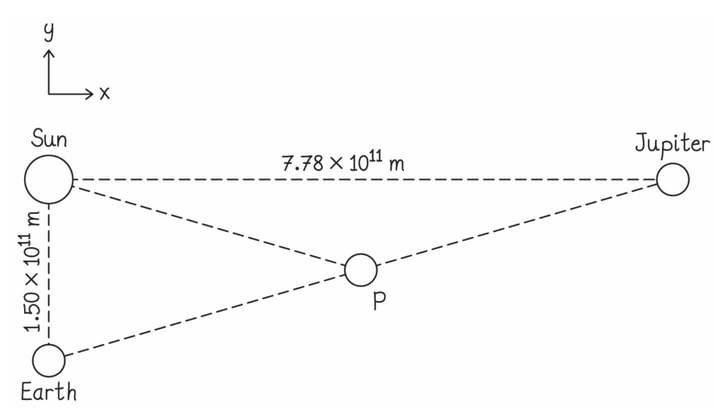




We can see in the top figure (that yields the shortest distance) $\Delta x = \ell(1+\cos(30°)+\cos(60°)+0)$, and since the x and y distances are equal based on symmetry, we find $d = \sqrt{2}\ell(1+\cos(30°)+\cos(60°)) = 3.3\ell$.
In the second figure (that yields the greatest distance) we find $\Delta x = 2\ell(1+\cos(30°))$ and $\Delta x = 2\ell(\sin(30°))$. Again using the Pythagorean Theorem yields a distance of $d = \left[(2\ell(1+\cos(30°)))^2 + (2\ell\sin(30°))^2\right]^{1/2} = 3.9\ell$.
If a combination of interior and exterior angles is used, there are even more possibilities. The shortest of these distances is 0 (parallelogram). It can be shown that other possible distances include 2.0ℓ, 2.2ℓ, 2.4ℓ.

1.32. Uncle, cousin, grandmother, aunt, grandfather, brother.

1.33. Consider the diagram below.



The distance from the Sun to point P is

$$\sqrt{\Delta x^2 + \Delta y^2} = (1/2)\sqrt{(1.50\times10^{11}\text{ m})^2 + (7.78\times10^{11}\text{ m})^2} = 3.96\times10^{11}\text{ m}$$

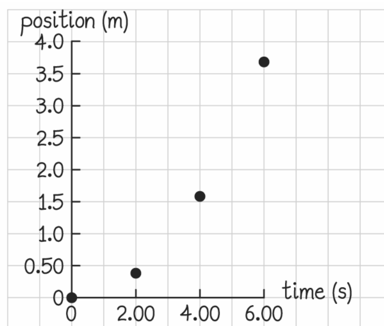
The time that light would require to cross this distance can be found using

$$\Delta t = d/v = (3.96\times10^{11}\text{ m})/(3.00\times10^{8}\text{ m/s}) = 1.32\times10^{3}\text{ s}$$

which is equivalent to about 22.0 minutes.

1.34.

Time (s)	Position (m)
0.00	0.00
2.00	0.40
4.00	1.59
6.00	3.64



1.35. (*a*) The position decreases linearly as a function of time, from an initial position of 4.0 m to a final position of zero at a time of 8.0 s, with a slope of -0.5 m/s. (*b*) $x(t) = mt + b$ where $m = -0.5$ m/s, and $b = 4.0$ m.

1.36. Forming a regular tetrahedron from these triangles will automatically satisfy all conditions.

1.37. $8.95\text{ m}\times\dfrac{1\text{ ft}}{0.3048\text{ m}}\times\dfrac{12\text{ inch}}{1\text{ ft}} = 352\text{ inches}$

1.38. We change units using known conversion factors:

$$35{,}000\text{ ft}\times\frac{1\text{ mile}}{5280\text{ ft}} = 6.629\text{ miles}$$

$$35{,}000\text{ ft}\times\frac{12\text{ in}}{1\text{ ft}}\times\frac{25.4\text{ mm}}{1\text{ in}}\times\frac{1\text{ m}}{10^3\text{ mm}}\times\frac{1\text{ km}}{10^3\text{ m}} = 10.7\text{ km}$$

1.39. (*a*) The density will be the same. (*b*) The density will be the same.

1.40. We change units using known conversion factors:

$$\frac{2.9979\times10^8\text{ m}}{1\text{ s}}\times\frac{1\text{ mile}}{1609\text{ m}} = 1.863\times10^5\text{ miles/s}$$

$$\frac{2.9979\times10^8\text{ m}}{1\text{ s}}\times\frac{1\text{ s}}{10^9\text{ ns}}\times\frac{10^3\text{ mm}}{1\text{m}}\times\frac{1\text{ in}}{25.4\text{ mm}} = 11.8\text{ inches/ns}$$

$$\frac{2.9979\times10^8\ \text{m}}{1\ \text{s}}\times\frac{1\ \text{km}}{1000\ \text{m}}\times\frac{3600\ \text{s}}{1\ \text{hr}}=1.079\times10^9\ \text{km/h}$$

1.41. If the two stones are made from the same material, they should have roughly the same density. We calculate the density of each stone and compare them:

$$\rho_1=\frac{m_1}{V_1}=\frac{2.9\times10^{-2}\ \text{kg}}{10.0\ \text{cm}^3}=2.9\times10^{-3}\ \text{kg/cm}^3$$

$$\rho_2=\frac{m_2}{V_2}=\frac{2.5\times10^{-2}\ \text{kg}}{7.50\ \text{cm}^3}=3.3\times10^{-3}\ \text{kg/cm}^3$$

No, it is not likely. Stone 2 has considerably higher density.

1.42. This length can be expressed as $1.000\ \text{mi}+440\ \text{yd}\times\frac{3\ \text{ft}}{1\ \text{yd}}\times\frac{1\ \text{mi}}{5{,}280\ \text{ft}}=1.250\ \text{mi}.$ We now convert this entirely to feet: $1.250\ \text{mi}\times\frac{5280\ \text{ft}}{1\ \text{mi}}=6{,}600\ \text{ft}.$

1.43. We can find the units of *A* from given information: $\text{units}[A]=\text{m/s}^2\cdot\text{s}^2=\text{m}.$ If *A* has units of meters, and we wish to add *A* and *B*, then *B* must also have units of meters.

1.44. Mass density is the ratio of mass per unit volume. While one could have any mass of a given substance, or any volume of that substance, the density tends to be a constant value for a given material (under certain conditions).

1.45. For all cases we find the order of magnitude of the mass of Earth using $m=\rho V=\rho\left(\frac{4}{3}\pi R_\text{E}^3\right).$

(*a*) $m=\rho_\text{air}\left(\frac{4}{3}\pi R_\text{E}^3\right)=(10^0\ \text{kg/m}^3)(1\times10\times(10^7\ \text{m})^3)=10^{22}\ \text{kg}$

(*b*) $m=\rho_{5515}\left(\frac{4}{3}\pi R_\text{E}^3\right)=(10^4\ \text{kg/m}^3)(1\times10\times(10^7\ \text{m})^3)=10^{26}\ \text{kg}$

(*c*) $m=\rho_\text{nucleus}\left(\frac{4}{3}\pi R_\text{E}^3\right)=(10^{18}\ \text{kg/m}^3)(1\times10\times(10^7\ \text{m})^3)=10^{40}\ \text{kg}$

1.46. We rearrange the given expression to solve for *y* and then write all units in terms of SI base units and powers of ten:

$$y=\left(\frac{x}{a}\right)^{2/3}=\left(\frac{61.7\ \text{Eg}\cdot\text{fm}^2/\text{ms}^3}{7.81\ \mu\text{g/Tm}}\right)^{2/3}$$

$$y=\left(\frac{61.7\ (10^{18}\ \text{g})\cdot(10^{-15}\ \text{m})^2/(10^{-3}\ \text{s})^3}{7.81\ (10^{-6}\ \text{g})/(10^{12}\ \text{m})}\right)^{2/3}$$

$$y=3.97\times10^{10}\ \text{m}^2/\text{s}^2$$

1.47. (*a*) $3.00\times10^8\ \text{m/s}$ (*b*) $8.99\times10^{16}\ \text{m}^2/\text{s}^2$ (*c*) No. There is a small difference because the answer to (*a*) was rounded before squaring. The answer to (*b*) was obtained using more digits of the speed of light, and only the result was rounded to three significant digits.

1.48. The given distance can also be written as 1.25 miles. We now convert to kilometers using known the known conversion factor:

$$1.25\ \text{mi}\times\frac{1.609\ \text{km}}{1\ \text{mi}}=2.012\ \text{km}$$

1.49. Your answer has four significant digits. When dividing a quantity by an integer, the number of significant digits should not change.

1.50. Yes, there is a difference in the precision. You will calculate your gas mileage by dividing the number of miles you travel by the gallons of gasoline used. Since the gas pump gives you thousandths (and most vehicles take 10 gal or more) you know the fuel used to five significant digits. Neither distance given has this many, making the precision of the distance value the limiting factor. Thus you can calculate mileage to three significant digits when you use 40.0 mi for distance and to four significant digits when you use 400.0 mi.

1.51. The odometer should still say 35,987.1 km. 47.00 m is only 0.04700 km. Because the odometer reading is only given to the nearest tenth of a kilometer, the sum (final odometer reading) must also be given only to the tenth of a kilometer.

1.52. We convert the given amount of ingested caffeine using known conversion factors:

$$\frac{34\text{ mg}}{\text{serving}}\times\frac{2\text{ servings}}{\text{day}}\times\frac{365.25\text{ days}}{1\text{ yr}}\times\frac{1\text{g}}{10^3\text{ mg}}\times\frac{1\text{ mol}}{194.19\text{ g}}\times\frac{6.02\times10^{23}\text{ molecules}}{1\text{mol}}=7.7\times10^{22}\text{ molecules/yr}$$

1.53. The molarity of the solution must be known to a precision of 1 part in 15. Since we can measure volume to arbitrary precision and we have the molar mass to very high precision, the limiting factor on the precision of our molarity is the precision of the mass we use. Hence the precision of the mass we use must be at least as good as the required precision of the molarity. We can only measure tenths of a gram, so we must have at least 1.5 grams in order to know the precision of our mass to 1 part in 15 or better. Mathematically:

$$\frac{m}{MV}\pm\frac{\Delta m}{MV}=(0.15\pm0.01)\text{ mol/L}$$
$$0.1\text{ g }\le\Delta m=0.01MV$$
$$V\ge\frac{0.1\text{ g}}{(0.01\text{ mol/L})(58.44\text{ g/mol})}$$
$$V\ge0.17\text{ L}$$
$$m=0.15(MV)=1.5\text{ g}$$

As can be seen from the above calculation, the answer could also be given in terms of the minimum volume of solution prepared. That minimum volume is 0.17 L.

1.54. The mass density is given by $\rho=m/V$. The volume is given in milliliters, which are equivalent to cubic centimeters. Hence

$$\rho=\frac{25.403\text{ g}}{23.42\text{ cm}^3}\times\frac{10^{-3}\text{ kg}}{1\text{ g}}\times\frac{10^6\text{ cm}^3}{1\text{ m}^3}=1.085\times10^3\text{ kg/m}^3$$

1.55. The mass evaporated each second is given by

$$\begin{aligned}\frac{|\Delta m|}{\Delta t}&=\frac{m_{\text{dish}}+m_{\text{liquid}}-m_{\text{final}}}{\Delta t}\\&=\frac{(145.67\text{ g})+(0.335\text{ g})-(145.82\text{ g})}{(25.01\text{ s})}\\&=7.4\times10^{-3}\text{ g/s}\end{aligned}$$

Note that we have only two significant digits. Because two of the masses were only known to the hundredths place, we only had two digits of precision after taking the sum (and difference) of masses.

1.56. There are too many significant digits. The volume is only given to two significant digits. The answer could have at most two significant digits.

1.57. We find the distance travelled by each hand in a day and we take the difference. The minute hand makes 24 full revolutions in one day, such that the distance travelled by its tip is

$$\frac{24\text{ rev}}{1\text{ day}}\times\frac{2\pi r}{1\text{ rev}}=\frac{24\text{ rev}}{1\text{ day}}\times\frac{2\pi(0.0113\text{ m})}{1\text{ rev}}=1.70\text{ m}$$

The hour hand makes only two revolutions, such that the distance travelled by its tip is

$$\frac{2\text{ rev}}{1\text{ day}}\times\frac{2\pi r}{1\text{ rev}}=\frac{2\text{ rev}}{1\text{ day}}\times\frac{2\pi(0.0080\text{ m})}{1\text{ rev}}=0.10\text{ m}$$

Clearly, the minute hand travels farther in one day by a distance of 1.60 m.

1.58. The path lengths are exactly the same along the straight sections of track. We need only consider the two curved ends. Along those sections, the runner in lane 1 moves through a circle of radius $R_1 = 36.80\text{ m}$. A runner in lane 8 moves through a circle of radius $R_8 = 36.80\text{ m} + 7(1.22\text{ m}) = 45.34\text{ m}$. The total path length difference will then be given by the difference in the circumferences of these two circles: $\Delta x = 2\pi(R_8 - R_1) = 2\pi(8.54\text{ m}) = 53.7\text{ m}$.

1.59. Place two coins on the balance, and hold the third in your hand. If they are balanced, the one coin in your hand is the counterfeit. If the two coins are not balanced, then one must be the counterfeit. Swap the lighter of the coins on the balance for the one in your hand. If the two coins now have equal mass, then the counterfeit coin is the one you just removed, and it is lighter than real coins. If they are still unbalanced, then the counterfeit is the one that left in place on the scale, and it is heavier than real coins.

1.60. (*a*) We proceed by assuming nearly perfect packing of the grains of rice (no unoccupied volume in the cup). This would almost certainly not be exactly true, but since the rice is not rigid after cooking, deformations could allow the rice to be tightly packed almost to this limit. With that assumption, we find the number of grains by dividing the total volume by the volume of a single grain:

$$N=\frac{V_{\text{cup}}}{V_{\text{grain}}}=\frac{V_{\text{cup}}}{\ell\pi R^2}=\frac{250\text{ cm}^3}{(0.6\text{ cm})\pi(0.1\text{ cm})^2}=1\times10^4\text{ grains}$$

(*b*) We simply divide the given energy by the number of grains we found in part (*a*): $\dfrac{785\text{ Cal}}{1.3\times10^4\text{ grains}}=0.06\text{ Cal/grain}.$

(*c*) The total food calories needed would be 8,000. Dividing this by the 785 food calories in one cup, we find that 10.2 cups of such rice would be required.

1.61. From *Principles* Figure 1.9 we see that there are approximately 10^{80} atoms in the universe. We also know that the number of atoms in a mole is of order 10^{24} atoms/Mole. Hence the number of Moles is given by

$$\frac{10^{80}\text{ atoms}}{10^{24}\text{ atoms/Mole}}=10^{56}\text{ Moles}$$

1.62. We divide the volume of the bread by the number of raisins in the bread and find:

$$\frac{V_{\text{bread}}}{N_{\text{raisin}}}=\frac{(1)(10\text{ in})^3}{10^2\text{ raisins}}=10\text{ in}^3\text{/raisin}$$

This volume could be treated as a spherical or cubic region of bread surrounding each raisin. The side length of such a cubic region would be of order 1 inch. This also means that 1 inch would be the order of magnitude of the distance between raisins.

1.63. We could divide the volume of the tree by the volume of the board to obtain the number of boards. But if a half-integer number of boards fits along the height of the tree, this does not help us and we have to round down. So, let us first find how many boards fit along the height.

$$h_{\text{tree}}=32\text{ m}\times\frac{10^3\text{ mm}}{1\text{ m}}\times\frac{1\text{ in}}{25.4\text{ mm}}\times\frac{1\text{ ft}}{12\text{ in}}=105\text{ ft or }17.5\text{ boards}$$

So we can only fit 17 entire boards along the height of the tree. Now if we look at a cross-section of the tree, we can calculate how many boards can fit across the tree. We will treat the tree as though it has a circular cross section.

$$\frac{A_{\text{tree}}}{A_{\text{board}}} = \frac{\pi(0.40\text{ m})^2}{(2\text{ in})^2} \times \left(\frac{10^3\text{ mm}^3}{1\text{ m}}\right)^2 \times \left(\frac{1\text{ in}}{25.4\text{ mm}}\right)^2 = 190\text{ boards}$$

Clearly, this overestimates the number of boards that can actually be cut, because of the curvature of the tree trunk. We might estimate how many boards will be affected by the curvature by determining how many boards would lie along the circumference of the trunk. We do this by dividing the circumference of the tree by the width of one board. Some might lie with an edge parallel to the trunk, whereas most will lie with their edges at some angle to the trunk. Still, as this is only an estimate, let us approximate the width of a single board as 2 in. Then the number of boards that would lie along the edge is

$$N = \frac{C_{\text{tree}}}{w_{\text{board}}} = \frac{2\pi r_{\text{tree}}}{w_{\text{board}}} = \frac{2\pi(0.40\text{ m})}{2\text{ in}} \times \frac{10^3\text{ mm}}{1\text{ m}} \times \frac{1\text{ in}}{25.4\text{ mm}} = 49\text{ boards}$$

So we estimate that we can fit a cross section of 140 complete boards in the cross section of the tree.
Finally, we have a total number of boards given by the number of boards that fit length-wise along the height of the tree times the number of complete boards in a cross section:

$$N_{\text{total}} = N_{\text{high}} N_{\text{across}} = 17 \times 140 = 2 \times 10^3$$

In the last step we have rounded to one significant figure. In addition to being given only one significant figure in the width of the board, we have also ignored secondary corrections such as wood that is lost to the sawing process. Hence our best estimate is 2×10^3 boards.

1.64. Estimates will vary, as there are many methods of estimating. One method of estimation would be to count the number of times it occurs on a few lines to obtain an average number of occurrences per line of text. One could then

multiply by the number of lines in a book. One could also estimate the number of letters in the book and assume that "d" occurs just as often as all other letters (not strictly true, but correct to an order of magnitude). The former method is crude, but requires no prior knowledge of the frequency of letters' occurrence in the English language. For this reason, we proceed with the former method. After counting the number of times "d" occurs on a few lines, I estimate that it occurs approximately twice on each line (of order 1). Each page of this book contains a number of lines that is of order 10^2. The book has a number of pages that is of order 10^3. Multiplying yields:

$$\frac{1\text{ "d"}}{1\text{ line}} \times \frac{10^2\text{ lines}}{1\text{ page}} \times \frac{10^3\text{ pages}}{1\text{ book}} = 10^5\text{ "d"/book}$$

One could obtain a much more reliable estimate if one happens to know that the letter "d" accounts for approximately 4% of the letters in a typical English text. But this method agrees with our crude method to the nearest order of magnitude.

1.65. The average full head of human hair has on the order of 10^5 hairs. Shoulder-length hair is right on the boundary between being of order 1 m and 0.1 m, but let us assume that the average human has hair that is shorter than 0.30 m. This yields a total length of all hairs of 10^4 m.

1.66. Not feasible during a four year stay at university, but perhaps possible by starting at age 5. $200K of university cost and 5 cents per can means the student must collect 4 million cans. Doing this during a 4-year university career would require almost 3000 cans every day. Because these must be pulled out of waste bins, even at one can every 10 seconds that uses more than 8 hours each and every day. This is not feasible. But if the student started collecting at age 5 and spent 12 full years at the job, at about 3 hours per day, by the time she reached university she would have her tuition money. Not a fun childhood, but perhaps feasible.

1.67. Call S the storage capacity and A the area required. We wish to obtain an order of magnitude by which the fraction S/A has increased. Hence we estimate

$$\frac{S_{\text{f}}/A_{\text{f}}}{S_{\text{i}}/A_{\text{i}}} = \frac{(10^{12}\text{ bytes})/\left((10^1\text{ plates})(10^{-2}\text{ m}^2/\text{plates})\right)}{(10^7\text{ bytes})/\left((10^2\text{ plates})(10^0\text{ m}^2/\text{plates})\right)} = 10^8$$

Hence storage per unit area has increased by a factor of approximately 10^8.

1.68. We convert to units of m/s using known conversion factors:

$$\frac{1.08243\times10^{19}\ \text{nm}}{1\ \text{yr}}\times\frac{1\ \text{m}}{10^{9}\ \text{nm}}\times\frac{1\ \text{yr}}{365.242\ \text{days}}\times\frac{1\ \text{day}}{24.0\ \text{h}}\times\frac{1\ \text{h}}{3600\ \text{s}}=343.009\ \text{m/s}$$

1.69. We first convert from inches to millimeters:

$$2.75\ \text{in}\times\frac{25.4\ \text{mm}}{1\ \text{in}}=69.9\ \text{mm}$$

We know this thickness is from 200 sheets, meaning $200t=69.9\ \text{mm}$ or $t=\dfrac{69.9\ \text{mm}}{200\ \text{sheets}}=0.349\ \text{mm/sheet}.$ Hence the thickness of each sheet is 0.349 mm.

1.70. (*a*) We use known conversion factors:

$$5.0\times10^{4}\ \text{L}\times\frac{1\ \text{m}^3}{10^3\ \text{L}}\times\frac{(10^3)^3\ \text{mm}^3}{1\ \text{m}^3}=5.0\times10^{10}\ \text{mm}^3$$

(*b*) The density of water is $\dfrac{1.0\times10^{3}\ \text{kg}}{\text{m}^3}\times\dfrac{10^6\ \text{mg}}{1\text{kg}}\times\dfrac{1\ \text{m}^3}{10^9\ \text{mm}^3}=1.0\ \text{mg/mm}^3.$ From here it follows trivially that the mass of the water is $5.0\times10^{10}\ \text{mg}.$

(*c*) Answers will vary based on assumptions about the size of a standard glass. A glass that holds a full liter would be very large, but a significant fraction of a liter would be plausible. Let us assume that 8 averaged sized glasses is approximately 5 L. Then we find

$$5.0\times10^{4}\ \text{L}\times\frac{1\ \text{day}}{5.0\ \text{L}}\times\frac{1\ \text{yr}}{365\ \text{days}}=30\ \text{yrs}$$

Hence the water would last about 30 years. Answers up to 45 years would also be plausible given different assumptions about the glass volume.

1.71. No, the model will not fit. Atoms are typically 10^5 times larger than nuclei. In order for the nucleus to be that large, the atom itself would need to be about 50 km across.

1.72. We calculate both sides of the equation and check for agreement.
(*a*) Initially keeping all the significant digits provided:

$$gR_E^2=GM_E$$

$$(9.80665\ \text{m/s}^2)(6.378140\times10^{6}\ \text{m})^2=(6.6738\times10^{-11}\ \text{m}^3/(\text{kg}\cdot\text{s}^2))(5.9736\times10^{24}\ \text{kg})$$

$$3.98941\times10^{14}\ \text{m}^3/\text{s}^2=3.98666\times10^{14}\ \text{m}^3/\text{s}^2$$

The results agree to three significant digits.
(*b*) Proceeding in exactly the same way as in part (*a*) one can round each quantity to various numbers of significant digits and calculate the two sides of the equation. We do this and look for a point at which the two answers no longer agree to three significant digits. One obtains:

$$4\ \text{sig. dig.}:3.98938\times10^{14}\ \text{m}^3/\text{s}^2=3.98705\times10^{14}\ \text{m}^3/\text{s}^2$$

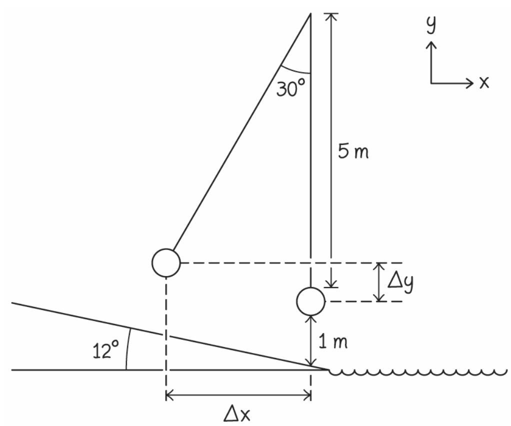
$$3\ \text{sig. dig.}:3.99310\times10^{14}\ \text{m}^3/\text{s}^2=3.98199\times10^{14}\ \text{m}^3/\text{s}^2$$

$$2\ \text{sig. dig.}:4.01408\times10^{14}\ \text{m}^3/\text{s}^2=4.02000\times10^{14}\ \text{m}^3/\text{s}^2$$

$$1\ \text{sig. dig.}:3.6\times10^{14}\ \text{m}^3/\text{s}^2=4.2\times10^{14}\ \text{m}^3/\text{s}^2$$

Clearly, the two sides of the equation agree to three significant digits unless the physical values are all rounded to only three significant digits or fewer.

1.73.



When the tire swing moves through an angle θ from vertical, it will have risen a vertical distance $L(1-\cos(\theta))$, and it will have moved over a horizontal distance $L\sin(\theta)$. Call the angle that the ground makes with the horizontal ϕ. Then $\tan(\phi) = \Delta y_{\text{ground}} / \Delta x_{\text{ground}}$. Hence we can write the amount by which the ground has risen, when the tire swing moves through a certain angle: $\Delta y_{\text{ground}} = \Delta x_{\text{ground}} \tan(\phi) = L\sin(\theta)\tan(\phi)$. The total vertical distance from the ground to the tire swing is then

$$y_{\text{i}} + \Delta y_{\text{swing}} - \Delta y_{\text{ground}} = (1.0\ \text{m}) + L(1-\cos(\theta)) - L\sin(\theta)\tan(\phi)$$

Not surprisingly, the maximum of this height occurs when the swing is at its maximum angle. This height is only about 1.1 m, which is a perfectly safe height. Of course, swinging out over the water (where the ground does not rise up to meet the swing) will result in a greater height above the water. But this is also only about 1.7 m, which is also not a dangerous height for falling into water. You might also have a concern that the tire swing may drag a child across the ground. We can find the minimum height of the tire swing above the ground by differentiating with respect to θ and requiring that the derivative be equal to zero. This yields the condition $\tan(\theta) = \tan(\phi)$, or $\theta = \phi$. This tells us that the minimum height of the tire swing above the ground is about 0.89 m. This is high enough that it is not likely that a child would unexpectedly strike the ground. There is no need for concern. The swing is perfectly safe.

1.74. We begin by calculating how much oxygen is actually needed per breath. Since oxygen is only 20.95% of the air we breathe, the volume of oxygen taken in during each breath is only 0.943 L. Since each breath only absorbs 25% of the oxygen present, the oxygen that actually gets used in each breath is only 0.236 L. This corresponds to approximately 2.36×10^{-4} kg of oxygen per breath. Finally we calculate how much oxygen is needed for an entire year using simple conversion factors:

$$\frac{2.36\times10^{-4}\ \text{kg}}{\text{breath}} \times \frac{15\ \text{breaths}}{1\ \text{min}} \times \frac{60\ \text{min}}{1\ \text{h}} \times \frac{24\ \text{h}}{1\ \text{day}} \times \frac{365\ \text{days}}{1\ \text{yr}} = 1900\ \text{kg}$$

The mass of oxygen required is around 1900 kg. This is reasonable. However, at atmospheric pressure and room temperature this air would occupy 1900 cubic meters, or a cubic room of side length 12 m. In an environment in which space is a precious commodity, this is probably not feasible. Alternatives would be to use highly compressed, cooled air to decrease storage requirements, or rely on plant life in the ship to replenish oxygen.

1.75. (*a*) The density is given by $\dfrac{m}{V} = \dfrac{10^{30}\ \text{kg}}{(1)(10)(10^4\ \text{m})^3} = 10^{17}\ \text{kg/m}^3$

(*b*) To the nearest order of magnitude, the density of Earth is $10^4\ \text{kg/m}^3$, and the density of water is $10^3\ \text{kg/m}^3$. Hence the neutron star is 13 orders of magnitude greater than the density of Earth and 14 orders of magnitude greater than the density of water. (*c*) The mass of a liter of water is 1.0 kg. Since the neutron star is 14 orders of magnitude denser than water, the mass contained in a 2-L container would be of the order 10^{14} kg.

2

Motion in One Dimension

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

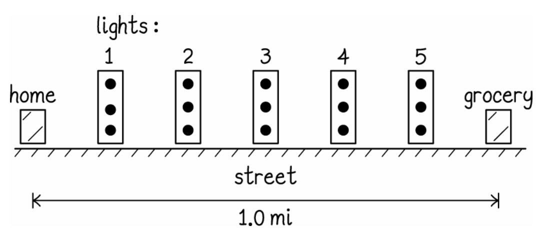
Developing a Feel

1. 10^2 m **2.** 10^{18} m **3.** 10^0 m; 10^1 m **4.** 10^0 s **5.** 10^5 s **6.** 10^2 m **7.** 10^2 m/s **8.** 10^0 m/s **9.** 10^5 s **10.** 10^7
11. 10^1 m/s **12.** 10^{-10} m

Guided Problems

2.2 City driving

1. Getting Started We start by drawing a diagram of the setup:



Between lights, your speed could not possibly be constant, since you have to stop and start at lights. But your average speed has already taken this stopping and starting into account. The average speed (while moving) is the distance between two lights divided by the time required to travel between those two lights. In terms of velocity, we can write $\vec{v}_{\text{av}} = \dfrac{\vec{x}_{\text{f}} - \vec{x}_{\text{i}}}{t_{\text{f}} - t_{\text{i}}}.$

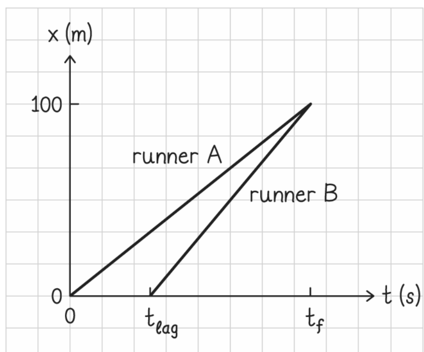
2. Devise Plan We can calculate the total time spent stopped a lights, because we know how much time is spent at each one. We can also easily calculate how much time is spent driving, because we know what distance must be covered, and we know what the average speed is while driving. The sum of these two times will be the total time for the trip. We already know the total distance travelled to get to the store, so knowing the total time will enable us to calculate the average speed using $v_{\text{av}} = d/t$. In general, average speeds and average velocities could be completely different. But in this case, the velocity is always in the same direction. In this case, the average speed will just be the magnitude of the average velocity.

3. Execute Plan (*a*) The time spent stopped at lights is $t_{\text{lights}} = (5\text{ lights}) \times \dfrac{(1\text{ min})}{\text{light}} \times \dfrac{1\text{ h}}{60\text{ min}} = 0.083\text{ h}.$ The time spent driving is $t_{\text{drive}} = \dfrac{d}{v} = \dfrac{1.0\text{ mi}}{20\text{ mi/h}} = 0.050\text{ h}.$ So the total time required for the trip is $t = 0.13\text{ h}.$ (*b*) The distance travelled is 1.0 mi. This is the same as the magnitude of your displacement in this case. Your displacement is 1.0 mi west. Hence the average velocity is $\vec{v}_{av} = \frac{\Delta\vec{x}}{t} = \frac{(1.0 \text{ mi}) \text{ west}}{0.13 \text{ h}} = 7.5 \text{ mi/h west}.$ (*c*) In this case the average speed is just the magnitude of the average velocity, so $v_{av} = 7.5 \text{ mi/h}.$

4. Evaluate Result The time required for the trip is 0.13 h or about 8 minutes. This is a reasonable time for a trip down the road. The speed and velocities do seem a little slow. But we know the answers must be significantly lower than 20 mi/h, because that is the average speed while moving. Therefore the speed and velocity also fit with expectations.

2.4 Race rematch

1. Getting Started We start by drawing a position vs time plot of the two runners:



We want the two runners to start at different times, but reach the 100 m mark at the same instant.

2. Devise Plan The runners are assumed to move at a constant speed in this problem. Certainly real runners require a few moments to reach their top speed. But after they reach their top speed, they may run at a constant speed to a very good approximation. If the runners move at a constant speed, we can express their positions as

$$x_f = x_i + v_x\Delta t = x_i + v_x(t_f - t_f) \tag{1}$$

We want to determine how long after runner A starts runner B should start. We call this time t_{lag}. We can use the speeds given to us in Worked Problem 2.3. We can set the final positions for the two runners at the final time equal to one another and solve for the lag time.

3. Execute Plan We write the position of each runner in the form of equation (1) and set the two final positions equal. We will call the position of the starting line $x_i = 0,$ and the time at which runner A starts $t = 0.$ To find the final time, let us consider runner A: $t_f = d/v_{Ax}.$

$$x_{Af} = v_{Ax}(t_f) = x_{Bf} = v_{Bx}(t_f - t_{lag})$$

$$t_{lag} = \left(1 - \frac{v_{Ax}}{v_{Bx}}\right)t_f$$

$$t_{lag} = \left(1 - \frac{v_{Ax}}{v_{Bx}}\right)\left(\frac{d}{v_{Ax}}\right)$$

$$t_{lag} = \left(1 - \frac{(8.00 \text{ m/s})}{(9.30 \text{ m/s})}\right)\left(\frac{100 \text{ m}}{8.00 \text{ m/s}}\right)$$

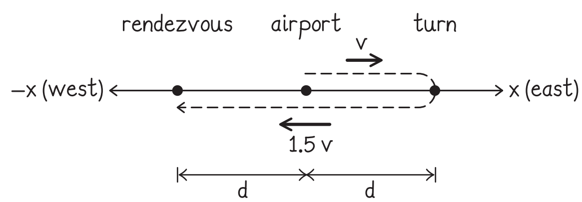
$$t_{lag} = 1.75 \text{ s}$$

Hence, runner B would have to wait 1.75 s before starting, in order for the race to end in a tie.

4. Evaluate Result This is a reasonable time to wait at the beginning of a 100 m race. We can compare this to the answer to Worked Problem 2.3, in which it was shown that a 2.0 s head start resulted in runner A winning by 2.4 m. Clearly, a 2.0 s head start is a little too long to end in a tie. But the result of Worked Problem 2.3 was so close to a tie that we would expect our answer to be fairly close to 2.0 s. Our answer of 1.75 s fits our expectations.

2.6 Wrong way

1. Getting Started We start by making a diagram of the setup. We show the initial velocity vectors of the helicopter pointing eastward, and the final velocities vectors as being larger and pointing westward. The distance between the airport and the turning point is d, as is the distance from the airport to the actual rendezvous point.



2. Devise a Plan This is similar to Worked Problem 2.5 in that it involves constant velocity motion in two separate intervals. The difference is that here the object turns around and heads in the opposite direction, whereas in Worked Problem 2.5, the object merely changed its speed.

Because no numbers are given, we expect an answer that contains given variables v and d. It also seems likely that the answer may be totally in terms of v, based on units (if d were in the expression, we would have no time variable to get that term in units of speed). We know the displacement vectors for each leg of the journey: $\Delta\vec{x} = d\,\hat{i}$ for the eastward leg and $\Delta\vec{x} = -2d\,\hat{i}$ for the westward leg. We can use the displacement and relative velocities to find expressions for the time required for each leg: $t_{\text{east}} = \dfrac{\Delta x}{v_x} = \dfrac{d}{v}$, and $t_{\text{west}} = \dfrac{\Delta x}{v_x} = \dfrac{-2d}{-1.5v} = \dfrac{4}{3}\left(\dfrac{d}{v}\right)$. Now that we have times in terms of the variables given, we can calculate expressions for the average x component of the velocity using $v_{x,\text{av}} = \dfrac{\Delta x}{t}$.

3. Execute Plan (*a*) The average x component of the velocity is given by

$$v_{x,\text{av}} = \frac{\Delta x}{t} = \frac{d - 2d}{\dfrac{d}{v} + \dfrac{4}{3}\left(\dfrac{d}{v}\right)} = -\frac{3}{7}v$$

(*b*) The velocity follows immediately from its x component: $\vec{v}_{\text{av}} = -\dfrac{3}{7}v\,\hat{i}$.

4. Evaluate Result Our result matches our expectations in terms of being written in terms of v. We also expect the final velocity to be in the $-x$ direction, because the final position of the helicopter is west of the airport. Finally, we expect the average velocity to have a magnitude that is significantly less than v because the helicopter changes directions. Our answer shows very good agreement with our expectations.

2.8 You're it!

1. Getting Started Child A runs eastward at a high and constant speed. Child B starts out very slowly but increases his speed in the eastward direction. Child C runs in the westward direction at a constant speed; his speed is not as great as Child A's. Child D starts our running east fairly quickly, but slows down. Child E remains stationary the entire time. Child F starts moving westward with the same initial speed as Child C, but Child F slows down almost to a stop.

2. Devise Plan We determine the direction of a child's motion by the sign of the slope on the $x(t)$ graph. If the slope is positive, the child is moving to the east; if the slope is negative, the child is moving to the west. Since the slope also tells us the magnitude of the child's velocity, and the sign tells us the direction, this information completely determines the velocity and speed of a child.

We can recognize a child moving at a constant velocity because the $x(t)$ curve will be a straight line. A child with a changing velocity will appear to have a curved path on the $x(t)$ graph (changing slope). Among the children who move at a constant velocity, the fastest child will have the greatest slope of the $x(t)$ line.

Whenever children pass each other, their position along the east-west direction must momentarily be the same. This corresponds to their $x(t)$ curves crossing each other.

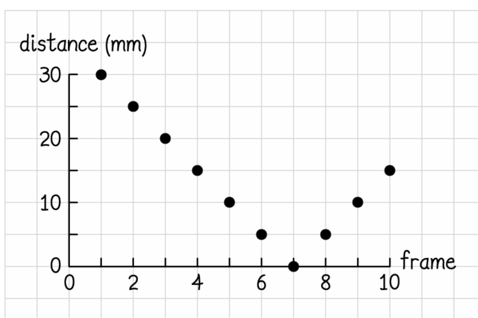
3. Execute Plan (*a*) Children A, B, and D are moving east; children C and F are moving west; child E is staying in the same place. (*b*) Children A, C, and E are all moving with constant velocity. The velocity of child A is positive, the velocity of child C is negative, and the velocity of child E is zero. (*c*) Children B, D, and F are changing their velocities as time passes. Children D and F are slowing down, while child B is speeding up. (*d*) Child A has the highest average speed. Child E has the lowest average speed. (*e*) Child B passes child D.

4. Evaluate Result These results are reasonable descriptions of playing children.

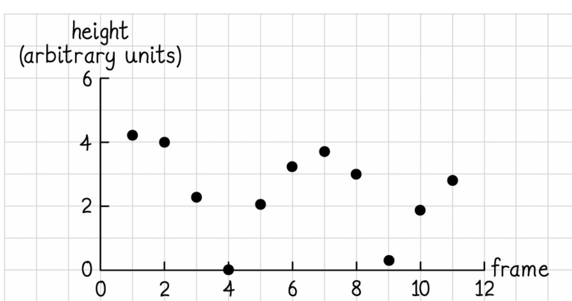
Questions and Problems

2.1. We must know the rate at which the camera took pictures (or equivalently the time interval between successive frames were captured), and something to provide a length scale (either the size of the object in the clip or a known distance between objects in the clip).

2.2.



2.3.



2.4. The object starts out 35 mm from the edge, moving toward the edge. This motion continues for 120 frames, at which point the object is 20 mm from the edge. The object remains in that position for 90 frames, before moving back away from the edge. It recedes back to the original distance (36 mm) between frames 210 and 270. The final motion away from the edge is faster than the initial motion toward the edge, assuming the frames were captured at a constant rate.

2.5. (*a*), (*c*), (*d*), (*e*), and (*f*) could all describe the same motion. All of these graphs show an object initially moving in one direction at a constant speed, then stopping, and then continuing on in the same direction as before at the same speed. They merely disagree on the location of the origin, which direction is positive, and the scales of the axes.

2.6. $\Delta x = x_f - x_i = (0.23\text{ m}) - (6.5\text{m}) = -6.3\text{ m}$

2.7. The distance is 6.4 km, whereas the displacement is zero (your initial and final positions are the same).

2.8. Zero. Since the distance of the race is an integer multiple of the track length $(\ell_{\text{race}} = 5\ell_{\text{track}})$ a runner will end the race at exactly the same position he or she started the race. Hence the displacement is zero.

2.9. (*a*) We could produce infinitely many graphs of the observations. We could put the origin anywhere, choose many possible timescales and guess many possible length scales. (*b*) We could still produce infinitely many graphs. We could still choose any timescale, and guess any length for the dog. (*c*) We could still choose any timescale and could therefore still make infinitely many graphs. (*d*) 2 (one for each choice of the positive direction).

2.10. (*a*) On your graph the person walking will always be somewhere on the positive x axis, whereas your friend's graph will show the person walking on the negative x axis. (*b*) Yes, both are equally good.

2.11. If numerical values of time and distance are converted, but the scale of each axis still uses the same numerical labels (that is, "0.40 m" becomes "0.40 in"), then the curve would be much narrower and much taller. This is just a matter of perspective, though. If the scale of each axis is also converted, so that "0.40 m" becomes "16 in", then the shape of the graph is not changed by the conversion of units.

2.12. (*a*) $\Delta\vec{x} = (x_\text{f} - x_\text{i})\ \hat{\imath} = ((+5\text{ m}) - (0))\ \hat{\imath} = (+5\text{m})\ \hat{\imath}$ (*b*) The object starts at the origin and moves in the $+x$ direction for 7 m before stopping. When the object starts moving again, it moves 2 m back in the $-x$ direction. Hence the object covers a total distance of 9 m.

2.13. In this case your total displacement in the x direction is made of several smaller displacements: $\Delta x = ((+4) + (-2) + (+1) + (+5) + (-7))\text{ blocks} = +1\text{ block}$.

2.14. Answers may vary depending on the height of the table. In the picture, the difference between the position of the bottom of the leg and the top of the table is $h = (+65\text{ mm}) - (+12\text{ mm}) = (+53\text{ mm})$. From this information, we can figure out the length scale in this picture. A standard table is approximately 0.75 m tall, though obviously there could be variations. Using this approximate height of 0.75 m, each millimeter of picture would correspond to a real life distance of $(0.75\text{ real m})/(53\text{ picture mm}) = 0.014\text{ real m/picture mm}$. Then the real length of the table would be given by this factor times the length of the table in the picture: $\left(0.014\text{ real m/picture mm}\right)\times ((+99\text{ mm}) - (+14\text{ mm})) = 1.2\text{ m}$.

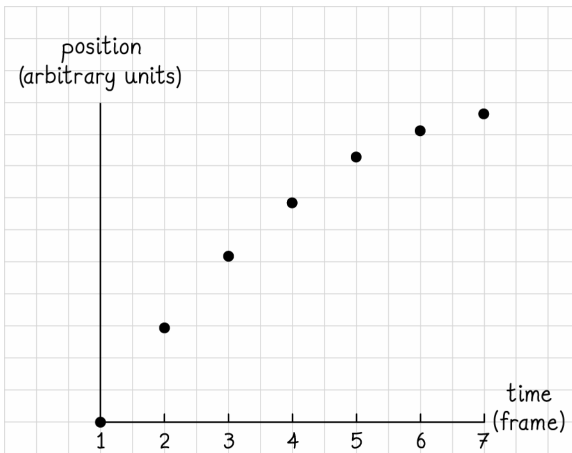
2.15. The swimmer swims in the positive x direction at a constant speed (left sloping leg of curve, increasing x values). She stops briefly (horizontal leg, most probably at end of her lane) and then returns to the starting point (right sloping leg, decreasing x values) at a speed slightly lower than her initial speed (this leg not as steep as left leg).

2.16. They are travelling along an essentially one-dimensional path. Because they travel in opposite directions over the course of the day, there must be an instant when they have the same position.

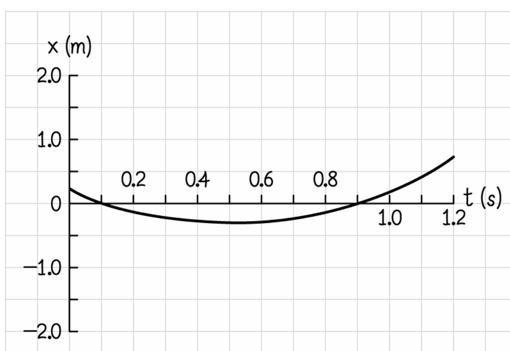
2.17. Interpolation always gives a continuous path, but there is no reason to expect that the path is accurate everywhere. Suppose you are photographing a clock's pendulum at 1.0-s intervals, collecting data to use in a graph showing the pendulum's position as a function of time. Suppose further that the pendulum takes 1.0 s to swing from left to right and back left again. At this swing speed, the pendulum has just enough time between photographs to swing and return to its initial position, so that the photographs make it appear that the pendulum does not move at all. An interpolation of data points collected from the photographs would show a continuous horizontal line on a position-versus-time graph, which is certainly not correct.

2.18. Starting from the earliest possible time, we see that the object first passes through $x = 2.0$ m at the time $t = 20$ s. After that, the object passes through the position $x = 3.0$ m three times: at $t = 30$ s, $t = 60$ s, and $t = 80$ s. Hence there are three correct answers: 10 s, 40 s, and 60 s.

2.19.



2.20. (*a*)



(*b*) We calculate the position of the object at the two times specified and take the difference:

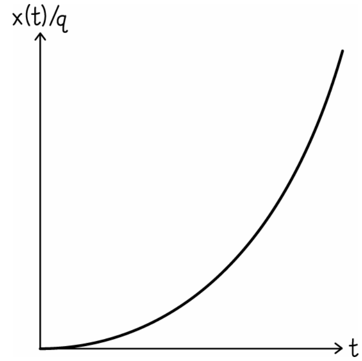
$$x(t = 0.50 \text{ s}) = (+0.20 \text{ m}) + (-2.0 \text{ m/s})(0.50 \text{ s}) + (+2.0 \text{ m/s}^2)(0.50 \text{ s})^2 = -0.30 \text{ m}$$

$$x(t = 0) = (+0.20 \text{ m}) + (-2.0 \text{ m/s})(0) + (+2.0 \text{ m/s}^2)(0)^2 = 0.20 \text{ m}$$

Hence $\Delta\vec{x} = (x_\text{f} - x_\text{i})\,\hat{\imath} = ((-0.30 \text{ m}) - (0.20 \text{ m}))\,\hat{\imath} = (-0.50 \text{ m})\,\hat{\imath}$.

(*c*) The farthest the object ever gets from the origin is its distance from the origin at the end of this time period (at $t = 1.2$ s). $x(t = 1.2 \text{ s}) = (+0.20 \text{ m}) + (-2.0 \text{ m/s})(1.2 \text{ s}) + (+2.0 \text{ m/s}^2)(1.2 \text{ s})^2 = (+0.68)$ m.

2.21. (*a*)



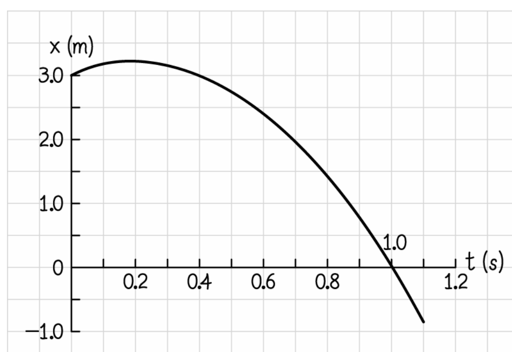
(*b*) $\Delta\vec{x} = (x_\text{f} - x_\text{i})\ \hat{\imath} = (q(3T)^3 - q(T)^3)\ \hat{\imath} = 26qT^3\ \hat{\imath}$

2.22. (*a*) $x(t=0) = (+3.0\text{ m}) + (+2.0\text{ m/s})(0) + (-5.0\text{ m/s}^2)(0)^2 = (+3.0)\text{ m}.$ (*b*) To find the time at which the position takes its maximum value, we take the derivative of the function with respect to time and solve for the time:

$$\frac{dx(t)}{dt} = q + 2rt = 0$$

$$t = -q/2r = \frac{-(+2.0\text{ m/s})}{2(-5.0\text{ m/s}^2)} = 0.20\text{ s}$$

Hence the time at which the function reaches a maximum value is 0.2 s. (*c*) Inserting the value obtained in part (*b*) into the position function yields $x(t=0.20\text{ s}) = (+3.0\text{ m}) + (+2.0\text{ m/s})(0.20\text{ s}) + (-5.0\text{ m/s}^2)(0.20\text{ s})^2 = (+3.2)\text{ m}.$ So the maximum value of the position is 3.2 m.
(*d*)



(*e*) The object starts at the position $x = 3.0\text{ m}$ and travels in the positive x direction with a decreasing speed. The object finally stops momentarily at $t = 0.2\text{ s}$ at $x = 3.2\text{ m}.$ The object turns around and accelerates steadily in the $-x$ direction, crossing the time axis at $t = 1.0\text{ s}.$ The motion is parabolic. (*f*) After 0.20 s, the object turns around. In the first 0.20 s, the object travels 0.20 m in the $+x$ direction. This initial distance must be added on to the distance the object moves in the $-x$ direction after 0.20 s. For the first interval:

$$x(t=0.50\text{ s}) = (+3.0\text{ m}) + (+2.0\text{ m/s})(0.50\text{ s}) + (-5.0\text{ m/s}^2)(0.50\text{ s})^2 = (+2.75)\text{ m}$$

Which corresponds to 0.20 m in the $+x$ direction and another 0.45 m in the $-x.$ Hence the total distance travelled is 0.65 m.

For the second interval:

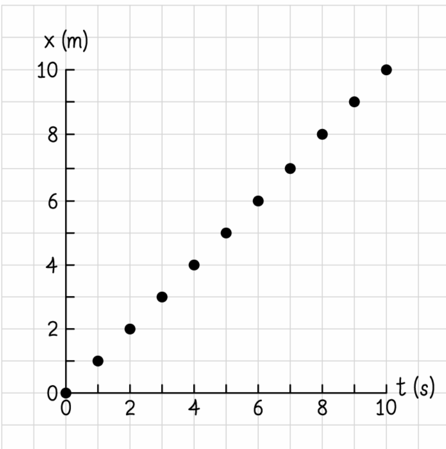
$$x(t=1.0\text{ s}) = (+3.0\text{ m}) + (+2.0\text{ m/s})(1.0\text{ s}) + (-5.0\text{ m/s}^2)(1.0\text{ s})^2 = (0)\text{ m}$$

This corresponds to an initial distance of 0.20 in the $+x$ direction and another 3.2 m in the $-x.$ Hence the total distance travelled is 3.4 m.

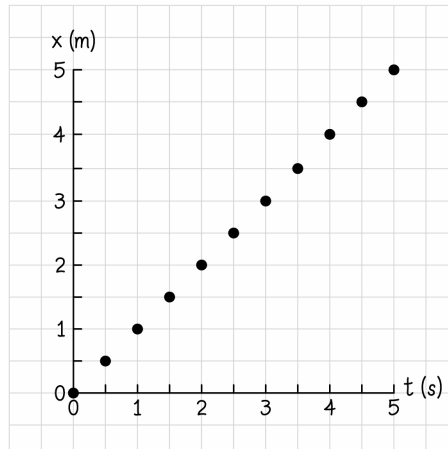
On the third interval, there is no change of direction. So in this case the distance is equal to the magnitude of the displacement. Since the position was 2.75 m at 0.50 s, and the final position is at 0.0 m after 1.0 s, the distance travelled is 2.75 m.

2.23.

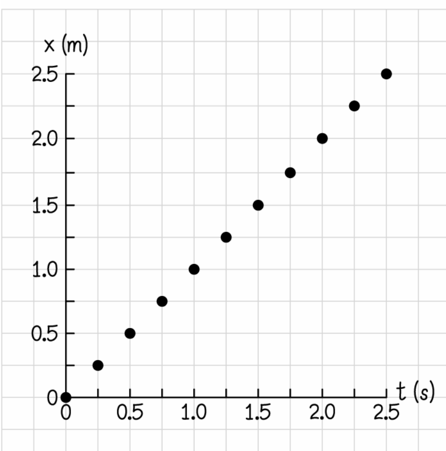
student A



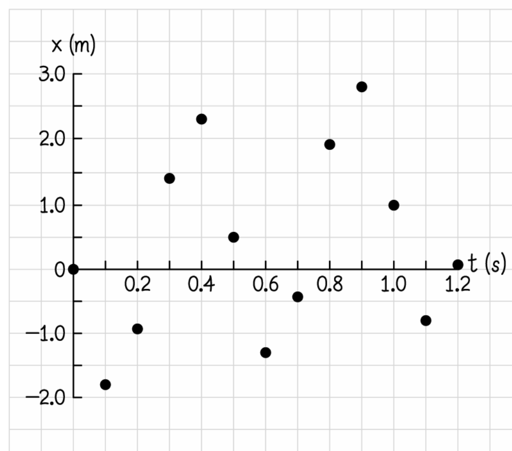
student B



student C



student D



Students A, B, and C agree that the motion is linear. Only student D has taken data points at time intervals short enough to reveal that the x values keep changing back and forth between positive and negative values.

2.24. A moving car can be filmed from the aircraft. Using the camera's frame rate and the known lengths of the marks, the police can calculate the car's speed.

2.25. In all cases we simply use $\text{speed} = \dfrac{\text{distance}}{\text{time}}$, converting units to SI as needed.

(*a*) $\text{speed} = \dfrac{100\text{ m}}{9.84\text{ s}} = 10.2\text{ m/s}$. (*b*) $\text{speed} = \dfrac{200\text{ m}}{19.32\text{ s}} = 10.4\text{ m/s}$. (*c*) $\text{speed} = \dfrac{400\text{ m}}{43.29\text{ s}} = 9.24\text{ m/s}$. (*d*) Here we first calculate the total time in seconds:

$$t = 3\text{ min} \times \frac{60\text{ s}}{1\text{ min}} + 27.37\text{ s} = 207.37\text{ s}$$

The speed is given as before.

$$\text{speed} = \frac{1500 \text{ m}}{207.37 \text{ s}} = 7.233 \text{ m/s}$$

(*e*) The total time is given by

$$t = 26 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} + 38.08 \text{ s} = 1598.08 \text{ s}$$

The speed is then

$$\text{speed} = \frac{10 \times 10^3 \text{ m}}{1598.08 \text{ s}} = 6.2575 \text{ m/s}$$

(*f*) The total distance is 138,435 ft, which we convert to meters.

$$(1.3844 \times 10^5 \text{ ft}) \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{0.0254 \text{ m}}{\text{in}} = 4.2195 \times 10^4 \text{ m}$$

The total time is

$$t = 2 \text{ h} \times \frac{3600 \text{ s}}{1 \text{ h}} + 6 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}} + 50 \text{ s} = 7610 \text{ s}$$

Hence the average speed is

$$\text{speed} = \frac{4.21950 \times 10^4 \text{ m}}{7610 \text{ s}} = 5.545 \text{ m/s}$$

2.26. (*a*) Yes. Speed is a scalar, the direction is irrelevant. (*b*) No. Velocity is a vector. The direction is different and so the velocity must be different.

2.27. (*a*) Because the images were created at equal time intervals, the spacing between adjacent images is a function of the ball's speed. That the spacing between adjacent images has one value in the first five frames and a different value in the final five frames tells you that the ball moved at one speed at the beginning of the motion and at a different speed at the end of the motion. (*b*) The ball had a higher speed in the final frames.

2.28. No. If the speed is constant, average speed and maximum speed are the same. If the speed is not constant, any speeds lower than the maximum contribute to the average and makes average speed lower than maximum speed.

2.29. (*a*) The cars are side by side at an instant just before t_2 and just after t_6. The graph shows that the cars have the same position at these times. (*b*) The cars have the same speed at (or very close to) t_4. Near t_4 there is a time at which the slopes of the two lines are equal, which is equivalent to the cars moving at the same speed.

2.30. The speeds of the pucks appear to the same. The spacing between images in (*a*) is about two-thirds the spacing between images in (*b*). But the frame rate is faster in (*a*) than in (*b*) by exactly two-thirds. Hence the distance per unit time is approximately the same.

2.31. B is closer to C than it is to A. Both segments take the same amount of time. Since the average speed from A to B was faster, the distance covered from A to B will be greater than the distance covered from B to C.

2.32. We find expressions for the time required in each case. Case 1 is travelling half the time at each speed, and case 2 is travelling half the distance at each speed. The distance from the starting point to the finish line *d* is the same in either case. We label the speeds, times, etc with subscripts that refer to the case being considered, and the speed travelled along a particular segment of the journey. For example, $t_{1,35}$ is the time required in case 1 for the segment travelled at 35 m/s.

Case 1: Call the time required in this case t_1.

$$d = v_{25}t_{1,25} + v_{35}t_{1,35} = (v_{25} + v_{35})\frac{t_1}{2} = (30 \text{ m/s})t_1$$

$$t_1 = \frac{d}{(30 \text{ m/s})}$$

Case 2: Call the time required in this case t_2.

$$t_2 = t_{1,25} + t_{1,35} = \frac{d/2}{v_{25}} + \frac{d/2}{v_{35}} = \frac{d}{2}\left(\frac{1}{25\text{ m/s}} + \frac{1}{35\text{ m/s}}\right)$$

$$t_2 = \frac{d}{(29.17\text{ m/s})}$$

Clearly $t_1 < t_2$, meaning that travelling half the time at each speed will yield a shorter time.

2.33. (*a*) The average speed is the distance covered in the total time: $\text{speed}_{\text{av}} = \frac{d}{t} = \frac{d}{t_{\text{there}} + t_{\text{back}}} = \frac{d}{\frac{d/2}{v_{\text{there}}} + \frac{d/2}{v_{\text{back}}}} =$ $2\left(\frac{1}{v_{\text{there}}} + \frac{1}{v_{\text{back}}}\right)^{-1} = 2\left(\frac{1}{10\text{ m/s}} + \frac{1}{16\text{ m/s}}\right)^{-1} = 12\text{ m/s}$ (*b*) He likely treated it as though the cyclist had ridden at each speed for equal times, rather than equal distances, and so just averaged the speeds: (10 m/s + 16 m/s) /2 = 13 m/s.

2.34. (*a*) The total distance that you cover is 2.5 km. The total time over which this trip was made is 100 minutes. The average speed is thus

$$\frac{2.50\times 10^3\text{ m}}{100\text{ min}} \times \frac{1\text{min}}{60\text{ s}} = 0.42\text{ m/s}$$

We could also express this in terms of km/h:

$$\frac{2.50\text{ km}}{100\text{ min}} \times \frac{60\text{min}}{1\text{ h}} = 1.5\text{ km/h}$$

(*b*) Zero. The initial and final positions are the same.

2.35. Given the distance you travel and your speed, we know that your trip requires a time $t = d/v = (500\text{ km})/(100\text{ km/h}) = 5.00\text{ h}$. Your brother leaves half an hour after you, meaning he makes the trip in only 4.50 h. Hence your brother's speed is $d/t = (500\text{ km})/(4.50\text{ h}) = 111\text{ km/h}$.

2.36. (*a*) Let us break the distance to the lake into three segments: the initial 3.0 km that you and your brother travel together, the additional distance that you travel before your brother reaches the lake, and the 5.0 km that remain for you to travel when your brother reaches the lake. Adding these three distances yields $d = (3.0\text{ km}) + \left((100\text{ km/h})(0.333\text{ h})\right) + (5.0\text{ km}) = 41\text{ km}$. (*b*) Your brother required one half hour to reach the lake, so his average speed is $\frac{d}{t} = \frac{(41.3\text{ km})}{(0.5\text{ h})} = 83\text{ km/h}$. (*c*) You have another 5.0 km to travel, at a speed of 100 km/h. The time is given by $t = \frac{d}{v} = \frac{(5.0\text{ km})}{(100\text{ km/h})} = 0.05\text{ h or }3.0\text{ min.}$

2.37. (1) No, because you could walk westward or eastward. (2) Yes, unless you also walk with some displacement north or south.

2.38. (*a*) 3 m (*b*) 3 m/s (*c*) –3 m/s

2.39. (*a*) 3 m (*b*) 3 m/s (*c*) 3 m/s

2.40. (*a*) –5 m for both (*b*) +5 m for both

2.41. (*a*) $A_x\hat{i}$ (*b*) $A_x\hat{i}$ Note that here A_x is a negative number (*c*) $A_x\hat{i}$

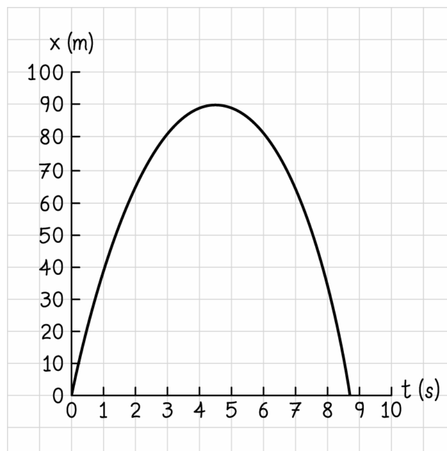
2.42. (*a*) $\vec{A}+\vec{B}=(+3.0\text{ m})\,\hat{\imath}+(-5.0\text{ m})\,\hat{\imath}=(-2.0\text{ m})\,\hat{\imath}$ (*b*) $\vec{A}-\vec{B}=(+3.0\text{ m})\,\hat{\imath}-(-5.0\text{ m})\,\hat{\imath}=(+8.0\text{ m})\,\hat{\imath}$

2.43. (*a*) The tip is 4.0 m above your head, meaning its *x* coordinate is $+4.0\text{m}$. (*b*) Expressed in terms of unit vectors, the position vector is $(+4.0\text{ m})\,\hat{\imath}$.

2.44. (*a*) We find the time using the quadratic equation:

$$t=\frac{-p\pm\sqrt{p^2-4(-q)(-x)}}{2(-q)}=\frac{-(42\text{ m/s})\pm\sqrt{(42\text{ m/s})^2-4(-4.9\text{ m/s}^2)(-20\text{ m})}}{2(-4.9\text{ m/s}^2)}=0.51\text{ s or }8.1\text{s}$$

(*b*) The rocket passes through a height of 20 m on its way up and again on its way down.
(*c*)

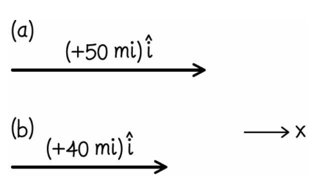


2.45. (*a*) 0.52 m (*b*) 0.8 m (*c*) 0.0 m (*d*) $\Delta\vec{x}=\vec{x}_\text{f}-\vec{x}_\text{i}=(0.8\text{ m }\hat{\imath})-(0.5\text{ m }\hat{\imath})=0.3\text{ m }\hat{\imath}$ (*e*) $\Delta\vec{x}=\vec{x}_\text{f}-\vec{x}_\text{i}=(0.0\text{ m }\hat{\imath})-(0.8\text{ m }\hat{\imath})=-0.8\text{ m }\hat{\imath}$ (*f*) $\Delta\vec{x}=\vec{x}_\text{f}-\vec{x}_\text{i}=(0.0\text{ m }\hat{\imath})-(0.5\text{ m }\hat{\imath})=-0.5\text{ m }\hat{\imath}$ (*g*) The object first travels a distance of 0.45 m in the positive *x* direction, followed by approximately 0.20 in the negative *x* direction. Hence the total distance travelled is 0.65 m (*h*) On this time interval, all motion is in the negative *x* direction. The object moves from an initial position of 0.8 m along the *x* axis to a final position of 0.0 m. Hence the object covers a distance of 0.80 m. (*i*) The object first moves 0.45 m in the positive *x* direction, followed by 1.0 m in the negative *x* direction. To the appropriate number of significant digits, this corresponds to a distance of 1.5 m.

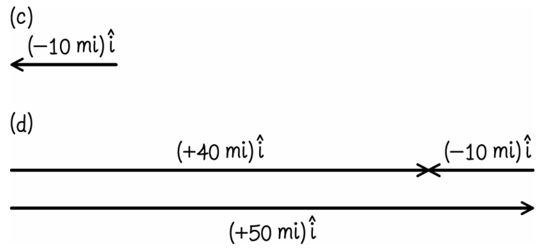
2.46. (*a*) $(+5\text{ m})\,\hat{\imath}+(-2\text{ m})\,\hat{\imath}+(+7\text{ m})\,\hat{\imath}$ (*b*) $(+5\text{ m})\,\hat{\imath}+(-2\text{ m})\,\hat{\imath}+(-7\text{ m})\,\hat{\imath}$ (*c*) $(+5\text{ m})\,\hat{\imath}+(+2\text{ m})\,\hat{\imath}+(-7\text{ m})\,\hat{\imath}$

2.47. $\vec{B}=-\vec{A}/2$

2.48. (*a*) $\vec{x}_\text{f1}=(+25\text{ mi/h})\,\hat{\imath}(2.0\text{ h})=(+50\text{ mi})\,\hat{\imath}$ (*b*) $\vec{x}_\text{f2}=\vec{x}_\text{f1}+(-20\text{ mi/h})\,\hat{\imath}(0.50\text{ h})=(+50\text{ mi})\,\hat{\imath}+(-10\text{ mi})\,\hat{\imath}=(+40\text{ mi})\,\hat{\imath}$ (*c*) $\Delta\vec{x}_2=\vec{x}_\text{f2}-\vec{x}_\text{f1}=(-20\text{ mi/h})\,\hat{\imath}(0.50\text{ h})=(-10\text{ mi})\,\hat{\imath}$ (*d*) $\Delta\vec{x}=\vec{x}_\text{f2}-\vec{x}_\text{f0}=(+40\text{ mi})\,\hat{\imath}-(0)=(+40\text{ mi})\,\hat{\imath}$
(*e*) You first travelled 50 mi in one direction, followed by 10 mi in the opposite direction. So the total distance travelled is 60 mi.
(*f*)

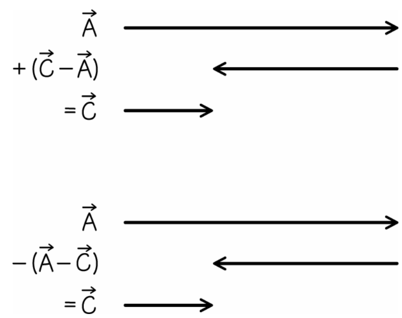


(*g*)



Yes, the values agree.

2.49. (*a*) $\vec{C}-\vec{A}$ (*b*) $\vec{A}-\vec{C}$

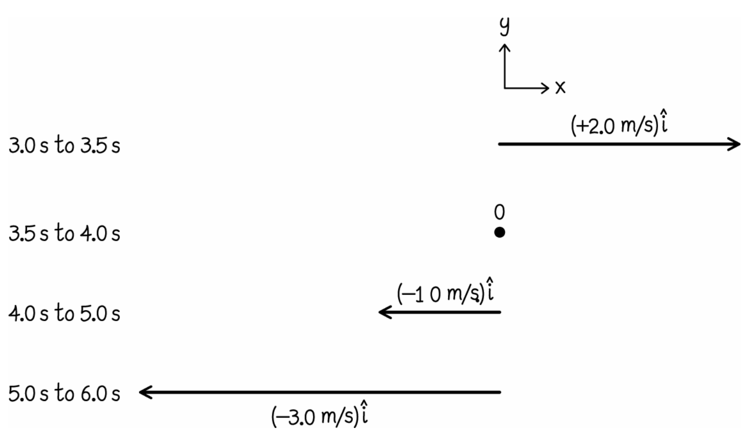


2.50. (*a*) 40 km/h (*b*) scalar (*c*) $d = vt = (40\text{ km/h})(2.0\text{ h}) = (80\text{ km})$. Distance is a scalar. (*d*) 80 km east (*e*) $\vec{v} = (40\text{ km/h})\,\hat{i}$.

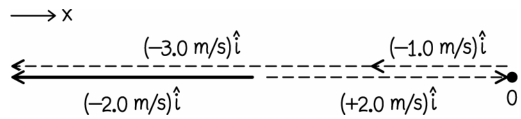
2.51. (*a*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_{\text{f}} - \vec{x}_{\text{i}}}{\Delta t} = \dfrac{(0.0\text{ m }\hat{i}) - (0.50\text{ m }\hat{i})}{1.2\text{ s}} = (-0.42\text{ m/s})\,\hat{i}$ (*b*) $\text{speed}_{\text{av}} = \dfrac{d}{\Delta t} = \dfrac{(1.5\text{ m})}{1.2\text{ s}} = (1.3\text{ m/s})$ (*c*) Some of the time the object was moving in the positive *x* direction, and sometimes it was moving in the negative *x* direction. Because velocity takes direction into account, this change of direction reduces the average velocity. In general because average velocity considers only actual distance between initial and final positions, but average speed considers distance traveled between these two positions; average speed is path-dependent, average velocity is not.

2.52. (*a*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_{\text{f}} - \vec{x}_{\text{i}}}{\Delta t} = \dfrac{(3.0\text{ m})\,\hat{i} - (0.0\text{ m})\,\hat{i}}{1.0\text{ s}} = (+3.0\text{ m/s})\,\hat{i}$ (*b*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_{\text{f}} - \vec{x}_{\text{i}}}{\Delta t} = \dfrac{(4.0\text{ m }\hat{i}) - (0.0\text{ m }\hat{i})}{1.0\text{ s}} = (+1.0\text{ m/s})\,\hat{i}$ (*c*) $\vec{v}_{\text{av}} = \dfrac{\vec{x}_{\text{f}} - \vec{x}_{\text{i}}}{\Delta t} = \dfrac{(0.0\text{ m }\hat{i}) - (3.0\text{ m }\hat{i})}{3.0\text{ s}} = (-1.0\text{ m/s})\,\hat{i}$ (*d*) Starting at time $t = 3.0$ s, the object first moves 1.0 m in the $+x$ direction, then turns around and moves 4.0 m in the $-x$ direction. Hence the object covers a total distance of 5.0 m between $t = 3.0$ s and $t = 6.0$ s. The average speed is then given by $d/t = (5.0\text{ m})/(3.0\text{ s}) = 1.7$ m/s.

(*e*)



(f) Adding the arrows from part (e) shown dashed below, we obtain the bold arrow:



2.53. We refer to the first two thirds of the trip as segment 1 and the last third as segment 2. Starting from the definition of average speed, we write

$$\frac{d}{t}=\frac{d_1+d_2}{t_1+t_2}=\frac{d_1+d_2}{d_1/v_1+d_2/v_2}$$

Solving this expression for v_2 yields

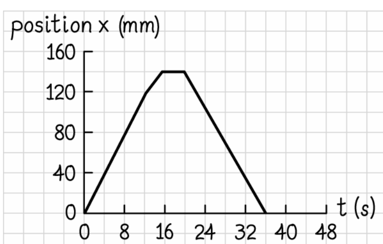
$$v_2=d_2\left(\frac{t(d_1+d_2)}{d}-d_1/v_1\right)^{-1}$$

$$v_2=d_2\left(\frac{(12\,\text{h})((2/3)d+(1/3)d)}{d}-(800\,\text{km})/(108\,\text{km/h})\right)^{-1}$$

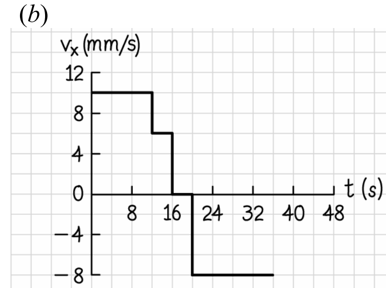
$$v_2=87\text{ km/h}$$

2.54. We find the final position from the given data using $x_\text{f}=x_\text{i}+v_xt=(-2.073\text{ m})+(-4.02\text{ m/s})(0.103\text{ s})$ $=-2.487$ m.

2.55. (a)



(b)



2.56. The distance covered by you added to the distance covered by your friend must equal four blocks. So we can write: $\Delta x=\Delta x_\text{you}+\Delta x_\text{friend}=v_\text{you}t+v_\text{friend}t.$ Assuming you leave your respective buildings simultaneously, the same amount of time passes for each of you. Solving for this time yields $t=\Delta x/(v_\text{you}+v_\text{friend})$. Since we know both speeds and the total distance covered, we can use this expression to find the distance that you cover:

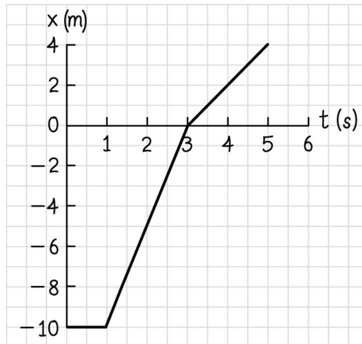
$$\Delta x_\text{you}=v_\text{you}t=v_\text{you}\Delta x/(v_\text{you}+v_\text{friend})=\frac{(1.6\text{ m/s})(4\text{ blocks})}{(1.6\text{ m/s})+(1.2\text{ m/s})}=2.3\text{ blocks}$$

Hence the restaurant must be 2.3 blocks from your building.

2.57. Object A has the greater displacement. The area under curve A is fairly large and positive, partly because the velocity is always in the $+x$ direction. The area under curve B appears to be approximately zero, as the velocity is negative for half the time shown and positive for half the time (with approximately equal magnitudes).

2.58. Object B has the greater displacement. The area under curve B is greater than the area under A.

2.59.



2.60. We simply calculate the travel time for each of you and compare. The time required for you to reach the school is given by $t_{\text{you}} = d/v_{\text{you}} = (1.5\times10^4 \text{ m})/(10 \text{ m/s}) = 1.5\times10^3 \text{ s}$. The time required for you friend to reach the school is the time spent riding added to the time spent fixing the tire, so $t_{\text{friend}} = d/v_{\text{friend}} + t_{\text{flat}} = (1.5\times10^4 \text{ m})/(15 \text{ m/s}) + (720 \text{ s}) = 1.7\times10^3 \text{ s}$. Hence, you get to the school first.

2.61. (*a*) Let us call the direction of your initial motion the $+x$ direction. We begin by finding how far from the house you were when you stopped to chat with a neighbor. You position was $x_{\text{neighbor}} = v_x t = (5.0 \text{ m/s})(600\text{s}) = 3.0\times10^3 \text{ m}$. So, after you turn around and make it halfway back to your house (when your friend passes you) you are 1.5×10^3 m from your house. This is the total displacement of your friend for the period of time in question. The total time elapsed depends on when you choose to mark the "beginning of the trip". Your friend remained at home for three minutes once you started riding. This could be treated as part of her trip (during which she had zero velocity) or one could start timing the trip once she finally starts moving. Here, we treat the trip as though it began at the same time for both of you, and you friend simply maintained zero velocity for three minutes. Then the total time is $\Delta t = \Delta t_{\text{away}} + \Delta t_{\text{neighbor}} + \Delta t_{\text{back}} = (600 \text{ s}) + (300 \text{ s}) + (1500\text{m})/(10 \text{ m/s}) = 1050 \text{ s}$. Then the magnitude of your friend's average velocity is $|v_{\text{av}}| = |(x_{\text{f}} - x_{\text{i}})|/\Delta t = (1500 \text{ m})/(1050 \text{ s}) = 1.4 \text{ m/s}$. (*b*) You and your friend have the same initial and final positions during this time interval. Hence your average velocity and hers must be the same. Hence the magnitude of her average velocity is also 10 m/s.

2.62. We start by finding the amount of time required for your roommate to arrive: $t = \Delta x/v = (320 \text{ mi})/(60 \text{ mi/h}) = 5 \text{ h, } 20 \text{ min}$. You wish to arrive after only 4h 50 min. Driving at 70 mi/h means you will have to actually be driving for $t = \Delta x/v = (320 \text{ mi})/(70 \text{ mi/h}) = 4 \text{ h, } 34 \text{ min}$. So you need 16 minutes less than you have, and can hence afford to stop for as much as 16 minutes.

2.63. The entire time of your trip can be written as $\Delta t = \Delta t_{\text{E}} + \Delta t_{\text{W}} + \Delta t_{\text{N}} + \Delta t_{\text{S}}$, where the subscripts indicate the direction of your run during different segments. Even though your westward return trip is split up, you still have to cross the same distance westward to return home as you initially jogged eastward. Since you jog east and west at 2.0 m/s, these two segments of your trip will take the same amount of time. Similarly, the distances and speeds for the northward and southward segments are the same. Hence we can write $\Delta t = 2\Delta t_{\text{E}} + 2\Delta t_{\text{N}} = 2d_{\text{E}}/v_{\text{E}} + 2d_{\text{N}}/v_{\text{N}}$. Solving for the distance in the northward direction yields

$$d_{\text{N}} = v_{\text{N}}(\Delta t - (2d_{\text{E}}/v_{\text{E}}))/2 = (3.0 \text{ m/s})((2400\text{s}) - 2(2000 \text{ m})/(2.0 \text{ m/s}))/2 = 6.0\times10^2 \text{ m}$$

Hence you travelled 6.0×10^2 m northward before turning around.

2.64. Position

2.65. The velocity is the rate of change of position, meaning $v_x(t) = \dfrac{dx(t)}{dt} = \dfrac{d}{dt}(bt^{3/2}) = \dfrac{3}{2}bt^{1/2}$. Hence

$$v_x(t = 1.0 \text{ s}) = \frac{3}{2}(30.2 \text{ m/s}^{3/2})(1.0 \text{ s})^{1/2} = 45 \text{ m/s}, \;\; v_x(t = 4.0 \text{ s}) = \frac{3}{2}(30.2 \text{ m/s}^{3/2})(4.0 \text{ s})^{1/2} = 91 \text{ m/s}$$

2.66. (*a*) It may be instrumental to first determine whether the direction of the mouse's velocity is always the same on the time interval specified. If the mouse changes direction, it must momentarily stop as its velocity passes from negative to positive or vice-versa. That is equivalent to saying

$$v_x(t_{\text{stop}}) = \left.\frac{dx(t)}{dt}\right|_{t_{\text{stop}}} = 2pt_{\text{stop}} + q = 0$$

This yields $t_{\text{stop}} = -q/2p = -(1.2 \text{ m/s})/(2(0.40 \text{ m/s}^2)) = 1.5$ s. So between $t = 0$ and $t = 1.0$ s the velocity of the mouse is always in the same direction. So here the speed and velocity should have the same magnitude. The average velocity is given by $\vec{v}_{\text{av}} = \dfrac{\vec{x}_{\text{f}} - \vec{x}_{\text{i}}}{\Delta t} = \dfrac{x(t = 1.0 \text{ s}) - x(t = 0)}{1.0 \text{ s}}\hat{i} = (-0.80 \text{ m/s})\,\hat{i}$, and the average speed is thus 0.80 m/s.

(*b*) The velocity can be calculated on this interval in the same way: $\vec{v}_{\text{av}} = \dfrac{\vec{x}_{\text{f}} - \vec{x}_{\text{i}}}{\Delta t} =$ $\dfrac{x(t = 4.0 \text{ s}) - x(t = 1.0 \text{ s})}{3.0 \text{ s}}\hat{i} = \dfrac{(1.6 \text{ m}) - (-0.80 \text{ m})}{3.0 \text{ s}}\hat{i} = (+0.80 \text{ m/s})\,\hat{i}$.

The average speed is not just the magnitude of the average velocity over this interval, because the velocity changes direction. To find the speed, we must consider that the mouse moves 0.10 m in the $-x$ direction before turning around and travelling 2.5 m in the $+x$ direction. Hence the average speed is $d/t = \big((2.5 \text{ m}) - (-0.1 \text{ m})\big)/(3.0 \text{ s}) = 0.87$ m/s.

2.67. Initially car A is moving faster than car B, because it passes car B. If car A continued moving faster than 30 m/s eastward the entire time, then there is no way that car B could ever have caught up with car A. Hence car A must have slowed from a velocity greater than 30 m/s eastward to some velocity that is less than 30 m/s eastward. The car cannot make discontinuous jumps in speed/velocity. So to pass smoothly from a velocity greater than 30 m/s eastward to a velocity less than 30 m/s eastward, it must have passed through the velocity of 30 m/s eastward at some point. Note that the speed could have always been greater than 30 m/s, but the velocity could not always have been greater than 30 m/s eastward.

2.68. For all cases, we use that $v_x(t) = \dfrac{dx(t)}{dt} = 3pt^2 + 2qt$

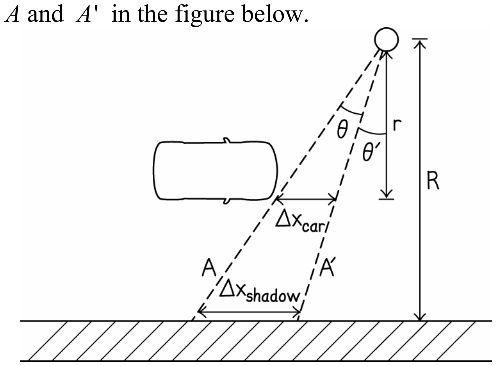
(*a*) $\vec{v}(\text{t} = 0) = 3(-2.0 \text{ m/s}^3)(0)^2 + 2(+1.0 \text{ m/s}^2)(0) = 0$

(*b*) $\vec{v}(\text{t} = 1.0 \text{ s}) = 3(-2.0 \text{ m/s}^3)(1.0 \text{ s})^2 + 2(+1.0 \text{ m/s}^2)(1.0 \text{ s}) = (-4.0 \text{m/s})\,\hat{i}$

(*c*) $\vec{v}(\text{t} = 1.0 \text{ s}) = 3(-2.0 \text{ m/s}^3)(2.0 \text{ s})^2 + 2(+1.0 \text{ m/s}^2)(2.0 \text{ s}) = (-20 \text{m/s})\,\hat{i}$

(*d*) $\vec{v}(\text{t} = 1.0 \text{ s}) = 3(-2.0 \text{ m/s}^3)(3.0 \text{ s})^2 + 2(+1.0 \text{ m/s}^2)(3.0 \text{ s}) = (-48 \text{m/s})\,\hat{i}$

2.69. (*a*) The average speed of the shadow's leading edge is greater. (*b*) No, speed of shadow is always greater. To see this, consider the rays of light A and A' in the figure below.



Ray A passes just in front of the front end of the car at time t_0 and ray A' makes the front edge at time $t_0 + dt$. In the short time interval dt, the car moves a distance Δx_{car} and the shadow moves a distance Δx_{shadow}. We can write expressions for each of these distances from the geometry shown in the figure:

$$\Delta x_{\text{car}} = r\tan(\theta) - r\tan(\theta')$$

$$\Delta x_{\text{shadow}} = R\tan(\theta) - R\tan(\theta')$$

We can write the ratio of the speed of the shadow v_{shadow} to the speed of the car v_{car} as

$$\frac{v_{\text{shadow}}}{v_{\text{car}}} = \frac{R(\tan(\theta) - \tan(\theta'))/dt}{r(\tan(\theta) - \tan(\theta'))/dt}$$

We can cancel everything but the relevant distances, provided that $\tan(\theta) - \tan(\theta') \neq 0$. This assumption is true on the interval $-90° < \theta, \theta' < +90°$ (note here that $\theta \neq \theta'$ as long as the speed is not zero, because we are considering a non-zero interval of time). Finally, we have $v_{\text{shadow}} = \dfrac{R}{r} v_{\text{car}}$ for all positions of the car. This means the speed of the shadow is always greater than the speed of the car. Note that this includes the moment when the car's leading edge is directly between the light and the wall.

2.70. (*a*) $v_{x,\text{av}} = \dfrac{x_\text{f} - x_\text{i}}{t} = \dfrac{(p + qt_\text{f}^2) - (p + qt_\text{i}^2)}{t_\text{f} - t_\text{i}} = \dfrac{(+2.00\ \text{m/s}^2)((3.00\ \text{s})^2 - (2.00\ \text{s})^2)}{(1.00\ \text{s})} = 10.0\ \text{m/s}$

(*b*) $v_{x,\text{av}} = \dfrac{x_\text{f} - x_\text{i}}{t} = \dfrac{(p + qt_\text{f}^2) - (p + qt_\text{i}^2)}{t_\text{f} - t_\text{i}} = \dfrac{(+2.00\ \text{m/s}^2)\left((2.10\ \text{s})^2 - (2.00\ \text{s})^2\right)}{(0.10\ \text{s})} = 8.20\ \text{m/s}$

(*c*) $v_{x,\text{av}} = \dfrac{x_\text{f} - x_\text{i}}{t} = \dfrac{(p + qt_\text{f}^2) - (p + qt_\text{i}^2)}{t_\text{f} - t_\text{i}} = \dfrac{(+2.00\ \text{m/s}^2)((2.01\ \text{s})^2 - (2.00\ \text{s})^2)}{(0.01\ \text{s})} = 8.02\ \text{m/s}$

(*d*)
$$\lim_{\Delta t\to 0}(\vec{v}_{\text{av}}) = \lim_{\Delta t\to 0}\left(\frac{\vec{x}(t+\Delta t) - \vec{x}(t)}{(t+\Delta t) - t}\right) = \lim_{\Delta t\to 0}\left(\frac{p + q(t+\Delta t)^2 - (p + q(t)^2)}{\Delta t}\right)\hat{i}$$
$$= \lim_{\Delta t\to 0}\left(\frac{q(t^2 + 2t\Delta t + \Delta t^2) - qt^2}{\Delta t}\right)\hat{i} = \lim_{\Delta t\to 0}(2qt + q\Delta t)\,\hat{i}$$
$$= 2qt\,\hat{i} = 2(2.00\ \text{m/s}^2)(2.00\ \text{s})\,\hat{i} = (+8.00\ \text{m/s})\,\hat{i}$$

(*e*) $v_x(t) = dx/dt = 2qt \Rightarrow v_x(t = 2.00\ \text{s}) = 8.00\ \text{m/s}$

2.71. (*a*) The average speed is the distance divided by time: $\dfrac{d}{\Delta t} = \dfrac{58\ \text{km}}{45\ \text{min}} \times \dfrac{1\ \text{min}}{60\ \text{s}} \times \dfrac{10^3\ \text{m}}{1\ \text{km}} = 21\ \text{m/s}$

(*b*) $\dfrac{d}{\Delta t} = \dfrac{58\ \text{km}}{45\ \text{min}} \times \dfrac{60\ \text{min}}{1\ \text{h}} \times \dfrac{1\ \text{mi}}{1.609\ \text{km}} = 48\ \text{mi/h}$

2.72. Yes, if the second place runner had a lower speed throughout most of the race, then the second place runner would be significantly behind the first place runner nearing the finish line. If the second place runner speeds up near the end, she could have a greater instantaneous speed and still fail to catch up with the first place runner.

2.73. (*a*) $x = -6.0\ \text{m}$ (*b*) $\vec{x} = (-6.0\ \text{m})\,\hat{i}$ (*c*) $x = 6.0\ \text{m}$

2.74. We use information about the distance and times for each runner to first calculate the time at which runner Q reaches the 1-km mark. Using that time, we calculate the positions of runners P and R. To do this we will need to know the speed of each runner:

$$v_\text{P} = \frac{d}{t_\text{P}} = \frac{5.000\ \text{km}}{15\ \text{min}} \times \frac{10^3\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{min}}{60\ \text{s}} = 5.555\ \text{m/s}$$

$$v_\text{Q} = \frac{d}{t_\text{Q}} = \frac{5.000\ \text{km}}{20\ \text{min}} \times \frac{10^3\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{min}}{60\ \text{s}} = 4.167\ \text{m/s}$$

$$v_R = \frac{d}{t_R} = \frac{5.000\ \text{km}}{25\ \text{min}} \times \frac{10^3\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{min}}{60\ \text{s}} = 3.333\ \text{m/s}$$

So, the time at which runner Q reaches the 1-km mark is given by $t = \frac{d}{v} = \frac{10^3\ \text{m}}{4.167\ \text{m/s}} = 240\ \text{s}.$ The positions of runners P and R after $240\ \text{s}$ are $d_P = v_P t = (5.555\ \text{m/s})(240\ \text{s}) = 1333\ \text{m},$ and $d_R = v_R t = (3.333\ \text{m/s})(240\ \text{s}) = 800\ \text{m},$ respectively. Hence runners P and R are $d_P - d_R = 533\ \text{m}$ apart as runner Q crosses the 1-km mark.

2.75. The problem only tells us by what distance one runner beats another. The problem says nothing about time, and therefore says nothing about absolute speed. We only know information about relative speeds, and can therefore only give an answer in terms of other variables in the problem. Let Δt_A be the time required for runner A to complete the race. When runner A crosses the finish line, runner B is only at the 90-m mark. Hence the speed of runner B can be written as

$$v_B = \frac{90\ \text{m}}{\Delta t_A} = \frac{100\ \text{m}}{\Delta t_B}$$

$$\Delta t_B = \frac{100\ \text{m}}{90\ \text{m}} \Delta t_A$$

Runner C is at the 90-m mark when runner B completes the race. Hence the time required for runner C to finish the 100-m race is

$$v_C = \frac{90\ \text{m}}{\Delta t_B} = \frac{100\ \text{m}}{\Delta t_C}$$

$$\Delta t_C = \frac{100\ \text{m}}{90\ \text{m}} \Delta t_B = \left(\frac{100\ \text{m}}{90\ \text{m}}\right)^2 \Delta t_A$$

The amount of time by which runner A beats runner C is

$$\Delta t_C - \Delta t_A = \left[\left(\frac{100\ \text{m}}{90\ \text{m}}\right)^2 - 1\right] \Delta t_A = 0.23 \Delta t_A$$

2.76. (*a*) At the instant car B passes the milepost, it is a certain distance behind car A. This distance is given by $v_A \Delta t.$ Car B is gaining on car A at a rate equal to the difference between their speeds: $v_{rel} = (v_B - v_A).$ The amount of time required for car B to catch up to car A is the distance separating them divided by the rate at which car B gains on car A: $t = \Delta t + v_A \Delta t / (v_B - v_A) = v_B \Delta t / (v_B - v_A).$ (*b*) The time interval found in part (*a*) began with car B right next to the mile post. So the distance from the milepost at which the two cars meet is equal to the distance travelled by car B in the interval found in (*a*). Hence $d = v_A v_B \Delta t / (v_B - v_A).$

2.77. (*a*) At the instant your friend crosses the starting line you are ahead by a distance of $d_{you} = v_{you} t = (4.0\ \text{m/s})(15\ \text{s}) = 60\ \text{m}.$ Once you increase your speed to 5.0 m/s, your friend is only gaining on you by 1.0 m/s (he is only moving 1.0 m/s faster than you are). So the time required for him to make up the distance between you is $t = \frac{d}{v} = \frac{60\ \text{m}}{1.0\ \text{m/s}} = 60\ \text{s}.$ (*b*) The 60 s time interval in part (*a*) began with your friend crossing the starting line. For that entire 60 s interval, your friend was running at a speed of 6.0 m/s. Hence your friend passes you a distance $d = vt = (6.0\ \text{m/s})(60\ \text{s}) = 3.6 \times 10^2\ \text{m}$ from the starting line.

2.78. The entire distance covered is 12 mi. The time required for the driving portion is given by $t = \frac{d}{v} = \frac{10\ \text{mi}}{45\ \text{mi/h}} = 0.22\ \text{h}.$ The time spent walking is equal to 0.67 h. Hence the average speed is $v_{av} = \frac{d}{t} = \frac{12\ \text{mi}}{0.89\ \text{h}} = 14\ \text{mi/h}.$

2.79. (*a*) $v_{\text{av}} = \dfrac{x_\text{f} - x_\text{i}}{t_\text{f} - t_\text{i}} = \dfrac{c(\sqrt{20\text{ s}} - \sqrt{10\text{ s}})}{10\text{ s}} = (0.131\text{ s}^{-1/2})c,$ and the speed at 15 s is $v(t = 15\text{ s}) = \dfrac{1}{2}ct^{-1/2}\Big|_{t=15\text{ s}} =$ $\dfrac{1}{2}c(15\text{ s})^{-1/2} = (0.129\text{ s}^{-1/2})c.$

Hence the average speed is greater than the instantaneous speed at 15 s. (*b*) This can be answered using intuition. The position is always increasing, but at a smaller and smaller rate. Hence the final speed should be smaller than the speed at all previous times. The average speed must be greater than the speed at 20 s. Again we confirm this explicitly. The average speed is the same as that calculated in part (*a*). The instantaneous speed at 20 s is $v(t = 20\text{ s}) = \dfrac{1}{2}ct^{-1/2}\Big|_{t=20\text{ s}} = \dfrac{1}{2}c(20\text{ s})^{-1/2} = (0.112\text{ s}^{-1/2})c.$ This confirms our argument that the average speed must be greater than the speed after 20 s.

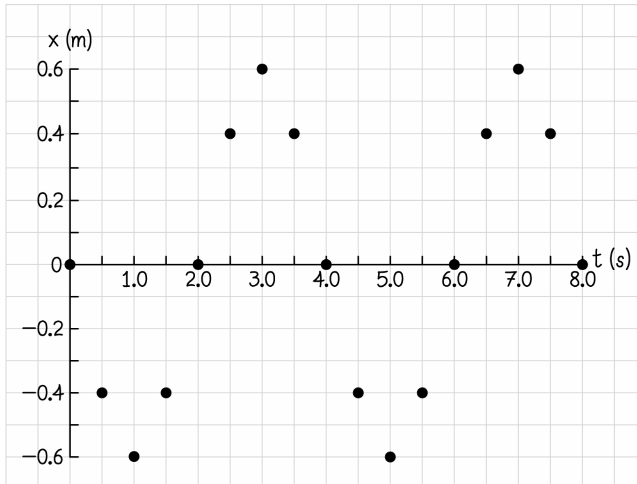
2.80. (*a*) These positions are all polynomials in time. The components of the velocities in the x direction are given by taking the derivative of each with respect to time. Terms that are linear in time will become time-independent. The only way for the velocity to have time dependence is for the position to have terms that are quadratic in time (t^2), or higher orders of time. Hence objects 3 and 4 have time-dependent velocities. (*b*) We solve each equation to find at what time the position is at the origin: Object 1 is never at the origin; it remains stationary at $x = 5$ m. Object 2 is at the origin when $bt + c = 0$ or $t = -c/b = -(-1\text{ m})/(+4\text{ m/s}) = 0.25\text{ s}.$ Object 3 is at the origin when $et^2 + ft = 0 \Rightarrow t = 0$ or $t = -f/e = -(-9\text{ m/s})/(+5\text{ m/s}^2) = 1.8\text{ s}.$ Finally, object 4 is at the origin when $gt^2 + h = 0$ or $t = \pm\sqrt{-h/g} = \pm\sqrt{-(+12\text{ m/s})/(-3\text{ m/s}^2)} = \pm 2\text{ s}.$ Hence, object 4 is at the origin at the earliest time, at $t = -2\text{ s}.$ (*c*) The *x* component of the object's velocity is given by the time derivative of the position: $v_x(t) = 2gt.$ So $v_x(-1\text{ s}) = 2(-3\text{ m/s}^2)(-1\text{ s}) = +6\text{ m/s}.$ The velocity is $(+6.0\text{ m/s})\,\hat{i}.$

2.81. (*a*) For the safe to move upward by a certain distance *d*, the length of rope on either side of the pulley attached to the safe must each decrease by *d*. Hence the mover must take up a length of rope 2*d* for every *d* of vertical rise of the safe. Hence the ratio of the vertical distance the safe moves to the length or rope pulled by the move is 1/2. (*b*) The relationship described in part (*a*) does not change in time. Hence, dividing any distances by times (to obtain speeds) will not affect the fraction found in part (*a*). The ratio is still 1/2.

2.82. The steam rollers approach each other at a combined relative speed of 2.0 m/s, and must cover 100 m between them. Hence, the steam rollers will meet after a time of $t = d/v = (100\text{ m})/(2.0\text{ m/s}) = 50\text{ s}.$ The fly is moving at a constant speed of 2.20 m/s. Hence the fly covers a distance of $d = vt = (2.20\text{ m/s})(50\text{ s}) = 110\text{ m}.$

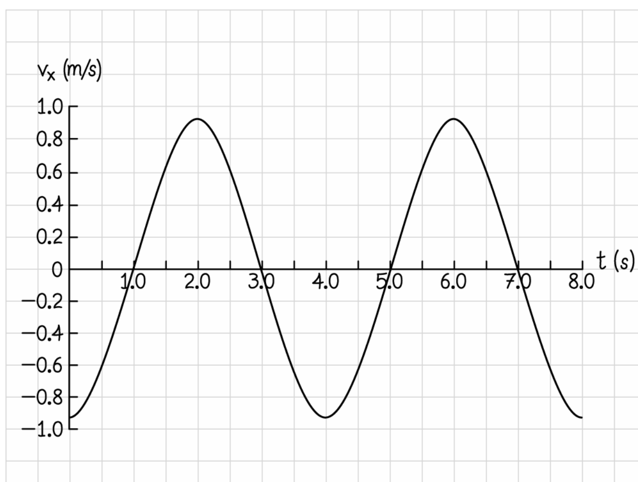
2.83. (*a*) $v_{x,\text{av}} = \dfrac{x_\text{f} - x_\text{f}}{t_\text{f} - t_\text{i}} = \dfrac{c(t_\text{f}^3 - t_\text{i}^3)}{t_\text{f} - t_\text{i}} = \dfrac{(+0.120\text{ m/s}^3)((1.50\text{ s})^3 - (0.500\text{ s})^3)}{(1.50\text{ s}) - (0.500\text{ s})} = (+0.39\text{ m/s})$ (*b*) $v_{x,\text{av}} = \dfrac{x_\text{f} - x_\text{f}}{t_\text{f} - t_\text{i}} =$ $\dfrac{c(t_\text{f}^3 - t_\text{i}^3)}{t_\text{f} - t_\text{i}} = \dfrac{(+0.120\text{ m/s}^3)((1.05\text{ s})^3 - (0.950\text{ s})^3)}{(1.05\text{ s}) - (0.950\text{ s})} = (+0.3603\text{ m/s})$ (*c*) Shortening the time interval by another factor of ten would make $t_\text{i} = 0.995\text{ s}$ and $t_\text{f} = 1.005\text{ s}.$ Then the same process as before yields $v_{x,\text{av}} = \dfrac{x(t = 1.005) - x(t = 0.995)}{0.01\text{ s}} = (+0.360003\text{ m/s})$ whereas $v_x = \dfrac{dx}{dt} = 3ct^2$ such that $v_x(t = 1.0\text{ s}) = 0.36\text{ m/s}.$

2.84. (*a*)



(*b*) These times could be read off of the plot in part (*a*) by looking for points where the slope of a continuous curve would be zero, or by solving $\frac{d}{dt}x(t)=0$. This equation becomes $\frac{d}{dt}(A\cos(pt+q))=-Ap\sin(pt+q)=0$. The sine function takes a value of zero whenever the argument is $n\pi$. Hence $pt+q=n\pi$ or $t=\frac{n\pi-q}{p}=\frac{(n-(1/2))\pi}{\pi/2\ \text{s}^{-1}}=2(n-(1/2))$ s. Hence, the x component of the velocity is zero at 1.0 s, 3.0 s, 5.0 s, 7.0 s.

(*c*)



2.85. (*a*) $d/2\Delta t$ (*b*) $2\Delta t$ (*c*) The distance traveled by the runner is an infinite series that requires infinitely many terms to approach *d*, but the time intervals that correspond to these distance intervals also get smaller and smaller in the series. At higher and higher terms in the series, the runner travels almost no distance in each term but does so in a time interval that is almost zero. Because the two effects cancel each other, the runner travels the distance from starting line to finish line in a finite time interval.

2.86. You start at the first light. The second light won't turn green for 10 s. So if you travel at a speed of $v=d/t=(300\text{ m})/(10\text{ s})=30\text{ m/s}$, you would have to stop at the second light momentarily. Hence there is an upper limit on your speed of 30 m/s. Since there is a 10-s lag between each light turning green, the last light will not turn green until 30 s after the first one turns green. That light will only stay on for 15 s, meaning you have a total of 45 s to get from the first light to through the fourth light. This requires that your speed be at least

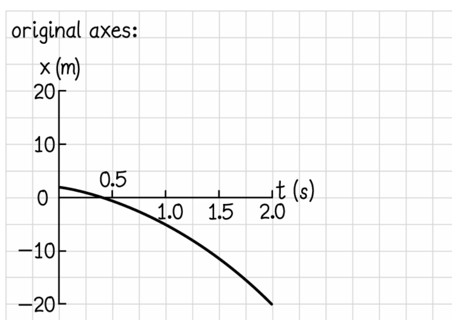
$v = d/t = (900\text{ m})/(45\text{ s}) = 20\text{ m/s}$. Hence the only constraints set by the lights are that your speed must satisfy $20\text{ m/s} < v < 30\text{ m/s}$. But if you also want to drive as slowly as possible, then $v = 20\text{ m/s}$ is best.

2.87. Since Hare can cover 1.0 mi in 10 minutes, when he falls asleep at after 5 minutes he must be at the half-way point (0.5 mi). This distance would take Tortoise 30 minutes, since he can only cover 1.0 mi in one hour. So 30 minutes into the race, Hare and Tortoise are both at the half-way point, with Hare only 25 minutes into his 40 minute nap, and with tortoise increasing his speed to (5/3) mi/h. From the half-way point, Tortoise will reach the finish line in an additional $t = d/v = (0.50\text{ mi})/(5/3\text{ mi/h}) = 0.3\text{ h}$. Hare will not wake up for another 0.25 h. This means Hare would have to cover the remaining 0.50 mi in just 0.05 h. Hare would have to run at 10 mi/h to win. This is not possible. Hare can only run at 6 mi/h. If Hare ran at that top speed, Hare could only cover $x = vt = (6\text{ mi/h})(0.05\text{ h}) = 0.30\text{ mi}$ before Tortoise crosses the finish line. So, Tortoise wins by 0.2 miles, even with Hare running at his top speed. Even if Hare can push himself a little faster than 6 mi/h, it is not likely that he could suddenly increase his top speed by 67% to reach the required 10 mi/h.

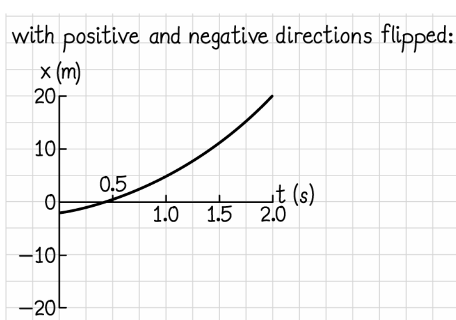
2.88. Runner B will require 13.5 s to complete the race. Runner A has an average speed of $v_{x,\text{av}} = d/t = (100\text{ m})/(12.0\text{ s}) = 8.33\text{ m/s}$. Hence, in the 13.5 s needed by runner B, runner A can cover a distance of $d = vt = (8.33\text{ m/s})(13.5\text{ s}) = 112.5\text{ m}$. Hence, runner A should add an extra 12.5 m to her distance, starting 12.5 m behind the starting line.

2.89. (*a*) Keeping the coefficients as they are defined in the problem, the position would become $x(t) = -p - qt - rt^2$. (*b*) Before any change in coordinates, the initial position of the car was at $x = (+2.0\text{ m})$. This must now be subtracted off, such that the car starts at the origin. This could be written as $x(t) = (p - 2.0\text{ m}) + qt + rt^2$ or simply as $x(t) = qt + rt^2$.

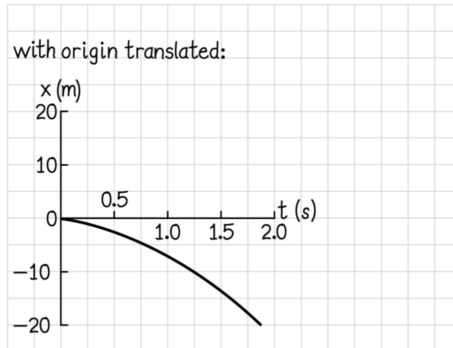
(*c*)



With the positive and negative directions flipped:



With the origin translated:



(*d*) Before axes were altered, the equation was $x(t) = p + qt + rt^2$, such that $x(t = 4.0\text{ s}) = (+2.0\text{ m}) + (-3.0\text{ m/s})(4.0\text{ s}) + (-4.0\text{ m/s}^2)(4.0\text{ s})^2 = -74\text{ m}$. After the direction of the axis is flipped, everything changes sign, such that the new displacement is +74 m. With the origin shifted, all that changes from the initial setup is that the position is shifted by –2.0 m, such that the position is now –76 m. (*e*) –35 m/s, 35 m/s, –35 m/s (*f*) You have only changed what position you choose to call zero, and which directions you choose to call positive.

2.90. The time required for light to travel from Earth to Mars is $t = d/v = (2.0\times10^{11}\text{ m})/(3.0\times10^{8}\text{ m/s}) = 670\text{ s}$ or just over 11 minutes. This means it would take 22 minutes for an image of a cliff to reach you and for you to send back a "stop" signal. In those 22 minutes, the rover can travel about 44 m. Hence, if the rover is travelling at top speed, you would need to be able to see 44 m ahead of the rover. However, it is probably not advisable to push the limits of this. It is more likely that you would send a command to move 25 m and then stop, rather than letting the rover continue until you specifically send a "stop" signal.

2.91. A loaded truck takes 7 hours to reach the mill, and an empty truck takes 6 hours to return from the mill. First consider your trip to the mill: Because the drivers from the mill should be spaced 30 minutes from the mine, 90 minutes from the mine, [etc.], there should be 6 other trucks already on the road. You must pass all 6 of these, plus any additional trucks that are sent out during your trip. Since your trip lasts 7 hours, 7 additional trucks will be sent out as you drive. This makes a total of 13 empty trucks that you pass along the way. The answer does not change on your return trip. One way to easily see this is that when you leave for the mine, there are 7 trucks already on the road. That is one more than we considered for your trip to the mill. But because you are driving faster now, you will require only 6 hours for the trip, meaning only 6 additional trucks will be sent out as you drive. That is one fewer than we considered on your trip to the mill. Hence you also pass 13 full trucks on the way back.w

3

ACCELERATION

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

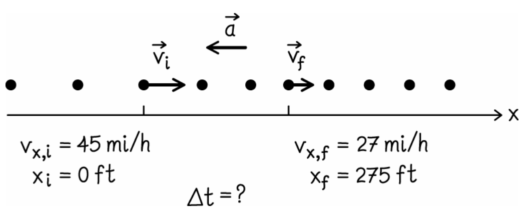
1. 10^2 s **2.** 10^2 m/s **3.** 10^3 m/s **4.** 10^1 m/s^2 **5.** 10^1 m/s^2 **6.** 10^2 m/s^2 **7.** 10^0 m/s^2 **8.** 10^0 m/s^2 **9.** 10^2 m/s^2 **10.** 10^0 m/s^2

Guided Problems

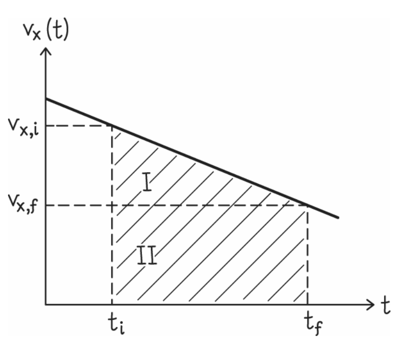
3.2 Slowing down

1. Getting Started This is a constant acceleration problem. Let the direction of the initial velocity of the car be the positive x direction. We know the initial speed in the x direction $v_{x,\mathrm{i}}$ and the final speed $v_{x,\mathrm{f}}$. Since the motion is always in the x direction, the distance traveled d is the same as the magnitude of the displacement $|\Delta x| = |x_\mathrm{f} - x_\mathrm{i}|$. We must find the change in time $\Delta t = t_\mathrm{f} - t_\mathrm{i}$.

2. Devise Plan We draw a motion diagram.



Just as in Worked Problem 3.1, we use our knowledge that the displacement is equal to the area under a constant-acceleration $v(t)$ graph. We construct a graph very similar to Principles Figure 3.17b, but with a decreasing speed:



As is Worked Problem 3.1, the area under the curve is most easily found by considering the triangular and rectangular regions separately. The result relates the elapsed time to the known variables.

3. Execute Plan The area under the curve is given piecewise by

$$I : \frac{1}{2}(v_{x,\mathrm{i}} - v_{x,\mathrm{f}})(t_\mathrm{f} - t_\mathrm{i})$$

$$II : (v_{x,\mathrm{i}} - v_{x,\mathrm{f}})(t_\mathrm{f} - t_\mathrm{i})$$

so that the total area can be written in terms of the elapsed time $\Delta t = t_\mathrm{f} - t_\mathrm{i}$ as

$$\Delta x = \frac{1}{2}(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})\Delta t$$

This can be easily rearranged to solve for time:

$$\Delta t = \frac{2\Delta x}{(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})} \tag{1}$$

In order to plug in values and obtain a numerical answer, we first have to convert all given quantities to SI units:

$$v_{x,\mathrm{i}} = \frac{45.0\ \mathrm{mi}}{\mathrm{h}} \times \frac{1609\ \mathrm{m}}{1\mathrm{mi}} \times \frac{1\ \mathrm{h}}{3600\ \mathrm{s}} = 20.1\ \mathrm{m/s}$$

$$v_{x,\mathrm{f}} = \frac{27.0\ \mathrm{mi}}{\mathrm{h}} \times \frac{1609\ \mathrm{m}}{1\mathrm{mi}} \times \frac{1\ \mathrm{h}}{3600\ \mathrm{s}} = 12.1\ \mathrm{m/s}$$

$$\Delta x = 275\ \mathrm{ft} \times \frac{0.3048\ \mathrm{m}}{1\ \mathrm{ft}} = 83.8\ \mathrm{m}$$

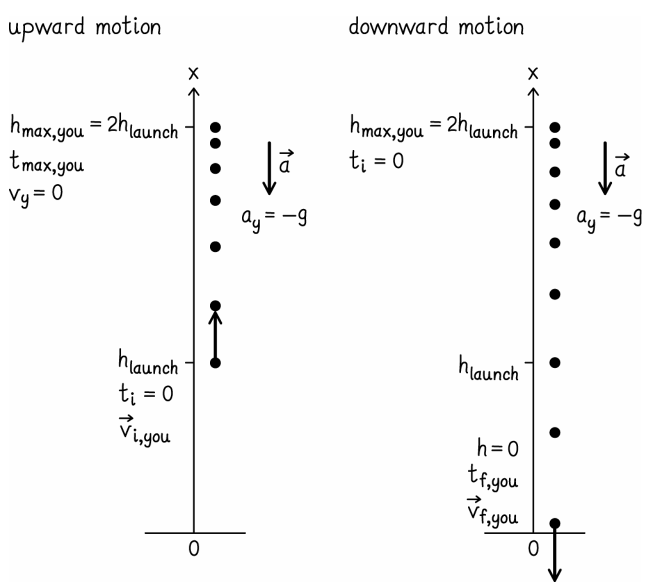
Inserting these values into equation (1) above yields

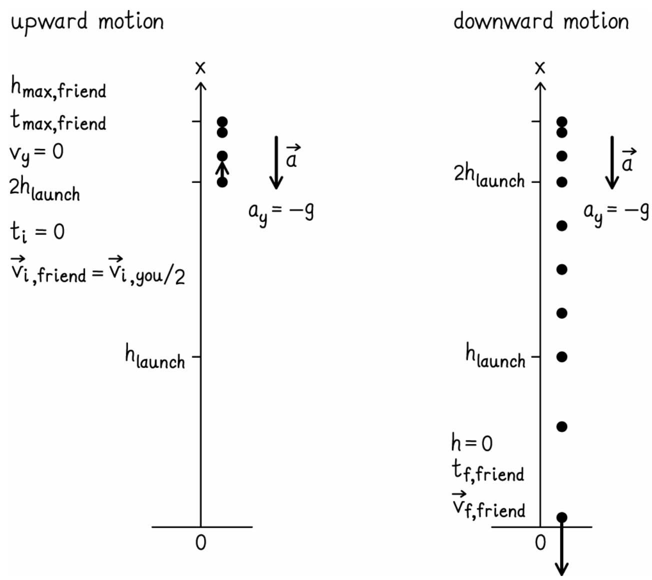
$$\Delta t = \frac{2(83.8\ \mathrm{m})}{((20.1\ \mathrm{m/s}) + (12.1\mathrm{m/s}))} = 5.2\ \mathrm{s}$$

4. Evaluate Result Requiring a few seconds to reduce your speed is reasonable, based on our experience driving cars. It is also reasonable that it could take a few seconds to cross the distance of 275 ft.

3.4 Double launch

1. Getting Started This is a constant acceleration problem, since the balls are being accelerated by gravity from the start of their vertical motion until they strike the ground. Let the positive y axis point vertically upward. We start by drawing motion diagrams for each ball. Note that some of the labels in the two diagrams will be the same (h_{launch} and t_i for example), but others (such as $h_{\max}$) will not.





2. Devise Plan Worked Problem 3.3 broke the motion down into parts. This could also be applied here, but the most straightforward way to answer all questions is to find expressions for the time at which each ball strikes the ground. This can be accomplished by solving equation 3.9 for time using the quadratic equation:

$$y_f - y_i = v_{y,i}\Delta t + \frac{1}{2}a_y \Delta t^2$$

$$\Rightarrow \Delta t = \frac{-v_{y,i} \pm \sqrt{v_{y,i}^2 + 2a_y(y_f - y_i)}}{a_y} \tag{1}$$

Applying this equation between the initial and final positions will give us two times for each ball. One time will be positive and will correspond to the physically correct time. The other solution will be a negative time. This clearly cannot be a valid answer because the ball was not even set in motion until time $t_i = 0$. This fact is not built into the above equation; we have to impose that constraint ourselves and take only the positive time as a valid solution. We can simplify equation (1) above by determining the y component of the initial velocity in terms of other variables. To do this, we apply PRIN equation (3.13) to the ball you throw, using the launch time as the initial time and the peak position as time 2:

$$\Rightarrow v_{y,i}^2 = v_{y,2}^2 - 2a_y \Delta y = 2gh_{launch} \tag{2}$$

The expressions for time will tell us which ball struck the ground first (the smaller time), and will enable us to obtain an expression for the difference between the two times. All that remains is to calculate the final velocities. Since both balls are accelerated by gravity and since we will obtain expressions for the times, the easiest way to calculate the final velocities is to use equation 3.4:

$$v_{y,f} = v_{y,i} + a_y \Delta t \tag{3}$$

3. Execute Plan (*a*) Inserting initial and final conditions of your ball into equation (1) and simplifying using equation (2) yields

$$\Delta t_{you} = \frac{\sqrt{2gh_{launch}} + \sqrt{2gh_{launch} + 2gh_{launch}}}{g} = (2 + \sqrt{2})\sqrt{\frac{h_{launch}}{g}} \tag{4}$$

Whereas applying equation (1) to your friend's ball yields

$$\Delta t_{friend} = \frac{\sqrt{\frac{gh_{launch}}{2}} + \sqrt{\frac{gh_{launch}}{2} + 4gh_{launch}}}{g} = \frac{4}{\sqrt{2}}\sqrt{\frac{h_{launch}}{g}} \tag{5}$$

It follows trivially that $\Delta t_{friend} < \Delta t_{you}$. Hence your friend's ball hits the ground first.

(*b*) Inserting the two times found in equations (4) and (5) into equation (3) we obtain

$$\begin{aligned} v_{\text{you},y,\text{f}} &= v_{\text{you},y,\text{i}} + a_y \Delta t_{\text{you}} = \sqrt{2gh_{\text{launch}}} - g(2+\sqrt{2})\sqrt{\frac{h_{\text{launch}}}{g}} \\ &= -2\sqrt{gh_{\text{launch}}} \end{aligned} \tag{6}$$

$$\begin{aligned} v_{\text{friend},y,\text{f}} &= v_{\text{friend},y,\text{i}} + a_y \Delta t_{\text{friend}} = \sqrt{\frac{gh_{\text{launch}}}{2}} - g\left(\frac{4}{\sqrt{2}}\right)\sqrt{\frac{h_{\text{launch}}}{g}} \\ &= -\frac{3}{\sqrt{2}}\sqrt{gh_{\text{launch}}} \end{aligned} \tag{7}$$

Thus the speeds are $v_{\text{you,f}} = 2\sqrt{gh_{\text{launch}}}$ and $v_{\text{friend,f}} = \frac{3}{\sqrt{2}}\sqrt{gh_{\text{launch}}}$, meaning your friend's ball has a greater speed.

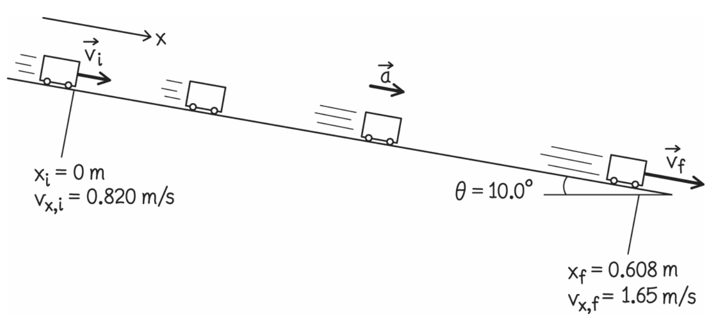
(*c*) To find how much time passes between your friend's ball striking the ground and your own ball striking the ground, we simply take the difference between the two times obtained in equations (4) and (5):

$$\Delta t_{\text{you}} - \Delta t_{\text{friend}} = ((2+\sqrt{2}) - 2\sqrt{2})\sqrt{\frac{h_{\text{launch}}}{g}} = (2-\sqrt{2})\sqrt{\frac{h_{\text{launch}}}{g}} \tag{8}$$

4. Evaluate Result For part (*a*), it seems reasonable that your ball covers so much more distance than your friend's that it ends up taking longer (despite your larger initial speed). One could have guessed the answer to part (*b*) correctly by symmetry arguments. When your ball reaches a height equal to twice the launch height, its speed is zero. When your friend's ball falls past that same point, it has a downward velocity. Since gravity will accelerate the two balls in the same way from that point to the ground, it is clear that your friend's ball must have the larger final speed. Also, both balls have a negative y component of their final velocity, as they should. The answer to part (*c*) is complicated enough that one might consider checking a known result. For example, if $v_{y,\text{i}} = 0$ (which necessarily implies $h_{\text{launch}} = 0$), then the time difference should be zero. Inserting this value into equation (8) does indeed yield a time difference of zero.

3.6 Another inclined track

1. Getting Started We start by drawing a motion diagram. Choose the x axis to lie along the inclined plane, and choose the origin to be at the initial position as indicated.



The acceleration is not equal to the gravitational acceleration for an object in freefall. But gravity is the underlying reason why the cart is accelerating and the angle made between the incline and the vertical direction does not change. So just as in Worked Problem 3.5, this is a constant acceleration problem.

2. Devise Plan The physical setup of this problem is the same as in Worked Problem 3.5. The principal difference is that we are collecting different information about the initial and final conditions. For example, in the current problem we have no knowledge of the time required for the cart to complete the described motion. One could solve for time, but a more direct route would be to express the acceleration along the x axis in terms of variables given. Equation 3.13 can be solved for the acceleration yielding

$$a_x = \frac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2\Delta x} \tag{1}$$

But we also know from equation 3.20 that the acceleration along the incline is given by

$$a_x = g\sin(\theta) \tag{2}$$

Equating the accelerations in equations (1) and (2), and solving for g yields

$$g = \frac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2\Delta x\sin(\theta)} \tag{3}$$

3. Execute Plan: Evaluating equation (3) for the given values yields

$$g = \frac{(1.65\ \text{m/s})^2 - (0.820\ \text{m/s})^2}{2(0.608\ \text{m})\sin(10.0°)} = 9.71\ \text{m/s}^2$$

4. Evaluate Result The value obtained $(9.71\ \text{m/s}^2)$ is very close to the actual average value of $g = 9.80\ \text{m/s}^2$. Small errors in the measurement of the initial velocity, final velocity, or distance travelled could account for the difference. Also, this treatment has not taken into account any effects of air resistance or friction, which will be treated in later chapters. It is reasonable to expect that the value obtained using this treatment would be slightly smaller than $9.80\ \text{m/s}^2$.

Questions and Problems

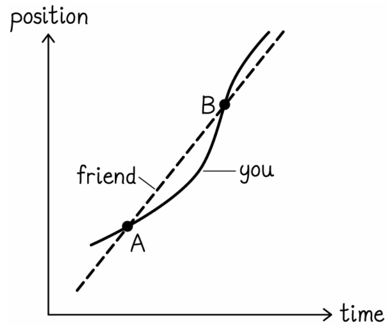
3.1. (*a*) The object is speeding up in the left half of the figure (from the first dot to the fifth dot). We know this because the spacing between dots is increasing, while the time interval between flashes remains constant. (*b*) The object is slowing down in the right half of the figure (from the fifth dot to the ninth dot). We know this because the spacing between dots is decreasing, while the time interval between flashes remains constant. (*c*) If we renumber the dots from right to left, then the object would still be speeding up from the first dot to the fifth dot, and the object would still be slowing down between the fifth dot and the ninth dot. If you gave your answers in terms of right and left sides of the picture, then obviously your answers are reversed.

3.2. (*a*) The velocity is north and the acceleration is north. (*b*) The velocity is north, but the acceleration is south.

3.3. The horse is not accelerating because it appears in the same position in each frame. The cameras were equally spaced and triggered at equal time intervals, which means that the horse's speed between each camera did not change.

3.4. A and B. The velocity in the x direction is given by the slope of the position graph. This slope is increasing at points A and B.

3.5. No, you are going faster than your friend. You must increase your speed to be greater than your friend's speed in order to catch up with him. But when you reach a speed adequate to catch up to your friend, you do not maintain that speed, because you accelerate at a constant rate. Your speed gets larger and larger, such that you are moving faster when you pass him. This is shown in the figure below. The friend passes you at point A. At that time you increase your speed, which is shown in the plot by the fact that the slope of your position curve increases. You catch up with your friend at point B. Note that the slope of your position vs. time curve is greater at this instant than the slope of your friend's position vs. time line.



3.6. It may be difficult in some areas to tell if the curve is a perfectly straight line or slightly bent. Acceptable answers are: (*a*) $0 < t <$ about 0.75 s or 1.0 s (*b*) 1.0 s or $1.25\text{ s} < t <$ about 3.2 s and $3.25\text{ s} < t < 4.00\text{ s}$ (*c*) yes, it is definitely zero on the interval $1.75\text{ s} < t < 3.25\text{ s}$, and it is zero somewhere around 1.00 s. A correct answer might include an interval as large as $0.75\text{ s} < t < 1.75\text{ s}$.

3.7. Initially you are at rest. When you start to move upward your velocity has changed; it has increased in the upward direction. If we take up to be the positive direction of motion, then the acceleration is positive as you speed up from rest. This upward acceleration only lasts a short a time. Soon, you reach the desired speed of the elevator. After this short time the acceleration is zero, as you are moving with a constant velocity upward. As you approach the 19$^{\text{th}}$ floor the elevator slows down. This corresponds to a negative acceleration. This negative acceleration continues until the elevator stops at your floor.

3.8. Cart B has the greater acceleration at point P. The slope of the position vs. time curve is constant for cart A, indicating that cart A has a constant velocity and therefore zero acceleration. The slope of the position vs. time curve for cart B is increasing at point P, indicating a positive acceleration.

3.9. The car that was initially faster is ahead. Throughout the 5.0 s interval the car that was initially faster has a greater speed than the car that was initially slower.

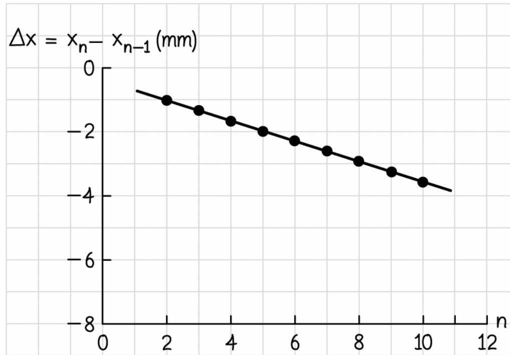
3.10. No. The pebble is in freefall with an acceleration of 9.8 m/s^2 downward. This means that the speed changes from 0.0 m/s to 9.8 m/s in 1.0 s. After the pebble falls for 1.0 s, its speed will be 9.8 m/s. But during that initial 1.0 s the speed was always less than 9.8 m/s. So it could not possibly have travelled 9.8 m in that 1.0 s. In fact, the pebble has only travelled 4.9 m in the first 1.0 s.

3.11. (*a*) $\Delta y = v_{y,\text{i}}t + \frac{1}{2}a_y t^2$ and since the pebble is released from rest $v_{y,\text{i}} = 0$. Hence the displacement in the y direction is $\Delta y = \frac{1}{2}(-9.8\text{ m/s}^2)(1.0\text{ s})^2 = -4.9\text{ m}$. The average speed is then $v_{\text{av}} = \Delta y/\Delta t = 4.9\text{ m}/1.0\text{ s} = 4.9\text{ m/s}$.

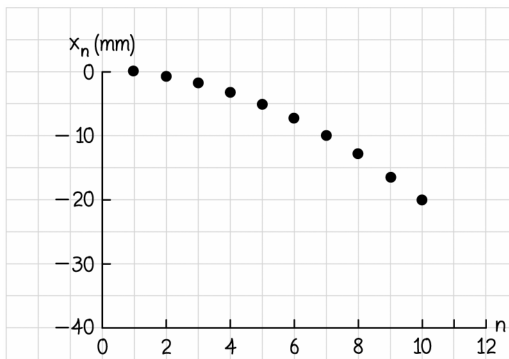
(*b*) Now $v_{y,\text{i}} \neq 0$ because the pebble has already been falling for 1.0 s. Now $v_{y,\text{i}} = a_y\,\Delta t = (-9.8\text{ m/s}^2)(1.0\text{ s}) = -9.8\text{ m/s}$. As before we use $\Delta y = v_{y,\text{i}}t + \frac{1}{2}a_y t^2$ and obtain $\Delta y = (-9.8\text{ m/s})(1.0\text{ s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(1.0\text{ s})^2 = -15\text{ m}$. $v_{\text{av}} = \Delta y/\Delta t = 15\text{ m}/1.0\text{ s} = 15\text{ m/s}$. (*c*) $\Delta y = \frac{1}{2}(-9.8\text{ m/s}^2)(2.0\text{ s})^2 = -19.6\text{ m}$ so the average speed is $v_{\text{av}} = \Delta y/\Delta t = 19.6\text{ m}/2.0\text{ s} = 9.8\text{ m/s}$.

3.12. More than 5 m above you. Both objects have the same acceleration, but the sandwich starts with a large initial speed and slows down, while the coin starts with zero initial speed and speeds up. This means that the sandwich travels the first 5 m faster than the coin. (Of course eventually the coin speed exceeds the sandwich speed, and the coin travels the last 5 m faster than the sandwich). Having reached 5 m first, the sandwich continues upward and passes the coin at some position higher than 5 m.

3.13. (*a*) Figure 3.6 a would still be curved downward, but not as much. Figure 3.6 b would still have a downward slope, but the slope would be about half what it was originally.



(*b*)



3.14. (*a*) The picture must be rotated 135 ° clockwise. It must be vertical since the ball is travelling downward. Further, the spacing between frames must be larger at the bottom since it will be start at rest and speed up as it falls. (*b*) The picture must be rotated 135 ° clockwise. The spacing between frames still must be larger at the bottom since its speed will start out large and decrease as it reaches its peak. (*c*) Downward

3.15. Let us call vertically upward the $+y$ direction. For parts *a–d* we find the velocity using $v_{y,\text{f}} = v_{y,\text{i}} + a_y t,$ and the speed is just the magnitude of the velocity.

(*a*) $v_{y,\text{f}} = (98 \text{ m/s}) + (-9.8 \text{ m/s}^2)(5.0 \text{ s}) = 49 \text{ m/s}.$ Hence the speed is 49 m/s; the velocity is $(+49 \text{ m/s})\hat{j}.$

(*b*) $v_{y,\text{f}} = (98 \text{ m/s}) + (-9.8 \text{ m/s}^2)(10 \text{ s}) = 0.0 \text{ m/s}.$ Hence the speed is 0; the velocity is 0.

(*c*) $v_{y,\text{f}} = (98 \text{ m/s}) + (-9.8 \text{ m/s}^2)(15 \text{ s}) = -49 \text{ m/s}.$ Hence the speed is 49 m/s; the velocity is $(-49 \text{ m/s})\hat{j}.$

(*d*) $v_{y,\text{f}} = (98 \text{ m/s}) + (-9.8 \text{ m/s}^2)(20 \text{ s}) = -98 \text{ m/s}.$ Hence the speed is 98 m/s; the velocity is $(-98 \text{ m/s})\hat{j}.$

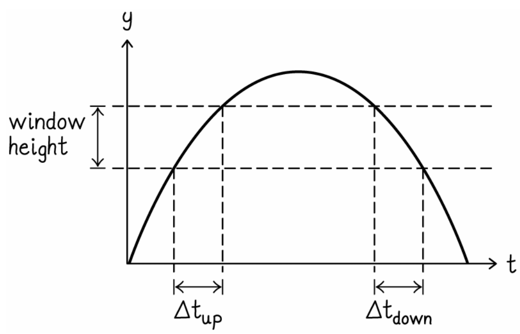
3.16. Earlier than 0.9 s. The coin moved through a greater distance on the way down than it did on the way up. Hence, more time was spent moving downward than moving upward, and the highest point must have been reached in less than half the total time.

3.17. (*a*) Yes. When you throw a ball perfectly upward it stops for an instant at the peak of its path. At this moment the velocity is zero, but the acceleration is still the acceleration due to gravity: 9.8 m/s^2 downward. (*b*) Yes. Any object moving at constant velocity has zero acceleration. For example, when you drive your car on the freeway at a constant velocity the acceleration is zero even though the velocity is certainly nonzero.

3.18. Figure 3.8*c* showing the ball's trajectory would still be parabolic, but the ball would reach a greater height (larger *x* value on graph) and remain in the air longer (*t* axis extended). Figure 3.8*d* would still be linear and have a negative slope, but the slope would be smaller by a factor of 1/6.

3.19. The two techniques cause the snowballs to hit the sidewalk at the same speed. Suppose you can throw a snowball at a maximum initial speed v_{max}. If you throw the snowball upward at v_{max} it will rise up to some maximum height and then fall past you on its way down to the sidewalk. Since we are ignoring air resistance, the acceleration on the way up will be the same as on the way down and the snowball will fall past you at a speed v_{max}. From then on the speed of the snowball will be exactly the same as if you had thrown it downward.

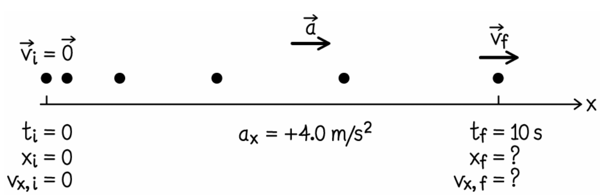
3.20. $\Delta t_{up} = \Delta t_{down}$. In the figure below, the horizontal dashed lines intersect the curve at the points (in space and in time) at which the ball becomes visible to you as it rises (leftmost dot), disappears (second dot from left), reappears as it falls (third dot from left), and vanishes below the window (rightmost dot). The two time intervals defined by where the vertical dashed lines intersect the time axis mark the lengths of the two time intervals during which you see the ball pass the window. Due to the symmetry of the ball's trajectory, the two intervals must be equal each other.



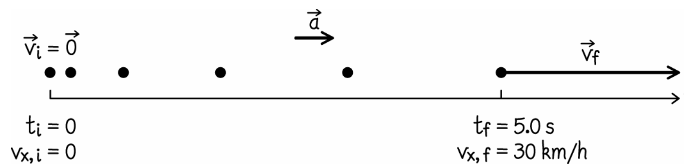
3.21. The curve would start out identical to the no-air-resistance case, with a slope of −9.8 m/s^2, but the magnitude of this slope would decrease with time. After some time interval, the curve would become horizontal, indicating a constant speed.

3.22. (*a*) During the ascent of the ball there is a downward acceleration due to gravity as always. But now the acceleration due to interaction with the air also acts downward (resisting the upward motion through the air). Hence these two sources of acceleration are in the same direction and the magnitude of the acceleration should be greater than in the idealized case. (*b*) During the descent of the ball the air resistance prevents gravity from accelerating the ball as it would in the idealized case. Hence the magnitude of the acceleration is less than in the idealized case. (*c*) The travel time will be less than in the idealized case. Air resistance increases the downward acceleration of the ball as it rises, causing a smaller travel time and a reduced maximum height. Air resistance decreases the downward acceleration of the ball as it falls, which would extend the travel time, but because it falls from a lower height the effect is not sufficient to cancel the time reduction on the way up. This results in a shorter travel time interval for the up-and-back trip than in the simplified case where air resistance is ignored.

3.23.

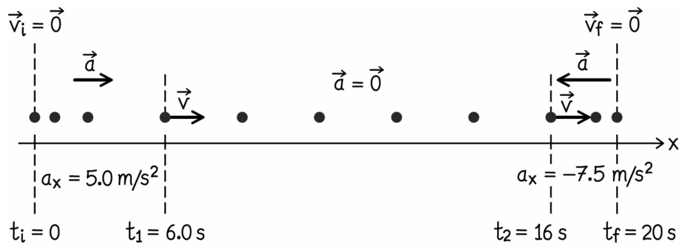


3.24.

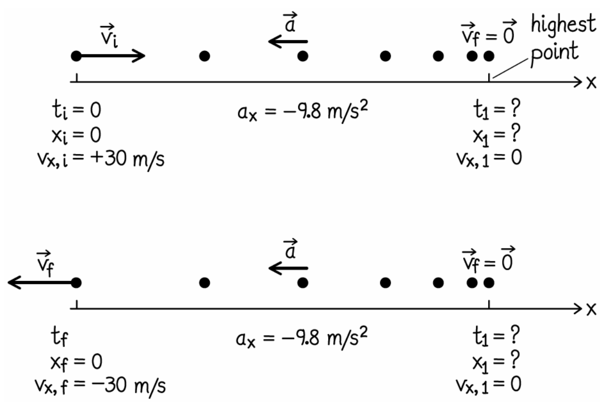


3.25. Graph (*b*). The cart appears to start from rest and come to rest at the end of the time period. This eliminates (*d*) as a possibility. The motion diagram appears to have roughly uniform spacing between several points around the middle of the time period. Hence the correct graph should have a period of constant speed as in (*b*).

3.26.



3.27.



3.28. (*a*) Car A has the greater speed at point 6. The spacing between adjacent dots is greater for car A than for car B. (*b*) Car B. The space between adjacent dots is greater than for car A. (*c*) Car B had the greater acceleration between points 6 and 11. It started slower than car A and ended up faster than car A, hence its increase in speed was greater than car A's, over the same amount of time.

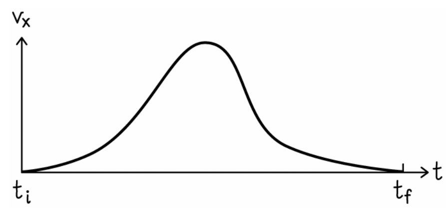
3.29. (*a*) If we choose the x direction to be the direction of the motion and acceleration, then $a_x = \dfrac{v_{x,\text{f}} - v_{x,\text{i}}}{\Delta t} = \dfrac{(+20\ \text{m/s}) - 0}{60\ \text{s}} = 0.33\ \text{m/s}^2$, such that $\vec{a} = 0.33\ \text{m/s}^2\ \hat{i}$. (*b*) Using the acceleration found in part (*a*) we can write $\Delta x = v_{x,\text{i}}\Delta t + \dfrac{1}{2}a_x\Delta t^2 = 0 + \dfrac{1}{2}(0.33\ \text{m/s}^2)(60\ \text{s})^2 = 6.0\times 10^2\ \text{m}$.

3.30. (*a*) The car is moving in the $-x$ direction toward your brother. Since the car is slowing down, its acceleration must be opposite its velocity. Hence the acceleration is in the $+x$ direction. (*b*) We can use the definition of average acceleration to obtain $a_x = \dfrac{v_{x,\text{f}} - v_{x,\text{i}}}{\Delta t} = \dfrac{0 - (-2.5\ \text{m/s})}{(3.0\ \text{s})} = +0.83\ \text{m/s}^2$. Hence $\vec{a} = +0.83\ \text{m/s}^2\ \hat{i}$.

3.31. (*a*) Let us choose the $+x$ direction to be the direction of motion of the electron. The electron is initially at rest, so the expression $\Delta x = \frac{1}{2}(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})t$ simplifies to $\Delta x = \frac{1}{2}(0 + 3.0\times10^{6}\ \mathrm{m/s})(5.0\times10^{-8}\ \mathrm{s}) = 75\ \mathrm{mm}$ (*b*) $a_x = \frac{v_{x,\mathrm{f}} - v_{x,\mathrm{i}}}{t}$ $= \frac{(3.0\times10^{6}\ \mathrm{m/s}) - 0}{(5.0\times10^{-8}\ \mathrm{s})} = 6.0\times10^{13}\ \mathrm{m/s^2}$, such that $\vec{a} = 6.0\times10^{13}\ \mathrm{m/s^2}\ \hat{\imath}$.

3.32. The comment is incorrect. That the velocity is always positive tells you the cart never turned around. The peak at $t = 2.0$ s means the cart reached its maximum velocity at that instant. After that, the velocity decreased linearly until the cart came to a stop at $t = 4.0$ s, at a position far from its starting position.

3.33. (*a*) You integrate. The area under the curve is equivalent to adding up differential amount of $v\delta t = \delta x$, and the sum of all such tiny steps is the displacement. (*b*) 1.0×10^{2} m (*c*) No. In Figure S3.XX, the initial and final velocities are both zero, a case in which the equation from Worked Problem 3.3 yields zero for $vx_{,\mathrm{av}}$. Yet the graph shows that the object has a positive velocity in the *x* direction for a significant time interval. Therefore the object must have a displacement in the *x* direction and its average velocity cannot be zero.



3.34. We choose the $+x$ direction to be in the direction of motion of the car. From the information about the first 500 m travelled, we can find the acceleration using $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x\Delta x$ or $a_x = \frac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2\Delta x} =$ $\frac{(+20\ \mathrm{m/s})^2 - (+5\ \mathrm{m/s})^2}{2(500\ \mathrm{m})} = +0.375\ \mathrm{m/s^2}$. Then we can use acceleration to determine the velocity at the end of the second 500 m: $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x\Delta x = (+20\ \mathrm{m/s})^2 + 2(0.375\ \mathrm{m/s^2})(+500\ \mathrm{m})$ or $v_{x,\mathrm{f}} = 28\ \mathrm{m/s}$, which is less than 35 m/s.

3.35. (*a*) We can find the position of ball at the times specified using $\Delta y = v_{y,\mathrm{i}}t + \frac{1}{2}a_y t^2$ and then take the difference between these positions to find the displacement. We choose the $+y$ direction to be vertically upward. Then $\Delta y(t = 0.15\ \mathrm{s}) = 0 + \frac{1}{2}(-9.8\ \mathrm{m/s^2})(0.15\ \mathrm{s})^2 = -0.110\ \mathrm{m}$ and $\Delta y(t = 0.25\ \mathrm{s}) = 0 + \frac{1}{2}(-9.8\ \mathrm{m/s^2})(0.25\ \mathrm{s})^2 = -0.306\ \mathrm{m}$, so the displacement between these two times is $-0.20\ \mathrm{m}\,\hat{\jmath}$ (*b*) Proceeding as in part (*a*) we have $\Delta y(t = 0.175\ \mathrm{s}) = 0 + \frac{1}{2}(-9.8\ \mathrm{m/s^2})(0.175\ \mathrm{s})^2 = -0.150\ \mathrm{m}$ and $\Delta y(t = 0.275\ \mathrm{s}) = 0 + \frac{1}{2}(-9.8\ \mathrm{m/s^2})(0.275\ \mathrm{s})^2 =$ $-0.371\ \mathrm{m}$ such that the displacement is $-0.22\ \mathrm{m}\,\hat{\jmath}$.

3.36. The car that accelerates from 10 m/s to 20 m/s in 50 m has greater acceleration. The two cars have the same change in velocity. Since this car is always moving faster than the car that starts from rest, it must cover the 50 m distance in less time than the car that starts from rest. Recall that acceleration is the change in velocity per unit time, not per unit distance. So the car that starts from a speed of 10 m/s changes its velocity in a shorter time and must have a greater acceleration than the car that starts from rest.

3.37. (*a*) We can calculate the acceleration by rearranging $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x\Delta x$ to obtain $a_x = \dfrac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2\Delta x}$ $= \dfrac{(+1.0\times10^7\ \mathrm{m/s})^2 - (+2.0\times10^5\ \mathrm{m/s})^2}{2(0.012\ \mathrm{m})} = 4.2\times10^{15}\ \mathrm{m/s^2}$. So the acceleration is $\vec{a} = 4.2\times10^{15}\ \mathrm{m/s^2}\ \hat{\imath}$ where $\hat{\imath}$ is in the direction of motion. (*b*) Using $\Delta x = \dfrac{1}{2}(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})t$ we obtain $t = \dfrac{2\Delta x}{(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})} = \dfrac{2(0.012\ \mathrm{m})}{(2.0\times10^5\ \mathrm{m/s} + 1.0\times10^7\ \mathrm{m/s})}$ $= 2.4\ \mathrm{ns}$.

3.38. (*a*) Choose the $+x$ direction to be in the direction of motion of the car. First we convert all speeds to basic SI units. $40\ \mathrm{km/h} = 40\ \dfrac{\mathrm{km}}{\mathrm{h}}\left(\dfrac{1000\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\dfrac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = 11.1\ \mathrm{m/s}$, and similarly $60\ \mathrm{km/h} = 16.7\ \mathrm{m/s}$. These speeds can be used to find the time using $\Delta x = \dfrac{1}{2}(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})t$ or $t = \dfrac{2\Delta x}{(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})} = \dfrac{2(250\ \mathrm{m})}{(11.1\ \mathrm{m/s} + 16.7\ \mathrm{m/s})} = 18\ \mathrm{s}$ (*b*) $a_x = \dfrac{(v_{x,\mathrm{f}} - v_{x,\mathrm{i}})}{t}$ $= \dfrac{(+16.7\ \mathrm{m/s}) - (+11.1\ \mathrm{m/s})}{18\ \mathrm{s}} = 0.31\ \mathrm{m/s^2}$, such that $\vec{a} = 0.31\ \mathrm{m/s^2}\ \hat{\imath}$.

3.39. (*a*) True. The fact that the curve is a straight line means the rate at which the velocity changes is constant. (*b*) Not necessarily true. The object could have started far enough from origin that it never passes through position $x = 0$ on a position-versus-time plot. (*c*) True. The velocity is zero at the instant the curve crosses the time axis. (*d*) Not true. The fact that the curve crosses from positive v values to negative v values means the direction of motion changed.

3.40. (*a*) Call the direction of motion the $+x$ direction. In terms of the units given $a_x = \dfrac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2\Delta x} = \dfrac{(+763\ \mathrm{mi/h})^2 - 0}{2(5.0\,\mathrm{mi})} = 5.8\times10^4\ \mathrm{mi/h^2}$. But it is much more common to express accelerations in SI units, in which case $\vec{a} = 7.2\ \mathrm{m/s^2}\,\hat{\imath}$ (*b*) Using $\Delta x = \dfrac{1}{2}(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})t$ and working again in the units given, we obtain $t = \dfrac{2\Delta x}{(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})} = \dfrac{2(5.0\ \mathrm{mi})}{(0 + 763\ \mathrm{mi/h})} = 0.0131\ \mathrm{h} = 47\ \mathrm{s}$.

3.41. (*a*) We must break this down into the three time intervals over which the acceleration is constant. Intervals 1, 2, and 3 will refer to the period of speeding up, riding at a constant speed, and slowing down, respectively. Call the direction of motion the $+x$ direction. In interval 1 we can use $\Delta x_1 = v_{1x,\mathrm{i}}t + \dfrac{1}{2}a_{1x}t^2 =$ $0 + \dfrac{1}{2}(+0.60\ \mathrm{m/s^2})(20\ \mathrm{s})^2 = 120\ \mathrm{m}$. Note that at the end of interval 1, your speed is $v_{1x,\mathrm{f}} = v_{1x,\mathrm{i}} + a_{1x}t =$ $0 + (0.60\ \mathrm{m/s^2})(20\mathrm{s}) = 12\ \mathrm{m/s}$. This is your speed at the beginning of interval 2, and now you have zero acceleration. You know you travel 200 m over interval 2 and 10 m over interval 3. Hence the total distance traveled is $\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 = (120\ \mathrm{m}) + (200\ \mathrm{m}) + (10\ \mathrm{m}) = 3.3\times10^2\ \mathrm{m}$. (*b*) We also calculate the time required for interval 2 using $\Delta t_2 = \Delta x_2 / v_2 = (200\mathrm{m})/(12\ \mathrm{m/s}) = 16.7\ \mathrm{s}$. At the beginning of interval 3 the speed is still 12 m/s. Hence we can find the time required to stop using $t = \dfrac{2\Delta x}{(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})} = \dfrac{2(10\ \mathrm{m})}{(12\ \mathrm{m/s} + 0)} = 1.67\ \mathrm{s}$. The total time required is just the sum of the times required for each interval: $\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3 = 20\ \mathrm{s} + 16.7\ \mathrm{s} + 1.67\ \mathrm{s} = 38\ \mathrm{s}$. (*c*) The average velocity is $\vec{v}_{\mathrm{av}} = \dfrac{\Delta\vec{x}}{\Delta t} = \dfrac{330\ \mathrm{m}\ \hat{\imath}}{38.3\ \mathrm{s}} = 8.6\ \mathrm{m/s}\ \hat{\imath}$.

3.42. Assume that you and the driver in front of you are initially moving at the same velocity (a safe assumption in rush hour traffic). Since the driver in front of you brakes first, his speed will always be less than yours. That means that over any time interval (before you both stop) you will cover more distance than the driver in front of you. Hence the spacing between your cars must decrease.

3.43. Call the direction of motion of both cars the +*x* direction. Consider that the first car is at the origin when the driver sees the deer and begins to brake at time $t = 0$ s. Let the position of the second car at this time be a distance *d* behind the first car. We can write the position of each car as $x_1 = v_{x,\text{i}}t + \frac{1}{2}a_x t^2$, and $x_2 = -d + v_{x,\text{i}}t + \frac{1}{2}a_x(t-\Delta t)^2$, where Δt is the 0.5 s delay for the second driver to respond. In order that the two cars do not collide, we require $x_1 - x_2 > 0$. Using the fact that the front car must time after some time t_f, we can write the acceleration as $a_x = -v_{x,\text{i}}/t_\text{f}$. This makes the stopping condition $x_1 - x_2 = \left(v_{x,\text{i}}t - \frac{1}{2}\frac{v_{x,\text{i}}}{t_\text{f}}t^2\right) - \left(-d + v_{x,\text{i}}t - \frac{1}{2}\frac{v_{x,\text{i}}}{t_\text{f}}(t-\Delta t)^2\right) > 0$

$$d > \frac{1}{2}\frac{v_{x,\text{i}}}{t_\text{f}}t^2 - \frac{1}{2}\frac{v_{x,\text{i}}}{t_\text{f}}(t-\Delta t)^2 = \frac{1}{2}\frac{v_{x,\text{i}}\Delta t^2}{t_\text{f}}\left(1 - \frac{2t}{\Delta t}\right)$$

This expression for the constraint on the distance is still a function of the variable t, and of the parameter t_f. If we want to determine what set of parameters and variables yields the minimum distance required, we can take the derivative of the above expression with respect to those variables and set that derivative equal to zero to check for local minima. We start by taking $\frac{\partial}{\partial t}(d(t))$:

$$\frac{\partial d_\text{min}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\frac{v_{x,\text{i}}\Delta t^2}{t_\text{f}}\left(1 - \frac{2t}{\Delta t}\right)\right) = -\frac{v_{x,\text{i}}\Delta t}{t_\text{f}}$$

But this quantity $-\frac{v_{x,\text{i}}\Delta t}{t_\text{f}}$ is not zero for any finite stopping time. Hence this function has no local extrema as a function of time, which means that the minimum of this function must be at one of the endpoints ($t = \Delta t$ or $t = t_\text{f} + \Delta t$). The minimum distances between cars at these two times are $d_\text{min} > d(\Delta t) = \left(\frac{1}{2}\frac{v_{x,\text{i}}\Delta t^2}{t_\text{f}}\right)$ and $d_\text{min} > d(t_\text{f} + \Delta t) = \left(\frac{1}{2}\frac{v_{x,\text{i}}\Delta t(2t_\text{f} + \Delta t)}{t_\text{f}}\right)$, clearly the latter is larger, so that is the constraint with which we proceed. There is still a continuum of possible stopping times (or possible accelerations, equivalently). We can find where the required distance is maximal by taking $\frac{\partial}{\partial t_\text{f}}(d(t_\text{f}))$ and setting the result equal to zero:

$$\frac{\partial d_\text{min}}{\partial t_\text{f}} = \frac{\partial}{\partial t_\text{f}}\left(\frac{1}{2}\frac{v_{x,\text{i}}\Delta t(2t_\text{f} + \Delta t)}{t_\text{f}}\right) = -\frac{v_{x,\text{i}}\Delta t^2}{2t_\text{f}^2} = 0$$

But this can only be satisfied in the limit as $t_\text{f} \to \infty$. Therefore the minimum required stopping distance is given by

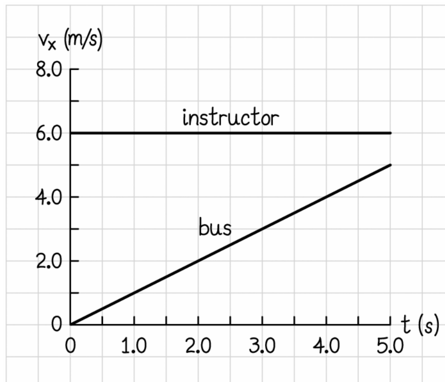
$$\underset{t_\text{f} \to \infty}{\text{Lim}}\left(\frac{1}{2}\frac{v_{x,\text{i}}\Delta t(2t_\text{f} + \Delta t)}{t_\text{f}}\right) = v_{x,\text{i}}\Delta t = (+26.9\,\text{m/s})(0.50\ \text{s}) = 13\ \text{m}.$$

3.44. (*a*) Let us call the initial position of the instructor the origin, and choose the +*x* direction to be in the direction that the instructor is running. The position of the instructor as a function of time can be written $x_\text{inst}(t) = v_{\text{inst}\,x,\text{i}}\Delta t$. The position of the bus can be written as $x_\text{bus}(t) = x_{\text{bus},i} + \frac{1}{2}a_{\text{bus}\,x}\Delta t^2$. When the instructor is next to the bus, these two expressions must be equal. Hence $x_\text{bus}(t) - x_\text{inst}(t) = x_{\text{bus},i} - v_{\text{inst}\,x,\text{i}}\Delta t + \frac{1}{2}a_{\text{bus}\,x}\Delta t^2 = 0$. This equation which is quadratic in Δt has solutions

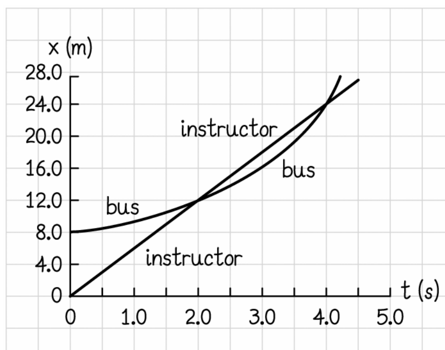
$$\Delta t = \frac{v_{\text{inst } x,\text{i}} \pm \sqrt{v_{\text{inst } x,\text{i}}^2 - 2a_{\text{bus } x}x_{\text{bus},i}}}{a_{\text{bus } x}} = \frac{(+6.0\text{ m/s}) \pm \sqrt{(+6.0\text{m/s})^2 - 2(2.0\text{m/s}^2)(8.0\text{ m})}}{(2.0\text{ m/s}^2)} = 2.0\text{s or } 4.0\text{ s}$$

Hence the first time the instructor is right next to the door is after 2.0 s. (*b*) The speed of the instructor is specified as being a constant 6.0 m/s. (*c*) The bus starts from rest and accelerates such that its speed is given by $v_{\text{bus}} = a_{\text{bus}}\Delta t = (+2.0\text{ m/s})(2.0\text{ s}) = 4.0\text{ m/s}.$

(*d*)



(*e*)



3.45. As explained in the solution to problem 3.44, the difference between the position of the instructor and the position of the bus can be written $x_{\text{inst}}(t) - x_{\text{bus}}(t) = -x_{\text{bus},i} + v_{\text{inst } x,\text{i}}\Delta t - \frac{1}{2}a_{\text{bus } x}\Delta t^2.$ To find when this difference reaches a local maximum we can take the time derivative of the expression and set it equal to zero.

$$\frac{\partial}{\partial \Delta t}\left(-x_{\text{bus},i} + v_{\text{inst } x,\text{i}}\Delta t - \frac{1}{2}a_{\text{bus } x}\Delta t^2\right) = v_{\text{inst } x,\text{i}} - a_{\text{bus } x}\Delta t = 0$$

This tells us that the time at which this maximum occurs is $\Delta t = \frac{v_{\text{inst } x,\text{i}}}{a_{\text{bus } x}} = \frac{6.0\text{m/s}}{2.0\text{ m/s}^2} = 3.0\text{ s}.$ Inserting this time into the expression for the difference in positions yields $x_{\text{inst}}(t) - x_{\text{bus}}(t) = -(8.0\text{ m}) + (6.0\text{ m/s})(3.0\text{ s}) - \frac{1}{2}(2.0\text{ m/s}^2)(3.0\text{ s})^2 = 1.0\text{ m}.$

3.46. (*a*) Call the downward direction of motion the –*y* direction. In freefall the acceleration is $(-9.8\text{ m/s}^2)\hat{j}$ and we know the pebble starts from rest, so we can write $\Delta y = v_{y,\text{i}}t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2$ or $t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-9.8\text{ m})}{(-9.8\text{ m/s}^2)}} = 1.4\text{s}.$

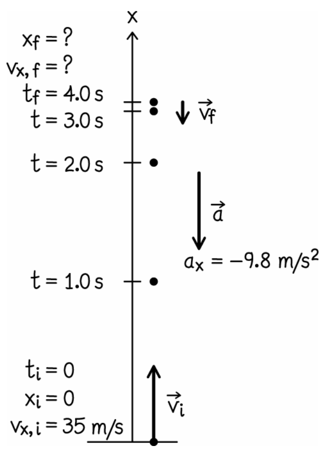
(*b*) We know $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t$ and we can use the time found in part (*a*) to say $v_{y,\mathrm{f}} = (-9.8\ \mathrm{m/s^2})(1.4\ \mathrm{s}) = -14\,\mathrm{m/s}$. But the question asks for a speed, rather than a component of velocity. The speed is 14 m/s.

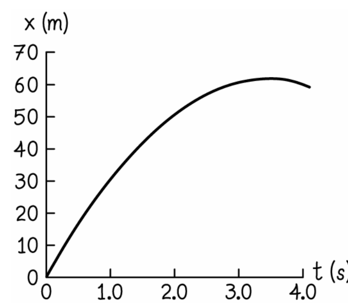
3.47. Let us call the original magnitude of acceleration $g = 9.8\,\mathrm{m/s^2}$, and this new magnitude of acceleration $g' = 4.9\,\mathrm{m/s^2}$. Call the new time $\Delta t'$. We can calculate expressions for the time required using the two different magnitudes of accelerations and compare them. We will use $\Delta y = v_{y,\mathrm{i}} t + \frac{1}{2} a_y t^2 = \frac{1}{2} a_y t^2$, since in each case the object is released from rest. It follows that

$$\frac{\Delta t'}{\Delta t} = \frac{\sqrt{2\Delta y/g'}}{\sqrt{2\Delta y/g}} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{9.8\,\mathrm{m/s^2}}{4.9\,\mathrm{m/s^2}}} = \sqrt{2}$$

Hence the time required for the fall would increase by a factor of $\sqrt{2}$.

3.48.





3.49. The ball reaches its maximum height at $t = 3.0$ s, making the trajectory symmetric around that point. Between t = 2.0 s and t = 3.0 s, the ball moves upward and has a downward acceleration *g*; it moves some distance in this time interval. Between t = 3.0 s and t = 4.0 s, the ball, after having zero velocity for an instant, moves downward and has a downward acceleration *g*. Because the time intervals are equal, the distance the ball moves downward in 1.0 s equals the distance it move upward in 1.0 s.

3.50. Consider only the half of the trip during which the ball is falling. Since the time to travel upward is the same as the time to travel downward, if we double the time required for this half of the trip we will double the total time. On this interval, the ball starts with zero initial speed. Hence we can write $-h = \frac{1}{2} a t^2$ for our initial throw and $-h' = \frac{1}{2} a (t')^2$. We require that $t'/t = 2$, so we obtain the condition $\frac{t'}{t} = \frac{\sqrt{2h'/g}}{\sqrt{2h/g}} = 2 \Rightarrow h' = 4h$.

3.51. (*a*) At the peak of the ball's trajectory its velocity in the y direction is momentarily zero. So from $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x \Delta x$ we can write $v_{y,\mathrm{i}} = \sqrt{-2a_y \Delta y} = \sqrt{-2(-9.8\,\mathrm{m/s^2})(25\,\mathrm{m})} = 22$ m/s. (*b*) To find time it takes to reach its peak, we can use $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t \Rightarrow t = \frac{v_{y,\mathrm{f}} - v_{y,\mathrm{i}}}{a_y} = \frac{0 - (+22\,\mathrm{m/s})}{(-9.8\,\mathrm{m/s^2})} = 2.3\mathrm{s}$.

3.52. Call vertically upward the $+y$ direction. This scenario consists of two parts, where each part has a different acceleration. During her fall the snowshoer's acceleration is purely due to gravity. As she lands in the snow, the acceleration comes largely from her interaction with the snow. First we examine her fall and find that the final y component of her velocity is given by $v_{y,\text{f}}^2 = v_{y,\text{i}}^2 + 2a_{\text{fall},y}\Delta y \Rightarrow v_{y,\text{f}} = -\sqrt{2a_{\text{fall},y}\Delta y} = -8.4 \text{ m/s}.$ Now we use this as the initial velocity when she strikes the snowbank. We can use the same kinematic equation: $a_{\text{snow},y} = \dfrac{v_{y,\text{f}}^2 - v_{y,\text{i}}^2}{2\Delta y} = \dfrac{0-(-8.4 \text{ m/s})^2}{2(-0.80 \text{ m})} = 44 \text{ m/s}^2.$ So the acceleration of the snowshoer while she is in contact with the snowbank is 44 m/s^2 upward.

3.53. (*a*) Call vertically upward the +y direction. We can begin this problem by finding the initial upward velocity of the brick. Using $v_{y,\text{f}}^2 = v_{y,\text{i}}^2 + 2a_y\Delta y$ with the final position chosen to be the peak height we obtain $v_{y,\text{i}} = \sqrt{-2a_y\Delta y} = \sqrt{-2(-9.8 \text{m/s}^2)(6.0 \text{ m})} = 10.8 \text{ m/s}.$ To find the amount of time that passes, we can use $\Delta y = v_{y,\text{i}}t + \dfrac{1}{2}a_y t^2$ and solve for time: $t = \dfrac{v_{y,\text{i}} \pm \sqrt{v_{y,\text{i}}^2 + 2a_y\Delta y}}{-a_y} = \dfrac{(+10.8 \text{ m/s}) \pm \sqrt{(+10.8 \text{m/s})^2 + 2(-9.8 \text{ m/s}^2)(5.0 \text{ m})}}{-(-9.8 \text{ m/s}^2)}.$ This yields two solutions at $t_1 = 0.65 \text{ s}$ and $t_2 = 1.6 \text{ s}.$ The brick will reach your hands the first time (on the way up) after 0.65 s. (*b*) In part (*a*) we found two solutions to the time at which the brick is at the height of your hands. The second (later) time corresponded to the brick passing you as it fell back to Earth. The time interval in which you can reach out and still catch the brick is the difference between the two times found in (a): $\Delta t = t_2 - t_1 = 0.90 \text{ s}.$

3.54. Call vertically upward the $+y$ direction. The acceleration of the rocket is different on the way up and on the way down. So we must treat these two steps separately. First we look at the upward acceleration. The rocket starts from rest so we can write $\Delta y_1 = \dfrac{1}{2}a_{1y}t_1^2 = 2gt_1^2,$ where t_1 is the time spent accelerating upward. This is the height at which the stage detaches. We also know that the upward velocity at that time is $v_{1\text{f}} = 4gt_1.$ This height and velocity will be our initial conditions for the period of downward acceleration. We can use $v_{y,\text{f}}^2 = v_{y,\text{i}}^2 + 2a_y\Delta y = (4gt_1)^2 - 2g(-2gt_1^2)$ or $v_{y,\text{f}} = \sqrt{20}gt_1 = \sqrt{20}(9.8 \text{ m/s}^2)(5.0 \text{ s}) = 2.2\times10^2 \text{ m/s}.$

3.55. Call vertically upward the $+y$ direction. Let $\Delta y_{\Delta t} = \frac{1}{2}g(\Delta t)^2$ be the distance traveled after a time interval Δt and $\Delta y_{\Delta t+1} = \frac{1}{2}g(\Delta t+1)^2$ be the distance traveled after time interval $\Delta t + 1.$ The difference between these two distances is the distance traveled in the 1-s interval between these two intervals. Call this distance $h_{\Delta t} = g(\Delta t + \frac{1}{2}),$ and because we are restricting ourselves to time intervals of 1 s, we could also write $h_N = g(N + \frac{1}{2})$ where N is an integer number of seconds. The ratio of the distance traveled in the first second to the distance traveled in the N^{th} second is

$$\frac{h_0}{h_N} = \frac{\frac{1}{2}}{N+\frac{1}{2}} = \frac{1}{2N+1}$$

Inserting the first few values for N yields 1/3, 1/5, 1/7, 1/9, . . .

3.56. Call vertically upward the $+y$ direction. At the end of the upward acceleration the sandbag has reached a final height given by $h = \Delta y = \dfrac{1}{2}a_y\Delta t^2 = \dfrac{g\Delta t^2}{8};$ this is the distance through which the sandbag must fall. Also at the end of the upward acceleration the sandbag has reached a final velocity given by $v_{y,\text{f}} = a_y\Delta t = \dfrac{g\Delta t}{4};$ this will be the initial velocity when the sandbag is released. When the sandbag is released it must fall back to earth according to $\Delta y = v_{y,\text{i}}t + \dfrac{1}{2}a_y t^2.$ Inserting the values obtained from the upward acceleration this equation becomes:

$-\frac{g\Delta t^2}{8} - \frac{g\Delta t}{4}t + \frac{1}{2}gt^2 = 0$. This quadratic equation in t has solutions $t = \Delta t\left(\frac{1 \pm \sqrt{5}}{4}\right)$. But we are only interested in the time that occurs after the initial upward acceleration (the positive answer). Hence $t = \Delta t\left(\frac{1+\sqrt{5}}{4}\right)$.

3.57. Call vertically upward the $+y$ direction. When the rock has fallen from the top of the building to the halfway point, its speed will be given by $v_{\text{half}} = \sqrt{gh}$ where h is the entire height of the building. We can now apply $\Delta y = v_{y,\text{i}}t + \frac{1}{2}a_y t^2$ using the halfway point as the initial position and the bottom of the building as the final position. This gives us $-h/2 = -\sqrt{gh}t - \frac{1}{2}gt^2$. Rearranging terms and squaring both sides of the equation yields $h^2 - 6ght^2 + g^2t^4 = 0$. The solutions to this quadratic equation are $h = (3 \pm 2\sqrt{2})gt^2$. Both roots are positive, but only the larger corresponds to a reasonable building height. Hence $h = (3 + 2\sqrt{2})(9.8 \text{ m/s}^2)(0.50 \text{ s})^2 = 14 \text{ m}$.

3.58. Call vertically upward the $+y$ direction. Let us first find the velocity of the camera just before it strikes the ground. We can use $v_{y,\text{f}}^2 = v_{y,\text{i}}^2 + 2a_y\Delta y$ which yields $v_{y,\text{f}} = -\sqrt{(12 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(-18 \text{ m})} = -22 \text{ m/s}$. Using this final velocity of the camera it is easy to find the time required for the camera to fall: $t = \frac{v_{y,\text{f}} - v_{y,\text{i}}}{a_y} = \frac{(-22 \text{ m/s}) - (12 \text{ m/s})}{(-9.8 \text{ m/s}^2)} = 3.5 \text{ s}$. During these 3.5 s, the railing will continue to rise from its original height of 18 m at a constant speed. Hence $y_\text{f} = y_\text{i} + v_y t = (18 \text{ m}) + (12 \text{ m/s})(3.5 \text{ s}) = 60 \text{ m}$.

3.59. (*a*) Call vertically upward the $+y$ direction. First we find the speed of the ball as it passes the bottom of the window using $\Delta y = v_{y,\text{bottom}}t + \frac{1}{2}a_y t^2$ or $v_{y,\text{bottom}} = \frac{\Delta y - \frac{1}{2}a_y t^2}{t} = \frac{(2.0 \text{ m}) - \frac{1}{2}(-9.8 \text{ m/s}^2)(0.20 \text{ s})^2}{(0.20 \text{ s})} = 11 \text{ m/s}$. This was the speed that the ball had 1.8 s after being thrown upward. The initial speed of the ball can then be found using $v_{y,\text{bottom}} = v_{y,\text{i}} + a_y t$ or $v_{y,\text{i}} = v_{y,\text{bottom}} - a_y t = (+11 \text{ m/s}) - (-9.8 \text{ m/s}^2)(1.8 \text{ s}) = 29 \text{ m/s}$. Hence the initial velocity of the ball was $\vec{v}_\text{i} = 29 \text{ m/s}\,\hat{j}$ (*b*) We can simply use the information obtained in part (*a*) to write $\Delta y = v_{y,\text{i}}t + \frac{1}{2}a_y t^2 = (+29 \text{ m/s})(1.8 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(1.8 \text{ s})^2 = 36 \text{ m}$. (*c*) Since we know the initial velocity, we can find the maximum height by setting the final y component of the velocity to be zero and using $v_{y,\text{f}}^2 = v_{y,\text{i}}^2 + 2a_y\Delta y$ or $\Delta y = \frac{v_{y,\text{f}}^2 - v_{y,\text{i}}^2}{2a_y} = \frac{(0)^2 - (29 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 42 \text{ m}$.

3.60. Choose the $+x$ direction to point down the incline along the direction of the skier's motion. The component of gravity that is parallel to the incline is $a_x = g\sin(\theta) = (9.8 \text{ m/s}^2)\sin(45°) = 6.9 \text{ m/s}^2$.

3.61. From the information given it is easy to find the acceleration of the cart using $\Delta x = v_{x,\text{i}}t + \frac{1}{2}a_x t^2$. He know the cart starts from rest, so this simplifies to $a_x = \frac{2\Delta x}{t^2} = \frac{2(1.80 \text{ m})}{(1.25 \text{ s})^2} = 2.30 \text{ m/s}^2$. This is the component of gravity that is along the track. Hence $g\sin(\theta) = a_x$ or $\theta = \arcsin\left(\frac{a_x}{g}\right) = 13.6°$.

3.62. Yes and no. The simple ratio of final position to the square of travel time would no longer be identical, but Galileo was a smart fellow who would have drawn the same conclusion about constant acceleration by understanding the more complicated function of time that describes this modified motion: $\Delta x = v_{x,\text{i}}t + (1/2)a_x t^2$.

3.63. Once the boxes leave the ramp they must have a speed given by $v = \dfrac{\Delta x}{\Delta t} = \dfrac{(10\text{ m})}{(2.0\text{ s})} = 5.0\text{ m/s}$. The ramp must be long enough for the boxes to reach this speed. Call the direction down the incline the $+x$ direction. Given that the boxes start from rest we can use $v_{x,\text{f}}^2 = v_{x,\text{i}}^2 + 2a_x\Delta x$ or $\Delta x = \dfrac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2a_x} = \dfrac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2g\sin(\theta)} = \dfrac{(5.0\text{ m/s})^2 - 0}{2(9.8\text{ m/s}^2)\sin(20°)} = 3.7\text{ m}$.

3.64. (*a*) The acceleration is just the component of gravity that is parallel to the driveway. Call this direction parallel to the driveway pointing down the incline the $+x$ direction. Then $a_x = g\sin(\theta) = (+9.8\text{ m/s}^2)\sin(20°) = 3.4\text{ m/s}^2$. So $\vec{a} = (+3.4\text{ m/s}^2)\,\hat{i}$. (*b*) Starting from rest we have $\Delta x = \dfrac{1}{2}a_x t^2$ or $t = \sqrt{\dfrac{2\Delta x}{a_x}} = \sqrt{\dfrac{2(4.0\text{ m})}{(3.4\text{ m/s}^2)}} = 1.5\text{ s}$. (*c*) If we proceed exactly as in part (*b*) we only need to change the value of Δx from half the length of the driveway to the full length. This yields $t = \sqrt{\dfrac{2\Delta x}{a_x}} = \sqrt{\dfrac{2(8.0\text{ m})}{(3.4\text{ m/s}^2)}} = 2.2\text{ s}$. (*d*) Since the briefcase starts from rest, the final speed is given by $v_{x,\text{f}} = a_x t = (+3.4\text{ m/s}^2)(2.2\text{ s}) = 7.3\text{ m/s}$.

3.65. (*a*) Choose the $+x$ direction to point down the slide. Since you start from rest we can write $\Delta x = \dfrac{1}{2}a_x t^2$ or $a_x = \dfrac{2\Delta x}{t^2} = \dfrac{2(100\text{ m})}{(10\text{ s})^2} = 2.0\text{ m/s}^2$. (*b*) Since the angle of inclination is the same for each of you and friction has the same effect for each of you, your friend's acceleration must also have a magnitude of 2.0 m/s^2. (*c*) Using $\Delta x = \dfrac{1}{2}a_x t^2$ yields $t = \sqrt{\dfrac{2\Delta x}{a_x}} = \sqrt{\dfrac{2(150\text{ m})}{(2.0\text{ m/s}^2)}} = 12\text{ s}$. (*d*) Again, since you start from rest, we can use $v_{x,\text{f}} = a_x t = (+2.0\text{ m/s}^2)(10\text{ s}) = 20\text{ m/s}$. (*e*) Just as in (d) we can write $v_{x,\text{f}} = a_x t = (+2.0\text{ m/s}^2)(12\text{ s}) = 24\text{ m/s}$.

3.66. (*a*) The child on the more steeply-inclined slide $(55\ °)$ has the greater acceleration. This follows directly from $a_x = g\sin(\theta)$, where we have chosen the $+x$ direction to point down the slide. (*b*) The children will have the same speed at a given height. This can most easily be seen by noting that the distance along the slide *d* that either child has moved is related the vertical distance Δy they have traverse by $d = \Delta y/\sin(\theta)$. The speed after moving that distance *d* along the slide is $v_{x,\text{f}} = \sqrt{2a_x d} = \sqrt{2g\sin(\theta)\Delta y/\sin(\theta)} = \sqrt{2g\Delta y}$, which is independent of the angle of inclination.

3.67. (*a*) First we use the given geometric information to determine the angle at which the table is now inclined: $\theta = \sin^{-1}\left(\dfrac{0.5\text{ m}}{2.4\text{ m}}\right) = 12°$. Now call the direction down the incline of the table the $+x$ direction. Since the puck starts from rest we can write $\Delta x = \dfrac{1}{2}a_x t^2$, or $t = \sqrt{\dfrac{2\Delta x}{g\sin(\theta)}} = \sqrt{\dfrac{2(+2.4\text{ m})}{(+9.8\text{ m/s}^2)\sin(12°)}} = 1.5\text{ s}$. (*b*) Again, since the puck starts from rest we can use $v_{x,\text{f}} = 0 + a_x t = g\sin(\theta)t = (+9.8\text{ m/s}^2)\sin(12°)(1.5\text{ s}) = 3.1\text{ m/s}$.

3.68. Call the direction pointing up along the incline the $+x$ direction. We know $a_x = -g\sin(\theta)$ and we can insert this expression into $v_{x,\text{f}}^2 = v_{x,\text{i}}^2 + 2a_x\Delta x$ such that we obtain $v_{x,\text{f}}^2 = v_{x,\text{i}}^2 - 2g\sin(\theta)\Delta x$ or $v_{x,\text{f}} = \sqrt{(+6.0\text{ m/s})^2 - 2(+9.8\text{ m/s}^2)\sin(37°)(+2.0\text{ m})} = 3.5\text{ m/s}$.

3.69. (*a*) Call the direction pointing up along the incline the $+x$ direction. If the worker gives the box the minimum necessary push, then the box will just come to rest at the top of the ramp. In this case the equation $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x\Delta x$ becomes $v_{x,\mathrm{i}}^2 = -2a_x\Delta x = 2g\sin(\theta)\ell$ or $v_{x,\mathrm{i}} = \sqrt{2g\sin(\theta)\ell}$. (*b*) We can proceed as in part (*a*), but now we insert only half the length of the ramp for the distance travelled and we use the initial speed found in (*a*). From $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x\Delta x$ this gives us $v_{x,\mathrm{f}}^2 = 2g\ell\sin(\theta) - 2g\sin(\theta)\ell/2$ or $v_{x,\mathrm{f}} = \sqrt{g\ell\sin(\theta)}$.

3.70. (*a*) Let us find out as much information as possible about the skier as he reaches the end of the first incline. Call the direction pointing down this incline the $+x$ direction. Simple geometry tells us that the angle of inclination is $\theta_1 = 45°$ and the length of the skier's path is 14 m. At the bottom of this first incline the skier's speed would be given by $v_{x,\mathrm{mid}}^2 = v_{x,\mathrm{i}}^2 + 2a_x\Delta x_1$, where $v_{x,\mathrm{i}}^2 = 0$ since the skier starts from rest. Hence $v_{x,\mathrm{mid}} = \sqrt{2g\sin(\theta_1)\Delta x_1} = \sqrt{2(+9.8\,\mathrm{m/s^2})\sin(45°)(14\ \mathrm{m})} = 14\ \mathrm{m/s}$. For the second incline, geometry shows the path length is 15.8 m and the angle of inclination is $\theta_2 = 18.4°$. The speed at the bottom of this incline would be given by $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{mid}}^2 + 2a_x\Delta x_2$ or $v_{x,\mathrm{f}} = \sqrt{v_{x,\mathrm{i}}^2 + 2g\sin(\theta_2)\Delta x_2} = \sqrt{(+14\ \mathrm{m/s})^2 + 2(+9.8\ \mathrm{m/s^2})\sin(18.4°)(15.8\ \mathrm{m})} = 17\ \mathrm{m/s}$. (*b*) One can immediately find the amount of time taken for this first part of the descent using $v_{x,\mathrm{mid}} = 0 + a_{1x}t_1 = g\sin(\theta_1)t_1$ such that $t = \dfrac{v_{x,\mathrm{mid}}}{g\sin(\theta_1)} = \dfrac{(14\ \mathrm{m/s})}{(+9.8\ \mathrm{m/s^2})\sin(45°)} = 2.0\ \mathrm{s}$. Similarly, for the second part of the descent $t_2 = \dfrac{v_{x,\mathrm{f}} - v_{x,\mathrm{mid}}}{g\sin(\theta_2)} = \dfrac{(17\ \mathrm{m/s}) - (14\ \mathrm{m/s})}{(+9.8\ \mathrm{m/s^2})\sin(18.4°)} = 1.0\ \mathrm{s}$. Hence the average acceleration is given by $a_{x,\mathrm{av}} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{17\ \mathrm{m/s}}{3.0\ \mathrm{s}} = 5.6\ \mathrm{m/s^2}$. The direction of this average acceleration must be along the direction down the second incline, as the final velocity is along that direction.

3.71. First consider the motion of the ball. Call the vertically upward direction the $+y$ direction. The initial speed of the ball can be found using $v_{y,\mathrm{i}}^2 = v_{y,\mathrm{f}}^2 - 2a_y\Delta y$ or $v_{y,\mathrm{i}} = \sqrt{v_{y,\mathrm{f}}^2 - 2a_y\Delta y} = \sqrt{(-15\ \mathrm{m/s})^2 - 2(-9.8\ \mathrm{m/s^2})(-5.0\ \mathrm{m})} = 11.3\ \mathrm{m/s}$. From here it is easy to determine the amount of time required for the ball to reach the ground using $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t$ or $t = \dfrac{v_{y,\mathrm{f}} - v_{y,\mathrm{i}}}{a_y} = \dfrac{(-15\ \mathrm{m/s}) - (11.3\mathrm{m/s})}{(-9.8\ \mathrm{m/s^2})} = 2.68\ \mathrm{s}$. This is the same amount of time required for the block to reach the bottom of the incline, and for the block to reach a speed of 15 m/s. Now consider the motion of the block down the incline and call the direction down the incline the $+x$ direction. One can write for the block $v_{x,\mathrm{f}} = v_{x,\mathrm{i}} + a_x t = 0 + g\sin(\theta)t$. Hence $\theta = \sin^{-1}\left(\dfrac{v_{x,\mathrm{f}}}{gt}\right) = \sin^{-1}\left(\dfrac{(+15\ \mathrm{m/s})}{(+9.8\ \mathrm{m/s})(2.68\ \mathrm{s})}\right) = 35°$.

3.72. Both objects start from rest. Call vertically upward the $+y$ direction, and call the direction down the incline the $+x$ direction. The time required for the ball to drop to the floor from a height h is given by $\Delta y = -h = \dfrac{1}{2}a_y t_{\mathrm{ball}}^2 = -\dfrac{1}{2}g t_{\mathrm{ball}}^2$ or $t_{\mathrm{ball}} = \sqrt{\dfrac{2h}{g}}$. The puck starts from rest and covers the same vertical distance, but the puck is only has an acceleration equal to $a_x = g\sin(\theta)$. The time required for the puck to move a distance Δx is given by $\Delta x = \dfrac{1}{2}a_x t_{\mathrm{puck}}^2 = \dfrac{1}{2}g\sin(\theta)t_{\mathrm{puck}}^2$. Here θ refers to the angle of inclination, which is 30 °. Note that the distance along the incline Δx and the vertical height of the incline h are related by $\Delta x = \dfrac{h}{\sin(\theta)}$. This yields $\dfrac{h}{\sin(\theta)} = \dfrac{1}{2}g\sin(\theta)t_{\mathrm{puck}}^2$ or $t_{\mathrm{puck}} = \sqrt{\dfrac{2h}{g\sin^2(\theta)}}$. Hence $\dfrac{t_{\mathrm{ball}}}{t_{\mathrm{puck}}} = \sin(\theta) = \dfrac{1}{2}$.

3.73. This problem must be broken into three parts, so that kinematic equations can be used during the three periods of constant acceleration. Call the starting point 0, the bottom of the first incline 1, the beginning of the second incline 2, and the final stopping position 3. Consider first the descent. Call the direction of motion down the incline the $+x$ direction. The speed at the bottom of this incline can be found using $v_{x,1}^2 = v_{x,0}^2 + 2a_x\Delta x_1$ or $v_{x,1} = \sqrt{v_{x,0}^2 + 2g\sin(\theta_1)\Delta x_1} = \sqrt{(+2.5\text{m/s})^2 + 2(9.8\text{ m/s}^2)\sin(15°)(10\text{ m})} = 7.55\text{ m/s}$. As the sled crosses the slushy region this speed changes according to $v_{x,2} = v_{x,1} + a_x t = (7.55\text{ m/s}) + (-1.5\text{ m/s}^2)(2.0\text{ s}) = 4.55\text{ m/s}$. In the last step, the sled slides up the second incline and finally stops. For this step let the $+x$ direction point up the second incline. This stopping point can be described by $v_{x,3}^2 = 0 = v_{x,2}^2 + 2a_x\Delta x_3 = v_{x,2}^2 - 2g\sin(\theta_2)\Delta x_3$ or $\Delta x_3 = \dfrac{v_{x,2}^2}{2g\sin(\theta_2)} = \dfrac{(4.55\text{ m/s})^2}{2(9.8\text{ m/s}^2)\sin(20°)} = 3.1\text{ m}$.

3.74. (*a*) The instantaneous acceleration is 9.8 m/s^2 downward for all of these instants. Even though the velocity changes the acceleration is constant. (*b*) The average acceleration over this interval is 9.8 m/s^2 downward.

3.75. The velocity is the rate of change of position and the acceleration is the rate of change of velocity. This means that the acceleration is the second time derivative of the position. Hence $\vec{a}(t) = \dfrac{d^2x(t)}{dt^2} = \dfrac{d^2}{dt^2}(bt^3\ \hat{i}) = \dfrac{d}{dt}(3bt^2\ \hat{i}) = 6bt\ \hat{i}$. Hence $\vec{a}(t) = 6bt\ \hat{i}$ where $b = 1.0\text{ m/s}^3$.

3.76. Since the acceleration is the rate of change of the velocity, the velocity is the integral of the acceleration. Graphically, this means the change in the velocity is the area under the acceleration curve. This area is slightly greater in case a than in case b. Hence the change in speed is greatest in case a.

3.77. (*a*) Yes, the cart is accelerating and the acceleration is constant. This can be seen by taking the second time derivative of the position: $a_x(t) = \dfrac{d^2x(t)}{dt^2} = \dfrac{d^2}{dt^2}(b + ct + et^2) = 2e$. This is a non-zero constant. (*b*) The average velocity between these two times is simply $v_{x\text{ av}} = \dfrac{x(t_\text{f}) - x(t_\text{i})}{t_\text{f} - t_\text{i}} = \dfrac{c(t_\text{f} - t_\text{i}) + e(t_\text{f}^2 - t_\text{i}^2)}{t_\text{f} - t_\text{i}} = \dfrac{(6.00\text{ m/s})(0.200\text{ s}) + (0.200\text{ m/s}^2)((0.400\text{ s})^2 - (0.200\text{ s})^2)}{(0.200\text{ s})} = 6.12\text{ m/s}$. Hence the average velocity is $6.12\text{ m/s}\ \hat{i}$. (*c*) The velocity is the first derivative of the position: $v_x(t) = \dfrac{dx(t)}{dt} = \dfrac{d}{dt}(b + ct + et^2) = c + 2et$. Evaluating these at the specified times yields $\vec{v}(t = 0.200\text{ s}) = 6.08\text{ m/s}\ \hat{i}$ and $\vec{v}(t = 0.400\text{ s}) = 6.16\text{ m/s}\ \hat{i}$. (*d*) The average acceleration between these two times is simply $a_{x\text{ av}} = \dfrac{v(t_\text{f}) - v(t_\text{i})}{t_\text{f} - t_\text{i}} = \dfrac{2e(t_\text{f} - t_\text{i})}{t_\text{f} - t_\text{i}} = \dfrac{2(0.200\text{ m/s}^2)(0.200\text{ s})}{(0.200\text{ s})} = 0.400\text{ m/s}^2$. Hence the average acceleration is $0.400\text{ m/s}^2\ \hat{i}$. (*e*) $0.400\text{ m/s}^2\ \hat{i}$ at both times. Remember we showed in part (*a*) that the acceleration is constant.

3.78. (*a*) The instantaneous values can be obtained by taking the first and second time derivatives: $v_x(t) = \dfrac{dx(t)}{dt} = \dfrac{d}{dt}(bt^3 + ct^2 + d) = 3bt^2 + 2ct$ and $a_x(t) = \dfrac{d^2x(t)}{dt^2} = \dfrac{d^2}{dt^2}(bt^3 + ct^2 + d) = \dfrac{d}{dt}(3bt^2 + 2ct) = 6bt + 2c$. Plugging in the given numbers yields $\vec{v}(t = 2.0\text{ s}) = 3bt^2 + 2ct\ \hat{i} = 3(4.0\text{ m/s}^3)(2.0\text{ s})^2 + 2(-10\text{ m/s}^2)(2.0\text{ s})\ \hat{i} = 8.0\text{ m/s}\ \hat{i}$ and $\vec{a}(t = 2.0\text{ s}) = 6(4.0\text{ m/s}^3)(2.0\text{ s}) + 2(-10\text{ m/s}^2)\ \hat{i} = 28\text{ m/s}^2\ \hat{i}$. (*b*) $v_{x\text{ av}} = \dfrac{x(t_\text{f}) - x(\text{t}_\text{i})}{t_\text{f} - t_\text{i}} = \dfrac{b(t_\text{f}^3 - t_\text{i}^3) + c(t_\text{f}^2 - t_\text{i}^2)}{(t_\text{f} - t_\text{i})} =$

$\frac{(4.0\text{ m/s}^3)(117\text{ s}^3)+(-10\text{ m/s}^2)(21\text{ s}^2)}{(3.0\text{ s})}=86\text{ m/s}$ such that $\vec{v}_{av}=86\text{ m/s }\hat{i}$. $a_{x\text{ av}}=\frac{v(t_f)-v(t_i)}{t_f-t_i}=$ $\frac{3b(t_f^2-t_i^2)+2c(t_f-t_i)}{(t_f-t_i)}=\frac{3(4.0\text{ m/s}^3)(21\text{ s}^2)+2(-10\text{ m/s}^2)(3.0\text{ s})}{(3.0\text{ s})}=64\text{ m/s}^2$. Hence $\vec{a}_{av}=64\text{ m/s}^2\ \hat{i}$.

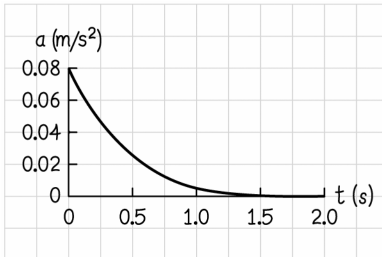
3.79. (*a*) The acceleration is given by $a_x(t)=bt$ such that $a_x(t=10.0\text{ s})=(1.00\text{ m/s}^3)(10.0\text{ s})=10.0\text{ m/s}^2$. Thus $\vec{a}=10.0\text{ m/s}^2\ \hat{i}$. (*b*) The change in velocity is given by integrating over the acceleration: $\Delta v_x=\int_i^f a_x(t)dt=\frac{1}{2}b(t_f^2-t_i^2)=\frac{1}{2}(1.00\text{ m/s}^3)((10.0\text{ s})^2-0)=50.0\text{ m/s}$. (*c*) Since the rocket started from rest at the beginning of this interval, this change in the x component of velocity is also the final x component of velocity: $v_{x,f}=50.0\text{ m/s }\hat{i}$. (*c*) $a_{x\text{ av}}=\frac{v_{x,f}-v_{x,i}}{t_f-t_i}=\frac{(50.0\text{ m/s})-0}{10.0\text{ s}}=5.00\text{ m/s}^2$, hence $\vec{a}=5.00\text{ m/s}^2\ \hat{i}$. (*d*) The change in position is given by the area under the velocity curve, or $\Delta x=\int_i^f v_x(t)dt=\int_i^f\frac{1}{2}bt^2dt=\frac{1}{6}b(t_f^3-t_i^3)=\frac{1}{6}(1.00\text{ m/s}^3)(10.0\text{ s})^3$ $=167\text{ m}$.

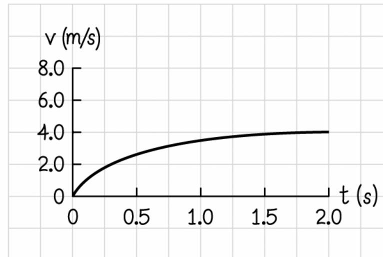
3.80. The change is speed is given by the area under the acceleration curve, or $\Delta v_x=\int_i^f a_x(t)dt=\frac{1}{2}b(t_f^2-t_i^2)$. Choosing the time at which the braking begins to be $t=0$, we can write $t_f=\sqrt{\frac{2\Delta v_x}{b}}=\sqrt{\frac{2(-50\text{ m/s})}{(-2.0\text{ m/s}^3)}}=7.1\text{ s}$. In the 7.1 s that the car needs to stop it will travel a distance given by integrating the area under the velocity curve, or $\Delta x=\int_i^f v_x(t)dt=\int_i^f v_{x,i}-\frac{1}{2}bt^2dt=v_{x,i}(t_f-t_i)-\frac{1}{6}b(t_f^3-t_i^3)=(+50\text{ m/s})(7.07\text{ s})-\frac{1}{6}(2.0\text{ m/s}^3)(7.1\text{ s})^3=2.4\times10^2\text{ m}$.

3.81. (*a*) Let us first examine the motion of the car. Call its direction of motion the $+x$ direction. Since the car starts from rest we can write $\Delta x=\frac{1}{2}a_xt^2$ or $t=\sqrt{\frac{2\Delta x}{a_x}}=\sqrt{\frac{2(12\text{ m})}{(2.5\text{ m/s}^2)}}=3.1\text{ s}$. This is the amount of time required for the car to reach the bottom of the ramp and also for the car to reach its final speed. This speed is given by $v_{x,f}=a_xt=(2.5\text{m/s}^2)(3.1\text{ s})=7.7\text{ m/s}$. The problem states that this is also the final speed of the crate. Let us now examine the motion of the crate as it slides down the incline (now call this direction $+x$). We know the acceleration down the incline will be $a_x=g\sin(\theta)$ and in this case $\theta=37°$. Since the crate starts from rest also, we can write $v_{x,f}^2=0+2a\Delta x$ or $d=\frac{v_{x,f}^2}{g\sin(\theta)}=\frac{(7.7\text{ m/s})^2}{(9.8\text{ m/s}^2)\sin(37°)}=5.1\text{ m}$. (*b*) The time required for the block to slide down the incline is given by $t=\frac{v_{x,f}-v_{x,i}}{a_x}=\frac{v_{x,f}}{g\sin(\theta)}=\frac{(7.7\text{m/s})}{(9.8\text{m/s}^2)\sin(37°)}=1.3\text{ s}$. Since the car requires 3.1 m/s to reach the bottom of the incline, the crate must have remained still for 1.8 s before starting its 1.3 s descent. Hence the crate was released 1.8 s after the car began accelerating.

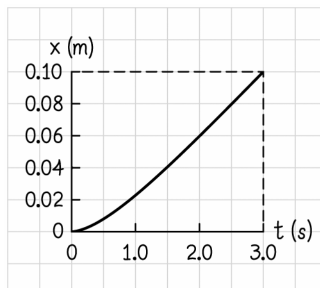
3.82. (*a*) We find an expression for the acceleration using $a(t)=\frac{dv(t)}{dt}=\frac{d}{dt}v_{\max}(1-e^{-t/\tau})=\frac{v_{\max}}{\tau}e^{-t/\tau}$.

(*b*)





(*c*) The position is given by integrating the expression for the speed: $\Delta x(t) = \int_0^{t_f} v_{\max}(1-e^{-t/\tau})dt = v_{\max}(t_f + \tau e^{-t_f/\tau})$. Dropping the subscript on the time and inserting the initial position yields $x(t) = v_{\max}(t + \tau e^{-t/\tau}) - x_0$ where $x_0 = 0.020$ m.



(*d*) 3.0 s

3.83. (*a*) The acceleration is the rate of change of the velocity, so we simply need to differentiate the expression for the velocity: $a_x(t) = \frac{d}{dt}(v_{\max}\cos(\omega t)) = -v_{\max}\omega\sin(\omega t)$. (*b*) The displacement is the area under the velocity curve, meaning $\Delta x = \int_0^{t_f} v_{\max}\cos(\omega t)dt = \frac{v_{\max}}{\omega}\sin(\omega t)$. Since the object starts out at the origin, we can write $x(t) = (v_{\max}/\omega)\sin(\omega t)$.

3.84. (*a*) $a_x = \frac{d^2}{dt^2}(bt^2 + ct + d) = 2b = 2(0.35\text{ m/s}^2) = 0.70\text{ m/s}^2$. Hence $\vec{a}(t) = 0.70\text{ m/s}^2\ \hat{i}$. (*b*) $v_x = \frac{d}{dt}(bt^2 + ct + d) = 2bt + c = 2(0.35\text{ m/s}^2)(10\text{ s}) + (6.0\text{ m/s}) = 13\text{ m/s}$. Hence $\vec{v}(t) = 13\text{ m/s}\ \hat{i}$.

3.85. Call the $+x$ the initial direction of motion. The acceleration can be found using $v_{x,f}^2 = v_{x,i}^2 + 2a_x\Delta x$ or $a_x = \frac{v_{x,f}^2 - v_{x,i}^2}{2\Delta x} = \frac{0-(9.0\text{ m/s})^2}{2(7.0\text{ m})} = -5.8\text{ m/s}^2$. The same kinematic equation can now be used to the find the stopping distance from the higher speed: $\Delta x = \frac{v_{x,f}^2 - v_{x,i}^2}{2a_x} = \frac{0-(27\text{ m/s})^2}{2(-5.8\text{ m/s}^2)} = 63\text{ m}$.

3.86. (*a*) We can find the initial velocity using $\Delta y = v_{y,i}t + \frac{1}{2}a_y t^2$ or $v_{y,i} = \left(\Delta y - \frac{1}{2}a_y t^2\right)/t = \left((-45\text{ m}) - \frac{1}{2}(-9.8\text{ m/s}^2)(2.0\text{ s})^2\right)/(2.0\text{ s}) = -13\text{ m/s}$. Hence the ball was thrown with an initial speed of 13 m/s. (*b*) From part (*a*) it can be seen that the initial velocity was downward.

3.87. (*a*) Call the $+x$ direction the initial direction of motion. The acceleration can be found using $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x\Delta x$ or $a_x = \dfrac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2\Delta x} = \dfrac{(320\text{ m/s})^2 - (480\text{ m/s})^2}{2(0.10\text{ m})} = -6.4\times10^5\text{ m/s}^2$. Hence $\vec{a}(t) = -6.4\times10^5\text{ m/s}^2\ \hat{i}$. (*b*) $t = \dfrac{v_{x,\mathrm{f}} - v_{x,\mathrm{i}}}{a_x} = \dfrac{(320\text{ m/s}) - (480\text{ m/s})}{(-6.4\times10^5\text{ m/s}^2)} = 0.25\text{ ms}$. (*c*) Now we require that the final speed be zero, such that $\Delta x = \dfrac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2a_x} = \dfrac{0 - (480\text{ m/s})^2}{2(-6.4\times10^5\text{ m/s}^2)} = 0.18\text{ m}$.

3.88. (*a*) Answers may vary a little based on estimates of the height of one floor of a building. 3.0 m is a reasonable estimate. Call the vertically upward direction the $+y$ direction. The displacement of the keys can be found using $\Delta y = \frac{1}{2}a_y t^2 = \frac{1}{2}(-9.8\text{ m/s}^2)(3.27\text{ s})^2 = -52\text{ m}$. This close to 17½ stories, meaning that you must be on the 18th floor. (*b*) The time is now split up between the descent and the sound travelling back up the elevator shaft. The time required for the fall is $t_{\text{fall}} = \sqrt{\dfrac{2\Delta y}{-g}}$ and the time required for the sound to travel is $t_{\text{sound}} = -\dfrac{\Delta y}{v_{\text{sound}}}$. The sum of these two times must be 3.27 s. This yields $-\dfrac{\Delta y}{v_{\text{sound}}} + \sqrt{\dfrac{2\Delta y}{-g}} - t_{\text{total}} = 0$. This is a quadratic equation with solutions $\Delta y = -\dfrac{v_{\text{sound}}\left[(gt_{\text{total}} + v_{\text{sound}}) \pm \sqrt{(2gt_{\text{total}} + v_{\text{sound}})v_{\text{sound}}}\right]}{g}$ or

$$\Delta y = -\frac{v_{\text{sound}}\left[((9.8\text{ m/s}^2)(3.27\text{ s}) + (343\text{ m/s})) \pm \sqrt{(2(9.8\text{ m/s}^2)(3.27\text{ s}) + (340\text{ m/s}))(340\text{ m/s})}\right]}{(9.8\text{ m/s}^2)} = -48\text{ m}.$$

This corresponds to 16 stories. If you are dropping the keys very near to your floor, you could be on the17th floor.

3.89. (*a*) The acceleration is most easily found using $a_x = \dfrac{v_{x,\mathrm{f}} - v_{x,\mathrm{i}}}{t} = \dfrac{(+31\text{ m/s}) - 0}{(14\text{ s})} = 2.2\text{ m/s}^2$. (*b*) Given the acceleration from part (*a*) we can write $v_{x,\mathrm{f}} = v_{x,\mathrm{i}} + a_x t = 0 + (2.2\text{m/s}^2)(9.5\text{ s}) = 21\text{ m/s}$. (*c*) Using the speed at this time found in part (*b*) we can write $\Delta x = \frac{1}{2}(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})t = \frac{1}{2}\big(0 + (+21\text{ m/s})\big)(9.5\text{ m/s}) = 1.0\times10^2\text{ m}$.

3.90. A dollar bill is approximately 0.156 m long. Let us assume that a person's reaction time is fast enough that they are very close to catching the dollar bill, but just barely miss it. In that case the reaction time is approximately equal to the time required for the top of the dollar bill to reach the fingers (initially at the bottom of the bill). Using the fact that the dollar bill starts from rest, we have $\Delta y = \frac{1}{2}a_y t^2$ or $t = \sqrt{\dfrac{2\Delta y}{a_y}} = \sqrt{\dfrac{2(-0.156\text{ m})}{(-9.8\text{ m/s}^2)}} = 0.18\text{ s}$. Hence the reaction time is around 0.18 s, although estimates may vary slightly based on the assumptions made about how close one is to catching the bill.

3.91. (*a*) When the shell reaches its peak height, the component of its velocity in the $+y$ direction will be momentarily zero. If we consider the launch as our initial condition and the peak height as the final state, we can write $v_{y,\mathrm{f}}^2 = v_{y,\mathrm{i}}^2 + 2a_y\Delta y$ or $v_{x,\mathrm{i}} = \sqrt{v_{y,\mathrm{f}}^2 - 2a_y\Delta y} = \sqrt{0 - 2(-9.80\text{ m/s}^2)(100\text{m})} = 44.3\text{ m/s}$. (*b*) Using the speed found in part (*a*) we can use $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t$ or $t = \dfrac{v_{y,\mathrm{f}} - v_{y,\mathrm{i}}}{a_y} = \dfrac{0 - (44.3\text{ m/s})}{(-9.80\text{ m/s}^2)} = 4.52\text{ s}$.

3.92. Let us call the vertically upward direction $+y$. We proceed by first finding the speed of the rock at the beginning of the last 1.0-second interval. Then we will use that speed to find how far the rock must have dropped prior to that 1.0-second interval. Looking only at the last second of freefall, we have $\Delta y = v_{y,\mathrm{i}}t + \frac{1}{2}a_y t^2$ or $v_{y,\mathrm{i}} = \left(\Delta y - \frac{1}{2}a_y t^2\right)/t = \left((-30\text{ m}) - \frac{1}{2}(-9.8\text{ m/s}^2)(1.0\text{ s})^2\right)/(1.0\text{ s}) = -25.1\text{ m/s}$. Now we know that the rock had to fall far enough for it to reach a speed of 25.1 m/s prior to this final second. Now looking at the motion between the release the start of this last 1.0-second interval, we can use $v_{y,\mathrm{f}}^2 = v_{y,\mathrm{i}}^2 + 2a_y\Delta y$ or $\Delta y = \frac{v_{y,\mathrm{f}}^2 - v_{y,\mathrm{i}}^2}{2a_y} = \frac{(-25.1\text{ m/s})^2 - 0}{2(-9.8\text{m/s}^2)} = -32\text{ m}$. So the rock had already fallen 32 m before the final second (in which it fell an additional 30 m to the bottom of the building). Hence the building must be 62 m tall.

3.93. (*a*) Call the vertically upward direction the $+y$ direction. An equation describing the height of the ball as a function of time is $\Delta y = v_{y,\mathrm{i}}t + \frac{1}{2}a_y t^2$. We know the acceleration due to gravity and the initial speed; all that remains is to plug in the appropriate times: $y(t) = (+24.5\text{ m/s})t + \frac{1}{2}(-9.8\text{ m/s}^2)t^2$. For example, at $t = 1.0\text{ s}$ the position is $y(t) = (+24.5\text{ m/s})(1.0\text{ s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(1.0\text{ s})^2 = 20\text{ m}$. Similarly, one obtains $y(t = 2.0\text{ s}) = 29\text{ m}$, $y(t = 3.0\text{ s}) = 29\text{ m}$, and $y(t = 4.0\text{ s}) = 20\text{ m}$. (*b*) An equation that describes the *y* component of the velocity is $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t$. Since the initial velocity and the acceleration are known, we only need to plug in the appropriate times. For example $v_{y,\mathrm{f}}(t = 1.0\text{ s}) = (+24.5\text{ m/s}) + (-9.8\text{ m/s}^2)(1.0\text{ s}) = 15\text{ m/s}$. Doing the same for the remaining times yields velocities of 4.9 m/s, –4.9 m/s, and –15 m/s in the vertical direction, respectively. (*c*) The average velocity is 0 m/s. This can be understood either by noting that the initial and final positions are the same, or by noting that the velocity is positive during the ascent and negative during the descent. (*d*) If one looks at the ascent only, one finds that the distance covered to the peak is 30 m. The ball accomplishes this is 2.5 seconds. Hence the average speed during the ascent is 12 m/s. Because the path is symmetric about the peak, the average speed during the descent must be the same. Hence the average speed during the entire trip is 12 m/s.

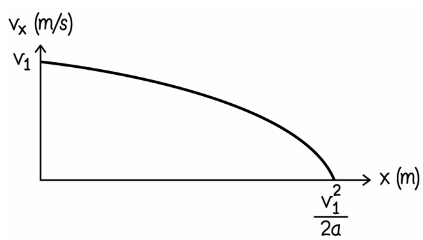
3.94. On Earth, we could express the fall time in terms of the acceleration due to gravity using $\Delta y = v_{y,\mathrm{i}}t + \frac{1}{2}a_y t^2$. Since the marker is dropped (not thrown) this expression simplifies to $\Delta y = -\frac{1}{2}g_\mathrm{E}t_\mathrm{E}^2$. For the marker on Mars, the expression would be $\Delta y = -\frac{1}{2}g_\mathrm{M}t_\mathrm{M}^2$. The accelerations and times are different on the two planets but the height of the module door above the ground is the same. Equating the two expressions for Δy yields $g_\mathrm{M} = g_\mathrm{E}t_\mathrm{E}^2/t_\mathrm{M}^2 = (9.8\text{ m/s}^2)(1.3\text{ s})^2/(2.1\text{ s})^2 = 3.8\text{ m/s}^2$.

3.95. We wish to know the acceleration during the collision between the ball and the floor. We could easily calculate that using $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t$ if we had the speed just prior to the collision and just after the collision. Let us therefore examine the fall and find the speed just before the ball hits the ground. Call the vertically upward direction the +*y* direction. Using $v_{y,\mathrm{f}}^2 = v_{y,\mathrm{i}}^2 + 2a_y\Delta y$ we find $v_{y,\mathrm{f}} = -\sqrt{v_{y,\mathrm{i}}^2 + 2a_y\Delta y} = -\sqrt{0 + 2(-9.8\text{ m/s}^2)(-2.1\text{m})} = -6.42\text{ m/s}$. We can do a similar calculation to obtain the speed at the beginning of the ball's ascent: $v_{y,\mathrm{i}} = \sqrt{v_{y,\mathrm{f}}^2 - 2a_y\Delta y} = \sqrt{0 - 2(-9.8\text{ m/s}^2)(0.88)(2.1\text{m})} = +6.02\text{ m/s}$. Using these two values for the y components of the velocity we can write $a_y = \frac{v_{y,\mathrm{f}} - v_{y,\mathrm{i}}}{t} = \frac{(+6.02\ \mathrm{m/s}) - (-6.42\ \mathrm{m/s})}{(0.013\ \mathrm{s})} = +9.6 \times 10^2\ \mathrm{m/s^2}$. Hence the average acceleration is $9.6 \times 10^2\ \mathrm{m/s^2}$ upward.

3.96. (*a*) Because the rock starts from rest, the equation $\Delta y = v_{y,\mathrm{i}} t + \frac{1}{2} a_y t^2$ simplifies and we can write $t_{\mathrm{fall}} = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-15.0\ \mathrm{m})}{(-9.8\ \mathrm{m/s^2})}} = 1.75\ \mathrm{s}$ as the amount of time for the rock to fall. Because this is an intermediate step, we are keeping one extra significant digit. There is also the short time required for the sound to reach our ears: $t_{\mathrm{sound}} = \frac{\Delta y}{v_{\mathrm{sound}}} = \frac{(15.0\ \mathrm{m})}{(343\ \mathrm{m/s})} = 0.0437\ \mathrm{s}$. Hence the total time is 1.8 s. (*b*) The speed is just given by $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t = 0 + (-9.8\ \mathrm{m/s^2})(1.75\ \mathrm{s}) = -17\ \mathrm{m/s}$. Hence the speed is17 m/s. (*c*) As shown in part (*b*), the velocity is 17 m/s downward.

3.97. Let us call the vertically upward direction the +*y* direction, and let us call the height of a single floor of the building *h*. The *y* component of the final velocity at the end of the freefall is given by $v_{y,\mathrm{f}} = -\sqrt{v_{y,\mathrm{i}}^2 + 2a_y \Delta y} = -\sqrt{0 - 2g(-4h)} = -\sqrt{8gh}$. Once the brakes kick in, the elevator goes from this speed to rest in just one floor. The acceleration over that one floor can be found using $a_y = \frac{v_{y,\mathrm{f}}^2 - v_{y,\mathrm{i}}^2}{2\Delta y} = \frac{0 - 8gh}{2(-h)} = 4g$. Hence the acceleration is $39\ \mathrm{m/s^2}$ upward.

3.98. (*a*)



(*b*) We first find an expression for the acceleration from the given information. We will then apply this acceleration to find at what point the car has reached half its initial speed. The acceleration is given by $a_x = \frac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2\Delta x} = -\frac{v_{x,\mathrm{i}}^2}{2\ell}$. Now we can use the same kinematic equation but looking for the displacement after which the speed is reduced by half. We solve $v_{x,\mathrm{f}}^2 = v_{x,\mathrm{i}}^2 + 2a_x \Delta x$ for $\Delta x = \frac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2a_x} = \frac{v_{x,\mathrm{i}}^2/4 - v_{x,\mathrm{i}}^2}{2(-v_{x,\mathrm{i}}^2/2\ell)} = \frac{3\ell}{4}$. So the car reaches half its original speed after a braking over a distance $3\ell/4$.

3.99. (*a*) The "free-rise" acceleration is given by $a_y = \frac{v_{y,\mathrm{f}} - v_{y,\mathrm{i}}}{t} = \frac{(+40\ \mathrm{m/s}) - (-20\ \mathrm{m/s})}{(10\ \mathrm{s})} = +6.0\ \mathrm{m/s^2}$ upward. (*b*) Again we use $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t$ or $t = \frac{v_{y,\mathrm{f}} - v_{y,\mathrm{i}}}{a_y} = \frac{0 - (-20\ \mathrm{m/s})}{(+6.0\ \mathrm{m/s^2})} = 3.3\ \mathrm{s}$.

3.100. (*a*) Call the vertically upward direction the +*y* direction. The final speed of $300\ \mathrm{km/h}$ is the same as 83.3 m/s. Hence we can find the average acceleration using $v_{y,\mathrm{f}} = v_{y,\mathrm{i}} + a_y t$ or $a_y = \frac{v_{y,\mathrm{f}} - v_{y,\mathrm{i}}}{t} =$

$\dfrac{(83.3\text{ m/s})-0}{(60\text{ s})}=+1.4\text{ m/s}^2$. Hence the average acceleration is 1.4 m/s^2 upward. (*b*) Again we can use $v_{y,\text{f}}=v_{y,\text{i}}+a_yt=0+(+1.4\text{ m/s}^2)(30\text{ s})=+42\text{ m/s}$. So the velocity is 42 m/s upward. (*c*) Given the acceleration found in part (*a*), the altitude can be found using $\Delta y=\dfrac{1}{2}a_yt^2=\dfrac{1}{2}(1.4\text{ m/s}^2)(60\text{ s})^2=2.5\text{ km}$.

3.101. (*a*) There are two relevant times in the problem: the time falling and the time for sound to reach our ears. We know the sum of these two times has to be 2.68 s. We can write each time symbolically as $t_{fall}+t_{\text{sound}}=\dfrac{v_{y,\text{f}}-v_{y,\text{i}}}{a_y}-\dfrac{\Delta y}{v_{\text{sound}}}=\dfrac{v_{y,\text{f}}-v_{y,\text{i}}}{a_y}-\dfrac{v_{y,\text{f}}^2-v_{y,\text{i}}^2}{2a_yv_{\text{sound}}}=t_{\text{total}}$. This equation is quadratic in $v_{y,\text{i}}$ and has a solution

$$v_{y,\text{i}}=v_{\text{sound}}-\sqrt{v_{y,\text{f}}^2+2a_yt_{\text{total}}v_{\text{sound}}-2v_{\text{f}}v_{\text{sound}}+v_{\text{sound}}^2}$$

$$v_{y,\text{i}}=(340\text{ m/s})-\sqrt{(-15.0\text{ m/s})^2+2(-9.80\text{ m/s}^2)(2.68\text{ s})(340\text{ m/s})-2(-15.0\text{ m/s})(340\text{ m/s})+(340\text{ m/s})^2}$$

$$v_{y,\text{i}}=+11.1\text{ m/s}$$

This means that the initial speed of the stone was 11.1 m/s. (*b*) As shown in part (*b*), the stone was thrown upward. (*c*) The height of the promontory can be calculated using $\Delta y=\dfrac{v_{y,\text{f}}^2-v_{y,\text{i}}^2}{2a_y}=\dfrac{(-15.0\text{ m/s})^2-(11.1\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}=-5.19\text{ m}$.

Hence the height is 5.19 m.

3.102. (*a*) Your initial speed of 100 km/h is equal to 27.8 m/s. Call the initial direction of motion of your car the $+x$ direction. Since you were obeying the 3.0 s rule, you started out with a distance of 83.3 m between you and the truck. But in the 0.75 s reaction time you reduced the distance between you and the box to 62.5 m. Now the problem reduces to simple kinematics. The acceleration is given by $a_x=\dfrac{v_{x,\text{f}}^2-v_{x,\text{i}}^2}{2\Delta x}=\dfrac{0-(27.8\text{ m/s})^2}{2(62.5\text{ m})}=-6.2\text{ m/s}^2$. Hence your minimum acceleration must be 6.2 m/s^2 against the direction of motion. (*b*) The solving process is exactly the same as in (*a*). The difference is that now the distance in which you have to stop (after reacting) is only 34.7 m. As before $a_x=\dfrac{v_{x,\text{f}}^2-v_{x,\text{i}}^2}{2\Delta x}=\dfrac{0-(27.8\text{ m/s})^2}{2(34.7\text{ m})}=-11\text{ m/s}^2$. Hence your minimum acceleration would be 11 m/s^2 against the direction of motion. The first acceleration is large, but possible. The second acceleration is greater in magnitude than gravitational acceleration.

3.103. Call the vertically upward direction the $+y$ direction. Using the information given, we can find how far the rock fell: $\Delta y=\dfrac{v_{y,\text{f}}^2-v_{y,\text{i}}^2}{2a_y}=\dfrac{(-16\text{ m/s})^2-0}{2(-9.8\text{ m/s}^2)}=-13.1\text{ m}$. We are asked about the final 12 m of the fall. At the beginning of this interval the rock has fallen only 1.1 m. That means that its speed at the beginning of the last 12 m interval is $v_{y,\text{f}}=-\sqrt{v_{y,\text{i}}^2+2a_y\Delta y}=-\sqrt{0+2(-9.8\text{ m/s}^2)(-1.1\text{m})}=-4.6\text{ m/s}$. Finally we can calculate the time required for the last 12 m of the fall using the initial and final speeds in $\Delta y=\dfrac{1}{2}(v_{y,\text{i}}+v_{y,\text{f}})t$ or

$$t=\frac{2\Delta y}{(v_{y,\text{i}}+v_{y,\text{f}})}=\frac{2(-12\text{ m})}{\left((-4.56\text{ m/s})+(-16\text{ m/s})\right)}=1.2\text{ s}.$$

3.104. The information given could be used to find the initial speed with which you threw the rock. We use $\Delta y=v_{y,\text{i}}t+\dfrac{1}{2}a_yt^2$ or $v_{y,\text{i}}=\left(\Delta y-\dfrac{1}{2}a_yt^2\right)/t=\left((-2.0\text{ m})-\dfrac{1}{2}(-9.8\text{ m/s}^2)(5.0\text{ s})^2\right)/(5.0\text{s})=24.1\text{ m/s}$. This is important because the path of the stone is not symmetric; the peak will not be attained after 2.5 seconds. We have to find the time at which the peak occurs. We do this by looking at the symmetric part of the rock's path: from the time it leaves your hand to the time it passes your hand on the way down. We can find the time required for this symmetric portion using $v_{y,\text{f}} = v_{y,\text{i}} + a_y t$ or $t = \dfrac{v_{y,\text{f}} - v_{y,\text{i}}}{a_y} = \dfrac{(-24.1\text{ m/s}) - (24.1\text{ m/s})}{(-9.8\text{ m/s}^2)} = 4.92\text{ s}.$ This means that the peak height will be reached by the stone 2.46 s after leaving your hand. This maximum height is given by $\Delta y = \dfrac{1}{2}\left(v_{y,\text{i}} + v_{y,\text{f}}\right)t = \dfrac{1}{2}\left((24.1\text{ m/s}) + 0\right)(2.46\text{ s}) = 30\text{ m}.$ Note that we measured positions relative to the release point, 2.0 m above the ground. Hence the tree is 32 m tall and will easily fit on the trailer.

3.105. At the end of the fuel burning period, the height of any rocket would be given by $\Delta y_{\text{burn}} = \dfrac{1}{2}a_{\text{burn}}t_{\text{burn}}^2.$ But after the fuel is spent, the rocket still has significant upward velocity equal to $a_{\text{burn}}t_{\text{burn}}$ and will continue to rise. The change in height during this period is $\Delta y = \dfrac{v_{y,\text{f}}^2 - v_{y,\text{i}}^2}{2a_y} = \dfrac{-(a_{\text{burn}}t_{\text{burn}})^2}{-2g}.$ Hence the final maximum height that any rocket will attain will be given by the sum $\Delta y_{\text{max}} = \dfrac{1}{2}a_{\text{burn}}t_{\text{burn}}^2\left(1 + \dfrac{a_{\text{burn}}}{g}\right).$ Evaluating this for the technical data given yields maximum heights for the "Cloud-scaper", the "Stratosphere" and the "Astronaut" equal to 600 m, 590 m, and 590 m, respectively. Hence the "Cloud-scraper" will go the highest (600 m).

3.106. Call the direction of motion of both cars the $+x$ direction. The speeder is moving at 37.5 m/s. So the speeder will only require $t = \dfrac{\Delta x}{v_x} = \dfrac{2000\text{ m}}{37.5\text{ m/s}} = 53.3\text{ s}$ to reach the state line. When you start driving, 5 seconds have already elapsed, meaning your cousin has 48.3 s to get to cover 2.0 km. In the first 15 s she will cover a distance $\Delta x = \dfrac{1}{2}\left(v_{x,\text{i}} + v_{x,\text{f}}\right)t = \dfrac{1}{2}\left(0 + (58.3\text{ m/s})\right)(15\text{ s}) = 438\text{ m}.$ The remaining distance of 1562 m will be covered moving at a constant 58.3 m/s. Hence your cousin will reach the state line in an additional $t = \dfrac{\Delta x}{v_x} = \dfrac{1562\text{ m}}{58.3\text{ m/s}} = 26.8\text{ s}.$ This time, along with the 15 seconds needed to accelerate makes a total time of 41.8 s. Recall that your cousin had 48.3 s, so she will be able to catch the speeder with about 6.5 s to spare. Don't take the bet.

3.107. (*a*) The three students have each calculated the acceleration using a different kinematic equation. The values are $a_{1x} = \dfrac{v_{x,\text{f}} - v_{x,\text{i}}}{t} = \dfrac{(96\text{ m/s}) - 0}{6.30\text{ s}} = 15.3\text{ m/s}^2,$ $a_{2x} = \dfrac{2\Delta x}{t^2} = \dfrac{2(402\text{ m})}{(6.30\text{ s})^2} = 20.3\text{ m/s}^2,$ and $a_{3x} = \dfrac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2\Delta x} = \dfrac{(96\text{ m/s}) - 0}{2(402\text{ m})} =$ 11.5 m/s^2 where all accelerations are in the direction of motion of the car. (*b*) a_{1x} is the geometric mean of a_{2x} and a_{3x}. That is, $a_{1x}^2 = a_{2x}a_{3x}$. (*c*) The drag racer does not have constant acceleration. Hence calculations that use data from different points in the race will yield different accelerations. The expression used for a_{1x} is the definition of the average acceleration, whereas a_{2x} and a_{3x} assume constant acceleration where it is invalid.

3.108. You will have different accelerations in freefall and when you turn on your jet-pack, so we must split the problem into two pieces accordingly. Call the vertically upward direction the $+y$ direction. Since you start from rest, during freefall you will cover a distance equal to $\Delta y = \dfrac{1}{2}a_{y,\text{fall}}t_{\text{fall}}^2 = \dfrac{1}{2}(0.65)(-9.80\text{ m/s}^2)(5.00\text{ s})^2 = -79.6\text{ m}.$ This means that during the use of your jet-pack you will need to descend an additional 420.4 m. Also, by the end of your freefall, you reach a speed equal to $v_{\text{f}} = a_{y,\text{fall}}t_{\text{fall}} = (0.65)(-9.80\text{ m/s}^2)(5.00\text{ s}) = -31.9\text{ m/s}.$ Now we can find the final speed at the end of the descent by using $\Delta y_{\text{jet}} = \dfrac{1}{2}(v_{y,\text{jet i}} + v_{y,\text{jet f}})t_{\text{jet}}$ or

$v_{y,\text{jet f}} = \dfrac{2\Delta y_{\text{jet}}}{t_{\text{jet}}} - v_{y,\text{jet i}} = \dfrac{2(-420.4\,\text{m})}{(26\text{ s})} - (-31.9\text{ m/s}) = -0.5\text{ m/s}.$ Hence you reach the ground with a velocity of 0.5 m/s downward.

3.109. The time required for sound to travel through the well is extremely small compared other time scales in the problem. We will ignore this. The depth of the well is given by $\Delta y = \dfrac{1}{2}a_y t^2 = \dfrac{1}{2}(-9.8\,\text{m/s}^2)(4.0\text{ s})^2 = -78\text{ m}.$ When you throw the stone downward you reduce this time, meaning the initial speed is given by $v_{y,\text{i}}\left(\Delta y - \dfrac{1}{2}a_y t^2\right)/t = \left((-78\,\text{m}) - \dfrac{1}{2}(-9.8\text{ m/ s}^2)(3.0\text{ s})^2\right)/(3.0\text{ s}) = -11\,\text{m/s}.$ This means that the greatest speed you can give a stone is 11 m/s. If you throw a stone upward at this speed, it will reawch a maximum height $\Delta y = \dfrac{v_{y,\text{f}}^2 - v_{y,\text{i}}^2}{2a_y} = \dfrac{0-(11\,\text{m/s})^2}{2(-9.8\text{ m/s}^2)} = 6.7\text{ m}.$ Don't take the bet. You can throw a stone to a maximum height of 6.7 m.

4

MOMENTUM

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

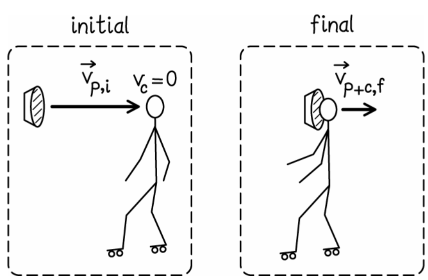
Developing a Feel

1. 10^2 kg **2.** 10^4 **3.** 10^0 kg·m/s **4.** 10^2 kg·m/s **5.** 10^1 kg·m/s **6.** 10^2 kg·m/s **7.** 10^1 kg·m/s **8.** 10^1 kg·m/s **9.** 10^5 kg·m/s **10.** 10^0 m/s

Guided Problems

4.2 A pie in the face

1. Getting Started We start by choosing an isolated system consisting of the clown and the pie. It is certainly true that as the clown rolls, interactions between the ground and the skates will slow him down. But the collision we wish to describe happens over a very short time interval, and the interaction with the ground will be negligible. We draw a system diagram before the collision and after the collision, including appropriate subscripts.



Initially, the clown is at rest relative to Earth, but the pie is moving to the right. Finally, the pie has stuck to the clown and both move to the right with a common speed.

2. Devise Plan Because momentum is a conserved quantity, and because our system is isolated we know that the initial momentum of the system must equal the final momentum of the system. We can simply write down expressions for the initial momentum of the pie and the final momentum of the clown and pie together, then equate the expressions. We use the symbol $\vec{v}_c$ for the velocity of the clown and $\vec{v}_p$ for the velocity of the pie. We can write

$$\vec{p}_{p,i} + \vec{p}_{c,i} = \vec{p}_{p,f} + \vec{p}_{c,f}$$

$$m_p\vec{v}_{p,i} + \vec{0} = (m_p + m_c)\vec{v}_{c,f}$$

We can solve the above relation for the final velocity of the clown.

3. Execute Plan Call the direction of the pie's initial velocity the $+x$ direction. Then the above statement of conservation of momentum could be rewritten in components as $m_\mathrm{p} v_{\mathrm{p}\,x,\mathrm{i}} + \vec{0} = (m_\mathrm{p} + m_\mathrm{c}) v_{\mathrm{c}\,x,\mathrm{f}}$. Solving this for the final x component of the clown's velocity yields

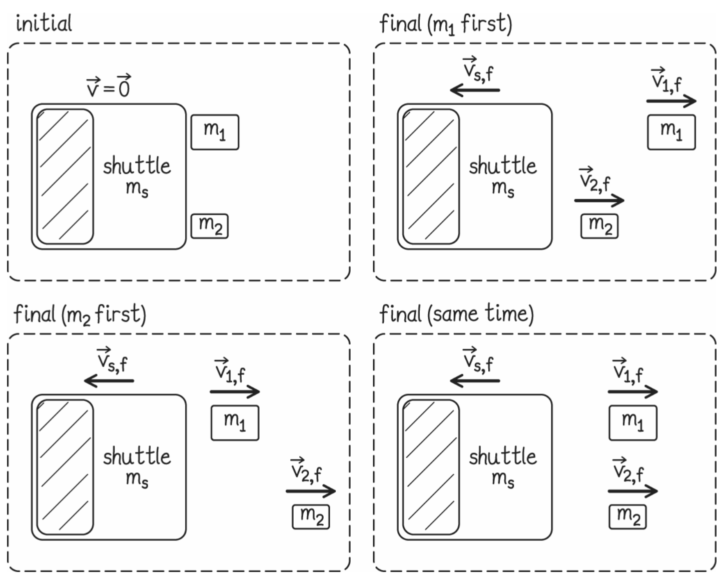
$$v_{\mathrm{c}\,x,\mathrm{f}} = \frac{m_\mathrm{p} v_{\mathrm{p}\,x,\mathrm{i}}}{(m_\mathrm{p} + m_\mathrm{c})} = \frac{(1.0\ \mathrm{kg})(5.0\ \mathrm{m/s})}{(61\ \mathrm{kg})} = 0.082\ \mathrm{m/s}$$

Since the clown's velocity has only an x component, this x component is the same as the speed. Hence the speed of the clown (and the pie) is $0.082\ \mathrm{m/s}$.

4. Evaluate Result Because the clown has so much more inertia than the pie, one would expect that the clown would move with a small fraction of the pie's initial speed. This is the case. A tenth of a meter per second is a reasonable speed for this scenario.

4.4 Space maneuvers

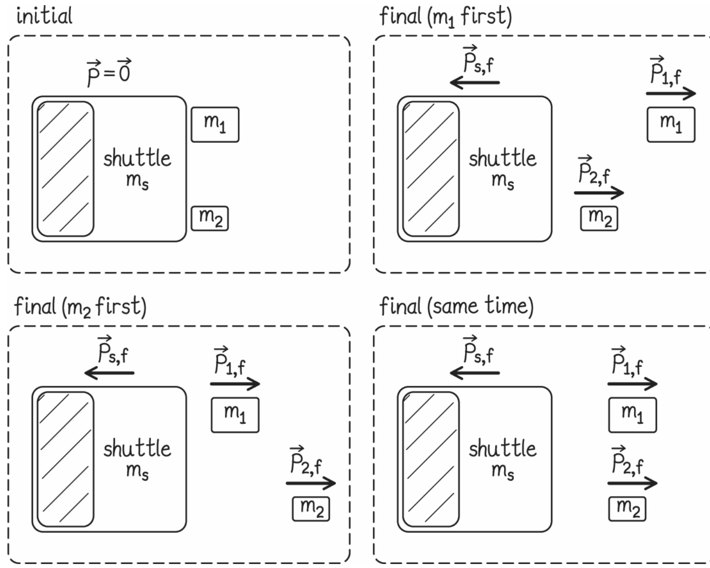
1. Getting Started We start by choosing an isolated system consisting of the shuttle, cargo pod 1 and cargo pod 2. It may not be obvious at the beginning whether or not multiple diagrams are necessary. So we show all three possible final diagrams here:



It is critical at this point that we notice that in the diagrams showing the pods launched at different times, the two pods have different velocity vectors. The reason for this is simple, but easy to miss. We are told that either pod can be launched from the Shuttle at a speed v, but this is the speed relative to what: the docking station, a distant star, Earth? It is the speed relative to the shuttle. After all, the launching mechanism pushes the pod relative to the shuttle. When the first pod is launched, its speed is v (as measured by Kirk, or someone on the docking station). But when the second pod is launched, the shuttle is already in motion. The speed could be v relative to the pod, but it would be moving at a speed less than v as seen by someone on the station.

We wish to use these diagrams and our understanding of momentum conservation to solve for the final velocity of the shuttle. Whichever scenario yields the largest final velocity will get Kirk to the docking station in the shortest amount of time.

2. Devise Plan We do expect the order in which the pods are launched to make a difference. In all three options we have three objects in motion: the two pods and the shuttle. Adding momentum vectors to the diagrams gives us some idea of the relative sizes of the three momenta involved. We also see that the momenta are collinear.



We can use the conservation of momentum to determine the final velocity of the shuttle in each case. Let us call the direction of motion of the pods the $+x$ direction, and let us choose to work in the reference frame of the docking station. In each case we write down the conservation of momentum statement for each launch separately, calling time 0 before the first launch, time 1 after the first launch, and time 2 after the second launch.

3. Execute Plan Case a: Pod 1 is launched first. The conservation of momentum can be written as

$$\vec{p}_{s,0}+\vec{p}_{m_1,0}+\vec{p}_{m_2,0}=\vec{p}_{s,1}+\vec{p}_{m_1,1}+\vec{p}_{m_2,1}$$

$$\vec{p}_{s,1}+\vec{p}_{m_1,1}+\vec{p}_{m_2,1}=\vec{p}_{s,2}+\vec{p}_{m_1,2}+\vec{p}_{m_2,2}$$

Or

$$0=m_s v_{s,1\,x}+m_1 v_{m_1,1\,x}+m_2 v_{s,1\,x} \tag{1}$$

$$m_s v_{s,1\,x}+m_1 v_{m_1,1\,x}+m_2 v_{s,1\,x}=m_s v_{s,2\,x}+m_1 v_{m_1,2\,x}+m_2 v_{m_2,2\,x} \tag{2}$$

Equation (1) can be rearranged to yield

$$m_s v_{s,1\,x}+m_2 v_{s,1\,x}=-m_1 v$$

$$v_{s,1\,x}=-\frac{m_1}{m_s+m_2}v$$

When the second pod is launched, it will move at a speed of v relative to the pod which is equal to $v-v_{s,1\,x}$ relative to the docking station. Hence we can insert for the final speed of the second pod

$$v_{m_2,2\,x}=\left(1-\frac{m_1}{m_s+m_2}\right)v$$

We insert the above expression into equation (2). Noting also that $v_{m_1,1\,x}=v_{m_1,2\,x}$, we can write

$$v_{s,2\,x}=-\left(\frac{m_1}{m_s}+\frac{m_2}{m_s}\left(1-\frac{m_1}{m_s+m_2}\right)\right)v=-\left(\frac{m_1}{m_s}+\frac{m_2}{m_s}-\frac{m_2 m_1}{m_s(m_s+m_2)}\right)v$$

Case b: The process is exactly the same as in case a, but pod 2 is launched first. A quick way to arrive at the final expression is to note that this merely exchanges the subscripts on the pods' inertias. Hence

$$v_{s,2\,x}=-\left(\frac{m_1}{m_s}+\frac{m_2}{m_s}-\frac{m_2 m_1}{m_s(m_s+m_1)}\right)v$$

The only difference is that the negative third term is now smaller. So the final velocity is larger in this case than in case a.

Case c: In this final case there is only one launch. In this case conservation of momentum yields $\vec{p}_{\text{s,i}}+\vec{p}_{m_1,\text{i}}+\vec{p}_{m_2,\text{i}}=\vec{p}_{\text{s,f}}+\vec{p}_{m_1,\text{f}}+\vec{p}_{m_2,\text{f}}$ or $0=m_\text{s}v_{\text{s,f}\,x}+m_1v_{m_1,\text{f}\,x}+m_2v_{\text{s,f}\,x}$. This can be rearranged to yield

$$m_\text{s}v_{\text{s},1\,x}=-(m_1+m_2)v$$

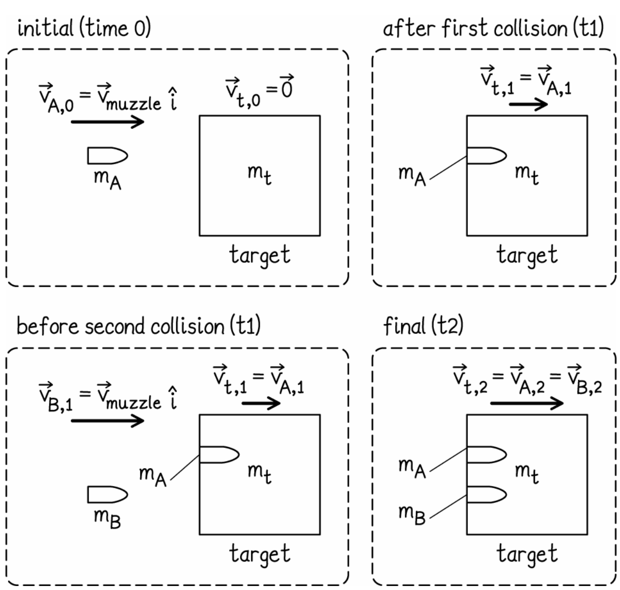
$$v_{\text{s},1\,x}=-\frac{m_1+m_2}{m_\text{s}}v$$

This shows that the greatest speed (and therefore minimum travel time) will be attained in case c, when both pods are launched simultaneously. Case b provides the second greatest speed, and case a provides the smallest speed.

4. Evaluate Result Recall that the speed v at which a pod is launched is the speed measured relative to the shuttle. The speed of a launched pod (relative to the docking station) will be greatest if it is launched when the shuttle is at rest. Once the shuttle is moving, anything launched at a speed v from the shuttle will move at a speed less than v as seen by the stationary docking station. This means that one can get all the cargo (pods) up to the greatest speed by launching it all at once while the shuttle is still stationary. That gives the cargo the greatest possible magnitude of momentum, and therefore the shuttle will have the greatest possible magnitude of momentum. This is consistent with our findings above.

4.6 Bullet impact

1. Getting Started We start by choosing a system that is isolated, so that we can use conservation of momentum. If the first bullet strikes the target before the second bullet is fired, then the target and first bullet do not make up a system that is isolated for the entire duration of interest. Neither would the target and both bullets make a system that is isolated throughout the duration of interest, as the gun interacts with the second bullet. In that case, two diagrams would be needed. In some cases, it could be possible for two shots to be fired before the first strikes a target (such as very long-distance shots with an automatic weapon). In that case, one could consider two bullets already flying through the air and a target to be an isolated system. These two methods of firing will yield the same final expression. Here we consider the former case, so that we need a set of before and after system diagrams for each collision.



Here we have set the initial direction in which the gun is fired to be the $+x$ direction. Now that we have isolated systems, we can make use of the conservation of momentum.

2. Devise a Plan We write down the statement of conservation of momentum for each of the two collisions. Rather than using "initial" and "final" subscripts, we refer to times 0, 1, and 2. We write

$$\vec{p}_{\mathrm{A},0} + \vec{p}_{\mathrm{t},0} = \vec{p}_{\mathrm{A},1} + \vec{p}_{\mathrm{t},1}$$
$$m_{\mathrm{A}}\vec{v}_{\mathrm{muzzle}} + \vec{0} = (m_{\mathrm{A}} + m_{\mathrm{t}})\vec{v}_{\mathrm{t},1}$$

for the first collision, and

$$\vec{p}_{\mathrm{B},1} + \vec{p}_{\mathrm{A},1} + \vec{p}_{\mathrm{t},1} = \vec{p}_{\mathrm{B},2} + \vec{p}_{\mathrm{A},2} + \vec{p}_{\mathrm{t},2}$$
$$m_{\mathrm{B}}\vec{v}_{\mathrm{muzzle}} + (m_{\mathrm{A}} + m_{\mathrm{t}})\vec{v}_{\mathrm{t},1} = (m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{t}})\vec{v}_{\mathrm{t},2}$$

for the second collision.

We can now combine these equations and solve for the final velocity of the target after each collision.

3. Execute Plan We can immediately solve the equation describing the first collision for the velocity of the target:

$$\vec{v}_{\mathrm{t},1} = \frac{m_{\mathrm{A}}}{(m_{\mathrm{A}} + m_{\mathrm{t}})}\vec{v}_{\mathrm{muzzle}}$$

The bullet subscripts have served to separate the collisions in our minds, but the bullets do have the same inertia. So we are free to write $m_{\mathrm{A}} = m_{\mathrm{B}} = m_{\mathrm{bullet}}$. Thus, we write

$$\vec{v}_{\mathrm{t},1} = \frac{m_{\mathrm{bullet}}}{(m_{\mathrm{bullet}} + m_{\mathrm{t}})}\vec{v}_{\mathrm{muzzle}}$$

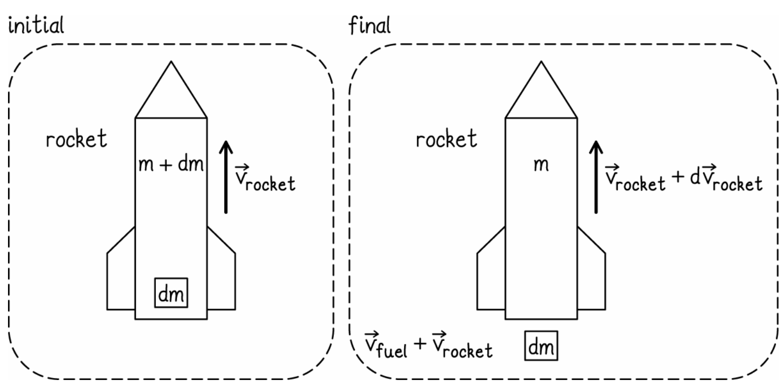
We then combine the equations for the first and second collisions, writing $m_{\mathrm{B}}\vec{v}_{\mathrm{muzzle}} + m_{\mathrm{B}}\vec{v}_{\mathrm{muzzle}} = (m_{\mathrm{A}} + m_{\mathrm{B}} + m_{\mathrm{t}})\vec{v}_{\mathrm{t},2}$. We simplify this by setting the inertia of each bullet to m_{bullet}. Solving for the final velocity of the target yields

$$\vec{v}_{\mathrm{t},2} = \frac{2m_{\mathrm{bullet}}}{(2m_{\mathrm{bullet}} + m_{\mathrm{t}})}\vec{v}_{\mathrm{muzzle}}$$

4. Evaluate Result Both results make intuitive sense. The direction of the final velocity of the target is in the same direction as the initial velocity of the bullet(s), which of course it must. The magnitude can be seen to be sensible by considering some limiting cases of a very light or very heavy target. If the target had approximately the same inertia as a bullet, we see that the final speed of the target after the first collision would be half the muzzle velocity. On the other hand, if the target were enormous, the final speed would be extremely small.

4.8 Rocket speed

1. Getting Started We wish to consider a short period of time dt in which a small quantity of inertia dm is expelled from the rocket. The figures below show this scenario before and after the fuel is expelled.



Note that the final velocity of the ejected fuel is not merely $\vec{v}_{\mathrm{fuel}}$, but $\vec{v}_{\mathrm{fuel}} + \vec{v}_{\mathrm{rocket}}$.

2. Devise Plan We wish to compare the momentum of the system before and after the fuel is ejected. Since the rocket and fuel make up an isolated system, we can set the initial and final momenta equal

$$(m + dm)\vec{v}_{\text{rocket}} = m(\vec{v}_{\text{rocket}} + d\vec{v}_{\text{rocket}}) + dm(\vec{v}_{\text{fuel}} + \vec{v}_{\text{rocket}})$$

Cancelling terms and rearranging yields

$$d\vec{v}_{\text{rocket}} = -\vec{v}_{\text{fuel}} \frac{dm}{m} \tag{1}$$

This equation describes a differential change in the velocity of the rocket that occurs over a very short time. In order to find the difference in speeds over some larger duration of time, we integrate both sides of equation (1).

3. Execute Plan Integrating the left hand side of equation (1) from the initial to final conditions yields the change in velocity:

$$\int_{\text{i}}^{\text{f}} d\vec{v}_{\text{rocket}} = \vec{v}_{\text{rocket,f}} - \vec{v}_{\text{rocket,i}}$$

Integrating the right hand side of equation (1) from the initial to the final conditions yields

$$-\int_{\text{i}}^{\text{f}} \vec{v}_{\text{fuel}} \frac{dm}{m} = -\vec{v}_{\text{fuel}} \ln\left(\frac{m_{\text{f}}}{m_{\text{i}}}\right) = \vec{v}_{\text{fuel}} \ln\left(\frac{m_{\text{i}}}{m_{\text{f}}}\right)$$

Hence,

$$\vec{v}_{\text{rocket,f}} - \vec{v}_{\text{rocket,i}} = \vec{v}_{\text{fuel}} \ln\left(\frac{m_{\text{f}}}{m_{\text{i}}}\right)$$

Note that the velocity of the rocket is always in the opposite direction as the velocity of the ejected fuel. Hence, if we wish to express the difference in y components we would pick up an additional negative sign on the right hand side:

$$v_{\text{rocket}\,y,\text{f}} - v_{\text{rocket}\,y,\text{i}} = -v_{\text{fuel}\,y} \ln\left(\frac{m_{\text{f}}}{m_{\text{i}}}\right) = v_{\text{fuel}\,y} \ln\left(\frac{m_{\text{i}}}{m_{\text{f}}}\right)$$

or

$$v_{\text{rocket,f}} - v_{\text{rocket,i}} = v_{\text{fuel}} \ln\left(\frac{m_{\text{i}}}{m_{\text{f}}}\right)$$

4. Evaluate Result The initial inertia of the rocket and fuel together is always larger than the final inertia. Hence the natural logarithm is always a positive number, and the speed of the rocket is always increasing. Note that at the very beginning, when no fuel has yet been ejected, the difference in speed is zero. This result makes logical sense.

Questions and Problems

4.1. Puck 2 had twice the initial speed of the puck 1.

4.2. The curve shape indicates that the horizontal surface is not of uniform smoothness. The portion over which the block slides from $t = 0$ to $t = 4$ s is rough, causing the speed to decrease. The portion over which the block slides from $t = 4$ s to $t = 8$ s is extremely smooth, so that there is no decrease in speed. The portion over which it slides from $t = 8$ s to $t = 12$ s is again rough, causing the speed to decrease until the block comes to a stop at $t = 12$ s.

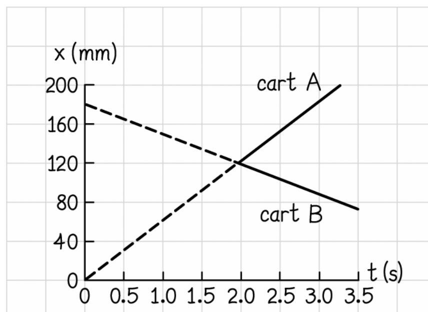
4.3. No for object 1, yes for object 2. Friction between two objects opposes relative motion between them, but object 1 speeds up as it travels across the surface, meaning friction cannot account for the change in *vx*. Note that in later chapters, we will learn more about friction including cases where an object could increase its speed due to friction with a surface that is itself moving (such as a box dropped onto a conveyor belt). But this had not yet been addressed. Object 2 slows down as it travels across the surface, meaning friction is one valid explanation for the change in v_x. (Though not the only explanation: a hockey stick slowing a puck sliding on ice, for instance, yields the same $v(t)$ curve as that shown for object 2.)

4.4. The motions described by the two graphs are very nearly the same. Each plot has a similar initial speed and a similar magnitude of acceleration, perhaps exactly the same. The effects of friction on each object certainly look very similar, since the decrease in speed is nearly identical in each case. These effects are much more visible in Figure P4.4 (*a*).

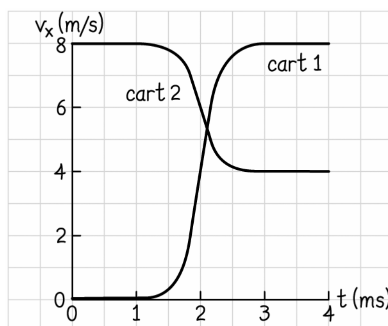
4.5. From Principles section 4.2, we know that when two objects collide in this way the ratios of the magnitude of their changes in velocity is the inverse of the ratio of their inertias. Since the change in the speed of object 1 is three times the change in the speed of object 2, object 2 has three times the inertia of object 1.

4.6. (*a*) The ratio of the inertias of the two carts is still one. Thus the ratio of velocity changes remains one. (*b*) The ratio of the inertias of the two carts is still one. Thus the magnitude of the ratio of velocity changes remains one.

4.7.



4.8.



Cart 2 slows down by 4 m/s. Since cart 2 has twice the inertia of cart 1, cart 1 should increase in speed by twice the amount that the speed of cart 2 was reduced. Hence the final speed of cart 1 should 8 m/s.

4.9. (*a*) The diagonal line is the continuation of the position of car A, and the horizontal line the continuation of the position of car B. (*b*) The passenger in car A has the greater inertia. We know this because car A has a very small change in its velocity, whereas car B has a relatively large change in velocity.

4.10. Call the direction of motion the $+x$ axis. Then $m_\mathrm{B}/m_\mathrm{A} = -\dfrac{\Delta v_{\mathrm{A}x}}{\Delta v_{\mathrm{B}x}} = -\dfrac{\big((1.4\text{ m/s})-(3.1\text{ m/s})\big)}{\big((2.0\text{ m/s})-(1.2\text{ m/s})\big)} = 2.1.$

4.11. Call the direction of motion the $+x$ axis. Then $m_\mathrm{B}/m_\mathrm{A} = -\dfrac{\Delta v_{\mathrm{A}x}}{\Delta v_{\mathrm{B}x}} = -\dfrac{((1.4\text{ m/s})-(0.6\text{ m/s}))}{((2.0\text{ m/s})-(0\text{ m/s}))} = 0.4.$ So $m_\mathrm{A} = 2.5m_\mathrm{B}$.

4.12. A quart of buckshot has the greater inertia. One way of seeing this is that a quart of buckshot resists changes to its velocity much more than a quart of feathers. If the two quart-sized containers were to collide head-on with identical initial speeds, the buckshot container would easily knock the feather container out of the way.

4.13. The inertia is unchanged. The same amount of the same type of matter is still present. It has simply changed shape.

4.14. For objects made of a given material, the inertia is determined by the amount of material present. Since all objects are made using the same volume of material, the shapes do not matter. The surface on which the objects sit also does not play a role in determining the inertia. Hence, the plastic block on ice and plastic pyramid on concrete have the same inertia. This is less than the inertia of the lead block on ice, lead sphere on ice, and lead pyramid on concrete. We know from experience that a lead block will resist a change in its velocity more than an identical block made of plastic. The inertias of the three lead objects are the same. So Plastic block ice = plastic pyramid concrete < lead block ice = lead sphere ice = lead pyramid concrete.

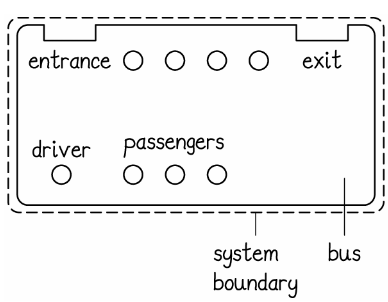
4.15. The inertia of just the bottle is the same in each case, but if the contents are included then the full bottle has the greater inertia because the water filling it has a greater inertia than the air filling the "empty" bottle.

4.16. The inertia of the tire does not change because you have changed neither the type of material of which it is made nor the amount of that material in the tire. What has changed is the inertia of the object made up of the tire plus whatever air it contains. On adding air, you have changed the amount of one of the materials making up this object.

4.17. (*a*) The track is icy. We know this because the objects do not appear to slow down much as time passes. One cart is wooden and one is plastic; the plastic cart is initially at rest and the wooden cart is initially moving. We know this because the cart initially at rest experiences such a large change in velocity compared to the other cart. (*b*) The carts are slowing down over time, but not very much. This indicates that the track is unpolished and dusty. The carts are made of the same material, but we cannot tell which material (wood or plastic) because have no information about friction between each cart and surface. The carts are initially moving in the same direction, one at 2.0 m/s and one at 5.0 m/s. (*c*) The track is damaged and rough, which we see in the obvious decrease in speed over time. The carts are made of the same material, but we cannot determine which material because have no information about friction between each cart and surface. Initially one cart is moving and the other cart is stationary.

4.18. The number of cookies is extensive, because the number of cookies scales with the size of the box of cookies. The serving size is intensive, because the serving size does not increase if the box of cookies increases in size. It is possible that different products and packages of different sizes may have different serving sizes printed on them. But the serving size does not increase as a result of increasing the system size. The number of calories per cookie is also intensive.

4.19. (*a*) The number of people in the bus is an extensive quantity.
(*b*)



(*c*) No, people can get on or off. In principle, people could also die or be born, but this is much less likely. (*d*) Yes, ignoring any deaths or births, the number of people on the bus is constant between stops.

4.20. (*a*) 2 quarts (*b*) No, the marbles have space between them that can be occupied by the water. The final volume will be somewhere between 1 quart and 2 quarts. (*c*) The total inertia of 2 quarts of water is the same before and after mixing. The total inertia of a quart of marbles and a quart of water is also the same before and after mixing. If the inertia of air occupying spaces between the marbles is included, inertia of mixed water and marbles is slightly smaller than combined inertia when in separate containers.

4.21. We are given 5 objects (truck, person, friend, ball, and Earth) from which to choose our system. Each of these objects could be included in our system or excluded from our system. That gives two possibilities for our system for every object, or $2^5 = 32$ ways. This assumes the 5 objects cannot be sub-divided, like dividing the truck into tires and doors, etc. These 32 systems include some obvious choices, such as a system containing the person, truck, and ball. Also included are systems that might be less useful, such as the empty system containing none of these things. But this is still a valid choice of a system.

4.22. In order for right side of Eq. 4.1 to be positive, velocity change must be negative for one cart and positive for other cart. Velocity is a vector, and its x component can be positive or negative. Hence, carts moving in opposite directions could both slow down, and this corresponds to one cart's velocity becoming less positive and the other becoming less negative. One change in velocity is positive, the other is negative, and both carts slow down.

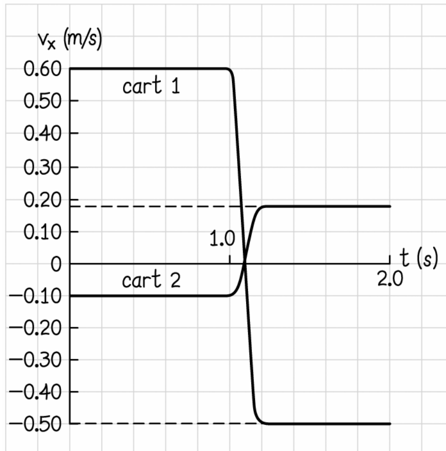
4.23. Call the initial direction of motion of cart B the $+x$ direction. Then $m_{\text{B}}/m_{\text{A}} = -\dfrac{\Delta v_{\text{A}x}}{\Delta v_{\text{B}x}} = -\dfrac{\left((+2.0\text{ m/s}) - (0)\right)}{\left((-3.0\text{ m/s}) - (+3.0\text{ m/s})\right)} = 0.33$. Hence $m_{\text{B}} = 0.33 m_{\text{A}} = 0.33(1.0\text{ kg}) = 0.33\text{ kg}$.

4.24. Call the 2.0-kg cart cart A, and call the 1.0-kg cart cart B. Choose the $+x$ direction to be to the right. We know $m_{\text{B}}/m_{\text{A}} = -\dfrac{v_{\text{A}x,\text{f}} - v_{\text{A}x,\text{i}}}{v_{\text{B}x,\text{f}} - v_{\text{B}x,\text{i}}}$. Solving for the initial velocity of cart A yields $v_{\text{A}x,\text{i}} = \left(\dfrac{m_{\text{B}}}{m_{\text{A}}}\right)(v_{\text{B}x,\text{f}} - v_{\text{B}x,\text{i}}) + v_{\text{A}x,\text{f}} = \left(\dfrac{1.0\text{ kg}}{2.0\text{kg}}\right)(0.40\text{ m/s} - 0) + (0.30\text{ m/s}) = (+0.50\text{ m/s})$. So cart A was initially moving at 0.50 m/s to the right.

4.25. Call the standard cart cart A, and call the 5-kg cart cart B. Choose the $+x$ direction to be to the right. We know $m_{\text{B}}/m_{\text{A}} = -\dfrac{v_{\text{A}x,\text{f}} - v_{\text{A}x,\text{i}}}{v_{\text{B}x,\text{f}} - v_{\text{B}x,\text{i}}}$. Solving for the initial velocity of cart A yields $v_{\text{A}x,\text{i}} = \left(\dfrac{m_{\text{B}}}{m_{\text{A}}}\right)(v_{\text{B}x,\text{f}} - v_{\text{B}x,\text{i}}) + v_{\text{A}x,\text{f}} = \left(\dfrac{1.0\text{ kg}}{5.0\text{ kg}}\right)(-0.20\text{ m/s} - 0) + (0) = (-1.0\text{ m/s})$. Hence the standard cart was initially moving at 1.0 m/s to the left.

4.26. (*a*) We calculate the velocities by reading off the approximate slopes from the graph: $\vec{v}_{1\text{i}} = +0.60\text{ m/s }\hat{\imath}$, $\vec{v}_{1\text{f}} = -0.50\text{ m/s }\hat{\imath}$, $\vec{v}_{2\text{i}} = -0.10\text{ m/s }\hat{\imath}$, $\vec{v}_{2\text{f}} = +0.17\text{ m/s }\hat{\imath}$ (*b*) $\vec{v}_{1\text{f}} - \vec{v}_{1\text{i}} = -1.1\text{ m/s }\hat{\imath}$ and $\vec{v}_{2\text{f}} - \vec{v}_{2\text{i}} = +0.27\text{ m/s }\hat{\imath}$ (*c*) $-\Delta v_1/\Delta v_2 = 4.1$ whereas $m_2/m_1 = 4.0$. The agreement is good; it is well within the error from reading the slopes from the graph.

(*d*)



(*e*) Yes, cart 1 has a nonzero acceleration during the collision, approximately from $t = 1.0$ s to $t = 1.2$ s. The acceleration is negative. (*f*) Yes, cart 2 has acceleration during the collision, approximately from $t = 1.0$ s to $t = 1.2$ s. The acceleration is positive.

4.27. (*a*) Since the velocity after the collision is likely constant, we can use the displacement and time: $\vec{v}_\text{f} = \dfrac{\Delta\vec{x}}{\Delta t}$ for either boat. This yields $\vec{v}_\text{1f} = \dfrac{-0.26\text{ m }\hat{\imath}}{2.0\text{ s}} = -0.13\text{ m/s }\hat{\imath}$ and $\vec{v}_\text{2f} = \dfrac{+2.3\text{ m }\hat{\imath}}{2.0\text{ s}} = +1.2\text{ m/s }\hat{\imath}$ (*b*) $\Delta\vec{v}_1 = \vec{v}_\text{1f} - \vec{v}_\text{1i} = -0.13\text{ m/s }\hat{\imath} - 1.5\text{ m/s }\hat{\imath} = -1.6\text{ m/s }\hat{\imath}$ and $\Delta\vec{v}_2 = \vec{v}_\text{2f} + \vec{v}_\text{2i} = +1.2\text{ m/s }\hat{\imath} + 0 = +1.2\text{ m/s }\hat{\imath}$ (*c*) We know $m_1 = -\dfrac{\Delta v_{2x}}{\Delta v_{1x}} m_2 = -\dfrac{\Delta v_{2x}}{\Delta v_{1x}}(m_\text{boat} + m_\text{father} + m_\text{mother} + m_\text{son}) = -\dfrac{(1.15\text{ m/s})}{(-1.63\text{ m/s})}(250\text{ kg}) = 176\text{ kg}$. This is the total inertia of everything in boat 1. Hence the man has inertia $m_\text{man} = m_1 - m_\text{boat} - m_\text{woman} = (176\text{ kg}) - (90\text{ kg}) - (45\text{ kg}) - (3.0\text{ kg}) = 38\text{ kg}$ (*d*) $\vec{a}_\text{1,av} = \dfrac{\Delta\vec{v}_1}{\Delta t} = \dfrac{-1.6\text{ m/s}^2\ \hat{\imath}}{0.50\text{ s}} = -3.2\text{ m/s}^2\ \hat{\imath}$ and $\vec{a}_2 = \dfrac{\Delta\vec{v}_2}{\Delta t} = \dfrac{-1.2\text{ m/s}^2\ \hat{\imath}}{0.50\text{ s}} = +2.4\text{ m/s}^2\ \hat{\imath}$.

4.28. (*a*) The velocities are arranged in the table below:

Time (s)	Velocity Cart A	Velocity Standard Cart
0	$(+5.0\text{ m/s})\hat{\imath}$	$(+1.0\text{ m/s})\hat{\imath}$
5.0	$(+4.0\text{ m/s})\hat{\imath}$	$(+0.0\text{ m/s})\hat{\imath}$
6.0	$(+2.0\text{ m/s})\hat{\imath}$	$(+4.3\text{ m/s})\hat{\imath}$
10	$(+1.2\text{ m/s})\hat{\imath}$	$(+3.5\text{ m/s})\hat{\imath}$

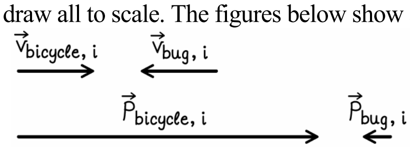
(*b*) The carts are slowing down when not colliding. (*c*) Yes, the slopes of velocity vs. time appear to be the same before and after the collision. (*d*) We know how the inertias are related to the changes in velocity, but we cannot simply read changes in velocity off from the table, because we must first account for the speed lost because of friction. Thus $m_\text{A} = -\dfrac{\Delta v_{\text{SC}x} - \Delta v_{\text{SC}x,\text{fr}}}{\Delta v_{\text{A}x} - \Delta v_{\text{A}x,\text{fr}}} m_\text{SC}$. The slope of the velocity curves while the carts are not colliding tell us that the speed of each cart would have been reduced by 0.2 m/s during the collision, just due to friction. Thus $m_\text{A} = -\dfrac{(+4.3\text{ m/s}) + (0.2\text{ m/s})}{(-2.0\text{ m/s}) + (0.2\text{ m/s})}(1\text{ kg}) = 2.5\text{ kg}$.

4.29. The magnitude of the momentum of the ball is $p_{\text{ball}} = m_{\text{ball}}v_{\text{ball}} = (0.14\text{ kg})(45\text{ m/s}) = 6.3\text{ kg}\cdot\text{m/s}$. The magnitude of the momentum of the bullet is $p_{\text{bullet}} = m_{\text{bullet}}v_{\text{bullet}} = (0.012\text{ kg})(480\text{ m/s}) = 5.8\text{ kg}\cdot\text{m/s}$. Hence, the baseball has greater momentum.

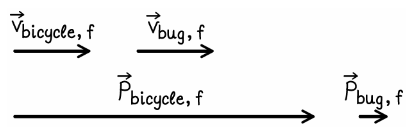
4.30. It is true that momentum is proportional to inertia, but momentum is also proportional to velocity. This means that when comparing the momenta of two objects of different inertias, only when their velocities are the same can you say for certain than the object having the greater inertia has the greater momentum. It is possible for an object that has a small inertia to have a greater momentum than an object that has a greater inertia. For example, a 0.012-kg bullet moving at 480 m/s has a much greater momentum than a 1.0-kg bowling ball moving at 0.10 m/s.

4.31. The two cars have the same change in momentum. They have the same inertia and the same change in speed. It is true that one car changed momentum at a greater rate than the other. But the overall change in momentum is the same for both.

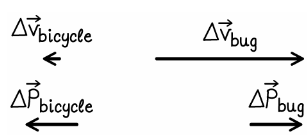
4.32. (*a*) Note that not all vectors can be drawn to scale in this problem. The magnitudes of the vectors are so different that one would need a huge piece of paper to draw all to scale. The figures below show approximate relative size of vectors.



(*b*)



(*c*)



4.33. Yes. Momentum is a vector. Therefore the system has zero momentum if two conditions are met: the carts move in opposite directions and the absolute value of the product of inertia and velocity is the same for the two carts.

4.34. (*a*) $\Delta p_{\text{A}x} = m_{\text{A}}(v_{\text{A}x,\text{f}} - v_{\text{A}x,\text{i}}) = (1.0\text{ kg})((0.50\text{ m/s}) - (0.60\text{ m/s})) = -0.1\text{ kg}\cdot\text{m/s}$ (*b*) $\Delta p_{\text{B}x} = m_{\text{B}}(v_{\text{B}x,\text{f}} - v_{\text{B}x,\text{i}}) = (0.10\text{ kg})((0.60\text{ m/s}) - (-0.40\text{ m/s})) = +0.1\text{ kg}\cdot\text{m/s}$ (*c*) $\Delta p_x = \Delta p_{\text{A}x} + \Delta p_{\text{B}x} = (-0.10\text{ kg}\cdot\text{m/s}) + (0.10\text{ kg}\cdot\text{m/s}) = 0$.

4.35. (*a*) The magnitude of the initial momentum is given by $|p_{1\text{i}}| = m_1 v_{1\text{i}}$, and we can find the inertia using the density given. We have $|p_{1\text{i}}| = \rho_1\left(\frac{4}{3}\pi R_1^3\right)v_{1\text{i}} = (1.00\times10^3\text{ kg/m}^3)\left(\frac{4}{3}\pi(0.0250\text{ m})^3\right)(3.00\text{ m/s}) = 0.196\text{ kg}\cdot\text{m/s}$. Similarly, for the final momentum we have $|p_{1\text{f}}| = \rho_1\left(\frac{4}{3}\pi R_1^3\right)|v_{1\text{f}}| = (1.00\times10^3\text{ kg/m}^3)\left(\frac{4}{3}\pi(0.0250\text{ m})^3\right)(2.00\text{ m/s}) = 0.131\text{ kg}\cdot\text{m/s}$. (*b*) Since ball 2 is initially at rest, we know $\vec{p}_{2\text{i}} = \vec{0}$. For the final momentum we cannot proceed as in part (*a*) because we are not given the density of ball 2. But we do know that $\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$, which tells us that $p_{2\text{f},x} = -(p_{1\text{f},x} - p_{1\text{f},x}) = -((-0.131\text{ kg}\cdot\text{m/s}) - (0.196\text{ kg}\cdot\text{m/s})) = +0.327\text{ kg}\cdot\text{m/s}$ (*c*) Given the results of part (*b*), we simply use $m_2 = \frac{p_{2\mathrm{f},x}}{v_{\mathrm{f},x}}$ and relate this to the density such that $\frac{4}{3}\pi R_2^3 \rho_2 = \frac{p_{2\mathrm{f},x}}{v_{\mathrm{f},x}}$. Thus $\rho_2 = \frac{3p_{2\mathrm{f},x}}{4\pi R_2^3 v_{\mathrm{f},x}} =$ $\frac{3(0.327\ \mathrm{kg\cdot m/s})}{4\pi(0.0400\ \mathrm{m})^3(1.00\ \mathrm{m/s})} = 1.22\times 10^3\ \mathrm{kg/m^3}.$

4.36. As the water falls, it accelerates due to gravity. The final velocity just before it strikes the bathtub is given by $v_{\mathrm{f}y} = -\sqrt{2\Delta y a_y} = -\sqrt{2(-2.0\ \mathrm{m})(-9.8\ \mathrm{m/s^2})} = -6.3\ \mathrm{m/s}.$ After striking the bathtub and splattering a little, the final momentum of the water should be zero. Hence the change in momentum is $\Delta p = \Delta p_y = p_{\mathrm{f}y} - p_{\mathrm{i},y} =$ $-mv_{\mathrm{i},y} = -(7.3\ \mathrm{kg})(-6.3\ \mathrm{m/s}) = 46\ \mathrm{kg\cdot m/s}.$

4.37. In order for a car to have the same momentum as when it drives at 30 m/s, it must be falling at a speed of 30 m/s. The inertia is the same in either case. In order to reach this speed the car must fall from a height given by $v_\mathrm{f}^2 = v_\mathrm{i}^2 + 2a_y\Delta y$ or $\Delta y = \frac{v_\mathrm{f}^2}{2a_y} = \frac{(30\ \mathrm{m/s})^2}{2(-9.8\ \mathrm{m/s^2})} = -46\ \mathrm{m}.$ So the initial height must be 46 m.

4.38. (*a*) We know the change in momentum is $\Delta p = m\Delta v.$ We can also write the change in speed as the magnitude of the average acceleration times time: $\Delta p = ma_\mathrm{av}\Delta t$ (*b*) If we rearrange our answer to part (*a*) we obtain $a_\mathrm{av} = \frac{\Delta p}{m\Delta t}.$ The change in momentum will be the same in either case. But the object hitting a hard floor will stop in a very short time, whereas an object hitting the soft bed will stop over a greater period of time. The acceleration of the object hitting the hard floor must be greater, because it stops in a shorter time. (*c*) Bringing a person to a stop over a longer period of time reduces the acceleration of the person. This can help prevent injuries that arise from being suddenly jerked in one direction.

4.39. We calculate the change in momentum using $\Delta p_y = m\Delta v_y.$ However, we do not know the speed of the ball just before impact or just after impact. We can find these speeds by using the kinematic equation $v_{\mathrm{f}y}^2 = v_{\mathrm{i}y}^2 + 2a_y\Delta y.$ Applying this equation to the fall toward the ground (dropped from rest) yields the y component of the velocity just prior to the ball striking the ground (call this $v_{\mathrm{prior},y}$): $v_{\mathrm{prior},y} = -\sqrt{2(-9.8\ \mathrm{m/s^2})(-2.0\ \mathrm{m})} = -6.3\ \mathrm{m/s}.$ Similarly, we find the y component of the velocity just after the ball strikes the ground (call this $v_{\mathrm{after},y}$): $v_{\mathrm{after},y} =$ $+\sqrt{-2(-9.8\ \mathrm{m/s^2})(1.6\ \mathrm{m})} = +5.6\ \mathrm{m/s}.$ Now finally we can write $\Delta p_y = m(v_{\mathrm{f}y} - v_{\mathrm{i}y}) = (0.15\ \mathrm{kg})((5.6\ \mathrm{m/s}) -$ $(-6.3\ \mathrm{m/s})) = +1.8\ \mathrm{kg\cdot m/s}.$

4.40. In a system consisting of the bullet and the rifle, the system momentum is zero before the firing. Then the bullet moves in one direction, and the rifle must move in the opposite direction in order to keep the system momentum constant at zero. This rifle motion is *recoil*.

4.41. Since this system is isolated (at least in the short time before the rifle makes strong contact the person's shoulder or other object) we know the change in the momentum of the bullet must be equal in magnitude and opposite in direction to the change in momentum of the rifle. That is $\Delta\vec{p}_\mathrm{rifle} = -\Delta\vec{p}_\mathrm{bullet}$, or $m_\mathrm{rifle}(\vec{v}_{\mathrm{rifle,f},y} - \vec{v}_{\mathrm{rifle,i},y}) = -m_\mathrm{bullet}(\vec{v}_{\mathrm{bullet,f},y} - \vec{v}_{\mathrm{bullet,i},y}).$ Both objects are initially at rest. Call the direction in which the bullet is fired the $+x$ direction. Solving for the final velocity of the rifle yields $\vec{v}_{\mathrm{rifle,f},y} = -\frac{m_\mathrm{bullet}}{m_\mathrm{rifle}}\vec{v}_{\mathrm{bullet,f},y} = -\frac{(0.010\ \mathrm{kg})}{(4.0\ \mathrm{kg})}(800\ \mathrm{m/s}\ \hat{i}) = -2.0\ \mathrm{m/s}\ \hat{i}.$ So the speed at which the rifle recoils is 2.0 m/s.

4.42. If all doors and windows are closed such that there is no draft and no net movement of air, the momentum of the system is zero.

4.43. (*a*) No. The car certainly has a change in momentum and it certainly has contact with other objects (the pole). (*b*) No. The car and pole have a combined momentum that definitely changes. It is also clear that the pole is held in place (roughly) by its contact with the Earth. (*c*) Yes, ignoring celestial motion the total momentum of the car-pole-Earth system is the same before and after the crash.

4.44. No, the momentum of my body changes. My body must have interacted with external objects.

4.45. Call the initial direction of the man's velocity the $+x$ direction. If there is no horizontal push on the ice, then the pair should constitute an isolated system. They should have the same momentum after the collision that they had before. Hence $\vec{p}_{\text{man,i}} + \vec{p}_{\text{woman,i}} = \vec{p}_{\text{man,f}} + \vec{p}_{\text{woman,f}}$. This can be simplified by noting that the woman is initially at rest and therefore has no momentum, and that after the collision they move together (with the same velocity). We can write $m_{\text{man}}\vec{v}_{\text{man,i}} = (m_{\text{man}} + m_{\text{woman}})\vec{v}_{\text{f}}$, or $\vec{v}_{\text{f}} = \dfrac{m_{\text{man}}}{(m_{\text{man}} + m_{\text{woman}})}\vec{v}_{\text{man,i}} = \dfrac{75\text{ kg}}{(75\text{ kg} + 50\text{ kg})}(+4.0\text{ m/s }\hat{\imath}) = +2.4\text{ m/s }\hat{\imath}$. Hence the final speed of the pair is 2.4 m/s.

4.46. (*a*) She will glide backwards, opposite the direction of the backpack and away from the bench. During the initial toss, she and the backpack constitute an isolated system, so the backpack's change in momentum must be accompanied by a change in her momentum in the opposite direction. (*b*) No, she does not stop at the same time. The system momentum is changed, because the system is no longer isolated when the backpack makes contact with the bench. Some momentum is transferred to the bench and ultimately to Earth.

4.47. (*a*) The total momentum of the system is just $6\text{ kg}\cdot\text{m/s}$ to the right, since the wall has no momentum. (*b*) The momentum of the wall is still zero. (*c*) No, it is not isolated. The wall is in contact with the ground or some structure beneath it. When the cart struck the wall, external interactions kept the wall from moving.

4.48. The momentum should be constant. This typically means that the velocity of the cart must also be constant, as long as the inertia does not change.

4.49. (*a*) We add up the momenta of all carts: $\vec{p}_{\text{i}} = \vec{p}_{1\text{i}} + \vec{p}_{2\text{i}} + \vec{p}_{3\text{i}} = (+6.0\text{ kg}\cdot\text{m/s }\hat{\imath}) + (-2.0\text{ kg}\cdot\text{m/s }\hat{\imath}) + (-3.0\text{ kg}\cdot\text{m/s }\hat{\imath}) = +1.0\text{ kg}\cdot\text{m/s }\hat{\imath}$ (*b*) As in part (*a*) we add up the momenta of all carts: $\vec{p}_{\text{f}} = \vec{p}_{1\text{f}} + \vec{p}_{2\text{f}} + \vec{p}_{3\text{f}} = (-4.0\text{ kg}\cdot\text{m/s }\hat{\imath}) + (+2.0\text{ kg}\cdot\text{m/s }\hat{\imath}) + (+3.0\text{ kg}\cdot\text{m/s }\hat{\imath}) = +1.0\text{ kg}\cdot\text{m/s }\hat{\imath}$ (*c*) Yes, the system appears to be isolated, based on these data.

4.50. On the low-friction track, the system of the two carts should be isolated to a good approximation. Hence we can write $\Delta\vec{p}_1 = -\Delta\vec{p}_2$, or $\vec{p}_{1\text{f}} - \vec{p}_{1\text{i}} = -(\vec{p}_{2\text{f}} - \vec{p}_{2\text{i}})$. Solving for the initial momentum of cart 2 yields $\vec{p}_{2\text{i}} = (\vec{p}_{2\text{f}} + \vec{p}_{1\text{f}} - \vec{p}_{1\text{i}}) = (-6.0\text{ kg}\cdot\text{m/s }\hat{\imath}) + (-2.0\text{ kg}\cdot\text{m/s }\hat{\imath}) - (10\text{ kg}\cdot\text{m/s }\hat{\imath}) = -18\text{ kg}\cdot\text{m/s }\hat{\imath}$.

4.51. (*a*) $\Delta\vec{p}_{\text{A}} = \vec{p}_{\text{A,f}} - \vec{p}_{\text{A,i}} = (+2.0\text{ kg}\cdot\text{m/s }\hat{\imath}) - (+10\text{ kg}\cdot\text{m/s }\hat{\imath}) = -8.0\text{ kg}\cdot\text{m/s }\hat{\imath}$ and $\Delta\vec{p}_{\text{B}} = \vec{p}_{\text{B,f}} - \vec{p}_{\text{B,i}} = +8.0\text{ kg}\cdot\text{m/s }\hat{\imath}$ (*b*) $\Delta\vec{p}_{\text{A}} + \Delta\vec{p}_{\text{B}} = (-8.0\text{ kg}\cdot\text{m/s }\hat{\imath}) + (8.0\text{ kg}\cdot\text{m/s }\hat{\imath}) = 0$ (*c*) Yes, this is consistent with the conservation of momentum because the change of momentum for the entire system was zero.

4.52. (*a*) $\Delta\vec{p}_{\text{A}} = \vec{p}_{\text{A,f}} - \vec{p}_{\text{A,i}} = (+1.0\text{ kg}\cdot\text{m/s }\hat{\imath}) - (+3.0\text{ kg}\cdot\text{m/s }\hat{\imath}) = -2.0\text{ kg}\cdot\text{m/s }\hat{\imath}$ and $\Delta\vec{p}_{\text{B}} = \vec{p}_{\text{B,f}} - \vec{p}_{\text{B,i}} = (-6.0\text{ kg}\cdot\text{m/s }\hat{\imath}) - (+2.0\text{ kg}\cdot\text{m/s }\hat{\imath}) = -8.0\text{ kg}\cdot\text{m/s }\hat{\imath}$ (*b*) $\Delta\vec{p}_{\text{A}} + \Delta\vec{p}_{\text{B}} = (-8.0\text{ kg}\cdot\text{m/s }\hat{\imath}) + (-2.0\text{ kg}\cdot\text{m/s }\hat{\imath}) = -10.0\text{ kg}\cdot\text{m/s }\hat{\imath}$ (*c*) No, the change in momentum for the entire system is not zero. Some external interactions must have been involved.

4.53. (*a*) No, provided that the two objects are isolated (no interactions with other objects). (*b*) Yes, the object that is initially moving could stop and impart all of its momentum to the second object initially at rest.

4.54. There are many possible answers here. The impulse delivered to the nail must be equal and opposite to the impulse delivered to the hammer. So our problem is reduced to finding the change in momentum of the hammer. We can assume that a hammer comes to a stop upon striking a nail, to a very good approximation. But a hammer could be anything from a small tack hammer to a sledge hammer. We choose a representative value of 1 kg for the inertia of the hammer. Also, hammers of different inertias and handle lengths will be swung at different speeds. But we assume for light hammering the maximum speed of the hammer just before impact is approximately 10 m/s. This would give the hammer a momentum just before impact of $10\ \text{kg}\cdot\text{m/s}$. Since this is equal in magnitude to the change in momentum of the hammer, it is also the impulse delivered to the nail. Answers up to $20\ \text{kg}\cdot\text{m/s}$ are reasonable.

4.55. (*a*) It is not possible to predict the velocities of both cars using momentum. The collisions discussed thus far have been collisions in which the objects do not deform or lose bits of material. You know from everyday experience that collisions between cars do not fit this description. Pieces of metal and glass may fly off the cars in question. Structural differences between the cars may cause one to spin off to the right or left, even if the cars strike each other head-on. In this type of collision, momentum is not sufficient to determine the final velocities of each car. (*b*) Yes, the combined momenta of both cars together must be the same afterward as before. More specifically, the momentum of both cars together after the collision must be $\vec{p}_\text{f} = \vec{p}_\text{i} = \vec{p}_{1\text{i}} + \vec{p}_{2\text{i}} = (1200\ \text{kg})(15\ \text{m/s}\ \hat{\imath}) + (1600\ \text{kg})(-10\ \text{m/s}\ \hat{\imath}) =$ $2000\ \text{kg}\cdot\text{m/s}\ \hat{\imath}$, where $\hat{\imath}$ is the direction of the 1200-kg car's initial motion.

4.56. (*a*) We refer to the 3.0-kg cart as cart A, and the 2.0-kg cart as cart B. Let the $+x$ direction point to the right. We use $\Delta\vec{p}_\text{A} = -\Delta\vec{p}_\text{B}$ or $m_\text{A}(v_{\text{A}x,\text{f}} - v_{\text{A}x,\text{i}}) = -m_\text{B}(v_{\text{B}x,\text{f}} - v_{\text{B}x,\text{i}})$. Rearranging to solve for the x component of the initial velocity of cart B yields $v_{\text{B}x,\text{i}} = \dfrac{m_\text{A}}{m_\text{B}}(v_{\text{A}x,\text{f}} - v_{\text{A}x,\text{i}}) + v_{\text{B}x,\text{f}} = \dfrac{(3.0\ \text{kg})}{(2.0\ \text{kg})}((5.0\ \text{m/s}) - (1.0\ \text{m/s})) + (3.0\ \text{m/s}) = +9.0\ \text{m/s}$. Hence the initial velocity of the 2.0-kg cart was $9.0\ \text{m/s}\ \hat{\imath}$. (*b*) We proceed exactly as in part (*a*), but changing the sign on the initial velocity of cart A. This yields $v_{\text{B}x,\text{i}} = \dfrac{m_\text{A}}{m_\text{B}}(v_{\text{A}x,\text{f}} - v_{\text{A}x,\text{i}}) + v_{\text{B}x,\text{f}} = \dfrac{(3.0\ \text{kg})}{(2.0\ \text{kg})}((5.0\ \text{m/s}) - (-1.0\ \text{m/s})) + (3.0\ \text{m/s}) = +12\ \text{m/s}\ \hat{\imath}$.

4.57. Remove the clothing and throw it in one direction. You should acquire a momentum equal and opposite to the thrown clothing and drift toward the shore.

4.58. Let us refer to the inertia of the larger moose as m_L and the inertia of the smaller moose as m_S. We will call their initial speed v. We know $m_\text{L}\Delta v_\text{L} = -m_\text{S}\Delta v_\text{S}$ or $m_\text{L}\left(\dfrac{v}{3} - v\right) = -m_\text{S}\left(\dfrac{v}{3} + v\right)$. Solving for the ratio of the larger moose to the smaller moose yields $\dfrac{m_\text{L}}{m_\text{S}} = -\left(\dfrac{4v/3}{-2v/3}\right) = 2$. Sothe ratio of the inertia of the larger moose to that of the smaller moose is 2/1.

4.59. We start with $\Delta\vec{p}_\text{P} = -\Delta\vec{p}_\text{F}$. Calling the initial direction of motion the $+x$ direction, we can write $m_\text{P}(v_{\text{P}x,\text{f}} - v_{\text{P}x,\text{i}}) = -m_\text{F}(v_{\text{F}x,\text{f}} - v_{\text{F}x,\text{i}})$. Solving for the inertia of the friend, we have $m_\text{F} = -m_\text{P}\dfrac{(v_{\text{P}x,\text{f}} - v_{\text{P}x,\text{i}})}{(v_{\text{F}x,\text{f}} - v_{\text{F}x,\text{i}})} = -(80\ \text{kg})\dfrac{((+4.0\ \text{m/s}) - (+7.0\ \text{m/s}))}{((+8.0\ \text{m/s}) - (5.0\ \text{m/s}))} = 80\ \text{kg}$.

4.60. (*a*) $\vec{p}_\text{car} = m_\text{car}\vec{v}_\text{car} = (1000\ \text{kg})(20\ \text{m/s}\ \hat{\imath}) = 2.0\times10^4\ \text{kg}\cdot\text{m/s}\ \hat{\imath}$ (*b*) $\vec{p}_\text{bug} = m_\text{bug}\vec{v}_\text{bug} = (9.0\times10^{-3}\ \text{kg})(-2.0\ \text{m/s}\ \hat{\imath}) =$ $-1.8\times10^{-2}\ \text{kg}\cdot\text{m/s}\ \hat{\imath}$ (*c*) Because of the significant digits in the data, the change of the car's velocity is zero, to two significant digits. It is worth mentioning however, that the change in velocity of the car cannot truly be exactly zero, because momentum is still conserved. We can estimate the change in velocity in the following way. To a very good approximation, the bug reverses direction and moves at about 20 m/s. From this we can calculate the change in the momentum of the bug: $\Delta\vec{p}_\text{bug} = m_\text{bug}\Delta\vec{v}_\text{bug} = (9.0\times10^{-3}\ \text{kg})((+20\ \text{m/s}\ \hat{\imath}) - (-2.0\ \text{m/s}\ \hat{\imath})) = 2.0\times10^{-1}\ \text{kg}\cdot\text{m/s}\ \hat{\imath}$. The bug and car together are at least very close to an isolated system (ignoring air resistance). So we can say that $\Delta\vec{p}_{\text{bug}} = -\Delta\vec{p}_{\text{car}}$. Hence the change in velocity of the car is $\Delta\vec{v}_{\text{car}} = \dfrac{\Delta\vec{p}_{\text{car}}}{m_{\text{car}}} = \dfrac{-2.0\times10^{-1}\ \text{kg}\cdot\text{m/s}\ \hat{\imath}}{1000\ \text{kg}} = -2.0\times10^{-4}\ \text{m/s}\ \hat{\imath}$. Of course, compared to the car's initial velocity, this change is approximately zero.

4.61. (*a*) We know $\Delta\vec{p}_{\text{boulder}} = -\Delta\vec{p}_{\text{sled}}$ since the ice is too slippery to interact with either the sled or the boulder. Calling the initial direction of the sled's motion the $+x$ direction, we write $m_{\text{boulder}}\Delta\vec{v}_{\text{boulder}} = -m_{\text{sled}}\Delta\vec{v}_{\text{sled}}$ or $\vec{v}_{\text{boulder,f}} = -\dfrac{m_{\text{sled}}}{m_{\text{boulder}}}\Delta\vec{v}_{\text{sled}} = -\dfrac{(5.0\ \text{kg})}{(200\ \text{kg})}((-6.0\ \text{m/s}\ \hat{\imath}) - (+10\ \text{m/s}\ \hat{\imath})) = +0.40\ \text{m/s}\ \hat{\imath}$. Hence the boulder moves at a speed of 0.40 m/s (*b*) Although you are on the sled, and it might be natural to think of you and the sled as one unit, the problem asks for your momentum. So, we exclude the inertia of the sled. $\vec{p}_{\text{you}} = m\vec{v}_{\text{you}} = (70\ \text{kg})(10\ \text{m/s}\ \hat{\imath}) = +7.0\times10^{2}\ \text{kg}\cdot\text{m/s}\ \hat{\imath}$. (*c*) We proceed finding the final speed of the boulder as in part (*a*) and then compare. $\vec{v}_{\text{boulder,f}} = -\dfrac{m_{\text{sled/person}}}{m_{\text{boulder}}}\Delta\vec{v}_{\text{sled/person}} = -\dfrac{(75.0\ \text{kg})}{(200\ \text{kg})}((+2.0\ \text{m/s}\ \hat{\imath}) - (+10\ \text{m/s}\ \hat{\imath})) = +3.0\ \text{m/s}\ \hat{\imath}$. So the speed in this case would be higher than calculated in part (*a*).

4.62. The impulse delivered by the water is equal in magnitude to the change in momentum of the water. We can write $\vec{J} = \Delta\vec{p} = m\Delta\vec{v}$. We ignore any splashing back of the water, and assume that the water basically comes to rest after striking the wall. But we still must write the inertia m in terms of the given variables. We know the inertia in a given volume of water is given by $m = V\rho$, so we need only consider what volume of water strikes the wall in a given time interval Δt. Since the stream of water will move toward the building a distance $v\Delta t$, we can say that the volume of water striking the surface that time is $Av\Delta t$. This tells us that the inertia of water striking the surface is $m = A\rho v\Delta t$. Hence $J = A\rho v^2\Delta t$.

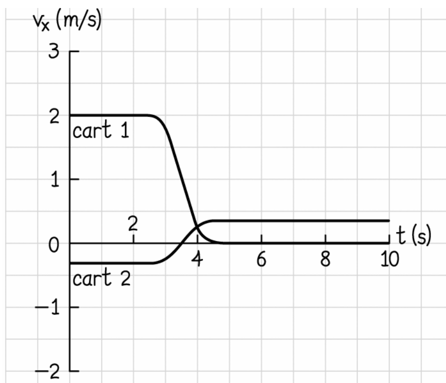
4.63. (*a*) Before collision, cue ball has momentum directed toward 8 ball and 8 ball has zero momentum. After collision, cue ball has zero momentum and 8 ball has momentum equal in magnitude and direction to momentum of cue ball before collision. (*b*) Call direction of motion the $+x$ direction. For the isolated system of two balls, $m_8(v_{8x,\text{f}} - v_{8x,\text{i}}) = -m_{\text{c}}(v_{\text{c}x,\text{f}} - v_{\text{c}x,\text{i}})$. With $v_{8x,\text{i}} = 0$, $v_{\text{c}x,\text{f}} = 0$, and $m_8 = m_{\text{c}}$, this yields $v_{8x,\text{f}} = v_{\text{c}x,\text{i}}$. Because $m_8 = m_{\text{c}}$, this corresponds to $p_{8x,\text{f}} = p_{\text{c}x,\text{i}}$, or zero change in system momentum, consistent with zero impulse and with the answer in (*a*). Considering each ball separately, the non-zero change in momentum is also equal to the non-zero impulse, because the impulse affecting each ball is equal in magnitude and in the opposite direction, just as the momentum changes of each ball are equal and opposite. Each of the four quantities has magnitude *mv*.

4.64. In either case the total initial momentum will end up being shared evenly by the three carts. Whichever system starts with the greatest momentum will result in the greatest final speed. Clearly, since case (*a*) has twice the initial momentum of case (*b*), the magnitude of the final velocity is greater in case (*a*).

4.65. The *x* component of the initial momentum and final momentum must be the same. Just before the collision the coal is not moving in the x direction. So we can write $p_{\text{car}\,x,\text{i}} + p_{\text{coal}\,x,\text{i}} = p_{\text{car}\,x,\text{f}} + p_{\text{coal}\,x,\text{f}}$ or $m_{\text{coal}} = \dfrac{p_{\text{car}\,x,\text{i}} - p_{\text{car}\,x,\text{f}}}{v_{\text{coal}\,x,\text{f}}} = \dfrac{m_{\text{car}}v_{\text{car}\,x,\text{i}} - m_{\text{car}}v_{\text{car}\,x,\text{f}}}{v_{\text{car}\,x,\text{f}}} = \dfrac{(3.0\times10^{4}\ \text{kg})(0.50\ \text{m/s} - 0.30\ \text{m/s})}{(0.30\ \text{m/s})} = 2.0\times10^{4}\ \text{kg}$.

4.66. Cases (*a*) and (*b*) both start with two carts initially moving at a velocity $\vec{v}$. This means the systems in (*a*) and (*b*) start with the same amount of momentum, initially. Assuming these systems are isolated, they must have the same final momentum also. In both cases all three carts are finally stuck together moving at one velocity. Hence, the only way for the carts to have the same final momentum in cases (*a*) and (*b*), is if the carts have the same final velocity in cases (*a*) and (*b*). Hence, the final speed of all three carts is the same in cases (*a*) and (*b*).

4.67. (*a*) In all cases, we simply read the x component of the momentum off of the graph and use $v_x = p_x/m$ to find the velocities in the x direction. We write this explicitly for the initial velocity of cart 1: $v_{1x,\text{i}} = \frac{p_{1x,\text{i}}}{m_1} = \frac{2.0\ \text{kg}\cdot\text{m/s}}{1.0\ \text{kg}} = 2.0\ \text{m/s}$, so $\vec{v}_{1\text{i}} = +2.0\ \text{m/s}\ \hat{\imath}$. In exactly the same way we see that $\vec{v}_{1\text{f}} = \vec{0}$ (*b*) Just as in part (*a*) we find $v_{2x,\text{i}} = \frac{p_{2x,\text{i}}}{m_2} = \frac{-1.0\ \text{kg}\cdot\text{m/s}}{3.0\ \text{kg}} = -0.33\ \text{m/s}$. So $\vec{v}_{2\text{i}} = -0.33\ \text{m/s}\ \hat{\imath}$. Similarly $\vec{v}_{2\text{f}} = +0.33\ \text{m/s}\ \hat{\imath}$ (*c*) We can simply read the momentum values off of the graph and take the difference: $\Delta\vec{p}_1 = \vec{p}_{1\text{f}} - \vec{p}_{1\text{i}} = (0) - (2.0\ \text{kg}\cdot\text{m/s}\ \hat{\imath}) = -2.0\ \text{kg}\cdot\text{m/s}\ \hat{\imath}$ and $\Delta\vec{p}_2 = \vec{p}_{2\text{f}} - \vec{p}_{2\text{i}} = (1.0\ \text{kg}\cdot\text{m/s}) - (-1.0\ \text{kg}\cdot\text{m/s}\ \hat{\imath}) = +2.0\ \text{kg}\cdot\text{m/s}\ \hat{\imath}$. (*d*) We assume that this system of carts is isolated, such that the impulse delivered to the system is zero. This means the total momentum of the system should not change. $\vec{J} = \Delta\vec{p} = \Delta\vec{p}_1 + \Delta\vec{p}_2 = (-2.0\ \text{kg}\cdot\text{m/s}\ \hat{\imath}) + (+2.0\ \text{kg}\cdot\text{m/s}\ \hat{\imath}) = \vec{0}$. So, yes this is consistent with the carts being an isolated system.
(*e*)



4.68. We find the momentum change in each scenario and then compare. Take the $+x$ direction to be to the right. (*a*) The system of two carts initially has a total momentum $p_{x,\text{i}} = p_{\text{A}\,x,\text{i}} = m_\text{A} v_{\text{A}\,x,\text{i}}$. This must also equal the final momentum, so we can equate this to $m_\text{A} v_{\text{A}\,x,\text{i}} = (m_\text{A} + m_\text{B}) v_{x,\text{f}}$. Here $v_{x,\text{f}}$ is the x component of the final velocity of both carts. Now, solving for the final velocity, we obtain $v_{x,\text{f}} = \frac{m_\text{A} v_{\text{A}\,x,\text{i}}}{(m_\text{A} + m_\text{B})} = \frac{m_\text{A} v_{\text{A}\,x,\text{i}}}{(m_\text{A} + 2m_\text{A})} = \frac{v_{\text{A}\,x,\text{i}}}{3} = \frac{1.0\ \text{m/s}}{3} = 0.33\ \text{m/s}$. Now, the change in momentum of cart B is given by $\Delta\vec{p}_{\text{B,a}} = \vec{p}_{\text{B,a,f}} - \vec{p}_{\text{B,a,i}} = m_\text{B}(0.33\ \text{m/s}\ \hat{\imath})$. So the magnitude of the momentum change can be written as $m_\text{B}(0.33\ \text{m/s})$. (*b*) This is very similar to part (*a*). The only difference is that now cart B (which has greater inertia than cart A) is the one initially moving. It can be shown just as in (*a*) that $v_{x,\text{f}} = \frac{m_\text{B} v_{\text{B}\,x,\text{i}}}{(m_\text{A} + m_\text{B})} = \frac{m_\text{B} v_{\text{B}\,x,\text{i}}}{((1/2)m_\text{B} + m_\text{B})} = \frac{2v_{\text{B}\,x,\text{i}}}{3} = \frac{2(-1.0\ \text{m/s})}{3} = -0.67\ \text{m/s}$. So the change in momentum of cart B in this case is $\Delta\vec{p}_{\text{B,b}} = \vec{p}_{\text{B,b,f}} - \vec{p}_{\text{B,b,i}} = m_\text{B}((-0.67\ \text{m/s}\ \hat{\imath}) - (-1.0\ \text{m/s}\ \hat{\imath})) = m_\text{B}(0.33\ \text{m/s}\ \hat{\imath})$. So again the magnitude of the change can be written as $m_\text{B}(0.33\ \text{m/s})$. (*c*) We start by finding the initial velocity of cart B. From conservation of momentum we know that $\Delta p_\text{A} = -\Delta p_\text{B}$ such that $m_\text{A}(v_{\text{A}\,x,\text{f}} - v_{\text{A}\,x,\text{i}}) = -m_\text{B}(v_{\text{B}\,x,\text{f}} - v_{\text{B}\,x,\text{i}})$. Solving for $v_{\text{B}\,x,\text{i}}$ yields $v_{\text{B}\,x,\text{i}} = v_{\text{B}\,x,\text{f}} + \frac{m_\text{A}}{m_\text{B}}(v_{\text{A}\,x,\text{f}} - v_{\text{A}\,x,\text{i}}) = (+0.67\ \text{m/s}) + \left(\frac{1}{2}\right)((-0.33\ \text{m/s}) - (1.0\ \text{m/s})) = 0$. Since the initial momentum of cart B is zero, we have $\Delta p_{\text{B,c}} = p_{\text{B,c,f}} = m_B(0.67\ \text{m/s})$ (*d*) We start by finding the final x component of the velocity of cart B. Using conservation of momentum, we have $m_\text{A}(v_{\text{A}\,x,\text{f}} - v_{\text{A}\,x,\text{i}}) = -m_\text{B}(v_{\text{B}\,x,\text{f}} - v_{\text{B}\,x,\text{i}})$, where

$v_{\mathrm{A}\,x,\mathrm{f}} = v_{\mathrm{B}\,x,\mathrm{f}} = v_{x,\mathrm{f}}$, because the cart stick together. Hence $v_{x,\mathrm{f}} = \dfrac{v_{\mathrm{B}\,x,\mathrm{i}} + \dfrac{m_\mathrm{A}}{m_\mathrm{B}} v_{\mathrm{A}\,x,\mathrm{i}}}{\left(\dfrac{m_\mathrm{A}}{m_\mathrm{B}} + 1\right)} = \dfrac{(-1.0\text{ m/s}) + \dfrac{1}{2}(1.0\text{ m/s})}{\left(\dfrac{1}{2}+1\right)} =$ (-0.33 m/s). Hence $\Delta p_{\mathrm{B}\,x} = m_\mathrm{B}(v_{\mathrm{B}\,x,\mathrm{f}} - v_{\mathrm{B}\,x,\mathrm{i}}) = m_\mathrm{B}((-0.33\text{ m/s}) - (-1.0\text{ m/s})) = m_\mathrm{B}(0.67\text{ m/s})$.

Hence $\Delta p_{\mathrm{B,c}} = \Delta p_{\mathrm{B,d}} > \Delta p_{\mathrm{B,a}} = \Delta p_{\mathrm{B,b}}$.

4.69. Let the $+x$ direction be to the right. We simply write the conservation of momentum in the x direction as

$$p_{\mathrm{R}\,x,\mathrm{i}} + p_{\mathrm{B}\,x,\mathrm{i}} + p_{\mathrm{G}\,x,\mathrm{i}} = p_{\mathrm{R}\,x,\mathrm{f}} + p_{\mathrm{B}\,x,\mathrm{f}} + p_{\mathrm{G}\,x,\mathrm{f}}$$
$$m_\mathrm{R} v + m_\mathrm{G}(-v) = m_\mathrm{R}(-v) + (m_\mathrm{G} + m_\mathrm{B})(-v/5)$$

Inserting $m_\mathrm{G} = 3m_\mathrm{R}$ we have

$$5m_\mathrm{R} = (3m_\mathrm{R} + m_\mathrm{B}) \text{ or } m_\mathrm{B} = 2m_\mathrm{R}$$

4.70. We wish to apply the conservation of momentum to the collision involving the putty. But first, we need to know the inertia of cart A. For this, we look at the first collision, which did not involve putty. For the first collision, we can write the expression for the conservation of momentum in the x direction as $p_{\mathrm{s}\,x,\mathrm{i}} + p_{\mathrm{A}\,x,\mathrm{i}} = p_{\mathrm{s}\,x,\mathrm{f}} + p_{\mathrm{A}\,x,\mathrm{f}}$, or equivalently $m_\mathrm{s} v_{\mathrm{s}\,x,\mathrm{i}} + m_\mathrm{A} v_{\mathrm{A}\,x,\mathrm{i}} = m_\mathrm{s} v_{\mathrm{s}\,x,\mathrm{f}} + m_\mathrm{A} v_{\mathrm{A}\,x,\mathrm{f}}$. Solving for the inertia of cart A yields

$$m_\mathrm{A} = m_\mathrm{s}\frac{(v_{\mathrm{s}\,x,\mathrm{f}} - v_{\mathrm{s}\,x,\mathrm{i}})}{(v_{\mathrm{A}\,x,\mathrm{i}} - v_{\mathrm{A}\,x,\mathrm{f}})} = (1.0\text{ kg})\frac{(0.40\text{ m/s}) - (0.20\text{ m/s})}{(+0.60\text{ m/s}) - 0} = 0.33\text{ kg}$$

We can now use this inertia of cart A to carry out the same process for the second collision. Now, the conservation of momentum in the x direction is written

$$p_{\mathrm{s}\,x,\mathrm{i}} + p_{\mathrm{A}\,x,\mathrm{i}} + p_{\mathrm{p}\,x,\mathrm{i}} = p_{\mathrm{s}\,x,\mathrm{f}} + p_{\mathrm{A}\,x,\mathrm{f}} + p_{\mathrm{p}\,x,\mathrm{f}}$$
$$m_\mathrm{s} v_{\mathrm{s}\,x,\mathrm{i}} + m_\mathrm{A} v_{\mathrm{A}\,x,\mathrm{i}} + m_\mathrm{p} v_{\mathrm{p}\,x,\mathrm{i}} = m_\mathrm{s} v_{\mathrm{s}\,x,\mathrm{f}} + m_\mathrm{A} v_{\mathrm{A}\,x,\mathrm{f}} + m_\mathrm{p} v_{\mathrm{p}\,x,\mathrm{f}}$$

There is some subtlety to the x component of the initial velocity of the putty. It is not the same as that of cart A. The problem states that the putty is "dropped", meaning it lands on cart A with no initial horizontal component to its velocity. This process could certainly be treated as an additional collision and could be treated separately. But nothing changes about the process if we consider the initial time to be before the putty strikes cart A, and the final time to be after the carts have collided. We solve for the inertia of the putty and find

$$\begin{aligned} m_\mathrm{p} &= \frac{m_\mathrm{s}(v_{\mathrm{s}\,x,\mathrm{f}} - v_{\mathrm{s}\,x,\mathrm{i}}) + m_\mathrm{A}(v_{\mathrm{A}\,x,\mathrm{f}} - v_{\mathrm{A}\,x,\mathrm{i}})}{(v_{\mathrm{p}\,x,\mathrm{i}} - v_{\mathrm{p}\,x,\mathrm{f}})} \\ &= \frac{(1.0\text{ kg})((-0.20\text{ m/s}) - (0.20\text{ m/s})) + (0.33\text{ kg})((0.40\text{ m/s}) - (-0.60\text{ m/s}))}{(0 - (0.40\text{ m/s}))} \\ &= 0.17\text{ kg} \end{aligned}$$

Hence the inertia of the putty is 0.17 kg.

4.71. Call the initial direction of motion of the cannonball the $+x$ direction. We write the conservation of momentum in the x direction, included some additional inertia to be added, Δm. Clearly the initial momentum of the system is zero, since all objects are initially at rest (at least relative to the deck).

$$p_{\mathrm{c}\,x,\mathrm{i}} + p_{\mathrm{b}\,x,\mathrm{i}} = 0 = p_{\mathrm{c}\,x,\mathrm{f}} + p_{\mathrm{b}\,x,\mathrm{f}}$$
$$0 = (m_\mathrm{c} + \Delta m)v_{\mathrm{c}\,x,\mathrm{f}} + m_\mathrm{b} v_{\mathrm{b}\,x,\mathrm{f}}$$

Solving this for the additional inertia yields

$$\Delta m = -\frac{m_\mathrm{b} v_{\mathrm{b}\,x,\mathrm{f}}}{v_{\mathrm{c}\,x,\mathrm{f}}} - m_\mathrm{c} = -\frac{(20\text{ kg})(60\text{ m/s})}{(-2.0\text{ m/s})} - (400\text{ kg}) = 200\text{ kg}$$

Hence, 2.0×10^2 kg must be added to the cannon.

4.72. Call the vertically upward direction the $+y$ direction. In order to apply the conservation of momentum to this problem, we must determine the velocity of the marble and tennis ball immediately after they collide. Since the marble and tennis ball rise under the influence of gravity only, they will have a constant acceleration, and we can use kinematics. We apply $v_{y,\mathrm{f}} = \sqrt{v_{y,\mathrm{i}}^2 + 2a_y\Delta y}$ to each object. Note that for this kinematic part, "initial" refers to the instant just after the objects have collided, and "final" refers to the instant the respective objects reach their maximum height. We find

$$v_{\mathrm{m}\,y,\mathrm{i}} = \sqrt{v_{\mathrm{m}\,y,\mathrm{f}}^2 - 2a_y\Delta y_\mathrm{m}} = \sqrt{0 - 2(-9.8\ \mathrm{m/s^2})(200\ \mathrm{m})} = 62.6\ \mathrm{m/s}$$

$$v_{\mathrm{t}\,y,\mathrm{i}} = \sqrt{v_{\mathrm{t}\,y,\mathrm{f}}^2 - 2a_y\Delta y_\mathrm{t}} = \sqrt{0 - 2(-9.8\ \mathrm{m/s^2})(54\ \mathrm{m})} = 32.5\ \mathrm{m/s}$$

Here we have kept one additional significant digit, since this is an intermediate step. Now we can use conservation of momentum in the y direction during the objects' collision. We now use "initial" to refer to the instant just before the collision, and "final" to refer to the instant just after. We have

$$p_{\mathrm{m}\,y,\mathrm{i}} + p_{\mathrm{t}\,y,\mathrm{i}} = p_{\mathrm{m}\,y,\mathrm{f}} + p_{\mathrm{t}\,y,\mathrm{f}}$$

$$0 + m_\mathrm{t}v_{\mathrm{t}\,y,\mathrm{i}} = m_\mathrm{m}v_{\mathrm{m}\,y,\mathrm{f}} + m_\mathrm{t}v_{\mathrm{t}\,y,\mathrm{f}}$$

Solving for the initial y component of the tennis ball's velocity yields

$$v_{\mathrm{t}\,y,\mathrm{i}} = \frac{m_\mathrm{m}v_{\mathrm{m}\,y,\mathrm{f}} + m_\mathrm{t}v_{\mathrm{t}\,y,\mathrm{f}}}{m_\mathrm{t}} = \frac{(0.010\ \mathrm{kg})(62.6\ \mathrm{m/s}) + (0.058\ \mathrm{kg})(32.5\ \mathrm{m/s})}{(0.058\ \mathrm{kg})} = 43\ \mathrm{m/s}$$

Hence the launch speed of the tennis ball is 43 m/s.

4.73. We write the ratio of the magnitudes of the momenta as

$$\frac{p_\mathrm{g}}{p_\mathrm{b}} = \frac{m_\mathrm{g}v_\mathrm{g}}{m_\mathrm{b}v_\mathrm{b}} = \frac{(m_\mathrm{b}/10)(5v_\mathrm{b})}{m_\mathrm{b}v_\mathrm{b}} = \frac{1}{2}$$

Hence $p_\mathrm{g}/p_\mathrm{b} = 1/2$.

4.74. We calculate the magnitude of the momentum of each object and compare them.

$$p_\mathrm{t} = m_\mathrm{t}v_\mathrm{t} = (30{,}000\ \mathrm{kg})(2.2\ \mathrm{m/s}) = 6.6\times10^4\ \mathrm{kg\cdot m/s}$$

$$p_\mathrm{v} = m_\mathrm{v}v_\mathrm{v} = (2{,}400\ \mathrm{kg})(30\ \mathrm{m/s}) = 7.2\times10^4\ \mathrm{kg\cdot m/s}$$

Hence, the minivan has a momentum of greater magnitude.

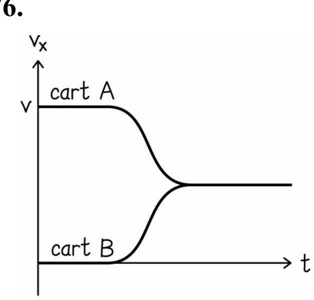
4.75. We equate the two momenta symbolically as $p_F = p_Y$ or equivalently $m_\mathrm{F}v_\mathrm{F} = m_\mathrm{Y}v_\mathrm{Y}$. Solving for the inertia of your ball yields

$$m_\mathrm{Y} = \frac{m_\mathrm{F}v_\mathrm{F}}{v_\mathrm{Y}} = \frac{(4.5\ \mathrm{kg})(10\ \mathrm{m/s})}{(8.0\ \mathrm{m/s})} = 5.6\ \mathrm{kg}$$

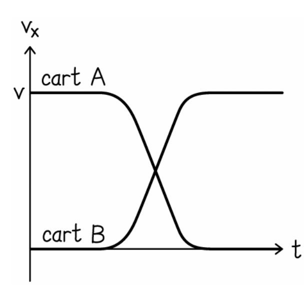
Hence, your ball must have an inertia of 5.6 kg for the two momenta to be equal.

4.76.

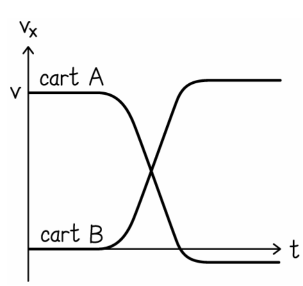
(*a*)



(*b*)

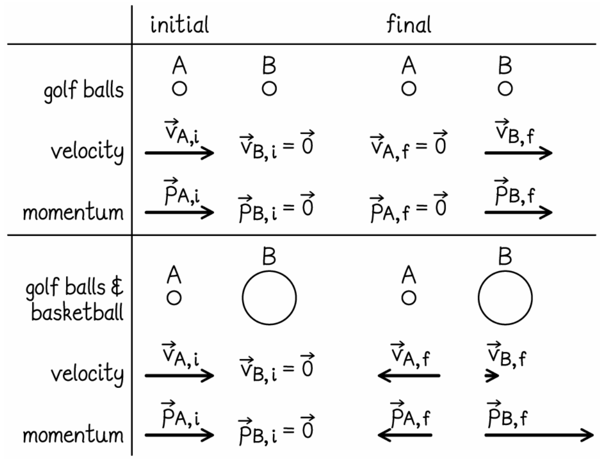


(*c*)



We will assume the velocity is along the x axis for our figures.

4.77. (*a*) and (*b*)



4.78. To find the magnitude of the momentum after 15 seconds, we need the final speed and the total inertia. The speed is obtained from the formula describing the position of the plane by taking the first derivative with respect to time:

$$v(t) = \frac{dx(t)}{dt} = bt$$

Hence, $v(t = 15\text{ s}) = (2.0\text{ m/s}^2)(15\text{ s}) = 30\text{ m/s}.$

Now we only need the inertia. The question refers to the momentum of the airplane. Since the inertia of the airplane itself is given separately from the inertia of the passengers, we might assume that it is truly only the airplane's inertia that is of interest to us. In that case, we have $p = mv = (35{,}000\text{ kg})(30\text{ m/s}) = 1.0\times10^6\text{ kg}\cdot\text{m/s}.$ This is the correct answer for the momentum of the airplane, only.

One the other hand, one might also be interested in the momentum of all the inertia that is moving together (including the plane and the passengers on it). In that case, we have

$$p = mv = (35{,}000\text{ kg} + 150(65\text{ kg}))(30\text{ m/s}) = 1.3\times10^6\text{ kg}\cdot\text{m/s}$$

4.79. Let the initial direction of motion of the bullet be the $+x$ direction. The conservation of the x component of momentum in this case reads

$$p_{\text{bullet }x,\text{i}} + p_{\text{block }x,\text{i}} = p_{\text{bullet }x,\text{f}} + p_{\text{block }x,\text{f}} \qquad \text{or}$$

$$m_{\text{bullet}}v_{\text{bullet }x,\text{i}} + m_{\text{block}}v_{\text{block }x,\text{i}} = m_{\text{bullet}}v_{\text{bullet }x,\text{f}} + m_{\text{block}}v_{\text{block }x,\text{f}}$$

Using the fact that $v_{\text{block }x,\text{i}} = 0,$ we can solve for $v_{\text{block }x,\text{f}}.$ We find $v_{\text{block,f}} = \dfrac{m_{\text{bullet}}}{m_{\text{block}}}(v_{\text{bullet,i}} - v_{\text{bullet,f}}).$

4.80. Let the initial direction of motion of the bullet be the $+x$ direction. The conservation of the x component of momentum in this case reads

$$p_{\text{bullet }x,\text{i}} + p_{\text{cart }x,\text{i}} = p_{\text{bullet }x,\text{f}} + p_{\text{cart }x,\text{f}} \qquad \text{or}$$

$$m_{\text{bullet}}v_{\text{bullet }x,\text{i}} + m_{\text{cart}}v_{\text{cart }x,\text{i}} = m_{\text{bullet}}v_{\text{bullet }x,\text{f}} + m_{\text{cart}}v_{\text{cart }x,\text{f}}$$

Using the fact that $v_{\text{cart }x,\text{i}} = 0$, we can solve for $v_{\text{bullet }x,\text{i}}.$ We find

$$v_{\text{bullet }x,\text{i}} = \frac{m_{\text{cart}}v_{\text{cart,f}} + m_{\text{bullet}}v_{\text{bullet }x,\text{f}}}{m_{\text{bullet}}} = \frac{(4.0\text{ kg})(1.8\text{ m/s}) + (0.012\text{ kg})(1.8\text{ m/s})}{(0.012\text{ kg})} = 6.0\times10^2\text{ m/s}$$

The initial speed of the bullet was 6.0×10^2 m/s.

4.81. When a rocket is launched or a cannonball is fired, there is significant forward momentum in the projectile. Since the total momentum of the system started out at zero, there must be some backward momentum after firing. In a bazooka, material (largely burnt fuel) can be expelled out the back of the tube. This means that the required backward momentum does not need to come entirely from a recoil of the bazooka. In a cannon, there is no such outlet for air or debris at the back. This means that the cannon itself must recoil with the same magnitude of momentum imparted to the cannonball.

4.82. Assume all the football players run at their top speeds and that the opponents run in a direction exactly opposite the runner with the ball. Call the initial direction of motion of the runner with the ball the $+x$ direction. When the players collide momentum must be conserved. If either of the opponents has a greater magnitude momentum than the runner with the ball, then the next momentum of the opponent and runner after collision will be negative. We calculate the momentum of each player to see if any opponent has a greater magnitude of momentum than the runner with the ball:

$$p_{\text{runner}\,x,\text{i}} = m_{\text{runner}} v_{\text{runner}\,x,\text{i}} = (95\ \text{kg})\frac{(50\ \text{m})}{(5.5\ \text{s})} = 8.6\times10^{2}\ \text{kg}\cdot\text{m/s}$$

$$p_{\text{heavy}\,x,\text{i}} = m_{\text{heavy}} v_{\text{heavy}\,x,\text{i}} = (110\ \text{kg})\frac{-(50\ \text{m})}{(6.6\ \text{s})} = -8.3\times10^{2}\ \text{kg}\cdot\text{m/s}$$

$$p_{\text{light}\,x,\text{i}} = m_{\text{light}} v_{\text{light}\,x,\text{i}} = (90\ \text{kg})\frac{-(50\ \text{m})}{(5.1\ \text{s})} = -8.8\times10^{2}\ \text{kg}\cdot\text{m/s}$$

Clearly, the lighter opponent (with and inertia of 90 kg) has a momentum that is greater in magnitude than the runner's. Hence this lighter opponent has the greatest chance of stopping the runner (and even driving him back slightly).

4.83. In space, the rocket expels a huge number of gas particles at very high speeds. When these gas particles are ejected from the rear of the rocket, the rocket must acquire a forward velocity in order for system momentum to remain constant.

4.84. Call the smaller object (the one that is initially moving) object A, and the larger object (that is initially stationary) object B. (*a*) Object A is initially moving in the positive direction and object B is initially stationary. (*b*) Object A is finally moving in the negative direction and object B is finally moving in the positive direction. (*c*) We can write the conservation of the x component of momentum as

$$p_{\text{A}\,x,\text{i}} + p_{\text{B}\,x,\text{i}} = p_{\text{A}\,x,\text{f}} + p_{\text{B}\,x,\text{f}}$$

$$m_{\text{A}} v_{\text{A}\,x,\text{i}} + m_{\text{B}} v_{\text{B}\,x,\text{i}} = m_{\text{A}} v_{\text{A}\,x,\text{f}} + m_{\text{B}} v_{\text{B}\,x,\text{f}}$$

Since object B is initially at rest, we can solve for the ratio of inertias to find

$$\frac{m_{\text{B}}}{m_{\text{A}}} = \frac{(v_{\text{A}\,x,\text{f}} - v_{\text{A}\,x,\text{i}})}{(v_{\text{B}\,x,\text{i}} - v_{\text{B}\,x,\text{f}})} = \frac{((1.0\ \text{m/s}) - (-0.80\ \text{m/s}))}{(0 - 0.20\ \text{m/s})} = 9$$

Hence the ratio of the greater inertia to the smaller inertia is 9 to 1. (*d*) No. We can see that both objects move at constant speeds, except during the collision. This indicates that the surface on which they collide must be relatively smooth.

4.85. In a collision, the change in momentum is the same in magnitude for each car. Hence the magnitude of the change in velocity is less for the object with the greater inertia. This means less acceleration for the car with greater inertia, which is preferable if you want to avoid injury.

4.86. Let the direction of the initial motion of the rocket be the $+x$ direction. We can express the conservation of momentum in the x direction as $p_{\text{rocket}\,x,\text{i}} = p_{\text{rocket}\,x,\text{f}} + p_{\text{gas}\,x,\text{f}}$, where the subscript "gas" refers to exhausted fuel. Equivalently

$$m_{\text{rocket,i}} v_{\text{rocket}\,x,\text{i}} = m_{\text{rocket,f}} v_{\text{rocket}\,x,\text{f}} + m_{\text{gas,f}} v_{\text{gas}\,x,\text{f}} \qquad (1)$$

It is important to note at this point that the magnitude of the final velocity of the gas is not the same thing as the "exhaust speed". The exhaust speed refers to the speed at which gas is ejected relative to the body of the rocket. If the rocket itself is moving with an x component of its velocity equal to $v_{\text{rocket}\,x,\text{i}}$, then we have $v_{\text{gas}\,x,\text{f}} = v_{\text{rocket}\,x,\text{i}} - v_{\text{exhaust}\,x}$. Inserting this result into equation (1) we obtain

$$m_{\text{rocket,i}} v_{\text{rocket}\,x,\text{i}} = \frac{2}{3} m_{\text{rocket,i}} v_{\text{rocket}\,x,\text{f}} + \frac{m_{\text{rocket,i}}}{3}(v_{\text{rocket}\,x,\text{i}} - v_{\text{exhaust}\,x})$$

Solving for the final x component of the rocket's velocity yields

$$v_{\text{rocket }x,\text{f}} = v_{\text{rocket }x,\text{i}} + \frac{1}{2}v_{\text{exhaust }x} = (2.0\times10^3\ \text{m/s}) + \frac{1}{2}(1.0\times10^3\ \text{m/s}) = 2.5\times10^3\ \text{m/s}$$

4.87. Call the vertically downward direction the $-y$ direction. If you were to drop the stereo, it would accelerate downward under the influence of gravity. We can use kinematics to find the final downward speed of the stereo, and use that to find the final momentum. The final speed will be given by the magnitude of the final y component of the velocity, given by $v_{y,\text{f}} = -\sqrt{v_{y,\text{i}}^2 + 2a_y\Delta y} = \sqrt{0 + 2(-9.8\ \text{m/s}^2)(-4.0\ \text{m})} = -8.9\ \text{m/s}$.

The final momentum is then given by $\vec{p}_\text{f} = m\vec{v}_\text{f} = (6.0\ \text{kg})(-8.9\ \text{m/s})\,\hat{j} = (-53\ \text{kg}\cdot\text{m/s})\,\hat{j}.$ Don't drop the stereo. The magnitude of its final momentum would be $53\ \text{kg}\cdot\text{m/s}$ downward when reaching your friend. This could be harmful to your friend (not to mention harmful to the stereo).

4.88. One might try to find what fraction of the fuel is required for the increase in speed, or one might calculate the maximum change in speed that could be produced by using all fuel. Either would give us a sense of whether this change in speed is likely to exhaust or nearly exhaust the remaining fuel. Let us begin by writing the conservation of momentum in the direction of the rocket's initial motion (taken to be the $+x$ direction).

$$m_{\text{rocket,i}}v_{\text{rocket }x,\text{i}} = m_{\text{rocket,f}}v_{\text{rocket }x,\text{f}} + m_{\text{gas,f}}v_{\text{gas }x,\text{f}}$$

Let us call the change in speed of the rocket Δv_x, and the inertia of the exhausted fuel Δm. Then we have

$$m_{\text{rocket,i}}v_{\text{rocket }x,\text{i}} = (m_{\text{rocket,i}} - \Delta m)(v_{\text{rocket }x,\text{i}} + \Delta v_x) + m_{\text{gas,f}}(v_{\text{rocket }x,\text{i}} - 800\ \text{m/s})$$

$$\Delta m(800\ \text{m/s} + \Delta v_x) = m_{\text{rocket,i}}\Delta v_x$$

We could express the fraction of the total inertia of the rocket that must be spent on the required speed increase as

$$\frac{\Delta m}{m_{\text{rocket,i}}} = \frac{\Delta v_x}{(800\ \text{m/s} + \Delta v_x)} = \frac{(5.2\ \text{m/s})}{(800\ \text{m/s} + 5.2\ \text{m/s})} = 0.0065$$

Since only 10% of the inertia of the rocket was initially made up of fuel, this means that the required speed increase would use 6.5% of the available fuel. This seems like a small correction that should leave plenty of fuel for landing maneuvers.

If one had instead calculated the total possible increase in speed that one could achieve by using all of the fuel on board, one would find

$$\Delta v_{x,\max} = \frac{\dfrac{\Delta m_{\max}}{m_{\text{rocket,i}}}(800\ \text{m/s})}{1 - \dfrac{\Delta m_{\max}}{m_{\text{rocket,i}}}} = 89\ \text{m/s}$$

This also shows that the fuel on board is capable of much more than a correction of 5.2 m/s.
Hence, it seems highly probable that enough fuel remains for minor adjustments.

4.89. Note first that it is necessary for you to do something. Your final downward speed is given by $v_{y,\text{f}} = -\sqrt{0 + 2(-1.5\ \text{m/s}^2)(-30\ \text{m})} = -9.49\ \text{m/s}^2$. If all sandbags remain in your balloon, the entire inertia upon landing will be 330 kg. This corresponds to a final momentum of magnitude $3.130\times10^3\ \text{kg}\cdot\text{m/s}$. This is enough to cause damage to your balloon.

If the sandbags are simply dropped from the basket, the inertia of you and all elements of the balloon will be only 280 kg. If you still drop at an acceleration of $1.50\ \text{m/s}^2$ downward, you will still hit the ground at a speed of 9.49 m/s. This corresponds to a momentum of $2.66\times10^3\ \text{kg}\cdot\text{m/s}$ downward. This is low enough to avoid damage to the basket. However, if you throw the sandbags downward you can increase their downward momentum and therefore decrease the downward momentum of yourself and the balloon. It would be smarter to throw them downward to ensure that you are nowhere close to the momentum that would damage the basket.

4.90. (*a*) The molecule initially moves in one direction (call it the $+x$ direction), and then reverses direction. Hence the change in momentum is $\Delta\vec{p} = \vec{p}_\text{f} - \vec{p}_\text{i} = m\vec{v} - (-m\vec{v}) = 2m\vec{v}$. Hence the magnitude of the change in momentum is $2mv$. (*b*) The particle has to cross the entire box from side B to side A, and then back again. So $\Delta t = \dfrac{\Delta x}{v} = \dfrac{2\ell}{v}$. (*c*) This is just the inverse of the answer to part (*b*): $v/2\ell$ (*d*) $\dfrac{\Delta p}{\Delta t} = \dfrac{2mv}{2\ell/v} = \dfrac{mv^2}{\ell}$.

4.91. (*a*) Let the direction of motion of the rocket be the $+x$ direction. Let time t_0 be the instant before firing the first stage; t_1 is an instant between the firing of the first and second stages, and t_2 is an instant just after firing the second stage. For the firing of the first stage, we can write the conservation of momentum as $p_{\text{rocket}\,x,0} + p_{\text{fuel}\,x,0} = p_{\text{rocket}\,x,1} + p_{\text{fuel}\,x,1}$. The initial momentum of the whole system is zero. Hence we can write

$$0 = 4mv_{\text{rocket}\,x,1} + mv_{\text{fuel}\,x,1}$$

The final x component of the velocity of the gas can be written $v_{\text{fuel}\,x,\text{f}} = v_{\text{rocket x,i}} + v_{\text{ex}\,x}$. We also cancel a factor of m to obtain.

$$0 = 4v_{\text{rocket}\,x,1} + (0 + v_{\text{ex}\,x})$$

$$v_{\text{rocket}\,x,1} = \frac{-v_{\text{ex}\,x}}{4}$$

Since $v_{\text{ex}\,x,\text{f}} < 0$, the sign of this answer is sensible.

Now consider the firing of the second stage. We have

$$3m\left(-\frac{v_{\text{ex}\,x}}{4}\right) = 2mv_{\text{rocket}\,x,2} + mv_{\text{fuel}\,x,2}$$

$$-\frac{3v_{\text{ex}\,x}}{4} = 2v_{\text{rocket}\,x,2} + \left(v_{\text{ex}\,x} + \left(\frac{-v_{\text{ex}\,x}}{4}\right)\right)$$

$$v_{\text{rocket}\,x,2} = \frac{1}{2}\left(-\frac{3v_{\text{ex}\,x}}{4} - \frac{3v_{\text{ex}\,x}}{4}\right) = -\frac{3}{4}v_{\text{ex}\,x}$$

Again, note that because $v_{\text{ex}\,x,\text{f}} < 0$, the sign of this answer is sensible. Written in terms of the exhaust speed (positive by definition), we have $v_{\text{rocket},2} = \dfrac{3}{4}v_{\text{ex}}$. (*b*) With only a single stage, we only need to apply conservation of momentum a single time:

$$p_{\text{rocket}\,x,0} + p_{\text{fuel}\,x,0} = p_{\text{rocket}\,x,1} + p_{\text{fuel}\,x,1}$$

The initial momentum of the whole system is zero. Hence we can write

$$0 = 3mv_{\text{rocket}\,x,1} + 2mv_{\text{fuel}\,x,1}$$

$$0 = 3v_{\text{rocket}\,x,1} + 2(0 + v_{\text{ex}\,x})$$

$$v_{\text{rocket}\,x,1} = \frac{-2v_{\text{ex}\,x}}{3}$$

In terms of speeds, $v_{\text{rocket}\,x,1} = \dfrac{2}{3}v_{\text{ex}}$. (*c*) The two stage rocket yields a higher payload speed. By detaching spent stages, the inertia that has to be sped up in the next stage decreases.

5

ENERGY

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^2 m/s east **2.** 10^3 m/s **3.** 10^2 J **4.** 10^6 J **5.** 10^9 J **6.** 10^1 m/s **7.** 10^2 J **8.** 10^3 J **9.** 10^3 J **10.** 10^7 J

Guided Problems

5.2 Throwing a punch

1. Getting Started The boxer's hand, glove, and forearm do not constitute an isolated system, nor a closed system, because they interact strongly with the opponent's head. Including the opponent's head in the system helps, but the forearm of the boxer is still being driven forward by the upper arm. If the jab is such that the upper arm doesn't move the lower arm forward much during actual contact, then one might estimate the hand, glove, forearm, and opponent's head system is isolated during the time interval over which the jab makes contact with the opponent's head. In other words, the momentum of that system should be roughly the same just before and just after contact. However, even with those assumptions, this system may still not be closed. For example, when the boxer jabs his opponent, the audience will likely hear a smacking sound. This means that some (typically very small) quantity of energy leaves the system in the form of sound.

A momentum diagram would be useful in determining the final velocities of the boxer's fist and his opponent's head. However, in this case we are assuming that half the initial kinetic energy is converted to internal energy of the opponent's head. So a momentum diagram is unnecessary.

2. Devise Plan We write the expression for the total kinetic energy of the boxer's fist, and require that it be large enough that half of it is sufficient to knock the opponent "wobbley-kneed".

$$K_{\text{fist}} = \frac{1}{2}mv_{\text{fist}}^2 = 2K_{\text{knockout}} \tag{1}$$

Now we can solve for the required speed of the fist.

3. Execute Plan Solving equation (1) above for the speed of the fist, we find

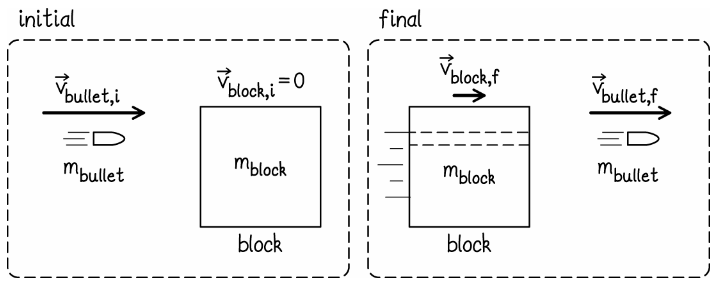
$$v_{\text{fist}} = \sqrt{\frac{4K_{\text{knockout}}}{m}} = \sqrt{\frac{4(20\text{ J})}{(3.0\text{ kg})}} = 5.2\text{ m/s}$$

4. Evaluate Result This seems like a plausible speed for a human fist. To determine how quantitatively correct this is, we could use kinematics to determine how long a jab would last if the final speed of the fist were 5.2 m/s. We expect a time that is much less than one second. Let us assume that as the boxer extends his arm (starting from rest) the fist covers a distance of 0.70 m at approximately constant acceleration. We can determine the time using

$\Delta x = \frac{1}{2}(v_{x,\mathrm{f}} + v_{x,\mathrm{i}})\Delta t$ or equivalently $\Delta t = \frac{2\Delta x}{v_{x,\mathrm{f}}} = \frac{2(0.70\ \mathrm{m})}{(5.2\ \mathrm{m/s})} = 0.27\ \mathrm{s}$. Clearly, the real time may vary according to arm length, and according to corrections from the acceleration not truly being constant. But this is a very reasonable duration for a jab. We conclude that our final speed is entirely achievable and reasonable.

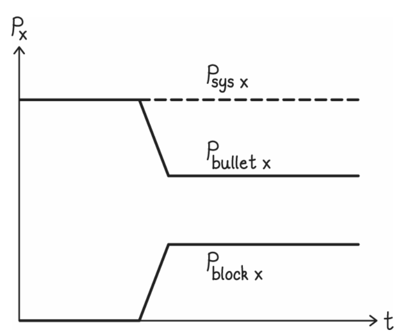
5.4 Burrowing bullet

1. Getting Started We define our system to include the bullet and the block, as shown in the system diagram below.

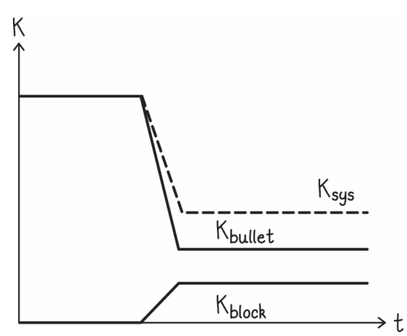


Because the system is isolated, the momentum will be constant. However, we expect that significant kinetic energy will be lost to breaking/heating of the wooden block. So we do not expect the kinetic energy to be constant.

Without calculating any values of energies or momenta, we can still draw qualitative representations of how these quantities will change over time. For example, we can show that momentum will be partially transferred from the bullet to the block, with the bullet retaining some momentum. The total momentum of the system should be the same before and after the collision. This is qualitatively shown in the graph below. We take the direction in which the bullet is fired to be the $+x$ direction. The dashed line shows the total momentum of the system.



We can make a similar graph for the kinetic energy of the system.



2. Devise Plan We write equations that relate the initial and final values of the momentum and the kinetic energy. Essentially we wish to express that $p_{x,\text{i}} = p_{x,\text{f}}$, and that $K_\text{f} - K_\text{f} = \Delta E_\text{internal}$. We write this in terms of the momenta and energies of each object. For momentum, we have

$$p_{\text{Block }x,\text{i}} + p_{\text{Bullet }x,\text{i}} = p_{\text{Block }x,\text{f}} + p_{\text{Bullet }x,\text{f}}$$

or

$$m_\text{Bullet} v_{\text{Bullet }x,\text{i}} = m_\text{Block} v_{\text{Block }x,\text{f}} + m_\text{Bullet} v_{\text{Bullet }x,\text{f}} \qquad (1)$$

For kinetic energy, we have

$$(K_\text{Block,f} + K_\text{Bullet,f}) - (K_\text{Block,i} + K_\text{Bullet,i}) = \Delta K = -\Delta E_\text{internal}$$

or

$$\left(\frac{1}{2} m_\text{Block} v_\text{Block,f}^2 + \frac{1}{2} m_\text{Bullet} v_\text{Bullet,f}^2\right) - \left(\frac{1}{2} m_\text{Bullet} v_\text{Bullet,i}^2\right) = -\Delta E_\text{internal} \qquad (2)$$

Equation (1) has only one unknown $(v_{\text{Bullet }x,\text{f}})$. So equation (1) can be used immediately to find the final speed of the bullet. Once that is known, all quantities on the left hand side of the (2) will be known, and the change in internal energy can be calculated.

3. Execute Plan We first use equation (1) to determine $v_{\text{Bullet }x,\text{f}}$:

$$v_{\text{Bullet }x,\text{f}} = \frac{m_\text{Bullet} v_{\text{Bullet }x,\text{i}} - m_\text{Block} v_{\text{Block }x,\text{f}}}{m_\text{Bullet}}$$

$$v_{\text{Bullet }x,\text{f}} = \frac{(1.20\times10^{-2}\text{ kg})(600\text{ m/s}) - (4.00\text{ kg})(1.2\text{ m/s})}{(1.20\times10^{-2}\text{ kg})}$$

$$v_{\text{Bullet }x,\text{f}} = 200\text{ m/s}$$

Now we can insert this value into equation (2) to determine the change in internal energy.

$$\Delta E_\text{internal} = \left(\frac{1}{2} m_\text{Bullet} v_\text{Bullet,i}^2\right) - \left(\frac{1}{2} m_\text{Block} v_\text{Block,f}^2 + \frac{1}{2} m_\text{Bullet} v_\text{Bullet,f}^2\right)$$

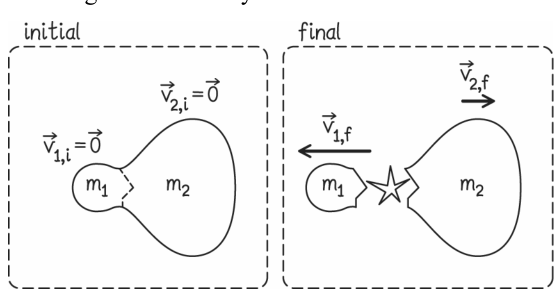
$$\Delta E_\text{internal} = \left(\frac{1}{2}(1.20\times10^{-2}\text{ kg})(600\text{ m/s})^2\right) - \left(\frac{1}{2}(4.00\text{ kg})(1.20\text{ m/s})^2 + \frac{1}{2}(1.20\times10^{-2}\text{ kg})(200\text{ m/s})^2\right)$$

$$\Delta E_\text{internal} = 1.92\times10^3\text{ J}$$

4. Evaluate Result We know that a great deal of kinetic energy is converted to heat, sound, and changes in the structure of the wood in such a collision. One might expect from the outset that the majority of the kinetic energy would be converted to other forms of energy. The number we found (1.92 kJ) corresponds to about 89% of the initial kinetic energy. This fits expectations.

5.6 Useful approximations

1. Getting Started The overall approach of Worked Problem 5.5 is applicable, in that momentum will be conserved and kinetic energy will not be conserved. We can essentially copy the initial procedure from Worked Problem 5.5, but we draw initial and final system diagrams for clarity.



2. Devise a Plan Because our system is isolated, we can use conservation of momentum. Kinetic energy will not be conserved, because we are told that some internal energy is converted to kinetic energy. We can still express the change in kinetic energy in terms of the kinetic energies of the two chunks of material. Note that initially, both momentum and kinetic energy have a magnitude of zero. This means there need never be a reference to initial speeds. Since the only speeds/velocities to which we will refer will be the final speeds/velocities, we can use a more compact notation. For example, we can write $v_{2\,x}$, rather than $v_{2\,x,\mathrm{f}}$, and so on. As shown in the system diagram, assume that object 1 initially moves in the $+x$ direction, and that object 2 initially moves in the $-x$ direction. We can write for momentum

$$p_{1\,x} + p_{2\,x} = 0$$

or

$$m_1 v_{1\,x} + m_2 v_{2\,x} = 0 \tag{1}$$

For kinetic energy, we can write

$$K_1 + K_2 = \Delta K$$

or

$$\frac{1}{2} m_1 v_{1\,x}^2 + \frac{1}{2} m_2 v_{2\,x}^2 = \Delta K \tag{2}$$

Equation (2) can be trivially re-written to isolate the final kinetic energy of the object with greater inertia:

$$\frac{1}{2} m_2 v_{2\,x}^2 = \Delta K - \frac{1}{2} m_1 v_{1\,x}^2 \tag{3}$$

We know that $m_1 \ll m_2$, or equivalently $\frac{m_1}{m_2} \ll 1$. We can use this to make approximations, particularly when subtracting small numbers from relatively large numbers.

3. Execute Plan We can use equation (1) to obtain an expression for v_{1x} and specifically, how it compares to v_{2x}. We obtain

$$v_{1\,x} = -\frac{m_2}{m_1} v_{2\,x} \tag{4}$$

If we insert equation (4) into equation (3), we obtain

$$\frac{1}{2} m_2 v_{2\,x}^2 = \Delta K - \frac{1}{2} m_1 \left(-\frac{m_2}{m_1} v_{2\,x} \right)^2$$

or

$$\frac{1}{2} m_2 v_{2\,x}^2 = \Delta K - \left(\frac{m_2}{m_1} \right) \left(\frac{1}{2} m_2 v_{2\,x}^2 \right)$$

or

$$\frac{1}{2} m_2 v_{2\,x}^2 = \frac{\Delta K}{\left(1 + \frac{m_2}{m_1} \right)} \tag{5}$$

Because we know that $\frac{m_1}{m_2} \ll 1$, we can see that the fraction in the denominator of equation (5) $\frac{m_2}{m_1} \gg 1$, such that $1 + \frac{m_2}{m_1} \approx \frac{m_2}{m_1}$. More precisely, if one is already comfortable using series expansions, one can write equation (5) as

$$\frac{1}{2} m_2 v_{2\,x}^2 = \frac{\Delta K}{\left(1 + \frac{1}{m_1/m_2} \right)}$$

and expand this around small $\frac{m_1}{m_2}$ in a Taylor series. Either way, the result is

$$\frac{1}{2}m_2v_{2x}^2 \approx \frac{m_1}{m_2}\Delta K$$

which is exactly equation (A). Inserting result (A) into equation (3), we have

$$\frac{1}{2}m_1v_{1x}^2 = \Delta K\left(1-\frac{m_1}{m_2}\right) \tag{6}$$

Again, we can either note that $\frac{m_1}{m_2} \ll 1$ implies that $1-\frac{m_1}{m_2} \approx 1$, or we can rigorously do the appropriate Taylor expansion. The result of either method applied to equation (6) is

$$\frac{1}{2}m_1v_{1x}^2 \approx \Delta K$$

which is exactly result (B).

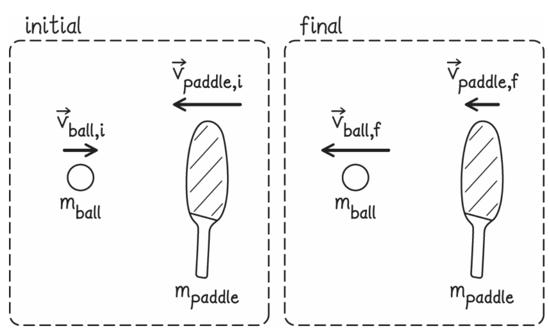
4. Evaluate Result Worked Problem 5.5 relates to a situation in which the change in kinetic energy is $\Delta K = 2.0\times10^{-3}$ J, the inertia of the lighter object is $m_1 = 5.0\times10^{-2}$ kg, and the speed of the heavier object must be of order $v_2 = 1.0\times10^{-4}$ m/s. Inserting these into equation (A) yields an inertia of

$$m_2 \approx \sqrt{\frac{2m_1}{v_{2x}^2}\Delta K} = \sqrt{\frac{2(5.0\times10^{-2}\ \text{kg})}{(1.0\times10^{-4}\ \text{m/s})^2}(2.0\times10^{-3}\ \text{J})} = 141.4\ \text{kg}$$

Here we have reported more significant digits than are warranted, merely to show the quality of the agreement to the answer to Worked Problem 5.5. To the correct number of significant digits this yields exactly the result of Worked Problem 5.5, which was 140 kg (prior to subtracting off the inertia of the patient). This approximation appears to be valid.

5.8 Fast service

1. Getting Started We choose our system to include the ball and the paddle.



Clearly, because the paddle is being held and moved by the player's hand, the system is not truly isolated, nor truly closed. However, the collision between the ball and the paddle is so brief, that over the interval of the collision the hand causes very little change in either momentum or kinetic energy. We expect the system to be isolated to a very good approximation during this collision. The system is not truly closed, since one hears a sound when a ball is struck by a paddle. But this sound energy that leaves the system is typically very small. So, to a very good approximation, one expects the system to be closed during the collision.

2. Devise Plan In a serve, the initial velocity of the ball $\vec{v}_{\text{Ball,i}}$ is very close to zero (the ball is typically released, or tossed upwards slightly). But we expect it to be large from experience, larger than the speed of the paddle in fact.

As mentioned above, the collision cannot be absolutely elastic because energy does leave the system in the form of sound. But a table-tennis ball is designed to collide in a mostly elastic way. One can check this by dropping such a ball from a small height (so that air resistance is minimal) and observing how many times the ball bounces. Compare this to other balls (basketball, tennis ball, etc). The collision should be very close to elastic, and for the purposes of this problem we will assume it is completely elastic.

We are not told any inertias explicitly, but we know from experience that the inertia of a table-tennis ball is orders of magnitude smaller than the inertia of a paddle and the player's hand and forearm. This means that the paddle will not slow down appreciably in the collision. It will continue on at roughly 20 m/s.

If we assume that the collision is elastic to a very good approximation, then the relative velocities must have the same magnitude before and after the collision. In other words:

$$(\vec{v}_{\text{Ball,i}} - \vec{v}_{\text{Paddle,i}}) = -(\vec{v}_{\text{Ball,f}} - \vec{v}_{\text{Paddle,f}})$$

Assuming the initial velocity of the paddle is in the $-x$ direction, we can write

$$(v_{\text{Ball }x,\text{i}} - v_{\text{Paddle }x,\text{i}}) = -(v_{\text{Ball }x,\text{f}} - v_{\text{Paddle }x,\text{f}})$$

We have enough information to use the relative velocity equation to determine $\vec{v}_{\text{Ball,f}}$.

3. Execute Plan We solve the relative velocity equation for $\vec{v}_{\text{Ball,f}}$.

$$v_{\text{Ball }x,\text{f}} = (v_{\text{Paddle }x,\text{f}}) - (v_{\text{Ball }x,\text{i}} - v_{\text{Paddle }x,\text{i}}) = (-20\text{ m/s}) - ((0\text{ m/s}) - (-20\text{ m/s})) = -40\text{ m/s}$$

4. Evaluate Result The final speed of the ball is twice as large as the initial speed of the paddle. This fits our expectations.

To solve this problem we assumed that the collision was perfectly elastic. We noted that this is not strictly true, but that it is a very reasonable approximation. If we used a coefficient of restitution that was smaller than one (inelastic collision), we would have obtained a slightly lower final speed for the ball. But this would be a small correction.

Questions and Problems

5.1. (*a*) The relative velocity of the truck as measured from your car is given by $\vec{v}_{\text{ct}} = \vec{v}_{\text{t}} - \vec{v}_{\text{c}} = (+22\text{ m/s})\,\hat{i} - (+25\text{ m/s})\,\hat{i} = (-3\text{ m/s})\,\hat{i}$. (*b*) The relative velocity of the motorcycle as measured from your car is given by $\vec{v}_{\text{cm}} = \vec{v}_{\text{m}} - \vec{v}_{\text{c}} = (+29\text{ m/s})\,\hat{i} - (+25\text{ m/s})\,\hat{i} = (+4\text{ m/s})\,\hat{i}$

5.2. Since the system is isolated, the final momentum will be equal to the initial momentum. Since the objects initially have equal and opposite momenta, the initial (and therefore also the final) momentum of the system is zero. In a totally inelastic collision, the objects stick together such that they have one common final velocity. The only way for the objects to have zero momentum while moving together is if the velocity of the objects is zero. Hence, the objects will be stationary.

5.3. (*a*) Call the object initially moving faster object A, and call the object initially moving more quickly object B. Let the direction of motion of both carts be the $+x$ direction. Momentum must be constant in this collision, so $m_{\text{A}}\vec{v}_{\text{A,i}} + m_{\text{B}}\vec{v}_{\text{B,i}} = m_{\text{A}}\vec{v}_{\text{A,f}} + m_{\text{B}}\vec{v}_{\text{B,f}}$ or equivalently $m_{\text{A}}(v_{\text{A }x,\text{f}} - v_{\text{A }x,\text{i}}) = -m_{\text{B}}(v_{\text{B }x,\text{f}} - v_{\text{B }x,\text{i}})$. The difference between the initial and final speeds of objects A and B appear to be roughly equal and opposite, meaning $(v_{\text{A }x,\text{f}} - v_{\text{A }x,\text{i}}) = -(v_{\text{B }x,\text{f}} - v_{\text{B }x,\text{i}})$. Comparing this with the above expression of momentum conservation, we see that the inertias of the two objects must be equal: $m_{\text{A}} = m_{\text{B}} = m$. Let us call the magnitude of the difference in the x components of the velocities Δv_x such that $v_{\text{A }x,\text{f}} = v_{\text{A }x,\text{i}} - \Delta v_x$ and $v_{\text{B }x,\text{f}} = v_{\text{B }x,\text{i}} + \Delta v_x$. Now we will write expressions for the initial and final kinetic energies and compare them: $K_{\text{i}} = \frac{1}{2}mv_{\text{A }x,\text{i}}^2 + \frac{1}{2}mv_{\text{B }x,\text{i}}^2$ and $K_{\text{f}} = \frac{1}{2}m(v_{\text{A }x,\text{i}} - \Delta v_x)^2 + \frac{1}{2}m(v_{\text{B }x,\text{i}} + \Delta v_x)^2$. The final kinetic energy can be rewritten as $K_{\text{f}} = \frac{1}{2}m(v_{\text{A }x,\text{i}}^2 + v_{\text{B }x,\text{i}}^2 + 2\Delta v_x(v_{\text{B }x,\text{i}} + \Delta v_x - v_{\text{A }x,\text{i}}))$.

Writing the difference between the final and initial kinetic energies yields $K_{\rm f} - K_{\rm i} = m\Delta v_x (v_{{\rm B}\,x,{\rm i}} + \Delta v_x - v_{{\rm A}\,x,{\rm i}}),$ which is clearly negative. Hence the final kinetic energy is less than the initial kinetic energy. So the collision is inelastic. Note that it is not totally inelastic because the objects do not have the same final velocity. (*b*) The objects stick together after the collision. So the collision is totally inelastic.

5.4. No. To a very good approximation the wall remains completely stationary. Hence all the energy in the system of the wall and the tennis ball is in the form of the kinetic energy of the ball. If the ball were to return with a greater speed than it had initially, the energy of the isolated system would have increased. This would violate the conservation of energy.

5.5. Since $\vec{p} = m\vec{v}$ doubling the magnitude of $\vec{v}$ would double the magnitude of $\vec{p}$. Kinetic energy is given by $K = \frac{1}{2}mv^2,$ such that doubling v would quadruple K.

5.6. Call the cart that doubles its kinetic energy cart A and the other cart will be cart B. The total energy of the system of two carts must remain constant because the collision is elastic. Hence $K_{\rm A,i} + K_{\rm B,i} = K_{\rm A,f} + K_{\rm B,f}$ or $K_{\rm A,i} + K_{\rm B,i} = 2K_{\rm A,i} + K_{\rm B,f}$. This can be rearranged to show $K_{\rm B,f} - K_{\rm B,i} = -K_{\rm A,i}$. The kinetic energy of cart B decreased by an amount equal to the initial kinetic energy of cart A.

5.7. We require the kinetic energy of the person be equal to the food energy in the donut: $K_{\rm p} = E_{\rm d}$, or $\frac{1}{2}m_{\rm p}v_{\rm p}^2 = E_{\rm d}$.

Hence we require $m_{\rm p} = \dfrac{2E_{\rm d}}{v_{\rm p}^2} = \dfrac{2\left(300\ \text{Cal} \times \dfrac{4186\ \text{J}}{\text{Cal}}\right)}{(1.0\ \text{m/s})^2} = 2.5 \times 10^6\ \text{kg}.$

5.8. (*a*) $p^2/2m$ (*b*) The expression in part (*a*) applies to one object that has a well-defined mass and momentum. If a system consists of only one such object then energy will be constant whenever momentum is constant. But if the system consists of multiple objects, then the momentum of the system would be a sum of momenta of the constituent objects. The sum of momenta $\sum_i \vec{p}_i$ would be constant. But the expression in part (*a*) for kinetic energy would have the individual momenta squared divided by respective masses $KE = \sum_i p_i^2/2m_i,$ which bears no obvious relationship to the sum of all momenta. Recall that the square of the sum is not (in general) equal to the sum of the squares.

5.9. We are told that $p_{\rm A} = p_{\rm B}$ and $K_{\rm A} = 4K_{\rm B}$. Equivalently we can write:

$$m_{\rm A}v_{\rm A} = m_{\rm B}v_{\rm B} \tag{1}$$

and

$$\frac{1}{2}m_{\rm A}v_{\rm A}^2 = 4\left(\frac{1}{2}m_{\rm B}v_{\rm B}^2\right) \tag{2}$$

Using equation (1), we can write $v_{\rm A} = \dfrac{m_{\rm B}}{m_{\rm A}}v_{\rm B}$. Inserting this into equation (2), we find $m_{\rm A}\left(\dfrac{m_{\rm B}}{m_{\rm A}}v_{\rm B}\right)^2 = 4m_{\rm B}v_{\rm B}^2$ or $m_{\rm A}/m_{\rm B} = 1/4$.

5.10. We are told that $p_{\rm A} = 4p_{\rm B}$ and $K_{\rm A} = K_{\rm B}$. Equivalently we can write:

$$m_{\rm A}v_{\rm A} = 4m_{\rm B}v_{\rm B} \tag{1}$$

and

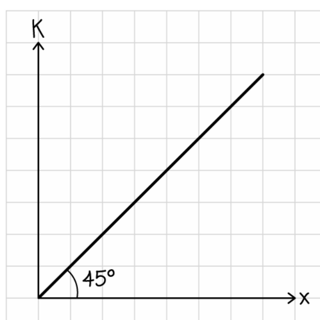
$$\frac{1}{2}m_{\rm A}v_{\rm A}^2 = \frac{1}{2}m_{\rm B}v_{\rm B}^2 \tag{2}$$

Using equation (1), we can write $v_{\mathrm{A}}=4\dfrac{m_{\mathrm{B}}}{m_{\mathrm{A}}}v_{\mathrm{B}}$. Inserting this into equation (2), we find $m_{\mathrm{A}}\left(4\dfrac{m_{\mathrm{B}}}{m_{\mathrm{A}}}v_{\mathrm{B}}\right)^2=m_{\mathrm{B}}v_{\mathrm{B}}^2$ or $m_{\mathrm{A}}/m_{\mathrm{B}}=16/1$.

5.11. If the objects have the same momentum then we can write $m_{\mathrm{X}}v_{\mathrm{X}}=m_{\mathrm{Y}}v_{\mathrm{Y}}$ or $v_{\mathrm{X}}=\dfrac{m_{\mathrm{Y}}}{m_{\mathrm{X}}}v_{\mathrm{Y}}$. We can use this to simplify the ratio of the kinetic energies: $\dfrac{K_{\mathrm{X}}}{K_{\mathrm{Y}}}=\dfrac{\frac{1}{2}m_{\mathrm{X}}v_{\mathrm{X}}^2}{\frac{1}{2}m_{\mathrm{Y}}v_{\mathrm{Y}}^2}=\dfrac{m_{\mathrm{X}}\left(\frac{m_{\mathrm{Y}}}{m_{\mathrm{X}}}v_{\mathrm{Y}}\right)^2}{m_{\mathrm{Y}}v_{\mathrm{Y}}^2}=\dfrac{m_{\mathrm{Y}}}{m_{\mathrm{X}}}$. Because we are told that object X has much larger inertia than object Y, the fraction $\dfrac{K_{\mathrm{X}}}{K_{\mathrm{Y}}}$ is very small. Hence the kinetic energy of Y is greater than the kinetic energy of X.

If the objects have the same kinetic energy then we can write $\frac{1}{2}m_{\mathrm{Y}}v_{\mathrm{Y}}^2=\frac{1}{2}m_{\mathrm{X}}v_{\mathrm{X}}^2$ or $v_{\mathrm{Y}}=\sqrt{\dfrac{m_{\mathrm{X}}}{m_{\mathrm{Y}}}}v_{\mathrm{X}}$. We can use this to simplify the ratio of the momenta:
$\dfrac{p_{\mathrm{X}}}{p_{\mathrm{Y}}}=\dfrac{m_{\mathrm{X}}v_{\mathrm{X}}}{m_{\mathrm{Y}}v_{\mathrm{Y}}}=\dfrac{m_{\mathrm{X}}v_{\mathrm{X}}}{m_{\mathrm{Y}}\sqrt{\frac{m_{\mathrm{X}}}{m_{\mathrm{Y}}}}v_{\mathrm{X}}}=\sqrt{\dfrac{m_{\mathrm{X}}}{m_{\mathrm{Y}}}}$. Because the inertia of X is larger than the inertia of Y, we know the fraction $\dfrac{p_{\mathrm{X}}}{p_{\mathrm{Y}}}$ is large. Hence the momentum of X is greater than the momentum of Y.

5.12.



We know from kinematics that the motion of the brick will obey $v_{y,\mathrm{f}}^2=v_{y,\mathrm{i}}^2+2a_y\Delta y$. Trivially multiplying both sides by $\frac{1}{2}m$ and noting that the brick starts from rest, we can write $K=\frac{1}{2}mv_{y,\mathrm{f}}^2=ma_y\Delta y$. As the brick falls, both a_y and Δy are negative, meaning the kinetic energy will increase. This means that the kinetic graph should show a linear increase that has a y intercept of zero.

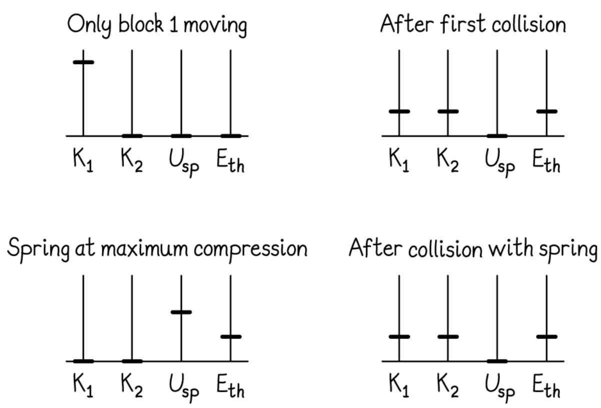
5.13. The book slows down due to friction, meaning that both the book and the table heat up slightly. This corresponds to kinetic energy being converted into thermal energy of the book and the table. You can also hear the sound made by the rubbing of the book against the table, though the amount of energy that escapes in the form of sound is typically much smaller than the amount of energy lost to heat.

5.14. A good way to identify whether a process is reversible or irreversible, is to do the following. Imagine a video is taken of the process. The video is then played for you, but in reverse. If you would immediately recognize that the video is being played in reverse, then the process is probably irreversible. If you can't tell that the video is being played in reverse, then the process is probably reversible. (*a*) Since the collision between billiard balls would look the same if the process were played backwards, the process is reversible. (*b*) A video of the toss could be played backwards and it would look like a hand catching a falling coin rather than throwing one upward. We could not be certain which process was filmed, so the process is reversible. (*c*) A collision between hockey players results in a loud sound, heating of the players' bodies and uniforms, and can even result in broken bones. There is no way for heat, sound and a spontaneously healing bone to throw two hockey players apart. Hence the process is irreversible. (*d*) There is no way for a cannonball to go down the barrel of the cannon in reverse in such a way as to reconstitute vapor into gunpowder. The process is irreversible. (*e*) There is no way for the light and heat given off by a match to re-create the chemicals in a match head. The process is irreversible.

5.15. Most obviously the velocities and momenta of the cannon and cannonball change $(v_{\text{cannon}}, v_{\text{ball}}, p_{\text{cannon}},$ and $p_{\text{ball}}).$ The cannon and the ball both heat up substantially, so T_{cannon} and T_{ball} increase. The powder or fuel used to fire the cannonball is typically turned into a gas, meaning that the chemical potential energy (E_{chemical}) of the system also changes.

5.16. (*a*) The momentum of the wood is less than the initial momentum of the bullet, assuming the block of wood was initially at rest. This is true because the total momentum of the block and imbedded bullet together must be the same as the initial momentum of the bullet. The momentum of the wood is just a part of that. (*b*) The total kinetic energy of the bullet and wood together is less than the initial kinetic energy of the bullet. This can understood by noting that energy leaves the bullet-block system in the form of sound during the collision. Much more energy is converted from kinetic to thermal energy as the block and bullet heat up during the collision.

5.17. (*a*) Here we choose the system consisting of the two blocks only (though other system choices could be equally valid). This system is not isolated; the blocks make contact with the spring outside the system, and this changes the momentum of the system.
(*b*)



(*c*) The system starts out with kinetic energy. During the collision of the two blocks, this kinetic energy is changed into thermal energy. As the warmer blocks strike the spring, they momentarily come to a stop as their kinetic energy is changed to internal energy in the spring. Once the spring returns to its equilibrium length, the blocks regain kinetic energy. (*d*) During the first collision between block 1 and block 2, the initial and final momenta must be equal. Call the initial direction of motion the $+x$ direction, such that $p_{1\,x,\text{i}} = m_1 v.$ We can write the conservation of the x component of the momentum as $m_1 v + 0 = (m_1 + m_2)v_\text{f}.$ It follows immediately that $v_\text{f} = \dfrac{m_1}{(m_1 + m_2)} v = \dfrac{1}{2} v.$ Note that the collision with the spring only reverses the direction of the blocks. The speed of the blocks is the same before striking the spring and after leaving the spring. Hence the final speed of the blocks is $\dfrac{1}{2} v.$

5.18. (*a*) Yes. Consider a piece of paper burning in a very well-insulated chamber.There are no interactions with anything outside the system and the momentum of the system is constant. (*b*) Yes. The example in (*a*) also works as an example here.

5.19. The kinetic energy of the car is being changed into thermal energy in the brakes. The brakes are designed to do this, but heating them up too much can cause them to malfunction.

5.20. Arguments can be made in favor of several answers. If the springs are not perfectly collinear with the momentum of both objects before the collision, there is strong possibility that the springs may entangle, and the objects could stick together, making it totally inelastic. If the springs are lined up in such a way that the objects bounce apart, real springs tend to continue to vibrate slightly in such a situation. If some energy is retained in the springs, then the kinetic energy of the objects has changed and the collision is inelastic. Finally, if springs are tuned just right and made to store all energy elastically and return that energy to the objects it could in principle be an elastic collision.

5.21. They convert the same amount of kinetic energy to internal energy. The two cars have the same initial and final kinetic energies. So the differences (kinetic energy converted to internal energy) must also be the same.

5.22. Much of the kinetic energy involved in the collision can be changed into internal energy through deformations of the car. The collisions become more inelastic, meaning there is less energy to affect people's bodies. The crumple zones also allow the transfer of energy to happen over of longer time.

5.23. In such an elastic collision, the ball simply reverses directions. Hence $\Delta\vec{p} = \vec{p}_\text{f} - \vec{p}_\text{i} = 2mv\,\hat{j}$ where $+\hat{j}$ is vertically upward. Hence the magnitude of the change in momentum is $2mv$. By definition of an elastic collision, $\Delta K = 0$. Yes, the two answers are consistent with each other. Because momentum is a vector it is affected by a change in the direction of a velocity. Kinetic energy is a scalar and only depends on the speed, not the direction of the velocity. Hence a change in the direction (but not the magnitude) of the velocity causes a change in momentum, but not in kinetic energy.

5.24. Since we know the inertia and speed of the moving car just before the collision, we can write its kinetic energy at this time. Since the collision is assumed to be elastic, this is the same as the kinetic energy of the two-car system immediately after the collision. Hence $K_\text{f} = K_\text{i} = \frac{1}{2}mv^2 = \frac{1}{2}(1200\text{ kg})(10\text{ m/s})^2 = 6.0\times10^4\text{ J}$.

5.25. Because the collision is elastic, we need only calculate the initial kinetic energy of the system of two toy cars. We will refer to the 0.020 kg toy car as car A, and the 0.016 kg car as car B. The first step is to calculate the speeds in SI base units.

$$v_\text{A} = (10\times10^2\text{ m/h})\times\frac{1\text{ h}}{3600\text{ s}} = 0.28\text{ m/s}$$

$$v_\text{A} = (20\times10^9\text{ mm/yr})\times\left(\frac{1\text{ yr}}{365\text{ days}}\right)\times\left(\frac{1\text{ day}}{24\text{ h}}\right)\times\left(\frac{1\text{ h}}{3600\text{ s}}\right)\times\frac{1\text{ m}}{10^3\text{ mm}} = 0.63\text{ m/s}$$

The kinetic energy of the system is then given by

$$K = K_\text{A} + K_\text{B} = \frac{1}{2}m_\text{A}v_\text{A}^2 + \frac{1}{2}m_\text{B}v_\text{B}^2 = \frac{1}{2}((0.080\text{ kg})(0.28\text{ m/s})^2 + (0.016\text{ kg})(0.63\text{ m/s}^2)) = 6.3\text{ mJ}$$

5.26. From kinematics, the final velocity of the ring after falling 1.0 m is $v_{y,\text{f}} = \sqrt{2g\Delta y} = \sqrt{2(9.8\text{ m/s}^2)(1.0\text{ m})}$ or 4.4 m/s. Then the kinetic energy is $K = (1/2)mv_\text{f}^2 = (0.5)(0.1\text{ kg})(4.4\text{ m/s})^2 = 1\text{ J}$. This also the energy needed to lift one penny to a height of approximately 4 m.

5.27. (*a*) In an elastic collision between two objects, the relative speed between the two objects is the same before and after the collision. Hence the relative speed between brothers is $v_\text{brother,you f} = v_\text{brother,you i} = \left|\vec{v}_\text{you,i} - \vec{v}_\text{brother,i}\right| = \left|\vec{0} - (-5.0\text{ m/s})\,\hat{\imath}\right| = 5.0\text{ m/s}$ after they collide. (*b*) Because the relative velocity does not change, we can write $v_{\text{you }x,\text{f}} - v_{\text{brother }x,\text{f}} = -(v_{\text{you }x,\text{i}} - v_{\text{brother }x,\text{i}})$ or $v_{\text{you }x,\text{f}} = v_{\text{brother }x,\text{f}} - (v_{\text{you }x,\text{i}} - v_{\text{brother }x,\text{i}}) = (-0.36\text{ m/s})\,\hat{\imath} - (5.0\text{ m/s})\,\hat{\imath} = -5.4\text{ m/s}\,\hat{\imath}$. (*c*) Yes, both answers are consistent with the collision being elastic. (*d*) By definition, the change in kinetic energy in an elastic collision is zero.

5.28. We first use the fact the relative velocity between two objects is unchanged in an elastic collision. Call the direction pointing to the right the $+x$ direction. Then we can write:

$$\vec{v}_{2,\mathrm{f}} - \vec{v}_{1,\mathrm{f}} = -(\vec{v}_{2,\mathrm{i}} - \vec{v}_{1,\mathrm{i}}),$$

or in terms of components:

$$\begin{aligned}
v_{2\,x,\mathrm{f}} &= v_{1\,x,\mathrm{f}} - (v_{2\,x,\mathrm{i}} - v_{1\,x,\mathrm{i}}) \\
v_{2\,x,\mathrm{f}} &= (-5.0\ \mathrm{m/s}) - (0 - 10\ \mathrm{m/s}) \\
v_{2\,x,\mathrm{f}} &= (+5.0\ \mathrm{m/s})
\end{aligned}$$

Hence the final velocity of cart 2 is $(+5.0\ \mathrm{m/s})\,\hat{i}$ or 5.0 m/s to the right.

We can find the inertia of cart 1 by using conservation of momentum:

$$\begin{aligned}
\vec{p}_{\mathrm{i}} &= \vec{p}_{\mathrm{f}} \\
\vec{p}_{1\mathrm{i}} + \vec{p}_{2\mathrm{i}} &= \vec{p}_{1\mathrm{f}} + \vec{p}_{2\mathrm{f}}
\end{aligned}$$

Or in terms of components:

$$\begin{aligned}
&m_1 v_{1\,x,\mathrm{i}} + m_2 v_{2\,x,\mathrm{i}} = m_1 v_{1\,x,\mathrm{f}} + m_2 v_{2\,x,\mathrm{f}} \\
&m_1 (v_{1\,x,\mathrm{i}} - v_{1\,x,\mathrm{f}}) = m_2 (v_{2\,x,\mathrm{f}} - v_{2\,x,\mathrm{i}}) \\
&m_1 = m_2 \frac{(v_{2\,x,\mathrm{f}} - v_{2\,x,\mathrm{i}})}{(v_{1\,x,\mathrm{i}} - v_{1\,x,\mathrm{f}})} = (6\ \mathrm{kg}) \frac{\big((+5.0\ \mathrm{m/s}) - 0\big)}{\big((+10\ \mathrm{m/s}) - (-5.0\ \mathrm{m/s})\big)} \\
&m_1 = 2\ \mathrm{kg}
\end{aligned}$$

5.29. We start by writing down a statement of conservation of energy and conservation of momentum.

$$(1/2) m_1 v_{1x,\mathrm{i}}^2 + (1/2) m_2 v_{2x,\mathrm{i}}^2 = (1/2) m_1 v_{1x,\mathrm{f}}^2 + (1/2) m_2 v_{2x,\mathrm{f}}^2 \tag{1}$$

$$m_1 v_{1x,\mathrm{i}} + m_2 v_{2x,\mathrm{i}} = m_1 v_{1x,\mathrm{f}} + m_2 v_{2x,\mathrm{f}} \tag{2}$$

Inserting $v_{2x,\mathrm{i}} = 0$ and solving eq.(2) for $v_{2x,\mathrm{f}}$ yields

$$v_{2x,\mathrm{f}} = \frac{m_1}{m_2}(v_{1x,\mathrm{i}} - v_{1x,\mathrm{f}}) \tag{3}$$

Inserting eq.(3) into eq.(1) yields

$$\begin{aligned}
m_1 (v_{1x,\mathrm{i}}^2 - v_{1x,\mathrm{f}}^2) &= m_2 \left(\frac{m_1 (v_{1x,\mathrm{i}} - v_{1x,\mathrm{f}})}{m_2} \right)^2 \\
m_1 (v_{1x,\mathrm{i}} + v_{1x,\mathrm{f}}) &= \frac{m_1^2}{m_2}(v_{1x,\mathrm{i}} - v_{1x,\mathrm{f}}) \\
v_{1x,\mathrm{f}} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1x,\mathrm{i}}
\end{aligned}$$

which is the first result. If we plug this result back into eq.(3), we obtain an expression for the final velocity of object 2:

$$\begin{aligned}
v_{2x,\mathrm{f}} &= \frac{m_1}{m_2} \left(v_{1x,\mathrm{i}} - \frac{m_1 - m_2}{m_1 + m_2} v_{1x,\mathrm{i}} \right) \\
v_{2x,\mathrm{f}} &= \frac{m_1}{m_2} \left(\frac{m_1 + m_2 - (m_1 - m_2)}{m_1 + m_2} \right) v_{1x,\mathrm{i}} \\
v_{2x,\mathrm{f}} &= \frac{2 m_1}{m_2 + m_1} v_{1x,\mathrm{i}}
\end{aligned}$$

which is the second result.

If $m_1 \ll m_2$ then object 1 reverses direction upon colliding with object 2, and continues at almost the same speed as before. Object 2 would have very little speed. Consider a ping-pong ball striking a bowling ball. If $m_1 = m_2$ then object 1 would come to a complete stop and object 2 would continue with the same speed as object 1 had initially. An example of this would be two pool balls with the same inertia. The cue ball can stop completely after imparting all of its energy and momentum into another ball. If $m_1 \gg m_2$, then object 1 would continue on with nearly the same speed in the same direction as before the collision. Object 2 would move at roughly twice the speed as object 1 and in the same direction. Picture a bowling ball rolling into a ping-pong ball.

5.30. Since the collision is elastic, we can write:

$$\frac{1}{2}mv_{\text{W,i}}^2 = \frac{1}{2}mv_{\text{W,f}}^2 + \frac{1}{2}mv_{\text{B,f}}^2$$

or

$$v_{\text{W,i}}^2 = v_{\text{W,f}}^2 + v_{\text{B,f}}^2 \tag{1}$$

Here we have implicitly used the fact that he pucks have identical inertias.

We also know that momentum is conserved, so we write:

$$mv_{\text{W}\,x,\text{i}} = mv_{\text{W}\,x,\text{f}} + mv_{\text{B}\,x,\text{f}}$$

or

$$v_{\text{W}\,x,\text{i}} = v_{\text{W}\,x,\text{f}} + v_{\text{B}\,x,\text{f}} \tag{2}$$

Squaring equation (2) yields

$$v_{\text{W}\,x,\text{i}}^2 = v_{\text{W}\,x,\text{f}}^2 + v_{\text{B}\,x,\text{f}}^2 + 2v_{\text{W}\,x,\text{f}}v_{\text{B}\,x,\text{f}} \tag{3}$$

Comparing equations (2) and (3) shows clearly that $v_{\text{W}\,x,\text{f}}v_{\text{B}\,x,\text{f}} = 0,$ meaning that either $v_{\text{W}\,x,\text{f}} = 0$ or $v_{\text{B}\,x,\text{f}} = 0$. Both of these are valid mathematical solutions. The latter solution $(v_{\text{B}\,x,\text{f}} = 0)$ means that the black puck never moves and the white puck simply keeps going (the white puck misses the black puck). This is certainly physically possible, and would leave momentum and kinetic energy conserved. But this is not the setup of the problem that we were given. Here, we want the solution that corresponds to the white puck actually striking the black puck. In that case, we have $v_{\text{W}\,x,\text{f}} = 0$. From this is trivially follows that $\vec{v}_{\text{B,f}} = \vec{v}_{\text{W,i}}$. This means the white puck comes completely to a stop $(\vec{v}_{\text{W,f}} = \vec{0})$ and the black puck starts moving with a velocity $\vec{v}_{\text{B,f}} = \vec{v}_{\text{W,i}}$.

5.31. (*a*) No, this would not violate the law of conservation of momentum. Call the inertia of a single ball m and call the velocity of the plucked ball just prior to the collision $\vec{v}$. Then the momenta of the initial state would be $m\vec{v}$ and the momentum of the final state would be $(2m)(\vec{v}/2) = m\vec{v}$. So momentum is still conserved. (*b*) Yes, this would violate the law of conservation of energy. Using the same variables as in (*a*), the initial kinetic energy would be $\frac{1}{2}mv^2$. The final kinetic energy would be $2\left(\frac{1}{2}m\left(\frac{v}{2}\right)^2\right) = \frac{1}{4}mv^2$. (*c*) Since we want to prove that the number of balls striking the left side is equal to the number of balls rising on the right side, we must not assume that and must therefore make up variables for each number. We hope in the end to show that the two numbers must be equal. We also cannot assume that the speeds of the balls are the same as they fall and as they rise on the other side. We assume only that all balls have equal inertia *m*. Call the number of balls that initially strike the left side N_i and the number of balls that finally rise on the right side N_f. We can write down a statement of conservation of momentum and of energy (cancelling factors of ½ for brevity):

$$N_\text{i}mv_\text{i} = N_\text{f}mv_\text{f} \tag{1}$$

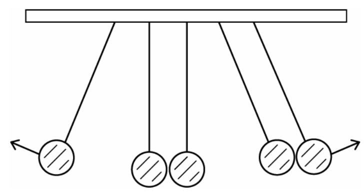
$$N_\text{i}mv_\text{i}^2 = N_\text{f}mv_\text{f}^2 \tag{2}$$

We can divide eq.(2) by eq(1) and immediately obtain $v_i = v_f$. We can therefore refer to the initial and final speeds of the balls by a single variable v. Inserting this into eq.(1) yields

$$N_i mv = N_f mv$$
$$N_i = N_f$$

This shows that the number of balls striking the left side of the array must be equal to the number of balls that rise on the other side.

5.32. It is shown in problem 5.31 that the number of balls moving to the right must be the same before the collision and after the collision. The same argument can be applied to the left moving ball. Finally, the speeds must be identical before and after the collision in order for conservation of energy to be satisfied.



5.33. Because the collision is elastic, we can use both conservation of momentum and conservation of kinetic energy. We can write:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \tag{1}$$

and

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \tag{2}$$

Inserting the given relationship $\vec{v}_{2i} = -0.5\vec{v}_{1i}$ into equation (1), we find

$$\vec{v}_{1f} = \left(1 - \frac{1}{2}\frac{m_2}{m_1}\right)\vec{v}_{1i} - \frac{m_2}{m_1}\vec{v}_{2f} \tag{3}$$

Inserting equation (3) into equation (2) yields

$$\left(m_1 + \frac{1}{4}m_2\right)v_{1i}^2 = m_1\left(\left(1 - \frac{m_2}{2m_1}\right)\vec{v}_{1i} - \frac{m_2}{m_1}\vec{v}_{2f}\right)^2 + m_2v_{2f}^2$$

or

$$-\left(m_2 + \frac{m_2^2}{m_1}\right)v_{2f}^2 + 2m_2\left(1 - \frac{m_2}{2m_1}\right)v_{1i}v_{2f} + \left(m_1 + \frac{1}{4}m_2 - m_1\left(1 - \frac{m_2}{2m_1}\right)^2\right)v_{1i}^2 = 0 \tag{4}$$

Equation (4) is just a quadratic equation in v_{2f}. One can solve using the quadratic equation to find

$$\vec{v}_{2f} = \frac{1}{2}\left(\frac{5m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{1i} \text{ or } \vec{v}_{2f} = \vec{v}_{2i} = -\vec{v}_{1i}/2$$

The latter solution would mean that the objects did not collide and object 2 would just continue on with its initial velocity. We want the former solution. Inserting the expression obtained for $\vec{v}_{2f}$ into equation (3) yields $\vec{v}_{1f} = -\frac{(m_1 - 2m_2)}{m_1 + m_2}\vec{v}_{1i}$.

Hence $\vec{v}_{1f} = -\frac{(m_1 - 2m_2)}{m_1 + m_2}\vec{v}_{1i}$ and $\vec{v}_{2f} = \frac{1}{2}\left(\frac{5m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{1i}$.

5.34. Call the initial speed of cart 2 v, so that the initial speed of cart 1 is $2v$. Call the inertia of cart 1 m and the inertia of cart 2 $2m$. If we call the initial direction of motion of both carts the $+x$ direction, then we can immediately write down a statement of conservation of momentum in the x direction:

$$m(2v) + (2m)v = mv_{1f} + (2m)v_{2f}, \tag{1}$$

and a statement of conservation of kinetic energy:

$$\frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}(2m)v_{2f}^2 \tag{2}$$

Solving equation (1) for v_{1f} yields

$$v_{1f} = 4v - 2v_{2f} \tag{3}$$

Inserting equation (3) into equation (2) and cancelling factors of m gives us

$$-6v_{2f}^2 + 16v - 10v^2 = 0$$

This can be easily solved using the quadratic equation, and we find $v_{2f} = v$ or $v_{2f} = \frac{5}{3}v$. Clearly, the former solution corresponds to object 1 missing object 2 (or it corresponds to a time prior to the collision). We are told that a collision occurs, so we want the second solution. Inserting this into equation (3) yields $v_{1f} = \frac{2}{3}v$.

Hence $v_{1f} = \frac{2}{3}v$ and $v_{2f} = \frac{5}{3}v$.

5.35. Recall that in an elastic collision the relative speed between the two colliding objects is constant. Initially the relative velocity is $\vec{v}_{AB,i} = \vec{v}_{B,i} - \vec{v}_{A,i} = \vec{0} - (2.2 \text{ m/s})\,\hat{\imath}$, where we have assumed the initial direction of motion of cart A to be in the $+x$ direction. After the collision the relative velocity is $\vec{v}_{AB,f} = \vec{v}_{B,f} - \vec{v}_{A,f} = (3.0 \text{ m/s})\,\hat{\imath} - (1.0 \text{ m/s})\,\hat{\imath} = (2.0 \text{ m/s})\,\hat{\imath}$. If the collision were elastic, the initial and final relative velocities would have the same magnitude. Since they do not, the collision is inelastic.

The inertia of cart B is most easily found using conservation of momentum. Writing only the momentum along the x axis, we have

$$m_A v_{A\,x,i} + m_B v_{B\,x,i} = m_A v_{A\,x,f} + m_B v_{B\,x,f}$$

or

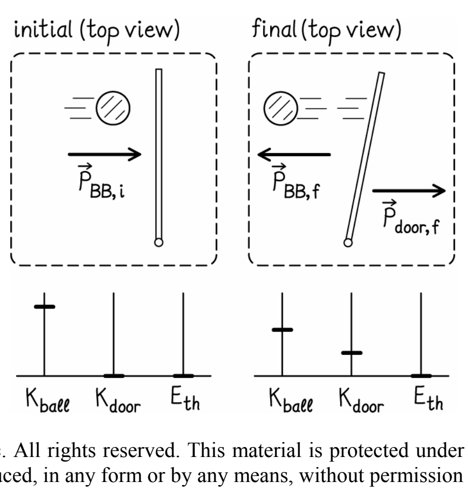
$$m_B = m_A \frac{(v_{A\,x,i} - v_{A\,x,f})}{(v_{B\,x,f} - v_{B\,x,i})} = (0.400 \text{ kg})\frac{(2.2 \text{ m/s})\,\hat{\imath} - (1.0 \text{ m/s})\,\hat{\imath}}{(3.0 \text{ m/s})\,\hat{\imath} - \vec{0}} = 0.16 \text{ kg}$$

5.36. We can solve this problem using conservation of momentum. Call the $+x$ direction the initial direction in which the physics student is driving. Call the student's car car A and the stationary car car B. Then we can write $m_A v_{A\,x,i} + m_B v_{B\,x,i} = m_A v_{A\,x,f} + m_B v_{B\,x,f}$. In this case the two cars are stuck together after the collision and move with one common final velocity. We use this and the fact that car B is initially at rest to write

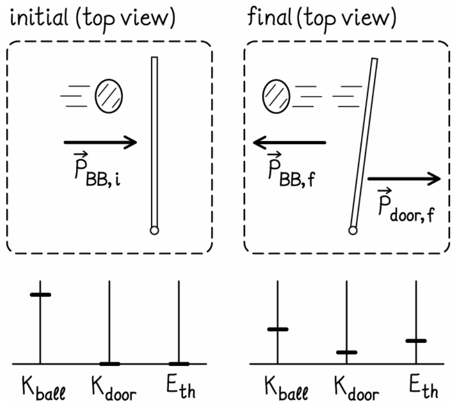
$$v_{A\,x,i} = \frac{(m_A + m_B)}{m_A} v_{x,f} = \frac{(1200 \text{ kg}) + (2000 \text{ kg})}{1200 \text{ kg}}(6.6 \text{ m/s}) = 18 \text{ m/s}$$

5.37.

(*a*)

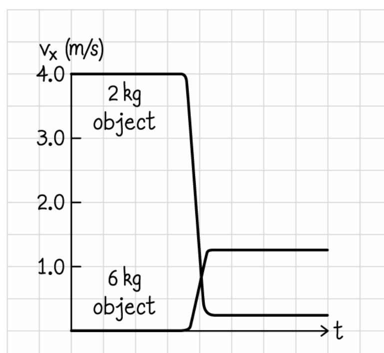


(*b*)



A key feature in these figures is that the door has less final kinetic energy when the ball is partially deflated. Since a partially deflated ball will tend to fall after the collision (as opposed to bouncing backwards) its change in momentum will be smaller than when a fully-inflated ball strikes the door. By momentum conservation, this means that the momentum imparted to the door will be smaller when the ball is partially deflated. Hence the door will have a smaller speed and a smaller kinetic energy when struck by the partially deflated ball.

5.38.



Note that the relative speed decreases by a factor of four.

5.39. First, note that all four cars will come to rest after the crash. This can be trivially seen by applying momentum conservation. This means that all the initial kinetic energy will be converted to internal energy. Hence it is sufficient for us to make a ratio of the initial kinetic energies:

$$\frac{K_{\text{fast}}}{K_{\text{slow}}} = \frac{\frac{1}{2}mv_{\text{fast,i}}^2}{\frac{1}{2}mv_{\text{slow,i}}^2} = \frac{(34\text{ m/s})^2}{(25\text{ m/s})^2} = 1.8$$

The collision between faster cars will convert 1.8 times as much kinetic energy to internal energy as the collision between slower cars.

5.40. Call the initial speed of m_1 simply v_i, and call the final speed of m_1 and m_2 combined v_f. Then conservation of momentum tells us $m_1v_\text{i} = (m_1 + m_2)v_\text{f}$, or $v_\text{f} = m_1v_\text{i}/(m_1 + m_2)$. We can insert this into an expression for the ratio of kinetic energies as follows:

$$\frac{K_\text{f}}{K_\text{i}} = \frac{(1/2)(m_1 + m_2)v_\text{f}^2}{(1/2)m_1v_\text{i}^2} = \frac{(m_1 + m_2)}{m_1v_\text{i}^2}\left(\frac{m_1v_\text{i}}{(m_1 + m_2)}\right)^2$$

$$= \frac{m_1}{(m_1 + m_2)}$$

This the fraction of kinetic energy that remains. What was asked for is the fraction that is converted. If we subtract this remaining fraction from one:

$$\frac{E_{\text{converted}}}{K_{\text{i}}} = 1 - \frac{m_1}{(m_1+m_2)} = \frac{(m_1+m_2)}{(m_1+m_2)} - \frac{m_1}{(m_1+m_2)}$$
$$= \frac{m_2}{(m_1+m_2)}$$

If $m_1 \gg m_2$ then this ratio is nearly one, meaning that almost all kinetic energy remains kinetic and almost none is converted. This makes sense because an object with huge inertia is barely slowed down by a collision with an object of very small inertia. In the case where $m_2 \gg m_1$, almost all the kinetic energy is converted.

5.41. First, write a statement of momentum conservation, relating the initial speed in the $+x$ direction to the final speeds in that direction:

$$mv = mv_{1\text{f}} + mv_{2\text{f}}$$

or

$$v = v_{1\text{f}} + v_{2\text{f}}$$

Now we write the ratio of initial and final kinetic energies and substitute in the above expression for v.

$$\frac{K_{\text{f}}}{K_{\text{i}}} = \frac{(1/2)mv_{1\text{f}}^2 + (1/2)mv_{2\text{f}}^2}{(1/2)mv^2} = \frac{v_{1\text{f}}^2 + v_{2\text{f}}^2}{(v_{1\text{f}} + v_{2\text{f}})^2}$$

To get this expression to be obviously equivalent to the one given, we will need to make a few algebraic steps that may seem contrived at first:

$$\frac{K_{\text{f}}}{K_{\text{i}}} = \frac{v_{1\text{f}}^2 + v_{2\text{f}}^2}{(v_{1\text{f}} + v_{2\text{f}})^2} = \frac{1}{2}\frac{2v_{1\text{f}}^2 + 2v_{2\text{f}}^2}{(v_{1\text{f}} + v_{2\text{f}})^2} = \frac{1}{2}\frac{v_{1\text{f}}^2 + 2v_{1\text{f}}v_{2\text{f}} + v_{2\text{f}}^2 + v_{1\text{f}}^2 - 2v_{1\text{f}}v_{2\text{f}} + v_{2\text{f}}^2}{(v_{1\text{f}} + v_{2\text{f}})^2}$$
$$= \frac{1}{2}\frac{(v_{1\text{f}} + v_{2\text{f}})^2 + (v_{1\text{f}} - v_{2\text{f}})^2}{(v_{1\text{f}} + v_{2\text{f}})^2} = \frac{1}{2}\left(1 + \frac{(v_{1\text{f}} - v_{2\text{f}})^2}{(v_{1\text{f}} + v_{2\text{f}})^2}\right) = \frac{1}{2}\left(1 + \frac{v_{12,\text{f}}{}^2}{v^2}\right) = \frac{1}{2}(1+e^2)$$

Which is the expression we were given.

5.42. (*a*) We solve this using momentum conservation. We call the initial direction of motion of the car the $+x$ direction. We can write

$$m_{\text{c}}v_{\text{c}\,x,\text{i}} + m_{\text{t}}v_{\text{t}\,x,\text{i}} = m_{\text{c}}v_{\text{c}\,x,\text{f}} + m_{\text{t}}v_{\text{t}\,x,\text{f}}$$

The truck is initially at rest, and the two final velocities are the same. This allows us to write

$$v_{x,\text{f}} = \frac{m_{\text{c}}v_{\text{c}\,x,\text{i}}}{(m_{\text{c}} + m_{\text{t}})} = \frac{(1000\text{ kg})(25\text{ m/s})}{(1000\text{ kg} + 2000\text{ kg})} = 8.3\text{ m/s}$$

So the car and truck are moving together at 8.3 m/s after the collision. (*b*) The initial kinetic energy is given by $K_{\text{i}} = K_{\text{c,i}} + K_{\text{t,i}} = \frac{1}{2}m_{\text{c}}v_{\text{c,i}}^2 + 0 = \frac{1}{2}(1000\text{ kg})(25\text{ m/s})^2 = 3.1\times10^5\text{ J}$. (*c*) The final kinetic energy is given by $K_{\text{f}} = K_{\text{c,f}} + K_{\text{t,f}} = \frac{1}{2}m_{\text{c}}v_{\text{c,f}}^2 + \frac{1}{2}m_{\text{t}}v_{\text{t,f}}^2 = \frac{1}{2}(1000\text{ kg} + 2000\text{ kg})(8.33\text{ m/s})^2 = 1.0\times10^5\text{ J}$. (*d*) Because the kinetic energy decreases, we know the collision must be inelastic. Furthermore, we know that when objects stick together, the collision is totally inelastic. (*e*) The coefficient of restitution is the ratio of final to initial relative speeds. Here, the final relative speed is zero because the two objects are stuck together. Hence $e = 0$. This matches our expectation for a totally inelastic collision.

5.43. The collision in which they have the same magnitude of momentum causes the greater conversion of energy from kinetic to internal. When the two objects have the same magnitude of momentum, but have their momenta in opposite directions, then conservation of momentum dictates that the two objects will come to rest after the collision. This means that they have no final kinetic energy. Hence when the momenta are equal and opposite we get 100% conversion of kinetic energy into internal energy.

5.44. (*a*) We can solve this portion using kinematics. We know $v_{x,\mathrm{f}} = v_{x,\mathrm{i}} + a_x t$, so $v_{x,\mathrm{i}} = v_{x,\mathrm{f}} - a_x t = (10\text{ m/s}) - (-4.9\times10^5\text{ m/s}^2)(1.0\times10^{-3}) = 5.0\times10^2\text{ m/s}$. (*b*) Since the bullet is the only thing that is moving, initially, the conservation of momentum in this case can be expressed

$$m_{\text{bullet}} v_{\text{bullet } x,\mathrm{i}} = m_{\text{bullet}} v_{\text{bullet } x,\mathrm{f}} + m_{\text{knot}} v_{\text{knot } x,\mathrm{f}} + m_{\text{block}} v_{\text{block } x,\mathrm{f}}$$

or

$$\begin{aligned} v_{\text{block } x,\mathrm{f}} &= (m_{\text{bullet}} v_{\text{bullet } x,\mathrm{i}} - m_{\text{bullet}} v_{\text{bullet } x,\mathrm{f}} - m_{\text{knot}} v_{\text{knot } x,\mathrm{f}})/m_{\text{block}} \\ &= ((0.0050\text{ kg})(5.0\times10^2\text{ m/s}) - (0.0050\text{ kg})(10\text{ m/s}) - (0.20\text{ kg})(10\text{ m/s}))/(1.8\text{ kg}) \\ &= 0.25\text{ m/s} \end{aligned}$$

Note that the inertia of the block in the above expression is really the inertia of the block with the knot removed. So the final velocity of the block (without the knot) is $(+0.25\text{ m/s})\,\hat{\imath}$. (*c*) Initially the only thing moving is the bullet. So $K_\mathrm{i} = K_{\text{bullet,i}} = \frac{1}{2} m_{\text{bullet}} v_{\text{bullet,i}}^2 = \frac{1}{2}(0.0050\text{ kg})(5.0\times10^2)^2 = 6.3\times10^2\text{ J}$. The final kinetic energy is

$$K_\mathrm{f} = K_{\text{bullet,f}} + K_{\text{knot,f}} + K_{\text{block,f}} = \frac{1}{2} m_{\text{bullet}} v_{\text{bullet,f}}^2 + \frac{1}{2} m_{\text{knot}} v_{\text{knot,f}}^2 + \frac{1}{2} m_{\text{block}} v_{\text{block,f}}^2$$

$$K_\mathrm{f} = \frac{1}{2}(0.0050\text{ kg})(10\text{ m/s})^2 + \frac{1}{2}(0.20\text{ kg})(10\text{ m/s})^2 + \frac{1}{2}(1.8\text{ kg})(0.25\text{ m/s})^2 = 10\text{ J}$$

The kinetic energy does change during the collision; it decreases substantially. (*d*) No, the coefficient of restitution depends on relative velocities between objects. In this problem one of the objects breaks into pieces and the relative velocities between the bullet and those two pieces are different. The collision is inelastic.

5.45. Call the amount of kinetic energy converted to thermal energy due to friction per meter of sliding *f*, such that we can say $K_\mathrm{f} = K_\mathrm{i} - fx$. Call the length of the entire bar ℓ. When the bartender slides the glass all the way to the end of the bar and it comes to a complete stop, we can write

$$K_\mathrm{f} = \frac{1}{2} m v_{\text{end}}^2 - fd = 0$$

Or

$$f = \frac{m v_{\text{end}}^2}{2d}$$

Now in order for the glass to stop 3/4 of the way down the bar, we require

$$K_\mathrm{f} = \frac{1}{2} m v^2 - f\frac{3}{4}d = 0$$

Inserting our expression for *f* and solving for *v* yields $v = \sqrt{\frac{3}{4}}\, v_{\text{end}}$.

5.46. The amount of kinetic energy converted to thermal energy depends linearly of the distance of the skid. Hence 50% of the total kinetic energy has been lost at the mid-point of the skid.

5.47. (*a*) We can solve this using conservation of momentum. We can call the direction of the initial motion of the croquet ball the +*x* direction, and write down the *x* components of the momenta:

$$\begin{aligned} m_\mathrm{c} v_{\mathrm{c}\,x,\mathrm{i}} + m_\mathrm{g} v_{\mathrm{g}\,x,\mathrm{i}} &= m_\mathrm{c} v_{\mathrm{c}\,x,\mathrm{f}} + m_\mathrm{g} v_{\mathrm{g}\,x,\mathrm{f}} \\ v_{\mathrm{g}\,x,\mathrm{f}} &= \frac{m_\mathrm{c}}{m_\mathrm{g}}(v_{\mathrm{c}\,x,\mathrm{i}} - v_{\mathrm{c}\,x,\mathrm{f}}) + v_{\mathrm{g}\,x,\mathrm{i}} \\ v_{\mathrm{g}\,x,\mathrm{f}} &= \frac{(0.25\text{ kg})}{(0.05\text{ kg})}\big((5.0\text{ m/s}) - (4.0\text{ m/s})\big) + 0 \\ v_{\mathrm{g}\,x,\mathrm{f}} &= 5.0\text{ m/s} \end{aligned}$$

So the final speed of the golf ball would be 5.0 m/s. (*b*) When a collision is elastic, the relative speed of the two objects colliding is constant. Here the objects initially have a relative speed of 5.0 m/s, and finally have a relative speed of 1.0 m/s. So, no, the collision cannot be elastic. (*c*) The increase in internal energy will be equal to the decrease in kinetic energy. We find the change in kinetic energy as follows:

$$\begin{aligned}K_\text{f} - K_\text{i} &= \frac{1}{2}m_\text{c}v_\text{c,f}^2 + \frac{1}{2}m_\text{g}v_\text{g,f}^2 - \frac{1}{2}m_\text{c}v_\text{c,i}^2 \\ &= \frac{1}{2}(0.25\text{ kg})(4.0\text{ m/s})^2 + \frac{1}{2}(0.050\text{ kg})(5.0\text{ m/s})^2 - \frac{1}{2}(0.25\text{ kg})(5.0\text{ m/s})^2 \\ &= -0.50\text{ J}\end{aligned}$$

So the internal energy must have increased by 0.50 J.

5.48. (*a*) In order to find the change in internal energy of the system, we need to know how much kinetic energy was converted into internal energy. We therefore need to know the final velocity of the car, so that we can calculate all kinetic energies. We find the final velocity of the car using the conservation of momentum. Let us call the direction in which the truck is driving the $+x$ direction. We can write $\vec{p}_\text{c,i} + \vec{p}_\text{t,i} = \vec{p}_\text{c,f} + \vec{p}_\text{t,f}$, and we can take the x components of these vectors:

$$m_\text{c}v_{\text{c}\,x,\text{i}} + m_\text{t}v_{\text{t}\,x,\text{i}} = m_\text{c}v_{\text{c}\,x,\text{f}} + m_\text{t}v_{\text{t}\,x,\text{f}}$$

$$v_{\text{c}\,x,\text{f}} = v_{\text{c}\,x,\text{i}} + \frac{m_\text{t}}{m_\text{c}}(v_{\text{t}\,x,\text{i}} - v_{\text{t}\,x,\text{f}})$$

$$v_{\text{c}\,x,\text{f}} = (-5.0\text{ m/s}) + \frac{(1800\text{ kg})}{(1200\text{ kg})}(3.0\text{ m/s} - (-1.5\text{ m/s}))$$

$$v_{\text{c}\,x,\text{f}} = +1.75\text{ m/s}$$

Now that we know the final velocity of the car, we can find the change in kinetic energy for the system of the car and the truck.

$$\Delta K = K_\text{f} - K_\text{i} = K_\text{c,f} + K_\text{t,f} - K_\text{c,i} - K_\text{t,i}$$

$$\Delta K = \frac{1}{2}m_\text{c}(v_\text{c,f}^2 - v_\text{c,i}^2) + \frac{1}{2}m_\text{t}(v_\text{t,f}^2 - v_\text{t,i}^2)$$

$$\Delta K = \frac{1}{2}(1200\text{ kg})((1.75\text{ m/s})^2 - (5.0\text{ m/s})^2) + \frac{1}{2}(1800\text{ kg})((1.5\text{ m/s})^2 - (3.0\text{ m/s})^2)$$

$$\Delta K = -19\text{ kJ}$$

If the kinetic energy decreases by 19 kJ, then the internal energy increases by 19 kJ. (*b*) $e = \dfrac{v_\text{ctf}}{v_\text{cti}} = \dfrac{(1.5\text{ m/s}) + (1.75\text{ m/s})}{(5.0\text{ m/s}) + (3.0\text{ m/s})} = 0.41.$

5.49. The officer can calculate how much friction would have been able to slow the cars after the accident. From this the officer can compute the speed of both vehicles after the collision. During the collision, kinetic energy will likely be lost to internal energy of the cars. But momentum will still be conserved in the collision. Using the inertias of the two cars, the officer can calculate the initial velocity of the moving car before the collision. The speed the officer estimates should be very close to your actual speed. But there could be complicating factors. For example, the skid marks could only account for part of the stopping distance. The cars could also have rolled part of the way without leaving skid marks. This would result in the officer estimating a speed that was lower than your actual speed.

5.50. (*a*) Call the direction of the initial motion of the wagon the $+x$ direction. Momentum will be constant during this collision, so we write a statement of the conservation of the x component of the momentum:

$$p_{\text{w}\,x,\text{i}} + p_{\text{p}\,x,\text{i}} = p_{\text{w}\,x,\text{f}} + p_{\text{p}\,x,\text{f}}$$

$$m_\text{w}v_{\text{w}\,x,\text{i}} + m_\text{p}v_{\text{p}\,x,\text{i}} = m_\text{w}v_{\text{w}\,x,\text{f}} + m_\text{p}v_{\text{p}\,x,\text{f}}$$

Initially, you have zero momentum in the x direction, and finally you and the wagon have the same velocity. This allows us to write

$$v_{x,\text{f}} = \frac{m_\text{w}v_{\text{w}\,x,\text{i}}}{(m_\text{w} + m_\text{p})} = \frac{(100\text{ kg})(5.00\text{ m/s})}{(100\text{ kg} + 50.0\text{ kg})} = 3.33\text{ m/s}$$

(b) If we assume that the kinetic energy in the system remains constant, we would write

$$\frac{1}{2}m_w v_{w,i}^2 + \frac{1}{2}m_p v_{p,i}^2 = \frac{1}{2}m_w v_{w,f}^2 + \frac{1}{2}m_p v_{p,f}^2$$

Using the fact that your initial speed is zero and that the final speed of you and the wagon are the same, we can write

$$v_f = \sqrt{\frac{m_w v_{w,i}^2}{(m_w + m_p)}} = \sqrt{\frac{(100\ \text{kg})(5.00\ \text{m/s})^2}{(100\ \text{kg} + 50.0\ \text{kg})}} = 4.08\ \text{m/s}$$

(c) Using conservation of momentum is correct. This is a perfectly inelastic collision, meaning kinetic energy is changed to internal energy. If one could conveniently quantify all internal energy, it would be possible to use conservation of energy. But assuming that just one type of energy (kinetic) is constant is not valid here.

5.50. It seems equally likely that the asteroid would have been moving with the motion of Earth around the sun or against Earth's motion, or anywhere in between. Therefore, let us approximate the initial speed of the asteroid as 3×10^4 m/s (the approximate speed of Earth around the Sun), toward Earth as measured from Earth. We assume further that the impact affected the kinetic energy of Earth very little, such that the vast majority of the initial kinetic energy of the asteroid went into internal energy of the system. In that case, the increase in internal energy is equal to the initial kinetic energy of the asteroid:

$$\begin{aligned}\Delta E_{int} &= K_i = (1/2)m_{ast}v_{ast}^2\\ &= \frac{1}{2}\frac{4\pi r_{ast}^3}{3}\rho_E v_{ast}^2 = \frac{2\pi}{3}(5\times10^3\ \text{m})^3(5500\ \text{kg/m}^3)(3\times10^4\ \text{m/s})^2\\ &= 1.3\times10^{24}\ \text{J}\\ &= 3\times10^8\ \text{megatons}\end{aligned}$$

Answers may vary as estimation is involved.

5.52. Yes. If the coefficient of restitution is greater than one, energy has been changed into kinetic energy. If the coefficient of restitution is less than one, then kinetic energy has been changed to some other form of energy. If the coefficient of restitution is equal to one, then the initial and final kinetic energies are identical.

5.53. The two coefficients of restitution would be each other's multiplicative inverse. The easiest way to see this is by noting that reversing the time order reverses the roles of "initial" and "final" in the definition of the coefficient of restitution, thus inverting it.

5.54. At the peak of the vertical path the momentum of the firework is momentarily zero. Let us call the direction of motion of particle 1 after the explosion the $+x$ direction. We can write down a statement of conservation of momentum for the explosive separation:

$$\begin{aligned}p_{x,i} &= p_{x,f}\\ (m_1 + m_2)(0) &= m_1 v_{1\,x,f} + m_2 v_{2\,x,f}\\ \frac{m_1}{m_2} &= -\frac{v_{2\,x,f}}{v_{1\,x,f}} = \frac{1}{3}\end{aligned}$$

Given the ratio of masses, the ratio of the final kinetic energies is simple:

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}m_1 v_{1,f}^2}{\frac{1}{2}m_2 v_{2,f}^2} = \frac{m_1}{m_2}\left(\frac{v_{1,f}}{v_{2,f}}\right)^2 = \frac{1}{3}(3)^2 = 3$$

5.55. (a) We solve this using conservation of momentum. Call the direction in which the first snowball is thrown (right) the $+x$ direction. We know

$$\begin{aligned}p_{x,i} &= p_{x,f}\\ (0) &= m_{snow}v_{snow\,x,f} + m_{skater}v_{skater\,x,f}\end{aligned}$$

where m_{skater} is the remaining inertia after the first snowball in thrown. Hence

$$v_{\text{skater }x,\text{f}} = -\frac{m_{\text{snow}}}{m_{\text{skater}}} v_{\text{snow }x,\text{f}}$$
$$v_{\text{skater }x,\text{f}} = -\frac{(1.0\text{ kg})}{(51\text{ kg})}(10\text{ m/s})$$
$$v_{\text{skater }x,\text{f}} = -0.20\text{ m/s}$$

So the skater's speed is 0.20 m/s and the direction is to the left. (*b*) Recall that $e = \frac{v_{\text{skater snow f}}}{v_{\text{skater skater i}}}$. Since the initial relative speed between the snowball and the skater was zero, this ratio is undefined. Taking the limit as the initial relative speed goes to zero would make the coefficient of restitution approach infinity. (*c*) Again, using conservation of momentum in the x direction yields

$$p_{x,\text{i}} = p_{x,\text{f}}$$
$$m_{\text{snow}} v_{\text{snow }x,\text{i}} + m_{\text{skater}} v_{\text{skater }x,\text{i}} = m_{\text{snow}} v_{\text{snow }x,\text{f}} + m_{\text{skater}} v_{\text{skater }x,\text{f}}$$

Initially the snow and skater are moving together, so we can write

$$v_{\text{skater }x,\text{f}} = \frac{(m_{\text{snow}} + m_{\text{skater}}) v_{x,\text{i}} - m_{\text{snow}} v_{\text{snow }x,\text{f}}}{m_{\text{skater}}}$$
$$v_{\text{skater }x,\text{f}} = \frac{(1.0\text{ kg} + 50\text{ kg})(-0.20\text{ m/s}) - (1.0\text{ kg})(-20\text{ m/s})}{(50\text{ kg})}$$
$$v_{\text{skater }x,\text{f}} = 0.20\text{ m/s}$$

So the skater is now moving at a speed of 0.20 m/s to the right. (*d*) This coefficient of restitution also goes to infinity for the same reasons outlined in part (*b*). (*e*) The change in kinetic energy is given by

$$K_\text{f} - K_\text{i} = \frac{1}{2} m_{\text{skater}} v^2_{\text{skater,f}} + \frac{1}{2} m_{\text{snow}} v^2_{\text{snow,f}} - \frac{1}{2} m_{\text{skater}} v^2_{\text{skater,i}} - \frac{1}{2} m_{\text{snow}} v^2_{\text{snow,i}}$$
$$K_\text{f} - K_\text{i} = \frac{1}{2}(50\text{ kg})((0.196\text{ m/s})^2 - (0.196\text{ m/s})^2) + \frac{1}{2}(1.0\text{ kg})((20\text{ m/s})^2 - (0.196\text{ m/s})^2)$$
$$K_\text{f} - K_\text{i} = 2.0\times 10^2\text{ J}$$

The kinetic energy of the system increased by 200 J. This excess energy came from chemical energy stored in the skater's body. (*f*) $E_{\text{food}} = (2.0\times 10^2\text{ J}) \times \frac{1\text{ Cal}}{4184\text{ J}} = 0.48\text{ Cal.}$

5.56. Momentum will be constant throughout this explosive separation. Kinetic energy obviously will not be constant, but we are told by exactly how much the kinetic energy can change. More specifically, we can write $K_\text{f} - K_\text{i} = E_{\text{firecracker}}$. We choose the direction of motion of the crate to be the $+x$ direction, and we refer to the inertia of the crate as m_c, and that of the block as m_b. Then we write down the conservation of momentum in the x direction, and the description of the change in kinetic energy:

$$m_\text{c} v_{\text{c }x,\text{i}} + m_\text{b} v_{\text{b }x,\text{i}} = m_\text{c} v_{\text{c }x,\text{f}} + m_\text{b} v_{\text{b }x,\text{f}}$$
$$\frac{1}{2} m_\text{c} v^2_{\text{c f}} + \frac{1}{2} m_\text{b} v^2_{\text{b f}} - \left(\frac{1}{2} m_\text{c} v^2_{\text{c i}} + \frac{1}{2} m_\text{b} v^2_{\text{b i}}\right) = E_{\text{firecracker}}$$

Since the crate and block are initially at rest, these simplify to

$$0 = m_\text{c} v_{\text{c }x,\text{f}} + m_\text{b} v_{\text{b }x,\text{f}} \quad (1)$$
$$\frac{1}{2} m_\text{c} v^2_{\text{c f}} + \frac{1}{2} m_\text{b} v^2_{\text{b f}} = E_{\text{firecracker}} \quad (2)$$

This system of two equations can easily be solved for the inertia of the crate. We can use equation (1) to find

$$v_{\text{b f}} = -\frac{m_\text{c}}{m_\text{b}} v_{\text{c f}}$$

and insert this into equation (2) to find

$$\frac{1}{2}m_c v_{c\,f}^2 + \frac{1}{2}m_b\left(-\frac{m_c}{m_b}v_{c\,f}\right)^2 = E_{\text{firecracker}}$$

$$\frac{m_c^2}{m_b} + m_c - \frac{2E_{\text{firecracker}}}{v_{c\,f}^2} = 0$$

This equation is quadratic in m_c and be solved to find

$$m_c = \frac{-m_b v_{c\,f}^2 + \sqrt{m_b v_{c\,f}^2}\sqrt{8E_{\text{firecracker}} + m_b v_{c\,f}^2}}{2v_{c\,f}^2}$$

$$m_c = \frac{-(0.60\text{ kg})(0.055\text{ m/s})^2 + \sqrt{(0.60\text{ kg})(0.055\text{ m/s})^2}\sqrt{8(9.0\text{ J}) + (0.60\text{ kg})(0.055\text{ m/s})^2}}{2(0.055\text{ m/s})^2}$$

$$m_c = 59\text{ kg}$$

5.57. (*a*) We can solve this part using conservation of momentum. If we call the direction of motion of the rocket the $+x$ axis, we can write down the x components of the initial and final momenta:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Of course, initially the two stages are moving together. We solve for the final speed of the second stage:

$$v_{2f} = \frac{(m_1 + m_2)v_i - m_1 v_{1f}}{m_2}$$

$$v_{2f} = \frac{(4500\text{ kg})(4000\text{ m/s}) - (3000\text{ kg})(2500\text{ m/s})}{1500\text{ kg}}$$

$$v_{2f} = 7000\text{ m/s}$$

So the second stage moves at a speed of 7000 m/s in the $+x$ direction (the same direction as the initial velocity). (*b*) We calculate the final kinetic energy and the initial kinetic energy. The difference must have been supplied by the explosion. Hence

$$E_{\text{exp}} = K_f - K_i$$

$$E_{\text{exp}} = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2 - \left(\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2\right)$$

$$E_{\text{exp}} = \frac{1}{2}(3000\text{ kg})(2500\text{ m/s})^2 + \frac{1}{2}(1500\text{ kg})(7000\text{ m/s})^2 - \frac{1}{2}(3000\text{ kg} + 1500\text{ kg})(4000\text{ m/s})^2$$

$$E_{\text{exp}} = 1.013\times 10^{10}\text{ J}$$

5.58. Clearly momentum will be constant throughout this explosive separation, and clearly kinetic energy will not be constant. But we know the difference in kinetic energies comes from the energy released from the spring. Hence we can write:

$$\vec{p}_i = \vec{p}_f$$

$$K_f - K_i = E_{\text{spring}}$$

Let cart A be the 4.0 kg cart, and let cart B be the 1.0 kg cart. Let us take the axis of the low-friction track to be the x axis. Then working only with the x components of momentum, we can write:

$$0 = m_A v_{A\,x,\,f} + m_B v_{B\,x,\,f}$$

or

$$v_{A\,x,\,f} = -\frac{m_B}{m_A}v_{B\,x,\,f} \tag{1}$$

We now write out the kinetic energies explicitly:

$$\frac{1}{2}m_A v_{A\,x,\,f}^2 + \frac{1}{2}m_B v_{B\,x,\,f}^2 - 0 = E_{\text{spring}} \tag{2}$$

Inserting equation (1) into equation (2) yields

$$\frac{1}{2}m_{\text{A}}\left(\frac{m_{\text{B}}}{m_{\text{A}}}v_{\text{B}\,x,\,\text{f}}\right)^2+\frac{1}{2}m_{\text{B}}v_{\text{B}\,x,\,\text{f}}^2=E_{\text{spring}}$$

or

$$v_{\text{B}\,x,\,\text{f}}=\sqrt{\frac{2E_{\text{spring}}}{\left(\frac{m_{\text{B}}^2}{m_{\text{A}}}+m_{\text{B}}\right)}}$$

$$v_{\text{B}\,x,\,\text{f}}=\sqrt{\frac{2\left(1.0\times10^3\text{ J}\right)}{\left(\frac{(1.0\text{ kg})^2}{4.0\text{ kg}}+1.0\text{ kg}\right)}}$$

$$v_{\text{B}\,x,\,\text{f}}=40\text{ m/s}$$

Inserting this value into equation (1) yields

$$v_{\text{A}\,x,\,\text{f}}=-\frac{1.0\text{ kg}}{4.0\text{ kg}}(40\text{ m/s})=10\text{ m/s}$$

The speed of the 4.0-kg cart is 10 m/s, and the speed of the 1.0 kg cart is 40 m/s.

5.59. Momentum is constant in this separation, and all motion is along a line, which we call the x axis. We can set the initial and final x components of momentum equal as follows:

$$mv_{\text{R}\,x,\text{f}}+9mv_{\text{B}\,x,\text{f}}=10mv_{x,\text{i}} \tag{1}$$

And we are also told that the relative final speed of the two objects is 100 m/s. In other words

$$v_{\text{R}\,x,\text{f}}-v_{\text{B}\,x,\text{f}}=100\text{ m/s} \tag{2}$$

Solving equation (2) for the final speed of the rocket and inserting this into equation (1) yields

$$m(v_{\text{B}\,x,\text{f}}+100\text{ m/s})+9mv_{\text{B}\,x,\text{f}}=10mv_{x,\text{i}}$$

or

$$v_{\text{B}\,x,\text{f}}=790\text{ m/s}$$

From this, it follows immediately from equation (2) that $v_{\text{R}\,x,\text{f}}=890$ m/s.

The final velocity of the shuttle is $(890\text{ m/s})\,\hat{i}$ and the velocity of the booster is $(790\text{ m/s})\,\hat{i}$.

5.60. In the separation of the nucleus, momentum is still constant. We can calculate the final speed of the α particle by equating the initial and final x components of the momentum:

$$0=m_{\text{th}}v_{\text{th}\,x,\text{f}}+m_{\alpha}v_{\alpha\,x,\text{f}}$$

or

$$v_{\alpha\,x,\text{f}}=\frac{m_{\text{th}}}{m_{\alpha}}v_{\text{th}\,x,\text{f}}$$

Given the final speeds of both particles, we can calculate the final kinetic energy.

$$K_{\mathrm{f}} = \frac{1}{2}m_{\mathrm{th}}v_{\mathrm{th}\,x,\mathrm{f}}^2 + \frac{1}{2}m_{\alpha}v_{\alpha\,x,\mathrm{f}}^2$$

$$K_{\mathrm{f}} = \frac{1}{2}m_{\mathrm{th}}v_{\mathrm{th}\,x,\mathrm{f}}^2 + \frac{1}{2}m_{\alpha}\left(\frac{m_{\mathrm{th}}}{m_{\alpha}}v_{\mathrm{th}\,x,\mathrm{f}}\right)^2$$

$$K_{\mathrm{f}} = \frac{1}{2}m_{\mathrm{amu}}\left((234)v_{\mathrm{th}\,x,\mathrm{f}}^2 + \frac{(234)^2}{4}v_{\mathrm{th}\,x,\mathrm{f}}^2\right)$$

$$K_{\mathrm{f}} = \frac{1}{2}(1.66\times10^{-27})\left((234) + \frac{(234)^2}{4}\right)(2.5\times10^5\ \mathrm{m/s})^2$$

$$K_{\mathrm{f}} = 7.22\times10^{-13}\ \mathrm{J}$$

$$= 7.2\times10^{-13}\ \mathrm{J}$$

Because the uranium atom was initially at rest, all of this kinetic energy came from the internal energy of the atom. Therefore the energy released in the breaking of the atom is also 7.22×10^{-13} J.

5.61. We could use conservation of momentum if we knew the velocity of the bullet just after the gun and bullet separate. We can find that initial velocity of the bullet using kinematics. But first we note that we need to consider three different times: before the gun is fired (time 0), just after the bullet leaves the gun (time 1) and when the bullet strikes the target (time 2). Calling the initial direction of motion of the bullet the $+x$ direction, we can write $v_{\mathrm{b}\,x,2}^2 = v_{\mathrm{b}\,x,1}^2 + 2a_{\mathrm{b}\,x}\Delta x$ or $v_{\mathrm{b}\,x,1} = \sqrt{v_{\mathrm{b}\,x,2}^2 - 2a_{\mathrm{b}\,x}\Delta x} = \sqrt{(299\ \mathrm{m/s})^2 - 2(-1.0\ \mathrm{m/s^2})(1000\ \mathrm{m})} = 302\ \mathrm{m/s}.$ Now we can set the initial and final x components of momentum equal: $0 = m_{\mathrm{g}}v_{\mathrm{g}\,x,1} + m_{\mathrm{b}}v_{\mathrm{b}\,x,1}$. Equivalently $v_{\mathrm{g}\,x,1} = -\frac{m_{\mathrm{b}}}{m_{\mathrm{g}}}v_{\mathrm{b}\,x,1} = -\left(\frac{0.010\ \mathrm{kg}}{5.0\ \mathrm{kg}}\right)(302\ \mathrm{m/s}) = -0.60\ \mathrm{m/s}.$

So the recoil velocity of the gun is $-0.60\ \mathrm{m/s}\ \hat{\imath}$.

5.62. We solve this problem using a combination of the conservation of momentum, and the known change in kinetic energy. Let us call the initial direction of motion the $+x$ direction. We can write

$$m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{i}} + m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{i}} = m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{f}} + m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{f}} \tag{1}$$

$$\frac{1}{2}m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{i}}^2 + \frac{1}{2}m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{i}}^2 + E_{\mathrm{spring}} = \frac{1}{2}m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{f}}^2 + \frac{1}{2}m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{f}}^2 \tag{2}$$

We know the initial velocities of the two objects are the same. We can solve equation (1) for the x component of the final velocity of cart A:

$$v_{\mathrm{A}\,x,\mathrm{f}} = \frac{(m_{\mathrm{A}} + m_{\mathrm{B}})v_{x,\mathrm{i}} - m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{f}}}{m_{\mathrm{A}}} \tag{3}$$

We can insert equation (3) into equation (2) to find

$$\frac{1}{2}\left(m_{\mathrm{A}} + m_{\mathrm{B}}\right)v_{x,\mathrm{i}}^2 + E_{\mathrm{spring}} = \frac{1}{2}m_{\mathrm{A}}\left(\frac{\left(m_{\mathrm{A}} + m_{\mathrm{B}}\right)v_{x,\mathrm{i}} - m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{f}}}{m_{\mathrm{A}}}\right)^2 + \frac{1}{2}m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{f}}^2$$

or

$$\left(m_{\mathrm{A}} + m_{\mathrm{B}} - \frac{(m_{\mathrm{A}} + m_{\mathrm{B}})^2}{m_{\mathrm{A}}}\right)v_{x,\mathrm{i}}^2 + 2E_{\mathrm{spring}} + 2\frac{m_{\mathrm{B}}}{m_{\mathrm{A}}}(m_{\mathrm{A}} + m_{\mathrm{B}})v_{x,\mathrm{i}}v_{\mathrm{B}\,x,\mathrm{f}} - \left(m_{\mathrm{B}} + \frac{m_{\mathrm{B}}^2}{m_{\mathrm{A}}}\right)v_{\mathrm{B}\,x,\mathrm{f}}^2 = 0$$

or

$$2E_{\mathrm{spring}} - \left(m_{\mathrm{B}} + \frac{m_{\mathrm{B}}^2}{m_{\mathrm{A}}}\right)(v_{x,\mathrm{i}} - v_{\mathrm{B}\,x,\mathrm{f}})^2 = 0 \tag{4}$$

This yields $v_{\mathrm{B}\,x,\mathrm{f}} = v_{x,\mathrm{i}} - \sqrt{\dfrac{2m_{\mathrm{A}}E_{\mathrm{spring}}}{m_{\mathrm{B}}(m_{\mathrm{A}} + m_{\mathrm{B}})}}$

Inserting this result into equation (3) yields

$$v_{\mathrm{A}\,x,\mathrm{f}} = v_{x,\mathrm{i}} + \sqrt{\frac{2m_{\mathrm{B}}E_{\mathrm{spring}}}{m_{\mathrm{A}}(m_{\mathrm{A}} + m_{\mathrm{B}})}}$$

5.63. Everything about this problem is exactly the same as problem 5.62, except that only a fraction of the spring's energy is converted to kinetic energy.

$$m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{i}} + m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{i}} = m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{f}} + m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{f}} \tag{1}$$

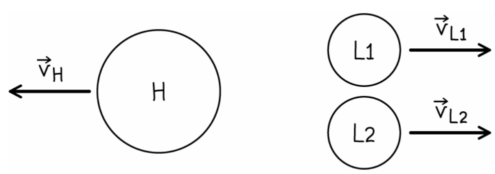
$$\frac{1}{2}m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{i}}^2 + \frac{1}{2}m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{i}}^2 + \frac{3E_{\mathrm{spring}}}{4} = \frac{1}{2}m_{\mathrm{A}}v_{\mathrm{A}\,x,\mathrm{f}}^2 + \frac{1}{2}m_{\mathrm{B}}v_{\mathrm{B}\,x,\mathrm{f}}^2 \tag{2}$$

We combine equations (1) and (2) exactly as in problem 5.62, and solve for the final speeds. The final speed of cart A is

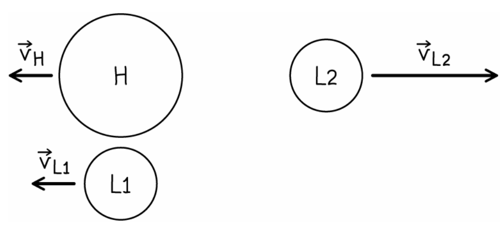
$$v_{\mathrm{A}\,x,\mathrm{f}} = v_{x,\mathrm{i}} + \sqrt{\frac{3m_{\mathrm{B}}E_{\mathrm{spring}}}{2m_{\mathrm{A}}(m_{\mathrm{B}} + m_{\mathrm{A}})}}$$

5.64. The initial momentum of the system is zero. If the tank is filled only with air, then only air can be ejected when the pressure is released. The air may move quickly, but the inertia of the air is so small that is will only carry a small amount of downward momentum as it is ejected. By the conservation of momentum, this means that the rocket will only have a small upward momentum. If water is added to the tank, then the water can be ejected at a high speed carrying a relatively large amount of downward momentum. This means that the rocket must have a large upward momentum.

5.65. (*a*)



(*b*)



5.66. We calculate each kinetic energy and compare. The kinetic energy of the baseball is $K_{\mathrm{ball}} = \frac{1}{2}m_{\mathrm{ball}}v_{\mathrm{ball}}^2 = \frac{1}{2}(0.14\text{ kg})(45\text{ m/s})^2 = 1.4\times10^2$ J. The kinetic energy of the bullet is $K_{\mathrm{bullet}} = \frac{1}{2}m_{\mathrm{bullet}}v_{\mathrm{bullet}}^2 =$ $\frac{1}{2}(0.012\text{ kg})(480\text{ m/s})^2 = 1.4\times10^3$ J. The bullet has more kinetic energy.

5.67. If the bat-ball collision is elastic, then the relative speeds between the bat and ball must be the same before and after the collision. Thus, if the ball is approaching the bat at a high speed, the ball must leave the bat at the twice the speed of the bat plus this high speed, so it is easier to hit a home run. If the initial speed of the ball is zero, the fastest the ball can leave the bat is at twice the bat speed.

5.68. Suggest that he knock it back elastically with his fists. The total momentum of the system consisting of your friend and the ball will be constant as the ball collides with your friend. If the change in momentum of the ball is very large, then the change in momentum of your friend will be very large, and your friend is more likely to fall backwards in that case. If the medicine ball is thrown back at the same speed with which it approached, its change in momentum will be twice as large as if the ball were merely caught.

5.69. We know that momentum will be constant in this collision. Considering only the momentum along the direction of the track (x axis), we can write $mv_\mathrm{i} = 2mv_\mathrm{f}$ or $v_\mathrm{f} = v_\mathrm{i}/2$. Now we can use this expression to write down the change in kinetic energy: $\Delta K = K_\mathrm{f} - K_\mathrm{i} = \frac{1}{2}(2m)\left(\frac{v_\mathrm{i}}{2}\right)^2 - \frac{1}{2}mv_\mathrm{i}^2 = -\frac{1}{4}mv_\mathrm{i}^2$. Since this change in kinetic energy is the energy that is absorbed by the coupling in the collision, we can write $\frac{1}{4}mv_\mathrm{i}^2 \le 1.0\times 10^4$ J. Solving for the initial speed, we find $v_\mathrm{i} \le \sqrt{\frac{4(1.0\times 10^4 \text{ J})}{2.0\times 10^4 \text{ kg}}} = 1.4$ m/s.

5.70. (*a*) The kinetic energy of sphere 1 is given by $K_1 = \frac{1}{2}mv_1^2$, such that $v_1 = \sqrt{\frac{2K_1}{m}} = \sqrt{\frac{2(0.098 \text{ J})}{(0.050 \text{ kg})}} = 2.0$ m/s. Since we are explicitly told that the sphere is moving to the right, we choose the $+x$ direction to point to the right. Then the initial velocity of sphere 1 is $(2.0 \text{ m/s})\,\hat{\imath}$. (*b*) Since sphere 1 is the only part of the system that is moving, the total kinetic energy is just 0.098 J. (*c*) Momentum is always conserved and will be constant in this collision. We are also told that the collision is elastic, so the kinetic energy will be constant. We start by writing down the initial and final x components of momentum, and the initial and final energies.

$$m_1 v_{1\,x,\mathrm{i}} = m_1 v_{1\,x,\mathrm{f}} + m_2 v_{2\,x,\mathrm{f}} \tag{1}$$

$$\frac{1}{2}m_1 v_{1\,x,\mathrm{i}}^2 = \frac{1}{2}m_1 v_{1\,x,\mathrm{f}}^2 + \frac{1}{2}m_2 v_{2\,x,\mathrm{f}}^2 \tag{2}$$

We solve equation (1) for the final x component of sphere 2:

$$v_{2\,x,\mathrm{f}} = \frac{m_1}{m_2}(v_{1\,x,\mathrm{i}} - v_{1\,x,\mathrm{f}}) \tag{3}$$

and we insert equation (3) into equation (4). This yields an equation that is quadratic in $v_{1\,x,\mathrm{f}}$:

$$m_1 v_{1\,x,\mathrm{i}}^2 = m_1 v_{1\,x,\mathrm{f}}^2 + m_2\left(\frac{m_1}{m_2}(v_{1\,x,\mathrm{i}} - v_{1\,x,\mathrm{f}})\right)^2$$

Solving yields

$$v_{1\,x,\mathrm{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1\,x,\mathrm{i}} = \frac{(0.050 \text{ kg}) - (0.10 \text{ kg})}{(0.050 \text{ kg}) + (0.10 \text{ kg})}(2.0 \text{ m/s}) = -0.66 \text{ m/s}$$

Inserting this value of $v_{1\,x,\mathrm{f}}$ into equation (3) yields

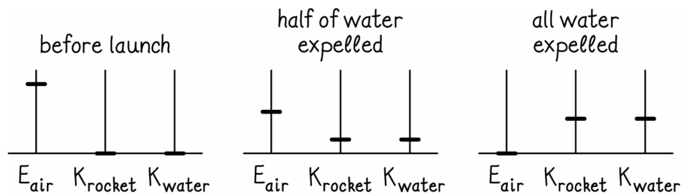
$$v_{2\,x,\mathrm{f}} = \frac{0.050 \text{ kg}}{0.10 \text{ kg}}\big((2.0 \text{ m/s}) - (-0.66 \text{ m/s})\big) = 1.3 \text{ m/s}$$

Hence $\vec{v}_{1\mathrm{f}} = 0.66$ m/s to the left, and $\vec{v}_{2\mathrm{f}} = 1.3$ m/s to the right. (*d*) We calculate the final kinetic energy and compare. It may be necessary to keep more significant digits than were reported for the final answers to part (*c*). $K_\mathrm{f} = \frac{1}{2}m_1 v_{1\mathrm{f}}^2 + \frac{1}{2}m_2 v_{2\mathrm{f}}^2 = \frac{1}{2}(0.050 \text{ kg})(-0.66 \text{ m/s})^2 + \frac{1}{2}(0.10 \text{ kg})(1.32 \text{ m/s})^2 = 0.098$ J. Yes, the final kinetic energy is still 0.098 J. The collision was elastic, meaning kinetic energy was constant. (*e*) $e = -\frac{v_{12\mathrm{f}}}{v_{12\mathrm{i}}} = -\frac{(1.32 \text{ m/s}) - (-0.66 \text{ m/s})}{(0) - (2.0 \text{ m/s})} = 1.0$ as expected for an elastic collision.

5.71. (*a*) The kinetic energy of sphere 1 is given by $K_1 = \frac{1}{2}mv_1^2$, such that $v_1 = \sqrt{\frac{2K_1}{m}} = \sqrt{\frac{2(0.098 \text{ J})}{(0.050 \text{ kg})}} = 2.0$ m/s. Since we are explicitly told that the sphere is moving to the right, we choose the $+x$ direction to point to the right. Then the initial velocity of sphere 1 is $(2.0 \text{ m/s})\,\hat{\imath}$. (*b*) Since sphere 1 is the only part of the system that is moving, the total kinetic energy is just 0.098 J. (*c*) Conservation of momentum along the x axis yields $m_1 v_{1\,x,\mathrm{i}} = (m_1 + m_2)v_{x,\mathrm{f}}$

or $v_{x,\mathrm{f}} = \dfrac{m_1 v_{1\,x,\mathrm{i}}}{(m_1 + m_2)} = \dfrac{(0.050\text{ kg})(2.0\text{ m/s})}{(0.15\text{ kg})} = 0.66\text{ m/s}.$ So, both spheres move at 0.66 m/s to the right. (*d*) $K_\mathrm{f} = \frac{1}{2}(m_1 + m_2)v_{x,\mathrm{f}}^2 = \frac{1}{2}(0.15\text{ kg})(0.66\text{ m/s})^2 = 0.033\text{ J},$ which is much less than the original kinetic energy from part (*b*). The collision was perfectly inelastic, so considerable kinetic energy was changed to internal energy of the system of two spheres. (*e*) $e = 0,$ as is always the case in a perfectly inelastic collision.

5.72. (*a*)



(*b*) We know the speed of the water moving through the opening. The volume of water passing out of the bottle each second is $V/s = \Delta x A/s = vA.$ Since the density of water is $\rho = \dfrac{m}{V} = 1000\text{ kg/m}^3,$ we can find the inertia leaving the bottle in one second. $m = \rho V = \rho v \pi r^2 t =$ $(1000\text{ kg/m}^3)(15.0\text{ m/s})\pi(5.0\times10^{-3}\text{ m})^2(1.0\text{ s}) = 1.2\text{ kg}.$ (*c*) No. The inertia of the rocket decreases as more water is expelled. The acceleration increases, assuming that the ejection speed of the water is constant.

5.73.

Let us call the proportionality constant between the kinetic energy converted and distance $\alpha,$ such that $K = K_\mathrm{i} - \alpha d.$ Clearly, at time $t = 0$ the kinetic energy must be $K_\mathrm{i},$ and after some total time the kinetic energy must drop to zero. The figure shows a curve that is concave up (as opposed to linear or some other shape). To motivate this, let us take the time derivative of both sides of the above equation describing the kinetic energy to see how the slope of this graph should look:

$$K = K_\mathrm{i} - \alpha d$$

$$\frac{\partial K}{\partial t} = \frac{\partial K_\mathrm{i}}{\partial t} - \alpha \frac{\partial d}{\partial t}$$

$$\frac{\partial K}{\partial t} = -\alpha v$$

We see that the slope of the curve should be negative and proportional to the speed $v.$ Since the speed is decreasing the entire time, this means that the slope should be steadily decreasing as the figure shows.

5.74. We can equate the initial and final x components of the momentum (treating $+x$ as the initial direction of the bowling ball's motion), and we can also equate the initial and final kinetic energies. We have

$$m_\mathrm{b} v_{\mathrm{b}\,x,\mathrm{i}} = m_\mathrm{b} v_{\mathrm{b}\,x,\mathrm{f}} + m_\mathrm{p} v_{\mathrm{p}\,x,\mathrm{f}} \tag{1}$$

$$\frac{1}{2}m_{\text{b}}v_{\text{b}\,x,\text{i}}^2 = \frac{1}{2}m_{\text{b}}v_{\text{b}\,x,\text{f}}^2 + \frac{1}{2}m_{\text{p}}v_{\text{p}\,x,\text{f}}^2 \quad (2)$$

We are also told that the bowling ball loses 40% of its speed during the collision. This means

$$v_{\text{b}\,x,\text{f}} = 0.60v_{\text{b}\,x,\text{i}} \quad (3)$$

We can insert equation (3) into equation (1) to find:

$$v_{\text{p}\,x,\text{f}} = (0.4)\left(\frac{m_{\text{b}}}{m_{\text{p}}}v_{\text{b}\,x,\text{i}}\right) \quad (4)$$

We now insert equation (4) and equation (3) into equation (2) to find:

$$m_{\text{p}} = \frac{(0.40)^2 m_{\text{b}}}{(1-(0.60)^2)} = \frac{(0.40)^2(6.5\text{ kg})}{(1-(0.60)^2)} = 1.6\text{ kg}$$

The inertia of one pin is 1.6 kg.

5.75. The two terms that we add together are $\frac{1}{2}(2m)\left[\frac{v_{\text{A}}+v_{\text{B}}}{2}\right]^2$ and $\frac{1}{2}(2m)\left[\frac{v_{\text{A}}-v_{\text{B}}}{2}\right]^2$. The first is like the kinetic energy of a car with twice the inertia of car A or car B, and moving at their average speed, $\frac{v_{\text{A}}+v_{\text{B}}}{2}$. The second term is like the kinetic energy of a car with twice the inertia of car A or car B, and moving at a speed equal to the relative speed between cars A and B, $\frac{v_{\text{A}}-v_{\text{B}}}{2}$.

$$\begin{aligned}K_{\text{sum}} + K_{\text{diff}} &= \frac{1}{2}(2m)\left[\frac{v_{\text{A}}+v_{\text{B}}}{2}\right]^2 + \frac{1}{2}(2m)\left[\frac{v_{\text{A}}-v_{\text{B}}}{2}\right]^2 \\ &= \frac{1}{2}(2m)\left[\frac{v_{\text{A}}^2+2v_{\text{A}}v_{\text{B}}+v_{\text{B}}^2}{4}\right] + \frac{1}{2}(2m)\left[\frac{v_{\text{A}}^2-2v_{\text{A}}v_{\text{B}}+v_{\text{B}}^2}{4}\right] \\ &= \frac{1}{2}(2m)\frac{2v_{\text{A}}^2}{4} + \frac{1}{2}(2m)\frac{2v_{\text{B}}^2}{4} \\ &= \frac{1}{2}mv_{\text{A}}^2 + \frac{1}{2}mv_{\text{B}}^2 = K_{\text{A}} + K_{\text{B}}\end{aligned}$$

5.76. (*a*) As usual, we equate initial and final x components of the momenta, and we relate the initial and final kinetic energies. In this case, we have:

$$m_{\text{c}}v_{\text{c}\,x,\text{i}} + m_{\text{t}}v_{\text{t}\,x,\text{i}} = m_{\text{c}}v_{\text{c}\,x,\text{f}} + m_{\text{t}}v_{\text{t}\,x,\text{f}} \quad (1)$$

$$\left(\frac{1}{2}m_{\text{c}}v_{\text{c}\,x,\text{i}}^2 + \frac{1}{2}m_{\text{t}}v_{\text{t}\,x,\text{i}}^2\right)(0.90) = \frac{1}{2}m_{\text{c}}v_{\text{c}\,x,\text{f}}^2 + \frac{1}{2}m_{\text{t}}v_{\text{t}\,x,\text{f}}^2 \quad (2)$$

The factor of 0.90 arises because 10% of the initial kinetic is converted to internal energy. We solve equation (1) for the final x component of the truck's velocity, and find

$$v_{\text{t}\,x,\text{f}} = \frac{m_{\text{c}}}{m_{\text{t}}}(v_{\text{c}\,x,\text{i}} - v_{\text{c}\,x,\text{f}}) + v_{\text{t}\,x,\text{i}} \quad (3)$$

which we then insert into equation (2).

$$(m_{\text{c}}v_{\text{c}\,x,\text{i}}^2 + m_{\text{t}}v_{\text{t}\,x,\text{i}}^2)(0.90) = m_{\text{c}}v_{\text{c}\,x,\text{f}}^2 + m_{\text{t}}\left(\frac{m_{\text{c}}}{m_{\text{t}}}(v_{\text{c}\,x,\text{i}} - v_{\text{c}\,x,\text{f}}) + v_{\text{t}\,x,\text{i}}\right)^2$$

The quadratic equation can be used to solve for $v_{\text{c}\,x,\text{f}}$. We find that there are two solutions:

$$\begin{aligned}v_{\text{c}\,x,\text{f}} = \frac{1}{2(m_{\text{c}}^2 + m_{\text{c}}m_{\text{t}})}\Big(&2m_{\text{c}}^2 v_{\text{c}\,x,\text{i}} + 2m_{\text{c}}m_{\text{t}}v_{\text{t}\,x,\text{i}} \\ &-\sqrt{(2m_{\text{c}}^2v_{\text{c}\,x,\text{i}} + 2m_{\text{c}}m_{\text{t}}v_{\text{t}\,x,\text{i}})^2 - 4(m_{\text{c}}^2 + m_{\text{c}}m_{\text{t}})(m_{\text{c}}^2v_{\text{c}\,x,\text{i}}^2 - (0.90)m_{\text{c}}m_{\text{t}}v_{\text{c}\,x,\text{i}}^2 + 2m_{\text{c}}m_{\text{t}}v_{\text{c}\,x,\text{i}}v_{\text{t}\,x,\text{i}} + m_{\text{t}}^2v_{\text{t}\,x,\text{i}}^2 - (0.90)m_{\text{t}}^2v_{\text{t}\,x,\text{i}}^2)}\Big)\end{aligned}$$

and

$$v_{\mathrm{c}\,x,\mathrm{f}} = \frac{1}{2(m_\mathrm{c}^2 + m_\mathrm{c} m_\mathrm{t})}\Big(2m_\mathrm{c}^2 v_{\mathrm{c}\,x,\mathrm{i}} + 2m_\mathrm{c} m_\mathrm{t} v_{\mathrm{t}\,x,\mathrm{i}}$$
$$+\sqrt{(2m_\mathrm{c}^2 v_{\mathrm{c}\,x,\mathrm{i}} + 2m_\mathrm{c} m_\mathrm{t} v_{\mathrm{t}\,x,\mathrm{i}})^2 - 4(m_\mathrm{c}^2 + m_\mathrm{c} m_\mathrm{t})(m_\mathrm{c}^2 v_{\mathrm{c}\,x,\mathrm{i}}^2 - (0.90) m_\mathrm{c} m_\mathrm{t} v_{\mathrm{c}\,x,\mathrm{i}}^2 + 2m_\mathrm{c} m_\mathrm{t} v_{\mathrm{c}\,x,\mathrm{i}} v_{\mathrm{t}\,x,\mathrm{i}} + m_\mathrm{t}^2 v_{\mathrm{t}\,x,\mathrm{i}}^2 - (0.90) m_\mathrm{t}^2 v_{\mathrm{t}\,x,\mathrm{i}}^2)}\Big)$$

Numerically, these correspond to $v_{\mathrm{c}\,x,\mathrm{f}} = 15$ m/s and $v_{\mathrm{c}\,x,\mathrm{f}} = 19$ m/s, respectively. Inserting these values into equation (3) yields correspond values of $v_{\mathrm{t}\,x,\mathrm{f}} = 13$ m/s and $v_{\mathrm{t}\,x,\mathrm{f}} = 9.4$ m/s.

The first set of solutions ($v_{\mathrm{c,f}} = 15$ m/s and $v_{\mathrm{t,f}} = 13$ m/s) corresponds to a hard, head-on collision in which each vehicle bounces backwards. The second set of solutions ($v_{\mathrm{c,f}} = 19$ m/s and $v_{\mathrm{t,f}} = 9.4$ m/s) corresponds to a glancing collision, in which the vehicles strike each other lightly (perhaps on the side) as they pass each other and continue in their initial directions of motion. (*b*) Everything about this part of the problem is exactly the same as part (*a*), except that we use a different value for the initial *x* component of the truck's velocity. Using exactly the same final expressions, we arrive again at two possible answers. First $v_{\mathrm{c,f}} = 12$ m/s and $v_{\mathrm{t,f}} = 15$ m/s, and second: $v_{\mathrm{c,f}} = 16$ m/s and $v_{\mathrm{t,f}} = 13$ m/s.

5.77. The collision cannot be completely elastic, because the ball is wet. At least some water sticks in a perfectly inelastic way. Much of the water bounces off, but the ongoing collision is not reliably elastic. We are better off using momentum. Assume that most of the water bounces off the basketball with very little speed, some even dripping down the ball to the ground. Then the initial momentum of the water would be transferred completely to the ball. The magnitude of the momentum delivered by the water each second is

$$\frac{\Delta p}{t} = \frac{\Delta m}{t} v = \frac{\Delta V \rho}{t} \frac{\Delta V}{A} = Q\rho \frac{Q}{\pi r^2}$$
$$\frac{\Delta p}{t} = \frac{Q^2 \rho}{\pi r^2}$$

5.78. (*a*) Intuition would suggest that when Pierre will have the higher initial kinetic energy (because he is lighter), and therefore will also travel farther before stopping. We check this rigorously. Since the kinetic energy decreases linearly with time, we can write

$$K(t) = K(t=0) - \alpha t$$

where α is some constant with units of J/s. We can use this to express the speed of a skater as a function of time:

$$v(t) = \sqrt{v^2(t=0) - \frac{2\alpha t}{m}} \tag{1}$$

Note that equation (1) is only valid while the skater is still moving, that is while $t \le \frac{v^2(t=0)m}{2\alpha}$. We can integrate both sides of the above equation over time to find an expression for the distance traveled by any skater as a function of time.

$$x(t) = \frac{mv^2(t=0)}{3\alpha} + \frac{2t\alpha - mv^2(t=0)}{3\alpha}\sqrt{v^2(t=0) - \frac{2\alpha t}{m}} \tag{2}$$

If we evaluate equation (2) at the time at which a skater stops $\left(t = \frac{v^2(t=0)m}{2\alpha}\right)$, we find

$$x_\mathrm{f} = \frac{mv^3(t=0)}{3\alpha}$$

We already know the ratios of the two skaters' inertias. We can use the conservation of momentum to find the ratio of their initial speeds. Call Pierre's inertia *m*, such that Jean-Claude's inertia is $1.5m$. Assume that Pierre is pushed in the $+x$ direction. Writing the x components of the initial and final momenta yields:

$$0 = mv_{\mathrm{P}\,x,\mathrm{f}} + (1.5)mv_{\mathrm{J}\,x,\mathrm{f}}$$

So the two skaters' speeds are related by $\dfrac{v_\mathrm{P}}{v_\mathrm{J}} = 1.5$.

Finally, we can write the ratio of the final distances travelled by each skater:

$$\frac{x_{\mathrm{P,f}}}{x_{\mathrm{J,f}}} = \frac{mv_\mathrm{P}^3}{(1.5)mv_\mathrm{J}^3} = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right)^3 = \frac{9}{4}$$

As we expected, Pierre travels a greater distance. (*b*) As shown above, Pierre travels 9/4 the distance travelled by Jean-Claude.

6

PRINCIPLE OF RELATIVITY

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

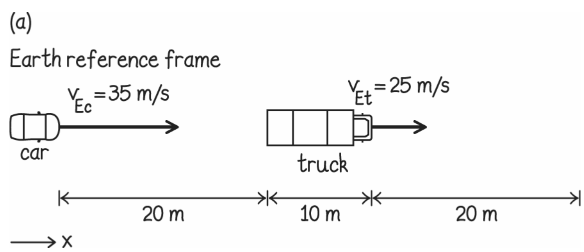
Developing a Feel

1. 10^1 m/s in direction truck is traveling **2.** 10^0 m/s in direction car is traveling **3.** 10^1 m/s in direction bird is flying **4.** 10^{23} kg **5.** center of first car behind locomotive **6.** 10^0 m above floor **7.** 10^0 m/s in original direction of cue ball **8.** 10^{-1} J **9.** 10^5 J **10.** 10^3 J

Guided Problems

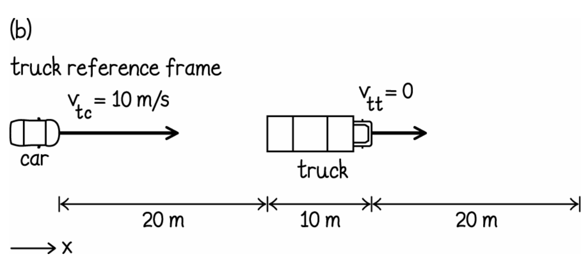
6.2 Safe passage

1. Getting Started Figure (*a*) shows the car and the truck, with the relevant distances indicated. The speeds are measured in the Earth reference frame.



In the reference frame of the truck driver, the truck is stationary. Also, you are interested in changing lanes when you are 20 m behind (or in front of) the truck. A point 20 m behind the truck is always changing in the Earth reference frame, as the truck continues moving. But in the truck reference frame, these points are stationary. This makes the truck reference frame much simpler for this problem.

Figure (*b*) shows the same car and truck, with the same spacing, but now with speeds measured in the reference frame of the truck. Let us call the direction of motion of the car the $+x$ direction, so that we can refer to velocities.



We assume that the car is significantly shorter than the truck. Very small smart cars may be 2.0 m long, and vans may be longer than 4.0 m. We will use 3.0 m for the length of the car.

2. Devise Plan For an object moving at a constant velocity, we can relate displacement and time using $\vec{v} = \frac{\Delta\vec{x}}{\Delta t}$. In this case, the velocity and displacement are along the $+x$ direction, so we can write

$$\Delta t = \frac{\Delta x}{v_x} \tag{1}$$

In the truck reference frame, the car simply has to move from the position 20 m behind the truck, past the 10 m long truck, and then to the position 20 m in front of the truck. Also, we want to begin our lane change with the truck 20 m from the front end of the car, and change back with the truck 20 m from the rear of the car. This means, we also have to move forward by one car-length. This means that the total distance the car must cover in the truck reference frame is

$$\Delta x = \Delta x_{\text{behind}} + \Delta x_{\text{truck}} + \Delta x_{\text{front}} + \Delta x_{\text{car-length}} \tag{2}$$

3. Execute Plan Inserting Equation. (2) into Equation. (1), we have

$$\Delta t = \frac{1}{v_{\text{tc}x}}(\Delta x_{\text{behind}} + \Delta x_{\text{truck}} + \Delta x_{\text{front}} + \Delta x_{\text{car-length}})$$

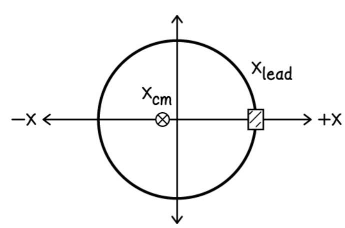
$$\Delta t = \frac{1}{(10\text{ m/s})}((20\text{ m}) + (10\text{ m}) + (20\text{ m}) + (3\text{ m}))$$

$$\Delta t = 5.3\text{ s}$$

4. Evaluate Result This seems like a plausible time for passing another vehicle on the freeway. One could convert the speeds to mi/h if one has a better intuition for those units. In that case we find that the speed of the car is near 80 mi/h, which is speeding on most freeways, and is likely to be comparable to the fastest cars in normal traffic. The truck is moving near 55 mi/h, meaning it is one of the slowest vehicles in normal traffic. Given those extremes, it is even more plausible that the passing can be safely done in only 5.3 s. The length of the car only affects the time slightly. For example, if we had assumed a longer car with a length 4.0 m, we would have obtained a passing time of 5.4 s.

6.4 Balancing a wheel

1. Getting Started We start by drawing a diagram of the tire and the positions of the initial center of mass and the lead bar. We choose our origin at the geometric center of the wheel.



2. Devise Plan With our origin chosen at the axle, the goal to make the balanced wheel have a center of mass at the position $x_{\text{cm,f}} = x_{\text{axle}} = 0$. We can write an expression for the center of mass using equation (6.24), in this special case where all locations of inertia are along the x axis. Thus

$$x_{\text{cm,f}} = \frac{x_{\text{lead}}m_{\text{lead}} + x_{\text{wheel}}m_{\text{wheel}}}{m_{\text{lead}} + m_{\text{wheel}}} = \frac{x_{\text{lead}}m_{\text{lead}} + x_{\text{cm,i}}m_{\text{wheel}}}{m_{\text{lead}} + m_{\text{wheel}}} \tag{1}$$

Here, x_{wheel} means the position of the center of mass of the wheel prior to balancing. All quantities in Equation. (1) are known, except m_{lead}, so we can solve for the unknown inertia of the lead.

3. Execute Plan Rearranging Equation. (1) to determine m_{lead}, we find

$$m_{\text{lead}} = \frac{x_{\text{cm,f}}(m_{\text{lead}} + m_{\text{wheel}}) - x_{\text{wheel}} m_{\text{wheel}}}{x_{\text{lead}}}$$

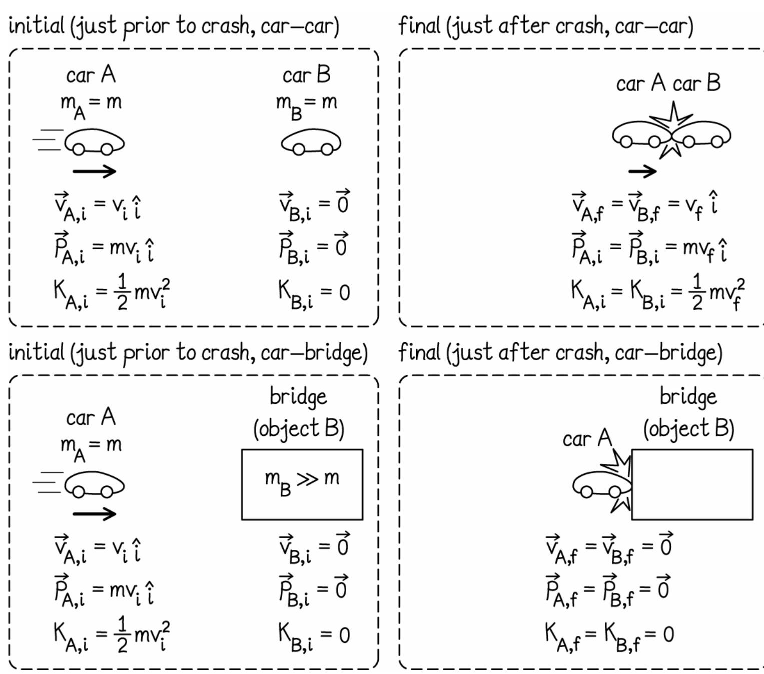
$$= \frac{(0) - (-1.0 \times 10^{-3}\ \text{m})(10\ \text{kg})}{(0.200\ \text{m})}$$

$$m_{\text{lead}} = 5.0 \times 10^{-2}\ \text{kg}$$

4. Evaluate Result The inertia of the lead bar must be positive. We were careful to note that the lead is one side of the tire, and the initial center of mass was on the opposite, one of the two positions had to be negative. Our choice of axes was such that the position of the initial center of mass was on the negative x axis. This made our final answer come out positive, as it must. The inertia of 50 g is very reasonable for a lead bar. For comparison, this is the inertia of about 20 pennies.

6.6 Crunch

1. Getting Started We draw a system diagram before and after each type of collision. Both momentum and energy may be useful, so we note both in diagrams:



In both cases, the amount of convertible kinetic energy is the total kinetic energy minus the kinetic energy that will be stored in the center of mass motion. Intuition tells us that this convertible kinetic energy will be much larger in the latter case of the collision with the bridge abutment. But we will demonstrate this below.

2. Devise Plan Because the collisions are totally inelastic, all available convertible kinetic energy is converted to internal energy of the car (through deformations, heating, etc).

Equation (6.40) tells us the amount of convertible kinetic energy is given by $K_{\text{conv}} = \frac{1}{2}\mu v_{12}^2$. So we must determine the relative speed v_{12}, and the reduced inertia μ. Since Car A has the same initial velocity in both cases, and since the object with which Car A will collide is at rest in both cases, $v_{12}^2 = v_{\rm i}^2$ in both cases. The reduced inertias, however, are different. Let us write the general reduced inertia as

$$\mu = \frac{m_{\rm moving} m_{\rm rest}}{m_{\rm moving} + m_{\rm rest}}$$

3. Execute Plan The converted kinetic energies in the two cases are:

$$(K_{\rm conv})_{\text{car-car}} = \frac{1}{2}\left(\frac{m_{\rm A} m_{\rm B}}{m_{\rm A} + m_{\rm B}}\right) v_{\rm i}^2 = \frac{1}{4} m_{\rm A} v_{\rm i}^2$$

$$(K_{\rm conv})_{\text{car-abutment}} = \frac{1}{2}\left(\frac{m_{\rm A} m_{\rm Bridge}}{m_{\rm A} + m_{\rm Bridge}}\right) v_{\rm i}^2$$

We are not given the inertia of the bridge abutment, but it is a safe bet that it is much larger than the inertia of any car. It is not infinite. But if we consider that the abutment is strongly connected to the Earth below it, and to move the abutment would require moving a significant amount of Earth as well, taking the limit as this inertia approaches infinity does seem appropriate. In that limit

$$\frac{m_{\rm A} m_{\rm Bridge}}{m_{\rm A} + m_{\rm Bridge}} = \frac{m_{\rm A}}{\dfrac{m_{\rm A}}{m_{\rm Bridge}} + \dfrac{m_{\rm Bridge}}{m_{\rm Bridge}}} \approx \frac{m_{\rm A}}{0+1} = m_{\rm A}$$

Thus, we have for the two converted energies

$$(K_{\rm conv})_{\text{car-car}} = \frac{1}{4} m_{\rm A} v_{\rm i}^2$$

$$(K_{\rm conv})_{\text{car-abutment}} = \frac{1}{2} m_{\rm A} v_{\rm i}^2$$

Although we do not know the initial velocity of the car, or the mass of the car, it is clear that twice as much energy will be converted to internal energy of the car in the car-abutment collision as in the car-car collision.

4. Evaluate Result Since twice as much energy goes into internal energy of the car and the people inside the car in the car-abutment collision, clearly it would be better to collide with a stationary car than a bridge abutment. This agrees with common sense. The only assumption that we made was that the abutment was much greater in inertia than the car. If the abutment was truly a flimsy wooden structure for a very old bridge, then of course a very heavy car might go right through it. But our assumption is very reasonable for a modern, well-anchored bridge abutment made of concrete and steel.

6.8 Slowing neutrons

1. Getting Started This problem is very similar to Worked Problem 6.7. Now we are interested in comparing the collisions between neutrons and light atoms (hydrogen and carbon) to collisions between neutrons and heavy atoms (lead or iron). Because we are not told to consider one particular target element, and because we would like to draw general results about the inertia of the target nuclei, we wish to work in terms of a variable target nuclei inertia. The goal of the collision with the moderator is to slow the neutrons. The physical variable associated with this slowing process is speed, not velocity. Changing the direction of the neutron does not help the reaction; slowing the neutron helps the reaction.

2. Devise Plan As is Worked Problem 6.7 we can write down the velocity of the center of mass of the neutron-moderator system, assuming the neutron has an initial velocity in the Earth reference frame of $\vec{v}_{\rm En} = v_{\rm n}\ \hat{\imath}$, and the initial velocity of the target moderator nucleus is $\vec{v}_{\rm ET} = -v_{\rm ET}\ \hat{\imath} = \vec{0}$. In the Earth reference frame the center of mass reference frame is given by

$$\vec{v}_{\rm EZ} = \vec{v}_{\rm cm} = \frac{m_{\rm n}\vec{v}_{\rm n} + m_{\rm T}\vec{v}_{\rm T}}{m_{\rm n} + m_{\rm T}}$$

In exactly the same manner as shown in Worked Problem 6.7, we write the velocities of the neutron and target in the zero-momentum frame in terms of the velocities in the Earth frame:

$$\vec{v}_{\text{Zn}} = \vec{v}_{\text{ZE}} + \vec{v}_{\text{En}} = -\vec{v}_{\text{cm}} + \vec{v}_{\text{En}}$$
$$\vec{v}_{\text{ZT}} = \vec{v}_{\text{ZE}} + \vec{v}_{\text{ET}} = -\vec{v}_{\text{cm}} + \vec{v}_{\text{ET}}$$

In general, the center of mass velocity given by

$$v_{\text{cm}\,x} = \frac{m_{\text{n}}v_{\text{n}} - m_{\text{T}}(0)}{m_{\text{n}} + m_{\text{T}}} = \frac{v_n}{\left(1 + \dfrac{m_{\text{T}}}{m_{\text{n}}}\right)}$$

From this we can immediately write down the general form for the x components of the initial velocities of the two particles in the zero-momentum reference frame:

$$v_{\text{Zn}\,x,\text{i}} = -v_{\text{cm}} + v_{\text{En}\,x,\text{i}} = -v_{\text{n}}\left(1 + \frac{m_{\text{T}}}{m_{\text{n}}}\right)^{-1} + v_{\text{n}} = v_{\text{n}}\left(1 + \frac{m_{\text{n}}}{m_{\text{T}}}\right)^{-1}$$
$$v_{\text{ZT}\,x,\text{i}} = -v_{\text{cm}} + v_{\text{ET}\,x,\text{i}} = -v_{\text{n}}\left(1 + \frac{m_{\text{T}}}{m_{\text{n}}}\right)^{-1}$$

As in Worked Problem 6.7, the minus sign on the x component of the velocity of the target is to be expected, since the target is stationary in the Earth reference frame. After the collision, we know the particles will reverse direction, but maintain the same speed in the zero momentum reference frame. Finally we transform back to the Earth reference frame using equation (6.14) to find

$$v_{\text{En}\,x,\text{f}} = v_{\text{cm}\,x} + v_{\text{Zn}\,x,\text{f}} = v_{\text{n}}\left(1 + \frac{m_{\text{T}}}{m_{\text{n}}}\right)^{-1} + -v_{\text{n}}\left(1 + \frac{m_{\text{n}}}{m_{\text{T}}}\right)^{-1} = v_{\text{n}}\frac{\left(1 - \dfrac{m_{\text{T}}}{m_{\text{n}}}\right)}{\left(1 + \dfrac{m_{\text{T}}}{m_{\text{n}}}\right)}$$
$$v_{\text{ET}\,x,\text{f}} = v_{\text{cm}\,x} + v_{\text{ZT}\,x,\text{f}} = v_{\text{n}}\left(1 + \frac{m_{\text{T}}}{m_{\text{n}}}\right)^{-1} + v_{\text{n}}\left(1 + \frac{m_{\text{T}}}{m_{\text{n}}}\right)^{-1} = 2v_{\text{n}}\left(1 + \frac{m_{\text{T}}}{m_{\text{n}}}\right)^{-1}$$

3. Execute Plan We can now insert various values of the target nucleus inertia to the equation obtained above:

$$v_{\text{En}\,x,\text{f}} = v_{\text{n}}\frac{\left(1 - \dfrac{m_{\text{T}}}{m_{\text{n}}}\right)}{\left(1 + \dfrac{m_{\text{T}}}{m_{\text{n}}}\right)}$$

The smallest value of m_{T} occurs in the case of a hydrogen target, in which case $m_{\text{T}} \approx m_{\text{n}}$, and we find $v_{\text{En}\,x,\text{f}} = v_{\text{n}}\dfrac{(1-1)}{(1+1)} = 0$. This is certainly a reduction in speed, but too much of a reduction. The neutron must have some speed in order to strike a nucleus and increase the rate of fission.

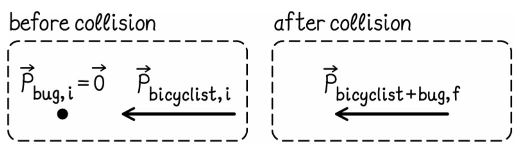
In the opposite extreme, we could consider very heavy targets. One might consider using something as heavy as the uranium itself, in which case $m_{\text{T}} \approx 235m_{\text{n}}$ or $m_{\text{T}} \approx 238m_{\text{n}}$ depending on the isotope. In either case, we obtain $v_{\text{En}\,x,\text{f}} = -(0.99)v_{\text{n}}$. This means the neutron essentially reverses direction with very little loss of speed. Using slightly lighter elements, such as iron yields $m_{\text{T}} \approx 56m_{\text{n}}$, and $v_{\text{En}\,x,\text{f}} = -(0.96)v_{\text{n}}$. One could easily plot $v_{\text{En}\,x,\text{f}}$ as a function of m_{T}, but it is clear that very light moderators may reduce the speed of the neutron too much, and very heavy moderators may cause the neutron to simply reverse its velocity with very little loss of speed. The desired reduction of neutron speed is achieved using somewhat lightweight moderators such as Carbon, for which $m_{\mathrm{T}} \approx 12m_{\mathrm{n}}$ and $v_{\mathrm{En}\,x,\mathrm{f}} = -(0.85)v_{\mathrm{n}}$.

4. Evaluate Result These results fit our intuition. In particular, in case of a hydrogen target, this is comparable to a moving billiard ball striking a stationary billiard ball. This can, indeed, leave the initially moving ball stationary as the initially stationary ball carries all momentum and energy forward. In the case of the much heavier uranium target, again this fit our intuition. The neutron reversed direction with almost no loss of speed. Thus is what we expect intuitively from everyday experience with objects the size/inertia of golf balls bouncing off of objects the size/inertia of bowling balls.

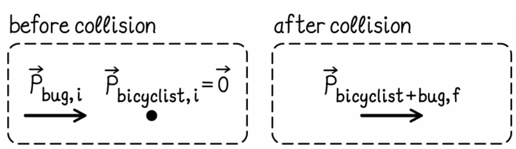
Questions and Problems

6.1. When getting on, you may want to walk quickly to try to match the speed of the moving sidewalk. When getting off, you may want to stop walking on the sidewalk to minimize your speed relative to the ground (to match the speed of the stationary ground as closely as possible).

6.2. The collision could be perfectly inelastic if the insect sticks to the helmet.
(*a*)



(*b*)

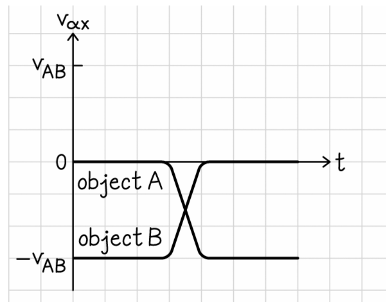


6.3. We determine the average velocity of each car, and then find the relative velocity of car B. Assume the motion of both cars is one-dimensional and call their direction of motion the $+x$ direction. Then $v_{\mathrm{A}\,x,\mathrm{av}} = \dfrac{x_{\mathrm{A,f}} - x_{\mathrm{A,i}}}{t_{\mathrm{A,f}} - t_{\mathrm{A,i}}} = \dfrac{(265\text{ m}) - (40\text{ m})}{(10\text{ s}) - (9\text{ s})} = 25\text{ m/s}$, and $v_{\mathrm{B}\,x,\mathrm{av}} = \dfrac{x_{\mathrm{B,f}} - x_{\mathrm{B,i}}}{t_{\mathrm{B,f}} - t_{\mathrm{B,i}}} = \dfrac{(290\text{ m}) - (20\text{ m})}{(10\text{ s}) - (9\text{ s})} = 30\text{ m/s}$. The x component of the relative velocity of car B as seen by car A is $v_{\mathrm{B}\,x,\mathrm{rel}} = v_{\mathrm{B}\,x,\mathrm{av}} - v_{\mathrm{A}\,x,\mathrm{av}} = (30\text{ m/s}) - (25\text{ m/s}) = 5\text{ m/s}$. Hence the average relative velocity of car B is $(+5\text{ m/s})\,\hat{\imath}$.

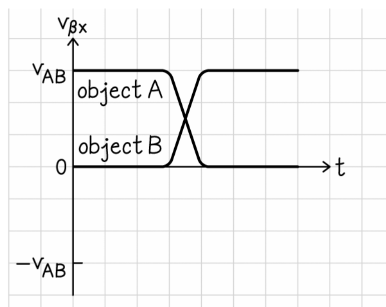
6.4. (*a*) No, observer A will measure the velocity of cart A to be zero. The observer on Earth will not (assuming cart A is actually moving relative to Earth). (*b*) No, since observer A does not collide, observer A is still moving relative to Earth. This means that observer A and an observer on Earth will still measure different velocities for any object, including cart B. (*c*) Yes, they will agree. In this case, each observer is measuring the velocity of one moving object relative to another moving object, not relative to themselves. To see how this works, consider observer A moving at $\vec{v}_{\mathrm{ObsA}}$ relative to Earth. Let us symbolically give the final velocities of carts A and B in the Earth reference frame as $\vec{v}_{\mathrm{Earth\,A,f}}$ and $\vec{v}_{\mathrm{Earth\,B,f}}$. An observer on Earth would measure the velocity of cart A relative to B as $\vec{v}_{\mathrm{Earth\,AB,f}} = \vec{v}_{\mathrm{Earth\,A,f}} - \vec{v}_{\mathrm{Earth\,B,f}}$. Observer A would measure the relative velocity to be $\vec{v}_{\mathrm{ObsA\,AB,f}} = \vec{v}_{\mathrm{ObsA\,A,f}} - \vec{v}_{\mathrm{ObsA\,B,f}} = (\vec{v}_{\mathrm{Earth\,A,f}} - \vec{v}_{\mathrm{ObsA}}) - (\vec{v}_{\mathrm{Earth\,B,f}} - \vec{v}_{\mathrm{ObsA}}) = \vec{v}_{\mathrm{Earth\,A,f}} - \vec{v}_{\mathrm{Earth\,B,f}}$. This is the same expression as the relative velocity measured by an observer in the Earth frame.

6.5. (*a*) You will roll toward the truck, as seen by a bystander on the road. When you hop off the back of the truck, your body continues moving forward at nearly the speed of the truck. As soon as you touch the ground and begin rolling, your interaction with the ground slows you relative to the truck, but you do not immediately stop. So you continue moving the in the direction of the truck for a short time. (*b*) An observer on the truck will observe you receding away from the truck. It is true that you still move in the direction of the truck, but at a lower speed than the truck. Hence, as you roll, the distance between you and the truck increases. Note that any reasonable observer on the truck knows that your speed relative to the Earth is still in the same direction as the truck. But that same reasonable observer must admit that since the distance between you and her is growing, you are moving away from the truck in the truck reference frame.

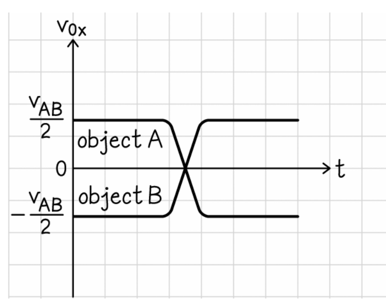
6.6. (*a*) Here, let the initial direction of motion of Cart B be the $-x$ direction, and call this reference frame α.



(*b*) Here, let the initial direction of motion of Cart A be the $+x$ direction, and call this reference frame β.



(*c*) Here, let the initial direction of motion of Cart A be the $+x$ direction, and this reference frame will be denoted by the standard "0".



6.7. The truck driver would see the car initially moving backwards ($-x$ direction) at 30 m/s. The car would then have a positive acceleration and end up moving at a speed of 0 m/s relative to the truck driver. If the truck were moving the opposite direction, the driver would see the car initially approaching him at 30 m/s in what we will now call the ($+x$ direction). The car would then accelerate in the $+x$ direction such that the car is finally approaching the truck at a speed of 60 m/s. No, the acceleration is not affected by the constant velocity of the truck driver.

6.8. Let us first express all speeds in the same units. The speed of the bus is $v_{\text{Earth Bus}\,x} = \dfrac{100\text{ km}}{1\text{ h}}\times\dfrac{10^3\text{ m}}{1\text{ km}}\times\dfrac{1\text{ h}}{3600\text{ s}} = 27.8\text{ m/s}$. Now, the x component of your velocity relative to the ground is $v_{\text{Earth you}\,x} = v_{\text{Earth Bus}\,x} + v_{\text{Bus you}\,x} = (27.8\text{ m/s}) + (-2.0\text{ m/s}) = 25.8\text{ m/s}$. Hence, you move relative to the ground at a speed of 25.8 m/s.

6.9. The cans are initially moving along with the truck through space. If they are not influenced by any other objects, they would continue moving at that speed forever. When the truck slows, the cans continue at their initial speed until they strike part of the truck or until friction slows them relative to the truck.

6.10. (*a*) No. The fact that the keys are initially moving upward in the elevator only affects the initial velocity of the keys; it does not affect the rate of change of the velocity. (*b*) Yes, keys will accelerate toward the floor of the elevator faster if it is moving at a constant speed than if it is accelerating downward. As in part (*a*), the acceleration is not affected by the elevator's motion at a constant velocity. But if the elevator is accelerating downward, then the downward change in velocity of the keys is at least partially matched by a downward change in the velocity of the elevator. As an extreme example, note that if the elevator were in freefall, the keys would not accelerate toward the floor at all, but hang suspended in air.

6.11. Assume all initial velocities are given relative to Earth. (*a*) The initial momenta are easy to calculate, as the inertias and initial velocities are given. Initially

$$\vec{p}_{\text{A i}} = m_{\text{A}}\vec{v}_{\text{A i}} = (1.50\times10^5\ \text{kg})\times\left(\frac{60\ \text{km}}{1\ \text{h}}\ \text{west}\right)\times\frac{10^3\ \text{m}}{1\ \text{km}}\times\frac{1\ \text{h}}{3600\ \text{s}} = 2.5\times10^6\ \text{kg}\cdot\text{m/s west, and}$$

$$\vec{p}_{\text{B i}} = m_{\text{B}}\vec{v}_{\text{B i}} = (1.00\times10^5\ \text{kg})\times\left(\frac{88\ \text{km}}{1\ \text{h}}\ \text{west}\right)\times\frac{10^3\ \text{m}}{1\ \text{km}}\times\frac{1\ \text{h}}{3600\ \text{s}} = 2.4\times10^6\ \text{kg}\cdot\text{m/s west}$$

For the final momenta, we must calculate the final (mutual) velocity using conservation of momentum. Call west the $+x$ direction. Because the trains stick together, we can write $m_{\text{A}}v_{\text{A}\,x,\text{i}} + m_{\text{B}}v_{\text{B}\,x,\text{i}} = (m_{\text{A}} + m_{\text{B}})v_{x,\text{f}}$. Solving for the final x component of the velocity yields $v_{x,\text{f}} = \dfrac{m_{\text{A}}v_{\text{A}\,x,\text{i}} + m_{\text{B}}v_{\text{B}\,x,\text{i}}}{(m_{\text{A}} + m_{\text{B}})} =$ $\dfrac{(1.50\times10^5\ \text{kg})(16.7\ \text{m/s}) + (1.00\times10^5\ \text{kg})(24.4\ \text{m/s})}{((1.50\times10^5\ \text{kg}) + (1.00\times10^5\ \text{kg}))} = 19.8\ \text{m/s}$. Here we have kept an additional significant digit, as this is an intermediate step in the calculation. We now find the momentum of each train separately, after the collision:

$$\vec{p}_{\text{A f}} = m_{\text{A}}\vec{v}_{\text{A f}} = (1.50\times10^5\ \text{kg})\times(19.8\ \text{m/s west}) = 3.0\times10^6\ \text{kg}\cdot\text{m/s west, and}$$

$$\vec{p}_{\text{B f}} = m_{\text{B}}\vec{v}_{\text{B f}} = (1.00\times10^5\ \text{kg})\times(19.8\ \text{m/s west}) = 2.0\times10^6\ \text{kg}\cdot\text{m/s west}$$

It might also be natural to give the final momentum of the two coupled trains together (rather than separately), which is $4.9\times10^6\ \text{kg}\cdot\text{m/s}$ west. (*b*) The process is identical to that of part (*a*), except that now all velocities are given relative to the car travelling 100 km/h west. Hence we have

$$\begin{aligned}
\vec{p}_{\text{A i}} = m_{\text{A}}\vec{v}_{\text{car A i}} &= m_{\text{A}}(\vec{v}_{\text{Earth A i}} - \vec{v}_{\text{Earth car i}}) = (1.50\times10^5\ \text{kg})\times((16.7\ \text{m/s west}) - (27.8\ \text{m/s west}))\\
&= -1.7\times10^6\ \text{kg}\cdot\text{m/s west or } 1.7\times10^6\ \text{kg}\cdot\text{m/s east}\\
\vec{p}_{\text{B i}} = m_{\text{B}}\vec{v}_{\text{car B i}} &= m_{\text{B}}(\vec{v}_{\text{Earth B i}} - \vec{v}_{\text{Earth car i}}) = (1.00\times10^5\ \text{kg})\times((24.4\ \text{m/s west}) - (27.8\ \text{m/s west}))\\
&= -3.3\times10^5\ \text{kg}\cdot\text{m/s west or } 3.3\times10^5\ \text{kg}\cdot\text{m/s east}\\
\vec{p}_{\text{A f}} = m_{\text{A}}\vec{v}_{\text{car A f}} &= m_{\text{A}}(\vec{v}_{\text{Earth A f}} - \vec{v}_{\text{Earth car f}}) = (1.50\times10^5\ \text{kg})\times((19.8\ \text{m/s west}) - (27.8\ \text{m/s west}))\\
&= -1.2\times10^6\ \text{kg}\cdot\text{m/s west or } 1.2\times10^6\ \text{kg}\cdot\text{m/s east}\\
\vec{p}_{\text{B f}} = m_{\text{B}}\vec{v}_{\text{car B f}} &= m_{\text{B}}(\vec{v}_{\text{Earth B f}} - \vec{v}_{\text{Earth car f}}) = (1.00\times10^5\ \text{kg})\times((19.8\ \text{m/s west}) - (27.8\ \text{m/s west}))\\
&= -8.0\times10^5\ \text{kg}\cdot\text{m/s west or } 8.0\times10^5\ \text{kg}\cdot\text{m/s east}
\end{aligned}$$

Again, it might be natural to list the final momentum of the two trains together, which would be $2.0\times10^6\ \text{kg}\cdot\text{m/s}$ east.

6.12. (*a*) To find the coefficient of restitution, we first need to find the final velocity of the car. For this, we use conservation of momentum. Call east the $+x$ direction, and use the given speeds in SI units: $v_{\text{c}\,x,\text{i}} =$ $\dfrac{50\ \text{km}}{1\ \text{h}}\times\dfrac{10^3\ \text{m}}{1\ \text{km}}\times\dfrac{1\ \text{h}}{3600\ \text{s}} = 13.9\ \text{m/s}$, and $v_{\text{t}\,x,\text{f}} = \dfrac{15\ \text{km}}{1\ \text{h}}\times\dfrac{10^3\ \text{m}}{1\ \text{km}}\times\dfrac{1\ \text{h}}{3600\ \text{s}} = 4.17\ \text{m/s}$. Then we have $m_{\text{c}}v_{\text{c}\,x,\text{i}} =$

$m_c v_{c\,x,f} + m_t v_{t\,x,f}$, such that $v_{c\,x,f} = \dfrac{m_c v_{c\,x,i} - m_t v_{t\,x,f}}{m_c} = \dfrac{(1000\text{ kg})(13.9\text{ m/s}) - (3000\text{ kg})(4.17\text{ m/s})}{(1000\text{ kg})} = 1.39\text{ m/s}$. Now, the coefficient of restitution is given by

$$e = -\frac{v_{ct\,x,f}}{v_{ct\,x,i}} = -\frac{v_{c\,x,f} - v_{t\,x,f}}{v_{c\,x,i} - v_{t\,x,i}} = -\frac{(1.39\text{ m/s}) - (4.17\text{ m/s})}{(13.9\text{ m/s}) - 0} = 0.20$$

Hence, the coefficient of restitution is 0.20. (*b*) We calculate the initial and final kinetic energies and take the difference. Initially, $K_i = K_{c\,i} = \frac{1}{2} m_c v_{c\,i}^2 = \frac{1}{2}(1000\text{ kg})(13.9\text{ m/s})^2 = 96.6\text{ kJ}$. Finally, $K_f = K_{c\,f} + K_{t\,f} = \frac{1}{2} m_c v_{c\,f}^2 + \frac{1}{2} m_t v_{t\,f}^2 = \frac{1}{2}(1000\text{ kg})(1.39\text{ m/s})^2 + \frac{1}{2}(3000\text{ kg})(4.17\text{ m/s})^2 = 27.0\text{ kJ}$. Thus the amount of kinetic energy that was converted to internal energy is just the difference $K_i - K_f = 69\text{ kJ}$. (*c*) The process is the same as in (*b*), except that now all speeds are measured relative to this moving observer. Initially, $K_{\text{obs i}} = K_{\text{obs c i}} + K_{\text{obs t i}} = \frac{1}{2} m_c v_{\text{obs c i}}^2 + \frac{1}{2} m_t v_{\text{obs t i}}^2 = \frac{1}{2}(1000\text{ kg})((13.9\text{ m/s}) - (5.0\text{ m/s}))^2 + \frac{1}{2}(3000\text{ kg})((0) - (5.0\text{ m/s}))^2 = 77.0\text{ kJ}$.

Finally, $K_{\text{obs f}} = K_{\text{obs c f}} + K_{\text{obs t f}} = \frac{1}{2} m_c v_{\text{obs c f}}^2 + \frac{1}{2} m_t v_{\text{obs c f}}^2 = \frac{1}{2}(1000\text{ kg})((1.39\text{ m/s}) - (5.0\text{ m/s}))^2 + \frac{1}{2}(3000\text{ kg})$ $((4.17\text{ m/s}) - (5.0\text{ m/s}))^2 = 7.56\text{ kJ}$.

The amount of kinetic energy converted to internal energy is thus $K_{\text{obs i}} - K_{\text{obs f}} = 69\text{ kJ}$. Note that the answers to (*b*) and (*c*) are the same, as they must be. The amount of internal energy in the deformations of the vehicles cannot depend on the reference frame.

6.13. Your speed relative to the ground is $v_{\text{Earth you}} = \dfrac{14.4\text{ km}}{1\text{ h}} \times \dfrac{10^3\text{ m}}{1\text{ km}} \times \dfrac{1\text{ h}}{3600\text{ s}} = 4.00\text{ m/s}$. The police officer's measurement of your kinetic energy tells you your speed relative to the officer: $v_{\text{officer you}} = \sqrt{2K/m} = \sqrt{2(1.632\times10^4\text{ J})/(510\text{ kg})} = 8.00\text{ m/s}$. However, we are told whether the officer is approaching you from behind, or from in front of you. Hence there are two possibilities for the officer's speed. Let east be the $+x$ direction. If the officer is approaching you from in front of you, then $v_{\text{Earth officer}\,x} = v_{\text{Earth you}\,x} - v_{\text{officer you}\,x} = (4.00\text{ m/s}) - (8.00\text{ m/s}) = -4.00\text{ m/s}$. If the officer is approaching you from behind, then we have $v_{\text{Earth officer}\,x} = v_{\text{Earth you}\,x} - v_{\text{officer you}\,x} = (4.00\text{ m/s}) - (-8.00\text{ m/s}) = 12.00\text{ m/s}$. Hence the officer's possible speeds are 12.00 m/s and 4.00 m/s.

6.14. The officer would calculate the same E_{def}. The energy required to deform a material cannot depend on your reference frame.

6.15. Call the partner on the moving walkway partner A, and the other partner B. Call their direction of motion the $+x$ direction, and call the length of the walkway Δx. We can write expressions for the x component of each partner's velocity relative to Earth as:

$$v_{\text{E A}\,x} = \frac{\Delta x}{t_A} = v_{\text{E W}\,x} + v_{\text{W A}\,x} \tag{1}$$

$$v_{\text{E B}\,x} = \frac{\Delta x}{t_B} \tag{2}$$

Here, $v_{\text{E W}\,x}$ is the x component of the velocity of the walkway, relative to Earth, and $v_{\text{W A}\,x}$ is the x component of the velocity of partner A relative to the walkway. We can solve equation (2) for the length of the walkway and insert this into equation (1) to obtain

$$v_{\text{E W}\,x} = \frac{v_{\text{E B}\,x} t_B}{t_A} - v_{\text{W A}\,x} = \frac{(1.5\text{ m/s})(90\text{ s})}{(60\text{ s})} - (1.5\text{ m/s}) = 0.75\text{ m/s}$$

Hence the walkway moves relative to Earth at 0.75 m/s.

6.16. No, the laws of the universe would remain the same. They sometimes depend on time intervals, but not on an absolute time. Indeed, the laws of physics in different time zones on Earth are identical.

6.17. (*a*) The momentum of the cart-ranger system must be the same before and after the collision. This must hold in any reference frame. Thus, we find the x component of the momentum of the cart-ranger system prior to the collision. The x component of the velocity of the cart in your friend's reference frame is $v_{\text{friend cart }x} = v_{\text{Earth cart }x} - v_{\text{Earth friend }x} = (1.0\text{ m/s}) - (-3.0\text{ m/s}) = 4.0\text{ m/s},$ and for the ranger we have $v_{\text{friend ranger }x} = v_{\text{Earth ranger }x} - v_{\text{Earth friend }x} = (0) - (-3.0\text{ m/s}) = 3.0\text{ m/s}.$ Thus the momentum is $\vec{p}_{\text{friend cart/ranger}} = m_{\text{cart}}\vec{v}_{\text{friend cart}} + m_{\text{ranger}}\vec{v}_{\text{friend ranger}} = (0.50\text{ kg})(4.0\text{ m/s})\ \hat{\imath} + (0.10\text{ kg})(3.0\text{ m/s})\ \hat{\imath} = 2.3\text{ kg}\cdot\text{m/s}\ \hat{\imath}.$ Thus the final momentum of the cart-ranger system is also $(+2.3\text{ kg}\cdot\text{m/s})\ \hat{\imath}.$ (*b*) Using the result from (*a*), we can write the final momentum as $p_{\text{friend cart/ranger }x} = (m_{\text{cart}} + m_{\text{ranger}})v_{\text{friend cart/ranger }x} = 2.3\text{ kg}\cdot\text{m/s},$ which yields $v_{\text{friend cart/ranger }x} = \dfrac{2.3\text{ kg}\cdot\text{m/s}}{0.60\text{ m/s}} = 3.8\text{ m/s}.$ Thus your friend's sonic ranger measures a final velocity of the cart equal to $(+3.8\text{ m/s})\ \hat{\imath}.$

6.18. (*a*) In the Earth reference frame each girl had an initial momentum of $p = mv = (20\text{ kg})(2.0\text{ m/s}) = 40\text{ kg}\cdot\text{m/s}$ in their respective directions, and finally each girl had zero momentum. (*b*) Let the mother's direction of motion be the $+x$ direction. In the mother's reference frame, the girl next to her had zero initial momentum. The other girl had an initial momentum of $\vec{p}_{\text{mother girl i}} = m_{\text{girl}}\vec{v}_{\text{mother girl i}} = m_{\text{girl}}(\vec{v}_{\text{Earth girl i}} - \vec{v}_{\text{Earth mother}}) = (20\text{ kg})((-2.0\text{ m/s}) - (2.0\text{ m/s})) = (-80\text{ kg}\cdot\text{m/s})\ \hat{\imath}.$ The girls each have a final momentum of $\vec{p}_{\text{mother girl f}} = m_{\text{girl}}\vec{v}_{\text{mother girl f}} = m_{\text{girl}}(\vec{v}_{\text{Earth girl f}} - \vec{v}_{\text{Earth mother}}) = (20\text{ kg})((0) - (2.0\text{ m/s})) = (-40\text{ kg}\cdot\text{m/s})\ \hat{\imath}.$ (*c*) The fathers have much more inertia than their daughters. When they collide there is a great deal more kinetic energy to be turned into internal energy. If each of the fathers has the same inertia, and they skate at the same initial speed, then they will also come to a complete stop after colliding. Hence all initial kinetic energy will be converted to internal energy. This greater internal energy may be sufficient to bruise muscle or even break a small bone.

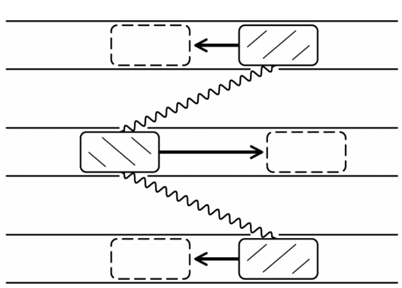
6.19. Yes, an observer in a reference frame that is moving at a speed v toward cart A would calculate the same kinetic energy. This observer would see cart A approaching it at a speed of $2v$ and would see cart B moving at only v.

6.20. With the cars travelling in opposite directions, the zero-momentum frame has a speed of 0 m/s. With the cars moving together in the same direction at a speed v, the zero-momentum frame would also have a speed v.

6.21. Not necessarily. If two objects are a rest in a certain reference frame, then both the momentum and the kinetic energy would be zero. But if two identical objects are approaching each other at the same speed (moving in opposite directions), an observer in the zero-momentum frame would still see the objects moving and would still calculate a non-zero kinetic energy.

6.22. In the zero momentum reference frame, the two objects must have the same initial speed (since they have identical inertias). Call this initial speed v. Since momentum is conserved, the two objects must have identical final speeds as well (in opposite directions). The problem states that the collision is elastic, meaning the system must have the same amount of kinetic energy after the collision. The only way this can be satisfied is if the final speed of each cart is again v. Hence we can say $v_{\text{A,f}}/v_{\text{A,i}} = v_{\text{B,f}}/v_{\text{B,i}} = 1.$

6.23.



6.24. Let the velocity of the zero momentum frame be $\vec{v}$. We can write the momentum in the zero-momentum frame as $m_\text{A}(\vec{v}_{0\text{A}} - \vec{v}) + m_\text{B}(\vec{v}_{0\text{B}} - \vec{v}) = 0$, and rearrange to find $\vec{v} = \dfrac{m_\text{A}\vec{v}_{0\text{A}} + m_\text{B}\vec{v}_{0\text{B}}}{m_\text{A} + m_\text{B}}$. We can now find expressions for the differences between the velocities of either individual object and the zero momentum reference frame:

$$\vec{v} - \vec{v}_\text{A} = \frac{m_\text{B}(\vec{v}_{0\text{B}} - \vec{v}_{0\text{A}})}{m_\text{A} + m_\text{B}} \tag{1}$$

$$\vec{v} - \vec{v}_\text{B} = \frac{m_\text{A}(\vec{v}_{0\text{B}} - \vec{v}_{0\text{A}})}{m_\text{A} + m_\text{B}} \tag{2}$$

Note that equations (1) and (2) differ only by the inertia in the numerator. The difference in equation (2) is clearly smaller, since $m_\text{A} \ll m_\text{A} + m_\text{B}$. Hence the velocity of the zero-momentum frame is closer to $\vec{v}_\text{B}$.

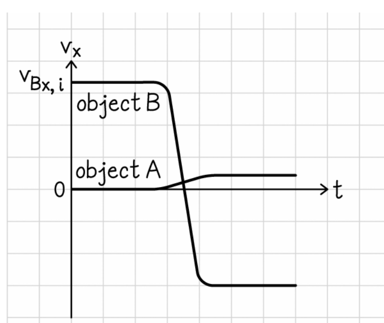
6.25. (*a*) Call the initial direction of motion of the truck the $+x$ axis. We can write the initial speed of the truck in SI units as $v_{\text{t,i}} = \dfrac{36\text{ km}}{1\text{ h}} \times \dfrac{10^3\text{ m}}{1\text{ km}} \times \dfrac{1\text{ h}}{3600\text{ s}} = 10\text{ m/s}$. We can write the conservation of momentum in the x direction, and solve for the final x component of the velocity as follows:

$$m_\text{t} v_{\text{t}\,x,\text{i}} + m_\text{c} v_{\text{c}\,x,\text{i}} = (m_\text{t} + m_\text{c}) v_{x,\text{f}}$$

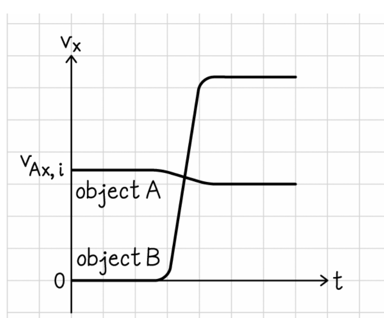
$$v_{x,\text{f}} = \frac{m_\text{t} v_{\text{t}\,x,\text{i}}}{m_\text{t} + m_\text{c}} = \frac{(4000\text{ kg})(10\text{ m/s})}{(4000\text{ kg} + 1000\text{ kg})} = 8.0\text{ m/s}$$

Hence the final speed of the truck and car together is 8.0 m/s. (*b*) If the jogger moves in the direction opposite the car and truck, then the car-truck combination approaches the jogger at a speed of 10 m/s. The momentum is then $\vec{p}_{\text{jogger car/truck}} = (m_\text{t} + m_\text{c})\vec{v}_{\text{jogger car/truck}} = ((4000\text{ kg}) + (1000\text{ kg}))(+10\text{ m/s})\,\hat{\imath} = (+5.0 \times 10^4\text{ kg}\cdot\text{m/s})\,\hat{\imath}$. (*c*) In order for the car-truck combination to be at rest in the jogger's frame, he must simply run at the same velocity as the car-truck combination is moving. Hence, his velocity must be $(+8.0\text{ m/s})\,\hat{\imath}$.

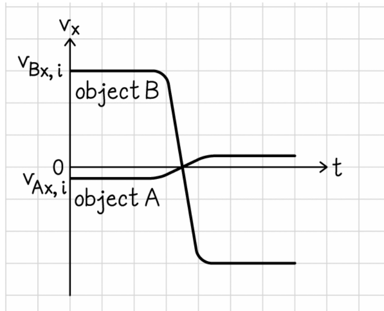
6.26. (*a*)



(*b*)



(*c*)



6.27. The answer certainly depends on the initial speeds. We will show this by determining a general expression for the ratio of the two kinetic energies, and verify that initial speeds do not somehow cancel by looking at a few example cases. Let the motion be only along the x axis. We can write the conservation of momentum and kinetic energy, respectively as

$$m_\mathrm{L}v_{\mathrm{L}\,x,\mathrm{i}} + m_\mathrm{H}v_{\mathrm{H}\,x,\mathrm{i}} = m_\mathrm{L}v_{\mathrm{L}\,x,\mathrm{f}} + m_\mathrm{H}v_{\mathrm{H}\,x,\mathrm{f}} \tag{1}$$

$$\frac{1}{2}m_\mathrm{L}v^2_{\mathrm{L}\,x,\mathrm{i}} + \frac{1}{2}m_\mathrm{H}v^2_{\mathrm{H}\,x,\mathrm{i}} = \frac{1}{2}m_\mathrm{L}v^2_{\mathrm{L}\,x,\mathrm{f}} + \frac{1}{2}m_\mathrm{H}v^2_{\mathrm{H}\,x,\mathrm{f}} \tag{2}$$

Solving equation (1) for the x component of the final velocity of the heavier object yields

$$v_{\mathrm{H}\,x,\mathrm{f}} = \frac{m_\mathrm{L}}{m_\mathrm{H}}(v_{\mathrm{L}\,x,\mathrm{i}} - v_{\mathrm{L}\,x,\mathrm{f}}) + v_{\mathrm{H}\,x,\mathrm{i}} \tag{3}$$

Inserting this into equation (2) yields

$$m_\mathrm{L}v^2_{\mathrm{L}\,x,\mathrm{i}} + m_\mathrm{H}v^2_{\mathrm{H}\,x,\mathrm{i}} = m_\mathrm{L}v^2_{\mathrm{L}\,x,\mathrm{f}} + m_\mathrm{H}\left(\frac{m_\mathrm{L}}{m_\mathrm{H}}(v_{\mathrm{L}\,x,\mathrm{i}} - v_{\mathrm{L}\,x,\mathrm{f}}) + v_{\mathrm{H}\,x,\mathrm{i}}\right)^2 \tag{4}$$

Equation (4) can be solved for $v_{\mathrm{L}\,x,\mathrm{f}}$:

$$v_{\mathrm{L}\,x,\mathrm{f}} = \frac{2m_\mathrm{H}v_{\mathrm{H}\,x,\mathrm{i}} - m_\mathrm{H}v_{\mathrm{L}\,x,\mathrm{i}} + m_\mathrm{L}v_{\mathrm{L}\,x,\mathrm{i}}}{m_\mathrm{H} + m_\mathrm{H}} \tag{4}$$

Re-inserting this result back into equation (3) yields a final expression for $v_{\mathrm{H}\,x,\mathrm{f}}$:

$$v_{\mathrm{H}\,x,\mathrm{f}} = \frac{m_\mathrm{H}v_{\mathrm{H}\,x,\mathrm{i}} - m_\mathrm{L}v_{\mathrm{H}\,x,\mathrm{i}} + 2m_\mathrm{L}v_{\mathrm{L}\,x,\mathrm{i}}}{m_\mathrm{H} + m_\mathrm{H}} \tag{5}$$

Using equations (4) and (5), we can easily write down the ratio of final kinetic energies:

$$\frac{K_{\mathrm{L\,f}}}{K_{\mathrm{H\,f}}} = \frac{\frac{1}{2}m_\mathrm{L}(2m_\mathrm{H}v_{\mathrm{H}\,x,\mathrm{i}} - m_\mathrm{H}v_{\mathrm{L}\,x,\mathrm{i}} + m_\mathrm{L}v_{\mathrm{L}\,x,\mathrm{i}})^2}{\frac{1}{2}m_\mathrm{H}(m_\mathrm{H}v_{\mathrm{H}\,x,\mathrm{i}} - m_\mathrm{L}v_{\mathrm{H}\,x,\mathrm{i}} + 2m_\mathrm{L}v_{\mathrm{L}\,x,\mathrm{i}})^2} \tag{6}$$

It may not be obvious that equation (6) does not reduce further to become independent of initial speeds. The easiest way to check this is through an example. Consider a collision between a heavy object with inertia 1.0 kg, and a lighter object with inertia 0.10 kg. Let us fix the initial x component of the velocity of the heavier object to be -1.0 m/s, and let us vary that of the lighter object. When $v_{\mathrm{L}\,x,\mathrm{i}} = 0.1$ m/s, we find $\frac{K_{\mathrm{L\,f}}}{K_{\mathrm{H\,f}}} = 0.56$. But when $v_{\mathrm{L}\,x,\mathrm{i}} = 1.1$ m/s, we find $\frac{K_{\mathrm{L\,f}}}{K_{\mathrm{H\,f}}} = 1.93$. Thus we can change which object carries away more kinetic energy just by changing one of the two initial speeds.

6.28. No. Let the objects' mutual speed be v, and let the direction of motion of one object (call it object A) be the $+x$ direction. Consider an observer who is initially moving in the $+x$ direction at a speed $3v$, and assume this observe is behind object A so that he is approaching both objects. Initially the observer would measure their speed of object A to be $2v,$ and the speed of the second object (B) to be $4v.$ Each object would be moving toward him, in the $-x$ direction. After the collision, the objects would switch speeds. Object A would be moving at a speed of $4v$ in the $-x,$ and object B would be moving at a speed of $2v$ in the $-x$ direction. Both objects move in the $-x$ direction before and after the collision, according to this observer.

6.29. The construction worker is moving upward relative to the ground a constant speed $v_{\text{const}}.$ In his reference frame, the ball is only moving upward as long as its upward speed relative to ground is greater than his own upward speed relative to the ground (while $v_{\text{ball}} > v_{\text{const}}$). Hence the construction worker would report that the ball has reached its maximum height (relative to him) when the ball's upward speed relative to the ground is reduced to $v_{\text{const}}.$ Clearly, it takes less time for the ball's upward speed to be reduced to $v_{\text{const}},$ than it takes for the ball to stop (relative to ground). Hence the construction worker reports a shorter time.

6.30. (*a*) We are given information in a mixture of reference frames. A good starting point would be express all quantities as measured in the Earth reference frame. Call the right the $+\hat{x}$ direction. In all cases, we can relate the velocity in the Earth reference frame to the velocity in the student reference frame using $\vec{v}_{\text{object E}} = \vec{v}_{\text{object S}} + \vec{v}_{\text{S E}}.$ Specifically we have $\vec{v}_{1\text{ E,i}} = 1.0 \text{ m/s}$ and $\vec{v}_{1\text{ E,f}} = 0.67 \text{ m/s } \hat{i}$ for the initial and final velocities of cart 1, and $\vec{v}_{2\text{ E,f}} = -1.0 \text{ m/s } \hat{i}$ for the final velocity of cart 2. We determine the initial velocity of cart 2 by using conservation of momentum in the $+\hat{x}$ direction:

$$m_1 v_{\text{E1 }x,\text{i}} + m_2 v_{\text{E2 }x,\text{i}} = m_1 v_{\text{E1 }x,\text{f}} + m_2 v_{\text{E2 }x,\text{f}}$$

$$v_{\text{E2 }x,\text{i}} = \frac{m_1}{m_2}(v_{\text{E1 }x,\text{f}} - v_{\text{E1 }x,\text{i}}) + v_{\text{E2 }x,\text{f}}$$

$$v_{\text{E2 }x,\text{i}} = \frac{0.36 \text{ kg}}{0.12 \text{ kg}}((0.67 \text{ m/s}) - (1.0 \text{ m/s})) + (-1.0 \text{ m/s})$$

$$v_{\text{E2 }x,\text{i}} = 0$$

(*b*) Note that in the students reference frame the initial velocity of cart 2 is $v_{\text{S2 }x,\text{i}} = -1.0 \text{ m/s}.$ Then the total initial momentum is $\vec{p}_{\text{S,i}} = \vec{p}_{\text{S1,i}} + \vec{p}_{\text{S2,i}} = (m_1 v_{\text{S1 }x,\text{i}} + m_2 v_{\text{S2 }x,\text{i}})\ \hat{i} = (0 + (0.12 \text{ kg})(-1.0 \text{ m/s}))\ \hat{i} = (-0.12 \text{ kg}\cdot\text{m/s})\ \hat{i}.$ (*c*) Using the velocities found in part (*a*) we have, $\vec{p}_{\text{E1,i}} = m_1 \vec{v}_{\text{E1,i}} = (0.36 \text{ kg})(1.0 \text{ m/s}) = (+0.36 \text{ kg}\cdot\text{m/s})\ \hat{i}$ and $\vec{p}_{\text{E2,i}} = m_2 \vec{v}_{\text{E2,i}} = 0.$

6.31. In Galilean relativity it is always true, because the amount of time that elapses between two events must be the same according to everyone. In special relativity, this is not the case.

6.32. No. If a frame is moving past the crash at a high speed, observers may see the two steps of the crash occur at drastically different places in their frame. Suppose for example that we are driving north on the highway and we witness such an accident across the median in a lane of traffic going south. We may look forward and to the left as we see the first collision. That is, the cars are slightly in front of us. We may continue craning our necks and see the second collision behind us. In our reference frame, the two collisions occurred in very different places. In the Earth frame, the car may only have moved a few centimeters.

6.33. We can answer this question intuitively, but we will also do a direct calculation. If our walking speed relative to the escalator is equal to the speed of the escalator relative to the ground, then walking up the escalator should double our speed relative to ground and cut our time in half. We do not quite succeed in cutting the time in half, so we do not walk relative to the escalator quite as quickly as the escalator moves relative to the ground. This means we cannot walk down the escalator; it will always move upward faster than we walk downward.

If one attempts to solve this problem algebraically, one should expect to obtain a nonsensical answer. We can write our speed relative to the ground in each of the cases described (standing still, walking upward, and walking downward, respectively) as follows:

$$v_{\text{E esc}} = \frac{d}{t_{\text{standing}}} \tag{1}$$

$$v_{\text{E esc}} + v_{\text{esc you}} = \frac{d}{t_{\text{upward}}} \tag{2}$$

$$v_{\text{you esc}} - v_{\text{E esc}} = \frac{d}{t_{\text{downward}}} \tag{3}$$

Inserting equation (1) into (2), and solving for $v_{\text{esc you}}$, allows us to write equation (3) as

$$\frac{d}{t_{\text{upward}}} - \frac{d}{t_{\text{still}}} = \frac{d}{t_{\text{downward}}} - \frac{d}{t_{\text{still}}}$$

or

$$t_{\text{downward}} = d\left(\frac{d}{t_{\text{upward}}} - \frac{2d}{t_{\text{still}}}\right)^{-1} = \left(\frac{1}{(20\text{ s})} - \frac{2}{(30\text{ s})}\right)^{-1} = -60\text{ s}$$

One could think of this negative time as being simply nonsense, or one could interpret it as meaning that if you are walking against the escalator and you are at the top of the escalator right now, you were at the bottom 60 seconds ago. Either way, we clearly see that we cannot walk down the "up" escalator.

6.34. In either case, we can relate the velocity of the woman in the Earth reference frame to her velocity in the train reference frame using $\vec{v}_{\text{E w}} = \vec{v}_{\text{E t}} + \vec{v}_{\text{t w}}$. In order for the woman to match her friend's velocity relative to Earth, we will set $\vec{v}_{\text{E w}} = \vec{v}_{\text{E f}}$. Call the train's direction of motion the $+x$ direction. (*a*) Here we have $\vec{v}_{\text{E t}} + \vec{v}_{\text{t w}} = \vec{v}_{\text{E f}}$ or $\vec{v}_{\text{t w}} = \vec{v}_{\text{E f}} - \vec{v}_{\text{E t}} =$ $(6.0\text{ m/s})\,\hat{\imath} - (4.0\text{ m/s})\,\hat{\imath} = (+2.0\text{ m/s})\,\hat{\imath}$. So the woman must walk at 2.0 m/s in the direction the train is moving. (*b*) As in part (*a*) we have $\vec{v}_{\text{t w}} = \vec{v}_{\text{E f}} - \vec{v}_{\text{E t}} = (6.0\text{ m/s})\,\hat{\imath} - (10\text{ m/s})\,\hat{\imath} = (-4\text{ m/s})\,\hat{\imath}$. So now the woman must walk at 4.0 m/s opposite the direction the train is moving.

6.35. (*a*) $T_{\text{calm}} = \dfrac{2d}{v}$ (*b*) $T_{\text{wind}} = T_{\text{headwind}} + T_{\text{tailwind}} = \dfrac{d}{v-w} + \dfrac{d}{v+w} = \dfrac{2dv}{(v^2 - w^2)}$ which can be related to T_{calm} using the following:

$$T_{\text{wind}} = \frac{2dv}{v^2 - w^2} = \frac{2d}{v}\left(\frac{v^2}{v^2 - w^2}\right) = \frac{2d}{v}\left(\frac{1}{1-(w/v)^2}\right)$$

(*b*) If we make the safe assumption that the wind speed is much less than the speed at which the plane can fly relative to the wind, then $w/v \ll 1$ and the above expression can be Taylor expanded around $w/v = 0$. This yields

$$T_{\text{wind}} = \frac{2d}{v}\left[1 + \left(\frac{w}{v}\right)^2 + \text{higher orders}\right]$$

$$T_{\text{wind}} \approx T_{\text{calm}}\left[1 + \left(\frac{w}{v}\right)^2\right]$$

6.36. The inertia would still be m for the moving observer. For an observer moving at a velocity $\vec{v}$ the momentum of the object would be $\vec{p} = -m\vec{v}$.

6.37. Let the origin of our coordinate system be at the center of Earth, and let the moon lie along the $+x$ axis. We can write the position of the center of mass along the x axis as:

$$x_{\text{cm}} = \frac{m_{\text{E}}x_{\text{E}} + m_{\text{M}}x_{\text{M}}}{m_{\text{E}} + m_{\text{M}}}$$

Note that $x_E = 0$ because the center of mass of Earth is at the origin of our coordinate system. Thus

$$x_{cm} = \frac{m_M x_M}{m_E + m_M} = \frac{(7.35\times10^{22}\ \text{kg})(3.84\times10^{8}\ \text{m})}{(7.35\times10^{22}\ \text{kg}) + (5.97\times10^{24}\ \text{kg})} = 4.66\times10^{6}\ \text{m}$$

Note that this distance of 4.66×10^{3} km is less than the radius of Earth, meaning the center of mass is inside Earth.

6.38. Let us work in axes that have their origin at the left-most point of the left-most object, and let the $+x$ axis point to the right. (*a*) We can find the center of mass of the collection of objects by treating each object like a small particle at its center of mass. For example, the center of mass of the left-most object would be at its geometric center, at the position along the x axis, $x = 0.50$ m. The centers of the middle and right-most objects are at $x = 2.0$ m and $x = 4.0$ m. We can write the inertia of any object as $m = \rho V$ where ρ is the density and V is the volume. In the case of these circular objects, cut from sheet metal of thickness t, we have for each object $m = \rho t \pi R^2$. The radius is different for each of the objects, but the material and thickness of the sheet is the same in each case. We will use subscripts ℓ, m, and r for "left-most", "middle", and "right-most". Finally, we can write the position of the center of mass as

$$x_{cm} = \frac{m_\ell x_\ell + m_m x_m + m_r x_r}{m_\ell + m_m + m_r}$$

$$x_{cm} = \frac{\rho t \pi (R_\ell^2 x_\ell + R_m^2 x_m + R_r^2 x_r)}{\rho t \pi (R_\ell^2 + R_m^2 + R_r^2)}$$

$$x_{cm} = \frac{((0.50\ \text{m})^2(0.50\ \text{m}) + (1.0\ \text{m})^2(2.0\ \text{m}) + (1.5\ \text{m})^2(4.50\ \text{m}))}{((0.50\ \text{m})^2 + (1.0\ \text{m})^2 + (1.50\ \text{m})^2)}$$

$$x_{cm} = 3.5\ \text{m}$$

Hence the center of mass is 3.5 m from the left edge of the left-most circle, along the line connected the centers of the three circles. (*b*) Now that we have spheres, the expression for the inertia changes to $m = \rho t \frac{4}{3}\pi R^3$. Otherwise the solving process is the same. Using the same axes and symbols as in part (*a*) we can write

$$x_{cm} = \frac{m_\ell x_\ell + m_m x_m + m_r x_r}{m_\ell + m_m + m_r}$$

$$x_{cm} = \frac{\rho t \frac{4}{3}\pi (R_\ell^3 x_\ell + R_m^3 x_m + R_r^3 x_r)}{\rho t \frac{4}{3}\pi (R_\ell^3 + R_m^3 + R_r^3)}$$

$$x_{cm} = \frac{((0.50\ \text{m})^3(0.50\ \text{m}) + (1.0\ \text{m})^3(2.0\ \text{m}) + (1.5\ \text{m})^3(4.50\ \text{m}))}{((0.50\ \text{m})^3 + (1.0\ \text{m})^3 + (1.50\ \text{m})^3)}$$

$$x_{cm} = 3.8\ \text{m}$$

Hence the center of mass is 3.8 m from the left edge of the left-most sphere, along the line connecting the centers of the three spheres.

6.39. (*a*) Because no outside forces are exerted on the system of the two children and their rafts, the initially stationary center of mass will remain stationary. Since the boy and his raft have half the inertia of the girl and her raft, the boy's raft has to move twice as far in any interval in order for the center of mass to remain in the same spot. Since the toy starts out halfway between them, and the boy will cover more distance in any time interval, clearly the boy will reach the toy first. (*b*) Since $v_b = 2v_g$, we can relate the distance covered by the children in a time interval Δt using $\Delta x_b = v_b \Delta t = 2v_g \Delta t = 2\Delta x_g$. Thus, when the boy reaches the toy (having travelled 5.0 m), the girl has moved only 2.5 m. The rafts started out 10 m apart. The boy covered 5.0 m of the distance between them and the girl covered 2.5 m. This leaves 2.5 m between the girl and the boy when he reaches the toy.

6.40. No. The same impulse is delivered in either case. This means the hammer causes the same change in momentum of the system of blocks in either case. Since the system is made up the same blocks in either case, equal final momenta indicates equal final velocities. Hence the center of mass of the system will move with the same velocity in either case.

6.41. (*a*) We can treat the entire baton as being made up of three objects with very simple geometry: the large sphere on the left, the thin rod in the middle and the smaller sphere on the right. We use subscripts ls, rod, and rs respectively for these objects. We can treat each object like a point with all its inertia concentrated at its center of mass. All objects are assumed to be homogeneous, such that their centers of mass correspond to their geometric centers. Then, with our origin at the center of left sphere and the $+x$ axis pointing to the right, we have

$$x_{\text{cm}} = \frac{m_{\text{ls}}x_{\text{cm ls}} + m_{\text{rod}}x_{\text{cm rod}} + m_{\text{rs}}x_{\text{cm rs}}}{m_{\text{ls}} + m_{\text{rod}} + m_{\text{rs}}}$$

$$x_{\text{cm}} = \frac{(0.20\text{ kg})(0) + (0.10\text{ kg})(0.50\text{ m}) + (0.10\text{ kg})(1.0\text{ m})}{(0.20\text{ kg}) + (0.10\text{ kg}) + (0.10\text{ kg})}$$

$$x_{\text{cm}} = 0.38\text{ m}$$

So the center of mass is 0.38 m to the right of the center of the large sphere. (*b*) One could repeat the steps in part (*a*) entering different positions for the centers of the objects (namely, $x_{\text{cm ls}} = -1.0\text{ m},$ $x_{\text{cm rod}} = -0.50\text{ m},$ and $x_{\text{cm rs}} = 0$). This would be a valid way of solving the problem, but this is not necessary. We have already found the position of the center of mass. That point in space is fixed. If we move our origin to the right, we simply have to translate the coordinates of the center of mass accordingly: $x_{\text{cm (b)}} = x_{\text{cm (a)}} - 1.0\text{ m}.$ Hence the position of the center of mass is now $-0.63\text{ m},$ or 0.63 m to the left of the center of the small sphere. (*c*) As in part (*b*), we simply translate our coordinates to find that the center of mass is 1.38 m to the right of the point indicated. The center of mass location must be calculated for only one origin choice; that location can then be expressed relative to the other two origin choices with almost no calculation.

6.42. Call the side length of the cubic box $\ell,$ and assume the bottom of the box lies in the xy plane, with the z axis pointing vertically upward. Call the inertia of one face of the box $m.$ We consider each face of the box to be homogeneous, such that the center of mass of each face lies at its geometric center. Thus the four vertical sides have their centers of mass at a height $\ell/2.$ The bottom of the box has its center of mass at a height 0. Thus the center of mass of the box is at a height given by

$$z_{\text{cm}} = \frac{4m(\ell/2) + m(0)}{5m} = \frac{2}{5}\ell$$

The center of mass is a height $\frac{2\ell}{5}$ above the bottom.

6.43. The center of mass of an isolated system should move at a constant velocity. If it does not, then you are measuring from a non-inertial reference frame.

6.44. As suggested, let us start by considering a complete cone, rather than the truncated cone. Rather than taking a discrete sum of the weighted positions of individual objects of inertia $m,$ in this case we integrate over a continuous distribution of inertia. Here

$$z_{\text{cm}} = \frac{1}{M}\int z\,dm \qquad (1)$$

Where M is the total inertia of the cue, z is the distance from the origin to an infinitesimal particle of inertia $dm.$ Let us choose our z axis to be on the axis of symmetry of the cone with the origin at the vertex, and call the angle between this z axis and the conical surface of the cue $\theta.$ With these choices, we can write the radius of a circular cross section of the cue a distance z from the vertex as $R = z\tan(\theta).$ We can write $dm = \rho dV = A\rho dz,$ where ρ is the density of the cue, and A is the cross-sectional area of the cue at a given distance from the end. Equivalently, $dm = \pi(z\tan(\theta))^2\rho dz.$ Inserting this into equation (1) yields

$$z_{\text{cm}} = \frac{\pi\rho\tan^2(\theta)}{M}\int z^3 dz \qquad (2)$$

We need two things before we can proceed. First, we need an expression for the total inertia M in terms of other variables used. This comes from integrating over all infinitesimal contributions to the inertia:

$$M = \int dm = \pi\rho\tan^2(\theta)\int z^2 dz \tag{3}$$

The second thing we need is appropriate limits on our integral. Remember that we want to integrate over an entire cone first. We will subtract off the end-piece later. The truncated cone has a length of 1.40 m, and a width of 0.040 m at the thick end. Note that the width of the cue changes by 15 mm on either side of the center over the length of 1.40 m. This shows us that $\tan(\theta) = \dfrac{0.015\text{ m}}{1.40\text{ m}} = \dfrac{0.020\text{ m}}{z_{\max}}$ or $z_{\max} = 1.87$ m. This means that the end piece we will later subtract from our answer has a length of 0.47 m. We will call this value $z_{\text{short}} = 0.47$. With these endpoints we can write the total inertia of the entire cone using equation (3) as

$$m_{\text{total}} = \pi\rho\tan^2(\theta)\int_0^{z_{\max}} z^2 dz = \frac{\pi}{3}\rho\tan^2(\theta)z_{\max}^3 \tag{4}$$

Similarly, the inertia of the truncated cone, and short end-piece are, respectively

$$m_{\text{truncated}} = \pi\rho\tan^2(\theta)\int_{z_{\text{short}}}^{z_{\max}} z^2 dz = \frac{\pi}{3}\rho\tan^2(\theta)(z_{\max}^3 - z_{\text{short}}^3) \tag{5}$$

and

$$m_{\text{short}} = \pi\rho\tan^2(\theta)\int_0^{z_{\text{short}}} z^2 dz = \frac{\pi}{3}\rho\tan^2(\theta)z_{\text{short}}^3 \tag{6}$$

Inserting equation (4) and the appropriate integration limits into equation (2) finally yields

$$z_{\text{cm total}} = \frac{3}{z_{\max}^3}\int_0^{z_{\max}} z^3 dz = \frac{3}{4}z_{\max}$$

Inserting equation (6) into (2) yields the position of the center of mass of the short end-piece

$$z_{\text{cm short}} = \frac{3}{z_{\text{short}}^3}\int_0^{z_{\text{short}}} z^3 dz = \frac{3}{4}z_{\text{short}}$$

Now we will write the center of mass of a complete cone (total) in terms of the centers of mass of the truncated cone and shorter end-piece:

$$z_{\text{cm total}} = \frac{m_{\text{truncated}}z_{\text{cm truncated}} + m_{\text{short}}z_{\text{cm short}}}{m_{\text{total}}}$$

Or equivalently

$$z_{\text{cm truncated}} = \frac{m_{\text{total}}z_{\text{cm total}} - m_{\text{short}}z_{\text{cm short}}}{m_{\text{truncated}}}$$

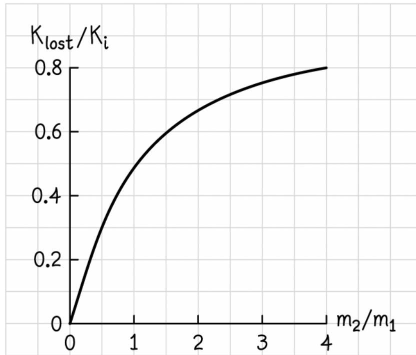
$$z_{\text{cm truncated}} = \frac{\left(\frac{\pi}{3}\rho\tan^2(\theta)z_{\max}^3\right)\left(\frac{3}{4}z_{\max}\right) - \left(\frac{\pi}{3}\rho\tan^2(\theta)z_{\text{short}}^3\right)\left(\frac{3}{4}z_{\text{short}}\right)}{\left(\frac{\pi}{3}\rho\tan^2(\theta)(z_{\max}^3 - z_{\text{short}}^3)\right)}$$

$$z_{\text{cm truncated}} = \left(\frac{3}{4}\right)\left(\frac{z_{\max}^4 - z_{\text{short}}^4}{z_{\max}^3 - z_{\text{short}}^3}\right)$$

$$z_{\text{cm truncated}} = \left(\frac{3}{4}\right)\left(\frac{(1.87\text{ m})^4 - (0.47\text{ m})^4}{(1.87\text{ m})^3 - (0.47\text{ m})^3}\right) = 1.42\text{ m}$$

This distance is measured from the vertex of the (imaginary) complete cone. It would be more sensible to measure the center of mass from the opposite end (the thick end of the cue). Thus we subtract the above distance from the length of the complete cone: $x_{\text{cm thick}} = (1.87\text{ m}) - (1.42\text{ m}) = 0.45\text{ m}$. Hence the distance from the thick end of the cue is 0.45 m.

6.45.



As m_2/m_1 approaches infinity, the fraction of kinetic energy converted to other forms approaches one, meaning that all kinetic energy is converted.

6.46. Equation 6.38 tells us that we can write the convertible kinetic energy as $K_{\text{conv}} = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)v_{12}^2$. The speed of object 2 (which we call the heavier object) relative to object 1 (lighter object) in this case is 3.0 m/s. Because object 2 is initially stationary in the Earth frame, the total kinetic energy in the Earth frame is clearly $K = \frac{1}{2}m_1 v_1^2$. Note that in this case the relative speed of the two objects is the same as the speed of object 1. So we have $\frac{K_{\text{conv}}}{K} = \left(\frac{m_2}{m_1 + m_2}\right)\frac{v_{12}^2}{v_1^2} = \left(\frac{m_2}{m_1 + m_2}\right) = \left(\frac{(7.0\times10^{-3}\ \text{kg})}{(3.0\times10^{-3}\ \text{kg}) + (7.0\times10^{-3}\ \text{kg})}\right) = 0.70$. Thus 70% of the kinetic energy is convertible.

6.47. The reduced inertia is given by $\mu = \frac{m_1 m_2}{m_1 + m_2}$. We can rewrite this as

$$\mu = \frac{m_1}{(m_1/m_2) + 1} \tag{1}$$

Now it becomes obvious that $\mu \le m_1$, since $(m_1/m_2) + 1 \ge 1$ with equality holding only if $m_1 = 0$. The reduced inertia cannot be larger than m_1 at all. We also have the stipulation that $m_1 < m_2$, meaning that $(m_1/m_2) + 1 < 2$. Inserting this into equation (1) shows that $\mu > \frac{m_1}{2}$. Thus, the bounds on the value of the reduced inertia can be summarized $\frac{m_1}{2} < \mu \le m_1$.

6.48. (*a*) Yes, the center of mass reference frame. One easy way to see this is by considering PRIN equation (6.35): $K = K_{\text{cm}} + K_{\text{conv}}$. K_{conv} depends only on the relative speed between particles, so cannot be reduced by changing the frame of the observer. So minimizing K_{cm} is equivalent to minimizing the total kinetic energy. Obviously K_{cm} is minimized in the center of mass reference frame (where it is zero). (*b*) No, the kinetic energy can be arbitrarily large. This is clearly the case, since K_{cm} will grow without bound as choose a reference frame with a velocity that differs from the center of mass velocity by a larger and larger amount.

6.49. Because we are only interested in the nonconvertible kinetic energy, we only need to find in which reference frame is the center of mass moving the most quickly. Because the bat has so much more inertia than the ball, most of the inertia of the system moves with the bat. This means that in the reference frame of the bat, the center of mass moves relatively slowly. To the ball, however, most of the inertia of the system is rushing toward it, meaning that the center of mass is moving very quickly. Thus the center of mass is moving more quickly in the ball's reference frame, and the system has greater nonconvertible energy in the reference frame in which the ball is at rest.

6.50. In case 1, the center of mass of the system is initially stationary, so there will be no kinetic energy after the collision (both cars will come to rest). Call the initial speed of each car v. Then the converted kinetic energy is equal to the initial kinetic energy such that $K_1 = 2\left(\frac{1}{2}mv^2\right) = mv^2$. In case 2, the initial speed of the center of mass is $\frac{v}{2}$, so that will be the final speed of the center of mass. We calculate the kinetic energy converted by taking the difference between the initial and final energies: $K_2 = \frac{1}{2}mv^2 - 2\left(\frac{1}{2}m\left(\frac{v}{2}\right)^2\right) = \frac{1}{2}mv^2 - \frac{1}{4}mv^2 = \frac{1}{4}mv^2$. Comparing the two cases, we can write $\frac{K_1}{K_2} = \frac{mv^2}{\frac{1}{4}mv^2} = 4.$

6.51. (*a*) Call the initial direction of the paintball the $+x$ direction. Because the disk is initially at rest, the velocity of the paintball relative to the disk is $K_2\ v_{\text{d p}} = (+11\text{ m/s})\ \hat{i}$. (*b*) We simply insert numerical values into PRIN equation (6.39):

$$\mu = \frac{m_\text{d} m_\text{p}}{m_\text{d} + m_\text{p}} = \frac{(0.075\text{ kg})(0.050\text{ kg})}{(0.075\text{ kg}) + (0.050\text{ kg})} = 0.030\text{ kg}$$

(*c*) There are several ways of going about this problem. One way would be to note that after the collision, the only kinetic energy left is the center of mass kinetic energy. So the amount of energy converted to interval energy is exactly equal to the initial convertible energy. That is, all convertible energy is actually converted in this case. Calculating the initial kinetic energy and the initial convertible energy allows us to calculate the ratio required:

$$\frac{K_\text{conv}}{K_\text{i}} = \frac{\frac{1}{2}\mu v_{\text{d p}}^2}{\frac{1}{2}m_\text{p} v_\text{p}^2} = \frac{m_\text{d} v_{\text{d p}}^2}{(m_\text{d} + m_\text{p})v_\text{p}^2} = \frac{(0.075\text{ kg})(11\text{ m/s})^2}{((0.075\text{ kg}) + (0.050\text{ kg}))(11\text{ m/s})^2} = 0.60$$

Thus 60% of the initial kinetic energy is converted to internal energy. This is the most efficient way of answering the question. But alternatively, one could determine the final velocity of the paint and disk together either by using the conservation of momentum, or by requiring that the center of mass of the system move at a constant velocity. An astute physicist will note that these two lines of thinking are completely equivalent. Either way, one finds a final velocity of $\vec{v}_\text{cm,f} = (+4.4\text{ m/s})\ \hat{i}$. Using this one can obtain the same result as above by calculating the initial and final kinetic energies explicitly. (*d*) The relative velocity of the objects does not depend on the reference frame. Thus the convertible kinetic energy is the same in all reference frames. Also, all observers agree that the two objects come completely to rest relative to each other after the collision. Thus in this new reference frame we can still say that the change in kinetic energy is exactly equal to the initial convertible kinetic energy:

$$|\Delta K| = K_\text{conv,i} = \frac{1}{2}\mu v_\text{dp,i}^2 = \frac{1}{2}\left(\frac{m_\text{d} m_\text{p}}{m_\text{d} + m_\text{p}}\right)v_\text{dp,i}^2 = \frac{1}{2}\left(\frac{(0.075\text{ kg})(0.050\text{ kg})}{(0.075\text{ kg}) + (0.050\text{ kg})}\right)(11\text{ m/s})^2 = 1.8\text{ J}$$

(*e*) In order to explicitly calculate the final kinetic energy, we need the final velocity of the system. This was stated in part (*c*), but not shown explicitly. Here we require that the initial and final velocities of the center of mass of the system be equal:

$$v_\text{cm,i} = v_\text{cm,f}$$

$$\frac{m_\text{d}\vec{v}_\text{d,i} + m_\text{p}\vec{v}_\text{p,i}}{m_\text{d} + m_\text{p}} = \frac{(m_\text{d} + m_\text{p})\vec{v}_\text{p,f}}{m_\text{d} + m_\text{p}}$$

$$\vec{v}_\text{p,f} = \frac{m_\text{d}\vec{v}_\text{d,i} + m_\text{p}\vec{v}_\text{p,i}}{m_\text{d} + m_\text{p}}$$

$$\vec{v}_\text{p,f} = \frac{(0.075\text{ kg})(0) + (0.050\text{ kg})(+11\text{ m/s})\ \hat{i}}{((0.075\text{ kg}) + (0.050\text{ kg}))}$$

$$\vec{v}_\text{p,f} = (+4.4\text{ m/s})\ \hat{i}$$

Now we simply calculate the initial and final kinetic energies:

$$K_\text{i} = \frac{1}{2}m_\text{p}v_\text{p,i}^2 = \frac{1}{2}(0.050\text{ kg})(11\text{ m/s})^2 = 3.03\text{ J}$$

$$K_\text{f} = \frac{1}{2}(m_\text{p}+m_\text{d})v_\text{f}^2 = \frac{1}{2}((0.050\text{ kg})+(0.075\text{ kg}))(4.4\text{ m/s})^2 = 1.21\text{ J}$$

Comparing the two energies shows that $|\Delta K| = |K_\text{f} - K_\text{i}| = (3.03\text{ J}) - (1.21\text{ J}) = 1.8\text{ J},$ which was exactly our result from part (*d*).

6.52. (*a*) The amount of kinetic energy that is not available to be converted is the center of mass kinetic energy. So, we want the ratio

$$\frac{K_\text{cm}}{K_\text{E,i}} = \frac{\frac{1}{2}(m_\text{moving}+m_\text{rest})v_\text{cm}^2}{\frac{1}{2}m_\text{moving}v_\text{moving,i}^2}$$

And the center of mass speed is just $v_\text{cm} = m_\text{moving}v_\text{moving}/(m_\text{moving}+m_\text{rest})$. Inserting this into the above equation yields

$$\frac{K_\text{cm}}{K_\text{E,i}} = \frac{\frac{1}{2}(m_\text{moving}+m_\text{rest})\left(\frac{m_\text{moving}v_\text{moving}}{m_\text{moving}+m_\text{rest}}\right)^2}{\frac{1}{2}m_\text{moving}v_\text{moving,i}^2}$$

$$\frac{K_\text{cm}}{K_\text{E,i}} = \frac{m_\text{moving}}{m_\text{moving}+m_\text{rest}}$$

(*b*) This ratio does not depend on the elasticity. This is not the fraction of kinetic energy that is actually converted to a different form. This is the fraction that is available for conversion. (*c*) This is the kinetic energy of the center of mass of the system. Since the center of mass of a closed system should continue moving at the same velocity, this center of mass kinetic energy must remain kinetic energy.

6.53. (*a*) Call the direction of motion in which we throw ball 2 the $+x$ direction. We can relate the x components of the initial and final velocities of ball 2 using the coefficient of restitution given:

$$v_{12\,x,\text{f}} = v_{1\,x,\text{f}} - v_{2\,x,\text{f}} = -(0.90)v_{12,\text{i}} = -(0.90)(v_{1\,x,\text{i}} - v_{2\,x,\text{i}})$$

or

$$v_{2\,x,\text{i}} = \frac{v_{1\,x,\text{f}} - v_{2\,x,\text{f}}}{(0.90)} + v_{1\,x,\text{i}} \qquad (1)$$

We can now use conservation of momentum in the x direction to determine the final and initial velocity of ball 2. We write

$$m_1v_{1\,x,\text{i}} + m_2v_{2\,x,\text{i}} = m_1v_{1\,x,\text{f}} + m_2v_{2\,x,\text{f}}$$

and insert equation (1) and solve for $v_{2\,x,\text{f}}$ to obtain

$$v_{2\,x,\text{f}} = \frac{1}{(1+(1/e))}\left(\frac{m_1}{m_2}(v_{1\,x,\text{i}} - v_{1\,x,\text{f}}) + \frac{v_{1\,x,\text{f}}}{e} + v_{1\,x,\text{i}}\right)$$

$$v_{2\,x,\text{f}} = \frac{1}{(1+(1/0.90))}\left(\frac{(0.500\text{ kg})}{(0.600\text{ kg})}((-10\text{ m/s})-(10\text{ m/s})) + \frac{(10\text{ m/s})}{(0.90)} + (-10\text{ m/s})\right)$$

$$v_{2\,x,\text{f}} = -7.37\text{ m/s}$$

Inserting this result into equation (1) above yields $v_{2\,x,\text{i}} = 9.3\text{ m/s}$.

Thus ball 2 must be travelling at 9.3 m/s toward ball 1 (in the $+x$ direction).

(*b*) $v_{2\,x,\mathrm{i}} - v_{1\,x,\mathrm{i}} = (9.3\text{ m/s}) - (-10\text{ m/s}) = 19\text{ m/s}.$ Thus the initial relative velocity of the two balls is 19 m/s toward each other. (*c*) The reduced inertia is given by $\mu = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{(0.500\text{ kg})(0.600\text{ kg})}{(0.500\text{ kg}) + (0.600\text{ kg})} = 0.273\text{ kg}.$ (*d*) The fraction of the initial kinetic energy that is convertible is

$$\frac{K_{\text{conv}}}{K_{\text{i}}} = \frac{1}{\dfrac{K_{\text{cm}}}{K_{\text{conv}}} + 1} = \frac{1}{\dfrac{\frac{1}{2}(m_1 + m_2)v_{\text{cm}}^2}{\frac{1}{2}\left(\dfrac{m_1 m_2}{m_1 + m_2}\right)v_{12}^2} + 1} = \left(\frac{(m_1 v_{1,\text{i}} + m_2 v_{2,\text{i}})^2}{m_1 m_2 v_{12}^2} + 1\right)^{-1}$$

$$= \left(\frac{((0.500\text{ kg})(-10\text{ m/s}) + (0.600\text{ kg})(9.30\text{ m/s}))^2}{(0.500\text{ kg})(0.600\text{ kg})(19.3\text{ m/s})^2} + 1\right)^{-1} = 0.997$$

So, 99.7% of the initial kinetic energy is convertible. (*e*) $\vec{v}_{1\text{f}} = -10\text{ m/s}\,\vec{i}$ or 10 m/s toward you, and $\vec{v}_{2\text{f}} = +7.4\text{ m/s}\,\vec{i}$ or 7.4 m/s away from you.

6.54. (*a*) We can use the same process as in problem 53. Call the direction of motion in which we throw ball 2 the $+x$ direction. We can relate the x components of the initial and final velocities of ball 2 using the coefficient of restitution given:

$$v_{12\,x,\mathrm{f}} = v_{1\,x,\mathrm{f}} - v_{2\,x,\mathrm{f}} = -(0.90)v_{12,\mathrm{i}} = -(0.90)(v_{1\,x,\mathrm{i}} - v_{2\,x,\mathrm{i}})$$

or

$$v_{2\,x,\mathrm{i}} = \frac{v_{1\,x,\mathrm{f}} - v_{2\,x,\mathrm{f}}}{(0.90)} + v_{1\,x,\mathrm{i}} \tag{1}$$

We can now use conservation of momentum in the x direction to determine the final and initial velocity of ball 2. We write

$$m_1 v_{1\,x,\mathrm{i}} + m_2 v_{2\,x,\mathrm{i}} = m_1 v_{1\,x,\mathrm{f}} + m_2 v_{2\,x,\mathrm{f}}$$

and insert equation (1) and solve for $v_{2\,x,\mathrm{f}}$ to obtain

$$v_{2\,x,\mathrm{f}} = \frac{1}{(1 + (1/e))}\left(\frac{m_1}{m_2}(v_{1\,x,\mathrm{i}} - v_{1\,x,\mathrm{f}}) + \frac{v_{1\,x,\mathrm{f}}}{e} + v_{1\,x,\mathrm{i}}\right)$$

$$v_{2\,x,\mathrm{f}} = \frac{1}{(1 + (1/0.90))}\left(\frac{(0.500\text{ kg})}{(0.600\text{ kg})}((+5.0\text{ m/s}) - (10\text{ m/s})) + \frac{(10\text{ m/s})}{(0.90)} + (5.0\text{ m/s})\right)$$

$$v_{2\,x,\mathrm{f}} = 5.7\text{ m/s}$$

Inserting this result into equation (1) above yields $v_{2\,x,\mathrm{i}} = 9.8\text{ m/s}.$ Thus ball 2 must be travelling at 9.8 m/s toward ball 1 (in the $+x$ direction). (*b*) We know $v_{2\,x,\mathrm{i}} - v_{1\,x,\mathrm{i}} = (9.8\text{ m/s}) - (5.0\text{ m/s}) = 4.8\text{ m/s},$ such that the relative velocity of the balls is 4.8 m/s toward each other. (*c*) The reduced inertia is given by $\mu = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{(0.500\text{ kg})(0.600\text{ kg})}{(0.500\text{ kg}) + (0.600\text{ kg})} = 0.273\text{ kg}.$ (*d*) The fraction of the initial kinetic energy that is convertible is

$$\frac{K_{\text{conv}}}{K_{\text{i}}} = \frac{1}{\dfrac{K_{\text{cm}}}{K_{\text{conv}}} + 1} = \frac{1}{\dfrac{\frac{1}{2}(m_1 + m_2)v_{\text{cm}}^2}{\frac{1}{2}\left(\dfrac{m_1 m_2}{m_1 + m_2}\right)v_{12}^2} + 1} = \left(\frac{(m_1 v_{1,\text{i}} + m_2 v_{2,\text{i}})^2}{m_1 m_2 v_{12}^2} + 1\right)^{-1}$$

$$= \left(\frac{((0.500\text{ kg})(+5.0\text{ m/s}) + (0.600\text{ kg})(9.8\text{ m/s}))^2}{(0.500\text{ kg})(0.600\text{ kg})(4.8\text{ m/s})^2} + 1\right)^{-1} = 0.090$$

So, 9.0% of the initial kinetic energy is convertible. (*e*) $\vec{v}_{1f} = +10$ m/s $\vec{i}$ or 10 m/s away from you, and $\vec{v}_{2f} = +5.7$ m/s $\vec{i}$, or 5.7 m/s away from you.

6.55. We write the expressions for the convertible energy in each case and compare. Call the case of equal inertias case 1, and the case of unequal inertias case 2.

$$K_{\text{conv},1} = \frac{1}{2}\frac{m^2}{2m}v_{12}^2$$
$$K_{\text{conv},2} = \frac{1}{2}\frac{m_1 m_2}{2m}v_{12}^2$$

Because the relative speed is the same in either case, comparing these two convertible kinetic energies clearly becomes a comparison between the quantities m^2 and $m_1 m_2$ where $m_1 + m_2 = 2m$. Perhaps you can see logically, that the quantity $m_1 m_2$ is greatest when each inertia is equal to m. If this is not clear from familiarity with squaring numbers, we could find the maximum possible value of the expression $m_1 m_2$, and compare that to m^2. To maximize $m_1 m_2$, we write it as $m_1(2m - m_1)$. When this expression is maximal, its derivative with respect to m_1 must be zero. So we differentiate with respect to m_1 and find:

$$\frac{\partial}{\partial m_1}(m_1(2m - m_1)) = 0$$
$$2m - 2m_1 = 0$$
$$m_1 = m$$

Hence the quantity $m_1 m_2$ is maximal when both $m_1 = m$ and $m_2 = m$. So for $m_1 < m_2$ the quantity m^2 is greater than $m_1 m_2$. This tells us that the convertible energy is greater in the system where both objects have inertia m.

6.56. (*a*) Since the skier and her cat are finally moving together, there is no convertible kinetic energy after the collision. Thus all the initial convertible kinetic energy is converted in this collision. Noting that the relative speed is initially 18.8 m/s, we can find the converted kinetic energy using:

$$K_{\text{conv}} = \frac{1}{2}\left(\frac{m_s m_c}{m_s + m_c}\right)v_{sc}^2 = \frac{1}{2}\left(\frac{(60\text{ kg})(5.0\text{ kg})}{(60\text{ kg}) + (5.0\text{ kg})}\right)(18.8\text{ m/s})^2 = 8.2 \times 10^2\text{ J}$$

(*b*) After the collision the speed of the cat is given by the speed of the center of mass of the system. Let us call "north" the $+x$ direction. Then the final x component of the cat's velocity is

$$v_{c,f} = \frac{m_s v_{s\,x,i} + m_c v_{c\,x,i}}{m_s + m_c} = \frac{(60\text{ kg})(15\text{ m/s}) + (5.0\text{ kg})(-3.8\text{ m/s})}{(60\text{ kg}) + (5.0\text{ kg})} = 13.6\text{ m/s}$$

Clearly, the cat's speed changes, so the only way for an observer to measure the cat's kinetic energy to be the same is if the cat moves southward relative to the observe initially, and northward relative to the observer after the collision. That is, the speed relative to the observer is the same initially and finally, but the observer is initially moving faster north than the cat, and the cat is finally moving faster north than the observer. This means

$$(v_{o\,x} - v_{c\,x,i}) = -(v_{o\,x} - v_{c\,x,f})$$
$$v_{o\,x} = \frac{v_{c\,x,i} + v_{c\,x,f}}{2} = \frac{(-3.8\text{ m/s}) + (13.6\text{ m/s})}{2}$$
$$v_{o\,x} = 4.9\text{ m/s}$$

So an observer walking 4.9 m/s to the north would measure the cat's kinetic energy to be the same before and after the collision.

6.57. (*a*) Call the direction in which you throw the ball the $+x$ direction. The velocity of the center of mass should not change. So, once the ball and the dog are moving together, their mutual velocity should still be $\vec{v}_{cm} = \dfrac{m_b \vec{v}_{b,i} + m_d \vec{v}_{d,i}}{m_b + m_d} = \dfrac{(0.40\text{ kg})(+9.0\text{ m/s})\,\hat{i} + (0)}{(0.40\text{ kg}) + (14\text{ kg})} = (+0.25\text{ m/s})\,\hat{i}$. Thus the dog will move at 0.25 m/s in the direction you threw the ball after the collision. (*b*) In order for the ball to have the same kinetic energy according to this observer, the ball must have the same speed before and after the collision. Since the velocity of the ball does change, this must mean that the observer was slower than the ball by some amount before the collision and was faster than the ball by the same amount after the collision. This means:

$$(v_{\mathrm{o}\,x} - v_{\mathrm{b}\,x,\mathrm{i}}) = -(v_{\mathrm{o}\,x} - v_{\mathrm{b}\,x,\mathrm{f}})$$

$$v_{\mathrm{o}\,x} = \frac{v_{\mathrm{b}\,x,\mathrm{i}} + v_{\mathrm{b}\,x,\mathrm{f}}}{2} = \frac{(9.0\text{ m/s}) + (0.25\text{ m/s})}{2}$$

$$v_{\mathrm{o}\,x} = 4.6\text{ m/s}$$

So, an observer moving 4.6 m/s in the direction the ball is thrown will measure its kinetic energy to be the same before and after the collision. (*c*) The convertible kinetic energy is given by $K_{\mathrm{conv}} = \frac{1}{2}\left(\frac{m_{\mathrm{b}}m_{\mathrm{d}}}{m_{\mathrm{b}} + m_{\mathrm{d}}}\right)v_{\mathrm{bd}}^2 = \frac{1}{2}\left(\frac{(0.40\text{ kg})(14\text{ kg})}{(0.40\text{ kg}) + (14\text{ kg})}\right)(9.0\text{ m/s})^2 = 16\text{ J}$. So 16 J of the original kinetic energy was convertible. (*d*) The relative velocity of objects does not depend on the reference frame in which the velocity is measured. Thus the convertible kinetic energy will be the same in every reference frame. The convertible kinetic energy in this reference frame is also 16 J.

6.58. Consider the expressions for the kinetic energy that we already know, and compare this to the new form. We require:

$$\frac{1}{2}(m_1 + m_2)v_{\mathrm{cm}}^2 + \frac{1}{2}(m_1 + m_2)v_{\mathrm{red}}^2 = \frac{1}{2}(m_1 + m_2)v_{\mathrm{cm}}^2 + \frac{1}{2}\frac{m_1 m_2}{(m_1 + m_2)}v_{12}^2$$

$$\frac{1}{2}(m_1 + m_2)(\vec{v}_{\mathrm{red}})^2 = \frac{1}{2}\frac{m_1 m_2}{(m_1 + m_2)}(\vec{v}_1 - \vec{v}_2)^2$$

$$\vec{v}_{\mathrm{red}} = \frac{\sqrt{m_1 m_2}}{(m_1 + m_2)}(\vec{v}_1 - \vec{v}_2)$$

6.59. The zero-kinetic energy reference frame may sometimes exist, but not always. More specifically, if all parts of a system have the same velocity, then there is a reference frame in which they are all at rest and have zero kinetic energy. However, if parts of a system have different velocities, then there is no reference frame that could be moving along with all the parts of the system. In this case all (or most) pieces of the system will have some non-zero velocity in all reference frames. Therefore there will be non-zero kinetic energy in all reference frames. Alternatively, there is always a reference frame in which K_{cm} is zero (the center of mass reference frame). However, K_{conv} is independent of the reference frame. So, it is non-zero in one reference frame, it is non-zero in all frames.

6.60. Zero. Since the collision is totally inelastic, all convertible kinetic energy is converted to internal energy. The remaining kinetic energy is zero, which must be equal to the center of mass kinetic energy. Hence the velocity of the center of mass is zero. If the quantity $\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$ is zero, then so is $m_1\vec{v}_1 + m_2\vec{v}_2$, which is exactly the expression for the momentum.

6.61. (*a*) Call the direction of the initial motion of the baby penguin the $+x$ direction. The conservation of momentum allows us to write for the x components of velocity:

$$m_b v_{b\,x,i} + m_m v_{m\,x,i} = m_b v_{b\,x,f} + m_m v_{m\,x,f}$$

$$m_b v_{b\,x,i} + 0 = m_b\left(-\frac{v_{b\,x,i}}{8}\right) + m_m\left(\frac{\Delta x}{\Delta t}\right)$$

$$v_{b\,x,i} = \frac{4}{\left(1+\frac{1}{8}\right)}\left(\frac{\Delta x}{\Delta t}\right)$$

$$v_{b\,x,i} = \frac{4}{\left(1+\frac{1}{8}\right)}\left(\frac{0.50\text{ m}}{0.40\text{ s}}\right)$$

$$v_{b\,x,i} = 4.4\text{ m/s}$$

So the baby penguin was initially sliding at 4.4 m/s. (*b*) Before the collision, the mother was at rest relative to the ice. So the velocity of the mother in the swimming penguin's reference frame is just 1.0 m/s toward the swimming penguin. The momentum is therefore $(1.0\text{ m/s})m_{\text{mother}}$ toward the swimming penguin. After the collision, the mother is moving relative to the ice at 1.3 m/s toward the swimming penguin. Thus, the mother's momentum in the reference frame of the swimming penguin is $(2.3\text{ m/s})m_{\text{mother}}$ after the collision.

6.62. (*a*) Call the initial direction of motion of the jumping skater the $+x$ direction, and call her skater 1. Her partner is skater 2. The conservation of momentum in the x direction tells us

$$m_1 v_{1\,x,i} + m_2 v_{2\,x,i} = m_1 v_{1\,x,f} + m_2 v_{2\,x,f}$$

$$m_1 v_{1\,x,i} + 0 = (m_1 + m_2) v_{x,f}$$

$$v_{x,f} = \frac{m_1 v_{1\,x,i}}{(m_1 + m_2)}$$

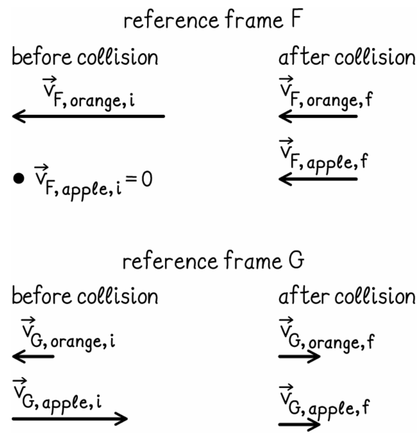
$$v_{x,f} = \frac{(50\text{ kg})(2.0\text{m/s})}{((50\text{ kg}) + (70\text{ kg}))}$$

$$v_{x,f} = 0.83\text{ m/s}$$

So after the collision, the pair will move at 0.83 m/s in the direction of the 50-kg skater's initial velocity. (*b*) The change in the x component of the skater's velocity was $v_{x,f} - v_{1\,x,i} = -1.17\text{ m/s}$. This change in speed is independent of the motion of the observer. Thus the change in momentum (in any reference frame) is given by $\Delta\vec{p} = m_1(\vec{v}_f - \vec{v}_{1,i}) = (50\text{ kg})(-1.17\text{ m/s})\,\hat{i} = (-58\text{ kg}\cdot\text{m/s})\,\hat{i}$. So the change in momentum is $58\text{ kg}\cdot\text{m/s}$ opposite the direction of the girl's initial velocity.

6.63. (*a*) Conservation of momentum tells us $m_o\vec{v}_o = (m_o + m_a)\vec{v}_f$, so we know that the final velocity of the orange (as well as the apple) is $\vec{v}_f = \dfrac{m_o\vec{v}_o}{(m_o + m_a)}$. The speed of the apple does change. So the only way for an observer to measure the same kinetic energy of the orange before and after the collision is if the speed of the apple initially exceeded that of the observer by some amount, and afterward the speed of the observer exceeded that of the apple by the same amount. This means $(\vec{v}_o - \vec{v}_{EF}) = (\vec{v}_{EF} - \vec{v}_f)$. Solving for $\vec{v}_{EF}$ yields , $\vec{v}_{EF} = \dfrac{\vec{v}_o}{2}\left(\dfrac{2m_o + m_a}{m_o + m_a}\right)$.

(*b*)



6.64. (*a*) Because the final velocity of the Earth would be unchanged to a very good approximation, the converted kinetic energy is essentially equal to the initial kinetic energy. $K_{\text{converted}} = \frac{1}{2} m_{\text{met}} v_{\text{met}}^2 \frac{1}{2}(50\text{ kg})(10^3\text{ m/s})^2 = 2.5\times10^7\text{ J}.$

(*b*) We can use the conservation of momentum to determine the extremely small final velocity of Earth. Call the direction of initial motion of the meteorite the $+x$ direction. Then we have

$$m_{\text{E}} v_{\text{E}\,x,\text{i}} + m_{\text{met}} v_{\text{met}\,x,\text{i}} = m_{\text{E}} v_{\text{E}\,x,\text{f}} + m_{\text{met}} v_{\text{met}\,x,\text{f}}$$

$$m_{\text{met}} v_{\text{met}\,x,\text{i}} = (m_{\text{E}} + m_{\text{met}}) v_{\text{E}\,x,\text{f}}$$

$$v_{\text{E}\,x,\text{f}} = \frac{m_{\text{met}} v_{\text{met}\,x,\text{i}}}{(m_{\text{E}} + m_{\text{met}})} = \frac{(50\text{ kg})(10^3\text{ m/s})}{((5.97\times10^{24}\text{ kg}) + (50\text{ kg}))}$$

$$v_{\text{E}\,x,\text{f}} = 8.38\times10^{-21}\text{ m/s}$$

Then the kinetic energy gained by Earth is just $K_{\text{f}} = \frac{1}{2} m_{\text{E}} v_{\text{E}\,x,\text{f}}^2 = \frac{1}{2}(5.97\times10^{24}\text{ kg})(8.38\times10^{-21}\text{ m/s})^2 = 2.1\times10^{-16}\text{ J}.$

(*c*) Nearly all the initial kinetic energy of the meteorite is converted to internal energy of the earth around the crash site. This energy takes the form of thermal energy, or heat.

6.65. (*a*) This problem can be solved by using both conservation of momentum and relating the initial and final kinetic energies. Call east the $+x$ direction. Then we have

$$m_{\text{S}} v_{\text{S}\,x,\text{i}} + m_{\text{R}} v_{\text{R}\,x,\text{i}} = m_{\text{S}} v_{\text{S}\,x,\text{f}} + m_{\text{R}} v_{\text{R}\,x,\text{f}} \tag{1}$$

$$(0.80)\left(\frac{1}{2} m_{\text{S}} v_{\text{S}\,x,\text{i}}^2 + \frac{1}{2} m_{\text{R}} v_{\text{R}\,x,\text{i}}^2\right) = \frac{1}{2} m_{\text{S}} v_{\text{S}\,x,\text{f}}^2 + \frac{1}{2} m_{\text{R}} v_{\text{R}\,x,\text{f}}^2 \tag{2}$$

We can solve equation (1) for $v_{\text{S}\,x,\text{f}}$ and insert the expression into equation (2). This could be solved algebraically, but it yields very complicated expressions. We can simplify our task by noting that the initial momentum of the system is zero. Requiring that the final momentum also be zero lets us write

$$v_{\text{S}\,x,\text{f}} = -\frac{m_{\text{R}}}{m_{\text{S}}} v_{\text{R}\,x,\text{f}} \tag{3}$$

To further simplify, the expressions, let us refer to the left hand side of equation (2) as K_{f} and note from the initial conditions given that its value is $K_{\text{f}} = 48\text{ J}.$ Then we have

$$K_f = \frac{1}{2}m_S\left(\frac{m_R}{m_S}v_{R\,x,f}\right)^2 + \frac{1}{2}m_R v_{R\,x,f}^2$$

$$v_{R\,x,f} = \sqrt{\frac{2K_f}{\frac{m_R^2}{m_S}+m_R}} = \sqrt{\frac{2(48\text{ J})}{\frac{(0.40\text{ kg})^2}{(0.20\text{ kg})}+(0.40\text{ kg})}}$$

$$v_{R\,x,f} = 8.9\text{ m/s}$$

Inserting this back into equation (3), we find

$$v_{S\,x,f} = -\frac{(0.40\text{ kg})}{(0.20\text{ kg})}(8.9\text{ m/s}) = -18\text{ m/s}$$

Thus, after the collision the rubber ball is moving 8.9 m/s east, and the softball is moving 18 m/s to the west. (*b*) We already noted in (*a*) that the initial kinetic energy is 60 J, 80% of which is 48 J. The 20% of the initial kinetic energy that was lost went into internal energy. Thus the change in internal energy is $\Delta E_{int} = K_i - K_f = (60\text{ J}) - (48\text{ J}) = 12\text{ J}.$ The internal energy has increased by 12 J. (*c*) This observer would also find that 12 J had been changed from kinetic energy into internal energy. It should be obvious that the change in internal energy (heat, deformations, etc) cannot depend on the reference frame. So all observers in all reference frame should agree that there was a 12 J increase in internal energy. Further, energy should be conserved in every reference frame, so everyone must agree that this 12 J was converted from kinetic energy to internal energy. (*d*) This observer would also agree that 12 J had been changed from kinetic energy into internal energy. See answer to (*c*) for more details.

6.66. We will demonstrate the claim for one object (object A), and this will be sufficient to demonstrate the claim for both objects, because we are free to switch subscripts (switch object A to B) without loss of generality. We assume the motion is confined to one dimension, and we call the direction of the initial motion of object A the $+x$ direction. We start by writing a statement of conservation of energy and of conservation of momentum for the two objects involved in the collision:

$$m_A v_{Ax,i} = -m_B v_{Bx,i} \tag{1}$$

$$m_A v_{Ax,f} = -m_B v_{Bx,f} \tag{2}$$

$$m_A v_{Ax,i}^2 + m_B v_{Bx,i}^2 = m_A v_{Ax,f}^2 + m_B v_{Bx,f}^2 \tag{3}$$

Here, eqs. (1) and (2) use the fact that the system is observed from the zero-momentum frame. The factors of one half have been cancelled in eq. (3). Solving eqs. (1) and (2) for the speeds involving object A, and inserting these into eq. (3) yields

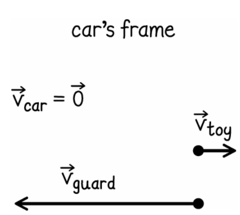
$$m_A\left(\frac{m_B}{m_A}\right)^2 v_{Bx,i}^2 + m_B v_{Bx,i}^2 = m_A\left(\frac{m_B}{m_A}\right)^2 v_{Bx,f}^2 + m_B v_{Bx,f}^2$$

$$v_{Bx,i}^2 = v_{Bx,f}^2$$

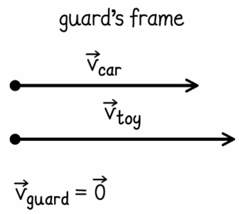
$$v_{Bx,i} = \pm v_{Bx,f}$$

In principle, either solution is correct. But if both objects have their velocities completely unchanged, then there was not really a collision. The only way the objects can collide and interact with each other is if we choose the "$-$" sign above. Hence $v_{Bx,i} = -v_{Bx,f}$, from which it follows directly that $\vec{p}_{B\,i} = -\vec{p}_{B\,f}$, which was the first claim we were asked to show. Now, the change in momentum is $\Delta\vec{p} = \vec{p}_{B\,f} - \vec{p}_{B\,i} = -2\vec{p}_{B\,i}$, which is the second claim.

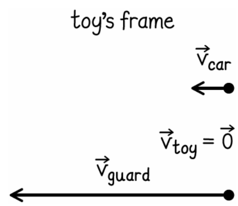
6.67. (*a*)



(*b*)



(*c*)



6.68. Call the child person 1, and the mother person 2. The relative velocity of the two is 5.0 m/s toward each other. The fraction of energy that is convertible is given by

$$\frac{K_{\text{conv}}}{K_{\text{i}}} = \frac{\frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)v_{12}^2}{\frac{1}{2}(m_1 v_1^2 + m_2 v_2^2)} = \frac{\left(\frac{(20\text{ kg})(68\text{ kg})}{(20\text{ kg})+(68\text{ kg})}\right)(5.0\text{m/s})^2}{((20\text{ kg})(3.0\text{ m/s})^2 + (68\text{ kg})(2.0\text{ m/s})^2)} = 0.85$$

Thus 85% of the initial kinetic energy is convertible.

6.69. Call the direction that the man and the dog are moving the $+x$ direction, and call the velocity of the observer in the Earth reference frame $\vec{v}_{\text{E o}}$. Then we require that

$$\vec{p}_{\text{o m}} = \vec{p}_{\text{o d}}$$

$$m_{\text{m}}(\vec{v}_{\text{E m}} - \vec{v}_{\text{E o}}) = m_{\text{d}}(\vec{v}_{\text{E d}} - \vec{v}_{\text{E o}})$$

$$\vec{v}_{\text{E o}} = \frac{\left(\frac{m_{\text{m}}}{m_{\text{d}}}\vec{v}_{\text{E m}} - \vec{v}_{\text{E d}}\right)}{\left(\frac{m_{\text{m}}}{m_{\text{d}}} - 1\right)}$$

$$\vec{v}_{\text{E o}} = \frac{\left(\left(\frac{80\text{ kg}}{10\text{ kg}}\right)(+2.0\text{ m/s})\ \hat{i} - (+10\text{ m/s})\ \hat{i}\right)}{\left(\left(\frac{80\text{ kg}}{10\text{ kg}}\right) - 1\right)} = (+0.86\text{ m/s})\ \hat{i}$$

The observer would have to walk 0.86 m/s in the same direction as the man and the dog in order for the man and the dog to have the same momentum in the observer's reference frame.

6.70. No. In either case the person would feel slightly heavier than usual because their bodies are being accelerated upward.

6.71. (*a*) The apparent acceleration of the two objects must be the same. (*b*) The apparent change in momentum of the objects will be related by their inertias according to $\Delta\vec{p}_{1,\text{app}}/\Delta\vec{p}_{2,\text{app}} = m_1/m_2$.

6.72. (*a*) $\vec{p}_{\text{sys,truck}} = -m_{\text{B}}\vec{v}_{\text{BT}}$ and $\vec{p}_{\text{sys,bug}} = m_{\text{T}}\vec{v}_{\text{BT}}$ (*b*) $\vec{p}_{\text{sys,bug}} = m_{\text{T}}\vec{v}_{\text{BT}}$ and $\vec{p}_{\text{sys,truck}} = -m_{\text{B}}\vec{v}_{\text{BT}}$ (*c*) No, nothing is wrong. You are adding a small or large amount to different reference frames.

6.73. Call the initial direction of the golf ball's motion the $+x$ direction. We first calculate the velocity of the center of mass of the system:

$$m_B\vec{v}_{B,i} + m_P\vec{v}_{P,i} = (m_B + m_P)\vec{v}_{cm}$$

$$\vec{v}_{cm} = \frac{m_B\vec{v}_{B,i} + m_P\vec{v}_{P,i}}{(m_B + m_P)} = \frac{(0.045\text{ kg})(50\text{ m/s})\ \hat{i}}{(0.045\text{ kg}) + (1.8\text{ kg})} = (+1.22\text{ m/s})\ \hat{i}$$

We will work in this center of mass reference frame, and convert back to the Earth reference frame at the end. In the center of mass reference frame, the final momentum must also be zero, so we can write

$$m_B\vec{v}_{cm\ B,f} = -m_P\vec{v}_{cm\ P,f} \tag{1}$$

The same coefficient of restitution holds in this reference frame, because relative velocities are independent of the reference frame. Hence we also have

$$(\vec{v}_{cm\ P,f} - \vec{v}_{cm\ B,f}) = -e(\vec{v}_{cm\ P,i} - \vec{v}_{cm\ B,i}) = (+25\text{ m/s})\,\hat{i} \tag{2}$$

Combining equations (1) and (2) yields $\vec{v}_{cm\ P,f} = (0.610\text{ m/s})\ \hat{i}$ and $\vec{v}_{cm\ B,f} = (-24.4\text{ m/s})\ \hat{i}$. Of course, now must still change reference frames back to the Earth frame according to $\vec{v}_{E\ object} = \vec{v}_{E\ cm} + \vec{v}_{cm\ object}$. This yields $\vec{v}_{E\ P,f} =$ $(1.8\text{ m/s})\ \hat{i}$ and $\vec{v}_{cm\ B,f} = (-23\text{ m/s})\ \hat{i}$.

6.74. (*a*) Let asteroid A1 be initially moving the $+x$ direction. The total momentum is given by

$$\vec{p}_i = m_{A1}\vec{v}_{A1,i} + m_{A2}\vec{v}_{A2,i}$$

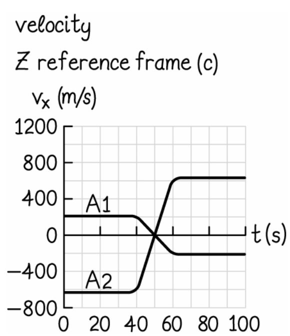
$$\vec{p}_i = (3.60\times10^6\text{ kg})(+528\text{ m/s})\ \hat{i} + (1.20\times10^6\text{ kg})(-315\text{ m/s})\ \hat{i}$$

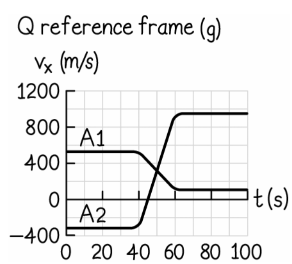
$$\vec{p}_i = (+1.52\times10^9\text{ kg}\cdot\text{m/s})\ \hat{i}$$

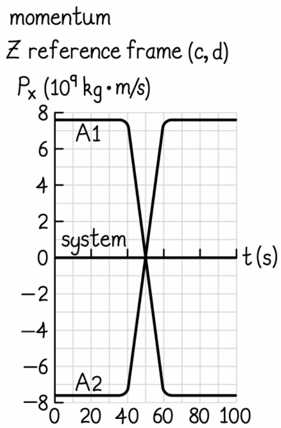
So the momentum of the entire system is $1.52\times10^9\text{ kg}\cdot\text{m/s}$ in the direction of asteroid A1. (*b*) Platform Z moves at the same velocity of the center of mass of the system. To find this velocity, we simply divide the answer to part (*a*) by the total inertia of the system.

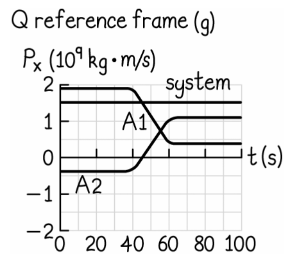
$$\vec{v}_{cm} = \frac{\vec{p}_i}{m_{A1} + m_{A2}} = \frac{(+1.52\times10^9\text{ kg}\cdot\text{m/s})\ \hat{i}}{(3.6\times10^6\text{ kg}) + (1.2\times10^6\text{ kg})} = (+317\text{ m/s})\ \hat{i}$$

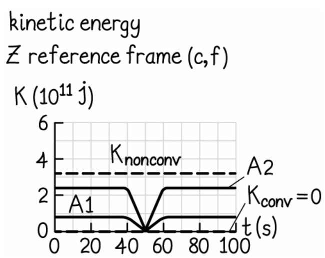
So platform Z is moving relative to Q at 317 m/s in the same direction as asteroid A1. (*c*) and (*d*) See the figure below. (*e*) Curves bend as kinetic energy is converted to other forms. (*f*) See the figure below. (*g*) See the figure below.

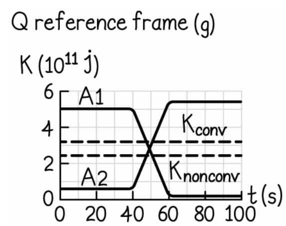




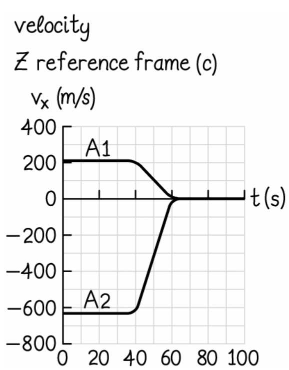


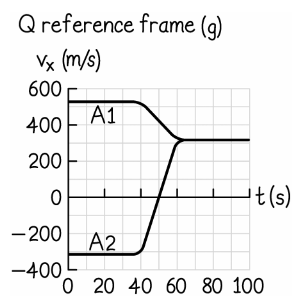


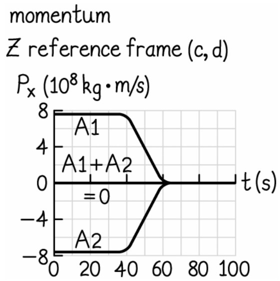


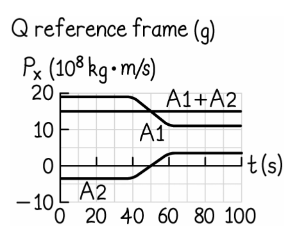


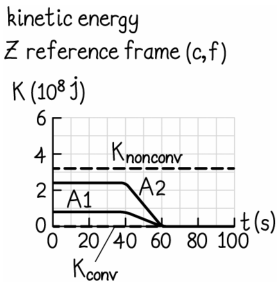
6.75. (*a*) Exactly the same process used in problem 6.75 (*a*) applies here and yields $1.52\times10^9\ \text{kg}\cdot\text{m/s}$. (*b*) As in 6.75 (*b*), we find the velocity of platform Z is 317 m/s in the same direction as asteroid A1. (*c*) and (*d*) See the figure below. (*e*) Curves bend as kinetic energy is converted to other forms. (*f*) See the figure below. (*g*) See the figure below.

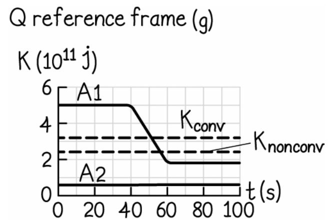












6.76. (*a*) One way of solving this problem is to work in the center of mass reference frame. Call the direction of motion of both cars before the collision the $+x$ direction. The velocity of the center of mass is given by

$$\vec{v}_{\text{cm}} = \frac{m_{\text{c}}\vec{v}_{\text{c,i}} + m_{\text{v}}\vec{v}_{\text{v,i}}}{m_{\text{c}} + m_{\text{v}}}$$

$$\vec{v}_{\text{cm}} = \frac{(1000\text{ kg})(25\text{ m/s})\ \hat{i} + (1500\text{ kg})(15\text{ m/s})\ \hat{i}}{(1000\text{ kg}) + (1500\text{ kg})}$$

$$\vec{v}_{\text{cm}} = (+19\text{ m/s})\ \hat{i}$$

In this reference frame, the final momentum must also be zero:

$$m_{\text{c}}\vec{v}_{\text{cm c,f}} = -m_{\text{v}}\vec{v}_{\text{cm v,f}} \tag{1}$$

We also have from the coefficient of restitution

$$(\vec{v}_{\text{cm v,f}} - \vec{v}_{\text{cm c,f}}) = -e(\vec{v}_{\text{cm v,i}} - \vec{v}_{\text{cm c,i}}) = -(0.70)((15\text{m/s}) - (25\text{m/s}))\ \hat{i}$$

$$\vec{v}_{\text{cm v,f}} = \vec{v}_{\text{cm c,f}} + (+7.0\text{ m/s})\ \hat{i}$$

Combining this result with equation (1) yields $\vec{v}_{\text{cm c,f}} = (-4.2\text{ m/s})\ \hat{i}$ and $\vec{v}_{\text{cm v,f}} = (2.8\text{ m/s})\ \hat{i}$. These are the velocities in the center of mass frame. We still need to convert back to the Earth frame using $\vec{v}_{\text{E object}} = \vec{v}_{\text{E cm}} + \vec{v}_{\text{cm object}}$. This yields $\vec{v}_{\text{E v,f}} = (+22\text{ m/s})\ \hat{i}$ and $\vec{v}_{\text{E c,f}} = (+15\text{ m/s})\ \hat{i}$. So in the Earth frame, both vehicles continue moving in their initial direction of motion, the van at a speed of 22 m/s. and the car at a speed of 15 m/s. (*b*) One can simplify take the difference between initial and final kinetic energies. One could also choose to work only with convertible energy expressions since the center of mass motion is unchanged. But this is not a great simplification.

$$\Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}m_{\text{c}}v_{\text{c,f}}^2 + \frac{1}{2}m_{\text{v}}v_{\text{v,f}}^2 - \left(\frac{1}{2}m_{\text{c}}v_{\text{c,i}}^2 + \frac{1}{2}m_{\text{v}}v_{\text{v,i}}^2\right)$$

$$\Delta K = \frac{1}{2}(m_{\text{c}}(v_{\text{c,f}}^2 - v_{\text{c,i}}^2) + m_{\text{v}}(v_{\text{v,f}}^2 - v_{\text{v,i}}^2))$$

$$\Delta K = \frac{1}{2}\Big((1000\text{ kg})\big((14.8\text{ m/s})^2 - (25\text{ m/s})^2\big) + \big((1500\text{ kg})((21.8\text{ m/s})^2 - (15\text{ m/s})^2\big)\Big)$$

$$\Delta K = -15\text{ kJ}$$

This means that 15 kJ of kinetic energy was converted to internal energy in the collision. (*c*) In this case, all convertible energy is converted to internal energy. Hence the amount converted can be written

$$\Delta K = K_{\text{conv,i}} = \frac{1}{2}\mu v_{\text{vc,i}}^2 = \frac{1}{2}\frac{(1000\text{ kg})(1500\text{ kg})}{(1000\text{ kg}) + (1500\text{ kg})}(10\text{ m/s})^2 = 30\text{ kJ}$$

6.77. Call the 0.30-kg object cart A and call the 0.5 kg object cart B, and call their initial direction of motion $+x$. We can describe the change in kinetic energy by

$$\left(\frac{1}{2}m_{\text{A}}v_{\text{A,i}}^2 + \frac{1}{2}m_{\text{B}}v_{\text{B,i}}^2\right) \times 1.3 = \frac{1}{2}m_{\text{A}}v_{\text{A,f}}^2 + \frac{1}{2}m_{\text{B}}v_{\text{B,f}}^2 \tag{1}$$

Momentum must also be conserved. So we can write

$$m_{\text{A}}\vec{v}_{\text{A,i}} + m_{\text{B}}\vec{v}_{\text{B,i}} = m_{\text{A}}\vec{v}_{\text{A,f}} + m_{\text{B}}\vec{v}_{\text{B,f}} \tag{2}$$

Solving equation (2) for the final velocity of cart A, and inserting this into equation (1) yields $\vec{v}_{\text{A,f}} = (0.17\text{ m/s})\,\hat{i}$ and $\vec{v}_{\text{B,f}} = (+2.1\text{ m/s})\,\hat{i}$.

6.78. We start from the second equation given, and we take the first derivative with respect to time. Note that time is the same in all reference frames discussed here, so there is no ambiguity about this time derivative. Also note that $\vec{a}_{\text{IN}}$ is a constant, and $\vec{v}_{\text{IN}}$ is a constant that describes the velocity difference at time $t = 0$ only. Thus

$$\frac{d}{dt}(\vec{r}_{\text{N}}) = \frac{d}{dt}\left(\vec{r}_{\text{I}} - \vec{v}_{\text{IN}}t - \frac{1}{2}\vec{a}_{\text{IN}}t^2\right)$$

$$\vec{v}_{\text{N}} = \vec{v}_{\text{I}} - \vec{v}_{\text{IN}} - \vec{a}_{\text{IN}}t \qquad (1)$$

This is our first expression, giving $\vec{v}_{\text{N}}$ in terms of given constants and $\vec{v}_{\text{I}}$. Now we take the derivative of this new equation (equation (1)) with respect to time:

$$\frac{d}{dt}(\vec{v}_{\text{N}}) = \frac{d}{dt}(\vec{v}_{\text{I}} - \vec{v}_{\text{IN}} - \vec{a}_{\text{IN}}t)$$

$$\vec{a}_{\text{N}} = \vec{a}_{\text{I}} - \vec{a}_{\text{IN}} \qquad (2)$$

This gives us $\vec{a}_{\text{N}}$ in terms of the constant $\vec{a}_{\text{IN}}$ and $\vec{a}_{\text{I}}$.

6.79. This object is made up of a large circle with a circular vacancy off-center. This is obviously what one gets if one starts with a large complete circle (the blue portion) and cuts out a smaller off-center circle. We will mathematically treat this removal of inertia by adding an off-center circle with negative inertia. Call the radius of the large circle R. Let the origin be at the center of the large circle and let the $+x$ axis point toward the center of the smaller cavity. We write the center of mass of our superposition as

$$x_{\text{cm}} = \frac{m_{\text{L}}x_{\text{L}} - m_{\text{S}}x_{\text{S}}}{m_{\text{L}} - m_{\text{S}}} = \frac{\rho\pi t\left(R^2 x_{\text{L}} - \left(\frac{R}{2}\right)^2 x_{\text{S}}\right)}{\rho\pi t R^2\left(1 - \left(\frac{1}{2}\right)^2\right)} = \frac{x_{\text{L}} - \frac{1}{4}x_{\text{S}}}{\left(\frac{3}{4}\right)} = \frac{(0) - \frac{1}{4}\left(\frac{R}{2}\right)}{\left(\frac{3}{4}\right)} = -\frac{R}{6}$$

Thus the center of mass of the entire medallion is at a position $x_{\text{cm}} = -\dfrac{R}{6}$.

6.80. On a calm day, you can travel 40 km in one hour, so your top speed (relative to the air) is 40 km/h. Your velocity relative to the air is related to your velocity relative to the ground according to $\vec{v}_{\text{E you}} = \vec{v}_{\text{E w}} + \vec{v}_{\text{w you}}$. Let us assume that you initially start riding in the $+x$ direction, such that the wind is blowing in the $-x$ direction. Let your maximum distance from home be Δx. Your total travel time can be written as

$$t = t_{\text{head}} + t_{\text{tail}} = \frac{\Delta x}{v_{\text{E you}\,x,\text{head}}} + \frac{-\Delta x}{v_{\text{E you}\,x,\text{tail}}} = \frac{\Delta x}{v_{\text{E w}\,x,\text{head}} + v_{\text{w you}\,x,\text{head}}} + \frac{-\Delta x}{v_{\text{E w}\,x,\text{tail}} + v_{\text{w you}\,x,\text{tail}}}$$

$$\Delta x = \left(\frac{1}{v_{\text{E w}\,x,\text{head}} + v_{\text{w you}\,x,\text{head}}} + \frac{-1}{v_{\text{E w}\,x,\text{tail}} + v_{\text{w you}\,x,\text{tail}}}\right)^{-1} t$$

$$\Delta x = \left(\frac{1}{(-20\text{ km/h}) + (40\text{ km/h})} + \frac{-1}{(-20\text{ km/h}) + (-40\text{ km/h})}\right)^{-1} (1\text{ h})$$

$$\Delta x = 15\text{ km}$$

On the windy day you can only ride 30 km total in one hour.

6.81. (*a*) Let the car initially be moving in the $+x$ direction. Because the grain has no initial horizontal velocity component, the momentum of the grain and car together must be the same as the initial momentum of the car. Thus we have

$$m_{\text{c}}\vec{v}_{\text{c,i}} = (m_{\text{c}} + m_{\text{g}})\vec{v}_{\text{f}}$$

or

$$\vec{v}_{\mathrm{f}} = \frac{m_{\mathrm{c}}\vec{v}_{\mathrm{c,i}}}{(m_{\mathrm{c}} + m_{\mathrm{g}})} = \frac{(20{,}000\ \mathrm{kg})(2.0\ \mathrm{m/s})\ \hat{i}}{((20{,}000\ \mathrm{kg}) + (4{,}000\ \mathrm{kg/s})(5.0\ \mathrm{s}))} = (+1.0\ \mathrm{m/s})\ \hat{i}$$

Thus the car and grain are moving together at a speed of 1.0 m/s after the grain is dumped into the car. (*b*) As grain is dumped in, the speed of the car changes. We can write the speed of the car in terms of how much grain has been added:

$$\vec{v}(t) = \frac{m_{\mathrm{c}}\vec{v}_{\mathrm{c,i}}}{(m_{\mathrm{c}} + m_{\mathrm{g}}(t))} \tag{1}$$

And of course, we can write the distance travelled as the integral over velocity:

$$x_{\mathrm{f}} - x_{\mathrm{i}} = \int_{\mathrm{i}}^{\mathrm{f}} v(t)dt = \int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} \frac{m_{\mathrm{c}}v_{\mathrm{c}\,x,\mathrm{i}}}{(m_{\mathrm{c}} + m_{\mathrm{g}}(t))}\left(\frac{dm}{dt}\right)dt$$

$$x_{\mathrm{f}} - x_{\mathrm{i}} = \int_{t_{\mathrm{i}}}^{t_{\mathrm{f}}} \frac{m_{\mathrm{c}}v_{\mathrm{c}\,x,\mathrm{i}}}{\left(m_{\mathrm{c}} + \left(\frac{dm}{dt}\right)t\right)}dt$$

$$x_{\mathrm{f}} - x_{\mathrm{i}} = m_{\mathrm{c}}v_{\mathrm{c}\,x,\mathrm{i}}\left(\frac{dm}{dt}\right)^{-1}\ln\left(\frac{m_{\mathrm{c}} + \left(\frac{dm}{dt}\right)t_{\mathrm{f}}}{m_{\mathrm{c}} + \left(\frac{dm}{dt}\right)t_{\mathrm{i}}}\right)$$

$$x_{\mathrm{f}} - x_{\mathrm{i}} = (20{,}000\ \mathrm{kg})(2.0\ \mathrm{m/s})\left(\frac{1.0\ \mathrm{s}}{4{,}000\ \mathrm{kg}}\right)\ln\left(\frac{(20{,}000\ \mathrm{kg}) + (4{,}000\ \mathrm{kg/s})(5.0\ \mathrm{s})}{(20{,}000\ \mathrm{kg}) + 0}\right)$$

$$x_{\mathrm{f}} - x_{\mathrm{i}} = 6.9\ \mathrm{m}$$

So, the car moves 6.9 m during the 5.0 s that the grain falls. (*c*) Let us assume that the grain is distributed along the entire length of the car (that is, that the car length is also 6.9 m). Finding the center of mass of the car is trivial, as we assume the car is symmetric. The center of mass of the car will be in the geometric center of the car, 3.47 m from the front end. But to find the center of mass of the grain, we need to integrate over all infinitesimal contributions to the inertia, weighted by their positions:

$$x_{\mathrm{cm}} = \frac{1}{m_{\mathrm{grain}}}\int_{\mathrm{i}}^{\mathrm{f}} x\,dm$$

Here we can insert the general expression for the position as a function of time. This is exactly the expression that resulted from the integral in part (*b*):

$$x(t) = m_{\mathrm{c}}v_{\mathrm{c}\,x,\mathrm{i}}\left(\frac{dm}{dt}\right)^{-1}\ln\left(\frac{m_{\mathrm{c}} + m_{\mathrm{g}}(t)}{m_{\mathrm{c}}}\right)$$

Inserting this expression above yields

$$x_{\mathrm{cm}} = \frac{m_{\mathrm{c}}v_{\mathrm{c}\,x,\mathrm{i}}}{m_{\mathrm{grain}}\left(\frac{dm}{dt}\right)}\int_{\mathrm{i}}^{\mathrm{f}}\ln\left(\frac{m_{\mathrm{c}} + m}{m_{\mathrm{c}}}\right)dm$$

$$x_{\mathrm{cm}} = \frac{m_{\mathrm{c}}v_{\mathrm{c}\,x,\mathrm{i}}}{m_{\mathrm{grain}}\left(\frac{dm}{dt}\right)}\left(-m + (m + m_{\mathrm{c}})\ln\left(\frac{m + m_{\mathrm{c}}}{m_{\mathrm{c}}}\right)\right)\Bigg|_{m_{\mathrm{i}}}^{m_{\mathrm{f}}}$$

$$x_{\mathrm{cm}} = \frac{(20{,}000\ \mathrm{kg})(2.0\ \mathrm{m/s})}{(20{,}000\ \mathrm{kg})(4{,}000\ \mathrm{kg/s})}\left(-(20{,}000\ \mathrm{kg}) + ((20{,}000\ \mathrm{kg}) + (20{,}000\ \mathrm{kg}))\ln\left(\frac{(20{,}000\ \mathrm{kg}) + (20{,}000\ \mathrm{kg})}{(20{,}000\ \mathrm{kg})}\right)\right)$$

$$x_{\mathrm{cm}} = 3.86\ \mathrm{m}$$

Now that we have the center of mass of the car itself, and the pile of grain, we can find the center of mass of the combination using the discrete sum

$$x_{\text{cm}} = \frac{m_{\text{c}}x_{\text{cm c}} + m_{\text{g}}x_{\text{cm g}}}{m_{\text{c}} + m_{\text{g}}} = \frac{(20{,}000\ \text{kg})}{(40{,}000\ \text{kg})}((3.47\ \text{m}) + (3.86\ \text{m})) = 3.7\ \text{m}$$

Thus the center of mass of the entire system is 3.7 m from the front end of the car. (*d*) This was calculated as part of the answer to part (*c*). The center of mass of the grain is 3.9 m from the front end of the car. (*e*) The speed changed as the grain was being added, so the grain was not evenly distributed over the length of the car.

6.82. (*a*) We can find the final speed of the roller blader using conservation of momentum. Call the initial direction of motion of the skateboarder $+x$. Then we know

$$m_{\text{s}}v_{\text{s}\,x,\text{i}} + m_{\text{r}}v_{\text{r}\,x,\text{i}} = m_{\text{s}}v_{\text{s}\,x,\text{f}} + m_{\text{r}}v_{\text{r}\,x,\text{f}}$$

$$v_{\text{r}\,x,\text{f}} = \frac{m_{\text{s}}}{m_{\text{r}}}(v_{\text{s}\,x,\text{i}} - v_{\text{s}\,x,\text{f}}) + v_{\text{r}\,x,\text{i}}$$

$$v_{\text{r}\,x,\text{f}} = \frac{(79\ \text{kg})}{(70\ \text{kg})}((5.0\ \text{m/s}) - (-2.0\ \text{m/s})) + 0$$

$$v_{\text{r}\,x,\text{f}} = 7.0\ \text{m/s}$$

Now we can find the coefficient of restitution:

$$e = -\frac{(v_{\text{s}\,x,\text{f}} - v_{\text{r}\,x,f})}{(v_{\text{s}\,x,\text{i}} - v_{\text{r}\,x,\text{i}})} = -\frac{((-2.0\ \text{m/s}) - (7.9\ \text{m/s}))}{((5.0\ \text{m/s}) - 0)} = 2.0$$

So the coefficient of restitution is 2.0. (*b*) The change in kinetic energy is given by

$$\Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}m_{\text{s}}v_{\text{s,f}}^2 + \frac{1}{2}m_{\text{r}}v_{\text{r,f}}^2 - \left(\frac{1}{2}m_{\text{s}}v_{\text{s,i}}^2 + \frac{1}{2}m_{\text{r}}v_{\text{r,i}}^2\right)$$

$$\Delta K = \frac{1}{2}((79\ \text{kg})(2.0\ \text{m})^2 + (70\ \text{kg})(7.9\ \text{m})^2 - (79\ \text{kg})(5.0\ \text{m})^2)$$

$$\Delta K = 1.4\ \text{kJ}$$

The kinetic energy decreases by 440 J, and the internal energy increases by 440 J. (*c*) The So the kinetic energy increases by 1.4 kJ. This increase in kinetic energy must come from a decrease in internal energy. So the internal energy decreases by 1.4 kJ. (*c*) Clearly the amount of chemical energy used by the skaters' muscles in the collision cannot depend on the reference frame of the observer. This means that the change in internal energy is the same in all reference frames, and is still a decrease of 1.4 kJ. Also, energy is conserved in all reference frames, so this internal energy must have been converted to kinetic energy. So this new observer would also agree that the kinetic energy increased by 1.4 kJ. (*d*) The energy lost would have been equal to all the kinetic energy that was initially convertible. Thus

$$\Delta K_{\text{conv}} = \frac{1}{2}\left(\frac{m_{\text{s}}m_{\text{r}}}{m_{\text{s}} + m_{\text{r}}}\right)v_{\text{sr,i}}^2 = \frac{1}{2}\left(\frac{(79\ \text{kg})(70\ \text{kg})}{((79\ \text{kg}) + (70\ \text{kg}))}\right)(1.0\ \text{m/s})^2 = 19\ \text{J}$$

In this perfectly inelastic collision, only 19 J of kinetic energy would have been converted to internal energy.

7

INTERACTIONS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

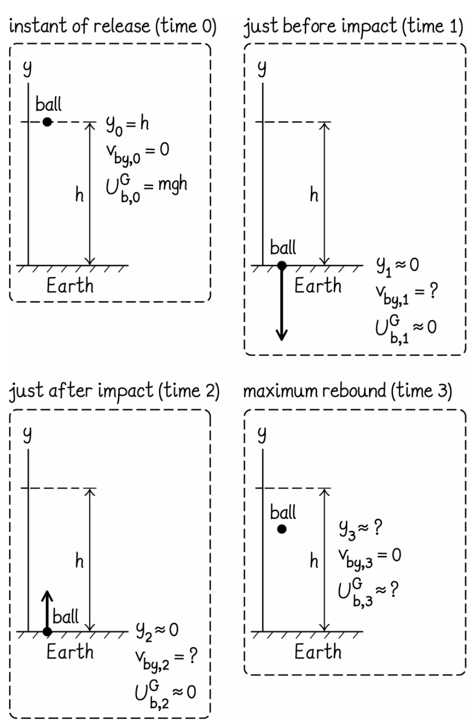
Developing a Feel

1. 10^0 J **2.** 10^1 J **3.** 10^8 J **4.** 10^{-22} m/s^2 **5.** 10^2 m/s^2 opposite direction bullet is fired **6.** 10^3 J **7.** 10^5 J **8.** 10^{-12} J **9.** 10^5 J **10.** 10^{10} J

Guided Problems

7.2 Safe play

1. Getting Started We start by making a system diagram. We could include the Earth in our system, in which case we would consider changes in gravitational potential energy, and conversion of kinetic energy partially into heat in the system. Alternatively, we could choose our system to consist of the ball only, and treat gravity and interaction with the ground as external forces. Here we adopt the former system, including Earth, although many choices of system may be valid. We draw the system at a few key instants: prior to release, just prior to the impact, just after the impact, and when the ball reaches its maximum rebound height.



2. Devise Plan At its highest point (both initial and after its first bounce) the ball has zero kinetic energy. As the ball falls toward the ground, gravitational potential energy is converted into kinetic energy. As the ball strikes the ground, some of the kinetic energy is converted to heat, and the rest is converted to elastic potential energy as the ball is compressed. When the ball bounces, the elastic potential energy is converted first back into kinetic energy, then as the ball rises, that energy is converted into gravitational potential energy. At its maximum height after the bounce, all energy has been converted back into gravitational potential energy, except for the energy that was converted into heat during the bounce.

We can use the above energy conversion to determine how much mechanical energy is left at the end to be converted to gravitational potential energy. From that, we can determine the maximum height.

3. Execute Plan The initial gravitational potential energy is $U^G_{\mathrm{b},0} = mgh.$ As the ball falls, all of this energy is converted to kinetic energy. Thus, we can write

$$mgh = \frac{1}{2}mv^2_{\mathrm{b}y,1} \tag{1}$$

or $v_{\mathrm{b}y,1} = -\sqrt{2gh}$. We know that the kinetic energy immediately after the bounce is somewhat less than the kinetic energy before the bounce, because of energy lost to heat. Specifically, $\frac{1}{2}mv^2_{\mathrm{b}y,1} - \Delta E_{\mathrm{th}} = \frac{1}{2}mv^2_{\mathrm{b}y,2}$, or equivalently we can insert equation (1) above to obtain

$$\Delta E_{\mathrm{th}} = mgh - \frac{1}{2}mv^2_{\mathrm{b}y,2} \tag{2}$$

Equation 7.26 from Principles gives us another expression for the energy converted into thermal energy in terms of the coefficient of restitution:

$$\Delta E_{\mathrm{th}} = \frac{1}{2}\mu v^2_{\mathrm{Eb},1}(1-e^2)$$

where the reduced inertia $\mu = \dfrac{m_{\mathrm{E}} m_{\mathrm{b}}}{m_{\mathrm{E}} + m_{\mathrm{b}}} \approx m_{\mathrm{b}}$ and $v_{\mathrm{Eb},1} = v_{\mathrm{b}y,1}$. Combining equation (2) above and equation (7.62) from Principles, we find the kinetic energy after the first bounce to be

$$\frac{1}{2}mv^2_{\mathrm{b}y,2} = mgh - \frac{1}{2}mv^2_{\mathrm{b}y,1}(1-e^2)$$

As the ball rises to its maximum height after rebound, all of this kinetic energy is converted to gravitational potential energy. So we can write

$$mgh_3 = \frac{1}{2}mv^2_{\mathrm{b}y,2} = mgh - \frac{1}{2}mv^2_{\mathrm{b}y,1}(1-e^2)$$

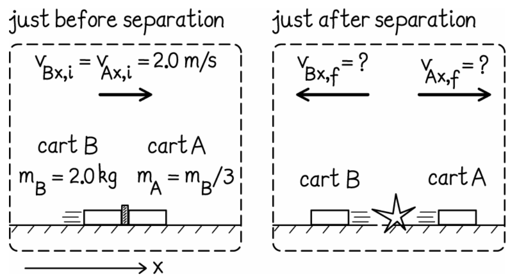
Inserting $\frac{1}{2}mv^2_{\mathrm{b}y,1} = mgh,$ we have $mgh_3 = mghe^2 \Rightarrow h_3 = e^2 h$ or $(0.0625)h.$

4. Evaluate Result This is a very small fraction of the initial height, but the coefficient of restitution is also fairly low. Such a small bounce is certainly possible for a ball made of very soft rubber, or for a ball that is not fully inflated.

7.4 An unequal match

1. Getting Started This problem is similar to Worked Problem 7.3, with a few differences. Two small differences are the numerical amount of energy released in the explosive charge, and the fraction of that energy converted into incoherent energy such as heat. But a major difference is that the two carts now have different inertias. This alters the symmetry of the problem.

Let us choose our system to consist of the two carts, and the explosive charge. A system diagram is shown below.



We know that the inertias of the carts and the velocity of the center of mass remain the same throughout the explosion. Only the velocity of each cart, and the overall mechanical energy of the system change.

2. Devise Plan We know that total energy will be conserved, even though mechanical energy is not. We account for the change in mechanical energy with the fraction f of the chemical potential energy U_{chem} released in the explosion. Thus we can write

$$\frac{1}{2}m_{\text{A}}v_{\text{EA}x,\text{i}}^2 + \frac{1}{2}m_{\text{B}}v_{\text{EB}x,\text{i}}^2 + fU_{\text{chem}} = \frac{1}{2}m_{\text{A}}v_{\text{EA}x,\text{f}}^2 + \frac{1}{2}m_{\text{B}}v_{\text{EB}x,\text{f}}^2 \tag{1}$$

We also know that momentum will be conserved during the explosion. Thus we can write

$$m_{\text{A}}v_{\text{EA}x,\text{i}} + m_{\text{B}}v_{\text{EB}x,\text{i}} = m_{\text{A}}v_{\text{EA}x,\text{f}} + m_{\text{B}}v_{\text{EB}x,\text{f}} \tag{2}$$

Here we have two equations in which all quantities except two are known. Thus we do have sufficient information to solve for the unknown final x components of velocity $v_{\text{EA}x,\text{f}}$ and $v_{\text{EB}x,\text{f}}$.

3. Execute Plan Rearranging equation (2) to determine $v_{\text{EA}x,\text{f}}$ in terms of other variables, we have

$$v_{\text{EA}x,\text{f}} = v_{\text{EA}x,\text{i}} + \frac{m_{\text{B}}}{m_{\text{A}}}(v_{\text{EB}x,\text{i}} - v_{\text{EB}x,\text{f}}) \tag{3}$$

Inserting this into equation (1) yields

$$\frac{1}{2}m_{\text{A}}v_{\text{EA}x,\text{i}}^2 + \frac{1}{2}m_{\text{B}}v_{\text{EB}x,\text{i}}^2 + fU_{\text{chem}} = \frac{1}{2}m_{\text{A}}\left[v_{\text{EA}x,\text{i}} + \frac{m_{\text{B}}}{m_{\text{A}}}(v_{\text{EB}x,\text{i}} - v_{\text{EB}x,\text{f}})\right]^2 + \frac{1}{2}m_{\text{B}}v_{\text{EB}x,\text{f}}^2$$

which we rearrange, collecting like powers of $v_{\text{EB}x,\text{f}}$ in order to use the quadratic equation. We find

$$\frac{1}{2}\left(m_{\text{B}} + \frac{m_{\text{B}}^2}{m_{\text{A}}}\right)v_{\text{EB}x,\text{f}}^2 - \left(m_{\text{B}}v_{\text{EA}x,\text{i}} + \frac{m_{\text{B}}^2}{m_{\text{A}}}v_{\text{EB}x,\text{i}}\right)v_{\text{EB}x,\text{f}} + \frac{1}{2}\left(m_{\text{A}}v_{\text{EA}x,\text{i}}^2 + \frac{m_{\text{B}}^2}{m_{\text{A}}}v_{\text{EB}x,\text{i}}^2 + 2m_{\text{B}}v_{\text{EA}x,\text{i}}v_{\text{EB}x,\text{i}}\right)$$
$$-\left(\frac{1}{2}m_{\text{A}}v_{\text{EA}x,\text{i}}^2 + \frac{1}{2}m_{\text{B}}v_{\text{EB}x,\text{i}}^2 + fU_{\text{chem}}\right) = 0$$

One can continue this evaluation analytically, but the expressions clearly become cumbersome. Let us insert known values at this point:

$$\frac{1}{2}\left((2.0\text{ kg}) + \frac{(2.0\text{ kg})^2}{(0.667\text{ kg})}\right)v_{\text{EB}x,\text{f}}^2 - \left((2.0\text{ kg})(2.0\text{ m/s}) + \frac{(2.0\text{ kg})^2}{(0.667\text{ kg})}(2.0\text{ m/s})\right)v_{\text{EB}x,\text{f}}$$
$$+\frac{1}{2}\left((0.667\text{ kg})(2.0\text{ m/s})^2 + \frac{(2.0\text{ kg})^2}{(0.667\text{ kg})}(2.0\text{ m/s})^2 + 2(2.0\text{ kg})(2.0\text{ m/s})^2\right)$$
$$-\left(\frac{1}{2}(0.667\text{ kg})(2.0\text{ m/s})^2 + \frac{1}{2}(0.667\text{ kg})(2.0\text{ m/s})^2 + \left(\frac{2}{3}\right)(27\text{ J})\right) = 0$$

or $-(2.0\text{ J}) - (16\text{ kg}\cdot\text{m/s})v_{\text{EB}x,\text{f}} + (4.0\text{ kg})v_{\text{EB}x,\text{f}}^2 = 0$.

The quadratic equation then yields solutions of -0.12 m/s and 4.1 m/s. Clearly, since cart B is pushed backward by the explosion and had an initial speed of only 2.0 m/s, the physically correct answer is $v_{\text{EB}x,\text{f}} = -0.12$ m/s. Inserting this back into equation (3) above, we find

$$v_{\text{EA}x,\text{f}} = v_{\text{EA}x,\text{i}} + \frac{m_\text{B}}{m_\text{A}}(v_{\text{EB}x,\text{i}} - v_{\text{EB}x,\text{f}}) = (2.0\,\text{m/s}) + \frac{(2.0\text{ kg})}{(0.667\text{ kg})}((2.0\text{ m/s}) - (-0.12\,\text{m/s})) = 8.4\,\text{m/s}$$

Thus $v_{\text{EA}x,\text{f}} = 8.4$ m/s and $v_{\text{EB}x,\text{f}} = -0.12$ m/s.

4. Evaluate Result One way of checking our answer is to verify if the velocity of the center of mass is the same as before the explosion:

$$v_{\text{cm}\,x,\text{f}} = \frac{m_\text{A}v_{\text{EA}x,\text{f}} + m_\text{B}v_{\text{EB}x,\text{f}}}{m_\text{A} + m_\text{B}} = \frac{(0.667\text{ kg})(8.4\text{ m/s}) + (2.00\text{ kg})(-0.12\text{ m/s})}{(0.667\text{ kg}) + (2.00\text{ kg})} = 2.0\text{ m/s}$$

Which is, indeed, the initial speed of the entire system when the two carts were linked, and therefore the initial x component of the velocity of the center of mass.

Another method of checking our answer would be to re-calculate the kinetic energy before and after the explosion and verify that the difference is accounted for by the chemical energy converted to mechanical energy in the explosion:

$$K_\text{i} = \frac{1}{2}m_\text{A}v_{\text{EA}x,\text{i}}^2 + \frac{1}{2}m_\text{B}v_{\text{EB}x,\text{i}}^2 = \frac{1}{2}(0.667\text{ kg})(2.0\text{ m/s})^2 + \frac{1}{2}(2.0\text{ kg})(2.0\text{ m/s})^2 = 5.3\text{ J}$$

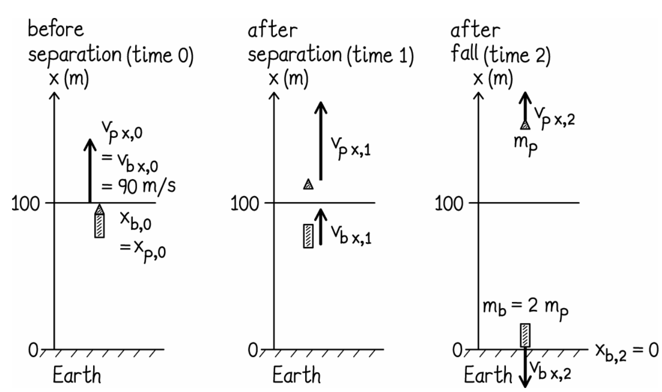
$$K_\text{f} = \frac{1}{2}m_\text{A}v_{\text{EA}x,\text{f}}^2 + \frac{1}{2}m_\text{B}v_{\text{EB}x,\text{f}}^2 = \frac{1}{2}(0.667\text{ kg})(8.4\text{ m/s})^2 + \frac{1}{2}(2.0\text{ kg})(-0.12\text{ m/s})^2 = 23.5\text{ J}$$

The difference (to the two significant digits given) is 18 J, which is in fact $\left(\frac{2}{3}\right)(27\text{ J})$, the amount of chemical potential energy converted into mechanical energy.

7.6 Booster impact

1. Getting Started The setup of this problem is very similar to that in Worked Problem 7.5. As in Worked Problem 7.5 we are given information about the rocket/booster system immediately before the explosive separation. Here, we are asked for information about the booster when it returns to Earth, as opposed to being asked about the payload as it continues to rise for some time. Another difference is that the explosive separation in Worked Problem 7.5 occurred at the peak height of the booster/payload system, such that the speed of the booster and payload was zero just before separation. In this problem, the explosive separation occurs when the fuel is exhausted, but while the booster and payload are still rising.

We draw a diagram similar to Figure WG7.5. Note that we need to include Earth in our system, if we plan to make use of gravitational potential energy (as we will). In this case, one could alternatively use kinematics and avoid the use of gravitational potential energy.



2. Devise Plan We know the initial velocity and inertia of the booster and payload, so we can determine the initial kinetic energy easily. The kinetic energy after the explosive separation is obtained by adding the fraction f of chemical potential energy U_{chem} in the explosive that is converted to coherent (kinetic) energy to the initial kinetic energy:

$$K_0 + fU_{\text{chem}} = K_1$$

or (1)

$$\frac{1}{2}(m_{\text{p}} + m_{\text{b}})v_{\text{p}x,0}^2 + fU_{\text{chem}} = \frac{1}{2}m_{\text{p}}v_{\text{p}x,1}^2 + \frac{1}{2}m_{\text{b}}v_{\text{b}x,1}^2$$

Equation (1) has two unknown final velocity components. We can obtain another relationship between these using momentum. We know the initial momentum of the system, and this cannot be changed by forces internal to the system (such as those exerted in the detonation of the explosive). Thus

$$\vec{p}_{\text{p},0} + \vec{p}_{\text{b},0} = \vec{p}_{\text{p},1} + \vec{p}_{\text{b},1}$$

Or, in terms of scalar components:

$$m_{\text{p}}v_{\text{p}x,0} + m_{\text{b}}v_{\text{b}x,0} = m_{\text{p}}v_{\text{p}x,1} + m_{\text{b}}v_{\text{b}x,1} \tag{2}$$

The above equations can be used to determine the velocity of the booster immediately after the explosive separation. At that instant, the booster is still at the position $x_{\text{b},0} = 100$ m. After falling, the booster will be at $x_{\text{b},2} = 0$ as it strikes the ground. This change in vertical position corresponds to a change in gravitational potential energy. As the booster falls, all gravitational potential energy is converted into kinetic energy. Thus, we can write

$$K_{\text{b},1} + U_{\text{b},1}^G = K_{\text{b},2}$$

or (3)

$$\frac{1}{2}m_{\text{b}}v_{\text{b}x,1}^2 + m_{\text{b}}gx_{\text{b},1} = \frac{1}{2}m_{\text{b}}v_{\text{b}x,2}^2$$

3. Execute Plan Equation (2) can be rearranged to yield

$$v_{\text{p}x,1} = v_{\text{p}x,0} + \frac{m_{\text{b}}}{m_{\text{p}}}(v_{\text{b}x,0} - v_{\text{b}x,1}) \tag{4}$$

We insert this into equation (1) to obtain

$$\frac{1}{2}(m_{\text{p}} + m_{\text{b}})v_{\text{p}x,0}^2 + fU_{\text{chem}} = \frac{1}{2}m_{\text{p}}\left(v_{\text{p}x,0} + \frac{m_{\text{b}}}{m_{\text{p}}}(v_{\text{b}x,0} - v_{\text{b}x,1})\right)^2 + \frac{1}{2}m_{\text{b}}v_{\text{b}x,1}^2$$

Which can be rearranged to group like powers of $v_{\text{b}x,1}$:

$$-\frac{1}{2}m_{\text{b}}v_{\text{p}x,0}^2 - fU_{\text{chem}} + m_{\text{b}}v_{\text{p}x,0}v_{\text{b}x,0} + \frac{1}{2}\frac{m_{\text{b}}^2}{m_{\text{p}}}v_{\text{b}x,0}^2 - \left(m_{\text{b}}v_{\text{p}x,0} - \frac{m_{\text{b}}^2}{m_{\text{p}}}v_{\text{b}x,0}\right)v_{\text{b}x,1} + \frac{1}{2}\left(m_{\text{b}} + \frac{m_{\text{b}}^2}{m_{\text{p}}}\right)v_{\text{b}x,1}^2 = 0$$

This is clearly quadratic in $v_{\text{b}x,1}$, and is most readily solved using the quadratic equation. Inserting the known values of initial components of velocity, and inertias, the above expression reduces to

$$(2.23\times10^4\ \text{J}) - (540\ \text{kg}\cdot\text{m/s})v_{\text{b}x,1} + (3\ \text{kg})v_{\text{b}x,1}^2 = 0$$

Which has solutions $v_{\text{b}x,1} = 64.2$ m/s and $v_{\text{b}x,1} = 116$ m/s. Clearly, because the explosion exerts downward forces on the booster, the speed must be reduced from the initial 90 m/s, meaning the physically correct solution is $v_{\text{b}x,1} = 64.2$ m/s (where we have kept one additional digit for this intermediate step). The other mathematical solution $v_{\text{b}x,1} = 116$ m/s corresponds to a situation in which the booster is on top of the payload and is push upward in the explosion (not the case here).

We can now rearrange equation (3) to solve for the final vertical component of the booster's velocity

$$v_{\text{b}x,2} = -\sqrt{v_{\text{b}x,1}^2 + 2gx_{\text{b},1}}$$

$$v_{\text{b}x,2} = -\sqrt{(64.2\ \text{m/s})^2 + 2(9.8\ \text{m/s}^2)(100\ \text{m})}$$

$$v_{\text{b}x,2} = -78\ \text{m/s}$$

Thus the booster strikes the ground with a speed of 78 m/s.

4. Evaluate Result This final speed is comparable to other speeds in the problem, and so is plausible. It is also reasonable that the final speed is greater than the speed of the booster just after explosive separation, since gravitational potential energy must be converted to kinetic energy during the fall. If we desire an additional check, we could go back and insert our results for the booster to see if we get a reasonable result for the vertical component of the velocity of the payload. We know we should expect something larger than $90\ \text{m/s}$ since the explosion should push the payload upward. We also know that the booster's speed was reduced by about $26\ \text{m/s}$, and it had twice the inertia of the payload. Thus we expect the speed of the payload to increase by more, close to $50\ \text{m/s}$. Inserting $v_{\text{bx},1} = 64.2\ \text{m/s}$ into equation (4) above, we find

$$v_{\text{px},1} = (90\ \text{m/s}) + \frac{(2.0\ \text{kg})}{(1.0\ \text{kg})}((90\ \text{m/s}) - (64.2\ \text{m/s})) = 1.4\times10^2\ \text{m/s}$$

This is consistent with our intuition.

7.8 Over the hill

1. Getting Started This problem is very similar to Worked Problem 7.7. However, some steps may be different, because we now have a cubic term $\sim x^3$.

We can determine the shape of the curve by simply calculating a number of points along the curve, but it may be particularly useful to know the local extrema. We find the locations at which the local extrama are reached by requiring $\dfrac{dU(x)}{dx} = 0.$

Another useful bit of information is the limiting behavior of the function $U(x)$ as x becomes very large in either the positive or negative x directions. In either case, as the magnitude of x becomes large, the cubic term will dominate, and for extremely large x, we expect $U(x) \approx cx^3$. Since $c < 0$, we expect the potential energy to decrease without bound as x becomes large and positive. Expressed differently: $\lim\limits_{x\to\infty} U(x) = -\infty$. Similarly, we expect $\lim\limits_{x\to-\infty} U(x) = +\infty$, meaning as x becomes large and negative, the potential energy increases without bound.

2. Devise Plan We begin by calculating the value of the potential at a few key points. One might choose any points at all, but we will specifically calculate the points needed for part (*e*), the value at the origin, and the local extrema. For the values of x referenced in part (*e*):

$$U(x = 1.0\ \text{m}) = (12\ \text{J/m})(1.0\ \text{m}) + (3.0\ \text{J/m})(1.0\ \text{m})^2 + (-2.0\ \text{J/m})(1.0\ \text{m})^3 = 13\ \text{J}$$

$$U(x = -1.0\ \text{m}) = (12\ \text{J/m})(-1.0\ \text{m}) + (3.0\ \text{J/m})(-1.0\ \text{m})^2 + (-2.0\ \text{J/m})(-1.0\ \text{m})^3 = -7.0\ \text{J}$$

$$U(x = 3.0\ \text{m}) = (12\ \text{J/m})(3.0\ \text{m}) + (3.0\ \text{J/m})(3.0\ \text{m})^2 + (-2.0\ \text{J/m})(3.0\ \text{m})^3 = 9.0\ \text{J}$$

Since all terms have at least one factor of x, one sees trivially that $U(x = 0) = 0$.

To find the local extrema, we require that

$$\frac{dU(x)}{dx} = a + 2bx + 3cx^2 = 0$$

$$\Rightarrow x_{\text{ext}} = \frac{-2b \pm \sqrt{(2)^2 b^2 - 4(3)ac}}{2(3)c}$$

$$x_{\text{ext}} = \frac{-2(3.0\ \text{J/m}) \pm \sqrt{(2)^2(3.0\ \text{J/m})^2 - 4(3)(12\ \text{J})(-2.0\ \text{J/m}^2)}}{2(3)(-2\ \text{J/m}^2)}$$

$$x_{\text{ext}} = -1.0\ \text{m or } 2.0\ \text{m}$$

The values of the first local extremum (a minimum) is already known from above: $U(x = -1.0\ \text{m}) = -7.0\ \text{J}$. The second local extremum is

$$U(x = 2.0\ \text{m}) = (12\ \text{J/m})(2.0\ \text{m}) + (3.0\ \text{J/m})(2.0\ \text{m})^2 + (-2.0\ \text{J/m})(2.0\ \text{m})^3 = 20\ \text{J}$$

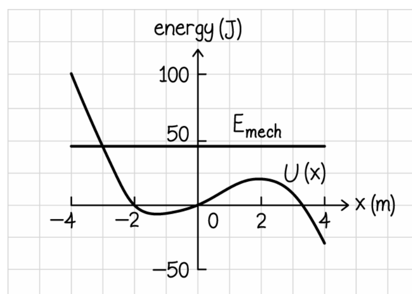
These points and the limiting behavior discussed above for large values of x are sufficient to draw a plot of the potential energy. But we are also asked to plot the kinetic energy of the particle. If no external forces are doing work on this system of two particles, the total energy should be constant. The initial kinetic energy at the moment/location of release is zero (released from rest). The potential energy at that position is

$$U(x=-3.0 \text{ m}) = (12 \text{ J/m})(-3.0 \text{ m}) + (3.0 \text{ J/m})(-3.0 \text{ m})^2 + (-2.0 \text{ J/m})(-3.0 \text{ m})^3 = 45 \text{ J}$$

Such that the total energy at any given time should also be 45 J. This is also useful for answering part (*e*), because once we know the potential energy at a particular point, the kinetic energy should follow immediately, using $K(x) = (45 \text{ J}) - U(x)$.

Finally, we can determine in which direction the object will start moving by considering in which direction the energy would be lowered. To see this, imaging a ball on an incline. The net force and the acceleration are in the direction that cause the ball to move down the incline, toward a lower potential energy.

3. Execute Plan Using the data points calculated above (and as many more as we feel we need) we can create a complete graph of the potential energy as a function of position. As discussed above, the total energy should be a horizontal line with constant energy of 45 J. We obtain our answer to part (*a*):



(*b*) Clearly, the object will lower its potential energy by moving to the right in the above picture, meaning it will move in the $+x$ direction. (*c*) As the object moves in the $+x$ direction, initially, the potential energy will decrease and the kinetic energy will increase. The object will continue to speed up until it reaches the point $x = -1.0$ m. At this position, the potential reaches a local minimum, so just beyond this point the potential is increasing. This means the object will slow down as it moves from $x = -1.0$ m to $x = +2.0$ m, the position of the local maximum in the potential energy. Beyond that point, the potential decreases again, and the kinetic energy will grow without bound as x increases. (*d*) As discussed above, the particle speeds up from $x = -3.0$ m to $x = -1.0$ m, slows down from $x = -1.0$ m to $x = +2.0$ m, and speeds up for $x > +2.0$ m. (*e*) Since we know the total mechanical energy and we know the potential energy at these positions, we simply write

$$K(x=1.0 \text{ m}) = (45 \text{ J}) - U(x=1.0 \text{ m}) = (45 \text{ J}) - (13 \text{ J}) = 32 \text{ J}$$
$$K(x=-1.0 \text{ m}) = (45 \text{ J}) - U(x=-1.0 \text{ m}) = (45 \text{ J}) - (-7.0 \text{ J}) = 52 \text{ J}$$
$$K(x=3.0 \text{ m}) = (45 \text{ J}) - U(x=3.0 \text{ m}) = (45 \text{ J}) - (9.0 \text{ J}) = 36 \text{ J}$$

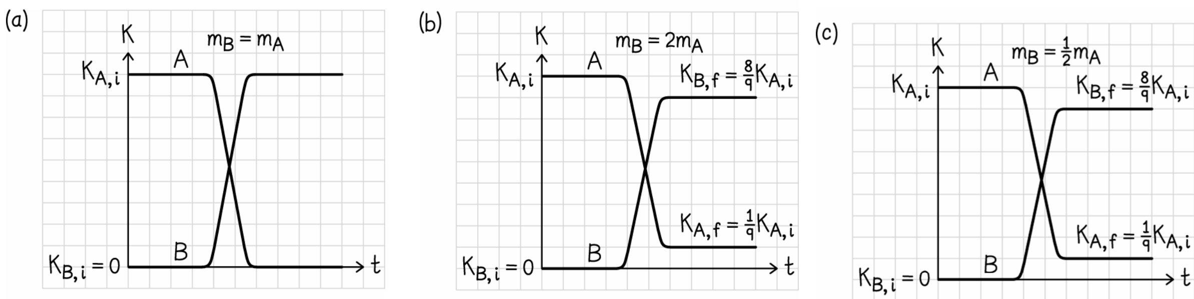
4. Evaluate Result Looking at the polynomial expression for $U(x)$, we see the highest power of x is x^3. Thus it follows that the derivative of $U(x)$ must be quadratic, and $U(x)$ must have two extrema. Thus our graph of $U(x)$ fits our intuition. Although we are not told the exact nature of the particle and the object with which it interacts, it might be instructive to think of the particle as a ball and the object with which it interacts as Earth. If we picture a ball rolling on a hill that has the shape of $U(x)$, then we see that the regions in which the particle is speeding up and slowing down do fit our intuition.

Questions and Problems

7.1. More than one. There are two interactions; one between each piece of bread and the salami.

7.2. When two objects interact, the magnitude of their acceleration is inversely proportional to their inertia. The huge inertia of Earth means that it will have an extremely small acceleration when interacting with anything on Earth, such as people. If an object's inertia were comparable to that of the Earth (a moon, another planet, or a very large asteroid) then it could give Earth a non-negligible acceleration through an interaction.

7.3.



7.4. (*a*) We know that any acceleration due to an interaction is inversely proportional to inertia. Since the objects started from rest, the final speeds after any time will also be inversely proportional to their inertias. Hence $v_{2\,x}/v_{1\,x} = -m_1/m_2$. (*b*) We write an expression for the ratio of kinetic energies, and then use the results from part (*a*):

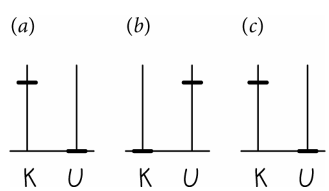
$$\frac{K_2}{K_1} = \frac{\frac{1}{2}m_2 v_{2\,x}^2}{\frac{1}{2}m_1 v_{1\,x}^2} = \frac{m_2}{m_1}\left(\frac{m_1}{m_2}\right)^2 = \frac{m_1}{m_2}$$

Hence $K_2/K_1 = m_1/m_2$.

7.5. We know that the magnitude of the acceleration caused by an interaction is inversely proportional to the inertia. Thus we can write for the magnitudes of the accelerations $\frac{a_1}{a_2} = \frac{m_2}{m_1}$ or $a_2 = \frac{m_1}{m_2}a_1$. Inserting the given numbers yields $a_2 = \frac{(0.200\text{ kg})}{(0.250\text{ kg})}(2.25\text{ m/s}^2) = 1.8\text{ m/s}^2$. We also know that the cars push each other in opposite directions, so $\vec{a}_{2.5\text{-kg}} = 1.8\text{ m/s}^2$ to the left.

7.6. As you climb, you increase your gravitational potential energy at the expense of your body's chemical energy. When you dive, you convert this potential energy to kinetic energy. When you hit the water, you retain some of this kinetic energy but most of it becomes kinetic energy of the water and thermal and sound energy.

7.7.



7.8. Initially, the mother is at rest at a height h off the ground. This means she initially has some gravitational potential energy U_i, but no kinetic energy. When she finally passes back through that height h on her way up, she momentarily has the same potential energy as before, but she also has some kinetic energy now. Hence the energy of the system made up of the mother and trampoline has increased. The change in energy could only have come from the teenager pulling on the trampoline. We write

$$\Delta E = K_\text{f} + U_\text{f} - (K_\text{i} + U_\text{i})$$

$$\Delta E = \frac{1}{2}mv_\text{f}^2 + U_\text{i} - (0 + U_\text{i})$$

$$\Delta E = \frac{1}{2}mv_\text{f}^2 = \frac{1}{2}(75\text{ kg})(3.0\text{ m/s})^2$$

$$\Delta E = 3.4 \times 10^2\text{ J}$$

Hence the son added 3.4×10^2 J to the system.

7.9. The change in gravitational potential energy is lower in case (*b*) where there is a counterweight. If there is no counterweight, then the magnitude of the change in energy when an elevator of inertia M moves a vertical distance Δy is $Mg\Delta y$. With a counterweight, whenever the elevator moves in one direction the counterweight moves the same distance in the opposite direction. In this case (with a counterweight of inertia m) the change in gravitational potential energy is $Mg\Delta y - mg\Delta y$, which is clearly smaller than without a counterweight.

7.10. No. Both figures illustrate potential energy changes in repulsive interactions; in attractive interactions, the potential energy is greatest when the interacting objects are farthest apart and least when they are closest to each other. Thus the energy conversion diagram sequence is the opposite of that in Figures 7.7 and 7.25.

7.11. Let us call the inertia of the lighter block m, such the inertia of the heavier block is $3m$. Recall that when two objects interact with each other, the acceleration of either object is inversely proportional to its inertia, meaning after the acceleration acts for any time interval we have $\frac{v_m}{v_{3m}} = \frac{3m}{m} = 3$. Thus we can write the ratio of kinetic energies as

$$\frac{K_\text{m}}{K_{3m}} = \frac{\frac{1}{2}mv_m^2}{\frac{1}{2}3mv_{3m}^2} = \frac{1}{3}\left(\frac{v_m}{v_{3m}}\right)^2 = \frac{9}{3} = 3$$

Thus the ratio of kinetic energies $\frac{K_m}{K_{3m}} = 3$.

7.12. (*a*) The energy that accounts for the bouncing is coherent. (*b*) The energy that accounts for the warmth is incoherent. (*c*) All the energy in the springs originally came from chemical potential energy in your body; you used this energy to drive the oscillation of the trampoline.

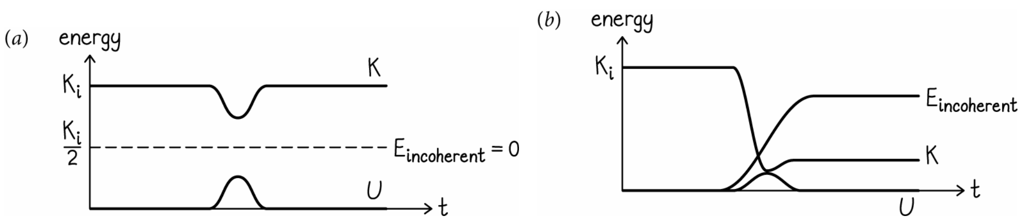
7.13. The trip down takes longer. One way to see this is to consider the tradeoff of potential and kinetic energy. The ball starts with an initial kinetic energy at a position that we will refer to as the ground. As the ball rises, kinetic energy is converted to gravitational potential energy. If there were no dissipation, at a given height the ball would have the same speed whether it was on its way up or down. This would make the upward path and the downward path totally symmetric. But since energy is dissipated through air resistance, at a given height the speed will be lower on the way down than it was at that same height on the way up. Thus at every point on along the downward path, the ball is moving more slowly than it did at that height on the way up. Thus the downward trip will take longer.

7.14. (*a*) Inelastic (*b*) No (*c*) Most kinetic energy initially in arrow converted to incoherent energy (mainly thermal energy but some sound energy) as arrow rubs against pipe interior; if pipe moves, some initial arrow kinetic energy converted to pipe kinetic energy.

7.15. (*a*) Into deforming metal (by breaking and re-forming chemical bonds) and heating it up. (*b*) No, in fact you have to put in more energy to do a similar rearrangement of molecules to fix the slinky.

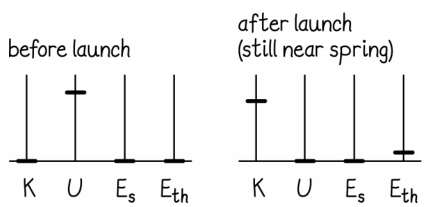
7.16. (*a*) Incoherent (*b*) A combination of coherent and incoherent (springs heat up as they compress and expand) (*c*) Mostly incoherent, although if the oscillation of the springs was correlated, some energy could be coherent (*d*) Incoherent.

7.17.

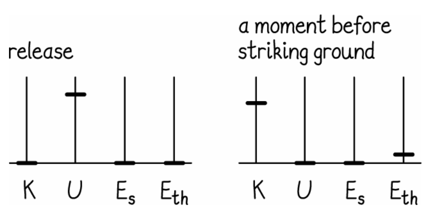


7.18. When the balls are dropped onto concrete the rubber ball is compressed a great deal. This allows the rubber ball to store coherent potential energy, but it also heats up the material of the ball. The steel ball is extremely rigid and goes through the collision with very little motion of its particles. It does not heat up as much as the rubber ball. So the steel ball retains more coherent energy than the rubber ball. This coherent energy is converted to gravitational potential energy, and the steel ball rises higher. When the balls are dropped into sand, the sand can be easily moved out of the way of either ball. When the steel ball strikes the sand, the sand is moved out of the way without compressing the steel very much. Hence, very little coherent energy is stored. But it is much easier to compress rubber. The rubber ball also moves sand out of the way, but the rubber is compressed some in the process. This compression of the rubber stores potential energy, which can cause the rubber ball to bounce higher.

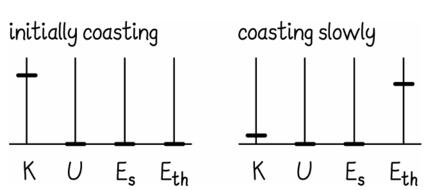
7.19. (*a*) The system is the ball, the spring, air, and Earth. Air resistance is not ignored.



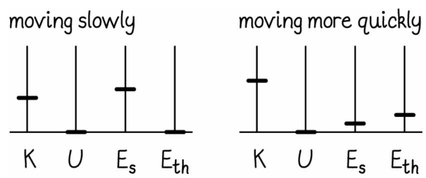
(*b*) The system is the ball, surrounding air, and Earth. Air resistance is not ignored.



(*c*) The system is the bicycle, cyclist, surrounding air, and the road beneath the bicycle. Air resistance and friction are not ignored.



(*d*) The system is the car, surrounding air, the road beneath the car, and the fuel (which contains the source energy). Air resistance and friction are not ignored.



7.20. We calculate the final kinetic energy of the car and the energy initially stored in the fuel used, and calculate the ratio. The final kinetic energy of the car is given by

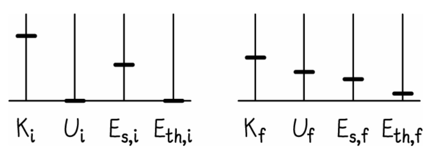
$$K_\mathrm{f} = \frac{1}{2}mv_\mathrm{f}^2 = \frac{1}{2}(800\ \mathrm{kg})(27\ \mathrm{m/s})^2 = 2.92\times10^5\ \mathrm{J}$$

Note that we have retained one additional significant digit, because this is an intermediate step. Similarly, the initial source energy in the gasoline is

$$E_\mathrm{s,i} = (0.0606\ \mathrm{L})\left(\frac{3.2\times10^7\ \mathrm{J}}{\mathrm{L}}\right) = 1.94\times10^6\ \mathrm{J}$$

Thus the efficiency is $\frac{K_\mathrm{f}}{E_\mathrm{s,i}} = \frac{(2.92\times10^5\ \mathrm{J})}{(1.94\times10^6\ \mathrm{J})} = 0.15$, so the efficiency of the car is 15%.

7.21. (*a*)



(*b*) You are on the Moon, where there is essentially no atmosphere and therefore no air resistance. You are initially moving at a fixed speed on your lunar rover such that you have 9 units of kinetic energy. You also have 5 units of source energy in the form of fuel. You have driven the rover at a slow pace so far, so the engine is slightly warm, containing two units of thermal energy. You see a lunar dune ahead and want to launch your rover into the air by going over it very quickly. You accelerate, hit the dune and fly upward. At one point in your trajectory, you have 7 units of kinetic energy, and 4 units of gravitational potential energy. You still have 3 units of fuel left. One additional unit of energy has been converted to thermal energy, because the engine of the rover is not perfect, and there is friction between the rover and the lunar dust.

7.22. (*a*) First, note that the final speed of the system is the center of mass speed:

$$v_{\mathrm{cm}\,x} = \frac{m_1 v_{1\,x,\mathrm{i}} + m_2 v_{2\,x,\mathrm{i}}}{m_1 + m_2} \quad (1)$$

The ratio of changes in kinetic energies can be written as

$$\frac{\Delta K_1}{\Delta K_2} = \frac{(1/2)m_1(v_{1\,x,\mathrm{i}}^2 - v_{\mathrm{cm}\,x}^2)}{(1/2)m_1(v_{1\,x,\mathrm{i}}^2 - v_{\mathrm{cm}\,x}^2)} = \frac{m_1(v_{1\,x,\mathrm{i}} - v_{\mathrm{cm}\,x})(v_{1\,x,\mathrm{i}} + v_{\mathrm{cm}\,x})}{m_2(v_{2\,x,\mathrm{i}} - v_{\mathrm{cm}\,x})(v_{2\,x,\mathrm{i}} + v_{\mathrm{cm}\,x})} \quad (2)$$

Inserting eq. (1) into eq. (2), and multiplying the top and bottom of the ratio by $(m_1 + m_2)$ yields

$$\frac{\Delta K_1}{\Delta K_2}=\frac{\dfrac{m_1}{m_2}(m_1+m_2)\left(v_{1\,x,\mathrm{i}}-\dfrac{m_1v_{1\,x,\mathrm{i}}+m_2v_{2\,x,\mathrm{i}}}{m_1+m_2}\right)(v_{1\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}{(m_1+m_2)\left(v_{2\,x,\mathrm{i}}-\dfrac{m_1v_{1\,x,\mathrm{i}}+m_2v_{2\,x,\mathrm{i}}}{m_1+m_2}\right)(v_{2\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}$$

$$\frac{\Delta K_1}{\Delta K_2}=\frac{\dfrac{m_1}{m_2}(m_1v_{1\,x,\mathrm{i}}+m_2v_{1\,x,\mathrm{i}}-m_1v_{1\,x,\mathrm{i}}-m_2v_{2\,x,\mathrm{i}})(v_{1\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}{(m_1v_{2\,x,\mathrm{i}}+m_2v_{2\,x,\mathrm{i}}-m_1v_{1\,x,\mathrm{i}}-m_2v_{2\,x,\mathrm{i}})(v_{2\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}$$

$$\frac{\Delta K_1}{\Delta K_2}=\frac{\dfrac{m_1}{m_2}(m_2v_{1\,x,\mathrm{i}}-m_2v_{2\,x,\mathrm{i}})(v_{1\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}{(m_1v_{2\,x,\mathrm{i}}-m_1v_{1\,x,\mathrm{i}})(v_{2\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}$$

$$\frac{\Delta K_1}{\Delta K_2}=\frac{-(-m_1v_{1\,x,\mathrm{i}}+m_1v_{2\,x,\mathrm{i}})(v_{1\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}{(m_1v_{2\,x,\mathrm{i}}-m_1v_{1\,x,\mathrm{i}})(v_{2\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}$$

$$\frac{\Delta K_1}{\Delta K_2}=-\frac{(v_{1\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}{(v_{2\,x,\mathrm{i}}+v_{\mathrm{cm}\,x})}$$

This was the claim to be proven. (*b*) It is not possible for both objects to gain kinetic energy. If both changes of kinetic energies were positive, this would violate the conservation of energy. It is, however, possible for both objects to lose kinetic energy. To fully understand the negative sign in the above expression, we must consider it in two cases. First, consider that objects 1 and 2 are moving in the same direction. In that case, all of the velocities in the expression have the same sign, such that the negative sign out front guarantees that the ratio of kinetic energy changes is negative. This means that if one object gains energy, the other must lose energy. Now consider the second case, in which the objects are moving in opposite directions. In that case, one object's velocity will be in the $-x$ direction and the other will be in the $+x$ direction. Both will be larger in magnitude than the center of mass speed. Hence, either the numerator or the denominator will be negative. The negative sign in front guarantees in this case that the ratio of changes in kinetic energies will be positive. This corresponds to both objects losing energy (each change in kinetic energy is negative).

7.23. Yes. The copper atoms are likely joined to one another by chemical bonds, whereas contact between macroscopic objects typically involves repulsion at a distance. So the copper atoms may be pictured as being more "in contact" than your shoe is with the ground, but only because the atoms in copper are closer together. However, even the chemical bonds between atoms are interactions over a distance.

7.24. Suppose you are standing on an ice hockey rink with your friend. You and your friend stand back to back. You throw the ball such that it bounces off of the wall in front of you, bounces across the rink and off of the far wall, and finally reaches your friend. In this way, the forward motion of the ball from your hands pushes you backward into your friend. Similarly, the collision between the ball and your friend pushes him closer to you. This interaction causes you both to move together and is hence an attractive interaction.

7.25. (*a*) At the beginning of the flight. This is the instant when you and the ball interact with each other. (*b*) At the end of the flight. This is the instant when your friend and the ball interact with each other.

7.26. (*a*) Finger touches remote buttons, short range; remote sends signal to television, long range. (*b*) Fingers touch phone keyboard, short range; phone sends signal to cell tower, long range.

7.27. (*a*) All charged particles in the universe would attract each other, and the charged particles would collapse into a very compact, charged center. Some fundamentally neutral objects (like neutrons) might remain elsewhere in the universe, but there could be no bonding or electrical interaction. (*b*) All charged particles would repel each other and spread out. The universe would become very diffuse, and continue to spread forever.

7.28. Friction between two materials is not a fundamental interaction because it is explained by the electromagnetic interactions of their atoms.

7.29. The strong interaction is responsible for holding atomic nuclei together against the electrical repulsion between protons. These interactions currently balance at a radius of about $10^{-14}-10^{-15}$ m. If the strong interaction increased in strength by 20 orders of magnitude, but the electrostatic repulsion did not increase, atomic nuclei would be crushed to much smaller radii, perhaps becoming black holes! Atomic size scales are determined by electrical interactions among electrons and nuclei, so atomic sizes would remain more or less unaffected. However, with such a large strength, the attraction between neutrons and protons would be large enough so that nuclei in adjacent atoms could be drawn together. All matter might therefore be crushed. This new strong interaction would still not be large enough to affect interplanetary distances, where gravity would still dominate.

7.30. Call r the distance between objects A and B. The interaction strength would vary like $1/r^2$, because the particles would be spread out over a spherical surface.

7.31. (*a*) One could use the accelerations of each object to determine the unknown inertia, since the acceleration of an object is a collision is inversely proportional to its inertia. But since the accelerations are calculated in part (*b*), we determine the unknown inertia here using conservation of momentum. Let the $+x$ direction point to the right. Call the 0.66 kg cart A and call the other cart B. Then we have

$$m_\text{A}v_{\text{A}\,x,\text{i}}+m_\text{B}v_{\text{B}\,x,\text{i}}=m_\text{A}v_{\text{A}\,x,\text{f}}+m_\text{B}v_{\text{B}\,x,\text{f}}$$

$$m_\text{B}=\frac{(v_{\text{A}\,x,\text{i}}-v_{\text{A}\,x,\text{f}})}{(v_{\text{B}\,x,\text{f}}-v_{\text{B}\,x,\text{i}})}m_\text{A}$$

$$m_\text{B}=\frac{((1.85\text{ m/s})-(-1.32\text{ m/s}))}{((3.22\text{ m/s})-(-2.17\text{ m/s}))}(0.66\text{ kg})=0.39\text{ kg}$$

So the unknown inertia is 0.39 kg.

(*b*) With the same choice of axes as in part (*a*) we have $a_{\text{A}\,x}=\dfrac{v_{\text{A}\,x,\text{f}}-v_{\text{A}\,x,\text{i}}}{\Delta t}=\dfrac{(-1.32\text{ m/s})-(1.85\text{ m/s})}{(0.010\text{ s})}=-3.2\times10^2\text{ m/s}^2$, such that $\vec{a}_{0.66\text{ kg}}=3.2\times10^2\text{ m/s}^2$ left. For the 0.39 kg object we have $a_{\text{B}\,x}=\dfrac{v_{\text{B}\,x,\text{f}}-v_{\text{B}\,x,\text{i}}}{\Delta t}=\dfrac{(3.22\text{ m/s})-(-2.17\text{ m/s})}{(0.010\text{ s})}=$ $5.4\times10^2\text{ m/s}^2$, such that $\vec{a}_{0.39\text{ kg}}=5.4\times10^2\text{ m/s}^2$ right.

(*c*) We know the magnitudes of the accelerations should be inversely proportional to the inertias. This means we should find that $a_{0.66\text{ kg}}/a_{0.39\text{ kg}}=m_{0.39\text{ kg}}/m_{0.66\text{ kg}}$. From parts (*a*) and (*b*) we can calculate

$$\frac{a_{0.66\text{ kg}}}{a_{0.39\text{ kg}}}=\frac{3.2\times10^2\text{ m/s}^2}{5.4\times10^2\text{ m/s}^2}=0.59$$

and

$$\frac{a_{0.39\text{ kg}}}{a_{0.66\text{ kg}}}=\frac{0.39\text{ kg}}{0.66\text{ kg}}=0.59$$

This validates our answers.

7.32. (*a*) When the two children interact with each other the magnitude of the acceleration of each child is inversely proportional to the child's inertia. We can therefore write

$$\frac{a_{30\text{ kg}}}{a_{25\text{ kg}}}=\frac{m_{25\text{ kg}}}{m_{30\text{ kg}}}\Rightarrow a_{25\text{ kg}}=a_{30\text{ kg}}\frac{m_{30\text{ kg}}}{m_{25\text{ kg}}}=(1.0\text{ m/s}^2)\left(\frac{30\text{ kg}}{25\text{ kg}}\right)=1.2\text{ m/s}^2$$

We used the above relationship to obtain the magnitude of the acceleration only. It is clear that the 25 kg child will move in the opposite direction as the 30 kg child. Hence, the acceleration of the 25 kg child is 1.2 m/s^2 right. (*b*) Both accelerations would decrease, but their ratio would remain the same.

7.33. (*a*) Call the direction in which the bullet is fired the $+x$ direction. We know the ratio of accelerations is related to the ratio of inertias via

$$\frac{a_{\text{gun}\,x}}{a_{\text{bullet}\,x}}=-\frac{m_\text{bullet}}{m_\text{gun}}=-\frac{(0.010\text{ kg})}{(5.0\text{ kg})}=-0.0020$$

(*b*) We know that the velocity of the bullet relative to the gun is 250 m/s forward (the muzzle velocity). We can write this in terms of the velocity components relative to ground as

$$v_{\text{bullet}\,x,\text{f}} - v_{\text{gun}\,x,\text{f}} = v_{\text{muzzle}} \qquad (1)$$

We can relate these final velocity components of the bullet and gun because we know $v_{x,\text{f}} = a_x \Delta t$ for either object. Hence

$$\frac{v_{\text{gun}\,x,\text{f}}}{v_{\text{bullet}\,x,\text{f}}} = \frac{a_{\text{gun}\,x}\Delta t}{a_{\text{bullet}\,x}\Delta t} = -0.0020 \text{ or } v_{\text{gun}\,x,\text{f}} = (-0.0020)v_{\text{bullet}\,x,\text{f}}$$

Inserting this into equation (1) yields

$$v_{\text{bullet}\,x,\text{f}} - (-0.0020)v_{\text{bullet}\,x,\text{f}} = v_{\text{muzzle}}$$

$$v_{\text{bullet}\,x,\text{f}} = \frac{v_{\text{muzzle}}}{1.0020} = \frac{(250 \text{ m/s})}{1.0020} = 250 \text{ m/s}$$

Clearly the ratio above is not exactly equal to 250 m/s, but to two significant digits, the speed of the bullet relative to the ground is 250 m/s.

(*c*) We relate the final speeds of the gun and bullet exactly as in part (*b*). Again we write

$$v_{\text{bullet}\,x,\text{f}} - v_{\text{gun}\,x,\text{f}} = v_{\text{muzzle}} \qquad (1)$$

And we insert $v_{\text{bullet}\,x,\text{f}} = \dfrac{v_{\text{gun}\,x,\text{f}}}{(-0.0020)}$ into (1) to obtain

$$\frac{v_{\text{gun}\,x,\text{f}}}{(-0.0020)} - v_{\text{gun}\,x,\text{f}} = v_{\text{muzzle}}$$

$$v_{\text{gun}\,x,\text{f}} = -\frac{v_{\text{muzzle}}}{\left(1+\dfrac{1}{0.0020}\right)} = -\frac{(250 \text{ m/s})}{\left(1+\dfrac{1}{0.0020}\right)} = -0.50 \text{ m/s}$$

So the gun recoils at a speed of 0.50 m/s.

7.34. Call the initial direction of motion of the 1500 kg vehicle $+x$. We begin by determining the final x component of the velocities of the vehicles by using conservation of momentum along the x direction.

$$m_\text{c} v_{\text{c}\,x,\text{i}} + m_\text{t} v_{\text{t}\,x,\text{i}} = m_\text{c} v_{\text{c}\,x,\text{f}} + m_\text{t} v_{\text{t}\,x,\text{f}} = (m_\text{c} + m_\text{t}) v_{x,\text{f}}$$

where $v_{x,\text{f}}$ is the final x component of the velocity common to both vehicles.

$$v_{x,\text{f}} = \frac{m_\text{c} v_{\text{c}\,x,\text{i}} + m_\text{t} v_{\text{t}\,x,\text{i}}}{(m_\text{c} + m_\text{t})} = \frac{(1500 \text{ kg})(6.32 \text{ m/s}) + (3000 \text{ kg})(0)}{((1500 \text{ kg}) + (3000 \text{ kg}))} = 2.107 \text{ m/s}$$

Here we have retained an additional significant digit because this is an intermediate step. To find the acceleration of each vehicle, we write

$$a_{\text{c}\,x} = \frac{v_{\text{c}\,x,\text{f}} - v_{\text{c}\,x,\text{i}}}{\Delta t} = \frac{(2.107 \text{ m/s}) - (6.32 \text{m/s})}{(0.203 \text{ s})} = -20.8 \text{ m/s}^2$$

$$a_{\text{c}\,x} = \frac{v_{\text{t}\,x,\text{f}} - v_{\text{t}\,x,\text{i}}}{\Delta t} = \frac{(2.107 \text{ m/s}) - (0)}{(0.203 \text{ s})} = +10.4 \text{ m/s}^2$$

Such that $\vec{a}_{1500 \text{ kg}} = -(20.8 \text{ m/s}^2)\,\hat{i}$ and $\vec{a}_{3000 \text{ kg}} = +(10.4 \text{ m/s}^2)\,\hat{i}$. Note that $\dfrac{a_{\text{c}\,x}}{a_{\text{t}\,x}} = -2 = -\dfrac{m_\text{t}}{m_\text{c}}$. This means that the ratio of the accelerations is the negative inverse of the ratio of the inertias, as expected.

7.35. (*a*) Call the initial direction of motion of the glob the $+x$ direction. We know the momentum must be conserved, so we can write

$$m_\text{g} v_{\text{g}\,x,\text{i}} + m_\text{c} v_{\text{c}\,x,\text{i}} = m_\text{g} v_{\text{g}\,x,\text{f}} + m_\text{c} v_{\text{c}\,x,\text{f}}$$

$$m_\text{g} = m_\text{c} \frac{(v_{\text{c}\,x,\text{f}} - v_{\text{c}\,x,\text{i}})}{(v_{\text{g}\,x,\text{i}} - v_{\text{g}\,x,\text{f}})} \qquad (1)$$

We also know the coefficient of restitution, so we can find the initial x component of the velocity of the glob by writing

$$e = -\frac{(v_{\mathrm{c}\,x,\mathrm{f}} - v_{\mathrm{g}\,x,\mathrm{f}})}{(v_{\mathrm{c}\,x,\mathrm{i}} - v_{\mathrm{g}\,x,\mathrm{i}})}$$

$$v_{\mathrm{g}\,x,\mathrm{i}} = v_{\mathrm{c}\,x,\mathrm{i}} + \frac{(v_{\mathrm{c}\,x,\mathrm{f}} - v_{\mathrm{g}\,x,\mathrm{f}})}{e}$$

$$v_{\mathrm{g}\,x,\mathrm{i}} = (0) + \frac{((1.0\text{ m/s}) - (-0.834\text{ m/s}))}{(0.64)} = 2.866\text{ m/s}$$

Here, we have kept some additional significant digits because this is an intermediate step. We can insert this result into equation (1) to obtain

$$m_\mathrm{g} = (0.500\text{ kg})\frac{((1.0\text{ m/s}) - (0))}{((2.866\text{ m/s}) - (-0.834\text{ m/s}))} = 0.14\text{ kg}$$

Thus the inertia of the glob is 0.14 kg. (*b*) Because we know the initial and final velocities of each object and the duration of the collision, we can calculate the accelerations directly. We have

$$a_{\mathrm{g}\,x} = \frac{v_{\mathrm{g}\,x,\mathrm{f}} - v_{\mathrm{g}\,x,\mathrm{i}}}{\Delta t} = \frac{(-0.834\text{ m/s}) - (2.866\text{ m/s})}{(0.15\text{ s})} = -25\text{ m/s}^2$$

And

$$a_{\mathrm{c}\,x} = \frac{v_{\mathrm{c}\,x,\mathrm{f}} - v_{\mathrm{c}\,x,\mathrm{i}}}{\Delta t} = \frac{(+1.0\text{ m/s}) - (0)}{(0.15\text{ s})} = +6.7\text{ m/s}^2$$

Thus $\vec{a}_\mathrm{g} = 25\text{ m/s}^2$ to the left and $\vec{a}_\mathrm{c} = 6.7\text{ m/s}^2$ to the right.

7.36. Call $+x$ the direction of the initial motion of the two chunks together. We will refer to the chunk at the front (the on that would be accelerated forward) as chunk A, and the chunk at the rear as chunk B. We know that momentum will be conserved in this explosive separation, and although kinetic energy is not conserved, we know by what amount it is increased. These facts allow us to write

$$(m_\mathrm{A} + m_\mathrm{B})v_{x,\mathrm{i}} = m_\mathrm{A} v_{\mathrm{A}\,x,\mathrm{f}} + m_\mathrm{B} v_{\mathrm{B}\,x,\mathrm{f}} \tag{1}$$

$$\frac{1}{2}(m_\mathrm{A} + m_\mathrm{B})v_{x,\mathrm{i}}^2 + \Delta E = \frac{1}{2}m_\mathrm{A} v_{\mathrm{A}\,x,\mathrm{f}}^2 + \frac{1}{2}m_\mathrm{B} v_{\mathrm{B}\,x,\mathrm{f}}^2 \tag{2}$$

We can solve equation (2) for $v_{\mathrm{A}\,x,\mathrm{f}}$ to obtain

$$v_{\mathrm{A}\,x,\mathrm{f}} = \sqrt{((m_\mathrm{A} + m_\mathrm{B})v_{x,\mathrm{i}}^2 + 2\Delta E - m_\mathrm{B} v_{\mathrm{B}\,x,\mathrm{f}}^2)/m_\mathrm{A}} \tag{3}$$

Inserting this into equation (1) yields

$$v_{\mathrm{B}\,x,\mathrm{f}} = \frac{1}{m_\mathrm{B}}\left((m_\mathrm{A} + m_\mathrm{B})v_{x,\mathrm{i}} - m_\mathrm{A}\sqrt{((m_\mathrm{A} + m_\mathrm{B})v_{x,\mathrm{i}}^2 + 2\Delta E - m_\mathrm{B} v_{\mathrm{B}\,x,\mathrm{f}}^2)/m_\mathrm{A}}\right)$$

With some algebra, this can be expressed as a quadratic equation with solutions

$$v_{\mathrm{B}\,x,\mathrm{f}} = v_{x,\mathrm{i}} \pm \sqrt{\frac{\Delta E}{m_\mathrm{B}}}$$

The $\pm$ sign in the solution simply indicates that there is nothing in our equation that dictates that chunk B is in the rear. Because of that choice, we know

$$v_{\mathrm{B}\,x,\mathrm{f}} = v_{x,\mathrm{i}} - \sqrt{\frac{\Delta E}{m_\mathrm{B}}} = (2.34\text{ m/s}) - \sqrt{\frac{16\text{ J}}{4.10\text{ kg}}} = +0.365\text{ m/s}$$

This can be inserted into equation (3) to yield the final $v_{\mathrm{A}\,x,\mathrm{f}}$.

With no additional algebra, we can argue that $v_{\mathrm{A}\,x,\mathrm{f}} = v_{x,\mathrm{i}} + \sqrt{\frac{\Delta E}{m_\mathrm{B}}}$. Because the inertias of the two chunks are equal, the change in speed over any time interval must be the same for each object. So, however much the speed of chunk B is reduced, the speed of chunk A must be increased by that same amount. Hence $v_{\mathrm{A}\,x,\mathrm{f}} = +4.32\text{ m/s}$.

Now that we have the initial and final velocities of each chunk, calculating the acceleration is trivial. We write

$$a_{\mathrm{A}\,x}=\frac{v_{\mathrm{A}\,x,\mathrm{f}}-v_{\mathrm{A}\,x,\mathrm{i}}}{\Delta t}=\frac{(4.32\ \mathrm{m/s})-(2.34\ \mathrm{m/s})}{(0.16\ \mathrm{s})}=+12\ \mathrm{m/s^2}$$

and

$$a_{\mathrm{B}\,x}=\frac{v_{\mathrm{B}\,x,\mathrm{f}}-v_{\mathrm{B}\,x,\mathrm{i}}}{\Delta t}=\frac{(0.365\ \mathrm{m/s})-(2.34\ \mathrm{m/s})}{(0.16\ \mathrm{s})}=-12\ \mathrm{m/s^2}$$

The acceleration of the rear chuck is $-12\ \mathrm{m/s^2}\ \hat{\imath}$ and the acceleration of the front chunk is $+12\ \mathrm{m/s^2}\ \hat{\imath}$.

7.37. Call right the $+x$ direction. We can first find the final x component of the velocity of the single railroad car. When we can write the acceleration in terms of the final velocity, the initial velocity and the elapsed time. We find the final x component of the velocity using conservation of momentum.

$$m_{\mathrm{car}}v_{\mathrm{car}\,x,\mathrm{i}}+m_{3\ \mathrm{car}}v_{3\ \mathrm{car}\,x,\mathrm{i}}=4m_{\mathrm{car}}v_{x,\mathrm{f}}$$

$$v_{x,\mathrm{f}}=\frac{1}{4}v_{\mathrm{car}\,x,\mathrm{i}}+\frac{3}{4}v_{3\ \mathrm{car}\,x,\mathrm{i}}$$

$$v_{x,\mathrm{f}}=\frac{1}{4}(4.25\ \mathrm{m/s})+\frac{3}{4}(2.09\ \mathrm{m/s})$$

$$v_{x,\mathrm{f}}=2.63\ \mathrm{m/s}$$

This is the final x component of the velocity of all cars. Now we know we can write the acceleration as $a_{\mathrm{car}\,x}=\dfrac{v_{\mathrm{car}\,x,\mathrm{f}}-v_{\mathrm{car}\,x,\mathrm{i}}}{\Delta t}$, or equivalently

$$\Delta t=\frac{v_{\mathrm{car}\,x,\mathrm{f}}-v_{\mathrm{car}\,x,\mathrm{i}}}{a_{\mathrm{car}\,x}}=\frac{(2.63\ \mathrm{m/s})-(4.25\ \mathrm{m/s})}{(-5.45\ \mathrm{m/s^2})}=0.297\ \mathrm{s}$$

So the duration of the collision is 0.297 s.

7.38. (*a*) Call west the $+x$ direction. We will use conservation of momentum to find the x component of their final mutual velocity. We will refer to the halfback as object H, and the opponent as object O.

$$m_{\mathrm{H}}v_{\mathrm{H}\,x,\mathrm{i}}+m_{\mathrm{O}}v_{\mathrm{O}\,x,\mathrm{i}}=(m_{\mathrm{H}}+m_{\mathrm{O}})v_{x,\mathrm{f}}$$

$$v_{x,\mathrm{f}}=\frac{m_{\mathrm{H}}v_{\mathrm{H}\,x,\mathrm{i}}+m_{\mathrm{O}}v_{\mathrm{O}\,x,\mathrm{i}}}{(m_{\mathrm{H}}+m_{\mathrm{O}})}$$

$$v_{x,\mathrm{f}}=\frac{(90\ \mathrm{kg})(10\ \mathrm{m/s})+(120\ \mathrm{kg})(-4.37\ \mathrm{m/s})}{((90\ \mathrm{kg})+(120\ \mathrm{kg}))}$$

$$v_{x,\mathrm{f}}=1.789\ \mathrm{m/s}$$

Now we can calculate the acceleration of each player.

$$a_{\mathrm{H}\,x}=\frac{v_{\mathrm{H}\,x,\mathrm{f}}-v_{\mathrm{H}\,x,\mathrm{i}}}{\Delta t}=\frac{(1.789\ \mathrm{m/s})-(10\ \mathrm{m/s})}{(0.207\ \mathrm{s})}=-40\ \mathrm{m/s^2}$$

and

$$a_{\mathrm{O}\,x}=\frac{v_{\mathrm{O}\,x,\mathrm{f}}-v_{\mathrm{O}\,x,\mathrm{i}}}{\Delta t}=\frac{(1.789\ \mathrm{m/s})-(-4.37\ \mathrm{m/s})}{(0.207\ \mathrm{s})}=+30\ \mathrm{m/s^2}$$

Thus $\vec{a}_{\mathrm{H}}=40\ \mathrm{m/s^2}$ east and $\vec{a}_{\mathrm{O}}=30\ \mathrm{m/s^2}$ west. (*b*) We simply find the difference between the initial and final kinetic energies of the two players combined, and we attribute the difference to the transformation of kinetic energy into incoherent energy.

$$K_\mathrm{i} - \Delta E_\mathrm{incoherent} = K_\mathrm{f}$$
$$\Delta E_\mathrm{incoherent} = K_\mathrm{i} - K_\mathrm{f}$$
$$\Delta E_\mathrm{incoherent} = \left(\frac{1}{2}m_\mathrm{H}v_\mathrm{H,i}^2 + \frac{1}{2}m_\mathrm{O}v_\mathrm{O,i}^2\right) - \left(\frac{1}{2}m_\mathrm{H}v_\mathrm{H,f}^2 + \frac{1}{2}m_\mathrm{O}v_\mathrm{O,f}^2\right)$$
$$\Delta E_\mathrm{incoherent} = \left(\frac{1}{2}(90\ \mathrm{kg})(10\ \mathrm{m/s})^2 + \frac{1}{2}(120\ \mathrm{kg})(4.37\ \mathrm{m/s})^2\right) - \left(\frac{1}{2}(90\ \mathrm{kg} + 120\ \mathrm{kg})(1.789\ \mathrm{m/s})^2\right)$$
$$\Delta E_\mathrm{incoherent} = 5.3\ \mathrm{kJ}$$

Thus 5.3 kJ of kinetic energy have been converted to incoherent energy.

7.39. (*a*) Let the direction of the puck's initial velocity be the $+x$ direction. We calculate the acceleration of the puck using $a_x = \dfrac{v_{x,\mathrm{f}}^2 - v_{x,\mathrm{i}}^2}{2\Delta x}$. We obtain

$$a_{\mathrm{puck}\ x} = \frac{(0.071\ \mathrm{m/s})^2 - (40\ \mathrm{m/s})^2}{2(0.15\ \mathrm{m})} = -5.3\times 10^3\ \mathrm{m/s^2}$$

One might be tempted to do the same for the goalie. However, what we know is that the goalie's glove moves a certain distance. This is not the same thing as the entire goalie moving that same distance. A better way to go about this, is to use the above acceleration for the puck to find the duration of the collision. That, of course, is the same for the puck and the goalie. We can then use the duration to determine the goalie's acceleration.

$$\Delta t = \frac{v_{\mathrm{puck}\ x,\mathrm{f}} - v_{\mathrm{puck}\ x,\mathrm{i}}}{a_{\mathrm{puck}\ x}}$$
$$\Delta t = \frac{(0.071\ \mathrm{m/s}) - (40\ \mathrm{m/s})}{(-5.33\times 10^3\ \mathrm{m/s^2})}$$
$$\Delta t = 7.49\times 10^{-3}\ \mathrm{s}$$

We can use this time as follows:

$$a_{\mathrm{goalie}\ x} = \frac{v_{\mathrm{goalie}\ x,\mathrm{f}} - v_{\mathrm{goalie}\ x,\mathrm{i}}}{\Delta t} = \frac{(0.071\ \mathrm{m/s}) - (0)}{(7.49\times 10^{-3}\ \mathrm{s})}$$
$$a_{\mathrm{goalie}\ x} = 9.5\ \mathrm{m/s^2}$$

Thus we have $\vec{a}_\mathrm{goalie\ av} = +9.5\ \mathrm{m/s^2}\ \hat{\imath}$ and $\vec{a}_\mathrm{puck\ av} = -5.3\times 10^3\ \mathrm{m/s^2}\ \hat{\imath}$. (*b*) The inertia can be found using the ratio of the accelerations.

$$m_\mathrm{goalie} = -m_\mathrm{puck}\left(\frac{a_{\mathrm{puck}\ x}}{a_{\mathrm{goalie}\ x}}\right) = -(0.16\ \mathrm{kg})\left(\frac{(-5.33\times 10^3\ \mathrm{m/s^2})}{(9.48\ \mathrm{m/s^2})}\right) = 90\ \mathrm{kg}$$

So the total inertia for the goalie with all equipment is 90 kg. (*c*) All loss of kinetic energy occurs because the energy is converted to thermal energy.

$$\Delta E_\mathrm{thermal} = K_\mathrm{i} - K_\mathrm{f}$$
$$\Delta E_\mathrm{thermal} = \left(\frac{1}{2}m_\mathrm{goalie}v_\mathrm{goalie,i}^2 + \frac{1}{2}m_\mathrm{puck}v_\mathrm{puck,i}^2\right) - \left(\frac{1}{2}m_\mathrm{goalie}v_\mathrm{goalie,f}^2 + \frac{1}{2}m_\mathrm{puck}v_\mathrm{puck,f}^2\right)$$
$$\Delta E_\mathrm{thermal} = \left(\frac{1}{2}(90\ \mathrm{kg})(0)^2 + \frac{1}{2}(0.16\ \mathrm{kg})(40\ \mathrm{m/s})^2\right) - \left(\frac{1}{2}(90\ \mathrm{kg})(0.071\ \mathrm{m/s})^2 + \frac{1}{2}(0.16\ \mathrm{kg})(0.071\ \mathrm{m/s})^2\right)$$
$$\Delta E_\mathrm{thermal} = 1.3\times 10^2\ \mathrm{J}$$

7.40. (*a*) Since the total momentum of the system must remain constant, the changes in momentum must be equal and opposite. This means $\Delta p_{\text{red }x}/\Delta p_{\text{yellow }x} = -1$. From PRIN equation (7.6) we know that $v_{\text{red }x}/v_{\text{yellow }x} = -m_{\text{yellow}}/m_{\text{red}}$. We write the kinetic energies in terms of the inertias and speeds.

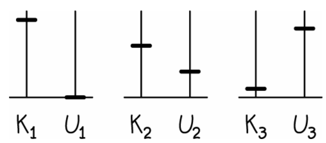
$$\frac{\Delta K_{\text{red}}}{\Delta K_{\text{yellow}}} = \frac{\frac{1}{2}m_{\text{red}}v_{\text{red}}^2}{\frac{1}{2}m_{\text{yellow}}v_{\text{yellow}}^2} = \frac{m_{\text{red}}}{m_{\text{yellow}}}\left(\frac{m_{\text{yellow}}}{m_{\text{red}}}\right)^2 = \frac{m_{\text{yellow}}}{m_{\text{red}}}$$

(*b*) The ratio of the x component of accelerations is $a_{\text{red }x}/a_{\text{yellow }x} = -m_{\text{yellow}}/m_{\text{red}}$. Comparing with the results from part (*a*), we see

$$\frac{\Delta p_{\text{red,x}}}{\Delta p_{\text{yellow,x}}} = -1 > \frac{a_{\text{red,x}}}{a_{\text{yellow,x}}},\quad \frac{\Delta v_{\text{red,x}}}{\Delta v_{\text{yellow,x}}} = -\frac{m_{\text{yellow}}}{m_{\text{red}}} = \frac{a_{\text{red,x}}}{a_{\text{yellow,x}}},\quad \frac{\Delta K_{\text{red}}}{\Delta K_{\text{yellow}}} = \frac{m_{\text{yellow}}}{m_{\text{red}}} = -\frac{a_{\text{red,x}}}{a_{\text{yellow,x}}}$$

(*c*) If we call the ball the red object and the earth the yellow object, we can readily see several things. The change in momentum of the ball and Earth are equal in magnitude and in opposite directions. The change in speed of the ball at some time is larger than the change in speed of the Earth by an enormous factor (the mass of Earth over the mass of the ball), the changes in velocity are in opposite directions. Similarly, the change in kinetic energy of the ball at some time is greater than the kinetic energy of Earth by that same enormous factor.

7.41.



7.42. (*a*) We solve this problem by requiring that all energy (all forms of energy added together) must be constant. We have kinetic and potential energy to consider. Initially, the potential energy is zero, and we are asked to find the position at which the speed (and therefore the kinetic energy) is zero. Thus

$$\begin{aligned}
K_{\text{i}} + U_{\text{i}} &= K_{\text{f}} + U_{\text{f}}\\
U_{\text{f}} &= K_{\text{i}}\\
ax_{\text{f}}^2 &= \frac{1}{2}mv_{\text{i}}^2\\
x_{\text{f}} &= \pm\sqrt{\frac{mv_{\text{i}}^2}{2a}} = \pm\sqrt{\frac{(0.82\ \text{kg})}{2(6.4\ \text{J/m}^2)}}(2.23\ \text{m/s})\\
x_{\text{f}} &= \pm 0.56\ \text{m}
\end{aligned}$$

So the object could cover a distance of 0.56 m before first stopping. (*b*) No. The velocity is squared in the expression for the kinetic energy, so any negative signs would disappear.

7.43. (*a*) Positive. Physically the elastic potential energy refers to energy that is stored in the spring. It cannot be negative, any more than kinetic energy can be negative. One way of thinking of this is that the elastic potential energy can be converted into motion (oscillations). And that motion corresponds to a positive kinetic energy, so there must have been some positive energy stored. Mathematically, one sees that in the equation for elastic potential energy the displacement from equilibrium is squared. (*b*) Positive, as explained in part (*a*).

7.44. Let the $+x$ axis point to the right. We can use conservation of momentum to determine the final velocity of the lighter cart. This is simplified by the fact that the carts are initially at rest. Call the heavier cart A, and the lighter cart B.

$$\begin{aligned}
m_{\text{A}}v_{\text{A }x,\text{f}} + m_{\text{B}}v_{\text{B }x,\text{f}} &= 0\\
v_{\text{B }x,\text{f}} &= -\frac{m_{\text{A}}}{m_{\text{B}}}v_{\text{A }x,\text{f}} = -\frac{(0.36\ \text{kg})}{(0.12\ \text{kg})}(1.1\ \text{m/s}) = -3.3\ \text{m/s}
\end{aligned}$$

Now we can determine the final kinetic energy of both carts together. All of that kinetic energy was provided by the conversion of elastic potential energy. Thus

$$U_\text{i} = K_\text{f} = \frac{1}{2}m_\text{A}v_\text{A,f}^2 + \frac{1}{2}m_\text{B}v_\text{B,f}^2$$

$$U_\text{i} = \frac{1}{2}(0.36\text{ kg})(1.1\text{ m/s})^2 + \frac{1}{2}(0.12\text{ kg})(3.3\text{ m/s})^2$$

$$U_\text{i} = 0.87\text{ J}$$

So 0.87 J was initially stored as elastic potential energy in the spring.

7.45. When the cart comes to rest against the spring, all the kinetic energy it had after the collision has been converted to elastic potential energy. So our problem reduces to finding the kinetic energy of the 0.25 kg cart immediately after the collision. We do this by using conservation of momentum and energy. Let the $+x$ axis point to the right, and call the 0.530 kg cart A, and the 0.25 kg cart B.

$$m_\text{A}v_{\text{A}\,x,\text{i}} + m_\text{B}v_{\text{B}\,x,\text{i}} = m_\text{A}v_{\text{A}\,x,\text{f}} + m_\text{B}v_{\text{B}\,x,\text{f}} \tag{1}$$

$$\frac{1}{2}m_\text{A}v_\text{A,i}^2 + \frac{1}{2}m_\text{B}v_\text{B,i}^2 = \frac{1}{2}m_\text{A}v_\text{A,f}^2 + \frac{1}{2}m_\text{B}v_\text{B,f}^2 \tag{2}$$

Solving (1) for $v_{\text{B}\,x,\text{f}}$ and inserting the expression into (2) yields

$$\frac{1}{2}m_\text{A}v_\text{A,i}^2 + \frac{1}{2}m_\text{B}v_\text{B,i}^2 = \frac{1}{2}m_\text{A}v_\text{A,f}^2 + \frac{1}{2}m_\text{B}\left(\frac{m_\text{A}}{m_\text{B}}(v_{\text{A}\,x,\text{i}} - v_{\text{A}\,x,\text{f}}) + v_{\text{B}\,x,\text{i}}\right)^2$$

This is a quadratic equation in $v_{\text{A}\,x,\text{f}}$, and can be solved using the quadratic equation or a calculator. This yields $v_{\text{A}\,x,\text{f}} = 0.331\text{ m/s}$ and $v_{\text{B}\,x,\text{f}} = 1.25\text{ m/s}.$ From this we can trivially calculate the kinetic energy of the lighter cart (B) after the collision.

$$K_\text{B,f} = \frac{1}{2}m_\text{B}v_\text{B,f}^2 = 0.20\text{ J}$$

As stated earlier, this means that the elastic potential energy that will be stored in the spring when cart B comes to rest is also 0.20 J.

7.46. (*a*) Let the $+x$ axis point to the right. Call the 2.0 kg cart A, and the 3.0 kg cart B. To calculate the coefficient of restitution we need the initial and final velocities of both carts. We can find the final velocity of cart B using conservation of momentum.

$$m_\text{A}v_{\text{A}\,x,\text{i}} + m_\text{B}v_{\text{B}\,x,\text{i}} = m_\text{A}v_{\text{A}\,x,\text{f}} + m_\text{B}v_{\text{B}\,x,\text{f}}$$

$$v_{\text{B}\,x,\text{f}} = \frac{m_\text{A}}{m_\text{B}}(v_{\text{A}\,x,\text{i}} - v_{\text{A}\,x,\text{f}}) + v_{\text{B}\,x,\text{i}}$$

$$v_{\text{B}\,x,\text{f}} = \frac{(2.0\text{ kg})}{(3.0\text{ kg})}((5.0\text{ m/s}) - (1.7\text{ m/s})) + (2.0\text{ m/s}) = 4.2\text{ m/s}$$

Now we can calculate e in the usual way

$$e = -\frac{(v_{\text{A}\,x,\text{f}} - v_{\text{B}\,x,\text{f}})}{(v_{\text{A}\,x,\text{i}} - v_{\text{B}\,x,\text{i}})} = -\frac{((1.7\text{ m/s}) - (4.2\text{ m/s}))}{((5.0\text{ m/s}) - (2.0\text{ m/s}))} = 0.83$$

(*b*) The 3.0 kg cart will strike the spring. At the maximum compression, the cart will stop and have no kinetic energy. Thus all the kinetic energy of cart B after the collision with cart A will be converted to spring potential energy. Thus

$$U_\text{max} = K_\text{B,f} = \frac{1}{2}m_\text{B}v_\text{B,f}^2 = \frac{1}{2}(3.0\text{ kg})(4.2\text{ m/s})^2 = 26\text{ J}$$

The spring stores 26 J of elastic potential energy at its maximum compression.

7.47. We solve both parts of this problem by requiring that the total energy of the object (kinetic and potential energy together) must be conserved. We can start by calculating the initial total energy of the object.

$$K_\text{i} + U_\text{i} = \frac{1}{2}mv_\text{i}^2 + (ax_\text{i} + bx_\text{i}^2) = \frac{1}{2}(10\text{ kg})(-3.0\text{ m/s})^2 + ((4.0\text{ J/m})(2.0\text{ m}) + (-2.0\text{ J/m}^2)(2.0\text{ m})^2)$$

$$K_\text{i} + U_\text{i} = 45\text{ J}$$

We will call this energy $E_\text{i} \equiv 45$ J.

(*a*) We require that the energy at this new position still be E_i. Thus

$$K_\text{f} + U_\text{f} = E_\text{i}$$

$$\frac{1}{2}mv_\text{f}^2 + (ax_\text{f} + bx_\text{f}^2) = E_\text{i}$$

$$v_\text{f} = \sqrt{\frac{2}{m}(E_\text{i} - (ax_\text{f} + bx_\text{f}^2))}$$

$$v_\text{f} = \sqrt{\frac{2}{(10\text{ kg})}((45\text{ J}) - ((4.0\text{ J/m})(1.0\text{ m}) + (-2.0\text{ J/m}^2)(1.0\text{ m})^2))} = 2.9\text{ m/s}$$

So the speed at this position is 2.9 m/s.

(*b*) We do the same process as before, but we insert $x_\text{f} = -1.0$ m. We obtain

$$v_\text{f} = \sqrt{\frac{2}{(10\text{ kg})}((45\text{ J}) - ((4.0\text{ J/m})(-1.0\text{ m}) + (-2.0\text{ J/m}^2)(-1.0\text{ m})^2))} = 3.2\text{ m/s}$$

So the speed at this position is 3.2 m/s.

7.48. (*a*) One may have read ahead to section 7.9, and may therefore already know the formula for gravitational potential energy. However, we will proceed without this. Let us start by requiring that the total energy (the sum of kinetic and gravitational potential energy) be constant. Further, let us measure our gravitational potential energy relative to ground at the bottom of the incline (that is $U^G(x = \ell) = 0).$ We can obtain an expression for the kinetic energy after moving down the incline a distance x using kinematics. Recall that $v_{x,\text{f}}^2 = v_{x,\text{i}}^2 + 2a_x\Delta x.$ Since the block starts from rest, we can write $\frac{1}{2}mv_{x,\text{f}}^2 = \frac{m}{2}(2a_x\Delta x) = mg\sin(\theta)x.$ In particular, let us consider the case when the block has slid all the way down the incline. At that moment, we know $K = mg\sin(\theta)\ell.$ We also know that the potential energy at the ground is zero (by assumption). This means that the total energy at the bottom is $mg\sin(\theta)\ell,$ and by the conservation of energy, this must always be the total energy. In particular, after the block has moved some distance $x,$ we can write

$$K(x) + U^G(x) = E_\text{total}$$

$$U^G(x) = mg\sin(\theta)\ell - K(x)$$

$$U^G(x) = mg\sin(\theta)\ell - mg\sin(\theta)x$$

$$U^G(x) = mg\sin(\theta)(\ell - x)$$

This is the expression for the gravitational potential energy as a function of x.

(*b*) It does not change. (*c*) No. (*d*) Starting with $K = \frac{1}{2}mv_{x,\text{f}}^2,$ we know $v_{x,\text{f}} = \sqrt{\frac{2K}{m}}.$ Thus $v_{x,\text{f}} = \sqrt{2g\sin(\theta)\ell}.$ From kinematics we know $v_{x,\text{f}}^2 = v_{x,\text{i}}^2 + 2a_x\Delta x,$ from which we obtain $v_{x,\text{f}}^2 = 2g\sin(\theta)x$ or $v_{x,\text{f}} = \sqrt{2g\sin(\theta)x}.$ We see the two results agree.

7.49. (*a*) We solve this problem using the conservation of energy and the conservation of momentum. Call the 0.36 kg cart A, and the 0.12 kg cart B. Le the $+x$ axis point to the right. For the first collision, we have

$$m_\text{A}v_{\text{A}\,x,\text{i}} + m_\text{B}v_{\text{B}\,x,\text{i}} = m_\text{A}v_{\text{A}\,x,\text{f}} + m_\text{B}v_{\text{B}\,x,\text{f}} \tag{1}$$

$$\frac{1}{2}m_\text{A}v_{\text{A,i}}^2 + \frac{1}{2}m_\text{B}v_{\text{B,i}}^2 = \frac{1}{2}m_\text{A}v_{\text{A,f}}^2 + \frac{1}{2}m_\text{B}v_{\text{B,f}}^2 \tag{2}$$

We can solve for the x components of both final velocities by solving equation (1) for $v_{\mathrm{B}\,x,\mathrm{f}}$.

$$v_{\mathrm{B}\,x,\mathrm{f}} = \frac{m_\mathrm{A}}{m_\mathrm{B}}(v_{\mathrm{A}\,x,\mathrm{i}} - v_{\mathrm{A}\,x,\mathrm{f}}) + v_{\mathrm{B}\,x,\mathrm{i}} \tag{3}$$

Inserting this into equation (2) yields an equation that is quadratic in $v_{\mathrm{A}\,x,\mathrm{f}}$:

$$m_\mathrm{A} v_{\mathrm{A,i}}^2 + m_\mathrm{B} v_{\mathrm{B,i}}^2 = m_\mathrm{A} v_{\mathrm{A,f}}^2 + m_\mathrm{B}\left(\frac{m_\mathrm{A}}{m_\mathrm{B}}(v_{\mathrm{A}\,x,\mathrm{i}} - v_{\mathrm{A}\,x,\mathrm{f}}) + v_{\mathrm{B}\,x,\mathrm{i}}\right)^2$$

This can be solved using the quadratic equation or a calculator. We find $v_{\mathrm{A}\,x,\mathrm{f}} = 0.96$ m/s and $v_{\mathrm{B}\,x,\mathrm{f}} = 3.14$ m/s. When cart B compresses the spring, kinetic energy is converted to elastic potential energy. At the maximum compression, no kinetic energy remains, and $U = \frac{1}{2} m_\mathrm{B} v_{\mathrm{B}\,x,\mathrm{f}}^2 = \frac{1}{2}(0.12 \text{ kg})(3.14 \text{ m/s})^2 = 0.59$ J.

(*b*) The spring will launch cart B back to the left toward cart A. The collision will proceed just as the first collision from part (*a*). Note that "final" and "initial" now refer to different moments than in part (*a*). "Initial" refers to the moment after cart B has bounced off the spring, just before the two carts collide for the second time. "Final" refers to the moment just after this second collision. For this collision $v_{\mathrm{A}\,x,\mathrm{i}} = +0.96$ m/s and $v_{\mathrm{B}\,x,\mathrm{i}} = -3.14$ m/s. As before, momentum and energy are both conserved. We solve the equivalent expression in equation (1) for $v_{\mathrm{B}\,x,\mathrm{f}}$ and insert it into the expression equivalent to equation (2), which yields a quadratic equation for $v_{\mathrm{A}\,x,\mathrm{f}}$:

$$m_\mathrm{A} v_{\mathrm{A,i}}^2 + m_\mathrm{B} v_{\mathrm{B,i}}^2 = m_\mathrm{A} v_{\mathrm{A,f}}^2 + m_\mathrm{B}\left(\frac{m_\mathrm{A}}{m_\mathrm{B}}(v_{\mathrm{A}\,x,\mathrm{i}} - v_{\mathrm{A}\,x,\mathrm{f}}) + v_{\mathrm{B}\,x,\mathrm{i}}\right)^2$$

Solving yields $v_{\mathrm{A}\,x,\mathrm{f}} = -1.1$ m/s, and inserting this into the analog of equation (3) yields $v_{\mathrm{B}\,x,\mathrm{f}} = +3.0$ m/s. So the final velocities are $\vec{v}_{\mathrm{A,f}} = 1.1$ m/s to the left, and $\vec{v}_{\mathrm{B,f}} = 3.0$ m/s to the right.

7.50. $\Delta U^G = U_\mathrm{f}^G - U_\mathrm{i}^G = mg(h_\mathrm{f} - h_\mathrm{i}) = (70 \text{ kg})(9.8 \text{ m/s}^2)(380 \text{ m}) = 2.6 \times 10^5$ J.

7.51. If air resistance ignored, cap reaches hands at speed *v*; if air resistance not ignored, cap reaches hands at speed slightly less than *v* because some initial kinetic energy is converted to incoherent energy of molecules in air. Both answers come from law of conservation of energy.

7.52. The change in gravitational potential energy is the same in all three cases. The change in gravitational potential energy depends only on the difference in distances from the Earth; it does not depend on the path taken.

7.53. We know the initial kinetic energy will be converted to gravitational potential energy as the bullet rises. When all kinetic energy is converted, the bullet will no longer have any kinetic energy and will stop rising. So treating the bullet at its maximum height as the final case, we can write

$$U_\mathrm{f}^G = K_\mathrm{i}$$
$$mgh_\mathrm{f} = \frac{1}{2}mv_\mathrm{i}^2$$
$$h_\mathrm{f} = \frac{v_\mathrm{i}^2}{2g} = \frac{(300 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 4.6 \text{ km}$$

So, if air resistance could truly be neglected, the bullet would rise to a maximum height of 4.6 km before stopping momentarily and returning to the ground. Note that neglecting air resistance for bullets is not a good approximation.

7.54. (*a*) The two answers turn out to have similar orders of magnitude. So it is reasonable for one to guess either; I happened to guess that getting the plane up to altitude takes more energy.

(*b*) $K_\mathrm{f} = \frac{1}{2}mv_\mathrm{f}^2 = \frac{1}{2}(2.1 \times 10^5 \text{ kg})(270 \text{ m/s})^2 = 7.7 \times 10^9$ J

(*c*) $U_\mathrm{f}^G = mgh_\mathrm{f} = (2.1 \times 10^5 \text{ kg})(9.8 \text{ m/s}^2)(10{,}400 \text{ m}) = 2.1 \times 10^{10}$ J

7.55. We can solve this problem using conservation of energy.

$$K_\mathrm{i} + U_\mathrm{i}^G = K_\mathrm{f} + U_\mathrm{f}^G$$
$$K_\mathrm{f} = U_\mathrm{i}^G - U_\mathrm{f}^G$$
$$\frac{1}{2}(2m)v_\mathrm{f}^2 = mg(h_\mathrm{f} - h_\mathrm{i})$$

Note that only one block falls, so only one block's inertia appears in the change in potential energy. However, both blocks are speeding up, so both blocks appear in the kinetic term. From this we obtain

$$v_\mathrm{f} = \sqrt{g(h_\mathrm{i} - h_\mathrm{f})} = \sqrt{(9.8\ \mathrm{m/s^2})(0.50\ \mathrm{m})} = 2.2\ \mathrm{m/s}$$

7.56. (*a*) The initial gravitational potential energy is $mgh = (30\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(2.0\ \mathrm{m}) = 590\ \mathrm{J}$, and the change in the kinetic energy is equal and opposite the change in gravitational potential energy (since the total energy must be conserved). The change in gravitational potential energy is $\Delta U = U_\mathrm{f} - U_\mathrm{i} = mg(h_\mathrm{f} - h_\mathrm{i}) =$ $(30\ \mathrm{kg})(9.8\,\mathrm{m/s^2})(0 - 2.0\ \mathrm{m}) = -590\ \mathrm{J}$. Hence the change in kinetic energy is $+5.9\times10^2$ J. (*b*) The initial gravitational potential energy is $0\ \mathrm{J}$, and the change in gravitational potential energy is $\Delta U =$ $U_\mathrm{f} - U_\mathrm{i} = mg(h_\mathrm{f} - h_\mathrm{i}) = (30\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(-2.0\ \mathrm{m} - 0) = -590\ \mathrm{J}$. Since total energy is conserved, all the potential energy that was lost must now be in the form of kinetic energy. Hence the change in kinetic energy is $+5.9\times10^2$ J.

7.57. (*a*) Using kinematics we can say $v_{y,\mathrm{i}}^2 = -v_{y,\mathrm{f}}^2 + 2a_y\Delta y$ so that if the ball stops exactly as it reaches your friend $v_{y,\mathrm{i}} = \sqrt{v_{y,\mathrm{f}}^2 - 2a_y\Delta y} = \sqrt{(0) - 2(-9.8\ \mathrm{m/s^2})(11\,\mathrm{m})} = 15\ \mathrm{m/s}$. (*b*) The initial energy when the ball is thrown must be the same as the final energy the instant before it strikes the ground. Thus $E_\mathrm{f} = K_\mathrm{i} + U_\mathrm{i} = \frac{1}{2}mv_\mathrm{i}^2 + mgh_\mathrm{i} =$ $\frac{1}{2}(0.12\ \mathrm{kg})(14.7\ \mathrm{m/s})^2 + (0.12\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(12\ \mathrm{m}) = 27\ \mathrm{J}$. (*c*) Since the kinetic energy only depends on the speed, not the direction of motion, nothing about the above expression changes, and we see that the energy is still equal to 27 J.

7.58. (*a*) Call the heavier block A, and the lighter block B. We can write the change in potential energy of the whole system as $\Delta U = U_\mathrm{f} - U_\mathrm{i} = (U_\mathrm{A,f} + U_\mathrm{B,f}) - (U_\mathrm{A,i} + U_\mathrm{B,i}) = (m_\mathrm{A}gh_\mathrm{A,f} + m_\mathrm{B}gh_\mathrm{B,f}) - (m_\mathrm{A}gh_\mathrm{A,i} + m_\mathrm{B}gh_\mathrm{B,i})$. Reorganizing this in terms of the height differences allows us to insert numerical values:

$$m_\mathrm{A}g(h_\mathrm{A,f} - h_\mathrm{A,i}) + m_\mathrm{B}g(h_\mathrm{B,f} - h_\mathrm{B,i}) = (3.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(-0.53\ \mathrm{m}) + (1.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(+0.53\ \mathrm{m}) = -10\ \mathrm{J}$$

(*b*) Because the total energy must be conserved, the kinetic energy must increase by +10 J. (*c*) Clearly the lighter block moves upward, and both blocks move at the same speed. We can find the speed by requiring that the total kinetic energy be equal to lost potential energy.

$$K = -\Delta U = \frac{1}{2}(m_\mathrm{A} + m_\mathrm{B})v^2$$
$$v = \sqrt{\frac{-2\Delta U}{(m_\mathrm{A} + m_\mathrm{B})}} = \sqrt{\frac{-2(-10.4\ \mathrm{J})}{(4.0\ \mathrm{kg})}} = 2.3\ \mathrm{m/s}$$

So the velocity of the 1.0 kg block is 2.3 m/s upward.

7.59. (*a*) Call the moment of release time 0, the moment just prior to striking the ground time 1, the most just after bouncing off the ground time 2, and the moment the ball reaches its maximum height after bouncing time 3. The speed of the ball just prior to impact is given by $v_1 = \sqrt{v_0^2 + 2a_y\Delta y} = \sqrt{(0) + 2(-9.8\ \mathrm{m/s^2})(-3.0\ \mathrm{m})} = 7.67\ \mathrm{m/s}$, and the ball is clearly moving downward. The speed of the ball just after bouncing is similarly given by $v_2 = \sqrt{v_3^2 - 2a_y\Delta y} = \sqrt{(0) - 2(-9.8\ \mathrm{m/s^2})(2.7\ \mathrm{m})} = 7.27\ \mathrm{m/s}$. Now we can calculate the coefficient of restitution using

$$e = -\frac{(v_{\mathrm{ball}\,y,\mathrm{f}} - v_{\mathrm{ground}\,y,\mathrm{f}})}{(v_{\mathrm{ball}\,y,\mathrm{i}} - v_{\mathrm{ground}\,y,\mathrm{i}})} = -\frac{((+7.27\ \mathrm{m/s}) - (0))}{((-7.67\ \mathrm{m/s}) - (0))} = 0.95$$

(*b*) The initial and final speeds should be related through the same coefficient of restitution. In order for the ball to rise to a height of 7.3 m/s, we need its speed at time 2 to be $v_2 = \sqrt{v_3^2 - 2a_y\Delta y} = \sqrt{(0) - 2(-9.8\ \text{m/s}^2)(7.3\ \text{m})} = 12.0\ \text{m/s}$. Using the same coefficient of restitution yields a speed at point 1 $v_1 = v_2/e = (12.0\ \text{m/s})/(0.95) = 12.6\ \text{m/s}$. All this kinetic energy must have come from the initial gravitational potential energy and the initial kinetic energy you give the ball. Hence

$$K_1 = K_0 + U_0$$

$$\frac{1}{2}mv_{y,0}^2 = \frac{1}{2}mv_{y,1}^2 - mgh_0$$

$$v_{y,0} = \pm\sqrt{v_{y,1}^2 - 2gh_0} = \pm\sqrt{(12.6\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(3.0\ \text{m})} = \pm 10\ \text{m/s}$$

So you can throw the ball either upward or downward with a speed of 10 m/s.

7.60. (*a*) We can use the conservation of energy to determine the speed of the target as it falls downward through the point 10 m above the ground. Although it is unlikely that the target was released exactly at ground level, we can assume that it was released low enough that the initial height is negligible. Hence $\frac{1}{2}m_\text{t}v_{\text{t}\,y,\text{i}}^2 = \frac{1}{2}m_\text{t}v_{\text{t}\,y,\text{f}}^2 + mgh_\text{f}$ or $v_{\text{t}\,y,\text{f}} = \pm\sqrt{v_{\text{t}\,y,\text{i}}^2 - 2gh_\text{f}} = \pm\sqrt{(15\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(10\ \text{m})} = \pm 5.4\ \text{m/s}$. Of course, because the target is falling back toward Earth, we know we want the negative solution. For the collision itself, we cannot use conservation of energy, because the collision is perfectly inelastic. However, we can still use the conservation of momentum. This yields $m_\text{t}v_{\text{t}\,y,\text{i}} + m_\text{a}v_{\text{a}\,y,\text{i}} = (m_\text{t} + m_\text{a})v_{y,\text{f}}$ or

$$v_{y,\text{f}} = \frac{m_\text{t}v_{\text{t}\,y,\text{i}} + m_\text{a}v_{\text{a}\,y,\text{i}}}{(m_\text{t} + m_\text{a})} = \frac{(0.52\ \text{kg})(-5.39\ \text{m/s}) + (0.323\ \text{kg})(25\ \text{m/s})}{((0.52\ \text{kg}) + (0.323\ \text{kg}))} = +6.3\ \text{m/s}$$

So the pair move at 6.3 m/s upward just after the collision.

(*b*) Here we can use conservation of energy. We know $K_\text{i} + U_\text{i} = K_\text{f}$ or $\frac{1}{2}m_\text{pair}v_{\text{pair}\,y,\text{f}}^2 = \frac{1}{2}m_\text{pair}v_{\text{pair}\,y,\text{i}}^2 + m_\text{pair}gh_\text{pair,i}$. We can solve this for the quantity $v_{\text{pair}\,y,\text{f}}$: $v_{\text{pair}\,y,\text{f}} = \pm\sqrt{v_{\text{pair}\,y,\text{i}}^2 + 2gh_\text{pair,i}} = \pm\sqrt{(6.26\ \text{m/s})^2 + 2(9.8\ \text{m/s})(10\ \text{m})} = \pm 15\ \text{m/s}$. For velocity components, clearly we would chose the negative answer. But the speed of the pair is 15 m/s just before striking the ground.

7.61. Let us choose to measure heights relative to the table (that is, let the table be the "ground"). The initial gravitational kinetic energy can be broken into two parts: the portion lying flat on the table and the portion hanging down. The portion lying horizontally is entirely at $h = 0$ and therefore contributes nothing to the potential energy. To find the potential energy of the hanging portion, we integrate over differential contributions to the potential energy:

$$U_\text{i} = \int dU = \int_{-\ell/4}^{0} gydm = \int_{-\ell/4}^{0} g\rho ydy = \frac{\rho}{2}y^2\Big|_{-\ell/4}^{0} = -\frac{mg}{2\ell}\left(\frac{-\ell}{4}\right)^2 = -\frac{mg\ell}{32}$$

Similarly, when the chain slides completely off the table, the gravitational potential energy is

$$U_\text{f} = \int dU = \int_{-\ell}^{0} gydm = \int_{-\ell}^{0} g\rho ydy = \frac{\rho}{2}y^2\Big|_{-\ell}^{0} = -\frac{mg}{2\ell}(\ell)^2 = -\frac{mg\ell}{2}$$

Now we know that any gravitational potential energy lost has been converted to kinetic energy, so we can write

$$U_\text{i} - U_\text{f} = K = \frac{1}{2}mv^2$$

$$-\frac{mg\ell}{32} - \left(\frac{-mg\ell}{2}\right) = \frac{1}{2}\left(\frac{15mg\ell}{16}\right) = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{15g\ell}{16}}$$

7.62. We know that when the chain has fallen a distance y, the conservation of energy tells us $K(y) = -\Delta U = U_\text{i} - U_\text{f} = 0 + mgy/2$. Or, equivalently $\frac{1}{2}mv^2 = \int_0^{y'} gydm = \int_0^{y'} gy\frac{m}{\ell}dy = \frac{gm}{2\ell}(y')^2$. This yields the speed as a function of position, though we have two possible solutions: $v(y) = \pm y\sqrt{\frac{g}{\ell}}$. As described in the problem, we know that $\int \frac{dy}{v} = \int dt$. Let us assume the chain begins to slip at time $t = 0$. Clearly, if none of the chain is initially hanging over the edge, the chain will not fall. But we are asked to consider a very tiny piece hanging over the edge. Let us call the length of the hanging piece ε. Let us choose to measure heights relative to the table (that is, let the table be the "ground").

Then we can write

$$\pm\sqrt{\frac{\ell}{g}}\int_\varepsilon^{y'}\frac{dy}{y} = \int_0^{t'} dt$$

$$\pm\sqrt{\frac{\ell}{g}}\ln(y)\Big|_\varepsilon^{y'} = t'$$

$$\pm\sqrt{\frac{\ell}{g}}\ln\left(\frac{y'}{\varepsilon}\right) = t'$$

Either solution (positive or negative) is acceptable. Earlier it was necessary to distinguish the integration variable t from the integration limit t', and similarly to distinguish y from y'. But having finished the integration, it is now simpler to drop the prime. The above expression can be rearranged to yield a the vertical position as a function of time, though the general solution will be a linear combination of both possible solutions:

$$y(t) = \varepsilon\left(Ae^{t\sqrt{\frac{g}{\ell}}} + Be^{-t\sqrt{\frac{g}{\ell}}}\right) \tag{1}$$

Differentiating both sides of equation (1) with respect to t' yields

$$v(t) = \varepsilon\sqrt{\frac{g}{\ell}}\left(Ae^{t\sqrt{\frac{g}{\ell}}} - Be^{-t\sqrt{\frac{g}{\ell}}}\right) \tag{2}$$

Finally, we can require that the initial speed be zero, and the initial length of the chain hanging off the edge be ε. Equation (1) shows us that for the initial length to be ε, we need $A + B = 1$. Equation (2) shows us that for the chain to be initially at rest, we must have $A - B = 0$. Thus $A = B = \frac{1}{2}$. Thus the speed of the chain after a time t is $v(t) = \frac{\varepsilon}{2}\sqrt{\frac{g}{\ell}}\left(e^{t\sqrt{\frac{g}{\ell}}} - e^{-t\sqrt{\frac{g}{\ell}}}\right)$. Note that this can also be written as $v(t) = \varepsilon\sqrt{\frac{g}{\ell}}\sinh\left(t\sqrt{\frac{g}{\ell}}\right)$.

7.63. Case (b) has the greater speed after the heavier block has descended a distance d. This can be understood, qualitatively as follows. In case (a), the blocks speed up as the heavier block descends. But not all of the gravitational potential energy lost by the heavier block can be converted to kinetic energy, because the lighter block must also be lifted. In other words, some of that gravitational potential energy lost by the heavier block, must be gained by the lighter block. The leftover change in gravitational potential energy can be converted to kinetic energy. In case (b), the elevation of the lighter block does not change. So all gravitational potential energy lost by the heavier block can be converted to kinetic energy.

There are infinitely many possible calculations. Consider when the inertia of the heavier block is 2.0 kg, and it descends 1.0 m. Assume further that the two blocks are at the same elevation at the moment they are released, and call their mutual elevation $y = 0$. We will calculate the kinetic energy after that descent in each case, and compare.

Case (*a*): The increase in kinetic energy of the system is equal to the loss in gravitational potential energy of the system.

$$K_f = U_i - U_f = 0 - (U_{heavy,f} + U_{light,f}) = -m_{heavy}gh_{heavy,f} - m_{light}gh_{light,f}$$

$$K_f = -(2.0\text{ kg})(9.8\text{ m/s}^2)(-1.0\text{ m}) - (1.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) = +9.8\text{ J}$$

Case (*b*): Similar to case (*a*), we have

$$K_f = U_i - U_f = 0 - (U_{heavy,f} + U_{light,f}) = -m_{heavy}gh_{heavy,f} - m_{light}gh_{light,f}$$

$$K_f = -(2.0\text{ kg})(9.8\text{ m/s}^2)(-1.0\text{ m}) - (1.0\text{ kg})(9.8\text{ m/s}^2)(0) = +19.6\text{ J}$$

Comparing the two cases, we see our predication was correct. The kinetic energy (and therefore the speed) is greater in case (*b*).

7.64. No subsequent hill could be higher than the height of the first hill. In reality, energy is dissipated and the subsequent hills must be shorter.

7.65. In the absence of dissipative interactions, the final kinetic energy should be equal to the initial gravitational potential energy:

$$K_{non\text{-}diss,f} = U_i - U_f = mgh_i \tag{1}$$

The actual, observed final kinetic energy is $K = \frac{1}{2}mv_f^2$. Finding the difference, and dividing by the final kinetic energy in the absence of dissipation, yields the percent difference:

$$\frac{K_{non\text{-}diss,f} - K}{K_{non\text{-}diss,f}} = \frac{mgh_i - \frac{1}{2}mv_f^2}{mgh_i} = \frac{gh_i - \frac{1}{2}v_f^2}{gh_i} = \frac{(9.8\text{ m/s}^2)(12\text{ m}) - \frac{1}{2}(14.6\text{ m/s})^2}{(9.8\text{ m/s}^2)(12\text{ m})} = 0.094$$

So 9.4% of the ball's kinetic energy was lost to dissipation.

7.66. Let us assume that dissipation in the air can be ignored compared to the dissipation in the sand. If we call the final position of the ball the "ground", then the initial gravitational potential energy of the ball is $mgh_i = (5.3\text{ kg})(9.8\text{ m/s}^2)(20.20\text{ m}) = 1.049\text{ kJ}.$ Finally, when in the sand, the ball has no energy. So 1.0×10^3 J of energy was dissipated in the sand. Note that this answer would have been exactly the same if we had treated the final position as being 0.20 m below the ground, and the initial height as only 20 m.

7.67. The kinetic energy is not the same in any reference frame. An observer moving very quickly (in a plane for example) might see the cars approaching the plane very quickly and report a completely different kinetic energy. The momentum is not the same for the same reason. Options (*c*) and (*d*) are the same in any frame.

7.68. (*a*) If we call the moment of release the initial time, and the moment at which the ball reaches its peak after bouncing the final time, then the energy lost can be written entirely in terms of the difference in gravitational potential energies (because the speed is zero in both cases). Then $\Delta E = U_f - U_i = mg(0.65h_i - h_i) = -(0.35)mgh_i$ $= -(0.35)(0.70\text{ kg})(9.8\text{ m/s}^2)(1.5\text{ m}) = 3.6\text{ J}.$ (*b*) Assuming the percent of energy lost in each bounce is roughly independent of the initial height, the maximum height between the third and fourth bounce will be $(0.65)^3h_i$ and the maximum height between the fourth and fifth bounce will be $(0.65)^4h_i$. Hence the energy change in the fourth bounce will be $\Delta E = U_f - U_i = mg((0.65)^4h_i - (0.65)^3h_i) = (0.70\text{ kg})(9.8\text{ m/s}^2)(1.5\text{ m})((0.65)^4 - (0.65)^3) =$ $-0.99\text{ J}.$ Hence 0.99 J are lost in the fourth bounce. (*c*) Most energy is converted to heat; some is converted to sound.

7.69. (*a*) Initially, the man and the ball are at rest. Finally, the ball has significant forward momentum, which means the man must have backwards momentum. We solve for the final velocity of the man relative to Earth, and then calculate the total kinetic energy of the system after the ball is thrown. Let the direction in which the ball is initially thrown be the $+x$ direction. Then we have

$$m_{\text{man}}v_{\text{man }x,\text{i}} + m_{\text{ball}}v_{\text{ball }x,\text{i}} = m_{\text{man}}v_{\text{man }x,\text{f}} + m_{\text{ball}}v_{\text{ball }x,\text{f}}$$
$$v_{\text{man }x,\text{f}} = (m_{\text{man}}v_{\text{man }x,\text{i}} + m_{\text{ball}}v_{\text{ball }x,\text{i}} - m_{\text{ball}}v_{\text{ball }x,\text{f}})/m_{\text{man}}$$
$$v_{\text{man }x,\text{f}} = (0+0-(0.500\text{ kg})(15\text{ m/s}))/(80\text{ kg})$$
$$v_{\text{man }x,\text{f}} = -0.094\text{ m/s}$$

So the total final kinetic energy of the system is

$$K_{\text{f}} = \frac{1}{2}m_{\text{man}}v_{\text{man,f}}^2 + \frac{1}{2}m_{\text{ball}}v_{\text{ball,f}}^2$$
$$K_{\text{f}} = \frac{1}{2}(80\text{ kg})(0.094\text{ m/s})^2 + \frac{1}{2}(0.500\text{ kg})(15\text{ m/s})^2$$
$$K_{\text{f}} = 57\text{ J}$$

This final kinetic energy came from source energy in the man's body. So 57 J of initial source energy were converted to kinetic energy.

(*b*) When the dog catches the ball momentum is still conserved. We can use this to find the final speed of the dog and ball together, and hence determine the final kinetic energy. We know

$$m_{\text{dog}}v_{\text{dog }x,\text{i}} + m_{\text{ball}}v_{\text{ball }x,\text{i}} = (m_{\text{dog}} + m_{\text{ball}})v_{x,\text{f}}$$
$$v_{x,\text{f}} = (m_{\text{dog}}v_{\text{dog }x,\text{i}} + m_{\text{ball}}v_{\text{ball }x,\text{i}})/(m_{\text{dog}} + m_{\text{ball}})$$
$$v_{x,\text{f}} = (0+(0.500\text{ kg})(15\text{ m/s}))/((20\text{ kg})+(0.500\text{ kg}))$$
$$v_{x,\text{f}} = +0.366\text{ m/s}$$

To find the amount of energy dissipated, we take the difference between the initial and final kinetic energy:

$$E_{\text{diss}} = K_{\text{i}} - K_{\text{f}} = \frac{1}{2}m_{\text{ball}}v_{\text{ball,i}}^2 - \frac{1}{2}(m_{\text{dog}} + m_{\text{ball}})v_{\text{f}}^2$$
$$E_{\text{diss}} = \frac{1}{2}(0.500\text{ kg})(15\text{ m/s})^2 - \frac{1}{2}((20\text{ kg})+(0.500\text{ kg}))(0.366\text{ m/s})^2$$
$$E_{\text{diss}} = 55\text{ J}$$

7.70. Let us call the vertically upward direction the $+y$ direction. We can write $\frac{dv_y(t)}{dt} = -ge^{-t/\tau}$ and we can integrate to obtain the velocity as a function of time:

$$v_y(t) = g\tau(e^{-t/\tau} - 1) \qquad (1)$$

Integrating a second time yields the vertical position as a function of time:

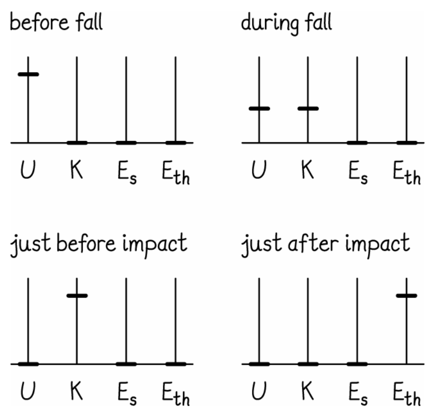
$$y(t) = (-g\tau^2e^{-t/\tau} - g\tau t) + C$$
$$y(t) = (-g\tau^2e^{-t/\tau} - g\tau t) + g\tau^2 + y_0 \qquad (2)$$

Here we have used $C = y_0$ is the initial height of the object. Using equations (1) and (2), we can write the decrease in gravitational potential energy as a function of time, as well as the kinetic energy as a function of time. If no energy were lost to air resistance, we could say $\Delta U = -\Delta K$. Instead, we say that $\Delta K + \Delta U = -\Delta E_{\text{thermal}}$. So

$$\Delta E_{\text{thermal}} = -(\Delta K + \Delta U) = -\frac{1}{2}m(v^2(t) - v_0^2) - mg(y(t) - y_0)$$
$$= -\frac{1}{2}mg^2\tau^2(e^{-t/\tau} - 1)^2 - mg((-g\tau^2e^{-t/\tau} - g\tau t) + g\tau^2)$$
$$= -\frac{mg^2\tau^2}{2}(e^{-2t/\tau} - 4e^{-t/\tau} - \frac{t}{\tau} + 3)$$

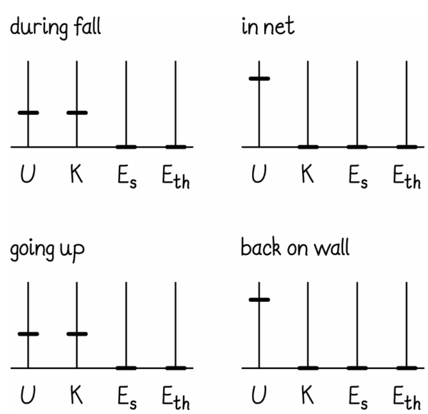
We are asked for the rate at which energy is converted to thermal energy. So we take the time derivative of the above expression, and find $\frac{dE_{\text{thermal}}}{dt} = -mg^2\tau(-e^{-2t/\tau} + 2e^{-t/\tau} - 1) = mg^2\tau(e^{-t/\tau} - 1)^2$. From this we can see that the maximum rate of energy conversion occurs in the limit as t becomes very large. Physically, what this means is that after a long time falling, the object will reach a terminal velocity. Once that happens, the object no longer speeds up, and so the rate of energy conversion no longer increases. This limiting rate of energy conversion is $\lim_{t\to\infty}\left(\frac{dE_{\text{thermal}}}{dt}\right) = \lim_{t\to\infty}[mg^2\tau(e^{-t/\tau} - 1)^2] = mg^2\tau = (2.2\text{ kg})(9.8\text{ m/s}^2)^2(5.68\text{ s}) = 1.2\times 10^3\text{ J/s}.$

7.71. (*a*) Before fall, system energy is all potential. During fall, system potential energy converted to kinetic energy. Just before impact, system energy is all kinetic. After impact, system kinetic energy has converted to thermal energy as HD breaks.



(*b*) It is not possible, because conversion of coherent energy (kinetic, potential) into incoherent energy (thermal) is irreversible. If men try to put HD together, they constitute source energy. Although this can raise his pieces up to gain (coherent) potential energy, it cannot reverse the irreversible conversion of coherent kinetic energy to incoherent thermal energy. No matter how much source energy men put in, HD cannot be put back together.

(*c*)



7.72. (*a*) We know that a slow, lightweight pellet will not be able to reverse the motion of the block, whereas a very fast, very heavy pellet will be able to. Let us first determine the condition for the pellet to reverse the block's motion at all. When the block has fallen a distance d, it will have a downward velocity component of $v_y = -\sqrt{2gd}$. The conservation of momentum during the collision tells us

$$m_{\text{block}} v_{\text{block } y,\text{i}} + m_{\text{pellet}} v_{\text{pellet } y,\text{i}} = (m_{\text{block}} + m_{\text{pellet}}) v_{y,\text{f}}$$

$$v_{y,\text{f}} = \frac{m_{\text{block}} v_{\text{block } y,\text{i}} + m_{\text{pellet}} v_{\text{pellet } y,\text{i}}}{(m_{\text{block}} + m_{\text{pellet}})} = \frac{m_{\text{block}}(-\sqrt{2gd}) + m_{\text{pellet}} v_{\text{pellet } y,\text{i}}}{(m_{\text{block}} + m_{\text{pellet}})}$$

In order for the pellet and block to move upward after the collision, the above expression for the y component of the final velocity must be greater than zero. That means the condition for the block and pellet to move upward is

$$m_{\text{pellet}} v_{\text{pellet } y,\text{i}} > m_{\text{block}}(\sqrt{2gd})$$

If the pellet's inertia and speed are sufficient to turn the block around, the block and pellet will start at the height $h-d$ moving upward at a speed of $v_{y,\text{f}} = \dfrac{m_{\text{block}}(-\sqrt{2gd}) + m_{\text{pellet}} v_{\text{pellet } y,\text{i}}}{(m_{\text{block}} + m_{\text{pellet}})}$. We can determine the additional vertical distance Δy the block and pellet will travel before stopping, using

$$v_{y,\text{f}}^2 = 0 = v_{y,\text{i}}^2 + 2(-g)\Delta y$$

$$\Delta y = \frac{-v_{y,\text{i}}^2}{-2g} = \frac{(m_{\text{block}}(-\sqrt{2gd}) + m_{\text{pellet}} v_{\text{pellet } y,\text{i}})^2}{2g(m_{\text{block}} + m_{\text{pellet}})^2}$$

So the final maximum height the pellet and block will reach after the collision is $h_{\max} = h - d + \dfrac{1}{2g}\left(\dfrac{m_{\text{pellet}} v_{\text{pellet}} - m_{\text{block}}\sqrt{2gd}}{m_{\text{pellet}} + m_{\text{block}}}\right)^2$.

(*b*) The dissipated energy will be the difference between the initial and final kinetic energy:

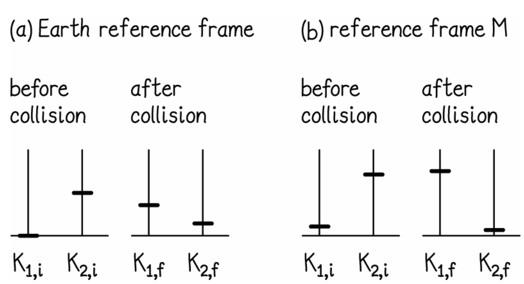
$$\Delta E_{\text{diss}} = K_{\text{i}} - K_{\text{f}} = \frac{1}{2} m_{\text{pellet}} v_{\text{pellet}}^2 + \frac{1}{2} m_{\text{block}}(2gd) - \frac{1}{2}(m_{\text{pellet}} + m_{\text{block}})\left(\frac{m_{\text{pellet}} v_{\text{pellet } y,\text{i}} - m_{\text{block}}\sqrt{2gd}}{(m_{\text{block}} + m_{\text{pellet}})}\right)^2$$

This can be somewhat simplified to

$$\frac{m_{\text{block}}}{2(m_{\text{block}} + m_{\text{pellet}})}(m_{\text{pellet}} v_{\text{block}}^2 + m_{\text{block}} v_{\text{block}}^2 + m_{\text{pellet}} v_{\text{pellet}}^2 + 2 m_{\text{pellet}} v_{\text{pellet}}\sqrt{2gd} - m_{\text{block}} 2gd)$$

7.73. No, the collision is not perfectly elastic. We hear sound, so energy escaped in that form. Also, the balls would heat up by a small amount.

7.74.



7.75. The minimum source energy provided by the man is equal to the increase in gravitational potential energy: $\Delta U = mg\Delta y = (80\text{ kg})(9.8\text{ m/s}^2)(152\text{ m}) = 1.19\times10^5\text{ J}$. This corresponds to $1.19\times10^5\text{ J}\left(\dfrac{1\text{ candy bar}}{1.3\times10^6\text{ J}}\right) = 0.092\text{ candy bars}$.

7.76. Above the midpoint. Air resistance would not change the answer. The speed increases like the square root of the distance fallen, so at the midpoint, the speed is $\frac{1}{\sqrt{2}}$ of the final speed, which is already more than half. The object must have reached half the final speed a little above this point. Air resistance increases with velocity, so it would have a more pronounced effect in the second half of the fall than in the first.

7.77. No, this is not impossible. In a steam engine, heat causes water to vaporize and expand. The expansion of the water vapor moves a piston, and causes mechanical components to move. Thus, one can use incoherent energy to obtain coherent energy.

7.78. If you ignore air resistance, the bullet will return to Earth with the same speed as it left the barrel of the gun. The initial kinetic energy is turned into gravitational potential energy as the bullet rises, but this is turned back into kinetic energy as the bullet falls again.

7.79. The center of mass of the system must always move at the same velocity, but the remaining kinetic energy can be converted to other forms, including energy stored in the magnetic fields. We calculate kinetic energy associated with the center of mass motion, and subtract it from the total initial kinetic energy. The difference is the maximum energy that could be stored in the magnetic field. The x component of the center of mass velocity is

$$v_{\text{cm}\,x} = \frac{m_1 v_{1\,x,\text{i}} + m_2 v_{2\,x,\text{i}}}{m_1 + m_2} = \frac{(1.2 \text{ kg})(0.323 \text{ m/s}) + (1.2 \text{ kg})(-0.147 \text{ m/s})}{((1.2 \text{ kg}) + (1.2 \text{ kg}))} = 0.088 \text{ m/s}$$

The convertible kinetic energy is thus

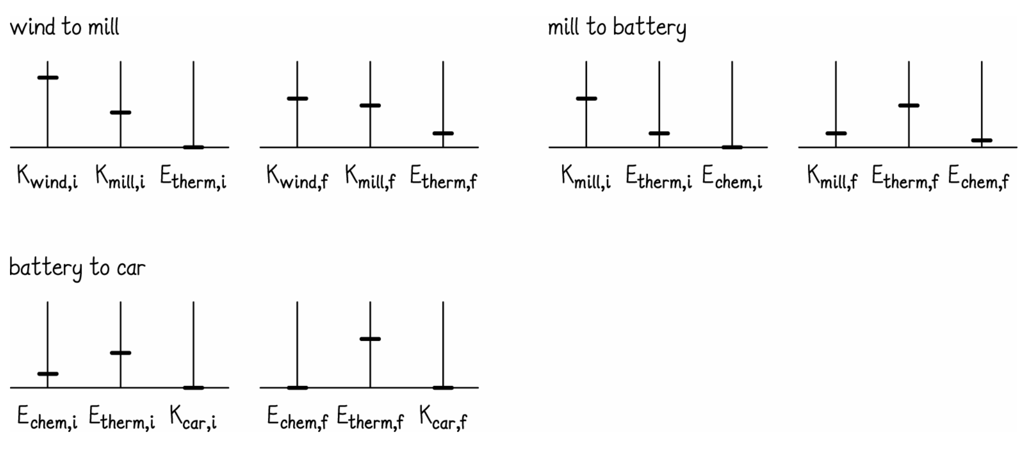
$$K_{\text{conv}} = K_1 - K_{\text{cm}} = \frac{1}{2} m_1 v_{1\,x,\text{i}}^2 + \frac{1}{2} m_1 v_{1\,x,\text{i}}^2 - \frac{1}{2}(m_1 + m_2) v_{\text{cm}}^2$$

$$K_{\text{conv}} = \frac{1}{2}(1.2 \text{ kg})(0.323 \text{ m/s})^2 + \frac{1}{2}(1.2 \text{ kg})(0.147 \text{ m/s})^2 - \frac{1}{2}((1.2 \text{ kg}) + (1.2 \text{ kg}))(0.088 \text{ m/s})^2$$

$$K_{\text{conv}} = 0.066 \text{ J}$$

This is the maximum amount of energy that can be stored in the magnets. The total kinetic energy in the carts is slightly larger than this (0.0756 J), but some of this must remain in the form of center-of-mass kinetic energy.

7.80.



7.81. In reality the acceleration may not have been constant. But since we only have information from before and after the collision (no data during the collision itself) we are forced to obtain an approximate value by assuming constant acceleration. In that case, we can find the acceleration of the bicycle, and use the ratio of inertias to determine the acceleration of the car. First, we find the final velocity of the bicycle. We do this by using the conservation of momentum. Let the initial direction of motion of the bicycle be the $+x$ direction. Then

$$m_{\rm B}v_{{\rm B}\,x,{\rm i}} + m_{\rm C}v_{{\rm C}\,x,{\rm i}} = m_{\rm B}v_{{\rm B}\,x,{\rm f}} + m_{\rm C}v_{{\rm C}\,x,{\rm f}}$$

$$v_{{\rm C}\,x,{\rm f}} = \frac{m_{\rm B}v_{{\rm B}\,x,{\rm i}} + m_{\rm C}v_{{\rm C}\,x,{\rm i}} - m_{\rm B}v_{{\rm B}\,x,{\rm f}}}{m_{\rm C}} = \frac{(80\text{ kg})(12\text{ m/s}) + (1800\text{ kg})(0)}{(1800\text{ kg})}$$

$$v_{x,{\rm f}} = 0.533\text{ m/s}$$

Now that we have the initial and final velocities of the car, as well as the distance over which the bicycle interacted with it, we can use $a_{{\rm C}\,x} = \dfrac{v^2_{{\rm C}\,x,{\rm f}} - v^2_{{\rm C}\,x,{\rm i}}}{2\Delta x} = \dfrac{(0.533\text{ m/s})^2 - (0)^2}{2(0.75\text{ m})/2} = 0.38\text{ m/s}^2$. Hence the acceleration of the car was 0.38 m/s^2 forward.

7.82. We know that the roller coaster must have enough kinetic energy at the beginning of the hill for it to reach the top. During this process, we will assume that friction is negligible such that we can require that the initial kinetic energy be greater than or equal to the final gravitational potential energy. Let us call the moment the coaster begins accelerating instant 0, the moment the acceleration stops instant 1, and the moment the coaster reaches its maximum height instant 2. Then

$$K_1 \ge U_2$$

$$\frac{1}{2}mv^2_{\min,1} = mgh_2$$

$$v_{\min,1} = \sqrt{2gh_2}$$

Now we know the acceleration of the roller coaster along the horizontal portion. We can use kinematics to determine over what distance the cars must have this acceleration in order to reach the above height:

$$\Delta x = \frac{v^2_{x,1} - v^2_{x,0}}{2a_x} = \frac{(2gh_2) - (0)}{2(0.85g)} = \frac{2g(66.4\text{ m})}{2g(0.85)} = 78\text{ m}$$

The required horizontal track would be 78 m, which is very long, but plausible.

7.83. One determines the final speed of each cart by using conservation of momentum, and using the known change in kinetic energy. Call the 1.00-kg cart object 1, and the 2.00-kg cart object 2. Let the initial direction of motion of cart 1 (to the right) be the $+x$ direction. We have

$$m_1v_{1\text{ x,i}} + m_2v_{2\text{ x,i}} = m_1v_{1\text{ x,f}} + m_2v_{2\text{ x,f}}$$

or

$$m_1v + m_2(-v) = m_1v_{1\text{ x,f}} + m_2v_{2\text{ x,f}} \qquad (1)$$

and

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{3E}{4} = \frac{1}{2}m_1v^2_{1\text{ x,f}} + \frac{1}{2}m_2v^2_{2\text{ x,f}} \qquad (2)$$

Equation (1) can be solved for the final speed of the 1.00-kg cart to yield

$$v_{1\text{ x,f}} = \left(1 - \frac{m_2}{m_1}\right)v - \frac{m_2}{m_1}v_{2\text{ x,f}}$$

or simply

$$v_{1\text{ x,f}} = -(v + 2v_{2\text{ x,f}}) \qquad (3)$$

Inserting equation (3) into equation (2) yields a quadratic equation for $v_{2\text{ x,f}}$:

$$\left(\frac{m_2}{m_1}\right)v^2_{2\text{ x,f}} + (v^2 + 4vv_{2\text{ x,f}} + 4v^2_{2\text{ x,f}}) - \frac{(m_1 + m_2)v^2 + \dfrac{3E}{2}}{m_1} = 0$$

or

$$6v_{2\,\mathrm{x,f}}^2 + 4vv_{2\,\mathrm{x,f}} - 2v^2 - \frac{3E}{2m} = 0 \tag{4}$$

One obtains the solutions to (4) from the quadratic equation, and finds $v_{2\,x,\mathrm{f}} = \left(-\frac{v}{3} \pm \sqrt{\frac{4}{9}v^2 + \frac{E}{4m}}\right)$, where we have introduced the parameter $m = 1.00\ \mathrm{kg}$ to ensure correct units throughout the expression. Here the physically correct root is $v_{2\,x,\mathrm{f}} = \left(-\frac{v}{3} = \sqrt{\frac{4}{9}v^2 + \frac{E}{4m}}\right)$. Inserting that root back into equation (3), we find the final speed of the 1.00-kg cart to be $v_{1\,x,\mathrm{f}} = \left(-\frac{v}{3} - \sqrt{\frac{16}{9}v^2 + \frac{E}{m}}\right)$. In terms of velocities $\vec{v}_{2,\mathrm{f}} = \left(-\frac{v}{3} = \sqrt{\frac{4}{9}v^2 + \frac{E}{4m}}\right)\hat{i}$ and $\vec{v}_{1,\mathrm{f}} = \left(-\frac{v}{3} - \sqrt{\frac{16}{9}v^2 + \frac{E}{m}}\right)\hat{i}$.

8

FORCE

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

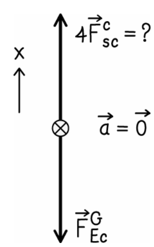
Developing a Feel

1. 10^5 N **2.** 10^3 N **3.** 10^6 N **4.** 10^5 N **5.** 10^6 N **6.** 10^3 N **7.** 10^3 N/m **8.** 10^4 N/m **9.** 10^5 N·s **10**. 10^8 N·s

Guided Problems

8.2 Car springs

1. Getting Started We start by considering the interactions in which the body of the car is involved. Earth exerts a gravitational force downward on the body of the car, and each of the four springs exerts an upward force on the body of the car. It is difficult to show the four vectors associated with the forces the springs exert on the car in a free body diagram where the tails of all vectors begin on the object's center of mass. For that reason, we simply label the force due to all four springs combined as $4\vec{F}^{c}_{sc}$. Thus our free body diagram is:



2. Devise Plan If the car is not bouncing up and down on its springs, but sitting still, the acceleration of the body of the car is zero. This is indicated in the free body diagram above. Since there is no direction of acceleration, either upward or downward would be perfectly acceptable choices for an axis. We choose the $+x$ axis to point upward as indicated in the free body diagram.

We can write the sum of all x components of the force:

$$\sum F_x = 4F^{c}_{sc} - F^{G}_{Ec} = 0 \tag{1}$$

3. Execute Plan Equation (1) above can be rearranged to yield the magnitude of the force exerted by each spring on the body of the car:

$$F^{c}_{sc} = \frac{1}{4}F^{G}_{Ec} = \frac{mg}{4} \tag{2}$$

We know that the force exerted by a spring is related to the spring constant according to $F^{c}_{scx} = -k\Delta x.$ Inserting this into equation (2), we have

$$-k\Delta x = \frac{mg}{4} \Rightarrow k = -\frac{mg}{4\Delta x} \tag{3}$$

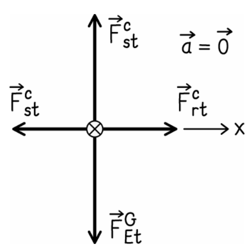
It is important here to note that when the body of the car sinks down and compresses the springs, the change in x is downward, meaning $\Delta x = -3.0\times10^{-3}$ m. Inserting numerical values, we obtain

$$k = -\frac{(640\ \text{kg})(9.8\ \text{m/s}^2)}{4(-3.0\times10^{-3}\ \text{m})} = 5.2\times10^{4}\ \text{N/m}$$

4. Evaluate Result We see from equation (3) above that increasing the inertia of the car results in a higher required spring constant, which is reasonable. In the solving process we have assumed that all springs carry an equal share of the weight of the car. In general, this is not true as a car tends to have its inertia concentrated around the engine.

8.4 Rubber-band leash

1. Getting Started Since the rubber band is being stretched between two objects (the child and the toy) we have to be very careful in our definition of the stretch. Let us treat the end of the rubber hand held by the child as a fixed end, and we consider the toy to be stretching the rubber band backwards (against the motion of the child) by a distance $\Delta x.$ We wish to draw the free body diagram for the toy. It is on the toy that the frictional force, and the spring force of the rubber band act. We note also that we have a gravitational force exerted by Earth on the toy and an upward contact force that the ground exerts on the toy. The toy is moving at a constant velocity, so the acceleration must be zero. The free body diagram is thus:



Here we have used the subscript "s" to refer to the surface across which the toy is being pulled, and "r" for the rubber band. Although the problem refers to the "tensile force", the force pulling the toy is being treated like a spring force, so we use the notation of a spring exerting a contact force on the toy.

2. Devise Plan The spring force being exerted on the toy is related to the stretch of the rubber band by $F^{c}_{st} = -k\Delta x.$ The sum of all forces in either the vertical or horizontal direction must be zero because the toy is not accelerating in any direction. However, the forces exerted in the vertical direction do not affect the motion we are describing (horizontal motion) and so we will not need them. We write for the x direction:

$$\sum F_x = F^{c}_{rt} + F^{c}_{st} = 0 \tag{1}$$

Inserting the expression for the spring force, we have

$$-k\Delta x + F^{c}_{st} = 0 \tag{2}$$

There is only one unknown, so we can solve for the stretch of the rubber band.

3. Execute Plan Rearranging equation (2) we find

$$\Delta x = \frac{F^{c}_{st}}{k} \tag{3}$$

Inserting numerical values into equation (3), we have

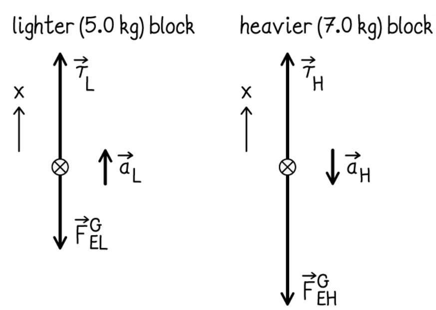
$$\Delta x = \frac{(-0.734\ \text{N})}{(20\ \text{N/m})} = -0.037\ \text{m}$$

This corresponds to a stretch of magnitude 0.037 m. The negative sign is an artifact of our definition of Δx. We chose it to be the backward stretch of the end of the rubber band attached to the toy. Since we called the forward direction of motion $+x$, a negative answer for Δx makes perfect sense.

4. Evaluate Result Our algebraic expression in equation (3) tells us that the stretch would be greater in magnitude if the frictional force were greater. Also, we see that a rubber band with a larger spring constant would require a smaller stretch to pull the toy. In the process of solving, we have assumed that the rubber band is perfectly horizontal. If the force the rubber band exerts on the toy were to have an upward or downward component, the vertical and horizontal directions would have become entangled and the solution would have been more difficult. This is the topic of Chapter 10.

8.6 Atwood machine

1. Getting Started We begin by making a free body diagram for each block. Although we do not know the acceleration at this point, we make a guess based on our life experience that the heavier block will accelerated downward, causing the lighter block to accelerate upward. Even though there are two different directions of acceleration for the two different objects, we still choose only one direction to call our $+x$ axis. We choose it to point upward, although it is perfectly acceptable to choose differently. We use the subscripts "L" and "H" to refer to the lighter and heavier blocks, respectively.



Note that the two acceleration vectors have the same magnitude. Since the two blocks are connected by a rope, they must move at the same speeds, and must hence have accelerations equal in magnitude. Clearly, since one moves upward and the other moves downward, we have $\vec{a}_H = -\vec{a}_L$. Similarly, the tensile forces have the same magnitude.

2. Devise Plan The free body diagrams involve four unknown quantities: $\vec{a}_H$, $\vec{a}_L$, $\vec{\mathcal{T}}_H$, and $\vec{\mathcal{T}}_L$. Solving the problem will require four equations, but two of these are the rather trivial relations already mentioned:

$$\vec{\mathcal{T}}_H = \vec{\mathcal{T}}_L \tag{1}$$

and

$$\vec{a}_H = -\vec{a}_L \tag{2}$$

The other two come from the two free body diagrams:

$$\sum F_{\text{on L}\,x} = \mathcal{T}_L - F^G_{EL} = m_L a_{Lx} \tag{3}$$

$$\sum F_{\text{on H}\,x} = \mathcal{T}_H - F^G_{EH} = m_H a_{Hx} \tag{4}$$

Inserting equations (1) and (2) into both equations (3) and (4), along with the known form of gravitational forces we obtain

$$\mathcal{T}_L - m_L g = m_L a_{Lx} \tag{5}$$

$$\mathcal{T}_L - m_H g = m_H(-a_{Lx}) \tag{6}$$

We have now reduced our initial set of four equations and four unknowns to only two equations involving only two unknowns.

3. Execute Plan Rearranging equation (5), we can obtain

$$\mathcal{T}_{\rm L} = m_{\rm L}a_{{\rm L}x} + m_{\rm L}g \tag{7}$$

which we insert into equation (6) to find

$$\begin{aligned}(m_{\rm L}a_{{\rm L}x} + m_{\rm L}g) - m_{\rm H}g &= -m_{\rm H}a_{{\rm L}x}\\ a_{{\rm L}x}(m_{\rm L} + m_{\rm H}) &= (m_{\rm H} - m_{\rm L})g\\ a_{{\rm L}x} &= \frac{g(m_{\rm H} - m_{\rm L})}{(m_{\rm L} + m_{\rm H})}\end{aligned} \tag{8}$$

Inserting numerical values, we have

$$a_{{\rm L}x} = \frac{((7.0\ {\rm kg}) - (5.0\ {\rm kg}))}{((7.0\ {\rm kg}) + (5.0\ {\rm kg}))}(9.8\ {\rm m/s^2})$$

$$a_{{\rm L}x} = 1.6\ {\rm m/s^2}$$

From this, it follows trivially that $a_{{\rm H}x} = -1.6\ {\rm m/s^2}$. If, instead we solve equation (5) for $a_{{\rm L}x}$ and insert the expression into equation (6), we obtain

$$\mathcal{T}_{\rm L} = \frac{2m_{\rm H}m_{\rm L}g}{m_{\rm H} + m_{\rm L}} \tag{9}$$

We could insert numbers into equation (9), or insert the value for the acceleration back into equation (7) to determine the tension. We find

$$\mathcal{T}_{\rm L} = (5.0\ {\rm kg})((1.6\ {\rm m/s^2}) + (9.8\ {\rm m/s^2}))$$

$$\mathcal{T}_{\rm L} = 57\ {\rm N}$$

meaning $\mathcal{T}_{\rm H} = 57\ {\rm N}$ also.

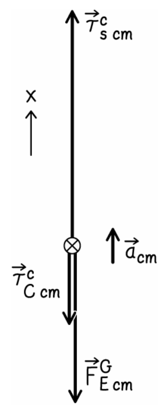
4. Evaluate Result The magnitudes of all quantities are positive. The only vector quantity we expected to be negative was the x component of the acceleration of the heavier block, and we found that it is negative. Equation (8) above shows us that the acceleration depends on the sum and the difference of the two inertias. For example, if the two inertias are equal, the acceleration will be zero regardless of the size of the inertias. If one inertia is very much smaller than the other, the acceleration will approach the acceleration due to gravity of a free object. This is consistent with our expectations. Equation (9) shows us that the tension increases as the inertias of the blocks increase. We also see that in the limit as one block becomes very light (practically zero inertia) the tension also approaches zero, as it should.

The numerical acceleration we found is significantly less than in freefall, which makes sense. The tension is right between the weight of the lighter and heavier blocks. This makes sense also, since the tension is overcoming the weight of the lighter block and being overcome by the weight of the heavier block.

8.8 Support force revisited

1. Getting Started We group the block being lifted, the pulleys, and rope segments A and B into one large system. The center of mass of this system is given by the inertias and locations of the block and the two pulleys, to a very good approximation, since we are treating the ropes as though they have approximately zero inertia. With this choice of our system, the tensions in ropes A and B are internal forces. We wish to show only external forces on our free body diagram. The external forces are the force that the support exerts on the system $\vec{\mathcal{T}}^{\rm c}_{\rm s\,cm}$, the tensile force in rope C $\vec{\mathcal{T}}^{\rm c}_{\rm C\,cm}$, and the gravitational force Earth exerts on the system $\vec{F}^{G}_{\rm E\,cm}$.

2. Devise Plan Including the external forces discussed above, our free body diagram is:



In the second-law equation $\sum F_{\text{ext}\,x} = ma_{\text{cm}\,x}$, m is the entire inertia of the system. In this case, because the ropes have approximately zero inertia, $m = 2m_{\text{p}} + m_{\text{b}}$. If we call $x_{\text{cm}} = \dfrac{x_{\text{b}}(m_{\text{p}} + m_{\text{b}}) + x_{\text{top p}}m_{\text{p}}}{2m_{\text{p}} + m_{\text{b}}}$, and note that the position of the top pulley does not change, it follows from taking two time derivatives that.
$a_{\text{cm}} = \dfrac{a(m_{\text{p}} + m_{\text{b}})}{2m_{\text{p}} + m_{\text{b}}}$. Thus we can write

$$\sum F_{\text{ext}\,x} = \mathcal{T}^{c}_{\text{s cm}} - \mathcal{T}^{c}_{\text{C cm}} - F^{G}_{\text{E cm}} = (m_{\text{p}} + m_{\text{b}})a \tag{1}$$

The tensile force in rope C is equal to the force exerted by the worker on rope C, which was found in Worked Problem 8.7, and is given in equation (2) of that problem as

$$\mathcal{T}^{c}_{\text{C cm}} = F^{c}_{\text{wC}} = \frac{1}{2}(m_{\text{p}} + m_{\text{b}})(a + g)$$

Inserting this into equation (1) above, as well as the expression for the gravitational force on the entire inertia of the system, we obtain

$$\mathcal{T}^{c}_{\text{s cm}} - \frac{1}{2}(m_{\text{p}} + m_{\text{b}})(a + g) - (2m_{\text{p}} + m_{\text{b}})g = (m_{\text{p}} + m_{\text{b}})a \tag{2}$$

This expression contains only known quantities and the tension in the support rope, which we wish to determine.

3. Execute Plan Rearranging equation (2) and simplifying, we find $\mathcal{T}^{c}_{\text{s cm}} = \dfrac{3}{2}(m_{\text{p}} + m_{\text{b}})(a + g) + m_{\text{p}}g$.

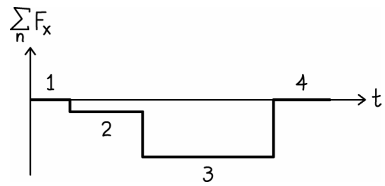
4. Evaluate Result This result does agree with that obtained in Worked Problem 8.7.

Questions and Problems

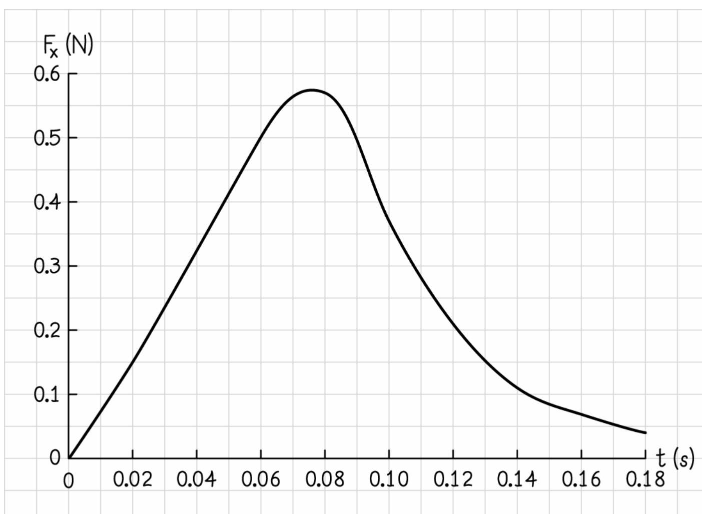
8.1. Neither. Each vehicle is moving at a constant speed. If we assume that the direction of each vehicle's motion is also constant, as in motion a straight stretch of highway, then both vehicles have zero change in their velocities. Hence the vector sum of all forces on each vehicle is zero.

8.2. (*a*) Yes, you must overcome some frictional force between the crate and the ground. (*b*) No, any non-zero vector sum of all forces on an object will produce a non-zero acceleration.

8.3.



8.4. The change in momentum is given precisely by $\Delta p_x = \int_{t_i}^{t_f} F_x dt,$ but with this discrete data we can approximate it by $\Delta p_x \approx \sum_n F_{x,n} \Delta t_n,$ meaning we add up $F_x \Delta t$ for all the time intervals shown. Note that every time interval is equal to 0.020 s. Thus $\Delta p_x \approx ((0)(0.020 \text{ s})) + ((0.15 \text{ N})(0.020 \text{ s})) + ((0.032 \text{ N})(0.020 \text{ s})) + \ldots + ((0.040 \text{ N})(0.020 \text{ s})) =$ $0.047 \text{ N} \cdot \text{s}.$ In base SI units $\Delta p_x \approx 0.047 \text{ kg} \cdot \text{m/s}.$



8.5. The object inertia is constant, which means the change in momentum came from a change in velocity. But knowing the change in velocity does not tell you values of initial and final velocities, both of which are needed in order to determine change in kinetic energy.

8.6. Yes. The constant force must increase the momentum, but that could be accomplished either by increasing the speed, or by increasing the amount of inertia that is moving. If inertia (coal) is added to the cart at just the right rate, all of the force could go toward speeding up that coal to make it move with the rest of the cart and cause no acceleration of the cart itself.

8.7. (*a*) The magnitudes are equal. (*b*) The magnitudes are equal. (*c*) The magnitudes are equal.

8.8. A force of equal magnitude and in opposite direction must be exerted by the object being moved on person.

8.9. (*a*)–(*g*) can all exert forces.

8.10. The magnitude of the force of the first player's racquet on the ball is twice the magnitude of the force exerted by the second player's hand on the ball. Let the direction of the ball after striking the racquet be the positive x direction. The force delivered by the racquet is sufficient to cause the ball to reverse direction, meaning

$F_{\text{racquet ball }x}\Delta t = -2mv_{\text{ball }x,\text{i}}$. When the other player catches the ball, the force exerted by her hand is only sufficient to stop the ball, meaning $F_{\text{hand ball }x}\Delta t = mv_{\text{ball }x,\text{i}}$. Since the time intervals are the same for each interaction, comparing the two expressions yields $\left|\dfrac{F_{\text{racquet ball }x}}{F_{\text{hand ball }x}}\right| = 2$.

8.11. (*a*) No. The force the pitcher exerted on the ball was made possible by "contact" between her hand and the ball. Once the ball leaves her hand, she is no longer able to exert forces on it. (*b*) There is likely substantial drag (the force from the ball's contact with the air) and the gravitational force of Earth acting on the ball.

8.12. The engine turns the wheels, relative to the car. If they are trying to spin, but grip the road, then the car moves forward.

8.13. No, the baby is still accelerating downward due to the influence of the gravitational force. The velocity of the baby is changing; it just happens to be passing through zero at that moment.

8.14. (*a*) Yes. For example two billiard balls colliding exert contact forces and comparatively weak gravitational (field) forces on each other. (*b*) No. Typically we think of contact forces as different from field forces, but contact forces are made up of field forces at a microscopic level.

8.15. There is some other force acting on the refrigerator such that the vector sum of all forces is still zero. This additional force could be the frictional force exerted by the floor, or if the refrigerator back is against a wall, it could be a contact force exerted by the wall.

8.16. Inertia is not something that must be overcome by a force; car does not move because the vector sum of forces exerted on it is nonzero. The magnitude of the force you exert on the car is less than the magnitude of the friction force exerted by the road on the car.

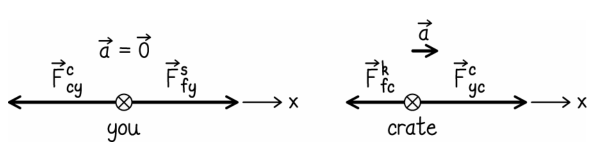
8.17. The contact force that you exert downward on the scale, and that it exerts upward on you.

8.18. The vase exerts a downward contact force on the table, $\vec{F}_{\text{by vase on table}}$, the floor exerts an upward contact force on the table, $\vec{F}_{\text{by floor on table}}$, and Earth exerts an downward gravitational force on the table, $\vec{F}_{\text{by Earth on table}}$. The vector sum of all these forces must be zero since the table is not accelerating.

8.19. The magnitude of the contact force that the floor exerts on you, $\vec{F}_{\text{by floor on you}}$, decreases.

8.20. In addition to feet exerting downward contact force, you also exert upward gravitational force on Earth. Earth does not move away from feet because magnitudes of these two forces are equal.

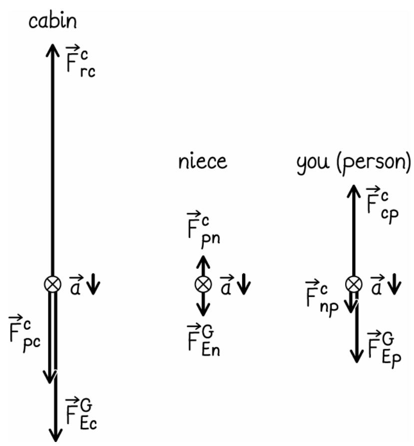
8.21.



The crate moves because the magnitude of the frictional force exerted by the floor on crate is smaller than magnitude of contact force exerted by you on the crate. You do not move because magnitude of frictional force exerted by the floor on you is equal to the magnitude of the contact force exerted by the crate on you (if you try to exert a force exceeding the available friction between your feet and the floor, your feet will slip).

8.22. The acceleration starts out equal to the acceleration due to gravity. As the raindrop speeds up, the force of air resistance increases and counteracts a larger and larger part of the downward gravitational force. This process continues until the upward force of air resistance is equal in magnitude to the downward gravitational force. At this point the vector sum of all forces is zero, and therefore the acceleration is also zero. The raindrop will fall the remaining distance at a constant speed.

8.23. (*a*)



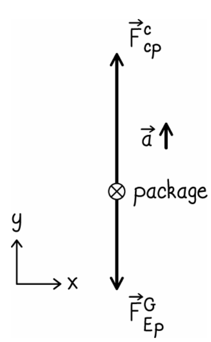
(*b*) $\vec{F}^G_{\text{En}} = 2.0\times10^2\ \text{N}$ downward, $\vec{F}^c_{\text{pn}} = 1.8\times10^2\ \text{N}$ upward, $\vec{F}^G_{\text{Ep}} = 4.9\times10^2\ \text{N}$ downward, $\vec{F}^c_{\text{np}} = 1.8\times10^2\ \text{N}$ downward, $\vec{F}^c_{\text{cp}} = 6.2\times10^2\ \text{N}$ upward, $\vec{F}^G_{\text{Ec}} = 9.8\times10^2\ \text{N}$ downward, $\vec{F}^c_{\text{pc}} = 6.2\times10^2\ \text{N}$ downward, and $\vec{F}^c_{\text{rc}} = 1.5\times10^3\ \text{N}$ upward. The magnitudes of the gravitational forces acting on each object are given by mg. For example, $F^G_{\text{En}} = m_{\text{n}}g = (20\ \text{kg})(9.8\ \text{m/s}^2) = 2.0\times10^2\ \text{N}$. The interaction pairs are $\vec{F}^c_{\text{pn}} - \vec{F}^c_{\text{np}}$, and $\vec{F}^c_{\text{cp}} - \vec{F}^c_{\text{pc}}$. Interaction pairs must be equal in magnitude and opposite in direction. For example $\vec{F}^c_{\text{pn}} = -\vec{F}^c_{\text{np}}$. Finally, the vector sum of all forces on each object must equal that object's inertia times the acceleration of the system $(-1.0\ \text{m/s}^2)$. So, for example, we can determine $\vec{F}^c_{\text{pn}}$ by writing $\sum \vec{F}_{\text{on n}} = m_{\text{n}}\vec{a}_{\text{n}}$ or $\sum F_{\text{on n}\,y} = m_{\text{n}}a_{\text{n}\,y}$. Inserting all the y components of the forces exerted on the niece yields

$$F^c_{\text{pn}} - F^G_{\text{En}} = m_{\text{n}}a_{\text{n}}$$

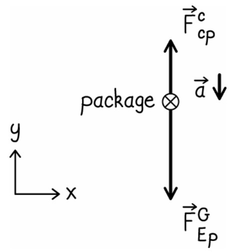
$$F^c_{\text{pn}} = m_{\text{n}}a_{\text{n}} + m_{\text{n}}g = (20\ \text{kg})((-1.0\ \text{m/s}^2) + (9.8\ \text{m/s}^2)) = 1.8\times10^2\ \text{N}$$

Other forces are found similarly.

8.24. (*a*) The cord increases in length. The force that the cord exerts on the package must increase, because now in addition to counteracting gravity, the cord must also accelerate the package upward.



(*b*) The cord decreases in length to even less than the initial length when the elevator was stationary. Now the cord does not have to entirely counteract gravity; gravity can succeed in causing a small downward acceleration.



8.25. No. The tension in the upper portion of the rope will be much higher than the tension in the lower part. The portion of rope near the top is exerting an upward force on a substantial inertia, and has to cancel out the gravitational force on the lower part of the rope. The lower portion of the rope supports very little inertia.

8.26. You say the student is wrong; what he describes (two equal-magnitude forces exerted on rope in opposite directions) is correct, but that situation does create nonzero tension in the rope.

8.27. The forces with which each team pulls on the rope are equal in magnitude. This can be understood most simply by Newton's third law (the two teams must exert equal and opposite force on one another). But this is discussed in the next section, so we provide a more fundamental explanation: since the gravitational force on the rope is negligible compared to the pulls of the two teams, the rope has a negligibly small inertia. If the vector sum of all forces on the rope were not zero, or negligibly small, the rope would achieve enormous acceleration. So the vector sum of all forces acting on the rope must be small, meaning the two oppositely directed pulling forces must be very close to being equal. The red team is losing because the forces between their feet and the ground are insufficient to overcome or cancel the tension, not because they are exerting less force on the rope.

8.28. (*a*) The greatest tensile force is exerted on the first coupling (the one right behind the engine). That coupling has to exert enough force to accelerate the ten trailing cars. (*b*) Stretching (*c*) The first coupling still exerts the greatest tensile force. (*d*) Compression

8.29. Here, there is only one force acting on the electron, so $\sum \vec{F}_{\text{e}} = \vec{F}^E_{E\text{e}} = m_{\text{e}}\vec{a}_{\text{e}}$, so $a_{\text{e}} = \dfrac{F^E_{E\text{e}}}{m_{\text{e}}} = \dfrac{(4.83\times10^{-13}\ \text{N})}{(9.11\times10^{-31}\ \text{kg})} =$ $5.30\times10^{17}\ \text{m/s}^2$. So the acceleration of the electron is $5.30\times10^{17}\ \text{m/s}^2$ in the direction the force is applied.

8.30. In each case the acceleration is given by $a_x(t) = \dfrac{F_x(t)}{m}$, which yields $a_x(t=0) = -8.7\ \text{m/s}^2$, $a_x(t=2.0) = 0.0\ \text{m/s}^2$, $a_x(t=4.0) = +8.7\ \text{m/s}^2$.

8.31. Yes. Knowing the history of all forces exerted on the object allows you to obtain the object's acceleration history and therefore the history of its velocity changes. Coupling this history with the known present position and velocity, you can work backwards to obtain the position and velocity at all previous instants.

8.32. Because we are speaking of the average force (rather than a time-dependent force), we can employ kinematic equations. Let the direction in which the bullet is fired be the $+x$ direction. Then we know $a_{\text{b}\,x} = \dfrac{v^2_{\text{b}\,x,\text{f}} - v^2_{\text{b}\,x,\text{i}}}{2\Delta x} = \dfrac{v^2_{\text{b}\,x,\text{f}}}{2\Delta x}$, where we have used the fact that the bullet starts from rest. Hence $v_{\text{b}\,x,\text{f}} = \sqrt{2\Delta x a_{\text{b}\,x}} = \sqrt{2\Delta x F_{\text{gb}x}/m}$. Since the final speed has to double, the force must increase by a factor of four. A force of $4F$ is needed.

8.33. (*a*) From Newton's second law, we know $\sum F_x(t) = ma_x(t) = m\dfrac{d^2x(t)}{dx^2} = m(2a-6bt) = (50\text{ kg})(2(15\text{m/s}^2)$ $-6(20\text{ m/s}^3)t)$. So $\sum F_x(t) = \alpha - \beta t$ where $\alpha = 1.5\times10^3$ N and $\beta = 6.0\times10^3$ N/s (*b*) For very small times, the first term will dominate and will be positive. For very large times, the second term will dominate and will be negative. We need only find where the force passes through zero: $m(2a-6bt_0) = 0 \Rightarrow t_0 = \dfrac{2a}{6b} = \dfrac{(15\text{ m/s}^2)}{3(20\text{ m/s}^3)} = 0.25$ s. Thus $\sum F_x > 0$ when $t < 0.25$ s, $\sum F_x = 0$ when $t = 0.25$ s, and $\sum F_x < 0$ when $t > 0.25$ s.

8.34. Call the direction of motion the $+x$ axis. Assuming the wheels are well-oiled such that friction can be ignored, the acceleration of the desk is given by $a_x = \dfrac{1}{m}\sum F_x = \dfrac{F_{\text{pd}x}}{m}$. Since this acceleration is constant, we can use kinematic equations to write $\Delta x(t) = v_{x,\text{i}}t + \dfrac{1}{2}a_x t^2$. Assuming the desk starts from rest, this yields a time of $t = \sqrt{\dfrac{2\Delta x}{a_x}} = \sqrt{\dfrac{2m\Delta x}{F_{\text{pd}x}}} = \sqrt{\dfrac{2(48\text{ kg})(5.9\text{ m})}{(19\text{ N})}} = 5.5$ s.

8.35. Call the direction of motion the $+x$ axis. We find the acceleration of the skier using $a_x = \dfrac{1}{m}\sum F_x = \dfrac{F^f_{\text{ice skier }x}}{m}$. Because the acceleration is constant we can use a kinematic equation to write $\Delta x = \dfrac{v^2_{\text{s }x,\text{f}} - v^2_{\text{s }x,\text{i}}}{2a_x} = \dfrac{m(v^2_{\text{s }x,\text{f}} - v^2_{\text{s }x,\text{i}})}{2F^f_{\text{ice skier }x}}$. Because we want to know at what point the skier stops, we insert $v_{\text{s }x,\text{f}} = 0$ and find $\Delta x =$ $\dfrac{(50\text{ kg})((0)-(9.72\text{ m/s})^2)}{2(-40\text{ N})} = 59$ m.

8.36. Let the initial motion of the cart be in the $+x$ direction. The x component of the velocity will change according to $v_{x,\text{f}} = v_{x,\text{i}} + \int_{t_\text{i}}^{t_\text{f}} a_x(t)dt = v_{x,\text{i}} + \int_0^{t_\text{f}} \dfrac{-at^2}{m}dt = v_{x,\text{i}} + \dfrac{-at_\text{f}^3}{3m}$. So, $v_{x,\text{f}}(t) = v_{x,\text{i}} + \dfrac{-at^3}{3m}$ and $v_{x,\text{f}}(t = 3.5\text{ s}) =$ $(0.23\text{ m/s}) + \dfrac{-(0.0200\text{ N/s}^2)(3.5)^3}{3(2.34\text{ kg})} = 0.11$ m/s. We can use the same expression to find the time at which the cart stops: $v_{x,\text{f}}(t) = v_{x,\text{i}} + \dfrac{-at^3}{3m} = 0 \Rightarrow t = \left(\dfrac{3mv_{x,\text{i}}}{a}\right)^{1/3} = \left(\dfrac{3(2.34\text{ kg})(0.23\text{ m/s})}{(0.0200\text{ N/s}^2)}\right)^{1/3} = 4.3$ s.

8.37. (*a*) $F^G_{\text{Eb}} = m_\text{b}g = (1.0\text{ kg})(9.8\text{ m/s}^2) = 9.8$ N (*b*) 9.8 N

8.38. In each case, we estimate the inertia of the specified object, then the magnitude of the force of gravity is given by $F^G = mg$. (*a*) If we take $m_{\text{sug}} \approx 3$ g, then $F^G_{\text{E sug}} = m_{\text{sug}}g = (3\times10^{-3}\text{ kg})(9.8\text{ m/s}^2) = 0.03$ N 0. (*b*) In the same way, if we take $m_\text{w} \approx 0.2$ kg, we find $F^G_{\text{Ew}} = 2$ N. (*c*) Taking $m_{\text{suit}} \approx 1\times10^1$ kg yields $F^G_{\text{E suit}} = 1\times10^2$ N. (*d*) Taking $m_\text{g} \approx 1\times10^2$ kg yields $F^G_{\text{Eg}} = 1\times10^3$ N. (*e*) Taking $m_\text{c} \approx 7\times10^2$ kg yields $F^G_{\text{Ec}} = 7\times10^3$ N.

8.39. (*a*) The force of your pull is transferred by the rope to the piano. Since that force is the only thing holding the piano up against the pull of gravity, it must also be equal to 1500 N. Explicitly, letting the $+y$ direction point upward, we have $\sum F_y = \mathcal{T} - F^G_{\text{Ep}y} = 0 \Rightarrow \mathcal{T} = F^G_{\text{Ep}y} = 1500$ N. (*b*) Note that now the rope is attached to the lower pulley in two places. Hence the tension can act twice, and the tension need only be half the gravitational force in order to hold up the piano. Now $\sum F_y = 2\mathcal{T} - F^G_{\text{Ep}y} = 0 \Rightarrow \mathcal{T} = F^G_{\text{Ep}y}/2 = 750$ N.

8.40. (*a*) Let the $+y$ axis point vertically upward. We write Newton's second law as $\sum F_y = \mathcal{T} - F^G_{\text{Ep}y} = ma_y = -mg/8 \Rightarrow \mathcal{T} = -mg/8 + F^G_{\text{Ep}y} = -(1500\text{ N})/8 + (1500\text{ N}) = 1313\text{ N}$. (*b*) Now Newton's second law becomes $\sum F_y = 2\mathcal{T} - F^G_{\text{Ep}y} = ma_y = -mg/8 \Rightarrow \mathcal{T} = -mg/16 + F^G_{\text{Ep}y}/2 = -(1500\text{ N})/16 + (1500\text{ N})/2 = 656.3\text{ N}$.

8.41. (*a*) We first use the data from Earth to obtain the inertia of the astronaut: $F^G_{\text{Ea}} = mg_{\text{E}} \Rightarrow m = \dfrac{F^G_{\text{Ea}}}{g_{\text{E}}} = \dfrac{(1960\text{ N})}{(9.8\text{ m/s}^2)} = 200\text{ kg}$. Now, on Jupiter we have $F^G_{\text{Ja}} = mg_{\text{J}} = (200\text{ kg})(25.9\text{ m/s}^2) = 5.2\times10^3\text{ N}$. (*b*) On the moon, we have $F^G_{\text{Ma}} = mg_{\text{M}} = (200\text{ kg})(1.6\text{ m/s}^2) = 3.2\times10^2\text{ N}$.

8.42. (*a*) It suffices to find the tension in the rope, because that is the same in magnitude as the force we exert downward on the rope. Using Newton's second law, we have $\sum F_y = \mathcal{T} - F^G_{\text{Ep}y} = ma_y \Rightarrow \mathcal{T} = mg + ma_y = (60\text{ kg})((9.8\text{ m/s}^2) + (1.5\text{ m/s}^2)) = 6.8\times10^2\text{ N}$. So the force we exert is 6.8×10^2 N downward (*b*) The same application of Newton's second law can also yield an expression for the maximum inertia: $\mathcal{T} = mg + ma_y \Rightarrow m_{\max} = \dfrac{\mathcal{T}_{\max}}{g + a_y} = \dfrac{(1225\text{ N})}{(9.8\text{ m/s}^2) + (1.5\text{ m/s}^2)} = 1.1\times10^2\text{ kg}$.

8.43. (*a*) The tension in the cord is equal to the gravitational force on the hanging block. This is clear because the hanging block is not accelerating. (*b*) The tension in the cord is less than the gravitational force on the hanging block. This is clear because the block is accelerating downward.

8.44. (*a*) Call the inertia of the hanging block m, and call this block 1 and the block on the table block 2. Choose the $+x$ axis to point to the right and the $+y$ axis to point upward. We write the vector sum of all forces on the hanging block first, and then on the block on the table:

$$\sum F_{1y} = \mathcal{T} - F^G_{\text{E}1y} = ma_{1y} \tag{1}$$

$$\sum F_{2x} = \mathcal{T} - F^f_{\text{t}2y} = 2ma_{2x} \tag{2}$$

Note that $a_{1y} = -a_{2x}$ because the two blocks move together. Now equations (1) and (2) can be combined to yield

$$-2ma_{1y} + F^f_{\text{t}2y} = ma_{1y} + F^G_{\text{E}1y} \quad (3)$$

Inserting $F^f_{\text{t}2y} = \dfrac{mg}{2}$ we find $a_{1y} = -\dfrac{g}{6} = -\dfrac{1}{6}(9.8\text{m/s}^2) = -1.6\text{m/s}^2$, or 1.6 m/s^2 downward.

(*b*) Here the solving process is identical to part (*a*), except that now since the initial motion is to the right friction will act to the left (in the positive x direction). Thus we have

$$\sum F_{1y} = \mathcal{T} - F^G_{\text{E}1y} = ma_{1y} \tag{1}$$

$$\sum F_{2x} = \mathcal{T} + F^f_{\text{t}2y} = 2ma_{2x} \tag{2}$$

We solve equations (1) and (2) for tension and set them equal, noting that $a_{1y} = -a_{2x}$ as before:

$$ma_{1y} + mg = -2ma_{1y} - \frac{mg}{2}$$

or equivalently $a_{1y} = -\dfrac{g}{2} = -4.9\text{ m/s}^2$. So the hanging block accelerates at 4.9 m/s^2 downward.

8.45. (*a*) Call the inertia of the hanging block m, and call this block 1 and the block on the table block 2. Choose the $+x$ axis to point to the right and the $+y$ axis to point upward. We write the vector sum of all forces on the hanging block first, and then on the block on the table:

$$\sum F_{1y} = \mathcal{T} - F^G_{\text{E}1y} = ma_{1y} \tag{1}$$

$$\sum F_{2x} = \mathcal{T} - F^f_{\text{t}2y} = \frac{m}{2}a_{2x} \tag{2}$$

Note that $a_{1y} = -a_{2x}$ because the two blocks move together. Now equations (1) and (2) can be combined to yield

$$-\frac{m}{2}a_{1y} + F^f_{t2y} = ma_{1y} + F^G_{E1y} \tag{3}$$

Inserting $F^f_{t2y} = \frac{1}{2}\left(\frac{m}{2}g\right)$ we find $a_{1y} = -\frac{g}{2} = -\frac{1}{2}(9.8\,\text{m/s}^2) = -4.9\,\text{m/s}^2$. We are asked about the acceleration of block 2 across the table, so $a_{2x} = 4.9\ \text{m/s}^2$ to the right. (*b*) Here the solving process is identical to part (*a*), except that now since the initial motion is to the right friction will act to the left (in the positive x direction). Thus we have

$$\sum F_{1y} = \mathcal{T} - F^G_{E1y} = ma_{1y} \tag{1}$$

$$\sum F_{2x} = \mathcal{T} + F^f_{t2y} = \frac{m}{2}a_{2x} \tag{2}$$

We solve equations (1) and (2) for tension and set them equal, noting that $a_{1y} = -a_{2x}$ as before:

$$ma_{1y} + mg = -\frac{m}{2}a_{1y} - \frac{mg}{4}$$

or equivalently $a_{1y} = -\frac{5g}{6} = -8.2\ \text{m/s}^2$. So the block on the table accelerates at $8.2\ \text{m/s}^2$ to the right.

8.46. We know that the concrete mix will accelerate downward, and the lumber will thus be lifted (and will have upward acceleration). Since the lumber is accelerating upward, tension must be overcoming the gravitational force exerted downward on the lumber. Thus the tension is greater in magnitude than the gravitational force on the lumber. Similarly, because the concrete mix will accelerate downward, the tension is smaller in magnitude than the gravitational force on the concrete mix.

8.47. Cases (*a*) and (*b*) will yield identical equations of motion for the block. In either case $\sum F_y = \mathcal{T} - mg = 0,$ so in either case $\mathcal{T} = mg$. In case (*c*), the equation of motion is similar, but with twice the inertia supported it yields $\mathcal{T} = 2mg$. In case (*d*), call the blocks on the right object 1, and the block on the left object 2. This case is slightly different because the blocks will not remain stationary. We write the sum of all forces in the y direction for each object:

$$\sum F_{1y} = \mathcal{T} - 2mg = 2ma_{1y} \tag{1}$$

$$\sum F_{2y} = \mathcal{T} - mg = ma_{2x} \tag{2}$$

Clearly if the blocks on the right move downward, the block on the left moves upward. It is also clear that they are constrained to have the same magnitude of acceleration because the length of the rope does not change. Hence $a_{1y} = -a_{2y}$. Solving equations (1) and (2) for accelerations and equating the accelerations (up to a sign) yields $\mathcal{T} = \frac{4mg}{3}$. Thus $a = b < d < c$.

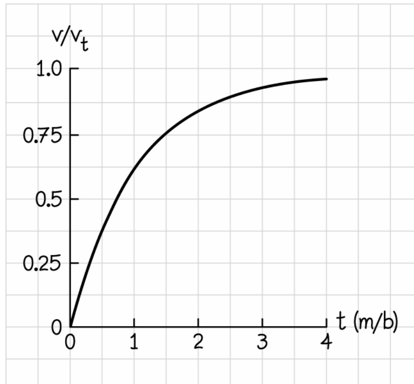
8.48. (*a*) Writing the sum of all forces in the y direction yields $\sum F_y = -mg + bv_y = ma_y$. Under the influence of gravity alone, the object would continue to speed up. But as it does so, the resistance from the air increases. This trade-off will continue until the acceleration is zero. At that time $mg = bv_y$, and because the acceleration is zero, the speed at this time will remain unchanged. We call it the terminal speed $v_t = mg/b$. (*b*) Starting from the sum of all forces on the object and rewriting the acceleration as the time derivative of the velocity yields $m\dot{v}(t) = -(b/m)v(t) + mg$. This is an ordinary differential equation that has the general solution $v(t) = \alpha e^{-\lambda t} + \beta$, where α, β, and λ are constant to be determined. If the object is dropped, then $v(t=0) = 0$, which tells us that $\alpha = -\beta$. In the limit as $t \to \infty$ we know the speed has to approach the terminal speed. Hence $\beta = mg/b$ and $\alpha = -mg/b$. Finally, at the instant of release, there should not be any air resistance yet, such that the acceleration should simply be that due to gravity. Inserting the results for α and β and taking the time derivative yields

$$v(t) = \frac{mg}{b}(e^{-\lambda t} - 1)$$

$$\dot{v}(t=0) = -\lambda\frac{mg}{b} = -g$$

$$\Rightarrow \lambda = \frac{m}{b}$$

Finally, putting all constants back in, one obtains $v(t) = (mg/b)(e^{-(b/m)t} - 1)$. (*c*) The units of m/b are seconds.



8.49. We write down the equations of motion for each of the three objects, and find an expression for each inertia in terms of the same variables.

$$\sum F_{\mathrm{P}y} = \mathcal{T} - m_{\mathrm{P}}g = m_{\mathrm{P}}a_{\mathrm{P}y} \quad (1)$$

$$\sum F_{\mathrm{M}y} = 2\mathcal{T} - m_{\mathrm{M}}g = m_{\mathrm{M}}a_{\mathrm{M}y} \quad (2)$$

$$\sum F_{\mathrm{W}y} = \mathcal{T} - m_{\mathrm{W}}g = m_{\mathrm{W}}a_{\mathrm{W}y} \quad (3)$$

We insert the given information $a_{\mathrm{M}y} = a_{\mathrm{P}y} = -g/3$ and we note that $a_{\mathrm{W}y} = -3a_{\mathrm{M}y} = +g$. This can be seen by picturing objects M and P moving downward some length ℓ and noting how much rope must be added in each segment. Solving equations (1-3) for the inertias in terms of the tensions yields

$$m_{\mathrm{P}} = \frac{3}{2}\frac{\mathcal{T}}{g} \quad (4)$$

$$m_{\mathrm{M}} = 3\frac{\mathcal{T}}{g} \quad (5)$$

$$m_{\mathrm{W}} = \frac{1}{2}\frac{\mathcal{T}}{g} \quad (6)$$

Expression equations (4-6) as a single ratio, we have $m_{\mathrm{W}} : m_{\mathrm{M}} : m_{\mathrm{P}} = \frac{1}{2} : 3 : \frac{3}{2}$.

8.50. The spring scale measures the contact force that the person exerts downward on the scale (which of course is equal to the upward force that the spring scale exerts on the person). Since the person jumped onto the scale, the scale didn't just have to counteract gravity; the scale had to accelerate the person upward to stop their downward motion onto the scale. Hence for a moment the upward force from the spring scale on the person has to greatly exceed the downward gravitational force on the person.

8.51. (*a*) We write the sum of all forces acting on the girl in the y direction, and solve for the spring constant: $\sum F_y = F^c_{\mathrm{Tg}y} - F^G_{\mathrm{Eg}y} = 0 \Rightarrow -k\Delta y - mg = 0$ so $k = -\frac{mg}{\Delta y} = -\frac{(20\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{(-0.11\ \mathrm{m})} = 1.8\times10^3\ \mathrm{N/m}$. (*b*) Using the same process, we write $\sum F_y = F^c_{\mathrm{TF}y} - F^G_{\mathrm{EF}y} = 0 \Rightarrow -k\Delta y - mg = 0$ so $\Delta y = -\frac{mg}{k} = -\frac{(75\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{(1.78\times10^3\ \mathrm{N/m})} =$ 0.41 m.

8.52. We know that $F_{1y} = -k_1\Delta y_1$ and $F_{2y} = -k_2\Delta y_2$ for each spring separately. Consider pulling down on the apparatus shown in Figure P8.52; this would stretch both springs a distance Δy producing a total upward force $F_{\text{combination}\,y} = F_{1y} + F_{2y} = -(k_1 + k_2)\Delta y = -k_{\text{combination}}\Delta y$. Hence $k_{\text{combination}} = k_1 + k_2$, which is always larger than either constituent spring constant.

8.53. The same force is applied to each spring (just as the tension everywhere along a rope is the same if the rope has negligible inertia). For each spring separately, we can write $F_{1y} = -k_1\Delta y_1$ and $F_{2y} = -k_2\Delta y_2$, where $\Delta y = \Delta y_1 + \Delta y_2$. Hence we can write

$$F_{2y} = -k_{\text{combination}}\Delta y = -k_{\text{combination}}(\Delta y_1 + \Delta y_2)$$
$$F_y = -k_{\text{combination}}\left(-\frac{F_{1y}}{k_1} + -\frac{F_{2y}}{k_2}\right)$$
$$F_y = F_y k_{\text{combination}}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

The last line can be rearranged to show $k_{\text{combination}} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1} = \frac{k_1 k_2}{k_1 + k_2}$. We see that $k_{\text{combination}} = \frac{k_1 k_2}{(k_1 + k_2)}$ is always smaller than either constituent spring constant.

8.54. (*a*) We know $F^c_{\text{Sb}y} = -k_{\text{combination}}\Delta y$, and since the brick comes to rest, we know $\sum F_{\text{b}y} = F^c_{\text{Sb}y} - F^G_{\text{Eb}y} = -k_{\text{combination}}\Delta y - mg = 0$. So $k_{\text{combination}} = -\frac{mg}{\Delta y} = -\frac{(4.0\text{ kg})(9.8\text{ m/s}^2)}{(-0.15\text{ m})} = 2.6\times10^2\text{ N/m}$. (*b*) If the top spring stretches 0.10 m, the bottom spring must stretch 0.05 m. This means that the spring constant of the bottom spring is twice that of the top spring. We know that the combination of two springs has a spring constant given by (see problem 8.54) $k_{\text{combination}} = \frac{k_{\text{top}}k_{\text{bottom}}}{(k_{\text{top}} + k_{\text{bottom}})}$. Inserting $k_{\text{bottom}} = 2k_{\text{top}}$ we see $k_{\text{combination}} = \frac{2k^2_{\text{top}}}{(3k_{\text{top}})} = \frac{2}{3}k_{\text{top}}$ so that $k_{\text{top}} = \frac{3}{2}k_{\text{combination}} = \frac{3}{2}(2.61\times10^2\text{ N/m}) = 3.9\times10^2\text{ N/m}$. This result combined with $k_{\text{bottom}} = 2k_{\text{top}}$ yields both spring constants: $k_{\text{top}} = 3.9\times10^2\text{ N/m}$ and $k_{\text{bottom}} = 7.8\times10^2\text{ N/m}$.

8.55. First we calculate the spring constant from data outside the elevator, then we use it with the data inside the elevator. Outside the elevator, the box comes to rest when hung, such that we can write

$$\sum F_{\text{b}\,y} = F^c_{\text{Sb}y} - mg = 0$$
$$-k\Delta y - mg = 0$$
$$k = -\frac{mg}{\Delta y} = -\frac{(5.0\text{ kg})(9.8\text{ m/s}^2)}{(0.050\text{ m})} = 9.8\times10^2\text{ N/m}$$

Now, inside the elevator, the box is accelerating at 2.0 m/s^2 upward. As before, the equation of motion yields $-k\Delta y - mg = ma$ such that $\Delta y = -\frac{m}{k}(a+g) = -\frac{(5.0\text{ kg})}{(9.8\times10^2\text{ N/m})}((2.0\text{ m/s}^2) + (9.8\text{ m/s}^2)) = 60\text{ mm}$.

8.56. Answer depends on your inertia and on mattress properties; values from 10^2 N/m to 10^3 N/m acceptable.

8.57. Call the two blocks on the right object 1, and the block on the left object 2, and left the $+y$ point vertically upward. We write the equations of motion for each object separately

$$\sum F_{1y} = \mathcal{T} - mg - F^c_{\text{S1}y} = ma_{1y} \tag{1}$$
$$\sum F_{2y} = \mathcal{T} - 2mg = 2ma_{2y} \tag{2}$$

and we note that $a_{1y} = -a_{2y}$ and the tension is the same throughout the rope. Equations (1) and (2) can then be combined to yield $2m(a_{2y} + g) = m(-a_{2y} + g) - k\Delta y$ or $a_{2y} = \dfrac{kd - mg}{3m}$. So $\vec{a} = \dfrac{kd - mg}{3m}$ downward.

8.58. Because the force exerted on each spring is the same (as the tension is the same throughout a rope of negligible inertia), we can equate the quantities $k_A \Delta y_A = k_B \Delta y_B$ for each pair. The data for A/B yields $k_B = k_A \dfrac{\Delta y_A}{\Delta y_B} = 1.9$. Similarly $k_C = k_A \dfrac{\Delta y_A}{\Delta y_C} = 2.3$ and $k_D = k_A \dfrac{\Delta y_A}{\Delta y_D} = 2.0$. Combining all these data allows us to write $k_A : k_B : k_C : k_D = 1:1.9:2.3:2.0$.

8.59. We write down the equations of motion for the books in the various cases, and solve for the quantity required. Call vertically upward the $+y$ direction.

$$\text{elevator: } \sum F_{Ry} = -k\Delta y_R - m_R g = m_R a_y \tag{1}$$

$$\text{elevator: } \sum F_{RYy} = -k\Delta y_{RY} - (m_R + m_Y)g = (m_R + m_Y)a_y \tag{2}$$

$$\text{floor: } \sum F_{RYy} = -k\Delta y_{RY\text{floor}} - (m_R + m_Y)g = 0 \tag{3}$$

This is a system of three equations with three unknowns, so we can solve for any of the three quantities needed. The quantities can be found in any order you like, but the algebra may be marginally easier if we solve for the inertia of the yellow book along with the spring constant. We can proceed by solving equation (3) for k:

$$k = -\frac{(m_R + m_Y)g}{\Delta y_{RY\text{floor}}} \tag{4}$$

and inserting this into equations (1) and (2). We then solve equation (1) to obtain an expression for a_y, namely

$$a_y = \frac{\Delta y_R}{m_R}\frac{(m_R + m_Y)g}{\Delta y_{RY\text{floor}}} - g \tag{5}$$

Inserting this and the expression for k into equation (2) yields

$$\left(\frac{(m_R + m_Y)g}{\Delta y_{RY\text{floor}}}\right)\Delta y_{RY} - (m_R + m_Y)g = (m_R + m_Y)\left(\frac{\Delta y_R}{m_R}\frac{(m_R + m_Y)g}{\Delta y_{RY\text{floor}}} - g\right)$$

which can be solved for the inertia of the yellow book:

$m_Y = \dfrac{-m_R \Delta y_R + m_R \Delta y_{RY}}{\Delta y_R} = \dfrac{-(5.0\text{ kg})(-0.071\text{ m}) + (5.0\text{ kg})(-0.110\text{ m})}{(-0.071\text{ m})} = 2.7\text{ kg}$. Inserting this into equation (4) yields the spring constant

$$k = -\frac{(m_R + m_Y)g}{\Delta y_{RY\text{floor}}} = -\frac{((5.0\text{ kg}) + (2.75\text{ kg}))(9.8\text{ m/s}^2)}{(-0.140\text{ m})} = 5.4\times 10^2\text{ N/m}.$$

The acceleration is now trivially found by inserting the values of m_Y and k into equation (5):

$$a_y = \left(\frac{(-0.071\text{ m})}{(5.0\text{ kg})}\frac{((5.0\text{ kg}) + (2.75\text{ kg}))}{(-0.140\text{ m})} - 1\right)(9.8\text{ m/s}^2) = -2.1\text{ m/s}^2$$

Collecting the results:
(*a*) $k = 5.4\times 10^2$ N/m (*b*) $m_Y = 2.7$ kg (*c*) $\vec{a} = 2.1\text{ m/s}^2$ downward.

8.60. The magnitude of the plate's momentum change Δp is the same in both cases, and so the magnitude $F = \Delta p/\Delta t$ of force exerted by carpet or tile on plate depends on Δt. When the plate drops on carpet, Δt is much greater than when the plate drops on tile, and so the magnitude $F_{\text{dropped on carpet}}$ is much smaller than the magnitude $F_{\text{dropped on tile}}$.

8.61. Let us assume the two hammers have identical initial momentum, such that stopping the hammers involves delivering identical impulses to them. In order to drive a nail into a board, a large force must be exerted on the nail by a hammer. This force is, of course, equal in magnitude to the force the nail exerts back on the hammer. In the case of the rubber mallet, the nail exerts a force over a long period of time as the rubber compresses, and the force exerted on the rubber mallet will thus be smaller. The steel hammer will not compress much at all, so the time over which forces can be exerted is very small. The forces are therefore much larger than in the case of the rubber mallet.

8.62. Call the direction in which the ball is hit the $+x$ direction. Then we know $F_{\text{av}\,x}\Delta t = \Delta p_x = m\Delta v_x$ or $F_{\text{av}\,x} = \dfrac{m}{\Delta t}(v_{\text{f}\,x} - v_{\text{i}\,x}) = \dfrac{(0.150\text{ kg})}{(5.0\times 10^{3}\text{ s})}((50.0\text{ m/s}) - (-44.4\text{ m/s})) = 2.8\times 10^3\text{ N}$. So the average force was 2.8×10^3 N toward the pitcher.

8.63. (*a*) The top is most likely to break first. The bottom rope must pull back up on your hand with an equal and opposite force, but the top of the yarn must counteract your pull and support the weight of the plant. (*b*) The bottom is most likely to break.

8.64. Values of $1\times 10^0\text{ N}\cdot\text{s}$ to $1\times 10^1\text{ N}\cdot\text{s}$ are acceptable. The direction is toward the nail.

8.65. (*a*) The impulse is given by $\vec{J} = \Delta\vec{p} = m\Delta\vec{v}$. So the impulse is $(1500\text{ kg})(27.8\text{ m/s}) = 4.2\times 10^4\text{ N}\cdot\text{s}$ forward. (*b*) Since the impulse is related to the force through $\vec{J} = \vec{F}_{\text{av}}\Delta t$, in this case $\vec{F}_{\text{av}} = \dfrac{\vec{J}}{\Delta t} = \dfrac{(4.17\times 10^4\text{ N}\cdot\text{s})}{(60\text{ s})}$ forward = 6.9×10^2 N forward.

8.66. (*a*) We can use $\Delta\vec{p} = \int_{t_\text{i}}^{t_\text{f}} \vec{F}dt$, and apply it in the direction of the push (call this the $+x$ direction). From the description we can write $F_x = \alpha t$ where $\alpha = 1.0$ N/s. So $m(v_{x,\text{f}} - v_{x,\text{i}}) = \alpha\int_{t_\text{i}}^{t_\text{f}} t\,dt = \alpha\dfrac{1}{2}(t_\text{f}^2 - t_\text{i}^2)$. Solving for the final x component of the velocity yields $v_{x,\text{f}} = \dfrac{\alpha}{m}\dfrac{1}{2}(t_\text{f}^2 - t_\text{i}^2) + v_{x,\text{i}} = \dfrac{(1.0\text{ N/s})}{(5.0\text{ kg})}\dfrac{1}{2}((5.0\text{ s})^2 - 0) + 0 = 2.5\text{ m/s}$. (*b*) We can use the same process and same expressions as in part (*a*), except that now $v_{x,\text{i}} = -3.0$ m/s. Hence $v_{x,\text{f}} = \dfrac{\alpha}{m}\dfrac{1}{2}(t_\text{f}^2 - t_\text{i}^2) + v_{x,\text{i}} = \dfrac{(1.0\text{ N/s})}{(5.0\text{ kg})}\dfrac{1}{2}((5.0\text{ s})^2 - 0) + (-3.0\text{ m/s}) = -0.50\text{ m/s}$. So the final speed of the box is 0.50 m/s.

8.67. (*a*) The final kinetic energy is half the initial kinetic energy, so the final speed is $\dfrac{1}{\sqrt{2}}$ times the initial speed. In component form (with $+y$ taken to be vertically upward) we have $v_{y,\text{f}} = -\dfrac{1}{\sqrt{2}}v_{y,\text{i}} = -\dfrac{1}{\sqrt{2}}(-3.30\text{ m/s}) =$ 2.33 m/s. So the final speed is 2.33 m/s. (*b*) We know that $\Delta\vec{p} = \vec{F}_{\text{av}}\Delta t$. So here $F_{y,\text{av}} = \dfrac{m}{\Delta t}(v_{y,\text{f}} - v_{y,\text{i}})$ $= \dfrac{(0.625\text{ kg})}{(9.25\times 10^{-3}\text{ s})}((2.333\text{ m/s}) - (-3.30\text{ m/s})) = 3.81\times 10^2\text{ N}$. The magnitude of the force is also 381 N.

8.68. (*a*) The impulse is just the change in momentum, so once we determine the initial and final velocities we will essentially be done. Call the moment of release time 0, the moment just before impact time 1, just after impact time 2, and the moment the ball reaches its peak height again time 3. Applying kinematics to the falling and rising of the ball separately, we find $v_{y,1}^2 = v_{y,0}^2 + 2a_y\Delta y_{0\to 1} \Rightarrow v_{y,1} = -\sqrt{2(-9.8\text{ m/s}^2)(-3.0\text{ m})} = -7.67\text{ m/s}$, and $v_{y,3}^2 = v_{y,2}^2 + 2a_y\Delta y_{2\to 3} \Rightarrow v_{y,2} = \sqrt{-2(-9.8\text{ m/s}^2)(+2.5\text{ m})} = +7.00\text{ m/s}$. Now $J_y = m(v_{y,\text{f}} - v_{y,\text{i}}) = (1.2\text{ kg})((+7.00\text{ m/s}) -$

$(-7.67\,\text{m/s})) = 18\ \text{N}\cdot\text{s}$. Thus the impulse is $18\ \text{N}\cdot\text{s}$ upward. (*b*) One object is the ball, and the other object is the steel plate, which does not move. Thus $e = -\dfrac{v_{\text{b}y,\text{f}} - v_{\text{s}y,\text{f}}}{v_{\text{b}y,\text{i}} - v_{\text{s}y,\text{i}}} = -\dfrac{7.00\ \text{m/s} - 0}{-7.67\ \text{m/s} - 0} = 0.91$.

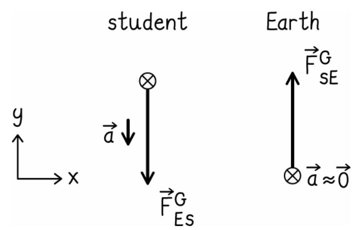
8.69. (*a*) Call vertically upward the positive y direction, and label the moment the egg is dropped as time 0, the moment just before the egg strikes the floor as time 1, and the moment just after the egg has come to rest on the floor as time 2. We can find the y component of the velocity just before impact using kinematics: $v_{y,1}^2 = v_{y,0}^2 + 2a_y\Delta y_{0\to1} \Rightarrow v_{y,1} = -\sqrt{0 + 2(-9.8\ \text{m/s}^2)(-1.0\ \text{m})} = -4.43\,\text{m/s}$. Now the y component of the impulse is given by $J_y = \Delta p_y = m(v_{y,2} - v_{y,1}) = (0.050\ \text{kg})((0) - (-4.43\,\text{m/s})) = 0.22\ \text{N}\cdot\text{s}$. Thus the impulse is $0.22\ \text{N}\cdot\text{s}$ upward. (*b*) This estimate could be made in several ways. We will assume here that the speed of the egg drops to zero linearly in time, over the duration of the impact, such that the average y component of the velocity is $v_{y,\text{av}} = \dfrac{1}{2}(v_{y,2} + v_{y,1}) = -2.22\ \text{m/s}$. Then $v_{y,\text{av}} = \dfrac{\Delta y}{\Delta t} \Rightarrow \Delta t = \dfrac{y_2 - y_1}{v_{y,\text{av}}} = \dfrac{-h/2}{v_{y,\text{av}}} = \dfrac{-0.020\ \text{m}}{-2.22\,\text{m/s}} = 9.0\ \text{ms}$. Other answers are possible depending on assumptions made about how the speed reduces to zero over time. (*c*) We know $\vec{J} = \vec{F}_{\text{av}}\Delta t$, so $F_{y,\text{av}} = \dfrac{m\Delta v_y}{\Delta t} = \dfrac{(0.050\ \text{kg})(4.43\ \text{m/s})}{(9.0\times10^{-3}\ \text{s})} = 25\ \text{N}$. The average force is then $25\ \text{N}$ upward. (*d*) Using the same expression as in part (*c*), but with a longer time, we have $F_{y,\text{av}} = \dfrac{m\Delta v_y}{\Delta t} = \dfrac{(0.050\ \text{kg})(4.43\ \text{m/s})}{4(9.0\times10^{-3}\ \text{s})} = 6.1\ \text{N}$. So the average force would be $6.1\ \text{N}$ upward.

8.70. (*a*) Let $+x$ be the initial direction of motion of the car. The x component of the impulse is just $J_x = m\Delta v_x = (70\ \text{kg})((0) - (22.2\ \text{m/s})) = 1.6\times10^3\ \text{N}\cdot\text{s}$. So $\vec{J} = 1.6\times10^3\ \text{N}\cdot\text{s}$ toward the rear of the car. (*b*) If we assume the speed of the car drops linearly in time, we can say that the average x component of velocity is $v_{x,\text{av}} = \dfrac{1}{2}(v_{x,\text{f}} + v_{x,\text{i}}) = \dfrac{1}{2}(0 + (22.2\ \text{m/s})) = 11.1\ \text{m/s}$. Relating this to the stopping distance yields $v_{x,\text{av}} = \dfrac{\Delta x}{\Delta t} \Rightarrow \Delta t = \dfrac{\Delta x}{v_{x,\text{av}}} = \dfrac{(1.23\ \text{m})}{(11.1\ \text{m/s})} = 0.11\ \text{s}$. Finally relating this to the impulse tells us $F^{\text{c}}_{\text{hb}x,\text{av}} = \dfrac{J_x}{\Delta t} = \dfrac{(-1.56\times10^3\ \text{N}\cdot\text{s})}{(0.111\ \text{s})} = -1.4\times10^4\ \text{N}$. So the average force is $1.4\times10^4\ \text{N}$ toward the rear of the car. (*c*) The change in momentum and therefore the impulse is the same as in part (*b*). The only thing that changes is the duration of the impact. Now we have $F^{\text{c}}_{\text{hb}x,\text{av}} = \dfrac{J_x}{\Delta t} = \dfrac{(-1.56\times10^3\ \text{N}\cdot\text{s})}{(0.0145\ \text{s})} = -1.1\times10^5\ \text{N}$. So the average force is $1.1\times10^5\ \text{N}$ toward the rear of the car.
(*d*) The driver is not likely to survive the crash without a seatbelt. The magnitude of the force exerted on the driver would be close to 160 times the magnitude of the gravitational force exerted by Earth on the driver.

8.71. (*a*) Let the direction in which the ball is hit be the $+x$ direction. Then $J_x = \displaystyle\int_0^{t_\text{f}} F(t)^{\text{c}}_{\text{hb}x,\text{av}}\,dt = \frac{1}{2}at_\text{f}^2 - \frac{1}{3}bt_\text{f}^3 = \frac{1}{2}(3.0\times10^5\ \text{N/s})(3.0\times10^{-3})^2 - \frac{1}{3}(1.0\times10^8\ \text{N/s}^2)(3.0\times10^{-3})^3 = 0.45\ \text{N}\cdot\text{s}$. So the magnitude of the impulse is $0.45\ \text{N}\cdot\text{s}$. (*b*) The average force in the x direction is related to the impulse by $F^{\text{c}}_{\text{hb}x,\text{av}} = \dfrac{J_x}{\Delta t} = \dfrac{(0.45\ \text{N}\cdot\text{s})}{(3.0\times10^{-3}\ \text{s})} = 1.5\times10^2\ \text{N}$. So the magnitude of the average force is $1.5\times10^2\ \text{N}$. (*c*) To find the time at which the maximum force is achieved, we differentiate the force function with respect to time, and set it equal to zero: $\dfrac{dF(t)^{\text{c}}_{\text{hb}x,\text{av}}}{dt} = a - 2bt = 0 \Rightarrow t_{\text{max}} = \dfrac{a}{2b} = \dfrac{(3.0\times10^5\ \text{N/s})}{2(1.0\times10^8\ \text{N/s}^2)} = 1.5\ \text{ms}$. Inserting this time into the expression for force yields $(F(t)^{\text{c}}_{\text{hb}x,\text{av}})_{\text{max}} = at_{\text{max}} - bt_{\text{max}}^2 = (3.0\times10^5\ \text{N/s})(1.5\times10^{-3}\ \text{s}) - (1.0\times10^8\ \text{N/s}^2)(1.5\times10^{-3}\ \text{s})^2 = 2.3\times10^2\ \text{N}$. (*d*) The x

component of the impulse is equal to the change in momentum, so $J_x = m(v_{x,\mathrm{f}} - v_{x,\mathrm{i}}) \Rightarrow$ $v_{x,\mathrm{f}} = \dfrac{J_x}{m} + v_{x,\mathrm{i}} = \dfrac{(0.45\ \mathrm{N\cdot s})}{(0.27\ \mathrm{kg})} + 0 = 1.7\ \mathrm{m/s}.$ So the final velocity is 1.7 m/s in the direction of the serve.

8.72. (*a*)



(*b*) The student is in freefall, so $\vec{a} = 9.8\ \mathrm{m/s^2}$ downward. The only force acting on the student is the gravitational force, so $\sum \vec{F} = -mg\,\hat{j} = 6.9\times10^2\ \mathrm{N}$ downward (*c*) We ignore other objects that may be interacting with Earth, and focus on the interaction with the student. We know $\vec{F}^{G}_{\mathrm{Es}} = -\vec{F}^{G}_{\mathrm{sE}}$, so $a_{\mathrm{E}} = \dfrac{F^{G}_{\mathrm{sE}}}{m_{\mathrm{E}}} = \dfrac{(6.86\times10^2\ \mathrm{N})}{(5.97\times10^{24}\ \mathrm{kg})} = 1.1\times10^{-22}\ \mathrm{m/s^2}.$

8.73. (*a*) The acceleration of the center of mass is the second time derivative of the position of the center of mass. Let the direction of the red cart's motion be the $+x$ direction. Then $x_{\mathrm{cm}} = \dfrac{m_{\mathrm{red}}x_{\mathrm{red}} + m_{\mathrm{green}}x_{\mathrm{green}}}{m_{\mathrm{red}} + m_{\mathrm{green}}}$, such that $\dfrac{d^2x_{\mathrm{cm}}}{dt^2} =$ $a_{\mathrm{cm}x}\dfrac{d^2x_{\mathrm{red}}}{dt^2}\left(\dfrac{m_{\mathrm{red}}}{m_{\mathrm{red}} + m_{\mathrm{green}}}\right) = a_{x,\mathrm{red}}\left(\dfrac{m_{\mathrm{red}}}{m_{\mathrm{red}} + m_{\mathrm{green}}}\right)$. The acceleration can be found from $a_{\mathrm{cm}x} = \dfrac{(F^{\mathrm{c}}_{\mathrm{h\ red}\ x} + F^{\mathrm{f}}_{\mathrm{floor\ red}\ x})}{(m_{\mathrm{red}} + m_{\mathrm{green}})} =$ $\dfrac{((12.0\ \mathrm{N}) + (-5.0\ \mathrm{N}))}{((5.00\ \mathrm{kg}) + (5.00\ \mathrm{kg}))} = 0.70\mathrm{m/s^2}$. So $\vec{a}_{\mathrm{cm}} = 0.70\ \mathrm{m/s^2}$ in direction of the push. (*b*) $\sum F_{\mathrm{red}x} = F^{\mathrm{c}}_{\mathrm{hred}x} + F^{\mathrm{f}}_{\mathrm{floor\ red}x} =$ $m_{\mathrm{red}}a_{\mathrm{red}x}$ so $a_{\mathrm{red}x} = \dfrac{1}{m_{\mathrm{red}}}(F^{\mathrm{c}}_{\mathrm{hred}x} + F^{\mathrm{f}}_{\mathrm{floor\ red}x}) = \dfrac{1}{(5.00\ \mathrm{kg})}((12.0\ \mathrm{N}) + (-5.0\ \mathrm{N})) = 1.4\ \mathrm{m/s^2}$. So $\vec{a}_{\mathrm{red}} = 1.4\ \mathrm{m/s^2}$ in the direction of the push. (*c*) Now that both carts are moving, the force of friction will act on each one. Writing the sum of all x components of forces on the system of two carts yields $\sum F_{\mathrm{both}\ x} = F^{\mathrm{c}}_{\mathrm{hred}x} + F^{\mathrm{f}}_{\mathrm{floor\ red}x} +$ $F^{\mathrm{f}}_{\mathrm{floor\ green}x} = (m_{\mathrm{red}} + m_{\mathrm{green}})a_{\mathrm{cm}x}$, so $a_{\mathrm{cm}x} = \dfrac{F^{\mathrm{c}}_{\mathrm{hred}x} + F^{\mathrm{f}}_{\mathrm{floor\ red}x} + F^{\mathrm{f}}_{\mathrm{floor\ green}x}}{(m_{\mathrm{red}} + m_{\mathrm{green}})} = \dfrac{(12.0\ \mathrm{N}) + (-5.0\ \mathrm{N}) + (-5.0\ \mathrm{N})}{((5.00\ \mathrm{kg} + (5.00\ \mathrm{kg}))} = 0.20\ \mathrm{m/s^2}.$

So $\vec{a}_{\mathrm{cm}} = 0.20\ \mathrm{m/s^2}$ in the direction of the push.

8.74. (*a*) The acceleration of the center of mass of the system of two carts is determined by the sum of all forces acting on the system. Call the cart that you push on cart 1, and the stationary cart cart 2. Let the direction of the push be the $+x$ direction. $\sum F_x = F^{\mathrm{c}}_{\mathrm{h}1x} = (m_1 + m_2)a_{\mathrm{cm}x}$. So $a_{\mathrm{cm}x} = \dfrac{F^{\mathrm{c}}_{\mathrm{h}1x}}{(m_1 + m_2)} = \dfrac{(2.00\ \mathrm{N})}{((0.500\ \mathrm{kg}) + (0.500\ \mathrm{kg}))} = 2.0\ \mathrm{m/s^2}$. So $\vec{a}_{\mathrm{cm}} = 2.0\ \mathrm{m/s^2}$ in the direction of the push. (*b*) The spring can only exert forces within the system. The motion of the center of mass cannot be affected by these internal forces. Thus $\vec{a}_{\mathrm{cm}} = 2.0\ \mathrm{m/s^2}$ in the direction of motion, still.

8.75. (*a*) The equation of motion in this case, yields $F_x = 3ma_x$ so $a = F/3m$. (*b*) Since the acceleration of the right block is $F/3m$, we know the force acting on it must equal $ma = F/3$. So $\vec{F}^{\mathrm{c}}_{\mathrm{LR}} = F/3$ to the right. (*c*) Clearly, $\vec{F}^{\mathrm{c}}_{\mathrm{RL}} = -\vec{F}^{\mathrm{c}}_{\mathrm{LR}}$, so the $\vec{F}^{\mathrm{c}}_{\mathrm{RL}} = F/3$ to the left. (*d*) Part (*a*) would be unchanged, and part (*b*) would become $2F/3$ to the right, and part (*c*) would become $2F/3$ to the left.

8.76. (*a*) Because the spring is the only thing exerting a force on the 2.0 kg block, we can say $-k\Delta x = m_2 a_{2x} \Rightarrow a_{2x} = \dfrac{-k\Delta x}{m_2} = \dfrac{-(300\ \text{N/m})(-0.100\ \text{m})}{(2.0\ \text{kg})} = 15\ \text{m/s}^2$. The equation of motion for the 8.0 kg cart reads $F^{\text{c}}_{\text{p}8x} + k\Delta x = m_8 a_{8x} = m_8 a_{2x}$, where we have equated the accelerations of the two carts in the last step. Hence $F^{\text{c}}_{\text{p}8x} = m_8 a_{2x} - k\Delta x = (8.0\ \text{kg})(15\ \text{m/s}^2) - (300\ \text{N/m})(-0.100\ \text{m}) = 1.5\times10^2\ \text{N}$. You exerted a force of $1.5\times10^2\ \text{N}$ in the direction of your pull. (*b*) Now the equation of motion for the 8.0 kg cart reads $-k\Delta x = m_8 a_{8x} \Rightarrow a_{8x} = \dfrac{-k\Delta x}{m_8} = \dfrac{-(300\ \text{N/m})(-0.100\ \text{m})}{(8.0\ \text{kg})} = 3.75\ \text{m/s}^2$. and the equation of motion for the 2.0 kg cart is $F^{\text{c}}_{\text{p}2x} + k\Delta x = m_2 a_{2x} = m_2 a_{8x}$. This yields a pulling force of $F^{\text{c}}_{\text{p}2x} = m_2 a_{8x} - k\Delta x = (2.0\ \text{kg})(3.75\ \text{m/s}^2) - (300\ \text{N/m})(-0.100\ \text{m}) = 38\ \text{N}$. So in this case you must exert a force of 38 N in the direction of your pull.

8.77. (*a*) There is zero center-of-mass acceleration, as no external forces act on the system made up of the car and truck. (*b*) Call the direction the car is facing the $+x$ direction. $\sum F_{\text{car}\,x} = m_{\text{car}} a_{\text{car}\,x} = (1000\ \text{kg})(1.2\ \text{m/s}^2) = 1200\ \text{N}$, so $\sum \vec{F}_{\text{car}} = 1200\ \text{N}$ in the direction the car is facing. For the truck $\sum F_{\text{truck}\,x} = -\sum F_{\text{car}\,x}$, so $\sum \vec{F}_{\text{truck}} = 1200\ \text{N}$ in the direction the truck is facing. (*c*) $\sum F_{\text{truck}\,x} = m_{\text{truck}} a_{\text{truck}\,x} \Rightarrow a_{\text{truck}\,x} = \dfrac{1}{m_{\text{truck}}} \sum F_{\text{truck}\,x} = \dfrac{(-1200\ \text{N})}{(1500\ \text{kg})} = -0.80\ \text{m/s}^2$. So $\vec{a}_{\text{truck}} = 0.8\ \text{m/s}^2$ in the direction the truck is facing.

8.78. (*a*) Call the direction of the snowmobile's initial motion the $+x$ direction. We use the kinematic equation $v^2_{\text{snow}\,x,\text{f}} = v^2_{\text{snow}\,x,\text{i}} + 2a_{\text{snow}\,x}\Delta y_{\text{snow}} \Rightarrow a_{\text{snow}\,x} = \dfrac{v^2_{\text{snow}\,x,\text{f}} - v^2_{\text{snow}\,x,\text{i}}}{2\Delta y_{\text{snow}}} = \dfrac{(0) - (10\ \text{m/s})^2}{2(10\ \text{m})} = -5.0\ \text{m/s}^2$, so $\vec{a}_{\text{snow}} = 5.0\ \text{m/s}^2$ opposite the original direction of motion. (*b*) The frictional force stopping the snowmobile is the only force acting in the horizontal direction on the system of the snowmobile and skier. Thus $\sum F_{\text{snow+skier}\,x} = (m_{\text{snow}} + m_{\text{skier}}) a_{\text{snow+skier}\,x}$, so that $a_{\text{snow+skier}\,x} = \dfrac{m_{\text{snow}} a_{\text{snow}\,x}}{(m_{\text{snow}} + m_{\text{skier}})} = \dfrac{(450\ \text{kg})(5.0\ \text{m/s}^2)}{(450\ \text{kg} + (70\ \text{kg}))} = 4.3\ \text{m/s}^2$. So $\vec{a}_{\text{snow+skier}} = 4.3\ \text{m/s}^2$ opposite the original direction of motion.

8.79. (*a*) Let the push on the 4.0 kg block be in the $+x$ direction. The equation of motion for the two-block system is $\sum F_{2\text{ and }4\,x} = F^{\text{c}}_{\text{p}4x} + F^{\text{c}}_{\text{p}2x} = m_{2\text{ and }4} a_{2\text{ and }4\,x}$, such that $a_{2\text{ and }4\,x} = \dfrac{F^{\text{c}}_{\text{p}4x} + F^{\text{c}}_{\text{p}2x}}{m_{2\text{ and }4}} = \dfrac{(50\ \text{N}) + (-20\ \text{N})}{((2.0\ \text{kg}) + (4.0\ \text{kg}))} = 5.0\ \text{m/s}^2$. So $\vec{a}_{2\text{ and }4} = 5.0\ \text{m/s}^2$ in the direction of the 50 N force. (*b*) First note that the center of mass of the system of two blocks is moving at $2.5\ \text{m/s}$ in the $+x$ direction after the forces are exerted for 0.50 s. But in the Earth reference frame, the 4.0 kg block is moving at a speed of 5.0 m/s in the $-x$ direction (away from the 2.0 kg block). This means that the 4.0 kg block is moving at 7.5 m/s in the $-x$ direction in the center of mass frame. That is, the 4.0 kg block is moving at $-7.5\ \text{m/s}\ \hat{\imath}$ relative to the center of mass. The only force that can make either block move in the center of mass reference frame is the internal spring force, which is the same in magnitude on either block. This means that in the center of mass reference frame $m_2 v_{2\text{ cm}\,x} = -m_4 v_{4\text{ cm}\,x}$. Thus $v_{2\text{ cm}\,x} = -\dfrac{m_4}{m_2} v_{4\text{ cm}\,x} = -\dfrac{(4.0\ \text{kg})}{(2.0\ \text{kg})}(-7.5\ \text{m/s}) = 15\ \text{m/s}$. This is the x component of the velocity in the center of mass reference frame, which is itself moving relative to Earth with $v_{\text{cm E}\,x} = 2.5\ \text{m/s}$. So $v_{2\text{ E}\,x} = v_{\text{cm E}\,x} + v_{2\text{ cm}\,x} = (2.5\ \text{m/s}) + (15\ \text{m/s}) = 18\ \text{m/s}$. So the velocity of the 2.0 kg block is 18 m/s away from the 4.0-kg block.

8.80. $\sum F_{\text{p}x} = F^{\text{c}}_{\text{hp}x} + F^{\text{f}}_{\text{bp}x} = m_{\text{p}} a_{\text{p}\,x} \Rightarrow a_{\text{p}\,x} = \dfrac{F^{\text{c}}_{\text{hp}x} + F^{\text{f}}_{\text{bp}x}}{m_{\text{p}}} = \dfrac{(10\ \text{N}) + (-6.0\ \text{N})}{(0.70\ \text{kg})} = 5.7\ \text{m/s}^2$. So $\vec{a}_{\text{p}} = 5.7\ \text{m/s}^2$ in the direction of your push.

8.81 As you accelerate the child upward, the magnitude of the upward force you exert on her is greater than the magnitude F^{G}_{Ec} of the gravitational force exerted by Earth on her. This means that the magnitude of the downward force she exerts on you is also greater than F^{G}_{Ec}. Hence, as you accelerate her upward, the scale reading increases for an instant or two. Once she is on your shoulders, however, the scale reading is the same as when the two of you stood side by side.

8.82. (*a*) Call vertically upward the $+x$ direction. The scale reading initially (with no spoon pushing) would just be $F^{c}_{smx} = -F^{G}_{Emx} = m_{m}g = (2.0\text{ kg})(9.8\text{ m/s}^2) = 19.6\text{ N}$. So, when the scale reading increases by 5%, it means the scale is exerting a force on the mashed potatoes equal to 20.6 N upward. So, while pushing with the spoon, the equation of motion of the center of mass of the mashed potatoes is $\sum F_{cmx} = F^{c}_{\text{scale m }x} + F^{c}_{\text{spoon m }x} + F^{G}_{Emx} = m_{m}a_{\text{cm }x} \Rightarrow a_{\text{cm }x} = \dfrac{F^{c}_{\text{scale m }x} + F^{c}_{\text{spoon m }x} + F^{G}_{Emx}}{m_{m}} = \dfrac{(20.6\text{ N}) + (-3.0\text{ N}) + (-19.6\text{ N})}{(2.0\text{ kg})} = -1.0\text{ m/s}^2$. So $\vec{a}_{cm} = 1.0\text{ m/s}^2$ downward. (*b*) As you push down, the dome of mashed potatoes becomes deformed. You are able to cause a depression in the peak of the dome, thus moving the center of mass downward. Since there is an acceleration of the center of mass, the vector sum of all forces is not zero. Eventually, the spoon would pass through the mashed potatoes and make contact with the scale. At that point, you would basically be pushing on the scale, and the reading on the scale would increase by 15%, instead of the initial 5%.

8.83. (*a*) The equation of motion of the entire system of three vehicles (denoted by a "3" subscript) is just $\sum F_{3x} = F = m_3 a_{3\,x} = (m_{\text{car}} + m_{\text{truck}} + m_{\text{trailer}})a_{3\,x}$. Thus $\vec{a}_3 = \vec{F}/3000\text{ kg}$ or $\vec{a}_3 = F/3000\text{ kg}$ toward the winch. (*b*) For any given acceleration, the rope that experiences the greatest tension is the rope that is pulling the greatest inertia. The rope attaching the truck to the car must accelerate both the car and the trailer. Thus when the tension in that rope is maximized, the equation of motion for the two vehicle system (car and trailer, denoted by a "2" subscript) is $\sum F_{2x} = \mathcal{T} = m_2 a_{2x} \Rightarrow (a_{2x})_{\max} = \dfrac{(\mathcal{T})_{\max}}{m_{\text{car}} + m_{\text{trailer}}} = \dfrac{(2000\text{ N})}{(1000\text{ kg}) + (500\text{ kg})} = 1.33\text{ m/s}^2$. In order to accelerate the entire three vehicle system at this rate, we must apply a pulling force equal to $(F^{c}_{\text{c truck }x})_{\max} = (m_3 a_{3x})_{\max} = m_3(a_{2x})_{\max} = (3000\text{ kg})(1.33\text{ m/s}^2) = 4.00\times10^3\text{ N}$. So $(\vec{F}^{c}_{\text{c truck}})_{\max} = 4.00\times10^3\text{ N}$ toward the winch. (*c*) Now the rope with the largest tension is the rope connecting the trailer to the car, which is responsible for accelerating both the car and the truck. The car and truck will now make up our system of two vehicles (subscript "2"). We proceed as before, finding the acceleration of the system when the tension is maximized: $\sum F_{2x} = \mathcal{T} = m_2 a_{2x} \Rightarrow (a_{2x})_{\max} = \dfrac{(\mathcal{T})_{\max}}{m_{\text{car}} + m_{\text{truck}}} = \dfrac{(2000\text{ N})}{(1000\text{ kg}) + (1500\text{ kg})} = 0.800\text{ m/s}^2$. In order to accelerate the entire three vehicle system at this rate, we must apply a pulling force equal to $(F^{c}_{\text{c truck }x})_{\max} = (m_3 a_{3x})_{\max} = m_3(a_{2x})_{\max} = (3000\text{ kg})(0.800\text{ m/s}^2) = 2.40\times10^3\text{ N}$. So $(\vec{F}^{c}_{\text{c truck}})_{\max} = 2.40\times10^3\text{ N}$ toward the winch.

8.84. Call the tension in the rope connecting the tugboat to the coal barge $\mathcal{T}_{\text{coal}}$, and that in the rope connecting the tugboat to the iron barge $\mathcal{T}_{\text{iron}}$. Call the direction in which the boat is accelerating the $+x$ direction. (*a*) Treat the two barges as one system, initially. Then the equation of motion for this system of two barges (subscript "2") is $\sum F_{2x} = F^{d}_{\text{w iron }x} + F^{d}_{\text{w coal }x} + \mathcal{T}_{\text{coal}} = m_2 a_{2x}$. Thus $\mathcal{T}_{\text{coal}} = m_2 a_{2x} - F^{d}_{\text{w iron }x} - F^{d}_{\text{w coal }x} = ((2.0\times10^5\text{ kg}) + (3.0\times10^5\text{ kg}))(0.40\text{ m/s}^2) - (1.0\times10^4\text{N}) - (-8.0\times10^3\text{N}) = 2.2\times10^5\text{ N}$. (*b*) The equation of motion for the barge carrying pig iron is $\sum F_{\text{iron }x} = \mathcal{T}_{\text{iron}} - F^{d}_{\text{w iron }x} = m_{\text{iron}}a_{\text{iron }x}$, so $\mathcal{T}_{\text{iron}} = m_{\text{iron}}a_{\text{iron }x} + F^{d}_{\text{w iron }x} = (3.0\times10^5\text{ kg})(0.40\text{ m/s}^2) + (1.0\times10^4\text{ N}) = 1.3\times10^5\text{ N}$. (*c*) Now that the tugboat is connected directly to the barge carrying pig iron, the rope connecting them is responsible for overcoming drag and accelerating the system of two barges. Thus this rope does exactly the same thing as the rope connecting the tugboat to the coal barge in part (*a*). Thus $\mathcal{T}_{\text{iron}} = 2.2\times10^5\text{ N}$. The equation motion for the coal barge reads $\sum F_{\text{coal }x} = \mathcal{T}_{\text{coal}} - F^{d}_{\text{w coal }x} = m_{\text{coal}}a_{\text{coal }x}$, so $\mathcal{T}_{\text{coal}} = m_{\text{coal}}a_{\text{coal }x} + F^{d}_{\text{w coal }x} = (2.0\times10^5\text{ kg})(0.40\text{ m/s}^2) + (8.0\times10^3\text{ N}) = 8.8\times10^4\text{ N}$.

8.85. (*a*) The tractor must exert sufficient force to overcome three frictional forces each with a magnitude of 900 N. Thus the tractor must exert at least $2.7\times10^3\ \text{N}$ forward. (*b*) Let the direction of the tractor's pull be the $+x$ direction. The rope from the first trailer to the second cannot have more than 2000 N of tension. The equation of motion for the system made up of the second and third trailers reads $\sum F_{\text{2nd and 3rd } x} = \mathcal{T} + 2F^{\text{f}}_{\text{mud t } x} = (m_2 + m_3)a_{\text{2nd and 3rd } x}$. So the maximum acceleration that the second and third trailers can have is $a_{\text{2nd and 3rd } x} = \dfrac{(\mathcal{T})_{\max} + 2F^{\text{f}}_{\text{mud t } x}}{(m_2+m_3)} = \dfrac{(2000\ \text{N}) + 2(-900\,\text{N})}{((500\ \text{kg}) + (500\ \text{kg}))} = 0.20\ \text{m/s}^2$. This must be the same as the acceleration of the entire system of three trailers, because the ropes remain taut at all times. Using this acceleration, we write the equation of motion for all three trailers $\sum F_{3\,x} = F^{\text{c}}_{\text{t}3x} + 3F^{\text{f}}_{\text{mud t } x} = (m_1 + m_2 + m_3)a_{3\,x}$, and find that $(F^{\text{c}}_{\text{t}3x})_{\max} = (m_1+m_2+m_3)(a_{3\,x})_{\max} - 3F^{\text{f}}_{\text{mud t } x} = (1500\ \text{kg})(0.20\ \text{m/s}^2) - 3(-900\ \text{N}) = 3.0\times10^3\ \text{N}$. So the maximum force is $3.0\times10^3\ \text{N}$ toward the tractor.

8.86. Note that the acceleration of the system of four cars will be $F/4m$ forward. The first coupling between the engine and the first car exerts enough force to give the entire system of four cars that acceleration. So the tension force in that coupling must be $\mathcal{T}_{\text{engine car1 } x} = (4m)a = (4m)F/(4m) = F$. Similarly, the coupling between the first and second car exerts enough force on the remaining three cars to accelerate them along with the engine. Thus $\mathcal{T}_{\text{car1 car2 } x} = (3m)a = (3m)F/(4m) = 3F/4$. Proceeding in a similar fashion, we find $\mathcal{T}_{\text{engine car1 } x} = F$, $\mathcal{T}_{\text{car1 car2 } x} = 3F/4$, $\mathcal{T}_{\text{car2 car3 } x} = F/2$, and $\mathcal{T}_{\text{car3 car4 } x} = F/4$.

8.87. (*a*) The equation of motion for the system of three carts reads $\sum F_{3x} = F^{\text{c}}_{\text{pc}x} = m_3 a_{\text{cm } x} \Rightarrow a_{\text{cm } x} = \dfrac{F^{\text{c}}_{\text{pc}x}}{m_3} = \dfrac{(10\ \text{N})}{(40\ \text{kg})} = 0.25\ \text{m/s}^2$. So $\vec{a}_{\text{cm}} = 0.25\ \text{m/s}^2$ in the direction of the push. (*b*) At the instant you begin to push, the spring has not yet had a chance to compress, so no forces are exerted in the horizontal direction on the 20 kg block, nor on the blue 10 kg block. The force of your push is exerted on the 10 kg red block, such that $a_{\text{10 kg red } x} = \dfrac{F^{\text{c}}_{\text{p 10 kg red}}}{m_{\text{10 kg red}}} = \dfrac{(10\ \text{N})}{(10\ \text{kg})} = 1.0\ \text{m/s}^2$. So $\vec{a}_{\text{10 kg red}} = 1.0\ \text{m/s}^2$ in the direction of the push and $\vec{a}_{\text{20 kg}} = \vec{a}_{\text{10 kg blue}} = \vec{0}$.

(*c*) $\sum F_{\text{10 kg red } x} = F^{\text{c}}_{\text{h 10 kg red}} + F_{\text{spring 10 kg red}} = F^{\text{c}}_{\text{h 10 kg red}} + -k\Delta x = (10\ \text{N}) - (60\ \text{N/m})(0.100\ \text{m}) = 4.0\ \text{N}$, so $\sum \vec{F}_{\text{10 kg red}} = 4.0\ \text{N}$ in the direction of the push. Since the spring exerts a force of $6.0\ \text{N}$ in the x direction on the 10 kg red cart, we know it will exert a force of $6.0\ \text{N}$ in the $+x$ direction on the 20 kg block. But this may not be the only force exerted on the 20 kg block. Because the 10 kg blue block is in contact with the 20 kg block, it may also exert a force on the 20 kg block. We know that the spring force is the only one exerted on the system of the 10 kg blue block, and the 20 kg block. Let us call this system 2. Then the equation of motion for system 2 reads $\sum F_{2\,x} = F_{\text{spring 2}} = -k\Delta x = m_2 a_{2x}$, such that $a_{2x} = \dfrac{-k\Delta x}{m_2} = \dfrac{-(60\ \text{N/m})(-0.100\ \text{m})}{(30\ \text{kg})} = 0.20\ \text{m/s}^2$. Now that we know the acceleration of each block, we can write $\sum \vec{F}_{\text{20 kg}} = m_{\text{20 kg}}\vec{a}_{\text{20 kg}} = (20\text{kg})(0.20\ \text{m/s})\ \text{forward} = 4.0\ \text{N}$ in the direction of the push, and similarly $\sum \vec{F}_{\text{10 kg blue}} = 2.0\ \text{N}$ all in the direction of your push.

8.88. The two forces in the interaction pair are equal, but they are acting on different objects (also with different inertias), and the two objects have a totally different vector sum of all forces. Frictional forces are able to keep the child's shoes from slipping, but frictional forces are not able to keep the toy's wheels from rolling.

8.89. For a car to stop from a given speed requires a fixed momentum change and therefore a fixed impulse. If we approximate the force magnitude as being constant in time, we can write $F = \Delta p/\Delta t$. If we allow a collision to occur over a greater distance (and therefore a longer time interval) the force magnitude at any given instant is smaller than the force magnitude when the car must stop in a shorter distance. The crumple zone allows more of the car to be destroyed, but it slows the passengers down over a greater distance, thereby exerting smaller-magnitude forces on them.

8.90. (*a*) Call vertically upward the $+x$ direction. We write the equation of motion for the boy $\sum F_{\text{b}x} = F^{\text{c}}_{\text{sb}x} + F^{G}_{\text{Eb}x} = m_{\text{b}}a_{\text{b}x}$. Since the velocity is constant, this yields $F^{\text{c}}_{\text{sb}x} = -F^{G}_{\text{Eb}x} = mg = (60\ \text{kg})(9.8\ \text{m/s}^2) = 5.9\times10^2\ \text{N}$. (*b*) The elevator is moving downward but slowing, meaning the acceleration is now $+2.0\ \text{m/s}^2$ in the $+x$ direction. So now the equation of motion yields $F^{\text{c}}_{\text{sb}x} = m_{\text{b}}a_{\text{b}x} + m_{\text{b}}g = (60\ \text{kg})((2.0\,\text{m/s}^2) + (9.8\,\text{m/s}^2)) = 7.1\times10^2\ \text{N}$. (*c*) The elevator is moving downward but speeding up, meaning the acceleration is now $2.0\ \text{m/s}^2$ in the $-x$ direction. So now the equation of motion yields $F^{\text{c}}_{\text{sb}x} = m_{\text{b}}a_{\text{b}x} + m_{\text{b}}g = (60\ \text{kg})((-2.0\,\text{m/s}^2) + (9.8\,\text{m/s}^2)) = 4.7\times10^2\ \text{N}$.

8.91. (*a*) Call vertically upward the $+x$ direction. We write the equation of motion for the hanging object: $\sum F_{\text{h}x} = \mathcal{T} + F^{G}_{\text{Eh}x} = \mathcal{T} - m_{\text{h}}g = m_{\text{h}}a_{\text{h}x}$. So $(m_{\text{h}})_{\max} = \dfrac{(\mathcal{T})_{\max}}{(a_{\text{h}x} + g)} = \dfrac{(45\ \text{N})}{((4.0\ \text{m/s}^2) + (9.8\ \text{m/s}^2))} = 3.3\ \text{kg}$. (*b*) The largest tension force is required when the elevator is accelerating upwards. This is the case when the elevator is moving upward and increasing its speed, or moving downward and decreasing its speed.

8.92. Call vertically upward the $+x$ direction. The equation of motion reads $\sum F_{\text{s}x} = \mathcal{T} + F^{G}_{\text{Es}x} = m_{\text{s}}a_{\text{s}x}$. When the tension is at its maximum magnitude, this equation of motion yields an x component of acceleration $a_{\text{s}x} = \dfrac{\mathcal{T} - mg}{m_{\text{s}}} = \dfrac{(42\ \text{kg})(9.8\ \text{m/s}^2) - (45\ \text{kg})(9.8\ \text{m/s}^2)}{(45\ \text{kg})} = -0.65\ \text{m/s}^2$. The acceleration downward must be larger in magnitude than $0.65\ \text{m/s}^2$ to ensure that the line does not break.

8.93. (*a*) Call the 10 kg block block 1, and the 20 kg block block 2. The equation of motion for the entire system of both blocks is $\sum F_{\text{both}\,x} = F^{\text{c}}_{\text{p}1x} = m_{\text{both}}a_{\text{cm}\,x} \Rightarrow a_{\text{cm}\,x} = \dfrac{F^{\text{c}}_{\text{p}1x}}{m_{\text{both}}} = \dfrac{(10\ \text{N})}{(30\ \text{m/s}^2)} = 0.33\ \text{m/s}^2$. So the acceleration of the center of mass is $0.33\ \text{m/s}^2$ in the direction of the push. (*b*) At the instant we begin to push, the spring has not yet had time to compress, so no force is exerted in the horizontal direction on the 20 kg block, nor does the spring push back on the 10 kg block. Thus $\vec{a}_{10\ \text{kg}} = 1.0\ \text{m/s}^2$ in the direction of the push, and $\vec{a}_{20\ \text{kg}} = \vec{0}$. (*c*) When the spring has its maximum compression, the two carts move together with the same acceleration. We know from part (*a*) that this acceleration must be $0.33\ \text{m/s}^2$ in the direction of the push.

8.94. (*a*) Since the system is not accelerating (being slowly lowered), the spring force is exactly supporting the block against the pull of gravity. $\vec{F}^{\text{c}}_{\text{spring b}} = -\vec{F}^{G}_{\text{Eb}} = mg = (5.0\ \text{kg})(9.8\ \text{m/s}^2) = 49\ \text{N}$. (*b*) The scale reading only 40 N means that the lower spring is exerting an upward force of $9.0\ \text{N}$. We know $F^{\text{c}}_{\text{lower b}} = k_{\text{lower}}|\Delta x| \Rightarrow k_{\text{lower}} = \dfrac{F^{\text{c}}_{\text{lower b}}}{|\Delta x|} = \dfrac{(9.0\ \text{N})}{(0.030\ \text{m})} = 3.0\times10^2\ \text{N/m}$. (*c*) At this point the equation of motion for the block would read $\sum F_{\text{b}x} = F^{\text{c}}_{\text{lower b}\,x} + F^{G}_{\text{Eb}x} = m_{\text{b}}a_{\text{b}\,x}$, such that $\Delta x = \dfrac{m_{\text{b}}g}{-k} = \dfrac{(5.0\ \text{kg})(9.8\ \text{m/s}^2)}{-(3.0\times10^2\ \text{N/m})} = -0.16\ \text{m}$. So the spring must be compressed by $1.6\times10^2\ \text{mm}$.

8.95. You realize that you can augment your lifting abilities even further by wrapping the long rope around the pulleys twice. You attach one pulley to the beam and one pulley to the load. You first wrap the rope around the lower pulley and up over the suspended pulley as shown in Worked Problem 8.7. In that case there were two lengths of rope connecting the two pulleys, such that if you pulled your end of the rope downward a distance d the heavy object would only be lifted a distance $d/2$. But the tension in the rope acts twice in lifting the load, which enables you to lift a greater weight than you normally could. If you now wrap the rope down around the second wheel on the lower pulley and up over the second wheel on the suspended pulley, you will have four lengths of rope connecting the two pulleys. Now, when you apply a tension force $\mathcal{T}$, the equation of motion for the hanging load is

$\sum F_{lx} = 4\mathcal{T} - F^G_{Ehx} = m_h a_{hx}$ where m_h is the inertia of everything hanging: the load and the light pulley. You require an upward acceleration with magnitude $g/8$, such that you have to pull with a force equal to $\mathcal{T} = \frac{1}{4}(m_\ell + m_p)(a_{hx} + g) = \frac{9g}{32}(m_\ell + m_p)$.

8.96. If you were just holding a fish steady on the line, you could hold a fish with an inertia of $m = \frac{(\mathcal{T})_{max}}{g} = \frac{(22\text{ N})}{(9.8\text{ m/s}^2)} = 2.2\text{ kg}$. But you need to give the fish some upward acceleration to get it out of the water. In that case, the equation of motion for the fish reads $\sum F_{\text{fish } x} = \mathcal{T} - mg = ma_x$ or $(m)_{max} = \frac{(\mathcal{T})_{max}}{(a_x + g)} = \frac{(22\text{ N})_{max}}{((1.0\text{ m/s}^2) + (9.8\text{ m/s}^2))} =$ 2.0 kg. So, you probably misjudged the maximum inertia of the fish by 0.20 kg. Depending on how the line is tested, there could also be issues with a "Buoyant Force", discussed in a later chapter.

8.97. There are at least two possibilities: (1) Use your arm strength to accelerate yourself up the free end of the rope. The tension in the rope would need to be at least $\mathcal{T} = m_{\text{friend}}g = (70\text{ kg})(9.8\text{ m/s}^2) = 6.9\times10^2\text{ N}$. If you pull with sufficient force to cause this tension, you will accelerate upward at $a_{\text{you}} = \frac{\mathcal{T} - m_{\text{you}}g}{m_{\text{you}}} =$ $\frac{(686\text{ N}) - (63\text{ kg})(9.8\text{ m/s}^2)}{(63\text{ kg})} = 1.1\text{ m/s}^2$. So, if you accelerate yourself upward at greater than 1.1 m/s^2, the tension in the rope will lift your friend off the ground. (2) Tie yourself to the same side of the rope as your friend, and pull on the other (free) end of the rope, lifting both of you with your amazing arm strength and the mechanical advantage of two rope segments to exert tension forces on your combined inertia.

8.98. (*a*) F_0 is the force at time $t = 0$. It is also the maximum force. (*b*) The momentum is given by $\int_0^\tau F_0 e^{-t/\tau} dt =$ $-F_0\tau(e^{-t/\tau})\Big|_0^\tau = F_0\tau\left(1 - \frac{1}{e}\right)$. So $\vec{p}_f = F_0\tau\left(1 - \frac{1}{e}\right)$ in the direction the dart is blown. (*c*) By exactly the same process as in part (*b*) we find $\vec{p}_f = F_0\tau\left(1 - \frac{1}{e^5}\right)$ in the direction the dart is blown. (*d*) The results from (*b*) can be generalized to $p(t) = F_0\tau(1 - e^{-t/\tau})$. In the limit as $t \to \infty$, $\vec{p}(t) \to F_0\tau$ in the direction of motion. (*e*) $p(t) = F_0\tau(1 - e^{-t/\tau}) =$ $(0.95)F_0\tau$, so $e^{-t/\tau} = 0.05$ or $t = -\tau\ln(0.05) = (0.50\times10^{-3}\text{ s})\ln(0.05) = 1.5\text{ ms}$.

8.99. The acceleration of the rocket will increase during this time, even through the force acting on the rocket remains constant. In this case $\vec{a} = \vec{F}_{\text{thrust}}/m_{\text{rocket}}$, but the inertia is the inertia of everything attached to the rocket. This includes the inertia of the payload and the inertia of the fuel itself. Since the fuel is being expelled, the total inertia is decreasing. This makes the effect of the constant force (the acceleration) increase. As an example, suppose that when the fuel tanks are full, the inertia of the fuel is the same as the inertia of the payload. In that case, $\vec{a}_i = \vec{F}_{\text{thrust, i}}/1.6m_{\text{payload}}$, and $\vec{a}_f = \vec{F}_{\text{thrust, f}}/1.1m_{\text{payload}}$. So, with that assumption about the inertias, the acceleration would increase from the initial to final state by about 50%.

9

WORK

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

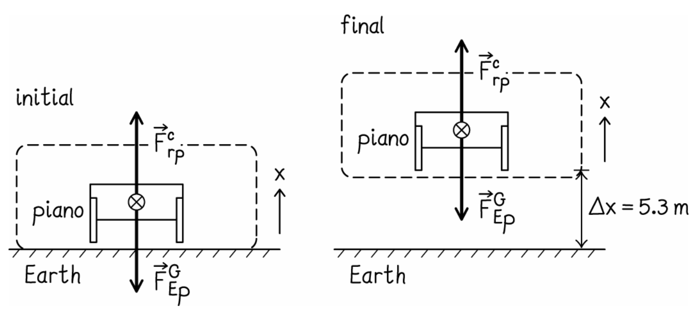
Developing a Feel

1. 10^2 J **2.** 10^2 J **3.** 10^7 J **4.** 10^2 N/m **5.** 10^3 W **6.** 10^4 W **7.** 10^3 W **8.** 10^{11} J **9.** 10^{20} J

Guided Problems

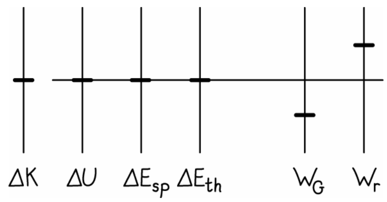
9.2 Delivering a piano

1. Getting Started We start by choosing the appropriate system. Clearly we must include the piano. Since we are asked to determine the work done by gravity, we must treat gravity as an external force doing work on our system. For this reason, we do not include Earth in our system. The system (consisting of the piano only) is shown in its initial and final state:



Ignoring rotation for the time being, the positions, velocities, and accelerations of the center of mass and of any piece of the piano should be the same. The piano is a rigid object.

Since the piano is at rest initially and finally, there is no change in kinetic energy. Since Earth is not in our system, there is not gravitational potential energy (gravity is treated as an external force that does work on our system). There are no springs nor any conversion to thermal energy. All that remains is work done on the system. Clearly the rope pulls in the direction of motion and therefore does positive work, whereas the force of gravity is exerted by Earth on the piano in the direction opposite the motion, and hence does negative work. Our energy diagram is thus



For both external forces (gravity and the force exerted by the rope) the force displacement is 5.3 m.

2. Devise Plan Any change in the energy of the system can only come from work done on the system by external forces. The equation that describes this is $\Delta E = W_{\text{ext}}$. In the energy diagram above, we have not included any friction in the pulley. Friction in pulleys is very often non-negligible, but we will consider here that the pulley is brand new, and very well lubricated so that energy converted to heat through friction in the pulleys is negligible compared to other energies in the problem.

From the energy diagram above, we know that the work done by all external force sums to zero, which allows us to equate the magnitudes of the work done by the rope and the work done by gravity. Since we know expressions for the work done by each force, determining them should be no problem. For part (*c*) we simply divide the work done by rope by the amount of time required for the work to be done to determine the average power.

3. Execute Plan The work done by gravity is

$$W_G = F^G_{\text{Ep}x}\Delta x = (-mg)\Delta x = -(150\text{ kg})(9.8\text{ m/s}^2)(5.3\text{ m}) = -7.8\text{ kJ}$$

We know from the energy diagram and arguments above the work done the rope is equal in magnitude to the work done by gravity, but opposite in sign. Thus $W_{\text{r}} = 7.8$ kJ.

These are the answers to parts (*a*) and (*b*). For part (*c*), we use

$$P_{\text{av}} = \frac{W_{\text{r}}}{\Delta t} = \frac{(7.79\times10^3\text{ J})}{(60\text{ s})} = 1.3\times10^2\text{ W}$$

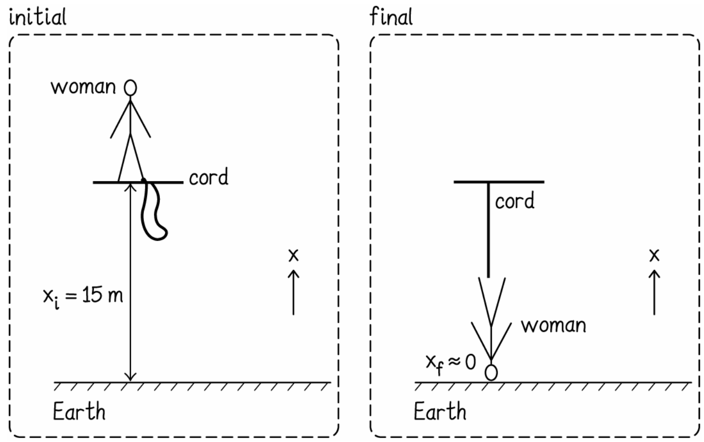
4. Evaluate Result For a person to walk up the stairs in the same time interval, they would need to do an amount of work that depends on their inertia. If we assume that a reasonable inertia for a human is 75 kg, we obtain

$$P_{\text{av}} = \frac{\Delta E}{\Delta t} = \frac{mg\Delta h}{\Delta t} = \frac{(75\text{ kg})(9.8\text{ m/s}^2)(5.3\text{ m})}{(60\text{ s})} = 65\text{ W}$$

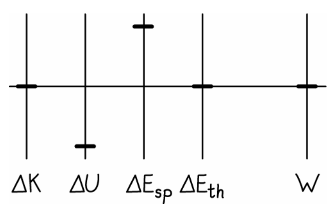
All calculations performed here have ignored friction and the efficiency with which the human body can convert chemical potential energy into mechanical work. It is reasonable that (ignoring friction) lifting a piano would require over twice the power that lifting a human would require. The power we found is reasonable for moving a piano. If it seems too low to fit your intuition, consider that friction might play a role in a real pulley, and it would certainly play a major role in trying to move a piano up stairs. Also, 150 kg is not the inertia of a large piano.

9.4 Thrill-seeking

1. Getting Started If we wish to solve the problem entirely using energy, then we must take care to include all necessary objects. For example, to treat gravity using potential energy, we must include Earth in our system. To treat the bungee cord using potential energy, we must include it in our system. The initial and final states of our system are



We will try energy methods to solve this problem. It is important to note that the woman's center of mass is initially somewhat higher than 15 m (15.9 m if we take her center of mass to be at her geometric mid-point). When her head is just above ground, this center of gravity is 0.90 m above ground. Thus, the change in vertical position is the same whether we measure from the lowest point on her body, or her center of mass. But using Earth as our zero point for gravitational potential energy, it is important to note that our system will have non-zero (though small) gravitational potential energy even in the final position. The initial and final kinetic energies are both zero. The initial gravitational potential energy is large, and the final one is small, so we expect a large negative change in gravitational potential energy. Similarly, the spring potential energy is initially zero, and is large in the final position. Thus we expect a large positive change in spring potential energy. As we have included all objects that interact with our system in the system itself, we expect no external forces will do work. Our energy diagram looks like



We have assumed that no mechanical energy is converted to thermal energy. This is not always strictly true; elastic bands heat up and they are stretched and relaxed. But we can assume that the mechanical energy converted to thermal energy is small.

2. Devise Plan For two arbitrary points 1 and 2 in the woman's descent, the system may have kinetic, gravitational potential, and spring potential energy. The total mechanical energy will be conserved, so $K_1 + U_1^G + U_1^{\text{sp}} = K_2 + U_2^G + U_2^{\text{sp}}$.

We are interested only in the initial and final cases drawn in our system diagrams above. In both of those cases, the kinetic energy is zero. Also, the initial spring potential energy is zero. Thus we can write $U_\text{i}^G = U_\text{f}^G + U_\text{f}^{\text{sp}}$, or, if x refers to the vertical position of the woman's center of mass, we can write

$$mgx_\text{i} = mgx_\text{f} + \frac{1}{2}k(\Delta x_{\text{spring}})^2 \qquad (1)$$

We must be careful in defining the quantity Δx_{spring} above. The spring potential energy depends on the stretch of the cord from its equilibrium length $\ell_0 = 5.5$ m. So Δx_{spring} is not the same as the position, nor even the distance fallen in the vertical direction. It is the total (stretched) length of the cord minus the equilibrium length of the cord: $\Delta x_{\text{spring}} = (x_\text{i} - x_\text{f} - h_{\text{woman}} - \ell_0)$, where we have corrected for the woman's height. Rewriting equation (1), we find

$$mgx_{\mathrm{i}} = mgx_{\mathrm{f}} + \frac{1}{2}k(x_{\mathrm{i}} - x_{\mathrm{f}} - h_{\mathrm{woman}} - \ell_0)^2 \tag{2}$$

The only unknown quantity in equation (2) is the spring constant k, so we have sufficient information to solve the problem.

3. Execute Plan Rearranging equation. (2) we find

$$k = \frac{2mg(x_{\mathrm{i}} - x_{\mathrm{f}})}{(x_{\mathrm{i}} - x_{\mathrm{f}} - h_{\mathrm{woman}} - \ell_0)^2} \tag{3}$$

And inserting values we obtain

$$k = \frac{2(50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})((15.9\ \mathrm{m}) - (0.90\ \mathrm{m}))}{((15.9\ \mathrm{m}) - (0.90\ \mathrm{m}) - (1.8\ \mathrm{m}) - (5.5\ \mathrm{m}))^2} = 2.5 \times 10^2\ \mathrm{N/m}$$

4. Evaluate Result Our algebraic expression in equation (3) tells us that the spring constant would need to be larger to stop a woman with a larger inertia. This makes sense. Equation (3) also shows us that a leap from a higher bridge would actually require a lower value of k. At first, this may seem contrary to experience and intuition. But we see that the gravitational potential energy in the initial state would increase linearly with the height of the bridge, and the spring potential energy stored in a cord stretching from the bridge to the ground would increase quadratically with the height. Hence, this dependence on height is reasonable. We also note that leaving all other variables unchanged and increasing the equilibrium length of the cord would mean the spring constant would have to be greater. This is also sensible, since increasing the equilibrium length decreases the "stretch" of the cord beyond its equilibrium length.

We have ignored conversion of mechanical energy to heat, and we have ignored air resistance. Of course, both of these effects do exist. But it is reasonable to treat them as small corrections. More importantly for the woman, both of these effects would reduce the mechanical energy of the system, meaning including these corrections could not increase the final spring potential energy. So, the cord could not stretch farther than what we have used in this calculation, and the woman will still be safe.

9.6 Atwood machine

1. Getting Started We expect that a faster bullet will make the block slide faster, which would compress the spring more. Because the collision is completely inelastic, the coefficient of restitution is zero. The initial kinetic energy of the bullet will be converted to kinetic energy of the bullet/block combination and a few forms of incoherent energy during the collision. The block will heat up as the bullet passes through it, bonds will be broken, and some energy (although a relatively small amount) will be converted to sound. Thus it is clear that mechanical energy of the bullet, block, and spring will not be conserved during the collision. However, we can break the problem up into two steps: the collision and the compression. The collision will occur very quickly, whereas the compression of the spring will happen more slowly. In the first very short time period, mechanical energy will not be conserved, but momentum will be. In the second, longer time interval, mechanical energy will be conserved as it is converted from kinetic to spring potential energy.

2. Devise Plan We can use conservation of momentum to determine the speed of the block with the bullet imbedded in it immediately after the collision. Let us call the direction in which the bullet is fired the $+x$ direction. We will denote the time just before impact time 1, the time just after impact time 2, and the time as the spring reaches maximum compression time 3. We use the subscript "b" for bullet and "block" for the block. Conservation of momentum tells us $p_{\mathrm{b}x,1} + p_{\mathrm{block}\,x,1} = p_{\mathrm{b}x,2} + p_{\mathrm{block}\,x,2}$, or equivalently $m_{\mathrm{b}}v_{\mathrm{b}x,1} + m_{\mathrm{block}}v_{\mathrm{block}\,x,1} = (m_{\mathrm{b}} + m_{\mathrm{block}})v_{\mathrm{block}\,x,2}$, where we have used the fact that the two objects move together with the same velocity at time 2. We also know that the block is initially at rest. Thus we can simplify the above expression to

$$m_{\mathrm{b}}v_{\mathrm{b}x,1} = (m_{\mathrm{b}} + m_{\mathrm{block}})v_{\mathrm{block}\,x,2} \tag{1}$$

Equation (1) can be used to determine the speed of the bullet and block after the collision. Then we can use energy conservation, provided that we include the bullet, block, and the spring in our system. Between times 2 and 3, there is no change in elevation. So gravitational potential energy will be the same at all times. Choosing our zero height to be at the position of the block/bullet system, there is no gravitational potential energy. At time 2 all mechanical energy is kinetic and at time 3, all mechanical energy is spring potential energy. Thus we can write

$$K_2 = U_2^{\text{spring}}$$
$$\frac{1}{2}(m_{\text{b}} + m_{\text{block}})v_{\text{block }x,2}^2 = \frac{1}{2}kd^2 \tag{2}$$

This can be used to relate the maximum compression d of the spring to the speed of the bullet and block right after the collision.

3. Execute Plan Rearranging equation (2), we can obtain

$$v_{\text{block }x,2} = \sqrt{\frac{k}{(m_{\text{b}} + m_{\text{block}})}}d \tag{3}$$

Inserting equation (3) into equation (1), we find

$$v_{\text{b}x,1} = \frac{d}{m_{\text{b}}}\sqrt{(m_{\text{b}} + m_{\text{block}})k} \tag{4}$$

which is the result we wanted.

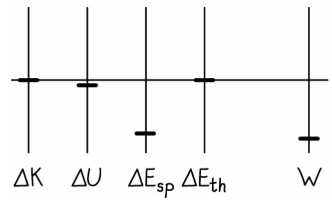
4. Evaluate Result Equation (4) above shows us that if the spring is depressed a greater distance, it means the speed of the bullet was greater. This is what we expect. It may not have been obvious from the outset that the initial speed of the bullet should depend linearly on d. But in retrospect, this is also reasonable, since the kinetic energy of the bullet depends on $v_{\text{b}x,1}^2$ and the spring potential energy depends on d^2. Similarly, a bullet with greater inertia can impart greater momentum and deliver more kinetic energy at a given speed, meaning the initial speed should decrease if the inertia of the bullet increases (but still compresses the spring the same amount d). Similarly, it is reasonable that the initial speed must increase if the same compression is achieved with either a stiffer spring or a block of greater inertia.

9.8 Darting about

1. Getting Started The first part of the question is obviously related to the kinetic energy. The second question is related to a force being exerted over some distance, which corresponds to work being done. The questions are related, in that the work done on the dart as it enters the target must be sufficient to reduce the kinetic energy of the dart to zero.

It seems like energy would be useful here. It would also be possible to use the force of the spring to determine an acceleration, and integrate to determine the final velocity. Note that it is not possible to use kinematic equations to describe the acceleration of the dart inside the dart gun, because the acceleration is not constant (the force depends on the compression of the spring). So energy seems like a simpler method. We can find the energy of the dart as it enters the target, and require that the work reduce the kinetic energy to zero.

Let us choose our system to consist of the spring, the dart, and Earth. It may not be obvious whether or not we must include Earth in our system, but we are certainly free to do so. In our system diagram, we note changes in energy from the moment of launch (time 1) to the moment just after the dart stops in the target (time 3). We call the moment just after the dart leaves the gun time 2. It is clear that the spring potential energy decreases. The kinetic energy first increases from zero as the dart is launched, but it is brought back to zero in the target, and so there is no net change in the kinetic energy of the system. It is also clear that the dart must fall some vertical distance during the flight, so there must be some decrease in gravitational potential energy. We ignore any changes in thermal energy or work done through air resistance between these two times.



From this diagram we can write $\Delta U^G + \Delta E_{\text{sp}} = W$.

2. Devise Plan Since we have included Earth in our system, there are no external forces acting on our system except the force exerted by the target on the dart. Whether or not gravity is important to the calculation depends on the level of precision one requires. For the purposes of this guided problem, and because we have not yet covered motion in two dimensions such as projectile motion, we shall assume that the change in gravitational potential energy is small enough to be neglected.

Our work energy equation becomes

$$\Delta E_{\text{sp}} = W \tag{1}$$

We could use an intermediate time (such as immediately before the dart strikes the target) to relate the speed to the work done by the target. If we did that, we would say that the work done by the target must reduce the speed, and therefore the kinetic energy to zero. But we know that the decrease in spring potential energy between times 1 and 2 was accompanied by an increase in kinetic energy, such that

$$\Delta E_{\text{sp}} = -K_2$$

or (2)

$$-\frac{1}{2}k(d)^2 = -\frac{1}{2}mv_2^2$$

where, d is the initial compression of the spring. Thus

$$-K_2 = W \tag{3}$$

The work done by the target on the dart is given by the scalar product of the average force and the displacement: $W = \vec{F}_{\text{av}} \cdot \Delta\vec{x}$. In this case, the force acts against the direction of motion of the dart, meaning $W = -F_{\text{av}}\Delta x$. Inserting this into equation (3) we have

$$\frac{1}{2}mv_2^2 = F_{\text{av}}\Delta x \tag{4}$$

We can obtain the speed of the dart immediately after launch from equation (2), and the average force from equation (4).

3. Execute Plan (*a*) Rearranging equation (2) we find

$$v_2 = \sqrt{\frac{k}{m}}d \tag{5}$$

Inserting the given values, we obtain our answer:

$$v_2 = \sqrt{\frac{(2000\ \text{N/m})}{(0.035\ \text{kg})}}(0.025\ \text{m}) = 6.0\ \text{m/s}$$

(*b*) Rearranging equation (4) yields

$$F_{\text{av}} = \frac{mv_2^2}{2\Delta x} \tag{6}$$

and inserting the above value for the speed of the dart, we have

$$F_{\text{av}} = \frac{(0.035\ \text{kg})(5.98\ \text{m/s})^2}{2(0.010\ \text{m})} = 63\ \text{N}$$

We are asked for the force (as opposed to the magnitude only), so we must specify: $\vec{F}_{\text{av}} = 63\ \text{N}$ opposite the direction in which the dart was fired.

4. Evaluate Result Equation (5) shows that the launch speed of the dart increases if a stiffer spring is used, or if the spring is compressed a greater distance. This makes sense. Also, if the dart has a greater inertia the launch speed will decrease.

If we rearrange equation (6) we can write the penetration depth as $\Delta x = \dfrac{mv_2^2}{2F_{\text{av}}}$. If we reinsert equation (5) for the speed, we see $\Delta x = \dfrac{kd^2}{2F_{\text{av}}}$. This means that the dart will penetrate deeper if a stiffer spring is used, or if the spring is compressed a greater distance. The result is independent of the inertia of the dart, because the dart only carries energy from the gun to the target; it is not the initial source. For a fixed speed, a dart with greater inertia should penetrate deeper. But here the speed is not fixed; a dart with greater inertia would not have reached as high a speed when fired from the gun.

Including gravity would correct the force by about 7%. Thus ignoring it still gives us a reasonable answer. But it is easy to imagine that some situations might require higher precision, meaning we would need to include the change in gravitational potential energy.

Questions and Problems

9.1. No. An impulse delivered means the system momentum changes. Assuming constant inertia, the momentum change must come from a velocity change. If the magnitude of the velocity changes, the system kinetic energy changes, which means work is done on the system. However, it may be that only the velocity *direction* changes, as for a ball attached to a string being spun at constant speed in a horizontal circle. If the system is the ball, an external force is exerted by the string on the system for some given time interval, delivering an impulse, but there is no change in the system's energy because its speed is constant. Therefore no work is done on the system.

9.2. Hitting either surface reduces the kinetic energy of your fist to zero, which means work is done on the fist to change its energy. For a given fist speed, the amount of work (force times distance) needed to reduce the speed to zero is the same regardless of which surface you hit. This work is done over a very short distance when you hit the door, and the force exerted on your fist must be large and thus painful. This work is done over a long distance when your fist sinks into the sofa cushion, and so a relatively small force is exerted and hurts less.

9.3. When you drop the brick from the greater height, the force of gravity is exerted on the brick over a greater distance, thus doing more work on the brick. When more work is done on the brick, it has more kinetic energy when it hits your foot and thus hurts more.

9.4. The work done on this object is zero.

9.5. Initially, your hand exerts an upward force on the ball. The force displacement is nonzero because the point of application of the force is at the ball and therefore moves. Hence this force does work on the ball. Once you release the ball, the only force exerted on it is the gravitational force. The point of application is again at the ball, which is moving; thus the force displacement is nonzero, and the gravitational force does work on the ball as it rises. At the top of its path the ball reverses its motion and the gravitational force again does work on it because again the force displacement is nonzero. Finally, the laundry exerts an upward force on the ball as the ball moves downward; this force does work on the ball because again the force displacement is nonzero.

9.6. We consider only work resulting from forces acting along the x axis. (*a*) For the work to be positive, the force must be parallel to the displacement. This means the object must speed up. This is the case from b to c and from d to e. (*b*) If the work is negative, the force is applied opposite the displacement, and the speed will decrease. This is the case from a to b, from c to d, and from e to f. (*c*) If the work is zero, there cannot be an increase in speed. This is the case from f to g.

9.7. No. You push down on the floor, but the floor doesn't move, and so the force displacement is zero; you do no work on the floor. You rise because your torso and hips push down on your legs, and your leg muscles push up on your torso and hips, lifting them. Your legs do positive work on your upper body.

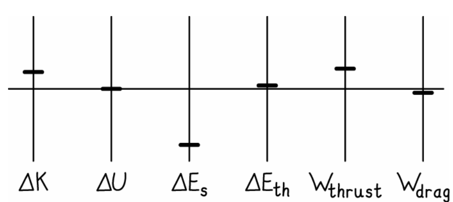
9.8. As you move down the hill, Earth exerts a downward gravitational force that has a component in your direction of motion and so does positive work on you. This work would continue increasing your kinetic energy until it became dangerous, except that you use your legs to stop yourself from tumbling down the hill. On level ground, you don't have a downward displacement over which the gravitational force could be exerted to speed you up. In walking down the hill, you must expend chemical energy to counteract the positive work done by gravity.

9.9. Both observers are right in their own reference frames.

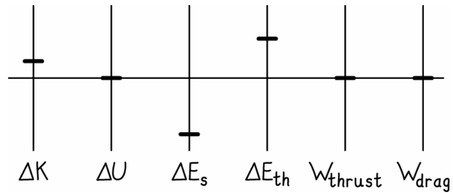
9.10. It is concave upward. If a displacement requires that positive work be done on the system, but does not increase the kinetic energy of the system, then it must increase the potential energy of the system. Because this occurs for displacement in either direction, the potential energy curve must increase in either direction.

9.11. No, work done on a system could also change its potential energy.

9.12. For the system of the jet plane alone, the drag force and the thrust are external forces that do work. The work done in each case is the product of force and force displacement, and the numbers given in the problem set a relative scale for the work by each force: force times force displacement *d* (the length of runway used). There is no potential energy to consider, so given the values of work we can compute the change in kinetic energy. However, there is a large amount of source energy converted (fuel being burned), and we have no way to determine the percentage dissipated as thermal energy, nor how much of that thermal energy is dissipated within the system and how much without. We suspect most of the dissipation is outside the system in the surrounding air, leaving an imbalance in the energy diagram (which is appropriate for a system that is not closed). But how much thermal energy is dissipated for a given amount of source energy? We must assume some energy efficiency for the jet engine. A quick check of the web suggests that roughly 30% of the source energy will go towards moving the plane, so we will assume that 70% of the source energy is dissipated. How much of that is dissipated in the plane and now much outside the system (in the air) is impossible to say, but we need that information to generate an energy diagram. To do so, we arbitrarily assume that 90% of the dissipation occurs outside the system. This is a blind assumption, with not much justification (it cannot be looked up on the web), so we add a question mark to the thermal energy change column in the energy diagram

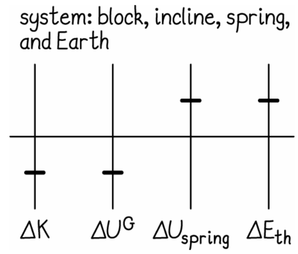


Air drag at a boundary is very much like kinetic friction at a boundary. This is why the plane alone is not a good choice of system! Perhaps a better choice of system would be the plane plus the surrounding air. This system is approximately closed as the plane taxis toward takeoff, so the energy changes should balance. Note that both forces are now internal, so that there is zero work done on this system, and that all of the thermal energy dissipation now takes place within the system. This time the diagram has no questionable bars as long as we believe that our assumption about the efficiency of the jet is reasonable.

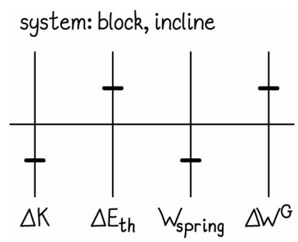


We could add other objects to this system, such as Earth, but would gain no useful information by doing so.

9.13. (*a*)



(*b*)



(*c*) The block alone should not be used as the system because friction occurs at the block-incline interface and it is not possible to know how much thermal energy produced by friction goes into the block (i.e., into the system) and how much goes into the incline (i.e., out of the system).

9.14. The only forces exerted on the block are exerted by objects in the system. Therefore no work is done by any external force, and the *W* bar should be empty. Then the number of units in the energy bars must be revised to show that the energy of the system remains constant. (Because no quantitative values are given, you can either remove one unit from the ΔU bar or add one unit to the ΔE_{th} bar.)

9.15. You push a block up an incline, with the block and incline part of a system that also includes Earth (but not you). The block increases in speed as you push (positive ΔK and positive ΔU), and some energy is lost to friction (positive ΔE_{th}). All these changes constitute work you do on the system (positive *W*).

9.16. (*a*) Yes. Positive work is done by the contact force you exert on the ball, and an equal amount of negative work is done by the gravitational force that Earth exerts on the ball. The algebraic sum of the positive and negative work is zero. (*b*) There is no change in the potential energy of the system because the ball alone is the system. In order for potential energy to have meaning, you must have two interacting objects. (*c*) Yes, positive work is done by the contact force that you hand exerts on the ball. (*d*) The potential energy of the system increases.

9.17. The downward trip takes longer because the ball's initial kinetic energy is converted to other forms, making the downward speed lower than the upward speed. System 1—ball: gravity does negative work as the ball rises but does an equal amount of positive work as the ball falls. The air does negative work on the ball both as it rises and as it falls, decreasing the system's energy. System 2—ball, Earth: as the ball rises, kinetic energy is converted to gravitational potential energy, and as the ball falls that potential energy is converted to kinetic energy. The air does negative work on the ball, making the system's final energy lower than its initial energy System 3—ball, Earth, air: the same kinetic-potential-kinetic interconversion as in system 2, plus collisions between the ball and molecules in the air cause the molecules and ball to heated up, converting some mechanical energy to thermal energy.

9.18. The following verbal description is summarized in the table below. (*a*) The spring potential energy is converted to kinetic energy in the tomato. This kinetic energy is converted to gravitational potential energy as the tomato rises, and the potential energy is converted back to kinetic energy as the tomato falls. As the tomato moves, collisions with molecules in the air cause the molecules and tomato to heat up, converting some mechanical energy to thermal energy. As the tomato lands, its kinetic energy is converted to thermal energy in it and in the pavement. (*b*) The spring potential energy is converted to kinetic energy in the tomato. Earth's gravitational force does negative work on the system as the tomato rises and does positive work on the system as the tomato falls. As the tomato moves, collisions with molecules in the air cause the molecules and tomato to heat up, converting some mechanical energy to thermal energy. As the tomato lands, its kinetic energy is converted to thermal energy in it and in the pavement. (*c*) The spring force does positive work on the system as the tomato rises through the barrel, giving the tomato kinetic energy as it is launched. Earth's gravitational force does negative work on the system as the tomato raises and positive work on the system as the tomato falls. As the tomato moves, collisions with molecules in the air cause the molecules and tomato to heat up, converting some mechanical energy to thermal energy. As the tomato lands, its kinetic energy is converted to thermal energy in it and in the pavement. (*d*) The spring force does positive work on the system as the tomato rises through the barrel, giving the tomato kinetic energy as it is launched. Earth's gravitational force does negative work on the system as the tomato rises and does positive work on the system as the tomato falls. As the tomato moves, molecules in the air do negative work on it. When the tomato lands, the pavement exerts an upward force and does negative work on it.

	system			
	(*a*) tomato, cannon, air, Earth, pavement	(*b*) tomato, cannon, air, pavement	(*c*) tomato, air, pavement	(*d*) tomato
during launch	$U_{\text{spring}} \longrightarrow K_{\text{tomato}}$	$U_{\text{spring}} \longrightarrow K_{\text{tomato}}$	$+W_{\text{by spring on sys}}$	$+W_{\text{by spring on sys}}$
tomato rises	$K_{\text{tomato}} \longrightarrow U_{\text{tomato}}$ $K_{\text{tomato}} \longrightarrow E_{\text{th, air/tomato}}$	$-W_{\text{by Earth on sys}}$ $K_{\text{tomato}} \longrightarrow E_{\text{th, air/tomato}}$	$-W_{\text{by Earth on sys}}$ $K_{\text{tomato}} \longrightarrow E_{\text{th, air/tomato}}$	$-W_{\text{by Earth on sys}}$ $K_{\text{tomato}} \longrightarrow -W_{\text{by air on sys}}$ and $E_{\text{th, tomato}}$
tomato falls	$U_{\text{tomato}} \longrightarrow K_{\text{tomato}}$ $K_{\text{tomato}} \longrightarrow E_{\text{th, air/tomato}}$	$+W_{\text{by Earth on sys}}$ $K_{\text{tomato}} \longrightarrow E_{\text{th, air/tomato}}$	$+W_{\text{by Earth on sys}}$ $K_{\text{tomato}} \longrightarrow E_{\text{th, air/tomato}}$	$+W_{\text{by Earth on sys}}$ $K_{\text{tomato}} \longrightarrow -W_{\text{by air on sys}}$ and $E_{\text{th, tomato}}$
tomato lands	$K_{\text{tomato}} \longrightarrow E_{\text{th, pavement/tomato}}$	$K_{\text{tomato}} \longrightarrow E_{\text{th, pavement/tomato}}$	$K_{\text{tomato}} \longrightarrow E_{\text{th, pavement/tomato}}$	$-W_{\text{by pavement on sys}}$ $K_{\text{tomato}} \longrightarrow E_{\text{th, tomato}}$

9.19. No, the work magnitudes are the same, but the work done is positive for one ball and negative for the other.

9.20. $W = F^G_{\text{E}dx}\Delta x = mg\Delta x = (2.0\times 10^{-6}\ \text{kg})(9.8\ \text{m/s}^2)(2000\ \text{m}) = 39\ \text{mJ}.$

9.21. Because the only force acting on the package in the horizontal direction is constant, the acceleration in the horizontal direction will also be constant. We assume that the initial speed of the package does not change. We can use a kinematic equation $a_x = \dfrac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2\Delta x}$ to see that if the stopping distance doubles, the acceleration is halved. Finally, using $t = \dfrac{v_{x,\text{f}} - v_{x,\text{i}}}{a_x}$, we see that halving the acceleration will double the time required to stop the package. Thus the time interval doubles.

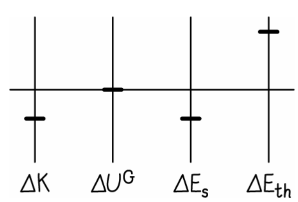
9.22. Because the only force acting on the package in the horizontal direction is constant, the acceleration in the horizontal direction will also be constant. We can use a kinematic equation $v_{x,\mathrm{i}}^2 = v_{x,\mathrm{f}}^2 - 2\Delta x a_x = -2\Delta x a_x$ to see that if the stopping distance doubles, it means the initial x component of the velocity increased by a factor of $\sqrt{2}$. Finally, using $t = \dfrac{v_{x,\mathrm{f}} - v_{x,\mathrm{i}}}{a_x} = \dfrac{-v_{x,\mathrm{i}}}{a_x}$, we see that increasing the initial x component of the velocity by $\sqrt{2}$ increases the stopping time by a factor of $\sqrt{2}$. Thus the time interval increases by a factor of $\sqrt{2}$.

9.23. (*a*) Treating the acrobat and Earth as the system, the work done by the acrobat's legs on his center of mass is equal to his change in gravitational potential energy. Call the vertical distance the acrobat moves while in contact with the ground $\Delta x_{\text{contact}}$, and call the total vertical distance that the acrobat rises until he reaches his maximum height Δx_{motion}. Thus $F^{\mathrm{c}}_{\text{legs cm}\,x,\text{av}}\Delta x_{\text{contact}} = mg\Delta x_{\text{motion}} \Rightarrow F^{\mathrm{c}}_{\text{legs cm}\,x,\text{av}} = \dfrac{mg\Delta x_{\text{motion}}}{\Delta x_{\text{contact}}} = \dfrac{(55\text{ kg})(9.8\text{ m/s}^2)((1.20\text{ m}) - (0.400\text{ m}))}{((0.900\text{ m}) - (0.400\text{ m}))}$ $= 8.6\times10^2$ N. (*b*) The maximum speed will occur just before his feet leave the ground. At this moment, we know the amount of work that has been done. So we can write $W = \Delta K + \Delta U = \dfrac{1}{2}mv_{x,\mathrm{f}}^2 + mg\Delta x_{\text{contact}} \Rightarrow v_{x,\mathrm{f}} =$ $\sqrt{\dfrac{2}{m}(W - mg\Delta x_{\text{contact}})} = \sqrt{\dfrac{2}{(55\text{ kg})}((55\text{ kg})(9.8\text{ m/s}^2)(0.8\text{ m}) - (55\text{ kg})(9.8\text{ m/s}^2)(0.500\text{ m}))} = 2.4\text{ m/s}.$

9.24. (*a*) Call vertically upward the $+x$ direction. The muscles do work on the beetle, giving it kinetic energy (and a small amount of gravitational potential energy), but as the beetle rises to its final height, all energy is converted to gravitational potential energy. Thus we can equate the work done on the beetle to that final gravitational potential energy: $W = F^{\mathrm{c}}_{\text{mb}x}\Delta x_{\text{contact}} = mg\Delta x_{\text{motion}}$. Here $\Delta x_{\text{contact}}$ is the vertical distance the beetle moves while in contact with the ground, and Δx_{motion} is the total vertical distance that the beetle rises until it reaches its maximum height. Thus $F^{\mathrm{c}}_{\text{mb}x} = mg\dfrac{\Delta x_{\text{motion}}}{\Delta x_{\text{contact}}} = (4.0\times10^{-6}\text{ kg})(9.8\text{ m/s}^2)\dfrac{(0.300\text{ m})}{(7.5\times10^{-4}\text{ m})} = 16\text{ mN}.$

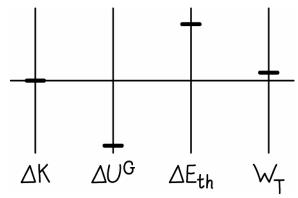
(*b*) $\sum F_{\text{b}x} = F^{\mathrm{c}}_{\text{mb}x} + F^{G}_{\text{Eb}x} = ma_{\text{b},x} \Rightarrow a_{\text{b},x} = \dfrac{F^{\mathrm{c}}_{\text{mb}x} - mg}{m} = \dfrac{(0.0157\text{ N}) - (4.0\times10^{-6}\text{ kg})(9.8\text{ m/s}^2)}{(4.0\times10^{-6}\text{ kg})} = 3.9\times10^3\text{ m/s}^2$. So the beetle's acceleration is $3.9\times10^3\text{ m/s}^2$ upward.

9.25. The system is the bicycle, the rider, Earth, the hill, and surrounding air.

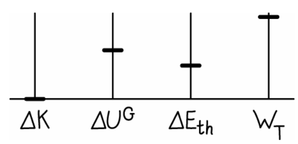


9.26. The work done by friction reduces the kinetic energy to zero, $mv_{\mathrm{f}}^2/2 = 0$, and so $|Fd| = mv_{\mathrm{i}}^2/2$, $d = mv_{\mathrm{i}}^2/2\mathrm{F}$. Thus for a constant force magnitude, increasing the initial speed from 10 m/s to 30 m/s increases the kinetic energy and thus the braking distance *d* by a factor of 9, making the new stopping distance 63 m.

9.27. (*a*) Yes (*b*) Gravitational force, tensile force exerted by rope, resistive force exerted by water, normal force exerted by bay floor. (*c*) We know that the change in gravitational potential energy is equal to $mg\Delta x = (4.0\text{ kg})(9.8\text{ m/s}^2)(-11.4\text{ m}) = -4.5\times10^2$ J. There is no change in kinetic energy, until the trap actually strikes the bottom of the bay. This means that the lost gravitational potential energy lost during the descent through air was lost because the tension in the rope did negative work on the system, and the gravitational potential energy lost during the descent through water was completely converted to thermal energy of the water and the trap. Thus $W_{\text{rope on trap}} = \Delta U^G_{\text{air}} = mg\Delta x_{\text{air}} = (4.0\text{ kg})(9.8\text{ m/s}^2)(-1.4\text{ m}) = 55\text{ J}$, and $\Delta E_{\text{th,trap/water}} = -\Delta U^G_{\text{water}} = -mg\Delta x_{\text{water}}$ $= -(4.0\text{ kg})(9.8\text{ m/s}^2)(-10\text{ m}) = 3.9\times10^2\text{ J}$. The energy diagram should approximately reflect these values.



9.28. (*a*) Yes (*b*) Gravitational force, tensile force exerted by rope, resistive force exerted by water. (*c*) Clearly the total change in gravitational potential energy is $mg\Delta x = (5.1\text{ kg})(9.8\text{ m/s}^2)(11.4\text{ m}) = 5.7\times10^2\text{ J}$. We might reasonably assume that the change in thermal energy of the trap and water will be similar to the change in thermal energy as the trap was lowered (from problem 9.27). In reality, resistive forces such as these would depend on the speed of the trap; but to a fair approximation we can say $\Delta E_{\text{th,trap/water}} = 3.9\times10^2\text{ J}$. We know all this energy came from work done by the tension in the rope, so $W_{\text{rope on trap}} = \Delta U^G + \Delta E_{\text{th,trap/water}} = (5.7\times10^2\text{ J}) + (3.9\times10^2\text{ J}) = 9.6\times10^2\text{ J}$. The energy diagram should approximately reflect these values.

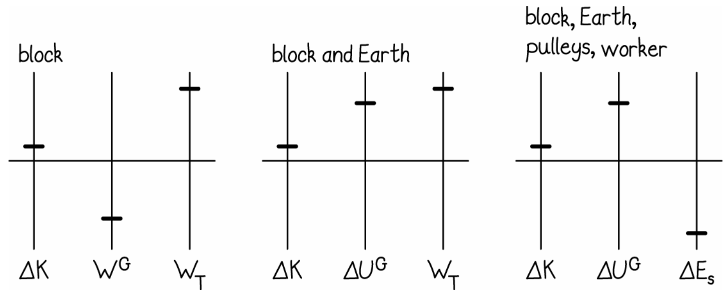


9.29. Choose a system consisting of the girl, water, slide, and Earth. No forces outside this system act on it, so we can say that the energy should be constant: $\Delta K + \Delta E_{\text{th}} + \Delta U = 0$ or $\Delta E_{\text{th}} = -\Delta K - \Delta U = K_{\text{i}} - K_{\text{f}} + U_{\text{i}} - U_{\text{f}} =$ $\frac{1}{2}mv_{\text{f}}^2 + mgh_{\text{i}} - mgh_{\text{f}} = (20\text{ kg})\left(\frac{1}{2}(1.5\text{ m/s})^2 + (9.8\text{ m/s}^2)((0.90\text{m}) - (0.95\text{ m}))\right) = 13\text{ J}$.

9.30. (*a*) The forces exerted on the block are the downward gravitational force and three upward tensile forces exerted by the ropes. Let the positive *y* direction be vertically upward. Because the block is not accelerating, $\sum F_y = -mg + 3\mathcal{T} = 0$ and $\mathcal{T} = mg/3$. (*b*) They have done the same amount of work, because the change in the block's energy is the same in both cases. In terms of forces, call the magnitude of the force exerted by the worker using the straight rope F_{s} and call the distance over which that force is exerted *d*. The magnitude of the force exerted by the worker using the block-and-tackle is only $F_{\text{s}}/3$, but because of the way the three ropes are strung, that force must be exerted over a distance 3*d*.

9.31. Call vertically upward the $+y$ direction throughout the problem. (*a*) The pulling rope makes contact with the lower pulley in two places. We momentarily treat the lower pulley and the hanging block as a single system. The sum of all forces on that system in the y direction is $\sum F_{\text{h}y} = 2\mathcal{T}_{\text{p}} + F^G_{\text{Eh}y} = 2\mathcal{T}_{\text{p}} - mg = ma_{\text{h},y}$, such that $\mathcal{T}_{\text{p}} = \dfrac{ma_{\text{h},y} + mg}{2} =$ $\dfrac{(50\text{ kg})((2.5\text{ m/s}^2) + (9.8\text{ m/s}^2))}{2} = 3.1\times10^2\text{ N}$. (*b*) Now we look at the hanging block only, and write the sum of all forces in the y direction: $\sum F_{\text{b}y} = \mathcal{T}_{\text{h}} + F^G_{\text{Eh}y} = \mathcal{T}_{\text{h}} - mg = ma_{\text{b},y}$ such that $\mathcal{T}_{\text{h}} = ma_{\text{b},y} + mg = (50\text{ kg})((2.5\text{ m/s}^2) +$ $(9.8\text{ m/s}^2)) = 6.2\times10^2\text{ N}$. (*c*) The pulling rope makes contact with the top pulley in three places. So the sum of all forces in the y direction on the top pulley is $\sum F_{\text{top }y} = 3\mathcal{T}_{\text{p}} + F^{\text{c}}_{\text{mount top }y} = 0$. So $F^{\text{c}}_{\text{mount top }y} = 3\mathcal{T}_{\text{p}} = 3(308\text{ N}) = 9.2\times10^2\text{ N}$. So the force exerted by the top mount on the top pulley is $9.2\times10^2\text{ N}$ upward. (*d*) Note that when the person pulls the rope down by an amount ℓ, the block only rises a distance $\ell/2$. So, the work done by the person is equal to $W = F^{\text{c}}_{\text{p rope }y}\Delta y_{\text{pulling rope}} = \mathcal{T}_{\text{p}}(2\Delta y_{\text{hanging block}}) = (308\text{ N})(2)(0.25\text{ m}) = 1.5\times10^2\text{ J}$. (*e*) The work done by the person in part (*d*) increases both the gravitational potential energy and the kinetic energy of the box. Thus $W = \Delta K + \Delta U$ or $\Delta K = W - mg\Delta y = (154\text{ J}) - (50\text{ kg})(9.8\text{ m/s}^2)(0.25\text{ m}) = 31\text{ J}$. (*f*) This is the same as the work done by the person: $1.5\times10^2\text{ J}$. (*g*) The contact point between the ceiling and the system never moves. So the ceiling does no work. (*h*) $W_{\text{E on block}} = F^{G}_{\text{Eb}y}\Delta y = -mg\Delta y = -(50\text{ kg})(9.8\text{ m/s}^2)(0.25\text{ m}) = -1.2\times10^2\text{ J}$.

(*i*)



9.32. (*a*) $W_{\text{by hand on gelatin}} = F^{\text{c}}_{\text{hg}x}\Delta x = (3.0\text{ N})(0.050\text{ m}) = 0.15\text{ J}$ (*b*) $\Delta K_{\text{cm}} = F^{\text{c}}_{\text{h cm }x}\Delta x_{\text{cm}} = (3.0\text{ N})(0.030\text{ m}) = 0.090\text{ J}$

9.33. (*a*) $W = F^{\text{c}}_{\text{U car }x,\text{av}}\Delta x_{\text{cm}} = \Delta K_{\text{cm}}$ so $F^{\text{c}}_{\text{U car }x,\text{av}} = -\dfrac{1}{2\Delta x_{\text{cm}}}mv^2_{\text{cm }x,\text{i}} = -\dfrac{1}{2(0.50\text{ m})}(1000\text{ kg})(5.0\text{ m/s})^2 = -2.5\times10^4\text{ N}$.

So the magnitude of the average force exerted on the car is $2.5\times10^4\text{ N}$. (*b*) The work done on the car's center of mass is $W = F^{\text{c}}_{\text{U car }x,\text{av}}\Delta x_{\text{cm}} = (-2.5\times10^4\text{ N})(0.50\text{ m}) = -1.3\times10^4\text{ J}$. (*c*) The change in kinetic energy is equal to the work done on the car, in this case. So $\Delta K = -1.3\times10^4\text{ J}$.

9.34. (*a*) $W = F^{\text{c}}_{\text{ws}x,\text{av}}\Delta x_{\text{cm}} = \Delta K_{\text{cm}} = \dfrac{1}{2}mv^2_{x,\text{f}}$ so $F^{\text{c}}_{\text{ws}x,\text{av}} = \dfrac{1}{2\Delta x_{\text{cm}}}mv^2_{x,\text{f}} = \dfrac{1}{2(0.50\text{ m})}(60\text{ kg})(3.0\text{ m/s})^2 = 5.4\times10^2\text{ N}$.

(*b*) The wall itself does no work on the skater, because the contact point between the wall and the skater does not move. The skater's muscles do work on his center of mass by pushing against the wall. This quantity of work is given by $W = F^{\text{c}}_{\text{ws}x,\text{av}}\Delta x_{\text{cm}} = (5.4\times10^2\text{ N})(0.50\text{ m}) = 2.7\times10^2\text{ J}$. (*c*) Here the change in kinetic energy is exactly equal to the work done in part (*b*). Hence $\Delta K = 2.7\times10^2\text{ J}$.

9.35. (*a*) $W = F^{\text{c}}_{\text{h red }x}\Delta x_{\text{red}} = (2.0\text{ N})(0.15\text{ m}) = 0.30\text{ J}$. (*b*) $\Delta x_{\text{cm}} = \dfrac{(m_{\text{red}}\Delta x_{\text{red}} + m_{\text{green}}\Delta x_{\text{green}})}{m_{\text{red}} + m_{\text{green}}} = \dfrac{(0.50\text{ kg})(0.15\text{ m})}{(1.0\text{ kg})} = 0.075\text{ m}$ or 75 mm. (*c*) The increase in kinetic energy is equal to the work done by you: 0.15 J.

9.36. (*a*) The work done on the system is just the sum of the work done on each cart separately: $W = W_{\text{on 1}} + W_{\text{on 2}} = F^{\text{c}}_{\text{h1}x}\Delta x_1 + F^{\text{c}}_{\text{h2}x}\Delta x_2 = (3.0\text{ N})(1.0\text{ m}) + (-3.0\text{ N})(-1.0\text{ m}) = 6.0\text{ J}$. (*b*) Here, all the work done on the system went into increasing its kinetic energy, so $\Delta K = 6.0\text{ J}$. (*c*) Both carts are moving at the same speed, but in opposite directions. So the kinetic energy of the center of mass of the system is zero. Another way of looking at this is that the vector sum of forces acting on the system of two carts as a whole was zero.

9.37. They require the same amount of work. Both procedures end up placing the same amount of inertia at the same heights, meaning each process corresponds to the same change in energy.

9.38. (*a*) $W = F^{\text{c}}_{\text{h 1.0 kg cart } x}\Delta x_{\text{1.0 kg cart}} = (2.0\ \text{N})(0.15\ \text{m}) = 0.30\ \text{J}$. (*b*) $\Delta x_{\text{cm}} = \dfrac{(m_{\text{1.0 kg cart}}\Delta x_{\text{1.0 kg cart}} + m_{\text{0.5 kg cart}}\Delta x_{\text{0.5 kg cart}})}{m_{\text{1.0 kg cart}} + m_{\text{0.5 kg cart}}} = \dfrac{(1.0\ \text{kg})(0.15\ \text{m})}{(1.5\ \text{kg})} = 0.10\ \text{m}$. (*c*) The change in kinetic energy of the center of mass of the system is $\Delta K = F^{\text{c}}_{\text{h cm } x}\Delta x_{\text{cm}} = (2.0\ \text{N})(0.10\ \text{m}) = 0.20\ \text{J}$.

9.39. (*a*) $W = F^{\text{c}}_{\text{hg}x}\Delta x_{\text{g}} = (2.0\ \text{N})(1.0\ \text{m}) = 2.0\ \text{J}$. (*b*) Here all work went to increasing the kinetic energy of the system, so $\Delta K = 2.0\ \text{J}$. (*c*) The center of mass of the system only moved $\Delta x_{\text{cm}} = \dfrac{(m_{\text{g}}\Delta x_{\text{g}} + m_{\text{t}}\Delta x_{\text{t}})}{m_{\text{g}} + m_{\text{t}}} = \dfrac{(1.0\ \text{kg})(1.0\ \text{m})}{(2.0\ \text{kg})} = 0.50\ \text{m}$ during the application of the force. Thus $\Delta K_{\text{cm}} = F^{\text{c}}_{\text{h cm } x}\Delta x_{\text{cm}} = (2.0\ \text{N})(0.50\ \text{m}) = 1.0\ \text{J}$. (*d*) This is exactly the calculation done to answer part (*c*). This quantity is 1.0 J. (*e*) When the two blocks collide and stick together, they move as one object with a velocity equal to the velocity of the center of mass. Thus, the energy of the system is then the kinetic energy of the center of mass, which is 1.0 J. This can be confirmed through conservation of momentum.

9.40. (*a*) Call right the $+x$ direction. The work done on the system is $W = F^{\text{c}}_{\text{hL}x}\Delta x_{\text{L}} = (5.0\ \text{N})(0.40\ \text{m}) = 2.0\ \text{J}$. (*b*) Since the left cart moves 0.40 m to the right, and the spring is compressed 0.20 m, the cart on the right moved only 0.20 m. So $\Delta x_{\text{cm}} = \dfrac{(m_{\text{L}}\Delta x_{\text{L}} + m_{\text{R}}\Delta x_{\text{R}})}{m_{\text{L}} + m_{\text{R}}} = \dfrac{(0.50\ \text{kg})(0.40\ \text{m}) + (0.50\ \text{kg})(0.20\ \text{m})}{(1.0\ \text{kg})} = 0.30\ \text{m}$. (*c*) $\Delta K = F^{\text{c}}_{\text{hL}x}\Delta x_{\text{cm}} = (5.0\ \text{N})(0.30\ \text{m}) = 1.5\ \text{J}$.

9.41. As you throw a snowball, you might be able to exert your maximum 100 N of force over a distance of 1.0 m. This would result in each snowball having a speed given by $v_{\text{s},x} = \sqrt{\dfrac{2F^{\text{c}}_{\text{hs}x}\Delta x_{\text{s}}}{m_{\text{s}}}} = \sqrt{\dfrac{2(100\ \text{N})(1.0\ \text{m})}{(1.0\ \text{kg})}} = 14.1\ \text{m/s}$. Suppose we throw N such snowballs at that speed, and we include the snowballs and the cooler in our system. The x component of the velocity of the center of mass is $v_{\text{cm},x} = \dfrac{Nm_{\text{s}}v_{\text{s},x} + m_{\text{cooler}}v_{\text{cooler},x}}{Nm_{\text{s}} + m_{\text{cooler}}}$. Once the snowballs collide inelastically with the cooler, this will also be the x component of the velocity of the cooler. Therefore we require $\dfrac{Nm_{\text{s}}v_{\text{s},x} + m_{\text{cooler}}v_{\text{cooler},x}}{Nm_{\text{s}} + m_{\text{cooler}}} \geq (v_{\text{cooler}})_{\text{min}} = 3.0\ \text{m/s}$. Solving for N yields $N \geq \dfrac{m_{\text{cooler}}((v_{\text{cooler}})_{\text{min}} - v_{\text{cooler},x})}{m_{\text{s}}(v_{\text{s},x} - (v_{\text{cooler}})_{\text{min}})} = \dfrac{(10\ \text{kg})((3.0\ \text{m/s}) - 0)}{(1.0\ \text{kg})((14.1\ \text{m/s}) - (3.0\ \text{m/s}))} = 2.7$. Thus 3 snowballs will be sufficient.

9.42. (*a*) We first look at conservation of momentum to determine the initial speed of the block. Call the direction in which the bullet is fired the $+x$ direction. Then $mv = mv/3 + 4mv_{\text{block } x,\text{f}} \Rightarrow v_{\text{block } x,\text{f}} = \dfrac{1}{6}v$. So the initial kinetic energy is $K_{\text{block,i}} = \dfrac{1}{2}4mv^2_{\text{block } x,\text{f}} = mv^2/18$. When the block comes to rest, it means that sufficient work has been done to reduce this energy to zero. In that case $W = F^{\text{f}}_{\text{surface block } x}d = -K_{\text{i}}$, or $F^{\text{f}}_{\text{surface block } x} = -mv^2/(18d)$. So the magnitude of the frictional force between the block and the surface is $mv^2/(18d)$. (*b*) We first find the difference between the energies before and after the collision. $\Delta K_{\text{collision}} = K_{\text{bullet,f}} + K_{\text{block,f}} - K_{\text{bullet,i}} = \dfrac{1}{2}m\left(\dfrac{v}{3}\right)^2 + \dfrac{1}{2}(4m)\left(\dfrac{v}{6}\right)^2 - \dfrac{1}{2}mv^2 = -\dfrac{7}{18}mv^2$. And the change in kinetic energy as the block slides across the surface is $-\dfrac{1}{18}mv^2$. So $K_{\text{collision}}/K_{\text{friction}} = 7$.

9.43. We first look at conservation of momentum to determine the initial speed of the block after the bullet has imbedded itself. Call the direction in which the bullet is fired the $+x$ direction. Then $mv = (m+4m)v_{\text{block }x,\text{f}} \Rightarrow v_{\text{block }x,\text{f}} = \frac{1}{5}v.$ So the initial kinetic energy is $K_{\text{block,i}} = \frac{1}{2}5mv^2_{\text{block }x,\text{f}} = mv^2/10.$ Some of this kinetic energy will turn into spring potential energy, but the rest is removed from the system through work done by friction between the block and the surface. Thus $W = K_{\text{block,i}} - U^{\text{spring}}_{\text{f}}$ or $F^{\text{f}}_{\text{surface block }x} = -\frac{1}{d}(K_{\text{block,i}} - U^{\text{spring}}_{\text{f}}) = -\frac{1}{d}\left(\frac{1}{10}mv^2 - \frac{1}{2}kd^2\right).$ So the magnitude of the frictional force between the block and the surface is $\frac{1}{2d}\left(\frac{mv^2}{5} - kd^2\right).$

9.44. (*a*) A given force F can compress a spring $\Delta x = F/k.$ The energy stored in a spring is $\frac{1}{2}k\Delta x^2,$ which we can also write as $\frac{1}{2}\frac{F^2}{k}.$ So if each spring is compressed using the same force, the spring with the smaller spring constant will store the most energy. A is less stiff, and therefore will store more potential energy. (*b*) Since the spring potential energy can be written as $\frac{1}{2}k\Delta x^2,$ it is clear that if both springs have the same compression, the spring with the greater spring constant will store the most energy. Since spring B is stiffer, spring B would store more energy in this case.

9.45. They require the same amount of work. The magnitude of a spring's restoring force does not depend on the direction of the displacement; it is the same whether you stretch or compress the spring.

9.46. The work done is simply the area under the curve, because $W = \int \vec{F}(\vec{x}) \cdot d\vec{x}.$ Here the area under the curve is approximately 4.2 J.

9.47. For the initial stretch, we can write $W = \Delta U^{\text{spring}} = \frac{1}{2}k\Delta x^2 \Rightarrow k = \frac{2W}{\Delta x^2} = \frac{2(18\text{ J})}{(0.10\text{ m})^2} = 3600\text{ N/m}.$ Now the work required to stretch the spring by 0.20 m is $W = \frac{1}{2}(3600\text{ N/m})(0.20\text{ m})^2 = 72\text{ J}.$ This is an additional 54 J.

9.48. No. In order to integrate the force over some distance, you must be able to write the time interval needed for the particle to travel that distance (and therefore the force) as a function of distance. You cannot do this because you do not know the particle's initial velocity. Nor do you know the particle's acceleration because you do not know its inertia.

9.49. (*a*) From the information given, we can determine the spring constant inside the dart gun: $k = F/\Delta x = \frac{(6.0\text{ N})}{(0.12\text{ m})} = 50\text{ N/m}.$ So as the dart is fired, spring potential energy is converted to kinetic energy of the dart, such that we can write $\frac{1}{2}k\Delta x_{\text{i}}^2 = \frac{1}{2}mv^2_{x,\text{f}} \Rightarrow v_{x,\text{f}} = \sqrt{\frac{k}{m}}\Delta x_{\text{i}} = \sqrt{\frac{(50\text{ N/m})}{(0.024\text{ kg})}}(0.12\text{ m}) = 5.5\text{ m/s}.$ (*b*) Yes, if the dart is fired vertically, some of the spring potential energy is converted to gravitational potential energy as the dart moves upward. Hence, not all the initial energy is converted to kinetic energy, making the vertical launch speed slightly slower than the horizontal launch speed.

9.50. Since (in one dimension) $W = \int F(x)dx,$ we can also write $F(x) = \dfrac{dW(x)}{dx} = \dfrac{d}{dx}(ax + bx^3) = a + 3bx^2.$ So $F(\Delta x) = a + 3b(\Delta x)^2.$

9.51. (*a*) We are always free to measure our gravitational potential energy relative to any height we like. As a result, answers involving gravitational potential energies may vary. Here, we choose to measure relative to the uncompressed bed. At each step we determine the gravitational potential energy using $U_\text{g}(\Delta x) = mg\Delta x,$ where Δx measures the change in vertical position relative to the height of the uncompressed mattress. The spring potential energy is determined using $U_\text{sp}(\Delta x) = \frac{1}{2}k\Delta x^2.$ Finally, because energy is conserved, we can determine the kinetic energy at each position by requiring $U_\text{sp}(\Delta x) + U_\text{g}(\Delta x) + K(\Delta x) = U_\text{sp}(\Delta x = 0) + U_\text{g}(\Delta x = 0) + K(\Delta x = 0) = K(\Delta x = 0)$ or equivalently $K(\Delta x) = (0.59\ \text{J}) - U_\text{sp}(\Delta x) - U_\text{g}(\Delta x),$ where we have determined the kinetic energy of the bowling ball after falling the initial 10 mm to reach the mattress. Thus we can determine all the values required, as shown in the following table.

Δx	0	0.050 m	0.10 m	0.15 m	0.20 m
K (J)	0.59	2.9	4.0	3.8	2.3
UG (J)	0	–2.9	–5.9	–8.8	–12
U (J)	0	0.63	2.5	5.6	10

(*b*) The bowling ball will reach its maximum compression when it stops, and therefore momentarily has zero kinetic energy. We use $K(\Delta x_\text{max}) = (0.59\ \text{J}) - U_\text{sp}(\Delta x_\text{max}) - U_\text{g}(\Delta x_\text{max}) = 0$ to find $\Delta x_\text{max}.$ This yields a quadratic equation $\frac{1}{2}k\Delta x^2 + mg\Delta x_\text{max} - (0.59\ \text{J}) = 0,$ which we solve to find $\Delta x = \dfrac{-mg \pm \sqrt{2kK_\text{i} + m^2g^2}}{k} = \dfrac{-(6.0\ \text{kg})(9.8\ \text{m/s}^2) \pm \sqrt{2(500\ \text{N/m})(0.59\ \text{J}) + (6.0\ \text{kg})^2(9.8\ \text{m/s}^2)^2}}{(500\ \text{N/m})}.$ The two solutions are $\Delta x = -0.24\ \text{m}$ and $\Delta x = 0.096\ \text{m}.$ The second (positive) solution is non-physical because it refers to a positive height above the mattress. At those positions, there was no compression of the mattress and so our equations are not applicable there. The physical solution is $\Delta x = -0.24\ \text{m},$ or a compression magnitude of 0.24 m.

9.52. (*a*) Call the distance through the haystack $d,$ the initial speed of the ball $v,$ and the frictional force between the hay and the ball $F.$ The work done by the friction will be equal to the change in kinetic energy of the ball, such that we can write $W = -Fd = \Delta K = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2.$ Solving for the speed yields $v = \sqrt{8dF/3m}.$ (*b*) We use $v = \sqrt{8dF/3m} = \sqrt{8(1.2\ \text{m})(6.0\ \text{N})/3(0.50\ \text{kg})} = 6.2\ \text{m/s}.$

9.53. In this one-dimensional case, we can write

$$W = \int_{x_\text{i}}^{x_\text{f}} F_x(x)\,dx = \int_{x_\text{i}}^{x_\text{f}} (ax^2 + bx^3)dx$$

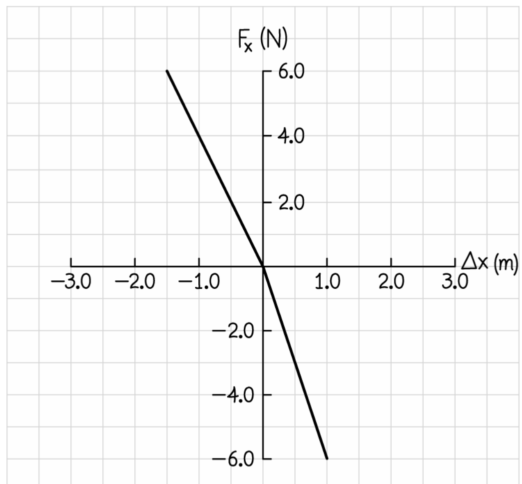
$$W = \left(\frac{1}{3}ax^3 + \frac{1}{4}bx^4\right)\Bigg|_{x_\text{i}}^{x_\text{f}} = \frac{a}{3}(x_\text{f}^3 - x_\text{i}^3) + \frac{b}{4}(x_\text{f}^4 - x_\text{i}^4)$$

$$= \frac{(3.0\ \text{N/m}^2)}{3}((2.0\ \text{m})^3 - (-0.40\ \text{m})^3) + \frac{(-0.50\ \text{N/m}^3)}{4}((2.0\ \text{m})^4 - (-0.40\ \text{m})^4)$$

$$= 6.1\ \text{J}$$

9.54. Accelerating from rest to a final speed of 20 m/s corresponds to an increase in kinetic energy of $\frac{1}{2}mv_\text{f}^2 = \frac{1}{2}(500\text{ kg})(20\text{ m/s})^2 = 1.0\times10^5\text{ J}$. We must be able to do this 50 times, meaning we must have a total energy of $5.0\times10^6\text{ J}$ at our disposal. This energy must initially be stored in spring potential energy. Thus we require $U_\text{i}^\text{sp} = \frac{1}{2}k\left(\frac{\ell}{2}\right)^2 = 5.0\times10^6\text{ J}$ or $k = \frac{8(5.0\times10^6\text{ J})}{(4.2\text{ m})^2} = 2.3\times10^6\text{ N/m}$.

9.55. (*a*)



(*b*) For this part, let us consider a system that includes the springs and the cart. If the cart is pushed to the left, the initial kinetic energy will turn into spring potential energy in the left spring. Thus $\frac{1}{2}mv_\text{i}^2 = \frac{1}{2}k_\text{l}\Delta x_\text{max}^2 \Rightarrow$ $\Delta x_\text{max} = \sqrt{\frac{m}{k_\text{l}}}v_\text{i} = \sqrt{\frac{(0.15\text{ kg})}{(4.0\text{ N/m})}}(4.0\text{ m/s}) = 0.77\text{ m}$. If the cart is initially pushed to the right, then the same principles apply, but it will be the right spring that is being compressed. Thus we have $\frac{1}{2}mv_\text{i}^2 = \frac{1}{2}k_\text{r}\Delta x_\text{max}^2 \Rightarrow$ $\Delta x_\text{max} = \sqrt{\frac{m}{k_\text{r}}}v_\text{i} = \sqrt{\frac{(0.15\text{ kg})}{(6.0\text{ N/m})}}(4.0\text{ m/s}) = 0.63\text{ m}$. (*c*) Here, we consider only the system of the cart, such that the springs exert external forces that do work. In either case, the work done by the cart is exactly enough to reduce the cart's kinetic energy to zero. Thus $W = \Delta K = -\frac{1}{2}mv_\text{i}^2 = -\frac{1}{2}(0.15\text{ kg})(4.0\text{ m/s})^2 = -1.2\text{ J}$.

9.56. Call the direction in which the ball is thrown the $+x$ direction. The initial x component of the ball's velocity is $v_{\text{b}\,x,\text{i}}$ and its inertia is m. Then the conservation of momentum in the x direction reads $mv_{\text{b}\,x,\text{i}} = 4mv_{x,\text{f}}$, where $v_{x,\text{f}}$ is the final x component of the velocity of the ball and mitt together. So the kinetic energy of the ball and mitt immediately after the collision can be written $K(\Delta x = 0) = \frac{1}{2}(4m)v_{x,\text{f}}^2 = \frac{1}{2}(4m)\left(\frac{v_{x,\text{i}}}{4}\right)^2 = \frac{1}{8}mv_{x,\text{i}}^2$. When the ball and mitt stop, all of this kinetic energy has been converted into spring potential energy. Thus $K(\Delta x = 0) = U^\text{sp}(\Delta x_\text{max})$ or $\frac{1}{8}mv_{x,\text{i}}^2 = \frac{1}{2}k\Delta x_\text{max}^2$. Solving for the initial speed yields $v_\text{i} = 2\Delta x_\text{max}\sqrt{\frac{k}{m}}$.

9.57. Call the direction of motion of the jogger the $+x$ direction. Then $P = \dfrac{\Delta E}{\Delta t} = \dfrac{F^{\rm d}_{{\rm aj}x}\Delta x}{\Delta t} = F^{\rm d}_{{\rm aj}x}v_x = (25\ \text{N})(5.0\ \text{m/s})$ $= 1.3\times 10^2\ \text{W}$.

9.58. Call vertically upward the $+y$ direction. And consider a system including the girl and Earth. Then $P = \dfrac{\Delta E}{\Delta t} = \dfrac{\Delta U^{\rm G}}{\Delta t} = \dfrac{mg\Delta y}{\Delta t} = \dfrac{(35\ \text{kg})(9.8\ \text{m/s}^2)(10\ \text{m})}{(25\ \text{s})} = 1.4\times 10^2\ \text{W}$.

9.59. Because the zigzag path is spread out over a greater distance, the climber expends the same quantity of energy over a greater time interval, for a lower average power rating.

9.60. The total work done on the car must equal its change in energy. In this case we have $W_{\text{engine}} + W_{\text{dissipative forces}} = \Delta K + \Delta U^G$. Inserting given information, we have $P_{\text{engine}}\Delta t - \frac{1}{2}K_{\rm i} - \frac{1}{2}P_{\text{engine}}\Delta t = K_{\rm f} - K_{\rm i} + U^G_{\rm f} - U^G_{\rm i}$ or

$$\Delta t = \frac{-\frac{1}{2}\left(\frac{1}{2}mv_{\rm i}^2\right) + mg\Delta y}{\frac{1}{2}P_{\text{engine}}} = \frac{-\left(\frac{1}{2}(1000\ \text{kg})(7.0\ \text{m/s})^2\right) + (2)(1000\ \text{kg})(9.8\ \text{m/s}^2)(20\ \text{m})}{(3.3\times 10^3\ \text{W})} = 1.1\times 10^2\ \text{s}.$$

9.61. No. The work increases with time. Work comes from a force being applied over a certain distance. As the particle speeds up, the distance covered each second increases, so the work done each second increases.

9.62. (*a*) Call the direction of motion the $+x$ direction. Then, ignoring friction we have $F^{\rm c}_{{\rm ds}x} = ma_x = m\left(\dfrac{v_{x,\rm f} - v_{x,\rm i}}{\Delta t}\right) = (200\ \text{kg})\left(\dfrac{(5.0\ \text{m/s}) - (0)}{(3.0\ \text{s})}\right) = 3.3\times 10^2\ \text{N}$. (*b*) The work done by the dogs on the sled is the same as the change in the sled's kinetic energy in this case. Thus $W = \Delta K_{\rm s} = \frac{1}{2}mv_{\rm f}^2 = \frac{1}{2}(200\ \text{kg})(5.0\ \text{m/s})^2 =$ $2.5\times 10^3\ \text{J}$. (*c*) The instantaneous power output is given by $P = \dfrac{\Delta E}{\Delta t} = \dfrac{F_x\Delta x}{\Delta t} = F_x v_x = (333\ \text{N})(5.0\ \text{m/s}) = 1.7\times 10^3\ \text{W}$. (*d*) If the acceleration is constant, it is given by $a_x = \left(\dfrac{v_{x,\rm f} - v_{x,\rm i}}{\Delta t}\right) = \left(\dfrac{(5.0\ \text{m/s}) - (0)}{(3.0\ \text{s})}\right) = 1.67\ \text{m/s}^2$. Then the speed after only 1.5 s is $v_{x,\rm f} = v_{x,\rm i} + a_x\Delta t = (0) + (1.67\ \text{m/s}^2)(1.5\ \text{s}) = 2.5\ \text{m/s}$. Now we can calculate the instantaneous power exactly as before: $P = \dfrac{\Delta E}{\Delta t} = \dfrac{F_x\Delta x}{\Delta t} = F_x v_x = (333\ \text{N})(2.5\ \text{m/s}) = 8.3\times 10^2\ \text{W}$.

9.63. (*a*) We cannot simply use the force and distance to calculate work because we do not immediately know the distance the child moves during this time interval. Instead we find the final speed of the child by using impulse: $\Delta p_x = mv_{x,\rm f} - mv_{x,\rm i} = F_x\Delta t$ so $v_{x,\rm f} = \dfrac{F_x\Delta t}{m} + v_{x,\rm i} = \dfrac{(5.0\ \text{N})(1.0\ \text{s})}{(20\ \text{kg})} + 0 = 0.25\ \text{m/s}$. Now, we know the child's final kinetic energy came from the work done by the water on the child, so $W = \frac{1}{2}mv_{x,\rm f}^2 = \frac{1}{2}(20\ \text{kg})(0.25\ \text{m/s})^2 = 0.63\ \text{J}$.
(*b*) The instantaneous power is given by $P = \dfrac{\Delta E}{\Delta t} = \dfrac{F_x\Delta x}{\Delta t} = F_x v_x = (5.0\ \text{N})(0.25\ \text{m/s}) = 1.3\ \text{W}$.

9.64. (*a*) The work done by the cog system equals the change in energy of the system of 25 cars. So $W = \Delta K + \Delta U^G = (25)\frac{1}{2}mv_{\rm f}^2 + (25)mg\Delta y = (25)(579\ \text{kg})\left(\frac{1}{2}(0.50\ \text{m/s}^2)^2 + (9.8\ \text{m/s}^2)(100\ \text{m})\right) = 1.4\times 10^7\ \text{J}$.

(*b*) $P = \dfrac{\Delta E}{\Delta t} = \dfrac{(1.42\times10^7\ \text{J})}{(60\ \text{s})} = 2.4\times10^5\ \text{W}$. (*c*) The only change in the energy requirement is that now the cars will have to move faster at the beginning. But as long as there is no dissipation, this higher kinetic energy can still be converted to gravitational potential energy as the cars rise. Thus, as long as the final speed does not change, there is no change in the total energy provided by the cog system. It remains 1.4×10^7 J.

9.65. (*a*) Let the positive x axis point to the right. The power from the child on the left is $P = \dfrac{F^{\text{c}}_{\text{Lb}x}\Delta x}{\Delta t} = F^{\text{c}}_{\text{Lb}x}v_x = (-3.0\ \text{N})(-3.0\ \text{m/s}) = 9.0\ \text{W}$. The power from the child on the right is $P = \dfrac{F^{\text{c}}_{\text{Rb}x}\Delta x}{\Delta t} = F^{\text{c}}_{\text{Rb}x}v_x = (+2.0\ \text{N})(-3.0\ \text{m/s}) = -6.0\ \text{W}$. (*b*) The power being delivered to the block by both forces is $P_{\text{L}} + P_{\text{R}} = (9.0\ \text{W}) + (-6.0\ \text{W}) = 3.0\ \text{W}$. (*c*) Yes. The box is accelerating because the vector sum of the forces exerted on it is nonzero. Because power is equal to the product of force and speed, if the box moves at a higher speed, then the power from the same forces will increase in magnitude.

9.66. (*a*) Choose a system that includes the elevator and Earth. First we note that the power is not constant. As the speed increases, the power needed to accelerate the elevator at a constant rate will increase. Also, as the speed increase, the gravitational potential energy will increase at an increasing rate. We will find the average power, and the peak power. The average power can be found by simply calculating the change in energy of the elevator and dividing by time. For this we need to know the height of the elevator as it reaches its cruising speed. We can use kinematics to find $\Delta y = \frac{1}{2}(v_{y,\text{f}} + v_{y,\text{i}})\Delta t = \frac{1}{2}((1.5\ \text{m/s}) + (0))(2.0\ \text{s}) = 1.5\ \text{m}$. To find the average power we write

$$P_{\text{av}} = \frac{\Delta E}{\Delta t} = \frac{\Delta K + \Delta U^G}{\Delta t} = \frac{\frac{1}{2}mv^2_{\text{cruising}} + mg\Delta y}{\Delta t} = \frac{\frac{1}{2}(1400\ \text{kg})(1.5\ \text{m/s})^2 + (1400\ \text{kg})(9.8\ \text{m/s}^2)(1.5\text{m})}{(2.0\ \text{s})} = 1.1\times10^4\ \text{W}.$$

The maximum power comes just as the elevator reaches its cruising speed. We can calculate the instantaneous power by taking the time derivative of the energy at that moment t_{f}. For this we will need the acceleration, which is given by $a_y = \dfrac{2\Delta y}{\Delta t^2} = \dfrac{2(1.5\ \text{m})}{(2.0\ \text{s})^2} = 0.75\ \text{m/s}^2$.

$$P_{\text{inst}}(t_{\text{f}}) = \left.\frac{dE(t)}{dt}\right|_{t_{\text{f}}} = \left.\frac{d}{dt}\left(\frac{1}{2}mv^2_{\text{cruising}} + mg\Delta y\right)\right|_{t_{\text{f}}} = mv_{\text{cruising}}\frac{dv}{dt} + mg\left.\frac{dy}{dt}\right|_{t_{\text{f}}} = mv_{\text{cruising}}a_y + mgv_{\text{cruising}}$$

$$= (1400\ \text{kg})(1.5\ \text{m/s})((0.75\ \text{m/s}^2) + (9.8\ \text{m/s}^2)) = 2.2\times10^4\ \text{W}$$

(*b*) Now the power goes entirely to increasing the gravitational potential energy of the elevator. Since the speed is constant, so is the power: $P_{\text{av}} = \dfrac{\Delta U^G}{\Delta t} = \dfrac{mg\Delta y}{\Delta t} = (1400\ \text{kg})(9.8\ \text{m/s}^2)(1.5\ \text{m/s}) = 2.1\times10^4\ \text{W}\ 15\ \text{kW}$.

9.67. If we use a system that includes the elevator and Earth, the elevator does work that increases the gravitational potential energy. So $W = \Delta U^G = mgh$.

9.68. Yes, if other forces are exerted on the object. For example, if a sled is being dragged across the snow at a constant speed by a rope, the direction of the tensile force exerted by the rope is the same as the sled's direction of motion. There is no change in kinetic energy, however, because the amount of (negative) work done by friction is the same as the (positive) amount done by the rope.

9.69. (*a*) Call the direction of motion of the cart the $+x$ direction. The center of mass moves 0.10 m as the force is applied to the grocery-cart system. We know the work changes the kinetic energy, so we can write $W = F^{\text{c}}_{\text{pc}x,\text{av}}\Delta x = \Delta K = -\frac{1}{2}mv^2_{\text{c}\,x,\text{i}}$. Solving for the force yields $F^{\text{c}}_{\text{pc}x,\text{av}} = -\dfrac{mv^2_{\text{c}\,x,\text{i}}}{2\Delta x} = -\dfrac{(50\ \text{kg})(2.0\ \text{m/s})^2}{2(0.10\ \text{m})} = -1.0\times10^3\ \text{N}$.

So the average force is 1.0×10^3 N toward the rear of the cart. (*b*) Zero, because the point of application does not move. (*c*) $\Delta K = F^{c}_{pcx,av}\Delta x = (-1.0\times10^3\ \text{N})(0.10\ \text{m}) = -1.0\times10^2\ \text{J}$.

9.70. Yes, when an object is spun in a circle, there is an inward centripetal force. However, because the object does not move radially, this force is not exerted over any distance and so does no work on the object.

9.71. (*a*) Call vertically upward the $+y$ direction. The work done by the man's legs on his center of mass must be sufficient for his center of mass to reach the specified height. So $W = F^{c}_{1\,\text{cm}\,y,\text{av}}\Delta y_{\text{pushing}} = mg\Delta y_{\text{to ladder}}$, so $F^{c}_{1\,\text{cm}\,y,\text{av}} = \dfrac{mg\Delta y_{\text{to ladder}}}{\Delta y_{\text{pushing}}} = \dfrac{(70\ \text{kg})(9.8\ \text{m/s}^2)(1.5\ \text{m})}{(0.55\ \text{m})} = 1.9\times10^3\ \text{N}$. So the minimum force is 1.9×10^3 N upward.

(*b*) The fraction of a bandy bar needed is $\dfrac{E_{\text{jump}}}{E_{\text{candy}}} = \dfrac{(1.87\times10^3\ \text{N})(0.55\ \text{m})}{(8.50\times10^5\ \text{J})} = 1.2\times10^{-3}$. So the jump requires 1.2×10^{-3} candy bars.

9.72. Estimates may vary. Here, we will assume the trash collector takes approximately 1 minute to lift a trash can, empty it and replace it, and move on to the next can, on average. Assume the average trash can has inertia of about 3×10^1 kg. Finally, assume that emptying a trash can requires lifting it to a height of 1 m, and that keeping the can aloft while trash falls out, and replacing the can adds an additional 50% to this energy. Then the total work is $W = N(1.5)(mg\Delta y) = (8\ \text{hr})\left(\dfrac{60\ \text{min}}{1\ \text{hr}}\right)\left(\dfrac{1\ \text{min}}{1\ \text{can}}\right)(1.5)(3\times10^1\ \text{kg})(9.8\ \text{m/s}^2)(1\ \text{m}) = 2\times10^5\ \text{J}$.

9.73. The block is moving at a constant speed. If it were speeding up or slowing down, this would change the power being delivered to it.

9.74. (*a*) Gravitational force exerted by Earth and spring restoring force. The gravitational force is always directed downward. The spring force is directed upward until the object passes through its equilibrium position. At that instant, the force direction changes to downward until the object passes through its equilibrium position again. At that instant, the force direction changes to upward again. (*b*) 0

9.75. (*a*) With no dissipation, it doesn't matter which way you roll the ball. (*b*) With energy dissipation, you want to roll the ball down hill A, so that it travels a shorter distance. The shorter the path, the less energy dissipated, so the ball has a better chance of having sufficient mechanical energy to get over hill B.

9.76. The dog can exert a maximum force of $2.5mg$ and can exert this force over a maximum of 0.20 m. From this we can find the maximum work that his legs can do on his center of mass, and equate this the dog's maximum change in gravitational potential energy. $W_{\max} = F^{c}_{1\,\text{cm}\,y,\max}\Delta y_{\max} = mg\Delta y_{\text{jump,max}}$, so $\Delta y_{\text{jump,max}} = \dfrac{F^{c}_{1\,\text{cm}\,y,\max}\Delta y_{\text{push,max}}}{mg} = \dfrac{(2.5)mg\Delta y_{\text{push,max}}}{mg} = 2.5(0.20\ \text{m}) = 0.50\ \text{m}$. Since the dog started at 0.10 m off the ground, this corresponds to a maximum height of 0.60 m.

9.77. We equate the work done by your legs to the required change in your gravitational potential energy: $W = F^{c}_{1\,\text{cm}\,y,\text{av}}\Delta y_{\text{push}} = mg\Delta y_{\text{jump}}$. In terms of the center of mass height at the moment your jump begins, $y_{\min}$, we have $(2.3)mg(y_{\text{push,max}} - y_{\min}) = mg(y_{\text{jump,max}} - y_{\min})$ or $y_{\min} = \dfrac{y_{\text{jump,max}} - (2.3)y_{\text{push,max}}}{(-1.3)} = \dfrac{(2.0\ \text{m}) - (2.3)(1.0\ \text{m})}{(-1.3)} = 0.23\ \text{m}$.
Your center of mass would have to be 0.23 m above the floor. This is not practical.

9.78. If we assume that the elevator is designed to move upward at that speed, then the only factor that is increasing the power output of the motor is the increased inertia. In that case, increasing the power output by 10% would correspond to increasing the inertia being moved by 10%. Here, we are 200 kg over the limit, where the limit is 3200 kg. This is only 6.3%. So, no, this should not be over the 10% tolerance.

9.79. In this problem we will ignore the inertia of the forearm itself, and focus on forces required to lift the cup, specifically. If we consider the system of the coffee cup and Earth, then we know it takes a certain amount of work to lift the cup a distance Δy_{cup}, which is given by $W = mg\Delta y_{\text{cup}}$. This work is done on the cup by the hand beneath it, so $F^{\text{c}}_{\text{hc}y}\Delta y_{\text{cup}} = mg\Delta y_{\text{cup}}$. As this motion occurs, you bend your elbow. The contact point where your bicep attaches to your forearm moves a much smaller distance upward as the cup is lifted. Specifically, the vertical distance that the bicep contact point moves $(\Delta y_{\text{bicep}})$ is related to Δy_{cup} through simple geometry, and one finds $\Delta y_{\text{bicep}} = \frac{d}{\ell}\Delta y_{\text{cup}}$. But the force exerted by the bicep on the forearm is ultimately the source of the work being done on the cup; forearm simply acts as a lever. So $F^{\text{c}}_{\text{bf}y}\Delta y_{\text{bicep}} = F^{\text{c}}_{\text{hc}y}\Delta y_{\text{cup}} = mg\Delta y_{\text{cup}}$ and we find $F^{\text{c}}_{\text{bf}y} = \frac{mg\Delta y_{\text{cup}}}{\Delta y_{\text{bicep}}} = \frac{mg\ell}{d}$. Coffee cups vary greatly in inertia. For a typical inertia around 0.30 kg, we find $F^{\text{c}}_{\text{bf}y} = \frac{(0.3\text{ kg})(9.8\text{ m/s}^2)(0.350\text{ m})}{(0.050\text{ m})} = 2\times10^1\text{ N}$. Answers in the range $20\text{ N} \pm 10\text{ N}$ are reasonable.

9.80. Consider a system of you, the bungee cord, and Earth. During your fall, you will first convert gravitational potential energy into kinetic energy alone, for the distance of the unstretched cord, which we will call ℓ_0. Then, you will convert gravitational potential energy and kinetic energy into spring potential energy, as you fall the remaining distance. If we call the height of the bridge h, then this second distance must be equal to $h - \ell_0$. Then we have $U^G_{\text{i}} = mgh = U^{\text{sp}}_{\text{f}} = \frac{1}{2}k(h-\ell_0)^2$, so $\ell_0 = h - \sqrt{\frac{2mgh}{k}}$. Let us assume that your inertia is somewhere between 70 kg and 80 kg. Using an inertia of 70 kg, we find $\ell_0 = (150\text{ m}) - \sqrt{\frac{2(70\text{ kg})(9.8\text{ m/s}^2)(150\text{ m})}{(40\text{ N/m})}} = 78\text{ m}$. Similarly, if we use an inertia of 80 kg, we find $\ell_0 = 73\text{ m}$. Any answer between these two values is reasonable.

10

MOTION IN A PLANE

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^7 m southwest **2.** 10^6 J **3.** 10^4 N **4.** 10^2 m **5.** 10^2 m **6.** 10^2 m **7.** 10^0 N **8.** 10^1 degrees or 10^0 rad **9.** 10^3 m

Guided Problems

10.2 Cannonball trajectory

1. Getting Started This is projectile motion, so we expect the cannonball to follow a parabolic path. Provided that we can neglect air resistance, there will be no forces exerted on the cannonball in the horizontal direction, such that $a_x = 0$. In the vertical direction, we expect a downward acceleration due to gravity. Choosing the positive y axis to point upward, we have $a_y = -9.8 \text{ m/s}^2 = -g$.

If we allow substitutions, we could in principle use any of the kinematic equations. But we are given information about the accelerations, components of initial velocity, and displacements in both directions. This means that one simple way of writing the positions as functions of time would be to use:

$$x_\text{f} = x_\text{i} + v_{x\text{i}}\Delta t + \frac{1}{2}a_x(\Delta t)^2 \tag{2}$$

$$y_\text{f} = y_\text{i} + v_{y\text{i}}\Delta t + \frac{1}{2}a_y(\Delta t)^2 \tag{3}$$

We will use these in the following section.

2. Devise Plan Equations (2) and (3) above give the displacements in the x and y directions as functions of time. Since motion in both directions begins and ends at the same times, the Δt s in the above expressions refer to the same variable. We would eventually like to have the vertical position as a function of horizontal position. So we could rearrange equation (2) to give us the time as a function of horizontal position, and insert this in place of Δt in equation (3). Rearranging equation (2), and using the fact that $a_x = 0$, we have

$$x_\text{f} = x_\text{i} + v_{x\text{i}}\Delta t$$

or (4)

$$\Delta t = \frac{x_\text{f} - x_\text{i}}{v_{x\text{i}}}$$

Inserting equation (4) into equation (3) yields

$$y_\text{f} = y_\text{i} + v_{y\text{i}}\left(\frac{x_\text{f} - x_\text{i}}{v_{x\text{i}}}\right) + \frac{1}{2}a_y\left(\frac{x_\text{f} - x_\text{i}}{v_{x\text{i}}}\right)^2 \tag{5}$$

This expression depends on the x and y components of the initial velocity. We can write both in terms of given quantities using simple trigonometry: $v_{xi} = v_i \cos(\theta)$ and $v_{yi} = v_i \sin(\theta)$. Inserting these into equation (5) yields

$$y_f = y_i + v_i \sin(\theta)\left(\frac{x_f - x_i}{v_i \cos(\theta)}\right) + \frac{1}{2}a_y\left(\frac{x_f - x_i}{v_i \cos(\theta)}\right)^2 \tag{6}$$

This can be rearranged to obtain equation (1) from the problem statement.

3. Execute Plan Combining sines and cosines and cancelling identical factors in equation (6) yields exactly the result we were asked to derive in part (*a*):

$$y_f = y_i + \tan(\theta)(x_f - x_i) - \frac{g}{2(v_i \cos(\theta))^2}(x_f - x_i)^2$$

If we replace y_f with $y(x)$, and x_f with x, this is identical to equation (1):

$$y(x) = y_i + \tan(\theta)(x - x_i) - \frac{g}{2(v_i \cos(\theta))^2}(x - x_i)^2 \tag{7}$$

For part (*b*) we are prompted to choose a different origin for our coordinates. In principle, one could use any origin, but we choose to place our origin right at the muzzle of the cannon, such that $y_i = x_i = 0$, $x = d$, and $y(d) = -h$. Inserting these values into equation (7) we find

$$h = -\tan(\theta)d + \frac{g}{2(v_i \cos(\theta))^2}d^2 \tag{8}$$

which gives us the height h in terms of other given variables.

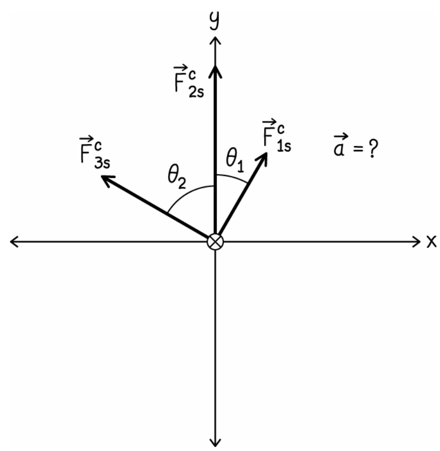
4. Evaluate Result For small x, the first term in equation (1) in the problem statement should be larger than the second term. Since that first term is positive, this means that when the cannonball has not moved forward very much yet (small x), its vertical position should be increasing. This makes sense since the cannon is oriented above horizontal; the cannonball must first rise to some maximum height and then fall. As x becomes large, the second term in equation (1), the negative quadratic term, should dominate. This means that after the cannonball has moved a large distance in x, its vertical position should be decreasing. This clearly corresponds to the cannonball falling toward Earth, and makes perfect sense.

Turning our attention to equation (8), the initial height increases as the distance travelled increases. This is reasonable. It is also reasonable that the height should be proportional to the horizontal distance squared. To see this, note that the speed in the horizontal direction is constant, whereas the speed in the vertical direction is changing due to gravity. This means that once the cannonball is descending, its speed will be increasing, such that doubling the initial height would not be sufficient to double the flight time (and therefore double the horizontal distance travelled). The height would need to increase by more than a factor of two. So this quadratic dependence on horizontal distance makes sense. It also makes sense that the required initial height should decrease as the horizontal component of the velocity increases, which it does. To check the dependence on the launch angle, let us consider a limiting case, where θ approaches $90°$. This corresponds to launching the cannonball straight up. As we approach that limit, the horizontal distance travelled should become small. In fact, as the launch angle gets extremely close to $90°$, we should require infinite height for any non-zero horizontal displacement. Since $\tan(\theta)$ and $1/\cos^2(\theta)$ both diverge as θ approaches $90°$, we see the equation obeys the expected divergence, unless $d = 0$ exactly. Thus equation (8) has the dependence that we expect on θ, d, and v_i.

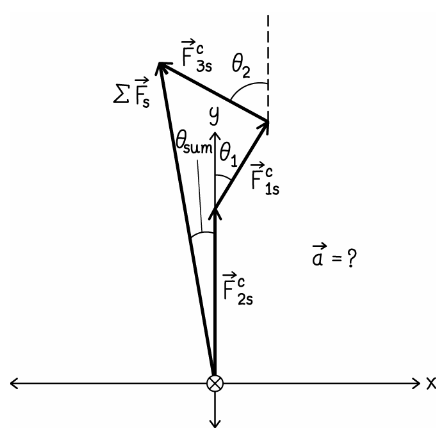
10.4 Push that shed

1. Getting Started It is certainly possible to obtain an answer using a carefully-drawn picture; the graphical method should work. However, we use it here only as a guide so that we know approximately what direction and magnitude to expect from our sum of all forces. We will use algebraic vector addition to obtain our numerical answer.

In the direction perpendicular to the ice, the forces exerted on the shed are gravity and the normal force. These counteract each other such that there is no acceleration vertically (perpendicular to the ice). Thus, we need not consider this direction in our free body diagram as it will not contribute to the acceleration of the shed. Including this direction would require a three-dimensional free body diagram, which would be complex, and it would be difficult to extract intuition from such a picture. Thus, we draw a free body diagram including only forces parallel to the ice:



At this early stage we can go ahead and add the force vectors graphically to gain intuition about our sum of all forces. We obtain:



From this figure, we expect our sum of all forces to be approximately 10^2 N slightly west of north, by approximately $10°$. We can use this picture to check our algebraic answer.

2. Devise Plan In constructing the diagrams above, we chose the $+x$ axis to point east, and the $+y$ axis to point north. Any choice of axis is possible, but it certainly makes sense to choose at least one axis parallel to a force. Since forces $\vec{F}^c_{1s}$ and $\vec{F}^c_{3s}$ are at a $90°$ angle to each other, one might choose the $+y$ and $+x$ axes parallel to these respective vectors. This would also have been a perfectly valid choice. We chose to align an axis with geographic north, simply because information is given relative to that direction.

We can write the sum of all components of forces acting on the shed in each of the directions defined.

$$\sum F_x = F^c_{1s\,x} + F^c_{3s\,x} = ma_x \tag{1}$$

$$\sum F_y = F^c_{1s\,y} + F^c_{2s\,y} + F^c_{3s\,y} = ma_y \tag{2}$$

Inserting the appropriate signs and trigonometric functions into equations (1) and (2), we obtain

$$\sum F_x = F^{c}_{1s}\sin(\theta_1) - F^{c}_{3s}\sin(\theta_2) = ma_x \tag{3}$$

$$\sum F_y = F^{c}_{1s}\cos(\theta_1) + F^{c}_{2s} + F^{c}_{3s}\cos(\theta_2) = ma_y \tag{4}$$

Since we have all force magnitudes, and directions, as well as the inertia of the shed, we have sufficient information to find the vector sum of all forces, and thus the acceleration.

3. Execute Plan Inserting the given numerical values to the sums of force components in equations (3) and (4), we obtain

$$\sum F_x = (32\text{ N})\sin(30°) - (41\text{ N})\sin(60°) = -19.5\text{ N} = ma_x \tag{5}$$

$$\sum F_y = (32\text{ N})\cos(30°) + (55\text{ N}) + (41\text{ N})\cos(60°) = 103.2\text{ N} = ma_y \tag{6}$$

At this point, we could either calculate the components of the acceleration and then combine them, or calculate the magnitude and direction of the sum of all forces and thus find the magnitude and direction of the acceleration. We choose to do the latter, simply because it will allow us to compare our calculated sum of all forces to our diagrammatic solution.

$$F = \left(\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2\right)^{1/2} = ((-19.5\text{ N})^2 + (103.2\text{ N})^2)^{1/2} = 105\text{ N}$$

and

$$\theta_{\text{sum}} = \tan^{-1}\left(\left|\sum F_x\right| / \left|\sum F_y\right|\right) = \tan^{-1}((19.5\text{ N})/(103.2\text{ N})) = 10.7°$$

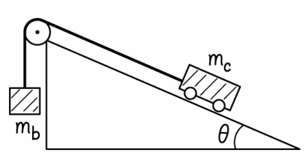
This is consistent with our expectations from our diagram.

Incidentally, if we were to calculate the components of acceleration, we would find $a_x = -0.163\text{ m/s}^2$ and $a_y = 0.860\text{ m/s}^2$. The acceleration will be in the same direction as the sum of all forces. The magnitude of the acceleration is given by $a = F/m = (105\text{ N})/(120\text{ kg}) = 0.875\text{ m/s}^2$, or equivalently by $a = (a_x^2 + a_y^2)^{1/2} = ((-0.163\text{ m/s}^2)^2 + (0.860\text{ m/s}^2)^2)^{1/2} = 0.875\text{m/s}^2$. Thus, to the precision given, the acceleration of the shed is 0.88 m/s^2 at $11°$ west of north.

4. Evaluate Result The main assumption we made was that friction could be ignored. Claiming that the ice is truly frictionless is not reasonable, since the fishermen must be able to gain some traction in order to push the shed. However, they may be wearing special shoes that break the otherwise smooth ice and allow them to gain more traction than the shed. It is certainly reasonable to claim that ice has a low coefficient of friction, and it is a fair guess that it might be ignored. More information would be required to determine how good of an approximation this is.
The final direction of the acceleration is in keeping with our diagrammatic expectation. The magnitude of the acceleration is also reasonable. Note that it is less than 1/10 the acceleration due to gravity.

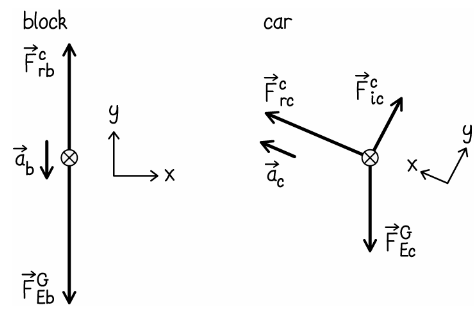
10.6 Moving a load

1. Getting Started The situation described is shown below.



In part (a) we are asked to find acceleration, which is a quantity most easily obtained by considering forces, as opposed to energy. When the acceleration of the coal car is zero the car will move with a constant speed. Thus inserting an acceleration of zero will allow us to solve for the relationship between variables required in part (b).
The motion of the block and that of the coal car are coupled, in that the rope connecting the two objects has a fixed length. Since the distance between objects along the length of the rope is constant, the speeds of the car and the block must be equal. Since that must be true at all times, their accelerations must also be equal in magnitude. They are, however moving along different directions, so we have to be careful to relate motion along one axis to motion along the other axis correctly.

The free body diagrams for the two objects are shown below.



2. Devise Plan If we treat the pulley as being ideal (zero inertia and no friction), then the tension everywhere in the rope is the same. This means that the force that the rope exerts on the coal car must be the same as in magnitude as the force exerted by the rope on the block.

We have chosen our $+y$ axis perpendicular to the incline along which the coal car moves. We could write Newton's second law in the y direction, but there is no reason to do so since the cart is not accelerating perpendicular to the incline. The acceleration we need is along the incline, meaning we need only consider motion in the x direction. Newton's second law yields

$$\sum F_{cx} = F^{c}_{rcx} + F^{G}_{Ecx} = m_{c}a_{cx}$$

Using the given geometry, and what we know about gravitational forces, we obtain

$$F^{c}_{rc} - m_{c}g\sin(\theta) = m_{c}a_{cx} \tag{1}$$

The case for the block is similar, in that the acceleration in the x direction is zero, and we need only consider motion in the y direction. Newton's second law applied to the block yields

$$\sum F_{by} = F^{c}_{rby} + F^{G}_{Eby} = m_{b}a_{by}$$

Inserting the known form of gravitational forces on Earth we obtain

$$F^{c}_{rb} - m_{b}g = m_{b}a_{by} \tag{2}$$

So far we have two equations and four unknowns. But we know that the tension is equal throughout the rope, allowing us to write $F^{c}_{rby} = F^{c}_{rcx}$. Note that both are positive. We also know that the accelerations of the two objects must be equal in magnitude, but note that as the block falls its acceleration in the y direction is negative, whereas the coal car must accelerate in the positive x direction. Thus $a_{cx} = -a_{by}$. We can now rewrite equations (1) and (2) in terms of only two variables:

$$F^{c}_{rc} - m_{c}g\sin(\theta) = m_{c}a_{cx} \tag{3}$$

$$F^{c}_{rc} - m_{b}g = -m_{b}a_{cx} \tag{4}$$

Now we have two equations and two unknowns such that we can solve for the acceleration of the coal car in terms of other given quantities. In the special case of constant speed for part (b), the acceleration is zero, and we simply find a relationship between the inertia of the block and other given quantities.

3. Execute Plan Rearranging equation (4) we find an expression for the force the rope exerts on the coal car:

$$F_{\text{rc}}^{\text{c}} = m_{\text{b}}(g - a_{\text{c}x}) \tag{5}$$

Inserting equation (5) into equation (3) and rearranging, we find

$$a_{\text{c}x} = \frac{g(m_{\text{b}} - m_{\text{c}}\sin(\theta))}{(m_{\text{b}} + m_{\text{c}})} \tag{6}$$

Equation (6) is the result required in part (*a*). To answer part (*b*), we set the left hand side of equation (6) to zero, such that the speed is constant. In order for the right hand side of equation (6) to be zero, we find

$$m_{\text{b}} = m_{\text{c}}\sin(\theta) \tag{7}$$

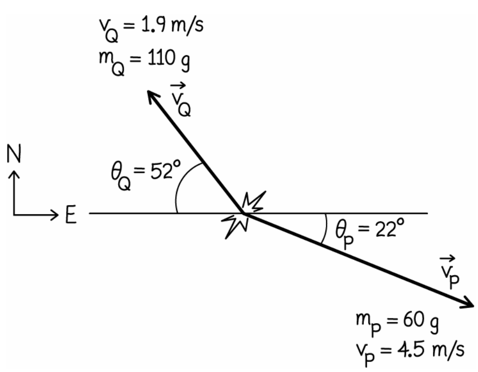
4. Evaluate Result Equation (6) above shows us the dependence of the acceleration of the coal car on the two inertias. We can see that if the inertia of the coal car becomes much larger than the inertia of the block, the acceleration will be negative, meaning the coal car will slide down the incline. If the inertia of the block is greater than that of the coal car, then the coal car will accelerate up the incline regardless of the angle. We also note that as the angle of inclination goes to zero, the block can accelerate the coal car regardless of the inertia of the coal car (only because we have ignored friction here). As θ approaches $90°$, the situation essentially becomes an Atwood's machine. In that case, we obtain an expression of the form obtained in Guided Problem 8.6:

$$a_{\text{c}x} = \frac{g(m_{\text{b}} - m_{\text{c}})}{(m_{\text{b}} + m_{\text{c}})}$$

10.8 Hockey with a bang

1. Getting Started We are asked to determine the velocity of the puck just before the firecracker explodes. If we ignored any frictional forces that the ice may exert on the puck, then no forces do work on the puck-firecracker system, so we choose our system to consist of those two objects only. One relatively simple way of attacking this problem is by using conservation of momentum. Since no external forces cause any acceleration of the system, the momentum of the whole puck before the explosion must equal the sum of the momenta of the two pieces after the explosion.

We can start by drawing a fairly accurate drawing of the velocities described after the explosion:



2. Devise Plan Using the diagram above, we can immediately write down an expression relating initial and final momenta in the east and north directions. Here, we choose east to be the $+x$ direction, and north to be the $+y$ direction, such that $p_{x\text{i}} = p_{x\text{f}}$ and $p_{y\text{i}} = p_{y\text{f}}$. Then we have

$$p_{\text{PQ}x\text{i}} = p_{\text{P}x\text{f}} + p_{\text{Q}x\text{f}}$$

$$p_{\text{PQ}y\text{i}} = p_{\text{P}y\text{f}} + p_{\text{Q}y\text{f}}$$

Rewriting in terms of velocity components and inertias, we find

$$(m_{\rm P}+m_{\rm Q})v_{{\rm PQ}x{\rm i}} = m_{\rm P}v_{{\rm P}x{\rm f}} + m_{\rm Q}v_{{\rm Q}x{\rm f}}$$

$$(m_{\rm P}+m_{\rm Q})v_{{\rm PQ}y{\rm i}} = m_{\rm P}v_{{\rm P}y{\rm f}} + m_{\rm Q}v_{{\rm Q}y{\rm f}}$$

Solving for the initial components of velocity, and inserting some of the geometry shown above, we can write

$$v_{{\rm PQ}x{\rm i}} = \frac{m_{\rm P}v_{\rm Pf}\cos(\theta_{\rm P}) - m_{\rm Q}v_{\rm Qf}\cos(\theta_{\rm Q})}{(m_{\rm P}+m_{\rm Q})} \qquad (1)$$

$$v_{{\rm PQ}y{\rm i}} = \frac{-m_{\rm P}v_{\rm Pf}\sin(\theta_{\rm P}) + m_{\rm Q}v_{\rm Qf}\sin(\theta_{\rm Q})}{(m_{\rm P}+m_{\rm Q})} \qquad (2)$$

We have all required values to determine the initial components of the velocity of the puck.

3. Execute Plan Inserting the given values into equations (1) and (2), we find

$$v_{{\rm PQ}x{\rm i}} = \frac{(0.060\text{ kg})(4.5\text{ m/s})\cos(22°) - (0.110\text{ kg})(1.9\text{ m/s})\cos(52°)}{((0.060\text{ kg})+(0.110\text{ kg}))} = 0.72\text{ m/s} \qquad (3)$$

$$v_{{\rm PQ}y{\rm i}} = \frac{-(0.060\text{ kg})(4.5\text{ m/s})\sin(22°) + (0.110\text{ kg})(1.9\text{ m/s})\sin(22°)}{((0.060\text{ kg})+(0.110\text{ kg}))} = 0.37\text{ m/s} \qquad (4)$$

These components completely define the initial velocity and might be an acceptable final answer: $\vec{v}_{\rm PQi} = (0.72\text{ m/s})\hat{\imath} + (0.37\text{ m/s})\hat{\jmath}$. But we can also express it in terms of the magnitude of the initial velocity and its direction as an angle $\theta_{\rm i}$ north of east:

$$v_{\rm PQi} = \sqrt{v^2_{{\rm PQ}x{\rm i}} + v^2_{{\rm PQ}y{\rm i}}} = \sqrt{(0.716\text{ m/s})^2 + (0.374\text{ m/s})^2} = 0.81\text{ m/s}$$

and

$$\theta_{\rm i} = \tan^{-1}\left(\frac{v_{{\rm PQ}y{\rm i}}}{v_{{\rm PQ}x{\rm i}}}\right) = \tan^{-1}\left(\frac{0.374\text{ m/s}}{0.716\text{ m/s}}\right) = 28°$$

So the initial velocity of the puck must have been 0.81 m/s at $28°$ north of east.

4. Evaluate Result The consideration of momentum conservation uniquely determined the initial velocity, so it is not possible that the puck had any other initial velocity. Specifically, it is not possible that the puck was initially at rest.

The initial velocity is reasonable. The fact that it is less than the magnitude of either final velocity is not problematic, because mechanical energy of the puck pieces is not conserved during the explosion. The pieces can be sped up upon conversion of chemical potential energy in the firecracker to kinetic energy.

Questions and Problems

10.1. The trajectory would be parabolic. One can prove this by writing out $x(t)$ and $y(t)$ and manipulating them to find the quadratic expression for $x(y)$. But it is easier to note that the motion of objects in freefall is parabolic, a situation in which the velocity is constant in one direction and the acceleration is constant in a perpendicular direction. The situation in this problem is equivalent, just turned on its side.

10.2. (*a*) Parabolic. The motion you observe is similar to the motion of a ball that is falling, but that also has a constant horizontal component to its velocity. (*b*) Yes, the slope would appear linear. To see this put the origin at the original position of the ball, and the let the train and ball have the same *x* position at time $t = 0$. If the train is accelerating from rest, we can write its position as $x(t) = \frac{1}{2}a_x t^2$, and the position of the falling ball in the Earth reference frame will be $y(t) = -\frac{1}{2}gt^2$. Solving the equation in x for time, and inserting it into the equation describing motion in *y* yields $y(x) = -\frac{g}{a_x}x$, which is of course linear.

10.3. 0.214. The shortest path any object could take between the initial and final positions of the ball is obvious a straight line path, which in this case would have a length of $\ell = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.0600\text{ m})^2 + (0.200\text{ m})^2} =$ 0.209 m. Clearly, the ball did not follow a straight line path, so the distance covered by the ball must be greater than 0.209 m. The only one of those answers that could be correct is 0.214 m.

10.4. Yes, in circular motion with a constant speed.

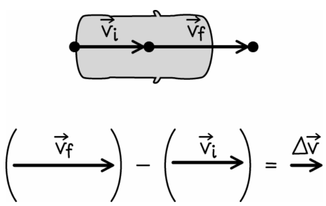
10.5. Ignoring air resistance, the acceleration is downward (toward the center of Earth). Not ignoring air resistance, the acceleration is mostly down and a little bit in the direction opposite the velocity.

10.6. Yes, if the direction of motion changes but not the speed. This is the case in circular motion at constant speed.

10.7. (*a*) No. If $\vec{A} + \vec{B} = 0$, then $\vec{A} = -\vec{B}$, and $A = B$. (*b*) Yes. As a simple example, consider the vectors $\vec{A} = (3.0\text{ units})\,\hat{\imath}$, $\vec{B} = (4.0\text{ units})\,\hat{\jmath}$, and $\vec{C} = (5.0\text{ units})$ at an angle of $233°$ counterclockwise from the $+x$ axis. None of these vectors have the same length, and $\vec{A} + \vec{B} + \vec{C} = \vec{0}$.

10.8. Swimmer A gets across first. The current only affects the component of a swimmer's motion along the direction of the river; it does not affect the motion perpendicular to the river (across). The swimmer that crosses fastest will be the swimmer who has the greatest component of his/her velocity perpendicular to the river, which in this case is swimmer A.

10.9. For each instant, we find the acceleration by looking at the dot immediately preceding and immediately following the instant of interest. In each case we draw a velocity arrow from the previous position to the current position, and from the current position to the next position. The change in these velocity vectors is the acceleration vector. (*a*) We draw this explicitly for the first instant of interest.



Acceleration to right as car speed increases. (*b*) Very small downward acceleration but almost constant speed. (*c*) Downward acceleration with small component to the left as car slows slightly. (*d*) Very small acceleration downward and left, perpendicular to velocity. (*e*) Acceleration upward and left, opposite velocity as car slows. (*f*) Acceleration upward and left, opposite velocity as car stops.

10.10. A car going downhill, because there is a component of the gravitational force pulling the car down the hill. Braking must overcome that force component as well as give the car a negative acceleration. Going uphill, that gravitational force component slows the car down, helping to bring it to a stop.

10.11. No. Earth exerts a gravitational force downward on the sweater. In order to keep the sweater from accelerating downward, there must be some other force, or component of a force acting upward. Since tensile force is exerted along the clothesline, this means that the clothesline cannot be completely horizontal. Increasing the tension would allow the clothesline to become closer to horizontal, but it would never be completely horizontal.

10.12. Call the tension on the left wire $\vec{\mathcal{T}}_{\text{left}}$ and the tension on the right wire $\vec{\mathcal{T}}_{\text{right}}$. We can write equations of motion for the sign in the x and y directions separately:

$$\sum F_x = \vec{\mathcal{T}}_{\text{left},x} + \vec{\mathcal{T}}_{\text{right},x} = ma_x = 0$$

$$\sum F_y = \vec{\mathcal{T}}_{\text{left},y} + \vec{\mathcal{T}}_{\text{right},y} + F^{c}_{\text{Es}y} = ma_y = 0$$

Inserting the relevant geometry, these two equations become

$$-\mathcal{T}_{\text{left}}\sin(40°)+\mathcal{T}_{\text{right}}\cos(20°)=0 \tag{1}$$

$$\mathcal{T}_{\text{left}}\cos(40°)+\mathcal{T}_{\text{right}}\sin(20°)-mg=0 \tag{2}$$

Solving equation (1) for $\mathcal{T}_{\text{right}}$ yields

$$\mathcal{T}_{\text{right}}=\mathcal{T}_{\text{left}}\frac{\sin(40°)}{\cos(20°)} \tag{3}$$

which we can insert into equation (2). Equation (2) can then be solved for $\mathcal{T}_{\text{left}}$. We find

$$\mathcal{T}_{\text{left}}=\frac{mg}{\left(\cos(40°)+\dfrac{\sin(40°)}{\cos(20°)}\sin(20°)\right)}=mg$$

So the tension in the left rope is equal in magnitude to the gravitational force that Earth exerts on the sign. Inserting this result into equation (3) gives us an expression for the tension on the right:

$$\mathcal{T}_{\text{right}}=mg\frac{\sin(40°)}{\cos(20°)}=0.68\,mg$$

So, the tension in the right rope is smaller in magnitude than the gravitational force that Earth exerts on the sign.

10.13. Five forces are exerted:

	parallel to roof ridge	normal to roof surface	tangential to roof surface
gravitational force by Earth, $\vec{F}^{\text{G}}_{\text{Ep}}$		X	X
normal force by roof, $\vec{F}^{\text{n}}_{\text{roof,p}}$		X	
tensile force by left rope, $\vec{F}^{\text{c}}_{\text{left,p}}$	X		X
tensile force by right rope, $\vec{F}^{\text{c}}_{\text{right,p}}$	X	X	X
force of static friction by roof, $\vec{F}^{\text{s}}_{\text{rp}}$	X		X

10.14. (*a*) The force of friction between the cloth and the dishes/glasses is not sufficient to accelerate them at the large acceleration of the cloth. (*b*) If the cloth's acceleration and speed are relatively small, the force of friction between it and the dishes/glasses is large enough to move them so that they eventually reach the table edge and fall off.

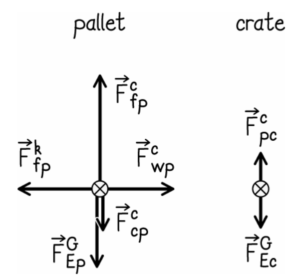
10.15. Static. Although the tires are moving, the surface of the tire and road that touch do not slide across each other.

10.16. (*a*) Both blocks convert the same quantities of kinetic and gravitational potential energies (although the conversions are opposite), and both experience the same amount of energy dissipation. The only difference is whether that dissipation has just begun at the top or bottom of the incline. Block (*b*) has a higher speed near the bottom of the incline than block (*a*). This means that at that position, block (*b*) has not yet experienced much dissipation and must have just begun its trip. So (*a*) is the falling block and (*b*) is the block pushed upward.

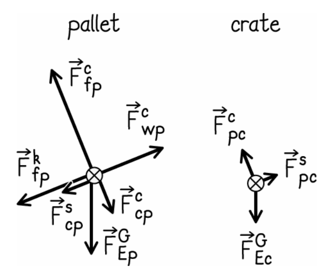
10.17. (*a*) The force of static friction. (*b*) The magnitude is equal to the gravitational force Earth exerts on the eraser. We know this because those are the only two forces acting in the y direction, and they must cancel each other since the eraser is not accelerating upward or downward. (*c*) No, unless you push with insufficient force to hold the eraser up at all. (*d*) It decreases to zero.

10.18. (*a*) Since all blocks are moving at a constant velocity, there is no acceleration. Since a force exerted by A on block C, $F^{\text{c}}_{\text{AC}x}$ would accelerate block C (no other forces are being exerted on C that could cancel counteract this force), that force must equal zero. If that force is zero, then the only remaining force component acting in the horizontal direction on A is $F^{\text{c}}_{\text{BA}x}$. This must also equal zero, otherwise block A would accelerate. Newton's third law tells us that $F^c_{BAx} = -F^c_{ABx}$. Thus $F^c_{ABx} = F^c_{ACx} = 0$. (*b*) Call the horizontal component of the acceleration of the system a_x. Since F^c_{ACx} is the only force acting on block CL, we can say $F^c_{ACx} = m_C a_x$. Block A must have the same acceleration, so the equation of motion for block A reads $F^c_{BAx} + F^c_{CAx} = m_A a_x$, or $F^c_{BAx} = m_A a_x + F^c_{ACx} > F^c_{ACx}$. So $F^c_{BAx} > F^c_{ACx}$.

10.19. (*a*)



(*b*)



10.20. (*a*) There is no frictional force being exerted on the cup. (*b*) Left. Friction opposes the relative motion between the cup and table, keeping the cup from moving as quickly across the table. (*c*) Right, if the table is being accelerated to the right. Friction opposes the relative motion between the cup and table, keeping the cup moving along with the table as the table is moved. If the table is moving to the right at a constant velocity, then there is no frictional force exerted on the cup.

10.21. The normal force does positive work on the saw as the board and saw are lifted, no work as they are transported horizontally, and negative work as they are lowered onto the sawhorses; the algebraic sum of these work values is positive, because the saw ends up at a greater elevation than it started. The force of static friction does no work on the saw as the board and saw are lifted, positive work as their speed increases from zero to carrying speed, negative work as their speed decreases from carrying speed to zero at the sawhorses, and no work as they are lowered onto the horses; the algebraic sum of these work values is zero.

10.22. (*a*) Let the $+x$ axis point up the incline, and the $+y$ axis be normal to the incline. Call the angle between the incline and horizontal ground θ. The equation of motion for the skier in the x direction reads $\sum F_x = F^c_{\text{rope s }x} + F^G_{\text{Es}x} = ma_x = 0$. So $F^c_{\text{rope s }x} = -F^G_{\text{Es}x} = +mg\sin(\theta) = (55\text{ kg})(9.8\text{ m/s}^2)\sin(15°) = 1.4\times10^2\text{ N}$.
(*b*) $W = F^c_{\text{rope s }x}\Delta x = (1.4\times10^2\text{ N})(100\text{ m}) = 14\text{ kJ}$.

10.23. (*a*) When the car accelerates, the string will have to accelerate the washer along with it. Simultaneously, the string is supporting the washer against the gravitational force exerted by Earth on the washer. Let the car be accelerating in the $+x$ direction, let the $+y$ direction be vertically upward, and call the angle between the string and a vertical line θ. We can write the equations of motion in the x and y separately:

$$\sum F_x = \mathcal{T}_{\text{rw}x} = ma_x \tag{1}$$

$$\sum F_y = F^G_{\text{Ew}y} + \mathcal{T}_{\text{rw}y} = ma_y = 0 \tag{2}$$

We can rewrite equations (1) and (2) using simple geometry, and rearrange slightly to obtain:

$$\mathcal{T}_{\text{rw}}\sin(\theta) = ma_x \tag{3}$$

$$\mathcal{T}_{\text{rw}}\cos(\theta) = mg \tag{4}$$

If we now divide equation (3) by equation (4), or equivalently, solve equation (3) for the magnitude of the tension and insert this into equation (4), we obtain $\tan(\theta) = \dfrac{a_x}{g}$ or

$$\theta = \arctan\left(\frac{a_x}{g}\right) = \arctan\left(\frac{v_{x,\text{f}} - v_{x,\text{i}}}{g\Delta t}\right) = \arctan\left(\frac{(26.8\text{ m/s}) - (0)}{(9.8\text{ m/s}^2)(5.1\text{ s})}\right) = 28°$$

(*b*) Most of this question was worked out in solving (*a*), but we give a complete description here. In the situation described, the frictional force would be the only force acting on the washer in the horizontal direction. So $F^{\text{f}}_{\text{dw}x} = \; ma_x = m\dfrac{(v_{x,\text{f}} - v_{x,\text{i}})}{\Delta t}$. We know the magnitude of the gravitational force that Earth exerts on the washer is mg. So the ratio of the magnitude of the frictional force to that of the gravitational force is $\dfrac{F^{\text{f}}_{\text{dw}x}}{F^{G}_{\text{Ew}y}} = \dfrac{m(v_{x,\text{f}} - v_{x,\text{i}})}{mg\Delta t} = \dfrac{((26.8\text{ m/s}) - (0))}{(9.8\text{ m/s}^2)(5.1\text{ s})} = 0.54.$

10.24. The force of static friction does positive work on the car whenever the car speeds up and negative work whenever it slows to a stop. So as you drive onto the ferry and stop, the force first does some amount of positive work and then an equal amount of negative work. When the ferry starts moving, the force does some amount of positive work on the car, and then an equal amount of negative work as the ferry slows to a stop. This is also true when you drive off the ferry and come to a stop at a stop sign. Thus, algebraic sum of all the work done by the force of static friction is zero.

10.25. (*a*) They have the same speed at the bottom of the slide, but the child on the steeper slide gets to the bottom first. To see that they have the same speed at the bottom of the slide, consider conservation of energy. Since we are initially ignoring dissipation, we have the simple case of converting initial gravitational potential energy into kinetic energy: $\Delta U^G = -\Delta K$ or $mg\Delta y = \frac{1}{2}mv_\text{f}^2$. Since the inertia appears on both sides and can be cancelled, and since both children traverse the same vertical distance, the final speed will be the same for the two children. However, for the child on the steeper slide, more of his velocity is directed downward at any given time. This will cause him to cross the vertical distance faster than the child on the less steep slide. Another way of thinking about this is that the steeper slide is actually shorter than the less steep slide, so the child on the steeper slide has a shorter distance (total, vertical and horizontal together) to cross. (*b*) We can approach this problem using energy again, but this time there is dissipation. We can write $\Delta U^G - \Delta E^{\text{th}} = -\Delta K$. We have already established that in the absence of dissipation both children will have the same speed. Thus the child that will now have the greatest speed at the bottom is the child that experiences the smallest amount of dissipation. We will assume the materials of the slides and of the children's clothing are identical, so that the same coefficient of kinetic friction applies for each child. Let the $+x$ axis point down the slide and parallel to it, while the $+y$ axis points upward and normal to the slide. The work done by friction will be equal to $W = \vec{F}^{\text{f}}_{\text{sc}} \cdot \Delta\vec{\ell} = -\mu_k F^n_{\text{sc}}\Delta\ell$, where $\Delta\ell$ is the length of the slide. This is different for the two slides; what is identical between them is the height. So we can write $\Delta\ell = \dfrac{\Delta y}{\cos(\theta)}$, where is the angle the slide makes with the vertical. We also know that the two children will likely experience different normal forces. We can check this by writing the equations of motion in *x* and *y*:

$$\sum F_x = F^G_{\text{Ec}x} + F^{\text{f}}_{\text{sc}x} = ma_x \tag{1}$$

$$\sum F_y = F^G_{\text{Ec}y} + F^n_{\text{sc}y} = ma_y = 0 \tag{2}$$

Equation (2) is the most important here. We can insert known values and rearrange to obtain $F^n_{\text{sc}y} = F^n_{\text{sc}} = -F^G_{\text{Ec}y} = mg\sin(\theta)$. Here we have used that the normal force is by definition entirely along the axis that we have chosen as our $+y$ axis, and we have used simple geometry to find that the component of gravity that lies along this axis is $-mg\sin(\theta)$. Now we can rewrite the work done by friction as $W = -\mu_k mg\Delta y\tan(\theta)$. Since $\tan(\theta)$

monotonically increases on the interval $0 < \theta < 90°$, clearly this work done by friction will be smaller for the smaller angle. So the child on the steeper slide will experience less dissipation and will have the greater speed at the bottom of the slide.

10.26. (*a*) $\vec{A}+\vec{B}=((2.0\ \hat{i})+(3.0\ \hat{j}))+((-4.0\ \hat{i})+(5.0\ \hat{j}))=-2.0\ \hat{i}+8.0\ \hat{j}$
(*b*) $\vec{A}-\vec{B}=((2.0\ \hat{i})+(3.0\ \hat{j}))-((-4.0\ \hat{i})+(5.0\ \hat{j}))=6.0\ \hat{i}-2.0\ \hat{j}$
(*c*) $\left|\vec{A}+\vec{B}\right|$ is largest when the two vectors point in exactly the same direction, such that their magnitudes can just be added. In that case $\left|\vec{A}+\vec{B}\right|=\left|((2.0\ \hat{i})+(3.0\ \hat{j}))\right|+\left|((-4.0\ \hat{i})+(5.0\ \hat{j}))\right|=\sqrt{((2.0)^2+(3.0)^2)}+\sqrt{((-4.0)^2+(5.0)^2)}$ $=1.0\times10^1$ units. The smallest value of $\left|\vec{A}+\vec{B}\right|$ occurs when the two vectors point in opposite directions, such that their magnitudes can simply be subtracted. In that case $\left|\vec{A}+\vec{B}\right|=-\left|((2.0\ \hat{i})+(3.0\ \hat{j}))\right|+\left|((-4.0\ \hat{i})+(5.0\ \hat{j}))\right|=$ $-\sqrt{((2.0)^2+(3.0)^2)}+\sqrt{((-4.0)^2+(5.0)^2)}=2.8$ units.

10.27. (*a*) $\vec{A}+\vec{B}=((3.0\ \hat{i})+(2.0\ \hat{j}))+((-2.0\ \hat{i})+(2.0\ \hat{j}))=1.0\ \hat{i}+4.0\ \hat{j}$ (*b*) $\left|\vec{A}+\vec{B}\right|=\left|1.0\ \hat{i}+4.0\ \hat{j}\right|=$ $\sqrt{(1.0)^2+(4.0)^2}=\sqrt{17}=4.1$ units.

10.28. (*a*) We find the radial distance from the origin to which the vector points by finding the magnitude of $\vec{A}$: $r=\left|\vec{A}\right|=\left|((3.0\ \hat{i})+(2.0\ \hat{j}))\right|=\sqrt{(3.0)^2+(2.0)^2}=3.6$ units. The angle from the positive x axis is found by taking $\theta=\arctan\left(\frac{A_y}{A_x}\right)=\arctan\left(\frac{2.0}{3.0}\right)=34°$. So the polar coordinates are $r=3.6$ units, and $\theta=34°$.
(*b*) As in part (*a*) we find $r=\left|\vec{B}\right|=\left|\left((-2.0\ \hat{i})+(2.0\ \hat{j})\right)\right|=\sqrt{(-2.0)^2+(2.0)^2}=2.8$ units. The angle from the positive x axis is slightly tricky here, although the solving process is the same. It is still true that $\theta=\arctan\left(\frac{A_y}{A_x}\right)=\arctan\left(\frac{2.0}{-2.0}\right)$. However, many calculators will return an answer of $-45°$. That is clearly wrong. It is in the wrong quadrant. What your calculator has done in that instance is calculate $\arctan\left(\frac{-2.0}{2.0}\right)$ (the minus sign on the other component). It's up to you to recognize that this answer is in the wrong quadrant and that you actually want the angle that yields the same size of the ratio $\frac{A_y}{A_x}$, that is $180°$ from this answer of $-45°$. The correct angle is $135°$. If this is troubling you, do a quick check with your calculator that indeed, $\tan(135°)=\frac{2.0}{-2.0}=\frac{A_y}{A_x}$. The polar coordinates are $r=2.8$ units and $\theta=135°$.

10.29. (*a*) This problem is far easier if we do part (*b*) first as part of solving part (*a*). We label the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in the order they are listed, and write them in terms of Cartesian coordinates such that $\vec{A}=(36\text{ m})\ \hat{i}+(0)\ \hat{j}$, $\vec{B}=(0)\ \hat{i}+(-42\text{ m})\ \hat{j}$, and $\vec{C}=(25\text{ m})\cos(135°)\ \hat{i}+(25\text{ m})\sin(135°)\ \hat{j}=(-17.7\text{ m})\ \hat{i}+(17.7\text{ m})\ \hat{j}$, where we have used that northwest refers to a direction $135°$ counterclockwise from the $+x$ axis. We add these vectors to find $\vec{A}+\vec{B}+\vec{C}=((36\text{ m})\ \hat{i}+(0)\ \hat{j})+((0)\ \hat{i}+(-42\text{ m})\ \hat{j})+((-17.7\text{ m})\ \hat{i}+(17.7\text{ m})\ \hat{j})=((36\text{ m})+(-17.7\text{ m}))\ \hat{i}+((-42\text{ m})$ $+(17.7\text{ m}))\ \hat{j}=(18.3\text{ m})\ \hat{i}+(-24.3\text{ m})\ \hat{j}$. Now that we have the sum of the vectors in component form, we find the magnitude of the vector sum. Let us call $\vec{r}\equiv\vec{A}+\vec{B}+\vec{C}$. Then $r=\left|\vec{A}+\vec{B}+\vec{C}\right|=\left|(18.3\text{ m})\ \hat{i}+(-24.3\text{ m})\ \hat{j}\right|=$

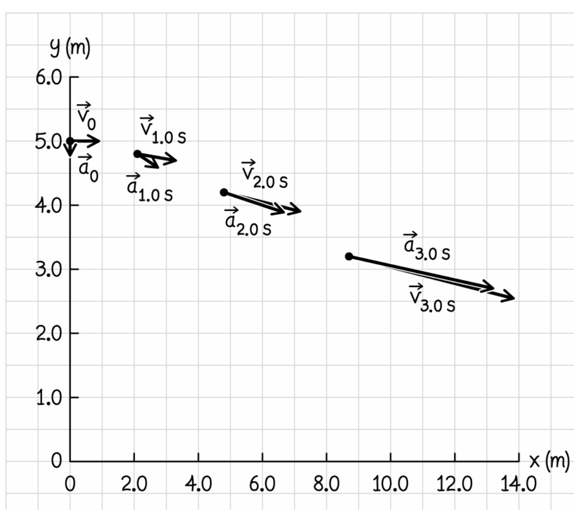
$\sqrt{(18.3\text{ m})^2+(-24.3\text{ m})^2}=30\text{ m}$. The angle from the positive $+x$ axis is found by $\theta=\arctan\left(\dfrac{A_y+B_y+C_y}{A_x+B_x+C_x}\right)$ or equivalently, just $\theta=\arctan\left(\dfrac{r_y}{r_x}\right)=\arctan\left(\dfrac{-24.3\text{ m}}{18.3\text{ m}}\right)=-53°$. So the polar coordinates are $r=30\text{ m}$ at $53°$ clockwise from east. (*b*) As shown in part (*a*) the Cartesian coordinates are $\vec{A}+\vec{B}+\vec{C}=(18\text{ m})\,\hat{\imath}+(-24\text{ m})\,\hat{\jmath}$ or equivalently 18 m east and 24 m south.

10.30. (*a*) Let the $+x$ axis point to the right, and the $+y$ axis point vertically upward. Since the platform is not accelerating in the horizontal direction, we can write $\sum F_x=\mathcal{T}_{\text{right p}\,x}+\mathcal{T}_{\text{left p}\,x}=ma_x=0$, or equivalently $\mathcal{T}_{\text{left p}}\cos(45°)=-\mathcal{T}_{\text{right p}}\cos(30°)\Rightarrow\mathcal{T}_{\text{left p}}=-(800\text{ N})\dfrac{\cos(30°)}{\cos(45°)}=9.8\times10^2\text{ N}$. (*b*) We can write the equation of motion for the y direction: $\sum F_y=\mathcal{T}_{\text{right p}\,y}+\mathcal{T}_{\text{left p}\,y}+F^G_{\text{Ep}y}=ma_y=0$ or $m=\dfrac{\mathcal{T}_{\text{right p}\,y}+\mathcal{T}_{\text{left p}\,y}}{g}=\dfrac{1}{(9.8\text{ m/s}^2)}((800\text{ N})\sin(30°)+(980\text{ N})\sin(45°))=1.1\times10^2\text{ kg}$.

10.31. We begin by writing your displacement vectors in terms of given quantities. Call the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ in the order they are listed, and write them in terms of Cartesian coordinates with $+x$ pointing east and $+y$ pointing north, such that $\vec{A}=v(1.0\text{ h})\,\hat{\imath}+(0)\,\hat{\jmath}$, $\vec{B}=v(1.5\text{ h})\cos(45°)\,\hat{\imath}+v(1.5\text{ h})\sin(45°)\,\hat{\jmath}$, $\vec{C}=(0)\,\hat{\imath}-v(1.0\text{ h})\,\hat{\jmath}$, and $\vec{D}=v(2.5\text{ h})\cos(225°)\,\hat{\imath}+v(2.5\text{ h})\sin(225°)\,\hat{\jmath}$. You final displacement from your house $\vec{R}$ will be the vector sum of all these displacements: $\vec{R}=\vec{A}+\vec{B}+\vec{C}+\vec{D}=v((1.0\text{ h})+(1.5\text{ h})\cos(45°)+(0)+(2.5\text{ h})\cos(225°))\,\hat{\imath}+v((0)+(1.5\text{ h})\sin(45°)+(-1.0\text{ h})+(2.5\text{ h})\sin(225°))\,\hat{\jmath}=v(0.293\text{ h})\,\hat{\imath}+v(-1.71\text{ h})\,\hat{\jmath}$. Thus the magnitude of your final displacement from home is $R=v\sqrt{(0.293\text{ h})^2+(-1.71\text{ h})^2}=v(1.73\text{ h})$. Since you walk directly home, this is the distance you will cover, and since you walk at the same speed as before, we can write $v\Delta t=R=v(1.73\text{ h})$. Thus you will require 1.7 h to reach your home.

10.32. (*a*) We simply take the first and second time derivatives of the position to obtain $v(t)=(A+3Bt^2)\hat{\imath}+(-2Dt)\hat{\jmath}$ and $\text{a}(t)=(6Bt)\hat{\imath}+(-2D)\hat{\jmath}$

(*b*)



10.33. Many people will be able to see by inspection that $r = A$ and $\theta = \omega t$. If you cannot, consider the standard procedure of finding the length of the vector r using the Pythagorean Theorem: $r = \sqrt{x^2 + y^2} = \sqrt{(A\cos(\omega t))^2 + (A\sin(\omega t))^2} = \sqrt{A^2(\cos^2(\omega t) + \sin^2(\omega t))} = \sqrt{A^2(1)} = A$. The angle θ from the $+x$ axis is typically found via $\theta = \arctan\left(\frac{r_y}{r_x}\right) = \arctan\left(\frac{A\sin(\omega t)}{A\cos(\omega t)}\right) = \arctan(\tan(\omega t)) = \omega t$.

10.34. Choose the $+x$ axis to point east, and the $+y$ axis to point north. Call the displacement vector of the plane $\vec{r}$. We can find the magnitude of $\vec{r}$ from the information given: $r = v_{\text{plane}}\Delta t_{\text{plane}} = (400 \text{ km/h})(65 \text{ min}) \frac{(1 \text{ h})}{(60 \text{ min})} = 433 \text{ km}$. Now, if we call the angle south of west at which the plane traveled θ, then we can write the components of the car's displacement as $\Delta x_{\text{car}} = -r\cos(\theta)$ and $\Delta y_{\text{car}} = -r\sin(\theta)$. We also know the total distance the car travelled was $d = 600 \text{ km}$, so we can write $|\Delta x_{\text{car}}| + |\Delta y_{\text{car}}| = r(\sin(\theta) + \cos(\theta)) = r(\sin(\theta) + \sqrt{1 - \sin^2(\theta)}) = d$. Equivalently, we can write $1 - \sin^2(\theta) = \left(\frac{d}{r} - \sin(\theta)\right)^2$. If we now change variables and use $\alpha \equiv \sin(\theta)$, then we have a quadratic equation: $2\alpha^2 - \frac{2d}{r}\alpha + \left(\frac{d}{r}\right)^2 - 1 = 0$. This can be solved using the standard quadratic formula to yield the two solutions $\alpha = 0.548$ or $\alpha = 0.836$. These correspond to values of $\theta = 33°$ and $\theta = 57°$. But we are told that the distance the car has to travel west is greater than the distance it has to travel south. So the angle south of west must be less than $45°$. Hence $\theta = 33°$.

10.35. (*a*) Choose the $+x$ axis to be east and the $+y$ axis to be north, and call her displacement vector $\vec{d}$. Then $\vec{d} = (\Delta x)\,\hat{i} + (\Delta y)\,\hat{j} = (5(160 \text{ m}))\,\hat{i} + (3(160 \text{ m}))\,\hat{j}$ such that $d = \sqrt{(5(160 \text{ m}))^2 + (3(160 \text{ m}))^2} = 9.3 \times 10^2 \text{ m}$ or 0.93 km. (*b*) Her average velocity is given by $\vec{v}_{\text{av}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{d}}{\Delta t}$. We know the magnitude of $\vec{d}$ already, so we can immediately find the magnitude of the average velocity: $v_{\text{av}} = \frac{d}{\Delta t} = \frac{(933 \text{ m})}{(15 \text{ min})} \times \frac{(1 \text{ min})}{(60 \text{ s})} = 1.0 \text{ m/s}$. To find the angle north of east, we write $\theta = \arctan\left(\frac{d_y}{d_x}\right) = \arctan\left(\frac{3(160 \text{ m})}{5(160 \text{ m})}\right) = 31°$. So $\vec{v}_{\text{av}} = 1.0 \text{ m/s}$ at $31°$ north of east.

(*c*) The average speed is the distance covered per unit time: $\frac{\Delta x + \Delta y}{\Delta t} = \left(\frac{5(160 \text{ m}) + 3(160 \text{ m})}{(15 \text{ min})}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 1.4 \text{ m/s}$.

10.36. (*a*) One may reasonably be able to see by inspection that $(x', y') = (x\cos(\varphi) + y\sin(\varphi), y\cos(\varphi) - x\sin(\varphi))$. If not, it may be easier to consider what happens when the point P happens to lie on the $+x$ axis itself. Then simple geometry shows that the distance x is related to the distance x' by $x' = x\cos(\varphi)$. Clearly, that is not sufficient in general, because if the point P had been along the $+y$ axis, this expression would give us nonsense. Clearly, for that special case $x' = y\sin(\varphi)$. In general $x' = x\cos(\varphi) + y\sin(\varphi)$. A similar analysis works for y'. (*b*) $(x', y') = ((5.0)\cos(30°) + (2.0)\sin(30°), (2.0)\cos(30°) - (5.0)\sin(30°)) = (5.3 \text{ m}, -0.77 \text{ m})$.

10.37. (*a*) There is only one such arrangement that we call "distinct". Whichever direction the first arrow points, the second must be directed exactly opposite. This could, of course, be done with the vectors lying along any line in two-dimensional space, but we do not consider these to be "distinct patterns". (*b*) There are two such distinct patterns: clockwise and counterclockwise equilateral triangles. (*c*) There are three such distinct patterns: clockwise and counterclockwise squares, and the four arrows lying directly on top of each other in alternating directions. (*d*) There are $N-1$ distinct patterns for N arrows.

10.38. We can determine how long the ball will be in the air by looking at the y component of the ball's motion. Because it is being accelerated by gravity alone in the y direction, we can use kinematic equations. We can write $\Delta y = v_{y,\mathrm{i}}\Delta t + \frac{1}{2}a_y\Delta t^2$. Here $v_{y,\mathrm{i}} = 0$ because the ball is thrown horizontally. Thus the time the ball is in the air is given by $\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-10\text{ m})}{(-9.8\text{ m/s}^2)}} = 1.43\text{ s}$. In the x direction there is no acceleration, so the same kinematic equation applied to motion in the x direction yields $\Delta x = v_{x,\mathrm{i}}\Delta t = (15\text{ m/s})(1.43\text{ s}) = 21\text{ m}$. So the ball will hit the ground 21 m from the building.

10.39. Path (*a*) is the best representation. Path (*b*) is a poor representation because it ends up pointing directly downward, indicating no horizontal component to the velocity; as long as it moves, the cantaloupe always has a horizontal velocity component. Path (*c*) has two big issues: no vertical acceleration at first, then abrupt acceleration at the upper right corner. The acceleration due to gravity should be constant.

10.40. They hit the ground at the same time. In both cases, the bullets have the same initial y component of their initial velocity. Both are accelerated by gravity at exactly the same rate. Everything about their motion in the y direction is identical. More specifically, one can see from $\Delta y = v_{y,\mathrm{i}}\Delta t + \frac{1}{2}a_y\Delta t^2$, that at a given time, the two bullets must have fallen the same vertical distance.

10.41. We can determine how long the bullet is in the air by examining the x component of its motion. If air resistance is ignored, then there is no acceleration of the bullet in the x direction and we can write $\Delta t = \frac{\Delta x}{v_{x,\mathrm{i}}} = \frac{(100\text{ m})}{(650\text{ m/s})} = 0.154\text{ s}$. During this time the bullet is being accelerated downward in the $-y$ direction by gravity. Thus $\Delta y = v_{y,\mathrm{i}}\Delta t + \frac{1}{2}a_y\Delta t^2 = (0) + \frac{1}{2}(-9.8\text{ m/s}^2)(0.154\text{ s})^2 = -0.116\text{ m}$. So the bullet misses the center by 0.116 m (striking the target below the center).

10.42. The shooter needs to compensate for the downward acceleration due to gravity by aiming slightly above the target. How high above depends on the shooter-target distance: the farther the bullet must travel, the greater vertical distance it is deflected. The telescope is angled very slightly down toward the barrel. That way when the telescope points directly at the target, the barrel points slightly upward. The bullet can rise, be pulled down by gravity, and strike the target.

10.43. (*a*) We can use $\Delta y = v_{y,\mathrm{i}}\Delta t + \frac{1}{2}a_y\Delta t^2$, and the fact that the initial velocity was entirely in the x direction, to obtain $\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-0.80\text{ m})}{(-9.8\text{ m/s}^2)}} = 0.40\text{ s}$. (*b*) Ignoring air resistance, there is no acceleration in the x direction. So we can write $\Delta x = v_{x,\mathrm{i}}\Delta t$. Using the time we found in part (*a*), we obtain $v_{x,\mathrm{i}} = \frac{\Delta x}{\Delta t} = \frac{(0.50\text{ m})}{(0.404\text{ s})} = 1.2\text{ m/s}$. So the initial speed of the ball as it rolled off the table was 1.2 m/s.

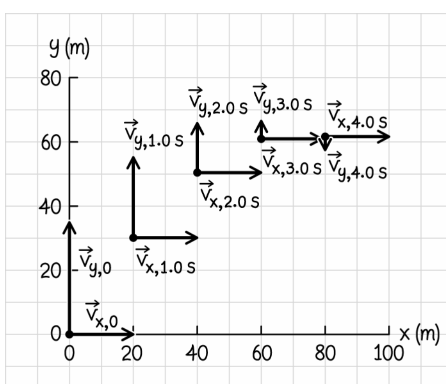
10.44. (*a*) Because the package falls under the influence of gravity and therefore has constant acceleration, we can use kinematic equations like $\Delta y = v_{y,\mathrm{i}}\Delta t + \frac{1}{2}a_y\Delta t^2$. Since the initial velocity upon release is entirely horizontal, we can rearrange this to obtain $\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-200\text{ m})}{(-9.8\text{ m/s}^2)}} = 6.4\text{ s}.$ (*b*) Ignoring air resistance, the package will have a constant x component of it velocity, since there is nothing to accelerate it in the horizontal direction. Thus $\Delta x = v_{x,\text{i}}\Delta t = (15\text{ m/s})(6.39\text{ s}) = 96\text{ m}.$ (*c*) This could be solved using either kinematics or energy methods. We use the former. The final y component of the velocity is given by $v_{y,\text{f}}^2 = v_{y,\text{i}}^2 + 2a_y\Delta y \Rightarrow v_{y,\text{f}} = -\sqrt{v_{y,\text{i}}^2 + 2a_y\Delta y} = -\sqrt{(0) + 2(-9.8\text{ m/s})(-200\text{ m})} = -62.6\text{ m/s}.$ The speed of the package as it lands is $v = \sqrt{v_{x,\text{f}}^2 + v_{y,\text{f}}^2} = \sqrt{(15\text{ m/s})^2 + (-62.6\text{ m/s})^2} = 64\text{ m/s}.$

10.45. (*a*) Never. The y component of displacement grows without bound as time passes. The y position will never again be zero. (*b*) Never. Again, the y component of the velocity is constant. No matter what happens with the x component, there will always be a non-zero y component to the velocity and therefore a nonzero speed. (*c*) The acceleration is given by $\vec{a} = \frac{d\vec{v}}{dt} = (a - 2bt)\,\hat{i} + 0\,\hat{j}.$ So for the acceleration to be zero, we require $t = \frac{a}{2b} = \frac{(14\text{ m/s}^2)}{2(10\text{ m/s}^3)} = 0.70\text{ s}.$

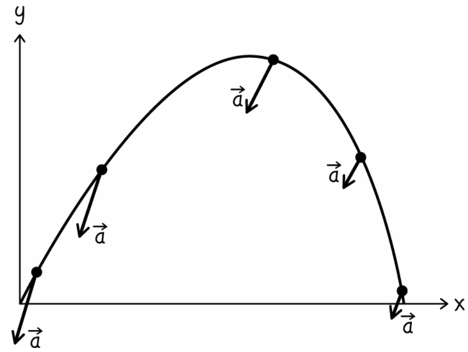
10.46. Example 10.6 in the *Principles* text shows that the expression for the range in such a situation is $\Delta x = v_{x,\text{i}}\left(\frac{2v_{y,\text{i}}}{g}\right).$ For this specific case we can write this as $\Delta x = \frac{2v^2}{g}\sin(\theta)\cos(\theta)$ or $\Delta x = \frac{v^2}{g}\sin(2\theta).$ But $\sin(2(35°)) = \sin(2(55°)),$ so the two shells have the same range. An expression for the time in the air can be found by considering the kinematic equation $v_{y,\text{f}} = v_{y,\text{i}} + a_y\Delta t,$ and noting that the path of the shell is symmetric about its highest point (meaning that $v_{y,\text{f}} = -v_{y,\text{i}}$. Thus $\Delta t = \frac{-2v_{y,\text{i}}}{a_y}.$ So the shell with the greatest initial y component of its velocity will be in the air the longer. Clearly, the shell fired at $55°$ will be in the air longer.

10.47. Air resistance is slowing the ball as it travels. So during the second half of the ball's trip, it will have a lower x component of its velocity, and will not travel as far. Hence the peak is at a position that is more than half the range.

10.48.



10.49.



10.50. If the pitcher threw the ball horizontally, and we ignore air resistance, then we can find the amount of time the ball is in the air by looking at the x direction. We can write $\Delta t = \dfrac{\Delta x}{v_{x,\mathrm{i}}} = \dfrac{(18.4\ \mathrm{m})}{(42\ \mathrm{m/s})} = 0.438\ \mathrm{s}$. The initial y component of the velocity would be zero, such that we can write $\Delta y = v_{y,\mathrm{i}}\Delta t + \dfrac{1}{2}a_y \Delta t^2 = (0) + \dfrac{1}{2}(-9.8\ \mathrm{m/s^2})(0.438\ \mathrm{s})^2$ $= -0.94\ \mathrm{m}$. So the ball will fall a vertical 0.94 m during its trip.

10.51. Let us first look at the horizontal direction. Because there is no acceleration (ignoring air resistance) we can write $\Delta x = v_{x,\mathrm{i}}\Delta t = v\cos(\theta)\Delta t$, or equivalently

$$\Delta t = \frac{\Delta x}{v\cos(\theta)} \qquad (1)$$

Now, if we look at the vertical direction, we have $\Delta y = v_{y,\mathrm{i}}\Delta t + \dfrac{1}{2}a_y\Delta t^2 = v\sin(\theta)\Delta t + \dfrac{1}{2}(-g)\Delta t^2$. The time is obviously the same regardless of which component we are looking at. So we can insert equation (1) to obtain $\Delta y = v\sin(\theta)\left(\dfrac{\Delta x}{v\cos(\theta)}\right) - \dfrac{g}{2}\left(\dfrac{\Delta x}{v\cos(\theta)}\right)^2$. Solving this expression for the initial speed v yields

$$v = \frac{\Delta x}{\cos(\theta)}\left[\left(\frac{-2}{g}\right)(\Delta y - \Delta x\tan(\theta))\right]^{-1/2} = \frac{(0.25\ \mathrm{m})}{\cos(63°)}\left[\left(\frac{-2}{9.8\ \mathrm{m/s^2}}\right)((0.45\ \mathrm{m}) - (0.25\ \mathrm{m})\tan(63°))\right]^{-1/2} = 6.0\ \mathrm{m/s}.$$

10.52. Let us assume that the burglar runs off the rooftop, meaning he is moving horizontally as opposed to jumping. Then we can find the amount of time it takes the burglar to fall the 3.0 meters to the height of the next rooftop by looking at the y direction. We have $\Delta y = v_{y,\mathrm{i}}\Delta t + \dfrac{1}{2}a_y\Delta t^2 \Rightarrow \Delta t = \sqrt{\dfrac{2\Delta y}{a_y}} = \sqrt{\dfrac{2(-3.0\ \mathrm{m})}{(-9.8\ \mathrm{m/s^2})}} = 0.782\ \mathrm{s}$.
Now, we look at the burglar's motion in the x direction to see if he made the jump. Ignoring air resistance, there is no force acting on the burglar to accelerate him in the x direction, so we can write $\Delta x = v_{x,\mathrm{i}}\Delta t = \left(\dfrac{50\ \mathrm{m}}{5.6\ \mathrm{s}}\right)(0.782\ \mathrm{s}) = 7.0\ \mathrm{m}$. So as the burglar falls 3.0 m to the height of the next rooftop he only covers a horizontal distance of 7.0 m. Since the rooftops are a horizontal distance of 8.0 m apart, no, the burglar does not make the jump.

10.53. (*a*) If we ignore air resistance, then we have for the x component of the ball's motion: $\Delta t = \dfrac{\Delta x}{v_{x,\mathrm{i}}} =$ $\dfrac{\Delta x}{v_\mathrm{i}\cos(\theta)} = \dfrac{(20\,\mathrm{m})}{(15\ \mathrm{m/s})\cos(30°)} = 1.5\ \mathrm{s}$. (*b*) We know the change in the vertical position is given by $\Delta y = v_{y,\mathrm{i}}\Delta t + \dfrac{1}{2}a_y\Delta t^2 =$

$v_i \sin(\theta)\Delta t - \frac{1}{2}g\Delta t^2 = (15\text{ m/s})\sin(30°)(1.54\text{ s}) - \frac{(9.8\text{ m/s}^2)}{2}(1.54\text{ s})^2 = -0.0678\text{ m}$. But this is relative to the initial height of 1.5 m. Thus the ball strikes the wall 1.4 m above the ground. (*c*) The horizontal component of the velocity does not change with time, so $v_{x,\text{f}} = v_{x,\text{i}} = v_i \cos(\theta) = (15\text{ m/s})\cos(30°) = 13\text{ m/s}$. The vertical component of the velocity can be found using $v_{y,\text{f}} = v_{y,\text{i}} + a_y\Delta t = v_i\sin(\theta) - g\Delta t = (15\text{ m/s})\sin(30°) - (9.8\text{ m/s}^2)(1.54\text{ s}) = -7.6\text{ m/s}$. (*d*) Yes. This is clear because the ball was initially thrown with some upward component, and when the ball strikes the wall it has a downward component.

10.54. (*a*) Since the paintball just barely gets over the billboard, the y component of the paintball's velocity is zero at the instant it passes the billboard. Thus we can use the kinematic equation $v_{y,\text{f}}^2 = v_{y,\text{i}}^2 + 2a_y\Delta y$ to find $v_{y,\text{i}} = v_i\sin(\theta) = \sqrt{-2a_y\Delta y} \Rightarrow \theta = \arcsin\left(\frac{\sqrt{-2a_y\Delta y}}{v_i}\right) = \arcsin\left(\frac{\sqrt{-2(-9.8\text{ m/s}^2)(66.0\text{ m})}}{(42\text{ m/s})}\right) = 59°$. Note that the change in height was only 66.0 m, because the paintball gun was fired at a height of 1.5 m. (*b*) Ignoring air resistance, the x component of the velocity should be constant, so $\Delta x_{\text{billboard}} = v_{x,\text{i}}\Delta t = v_i\cos(\theta)\Delta t$. But we still need the time. Returning to the y component, we can write $v_{y,\text{f}} = v_{y,\text{i}} + a_y\Delta t$ or $\Delta t = \frac{v_{y,\text{f}} - v_{y,\text{i}}}{a_y} = \frac{(0) - (42\text{ m/s})\sin(58.9°)}{(-9.8\text{ m/s}^2)} = 3.67\text{ s}$. So the horizontal distance is $\Delta x_{\text{billboard}} = v_i\cos(\theta)\Delta t = (42\text{ m/s})\cos(58.9°)(3.67\text{ s}) = 80\text{ m}$. (*c*) The height difference between the billboard and roof is 15.7 m. So, we simply calculate how long a projectile requires to fall that distance. Note that since the paintball begins at its highest point the initial y component of the velocity is zero. So $\Delta y = v_{y,\text{i}}\Delta t + \frac{1}{2}a_y\Delta t^2 \Rightarrow \Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-15.7\text{ m})}{(-9.8\text{ m/s}^2)}} = 1.8\text{ s}$. (*d*) In the 1.8 s that the paintball requires to fall to the height of the roof it will travel a distance in the x direction given by $\Delta x_{\text{toward roof}} = v_i\cos(\theta)\Delta t_{\text{toward roof}} =$ $(42\text{ m/s})\cos(58.9°)(1.79\text{ s}) = 38.8\text{m}$. This is greater than the 20 m required to reach the roof and less than the 40 m that would result in overshooting the roof. So, yes, the paintball does strike the roof. (*e*) The x component of the velocity is still the same as the initial value. The y component can be found using $v_{y,\text{f}} = v_{y,\text{i}} + a_y\Delta t = 0 + (-9.8\text{ m/s}^2)(1.79\text{ s}) = -17.5\text{ m/s}$. So the speed as it lands is $v = \sqrt{v_x^2 + v_y^2} =$ $\sqrt{((42)\cos(58.9°))^2 + (-17.5\text{ m/s})^2} = 28\text{ m/s}$.

10.55. Ignoring air resistance, the x component of the velocity will be constant, such that $v_{x,\text{i}} = v_i\cos(\theta) = \frac{\Delta x}{\Delta t}$ or equivalently

$$\Delta t = \frac{\Delta x}{v_i\cos(\theta)} \tag{1}$$

Looking at the y direction, we can write $\Delta y = v_i\sin(\theta)\Delta t + \frac{1}{2}a_y\Delta t^2$. Inserting equation (1) here yields $\Delta y =$ $v_i\sin(\theta)\left(\frac{\Delta x}{v_i\cos(\theta)}\right) + \frac{1}{2}a_y\left(\frac{\Delta x}{v_i\cos(\theta)}\right)^2$, which we can rearrange to obtain $v_i = \frac{\Delta x}{\cos(\theta)}\left[\frac{2}{a_y}(\Delta y - \Delta x\tan(\theta))\right]^{-1/2} =$ $\frac{(30\text{ m})}{\cos(35°)}\left[\frac{2}{(-9.8\text{ m/s}^2)}((-8.5\text{ m}) - (30\text{ m})\tan(35°))\right]^{-1/2} = 15\text{ m/s}$.

10.56. The vertical height of the ground rises as the horizontal position moves to the right, according to

$$y_{\text{slope}}(x) = x\tan(\theta_{\text{slope}}) \tag{1}$$

The vertical position of the ball is given by $y(t) = y_\text{i} + v_{y,\text{i}}\Delta t + \frac{1}{2}a_y\Delta t^2 = y_\text{i} + v_\text{i}\sin(\theta)\Delta t - \frac{1}{2}g\Delta t^2$. We wish to relate this to the x direction. Ignoring air resistance the x component of the velocity is constant, so $\Delta t = \frac{x}{v_{x,\text{i}}} = \frac{x}{v_\text{i}\cos(\theta)}$. Inserting this into the equation we obtained from the y direction, we have

$$y(x) = y_\text{i} + v_\text{i}\sin(\theta)\left(\frac{x}{v_\text{i}\cos(\theta)}\right) - \frac{1}{2}g\left(\frac{x}{v_\text{i}\cos(\theta)}\right)^2$$

or

$$y(x) = y_\text{i} + x\tan(\theta) - \frac{1}{2}g\left(\frac{x}{v_\text{i}\cos(\theta)}\right)^2 \tag{2}$$

The ball will strike the ground when $y(x) = y_\text{slope}(x)$. Setting the expressions in equations (1) and (2) equal to each other yields

$$\frac{g}{2v_\text{i}^2\cos^2(\theta)}x^2 - x(\tan(\theta) - \tan(\theta_\text{slope})) - y_\text{i} = 0$$

Which can be solved for x using the quadratic formula, to obtain $x = -\frac{\cos^2(\theta)}{2g}\left[2v_\text{i}^2(\tan(\theta_\text{slope}) - \tan(\theta)) - \sqrt{\frac{8gv_\text{i}^2}{\cos^2(\theta)} + \left[2v_\text{i}^2(\tan(\theta_\text{slope}) - \tan(\theta))\right]^2}\,\right] = 14.6\text{ m}$. This is the horizontal distance that the ball travels, but we are asked for the distance d along the incline: $d = \frac{x}{\cos(\theta_\text{slope})} = \frac{(14.6\text{ m})}{\cos(20°)} = 16\text{ m}$.

10.57. (*a*) The force that acts to accelerate the block is the gravitational force that Earth exerts on it. But the component of this force that is perpendicular to the xy plane is canceled out by the normal force that the plane exerts up on the block. Only the component along the y axis survives. Thus $\vec{a} = -g\sin\phi\ \hat{i}$. (*b*) The x component of the velocity is constant, and the y component changes due to the acceleration found in part (*a*). Thus $\vec{v}(t) = (v\cos(\theta))\ \hat{i} + (v\sin(\theta) - gt\sin(\phi))\ \hat{j} + 0\ \hat{k}$. (*c*) One expression for the y component of the displacement is $\Delta y = \frac{v_{y,\text{f}}^2 - v_{y,\text{i}}^2}{2a_y}$. At the maximum displacement in y the y component of the velocity will momentarily be zero. Inserting this condition yields $\Delta y_\text{max} = \frac{(0) - (v_\text{i}\sin(\theta))^2}{2(-g\sin(\phi))} = \frac{v_\text{i}^2\sin^2(\theta)}{2g\sin(\phi)}$. (*d*) We can use the y direction to find an expression for the time. Start with $v_{y,\text{f}} = v_{y,\text{i}} + a_y\Delta t$. If the initial time the time of launch, and the final time is the moment when the block returns to the x axis, then the symmetry of the path about its peak Δy lets us write $v_{y,\text{i}} = -v_{y,\text{f}}$. So $\Delta t_\text{max} = \frac{-2v_{y,\text{i}}}{a_y} = \frac{2v_\text{i}\sin(\theta)}{g\sin(\phi)}$. Inserting this into the expression for the x component of displacement $\Delta x = v_{x,\text{i}}\Delta t$ yields $\Delta x_\text{max} = v_\text{i}\cos(\theta)\Delta t_\text{max} = \frac{2v_\text{i}^2\cos(\theta)\sin(\theta)}{g\sin(\phi)}$.

10.58. The center of mass will continue on with the same velocity that the shell had before the explosion.

10.59. (*a*) The original kinetic energy of $\frac{1}{2}mv^2$, should all remain in the form of kinetic energy because the collision was elastic. Thus the average kinetic energy of each of the 16 balls is $K_\text{av} = \frac{1}{2}mv_\text{av}^2 = \frac{1}{16}\left(\frac{1}{2}mv^2\right)$. So the average speed of each ball is $v_{\text{av}} = \frac{1}{4}v.$ (*b*) Because momentum is conserved, we can write $mv_{x,\text{i}} = mv = 16mv_{x,\text{f,av}} \Rightarrow mv_{x,\text{f,av}} = mv/16.$ So the average momentum of each ball after the collision is $mv/16$ in the direction of the cue ball's original velocity.

10.60. Each momentum component remains unchanged,

$$mv_{\text{Ai}} = mv_{\text{Af}} \cos\theta_{\text{A}} + mv_{\text{Bf}} \cos\theta_{\text{B}} \tag{1}$$

$$0 = mv_{\text{Af}} \sin\theta_{\text{A}} - mv_{\text{Bf}} \sin\theta_{\text{B}} \tag{2}$$

and the system energy remains constant,

$$v_{\text{Ai}}^2 = v_{\text{Af}}^2 + v_{\text{Bf}}^2 \tag{3}$$

Solving Eq. 2 for v_{Af} and using that result in Eqs. 1 and 3 yield

$$v_{\text{Ai}} = v_{\text{Bf}}\left(\frac{\cos\theta_{\text{A}}\sin\theta_{\text{B}}}{\sin\theta_{\text{A}}} + \cos\theta_{\text{B}}\right) \tag{4}$$

$$v_{\text{Ai}}^2 = v_{\text{Bf}}^2\left(\frac{\sin^2\theta_{\text{B}}}{\sin^2\theta_{\text{A}}} + 1\right) \tag{5}$$

Squaring Eq. 4 and setting the result equal to the right in Eq. 5 yield

$$(\cos\theta_{\text{A}}\sin\theta_{\text{B}} + \cos\theta_{\text{B}}\sin\theta_{\text{A}})^2 = \sin^2\theta_{\text{B}} + \sin^2\theta_{\text{A}}$$

which simplifies to

$$\cos\theta_{\text{A}}\cos\theta_{\text{B}} - \sin\theta_{\text{A}}\sin\theta_{\text{B}} = 0$$
$$\cos(\theta_{\text{A}} + \theta_{\text{B}}) = 0$$
$$\Rightarrow \theta_{\text{A}} + \theta_{\text{B}} = 90°$$

10.61. (*a*) Call the initial direction of motion of disk 1 the $+x$ axis. Then the initial momentum of the system of the two disks is entirely in the x direction, meaning the final momentum cannot have any component in the y direction. Thus $mv_{1y,\text{f}} + 2mv_{2y,\text{f}} = 0$ or

$$mv_{1,\text{f}}\sin(\theta_1) - 2mv_{2,\text{f}}\sin(\theta_2) = 0 \tag{1}$$

Momentum must also be conserved in the x direction, meaning $mv_{1x,\text{i}} = mv_{1x,\text{f}} + 2mv_{2x,\text{f}}$ or

$$mv_{1,\text{i}} = mv_{1,\text{f}}\cos(\theta_1) + 2mv_{2,\text{f}}\cos(\theta_2) \tag{2}$$

Solving equation (1) for $v_{1,\text{f}}$ yields

$$v_{1,\text{f}} = 2v_{2,\text{f}}\frac{\sin(\theta_2)}{\sin(\theta_1)} \ (3)$$

Inserting equation (3) into equation (2) leaves us with an equation that can be numerically solved for $v_{2,\text{f}}$:

$$v_{2,\text{f}} = \frac{v_{1,\text{i}}}{2\left(\frac{\sin(\theta_2)}{\sin(\theta_1)}\cos(\theta_1) + \cos(\theta_2)\right)} = \frac{(1.0\text{ m/s})}{2\left(\frac{\sin(55°)}{\sin(15°)}\cos(15°) + \cos(55°)\right)} = 0.14\text{ m/s}$$

Inserting this result back into equation (3) yields

$$v_{1,\text{f}} = 2(0.138\text{ m/s})\frac{\sin(55°)}{\sin(15°)} = 0.87\text{ m/s}$$

(*b*) The initial kinetic energy per unit inertia m is $\frac{K_\text{i}}{m} = \frac{1}{2}v_{1,\text{i}}^2 = 0.50\text{ J/kg}.$ After the collision, $\frac{K_\text{f}}{m} = \frac{1}{2}v_{1,\text{f}}^2 + 2\frac{1}{2}v_{2,\text{f}}^2 = \frac{1}{2}(0.138\text{ m/s})^2 + (0.872\text{ m/s})^2 = 0.77\text{ J/kg}.$ So, no, the collision is not elastic. In fact, some energy has been added to the system (such as tiny explosive packs on the outer edges of the disks).

10.62. If the projectile rises to a maximum of 2.0 m, then the initial velocity in the y direction is given by $v_{py,f}^2 = v_{py,i}^2 + 2a_y\Delta y \Rightarrow v_{py,i} = \sqrt{-2a_y\Delta y_{max}} = \sqrt{-2(-9.8\ \text{m/s}^2)(2.0\ \text{m})} = 6.26\ \text{m/s}$. Since the projectile is initially fired at an angle of $40°$, we can say $v_{px,i} = \dfrac{v_{py,i}}{\tan(\theta)} = \dfrac{(6.26\ \text{m/s})}{\tan(40°)} = 7.46\ \text{m/s}$. Prior to firing the gun, the momentum of the cart-gun-projectile system was zero. So we require that the x component of the final momentum also be zero: $m_p v_{px,f} + m_c v_{cx,f} = 0 \Rightarrow v_{cx,f} = -\dfrac{m_p}{m_c} v_{px,f} = -\dfrac{(0.050\ \text{kg})}{(0.50\ \text{kg})}(7.46\ \text{m/s}) = -0.75\ \text{m/s}$. So the cart recoils with a speed of $0.75\ \text{m/s}$.

10.63. (*a*) Initially the momentum of the system of two blocks is entirely along the x axis, so the final momentum of the system must also be along the x axis, such that $p_{\text{both }y,f} = 0$. So we can write $m_1 v_{1y,f} + m_2 v_{2y,f} = -m_1 v_{1,f}\sin(\theta) + m_2 v_{2,f}\sin(\theta_2) = 0$. We rearrange this to obtain

$$v_{1,f} = \left(\frac{m_2}{m_1}\right)\left(\frac{\sin(\theta_2)}{\sin(\theta)}\right)v_{2,f} \tag{1}$$

Turning our attention to the conservation of momentum in the x direction, we write

$$(m_1 + m_2)v_{\text{both }x,i} = m_1 v_{1x,f} + m_2 v_{2x,f} \quad \text{or}$$

$$(m_1 + m_2)v_{\text{both }x,i} = m_1 v_{1,f}\cos(\theta) + m_2 v_{2,f}\cos(\theta_2) \tag{2}$$

Inserting equation (1) into equation (2), we find

$$\begin{aligned}\theta &= \arctan\left(\left[\frac{(m_1+m_2)}{m_2}\frac{v_{\text{both }x,i}}{v_{2,f}}\frac{1}{\sin(\theta_2)} - \frac{1}{\tan(\theta_2)}\right]^{-1}\right)\\ &= \arctan\left(\left[\frac{((1.40\ \text{kg})+(2.00\ \text{kg}))}{(2.00\ \text{kg})}\left(\frac{(2.90\ \text{m/s})}{()3.50\ \text{m/s}}\right)\frac{1}{\sin(34.0°)} - \frac{1}{\tan(34.0°)}\right]^{-1}\right) = 44.0°\end{aligned} \tag{3}$$

Inserting this result into equation (1) yields $v_{1,f} = \left(\dfrac{(2.00\ \text{kg})}{(1.40\ \text{kg})}\right)\left(\dfrac{\sin(34.0°)}{\sin(44.0°)}\right)(3.50\ \text{m/s}) = 4.03\,\text{m/s}$. Thus $\vec{v} = 4.03\ \text{m/s at } \theta = 44.0°$.

(*b*) Energy must be conserved here, although kinetic energy clearly is not. We write $K_i + U_i^{sp} = K_f = K_{1,f} + K_{2,f}$ or

$$\begin{aligned}&\frac{1}{2}k(y-y_0)^2 = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2 - \frac{1}{2}(m_1+m_2)v_{\text{both},i}^2\\ &\Rightarrow (y-y_0) = \sqrt{\frac{1}{k}\left[m_1 v_{1,f}^2 + m_2 v_{2,f}^2 - (m_1+m_2)v_{\text{both},i}^2\right]}\\ &= \sqrt{\frac{1}{(3800\ \text{N/m})}\left[(1.40\ \text{kg})(4.03\ \text{m/s})^2 + (2.00\ \text{kg})(3.50\text{m/s})^2 - ((1.40\ \text{kg})+(2.00\ \text{kg}))(2.90\ \text{m/s})^2\right]}\\ &= 0.0700\ \text{m}\end{aligned}$$

10.64. (*a*) We use conservation of momentum in the x and y directions separately:

$$m_P v_{Px,i} = m_P v_{Px,f} + m_Q v_{Qx,f}$$

$$m_P v_{Py,i} = m_P v_{Py,f} + m_Q v_{Qy,f}$$

or

$$v_{Px,i} = v_{Px,f} + \frac{m_Q}{m_P}v_{Qx,f} = v_{P,f}\cos(\theta_P) + \frac{m_Q}{m_P}v_{Q,f}\cos(\theta_Q) = (1.4\text{m/s})\cos(20°) + \frac{(0.70\ \text{kg})}{(0.40\ \text{kg})}(0.96\ \text{m/s})\cos(50°) = 2.40\ \text{m/s}$$

$$v_{Py,i} = v_{Py,f} + \frac{m_Q}{m_P}v_{Qy,f} = v_{P,f}\sin(\theta_P) - \frac{m_Q}{m_P}v_{Q,f}\sin(\theta_Q) = (1.4\text{m/s})\sin(20°) - \frac{(0.70\ \text{kg})}{(0.40\ \text{kg})}(0.96\ \text{m/s})\sin(50°) = -0.808\ \text{m/s}$$

We combine these initial components to find $v_{\mathrm{P,i}} = \sqrt{v_{\mathrm{P}x,\mathrm{i}}^2 + v_{\mathrm{P}y,\mathrm{i}}^2} = \sqrt{(2.40\ \mathrm{m/s})^2 + (-0.808\ \mathrm{m/s})^2} = 2.5\ \mathrm{m/s}$, and $\theta = \arctan\left(\dfrac{v_{\mathrm{P}y,\mathrm{i}}}{v_{\mathrm{P}x,\mathrm{i}}}\right) = \arctan\left(\dfrac{(-0.808\ \mathrm{m/s})}{(2.40\ \mathrm{m/s})}\right) = -19°$. So $\vec{v}_{\mathrm{P,i}} = 2.5\ \mathrm{m/s}$ at $\theta = 19°$ clockwise from the $\arctan\left(\dfrac{v_{\mathrm{P}y,\mathrm{i}}}{v_{\mathrm{P}x,\mathrm{i}}}\right) + x$ axis.

(*b*) We find the initial and final kinetic energies and take the ratio.

$$K_{\mathrm{i}} = K_{\mathrm{P,i}} = \frac{1}{2}m_{\mathrm{p}}v_{\mathrm{P,i}}^2 = \frac{1}{2}(0.40\ \mathrm{kg})(2.53\ \mathrm{m/s})^2 = 1.28\ \mathrm{J}$$

$$K_{\mathrm{f}} = K_{\mathrm{P,f}} + K_{\mathrm{Q,f}} = \frac{1}{2}m_{\mathrm{P}}v_{\mathrm{P,f}}^2 + \frac{1}{2}m_{\mathrm{Q}}v_{\mathrm{Q,f}}^2 = \frac{1}{2}(0.40\ \mathrm{kg})(1.4\ \mathrm{m/s})^2 + \frac{1}{2}(0.70\ \mathrm{kg})(0.96\ \mathrm{m/s})^2 = 0.715\ \mathrm{J}$$

So the fraction of the kinetic energy that was converted is $\dfrac{K_{\mathrm{i}} - K_{\mathrm{f}}}{K_{\mathrm{i}}} = \dfrac{(1.28\ \mathrm{J}) - (0.715\ \mathrm{J})}{(1.28\ \mathrm{J})} = 0.44.$

10.65. $\vec{A}\cdot\vec{B} = AB\cos(\theta) = (3.0\ \mathrm{m})(2.5\ \mathrm{m})\cos(165°) = -7.2\ \mathrm{m}^2$

10.66. $\vec{A}\cdot\vec{B} = AB\cos(\theta)$ so $\theta = \arccos\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\dfrac{22.4\ \mathrm{units}^2}{(3.5\ \mathrm{units})(11\ \mathrm{units})}\right) = 54°$

10.67. (*a*) The vector $\vec{A}$ points along the $-y$ axis. The vector $\vec{B}$ makes an angle with the $+x$ axis given by $\theta = \arctan\left(\dfrac{B_y}{B_x}\right) = \arctan\left(\dfrac{(-7.0\ \mathrm{units})}{(3.0\ \mathrm{units})}\right) = -66.8°$, meaning the angle that $\vec{B}$ makes with the $-y$ axis (and therefore with vector $\vec{A}$) is $23°$.

(*b*) $\vec{A}\cdot\vec{B} = AB\cos(\theta) = (5.0\ \mathrm{m})\sqrt{(3.0\ \mathrm{units})^2 + (-7.0\ \mathrm{units})^2}\cos(23.2°) = 35\ \mathrm{units}^2$.

10.68. (*a*) Because the work is $W = \int \vec{F}\cdot d\vec{r}$ only the components of the force that are parallel to the displacement will do work. Thus, in this case of a constant force, we have $W = F_x\Delta x = (50\ \mathrm{N})(-6.0\ \mathrm{m}) = -3.0\times 10^2\ \mathrm{J}$. (*b*) We know that for a constant force $W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos(\theta)$, so $\theta = \arccos\left(\dfrac{W}{F\Delta r}\right) = \arccos\left(\dfrac{-300\ \mathrm{J}}{\sqrt{(50\ \mathrm{N})^2 + (12\ \mathrm{N})^2}(6.0\ \mathrm{m})}\right) = 167°$.

10.69. (*a*) For a force that is constant in time, we can write the work done as $W = \vec{F}\cdot\Delta\vec{D} = F_x\Delta x + F_y\Delta y = (3.0\ \mathrm{N})(2.0\ \mathrm{m}) + (2.0\ \mathrm{N})(-2.0\mathrm{m}) = 2.0\ \mathrm{J}$.

(*b*) $W = \vec{F}\cdot\Delta\vec{D} = FD\cos(\theta) \Rightarrow \theta = \arccos\left(\dfrac{W}{FD}\right) = \arccos\left(\dfrac{W}{\sqrt{F_x^2 + F_y^2}\sqrt{D_x^2 + D_y^2}}\right)$

$$= \arccos\left(\frac{2.0\ \mathrm{J}}{\sqrt{(3.0\ \mathrm{N})^2 + (2.0\ \mathrm{N})^2}\sqrt{(2.0\ \mathrm{m})^2 + (-2.0\ \mathrm{m})^2}}\right) = 79°$$

10.70. For brevity, let us define $\vec{D} = (\vec{B} - \vec{A}) = (-2.0)\,\hat{\imath} + (-3.0)\,\hat{\jmath}$. Then $\vec{C}\cdot\vec{D} = C_xD_x + C_yD_y = (2.0)(-2.0) + (2.0)(-3.0) = -10$. No units are given, so we will simply call this $-10\ \mathrm{units}^2$.

10.71. (*a*) Let "initial refer to the instant after the marble is launched, and let "final" refer to the moment the marble reaches the top of the loop. As the ball rises, kinetic energy will be partially converted to gravitational potential energy. So we write $K_{\mathrm{i}} = U_{\mathrm{f}}^G + K_{\mathrm{f}}$ or $\dfrac{1}{2}mv_{\mathrm{i}}^2 = mg\Delta y + \dfrac{1}{2}mv_{\mathrm{f}}^2 \Rightarrow v_{\mathrm{i}} = \sqrt{2g\Delta y + v_{\mathrm{f}}^2} =$

$\sqrt{2(9.8\text{ m/s}^2)(0.30\text{ m})+(0.70\text{m/s})^2}=2.5\text{ m/s}.$ (*b*) The kinetic energy that the marble has at the bottom of the track was initially stored as spring potential energy. Let "initial" now refer to the instant before the marble was launched, and let "final" refer to the instant just after launch. $U_\text{i}^\text{sp}=K_\text{f}$ or $\frac{1}{2}k\Delta x^2=\frac{1}{2}mv_\text{f}^2\Rightarrow\Delta x=\sqrt{\frac{m}{k}}v_\text{f}=\sqrt{\frac{(5.0\times10^{-3}\text{ kg})}{(13\text{ N/m})}}(2.52\text{ m/s})=49\text{ mm}.$

10.72. The angle the ramp makes with the horizontal is $\theta=\arcsin\left(\frac{1.4\text{ m}}{5.0\text{ m}}\right)=16.3°.$ So as the refrigerator slides down the incline at an able of $16.3°$ below the $+x$ axis, and you push directly along the $-x$ axis, the angle between your force and the force displacement is $163.7°.$ So $W=\vec{F}\cdot\Delta\vec{r}=F\Delta r\cos(\theta)=(300\text{ N})(5.0\text{ m})\cos(163.7°)=-1.4\text{ kJ}.$

10.73. (*a*) Treating the boy and sled as the system, the only work is done by the gravitational force that Earth exerts on the boy. All of this work goes to increasing the boy's kinetic energy. So we write $W=\vec{F}_\text{Eb}^G\cdot\Delta\vec{r}=mg\Delta r\cos(\theta)$ where θ is the angle between the gravitational force and the displacement (in this case $\theta=80°$). Equating this to the final kinetic energy yields $mg\Delta r\cos(\theta)=\frac{1}{2}mv_\text{f}^2\Rightarrow v_\text{f}=\sqrt{2g\Delta r\cos(\theta)}=\sqrt{2(9.8\text{ m/s}^2)(50\text{ m})\cos(80°)}=13\text{ m/s}.$
(*b*) In part (*a*) we found the work done by gravity as the boy descended the slope. Clearly, in ascending the slope, the only thing that changes is that the work done by gravity is now negative: $W_\text{Eb}=mg\Delta r\cos(\theta_\text{ascending})=(20\text{ kg})(9.8\text{ m/s}^2)(50\text{ m})\cos(80°+180°)=-1.70\text{ kJ}.$ The sister must do enough work to compensate for this negative work done by gravity (just to get her brother up the hill). She must also do enough work to increase his kinetic energy as described. So $W_\text{sister}=-W_\text{Eb}+\frac{1}{2}mv_\text{f}^2=(1.70\times10^3\text{ J})+\frac{1}{2}(20\text{ kg})(0.50\text{ m/s})^2=(1.70\times10^3\text{ J})$ to three significant digits. The force she applies to the sled is presumably directed parallel to the slope, meaning the angle between her force and the displacement of the sled is zero. Thus $F_\text{av}=\frac{W}{\Delta r}=\frac{(1.7\times10^3\text{ J})}{(50\text{ m})}=34\text{ N}.$ So the sister exert an average of 34 N up the slope and parallel to it.

10.74. (*a*) Call the direction of "straight ahead" motion the $+x$ direction. In order to move in that direction, the vector sum of all forces cannot have a component perpendicular to that, along what we will call the y axis. Let the first child's force have a component along the $+y$ axis. The sum of all y components of forces is then $\sum F_y=\mathcal{T}_{\text{child1 tricycle }y}+\mathcal{T}_{\text{child2 tricycle }y}=ma_y=0\Rightarrow\theta=\arcsin\left(\frac{\mathcal{T}_\text{child1 tricycle}}{\mathcal{T}_\text{child2 tricycle}}\sin(\theta_1)\right)=\arcsin\left(\frac{(100\text{ N})}{(80\text{ N})}\sin(45°)\right)=62°.$
(*b*) By construction, the motion will be entirely along the x axis, so we need only consider the sum of the x components of all forces: $\sum F_x=\mathcal{T}_{\text{child1 tricycle }x}+\mathcal{T}_{\text{child2 tricycle }x}=ma_x\Rightarrow a_x=\frac{1}{m}(\mathcal{T}_\text{child1 tricycle}\cos(\theta_1)+\mathcal{T}_\text{child2 tricycle}\cos(\theta_2))=$ $\frac{1}{(35\text{ kg})}((100\text{ N})\cos(45°)+(80\text{ N})\cos(62.1°))=3.1\text{ m/s}^2.$ So $\vec{a}=3.1\text{ m/s}^2\ \hat{i}.$ (*c*) This could be solved using kinematics, but we will use work here. Since the vector sum of all forces is constant, we can write $W=\vec{F}\cdot\Delta\vec{r}=F_{\text{child1 tricycle }x}\Delta x+F_{\text{child2 tricycle }x}\Delta x=(100\text{ N})\cos(45°)(2.0\text{ m})+(80\text{ N})\cos(62.1°)(2.0\text{ m})=2.2\times10^2\text{ J}.$

10.75.
$$\begin{aligned}\vec{A}\cdot\vec{B}&=(A_x\hat{i}+A_y\hat{j})\cdot(B_x\hat{i}+B_y\hat{j})\\&=A_xB_x\cos(0°)+A_xB_y\cos(90°)+A_yB_x\cos(90°)+A_yB_y\cos(0°)\\&=A_xB_x+A_yB_y\end{aligned}$$

10.76. (*a*) $W = F_x\Delta x = F\Delta x\cos(\theta) = (80\text{ N})(15\text{ m})\cos(35°) = 9.8\times10^2\text{ J}$. (*b*) Since we are ignoring friction, all the work done in part (*a*) goes to increasing the kinetic energy of the gurney. Thus $\frac{1}{2}mv_{\text{gurney,f}}^2 = W_{\text{intern gurney}} \Rightarrow$ $v_{\text{gurney,f}} = \sqrt{\frac{2W_{\text{intern gurney}}}{m}} = \sqrt{\frac{2(9.83\times10^2\text{ J})}{(45\text{ kg})}} = 6.6\text{ m/s}$. (*c*) We can use $\Delta x = \frac{1}{2}(v_{x,\text{i}} + v_{x,\text{f}})\Delta t \Rightarrow \Delta t = \frac{2\Delta x}{(v_{x,\text{i}} + v_{x,\text{f}})} =$ $\frac{2(15\text{ m})}{(0+(6.61\text{ m/s}))} = 4.5\text{ s}$.

10.77. (*a*) Assuming we apply a constant power, we can write $P_{\max} = \left(\frac{\Delta E}{\Delta t}\right)_{\max} = \left(\frac{mg\Delta y}{\Delta t}\right)_{\max} = mgv_{y,\max}$, since all the work we do goes into increasing the gravitational potential energy of the safe. Thus $v_{y,\max} = \frac{P_{\max}}{mg} = \frac{(500\text{ W})}{(50\text{ kg})(9.8\text{ m/s}^2)} = 1.0\text{ m/s}$. (*b*) All the work goes into changing the gravitational potential energy, so $W = mg\Delta y = (50\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 4.9\text{ kJ}$. (*c*) Proceeding exactly as in (*a*), we write $P_{\max} = \left(\frac{\Delta E}{\Delta t}\right)_{\max} = \left(\frac{mg\Delta y}{\Delta t}\right)_{\max} = mgv_{y,\max}$. But now we note that on this incline $v_{y,\max} = v_{\max}\sin(30°)$. So now $v_{y,\max} = \frac{P_{\max}}{mg\sin(30°)} = \frac{2(500\text{ W})}{(50\text{ kg})(9.8\text{ m/s}^2)} = 2.0\text{ m/s}$. (*d*) Now that we push it up an incline, we can move it at a greater speed, but we are still increasing the gravitational potential energy at the same rate. Also, the overall change in gravitational potential energy is the same as when lifting it straight up. So, clearly the work done on the safe is still 4.9 kJ.

10.78. The work done by friction will go entirely into reducing the kinetic energy of the mug of beer. We write $W_{\text{friction mug}} = F_{\text{tm}x}^{\text{fr}}\Delta x = \mu_{\text{k}}mg\Delta x = \frac{1}{2}mv_{\text{m,i}}^2$. Clearly $v_{\text{m,i}} = \sqrt{2\mu_{\text{k}}g\Delta x}$, which is independent of the inertia of the mug. Thus $v_{\text{half}} = v_{\text{full}}$.

10.79. Pulling is easier. When you pull, there is a component of the force forward, and also a component of the force upward. The upward component reduces the normal force that the floor exerts on the skid plates, thereby reducing the friction. If you push, you have the same horizontal component, but now you are also exerting a downward force. This increases the normal force exerted by the floor on the skid plates, thereby increasing friction.

10.80. Let the $+x$ axis point down the driveway, and the $+y$ axis point upward, perpendicular to the driveway. We look at the x and y equations of motion:

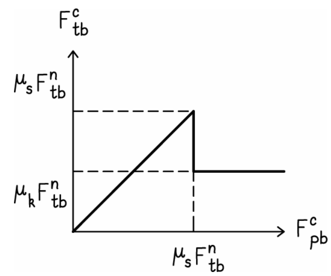
$$\sum F_x = F_{\text{E}px}^G + F_{\text{ice p}\,x}^{\text{f}} = ma_x = 0 \qquad (1)$$

$$\sum F_y = F_{\text{E}py}^G + F_{\text{ice p y}}^n = ma_y = 0 \qquad (2)$$

If the coefficient of static friction is truly at its minimum value, then the shoes are just on the verge of slipping across the ice. When the force of static friction is at its maximum as in this case, we can equate $F_{\text{ice p}\,x}^{\text{f}} = -\mu_{\text{s}}F_{\text{ice p}\,y}^n$. Equation (2) can be rewritten as $F_{\text{ice p y}}^n = mg\cos(15°)$. Inserting this into equation (1) yields $mg\sin(15°) - \mu_{\text{s,min}}mg\cos(15°) = 0$ or $\mu_{\text{s,min}} = \tan(15°) = 0.27$.

10.81. When you are just overcoming the force of static friction, you can write $F_{\text{pb}x}^{\text{c}} = (F_{\text{fb}x}^s)_{\max} = \mu_{\text{s}}F_{\text{fb}}^n =$ $\mu_{\text{s}}mg \Rightarrow \mu_{\text{s}} = \frac{F_{\text{pb}x}^{\text{c}}}{mg} = \frac{(200\text{ N})}{(51\text{ kg})(9.8\text{ m/s}^2)} = 0.40$. Similarly, when overcoming the force of kinetic friction, we have $F_{\text{pb}x}^{\text{c}} = F_{\text{fb}x}^k = \mu_k F_{\text{fb}}^n = \mu_k mg \Rightarrow \mu_k = \frac{F_{\text{pb}x}^{\text{c}}}{mg} = \frac{(100\text{ N})}{(51\text{ kg})(9.8\text{ m/s}^2)} = 0.20$. So $\mu_s = 0.40$ and $\mu_k = 0.20$.

10.82.



10.83. (*a*) Let the $+x$ axis point up the slope, and the $+y$ axis point upward, perpendicular to the slope. Because the speed is constant, we can write $\sum F_x = F^G_{\text{Es}x} + F^k_{\text{snow s}\,x} + \mathcal{T}_{\text{rope s}\,x} = ma_x = 0$ or $\mathcal{T}_{\text{rope s}\,x} = mg\sin(\theta) + \mu_k F^n_{\text{snow s}} = mg(\sin(\theta) + \mu_k\cos(\theta)) = (55\text{ kg})(9.8\text{m/s}^2)(\sin(40°) + 0.20\cos(40°)) = 4.3\times10^2\text{ N}.$

(*b*) For this constant force, the work is simply $W = \vec{F}\cdot\Delta\vec{r} = F_x\Delta x\cos(0°) = (429\text{ N})(100\text{m}) = 43\text{ kJ}.$

10.84. (*a*) Choose the wax that makes the coefficient of static friction as high as possible. You want to be able to push against the snow to increase your speed. (*b*) Choose the wax that makes the coefficient of kinetic friction as small as possible. You would like to glide across the snow with minimal dissipation.

10.85. (*a*) Call the direction of his push the $+x$ direction. Pushing horizontally, $\sum F_x = F^c_{\text{pt}x} + F^k_{\text{ft}x} = ma_x = 0$, so $F^c_{\text{pt}x} = -F^k_{\text{ft}x} = \mu_k F^n_{\text{ft}} = \mu_k mg = (0.10)(11\text{ kg})(9.8\text{ m/s}^2) = 11\text{ N}.$

(*b*) Now we need to look at both the x and y directions. Consider the sum of all components of forces:

$$\sum F_x = F^c_{\text{pt}x} + F^k_{\text{ft}x} = ma_x = 0 \tag{1}$$

$$\sum F_y = F^c_{\text{pt}y} + F^n_{\text{ft}} + F^G_{\text{E}ty} = ma_y = 0 \tag{2}$$

or equivalently

$$F^c_{\text{pt}}\cos(\theta) - \mu_k F^n_{\text{ft}} = 0 \tag{3}$$

$$-F^c_{\text{pt}}\sin(\theta) + F^n_{\text{ft}} - mg = 0 \tag{4}$$

Solving equation (4) for the normal force yields

$$F^n_{\text{ft}} = F^c_{\text{pt}}\sin(\theta) + mg \tag{5}$$

And inserting equation (5) into equation (3) yields

$$F^c_{\text{pt}} = \frac{\mu_k mg}{(\cos(\theta) - \mu_k\sin(\theta))} = \frac{(0.10)(11\text{ N})(9.8\text{ m/s}^2)}{(\cos(30°) - (0.10)\sin(30°))} = 13\text{ N}$$

10.86. (*a*) The work being done by friction is reducing the kinetic energy of the puck. So $W = F^k_{\text{ice p}\,x}\Delta x = -\mu_k F^n_{\text{ice p}}\Delta x = \Delta K$ or $-\mu_k mg\Delta x = \frac{1}{2}m(v^2_{\text{p,f}} - v^2_{\text{p,i}}) \Rightarrow \mu_k = -\frac{(v^2_{\text{p,f}} - v^2_{\text{p,i}})}{2g\Delta x} = -\frac{((10.39\text{ m/s})^2 - (10.50\text{ m/s})^2)}{2(9.8\text{ m/s}^2)(40.00\text{ m})} = 0.0029.$

(*b*) 10.39 m/s. The frictional force will be twice as strong, due to the increased normal force that the ice exerts on the lower puck. But this frictional force will also have to slow down twice as much inertia.

10.87. (*a*) Call your initial direction of motion the $+x$ direction. Then $\sum F_x = F^k_{\text{Ec}x} = -\mu_k mg = ma_x \Rightarrow a_x = -\mu_k g.$ Since this is a constant, we can use the kinematic equation $\Delta t = \frac{(v_{\text{c}\,x,\text{f}} - v_{\text{c}\,x,\text{i}})}{a_x} = \frac{((0) - (27\text{ m/s}))}{-(0.80)(9.8\text{ m/s}^2)} = 3.4\text{ s}.$

(*b*) Following the same procedure as in part (*a*) we have $\Delta t = \frac{(v_{\text{c}\,x,\text{f}} - v_{\text{c}\,x,\text{i}})}{a_x} = \frac{((0) - (27\text{ m/s}))}{-(0.25)(9.8\text{ m/s}^2)} = 11\text{ s}.$

(*c*) We use $v_{\text{c}\,x,\text{f}}^2 = v_{\text{c}\,x,\text{i}}^2 + 2a_x\Delta x$ and find $\Delta x = \dfrac{-v_{\text{c}\,x,\text{i}}^2}{2a_x} = \dfrac{-v_{\text{c}\,x,\text{i}}^2}{-2\mu_k g} = \dfrac{(27\text{ m/s})^2}{2(0.25)(9.8\text{ m/s}^2)} = 1.5\times10^2\text{ m.}$

10.88. (*a*) Choose the $+x$ axis to point up the incline, and the $+y$ axis to point perpendicular to the incline with an upward component. We begin by using the equations of motion in x and y to determine the acceleration down the incline.

$$\sum F_x = F_{\text{Eb}x}^G + F_{\text{sb}x}^k = ma_x \qquad (1)$$

$$\sum F_y = F_{\text{sb}}^n + F_{\text{Eb}y}^G = ma_y = 0 \qquad (2)$$

or

$$-mg\sin(\theta) - \mu_k F_{\text{sb}}^n = ma_x \qquad (3)$$

$$F_{\text{sb}}^n - mg\cos(\theta) = 0 \qquad (4)$$

Solving equation (4) for the normal force and inserting it into equation (3) yields

$$a_x = -g(\sin(\theta) + \mu_k\cos(\theta)) \qquad (5)$$

When the block reaches its maximum distance along the incline, $v_{\text{b}x,\text{f}} = 0$ momentarily before it begins sliding back down. Thus we can use $v_{\text{b}x,\text{f}}^2 = v_{\text{b}x,\text{i}}^2 + 2a_x\Delta x$ to write $\Delta x_{\text{max}} = \dfrac{0 - v_{\text{b}x,\text{i}}^2}{2a_x} = \dfrac{v^2}{2g(\sin(\theta) + \mu_k\cos(\theta))}$. (*b*) In the special case where μ_s is as small as possible, the force of static friction that is being exerted on the block as large as possible, and the block is just on the verge of slipping. In that case, $F_{\text{sb}}^s = \mu_s F_{\text{sb}}^n$. Also note that if the block is just on the verge of slipping down the incline, friction must be acting up the incline. With these changes, the equation of motion in x becomes

$$-mg\sin(\theta) + \mu_s F_{\text{sb}}^n = ma_x \qquad (6)$$

There are no changes to the equation of motion in y, so the same normal force can be extracted from equation (4) as before. The expression for the acceleration is then given by $a_x = g(-\sin(\theta) + \mu_s\cos(\theta))$. In order for the block to remain stationary, we require $a_x = g(-\sin(\theta) + \mu_s\cos(\theta)) = 0 \Rightarrow \tan(\theta) = \mu_s$ or $\theta_{\text{max}} = \tan^{-1}(\mu_s)$.

10.89. Initially the energy of the system is entirely spring potential energy. After work is done by friction as the block slides, the final energy of the system is zero. Thus $W = \Delta U^{\text{sp}} = -\frac{1}{2}k\Delta x_{\text{spring}}^2$. We also know the work done by friction can be written $W = \vec{F}_{\text{sb}}^k \cdot \Delta\vec{r} = -\mu_k F_{\text{sb}}^n \Delta r = -\mu_k mg\Delta r$, where Δr is the maximum distance the block travels before stopping. Equating the two expressions for work, we obtain $\Delta r = \dfrac{k\Delta x_{\text{spring}}^2}{2\mu_k mg} = \dfrac{(100\text{ N/m})(0.20\text{ m})^2}{2(0.20)(1.0\text{ kg})(9.8\text{ m/s}^2)} =$ 1.0 m.

10.90. Call the direction in which the box moves the $+x$ direction, and call vertically upward the $+y$ direction. The equations of motion read

$$\sum F_x = F_{\text{pb}x}^{\text{c}} + F_{\text{sb}x}^k = ma_x = 0$$

$$\sum F_y = F_{\text{sb}}^n + F_{\text{pb}y}^{\text{c}} + F_{\text{Eb}y}^G = ma_y = 0$$

Or equivalently

$$F_{\text{pb}}^{\text{c}}\cos(\theta) - \mu_k F_{\text{sb}}^n = 0 \qquad (1)$$

$$F_{\text{sb}}^n + F_{\text{pb}}^{\text{c}}\sin(\theta) - mg = 0 \qquad (2)$$

Solving equation (2) for the normal force and inserting this into equation (1) yields

$$F_{\text{pb}}^{\text{c}} = \frac{\mu_k mg}{(\cos(\theta) + \mu_k\sin(\theta))} = \frac{(0.10)(50\text{ kg})(9.8\text{ m/s}^2)}{(\cos(36.9°) + (0.10)\sin(36.9°))} = 57.0\text{ N}$$

Now that we know the force the man exerts on the box, finding the work is simple: $W = \vec{F}^{\mathrm{c}}_{\mathrm{pb}} \cdot \Delta\vec{r} = F^{\mathrm{c}}_{\mathrm{pb}x}\Delta x\cos(\theta) = (56.0\text{ N})(10\text{ m})\cos(36.9°) = 4.6\times10^2\text{ J}.$

10.91. (*a*) This can be solved most easily by using energy. $U^G_{\mathrm{i}} = U^{\mathrm{sp}}_{\mathrm{f}} \Rightarrow mg\Delta y = \frac{1}{2}k\Delta x^2$ so $\Delta y = \frac{k\Delta x^2}{2mg} = \frac{(5400\text{ N/m})(0.0240\text{ m})^2}{2(2.2\text{ kg})(9.8\text{ m/s}^2)} = 0.0721\text{ m}.$ This is related to the distance along the incline by $d = \frac{\Delta y}{\sin(\theta)} = \frac{(0.0721\text{ m})}{\sin(30°)} = 0.14\text{ m}.$

(*b*) Now there is an additional term: $U^G_{\mathrm{i}} + W = U^{\mathrm{sp}}_{\mathrm{f}}$ or

$$mg\Delta y - \mu_k F^n_{\mathrm{sb}}\Delta d = \frac{1}{2}k\Delta x^2_{\mathrm{spring,f}} \qquad (1)$$

where $\Delta x_{\mathrm{spring,f}}$ is the final compression of the spring, and d is the distance along the incline that the block travels. Note that $\Delta y = d\sin\theta,$ and by looking at the sum of y components of forces, one case see that $F^n_{\mathrm{sb}} = mg\cos(\theta).$ Finally inserting this into equation (1), we have

$$d = \frac{k\Delta x^2_{\mathrm{spring,f}}}{2mg(\sin(\theta) - \mu_k\cos(\theta))} = \frac{(5400\text{ N/m})(0.0240\text{ m})^2}{2(2.2\text{ kg})(9.8\text{ m/s}^2)(\sin(30°) - (0.10)\cos(\theta))} = 0.17\text{ m}$$

10.92. Choose the x axis to be horizontal such that the push we exert has a component that is in the $+x$ direction and let $+y$ point vertically upwards. We write the sum of all x and y components of forces separately:

$$\sum F_x = F^{\mathrm{c}}_{\mathrm{pb}x} + F^s_{\mathrm{sb}x} = ma_x$$
$$\sum F_y = F^n_{\mathrm{sb}} + F^{\mathrm{c}}_{\mathrm{pb}y} + F^G_{\mathrm{E}by} = ma_y = 0$$

Or equivalently, assuming we are very close to getting the book to slip such that $F^s_{\mathrm{sb}} = \mu_s F^n_{\mathrm{sb}}$,

$$F^{\mathrm{c}}_{\mathrm{pb}}\sin(\theta) - \mu_s F^n_{\mathrm{sb}} = ma_x \qquad (1)$$
$$F^n_{\mathrm{sb}} - F^{\mathrm{c}}_{\mathrm{pb}}\cos(\theta) - mg = 0 \qquad (2)$$

Solving equation (2) for the normal force and inserting this into equation (1), we obtain

$$a_x = \frac{F^{\mathrm{c}}_{\mathrm{pb}}}{m}(\sin(\theta) - \mu_s\cos(\theta)) - \mu_s g \qquad (3)$$

A negative acceleration in the above equation is, of course, nonsense because from the beginning we defined the $+x$ axis such that the push had some component along it. A negative acceleration in the equation above means we are breaking one of the assumptions that we made, namely that we are maxing out the force of static friction. Clearly, if $\theta = 0$ (for example) the surface could not possibly be exerting the maximum possible force of static friction. Positive accelerations or an acceleration of zero make perfect sense. So clearly the term $(\sin(\theta) - \mu_s\cos(\theta)) \geq 0,$ with equality only holding if the coefficient of static friction is zero. The problem says we can push with an arbitrarily large force, and the book will not move unless the angle is larger than some minimum value. If we take the limit as our force becomes extremely large, the quantity $(\sin(\theta) - \mu_s\cos(\theta))$ can be arbitrarily small and the acceleration will still be positive. So in this limiting case we set $(\sin(\theta) - \mu_s\cos(\theta)) = 0 \Rightarrow \theta = \tan^{-1}(\mu_s).$ Thus $\theta \geq \tan^{-1}(\mu_s)$ or no magnitude of pushing force will make the book move.

10.93. (*a*) Call the direction up the incline $+x.$ The work you do goes to increasing the block's gravitational potential energy and the kinetic energy: $K_{\mathrm{i}} + W_{\mathrm{p}} = K_{\mathrm{f}} + U^G_{\mathrm{f}} \Rightarrow \frac{1}{2}mv_{\mathrm{i}}^2 + F^{\mathrm{c}}_{\mathrm{pb}x}\Delta x = \frac{1}{2}mv_{\mathrm{f}}^2 + mg\Delta x\sin(\theta)$ such that $v_{\mathrm{f}} = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_{\mathrm{i}}^2 + F^{\mathrm{c}}_{\mathrm{pb}x}\Delta x - mg\Delta x\sin(\theta)\right)} = \sqrt{\frac{2}{(2.0\text{ kg})}\left(\frac{1}{2}(2.0\text{ kg})(2.0\text{ m/s})^2 + (100\text{ N})(2.0\text{ m}) - (2.0\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m})\sin(30°)\right)}$ $= 14\text{ m/s}.$

(*b*) The process as in part (*a*) applies, except that now work will be done by friction as well. Note that the sum of all forces in the direction perpendicular to the incline shows us that $F^n_{\text{sb}} = mg\cos(\theta)$. We can write $K_\text{i} + W_\text{p} + W_\text{surface} = K_\text{f} + U^G_\text{f} \Rightarrow \frac{1}{2}mv_\text{i}^2 + F^\text{c}_{\text{pb}x}\Delta x - F^k_{\text{sb}x}\Delta x = \frac{1}{2}mv_\text{f}^2 + mg\Delta x\sin(\theta)$ such that

$$v_\text{f} = \sqrt{\frac{2}{m}\left(\frac{1}{2}mv_\text{i}^2 + F^\text{c}_{\text{pb}x}\Delta x - \mu_k mg\cos(\theta)\Delta x - mg\Delta x\sin(\theta)\right)}$$

$$= \sqrt{\frac{2}{(2.0\text{ kg})}\left(\frac{1}{2}(2.0\text{ kg})(2.0\text{ m/s})^2 + (100\text{ N})(2.0\text{ m}) - (0.25)(2.0\text{ kg})(9.8\text{ m/s}^2)\cos(30°)(2.0\text{ m}) - (2.0\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m})\sin(30°)\right)}$$

$$= 13\text{ m/s}$$

(*c*) Once you release the block, the only work will be that done by friction. Starting at the moment you release the block and with the final time being the instant the block stops, we have $K_\text{i} + W_\text{surface} = U^G_\text{f} \Rightarrow \frac{1}{2}mv_\text{i}^2 - \mu_k mg\Delta x\cos(\theta)$ $= mg\Delta x\sin(\theta)$ or $\Delta x = \dfrac{v_\text{i}^2}{2g(\sin(\theta) + \mu_k\cos(\theta))} = \dfrac{(13.3\text{ m/s})^2}{2(9.8\text{m/s}^2)(\sin(30°) + (0.25)\cos(30°))} = 13\text{ m}.$

10.94. (*a*) The load is most likely to fall off when the platform under it is being given the largest magnitude acceleration. This occurs when the magnitude of the spring force is maximal. Thus the load is most likely to fall off when the spring is at either maximum compression or maximum extension. (*b*) Call the axis along which the spring is oriented the x axis. The maximum force that the spring will ever exert on the platform is $(F^\text{c}_{\text{sp}x})_\text{max} = k(\Delta x)_\text{max} = (m_\text{p} + m_\ell)(a_x)_\text{max}$, or trivially rearranging: $(a_x)_\text{max} = \dfrac{k(\Delta x)_\text{max}}{(m_\text{p} + m_\ell)}$. But if we now look at the forces acting in the x direction on the load only, we find $(F_\text{on load})_\text{max} = (F^s_{\text{p}\ell x})_\text{max} = \mu_s m_\ell g = m_\ell(a_x)_\text{max}$ or $(a_x)_\text{max} = \mu_s g$. The acceleration of the platform and load must be the same if they are to move together. So we have equate these two expressions for the maximum acceleration to obtain $\Delta x_\text{max} = (m_\text{p} + m_\ell)g\mu_s / k$.

10.95. The tow cable is doing work that is increasing the skiers' gravitational potential energy and overcoming work done by friction. We can write $W_\text{tow} + W_\text{friction} = \Delta U^\text{G} \Rightarrow \dfrac{W_\text{tow}}{\Delta t} - \dfrac{\mu_k mg\cos(\theta)\Delta x}{\Delta t} = \dfrac{mg\Delta x\sin(\theta)}{\Delta t}$ so $P_\text{tow} = mgv(\sin(\theta) + \mu_k\cos(\theta)) = (20)(60\text{ kg})(9.8\text{ m/s}^2)(3.0\text{ m/s})(\sin(32°) + (0.12)\cos(32°)) = 22\text{ kW}.$

10.96. There are many cases to consider here.

Case 1: This is the simplest case. Suppose the values of k, d, θ, μ and m are such that the spring fully extends, but the block never goes higher than $2d$. In that case there is friction the entire time. Then we can write $U^\text{sp}_\text{i} + W_\text{friction} = U^G_\text{f} \Rightarrow \frac{1}{2}kd^2 - \mu mg\cos(\theta)\Delta x_\text{AB} = mg\Delta x_\text{AB}\sin(\theta)$ or $\Delta x_\text{AB} = \dfrac{kd^2}{2mg(\sin(\theta) + \mu\cos(\theta))}$.

Case 2: Suppose the values of the variables are such that the block goes farther than $2d$ up the ramp. The block can't stop where there is no friction, but we can find the kinetic energy at the moment the block leaves the rough patch. The block will rise without dissipation and then return to the rough patch with the same kinetic energy it had when it left. Then additional work will be done by friction. We assume the block must stop before striking the spring again, in order to fit the description in the problem. For the first trip up the rough patch, followed by a partial return down the patch, we can write $U^\text{sp}_\text{i} + W_\text{friction,1} + W_\text{friction,2} = U^G_\text{f} \Rightarrow \frac{1}{2}kd^2 - \mu mg\cos(\theta)2d - \mu mg\cos(\theta)(2d - \Delta x_\text{AB}) =$ $mg\Delta x_\text{AB}\sin(\theta)$ or $\Delta x_\text{AB} = \dfrac{\frac{1}{2}kd^2 - 4\mu mgd\cos(\theta)}{mg(\sin(\theta) - \mu\cos(\theta))}$.

Case 3: The spring does not fully extend. Note that the spring must partially extend, because the problem statement says the block moves up the incline. If the spring does not fully extend, then $U^\text{sp}_\text{i} + W_\text{friction} =$

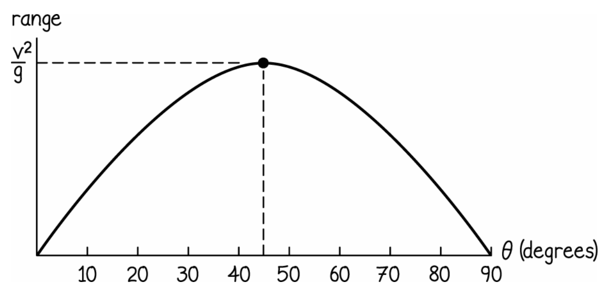
$U_f^{sp} + U_f^G \Rightarrow \frac{1}{2}kd^2 - \mu mg\cos(\theta)\Delta x_{AB} = mg\Delta x_{AB}\sin(\theta) + \frac{1}{2}k(d - \Delta x_{AB})^2$. This quadratic equation has the solution $x = \frac{2}{k}\left[kd - mg(\mu\cos(\theta) - \sin(\theta))\right]$.

There may be other intermediate cases of interest.

10.97. The work done by friction reduces the kinetic energy to zero, so $W = -mg\mu\Delta x = \Delta K = -\frac{1}{2}mv_i^2 \Rightarrow v_i = \sqrt{2g\mu\Delta x} = \sqrt{2(9.8\,\text{m/s}^2)(0.5)(290\text{ m})} = 53\text{ m/s}$.

10.98. Assume the highest point on the Empire State building where a person can stand is 380 m. We can write for the vertical component of the penny's motion $\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-380\text{ m})}{(-9.8\text{ m/s}^2)}} = 8.8\text{ s}$. Estimates of the maximum initial speed with which a person can throw a penny will vary greatly. Let us assume the order of magnitude is $v_x = 1\times10^1\text{ m/s}$. Then the horizontal distance from the building at which the penny lands is of order $\Delta x = v_x\Delta t = (1\times10^1\text{ m/s})(8.8\text{ s}) = 1\times10^2\text{ m}$. Answers close to this are acceptable.

10.99.



10.100. As you increase the angle, the initial velocity has a greater initial vertical component. This increase in the vertical component of the initial velocity will increase the time that the water is in the air. For small angles above horizontal this increases the range of the water. But as you increase this angle you also decrease the component of the initial velocity parallel to the ground. So there is a tradeoff between the amount of time the water is in the air and how fast it is moving parallel to the ground. This tradeoff results in an optimum angle that gives the maximum range. In the absence of air resistance, this optimum angle is $45°$. Increasing the angle up to $45°$ will increase the range, but increasing the angle beyond $45°$ will reduce the range again.

10.101. (*a*) The maximum x coordinate will be reached when v_x is momentarily zero. So we can write $v_{x,f} = v_{x,i} + a_x\Delta t_{max} \Rightarrow \Delta t_{max} = \frac{v_{x,f} - v_{x,i}}{a_x} = \frac{(0) - (40\text{ m/s})}{(-1.0\text{ m/s}^2)} = 40\text{ s}$. Then the maximum x coordinate is $x_{max} = \Delta x_{max} = v_{x,i}\Delta t_{max} + \frac{1}{2}a_x\Delta t_{max}^2 = (40\text{ m/s})(40\text{ s}) + \frac{1}{2}(-1.0\text{ m/s}^2)(40\text{ s})^2 = 8.0\times10^2\text{ m}$. (*b*) As stated in part (*a*), $v_x(\Delta t_{max}) = 0$. So we need only consider the y component: $v_{y,f} = v_{y,i} + a_y\Delta t_{max} = (0) + (-0.50\text{ m/s}^2)(40\text{ s}) = -20\text{ m/s}$. So $\vec{v}(\Delta t_{max}) = -20\text{ m/s}\,\hat{j}$.
(*c*) $y_f = \Delta y = v_{y,i}\Delta t_{max} + \frac{1}{2}a_y\Delta t_{max}^2 = (0) + \frac{1}{2}(-0.50\text{ m/s}^2)(40\text{ s})^2 = -4.0\times10^2\text{ m}$.

10.102. (*a*) $\vec{B} = 0$ (*b*) $\vec{A}$ and $\vec{B}$ are parallel.

10.103. (*a*) Label the forces F_1, F_2, and F_3 in the order they are listed in the problem. We take the sum of each component of the forces separately:

$$\sum F_x = F_{1x} + F_{2x} + F_{3x} = (100\text{ N}) + (50\text{ N})\cos(30°) + (144\text{ N})\cos(190°) = 1.49\text{ N}$$
$$\sum F_y = F_{1y} + F_{2y} + F_{3y} = (0) + (50\text{ N})\sin(30°) + (144\text{ N})\sin(190°) = -5.34\times10^{-3}\text{ N}$$

So the vector sum of all forces is $1.49\text{ N}\,\hat{i},\ 5.34\times10^{-3}\text{ N}\,\hat{j}$.

(*b*) The magnitude of the acceleration is $a = \dfrac{F}{m} = \dfrac{\sqrt{(1.49\text{ N})^2 + (5.34\times10^{-3}\text{ N})^2}}{(2.00\text{ kg})} = 0.744\text{ m/s}^2$. So the final speed of the block will be $v_\text{f} = v_\text{i} + a\Delta t = (0) + (0.744\text{ m/s}^2)(10.0\text{ s}) = 7.44\text{ m/s}$. Finally, the work will be equal to the change in kinetic energy, so $W = \Delta K = \dfrac{1}{2}mv_\text{f}^2 = \dfrac{1}{2}(2.00\text{ kg})(7.44\text{ m/s})^2 = 55.4\text{ J}$.

10.104. Let the $+x$ axis point east, and the $+y$ axis point north. Flying somewhat into the wind (westward) such that she ends up in Duluth means the westward component of her velocity $v_y = -40$ m/s relative to Earth. This means she must fly at an angle from north such that $\theta = \sin^{-1}\left(\dfrac{|v_x|}{v}\right) = 8.85°$. So the time it will take her to get to Duluth is

$$\Delta t_\text{angled} = \frac{\Delta y}{v_y} = \frac{\Delta y}{v\cos(\theta)} = \frac{(1.500\times10^6\text{ m})}{(260\text{ m/s})\cos(8.85°)} = 5.84\times10^3\text{ s or }1.62\text{ h}.$$

Flying north will take $\Delta t_\text{north} = \dfrac{\Delta y}{v_y} = \dfrac{\Delta y}{v} = \dfrac{(1.500\times10^6\text{ m})}{(260\text{ m/s})} = 5.77\times10^3$ s or 1.60 h. After this amount of time, the plane will have drifted westward a distance $\Delta x = v_{\text{wind},x}\Delta t_\text{north} = (40\text{ m/s})(5{,}769\text{ s}) = 2.31\times10^5\text{ m}$. So flying west will require addition time given by $\Delta t_\text{west} = \dfrac{\Delta x}{v - v_\text{wind}} = \dfrac{(2.31\times10^5\text{ m})}{(260\text{ m/s}) - (40\text{ m/s})} = 1.05\times10^3$ s or 0.291 h. Note that you are flying against the wind, so your speed relative to Earth is only 240 m/s. The total flight time using this path is 1.89 h. Flying angled somewhat into the wind (westward) takes less time.

10.105. Choose the $+x$ axis to point down the incline, and the $+y$ axis to be perpendicular to the incline, with an upward component. We refer to the top layer of snow with a subscript "t", and the surface of snow beneath it with a subscript "s". The sum of the x and y components of all forces yields

$$\sum F_x = F^G_{\text{Et}x} + F^s_{\text{st}x} = ma_x = 0 \Rightarrow mg\sin(\theta) = \mu_s F^n_\text{st} \quad (1)$$
$$\sum F_y = F^n_\text{st} + F^G_{\text{Eb}y} = ma_y = 0 \Rightarrow F^n_\text{st} = mg\cos(\theta) \quad (2)$$

In equation (1), in setting the force of static friction equal to $\mu_s F^n_\text{st}$, we have assumed that the force of friction is as large as it can get before the top layer slips. So this condition does correspond to the largest possible angle before an avalanche occurs. Combining equations (1) and (2) yields $\sin(\theta) = \mu_s\cos(\theta)$ or $\theta = \tan^{-1}(\mu) = \tan^{-1}(3.7) = 75°$.

10.106. The time required for the object to fall the distance h is given by the kinematic equation $\Delta y = v_{y,\text{i}}\Delta t + \dfrac{1}{2}a_y\Delta t^2 \Rightarrow \Delta t = \dfrac{-v_{y,\text{i}} \pm \sqrt{v_{y,\text{i}}^2 + 2a_y\Delta y}}{a_y}$. In this case, the physically correct solution (putting in the given variables) is $\Delta t = \dfrac{v_\text{i}\sin(\theta) + \sqrt{v_\text{i}^2\sin^2(\theta) + 2gh}}{g}$. Ignoring air resistance the x component of the velocity is constant, so $\Delta x = v_{x,\text{i}}\Delta t = v_\text{i}\cos(\theta)\left(\dfrac{v_\text{i}\sin(\theta) + \sqrt{v_\text{i}^2\sin^2(\theta) + 2gh}}{g}\right)$.

10.107. For the counterweight, let us call vertically upward the $+y$ direction. Then the equation of motion for the counterweight in the y direction is

$$\sum F_y = \mathcal{T}_{\text{rope}} - m_{\text{c}}g = m_{\text{c}}a_{\text{c},y} \Rightarrow \mathcal{T}_{\text{rope}} = m_{\text{c}}(g + a_{\text{c},y}) \tag{1}$$

For the people being lifted, let us choose the $+x$ axis to point up the inline. The equation of motion for the people in the x direction is

$$\sum F_x = \mathcal{T}_{\text{rope}} + F^G_{\text{Ep}x} + F^k_{\text{sp}x} = m_{\text{p}}a_{x,\text{p}} \Rightarrow \mathcal{T}_{\text{rope}} = m_{\text{p}}a_{x,\text{p}} + m_{\text{p}}g\left(\sin(\theta) + \mu_k\cos(\theta)\right) \tag{2}$$

The tension is the same regardless of which system we are considering, so we can combine equations (1) and (2). Note also that in order for the rope to be of a fixed length, we know $a_{\text{p,x}} = -a_{\text{c,y}}$. We can find a value for this acceleration by requiring that the final velocity at the top not exceed 5.0 m/s. We can write $a_{\text{p},x} = \dfrac{v_{x,\text{f}}^2}{2\Delta x}$. Thus we obtain

$$m_{\text{c}} = \frac{m_{\text{p}}a_{x,\text{p}} + m_{\text{p}}g(\sin(\theta) + \mu_k\cos(\theta))}{(g - a_{\text{p},x})} = \frac{m_{\text{p}}\left(\dfrac{v_{x,\text{f}}^2}{2\Delta x}\right) + m_{\text{p}}g(\sin(\theta) + \mu_k\cos(\theta))}{\left(g - \left(\dfrac{v_{x,\text{f}}^2}{2\Delta x}\right)\right)} \tag{3}$$

Inserting the maximum speed at the top of the hill for the lightest pair of passengers will give us the maximum inertia of the counterweight:

$$(m_{\text{c}})_{\text{max}} = \frac{(100\text{ kg})\left(\dfrac{(5.0\text{ m/s})^2}{2(400\text{ m})}\right) + (100\text{ kg})(9.8\text{ m/s}^2)(\sin(35°) + (0.10)\cos(35°))}{\left((9.8\text{ m/s}^2) - \left(\dfrac{(5.0\text{ m/s})^2}{2(400\text{ m})}\right)\right)} = 66\text{ kg}$$

Now, if you insert 66 kg as the inertia in the case of the heaviest pair of skiers, you will find that the counterweight is not massive enough to get them moving at all.

In order for a 100-kg pair to reach the top at a maximum speed of 5.0 m/s, the counterweight must have a maximum inertia of 66 kg. For a 200-kg pair, the counterweight inertia must be 1.3×10^2 kg. The plan won't work. Either the 200-kg pair will not be accelerated or the 100-kg pair will be accelerated to a speed higher than 5.0 m/s.

10.108. Choose the $+x$ axis to point to the east and the $+y$ axis to point to the north. Call the moment just before impact time 1, the moment after impact time 2, and the moment the vehicles came to rest time 3. First, we use the information about the skid marks to determine the velocity of the two vehicles immediately after the crash. The direction is given by the skid marks themselves. The work done by friction as the cars skidded will tell use about the initial kinetic energy:

$$\begin{aligned}
W_{\text{friction}} &= \vec{F}^k_{\text{road vehicles}} \cdot \Delta\vec{r} \\
&= -\mu_k(m_{\text{car}} + m_{\text{van}})gd \\
&= \Delta K = -\frac{1}{2}(m_{\text{car}} + m_{\text{van}})v_2^2 \Rightarrow v_2 \\
&= \sqrt{2\mu_k g d} \\
&= \sqrt{2(0.70)(9.8\text{ m/s}^2)(14\text{ m})} \\
&= 13.9\text{ m/s.}
\end{aligned}$$

So the velocity of the car-van system immediately after the collision was $13.9\,\text{m/s}$ at an angle $76°$ south of west.

We can now use this velocity along with conservation of momentum to determine the initial speed of each car. We assume that the velocity of the car was entirely westward, and the velocity of the car was entirely southward. Then the conservation of momentum in the x direction tells us

$$p_{x,1} = p_{x,2} \Rightarrow m_{\text{c}}v_{\text{c}x,1} = (m_{\text{c}} + m_{\text{v}})v_{x,2}$$

$$v_{c,1} = -\frac{(m_{\text{c}} + m_{\text{v}})}{m_{\text{c}}}v_2\cos(\theta) = -\frac{((1400\text{ kg}) + (6500\text{ kg}))}{(1400\text{ kg})}(13.9\text{ m/s})\cos(76°) = 19\text{ m/s}$$

Now, looking at the y direction, we find

$$p_{y,1} = p_{y,2} \Rightarrow m_{\mathrm{v}} v_{\mathrm{v}y,1} = (m_{\mathrm{c}} + m_{\mathrm{v}}) v_{y,2}$$

$$v_{\mathrm{v},1} = -\frac{(m_{\mathrm{c}} + m_{\mathrm{v}})}{m_{\mathrm{v}}} v_2 \sin(\theta) = -\frac{((1400\ \mathrm{kg}) + (6500\ \mathrm{kg}))}{(6500\ \mathrm{kg})}(13.9\ \mathrm{m/s})\sin(76°) = 16\ \mathrm{m/s}$$

The 35 mi/h speed limit is equal to $\left(\dfrac{35\ \mathrm{mi}}{1\ \mathrm{h}}\right)\times\left(\dfrac{1609\ \mathrm{m}}{1\ \mathrm{mi}}\right)\left(\dfrac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = 16\ \mathrm{m/s}$. If you kept extra digits in your calculator, you may find that the van was speeding. But to the correct number of significant digits the van was driving at the speed limit. The car was clearly speeding.

10.109. (*a*) Kinematics tells us $\Delta y = v_{y,\mathrm{i}}\Delta t + \frac{1}{2}a_y \Delta t^2 = v_{\mathrm{i}}\sin(\theta)\Delta t - \frac{g}{2}\Delta t^2$. Also, since there is no acceleration in the x direction, we can write $\Delta x = v_{x,\mathrm{i}}\Delta t$ or equivalently $\Delta t = \dfrac{\Delta x}{v_{\mathrm{i}}\cos(\theta)}$. We assume the kick occurs at the origin, so that we can let $\Delta x \to x$. Inserting this expression for time into the equation for the vertical distance yields $\Delta y(x) = \tan(\theta)x - \dfrac{g}{2v^2\cos^2(\theta)}x^2$. (*b*) The expression obtained in part (*a*) can be rearranged to yield $v = \sqrt{\left(\dfrac{g}{\tan(\theta)x - y}\right)\dfrac{x^2}{2\cos^2(\theta)}} = \sqrt{\dfrac{1}{2}\left(\dfrac{(9.8\ \mathrm{m/s^2})}{\tan(30°)(48\,\mathrm{m}) - (0)}\right)\dfrac{(48\ \mathrm{m})}{\cos(30°)}} = 23\ \mathrm{m/s}$. (*c*) $F_{\mathrm{av}} = ma_{\mathrm{av}} = m\dfrac{(v_{\mathrm{f}} - v_{\mathrm{i}})}{\Delta t} = (0.45\ \mathrm{kg})$ $\dfrac{((23.3\ \mathrm{m/s}) - (0))}{(0.15\ \mathrm{s})} = 70\ \mathrm{N}$.

10.110. (*a*) Call the moment the bullet is fired time 1, the moment just after the bullet strikes the block time 2, and the moment when the block reaches its maximum height time 3. We first consider the conservation of energy after the block was struck.

$$K_2 = U_3^G \Rightarrow \frac{1}{2}(m_{\mathrm{block}} + m_{\mathrm{bullet}})v_2^2 = (m_{\mathrm{block}} + m_{\mathrm{bullet}})g\Delta y_3$$

$$v_2 = \sqrt{2g\Delta y_3} = \sqrt{2(9.8\ \mathrm{m/s^2})(0.060\ \mathrm{m})} = 1.08\ \mathrm{m/s}$$

Now for the collision we will use conservation of momentum. We know $p_{x,1} = p_{x,2}$ so

$$m_{\mathrm{bullet}} v_{x,1} = (m_{\mathrm{block}} + m_{\mathrm{bullet}})v_{x,2}$$

$$v_{x,1} = \frac{(m_{\mathrm{block}} + m_{\mathrm{bullet}})}{m_{\mathrm{bullet}}} v_{x,2} = \frac{((5.0\ \mathrm{kg}) + (0.0095\ \mathrm{kg}))}{(0.0095\ \mathrm{kg})}(1.08\ \mathrm{m/s}) = 5.7\times 10^2\ \mathrm{m/s}$$

(*b*) We calculate the kinetic energies before and after the collision:

$$K_1 = \frac{1}{2}m_{\mathrm{bullet}}v_1^2 = \frac{1}{2}(0.0095\ \mathrm{kg})(572\ \mathrm{m/s})^2 = 1.553\ \mathrm{kJ}$$

$$\mathrm{K}_2 = \frac{1}{2}(m_{\mathrm{bullet}} + m_{\mathrm{block}})v_2^2 = \frac{1}{2}(5.0095\ \mathrm{kg})(1.08\ \mathrm{m/s})^2 = 2.92\ \mathrm{J}$$

So 1.6 kJ of energy was dissipated in the collision. This energy went mostly into breaking bonds in the wood, and heat, although a small amount also turned into sound.

10.111. Call the moment the bullet is fired time 1, the moment just after the bullet strikes the block time 2, and the moment when the block reaches its maximum height time 3. Call the direction in which the bullet is fired the $+x$ axis. We start by treating the collision using conservation of momentum:

$$m_{\mathrm{bullet}} v_{x,1} = (m_{\mathrm{block}})v_{x,2} + m_{\mathrm{bullet}}\frac{v_{x,1}}{3}$$

$$v_{x,2} = \frac{2m_{\mathrm{bullet}}v_{x,1}}{3(m_{\mathrm{block}})} = \frac{2(0.010\ \mathrm{kg})(300\ \mathrm{m/s})}{3((2.0\ \mathrm{kg}))} = 1.0\ \mathrm{m/s}$$

Now we can use conservation of energy as the block rises:

$$K_2 = U_3^G \Rightarrow \frac{1}{2}(m_{\text{block}} + m_{\text{bullet}})v_2^2 = (m_{\text{block}} + m_{\text{bullet}})g\Delta y_3$$

$$\Delta y_3 = \frac{v_2^2}{2g} = \frac{(1.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 0.051\text{ m}$$

So the block rises 51 mm.

11

MOTION IN A CIRCLE

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^3 m/s **2.** 10^{-1} $\text{kg} \cdot \text{m}^2$ **3.** 10^0 $\text{kg} \cdot \text{m}^2$ **4.** 10^2 $\text{kg} \cdot \text{m}^2/\text{s}$ **5.** 10^2 $\text{kg} \cdot \text{m}^2/\text{s}$ **6.** 10^4 m/s **7.** 10^{23} N **8.** 10^{29} J
9. 10^6 $\text{kg} \cdot \text{m}^2/\text{s}$ **10.** 10^1 J

Guided Problems

11.2 Swift shuttle

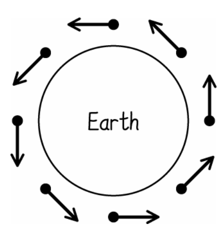
1. Getting Started The rotational speed of the shuttle is related to the period and the radius of its path by

$$\omega = \frac{2\pi}{\Delta t} = \frac{2\pi}{T} \tag{1}$$

The above expression holds, simply because there is no acceleration in the direction of motion; the acceleration is always directed radially inward toward the center of Earth.

We know we can write the tangential speed and the centripetal acceleration in terms of this rotational speed. These relationships were obtain in the text using purely kinematic arguments. We do not need to use forces to obtain the centripetal acceleration.

The setup described in the question is shown below in a motion diagram. Note that all dots are evenly spaced around the circular orbit. This must be the case since the motion is tangential, and the acceleration is purely radial.



2. Devise Plan We can use equation (1) above to determine the rotational speed, and we can relate that to the tangential speed using

$$v = \omega R_{\text{orbit}} \tag{2}$$

It is important to note that the radius of the shuttle's path R_{orbit} is not the altitude given, but the altitude plus the radius of Earth. The exact radius of Earth varies slightly depending on latitude, but we use the equatorial radius to obtain $R_{\text{orbit}} = R_{\text{E}} + h = (6.38\times10^6\ \text{m}) + (3.00\times10^5\ \text{m}) = 6.68\times10^6\ \text{m}$. This is all that is required to use equation (2). The magnitude of the centripetal acceleration can be determined from

$$a_{\text{c}} = \frac{v^2}{r} = \frac{v^2}{R_{\text{orbit}}} \tag{3}$$

We are not asked for the direction of the centripetal acceleration, because it must always point radially inward toward the center of the circular path (in this case the center of Earth).

3. Execute Plan Inserting the given values into equation (1), we obtain our answer to part (*a*):

$$\omega = \frac{2\pi}{(90.5\ \text{min})}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 1.16\times10^{-3}\ \text{s}^{-1}$$

Inserting this value for the rotational speed, and the numerical value obtained above for the radius of the orbit into equation (2) yields the answer to part (*b*):

$$v = (1.16\times10^{-3}\ \text{s}^{-1})(6.68\times10^6\ \text{m/s}) = 7.73\times10^3\ \text{m/s}$$

Finally, inserting this numerical value into equation (3) yields our centripetal acceleration for part (*c*):

$$a_{\text{c}} = \frac{v^2}{R_{\text{orbit}}} = \frac{(7.73\times10^3\ \text{m/s})^2}{(6.68\times10^6\ \text{m})} = 8.94\ \text{m/s}^2$$

4. Evaluate Result Looking back at equation (1), we note that a longer period would correspond to a lower rotational speed and a shorter period would correspond to a higher rotational speed. This is sensible. Inserting equation (1) into equation (2), we see similar behavior for the speed:

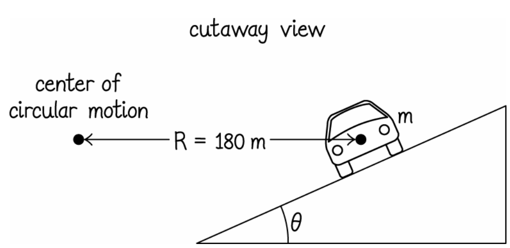
$$v = \frac{2\pi R_{\text{orbit}}}{T}$$

Clearly, a smaller period corresponds to a greater speed.

We know that an object on the surface of Earth experiences an acceleration due to gravity of $9.8\ \text{m/s}^2$, toward the center of Earth. We will not have the necessary information to exactly determine the acceleration due to gravity from forces until Chapter 13. But it is very reasonable to guess that the acceleration due to gravity in orbit 300 km above Earth's surface might be somewhat smaller than on the surface of Earth. Thus, our answer of $8.94\ \text{m/s}^2$ is quite sensible.

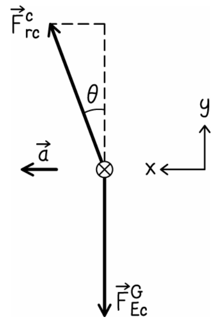
11.4 It's in the bank

1. Getting Started We begin by drawing a cutaway section of the road, with the car on it:



This is similar to Worked Problem 11.3 in that there is a component of acceleration in toward the center of circular motion. This case is diagrammatically similar, in that we have not drawn the actual circular path, just the state of the car at one moment in its circular motion. This is also similar to Worked Problem 11.3 and *Principles* Example 11.4 in that the centripetal force is provided by contact forces with the ground. The main difference is that here, we require that the force of friction be zero. The contact force providing centripetal force is the normal force that the ground exerts on the car, which has a component in the radial direction.

2. Devise Plan Because we require that the force of friction be zero, the only forces being exerted on the car are gravity and the normal force. We construct a free body diagram:



The car is driving in a circle as seen from above, meaning that the center of the circle is to the left in the diagram above. This is how we obtain the direction of the acceleration in the free-body diagram. One can choose whatever axes one likes, but it is a good habit to choose one axis along the direction of the acceleration. In the above diagram we have called this direction $+x$, although it would have been perfectly acceptable to call it $+r$ to remind us that this is the same direction as radially inward toward the center of the circular path. We have chosen the $+y$ direction to be perpendicular to that direction, and point vertically upward.

From the free-body diagram, we can write the equations of motion given by Newton's second law in the x and y directions:

$$\sum F_x = F^{c}_{rc}\sin(\theta) = ma_x \tag{1}$$

$$\sum F_y = F^{c}_{rc}\cos(\theta) - F^{G}_{Ec} = ma_y \tag{2}$$

Since the acceleration in the x direction is the centripetal acceleration, given by $a_c = v^2/r$,and the acceleration in the y direction is zero, equations (1) and (2) become

$$F^{c}_{rc}\sin(\theta) = mv^2/r \tag{3}$$

$$F^{c}_{rc}\cos(\theta) = mg \tag{4}$$

We have three unknowns: F^{c}_{rc}, θ, and m. We have only two equations, which could be a problem. However, we recognize that it is always possible that one unknown may cancel in our equations and may not be needed. We may even recognize that the inertia m will drop out, as it occurs linearly in both equations.

3. Execute Plan Rearranging equation (4) to solve for F^{c}_{rc} in terms of other variables, and inserting the result into equation (3), we obtain $\tan(\theta) = v^2/gr$ or

$$\theta = \tan^{-1}\left(\frac{v^2}{gr}\right) \tag{5}$$

The posted speed is $v = \left(\frac{100\text{ km}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right) = 27.8\text{ m/s}.$ So equation (5) yields

$$\theta = \tan^{-1}\left(\frac{(27.8\text{ m/s})^2}{(9.8\text{ m/s}^2)(180\text{ m})}\right) = 23.6°$$

4. Evaluate Result The function $\tan(\theta)$ increases monotonically as θ increases on the physically relevant interval: $0 \le \theta < 90°$. So increasing the angle θ means increasing $\tan(\theta)$. Thus equation (5) shows that increasing the speed increases the required banking angle. This is perfectly sensible, since a very fast car is more likely to fly off a road, and may require a greater banking angle to keep it on the road. We also see that the required banking angle decreases if the radius of curvature increases. This is also reasonable. Consider the limit in which r becomes very large, on the order of kilometers. In that limit, one would not even notice the curvature of the road, and the banking angle should approach zero. We see this behavior in equation (5).

The banking angle does not depend on the inertia of the vehicle. This can be understood by noting which forces are proportional to that inertia m. Because the normal force counteracts a component of gravity, it is clear that the magnitude of F_{rc}^{c} must be proportional to m. Thus the component of F_{rc}^{c} responsible for the centripetal force must also be proportional to m. Dividing by m to obtain the centripetal acceleration yields no dependence on m.

This is a rather steep angle, but it is also for a fairly sharp curve. It is within the range of possibility.

11.6 Hit that door

1. Getting Started Let us choose a system consisting of the door and the ball. This system is both isolated and closed, provided that gravity is not able to do any work on the system. Since our collision can be assumed to occur over a short time interval, and since the directions of motion are all perpendicular to the direction in which gravity would act, the system is isolated and closed to a very good approximation.

Figure WG 11.7 shows the instant just before the collision and the instant just after the collision. Although the clockwise direction is often chosen to be negative, here we have chosen the initial motion of the door around the hinge to be clockwise, and thus make the clockwise direction positive.

2. Devise Plan In part (*a*) we seek the final rotational speed of the door ω_{f}, in terms of the other variables given. In part (*b*), we require that this final rotational speed be negative (reversing direction). We can rearrange the expression obtained in part (*a*) to determine the minimum require speed of the ball in order to achieve this. So, (*a*) and (*b*) do ask for different things, but we should be able to obtain both answers from the same relation.

The most useful quantity that is conserved in this collision is rotational momentum. We can say

$$\vec{L}_{\text{i}} = \vec{L}_{\text{f}}$$
$$\vec{L}_{\text{d,i}} + \vec{L}_{\text{b,i}} = \vec{L}_{\text{d,f}} + \vec{L}_{\text{b,f}}$$

We choose the axis of the system to be the hinge of the door, and note the direction of each angular momentum around that axis. Writing out each angular momentum in terms of variables inserting the appropriate signs, we obtain $I_{\text{d}}\omega_{\text{i}} - m_{\text{b}}v_{\text{b,i}}r_{\text{b}} = I_{\text{d}}\omega_{\text{f}} + m_{\text{b}}v_{\text{b,f}}r_{\text{b}}$.

The rotational inertia of the door is $I_{\text{d}} = \frac{1}{3}m_{\text{d}}\ell_{\text{d}}^2$ and the radial distance from the axis to the point at which the ball strikes is $r_{\text{b}} = d$. Thus $\frac{1}{3}m_{\text{d}}\ell_{\text{d}}^2\omega_{\text{i}} - m_{\text{b}}v_{\text{b,i}}d = \frac{1}{3}m_{\text{d}}\ell_{\text{d}}^2\omega_{\text{f}} + m_{\text{b}}v_{\text{b,f}}d$. Inserting the relationship between the initial and final speed of the ball, we find

$$\frac{1}{3}m_{\text{d}}\ell_{\text{d}}^2\omega_{\text{i}} - m_{\text{b}}v_{\text{b}}d = \frac{1}{3}m_{\text{d}}\ell_{\text{d}}^2\omega_{\text{f}} + m_{\text{b}}\left(\frac{v_{\text{b}}}{4}\right)d \tag{1}$$

3. Execute Plan (*a*) Rearranging equation (1) we find an expression for the final rotational velocity of the door around its hinge:

$$\omega_{\text{f}} = \omega_{\text{i}} - \frac{15}{4}\frac{m_{\text{b}}v_{\text{b}}d}{m_{\text{d}}\ell_{\text{d}}^2} \tag{2}$$

(*b*) In order for the door to reverse direction, this component of the final rotational velocity around the hinge must be negative. Thus we require

$$\omega_{\text{i}} < \frac{15}{4}\frac{m_{\text{b}}v_{\text{b}}d}{m_{\text{d}}\ell_{\text{d}}^2}$$

or

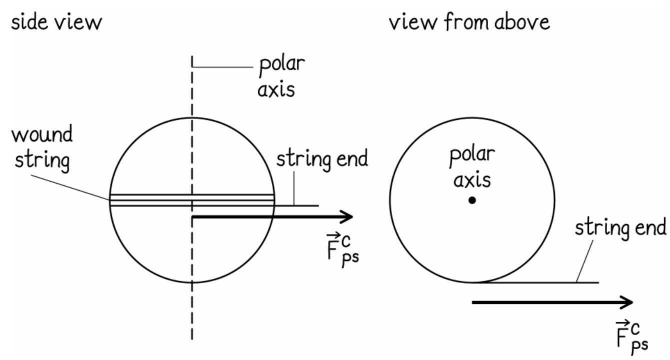
$$v_{\text{b}} > \frac{4\omega_{\text{i}}m_{\text{d}}\ell_{\text{d}}^2}{15m_{\text{b}}d} \tag{3}$$

4. Evaluate Result Our result for the final rotational velocity of the door around its hinge makes sense. Equation (2) shows us that if the ball is extremely light weight $(m_b \to 0)$ or if it is moving very slowly $(v_b \to 0)$ the rotational speed of the door would not change. Also if the ball strikes directly at the hinge $(d = 0)$ the ball would have no effect on the rotational speed of the door. Similar behavior is noted if the inertia of the door or its length become extremely large.

Our answer to part (*b*) also shows the scaling we expect. Equation (3) shows us that if the door has very large inertia, the speed of the ball will need to be very large in order for the door to swing the other way. This fits our expectation. Clearly, if the inertial of the ball increases, we will not require as high of a speed to reverse the direction of the door.

11.8 Globe pulling

1. Getting Started The figure below shows a side view and aerial view of the globe with string wrapped around it:



Since the rope is wrapped around the globe, we will always be pulling tangential to the surface of the globe. This means we will cause a torque on the globe, and cause its rotational momentum, and rotational kinetic energy to increase. We are asked to get the globe up to a specific final rotational speed, and we know that the torque and rotational speed can be related in a simple way through the rotational kinetic energy. Thus we focus on the rotational kinetic energy.

If the string is slipping across the surface of the globe, we don't have enough information about the materials to proceed (coefficients of friction, etc). Thus, we assume that the string does not slip.

2. Devise a Plan In this situation, we are exerting a constant force on the string, and this is causing a constant torque on the globe. Thus, we can write

$$\tau \Delta \vartheta = \Delta K_{\text{rot}} = \frac{1}{2} I_g \omega_{g,f}^2 - \frac{1}{2} I_g \omega_{g,i}^2 \qquad (1)$$

This involves both the torque we apply and the rotational speed we want.

The rotational inertia of a globe is closest to that of a hollow spherical shell: $I_g = \frac{2}{3} m_g R_g^2$. Inserting this expression into equation (1) and using the fact that the globe starts from rest, we find

$$\tau \Delta \vartheta = \frac{1}{3} m_g R_g^2 \omega_{g,f}^2 \qquad (2)$$

As we exert a constant force at the edge of the globe and tangential to it, the torque is equal to $\tau = F^c_{ps} R_g$. This is the same torque that would be caused by a force being exerted on a single point on the globe's surface. Also, as the globe rotates through an angle $\Delta \vartheta$, a length of string is unwound that is equal to $\Delta \ell = R_g \Delta \vartheta$. Thus equation (2) can be written

$$F_{\text{ps}}^{\text{c}} R_{\text{g}}\left(\frac{\Delta\ell}{R_{\text{g}}}\right) = \frac{1}{3} m_{\text{g}} R_{\text{g}}^2 \omega_{\text{g,f}}^2$$

$$\Delta\ell = \frac{1}{3}\frac{m_{\text{g}} R_{\text{g}}^2 \omega_{\text{g,f}}^2}{F_{\text{ps}}^{\text{c}}} \quad (3)$$

3. Execute Plan We require that the globe reach a rotational speed described by 0.50 rev/s. In SI units, this is $\omega_{\text{f}} = (0.50 \text{ rev/s})(2\pi \text{ rev}^{-1}) = 3.14 \text{ s}^{-1}$. Inserting the given values into equation (3), we find

$$\Delta\ell = \frac{1}{3}\frac{(20 \text{ kg})(0.50 \text{ m})^2(3.14 \text{ s}^{-1})^2}{(2.0 \text{ N})} = 8.2 \text{ m}$$

So you would need to wrap 8.2 m of rope around the globe.

4. Evaluate Result Half of a rotation each second is a reasonable speed to achieve. So it makes sense that the string would have to be wound several times around the globe, but not an extremely large number of times. The length we found corresponds to winding the string about 2.6 times completely around the globe. This is plausible.

Questions and Problems

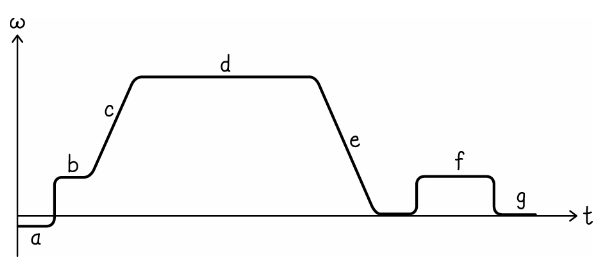
11.1. The maximum density of "bits" is on the inner track. The entire disk spins at one rotational speed, this means that a bit on the outer track will have a higher translational speed than a bit on the inner track. So for bits to be read at the same rate (while moving at different translational speeds) the spacing between the bits must be different on the inner and outer tracks. The spacing between bits on the outer track must be larger, meaning the maximum density is on the inner track.

11.2. The daily rotation carries Earth through an angle of 2π radians in just one day. The annual motion of Earth around the sun requires 365 days to go through 2π radians. Thus the ratio of daily rotational speed to annual rotational speed is 365:1.

11.3. The distance around the curved track is shorter in the inner lane, meaning the distance the passing car has to travel in order pass is shorter in the inner lane than it would be in an outer lane.

11.4. (*a*) With t representing time interval required for car in innermost lane to complete 20 laps, required time intervals are $1.10t$ for car in middle lane and $1.20t$ for car in outermost lane. (*b*) With v representing speed of car in innermost lane, speeds are $1.10v$ for car in middle lane and $1.20v$ for car in outermost lane.

11.5.

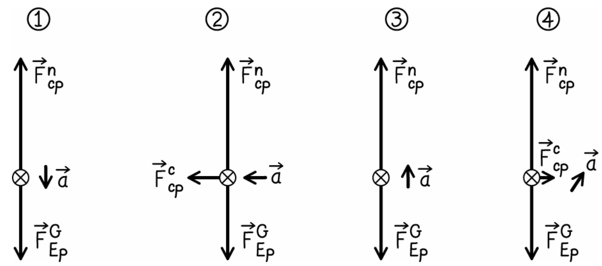


11.6. The center of the moving quarter revolves around the stationary quarter, tracing out a circle that is concentric with the stationary quarter and has a radius equal to the sum of the radii of the two coins. As it moves once around the stationary quarter, the moving quarter completes one rotation about its center. The direction of this rotation is the same as the direction of the revolution.

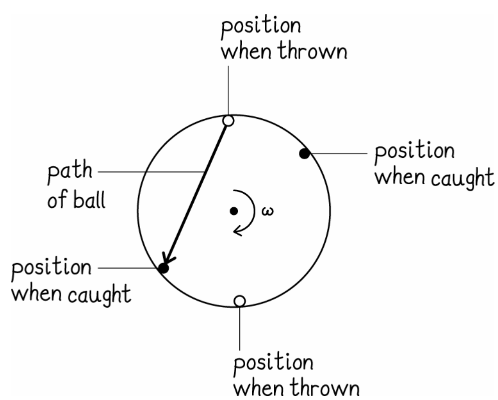
11.7. The units of acceleration are $\mathrm{m/s^2}$. Since units of time only appear in v, not in r, we know $p = 2$ so that we have two powers of s. Just having v^2 gives us units of $\mathrm{m^2/s^2}$, which is one too many powers of m. Thus $q = -1$. That way we have $\dfrac{v^2}{r}$, the units of which are $\mathrm{m/s^2}$. So $p = 2$ and $q = -1$.

11.8. Point b. The stone will follow a trajectory that starts out tangential to the initial circular motion.

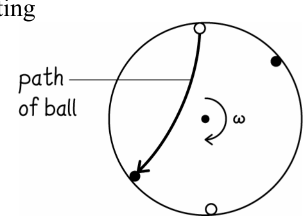
11.9.



11.10. (*a*) The path is a straight line across the merry-go-round, from you to your friend, passing near the center of the circle but slightly to one side.



(*b*) To you, the path does not follow a straight line; it curves either left or right away from the center depending on in which direction the merry-go-round is rotating

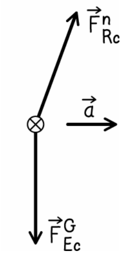


(*c*) To your friend, the path curves in the direction opposite the direction you observe. (*d*) You would need to identify a force that can cause the observed motion. Such a force is fictitious (does not actually exist), but would be required to explain why the ball apparently accelerates.

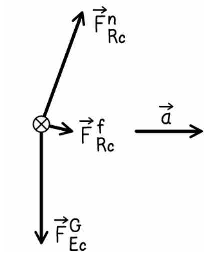
11.11. As the spinner rotates, the water and lettuce, because of their inertia, tend to move in a straight line tangent to the spinner circumference. Because it cannot fit through the holes in the spinner wall, the lettuce can't move in a straight line. The contact force exerted on it by the wall keeps it inside. Water drops can pass through the holes and do so, separating from the lettuce and leaving it dry. Initially, the drops might be held to the lettuce by cohesive forces, but as the spinner speeds up, these forces are insufficient to keep the drops moving a circle, and they spin out of the spinner.

11.12.

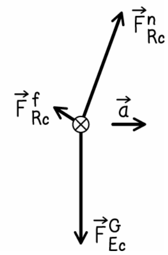
(a)



(b)

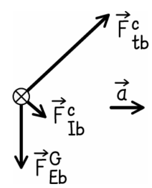


(c)

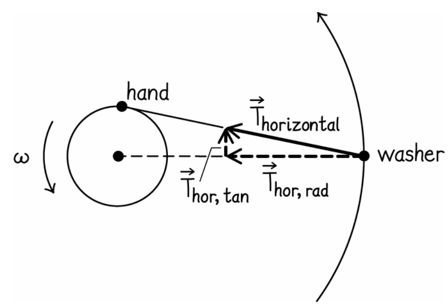


11.13. Ignoring friction between the car and track, the car climbs up the right wall as it enters the left turn. Because the student's body and the metal chunk climb this wall by the same amount as the car, the chunk stays between his knees, just as it was before the ride began.

11.14. (*a*) The top string breaks first. The bottom string and gravity both exert forces that have a downward component. The top string is the only thing exerting a force with an upward component. So that vertical component of the tensile force in the top string must be larger than that in the bottom string. Since the strings are at the same angle from vertical, this means the total tension in the top rope is greater.
(*b*)



11.15. (*a*)



(*b*) The tensile force exerted by string is responsible for the acceleration. This is possible because the hand is not at the center of the circular motion. The acceleration has a radial and a tangential component. (*c*) Ignoring air resistance, your hand stays at the center of the circle; the tensile force is directed toward the circle center and the acceleration has no tangential component.

11.16. You should hold the lamp close to the base. The center of mass will be closer to the base than to the top.

11.17. Because the body's rotational inertia is greater in layout position than in tucked position. The radius of the circular path the head and feet travel during the flip is larger in layout position than in tucked position. Because the body remains airborne for roughly the same time interval in either position, the gymnast must have much greater kinetic energy in layout position to complete the backflip.

11.18. The ice cube will reach the bottom first. As the ball rolls down the cookie sheet, it gains translational kinetic energy, but it also spins around its center meaning it gains rotational kinetic energy. In contrast, the ice cube does not roll, so all the energy converted to kinetic energy is purely translational. This means the ice cube will have a higher translational speed after falling a given distance, and will thus reach the bottom first.

11.19. (*a*) No, the inertia of the bowling ball is so much greater than that of the baseball that even placing the rotation axis on the outer surface of the baseball would not make it as difficult to spin as the bowling ball. (*b*) Yes. You could choose to revolve the baseball around an axis that passes no closer than, say, 5 m to the baseball, while choosing an axis passing just centimeters from the edge of the bowling ball. The rotational inertias are $I_{\text{bowl}} = m_{\text{bowl}}r^2_{\text{bowl}}$ and $I_{\text{base}} = m_{\text{base}}r^2_{\text{base}}$, where r is distance from either ball to the axis of revolution. Choose an arbitrary location for this axis and place the two balls at distances r_{bowl} and $r_{\text{base}} >> r_{\text{bowl}}$ from the axis. Once the inequality $r_{\text{base}} >> r_{\text{bowl}}$ is sufficiently large to makes the difference between m_{bowl} and m_{base} insignificant, $I_{\text{base}} > I_{\text{bowl}}$.

11.20. (*a*) A vertical axis going through the center of the door would yield the smallest rotational inertia for the door. (*b*) This axis is not practical, as people would have great difficulty fitting through such a door.

11.21. (*a*) The rotational inertia decreases. As you move toward the center, there is less inertia far from the axis of rotation. (*b*) The carousel should rotate faster. As you decrease the rotational inertia of the system, it becomes easier to rotate.

11.22. $\omega = \dfrac{\Delta\vartheta}{\Delta t} = \left(\dfrac{6000 \text{ rev}}{(1 \text{ min})}\right)\left(\dfrac{2\pi}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = 6.283\times10^2 \text{ s}^{-1}$

11.23. $\omega_{\text{second}} = \dfrac{\Delta\vartheta}{\Delta t} = \left(\dfrac{1 \text{ rev}}{(1 \text{ min})}\right)\left(\dfrac{2\pi}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right) = 0.10 \text{ s}^{-1}$ and $\omega_{\text{hour}} = \dfrac{\Delta\vartheta}{\Delta t} = \left(\dfrac{1 \text{ rev}}{(12 \text{ h})}\right)\left(\dfrac{2\pi}{1 \text{ rev}}\right)\left(\dfrac{1 \text{ h}}{60 \text{ min}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)$ $= 1.5\times10^{-4} \text{ s}^{-1}$

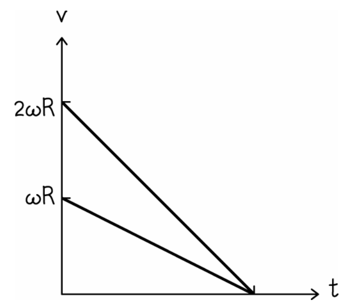
11.24. The odometer would overestimate the distance travelled. The tires would slip a little as the car drives, so that some of the tire rotation measured by the odometer would not actually correspond to moving the car forward.

11.25. $\omega_{\text{f}}^2 = \omega_{\text{i}}^2 + 2\alpha\Delta\vartheta \Rightarrow \alpha = \dfrac{\omega_{\text{f}}^2 - \omega_{\text{i}}^2}{2\Delta\vartheta} = \dfrac{(0) - \left(\dfrac{2\pi}{12 \text{ s}}\right)^2}{2(2.5)(2\pi)} = -8.7\times10^{-3} \text{ s}^{-1}$

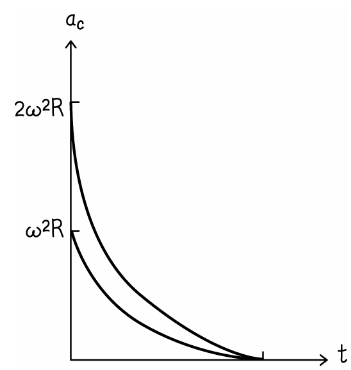
11.26. Call the horizontal direction radially in toward the center of the circular path the $+r$ axis, and call the direction perpendicular to the incline with some upward component $+y$. In order for the car to move in a circle, the vector sum of all forces must point inward toward the center of the circular path, along $\vec{r}$. The only force with a component along $\vec{r}$ is the normal force that the road exerts on a car. Call the radius of curvature of the road R. We can write $\sum F_r = F^n_{\text{road car }r} = ma_r = \dfrac{mv^2}{R}$. The sum of force components perpendicular to the incline is $\sum F_y =$ $F^n_{\text{road car}} + F^G_{\text{Ec}} = ma_y \Rightarrow F^n_{\text{road car}} = mg\cos(\theta) + ma_r\sin(\theta)$. Combining the two sums of forces yields $\dfrac{v^2}{R}(1-\sin^2(\theta)) =$ $\dfrac{v^2}{R}\cos(\theta) = g\cos(\theta)\sin(\theta) \Rightarrow v = \sqrt{gR\sin(\theta)} = \sqrt{(9.8 \text{ m/s}^2)(400 \text{ m})\sin(7.0°)} = 22 \text{ m/s},$ which is roughly 50 mi/h.

11.27. (*c*)

11.28. (*a*) If a "smooth" decrease in speed, means a decrease in speed that is linear in time, then the speed of both horses should be drawn as straight lines with negative slopes that end up with a speed of zero. Note that since the horse at $2R$ has the same rotational speed as the other horse, but twice the distance from the axis of rotation, the horse at $2R$ must have twice the initial speed of the horse at R.



(*b*) The centripetal acceleration of either horse is just $a_r = \dfrac{v^2}{R} = \omega^2 R.$ Clearly the horse at $2R$ has twice the initial centripetal acceleration of the horse at R. Again, both curves end at a value of zero. The primary difference between these curves and those shown in part (*a*), is that here the decrease is not linear, but quadratic.



11.29. (*a*) Let the unit vector $\hat{r}$ point radially inward toward the pivot. The sum of all forces in the radial direction is

$$\sum F_r = F^{\mathrm{c}}_{\mathrm{sb}} + F^{G}_{\mathrm{Eb}r} = ma_r \qquad (1)$$

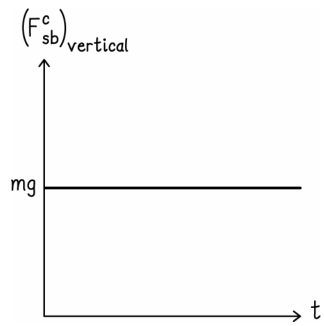
Since the ball is moving in a circle, we know the radial acceleration must be given by $a_r = \dfrac{v^2}{R},$ but to find the speed we must consider energy. Let the moment of release be the initial time, and the moment the string makes an angle θ with the horizontal the final time. Then $U^G_{\mathrm{i}} = U^G_{\mathrm{f}} + K_{\mathrm{f}}$ or $-mg\Delta y = mg\ell\sin(\theta) = \dfrac{1}{2}mv^2.$ From this it follows that $a_r = 2g\sin(\theta).$ Inserting this into equation (1) yield an expression for the tensile force $F^{\mathrm{c}}_{\mathrm{sb}} = 3mg\sin(\theta).$ So, the tension is also given by $\mathcal{T}(\theta) = 3mg\sin(\theta).$ (*b*) The largest value that the tension achieves is $3mg,$ which it reaches when $\theta = \pi/2.$ The string should be able to support at least this much tension.

11.30. We can find the translational speed using conservation of energy, and then relate the translational speed to the rotational speed. Let the $+y$ direction point vertically upward. We have $U^G_{\mathrm{i}} = U^G_{\mathrm{f}} + K_{\mathrm{f}} \Rightarrow -mg\Delta y = \dfrac{1}{2}mv^2.$ Using simple trigonometry, we can rewrite this as $v = \sqrt{2g\ell\left[\cos(\theta) - \cos(\theta_{\mathrm{i}})\right]}.$ Since rotational and translational speeds are related through $v = \omega R,$ we can write $\omega = \sqrt{\dfrac{2g\left[\cos(\theta) - \cos(\theta_{\mathrm{i}})\right]}{\ell}}.$

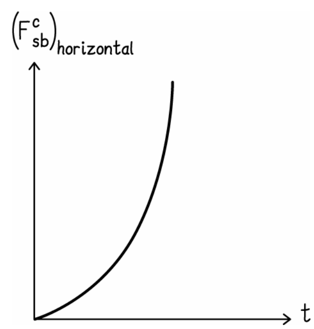
11.31. (*a*) $a_r = \dfrac{v^2}{R} = \dfrac{(2\pi R/T)^2}{R} = \dfrac{4\pi^2(1.50\times10^{11}\ \mathrm{m})}{\left[(365.26\ \mathrm{days})\left(\dfrac{24\ \mathrm{h}}{1\ \mathrm{day}}\right)\left(\dfrac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right)\right]^2} = 5.95\times10^{-3}\ \mathrm{m/s^2}.$ We have explicitly state this as the radial component of the acceleration, but in this case it is also the magnitude of the entire acceleration:

$a = 5.95\times10^{-3}$ m/s^2. (*b*) The radial component of the force that causes this acceleration must be given by $\sum F_r = ma_r = (5.97\times10^{24}\text{ kg})(5.946\text{ m/s}^2) = 3.55\times10^{22}\text{ N}$. So the force is 3.55×10^{22} N toward the center of the Earth-sun system, which is essentially the center of the sun.

11.32. (*a*) The vertical component of tensile force will slightly exceed the gravitational force slightly at some times, and be slightly smaller than the gravitational force at other times, as the ball rises and then settles into a circular motion with the string very nearly horizontal after a large time interval has passed. Because the rotational acceleration is small, these deviations are too small to illustrate.



(*b*) The rotational speed will increase linearly with time, and the radial component of acceleration increases quadratically with the rotational speed.



11.33. Let our gravitational potential energy be zero at the bottom of the bowl. Ignoring dissipation, we can equate the initial potential energy to the final kinetic energy. From this we obtain $v^2 = 2gR$. Thus the radial acceleration is $a_r = \dfrac{v^2}{R} = 2g = 19.6\text{ m/s}^2$. Since there is no force in the horizontal plane, there is no other component to acceleration at this instant. Thus $a = 19.6\text{ m/s}^2$.

11.34. (*a*) Clearly at point 1, the tensile force is supporting the ball against the force of gravity and accelerating the ball. At positions 2 and 4, there is no component of gravity along the rod. At position 3, the force of gravity is actually helping accelerate the ball in toward the center of the circular path. This tells us $\mathcal{T}_1 > \mathcal{T}_2 = \mathcal{T}_4 > \mathcal{T}_3$. (*b*) At position 3, we can write $\sum F_r = F^c_{\text{wb}y} + F^G_{\text{Eb}y} = F^c_{\text{wb}} + F^G_{\text{Eb}} = m\omega^2 R$. So $\omega = \sqrt{\dfrac{F^c_{\text{wb}} + F^G_{\text{Eb}}}{mR}}$. We require that the tensile force be greater than zero, but by any infinitesimal amount. So we take the limit as F^c_{wb} approaches zero to obtain $\omega_{\min} = \sqrt{\dfrac{F^G_{\text{Eb}}}{mR}} = \sqrt{\dfrac{g}{R}} = \sqrt{\dfrac{(9.8\text{ m/s}^2)}{(0.20\text{ m})}} = 7.0\text{ s}^{-1}$. (*c*) At position 1, $\sum F_r = F^c_{\text{wb}y} + F^G_{\text{Eb}y} = F^c_{\text{wb}} - F^G_{\text{Eb}} = m\omega^2 R$. So $F^c_{\text{wb}} =$ $m\omega^2 R + mg = (0.100\text{ kg})((7.0\text{ s}^{-1})^2(0.20\text{ m}) + (9.8\text{ m/s}^2)) = 2.0\text{ N}$.

11.35. (*a*) $\omega_{\rm f} = \omega_{\rm i} + \alpha\Delta t = (0) + (5.8\ {\rm s}^{-2})(10\ {\rm s}) = 58\ {\rm s}^{-1}$. (*b*) $\Delta\vartheta = \omega_{\rm i}\Delta t + \frac{1}{2}\alpha\Delta t^2 = (0) + \frac{1}{2}(5.8\ {\rm s}^{-2})(10\ {\rm s})^2 = 2.9\times10^2$. (*c*) $v = \omega R = (58\ {\rm s}^{-1})(0.33\ {\rm m}) = 19$ m/s. (*d*) $\Delta x = \Delta\vartheta R = (2.9\times10^2)(0.33\ {\rm m}) = 96$ m.

11.36. The final centripetal acceleration is given by $\omega_{\rm f}^2 R,$ and the rotational acceleration is $\frac{\omega_{\rm f}}{\Delta t}$. So, the ratio of the radial to tangential acceleration is $\frac{a_r}{\alpha R} = \omega_{\rm f}\Delta t.$ Estimates will vary, but $\omega_{\rm f}\Delta t$ should be of order 10^2. The centripetal acceleration is much larger.

11.37. (*a*) Call $+y$ vertically upward. Ignore dissipation. Energy conservation tells us $U_{\rm i}^G = U_{\rm f}^G + K_{\rm f} \Rightarrow -mg\Delta y = \frac{1}{2}mv^2,$ or $v = \sqrt{2g(h-d)}$. (*b*) Call the direction in toward the center of the circular path the $+r$ direction. Then at the bottom of the path $\sum F_r = F_{\rm tc}^n - F_{\rm Ec}^G = ma_r \Rightarrow F_{\rm tc}^n = m\left(\frac{v^2}{R} + g\right) = (mg)\left(\frac{2(h-d)}{R} + 1\right)$. (*c*) We can apply energy conservation again, as in part (*a*). The only difference is that the height of the car is greater by R. So $v = \sqrt{2g(h-d-R)}$. (*d*) At this position, the normal force is the only force acting in the radial direction. So $F_{\rm tc}^n = ma_r = \frac{mv^2}{R} = mg\left(\frac{2(h-d-R)}{R}\right)$. (*e*) This was found in part (*d*): $a_r = g\left(\frac{2(h-d-R)}{R}\right)$.

11.38. (*a*) We begin by writing the sum of all forces on the car at the top of its path and at the bottom of its path:

$$\sum F_{{\rm top},r} = F_{\rm tc,top}^n + mg = \frac{mv_{\rm top}^2}{R} \tag{1}$$

$$\sum F_{{\rm bottom},r} = F_{\rm tc,bottom}^n + mg = \frac{mv_{\rm bottom}^2}{R} \tag{2}$$

Solving eqs. (1) and (2) for the normal forces and taking the difference yields:

$$F_{\rm tc,bottom}^n - F_{\rm tc,top}^n = 2mg + \frac{m(v_{\rm bottom}^2 - v_{\rm top}^2)}{R} \tag{3}$$

In order to determine the difference of the squares of the speeds, we need to turn to energy. Applying energy conservation between the bottom and the top yields:

$$\frac{1}{2}mv_{\rm bottom}^2 = mgh + \frac{1}{2}mv_{\rm top}^2$$

$$v_{\rm bottom}^2 - v_{\rm top}^2 = 2gh = 4gR \tag{4}$$

Inserting eq. (4) into eq. (3) yields finally:

$$F_{\rm tc,bottom}^n - F_{\rm tc,top}^n = 2mg + \frac{m(4gR)}{R} = 6mg$$

which was the required result. (*b*) A number of expressions are possible, depending on what variables we decide to use, and which we eliminate. We require that the centripetal acceleration at the bottom of the loop, $a_{r,\rm bottom} = \frac{v_{\rm bottom}^2}{R_{\rm bottom}}$, be the same as the centripetal acceleration at height h, $a_{r,h} = \frac{v_h^2}{R(h)}$. We know the speeds are related through the conservation of energy: $v_h^2 = v_{\rm bottom}^2 - 2gh$. So equating the centripetal accelerations at each position yields $\frac{v_{\rm bottom}^2 - 2gh}{R(h)} = \frac{v_{\rm bottom}^2}{R_{\rm bottom}} \Rightarrow R(h) = \frac{(v_{\rm bottom}^2 - 2gh)R_{\rm bottom}}{v_{\rm bottom}^2}$.

11.39. (*a*) $a_r = \dfrac{v^2}{R} = \dfrac{(3.0\text{ m/s})^2}{(0.500\text{ m})} = 18\text{ m/s}^2$. (*b*) The problem states that the ball is moving at a constant speed. Thus $a_\text{t} = 0\text{ m/s}^2$. (*c*) The vertical component of the normal force exerted by the cone on the ball. (*d*) Call the angle that one side of the cone makes with the vertical θ. Call the direction in toward the center of the circular path $+\hat{r}$, and call vertically upward $+\hat{y}$. We write the sum of all force components in each of these directions.

$$\sum F_r = F^\text{c}_{\text{cb}r} = ma_r \Rightarrow F^\text{c}_\text{cb}\cos(\theta) = \frac{mv^2}{R} \tag{1}$$

$$\sum F_y = F^\text{c}_{\text{cb}y} + F^G_{\text{Eb}y} = 0 \Rightarrow F^\text{c}_\text{cb}\sin(\theta) - mg = 0 \tag{2}$$

Combining equations (1) and (2) yields $\tan(\theta) = \dfrac{gR}{v^2}$. Note from the geometry of the cone that $\tan(\theta) = \dfrac{R}{h}$ also. Equating the two expressions for $\tan(\theta)$ yields $h = \dfrac{v^2}{g} = \dfrac{(3.0\text{ m/s})^2}{(9.8\text{ m/s}^2)} = 0.92\text{ m}$.

11.40. (*a*) Call the direction in toward the center of the circular path $+\hat{r}$, and call vertically upward $+\hat{y}$. We write the sum of all force components in each of these directions, assuming no frictional force.

$$\sum F_r = F^n_{\text{rc}r} = ma_r \Rightarrow F^n_\text{rc}\sin(\theta) = \frac{mv^2_\text{critical}}{R} \tag{1}$$

$$\sum F_y = F^n_{\text{rc}y} + F^G_{\text{Ec}y} = 0 \Rightarrow F^n_\text{rc}\cos(\theta) - mg = 0 \tag{2}$$

Combining equations (1) and (2) yields $\tan(\theta) = \dfrac{v^2_\text{critical}}{Rg} \Rightarrow v_\text{critical} = \sqrt{Rg\tan(\theta)}$. (*b*) Again, we write the sum of all force components in r and y, this time assuming that friction acts with a component in toward the center of the circular path, and we assume the car is just on the verge of slipping.

$$\sum F_r = F^n_{\text{rc}r} + F^s_{\text{rc}r} = ma_r \Rightarrow F^n_\text{rc}\sin(\theta) + \mu F^n_\text{rc}\cos(\theta) = \frac{mv^2_\text{max}}{R} \tag{1}$$

$$\sum F_y = F^n_{\text{rc}y} + F^G_{\text{Ec}y} + F^s_{\text{rc}r} = 0 \Rightarrow F^n_\text{rc}\cos(\theta) - mg - \mu F^n_\text{rc}\sin(\theta) = 0 \tag{2}$$

Equation (2) can be rearranged to find

$$F^n_\text{rc} = \frac{mg}{\cos(\theta) - \mu\sin(\theta)} \tag{3}$$

Inserting equation (3) into equation (1) and solving for the maximum speed yields

$$v_\text{max} = \sqrt{\frac{Rg(\sin(\theta) + \mu\cos(\theta))}{\cos(\theta) - \mu\sin(\theta)}}.$$

11.41. $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m(\omega r)^2 = \dfrac{1}{2}(0.25\text{ kg})(1.0\text{ m})^2\left[\left(\dfrac{2.5\text{ rev}}{1\text{ s}}\right)\left(\dfrac{2\pi}{1\text{ rev}}\right)\right]^2 = 31\text{ J}$

11.42. (*a*) $K_\text{rot} = 3\left(\dfrac{1}{2}mr^2\omega^2\right) = \dfrac{3}{2}(1.0\text{ kg})(0.50\text{ m})^2(3.0\text{ s}^{-1})^2 = 3.4\text{ J}$. (*b*) $L_\vartheta = 3I\omega = 3mr^2\omega = 3(1.0\text{ kg})(0.50\text{ m})^2$ $(3.0\text{ s}^{-1}) = 2.3\text{ kg}\cdot\text{m}^2/\text{s}$.

11.43. (*a*) When the bird passes directly over your head, it is a distance $r = 3.0\text{ m}$ above the reference point, moving with a speed of 2.0 m/s. So $L_\vartheta = mvr = (2.0\text{ kg})(2.0\text{ m/s})(3.0\text{ m}) = 12\text{ kg}\cdot\text{m}^2/\text{s}$. (*b*) When the bird passes directly below your friend's head, it is a distance $r = 1.0\text{ m}$ from his head. Thus $L_\vartheta = mvr =$ $(2.0\text{ kg})(2.0\text{ m/s})(1.0\text{ m}) = 4.0\text{ kg}\cdot\text{m}^2/\text{s}$.

11.44. (*a*) We calculate the angular momentum of each puck in terms of the variables given. $L_{A\vartheta} = m_A v r_A = mv\ell$ and $L_{B\vartheta} = m_B v r_B = 8mv\frac{\ell}{2} = 4mv\ell$. So $L_{B\vartheta} = 4L_{A\vartheta}$. (*b*) We calculate the rotational kinetic energy of each puck in terms of the variables given. $K_{\text{rot,A}} = \frac{1}{2}m_A r_A^2 \omega_A^2 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}mv^2$ and $K_{\text{rot,B}} = \frac{1}{2}m_B r_B^2 \omega_B^2 = \frac{1}{2}m_B v_B^2 = \frac{1}{2}8mv^2$. So $K_{\text{rot,B}} = 8K_{\text{rot,A}}$.

11.45. In order to continue moving in a circle at constant speed along with the bucket, the water must have centripetal acceleration. This acceleration must be due to some force directed toward the center of the circle. When you whirl the bucket rapidly, the forces exerted on the water are the force of gravity and a large contact force exerted by the bottom of the bucket. When you whirl the bucket at just the right speed, the gravitational force alone is sufficient to keep the water from spilling out. If you whirl too slowly, the acceleration due to gravity is larger than the necessary centripetal acceleration and the water spills out.

11.46. Let us first look at the sum of all forces in toward the center of the circular path ($+\hat{r}$) at the moment the car is at the top of the circular loop: $\sum F_r = F^n_{\text{tc}r} + F^G_{\text{Ec}r} = \frac{mv_{\text{top}}^2}{R}$. If the car starts from a very great height, it could have an enormous speed at the top of the loop, requiring that a significant force be exerted by the track on the car to keep it moving in a circle. But we are asked for the minimum starting height. At that minimum, gravity alone is sufficient to maintain the circular motion, and the car barely makes contact with the tracks at the top of the loop. Thus, in that limit: $mg = \frac{mv_{\text{top,min}}^2}{R}$. Now, let us consider the conservation of energy, treating the highest position on the track as the initial position, and the top of the loop as the final position. We have $U_i^G = U_f^G + K_f \Rightarrow mgh_{\min} = mg(d+2R) + \frac{1}{2}mv_{\text{top,min}}^2$. Inserting our result from the consideration of radial forces, we have $h_{\min} = d + \frac{5}{2}R$.

11.47. (*a*) Let us use conservation of angular momentum, and kinetic energy:

$$L_{A\vartheta,i} = L_{A\vartheta,f} + L_{B\vartheta,f} + L_{C\vartheta,f} \Rightarrow m_A\frac{\ell}{2}v_{A,i} = m_A\frac{\ell}{2}v_v + (m_B + m_C)\frac{\ell}{2}\omega_f \tag{1}$$

$$K_{A,i} = K_{A,f} + K_{\text{rot,BC,f}} \Rightarrow \frac{1}{2}m_A v_{A,i}^2 = \frac{1}{2}m_A v^2 + \frac{1}{2}(m_B + m_C)\left(\frac{\ell}{2}\right)^2\omega^2 \tag{2}$$

Combining these equations, we can solve for the two unknowns to obtain

$$v = \frac{(m_A - m_B - m_C)v_{A,i}}{(m_A + m_B + m_C)} \tag{3}$$

$$\omega = \frac{4m_A v_{A,i}}{\ell(m_A + m_B + m_C)} \tag{4}$$

Inserting the inertias given into equation (4), we have

$$\omega = \frac{4m_A v_{A,i}}{\ell(m_A + m_B + m_C)} = \frac{4v_i}{5\ell}$$

(*b*) Inserting the given values into equation (3) obtained in part (*a*), we have $v = \frac{(m_A - m_B - m_C)v_{A,i}}{(m_A + m_B + m_C)} = -\frac{3v_i}{5}$, where the negative sign indicates that puck A has reversed directions. Thus $\vec{v} = -\frac{3}{5}\vec{v}_i$.

11.48. We require that the angular momentum be constant, so $L_{\vartheta,i} = L_{\vartheta,f} \Rightarrow mr^2\omega = m\left(\frac{r}{2}\right)^2\omega_f \Rightarrow \omega_f = 4\omega$. So, the rotational speed has increased by a factor of four. (*b*) From part (*a*), we know that the rotational speed has increased by a factor of four. Thus we know that $v_{\mathrm{f}} = \frac{r}{2}\omega_{\mathrm{f}} = 2r\omega,$ and we know the initial speed $v_{\mathrm{i}} = r\omega.$ So, the speed doubled.

11.49. After the collision, the pair will rotate around their center of mass. Thus the center of mass makes a good reference point. We refer to the heavier skater with a subscript h, and to the lighter skater with a subscript l. Placing the origin at the heavier skater, and letting the positive x axis point in the direction of the lighter skater, the center of mass at the moment of collision is $x_{\mathrm{cm}} = \frac{m_{\mathrm{h}}x_{\mathrm{h}} + m_{\mathrm{l}}x_{\mathrm{l}}}{(m_{\mathrm{h}} + m_{\mathrm{l}})} = \frac{(0) + (48\text{ kg})(2.0\text{ m})}{((75\text{ kg}) + (48\text{ kg}))} = 0.78\text{ m}$ from the heavier skater.

Now, angular momentum will be constant during this collision, so we can write $L_{\vartheta,\mathrm{i}} = L_{\vartheta,\mathrm{f}} \Rightarrow$ $m_{\mathrm{h}}r_{\mathrm{h}}v_{\mathrm{h,i}} + m_{\mathrm{l}}r_{\mathrm{l}}v_{\mathrm{l,i}} = (m_{\mathrm{h}}r_{\mathrm{h}}^2 + m_{\mathrm{l}}r_{\mathrm{l}}^2)\omega_{\mathrm{f}}.$ Solving for the final rotational speed yields $\omega_{\mathrm{f}} = \frac{m_{\mathrm{h}}r_{\mathrm{h}}v_{\mathrm{h,i}} + m_{\mathrm{l}}r_{\mathrm{l}}v_{\mathrm{l,i}}}{(m_{\mathrm{h}}r_{\mathrm{h}}^2 + m_{\mathrm{l}}r_{\mathrm{l}}^2)} =$ $\frac{(75\text{ kg})(0.78\text{ m})(3.3\text{ m/s}) + (48\text{ kg})(1.22\text{ m})(3.3\text{ m/s})}{((75\text{ kg})(0.78\text{ m})^2 + (48\text{ kg})(1.22\text{ m})^2)} = 3.3\text{ s}^{-1}.$

11.50. (*a*) This can be answered using rotational kinematics. Using the information about the first five seconds, we can write $\omega_{\mathrm{f}} = \omega_{\mathrm{i}} + \alpha\Delta t \Rightarrow \alpha = \frac{\omega_{\mathrm{f}} - \omega_{\mathrm{i}}}{\Delta t} = \frac{-0.25\omega_{\mathrm{i}}}{\Delta t} = \frac{-0.25}{(5.0\text{ s})}\left(\frac{33\frac{1}{3}\text{ rev}}{\text{min}}\right)\left(\frac{\text{min}}{60\text{ s}}\right)\left(\frac{2\pi}{1\text{ rev}}\right) = -0.175\text{ s}^{-2}.$ Now we can use this rotational acceleration to determine the stopping time: $\Delta t = \frac{\omega_{\mathrm{f}} - \omega_{\mathrm{i}}}{\alpha} = \frac{(0) - (3.49\text{ s}^{-1})}{(0.175\text{ s}^{-2})} = 20\text{ s}.$ (*b*) The work done will be the change in kinetic energy: $W = K_{\mathrm{f}} - K_{\mathrm{i}} = -\frac{1}{2}I\omega_{\mathrm{i}}^2 = -\frac{1}{2}(0.020\text{ kg}\cdot\text{m}^2)(3.49\text{ s}^{-1})^2 = -0.12\text{ J}.$

11.51. Because there is friction between the disks, we cannot say that the rotational energy is constant during this process. But we can still say that the rotational momentum is constant. Thus $L_{\vartheta,\mathrm{i}} = L_{\vartheta,\mathrm{f}} \Rightarrow I\omega_{\vartheta,\mathrm{i}} = 2I\omega_{\vartheta,\mathrm{f}} \Rightarrow$ $\omega_{\vartheta,\mathrm{f}} = \frac{\omega_{\vartheta,\mathrm{i}}}{2}.$

11.52. (*a*) We will break this problem into two steps. When the child collides with the ride, rotational momentum is conserved. After the collision, as the child rises, energy is constant. Call the moment just before the collision time 1, the moment just after the collision time 2, and the moment that the child reaches the top of the circle time 3. Then we can write

$$L_{\text{child}\,\vartheta,1} = L_{\text{child}\,\vartheta,2} + L_{\text{ride}\,\vartheta,2} \Rightarrow m_{\text{child}}Rv_{\text{child},1} = (m_{\text{child}}R^2 + I_{\text{ride}})\omega_2 \tag{1}$$

$$K_{\text{rot},2} + U_2^G = U_3^G \Rightarrow \frac{1}{2}(m_{\text{child}}R^2 + I_{\text{ride}})\omega_2^2 + m_{\text{child}}gh_2 = m_{\text{child}}gh_3 \tag{2}$$

In writing equation (2), we have assumed that the child and ride stop when the child reaches the maximum height, because we are looking for the minimum initial speed of the child (meaning all energy is required to get the child up to that height, and no energy is left in the form of kinetic energy). Solving equation (2) for the rotational speed, and inserting this into equation (1), we obtain

$$\begin{aligned} v_{\text{child},1} &= \frac{(m_{\text{child}}R^2 + I_{\text{ride}})}{m_{\text{child}}R}\sqrt{\frac{4m_{\text{child}}gR}{(m_{\text{child}}R^2 + I_{\text{ride}})}} \\ &= \frac{((30.0\text{ kg})(1.00\text{ m})^2 + (200\text{ kg}\cdot\text{m}^2))}{(30.0\text{ kg})(1.00\text{ m})}\sqrt{\frac{4(30.0\text{ kg})(9.8\text{ m/s}^2)(1.00\text{ m})}{((30.0\text{ kg})(1.00\text{ m})^2 + (200\text{ kg}\cdot\text{m}^2))}} \\ &= 17.3\text{ m/s} \end{aligned}$$

(*b*) No, this is not a reasonable speed for a child. This is likely desirable from a safety standpoint.

11.53. From left to right in Figure P11.53, number the particles 1, 2, and 3. The rotational inertia is $I = I_1 + I_2 + I_3 =$ $m(r\sin(\theta))^2 + m(0) + 4m\left(\frac{r}{2}\sin(\theta)\right)^2 = 2mr^2\sin^2(\theta).$

11.54. Two balls are on the axis and will not contribute to the rotational inertia. Four balls are exactly one edge-length away from the axis of rotation, and the remaining two are a distance $\sqrt{2}\ell$ from the axis of rotation. Thus $I = 2m(0)^2 + 4m(\ell^2) + 2m(2\ell^2) = 8m\ell^2 = 8(0.20\text{ kg})(0.25\text{ m})^2 = 0.10\text{ kg}\cdot\text{m}^2.$

11.55. According to equation (11.53) in *PRIN*, the rotational inertia of an object around some axis a distance d from the object's center of mass is given by $I = I_{\text{cm}} + md^2.$ So, in this case we can read the rotational inertia of the spherical shell around its center of mass from table 11.3, and we move the axis of rotation by exactly R. Thus $I = \frac{2}{3}mR^2 + mR^2 = \frac{5}{3}mR^2.$

11.56. Running quickly requires swinging the legs back and forth rapidly. If muscles were distributed evenly along the legs, the rotational inertia of the legs would be relatively large, making it difficult for the deer to swing them rapidly. Having most of the muscle near the hip gives the legs a lower rotational inertia and makes it easier for the deer to run quickly.

11.57. During the collision between the child and the merry-go-round, angular momentum must be constant. Thus we can write $L_{\text{child}\,\vartheta,\text{i}} = L_{\text{child}\,\vartheta,\text{f}} + L_{\text{merry}\,\vartheta,\text{f}} \Rightarrow m_{\text{child}}Rv_{\text{child,i}} = m_{\text{child}}R^2\omega_{\text{f}} + \frac{1}{2}m_{\text{merry}}R^2\omega_{\text{f}}.$ Rearranging to solve for the final rotational speed, we find $\omega_{\text{f}} = \dfrac{m_{\text{child}}v_{\text{child,i}}}{\left(m_{\text{child}} + \frac{1}{2}m_{\text{merry}}\right)R} = \dfrac{(20\text{ kg})(1.4\text{ m/s})}{\left((20\text{ kg}) + \frac{1}{2}(180\text{ kg})\right)(1.6\text{ m})} = 0.16\text{ s}^{-1}.$

11.58. (*a*) Using the formula from table 11.3, we can write $I_{\text{Earth}} = \frac{2}{5}M_{\text{Earth}}R_{\text{Earth}}^2 = \frac{2}{5}(5.97\times10^{24}\text{ kg})$ $(6.37\times10^6\text{ m})^2 = 9.69\times10^{37}\text{ kg}\cdot\text{m}^2.$ (*b*) The fact that we get a different number means that the density of Earth must not be uniform. The fact that the measured rotational inertia is smaller than the calculated value, means that the density must be greater near the center of Earth, than near the outer crust.

11.59. (*a*) If one axis in the plane of the hoop is along a diameter, then choose our second axis in the plane of the hoop to cross this first axis at its midpoint such that the second axis also lies along a diameter of the hoop. In this way, we have chosen two axes in the plane of the hoop that yield identical rotational inertias. Using the perpendicular axis theorem, we know the third axis (out of plane) must pass directly through the center of the hoop, and the rotational inertia around this point is given by table 11.3 $I_{\text{out-of-plane}} = mR^2.$ Thus $2I_{\text{in-plane}} = I_{\text{out-of-plane}} \Rightarrow I_{\text{in-plane}} =$ $\frac{1}{2}I_{\text{out-of-plane}} = \frac{1}{2}mR^2.$ (*b*) We know from table 11.3 that the rotational inertia of a square sheet of side length a about an axis passing perpendicularly through its center is $I_{\text{out-of-plane}} = \frac{1}{6}ma^2.$ If our first in-plane axis of rotation goes from the center of one side through the center of the square and to the center of the opposite side, then there is only one choice for an in-plane axis that is perpendicular to this. Further, this perpendicular axis is identical to the first. Thus the sum of the two in-plane rotational inertias can be written $2I_{\text{in-plane}} = I_{\text{out-of-plane}} \Rightarrow I_{\text{in-plane}} = \frac{1}{2}I_{\text{out-of-plane}} = \frac{1}{12}ma^2.$

11.60. Call the inertia of each rod m and the length of each rod ℓ. We calculate the rotational inertia around each axis, and then compare them.

$$I_{(a)} = \frac{1}{3}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 + \frac{1}{3}m\ell^2 = \frac{11}{12}m\ell^2$$

$$I_{(b)} = \frac{1}{3}m\ell^2 + (0) + m\ell^2 = \frac{4}{3}m\ell^2$$

$$I_{(c)} = \frac{1}{12}m\ell^2 + (0) + \frac{1}{12}m\ell^2 = \frac{1}{6}m\ell^2$$

$$I_{(d)} = \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{7}{12}m\ell^2$$

Thus $(c) < (d) < (a) < (b)$.

11.61. The melting of Antarctica would lengthen the day. Water that is currently confined close to the poles (near the axis of rotation) would flow into the ocean and redistribute over the entire surface. Some of this water would end up far from the pole. This would increase the rotational inertia of Earth, meaning the rotational speed of Earth would have to decrease in order for rotational momentum to be unchanged. A slower rotation would correspond to a longer day. Incidentally, the melting of the Arctic would not have the same effect. You may want to revisit this problem after you have studied Archimedes Principle in Chapter 18.

11.62. (*a*) The rotational kinetic energy can be written as $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}cmR^2\omega^2$. Now, if the object rolls without slipping, then $v_{\text{cm}} = \omega R$. In that case the rotational kinetic energy becomes $K_{\text{rot}} = \frac{c}{2}mv_{\text{cm}}^2$. The total kinetic energy is $K = \frac{1}{2}mv_{\text{cm}}^2 + \frac{c}{2}mv_{\text{cm}}^2$. So clearly, the object with the greatest shape factor c will have the greatest fraction of kinetic energy in rotational form. Thus billiard ball < solid cylinder < hollow cylinder. (*b*) If all objects start from the same height, then all objects will have the same amount of total kinetic energy when they reach the bottom. We can write, for any of the objects $K_{\text{tot,f}} = \frac{1}{2}mv_{\text{cm}}^2(1+c) = U_{\text{i}}^G \Rightarrow v_{\text{cm}} = \sqrt{\frac{2gh_{\text{i}}}{(1+c)}}$. This expression is the same for all objects, except for the shape factor. We can relate the final speeds of the objects:

$$\frac{v_{\text{hollow cylinder}}}{v_{\text{solid cylinder}}} = \sqrt{\frac{(1+c_{\text{solid cylinder}})}{(1+c_{\text{hollow cylinder}})}} = \sqrt{\frac{\left(1+\frac{1}{2}\right)}{(1+1)}} = 0.87$$

$$\frac{v_{\text{hollow cylinder}}}{v_{\text{billiard ball}}} = \sqrt{\frac{(1+c_{\text{billiard ball}})}{(1+c_{\text{hollow cylinder}})}} = \sqrt{\frac{\left(1+\frac{2}{5}\right)}{(1+1)}} = 0.84$$

and $$\frac{v_{\text{solid cylinder}}}{v_{\text{billiard ball}}} = \sqrt{\frac{(1+c_{\text{billiard ball}})}{(1+c_{\text{solid cylinder}})}} = \sqrt{\frac{\left(1+\frac{2}{5}\right)}{\left(1+\frac{1}{2}\right)}} = 0.97$$

11.63. The human body can be approximated as a cylinder. Table 11.3 gives us an equation for the rotational inertia of a cylinder about an axis pass through its center and through the curved face. Let us estimate the athlete's average radius as 0.20 m, and her height as 1.8 m. Then $I = \frac{1}{4}mR^2 + \frac{1}{12}m\ell^2 = \frac{1}{4}(70\text{ kg})(0.20\text{ m})^2 + \frac{1}{12}(70\text{ kg})(1.8\text{ m})^2$ $= 20\text{ kg}\cdot\text{m}^2$. This is only an estimate; answers may vary.

11.64. We can solve this problem using conservation of energy. Let us assume that the rope does not slip across the pulley, meaning the translational speed of either block is the same as the speed of a point on the surface of the pulley. We refer to the lighter block using a subscript l, and to the heavier block with a subscript h. We can write $U_{\text{i}}^{G} = U_{\text{f}}^{G} + K_{\text{f}} \Rightarrow U_{\text{l,i}}^{G} + U_{\text{h,i}}^{G} = U_{\text{l,f}}^{G} + U_{\text{h,f}}^{G} + K_{\text{l,f}} + K_{\text{h,f}} + K_{\text{p,f}}$. Inserting expressions for the various types of energy yields

$$m_{\text{l}}gh_{\text{l,i}} + m_{\text{h}}gh_{\text{h,i}} = m_{\text{l}}gh_{\text{l,f}} + m_{\text{h}}gh_{\text{h,f}} + \frac{1}{2}m_{\text{l}}v_{\text{l,f}}^2 + \frac{1}{2}m_{\text{h}}v_{\text{h,f}}^2 + \frac{1}{2}I\omega_{\text{p,f}}^2$$

Because the rope does not slip and because the rope does not stretch, the two translational speeds are the same and $\omega_{\text{p,f}} = \dfrac{v}{R}$. The expression above becomes

$$(m_{\text{l}} - m_{\text{h}})g\Delta h = \frac{1}{2}(m_{\text{l}} + m_{\text{h}})v_{\text{f}}^2 + \frac{1}{4}m_{\text{p}}v_{\text{f}}^2 \Rightarrow v_{\text{f}} = \sqrt{\frac{(m_{\text{l}} - m_{\text{h}})g\Delta h}{\frac{1}{2}(m_{\text{l}} + m_{\text{h}}) + \frac{1}{4}m_{\text{p}}}}$$

$$= \sqrt{\frac{\big((0.20\text{ kg}) - (0.25\text{ kg})\big)(9.8\text{ m/s}^2)(-0.30\text{ m})}{\frac{1}{2}\big((0.20\text{ kg}) + (0.25\text{ kg})\big) + \frac{1}{4}(0.50\text{ kg})}} = 0.65\text{ m/s}$$

11.65. All the initial gravitational potential energy will be turned into kinetic energy: $U_{\text{i}}^{G} = U_{\text{f}}^{G} + K_{\text{f}} \Rightarrow U_{\text{b,i}}^{G} = U_{\text{b,f}}^{G} + K_{\text{b,f}} + K_{\text{rot,h,f}}$. Inserting expressions for the energies yields

$$m_{\text{b}}gh_{\text{b,i}} = m_{\text{b}}gh_{\text{b,f}} + \frac{1}{2}m_{\text{b}}v_{\text{b,f}}^2 + \frac{1}{2}I_{\text{c}}\omega_{\text{c,f}}^2$$

If we assume the rope does not slip across the cylinder, then we can write $v_{\text{b,f}} = \omega_{\text{c,f}}R$. Rearranging the above equation and inserting the rotational inertia of a solid cylinder tells us

$$2m_{\text{b}}g(h_{\text{b,i}} - h_{\text{b,f}}) = m_{\text{b}}R^2\omega_{\text{c,f}}^2 + \frac{1}{2}m_{\text{c}}R^2\omega_{\text{c,f}}^2 \Rightarrow \omega_{\text{c,f}} = \sqrt{\frac{2m_{\text{b}}g(h_{\text{b,i}} - h_{\text{b,f}})}{\left(m_{\text{b}} + \frac{1}{2}m_{\text{c}}\right)R^2}}$$

$$= \sqrt{\frac{2(12\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})}{\left((12\text{ kg}) + \frac{1}{2}(4.0\text{ kg})\right)(0.080\text{ m})^2}} = 2.0\times10^2\text{ s}^{-1}$$

11.66. Gravitational potential energy is being converted to kinetic energy as the lighter block rises. Let us refer to the lighter block with a subscript l, and to the heavier block with a subscript h. Let us assume that the string does not slip over the surface of the pulley, such that the speed of the block is related to the rotational speed of the cylinder through $v_{\text{h,f}} = \omega_{\text{f}}R$. Then we can write $U_{\text{i}}^{G} = U_{\text{f}}^{G} + K_{\text{f}} \Rightarrow U_{\text{l,i}}^{G} + U_{\text{h,i}}^{G} = U_{\text{l,f}}^{G} + U_{\text{h,f}}^{G} + K_{\text{l,f}} + K_{\text{h,f}} + K_{\text{p,f}}$. Inserting the expressions for the various types of energies yields $m_{\text{l}}gh_{\text{l,i}} + m_{\text{h}}gh_{\text{h,i}} = m_{\text{l}}gh_{\text{l,f}} + m_{\text{h}}gh_{\text{h,f}} + \frac{1}{2}m_{\text{l}}v_{\text{l,f}}^2 + \frac{1}{2}m_{\text{h}}v_{\text{h,f}}^2 + \frac{1}{2}I\omega_{\text{p,f}}^2$.

We have written the final translational speeds of the blocks using different subscripts, but we know they must have the same final speed. Inserting the moment of inertia of a solid cylinder and rearranging, we find

$$v_{\text{f}} = \sqrt{\frac{2(m_{\text{h}} - m_{\text{l}})g(h_{\text{h,i}} - h_{\text{h,f}})}{m_{\text{l}} + m_{\text{h}} + \frac{1}{2}m_{\text{p}}}} = \sqrt{\frac{2(m)g(h)}{3m + \frac{3}{2}m}} = \sqrt{\frac{4gh}{9}}$$

11.67. Using the equation for the rotational inertia about the symmetry axis of the cylinder (I_{cm}) given in table 11.3, we find $I_{\text{cm}} = \frac{1}{2}m(R_{\text{outer}}^2 + R_{\text{inner}}^2) = \frac{1}{2}m\left(\frac{5}{4}R_{\text{outer}}^2\right)$. Equation 11.53 relates the rotational inertia of an object about an axis through its center of mass to the rotational inertia about some parallel axis a distance d from the center of mass.

In this case, the axis on the outer edge of the cylinder is a distance R_{outer} from the axis of symmetry. So, $I = I_{\text{cm}} + mR_{\text{outer}}^2 = \frac{5}{8}mR_{\text{outer}}^2 + mR_{\text{outer}}^2 = \frac{13}{8}MR_{\text{outer}}^2$.

11.68. (*a*) We have a formula in table 11.3 for the rotational inertia of a rectangular sheet about an axis perpendicular to the sheet and passing through its center of mass. Let a be the length of the longer side and b be the length of the shorter side. We can relate this to the axis specified using the parallel axis theorem: $I = I_{\text{cm}} + md^2$. In this case, the axis is translated from the center of mass to the center of a long edge, which is a distance $d = b/2$. Thus $I = I_{\text{cm}} + md^2 = \frac{1}{12}m(a^2+b^2) + m\left(\frac{b}{2}\right)^2 = \frac{1}{12}m(a^2+4b^2) = \frac{1}{12}(0.15\text{ kg})((0.080\text{ m})^2 + 4(0.040\text{ m})^2)$ $= 1.6\times10^{-4}\text{ kg}\cdot\text{m}^2$. (*b*) We proceed exactly as in part (*a*), but now the axis of rotation is translated a distance $d = a/2$ from the center of mass to the center of a short side. Thus $I = I_{\text{cm}} + md^2 = \frac{1}{12}m(a^2+b^2) + m\left(\frac{a}{2}\right)^2 =$ $\frac{1}{12}m(4a^2+b^2) = \frac{1}{12}(0.15\text{ kg})(4(0.080\text{ m})^2 + (0.040\text{ m})^2) = 3.4\times10^{-4}\text{ kg}\cdot\text{m}^2$. (*c*) Now the axis of rotation is translated a distance $d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$ from the center of mass to one corner. Thus $I = I_{\text{cm}} + md^2 =$ $\frac{1}{12}m(a^2+b^2) + m\left[\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2\right] = \frac{1}{12}m(4a^2+4b^2) = \frac{1}{3}(0.15\text{ kg})((0.080\text{ m})^2 + (0.040\text{ m})^2) = 4.0\times10^{-4}\text{ kg}\cdot\text{m}^2$.

11.69. This is the same as the rotational inertia of a hoop, and four rods about the axis specified. We can read off the rotational inertias for each from table 11.3. Thus $I_{\text{wagon wheel}} = 4I_{\text{rod}} + I_{\text{hoop}} = 4\left[\frac{1}{12}m_{\text{rod}}(2R)^2\right] + m_{\text{hoop}}R^2 =$ $\frac{4}{3}(0.80\text{ kg})(0.30\text{ m})^2 + (2.0\text{ kg})(0.30\text{ m})^2 = 0.28\text{ kg}\cdot\text{m}^2$.

11.70. The initial rotational kinetic energy will be converted to gravitational potential energy as the center of mass of the rod rises. Thus we can write $K_{\text{rot,i}} = U_{\text{f}}^G \Rightarrow \frac{1}{2}I_{\text{rod}}\omega_{\text{i}}^2 = mg\ell$, where we have used the fact that the center of mass of the rod will one full length of the rod. Inserting the rotational inertia of a rod about one end yields $\left(\frac{1}{3}\ell^2\right)\omega_{\text{i}}^2 = 2g\ell$. Finally, note that the initial speed of the free end is related to the rotational speed by $\omega_{\text{i}}\ell = v_{\text{i}}$. Finally, we have required initial speed of the free end $v_{\text{i}} = \sqrt{6g\ell} = \sqrt{6(9.8\text{ m/s}^2)(0.83\text{ m})} = 7.0\text{ m/s}$.

11.71. We first calculate the rotational speed of each of these hands:

$$\omega_{\text{hour}} = \left(\frac{2\pi}{12\text{ h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 1.454\times10^{-4}\text{ s}^{-1}$$

$$\omega_{\text{minute}} = \left(\frac{2\pi}{1\text{ h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 1.745\times10^{-3}\text{ s}^{-1}$$

Now the rotational kinetic energy is given by

$$K_{\text{rot}} = K_{\text{rot,hour}} + K_{\text{rot,minute}} = \frac{1}{2}I_{\text{rod,hour}}\omega_{\text{hour}}^2 + \frac{1}{2}I_{\text{rod,minute}}\omega_{\text{minute}}^2 = \frac{1}{2}\left(\frac{1}{3}m_{\text{hour}}R_{\text{hour}}^2\right)\omega_{\text{hour}}^2 + \frac{1}{2}\left(\frac{1}{3}m_{\text{minute}}R_{\text{minute}}^2\right)\omega_{\text{minute}}^2$$

$$K_{\text{rot}} = \frac{1}{2}\left(\frac{1}{3}(300\text{ kg})(2.7\text{ m})^2\right)(1.545\times10^{-4}\text{ s}^{-1})^2 + \frac{1}{2}\left(\frac{1}{3}(100\text{ kg})(4.2\text{ m})^2\right)(1.745\times10^{-3}\text{ s}^{-1})^2 = 9.0\times10^{-4}\text{ J}$$

11.72. (*a*) We know the rotational inertia of an actual disk is $I_{\text{disk}} = \frac{1}{2}mR^2,$ and we require that this be the same rotational inertia as a single particle of the same inertia located at R_{gyr}. Thus $mR_{\text{gyr}}^2 = \frac{1}{2}mR^2 \Rightarrow R_{\text{gyr}} = \frac{R}{\sqrt{2}}.$ (*b*) We proceed exactly as in part (*a*), but now we use the rotational inertia of a solid sphere. So $mR_{\text{gyr}}^2 = \frac{2}{5}mR^2 \Rightarrow R_{\text{gyr}} = \sqrt{\frac{2}{5}}R.$

(*c*) The process is the same as in (*a*) and (*b*), but now we use the rotational inertia of a solid sphere about an axis tangent to its surface. Using the parallel axis theorem, we have $I_{\tan} = I_{\text{cm}} + md^2 = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2.$ So $mR_{\text{gyr}}^2 = \frac{7}{5}mR^2 \Rightarrow R_{\text{gyr}} = \sqrt{\frac{7}{5}}R.$

11.73. (*a)* We define the density of the materials as $\sigma = \frac{m}{A} = \frac{m}{\frac{1}{2}(2\ell^2)} = \frac{m}{\ell^2}.$ We integrate over differential contributions to the rotational inertia as follows. Let the short side lie along the x axis such that the distance of any particle from the axis is simply equal to its position along the y axis. Then

$$I_{\text{short}} = \int_{\text{triangle}} dI = \int_{\text{triangle}} r^2 dm = \int_0^{\ell}\int_0^{2x} y^2 \sigma dx dy$$

$$I_{\text{short}} = \sigma\int_0^{\ell}\left(\frac{8x^3}{3}\right)dx = \sigma\frac{2}{3}\ell^4 = \frac{2m\ell^2}{3}$$

(*b*) Here the process is the same as in part (*a*), except that we alter our choice of axes to let the long side lie along the x axis, and we change our integration limits, accordingly.

$$I_{\text{long}} = \int_{\text{triangle}} dI = \int_{\text{triangle}} r^2 dm = \int_0^{2\ell}\int_0^{x/2} y^2 \sigma dx dy$$

$$I_{\text{long}} = \sigma\int_0^{2\ell}\left(\frac{x^3}{24}\right)dx = \sigma\frac{1}{6}\ell^4 = \frac{m\ell^2}{6}$$

11.74. The initial spring potential energy and gravitational potential energy will be converted to spring potential energy, gravitational potential energy and kinetic energy. We can write

$$U_{\text{b,i}}^G + U_{\text{i}}^{\text{sp}} = U_{\text{b,f}}^G + U_{\text{f}}^{\text{sp}} + K_{\text{b,f}} + K_{\text{rot,f}}$$

$$mgh_{\text{i}} + \frac{1}{2}kd^2 = mgh_{\text{f}} + \frac{1}{2}k\left(\frac{\ell}{2}\right)^2 + \frac{1}{2}mv_{\text{b,f}}^2 + \frac{1}{2}I_{\text{cyl}}\omega_{\text{cyl,f}}^2$$

This can be simplified somewhat by noting that the distance down the incline that the block has moved is $d + \ell/2$, which is related to the vertical distance Δy that the block has moved according to $\Delta y = (d + \ell/2)\sin(\theta)$. We assume further that the rope does not slip on the surface of the cylinder, such that $\omega_{\text{cyl,f}}R = v_{\text{b,f}}$. Then

$$mg\left(d + \frac{\ell}{2}\right)\sin(\theta) + \frac{1}{2}k\left[d^2 - \left(\frac{\ell}{2}\right)^2\right] = \frac{1}{2}mv_{\text{b,f}}^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_{\text{cyl,f}}^2$$

11.75. Let us call the central axis of symmetry of the bat the z axis. We start by finding the volume of the bat, so that we can obtain an expression for the density. We work in cylindrical coordinates. We integrate over z from 0 to L. The radius R of the bat varies from 0.020 m to 0.035 m.

$$V = 2\pi \int_0^L \int_0^{R_{\min} + \frac{R_{\max} - R_{\min}}{L} z} r dr dz = 2\pi \int_0^L \frac{1}{2} \left(R_{\min} + \frac{R_{\max} - R_{\min}}{L} z \right)^2 dz = \pi \int_0^L \left(R_{\min}^2 + \frac{R_{\min}}{L} (R_{\max} - R_{\min}) z + \left(\frac{R_{\max} - R_{\min}}{L} \right)^2 z^2 \right) dz$$

$$V = \pi \left(R_{\min}^2 L + \frac{R_{\min}}{L} (R_{\max} - R_{\min}) \frac{L^2}{2} + \left(\frac{R_{\max} - R_{\min}}{L} \right)^2 \frac{L^3}{3} \right)$$

$$= \pi \left((0.020 \text{ m})^2 (0.84 \text{ m}) + (0.020 \text{ m}) \left(\frac{(0.035 \text{ m}) - (0.020 \text{ m})}{(0.84 \text{ m})} \right) \frac{(0.84 \text{ m})^2}{2} + \left(\frac{(0.035 \text{ m}) - (0.020 \text{ m})}{(0.84 \text{ m})} \right)^2 \frac{(0.84 \text{ m})^3}{3} \right)$$

$$V = 2.045 \times 10^{-3} \text{ m}^3$$

From this volume we can write the density as $\rho = \frac{m}{V} = \frac{(0.85 \text{ kg})}{2.045 \times 10^{-3} \text{ m}^3} = 415.6 \text{ kg/m}^3$. Now we calculate the rotational inertia as follows:

$$I = \int_{\text{bat}} dI = \int_{\text{bat}} d^2 dm = 2\pi \int_0^L \int_0^{\left(R_{\min} + \frac{R_{\max} - R_{\min}}{L} z \right)} z^2 \sigma r dr dz = \pi\sigma \int_0^L z^2 \left(R_{\min} + \frac{R_{\max} - R_{\min}}{L} z \right)^2 dz$$

$$I = \pi\sigma \int_0^L z^2 \left(R_{\min} + \frac{R_{\max} - R_{\min}}{L} z \right)^2 dz = \pi\sigma L^3 \left(\frac{R_{\min}^2}{30} + \frac{R_{\min} R_{\max}}{10} + \frac{R_{\max}^2}{5} \right)$$

$$I = \pi (415.6 \text{ kg/m}^3)(0.84 \text{ m})^3 \left(\frac{(0.020 \text{ m})^2}{30} + \frac{(0.020 \text{ m})(0.035 \text{ m})}{10} + \frac{(0.035 \text{ m})^2}{5} \right) = 0.25 \text{ kg} \cdot \text{m}^2$$

11.76. No, because the translational speed v depends on radial distance from the rotation axis. All the pennies have the same rotational speed ω, but v for pennies on the outside rim is greater than v for pennies near the rotation axis. Because v increases linearly with radial distance from the center and because v is squared in the equation for centripetal acceleration, pennies near the center stay on better.

11.77. (*a*) There may be several possible answers depending on which numbers you choose to use. One might use the equatorial diameter of the sun, or the polar diameter (they are very similar). Also, the distance between Earth and the sun varies throughout the year. Here, we take the diameter of the sun to be 1.393×10^9 m, and we assume it is 1.496×10^{11} m from Earth on average. Thus, the angle from the center of the sun to its outer edge is $\theta_{\text{half}} = \tan^{-1} \left(\frac{(1.393 \times 10^9 \text{ m})/2}{(1.496 \times 10^{11} \text{ m})} \right) = 0.267°$, so the full angular size of the sun is $\theta = 2\theta_{\text{half}} = 0.53°$. (*b*) The radius of the moon is 1.738×10^6 m and its average distance from Earth is 3.84×10^8 m. So $\theta_{\text{half}} = \tan^{-1} \left(\frac{(1.738 \times 10^6 \text{ m})}{(3.84 \times 10^8 \text{ m})} \right) = 0.259°$, so the full angular size of the moon is $\theta = 2\theta_{\text{half}} = 0.52°$.

11.78. Centripetal acceleration is given by $a_c = v^2/r$, and v can be expressed distance traveled in a given time interval: $v = 2\pi r/T$. Inserting this expression in the expression for a_c yields $a_c = 4\pi^2 r/T^2$.

11.79. The liquid inside the egg continues to rotate once you stop the egg. When stopped and released, viscous forces between the liquid and shell soon cause the shell to move along with the liquid, causing the entire egg to spin. In a hard-boiled egg, there is no liquid to continue rotating once you stop the egg. When you stop the outside of the hard-boiled egg, you stop everything.

11.80. Let us assume that the distance from your eye to your thumb is approximately 1.0 m. Then your finger must move in a half-circle over a 12-hour period. Thus $v = \left(\frac{\pi (1.0 \text{ m})}{12 \text{ h}} \right) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \left(\frac{10^3 \text{ mm}}{1 \text{ m}} \right) = 4.4 \text{ mm/min}$.

11.81. (*a*) No work is done. The tensile force is always radially inward toward the point at which the vine is anchored. Jane's motion is always tangential to the circle. Hence $\vec{F}\cdot\Delta\vec{r}=0$. (*b*) The tensile force is greatest when Jane is at the lowest point. This is the position at which the speed is greatest; it is also the position at which gravity is directed exactly opposite the tensile force. Thus at this position the tensile force must overcome the entire gravitational force (rather than just one component of it), and it must provide a centripetal force that is larger than at any other point on Jane's path.

11.82. The speed of the Earth-Moon system around the sun is given by $v_{\rm E}=\dfrac{2\pi R_{\rm ES}}{T_{\rm E}}=\left(\dfrac{2\pi(1.496\times10^{11}\ {\rm m})}{(365.24\ {\rm days})}\right)\left(\dfrac{1\ {\rm day}}{24\ {\rm h}}\right)\left(\dfrac{1\ {\rm h}}{3600\ {\rm s}}\right)=2.979\times10^{4}\ {\rm m/s}$. The speed of the moon around Earth is given by $v_{\rm m}=\dfrac{2\pi R_{\rm Em}}{T_{\rm m}}=\left(\dfrac{2\pi(3.84\times10^{8}\ {\rm m})}{(2.36\times10^{6}\ {\rm s})}\right)=1.02\times10^{3}\ {\rm m/s}$. So the moon's speed around Earth is far too low to cause the moon to appear retrograde. No, the Moon never appears retrograde.

11.83. (*a*) We know that the force $F=\dfrac{mv^2}{r}$, and we can relate this to the period of the motion through $v=\dfrac{2\pi r}{T}$. This shows us that $F=\dfrac{4m\pi^2 r}{T^2}$. Now, if the period depends on $\sqrt{r}$ (we can express this as $T=C\sqrt{r}$ where C is a constant that has no r dependence), then $F=\dfrac{4m\pi^2}{C^2}$. So, in this case the force F is independent of the distance r. (*b*) From part (*a*) we already have $F=\dfrac{4m\pi^2 r}{T^2}$. Now we assume $T=Cr^{3/2}$ and obtain $F=\dfrac{4m\pi^2}{C^2r^2}$, so in this case the force is proportional to $1/r^2$. (*c*) Again we start with $F=\dfrac{4m\pi^2 r}{T^2}$, and now we assume $T=C$ (independent of *r*). In this case we find the force is proportional to r.

11.84. We will use the conservation of rotational momentum to solve this. We can write $L_{\vartheta,{\rm i}}=L_{\vartheta,{\rm f}}$. If we assume no matter is ejected from the sun in the process, such that its inertia remains constant, we can write $R^2_{\rm sun,i}\omega_{\rm sun,i}=R^2_{\rm sun,f}\omega_{\rm sun,f}\Rightarrow$ $\dfrac{\omega_{\rm sun,i}}{\omega_{\rm sun,f}}=\dfrac{R^2_{\rm sun,f}}{R^2_{\rm sun,i}}$ or $T_{\rm sun,f}=T_{\rm sun,i}\left(\dfrac{R^2_{\rm sun,f}}{R^2_{\rm sun,i}}\right)=(25.4\ {\rm days})\left(\dfrac{24\ {\rm h}}{1\ {\rm day}}\right)\left(\dfrac{60\ {\rm min}}{1\ {\rm h}}\right)\left(\dfrac{(6.38\times10^{6}\ {\rm m})}{(6.96\times10^{8}\ {\rm m})}\right)^2=3.1\ {\rm minutes}.$

11.85. We will use the conservation of rotational momentum to solve this. We know $L_{\vartheta,{\rm i}}=L_{\vartheta,{\rm f}}\Rightarrow L_{{\rm clay}\,\vartheta,{\rm i}}=L_{{\rm clay}\,\vartheta,{\rm f}}+L_{{\rm door}\,\vartheta,{\rm f}}$. Inserting expressions for each rotational momentum yields

$$m_{\rm b}r_{\rm b}v_{\rm b}=m_{\rm b}r_{\rm b}v_{\rm b,f}+I_{\rm d}\omega_{\rm d,f}=\left(m_{\rm b}r_{\rm b}^2+\frac{1}{3}m_{\rm d}\ell_{\rm d}^2\right)\omega_{\rm d,f}$$

$$\omega_{\rm d,f}=\frac{m_{\rm B}r_{\rm b}v_{\rm b}}{\left(m_{\rm b}r_{\rm b}^2+\frac{1}{3}m_{\rm d}\ell_{\rm d}^2\right)}=\frac{m_{\rm b}\left(\frac{2}{3}\ell_{\rm d}\right)v_{\rm b}}{\left(m_{\rm b}\left(\frac{2}{3}\ell_{\rm d}\right)^2+\frac{1}{3}m_{\rm d}\ell_{\rm d}^2\right)}=\frac{6m_{\rm b}v_{\rm b}}{(4m_{\rm b}+3m_{\rm d})\ell_{\rm d}}$$

11.86. Call the starting height of the roller coaster cars the initial position, and call the top of the loop the final position. At the top of the loop, we will call the direction radially in toward the center of the loop the $+r$ direction. In general, the sum of all radial components of forces at this final position is

$$\sum F_r=F^n_{{\rm tc}r}+F^G_{{\rm Ec}r}=ma_r\Rightarrow F^n_{{\rm tc}r}+mg=\frac{mv_{\rm f}^2}{R}\qquad(1)$$

We want this loop to be as high as possible, meaning there will be as little energy left in the form of kinetic energy as possible. Minimizing the right hand side of equation (1) means taking the limit as the normal force exerted by the track on the car goes to zero. This means the cars are just barely maintaining contact with the track. Thus equation (1) yields $mgR = mv_{\mathrm{f}}^2$. Now we can use conservation of energy. Some of the initial gravitational potential energy is converted to kinetic energy. We can write $U_{\mathrm{i}}^G = U_{\mathrm{f}}^G + K_{\mathrm{f}} \Rightarrow mgh_{\mathrm{i}} = mg(2R_{\max}) + \frac{1}{2}mv_{\mathrm{f}}^2 = \frac{5}{2}mgR_{\max}$. Solving for the maximum radius of the loop yields $R_{\max} = \frac{2}{5}h_{\mathrm{i}} = \frac{2}{5}(55\text{ m}) = 22\text{ m}.$

11.87. Let us assume the astronaut is approximately 1.8 m tall. If we assume the apparent gravity as the astronaut's feet is equal to the acceleration due to gravity on Earth, then we know $g = \omega^2 R = \omega^2 3h_{\mathrm{ast}}$. But at the astronaut's head, the apparent gravity would be only $g_{\mathrm{app,head}} = \omega^2 2h = \frac{2}{3}\omega^2 3h = \frac{2}{3}g = 6.5\text{ m/s}^2$. So the apparent acceleration due to gravity at the position of the astronauts' heads would only be about 6.5 m/s^2. Suppose you wanted to ensure that the acceleration due to gravity at the astronauts' heads is within 5% of the value at their feet. Then we would write $\frac{g - g_{\mathrm{app,head}}}{g} < 0.05 \Rightarrow \frac{\omega^2 R - \omega^2(R-h)}{\omega^2 R} = \frac{h}{R} < 0.05$ or $R > \frac{h}{0.05} \Rightarrow R > 36\text{ m}$ in the case of a 1.8 m astronaut. Astronauts may reasonably be as tall as 2 m, so you would build a circular section of the spacecraft having a radius of no less than 40 m.

11.88. The rod would fly off of the turntable, provided there is no friction between them. The center of mass of the rod would move off of the turntable in a straight line, tangential to the circle it made when it was attached to the turntable. In the figure, this would be the $+y$ direction. At the moment of release, the center of mass of the rod was a distance 0.080 m from the center of the disk, and had been moving at a speed of $v = \frac{2\pi r}{T} = \frac{2\pi(0.080\text{ m})}{(0.360\text{ s})} = 1.40\text{ m/s}$. Immediately after release, the speed of the center of mass would still be 1.40 m/s. The rod will also rotate about its center of mass so as to satisfy conservation of angular momentum. Assuming there is no friction and that the angular momentum of the turntable is unchanged, the angular momentum of the rod would also be unchanged. Thus we can write $\frac{1}{3}m\ell^2\omega_{\mathrm{i}} = \frac{1}{12}m\ell^2\omega_{\mathrm{f}} \Rightarrow \omega_{\mathrm{f}} = 4\omega_{\mathrm{i}} = \frac{4(2\pi)}{T} = \frac{8\pi}{0.360\text{ s}} = 69.8\text{ s}^{-1}$. Note that the rotational inertia has changed because initially the rod was constrained to rotate around one end, but after release the rod will rotate around its center of mass. So the rod will rotate with a rotational speed of 69.8 s^{-1} in the counterclockwise direction.

11.89. The energy involved in the desired final rotational motion of the top is

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(5.0\times10^{-3}\text{ kg}\cdot\text{m}^2)[(1800\text{ rev/min})(2\pi\text{ rad/rev})(1\text{ min/60 s})]^2$$
$$= 9.0\pi^2\text{ J} = 89\text{ J}$$

This energy must come from the work done as you pull on the string. If we assume no dissipation, and that you pull with a constant force F on the string, then we can compute the work you do as the product of F and the displacement of your hand. Here we must also assume that the center of mass of the top moves by a negligible distance as the string unwinds, so that all of your effort goes into producing spin. Your hand will then move by the length of the string during the time that the string unwraps. Putting this together, we find

$$K = W = F\Delta r$$
$$F = K/\Delta r = (9.0\pi^2\text{ J})/(1.2\text{ m}) = 74\text{ N}$$

This is enough force to lift a large bowling ball, so you really have to pop that string to make the top sing!

11.90. (*a*) We use conservation of angular momentum. We have $L_{\mathrm{t}\vartheta,\mathrm{i}} + L_{\mathrm{d}\vartheta,\mathrm{i}} = L_{\mathrm{t}\vartheta,\mathrm{f}} + L_{\mathrm{d}\vartheta,\mathrm{f}} \Rightarrow -I_{\mathrm{t}}\omega_{\mathrm{i}} + m_{\mathrm{d}}R_{\mathrm{t}}v_{\mathrm{d}} = I_{\mathrm{t}}\omega_{\mathrm{f}} + m_{\mathrm{d}}R_{\mathrm{t}}v_{\mathrm{d,f}}$. After the collision, we know $\omega_{\mathrm{f}}R_{\mathrm{t}} = v_{\mathrm{d,f}}$. Inserting this, and the rotational inertia of a disk, we have

$-\frac{1}{2}m_{\mathrm{t}}R_{\mathrm{t}}^2\omega + m_{\mathrm{d}}R_{\mathrm{t}}v_{\mathrm{d}} = \left(\frac{1}{2}m_{\mathrm{t}} + m_{\mathrm{d}}\right)R_{\mathrm{t}}^2\omega_{\mathrm{f}}.$ Solving for the final rotational speed of the target yields $\omega_{\mathrm{f}} = \dfrac{-\frac{1}{2}m_{\mathrm{t}}R_{\mathrm{t}}\omega + m_{\mathrm{d}}v_{\mathrm{d}}}{\left(\frac{1}{2}m_{\mathrm{t}} + m_{\mathrm{d}}\right)R_{\mathrm{t}}}.$ (*b*) Again we can use conservation of angular momentum, but now the dart has zero initial angular momentum. Now we have $L_{\mathrm{t}\vartheta,\mathrm{i}} + L_{\mathrm{d}\vartheta,\mathrm{i}} = L_{\mathrm{t}\vartheta,\mathrm{f}} + L_{\mathrm{d}\vartheta,\mathrm{f}} \Rightarrow -I_{\mathrm{t}}\omega_{\mathrm{i}} + m_{\mathrm{d}}(0)v_{\mathrm{d}} = I_{\mathrm{t}}\omega_{\mathrm{f}} + m_{\mathrm{d}}R_{\mathrm{t}}v_{\mathrm{d,f}}.$ Inserting $\omega_{\mathrm{f}}R_{\mathrm{t}} = v_{\mathrm{d,f}}$ as before, we find $-\frac{1}{2}m_{\mathrm{t}}R_{\mathrm{t}}^2\omega = \left(\frac{1}{2}m_{\mathrm{t}} + m_{\mathrm{d}}\right)R_{\mathrm{t}}^2\omega_{\mathrm{f}}.$ Solving for the final rotational speed yields $\omega_{\mathrm{f}} = \dfrac{-\frac{1}{2}m_{\mathrm{t}}\omega}{\left(\frac{1}{2}m_{\mathrm{t}} + m_{\mathrm{d}}\right)}.$

11.91. Call the moment just before the cube strikes the threshold time 1, the moment just after time 2, and the moment the center of the cube reaches a height h time 3. When the cube strikes the door threshold, rotational kinetic energy will not be constant, but angular momentum will. We can determine the rotational speed of the cube just after the collision by writing $L_{\vartheta,1} = L_{\vartheta,2} \Rightarrow m\frac{d}{2}v = \frac{2}{3}md^2\omega_2 \Rightarrow \omega_2 = \frac{3v}{4d}.$ Now as the block's center rises the energy of the block will be constant. We can write $K_{\mathrm{rot},2} = K_{\mathrm{rot},3} + U_3^G \Rightarrow \frac{1}{2}I\omega_2^2 = \frac{1}{2}I\omega_3^2 + mgh.$ Solving for the final rotational speed, and inserting our known expression for ω_2 yields $\omega_3 = \sqrt{\left(\frac{3v}{4d}\right)^2 - \frac{2mgh}{I}}.$ Now to write the height h in terms of the horizontal displacement $x,$ consider the diagonal line from the center of the cube to the center of the edge that struck the threshold. This diagonal line has a length $\ell = \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2} = \frac{d}{\sqrt{2}}.$ This length will be the same no matter where the center of the cube is. Take for example the moment when the cube's center has reached $x_{\mathrm{f}} = \frac{d}{2} - x,$ and $y_{\mathrm{f}} = \frac{d}{2} + h.$ We know $x_{\mathrm{f}}^2 + y_{\mathrm{f}}^2 = \frac{d^2}{2} \Rightarrow h = \sqrt{\frac{d^2}{2} - \left(\frac{d}{2} - x\right)^2} - \frac{d}{2}.$ Inserting this into our expression for the final rotational speed yields

$$\omega_3(x) = \sqrt{\left(\frac{3v}{4d}\right)^2 - \frac{3g}{d^2}\left(\sqrt{\frac{d^2}{2} - \left(\frac{d}{2} - x\right)^2} - \frac{d}{2}\right)}$$

or $\omega(x) = \frac{1}{d}\sqrt{\left(\frac{3v}{4}\right)^2 - \frac{3g}{2}(\sqrt{d^2 + 4dx - 4x^2} - d)}$

12

TORQUE

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^1 N·m **2.** 10^3 N·m **3.** 10^2 N·m **4.** 10^1 m **5.** 10^1 **6.** 10^1 **7.** 10^3 N·m **8.** 10^4 J **9.** 10^4 N·m

Guided Problems

12.2 At the grindstone

1. Getting Started The force exerted by the knife on the stone that is normal to the surface of the stone cannot cause a torque. It is directed radially inward toward the axis around which the stone is spinning. However, this normal force is associated with a frictional force between the knife and the stone. This frictional force is tangential to the surface of the grindstone, and therefore is capable of causing a torque.

2. Devise Plan The vector sum of all torques acting on an object is equal to the object's rotational inertia times its rotational acceleration, $\sum \vec{\tau} = I\alpha$. In this case we want the grindstone to spin with a constant rotational speed, meaning the rotational acceleration must be zero. We therefore want the vector sum of all torques to be zero: $\sum \vec{\tau} = I(\vec{0}) = \vec{0}$. The two torques involved are the torque caused by the frictional force that the knife exerts on the grindstone, and the torque provided by the motor. Any torque can be written $\vec{\tau} = \vec{F} \times \vec{r}$. Calling the direction of the rotational motion the positive $\hat{\vartheta}$ direction, one can write $\sum \vec{\tau} = (\tau_{\text{motor}} - \tau_{\text{kin. friction}})\hat{\vartheta} = \vec{0}$. We need to set the magnitudes of the two torques equal, such that they cancel. Then, an expression for the torque from friction can be inserted. The normal force will be used in the calculation of the force of kinetic friction, according to $F^k_{\text{kg}} = \mu_k F^n_{\text{kg}}$. but the normal force itself does not cause any torque.

3. Execute Plan Because the vector sum of torques must be zero, and the two torques are directed opposite one another, we can set their magnitudes equal: $\tau_{\text{motor}} = \tau_{\text{kin. friction}}$. Now inserting the expression for the torque from kinetic friction yields: $\tau_{\text{motor}} = F^k_{kg} R\sin(\theta)$. In this case, R is the radius of the grindstone, and $\theta = 90°$ because the force of kinetic friction is perpendicular to the radius. Inserting $F^k_{\text{kg}} = \mu_k F^n_{\text{kg}}$ yields:

$$\begin{aligned}\tau_{\text{motor}} &= \mu_k F^n_{kg} R\sin(\theta)\\ &= 0.54(20\text{ N})(0.17\text{ m})\sin(90°)\\ &= 1.8\text{ N}\cdot\text{m}\end{aligned}$$

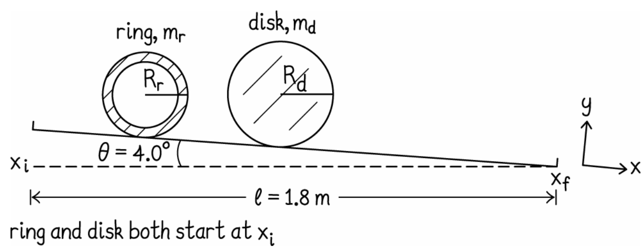
Of course, since the motor must keep the grindstone spinning, this torque must be in the direction of the rotational velocity, so $\tau_{\text{motor}} = 1.8\text{ N}\cdot\text{m}\ \hat{\vartheta}$.

4. Evaluate Result The expression for the torque from the motor seems correct in that the required torque would increase if either the coefficient of kinetic friction or the normal force increased. The magnitude of this torque is very reasonable since a person can easily deliver this torque. A motor should have no trouble providing this torque. If it is more comfortable to think in terms of forces, one could estimate that the radius of the axle is no more than 0.03 m. That would mean that the motor would somehow have to exert a force tangential to the surface of the axle rod (possible using a belt or chain) of about 60 N. This is a perfectly reasonable force for a motor to exert.

12.4 Inclined race

1. Getting Started As the objects move down the incline, they have translational and rotational motion. We could solve this problem in several ways. One way to solve part (*a*) would be to use the conservation of energy. There would be gravitational potential energy initially, and finally there would be rotational and translational kinetic energy. This could be used to the find the final velocity of each object, and would effectively determine which reaches the end first. A second possibility would be to relate the translational and rotational accelerations and use kinematics. For part (*b*), the conservation of energy would not be sufficient to determine the time difference. Kinematics would be necessary. Hence, here, we will adopt the kinematic approach. Energy conservation could be used to check one's answer to part (*a*), and to show qualitative agreement with (*b*).

2. Devise Plan The figure below defines the quantities we will use.



The acceleration of any object down an incline is given in the Principles text in equation (12.25), repeated here for convenience:

$$a_{\mathrm{cm},x} = \frac{g\sin(\theta)}{1+c} \tag{1}$$

Here c is the shape factor $c \equiv I/mR^2$. For a ring $c=1$, whereas for a disk $c=1/2$. Note that this expression is independent of the masses and radii. Before any equations have been solved, we note that the acceleration for the ring will be smaller at any moment, meaning its speed will always be less than the speed of the disk. Therefore we intuitively expect the disk to reach the ground first. In order to find exact times we can use the kinematic equation:

$$\Delta x = v_{x,\mathrm{i}}t + (1/2)a_x t^2 \tag{2}$$

to find the time in terms of the acceleration given above and the distance travelled.

3. Execute Plan Solving equation (2) for time, and inserting the acceleration from equation (1) yields

$$t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2\ell(1+c)}{g\sin(\theta)}} \tag{3}$$

From here, we insert the shape factors for the two objects and find the times at which each reaches the ground:

$$t_\mathrm{r} = \sqrt{\frac{2(1.8\text{ m})(1+1)}{(9.8\text{ m/s}^2)\sin(4°)}} = 3.25\text{ s}$$

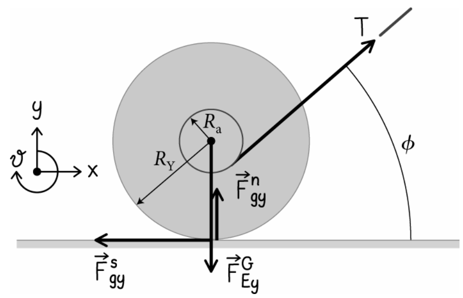
$$t_\mathrm{d} = \sqrt{\frac{2(1.8\text{ m})(1+0.5)}{(9.8\text{ m/s}^2)\sin(4°)}} = 2.81\text{ s}$$

This answers part (*a*) in that the time for the disk is less than the time for the ring, so the disk reaches the ground first. It also answers part (*b*), in that the time difference is $t_\mathrm{r} - t_\mathrm{d} = 0.43$ s. So the disk arrives at the ground 0.43 s before the ring.

4. Evaluate Result The final expression for the time in equation (3) contains reasonable dependence on several variables. For example, if the length ℓ of the ramp is increased, then the time also increases. The time does not increase linear with the length, because the object is always speeding up. The square root dependence is reasonable. Also, as the incline becomes steeper the time decreases. This also makes sense. The order of magnitude of these answers is reasonable. A few seconds for objects to roll down an incline seems right.

12.6 Fickle yo-yo

1. Getting Started We should begin by making a good extended free body diagram.



Note the unconventional choice of direction for the positive rotational motion. This was done so that positive angular acceleration and positive translational acceleration correspond to the same direction of motion. The forces acting on the yo-yo are shown in the figure below. Note for part (*b*) that when the yo-yo changes the direction of its acceleration, the value of the acceleration must momentarily be zero. This is the critical angle: the angle at which the acceleration is zero.

2. Devise Plan We can write the sum of all forces acting on the yo-yo, and the sum of all torques around the center of its axle. Writing only the sum of forces in the x direction, we have

$$\sum F_x = \mathcal{T}\cos(\phi) - F^s_{\text{Ey}} = ma_{\text{cm},x} \tag{1}$$

$$\sum \tau = F^s_{\text{Ey}} R_{\text{y}} - \mathcal{T}R_{\text{a}} = I\alpha \tag{2}$$

Note that the gravitational and normal forces do not cause any torque. The rotational inertia for the yo-yo is given by $I = cmR_{\text{y}}^2$, but it is not known exactly what the shape factor c is. If we approximate the yo-yo to be made up of massive disks connected by a very lightweight axle, then the shape factor is approximately $1/2$. Inserting the general expression for the rotational inertia and the general relationship $\alpha = a/r$ into equation (2) yields:

$$F^s_{\text{Ey}} R_{\text{y}} - \mathcal{T}R_{\text{a}} = cmR_{\text{y}} a_{\text{cm},x} \tag{3}$$

Equations (1) and (3) can now be combined to solve for the acceleration in the x direction, as a function of the angle ϕ. The critical angle will then be determined by setting the acceleration equal to zero and solving for ϕ.

3. Execute Plan Solving equation (1) for the force of friction yields

$$F^s_{\text{Ey}} = \mathcal{T}\cos(\phi) - ma_{\text{cm},x}$$

and inserting this into equation (3) yields

$$(\mathcal{T}\cos(\phi) - ma_x)R_{\text{y}} - \mathcal{T}R_{\text{a}} = cmR_{\text{y}} a_{\text{cm},x}$$

Finally, solving for the acceleration we obtain

$$a_{\text{cm},x} = \frac{\mathcal{T}\,[\cos(\phi) - R_{\text{a}}/R_{\text{y}}]}{(c+1)m} \tag{4}$$

If we assume that the shape factor is approximately $1/2$, then equation (4) becomes our final answer to part (*a*)

$$a_{\text{cm},x} = \frac{2\mathcal{T}\,[\cos(\phi) - R_{\text{a}}/R_{\text{y}}]}{3m} \tag{5}$$

For part (*b*), we must determine at what angle the acceleration in equation (5) changes sign. This can be determined trivially by setting equation (5) equal to zero, and obtaining

$$\cos(\phi_{\text{crit}}) - R_{\text{a}}/R_{\text{y}} = 0$$
$$\phi_{\text{crit}} = \cos^{-1}(R_{\text{a}}/R_{\text{y}})$$

4. Evaluate Result We have assumed that the coefficient of static friction between the yo-yo and the ground is sufficient to allow rolling without slipping. But if that is satisfied, the result appears to be independent of the coefficient of friction.

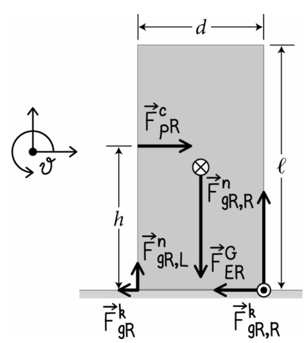
One would expect that pulling directly upward on the yo-yo should always make it move in the $-x$ direction. Indeed, inserting $\phi = \pi/2$ in equation (5) ensures that the acceleration is always negative, regardless of the ratio of the radii. Similarly, one would expect that pulling in the $+x$ direction would cause the yo-yo to move in the $+x$ direction. Inserting $\phi = 0$ into equation (5) shows that the acceleration is positive, and could only become negative if the axle had a larger radius than the yo-yo itself, which violates assumptions that went into the equations. Hence the answer to part (*a*) agrees with physical intuition.

The torques that serve to spin the yo-yo have the radii as their respective lever arms. It makes perfect sense that the ratio of these lever arms would determine when one torque wins out over the other. It also makes sense that if the ratio approached unity $R_{\text{a}}/R_{\text{y}} \approx 1$, then the critical angle would approach zero. This would mean that pulling at any angle would result in the yo-yo rolling in the $-x$ direction. This makes sense. Consider the opposite extreme, at which the axle is so thin that the tensile force can barely cause any torque at all. Then $R_{\text{a}}/R_{\text{y}} \approx 0$ and pulling at any angle would cause the yo-yo to accelerate in the $+x$ direction. Hence the answer to part (*b*) also fits with intuition.

12.8 Moving a refrigerator

1. Getting Started In order for the refrigerator not to tip, the sum of all torques on the refrigerator (around any axis) must be zero. Also, we know that the force with which you push the refrigerator must be exactly equal to the force of kinetic friction, such that the speed of the refrigerator across the floor is constant. We can set our axis to be the bottom edge of the refrigerator on the side opposite you as you push. Now we can determine which forces cause torques that would tend to spin the refrigerator clockwise and counterclockwise. Your push would cause a clockwise torque, whereas the force of gravity on the refrigerator would cause a counterclockwise torque.

2. Devise Plan The forces acting on the refrigerator are shown in the extended free body diagram below.



The effective lever arm of the normal force acting on the refrigerator is a component of the distance between the axis of rotation and the point of application of the normal force, the component perpendicular to the force itself. If the refrigerator were at rest, then the normal force would be distributed evenly over all of the surface of the refrigerator that is in contact with the floor. But in this case, we require that the sum of all torques be zero, and there is a torque from our push on the side of the refrigerator. The normal forces on the left and right bottom edges of the refrigerator must be different, so as to compensate for the torque from our push. As drawn, the force of our push is exerted above the center of mass and would serve to spin the refrigerator clockwise. So, in this case, the normal force must be larger on the right than on the left. This can be seen in various ways. We could reach this conclusion by looking at the sum of all torques around the center of mass, or we could simply consider the limiting case where the refrigerator is just teetering on the edge of tipping over. At that time the left edge of the refrigerator would be just on the verge of lifting up off the ground, meaning the normal force of the ground acting on the left edge would be zero. Hence, when the refrigerator is just on the verge of tipping, the normal force is entirely on the right bottom edge of the refrigerator.

3. Execute Plan If the refrigerator is not spinning, then one can say it is not spinning around any axis at all. But if the refrigerator is just on the verge of tipping, then there is only one axis around which it is about to tip. This is the bottom right edge. We have established that the normal force also acts entirely at the lower right edge. Hence there is no distance at all between the axis and the point of application of the force. The answer to part (*a*) is zero.

For part (*b*) we set the sum of all torques equal to zero at the moment just before tipping:

$$\begin{aligned}\sum\tau &= -\tau_{\mathrm{p}} + \tau_{\mathrm{g}} = 0\\ &= -F^{c}_{\mathrm{pr}}h + F^{G}_{\mathrm{Er}}d/2\end{aligned}$$

Recall that the magnitude of the force of the push must equal that of the force of kinetic friction for the refrigerator to be moving at a constant speed. Hence

$$\mu_k mgh = mgd/2$$

or

$$h_{\max} = d/2\mu_k \tag{6}$$

4. Evaluate Result The result in equation (6) shows that one can push higher on a wider refrigerator without tipping it. This fits with physical intuition. Also, equation (6) suggests that the greater the coefficient of friction, the lower you must push. This makes sense because the friction is what makes the corner catch, but it makes even more sense when we recall the condition that the push had to equal the force of kinetic friction in magnitude. Increasing the coefficient of friction would also mean you are pushing harder. So, certainly the push would be more likely to tip the refrigerator in that case, unless we push lower.

Questions and Problems

12.1. You can exert a greater torque by exerting a greater force tangential to the lid. The coefficient of static friction between the rubber and the lid is likely to be greater than the coefficient of static friction between your hand and the lid.

12.2. Move the small stone (the fulcrum) closer to the large stone. This gives you a greater lever arm.

12.3. Torque is a better tightening specification because giving the specification in terms of the force exerted on the wrench you use does not provide a unique specification for how tight you fasten the bolts. You could use a wrench of any length, and the force necessary to get the correct tightness would be different for each different wrench length.

12.4. The additional length of pipe effectively lengthens the wrench. You can exert the same force to the end of the pipe, and get a greater torque because of the longer lever arm.

12.5. Options a, b, and f all have the forces being exerted along the body of the fence. This means that the angle between the radial vector and the force is $0°$ (or $180°$) and vector product in $\vec{\tau} = \vec{F} \times \vec{r}$ will give us zero. In option d, the force is exerted on the axis of rotation. Physically this means there is no lever arm and the force will not yield a torque; mathematically this means $\vec{r} = 0$. Options c and e yield nonzero torques, as the forces are exerted at nonzero distances from the axis and at angles such that the vector product is nonzero.

12.6. Starting from $\tau = Fr\sin(\theta)$ it is clear that the maximum torque will be achieved by exerting a force perpendicular to the wrench. This makes it easy to rank options (*c*) through (*f*). In all of those options, the forces are applied at right angles to the wrench, so that the torque will scale with the distance from the nut to the point at which the force is exerted. Hence *(f)* > (*d*) = (*c*) > (*e*). Note that the lever are (and hence the torque) is approximately twice as large in options (*c*) and (*d*) as in option (*e*). Option (*a*) shows a force being applied at the same point as in option (*e*), but not at a right angle. Hence the torque from option (*a*) must be less than from option (*e*). To correctly place option (*b*), one must estimate the angle away from the length of the wrench, which is approximately $45°$. Hence the torque from option (*b*) is less than the torque from options (*c*) and (*d*) by a factor of $\sin(45°) = \sqrt{2}/2$. This also means that it is not quite as small as the torque from option (*e*). Hence the relationship between all the options is $(f) > (d) = (c) > (b) > (e) > (a)$.

12.7. Option *a*. The baton's center of mass is closer to the larger sphere, making the lever arm distance between your hand and the center of mass longer in option a than in option *b*. The longer lever arm distance gives you finer control.

12.8. The car turns because of the force exerted by the road on the tires, preventing them from slipping. The car itself would tend to move in a straight line if not for that force. As the road pulls the car in toward the center of the turn, it also exerts a torque on the car (because it makes contact with the tires, not the center of mass of the car). In extreme cases, this torque would cause the car to flip over (away from the center of the circle). In safer turning, the inner tires will still lift slightly.

12.9. The simplest way may be to calculate the torque from each parent. Use $\tau = Fr\sin(\theta)$ for each parent. $\tau_A =$ $(100\text{ N})\ (0.80\text{ m})\sin(90°) = 80\text{ N}\cdot\text{m}$, $\tau_B = (300\text{ N})\ (1.0\text{ m})\sin(180°) = 0\text{ N}\cdot\text{m}$, $\tau_C = (200\text{ N})\ (0.50\text{ m})\sin(60°) =$ $87\text{ N}\cdot\text{m}$, and $\tau_D = (250\text{ N})\ (1.0\text{ m})\sin(60°) = 220\text{ N}\cdot\text{m}$. Hence $\tau_B < \tau_A < \tau_C < \tau_D$.

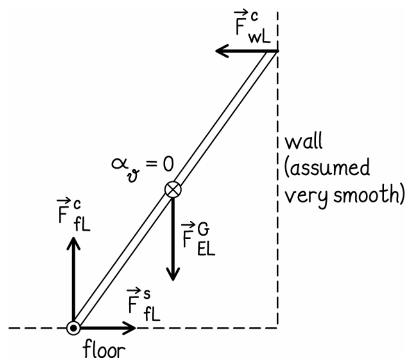
12.10. In order for the object to remain horizontal, the sum of all torques must be zero. One could try several combinations, but notice that when the $9M$ object is placed on one of the outer hooks, it causes a torque that is so great that it cannot be canceled by any combination. Place the $3M$ object on the far left, and the $5M$ object on the left, closest to the axis. Place the $9M$ object on the right closest to the axis, followed by the $1M$ object on the far right. Then the sum of all torques would be $\sum\tau = (3M)(2d) + (5M)(d) - (9M)(d) - (1M)(2d) = 11Md - 11Md = 0$, and the system is indeed in rotational equilibrium.

12.11. When this asymmetric baton spins, it will spin about its center of mass. The location of the center of mass is given by (assume the baton is lying along the x axis, with the $4m$ object at the origin) $x_{cm} = ((4m)(0\text{ m}) + (m)(\ell))/5m = \ell/5$. So object A will spin in a circle of radius $\ell/5$, whereas object B will spin with a radius of $4\ell/5$. Hence $C_A/C_B = 1/4$.

12.12. If the system is unconstrained, then it will spin around its center of mass. Assume the rigid rod is oriented along the x axis and object A is at the origin, then $x_{cm} = ((3m)(0) + (m)(\ell))/4m = \ell/4$. The speed at which either object rotates around the center of mass is given by $v = \omega r$. This corresponds to speeds of $v_{cm,A} = \omega\ell/4 = v/2$ and $v_{cm,B} = 3\omega\ell/4 = 3v/2$. The entire system is moving to the right, and is rotating counterclockwise. This means that at the moment shown, the velocity of object A around the center of mass is in the same direction as the translational velocity, and the two will add. The two velocities will subtract for object B. This leads to total speeds of $v_A = 3v/2$ and $v_B = v/2$. Hence the ratio $v_A/v_B = 3$.

12.13. The stick does not rotate, which means the torques caused by the contact forces exerted on it by your two fingers cancel each other. The farther a finger is from the stick's center of mass, the greater its lever arm distance and the smaller the contact force it exerts on the stick. So as you begin sliding your fingers, any slight difference in lever arm distance causes the finger farther from the center of mass—the left finger, say—to exert a smaller contact force and so slide more easily than the right finger. Once your left finger moves far enough toward the center of the stick to make the right lever arm distance greater than the left lever arm distance, the right finger exerts the smaller contact force and so speeds up, but only until its lever arm distance is less than that of the left finger. This tradeoff continues until the fingers meet, always at the 0.5-m mark.

12.14.

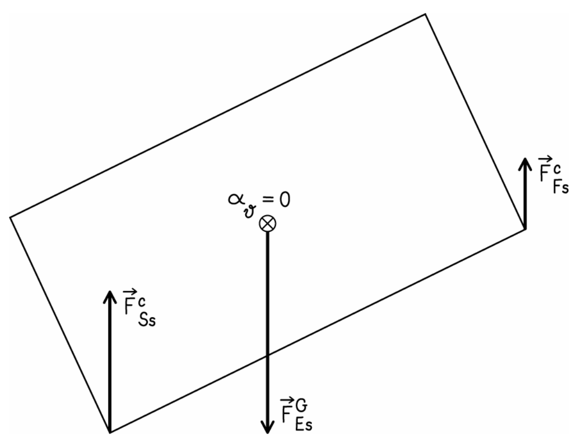


Because you are standing on the ladder (and not falling to the ground), make sure that the sum of all forces is zero in each direction, as well as the sum of all torques around any axis.

12.15. The refrigerator is oriented in such a way that if it rotates, it will be around its lower right edge. In figure a, the center of mass of the refrigerator is to the left of the axis, meaning the torque on the refrigerator (due to gravity) will be positive (counterclockwise) and the refrigerator should return to a standing position. In figure b, the center of mass is directly over the axis. In this case there is no torque in either direction and the refrigerator will teeter over the axis until some other force (a gust of wind perhaps) determines which way the refrigerator will fall. In figure c, the center of mass is to the right of the axis. This means that gravity will cause a negative (clockwise) torque and the refrigerator will certainly rotate and fall. Hence it is most in danger of falling in case c.

12.16. The left configuration is easier to handle; it has a lower center of mass.

12.17. (*a*)

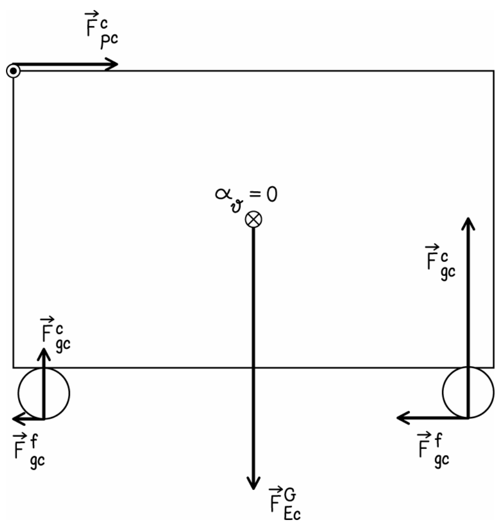


Note that the sum of all forces in the y direction must be zero, because the sofa is not falling. Also, the sum of all torques must be zero because the sofa is not spinning. Be sure that your arrow lengths reflect those facts. (*b*) Setting the sum of all forces to zero only tells you that the father and son together must exert enough force to cancel out the gravitational force on the sofa. To see who exerts more force, look at the sum of all torques on the sofa, around the sofas center of mass. As the sofa is tilted down the stairs, the vector $\vec{r}$ (which always points from the axis to the point at which a force acts) associated with the father's force moves closer to horizontal. The direction of $\vec{r}$ is farther from horizontal for the son. This means that for the sum of all torques to be zero, the son must exert more force on the sofa because he has a smaller effective lever arm. The son carries more of the load than the father. (*c*) If the board were very thin, then $\vec{r}$ would lie essentially along the body of the board, and there would be very little difference in the angles at which the two forces (father and son) act. The son and father would carry roughly equal loads in the case of a sheet of plywood.

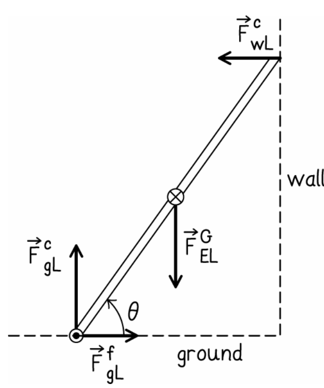
12.18. In order for the giraffe and monkey to the level, the sum of all torques must be zero around the point where the string connects to the rod that supports them. Hence $\sum\tau = (m_{\rm m}g)(3d) - (m_{\rm g}g)(d) = 0$, or $m_{\rm g} = 3m_{\rm m}$. Similarly, the sum of all torques on the upper rod around the point where the utmost string holds it must also be zero. This means $\sum\tau = (4m_{\rm m}g)(2d) - (m_{\rm e}g)(4d) = 0$, or $m_{\rm e} = 2m_{\rm m}$.

12.19. As the painter climbs, he moves farther from the point where the ladder contacts the carpeting. This gives him a greater lever arm distance, meaning that as he climbs he causes a greater torque around the axis running through the contact point.

12.20. Because the cart is in rotational equilibrium, one can pick any axis of rotation. For example, the sum of all torques around the center of mass of the cart must be zero. The force of gravity is exerted at the center of mass of the cart, and cannot cause a torque around the center of mass. The forces that could cause a torque are the contact forces between the front and rear wheels and the ground, and the force from your hands pushing on the top. Your hands, both frictional forces and the normal force that the floor exerts on the rear wheels all cause a negative (clockwise) torque on the cart. The only force that can cancel all of these is the normal force that the ground exerts on the front wheels. Because the force on the front wheels must cancel the force on the rear wheels plus additional forces, the force on the front wheels must be greater than the force on the rear wheels.



12.21. In order for the ladder not to slip, the sum of all forces in both the x and y directions must be zero, as must the sum of all torques. Begin by drawing an extended free body diagram.



Proceed by writing out the corresponding equations:

$$\sum F_x = F_{\text{gL}}^{\text{f}} - F_{\text{wL}}^{\text{c}} = 0 \tag{1}$$

$$\sum F_y = F_{\text{gL}}^{\text{c}} - mg = 0$$

$$\sum \tau = F_{\text{wL}}^{\text{c}} \ell \sin(\theta) - mg\cos(\theta)\ell/2 = 0 \tag{2}$$

The question asks for the minimum angle the ladder can have before it slips, meaning we exert the maximum static force of friction. Therefore we can replace F_{gL}^{f} with $mg\mu_s$ in (1). Solving (2) for F_{wL}^{c} and equating it to the frictional force through (1) yields $mg\mu_s = \dfrac{1}{2\tan(\theta)}$ or $\theta = \tan^{-1}\left(\dfrac{1}{2\mu_s}\right)$.

12.22. The sum of all torques around any axis must be zero, but it is convenient to choose the axis at be at the point where the clock meets the nail. In that case, the only forces that cause torques on the clock are the gravitational force and the normal force that the wall exerts on the clock. Call the angle between the wall and the back of the clock θ, and call $\alpha = \tan^{-1}(w/\ell)$. Call the vector from the axis to the center of mass $\vec{r}_{\text{cm}}$ and the vector from the axis to the point of contact with the wall $\vec{r}_{\text{w}}$. The angle between $\vec{F}_{\text{Ec}}^{\text{G}}$ and $\vec{r}_{\text{cm}}$ is $\alpha - \theta$ which can be seen from the geometry of the clock. The angle between $\vec{F}_{\text{wc}}^{\text{c}}$ and $\vec{r}_{\text{w}}$ is $90° - \alpha$. Writing the sum of all torques about the chosen axis yields

$$\sum \tau = F_{\text{wc}}^{\text{c}} \ell \sin(90° - \theta) - mgr_{\text{cm}} \sin(\alpha - \theta) = 0$$

$$F_{\text{wc}}^{\text{c}} = \frac{mg}{2\ell}\sqrt{\ell^2 + w^2}\,(\sin(\alpha)\cos(\theta) - \sin(\theta)\cos(\alpha))/\cos(\theta)$$

Inserting $\sin(\alpha) = w/\sqrt{\ell^2 + w^2}$, $\cos(\alpha) = \ell/\sqrt{\ell^2 + w^2}$, and $\tan(\theta) = d/\sqrt{\ell^2 - d^2}$, one obtains

$$F_{\text{wc}}^{\text{c}} = \frac{mg}{2}\left(\frac{w}{\ell} - \frac{d}{\sqrt{\ell^2 - d^2}}\right).$$

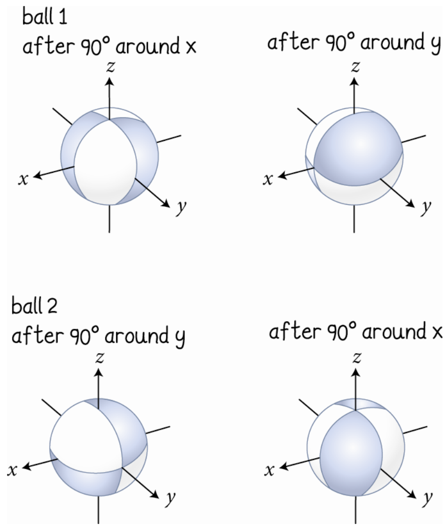
12.23. In the United States, the threading of nearly all bolts, screws, and nuts is such that a clockwise rotation tightens and a counterclockwise rotation loosens. From the perspective of the mechanic looking at his or her work, clockwise rotation is comparable to rightward motion and counterclockwise rotation is comparable to leftward motion. (Picture your hand placed on the 12 on a clock face: moving to the 1 is to the right, moving to the 11 is to the left.) Note that this clockwise-counterclockwise convention is consistent with the right-hand rule. With a nut on a bolt, for instance, the direction of the torque is the direction that the nut should move (down the body of the bolt to tighten, for example).

12.24. Use the right hand rule. As your fingers curl around the jar in a counterclockwise fashion, your thumb points upward, away from the jar. So, the rotational velocity is upward (away from the jar).

12.25. Clearly, gear A causes gear B to rotate in the opposite direction. Using the right hand rule verifies that the rotational velocity of gear B is in the positive x direction.

12.26. Path c. The ball would reflect along path b if the ball had not been made to spin. If the ball is spinning clockwise when viewed from above, and the point on the ball in contact with the table edge momentarily stops during the collision, the angular momentum of the ball will force it to move more to the left. In terms of the rotational velocity, it was initially downward because the cue caused the ball to rotate clockwise when viewed from above. In order for the ball to spin less, there must be some torque applied to the ball that would cause an acceleration in the counterclockwise direction. This can only come from the edge of the table, meaning the edge of the table must exert a force on the ball that has a component to the left (when viewed from the player's perspective, or to the right in the top view shown).

12.27.



The results of the proposed rotations are shown in the figure. An additional rotation of $90°$ around the y axis followed by $-90°$ around x axis will put the second ball in the same orientation as the first.

12.28. The rotation of the football about its long axis helps stabilize the ball, ensuring that it maintains one orientation during flight, making it easier to catch. If it were not spinning, then small fluctuations in air density or gusts of wind could easily cause the ball to tumble along any axis. The fact that the ball is spinning means that it has significant angular momentum along the long axis. Accordingly, significant torque would have to be sustained in order to cause the ball to tumble away from that axis. It is also true that this is the most aerodynamic axis along which it could spin during flight.

12.29. (*a*) The torque around the pivot must be zero in order for it to remain in rotational equilibrium. Because the buckets are identical, Earth will exert the same amount of gravitational force on each, and that force will be exerted directly downward on each. Hence in order for the torques to be equal (and in opposite directions), the lever arm must be equal on either side. Hence the pivot must be at the midpoint, 0.75 m from either end of the rod. (*b*) The same arguments apply, except that now the masses of the buckets are not equal, and so the lever arms are also not equal. Call the distance from the heavier bucket to the pivot x, and the length of the rod ℓ. Then the sum of all torques is $\sum \tau = mgx - mg(\ell - x)/4 = 0$, or $x = \ell/5 = 1.50\ \text{m}/5$. So the pivot is now 0.30 m from the heavier bucket.

12.30. (*a*) $\alpha = (\omega_\text{f} - \omega_\text{i})/\text{t} = ((28\ \text{rev/s})(2\pi\ \text{rad/rev}) - 0\ \text{rad/s})/8.0\ \text{s}$, so $\alpha = 22\ \text{rad/s}^2$. (*b*) Use $\tau = I\alpha$, where the rotational inertia for the disk $I = (1/2)mr^2$. Then the torque is given by $\tau = (1/2)mr^2\alpha = (1/2)(0.2\ \text{kg})(0.2\ \text{m})^2(22\ \text{rad/s}^2)$. The torque $\tau = 8.8\times10^{-2}\ \text{N}\cdot\text{m}$.

12.31. Because there is a collision in which objects stick together, there is no reason to think that energy might be constant here. But because there is no external torque applied to the merry-go-round and child, angular momentum will be conserved. Set the initial and final angular momenta equal: $L_\text{p,i} + L_\text{m,i} = L_{(\text{m+p}),\text{f}}$. Inserting $L = I\omega = (1/2)mr^2\omega$ for the disk and $L = mvr$ for the child (treated like a particle), one obtains $m_\text{p}v_\text{p,i}r = (1/2)(m_\text{m}rv_\text{m,f}) + mv_\text{p,f}r$. The final velocities of the child and of the edge of the merry go round are the same, so the object subscripts can be dropped. Finally,

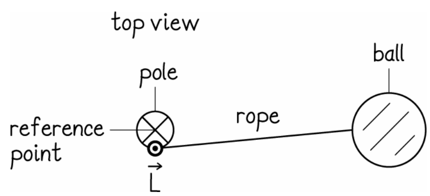
$$v_\text{f} = \frac{m_\text{p}v_\text{p,i}}{\frac{1}{2}m_\text{m} + m_\text{p}} = \frac{(44.0\ \text{kg})(3.0\ \text{m/s})}{\frac{1}{2}(180\ \text{kg}) + 44\ \text{kg}} = 0.99\ \text{m/s}$$

or $\omega = v/r = (0.99\ \text{m/s})/(1.2\ \text{m})$, so $\omega = 8.2\times10^{-1}\ \text{s}^{-1}$.

12.32. (*a*) The angular momentum of your body and the carousel together must be constant. You start out moving in a large circle (at the outer edge) and end up moving in a small circle (at the inner edge). By moving in toward the center of the carousel you have decreased the moment of inertia of the system as a whole. The only way angular momentum can be constant is if the rotational velocity increased. The rotational velocity must increase. (*b*) First, the path you followed in free space was not radial, but a spiral toward the center. Second, consider your tangential speeds at different positions. Near the outside you have a high tangential speed. As you step in toward the center, you are moving in a circle of smaller radius but at (roughly) the same rotational speed, assuming the carousel has a much greater mass than you. Hence your tangential speed must decrease. This is an acceleration, and can only come from a force acting on your body: the contact force between your foot and the carousel. Hence the force of the carousel acting on your foot has some component that is opposite the direction of motion. This means that the force of your foot on the carousel must have a component that is in the direction of motion. This is how your foot is able to increase the rotational speed of the carousel.

12.33. You fall backwards. The axis around which you are free to rotate (your toes) is right up against the wall. Your center of mass is not right up against the wall; it is near the (back-to-front) center of your body. Hence the force of gravity exerted on you causes a torque that tends to rotate you backwards about your toes.

12.34.



The angular momentum is not constant. The tension causes the torque that changes the angular momentum. The tension is not acting perfectly perpendicular to the path of the ball since it has a small lever arm around the center of the pole.

12.35. Many assumptions must be made in this problem. Let us assume that the asteroid has the same density as Earth, and that it strikes Earth moving nearly tangential to the surface and with a velocity that takes it against the rotation of the earth. Let us further assume that the speed of Earth around the Sun (approximately 30 km/s) is a reasonable speed for an astronomical object, and take that to be the speed of the asteroid. We then use conservation of angular momentum:

$$L_{\mathrm{E,i}} + L_{\mathrm{A,i}} = L_{\mathrm{f}}$$

$$\frac{2}{5}M_{\mathrm{E}}R_{\mathrm{E}}^2\omega_{\mathrm{E}} + M_{\mathrm{A}}R_{\mathrm{E}}v_{\mathrm{A}} = \left(\frac{2}{5}M_{\mathrm{E}} + M_{\mathrm{A}}\right)R_{\mathrm{E}}^2\omega_{\mathrm{f}} \tag{1}$$

Where $M_{\mathrm{A}} = \frac{4}{3}\pi r_{\mathrm{A}}^3\rho_{\mathrm{E}} = \frac{4}{3}\pi(500\ \mathrm{m})^3(5500\ \mathrm{kg/m^3}) = 2.9\times10^{12}\ \mathrm{kg}$. Solving Eq. (1) for the final rotational velocity yields

$$\frac{2}{5}M_{\mathrm{E}}R_{\mathrm{E}}^2\omega_{\mathrm{E}} - M_{\mathrm{A}}R_{\mathrm{E}}v_{\mathrm{A}} = \left(\frac{2}{5}M_{\mathrm{E}} + M_{\mathrm{A}}\right)R_{\mathrm{E}}^2\omega_{\mathrm{f}} \tag{2}$$

$$\omega_{\mathrm{f}} = \frac{\frac{2}{5}M_{\mathrm{E}}}{\left(\frac{2}{5}M_{\mathrm{E}} + M_{\mathrm{A}}\right)}\omega_{\mathrm{E}} - \frac{M_{\mathrm{A}}}{\left(\frac{2}{5}M_{\mathrm{E}} + M_{\mathrm{A}}\right)}\frac{R_{\mathrm{E}}v_{\mathrm{A}}}{R_{\mathrm{E}}^2}$$

$$\omega_{\mathrm{f}} \approx (1 - 1.2\times10^{-12})\omega_{\mathrm{E}} - 5.7\times10^{-15}\ \mathrm{rad/s}$$

$$\omega_{\mathrm{f}} \approx (1 - 7.9\times10^{-11})\omega_{\mathrm{E}}$$

We will round this number to the nearest order of magnitude. This means that the rotational velocity of Earth could be changed by as much as 10^{-10} times the current rotational velocity. If the rotational velocity were decreased by this amount, the period of the rotation of Earth could increase by as much as $10^{-10}\ T_\text{E}$, where T_E is the period of Earth's rotation, 24 hours.

12.36. (*a*) Start by writing the sum of all torques on the rod, around the pivot at the base of the rod: $\sum\tau = -m_\text{s}g\ell\sin(\theta) - m_\text{b}g(\ell/2)\sin(\theta) + T\ell\sin(90°) = 0$ or $T = (m_\text{s}g + m_\text{b}g/2)\sin(\theta) = (10\text{ kg} + 2.5\text{ kg})(9.8\text{ m/s}^2)\sin(37°) =$ 74 N. Hence the tensile strength of the cable must be at least 74 N. (*b*) The force exerted by the pivot is now the only unknown force, so it can be found by setting the sum of all forces to zero in each of the two Cartesian coordinates:

$$\sum F_x = F^\text{c}_{\text{pr},x} - T\cos(37°) = 0 \Rightarrow F^\text{c}_{\text{pr},x} = (73.7\text{ N})\cos(37°) = 59\text{ N}$$

$$\sum F_y = F^\text{c}_{\text{pr},y} + T\sin(37°) - m_\text{r}g - m_\text{s}g = 0 \Rightarrow F^\text{c}_{\text{pr},y} = (15\text{ kg})(9.8\text{ m/s}^2) - (73.7\text{ N})\cos(37°) = 100\text{ N}$$

Hence, $F^\text{c}_{\text{pr},x} = 59\text{ N}$ and $F^\text{c}_{\text{pr},y} = 100\text{ N}$.

12.37. (*a*) Because the wall is smooth, we neglect the frictional forces between the wall and the ladder. This means the only vertical forces are the gravitational force of Earth on the board, and the upward normal force that the ground exerts on the board. Because the board is in translational equilibrium, we can write $\sum F_y = F^\text{c}_\text{gb} - F^\text{G}_\text{Eb} = 0$, or $F^\text{c}_\text{gb} = m_\text{b}g = (4.6\text{ kg})(9.8\text{ m/s}^2) = 45\text{ N}$. So the normal force that the ground exerts upward on the board is 45 N. (*b*) The normal force that the wall exerts on the board cannot be found as simply, because there is another unknown force in the x direction, the force of friction exerted by the ground on the board. But one can still say that the sum of all torques around any axis must be zero, because the board is in rotational equilibrium. Choosing the axis at the point where the ladder touches the ground eliminates the force of friction from our equation, because it acts at the axis and has no lever arm. The remaining sum of torques can be written as: $\sum\tau = F^\text{c}_\text{wL}\ell\sin(37°) - m_\text{L}g(\ell/2)\sin(53°) = 0$, or, solving for the normal force $F^\text{c}_\text{wL} = m_\text{L}g\sin(53°)/(2\sin(37°)) = 30\text{ N}$. Hence the normal force exerted by the wall on the ladder is 30 N.

12.38. You will spin on the turntable in accordance with the conservation of angular momentum; your angular momentum vector around the turntable axis will be upward.

12.39. If there are no external torques on the helicopter, its angular momentum must remain constant. With only a main rotor, the body of the helicopter would therefore rotate in the direction opposite the direction in which the rotor rotates. The purpose of the tail rotor is to exert a force on the air surrounding the craft, causing an external torque that keeps the body from rotating.

12.40. Angular momentum is constant in this situation, so we can set the initial and final angular momenta equal to each other. The initial angular momentum can be written as $L_\text{i} = I_\text{m,i}\omega_\text{m,i} + I_\text{p,i}\omega_\text{p,i} = (1/2)m_\text{m}r_\text{m}^2\omega_\text{i} + m_\text{p}r_\text{p,i}^2\omega_\text{i}$, and the final angular momentum as $L_\text{f} = (1/2)m_\text{m}r_\text{m}^2\omega_\text{f} + m_\text{p}r_\text{p,f}^2\omega_\text{f}$. Initially the radius of the merry-go-round and the radius at which the child is standing are the same. With that simplification, equating the two momenta yields $\omega_\text{f} = \dfrac{((1/2)m_\text{m} + m_\text{p})r_\text{m}^2\omega_\text{i}}{(1/2)m_\text{m}r_\text{m}^2 + m_\text{p}r_\text{p,f}^2} = \dfrac{((1/2)(400\text{ kg}) + 35\text{ kg})(1.5\text{ m})^2(2.2\text{ rev/s})(2\pi\text{ rad/rev})}{(1/2)(400\text{ kg})(1.5\text{ m})^2} = 16\text{ rad/s}$. The final rotational velocity of the merry-go-round is 16 s^{-1}.

12.41. (*a*) Angular momentum is conserved, so $I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f}$ or $\omega_\text{f} = I_\text{i}\omega_\text{i}/I_\text{f}$. Plugging in the given data yields $\omega_\text{f} = (3.6\text{ kg}\cdot\text{m}^2)(0.85\text{ rev/s})(2\pi\text{ rad/rev})/(1.1\text{ kg}\cdot\text{m}^2) = 17\text{ s}^{-1}$. So the final rotational speed is 17 s^{-1}. (*b*) We want the difference between the final and initial rotational energies, or $(1/2)I_\text{f}\omega_\text{f}^2 - (1/2)I_\text{i}\omega_\text{i}^2 = (1/2)((1.1\text{ kg}\cdot\text{m}^2)(17.5\text{ rad/s})^2 - (3.6\text{ kg}\cdot\text{m}^2)(5.3\text{ rad/s})^2) = 1.2\times10^2\text{ J}$. So the final rotation has 1.2×10^2 J more energy than the initial rotation. (*c*) The additional energy comes from work done by the skater's muscles as she pulls her arms and weighted hands in toward her body. This force is not perpendicular to the motion because the hands and weights spiral in toward the center of her body.

12.42. Angular momentum is constant in this problem. Setting the initial and final angular momenta equal yields: $(I_{\text{m}} + I_{\text{p,i}})\omega_{\text{i}} = (I_{\text{m}} + I_{\text{p,f}})\omega_{\text{f}}$. Since the child is initially standing at the axis of rotation, the rotational inertia for the child, $I_{\text{p,i}} = 0$. The rotational inertia for the merry-go-round is simply that of a disk, and we will treat the child like a particle. Inserting the appropriate expressions for rotational inertia and solving for the final rotational velocity yields:

$$(I_{\text{m}} + I_{\text{p,i}})\omega_{\text{i}} = (I_{\text{m}} + I_{\text{p,f}})\omega_{\text{f}}$$

$$\omega_{\text{f}} = \frac{I_{\text{m}}\omega_{\text{i}}}{I_{\text{m}} + m_{\text{p}}r_{\text{m}}^2} = \frac{(500\ \text{kg}\cdot\text{m}^2)(0.2\ \text{rev/s})(2\pi\ \text{rad/rev})}{(500\ \text{kg}\cdot\text{m}^2) + (25\ \text{kg})(2.0\ \text{m})^2} = 1.0\ \text{s}^{-1}$$

So, the final rotational speed is $\omega_{\text{f}} = 1.0\ \text{s}^{-1}$.

12.43. (*a*) There is no external torque applied, so the angular momentum remains the same. (*b*) Because the angular momentum is constant and the radius is decreasing, the rotational speed must increase to compensate. The rotational speed increases. (*c*) In order for rotational kinetic energy to increase, some external force must do work on the ball. At first glance, it may seem like no force can do work on the ball, because it seems as though the tensile force is perpendicular to the motion. But this is not true. The ball no longer moves in a perfect circle, but spirals in toward the center. Since there is a component of the ball's displacement that is radially inward, the tensile force can and does add energy to the system. The rotational kinetic energy increases.

12.44. Note that in flipping the wheel, you flip the direction of the wheel's angular momentum. But because the angular momentum of you and the wheel together must be constant, you must acquire some angular momentum to compensate for the change you imposed. Assume the wheel is initially spinning counterclockwise when viewed from above, such that its angular momentum vector points upward. The total angular momentum of your body and the wheel together is upward (because you are initially stationary). When you flip the wheel, its angular momentum vector points downward. As a result, you must acquire a significant angular momentum directed upward, such that the total angular momentum is constant. In this case, you would spin around in the chair, counterclockwise when viewed from above.

12.45. (*a*) Assume the rod is lying along the x axis, and the ball is at the origin. The center of mass is given by $x_{\text{cm}} = (1/M_{\text{tot}})\sum_i m_i x_i = (1/3m)((m)(0\ \text{m}) + (2m)(\ell/2)) = \ell/3$. So, the center of mass is a distance $\ell/3$ from the ball. (*b*) Because the surface is "slick", we will neglect any frictional forces, in which case linear momentum must be constant. This means $mv_{\text{i}} = 3mv_{\text{f}}$, or $v_f = v_i/3$ in the direction of the ball's initial motion. (*c*) In the absence of friction, angular momentum must also be constant, such that we can write $m_{\text{b}}v_{\text{i}}r = (I_{\text{r}} + m_{\text{b}}r^2)\omega_{\text{f}}$, where v_{i} is the initial speed of the ball, and all distances r are measured from the center of mass of the system after the collision. Note that I_{r} refers to the rotational inertia of the rod around the center of mass of the ball-rod system, not the center of the rod itself. This requires the use of the parallel axis theorem to obtain $I_{\text{r}} = (1/3)m_{\text{r}}\ell^2 - m_{\text{r}}(\ell/3)^2 = (2/9)m_{\text{r}}\ell^2$. Inserting this result and solving for the final rotational speed, one finds $\omega_{\text{f}} = \dfrac{m_{\text{b}}v_{\text{i}}\ell/3}{(2/9)m_{\text{r}}\ell^2 + m_{\text{b}}(\ell/3)^2} = \dfrac{v_{\text{i}}/3}{(2/9)\ell + \ell/9} = \dfrac{v_{\text{i}}}{\ell}$.

So, the final rotational speed is $\omega_f = v/\ell$.

12.46. If you are exactly on the verge of causing the ball to slip, then both forces of static friction are being maximized, and can be written as the corresponding normal forces times the coefficient of friction. We start by writing the sum of all forces in the x and y directions as well as the sum of all torques on the ball:

$$\sum F_x = F_{\text{wb}}^n - F_{\text{lb}}^{\text{f}} = F_{\text{wb}}^n - \mu F_{\text{lb}}^n = 0 \tag{1}$$

$$\sum F_y = F_{\text{lb}}^n + \mu F_{\text{wb}}^n + F_{\text{pb}}^{\text{c}} - F_{\text{Eb}}^{\text{G}} = 0 \tag{2}$$

$$\sum \tau = F_{\text{pb}}^{\text{c}}r - \mu F_{\text{wb}}^n r - \mu F_{\text{lb}}^n r = 0 \tag{3}$$

Solving Eq. (1) for the normal force of the wall on the ball and plugging this result into equations (2) and (3) yields

$$-F^{\rm c}_{\rm pb} + F^{\rm G}_{\rm Eb} = (\mu^2+1)F^n_{\rm lb} = (\mu+1)(\mu-1)F^n_{\rm lb} \tag{4}$$

$$F^{\rm c}_{\rm pb} = (\mu^2+\mu)F^n_{\rm lb} = \mu(\mu+1)F^n_{\rm lb} \tag{5}$$

Equations (4) and (5) can each be solved for $F^n_{\rm lb}$ and the two expressions can be equated. The result is

$$\frac{-F^{\rm c}_{\rm pb} + F^{\rm G}_{\rm Eb}}{\mu^2+1} = \frac{F^{\rm c}_{\rm pb}}{\mu(\mu+1)}$$

Which can be solved for the contact force that the person exerts on the ball:

$$F^{\rm c}_{\rm pb} = \left(\frac{\mu^2+1}{\mu(\mu+1)}+1\right)^{-1} F^{\rm G}_{\rm Eb} = \left(\frac{1.25}{0.75}+1\right)^{-1}(4.5\ \text{kg})(9.8\ \text{m/s}^2) = 17\ \text{N}$$

So the maximum upward force you could exert on the ball is 17 N.

12.47. (*a*) Use conservation of angular momentum to find the final rotational speed. Initially disk A is spinning clockwise, and disk B is spinning counterclockwise, so the initial angular momentum is $L_{\rm i} = -(1/2)m_{\rm A}R_{\rm A}^2(\omega_{\rm i}/2) + (1/2)m_{\rm B}R_{\rm B}^2\omega_{\rm i}$. We can equate this to the expression for the angular momentum of both disks spinning together. But first, let us expression the mass of disk B in terms of the mass of disk A. Since they are made from the same material, their densities must be the same. We can write $m_{\rm B} = (R_{\rm B}/R_{\rm A})^2 m_{\rm A} = m_{\rm A}/4$. Now the initial momentum can be written as $L_{\rm i} = -(7/32)m_{\rm A}R_{\rm A}^2\omega_{\rm i}$, which we equate to the final angular momentum $L_{\rm i} = (17/32)m_{\rm A}R_{\rm A}^2\omega_{\rm f}$. Finally, both disks will spin with a rotational speed of $-(7/17)\ \omega$, where the negative sign indicates clockwise motion.

(*b*) The fraction of kinetic energy remaining is given by

$$\frac{K_{\rm f}}{K_{\rm i}} = \frac{K_{\rm A,f}+K_{\rm B,f}}{K_{\rm A,i}+K_{\rm B,i}} = \frac{\frac{1}{2}\left(\frac{1}{2}m_{\rm A}R_{\rm A}^2 + \frac{1}{2}m_{\rm B}R_{\rm B}^2\right)\omega_{\rm f}^2}{\frac{1}{2}\left(\frac{1}{2}m_{\rm A}R_{\rm A}^2\frac{\omega_{\rm I}^2}{4} + \frac{1}{2}m_{\rm B}R_{\rm B}^2\omega_{\rm I}^2\right)}$$

$$\frac{K_{\rm f}}{K_{\rm i}} = \frac{\left(m_{\rm A}R_{\rm A}^2 + \frac{m_{\rm A}}{4}\frac{R_{\rm A}^2}{4}\right)\left(\frac{7}{17}\right)^2}{\left(\frac{m_{\rm A}R_{\rm A}^2}{4} + \frac{m_{\rm A}}{4}\frac{R_{\rm A}^2}{4}\right)} = \frac{49}{85}$$

Thus the fractional change of kinetic energy is $-\dfrac{36}{85} \approx -0.42$.

12.48. Because the wall is smooth, we will neglect friction along its surface. As long as the ladder is stationary, we know that the sum of all forces in the x and y directions must be zero, as must the sum of all torques around any axis. Using the sum of all forces in y, one can see that the normal force exerted by the ground on the ladder must cancel the downward gravitational forces on the ladder and the person, $F^n_{\rm gl} = m_{\rm l}\,{\rm g} + m_{\rm p}g$. Choose the axis of rotation to be where the ladder meets the wall. The sum of all torques around this point can be written as

$$\sum\tau = -F^n_{gL}\ell\sin(90° - \theta) - F^s_{gL}\ell\sin(\theta) + F^{\rm G}_{\rm Ep}(\ell - d)\sin(90° - \theta) + F^{\rm G}_{\rm El}(\ell/2)\sin(90° - \theta) = 0$$

Where d is the distance the person has walked along the ladder and θ is the angle from the ground to the ladder. Writing the gravitational forces explicitly and assuming the friction is being maximized yields

$$\sum\tau = -(m_{\rm p} + m_{\rm l})\,{\rm g}\,\ell\cos(\theta) - \mu_s(m_{\rm p} + m_{\rm l})\,{\rm g}\,\ell\sin(\theta) + m_{\rm p}g(\ell - d)\cos(\theta) + m_{\rm l}g(\ell/2)\cos(\theta) = 0$$

Finally, solving for d:

$$d = \ell - \frac{(m_p + m_l)\ell(\cos(\theta) - \mu_s \sin(\theta)) - m_l \ell \cos(\theta)/2}{m_p \cos(\theta)}$$

$$= 5.0 \text{ m} - \frac{(100 \text{ kg})(5.0 \text{ m})(\cos(50°) - (0.5)\sin(50°)) - (25 \text{ kg})(2.5 \text{ m})\cos(50°)}{(75 \text{ kg})\cos(50°)}$$

$$= 3.1 \text{ m}$$

So the person can walk 3.1 m along the ladder before it begins to slip.

12.49. The object continues to roll without slipping. Nothing changes when the coefficient of friction increases. Only the maximum possible force of static friction increases, not the actual force of static friction exerted.

12.50. Buy the lightweight wheel rims. The seat post only has to be translated through the space. The pedals and wheel rims also have to be made to spin. That means you have to put translational energy and rotational energy into them. Also, since the rims have the bulk of their mass concentrated far from their axis of rotation (compared to the pedals) the rims will require more significant rotational energy. Hence, you will save on the amount of energy needed by the greatest factor by decreasing the mass of the wheel rims.

12.51. (*a*) Start by writing the sum of all forces along the incline (called x here), and the sum of all torques acting on the ball:

$$\sum F_x = mg\sin(\theta) - F^{\text{f}}_{\text{gb}} = ma_x \tag{1}$$

$$\sum \tau = F^{\text{f}}_{\text{gb}} r = I\alpha = (2/5)mr^2\alpha = (2/5)mra_x \tag{2}$$

Inserting the results from (2) into (1) and solving for a_x, one obtains $a_x = 5g\sin(\theta)/7$ or $a_x = 5(9.8 \text{ m/s}^2)\sin(30°)/7 = 3.5 \text{ m/s}^2$. The acceleration of the center of mass of the ball down the incline is 3.5 m/s^2. (*b*) Now that the acceleration is known, the force of friction can be obtained directly through eq. (2). $F^{\text{f}}_{\text{gb}} = (2/5)ma_x = (2/5)(3.0 \text{ kg})(3.5 \text{ m/s}) = 4.2 \text{ N}$.

12.52. (*a*) The steady force from the string acting on the cylinder will cause a torque and hence an angular acceleration according to $F^{\text{c}}_{\text{sc}} r = I\alpha = (1/2)mr^2\alpha$, or $\alpha = 2F^{\text{c}}_{\text{sc}}/(mr)$. That angular acceleration will give a final rotational speed according to $\omega_{\text{f}} = \omega_{\text{i}} + \alpha t = 2F^{\text{c}}_{\text{sc}} t/(mr)$. Inserting the numerical values yields $\omega_{\text{f}} = 2(20 \text{ N})(5.0 \text{ s})/((5.0 \text{ kg})(0.25 \text{ m})) = 160 \text{ s}^{-1}$. (*b*) The angle through which the cylinder has spin can be found using $\theta_{\text{f}} - \theta_{\text{i}} = \omega_{\text{i}} t + (1/2)\alpha t^2$. Using the expression for the rotational acceleration from part a yields $\theta_{\text{f}} - \theta_{\text{i}} = (1/2)2F^{\text{c}}_{\text{sc}} t^2/(mr) = (1/2)(2)(20 \text{ N})(5.0 \text{ s})^2/((5.0 \text{ kg})(0.25 \text{ m})) = 400 \text{ radians}$.

12.53. Equation (12.26) in the reading gives a formula for the friction required to keep an object spinning as it is accelerated down an incline. In this case we have two objects, a disk and a hoop which have shape factors of $1/2$ and 1, respectively. Assuming the frictional force is being maximized, we can write $\mu mg\cos(\theta) = \dfrac{mg\sin(\theta)}{1+(1/c)}$, or $\mu = \dfrac{\tan(\theta)}{1+(1/c)}$. Inserting the values of c for the two shapes yields $\mu_{\text{disk}} = \tan(\theta)/3$, and $\mu_{\text{hoop}} = \tan(\theta)/2$. The coefficient of friction must be greater than or equal to the larger of these two values to ensure that neither object slips. Hence $\mu_{\min} = \tan(30)/2 = \sqrt{3}/6$ or $\mu_{\min} \approx 0.29$.

12.54. A force from the cue stick can accelerate the center of mass of the cue ball, but it can also cause a torque and hence cause the ball to spin, provided the force acts with some lever arm above the center of mass. We proceed by writing the sum of all forces and the sum of all torques, and then by requiring that the linear and rotational accelerations be related by $a = \alpha R$ (as in rolling without slipping).

$$\sum F_x = F^{\text{c}}_{\text{sb}} = m_{\text{b}} a_x \tag{1}$$

$$\sum \tau = F^{\text{c}}_{\text{sb}} h = I\alpha = (2/5)m_{\text{b}} R^2\alpha = (2/5)m_{\text{b}} R a_x \tag{2}$$

Solving eq. (2) for the force that the stick exerts on the ball, and inserting the expression into (1) yields $2m_b Ra_x/5h = m_b a_x$ or $h = 2R/5$.

12.55. (*a*) Equation (12.25) in the reading gives an expression for the acceleration of an object that rolls down an incline without slipping. Inserting the shape factor for a cylinder yields $a_x = g\sin(\theta)/(1+(1/2)) = (9.8\text{ m/s}^2)\sin(30°)/1.5 = 3.3\text{ m/s}^2$. (*b*) Because it is rolling without slipping, $\alpha = a_x/r = (3.27\text{ m/s}^2)/(0.45\text{ m}) = 7.3\text{ s}^{-2}$. (*c*) Using kinematics, we can write $t = \sqrt{2\Delta x/a_x} = \sqrt{2(35\text{ m})/(3.27\text{ m/s}^2)} = 4.6\text{ s}$. (*d*) Using rotational kinematics, we can write $\omega_f = \omega_i + \alpha t = (7.26\text{ s}^{-2})(4.63\text{ s}) = 34\text{ s}^{-1}$.

12.56. Equation (12.25) gives an equation for the acceleration of the center of mass of an object that rolls down an incline without slipping. Note that it depends only on the shape factor of the object. Both cans of pumpkins have the same shape factor, because both are cylinders. Hence the two objects have the same acceleration, and will require the same amount of time to reach the bottom. The cans reach the ground at the same time.

12.57. Basic kinematics tells us that $t = \sqrt{2\Delta x/a_x}$, and the acceleration of each object can be found using eq. (12.25) from the reading. This yields

$$\frac{t_h}{t_s} = \frac{\sqrt{2\Delta x/a_{h,x}}}{\sqrt{2\Delta x/a_{s,x}}} = \sqrt{\frac{g\sin(\theta)/(1+c_s)}{g\sin(\theta)/(1+c_h)}} = \sqrt{\frac{1+c_h}{1+c_s}} = \sqrt{\frac{1+(2/3)}{1+(2/5)}} = \sqrt{\frac{25}{21}}$$

12.58. (*a*) Let us call the rotation of the entire system toward the left (the 50 kg block falling) the positive x direction. We will write the sum of all forces on each of the two blocks and the sum of all torques on the pulley. L and R will refer to the left and right block, respectively, whereas r will refer to the radius of the pulley.

$$\sum F_L = -T_L + m_L g = m_L a_x \tag{1}$$

$$\sum F_R = T_R - m_R g = m_R a_x \tag{2}$$

$$\sum \tau = T_L r - T_R r = I\alpha = Ia_x/r \tag{3}$$

Solving eqs. (1) and (2) for $\mathcal{T}_L$ and $\mathcal{T}_R$, respectively and plugging them into eq. (3) yields $m_L(g - a_x)r - m_R(g + a_x)r = Ia_x/r$, or solving for the acceleration: $a_x = \dfrac{(m_L - m_R)g}{(I/r^2 + m_L + m_R)} = \dfrac{(30\text{ kg})(9.8\text{ m/s}^2)}{(0.15\text{ kg}\cdot\text{m}^2)/(0.10\text{ m})^2 + (70\text{ kg})} = 3.5\text{ m/s}^2$.

(*b*) Now that the acceleration is known, the tension in the left cord is given by eq. (1) above: $\mathcal{T}_L = m_L(\text{g} - \text{a}) = (50\text{ kg})(9.8\text{ m/s}^2 - 3.46\text{ m/s}^2) = 3.2\times 10^2\text{ N}$.

(*c*) Using eq. (2) yields $\mathcal{T}_R = m_R(g + a) = (30\text{ kg})(9.8\text{ m/s}^2 + 3.46\text{ m/s}^2) = 2.7\times 10^2\text{ N}$.

12.59. (*a*) Let us say that motion of the ball dropping (or the block moving to the right) is in the positive x direction. Neglecting friction, the tension in the horizontal cord is simply $\mathcal{T}_h = ma_x$. The tension in the vertical string is $\mathcal{T}_v = m(g - a_x)$. We now write the sum of all torques and solve for a_x:

$$\sum \tau = \mathcal{T}_v r - \mathcal{T}_h r = m(g - a_x)r - ma_x = I\alpha = (1/2)3mr^2\alpha = 3mra_x/2$$
$$mgr = (3mr/2 + 2mr)a_x$$
$$a_x = 2g/7$$

(*b*) The process would be the same as above, except that the tension in the vertical cord would simply be mg. Solving in the same way:

$$\sum \tau = \mathcal{T}_v r - \mathcal{T}_h r = mgr - ma_x = 3mra_x/2$$
$$mgr = (3mr/2 + mr)a_x$$
$$a_x = 2g/5$$

12.60. (*a*) $I_{\text{cylinder}} = (1/2)mr^2 = (1/2)(0.215\text{ kg})(0.0319\text{ m})^2 = 1.09\times10^{-4}\text{ kg}\cdot\text{m}^2$. (*b*) The time for the object to reach the bottom is related to its center of mass acceleration by $a_x = 2\Delta x/t^2$. Equation (12.25) from the reading gives an expression for the center of mass acceleration of an object that rolls without slipping down an incline. Equating the two expressions yields $g\sin(\theta)/(1+c) = 2\Delta x/t^2$. Solving this equation for c and recalling that $c = I/mr^2$ yields

$$I = cmr^2 = \left(\frac{gt^2\sin(\theta)}{2\Delta x} - 1\right)mr^2$$

$$I = \left(\frac{(9.80\text{ m/s}^2)(1.40\text{ s})^2\sin(25°)}{2(3.00\text{ m})} - 1\right)(0.215\text{ kg})(0.0319\text{ m})^2 = 7.72\times10^{-5}\text{ kg}\cdot\text{m}^2$$

(*c*) The can is not completely full. There is a cavity of empty space. When the can is at rest, this cavity will be at the top which will be radially far from the center. This means that not as much mass is concentrated far from the center as was estimated. Hence the rotational inertia is less than estimated. Note that if the can were to spin fast enough, the cavity could end up being at the center and the rotational inertia could actually increase.

12.61. The initial energy is purely gravitational potential energy, and the final energy is in the form of rotational and translational kinetic energy. Setting these energies equal yields

$$mgh_{\text{i}} = (1/2)mv_{\text{f}}^2 + (1/2)I\omega_{\text{f}}^2 = (3/4)mv_{\text{f}}^2$$

$$v_{\text{f}} = \sqrt{4gh_{\text{i}}/3} = \sqrt{4gL\sin(\theta)/3} = \sqrt{4(9.8\text{ m/s}^2)(1.0\text{ m})\sin(15°)/3} = 1.8\text{ m/s}$$

$$KE_{\text{rot}} = (1/2)I\omega_{\text{f}}^2 = (1/4)(0.2\text{ kg})(1.84\text{ m/s})^2 = 1.7\times10^{-1}\text{ J}$$

12.62. $W = \Delta KE = (1/2)mv_{\text{f}}^2 + (1/2)I\omega_{\text{f}}^2 = (3/4)mr^2\omega^2 = (3/4)(50\text{ kg})(0.10\text{ m})^2(20\times2\pi\text{ rad/s})^2$

$W = 5.9\text{ kJ}$

12.63. This problem can be solved using conservation of energy, but first the rotational inertia of the ring is needed. We can accomplish this by subtracting a solid disk with the smaller radius from a solid disk with the larger radius: $I_{\text{ring}} = I_{\text{large disk}} - I_{\text{small disk}} = (1/2)(m_{\text{l}}r_{\text{l}}^2 - m_{\text{s}}r_{\text{s}}^2)$. To find the appropriate masses, we use the density of the ring $\rho = m/(h\pi(r_{\text{l}}^2 - r_{\text{s}}^2))$. The final expression for the rotational inertia of the ring is $I_{\text{ring}} = m(r_{\text{l}}^2 + r_{\text{s}}^2)/2$. Now we use conservation of energy:

$$mgh_{\text{f}} = (1/2)mv_{\text{i}}^2 + m(r_{\text{l}}^2 + r_{\text{s}}^2)\omega_{\text{f}}^2/4$$

$$h_{\text{f}} = \frac{v_{\text{i}}^2}{g}\left(\frac{1}{2} + \frac{r_{\text{l}}^2 + r_{\text{s}}^2}{4r_{\text{l}}^2}\right) = \frac{(2.8\text{ m/s})^2}{9.8\text{ m/s}^2}\left(\frac{1}{2} + \frac{(0.08\text{ m})^2 + (0.06\text{ m})^2}{4(0.08\text{ m})^2}\right)$$

$$h_{\text{f}} = 0.713\text{ m}$$

The question asks for the distance along the incline. This distance is related to the height of the ring by

$$d = h_{\text{f}}/\sin(30°) = 1.4\text{ m}$$

12.64. $P = W/\Delta t = \tau\Delta\theta/\Delta t = \tau\omega = (380\text{ N}\cdot\text{m})(3200\times2\pi\text{ rad/min})(1\text{ min}/60\text{ s}) = 0.13\text{ MW}$.

12.65. The final rotational speed of the disk is given by $\omega_{\text{f}} = \omega_{\text{i}} + \alpha t = (4.5\text{ rev/s}) + (0.30\text{ rev/s}^2)(5.0\text{ s}) = 6.0\text{ rev/s} = 37.7\text{ rad/s}$. We can now calculate the difference between final and initial energies:

$$W = (1/2)\text{I}(\omega_{\text{f}}^2 - \omega_{\text{i}}^2) = (1/2)(1/2)(680\text{ kg})(1.2\text{ m})^2((37.7\text{ rad/s})^2 - (28.3\text{ rad/s})^2) = 0.15\text{ MJ}$$

12.66. (*a*) $\sum\tau = mg\ell/2 = I\alpha = (1/3)\text{m}\ell^2$, such that $\alpha = \dfrac{3g}{2\ell}$. (*b*) The penny will accelerate downward at exactly g, the end of the rod farthest from the axis will have an initial acceleration given by $a = \alpha\ell = 3g/2$. Hence the rod will accelerate faster and will drop away from the penny. They will not stay in contact.

12.67. We start by writing the sum of all forces on the block directed down the incline (x direction), and the sum of all torques on the disk:

$$\sum F_{b,x} = -T + m_b g\sin(\theta) = m_b a_x = m_b \alpha r \tag{1}$$

$$\sum \tau = Tr = I\alpha = (1/2)m_d r^2\alpha \tag{2}$$

Solving eq. (2) for tension and substituting the expression into eq. (1) yields $m_b g\sin(\theta) = (m_b r + m_d r/2)\alpha,$ or $\alpha = m_b g\sin(\theta)/(m_b r + m_d r/2) = (2.0 \text{ kg})(9.8 \text{ m/s}^2)\sin(37°)/((2.0 \text{ kg})(0.5 \text{ m})) = 5.9 \text{ s}^{-2}.$

12.68. (*a*) $W = \tau\Delta\theta = (50 \text{ N})(0.25 \text{ m})(1000 \text{ rad}) = 1.3\times10^4 \text{ J}.$ (*b*) The work is related to the final rotational speed by $W = \Delta KE_{\text{rot}} = (1/2)I\omega_f^2 = (1/2)(mr^2)\omega_f^2.$ Solving for the final rotational speed gives us $\omega_f = \sqrt{2W/mr^2} = \sqrt{2(1.25\times10^4 \text{ J})/((5.0 \text{ kg})(0.25 \text{ m})^2)} = 2.8\times10^2 \text{ s}^{-1}.$

12.69. Start with the conservation of energy and solve for the final rotational speed:

$$mgh_i = (1/2)mv_f^2 + (1/2)(2mr^2/5)\omega_f^2$$

$$gR(1-\cos(\theta)) = \omega_f^2 r^2(7/10)$$

$$\omega_f = \sqrt{10gR(1-\cos(\theta))/(7r^2)}$$

$$\omega_f = \sqrt{10(9.8 \text{ m/s}^2)(0.10 \text{ m})(1-\cos(30°))/(7(0.01 \text{ m})^2)}$$

$$\omega_f = 43 \text{ s}^{-1}$$

12.70. Note first that the force of friction needed in order for the disk to roll without slipping (given by eq. (12.28)) is $mg\sin(\theta)/(c^{-1}+1),$ which in the case of a disk is $mg\sin(\theta)/3.$ The maximum force of static friction that the incline can actually exert on the disk is $\mu mg\cos(\theta).$ Since $\tan(62°)/3 = 0.63 > \mu,$ one can see that it is not possible for this disk to roll down the incline without slipping. Instead, the disk will slip down the incline. But the force of friction being exerted on the disk will always cause a torque on the disk.

$$\sum \tau = \mu mg\cos(\theta)r = I\alpha = (1/2)mr^2\alpha$$

$$\alpha = 2\mu g\cos(\theta)/r = 2(0.5)(9.8 \text{ m/s}^2)\cos(62°)/(0.050 \text{ m})$$

$$\alpha = 92.0 \text{ s}^{-2} \tag{1}$$

In order to find the time taken for the disk to slip, consider the linear motion along the incline

$$\sum F_x = mg\sin(\theta) - \mu mg\cos(\theta) = ma_x$$

$$a_x = g(\sin(\theta) - \mu\cos(\theta)) = (9.8 \text{ m/s}^2)(\sin(62°) - (0.5)\cos(62°))$$

$$a_x = 6.35 \text{ m/s}^2$$

The time required to travel 1.5 m at the above acceleration is given by

$$t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2(1.5 \text{ m})}{6.35 \text{ m/s}^2}} = 0.687 \text{ s} \tag{2}$$

Finally, using the rotational acceleration in eq. (1) and the time found in eq. (2), the final rotational speed is

$$\omega_f = \omega_i + \alpha t = (92.0 \text{ s}^{-2})(0.687 \text{ s}) = 63 \text{ s}^{-1}$$

12.71. (*a*) $W = \tau\Delta\theta = Fr\Delta\theta = F\Delta x = (5.0 \text{ N})(1.0 \text{ m}) = 5.0 \text{ J}$ (*b*) $W = KE_f = 5.0 \text{ J}$ (*c*) $KE_f = (1/2)I\omega_f^2 = (1/2)(7/20)mr^2\omega_f^2,$ so $\omega_f = \sqrt{40KE_f/(7mr^2)}$

$$\omega_f = \sqrt{40(5.0 \text{ J})/(7(0.125 \text{ kg})(0.020 \text{ m})^2)} = 7.6\times10^2 \text{ s}^{-1}$$

12.72. $W = \tau\Delta\theta = mgh$ or $\Delta\theta = mgh/W = (44.0 \text{ kg})(9.8 \text{ m/s}^2)(2.5 \text{ m})/(12 \text{ N}\cdot\text{m}).$ Hence $\Delta\theta = 89.8 \text{ rad} = 14 \text{ rev}.$ You have to turn the handle 14 times.

12.73. (*a*) $L_i = L_f$ such that $mv_i r_i = mv_f r_f$, or $v_f = v_i r_i / r_f = (1.5\text{ m/s})(0.45\text{ m})/(0.20\text{ m}) = 3.4\text{ m/s}$ (*b*) $\mathcal{T} = mv^2/r = (0.050\text{ kg})(3.38\text{ m/s})^2/(0.20\text{ m}) = 2.8\text{ N}$ (*c*) $W = KE_f - KE_i$ $W = (1/2)m(v_f^2 - v_i^2) = (1/2)(0.050\text{ kg})((3.38\text{ m/s})^2 - (1.5\text{ m/s})^2) = 2.3\times 10^{-1}\text{ J}$.

12.74. (*a*) In order to stop the ball without causing any slipping, the torque will have to be sufficient to stop both rotational and translational motion while preserving $\omega r = v$. One way to solve this is to require that the work done by the torque reduce all kinetic energy to zero:

$$W = -\tau\Delta\vartheta = -K_{\text{trans}} - K_{\text{rot}}$$

$$\tau = \frac{1}{\Delta\vartheta}\left[\frac{1}{2}mv_i^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega_i^2\right] = \frac{7mv_i^2}{10\Delta\vartheta}$$

$$\tau = \frac{7(30\text{ kg})(2.0\text{ m/s})^2}{10(5\cdot 2\pi)} = 2.7\text{ N}\cdot\text{m}$$

(*b*) Given the torque, it is trivial to find the force acting on the outer edge of the ball, tangential to its surface: $F = \dfrac{\tau}{R} = \dfrac{(2.67\text{ N}\cdot\text{m})}{(0.12\text{ m})} = 22\text{ N}$.

12.75. The marble flies off the globe when the normal force from the globe on the marble goes to zero.

$$\sum F_r = mg\cos(\theta) - F_{\text{gm}}^n = mv^2/R$$

$$\cos(\theta) = v^2/Rg \qquad (1)$$

In order to determine the final speed of the marble when it flies off, use conservation of energy

$$mgh_i = (1/2)mv_f^2 + (1/2)I\omega_f^2$$

$$mgR(1-\cos(\theta)) = \frac{m}{2}\left(1+\frac{2}{5}\right)v_f^2$$

$$v_f^2 = 10gR(1-\cos(\theta))/7 \qquad (2)$$

Inserting eq.(2) into eq.(1) yields $\cos(\theta) = 10/17$ or $\theta = 54°$.

12.76. (*a*) The friction will cause a torque on the ball as it slides across the ground. Since it is slipping, we have no information about the angle through which the ball rolls, as its translational motion slows. However, we know that the slowing of the translational motion and the rotational acceleration must happen over the same period of time. $\tau t = \Delta L \Rightarrow \mu mgrt = (2/5)mr^2\Delta\omega$ such that the rotational speed of the ball after a time t is given by $\omega_f = 5\mu gt/2r$. Similarly, the translational speed of the ball will be given by $v_f = v_i + at = v_i - \mu gt$. When $v_f = \omega_f r$, the ball will be rolling without slipping. This yields $v_i - \mu gt = 5\mu gt/2 \Rightarrow t = v_i/(\mu g + 5\mu g/2)$, and so the ball is rolling without slipping after $t = 1.5\text{ s}$. (*b*) Plugging the time from part (*a*) into the above expression yields $v_f = 10\text{ m/s} - (0.2)(9.8\text{ m/s}^2)(1.46\text{ s}) = 7.1\text{ m/s}$.

12.77. (*a*) Because $L = mrv_i$, treat the cube as a particle located at the cube's center, and insert $r = d/2$. This yields $L = mdv_i/2$. (*b*) At the instant of collision, the point of application of the contact force exerted by the lip on the cube is at the axis of rotation. The lever arm distance for this force is zero and thus this force cannot cause a torque about the axis. Hence there is no external torque on the cube and its angular momentum must be constant. (*c*) We know $\tau = I\alpha$, and the torque comes from the gravitational force. The rotational inertia can be found from the parallel axis theorem: $I_{\text{edge}} = I_{\text{cm}} + m\ell^2$, where ℓ is the distance by which the axis is shifted ($d/\sqrt{2}$ in this case). Hence $I_{\text{edge}} = md^2/6 + md^2/2 = 2md^2/3$. Inserting this expression yields

$$\tau = Fr\sin(\theta) = mg\frac{d}{\sqrt{2}}\frac{\sqrt{2}}{2} = \frac{2md^2}{3}\alpha$$

$$\alpha = \frac{3g}{4d}$$

(*d*) After the collision, energy is conserved as the box tips. First, using the expression for the angular momentum $L = mvd/2 = I\omega$, one can obtain an expression for the initial rotational speed $\omega = mvd/2I$. Now, using energy conservation and assuming that the block has exactly enough energy to stand up on its edge, but no more, one obtains

$$mgh_{\text{f}} = KE_{\text{rot,i}}$$

$$\frac{mgd}{\sqrt{2}} = \frac{1}{2}I\omega_{\text{i}}^2 = \frac{1}{2}I\left(\frac{mvd}{2I}\right)^2 = \frac{3mv^2}{16}$$

$$v = \sqrt{\frac{16gd}{3\sqrt{2}}}$$

12.78. (*a*) and (*c*) are sensible. (*b*) is not because you cannot perform a vector product with a scalar.

12.79. To change your angular momentum, there must be a torque on you. Otherwise your body continues moving in a straight line and you fall off the bike.

12.80. $\vec{A}\cdot\vec{B} = AB\cos(\theta)$ and $|\vec{A}\times\vec{B}| = AB\sin(\theta)$. Equating the two yields $\theta = 45°$.

12.81. For all parts use $\tau = Fr\sin(\theta)$. (*a*) $\tau = (150\text{ N})(0.20\text{ m})\sin(90°) = 30\text{ N}\cdot\text{m}$ (*b*) $\tau = (150\text{ N})(0.20\text{ m})\sin(60°) = 26\text{ N}\cdot\text{m}$ (*c*) $\tau = (150\text{ N})(0.20\text{ m})\sin(45°) = 21\text{ N}\cdot\text{m}$ (*d*) $\tau = (150\text{ N})(0.20\text{ m})\sin(30°) = 15\text{ N}\cdot\text{m}$ (*e*) $\tau = (150\text{ N})(0.20\text{ m})\sin(0°) = 0$. (*f*) We know the expression for the torque at a given angle. To find the total torque in half a revolution we integrate

$$\tau_{\text{av}} = \frac{1}{\pi}\int_0^{\pi} Fr\sin(\theta)\,\text{d}\theta = \frac{(150\text{ N})(0.20\text{ m})}{\pi}[-\cos(\theta)]_0^{\pi}$$

$$\tau_{\text{av}} = 19\text{ N}\cdot\text{m}$$

12.82. You want to exert a force that is perpendicular to the radial vector. In this case the radial vector is directed $\tan^{-1}(1/\sqrt{3}) = 30°$ above the horizontal. Hence the force should be applied $30°$ from vertical. Note that in the figure, θ is shown to the left of the vertical dashed line. But the maximum torque is achieved when the angle is $30°$ to the right of the vertical line.

12.83. Writing $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$, you can compute the vector product one component at a time, noting that for any two components along the same axis the vector product is always zero. Thus you have $\vec{A}\times\vec{B} = A_xB_y\hat{k} + A_yB_x(-\hat{k}) = (A_xB_y - A_yB_x)\hat{k}$.

12.84. (*a*) $\sum\tau = m_{\text{L}}g\ell/2\sin(\theta) - m_{\text{R}}g\ell/2\sin(\theta) = (-1.0\text{ kg})(9.8\text{ m/s}^2)(0.75\text{ m})\sin(60°) = -6.4\text{ N}\cdot\text{m}$ where the negative sign indicates clockwise torque. (*b*) $\alpha = \tau/I$, but the rotational inertia is the sum of the rod and the blocks together: $I = (1/12)m_{\text{rod}}\ell^2 + (m_{\text{L}} + m_{\text{R}})(\ell/2)^2 = 2.25\text{ kg}\cdot\text{m}^2$. Inserting this value yields $\alpha = (-6.37\text{ N}\cdot\text{m})/(2.25\text{ kg}\cdot\text{m}^2) = -2.8\text{ s}^{-2}$.

12.85. (*a*) In order to know how the friction slows the disk, we must first know the rotational speed of the disk at the moment the motor is shut off. Using $\Delta\theta = (1/2)(\omega_{\text{f}} + \omega_{\text{i}})t$ yields $\omega_{\text{f}} = 2\Delta\theta/t = 2(23.9\times 2\pi)/(5.0\text{ s}) = 60.1\text{ s}^{-1}$. The force of friction causes a torque that changes this rotational speed to zero in 12 s, meaning $|\alpha| = (60.1\text{ s}^{-1})/(12\text{ s}) = 5.01\text{ s}^{-2}$. Now the torque from friction is given by $\tau^f = I\alpha = (4.0\text{ kg}\cdot\text{m}^2)(5.01\text{ s}^{-2}) = 20\text{ N}\cdot\text{m}$ in the direction opposite the direction of motion. (*b*) As the motor sped up the disk, $\sum\tau = \tau_{\text{m}} - \tau^f = I\alpha = I(\omega_{\text{f}}/\text{t})$ or $\tau_{\text{m}} = I(\omega_{\text{f}}/\text{t}) + \tau^f = (4.0\text{ kg}\cdot\text{m}^2)(60.1\text{ s}^{-1})/(5.0\text{ s}) + 20.0\text{ N}\cdot\text{m} = 68\text{ N}\cdot\text{m}$ in the direction of motion.

12.86. (*a*) clockwise (*b*) The nose of the plane would veer to the right. The angular momentum of the propellers starts out horizontal. As the plane climbs, this angular momentum vector is changed to have a vertical component. By conservation of angular momentum, this must be accompanied by some part of the object gaining an angular momentum with a downward component. For the plane to have a downward angular momentum, it must tend to spin clockwise when viewed from above. This corresponds to the nose of the plane veering to the right.

12.87. (*a*) $\tau = F^f_{\text{a,w}} r = I\alpha = (1/2)mr^2(\omega_\text{i}/t)$ or $t = (1/2)mr^2\omega_\text{i}/(\mu F^n_{\text{a,w}}\,\text{r})$. Plugging in numbers yields $t = (1/2)(40\text{ kg})(0.10\text{ m})(3.3\times 2\pi\text{ s}^{-1})/(0.35(40\text{ N})) = 3.0\text{ s}$. (*b*) Using $\Delta\theta = (1/2)(\omega_\text{i} + \omega_\text{f})t$, and the time from (*a*), one finds $\Delta\theta = (1/2)(3.3\text{ rev/s} + 0)(2.96\text{ s}) = 4.9\text{ rev}$.

12.88. (*a*) The wheel has angular momentum that is directed horizontally to the right. The gravitational force would serve to rotate the wheel about the point where it is fixed to the string and cause this angular momentum to point entirely downward. But the conservation of momentum demands that this change in momentum be accompanied by an upward angular momentum. The wheel will have a net upward angular momentum around the string if the wheel processes counterclockwise around the string (as viewed from above). (*b*) As time passes, the wheel tilts farther. This means its angular momentum is directed increasingly downward, and there must be an increasing upward angular momentum from the precession of the wheel. Also, the mass of the wheel is located closer and closer to the axis of the string as it drops, meaning that the angular momentum can only be increased by increasing the speed of precession.

12.89. $\tau t = \Delta L = -I\omega_\text{i}$, or $|\tau| = mr^2\omega_\text{i}/(2t) = (1.0\text{ kg})(0.10\text{ m})^2(14{,}200\times 2\pi/60\text{ s})/(2(20\text{ s})) = 0.37\text{ N}\cdot\text{m}$

12.90. (*a*) The equilibrium is disrupted and the seesaw will spin. Whichever side moves downward will rotate such that the force of gravity on that side is closer to perpendicular to the seesaw. Hence that side will exert a greater downward torque and the system will rotate in that direction. (*b*) The system returns to equilibrium. The side that moves downward will experience less downward torque due to gravity, and the system will rotate back to equilibrium.

12.91. $\hat{i}\times\left\{\hat{i}\times[\hat{i}\times(\hat{i}\times\hat{j})]\right\} = \hat{i}\times\left\{\hat{i}\times[\hat{i}\times\hat{k}]\right\} = \hat{i}\times\left\{\hat{i}\times -\hat{j}\right\} = \hat{i}\times -\hat{k} = \hat{j}$

12.92. Walk along the edge of the raft in the direction opposite the direction you wish to rotate the raft. You will traverse the entire circumference of the raft, since you will walk 180 degrees in one direction and the raft will rotate 180 degrees in the other. This assumes that frictional/adhesive forces between the raft and the water are negligible.

12.93. (*a*) The engine is connected to the rear wheels, and so the force that accelerates the car forward is exerted at the bottom of the rear wheels. Because the line of action of this force goes below the center of mass of the car, the force causes a torque on the car that tends to lift the front wheels off the road. (*b*) With front-wheel drive, the force that accelerates the car forward is exerted at the bottom of the front wheels. The line of action of this force also goes below the center of mass of the car causing a torque on the car, but now the rear wheels are pushed into the road.

12.94. The angular momentum would be conserved. We will write down the angular momentum of Earth and the people on its surface separately.

$$L_{\text{i,E}} + L_{\text{i,p}} = L_{\text{f,E}} + L_{\text{f,p}}$$

$$\frac{2}{5}m_\text{E}R_\text{E}^2\omega_\text{i} + m_\text{p}R_\text{E}^2\omega_\text{i} = \frac{2}{5}m_\text{E}R_\text{E}^2\omega_\text{f} + m_\text{p}R_\text{E}^2(\omega_\text{f} + v/R_\text{E})$$

$$\frac{\omega_\text{i} - \omega_\text{f}}{\omega_\text{i}} = \frac{5m_\text{p}v}{2m_\text{E}R_\text{E}\omega_\text{i}}$$

We may approximate a brisk human walking speed (relative to the surface of Earth) as 2 m/s. A of this writing, there are approximately seven billion humans on Earth, and we will assume an average mass of 70 kg. Inserting these values yields

$$\frac{\omega_{\mathrm{i}}-\omega_{\mathrm{f}}}{\omega_{\mathrm{i}}}=\frac{5(70\ \mathrm{kg})(7\times10^{9})(2\ \mathrm{m/s})(24)(3600)}{2(5.98\times10^{24}\ \mathrm{kg})(6.37\times10^{6}\ \mathrm{m})2\pi}=8.4\times10^{-16}$$

But there are several approximations here. Anything close to $\Delta\omega/\omega=10^{-15}$ is acceptable.

12.95. (*a*) The sum of all torques must equal zero, meaning $\sum\tau=\mathcal{T}\ell\sin(40°)-mg\ell\sin(90°)=0$. Solving for the tensile forceyields $\mathcal{T}=mg\sin(90°)/\sin(40°)$, or $\mathcal{T}=(51\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(1)/\sin(40°)=7.8\times10^{2}\ \mathrm{N}$. (*b*) The *y* components of forces cancel without any help from the pin (because the sum of all torques is zero, and the *y* component of $\mathcal{T}$ is perpendicular to the rod). But the sum of all forces along the *x* axis yields $\sum F_{\mathrm{rod},x}=F_{\mathrm{pin},x}-\mathcal{T}\cos(40°)=0$, meaning that $F_{\mathrm{pin}}=F_{\mathrm{pin},x}=\mathcal{T}\cos(40°)=6.0\times10^{2}\ \mathrm{N}$. (*c*) Rewriting the sum of all torques gives us $\sum\tau=\mathcal{T}\ell\sin(40°)-m_{\mathrm{b}}g\ell\sin(90°)-m_{\mathrm{r}}g\ell\sin(90°)/2=0$, and solving again for tension yields $\mathcal{T}=(51\ \mathrm{kg}+10.2\ \mathrm{kg}/2)(9.8\ \mathrm{m/s^2})(1)/\sin(40°)=8.6\times10^{2}\ \mathrm{N}$. The force from the pin now has an *x* and a *y* component. From the sum of all forces in each direction one obtains

$$F_{\mathrm{pin},x}=\mathcal{T}\cos(40°)=(855\ \mathrm{N})\cos(40°)=655\ \mathrm{N}$$

and

$$F_{\mathrm{pin},y}=(m_{\mathrm{r}}+m_{\mathrm{b}})g-\mathcal{T}\sin(40°)=(61.2\ \mathrm{kg})(9.8\ \mathrm{m/s^2})-(855\ \mathrm{N})\sin(40°)=50\ \mathrm{N}$$

such that the final magnitude of the reaction force from the pin is $F_{\mathrm{pin}}=6.6\times10^{2}\ \mathrm{N}$.

12.96. Setting $L_{\mathrm{i}}=L_{\mathrm{m,i}}+L_{\mathrm{p,i}}=L_{\mathrm{f}}=L_{\mathrm{m,f}}=L_{\mathrm{p,f}}$ yields $I_{\mathrm{m}}\omega_{\mathrm{m,i}}+m_{\mathrm{m}}r^{2}\omega_{\mathrm{p,i}}=I_{\mathrm{m}}\omega_{\mathrm{m,f}}+m_{\mathrm{m}}r^{2}\omega_{\mathrm{p,f}}$, where all speeds are measured relative to the ground. Solving for the final rotational speed of the child

$$\begin{aligned}\omega_{\mathrm{p,f}}&=(I/m_{\mathrm{p}}r^{2})(\omega_{\mathrm{m,i}}-\omega_{\mathrm{m,f}})+\omega_{\mathrm{m,i}}\\&=((500\ \mathrm{kg\cdot m^2})/(35\ \mathrm{kg})(2.0\ \mathrm{m})^{2})(-0.05\times2\pi\ \mathrm{s^{-1}})+(0.2\times2\pi\ \mathrm{s^{-1}})=0.13\ \mathrm{s^{-1}}\end{aligned}$$

This means that the rotational speed relative to the merry-go-round is $\omega_{\mathrm{rel,p}}=\omega_{\mathrm{p,f}}+\omega_{\mathrm{m,f}}=0.1346\ \mathrm{s^{-1}}+1.571\ \mathrm{s^{-1}}=1.71\ \mathrm{s^{-1}}$. Using $v=\omega r$ yields $v=(1.71\ \mathrm{s^{-1}})(2.0\ \mathrm{m})=3.4\ \mathrm{m/s}$, and the child is walking opposite the direction of the merry-go-round.

12.97. Using conservation of angular momentum $L_{\mathrm{i}}=L_{\mathrm{f}}\Rightarrow(2/5)m\mathrm{r}_{\mathrm{i}}^{2}\,\omega_{\mathrm{i}}=(2/5)m\mathrm{r}_{\mathrm{f}}^{2}\,\omega_{\mathrm{f}}$, and inserting $\omega=2\pi/T$ and solving for T_{f} yields $\omega_{\mathrm{f}}=(r_{\mathrm{i}}^{2}/r_{\mathrm{f}}^{2})\omega_{\mathrm{i}}\Rightarrow T_{\mathrm{f}}=(r_{\mathrm{f}}^{2}/r_{\mathrm{i}}^{2})T_{\mathrm{i}}$. Inserting the given values yields $T_{\mathrm{f}}=((2.0\times10^{4}\ \mathrm{m})^{2}/(13\times10^{8}\ \mathrm{m})^{2})(5\ \mathrm{days})=1.0\times10^{-4}\ \mathrm{s}$. Hence the star spins at a rate of 1 rev/0.10 ms, or 1.0×10^{4} times each second.

12.98. (*a*) If the system is in rotational equilibrium, then it is not undergoing a rotational acceleration around any axis. We are free to write the sum of all torques around any axes and set the sum equal to zero. Because there are two unknowns (the force and the position at which it is exerted), we must write two such equations. We choose our first axis to be at the left edge of the rod, and the second axis to be at the center of the rod.

$$\sum\tau_{1}=-m_{\mathrm{r}}g\ell/2-m_{\mathrm{Rb}}g\ell+Fx=0 \quad (1)$$

$$\sum\tau_{2}=m_{\mathrm{Lb}}g\ell/2-m_{\mathrm{Rb}}g\ell/2+F(x-1)=0 \quad (2)$$

Here x is the distance from the left end at which the external force F is exerted. Solving eq. (1) for Fx yields $Fx=(m_{\mathrm{Rb}}+m_{\mathrm{r}}/2)g\ell$, and inserting this into eq. (2) yields $F=(m_{\mathrm{Lb}}+m_{\mathrm{Rb}}+m_{\mathrm{r}})g\ell/2=(14\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(1.0\ \mathrm{m})=1.4\times10^{2}\ \mathrm{N}$. (*b*) Going back and inserting the answer into the expression for Fx and solving for x yields $x=(m_{\mathrm{Rb}}+m_{\mathrm{r}}/2)g\ell/F=(5\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(2.0\ \mathrm{m})/(137\ \mathrm{N})=0.71\ \mathrm{m}$ from the left end.

12.99. The torque is the gravitational force exerted on the cyclist times the lever arm distance, which is the length of the metal rod attaching each pedal to the bicycle. For a typical human (m = 75 kg) and bicycle (lever arm distance 0.3 m), this yields a torque of $10^2\ \text{N}\cdot\text{m}$.

12.100. (*a*) Equation (12.26) in the reading gives a formula for the friction needed to keep an object rolling down an incline without slipping. Using $I = (2/5)mr^2$ in eq. (12.26) yields

$$F^s_{\text{gb}} = \frac{mg\sin(\theta)}{5/2+1} = 2mg\sin(\theta)/7$$

(*b*) If we assume that the force of static friction is being pushed to its limit, we can set the expression from part (*a*) equal to the coefficient of friction times the normal force, and solve for the coefficient.

$$F^s_{\text{gb}} = 2mg\sin(\theta)/7 = \mu mg\cos(\theta)$$

$$\mu = 2\tan(\theta)/7$$

12.101. (*a*) Start by writing the sum of all forces on the lawn roller in the horizontal direction (call it x) and the sum of all torques on the lawn roller.

$$\sum F_x = 2F_\text{p} - F^s_{\text{gr}} = m_\text{r}a_x \qquad (1)$$

$$\sum \tau = F^s_{\text{gr}}r = I\alpha = (1/2)m_\text{r}r^2(a_x/r) \qquad (2)$$

Solving eq. (2) for the force of friction and inserting it into eq. (1) yields $a = 4F_\text{p}/3m_\text{r} = 4000\ \text{N}/(3(250\ \text{kg})) = 5.3\ \text{m/s}^2$. (*b*) Plugging in the acceleration from part (*a*) into eq. (2) yields an expression for the force of friction, which we can set equal to μmg, assuming that the static force of friction is maximized. This yields $F^s_{\text{gr}} = m_\text{r}a_x/2 = \mu m_\text{r}g$ or $\mu = a_x/2g = (5.33\ \text{m/s}^2)/(2(9.8\ \text{m/s}^2)) = 0.27$.

12.102. We want the ratio of $v_{\text{cm}}/\omega r$. Equating the two types of energy yields

$$\frac{1}{2}I\omega^2 = \frac{1}{2}mv^2_{\text{cm}}$$

$$\frac{2}{5}r^2\omega^2 = v^2_{\text{cm}}$$

$$\frac{v_{\text{cm}}}{r\omega} = \sqrt{\frac{2}{5}}$$

12.103. First, use energy conservation to find the speed of the cylinder at the bottom of the ramp, then use kinematics to find the distances.

$$mgh_\text{i} = \frac{1}{2}mv^2_\text{f} + \frac{1}{2}I\omega^2_\text{f}$$

$$2gh_\text{i} = \left(1+\frac{1}{2}\right)v^2_\text{f}$$

$$v_\text{f} = \sqrt{\frac{4gh_\text{i}}{3}} = \sqrt{\frac{4(9.8\ \text{m/s}^2)(3.0\ \text{m})}{3}} = 6.26\ \text{m/s}$$

$$v_{x,\text{f}} = 5.42\ \text{m/s}$$

$$v_{y,\text{f}} = -3.13\ \text{m/s}$$

Now for the freefall portion, we treat the above speeds at the bottom of the incline as the initial speeds. Solve first for the amount of time required for the cylinder to reach the ground, then use that time to find the distance in the x direction.

$$\Delta y = v_{y,\text{i}}t + \frac{1}{2}a_yt^2$$

$$-5.0\ \text{m} = (-3.13\ \text{m/s})t + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$$

$$\Rightarrow t = 0.74\ \text{s}$$

We have used the quadratic equation in the last step. Now $\Delta x = v_{x,\text{i}}t + (1/2)a_x t^2$, and since there is no horizontal acceleration $\Delta x = v_{x,\text{i}}t = (5.42\text{ m/s})(0.74\text{ s}) = 4.0\text{ m}$.

12.104. The child and the equipment stick together, so the collision is definitely inelastic. The coefficient of restitution was defined in terms of linear motion, but one could easily say that the collision between the child and a piece of the edge of the playground equipment occurred in a single line.

12.105. (*a*) $Fr = \tau = I\alpha \Rightarrow F = I\alpha/r = (15\text{ kg}\cdot\text{m}^2)(2\pi\text{ s}^{-1})/(0.30\text{ m}) = 3.1\times10^2\text{ N}$ (*b*) Your force would have to cause a torque that overcomes the torque coming from the gravitational force, and still cause an angular acceleration. Your force must be larger. Numerically,

$$\begin{aligned}\sum\tau &= F^c_{\text{pp}}r_\text{p} - F^\text{G}_{\text{Ep}}r_\text{E} = I\alpha \Rightarrow F^c_{\text{pp}} = (I\alpha + m_\text{p}gr_\text{E})/r_\text{p}\\ &= ((15\text{ kg}\cdot\text{m}^2)(2\pi\text{ s}^{-1}) + (60\text{ kg})(9.8\text{ m/s}^2)(0.12\text{ m}))/(0.30\text{ m}) = 5.5\times10^2\text{ N}\end{aligned}$$

12.106. From the density of steel one can find a mass of such a flywheel: $m = \rho V = \rho\pi r^2 w$. The density of steel varies slightly by alloy, but is around $7{,}900\text{ kg/m}^3$. You might choose to maximize the radius of the flywheel so as to maximize the energy that can be stored in its rotation. Inserting that value gives $m = (7900\text{ kg/m}^3)\pi(0.20\text{ m})^2(0.050\text{ m}) = 49.6\text{ kg}$. Assume each side of the braking surface exerts a force F^c_{bd} on the outer edge of the disk. The force that will actually give a torque will be the force of friction. Assume there is a very high coefficient of static friction between the shoe and the disk, of order $\mu = 1$. Writing the sum of all torques and solving for the force yields

$$\sum\tau = 2\mu F^\text{c}_{\text{bd}}r = I\alpha = (1/2)mr^2\omega^2/(2\Delta\vartheta)$$

$$\Rightarrow F^\text{c}_{\text{bd}} = \frac{mr\omega}{8\mu\Delta\vartheta} = \frac{(49.6\text{ kg})(0.20\text{ m})(200\pi\text{ s}^{-1})^2}{8(1)(12\times\pi)} = 13\text{ kN}$$

This is a very large force. The force needed could be reduced by reducing the radius of the wheel. Note that this force would not simply drop off linearly as the radius is decreased. The radius also appeared in the calculation of the mass from the density of steel. One could require quite a bit less force by decreasing the radius only slightly. This reduction of the force must necessarily come at the expense of storing less energy in the flywheel. Your exact choice would depend on the energy requirements of the car.

12.107. (*a*) The sum of all forces downward ($+y$ direction) on the block is $\sum F_y = mg - \mathcal{T} = ma_y$. The sum of all torques on the cylinder is $\sum\tau = \mathcal{T}r = I\alpha \Rightarrow \mathcal{T} = Ia_y/r^2$. Inserting this into the force equation and solving for the acceleration yields

$$a = \frac{mg}{m + \dfrac{I}{r^2}} = \frac{(3.0\text{ kg})(9.8\text{ m/s}^2)}{(3.0\text{ kg}) + \dfrac{0.80\text{ kg}\cdot\text{m}^2}{(0.30\text{ m})^2}} = 2.5\text{ m/s}^2$$

(*b*) The speed of the block after it drops 1.5 m is given by kinematics: $v_{y,\text{i}} = \sqrt{2a_y\Delta y} = \sqrt{2(2.47\text{ m/s}^2)(1.5\text{ m})} =$ 2.72 m/s. Using $\omega = v/r$ yields $\omega_\text{f} = 9.1\text{ s}^{-1}$.

(*c*) Using energy conservation

$$\begin{aligned}mgh_\text{i} &= \frac{1}{2}mv_\text{f}^2 + \frac{1}{2}I\omega_\text{f}^2\\ 2mgh_\text{i} &= (mr^2 + I)\omega_\text{f}^2\\ \omega_\text{f} &= \sqrt{\frac{2mgh_\text{i}}{(mr^2 + I)}} = \sqrt{\frac{2(3.0\text{ kg})(9.8\text{ m/s}^2)(1.5\text{ m})}{(3.0\text{ kg})(0.30\text{ m})^2 + 0.80\text{ kg}\cdot\text{m}^2}} = 9.1\text{ s}^{-1}\end{aligned}$$

(*d*) The tensile force is given by $\mathcal{T} = Ia_y/r^2$ regardless of position: $\mathcal{T} = (0.8\text{ kg}\cdot\text{m}^2)(2.47\text{ m/s}^2)/(0.30\text{ m})^2 = 22\text{ N}$.

(*e*) $P = \partial W/\partial t = \mathcal{T}\partial y/\partial t = \mathcal{T}v_y = (22\text{ N})(2.72\text{ m/s}) = 60\text{ W}$

12.108. The force from the wall on the ball will cause a certain acceleration. The torque from that force acting a distance h above the center of mass will cause some angular acceleration. In order for there to be no slipping, these quantities must obey $a = \alpha r.$ Writing the sum of all torques gives us $\sum \tau = Fh = I\alpha = (2/5)mr^2\alpha = (2/5)mra$ or $F = 2mra/(5h).$ But we also know that $F = ma.$ Setting the two expressions for F equal to one another yields $h = 2r/5.$ This was the height that the wall strikes above the ball's center of mass. The total height of the wall $h' = h + r = 7r/5.$

12.109. The sum of all forces acting downward ($+y$ direction) is $\sum F_y = mg - \mathcal{T} = ma_y.$ The sum of all torques on the yo-yo is $\sum \tau = \mathcal{T}b = I\alpha = ma^2\alpha/2 = ma^2 a_y/2b,$ or $\mathcal{T} = ma^2 a_y/2b^2.$ Inserting this into the sum of all forces yields $a_y = \dfrac{g}{1 + a^2/2b^2}$ downward.

13

GRAVITY

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

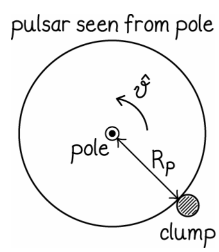
Developing a Feel

1. 10^{-6} N **2.** 10^{-10} N **3.** 10^{-4} N **4.** 10^{13} m **5.** 10^{3} m/s **6.** 10^{3} m **7.** 10^{-2} m **8.** 10^{35} J **9.** zero and 10^{3} N **10.** 10^{40} $\text{kg}\cdot\text{m}^2/\text{s}$

Guided Problems

13.2 Disruptive spin

1. Getting Started When a millisecond pulsar spins, matter on the outer surface of the star is held in place by gravity. The force of gravity acting at the surface of the pulsar depends on the radius of the pulsar. A small clump of material at the surface of a millisecond pulsar moves in a circle around the axis of rotation. If the clump is very near the pole, it may move through a small circle. If the clump is near the equator, it would move through a very large circle with a radius equal to the radius of the spherical pulsar. Thus the radius is involved in both the circular motion of the clump, and in the gravitational force holding the clump in place. This set up is shown below.



2. Devise Plan The magnitude of the gravitational force holding the clump in place is given by

$$F^G_{\text{pc}} = \frac{Gm_\text{c}m_\text{p}}{R_\text{p}^2} \tag{1}$$

This force is exerted on the clump inward toward the center of the pulsar, and it accounts for the centripetal force holding the clump to the surface. Note that if the pulsar were not just on the verge of losing matter due to its spin, we would have to say that the sum of the gravitational force and the normal force that the pulsar exerts on the clump yield the centripetal force. But in our case, the clump is just barely being held in place, such that the normal force approaches zero. Thus we can equate the gravitational force to the centripetal force given by $\dfrac{m_\text{c}v_\text{c}^2}{R_\text{p}} = m_\text{c}R_\text{p}\omega_\text{p}^2 = m_\text{c}R_\text{p}\left(\dfrac{2\pi}{T_\text{p}}\right)^2$.

We need only rearrange terms to solve for the radius of the pulsar.

3. Execute Plan Inserting our expression for the centripetal force into equation (1), we find

$$\frac{Gm_c m_p}{R_p^2} = m_c R_p \left(\frac{2\pi}{T_p}\right)^2$$

Or

$$R_p = \left(\frac{Gm_p T_p^2}{4\pi^2}\right)^{1/3} = \left(\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)2(2.0\times10^{30}\ \text{kg})(1.0\times10^{-3}\ \text{s})^2}{4\pi^2}\right)^{1/3} = 1.9\times10^4\ \text{m}$$

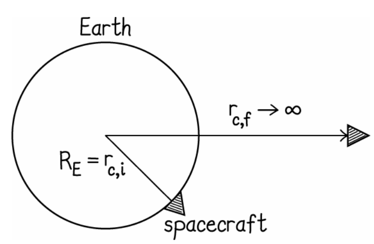
4. Evaluate Result In solving for the radius of the pulsar, we assumed that the pulsar remains spherical. This is not the case in reality. Such millisecond pulsars are deformed into ellipsoids. Even Earth is slightly ellipsoidal, having a greater equatorial radius than volumetric radius.

Despite this simplification, our answer is not far off. The fastest millisecond pulsar, PSR J1748-2446ad, has a radius of less than 1.6×10^4 m. This particular pulsar has a rotational period of 1.40 ms. The value we obtained is very reasonable for a millisecond pulsar.

13.4 Spring to the stars

1. Getting Started A spacecraft is initially bound by gravitational forces to the surface of Earth. This corresponds to a negative gravitational potential energy. If sufficient energy could be stored in a spring, it could launch the spacecraft free of Earth (a total energy of zero or more). This is similar to Worked Problem 13.3 in that we are asked to help a craft escape a planetary body using a force that is position-dependent. This problem differs from Worked Problem 13.3 in that we are not asked anything about intermediate kinetic energies.

This situation is shown below:



The spacecraft gains its escape speed as spring potential energy is traded for kinetic and gravitational potential energies.

2. Devise Plan One could use Newton's second law to determine the acceleration of a spacecraft due to the sum of forces exerted on it. However, both the spring force and the gravitational force are position-dependent (not constant). Using Newton's second law would be unnecessarily complicated. The simplest approach is to use the conservation of energy.

Since the spacecraft must be launched vertically, compressing the spring means moving the spacecraft to a lower position. Before solving the problem we may not have a sense of how great of a distance this compression will be, so we may not know whether this will be negligible or not. The safest way to proceed is to assume that a cavity is dug out of Earth for the spring such that when it is compressed one end is even with the surface of Earth. This way the spacecraft really begins at the surface of Earth, as opposed to sitting atop a high spring that is resting on the surface of Earth.

We are now free to use conservation of energy to relate the initial energy (at the instant the spacecraft is launched) and the final energy (when the spacecraft is so far away from Earth that the gravitational potential energy is zero to a very good approximation). We include Earth and the spring in our system, along with the spacecraft. Thus $U_i^{sp} + U_i^G + K_i = U_f^{sp} + U_f^G + K_f$.

We do not know exactly what the initial kinetic energy of the spacecraft is. If it were launched from either pole of Earth, its initial speed would be zero. However, most spacecraft are launched from facilities somewhere in the tropics, so that they start with some initial kinetic energy. Because we are not told the launch point, for simplicity, we ignore the initial kinetic energy as though the spacecraft is being launched from a point far to the north or south. The final spring potential energy is zero, as is the final gravitational potential energy (to a very good approximation). One could escape Earth and have plenty of energy left over in the form of kinetic energy. But we are asked for the limiting case where the spacecraft has just enough energy to break free of Earth. Thus the final kinetic energy is also zero. Thus

$$U_{\mathrm{i}}^{\mathrm{sp}} + U_{\mathrm{i}}^{\mathrm{G}} = 0 \tag{1}$$

Inserting the form of the two potential energies into equation (1), we have

$$\frac{1}{2}k(\Delta x)^2 + \left(-\frac{Gm_{\mathrm{E}}m_{\mathrm{s}}}{R_{\mathrm{E}}}\right) = 0 \tag{2}$$

3. Execute Plan Solving equation (2) for the compression Δx, we find

$$\Delta x = \sqrt{\frac{2}{k}\frac{Gm_{\mathrm{E}}m_{\mathrm{s}}}{R_{\mathrm{E}}}} \tag{3}$$

Inserting the given quantities into equation (3), we find

$$\Delta x = \sqrt{\frac{2(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.97\times10^{24}\ \mathrm{kg})(1.00\times10^{4}\ \mathrm{kg})}{(1.00\times10^{5}\ \mathrm{N/m})(6.378\times10^{6}\ \mathrm{m})}} = 3.53\ \mathrm{km} \tag{4}$$

We could have obtained a somewhat smaller distance if we had launched from the equator, where the spacecraft would have had an initial kinetic energy.

4. Evaluate Result The final expression for the compression in equation (3) contains reasonable dependence on several variables. For example, if the inertia of the spacecraft increases, the required compression increases. If the spring constant increases, then we don't require as much compression. If the radius of Earth increases (or more reasonably, the if the distance from Earth's center from which the craft is launched increases) the required compression decreases.

We would certainly not recommend this as a launch method. Not only is it implausible to build any launch device 3.53 km high, it is also unreasonable to treat such a spring as ideal. The inertia of the upper part of the spring would seriously affect the way the bottom part of the spring functions.

13.6 More rocket launches

1. Getting Started In each part of the problem, we wish to find r_{max}, the greatest distance from the center of Earth that any of the rockets can achieve. Clearly, rocket (*a*) will fly straight upward and return straight down. It may help to imagine launching the rocket from the North Pole, such that Earth's rotation would truly be negligible. Rocket (*b*) will skim the surface of Earth on one side, but reach a great distance from Earth on the other side, before returning to the launch point. Rocket (*c*) would reach its most distant point along a diagonal as shown in Figure WG 13.5, and then return to Earth, striking the surface at a different location.

2. Devise Plan Once the rockets are launched, if we include Earth in our system, the energy of each rocket should remain constant. Also, since no external forces exert any torque on the system, the rotational momentum about the center of Earth should be constant. Thus we can use conservation of energy and conservation of rotational momentum to relate initial and final states.

Let θ be the angle from the line normal to the Earth's surface. Then the rotational momentum at the moment of launch in any of the three cases can be written

$$L = mv_{\mathrm{launch}}R_{\mathrm{E}}\sin(\theta) \tag{1}$$

The rotational momentum at any other time after launch can be written as

$$L = mvr\sin(\vartheta) \tag{2}$$

Here ϑ is the angle the velocity makes from the radial direction. Equating (1) and (2), we find

$$v_{\text{launch}} R_{\text{E}} \sin(\theta) = vr\sin(\vartheta) \tag{3}$$

Eventually, we will use equation (3) to relate the speed, angle, and distance from Earth's center. But let us write down the initial and final energies first. We know that there is initial kinetic energy from the launch of the rocket. There is also gravitational potential energy, since the rocket is initially on Earth's surface. In the final case, there will certainly be gravitational potential energy, but there may or may not be kinetic energy. In case (*a*), we expect the rocket to rise to a peak height, stop momentarily, and return to Earth. But in the other cases, the rocket does not come to a complete stop at its most distant point. So, in general, we can only write

$$\frac{1}{2}mv_{\text{launch}}^2 - \frac{Gm_{\text{E}}m}{R_{\text{E}}} = \frac{1}{2}mv^2 - \frac{Gm_{\text{E}}m}{r} \tag{4}$$

Equations (3) and (4) contain the unknown variables v and r, but equation (3) also involves a third unknown variable ϑ. This makes sense, because as this angle of motion changes, so will the distance from Earth to the rocket and the speed of the rocket. But we must consider what value ϑ has at our points of interest (the maximum distance of the rocket from Earth). In case (*a*), we have the trivial solution that $\vartheta = 0$ at all times, because the rocket is launched with zero rotational momentum. But in cases (*b*) and (*c*) we have to be a bit more clever. Consider this: if $\vartheta \neq 90°$, then there is a component of the velocity that is in the radial direction. That would mean that the rocket must be moving toward Earth's center or away from it. We want to find the position at which the rocket has moved as far away as it ever will, and is just between moving farther outward and coming back in to Earth. We want there to be momentarily be zero component of the velocity in the radial direction, and this is only satisfied if $\vartheta = 90°$ (or if the velocity itself is zero at that moment as in case a). So for part (*b*) and (*c*), we insert $\vartheta = 90°$ to obtain the maximum distance from Earth to the rocket. But, to be careful, we note that $\vartheta = 90°$ is really just the condition for a radial extremum to be found. We may find a maximum, or a minimum. We'll remember this when we obtain numerical answers.

3. Execute Plan Now that we have considered the value of ϑ in each case separately, we can simply solve equation (3) for the variable speed of the rocket along its path, in terms of other variables:

$$v = \frac{v_{\text{launch}} R_{\text{E}} \sin(\theta)}{r_{\text{max}} \sin(90°)}$$

$$v = \frac{v_{\text{launch}} R_{\text{E}} \sin(\theta)}{r_{\text{max}}} \tag{5}$$

We can insert equation (5) into equation (4) and solve for the maximum distance.

Case (*a*): The speed at the maximum distance is zero. So equation (4) yields

$$r_{\text{max}} = \left(\frac{1}{R_{\text{E}}} - \frac{1}{2}\frac{v_{\text{launch}}^2}{Gm_{\text{E}}}\right)^{-1}$$

$$r_{\text{max}} = \left(\frac{1}{(6.378\times10^6\ \text{m})} - \frac{1}{2}\frac{\left(\frac{3}{4}\times1.12\times10^4\ \text{m/s}\right)^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}\right)^{-1}$$

$$r_{\text{max}} = 1.47\times10^7\ \text{m} = 2.29\ R_{\text{E}}$$

Case (*b*): Inserting equation (5) into equation (4), and rearranging yields

$$\frac{1}{2}v_{\text{launch}}^2 - \frac{Gm_{\text{E}}}{R_{\text{E}}} = \frac{1}{2}\left(\frac{v_{\text{launch}} R_{\text{E}} \sin(\theta)}{r_{\text{max}}}\right)^2 - \frac{Gm_{\text{E}}}{r_{\text{max}}}$$

$$\left(\frac{1}{2}v_{\text{launch}}^2 - \frac{Gm_{\text{E}}}{R_{\text{E}}}\right)r_{\text{max}}^2 + Gm_{\text{E}}r_{\text{max}} - \frac{1}{2}(v_{\text{launch}}R_{\text{E}})^2 = 0$$

This is clearly a quadratic equation. Although the algebraic solution is cumbersome to write out, it is not difficult to solve. One might plug in numbers at this point to make the written solution more tractable.

Inserting $\theta = 90°$, and other variables, we can write the above quadratic equation as

$$(-2.715\times10^7\ \text{m}^2/\text{s}^2)r_{\text{max}}^2 + (3.982\times10^{14}\ \text{m/s}^2)r_{\text{max}} - (1.435\times10^{21}\ \text{s}^{-2}) = 0$$

which yields solutions: $r_{\text{max}} = 8.29\times10^6$ m and $r_{\text{max}} = 6.37\times10^6$ m. Note that the second solution is just the radius of Earth. This fits with the picture shown in Figure WG 13.5. This is the minimum distance from the center of Earth, not the maximum. The maximum distance is $r_{\text{max}} = 8.29\times10^6\ \text{m} = 1.29R_{\text{E}}$.

Case (*c*): We proceed exactly as in (*b*), except we insert $\theta = 45°$. We find

$$\frac{1}{2}v_{\text{launch}}^2 - \frac{Gm_{\text{E}}}{R_{\text{E}}} = \frac{1}{2}\left(\frac{v_{\text{launch}}R_{\text{E}}\sin(\theta)}{r_{\text{max}}}\right)^2 - \frac{Gm_{\text{E}}}{r_{\text{max}}}$$

$$\left(\frac{1}{2}v_{\text{launch}}^2 - \frac{Gm_{\text{E}}}{R_{\text{E}}}\right)r_{\text{max}}^2 + Gm_{\text{E}}r_{\text{max}} - \frac{1}{2}(v_{\text{launch}}R_{\text{E}}\sin(\theta))^2 = 0$$

Again, this quadratic equation may be easier to solve if one first inserts numbers. We find

$$(-2.715\times10^7\ \text{m}^2/\text{s}^2)r_{\text{max}}^2 + (3.982\times10^{14}\ \text{m/s}^2)r_{\text{max}} - (7.1758\times10^{20}\ \text{s}^{-2}) = 0$$

This yields two solutions: $r_{\text{max}} = 1.26\times10^7$ m and $r_{\text{max}} = 2.10\times10^6$ m. Note that the latter is less than the radius of Earth, and so cannot be the maximum distance from the center of Earth that is attained. The first solution is correct. The second corresponds to a minimum distance from the center of Earth, if the rocket's trajectory were able to continue on through the material of the Earth once it returns to the surface. So, this latter solution does have meaning, but the rocket will crash into the surface of Earth and prevent that solution from being useful. The maximum distance the rocket will reach is $r_{\text{max}} = 1.26\times10^7\ \text{m} = 1.96R_{\text{E}}$.

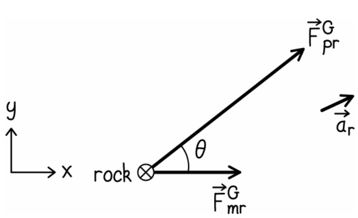
4. Evaluate Result Our answers are all reasonable numbers for rockets launched at a speed close to the escape speed. It is very reasonable that we found the greatest distance in the case where the rocket was fired normal to the surface of Earth. In this case, the rocket had zero speed at its farthest point, meaning it was able to convert all of its initial kinetic energy to gravitational potential energy. It also follows that case (*b*), which has the highest speed at its most distant point, should reach a maximum distance much lower than in case (*a*) or (*c*).

From the algebraic solution in part (*a*), we see that there is not simple scaling with the mass of Earth or radius of Earth. But small increases in the mass of Earth would decrease the maximum distance, and small increases in the radius of Earth would increase the maximum distance.

13.8 Heavenly rock

1. Getting Started We will calculate the vector sum of all forces and equate that to the mass of the rock times its acceleration. We won't worry yet that we don't know the mass of the rock. We know that the force of gravity acting on the rock will depend on the rock's mass. It is plausible that the mass will cancel out later in the calculation.

Only two forces are exerted on the rock: the gravitational forces exerted on the rock by the planet and by the moon. They are shown below:



2. Devise Plan The gravitational force that each body exerts on the rock depends on the distance from the body to the rock. We can write each force in terms of given variables:

$$\vec{F}^G_{\text{pr}} = \frac{Gm_{\text{p}}m_{\text{r}}}{r^2_{\text{pr}}}\hat{r}_{\text{pr}} \text{ and } \vec{F}^G_{\text{mr}} = \frac{Gm_{\text{m}}m_{\text{r}}}{r^2_{\text{mr}}}\hat{r}_{\text{mr}}$$

Where $\hat{r}_{\text{pr}}$ is the direction of the force the planet exerts on the rock (pointing from the rock to the planet), and $\hat{r}_{\text{mr}}$ is the direction of the force the moon exerts on the rock. Here, we are told

$$r_{\text{pr}} = \sqrt{(16R_{\text{E}})^2 + (12R_{\text{E}})^2} = 20R_{\text{E}}, \text{ and } r_{\text{mr}} = 16R_{\text{E}}$$

Simple trigonometry tells us that $\theta = \tan^{-1}(12/16) = 36.9°.$ We now have everything we need to find the vector sum of all forces being exerted on the rock.

3. Execute Plan We write the sums of forces in the x and y directions separately:

$$\sum F_x = F^G_{\text{mr}} + F^G_{\text{pr}}\cos(\theta) = m_{\text{r}}a_{\text{r}x}$$

$$\frac{Gm_m m_{\text{r}}}{(16R_{\text{E}})^2} + \frac{Gm_p m_{\text{r}}}{(20R_{\text{E}})^2}\cos(\theta) = m_{\text{r}}a_{\text{r}x}$$

$$a_{\text{r}x} = \frac{Gm_{\text{E}}}{R^2_{\text{E}}}\left(\frac{8}{16^2} + \frac{25}{20^2}\left(\frac{16}{20}\right)\right)$$

$$a_{\text{r}x} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{(6.378\times10^6\ \text{m})^2}\left(\frac{8}{16^2} + \frac{25}{20^2}\left(\frac{16}{20}\right)\right)$$

$$a_{\text{r}x} = 0.795\ \text{m/s}^2$$

and

$$\sum F_y = F^G_{\text{pr}}\sin(\theta) = m_{\text{r}}a_{\text{r}y}$$

$$\frac{Gm_p m_{\text{r}}}{(20R_{\text{E}})^2}\sin(\theta) = m_{\text{r}}a_{\text{r}y}$$

$$a_{\text{r}y} = \frac{Gm_{\text{E}}}{R^2_{\text{E}}}\left(\frac{25}{20^2}\left(\frac{12}{20}\right)\right)$$

$$a_{\text{r}y} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{(6.378\times10^6\ \text{m})^2}\left(\frac{25}{20^2}\left(\frac{12}{20}\right)\right)$$

$$a_{\text{r}y} = 0.367\ \text{m/s}^2$$

The acceleration of the rock can be written as a magnitude and direction by using the Pythagorean Theorem:

$$a_{\text{r}} = \sqrt{a^2_{\text{r}x} + a^2_{\text{r}y}} = \sqrt{(0.795\ \text{m/s}^2)^2 + (0.367\ \text{m/s}^2)^2} = 0.876\,\text{m/s}^2$$

and finding the angle at which it accelerates:

$$\phi = \tan^{-1}(a_{\text{r}y}/a_{\text{r}x}) = \tan^{-1}\left(\frac{0.367\ \text{m/s}^2}{0.795\ \text{m/s}^2}\right) = 24.8°$$

So the acceleration of the rock is $0.876\ \text{m/s}^2$ at an angle $24.8°$ above the line connecting the rock and moon.

4. Evaluate Result The acceleration is less than the gravitational acceleration on Earth. This is reasonable because the distances are so much greater than the radius of Earth. The masses of the planet and moon are also much larger than the mass of Earth. But the radii and masses exceed those of Earth by comparable factors, and the gravitational force depends linearly on mass and drops off like the radial distance squared. So we expect the effect due to the increase in distance to dominate. We see that it does.

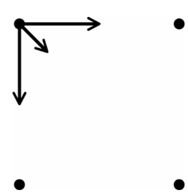
Questions and Problems

13.1. In Chapter 13 of the *Principles* text we learned $F_{21}^G \propto \frac{m_1 m_2}{r_{12}^2}$, where we will assume that object 1 is the planet in question, and object 2 is at the surface of the planet. We can write the mass of the planet as $m_1 = \rho V = \rho\left(\frac{4}{3}\pi R_\text{p}^3\right)$, such that the force $F_{21}^G \propto \frac{\rho\left(\frac{4}{3}\pi R_\text{p}^3\right)m_2}{R_\text{p}^2} \propto R_\text{p}$. Thus the acceleration due to gravity would increase linearly with the radius.

13.2. (*a*) As discussed in the *Principles* text Section 13.1, $\frac{m_1}{m_2} = \frac{r_2}{r_1}$. Rearranging, we have $\frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{1}{3}$. (*b*) The ratio is still $\frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{1}{3}$. The proportionality arose from the definition of torque and from the fact that gravitational force is proportional to the masses attracting each other. It does not require that objects be on Earth.

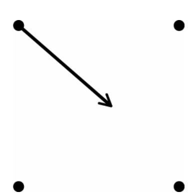
13.3. Because this is a scale model, we require that $\frac{r_\text{scale Mercury}}{r_\text{real Mercury}} = \frac{d_\text{scale outer}}{d_\text{real outer}} \Rightarrow d_\text{scale outer} = \left(\frac{r_\text{scale Mercury}}{r_\text{real Mercury}}\right) d_\text{real outer}$. We insert astronomical data from Table 13.1 (for the radius of Mercury and the semimajor axis of Neptune) to find $d_\text{scale outer} = \left(\frac{(0.010\ \text{m})}{(2.440\times10^6\ \text{m})}\right)(4.498\times10^{12}\ \text{m}) = 18\ \text{km}.$

13.4. (*a*)



We choose to draw all gravitational forces acting on the upper left object. It should feel an attractive force with all three other objects. The force along the diagonal of the square will be weaker than the other two forces, because of the larger distance involved.

(*b*)



This results from simple vector addition.

13.5. We use the proportionality relation $F_{21}^G \propto \frac{m_1 m_2}{r_{12}^2}$ for each pair of objects and in each case, we use the center-to-center distance for the objects' separation. Thus $\frac{F_\text{BB}^G}{F_\text{gg}^G} = \frac{r_\text{gg}^2 m_\text{B}^2}{r_\text{BB}^2 m_\text{g}^2} = \frac{(44\times10^{-3}\ \text{m})^2(0.60\ \text{kg})^2}{(0.24\ \text{m})^2(0.045\ \text{kg})^2} = 6.0.$

13.6. In the reference frame of the space station, the satellite moves very little. It appears to hold the same position except for the satellite's very small initial velocity and a tiny acceleration due to the gravitational force exerted on it by the space station. In the Earth reference frame, the satellite orbits with the space station, again excepting the very small difference in the initial velocities of the two objects and the tiny acceleration toward the space station due to their tiny mutual gravitational attraction.

13.7. We know the gravitational force attracting the two pieces will obey $F_{21}^G \propto \dfrac{m_1 m_2}{r_{12}^2}$. So maximizing the product $m_1 m_2 = m_1(m - m_1)$ will also maximize the attractive force. To find the value of m_1 that maximizes this product, we take the derivative of the product with respect to m_1 and require that it be zero: $\dfrac{d}{dm_1}\left[m_1(m-m_1)\right] = m - 2m_1 = 0 \Rightarrow m_1 = \dfrac{m}{2}$, which of course means $m_2 = \dfrac{m}{2}$ as well. Thus $\dfrac{m_1}{m_2} = 1$.

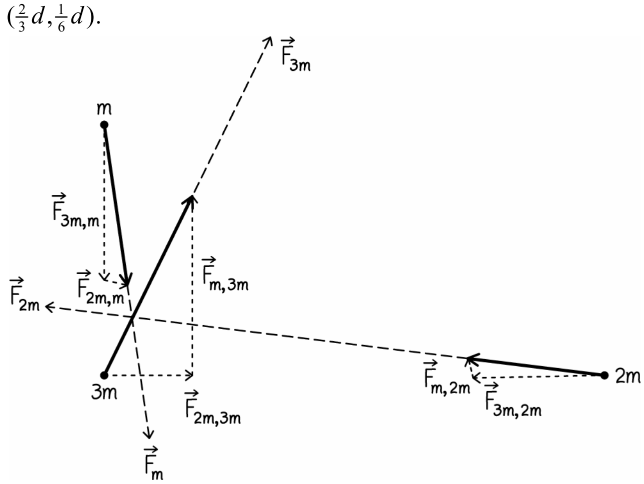
13.8. (*a*) Let the particle of mass 3*m* be at the origin, and calculate the locations of the centers of mass of the three two-particle systems: 3*m*-2*m*, center of mass at $(\frac{4}{5}d, 0)$; *m*-2*m*, center of mass at $(\frac{4}{3}d, \frac{1}{3}d)$; *m*-3*m*, center of mass at $(0, \frac{1}{4}d)$. The line of action of the force exerted by a given particle runs from the particle to the center of mass of the system comprising the other two particles: particle of mass *m*, force acts along line $y = d - \frac{5}{4}x$ (call it line *m*); particle of mass 2*m*, force acts along line $y = \frac{1}{4}d - \frac{1}{8}x$ (line 2*m*); particle of mass 3*m*, force acts along line $y = \frac{1}{4}x$ (line 3*m*). Line *m* and line 2*m* intersect at

$$d - \tfrac{5}{4}x = \tfrac{1}{4}d - \tfrac{1}{8}x \quad \Rightarrow \quad x = \tfrac{2}{3}d \quad \Rightarrow \quad y = \tfrac{1}{6}d$$

Line 2*m* and line 3*m* intersect at

$$\tfrac{1}{4}d - \tfrac{1}{8}x = \tfrac{1}{4}x \quad \Rightarrow \quad x = \tfrac{2}{3}d \quad \Rightarrow \quad y = \tfrac{1}{6}d$$

The three lines intersect at $(\frac{2}{3}d, \frac{1}{6}d)$.



(*b*) Yes (*c*) The system is isolated. If released from rest, each particle moves in a straight line toward the common center of mass. The accelerations are not constant, however, so the motion is difficult to describe completely.

13.9. Torque can be interpreted as the second time derivative of the same area whose first derivative represents angular momentum. This follows trivially from the fact that torque is the rate of change of angular momentum.

13.10. The device tilts clockwise. When the elevator is at rest, the plank only needs to exert an upward force on the block that is equal to its mass times the acceleration due to gravity. But once the elevator is accelerating upward, the plank must exert a greater upward force on the block to accelerate it along with the elevator. This means the block will exert a greater force (and therefore a greater torque also) on the plank. The spring must stretch in order to exert a greater force on the plank so that the torques will be balanced. Stretching the spring corresponds to a clockwise rotation.

13.11. (*a*) Yes, when you jump the scale shows a higher number at first. The reading then oscillates and settles to the same value as when you gently step onto the scale. (*b*) No, the magnitude of the gravitational force exerted by Earth on you has not changed. The only quantity the scale can measure is the contact force your body exerts on the scale. What has changed when you go from stepping gently to jumping onto the scale is the normal force exerted by the scale on you. When you jump, the normal force first had to be larger than the gravitational force exerted on you in order to accelerate you upward and decrease your downward velocity.

13.12. The balance arm remains level. Everything accelerates at the same rate due to gravity.

13.13. (*a*) The spring scale would only indicate a mass of zero if no force at all were being exerted downward on it. This is only the case if everything (the brick, and spring scale) are in freefall. Thus the spring scale will read zero if the elevator accelerates at 9.8 m/s^2 downward. (*b*) The same criteria apply as in part (*a*), but on the moon the acceleration is 1.6 m/s^2 downward. So the scale will read zero if the elevator accelerates at 1.6 m/s^2 downward. (*c*) The answers would not change. In order to read zero the spring scale cannot be exerting any forces on the brick. If that is the case, then the brick will have only gravity acting on it and must experience acceleration due to gravity. This is independent of how the spring and brick are connected.

13.14. The force measured by the spring will be zero when you are in the air, rising smoothy to a value considerably larger than the gravitational force exerted on you by Earth when you bounce. This would also be true for the pogo stick or the diving board used alone. When you and the pogo stick are not in contact with any other object, the spring exerts no force and the reading is zero. When you come into contact with the diving board the spring in the pogo stick begins to compress, and the scale reading increases. In order to stop your downward motion, at some point the spring force must exceed the gravitational force for a short time interval. The diving board compounds this effect, since it acts like a second spring in series with the spring in the pogo stick. At the low point when you're in contact with the diving board, both the spring and diving board are maximally compressed so your upward acceleration is maximum and the sensor reading is much greater than the gravitational force exerted by Earth. As you rise, both the compression and the associated scale reading diminish. The result is a sensor reading that is periodically zero as you fly through the air, then rises to maximum and falls again to zero as you bounce and again lose contact with the board.

13.15. The airplane moves downward at $2g$. During the time interval in which this motion takes place, the person is in free fall, accelerating downward at g. This means that, relative to the plane, he is accelerating *upward* at g! This relative motion of plane and person is exactly like the person falling onto his head from an upside-down position on solid ground.

13.16. There should be no tilt. There should only be upward acceleration combined with tilt of viewing image to right. If the curve is banked and the bobsled exerts no frictional forces on the ice, then the bobsled will swing out such that all force required to keep the bobsled moving through the turn is exerted normal to the ice, on the bottom of the sled. Thus, you will feel no rightward or leftward acceleration. You will feel an acceleration normal to the bottom of your bobsled. Of course, as the bobsled slides up the banked curve, the rest of the world will appear sideways to you. Thus the required tilting of the viewing image.

13.17. There may be several answers. One could start by holding the tube horizontally. The rubber band should compress and pull the metal bob toward the cork. Jiggle the tube a bit to make that friction is not preventing the bob from sliding in the tube. Mark this position as the 0 g position. You now have marks for 0 g and 1 g, so the distance between these marks should also be the distance from the 1 g to 2 g marks and from 2 g to 3 g. Alternatively, one could add inertia to the metal bob causing the rubber band to stretch farther downward. Suppose the inertia of the metal bob is m. Then adding a second object of inertia m would require the rubber band to exert an upward force of $2mg$, which is exactly the force that the rubber band would have to exert if a single bob were being accelerated at $2g$. The process can be repeated with multiple bobs for higher accelerations.

13.18. (*a*) Seen from Earth, the satellites move in circular orbits at fairly constant speed, with their paths intersecting twice. An observer on satellite 1 sees satellite 2 approach satellite 1, then drift away, then return. (*b*) No, to the extent that the very tiny gravitational force exerted by each satellite on the other can be ignored.

13.19. (*a*) The milk climbs the sides of the bowl, and the milk surface becomes concave. (*b*) Either a massive hemisphere placed near the milk or the combination of a toroid around the bowl and a massive object beneath the bowl.

13.20. (*a*) We know $F^G_{\text{SM}} = \dfrac{Gm_{\text{S}}m_{\text{M}}}{r^2_{\text{SM}}} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\ \text{kg})(6.42\times10^{23}\ \text{kg})}{(2.279\times10^{11}\ \text{m})^2} = 1.6\times10^{21}\ \text{N}$. (*b*) One could calculate this exactly as in part (*a*), or simply use Newton's Third Law to write $F^G_{\text{MS}} = 1.64\times10^{21}\ \text{N}$.

13.21. Starting from equation (13.1) we have

$$F^G_{12} = \frac{Gm_1m_2}{r^2_{12}} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.0\times10^{-3}\ \text{kg})^2}{(0.100\ \text{m})^2} = 6.7\times10^{-15}\ \text{N}$$

13.22. Starting from equation (13.1) we have

$$F^G_{12} = \frac{Gm_1m_2}{r^2_{12}} \Rightarrow r_{12} = \sqrt{\frac{Gm_1m_2}{F^G_{12}}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2\times10^{12}\ \text{kg})(5\times10^{20}\ \text{kg})}{(3\times10^{7}\ \text{N})}} = 5\times10^7\ \text{m}$$

13.23. Using equation (13.1) we can write the ratio of accelerations as

$$\frac{g_{\text{Mars}}}{g_{\text{Earth}}} = \frac{F^G_{\text{Mars object}}/m_{\text{object}}}{F^G_{\text{Earth object}}/m_{\text{object}}} = \frac{\dfrac{\text{G}\,m_{\text{Mars}}}{R^2_{\text{Mars}}}}{\dfrac{\text{G}\,m_{\text{Earth}}}{R^2_{\text{Earth}}}} = \left(\frac{m_{\text{Mars}}}{m_{\text{Earth}}}\right)\left(\frac{R^2_{\text{Earth}}}{R^2_{\text{Mars}}}\right) = \left(\frac{1}{9}\right)(2)^2 = \frac{4}{9}$$

13.24. (*a*) Earth has an enormous amount of mass, making the relatively weak gravitational interaction very noticeable on a human scale. All matter participates in the gravitational interaction, which is of much longer range than the strong and weak nuclear interactions. Ordinary matter is also electrically neutral, which means that the electromagnetic interaction (also long range) generally cancels on a human scale. (*b*) The fact that gravity is long range is also important in the rest of the universe. There is a lot of mass out there, and all of it participates in gravitational attraction. The orbits of the planets and other bodies in the solar system are well-predicted using the gravitational interaction. Also, matter in bulk seems to be electrically neutral even on a cosmic scale, so the other long-range interaction (electromagnetism) is much less important for interactions between stars and/or planets. Thus on a scale from human to galactic and beyond: gravity rules.

13.25. Using equation (13.1) we can express the ratio of accelerations as $\dfrac{g_{\text{star}}}{g_{\text{Earth}}} = \dfrac{F^G_{\text{star object}}/m_{\text{object}}}{F^G_{\text{Earth object}}/m_{\text{object}}} = \dfrac{\dfrac{\text{G}\,m_{\text{star}}}{R^2_{\text{star}}}}{\dfrac{\text{G}\,m_{\text{Earth}}}{R^2_{\text{Earth}}}} =$

$\left(\dfrac{m_{\text{star}}}{m_{\text{Earth}}}\right)\left(\dfrac{R^2_{\text{Earth}}}{R^2_{\text{star}}}\right) = \left(\dfrac{2(2.0\times10^{30}\ \text{kg})}{(5.97\times10^{24}\ \text{kg})}\right)\left(\dfrac{(6.378\times10^6\ \text{m})^2}{(1.0\times10^4\ \text{m})^2}\right) = 3\times10^{11}$. So $g_{\text{star}} = 3\times10^{11}g_{\text{Earth}}$.

13.26. We can simply calculate the magnitude of each force by using equation (13.1) and the astronomical data provided in Table 13.1. $F^G_{\text{Em}} = \dfrac{Gm_{\text{E}}m_{\text{m}}}{r^2_{\text{Em}}} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(7.3\times10^{22}\ \text{kg})}{(3.84\times10^8\ \text{m})^2} = 2.0\times10^{20}\ \text{N}$ and $F^G_{\text{Sm}} = \dfrac{Gm_{\text{S}}m_{\text{m}}}{r^2_{\text{Sm}}} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\ \text{kg})(7.3\times10^{22}\ \text{kg})}{(1.496\times10^{11}\ \text{m})^2} = 4.4\times10^{20}\ \text{N}$. Thus the Sun pulls harder on the moon than the Earth does.

13.27. (*a*) We require $\frac{F^G_{\text{E obj}}}{m_{\text{obj}}} = \frac{Gm_{\text{E}}}{r^2_{\text{E obj}}} = (0.999)\frac{Gm_{\text{E}}}{r^2_{\text{E}}} \Rightarrow r_{\text{E obj}} = \sqrt{\frac{1}{0.999}}r_{\text{E}}$. So the height above Earth's surface would be $h = r_{\text{E obj}} - r_{\text{E}} = \left(\sqrt{\frac{1}{0.999}} - 1\right)r_{\text{E}} = \left(\sqrt{\frac{1}{0.999}} - 1\right)(6.378\times10^6\ \text{m}) = 3.2\ \text{km}$. (*b*) We proceed as in part (*a*). Now we require $\frac{F^G_{\text{E obj}}}{m_{\text{obj}}} = \frac{Gm_{\text{E}}}{r^2_{\text{E obj}}} = (0.99)\frac{Gm_{\text{E}}}{r^2_{\text{E}}} \Rightarrow r_{\text{E obj}} = \sqrt{\frac{1}{0.99}}r_{\text{E}}$. So the height above Earth's surface would be $h = r_{\text{E obj}} - r_{\text{E}} = \left(\sqrt{\frac{1}{0.99}} - 1\right)r_{\text{E}} = \left(\sqrt{\frac{1}{0.99}} - 1\right)(6.378\times10^6\ \text{m}) = 32\ \text{km}$. (*c*) Again, the calculation is the same as in part (*a*). Now we require $\frac{F^G_{\text{E obj}}}{m_{\text{obj}}} = \frac{Gm_{\text{E}}}{r^2_{\text{E obj}}} = (0.90)\frac{Gm_{\text{E}}}{r^2_{\text{E}}} \Rightarrow r_{\text{E obj}} = \sqrt{\frac{1}{0.90}}r_{\text{E}}$. So the height above Earth's surface would be $h = r_{\text{E obj}} - r_{\text{E}} = \left(\sqrt{\frac{1}{0.90}} - 1\right)r_{\text{E}} = \left(\sqrt{\frac{1}{0.90}} - 1\right)(6.378\times10^6\ \text{m}) = 3.5\times10^2\ \text{km}$.

13.28. (*a*) To find this distance from Earth, we simply equate the two gravitational forces: We require $F^G_{\text{Ea}} = F^G_{\text{ma}} \Rightarrow \frac{GM_{\text{E}}}{r^2_{\text{Ea}}} = \frac{GM_{\text{m}}}{r^2_{\text{ma}}} = \frac{GM_{\text{m}}}{(r_{\text{Em}} - r_{\text{Ea}})^2}$. Rearranging this we find $M_{\text{E}}(r_{\text{Em}} - r_{\text{Ea}})^2 = M_{\text{m}}r^2_{\text{Ea}} \Rightarrow r_{\text{Ea}} = \frac{r_{\text{Em}}}{\left(1 + \sqrt{\frac{M_{\text{m}}}{M_{\text{E}}}}\right)} = \frac{(3.84\times10^8\ \text{m})}{\left(1 + \sqrt{\frac{(7.3\times10^{22}\ \text{kg})}{(5.97\times10^{24}\ \text{kg})}}\right)} = 3.5\times10^8\ \text{m}$. (*b*) As the spacecraft approaches this location, the acceleration due to Earth's gravity is around $10^{-4}g$ directed toward Earth, and so the passengers feel very light but not quite weightless. At this location, the force of gravity exerted on them is zero. After the craft passes this location, the passengers' acceleration is about $10^{-4}g$ directed toward the Moon, making them feel very light but pulled slightly toward the Moon. This problem has not addressed any gravitational attraction they may feel toward the Sun.

13.29. We know that a particle making up the rings will move in uniform circular motion because of the particle's gravitational attraction to Saturn. Thus we can write

$$\frac{Gm_{\text{S}}m_{\text{p}}}{r^2_{\text{Sp}}} = \frac{m_{\text{p}}v^2_{\text{p}}}{r^2_{\text{p}}} \tag{1}$$

We also know we can relate the speed of the particle to the period using

$$v_{\text{p}} = \frac{2\pi r_{\text{p}}}{T} \tag{2}$$

Inserting equation (2) into equation (1), and rearranging, we find $T = \sqrt{\frac{4\pi^2 r^3_{\text{p}}}{GM_{\text{S}}}}$. Table 13.1 tells us that the radius of Saturn is 6.027×10^7 m. The rings have substantial width, so there is no single correct radius. But it appears as though a radius equal to twice that of Saturn itself would describe a ring roughly in the middle of the range. We assume $r_{\text{p}} = 2r_{\text{S}}$. Thus $T = \sqrt{\frac{4\pi^2(8)(6.027\times10^7\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.68\times10^{26}\ \text{kg})}} = 1\times10^1$ h.

13.30. (*a*) We can simply calculate the magnitude of each force by using equation (13.1) and the astronomical data provided in Table 13.1. $F^G_{\text{mE}} = \frac{Gm_{\text{m}}m_{\text{E}}}{r^2_{\text{mE}}} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(7.3\times10^{22}\ \text{kg})}{(3.84\times10^8\ \text{m})^2} = 2.0\times10^{20}$ N and

$F_{\text{SE}}^{G} = \dfrac{Gm_{\text{S}}m_{\text{E}}}{r_{\text{SE}}^2} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\ \text{kg})(5.97\times10^{24}\ \text{kg})}{(1.496\times10^{11}\ \text{m})^2} = 3.5\times10^{22}\ \text{N}.$ Thus the Sun exerts a greater force on Earth than the Moon does. (*b*) Using our results from part (*a*) (but using one additional significant digit for the intermediate calculation), we find $\dfrac{F_{\text{SE}}^{G}}{F_{\text{mE}}^{G}} = \dfrac{3.54\times10^{22}\ \text{N}}{1.97\times10^{20}\ \text{N}} = 1.8\times10^2.$

13.31. Using equation 13.1 we can write the acceleration due to gravity at the asteroid's surface as $g_{\text{A}} = \dfrac{F_{\text{A obj}}^{G}}{m_{\text{obj}}} = \dfrac{Gm_{\text{A}}}{r_{\text{A}}^2}.$ Rearranging, we have $m_{\text{A}} = \dfrac{g_{\text{A}}r_{\text{A}}^2}{G} = \dfrac{(0.0500\ \text{m/s}^2)(3.75\times10^4\ \text{m})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 1.05\times10^{18}\ \text{kg}.$

13.32. We require that $\dfrac{F_{\text{p obj}}^{G}}{m_{\text{obj}}} = \dfrac{Gm_{\text{p}}}{r^2} = \left(\dfrac{1}{2}\right)\left(\dfrac{Gm_{\text{p}}}{r_{\text{p}}^2}\right) \Rightarrow r = \sqrt{2}r_{\text{p}}.$ Thus the height above Pluto's surface is $h = r - r_{\text{p}} = (\sqrt{2}-1)r_{\text{p}} = (\sqrt{2}-1)(1.151\times10^6\ \text{m}) = 4.768\times10^5\ \text{m}.$

13.33. We must break the attractive forces into components.

$$\sum F_{\text{test}\,x}^{G} = F_{1\text{ test}}^{G} - F_{2\text{ test}}^{G}\cos\left(\tan^{-1}\left(\frac{\ell}{d}\right)\right) = \frac{Gm_{\text{test}}^2}{d^2} - \left(\frac{Gm_{\text{test}}^2}{d^2+\ell^2}\right)\left(\frac{d}{\sqrt{d^2+\ell^2}}\right)$$

$$\sum F_{\text{test}\,y}^{G} = F_{2\text{ test}}^{G}\sin\left(\tan^{-1}\left(\frac{\ell}{d}\right)\right) = \left(\frac{Gm_{\text{test}}^2}{d^2+\ell^2}\right)\left(\frac{2\ell}{\sqrt{d^2+\ell^2}}\right)$$

We add these two components to find the magnitude of the total force using the Pythagorean Theorem, and obtain

$$F = Gm^2\sqrt{\left(\frac{1}{d^2} - \frac{2d}{(d^2+\ell^2)^{3/2}}\right)^2 + \left(\frac{2\ell}{(d^2+\ell^2)^{3/2}}\right)^2}$$

13.34. The precise expression (from equation 13.1) is $g = \dfrac{Gm_{\text{E}}}{(R_{\text{E}}+h)^2}.$ If we assume that h is still small compared to the radius of Earth, but not completely negligible, then we can expand this expression in a Taylor series to obtain $g(h) = g_0\left[1 - 2\dfrac{h}{R_{\text{E}}} + 3\dfrac{h^2}{R_{\text{E}}^2}\right].$

13.35. Suppose that an object is a distance r from the center of Earth. One could picture this object as being at the surface of a sphere of radius r, and inside a thick spherical shell of inner radius r and outer radius R_{E}. The object will experience no net gravitational force due to mass in the thick spherical shell. But the object will be attracted to the center of Earth by the mass inside the sphere of radius r. Thus, the mass that effectively pulls the object toward the center of Earth is $m = \left(\dfrac{4}{3}\pi r^3\right)\rho.$ The total mass of Earth is, of course, $m_{\text{E}} = \left(\dfrac{4}{3}\pi R_{\text{E}}^3\right)\rho.$ Thus $g = \dfrac{F_{\text{E obj}}^{G}}{m_{\text{obj}}} = \dfrac{Gm}{r^2} = \dfrac{G\rho\left(\frac{4}{3}\pi r^3\right)}{r^2} = \dfrac{Gm_{\text{E}}\rho\left(\frac{4}{3}\pi r^3\right)}{r^2\rho\left(\frac{4}{3}\pi R_{\text{E}}^3\right)} = \dfrac{Gm_{\text{E}}r}{R_{\text{E}}^3}.$

13.36. Using equation (13.11) $U = -\dfrac{Gm_{\text{S}}m_{\text{E}}}{r_{\text{SE}}} = -\dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\ \text{kg})(5.97\times10^{24}\ \text{kg})}{(1.496\times10^{11}\ \text{m})} = -5.3\times10^{33}\ \text{J}.$

13.37. No. A 70-kg person can jump high enough to bring his center of mass to a height of 1.0 m to 1.5 m. That means the legs can exert forces capable of increasing the gravitational potential energy by at most about

$\Delta U^G = -Gm_E m_p\left(\frac{1}{r_E+h}-\frac{1}{r_E}\right) \approx \frac{Gm_E}{R_E^2}m_p h = (9.8\text{ m/s}^2)(70\text{ kg})(1.5\text{ m})$ which is about 10^3 J. On the surface of Toro, this person's gravitational potential energy is $U^G = -\frac{Gm_T m_p}{R_T} = -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{15}\text{ kg})(70\text{ kg})}{(5.0\times10^3\text{ m})} = -1.9\text{ kJ},$ meaning he is bound to Toro by more energy than he can deliver in a jump.

13.38. (*a*) $U^G = -\frac{Gm_E m_{obj}}{R_{obj}} = -\frac{Gm_E m_{obj}}{2R_E} = -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})(100\text{ kg})}{2(6.378\times10^6\text{ m})} = -3.12\times10^9\text{ J}$ (*b*) We require that the kinetic energy be equal in magnitude to the gravitational potential energy found in part (*a*): $\frac{1}{2}m_{obj}v_{obj}^2 = |U^G| \Rightarrow v_{obj} = \sqrt{\frac{2|U^G|}{m_{obj}}} = \sqrt{\frac{2(3.12\times10^9\text{ J})}{(100\text{ kg})}} = 7.90\times10^3\text{ m/s}.$

13.39. We write an expression for the gravitational potential energy of the system in terms of the distance d_1 from object 3 to object 1: $U^G = U_{12}^G + U_{13}^G + U_{23}^G = -Gm_1\left(\frac{2m_1}{d}+\frac{m_3}{d_1}+\frac{2m_3}{(d-d_1)}\right).$ To find the value of d_1 that maximizes this expression (minimizes the magnitude) we take the derivative of the expression with respect to d_1 and require that the derivative be zero: $\frac{d}{d(d_1)}\left[-Gm_1\left(\frac{2m_1}{d}+\frac{m_3}{d_1}+\frac{2m_3}{(d-d_1)}\right)\right] = Gm_1 m_3\left(\frac{1}{d_1^2}-\frac{2}{(d-d_1)^2}\right) = 0.$ Thus $d_1 = \frac{d}{(1+\sqrt{2})},$ which is less than $\frac{d}{2}.$ Thus we see that object 3 should be closer to object 1 than to object 2. More specifically $d_1 = (\sqrt{2}-1)d$ and $d_1 = (2-\sqrt{2})d.$

13.40. We equate initial and final total energies: $K_i + U_i^G = 0 = K_f + U_f^G.$ Since both objects have the same mass, they will have the same final speed. Thus we can write $2\left(\frac{1}{2}mv_f^2\right) = \frac{Gm^2}{d} \Rightarrow v_f = \sqrt{\frac{Gm}{d}}.$ The objects will each have this speed, and they will be moving toward each other, meaning that their speed of approach (relative to each other) will be $v = 2\sqrt{\frac{Gm}{d}}.$

13.41. (*a*) We simply integrate the force: $W = \int_1^2 \vec{F}\cdot d\vec{r} = -Cm_m m_E \int_\infty^{R_E+h}\frac{dr}{r^3} = \frac{Cm_m m_E}{2(R_E+h)^2}.$ (*b*)We set the change in kinetic energy equal to the work done by gravitational forces: $\frac{1}{2}m_m v_m^2 = \frac{Cm_m m_E}{2(R_E+h)^2} \Rightarrow v_m = \sqrt{\frac{Cm_E}{(R_E+h)^2}}.$

13.42. Since the gravitational attraction holds the satellite in its circular orbit, we can equate $\frac{Gm_s m_p}{a^2} = \frac{m_s v_s^2}{a} \Rightarrow v_s = \sqrt{\frac{Gm_p}{a}}.$ To find the energy of the system, we calculate $E = K + U^G = \frac{1}{2}m_s v_s^2 - \frac{Gm_s m_p}{a} = \frac{m_s}{2}\left(\frac{Gm_p}{a}\right) - \frac{Gm_s m_p}{a} = \frac{-Gm_p m_s}{2a}.$

13.43. If we ignore the rotation of the Earth (such as when the projectile is launched from either pole) then we simply equate $U_i^G + K_i = U_f^G \Rightarrow -\frac{Gm_E m_p}{R_E} + \frac{1}{2}mv_{p,i}^2 = -\frac{Gm_E m_p}{R_E+h}.$ Rearranging, we find

$$h=\left(\frac{1}{R_E}-\frac{1}{2}\frac{v_{p,i}^2}{Gm_E}\right)^{-1}-R_E=\left(\frac{1}{(6.378\times10^6\text{ m})}-\left(\frac{1}{2}\right)\left(\frac{(4.0\times10^3\text{ m/s})^2}{(6.67\times10^{-11}\text{ Nm}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}\right)\right)^{-1}-(6.378\times10^6\text{ m})$$
$$=9.4\times10^2\text{ km}$$

13.44. (*a*) We simply set the initially energy equal to the final energy. Call the energy we put in to launch the projectile E_i. This input energy could be kinetic, chemical, spring potential energy, etc. $E_i+U_i^G=K_f+U_f^G$. Since the initial position is on the surface of Earth, and at the final position the object stops momentarily, we can rewrite this as $E_i-\frac{Gm_Em_o}{R_E}=-\frac{Gm_Em_o}{R_E+h}\Rightarrow E_i=Gm_Em_o\left(\frac{1}{R_E}-\frac{1}{R_E+h}\right)$. (*b*) In order to put the object into orbit, we require that it have some nonzero kinetic energy when it reaches the specified height. Specifically, when in orbit, we know the gravitational force provides the necessary centripetal force to maintain uniform circular motion with a radius r. Thus $\frac{Gm_Em_o}{r^2}=\frac{m_ov_o^2}{r}\Rightarrow K_f=\frac{1}{2}mv_o^2=\frac{Gm_Em_o}{2r}$. Of course, for a satellite a distance h above the surface of Earth $r=R_E+h$. Thus, our initial energy must now satisfy $E_i+U_i^G=K_f+U_f^G$ or $E_i-\frac{Gm_Em_o}{R_E}=\frac{Gm_Em_o}{2(R_E+h)}-\frac{Gm_Em_o}{R_E+h}\Rightarrow$ $E_i=Gm_Em_o\left(\frac{1}{R_E}-\frac{1}{2(R_E+h)}\right)$.

13.45. The work done by this force will be $W=\int_{r_i}^{r_f}\vec{F}\cdot d\vec{r}=C\int_{r_i}^{r_f}\frac{dr}{r^2}=C\left(\frac{1}{r_i}-\frac{1}{r_f}\right)$. If we take the limit as the final distance between the Sun and the ship approaches infinity, this reduces to $W=\frac{C}{r_i}$. We require that this work be sufficient to increase the speed to $0.10\ c$ such that

$$W=\frac{C}{r_i}=\frac{1}{2}mv_f^2\Rightarrow C=\frac{mv_f^2r_i}{2}=\frac{(5.0\times10^4\text{ kg})\left[(0.10)(3.0\times10^8\text{ m/s})\right]^2(1.0\times10^{11}\text{ m})}{2}=2.3\times10^{30}\text{ N}\cdot\text{m}^2$$

13.46. (*a*) If the shell has sufficient initial kinetic energy to completely escape Earth, then it certainly has enough to reach the Moon. It turns out that this treatment will give you the correct answer to the given precision. However, here we will be very precise and note that the ship does not need to completely escape the Earth-Moon system. There is a point between Earth and the Moon along a line connecting their centers at which the gravitational force that the Moon exerts on the shell would match the gravitational force that Earth exerts on the shell. If the shell just barely reaches that point then gravity will take over and pull it the rest of the way to the Moon. We set the forces equal to each other: $F_{Es}^G=F_{ms}^G\Rightarrow\frac{GM_E}{r_{Es}^2}=\frac{GM_m}{r_{ms}^2}=\frac{GM_m}{(r_{Em}-r_{Es})^2}$. Rearranging this we find $M_E(r_{Em}-r_{Es})^2=M_mr_{Es}^2\Rightarrow r_{Es}=$ $\frac{r_{Em}}{\left(1+\sqrt{\frac{M_m}{M_E}}\right)}=\frac{(3.84\times10^8\text{ m})}{\left(1+\sqrt{\frac{(7.3\times10^{22}\text{ kg})}{(5.97\times10^{24}\text{ kg})}}\right)}=3.46\times10^8\text{ m}$. Call this distance to the point at which the two forces cancel R_0. Thus, the shell must have sufficient initial kinetic energy to reach that distance from Earth. Now we use the conservation of energy to write $K_i+U_i^G=K_f+U_f^G\Rightarrow K_i+U_{\text{E shell,i}}^G+U_{\text{M shell,i}}^G=K_f+U_{\text{E shell,f}}^G+U_{\text{M shell,f}}^G$. Inserting expressions for the various energy types and letting the final kinetic energy go to zero, we are left with $\frac{1}{2}m_{\text{shell}}v_{\text{shell,i}}^2-\frac{Gm_Em_{\text{shell}}}{R_E}-\frac{Gm_Mm_{\text{shell}}}{R_{EM}-R_E}=-\frac{Gm_Em_{\text{shell}}}{R_0}-\frac{Gm_Mm_{\text{shell}}}{R_{EM}-R_0}$. Rearranging, we find

$$v_{\text{shell,i}} = \sqrt{2G\left(\frac{m_{\text{E}}}{R_{\text{E}}} + \frac{m_{\text{M}}}{R_{\text{EM}} - R_{\text{E}}} - \frac{m_{\text{E}}}{R_0} - \frac{m_{\text{M}}}{R_{\text{EM}} - R_0}\right)}$$

$$v_{\text{shell,i}} = \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{(5.97\times10^{24}\ \text{kg})}{(6.378\times10^6\ \text{m})} + \frac{(7.3\times10^{22}\ \text{kg})}{\left((3.84\times10^8\ \text{m}) - (6.378\times10^6\ \text{m})\right)} - \frac{(5.97\times10^{24}\ \text{kg})}{(3.46\times10^8\ \text{m})} - \frac{(7.3\times10^{22}\ \text{kg})}{\left((3.84\times10^8\ \text{m}) - (3.46\times10^8\ \text{m})\right)}\right)}$$
$$= 11\ \text{km/s}$$

(*b*) If the acceleration is constant, we can use basic kinematics: $\Delta x = \dfrac{v_{x,\text{f}}^2 - v_{x,\text{i}}^2}{2a_x} = \dfrac{(1.13\times10^4\ \text{m/s})^2}{12(9.8\ \text{m/s}^2)} = 1.1\times10^6\ \text{m}.$

(*c*) No, this method is not practical. It requires a gun barrel that is as long as a significant fraction of Earth's radius. It would also be very difficult to maintain a truly constant acceleration (impossible if fired like a gun), and a variation in accelerations of that size would likely kill the astronauts.

13.47. (*a*) Call the mass of the heavier sphere m_{h} and the mass of the lighter sphere m_{l}. Call the distance from the heavier lead sphere to the object d_{h}, and from the lighter sphere to the object d_{l}. The net force is $F = \dfrac{Gm_{\text{h}}m_{\text{obj}}}{d_{\text{h}}^2} - \dfrac{Gm_{\text{l}}m_{\text{obj}}}{d_{\text{l}}^2} = Gm_{\text{obj}}\left(\dfrac{m_{\text{h}}}{d_{\text{h}}^2} - \dfrac{m_{\text{l}}}{d_{\text{l}}^2}\right) = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.00\times10^{-3}\ \text{kg})\left(\dfrac{(8.00\times10^{-1}\ \text{kg})}{(0.0800\ \text{m})^2} - \dfrac{(2.00\times10^{-1}\ \text{kg})}{(0.0400\ \text{m})^2}\right) = 0.$

(*b*) $\dfrac{U^G}{m_{\text{obj}}} = -\dfrac{Gm_{\text{h}}}{d_{\text{h}}} - \dfrac{Gm_{\text{l}}}{d_{\text{l}}} = -G\left(\dfrac{m_{\text{h}}}{d_{\text{h}}} + \dfrac{m_{\text{l}}}{d_{\text{l}}}\right) = -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\dfrac{(8.00\times10^{-1}\ \text{kg})}{(0.0800\ \text{m})} + \dfrac{(2.00\times10^{-1}\ \text{kg})}{(0.0400\ \text{m})}\right)$

$$= -1.00\times10^{-9}\ \text{J/kg} = -1.00\times10^{-12}\ \text{J/g}$$

(*c*) The potential energy in the initial configuration is

$$U_{\text{i}}^G = -\frac{Gm_{\text{h}}m_{\text{obj}}}{d_{\text{h}}} - \frac{Gm_{\text{l}}m_{\text{obj}}}{d_{\text{l}}} = -Gm_{\text{obj}}\left(\frac{m_{\text{h}}}{d_{\text{h}}} + \frac{m_{\text{l}}}{d_{\text{l}}}\right)$$
$$= -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.00\times10^{-3}\ \text{kg})\left(\frac{(8.00\times10^{-1}\ \text{kg})}{(0.0800\ \text{m})} + \frac{(2.00\times10^{-1}\ \text{kg})}{(0.0400\ \text{m})}\right) = -1.00\times10^{-12}\ \text{J}$$

In the final configuration the potential energy is

$$U_{\text{i}}^G = -Gm_{\text{obj}}\left(\frac{m_{\text{h}}}{d_{\text{h}}} + \frac{m_{\text{l}}}{d_{\text{l}}}\right) = -(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.00\times10^{-3}\ \text{kg})\left(\frac{(8.00\times10^{-1}\ \text{kg})}{(0.0400\ \text{m})} + \frac{(2.00\times10^{-1}\ \text{kg})}{(0.0800\ \text{m})}\right)$$
$$= -1.50\times10^{-12}\ \text{J}$$

The work necessary is $W = U_{\text{f}}^G - U_{\text{i}}^G = -5.0\times10^{-13}\ \text{J}$, where the negative sign means that this could happen spontaneously. No energy input is required.

13.48. We get the potential energy by setting it equal to the work done by the gravitational force in moving the brick from an infinite distance to the center of Earth, by integrating the force over distance. Outside of Earth the gravitational force is $F_{\text{Eb}}^G = \dfrac{Gm_{\text{E}}m}{r^2}$, whereas inside Earth, the force is $F_{\text{Eb}}^G = \dfrac{Gm_{\text{E}}mr}{R_{\text{E}}^2}$. Thus $U^G =$ $\displaystyle\int_{\infty}^{R_{\text{E}}} \frac{Gm_{\text{E}}m}{r^2}dr + \int_{R_{\text{E}}}^{0} \frac{Gm_{\text{E}}mr}{R_{\text{E}}^3}dr = Gm_{\text{E}}m\left(-\frac{1}{R_{\text{E}}} - \frac{1}{2R_{\text{E}}}\right) = \frac{-3Gm_{\text{E}}m}{2R_{\text{E}}}.$

13.49. The approximation is valid for a given body b as long as $\Delta x \ll R_b$. Thus, because the Sun has a much larger radius than Earth, the approximation will be valid over a larger range on the Sun.

13.50. (*a*) A tiny segment of the rod should behave as any other particle. Thus $dU_g = -\dfrac{G(dm_{\text{rod}})m_{\text{ball}}}{(x_{\text{ball}} - x_{dm})}$. One may also choose to write this in terms of a differential length dx_{dm}, in which case one finds $-\dfrac{Gm_{\text{rod}}m_{\text{ball}}dx_{dm}}{\ell_{\text{rod}}(x_{\text{ball}} - x_{dm})}$.

(*b*) $U^G = \int_{\text{rod}} dU^G = \int_{-\ell_{\text{rod}}/2}^{\ell_{\text{rod}}/2} -\frac{Gm_{\text{rod}}m_{\text{ball}}dx_{dm}}{\ell_{\text{rod}}(x_{\text{ball}} - x_{dm})} = -\frac{Gm_{\text{rod}}m_{\text{ball}}}{\ell_{\text{rod}}}\ln\left[\frac{x_{\text{ball}} - \frac{\ell_{\text{rod}}}{2}}{x_{\text{ball}} + \frac{\ell_{\text{rod}}}{2}}\right]$. (*c*) By differentiating, we see $F_x =$ $-\frac{dU^G}{dx_{\text{ball}}} = \left(\frac{Gm_{\text{rod}}m_{\text{ball}}}{\ell_{\text{rod}}}\right)\frac{d}{dx_{\text{ball}}}\ln\left[\frac{x_{\text{ball}} - \frac{\ell_{\text{rod}}}{2}}{x_{\text{ball}} + \frac{\ell_{\text{rod}}}{2}}\right] = -\frac{4Gm_{\text{rod}}m_{\text{ball}}}{\ell_{\text{rod}}^2 - 4x_{\text{ball}}^2}$, such that $\vec{F}^{\text{G}}_{\text{rod ball}} = -\frac{4Gm_{\text{rod}}m_{\text{ball}}}{\ell_{\text{rod}}^2 - 4x_{\text{ball}}^2}\,\hat{i}$.

13.51. Elastic, since kinetic energy is conserved during the collision.

13.52. (*a*) We set the initial and final energies equal to each other. We note that if the particle reaches an infinite distance from the Sun, the final potential energy will be zero, and if the particle just barely escapes the final velocity is also zero. Thus $K_{\text{i}} + U_{\text{i}}^G = K_{\text{f}} + U_{\text{f}}^G = 0$, or $\frac{1}{2}mv_{\text{i}}^2 - \frac{Gm_{\text{S}}m}{R_{\text{S}}} = 0 \Rightarrow v_{\text{i}} = \sqrt{\frac{2Gm_{\text{S}}}{R_{\text{S}}}}$. Note, before inserting numbers, that this answer is independent of the mass of the particle. $v_{\text{i}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\ \text{kg})}{(7\times10^8\ \text{m})}} =$ 6×10^5 m/s. (*b*) As noted in (*a*), the answer is independent of the mass of the object. Thus the required initial speed is still 6×10^5 m/s.

13.53. We know that when the probe has the escape speed, it has just enough kinetic energy to escape Earth's gravity, meaning $K_{\text{esc}} + U_{\text{launch}}^G = 0$. If the probe leaves Earth at twice its escape speed, then it will have initial kinetic energy equal to $4K_{\text{esc}}$, and we can write $4K_{\text{esc}} + U_{\text{launch}}^G = K_{\text{f}} + U_{\text{f}}^G \Rightarrow \frac{1}{2}m_{\text{p}}v_{\text{p,f}}^2 = -3U_{\text{launch}}^G - U_{\text{f}}^G$. Since the probe is finally "very far" from Earth, we take the limit as the probe-Earth distance approaches zero, such that the final gravitational potential energy is zero. Thus we find $\frac{1}{2}m_{\text{p}}v_{\text{p,f}}^2 = 3\frac{Gm_{\text{E}}m_{\text{p}}}{R_{\text{E}}} \Rightarrow v_{\text{p,f}} = \sqrt{6\frac{Gm_{\text{E}}}{R_{\text{E}}}} =$ $\sqrt{\frac{6(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{(6.378\times10^6\ \text{m})}} = 19.4$ km/s.

13.54. It should be clear that an object on Earth is more energetically bound to Earth than an object on the Moon is energetically bound to the Moon. One might (correctly) jump from this fact to the conclusion that a trip from Earth to the Moon requires more energy than a trip from the Moon to Earth. However, here we make a more complete argument. There is a point along a line connecting the centers of Earth and the Moon at which the net gravitational force on an object is zero. Once an object passes that point (going either direction and with an arbitrarily small speed), the object can simply drift the rest of the way to its destination. We calculate the energy required to reach that point from either celestial body, and see which is lower. First we calculate the point at which the gravitational force due to the Moon and Earth cancel each other. Let us assume our object is a spaceship. We set the two gravitational forces exerted on the spaceship equal to each other: $F_{\text{Es}}^G = F_{\text{ms}}^G \Rightarrow \frac{GM_{\text{E}}}{r_{\text{Es}}^2} = \frac{GM_{\text{m}}}{r_{\text{ms}}^2} = \frac{GM_{\text{m}}}{(r_{\text{Em}} - r_{\text{Es}})^2}$. Rearranging this we find $M_{\text{E}}(r_{\text{Em}} - r_{\text{Es}})^2 = M_{\text{m}}r_{\text{Es}}^2 \Rightarrow r_{\text{Es}} = \frac{r_{\text{Em}}}{\left(1 + \sqrt{\frac{M_{\text{m}}}{M_{\text{E}}}}\right)} = \frac{(3.84\times10^8\ \text{m})}{\left(1 + \sqrt{\frac{(7.3\times10^{22}\ \text{kg})}{(5.97\times10^{24}\ \text{kg})}}\right)} = 3.46\times10^8\ \text{m}$. Call this distance to the point at which the two forces cancel R_0. Thus in order to make a trip to the Moon, the spaceship must have sufficient initial kinetic energy to reach that distance from Earth. Now we use the conservation of energy to determine how much energy (likely chemical potential energy in the fuel) is required to change the gravitational potential energy of the system. We write

$$E_{\text{chem}} = U^G_{\text{ship,f}} - U^G_{\text{ship,i}} = -\frac{Gm_{\text{E}}m_{\text{ship}}}{R_0} - \frac{Gm_{\text{M}}m_{\text{ship}}}{R_{\text{EM}} - R_0} + \frac{Gm_{\text{E}}m_{\text{ship}}}{R_{\text{E}}} + \frac{Gm_{\text{M}}m_{\text{ship}}}{R_{\text{EM}} - R_{\text{E}}} = Gm_{\text{ship}}\left(-\frac{m_{\text{E}}}{R_0} - \frac{m_{\text{M}}}{R_{\text{EM}} - R_0} + \frac{m_{\text{E}}}{R_{\text{E}}} + \frac{m_{\text{M}}}{R_{\text{EM}} - R_{\text{E}}}\right)$$

Inserting values we find

$$E_{\text{chem}} = Gm_{\text{ship}}\left(\frac{(5.97\times10^{24}\ \text{kg})}{(6.378\times10^{6}\ \text{m})} + \frac{(7.3\times10^{22}\ \text{kg})}{\left((3.84\times10^{8}\ \text{m}) - (6.378\times10^{6}\ \text{m})\right)} - \frac{(5.97\times10^{24}\ \text{kg})}{(3.46\times10^{8}\ \text{m})} - \frac{(7.3\times10^{22}\ \text{kg})}{\left((3.84\times10^{8}\ \text{m}) - (3.46\times10^{8}\ \text{m})\right)}\right)$$

$$= Gm_{\text{ship}}(9.2\times10^{17}\ \text{kg/m})$$

We can carry out the same process for a trip from the Moon to Earth. We need only change the initial conditions such that the object is initially on the surface of the Moon. This yields

$$E_{\text{chem}} = U^G_{\text{ship,f}} - U^G_{\text{ship,i}} = -\frac{Gm_{\text{E}}m_{\text{ship}}}{R_0} - \frac{Gm_{\text{M}}m_{\text{ship}}}{R_{\text{EM}} - R_0} + \frac{Gm_{\text{M}}m_{\text{ship}}}{R_{\text{M}}} + \frac{Gm_{\text{E}}m_{\text{ship}}}{R_{\text{EM}} - R_{\text{M}}} = Gm_{\text{ship}}\left(-\frac{m_{\text{E}}}{R_0} - \frac{m_{\text{M}}}{R_{\text{EM}} - R_0} + \frac{m_{\text{M}}}{R_{\text{M}}} + \frac{m_{\text{E}}}{R_{\text{EM}} - R_{\text{M}}}\right)$$

or

$$E_{\text{chem}} = Gm_{\text{ship}}\left(\frac{(7.3\times10^{22}\ \text{kg})}{(1.738\times10^{6}\ \text{m})} + \frac{(5.97\times10^{24}\ \text{kg})}{\left((3.84\times10^{8}\ \text{m}) - (1.738\times10^{6}\ \text{m})\right)} - \frac{(5.97\times10^{24}\ \text{kg})}{(3.46\times10^{8}\ \text{m})} - \frac{(7.3\times10^{22}\ \text{kg})}{\left((3.84\times10^{8}\ \text{m}) - (3.46\times10^{8}\ \text{m})\right)}\right)$$

$$= Gm_{\text{ship}}(3.9\times10^{16}\ \text{kg/m})$$

Clearly, the trip from Earth to the Moon requires more fuel by more than an order of magnitude.

13.55. The satellite is in very low orbit, meaning that its radial distance from the center of the moon is approximately the radius of the moon R_{moon}. We can find an expression for this speed by considering that it is the gravitational force that keeps the projectile moving in uniform circular motion: $\frac{Gm_{\text{moon}}m_{\text{p}}}{R^2_{\text{moon}}} = \frac{m_{\text{p}}v^2}{R_{\text{moon}}} \Rightarrow v = \sqrt{\frac{Gm_{\text{moon}}}{R_{\text{moon}}}}$. Now we can use energy conservation to determine the final height if the projectile is launched upward. We have $K_{\text{i}} + U^G_{\text{i}} = U^G_{\text{f}} \Rightarrow \frac{1}{2}m_{\text{p}}v^2 - \frac{Gm_{\text{moon}}m_{\text{p}}}{R_{\text{moon}}} = \frac{Gm_{\text{moon}}m_{\text{p}}}{r_{\text{f}}}$, where we have used that the final speed of the projectile at its maximum height is zero. Rearranging, we have $\frac{Gm_{\text{moon}}m_{\text{p}}}{2R_{\text{moon}}} - \frac{Gm_{\text{moon}}m_{\text{p}}}{R_{\text{moon}}} = \frac{Gm_{\text{moon}}m_{\text{p}}}{r_{\text{f}}} \Rightarrow r_{\text{f}} = 2R_{\text{moon}}$. Thus the projectile launched vertically at speed v will reach a distance $2R_{\text{moon}}$ from the moon's center, or a height of R_{moon} above the surface of the moon.

13.56. We know we can write the total energy of the system as $K + U^G = \frac{1}{2}m_{\text{s}}v_{\text{s}}^2 - \frac{Gm_{\text{E}}m_{\text{s}}}{r} = \frac{L^2}{2m_{\text{s}}r^2} - \frac{Gm_{\text{E}}m_{\text{s}}}{r}$. As the position of the satellite changes, this total energy must remain constant, as must the angular momentum. Thus, equating the energies at apogee and perigee yields $\frac{L^2}{2m_{\text{s}}r_{\text{a}}^2} - \frac{Gm_{\text{E}}m_{\text{s}}}{r_{\text{a}}} = \frac{L^2}{2m_{\text{s}}r_{\text{p}}^2} - \frac{Gm_{\text{E}}m_{\text{s}}}{r_{\text{p}}}$, which is quadratic in r_{a}. Solving, using the quadratic equation, we find the two solutions are $r_{\text{a}} = r_{\text{p}}$ (circular orbit) and $r_{\text{a}} = \frac{-L^2r_{\text{p}}}{L^2 - 2Gm_{\text{E}}m_{\text{s}}^2r_{\text{p}}}$. We use the latter, nontrivial solution. The distance above the ground will, of course, be

$$h = r_{\text{a}} - R_{\text{E}} = \frac{-L^2r_{\text{p}}}{L^2 - 2Gm_{\text{E}}m_{\text{s}}^2r_{\text{p}}} - R_{\text{E}} = \frac{-v_{\text{s,p}}^2r_{\text{p}}^3}{v_{\text{s,p}}^2r_{\text{p}}^2 - 2Gm_{\text{E}}r_{\text{p}}} - R_{\text{E}}$$

$$h = \frac{-(8032\ \text{m/s})^2\left((6.378\times10^{6}\ \text{m}) + (1.12\times10^{5}\ \text{m})\right)^3}{(8032\ \text{m/s})^2\left((6.378\times10^{6}\ \text{m}) + (1.12\times10^{5}\ \text{m})\right)^2 - 2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})\left((6.378\times10^{6}\ \text{m}) + (1.12\times10^{5}\ \text{m})\right)} - (6.378\times10^{6}\ \text{m})$$

$$= 816\ \text{km}$$

13.57. For a parabolic approach, we know the total mechanical energy is zero. Thus $\frac{1}{2}m_c v_c^2 - \frac{Gm_S m_c}{R_M} = 0 \Rightarrow v_c = \sqrt{\frac{2Gm_S}{R_M}}$. Since Mercury is held in (assume circular) orbit by its gravitational attraction to the Sun, we can write $\frac{Gm_S m_M}{R_M^2} = \frac{m_M v_M^2}{R_M} \Rightarrow v_M = \sqrt{\frac{Gm_S}{R_M}}$. Comparing the two speeds, we see $v_c/v_M = \sqrt{2}$.

13.58. Call the objects 1 and 2, their masses m_1 and m_2, and the distance between them ℓ. Call the distance from object 1 to the system's center of mass r_1 and the distance from object 2 to the center of mass r_2, so that $r_1 + r_2 = \ell$. The gravitational potential energy of the system is

$$U^G = -\frac{Gm_1 m_2}{\ell} \tag{1}$$

For the system's kinetic energy, we note that the objects are held in place by the gravitational force they exert on each other. This means that this force supplies the centripetal force necessary to hold them in circular orbits. Thus the centripetal force exerted on each object, $m_1 v_1^2/r_1$ and $m_2 v_2^2/r_2$, is equal to the gravitational force:

$$\frac{m_1 v_1^2}{r_1} = \frac{Gm_1 m_2}{\ell^2} = \frac{m_2 v_2^2}{r_2}$$

This equality gives the squares of the speeds: $v_1^2 = Gm_2 r_1/\ell^2$, $v_2^2 = Gm_1 r_2/\ell^2$. Inserting these expressions into the equation for kinetic energy yields

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{Gm_1 m_2}{2\ell^2}(r_1 + r_2) = \frac{Gm_1 m_2}{2\ell} \tag{2}$$

The energy of the system is:

$$E = U^G + K = -\frac{Gm_1 m_2}{\ell} + \frac{Gm_1 m_2}{2\ell} = -\frac{Gm_1 m_2}{2\ell} \tag{3}$$

Comparing Eqs. 1, 2, and 3 shows that $E = U/2 = -K$.

13.59. Note first that this attractive force could very well be gravity, assuming $C = Gm_{\text{source}}$. We proceed to use results from Chapter 13 as though this is indeed the gravitational force. The mathematical results would not change, however, if the constant had a different value. In Example 13.5, equation (2) gives us an expression for the angular momentum of an object in orbit: $L^2 = Gm^2 m_{\text{source}} a(1-e^2) = Cm^2 a(1-e^2)$. We can obtain a similar condition by simply considering the angular momentum described at apogee (maximal object-source separation): $\left(L_{\text{apogee}}^2 = m^2 v_{\text{apogee}}^2 r_{\text{apogee}}^2 = m^2 v_{\text{apogee}}^2 a^2 (1+e)^2\right)$. Equating the two expressions yields

$$\frac{(1+e)}{(1-e)} = \frac{C}{a v_{\text{apogee}}^2} \tag{1}$$

We also know that the angular momentum must be constant, so we can equate the square of the angular momentum at apogee (maximal object-source separation): $\left(L_{\text{apogee}}^2 = m^2 v_{\text{apogee}}^2 r_{\text{apogee}}^2 = m^2 v_{\text{apogee}}^2 a^2 (1+e)^2\right)$ and perigree (minimal object-source separation): $L_{\text{perigree}}^2 = m^2 v_{\text{perigree}}^2 r_{\text{perigree}}^2 = m^2 v_{\text{perigree}}^2 a^2 (1-e)^2$, to obtain the condition

$$v_{\text{perigree}} = \frac{(1+e)}{(1-e)} v_{\text{apogee}} \tag{2}$$

Inserting equation (1) into equation (2) yields $v_{\text{perigree}} = \frac{C}{a v_{\text{apogee}}} = \frac{C}{a}\sqrt{\frac{2a}{C}} = \sqrt{\frac{2C}{a}}$.

13.60. We use energy conservation and allow that the particle's final distance from the Moon (and Earth) will be so large that the gravitational potential energy will be approximately zero, and that the particle's final speed can be arbitrarily close to zero also. Thus $K_\text{i} + U_\text{i}^G = K_\text{f} + U_\text{f}^G = 0 \Rightarrow \frac{1}{2}mv_\text{i}^2 = \frac{Gm_\text{M}m}{R_\text{M}} + \frac{Gm_\text{E}m}{R_\text{EM}}$ or

$$v_\text{i} = \sqrt{\frac{2Gm_\text{M}}{R_\text{M}} + \frac{2Gm_\text{E}}{R_\text{EM}}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.3\times10^{22}\ \text{kg})}{(1.738\times10^6\ \text{m})} + \frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{(3.84\times10^8\ \text{m})}}$$
$$= 2.8\ \text{km/s}$$

13.61. We use conservation of energy and we assume that the initial speed is negligible. Note that 19 Earth radii above Earth's surface is 20 Earth radii from Earth's center (which is the relevant quantity). Thus

$$U_\text{i}^G = K_\text{f} + U_\text{f}^G \Rightarrow \frac{1}{2}mv_\text{i}^2 = -\frac{Gm_\text{E}m}{20R_\text{E}} + \frac{Gm_\text{E}m}{2R_\text{E}} \Rightarrow v_\text{f} = \sqrt{\left(\frac{1}{2} - \frac{1}{20}\right)\frac{2Gm_\text{E}}{R_\text{E}}}$$
$$= \sqrt{\left(\frac{1}{2} - \frac{1}{20}\right)\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{(6.378\times10^6\ \text{m})}} = 7.5\ \text{km/s}$$

13.62. (*a*) We use energy conservation and allow that the probe's final distance from Earth will be so large that the gravitational potential energy will be approximately zero, and that the probe's final speed can be arbitrarily close to zero also. Thus $K_\text{i} + U_\text{i}^G = K_\text{f} + U_\text{f}^G = 0 \Rightarrow \frac{1}{2}m_\text{p}v_\text{i}^2 = \frac{Gm_\text{E}m_\text{p}}{R_\text{E} + h}$ or $v_\text{i} = \sqrt{\frac{2Gm_\text{E}}{R_\text{E} + h}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{(6.378\times10^6\ \text{m}) + (1.80\times10^5\ \text{m})}} = 11.0\ \text{km/s}$. (*b*) We proceed as in part (*a*), but this time with the probe initially on Earth's surface. We have $K_\text{i} + U_\text{i}^G = K_\text{f} + U_\text{f}^G = 0 \Rightarrow \frac{1}{2}m_\text{p}v_\text{i}^2 = \frac{Gm_\text{E}m_\text{p}}{R_\text{E}}$ or $v_\text{i} = \sqrt{\frac{2Gm_\text{E}}{R_\text{E}}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{(6.378\times10^6\ \text{m})}} = 11.1\ \text{km/s}$.

13.63. (*a*) The stars move in uniform circular motion which is maintained by their mutual gravitational attraction. So we can write $\frac{Gm_\text{star}^2}{(2R_\text{star})^2} = \frac{m_\text{star}v_\text{star}^2}{R_\text{star}} \Rightarrow v_\text{star} = \sqrt{\frac{Gm_\text{star}}{4R_\text{star}}}$ or $\omega = \frac{v_\text{star}}{R_\text{star}} = \sqrt{\frac{Gm_\text{star}}{4R_\text{star}^3}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.0\times10^{30}\ \text{kg})}{4(1.0\times10^{11}\ \text{m})^3}} = 2.2\times10^{-13}\ \text{s}^{-1}$. (*b*) We use energy conservation and allow that the meteoroid's final distance from the stars will be so large that the gravitational potential energy will be approximately zero, and that the meteoroid's final speed can be arbitrarily close to zero also. Thus $K_\text{i} + U_\text{i}^G = K_\text{f} + U_\text{f}^G = 0 \Rightarrow \frac{1}{2}m_\text{meteoroid}v_\text{i}^2 = \frac{2Gm_\text{star}m_\text{meteoroid}}{R_\text{star}}$ or $v_\text{i} = \sqrt{\frac{4Gm_\text{star}}{R_\text{star}}} = \sqrt{\frac{4(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.0\times10^{30}\ \text{kg})}{(1.0\times10^{11}\ \text{m})}} = 89\ \text{km/s}$.

13.64. Kepler's second law says that a straight line from the Sun to any planet sweeps out the same area in any given time interval. Consider a very small time interval, such that a section of area traced by such a line is roughly triangular in shape. More precisely: $\delta A = \frac{1}{2}r_\text{planet}(v_\text{planet}\delta t) \Rightarrow \frac{dA}{dt} = \frac{r_\text{planet}v_\text{planet}}{2} = \frac{L_\text{planet}}{2m_\text{planet}}$. Using the expression for the angular momentum in equation (2) of Example 13.5, we can write this rate as

$$\frac{dA}{dt} = \frac{1}{2m_\text{planet}}\sqrt{Gm_\text{planet}^2 m_\text{Sun} a(1-e^2)} \qquad (1)$$

Clearly, the period of time required for one full revolution of the planet can be written as $T = \frac{A}{dA/dt}$. Inserting the expression for the area of an ellipse, we have $T = \frac{\pi ab}{dA/dt} = \frac{\pi a^2\sqrt{1-e^2}}{dA/dt}$. Finally, inserting the rate given by equation (1), we have

$$T = \frac{2m_{\text{planet}}\pi a^2\sqrt{1-e^2}}{\sqrt{Gm_{\text{planet}}^2 m_{\text{Sun}} a(1-e^2)}} = \frac{2m_{\text{planet}}\pi a^{3/2}\sqrt{1-e^2}}{\sqrt{Gm_{\text{planet}}^2 m_{\text{Sun}}(1-e^2)}} = \frac{2\pi a^{3/2}}{\sqrt{Gm_{\text{Sun}}}} \quad (2)$$

Squaring equation (2) we find

$$T^2 = \frac{4\pi^2 a^3}{Gm_{\text{Sun}}}$$

which shows the required proportionality.

13.65. Call the distance from object one to the center of mass r_1, and call the distance from object two to the center of mass r_2. Clearly $r_1 + r_2 = d$. Let us define a coordinate frame with its origin at the center of mass. Then $r_{\text{cm}} = \frac{r_1 m_1 - r_2 m_2}{m_1 + m_2} = 0 \Rightarrow r_2 = \left(\frac{m_1}{m_2}\right) r_1$. Thus

$$r_1 + \left(\frac{m_1}{m_2}\right) r_1 = d \Rightarrow r_1 = \frac{m_2 d}{m_1 + m_2} \quad (1)$$

Let us write the sum of all forces on object one in the radial direction:

$$\sum F_{1,r} = \frac{Gm_1 m_2}{d^2} = \frac{m_1 v_1^2}{r_1} \Rightarrow v_1 = \sqrt{\frac{Gm_2 r_1}{d^2}} \quad (2)$$

But we also know that the speed in orbit is the distance over the time, or

$$v_1 = \frac{2\pi r_1}{T} \Rightarrow T = \frac{2\pi r_1}{v_1} \quad (3)$$

Squaring eq. (3) and inserting the expression for the speed from eq. (2) yields

$$T^2 = \frac{4\pi^2 d^2 r_1}{Gm_2}$$

Now inserting equation (1) yields

$$T^2 = \frac{4\pi^2 d^3}{(m_1 + m_2)G}$$

This was the relationship that was to be shown.

13.66. (*a*) From Example 13.5 we know

$$E_{\text{mech}} = -\frac{Gm_{\text{sun}} m_{\text{probe}}}{2a} = -\frac{Gm_{\text{sun}} m_{\text{probe}}}{a_{\text{Earth}} + a_{\text{Mars}}} = -\frac{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times 10^{30}\ \text{kg})(1.00\times 10^3\ \text{kg})}{(1.50\times 10^{11}\ \text{m}) + (2.28\times 10^{11}\ \text{m})} = -3.51\times 10^{11}\ \text{J}.$$

(*b*) Setting $E_{\text{mech}} = K + U^G \Rightarrow \frac{1}{2} m_{\text{probe}} v_{\text{probe}}^2 = E_{\text{mech}} + \frac{Gm_{\text{Sun}} m_{\text{probe}}}{r}$ or $v_{\text{probe}} = \sqrt{\left(\frac{2E_{\text{mech}}}{m_{\text{probe}}} + \frac{2Gm_{\text{Sun}}}{r}\right)}$.

(*c*) The probe enters the transfer orbit at the position of Earth, thus $v_{\text{probe}} = \sqrt{\left(\frac{2E_{\text{mech}}}{m_{\text{probe}}} + \frac{2Gm_{\text{Sun}}}{R_{\text{Earth Sun}}}\right)} = \sqrt{\left(\frac{2(-3.51\times 10^{11}\ \text{J})}{(1.00\times 10^3\ \text{kg})} + \frac{2(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times 10^{30}\ \text{kg})}{(1.50\times 10^{11}\ \text{m})}\right)} = 3.27\times 10^4\ \text{m/s}.$

(*d*) The process is the same as in part (*c*), but now the probe reaches Mars, such that $v_{\text{probe}} = \sqrt{\left(\frac{2E_{\text{mech}}}{m_{\text{probe}}} + \frac{2Gm_{\text{Sun}}}{R_{\text{Mars Sun}}}\right)} = \sqrt{\left(\frac{2(-3.51\times10^{11}\ \text{J})}{(1.00\times10^3\ \text{kg})} + \frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(2.28\times10^{11}\ \text{m})}\right)} = 21.5\times10^4\ \text{m/s}.$

(*e*) Assuming launch in directions of Earth's solar orbit $v_{\text{additional}} = v_{\text{transfer}} - v_{\text{Earth}} = (3.27\times10^4\ \text{m/s}) - (2.98\times10^4\ \text{m/s}) = 2.9\ \text{km/s}.$ (*f*) The probe must be able to break free of Earth and still have the required speed for the transfer orbit. Assuming launch in directions of Earth's solar orbit $v_{\text{launch}} = v_{\text{escape}} + v_{\text{transfer}} - v_{\text{Earth}} = (1.11\times10^4\ \text{m/s}) + (3.27\times10^4\ \text{m/s}) - (2.98\times10^4\ \text{m/s}) = 14\ \text{km/s}.$

13.67. To the right in the figure.

13.68. No, this is not valid. To see why, consider the limit in which almost all of the sphere's mass is concentrated at the location on the sphere closest to you, with the remainder of the sphere holding virtually no mass.

13.69. Any tiny segment of the ring having mass dm_{ring} would exert an attractive force on the object $dF = \frac{Gdm_{\text{ring}}m_o}{r^2}$. The magnitudes of all such forces will not add, because the forces are at different angles. If we call the dashed axis of symmetry the x axis with the $+x$ axis pointing to the right, then $dF_x = -\frac{Gdm_{\text{ring}}m_o s}{r^3}$. Now we can integrate over the ring to find the total attractive force $F_x = \int_{\text{ring}} dF_x = -\int_{\text{ring}} \frac{Gdm_{\text{ring}}m_o s}{r^3} = -\frac{Gm_{\text{ring}}m_o s}{2\pi R(R^2+s^2)^{3/2}} \int_0^{2\pi R} d\ell = -\frac{Gm_{\text{ring}}m_o s}{(R^2+s^2)^{3/2}}$. Thus $F^G_{\text{ring o}} = \frac{Gm_{\text{ring}}m_o s}{(R^2+s^2)^{3/2}}$.

13.70. (*a*) A spherical shell will exert no force on an object inside the shell. But an object outside the shell will feel the same force due to the shell that it would feel if all the mass of the shell were concentrated at the shell's center. In this case, when $x = 3.0R$, the attractive forces on the object due to the two shells will be in the same direction, so we can simply add the magnitudes of the forces. Thus $\sum F^G_{\text{obj}}(x = 3.0R) = \frac{G(3m_{\text{inner}})m_{\text{obj}}}{(3.0R)^2} + \frac{Gm_{\text{inner}}m_{\text{obj}}}{(2.2R)^2} = (0.54)\left(\frac{Gm_{\text{inner}}m_{\text{obj}}}{R^2}\right)$. (*b*) When $x = 1.9R$ the object is inside the outer shell, and so the object feels no force due to the outer spherical shell. Thus $\sum F^G_{\text{obj}}(x = 3.0R) = \frac{Gm_{\text{inner}}m_{\text{obj}}}{(1.1R)^2} = (0.83)\left(\frac{Gm_{\text{inner}}m_{\text{obj}}}{R^2}\right)$. (*c*) When $x = 0.9R$, the object is inside both shells, so neither exerts a force on the object. The sum of all forces in this case is zero.

13.71. (*a*) Consider the first the gravitational force that a ring of radius r and width dr would exert on the particle. The distance from any segment of the ring of length $d\ell$ (with mass $dm = \frac{m_{\text{ring}}}{2\pi r}(d\ell)$) to the particle would be $d = \sqrt{r^2 + y^2}$. So the gravitational force that any such segment exerts on the particle is $F^G_{\text{segment particle}} = \frac{Gm_{\text{ring}}m_{\text{part}}}{2\pi r(r^2+y^2)}(d\ell)$. To find the force the ring as a whole exerts on the particle, we cannot simply integrate because all the contributions from all segments are not in the same direction. Note that contributions from various segments in the radial direction will cancel and contributions in the $-y$ direction will add. Thus $F^G_{\text{ring particle},x} = \int_{\text{ring}} F^G_{\text{segment part},x} = -\int_0^{2\pi r} \frac{Gm_{\text{ring}}m_{\text{part}}y(d\ell)}{2\pi r(r^2+y^2)^{3/2}} = -\frac{Gm_{\text{ring}}m_{\text{part}}y}{(r^2+y^2)^{3/2}}$. Now, consider that the disk is made up of a continuum of many such rings, with $m_{\text{ring}} = \sigma A_{\text{ring}} = \left(\dfrac{m_{\text{disk}}}{A_{\text{disk}}}\right) A_{\text{ring}} = \left(\dfrac{m_{\text{disk}}}{\pi R^2}\right) 2\pi r dr.$ We integrate the expression for the force due to a ring of radius r, letting r run from 0 to R. $\vec{F}^G_{\text{disk particle}} = -\dfrac{2Gm_{\text{disk}} m_{\text{part}} y}{R^2} \displaystyle\int_0^R \dfrac{(rdr)}{(r^2+y^2)^{3/2}}\, \hat{j} =$ $-\dfrac{2Gm_{\text{disk}} m_{\text{part}} y}{R^2}\left(\dfrac{1}{y} - \dfrac{1}{\sqrt{R^2 - y^2}}\right)\hat{j}$ or $F^G_{\text{disk particle}} = \dfrac{2Gm_{\text{disk}} m_{\text{part}}}{R^2}\left(1 - \dfrac{y}{\sqrt{R^2+y^2}}\right).$ (*b*) Rewrite the expression from part (*a*) in terms of a dimensionless parameter $\varepsilon \equiv R/y$: $\dfrac{2Gm_{\text{part}} m_{\text{disk}} y}{R^3}\left(\varepsilon - \dfrac{1}{\sqrt{1+\varepsilon^{-2}}}\right).$ To obtain an expression showing how the force magnitude behaves as y becomes much larger than R, Taylor expand the expression in parentheses around $\varepsilon = 0$ and obtain $\dfrac{2Gm_{\text{part}} m_{\text{disk}} y}{R^3}\left(\dfrac{\varepsilon^3}{2} + O[\varepsilon^5]\right).$ In the limit as ε becomes very small, you can ignore terms of order ε^5 and keep only the first term in the parentheses. This yields for the gravitational force exerted by the disk on the particle $\dfrac{2Gm_{\text{part}} m_{\text{disk}} y}{R^3}\left(\dfrac{R^3}{y^3 2}\right) = \dfrac{Gm_{\text{part}} m_{\text{disk}}}{y^2}$ which is the expression for two particles separated by a distance y.

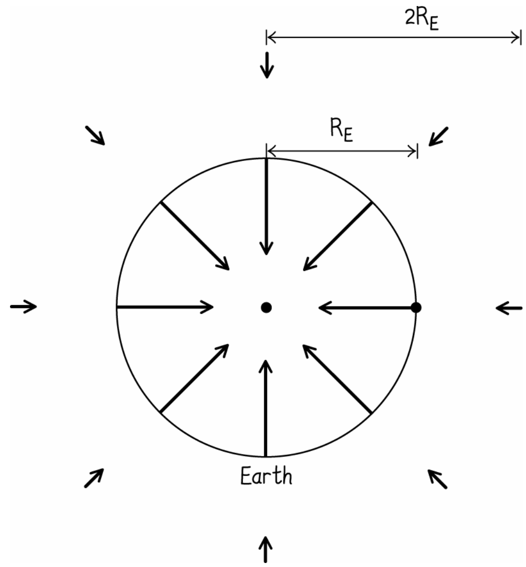
13.72. There would be $(N-1)N/2$ terms. To see this, note that every one of the N particles interacts with its $N-1$ neighbors, meaning there are $N(N-1)$ potential energies to consider. But that double-counts interactions. We have counted U^G_{1n} when we considered all the $N-1$ neighbors that particle 1 interacts with, and we counted U^G_{1n} (or U^G_{n1} depending on your labeling convention) a second time when we considering all the $N-1$ neighbors that particle n interacts with. But there is really only one such (U^G_{1n}) potential energy term. Thus we need a factor of $1/2$ to remove this double-counting.

13.73. From a theoretical standpoint, it does not matter in terms of the changes in gravitational potential energy. However, there may be other logistical considerations like how much atmosphere you must traverse before reaching space where air resistance is negligible. With these secondary considerations it is best to launch vertically.

13.74. $g_{\text{Venus}} = \dfrac{F^G_{\text{Venus obj}}}{m_{\text{obj}}} = \dfrac{Gm_{\text{Venus}}}{R^2_{\text{Venus}}} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.87\times10^{24}\ \text{kg})}{(6.052\times10^6\ \text{m})^2} = 8.87\ \text{m/s}^2$

13.75. The total acceleration is due to the gravitational attraction between the satellite and the source at one focus. Thus the total acceleration always points from the orbital position toward one focus. Let θ be the angle between a line from the focus to the orbital position, and a line tangential to the orbit. Then $a\cos(\theta) = a_t$, and $a\sin(\theta) = a_\perp$. Clearly if $\tan(\theta) < 1$, then $a_t < a_\perp$. The minimum angle between a foci line and a the tangent to the orbit occurs when the satellite is crossing the semiminor axis. At this point $\tan(\theta) = \dfrac{b}{ae} = \dfrac{\sqrt{1-e^2}}{e}.$ If the criterion $\tan(\theta) < 1$ is not satisfied when the satellite crosses the semiminor axis, then it is never satisfied. Thus, we require $\dfrac{\sqrt{1-e^2}}{e} < 1 \Rightarrow e > \dfrac{1}{\sqrt{2}}.$ Thus we see the tangential acceleration could be greater than the radial acceleration when it is along the semi-minor axis, provided that $e^2 > 1/2$. If the eccentricity is greater than one half, then this condition could be satisfied elsewhere in the orbit. It can never be satisfied though, when the satellite is along a semi-major axis.

13.76. (*a*)



(*b*) $\vec{F}^G_{\text{Eo}}(\vec{r}) = m_\text{o}\vec{g}(\vec{r})$

13.77. Let us assume $R_\text{planetoid} \approx 5$ m. Then we require $F^G_\text{Earth person} = F^G_\text{planetoid person}$ for a person on the surface of either object. However, with such a tiny radius, there will be a significant difference between the force of gravity acting at the person's feet and at the person's head. Here, we require that the gravitational force exerted on the person's center of mass (roughly 6 m from the planetoid center) match that on Earth. Call this distance $R_\text{planetoid person} = 6$ m. Thus $\dfrac{Gm_\text{Earth}m_\text{person}}{R^2_\text{Earth}} = \dfrac{Gm_\text{planetoid}m_\text{person}}{R^2_\text{planetoid person}} \Rightarrow m_\text{planetoid} = \left(\dfrac{R^2_\text{planetoid person}}{R^2_\text{Earth}}\right)m_\text{Earth}$. Dividing by the volume of the planetoid we have

$$\rho_\text{planetoid} = \frac{m_\text{planetoid}}{V_\text{planetoid}} = \frac{3}{4\pi R^3_\text{planetoid person}}\left(\frac{R^2_\text{planetoid person}}{R^2_\text{Earth}}\right)m_\text{Earth} = \frac{3m_\text{Earth}}{4\pi R_\text{planetoid person}R^2_\text{Earth}} = \frac{3(5.97\times10^{24}\text{ kg})}{4\pi(6\text{ m})(6.378\times10^6\text{ m})^2}$$
$$= 6\times10^9\text{ kg/m}^3$$

13.78. (*a*) We calculate both quantities and compare. The energy required per kilogram to move the satellite to the orbital position is

$$\frac{\Delta E_\text{launch}}{m_\text{sat}} = \frac{U^G_\text{f}}{m_\text{sat}} - \frac{U^G_\text{i}}{m_\text{sat}} = Gm_\text{Earth}\left(-\frac{1}{R_\text{Earth}+h} + \frac{1}{R_\text{Earth}}\right)$$
$$= (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})\left(-\frac{1}{(6.378\times10^6\text{ m})+(1.60\times10^6\text{ m})} + \frac{1}{(6.378\times10^6\text{ m})}\right)$$
$$= 1.25\times10^7\text{ J/kg}$$

In order for the satellite to be in a stable orbit, we require $\dfrac{Gm_\text{Earth}m_\text{sat}}{R^2_\text{orbit}} = \dfrac{m_\text{sat}v^2_\text{sat}}{R_\text{orbit}} \Rightarrow \dfrac{1}{2}m_\text{sat}v^2_\text{sat} = \dfrac{Gm_\text{Earth}m_\text{sat}}{2R_\text{orbit}}$, meaning the energy required to get the satellite up to orbital speed is $\dfrac{\Delta E_\text{orbit}}{m_\text{sat}} = \dfrac{Gm_\text{Earth}}{2R_\text{orbit}} = \dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{2((6.378\times10^6\text{ m})+(1.60\times10^6\text{ m}))} = 2.50\times10^7\text{ J/kg}$. Thus more energy is required to put the satellite into orbit once it is already at the appropriate position than is required to get the satellite up to the appropriate height.
(*b*) We proceed exactly as in part (*a*), but with a different height. The energy required per kilogram to move the satellite to the orbital position is

$$\frac{\Delta E_{\text{launch}}}{m_{\text{sat}}}=\frac{U_{\text{f}}^{G}}{m_{\text{sat}}}-\frac{U_{\text{i}}^{G}}{m_{\text{sat}}}=Gm_{\text{Earth}}\left(-\frac{1}{R_{\text{Earth}}+h}+\frac{1}{R_{\text{Earth}}}\right)$$
$$=(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})\left(-\frac{1}{(6.378\times10^6\ \text{m})+(3.20\times10^6\ \text{m})}+\frac{1}{(6.378\times10^6\ \text{m})}\right)$$
$$=2.09\times10^7\ \text{J/kg}$$

In order for the satellite to be in a stable orbit, we require $\frac{Gm_{\text{Earth}}m_{\text{sat}}}{R_{\text{orbit}}^2}=\frac{m_{\text{sat}}v_{\text{sat}}^2}{R_{\text{orbit}}}\Rightarrow\frac{1}{2}m_{\text{sat}}v_{\text{sat}}^2=\frac{Gm_{\text{Earth}}m_{\text{sat}}}{2R_{\text{orbit}}}$, meaning the energy required to get the satellite up to orbital speed is $\frac{\Delta E_{\text{orbit}}}{m_{\text{sat}}}=\frac{Gm_{\text{Earth}}}{2R_{\text{orbit}}}=\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{2((6.378\times10^6\ \text{m})+(3.20\times10^6\ \text{m}))}=2.08\times10^7\ \text{J/kg}$. These energies are nearly the same, but slightly more energy is required to get the satellite up to the appropriate height, than is required to put the satellite in orbit once it is at the appropriate height. (*c*) The process is the same as in parts (*a*) and (*b*). The energy required per kilogram to move the satellite to the orbital position is

$$\frac{\Delta E_{\text{launch}}}{m_{\text{sat}}}=\frac{U_{\text{f}}^{G}}{m_{\text{sat}}}-\frac{U_{\text{i}}^{G}}{m_{\text{sat}}}=Gm_{\text{Earth}}\left(-\frac{1}{R_{\text{Earth}}+h}+\frac{1}{R_{\text{Earth}}}\right)$$
$$=(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})\left(-\frac{1}{(6.378\times10^6\ \text{m})+(4.80\times10^6\ \text{m})}+\frac{1}{(6.378\times10^6\ \text{m})}\right)$$
$$=2.68\times10^7\ \text{J/kg}$$

In order for the satellite to be in a stable orbit, we require $\frac{Gm_{\text{Earth}}m_{\text{sat}}}{R_{\text{orbit}}^2}=\frac{m_{\text{sat}}v_{\text{sat}}^2}{R_{\text{orbit}}}\Rightarrow\frac{1}{2}m_{\text{sat}}v_{\text{sat}}^2=\frac{Gm_{\text{Earth}}m_{\text{sat}}}{2R_{\text{orbit}}}$, meaning the energy required to get the satellite up to orbital speed is $\frac{\Delta E_{\text{orbit}}}{m_{\text{sat}}}=\frac{Gm_{\text{Earth}}}{2R_{\text{orbit}}}=\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{2((6.378\times10^6\ \text{m})+(4.80\times10^6\ \text{m}))}=1.78\times10^7\ \text{J/kg}$. Thus, more energy is required to get the satellite up to the appropriate height, than is required to put the satellite in orbit once it is at the appropriate height.

13.79. We require that the force you experience at a distance r from the center of Jupiter be equal to the force you experience on the surface of Earth: $\frac{Gm_{\text{Jupiter}}m_{\text{you}}}{r^2}=\frac{Gm_{\text{Earth}}m_{\text{you}}}{R_E^2}\Rightarrow r=R_{\text{E}}\sqrt{\frac{m_{\text{Jupiter}}}{m_{\text{Earth}}}}$. Thus your height above Jupiter's surface would be

$$h=r-R_{\text{Jupiter}}=R_{\text{E}}\sqrt{\frac{m_{\text{Jupiter}}}{m_{\text{Earth}}}}-R_{\text{Jupiter}}=(6.378\times10^6\ \text{m})\sqrt{\frac{(1.90\times10^{27}\ \text{kg})}{(5.97\times10^{24}\ \text{kg})}}-(7.149\times10^7\ \text{m})=4.23\times10^7\ \text{m}$$

13.80. Call a time at which the asteroid is very far away and moving very slowly time 0. Call the moment the asteroid passes the moon time 1, and call the moment of impact time 2. The speed of the asteroid as it passes the moon can be found by simple conservation of energy: $K_0+U_0^G=0=K_1+U_1^G\Rightarrow\frac{1}{2}m_{\text{ast}}v_{\text{ast 1}}^2=\frac{Gm_{\text{Earth}}m_{\text{ast}}}{R_{\text{Earth-moon}}}\Rightarrow v_{\text{ast 1}}=\sqrt{\frac{2Gm_{\text{Earth}}}{R_{\text{Earth-moon}}}}$. Generalizing this, we can express the velocity at any distance from the center of Earth as $v_{\text{ast}}(r)=\sqrt{\frac{2Gm_{\text{Earth}}}{r}}$. Clearly, since the radial distance is decreasing with time the speed can be written $v=-\frac{dr}{dt}$, and we can write

$$dt=-\frac{dr}{v}=-dr\sqrt{\frac{r}{2Gm_{\text{Earth}}}} \quad\text{or}\quad \int_{t_1}^{t_2}dt=\Delta t=-\int_{R_{\text{Earth-Moon}}}^{R_{\text{Earth}}}\sqrt{\frac{r}{2Gm_{\text{Earth}}}}dr=-\frac{2}{3\sqrt{2}}\left(R_{\text{Earth}}\sqrt{\frac{R_{\text{Earth}}}{Gm_{\text{Earth}}}}-R_{\text{Earth-Moon}}\sqrt{\frac{R_{\text{Earth-Moon}}}{Gm_{\text{Earth}}}}\right).$$

Thus the time interval between the asteroid passing the moon and the asteroid striking Earth is

$$\Delta t=-\frac{2}{3\sqrt{2}}\left(R_{\text{Earth}}\sqrt{\frac{R_{\text{Earth}}}{Gm_{\text{Earth}}}}-R_{\text{Earth-Moon}}\sqrt{\frac{R_{\text{Earth-Moon}}}{Gm_{\text{Earth}}}}\right)$$

$$=-\frac{2}{3\sqrt{2}}\left((6.378\times10^6\text{ m})\sqrt{\frac{(6.378\times10^6\text{ m})}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}}-(3.84\times10^8\text{ m})\sqrt{\frac{(3.84\times10^8\text{ m})}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}}\right)$$

$=1.77\times10^5$ s or 49 hours

Humanity would have 49 hours to prepare.

13.81. The satellite will complete circular orbits around Earth while Earth spins beneath it. If the satellite is to cover the entire Earth with its field of view, then (in particular to cover the equator) it must circle Earth $N=\frac{2\pi R_{\text{Earth}}}{2w_{\text{view}}}=\frac{2\pi(6.378\times10^6\text{ m})}{2(2.500\times10^6\text{ m})}=8.01\text{ times}$ in one day. This tells us the period of the orbit must be $T=1.08\times10^4\text{ s}.$ We can relate this period to other known quantities by noting first that

$$v=\frac{2\pi r}{T} \tag{1}$$

where r is the radial distance from the center of Earth to the satellite, and then by using the centripetal force:

$$\frac{Gm_{\text{Earth}}m_{\text{sat}}}{r^2}=\frac{m_{\text{sat}}v_{\text{sat}}^2}{r} \tag{2}$$

Inserting equation (1) into equation (2) and rearranging, we find

$$r=\left(\frac{Gm_{\text{Earth}}T^2}{4\pi^2}\right)^{1/3}=\left(\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})(1.08\times10^4\text{ s})^2}{4\pi^2}\right)^{1/3}=1.06\times10^7\text{ m}$$

from Earth's center. This corresponds to a height above the surface of Earth equal to $h=r-R_{\text{E}}=(1.06\times10^7\text{ m})-(6.378\times10^6\text{ m})=\ 4.18\times10^6\text{ m}.$

13.82. Call the distance between stars d such that the radial distance of either star from the center of mass of the system is $r=\frac{d}{2}.$ We can relate the observed period to other known quantities by noting first that

$$v=\frac{2\pi r}{T} \tag{1}$$

and then by using the centripetal force:

$$\frac{Gm_{\text{star 2}}m_{\text{star 1}}}{d^2}=\frac{Gm_{\text{star 2}}m_{\text{star 1}}}{4r^2}=\frac{m_{\text{star 1}}v_{\text{star}}^2}{r} \tag{2}$$

Inserting equation (1) into equation (2) and rearranging, we find

$$r=\left(\frac{Gm_{\text{star 2}}T^2}{16\pi^2}\right)^{1/3}=\left(\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\text{ kg})(2.10\times10^6\text{ s})^2}{16\pi^2}\right)^{1/3}=1.55\times10^{10}\text{ m}$$

Thus the stars are a distance $d=2r=3.1\times10^{10}\text{ m}$ apart.

13.83. (*a*) In order for that galaxy to have sufficient energy to escape the gravitational pull of that sphere, we require $K_{\text{galaxy,i}}\ge U_{\text{f}}^G-U_{\text{i}}^G.$ Consider the limiting case where there is just barely enough kinetic energy for the system not to be closed. This is equivalent to requiring $v_{\text{galaxy}}^2=\frac{2Gm_{\text{sphere}}}{R_{\text{sphere}}}\Rightarrow\frac{4}{3}\pi R_{\text{sphere}}^3\rho=\frac{v_{\text{galaxy}}^2R_{\text{sphere}}}{2G}$ or $\rho=\frac{3H^2}{8\pi G}.$ If the density is any higher than this, the system will be closed. Thus $\rho_{\max} = \dfrac{3H^2}{8\pi G} = \dfrac{3(2.2\times10^{-18}\ \mathrm{s}^{-1})^2}{8\pi(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})} = 8.7\times10^{-27}\ \mathrm{kg/m^3}$.

(*b*) As it is, the universe seems balanced between being open and being closed. If there were no dark matter or dark energy in the universe, it would be much less dense than estimated, and we would expect it to be open.

14

SPECIAL RELATIVITY

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

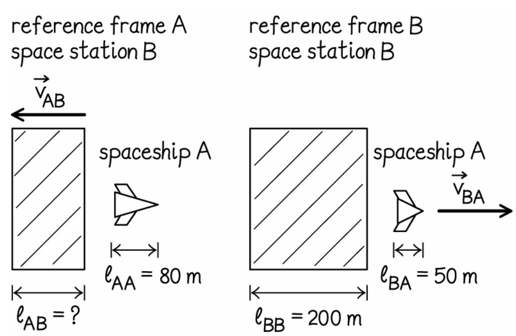
Developing a Feel

1. 10^{-11} **2.** 10^{-10} s **3.** 10^{-7} s **4.** 10^{-3} m **5.** 10^{2} m **6.** 10^{-11} kg **7.** 10^{10} kg **8.** 10^{1}

Guided Problems

14.2 Relativistic fly-by

1. Getting Started We wish to compare measurements made in reference frame A and reference frame B. We make a diagram of the relevant objects (space station and spaceship) in each reference frame, showing their approximate sizes as measured in that reference frame.



2. Devise Plan The two lengths of the ship are different because of length contraction. Length contraction depends on the relative speed between two objects according to

$$\ell_v = \frac{\ell_{\text{proper}}}{\gamma} = \sqrt{1-(v/c_0)^2}\,\ell_{\text{proper}} \tag{1}$$

Here ℓ_{proper} is the length of an object measured from a reference frame in which the object is at rest, ℓ_v is the length of the same object but measured from a reference frame in which the object is not at rest. The relative speed between these two reference frames is v. In this case, v is the speed to be determined, ℓ_{proper} is the length of the ship measured in dry dock, when the ship was at rest in the dry dock, and ℓ_v is the length of the ship measured by the space station now that the ship and space station are moving relative to each other.

We can rearranged equation (1) to solve for the speed v. Once this speed is known, we can apply the same length contraction equation to the space station to determine its length according to observers in A (the ship).

3. Execute Plan (*a*) The relative speed of frames A and B is found by rearranging equation (1):

$$v = c_0\sqrt{1-\left(\frac{\ell_v}{\ell_{\text{proper}}}\right)^2} \tag{2}$$

And inserting numbers, we find

$$v = c_0\sqrt{1-\left(\frac{50.0\text{ m}}{80.0\text{ m}}\right)^2} = 0.781c$$

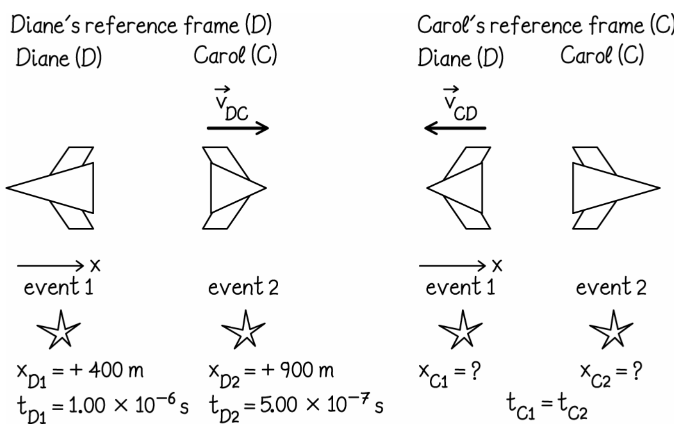
(*b*) We can insert this relative speed into equation (1) and apply equation (1) to describe the length contraction of the station as measured from the ship:

$$\ell_v = \sqrt{1-(v/c_0)^2}\,\ell_{\text{proper}} = \sqrt{1-(0.7806)^2}\,(200\text{ m}) = 125\text{ m}$$

4. Evaluate Result We note that in both cases, the proper lengths are greater than the corresponding non-proper lengths.

14.4 Lorentz would love it

1. Getting Started We draw a diagram for each reference frame: Diane (D) and Carol (C):



Neither observer measures the proper time nor the proper length for the two events.

2. Devise Plan The Lorentz transformations relate the time and space coordinates of events measured in different reference frames. Using equation (14.29) from Principles, we can relate the measurements made by Carol to those made by Diane:

$$t_{\text{C1}} = \gamma\left(t_{\text{D1}} - \frac{v_{\text{DC}x}}{c_0^2}x_{\text{D1}}\right)$$

$$t_{\text{C2}} = \gamma\left(t_{\text{D2}} - \frac{v_{\text{DC}x}}{c_0^2}x_{\text{D2}}\right)$$

Carol measures the two events to be simultaneous. So the difference of the two equations above must be zero. Thus

$$t_{\text{D1}} - \frac{v_{\text{DC}x}}{c_0^2}x_{\text{D1}} = t_{\text{D2}} - \frac{v_{\text{DC}x}}{c_0^2}x_{\text{D2}} \tag{1}$$

This can be rearranged to determine the relative x component of velocity $v_{\text{DC}x}$, giving us our answer to part (*a*). Once we know the relative velocity, we can insert it back into the Lorentz transformations to determine the time at which Carol observes the two events. This will be our answer to (*b*).

3. Execute Plan (*a*) Rearranging equation (1) above, we find

$$v_{\text{DC}x} = \frac{(t_{\text{D1}} - t_{\text{D2}})}{(-x_{\text{D2}} + x_{\text{D1}})} c_0^2 = \frac{((1.00\times10^{-6}\ \text{s}) - (7.00\times10^{-7}\ \text{s}))}{(-(900\ \text{m}) + (400\ \text{m}))}(3.00\times10^{8}\ \text{m/s})^2$$

$$v_{\text{DC}x} = -0.300\ c_0$$

Thus the relative velocity of Carol to Diane $\vec{v}_{\text{DC}} = -0.300 c_0\ \hat{\imath}$. (*b*) Using the relative velocity obtained in part (*a*), we can now use the Lorentz transformations (either of those we had before) to write

$$t_{\text{C1}} = \gamma\left(t_{\text{D1}} - \frac{v_{\text{DC}x}}{c_0^2} x_{\text{D1}}\right) = \frac{1}{\sqrt{1 - \frac{v_{\text{DC}x}^2}{c_0^2}}}\left(t_{\text{D1}} - \frac{v_{\text{DC}x}}{c_0^2} x_{\text{D1}}\right)$$

$$t_{\text{C1}} = \frac{1}{\sqrt{1 - (-0.300)^2}}\left((1.00\times10^{-6}\ \text{s}) - \frac{(-0.300)}{(3.00\times10^{8}\ \text{m/s})}(400\ \text{m})\right)$$

$$t_{\text{C1}} = 1.47\times10^{-6}\ \text{s}$$

So Carol measures both events occurring at $t_{\text{C1}} = 1.47\ \mu\text{s}$.

4. Evaluate Result We can apply equation (14.30) from Principles to determine the positions at which Carol measures the two events.

$$x_{\text{C1}} = \gamma(x_{\text{D1}} - v_{\text{DC}x} t_{\text{D1}}) = \frac{1}{\sqrt{1 - (0.300)^2}}(400\ \text{m} - (-0.300)(3.00\times10^{8}\ \text{m/s})(1.00\times10^{-6}\ \text{s})) = 513.66\ \text{m}$$

$$x_{\text{C2}} = \gamma(x_{\text{D2}} - v_{\text{DC}x} t_{\text{D2}}) = \frac{1}{\sqrt{1 - (0.300)^2}}(900\ \text{m} - (-0.300)(3.00\times10^{8}\ \text{m/s})(5.00\times10^{-7}\ \text{s})) = 990.63\ \text{m}$$

So the spatial distance between the two events according to Carol is 476.97 m. With that information, we can determine the space-time interval according to each observer and ensure that they match. The spacetime interval should be invariant. So the two observers should agree on this, if we have made no mistakes. Using equation (14.18) from Principles, we write

$$s_{\text{C}}^2 = (c_0 \Delta t_{\text{C}})^2 - (\Delta x_{\text{C}})^2 = 0 - (476.97\ \text{m})^2 = -2.275\times10^{5}\ \text{m}^2$$

$$s_{\text{D}}^2 = (c_0 \Delta t_{\text{D}})^2 - (\Delta x_{\text{D}})^2 = ((3.00\times10^{8}\ \text{m/s})(5.00\times10^{-7}\ \text{s}))^2 - (500\ \text{m})^2 = -2.275\times10^{5}\ \text{m}^2$$

The space-time intervals are the same. It seems likely that we have done our calculation correctly.

14.6 Space-time interval

1. Getting Started The events that bracket the space-time interval of interest are (1) The entry of the particle into the detector, and (2) the decay of the particle. This makes the length of the track relevant, because that gives us a spatial separation between the two events in the reference frame of the detector. The speed of the particle is also relevant, because this allows us to calculate the separation of the two events in time, in the reference frame of the detector.

2. Devise Plan We can determine the time interval in reference frame D by using the speed of the particle:

$$\Delta t_{\text{D}} = \frac{\ell_{\text{D}}}{v} \tag{1}$$

We are given the length in the detector frame explicitly, so we have a complete set of intervals for the detector reference frame. If we could obtain such intervals for the P reference frame, we could compare space-time intervals using equation (14.18) from Principles.

The length of the track measured in the D reference frame is the proper length of the path; the track is at rest relative to the detector. This means we can relate the spatial distance covered by the particle in a simple way:

$$\ell_v = \frac{\ell_{\text{proper}}}{\gamma}$$

$$\ell_{\text{P}} = \sqrt{1 - (v/c_0)^2}\,\ell_{\text{D}} \tag{2}$$

Similarly, the time interval measured in the particle reference frame P is the proper time interval. Thus

$$\Delta t_v = \gamma \Delta t_{\text{proper}}$$

$$\Delta t_{\text{P}} = \Delta t_{\text{D}} \sqrt{1-(v/c_0)^2} \tag{3}$$

This will give us sufficient information to determine the space-time interval in both reference frames.

3. Execute Plan Inserting the given values into equation (1), we have

$$\Delta t_{\text{D}} = \frac{\ell_{\text{D}}}{v} = \frac{(1.05\times 10^{-3}\ \text{m})}{0.992(3.00\times 10^{8}\ \text{m/s})} = 3.528\times 10^{-12}\ \text{s}$$

Inserting this value into equation (3) yields

$$\Delta t_{\text{P}} = \Delta t_{\text{D}} \sqrt{1-(v/c_0)^2} = \sqrt{1-(0.992)^2}(3.528\times 10^{-12}\ \text{s}) = 4.454\times 10^{-13}\ \text{s}$$

Finally inserting given numbers into equation (2), we have

$$\ell_{\text{P}} = \sqrt{1-(v/c_0)^2}\,\ell_{\text{D}} = \sqrt{1-(0.992)^2}(1.05\times 10^{-3}\ \text{m}) = 1.3255\times 10^{-4}\ \text{m}$$

Now we insert these values for time and space intervals into equation (14.18) to determine the space-time interval in each reference frame:

$$s_{\text{P}}^2 = (c_0\Delta t_{\text{P}})^2 - (\ell_{\text{P}})^2 = ((3.00\times 10^{8}\ \text{m/s})(4.454\times 10^{-13}\ \text{s}))^2 - (1.3255\times 10^{-4}\ \text{m})^2 = 1.78\times 10^{-8}\ \text{m}^2$$

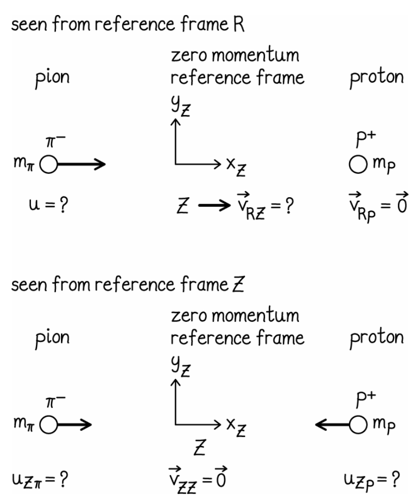
$$s_{\text{D}}^2 = (c_0\Delta t_{\text{D}})^2 - (\Delta x_{\text{D}})^2 = ((3.00\times 10^{8}\ \text{m/s})(3.528\times 10^{-12}\ \text{s}))^2 - (1.05\times 10^{-3}\ \text{m})^2 = 1.78\times 10^{-8}\ \text{m}^2$$

4. Evaluate Result Clearly, the space-time interval between the events is the same measured from either reference frame, as it should be. We also see that $s^2 > 0$, meaning the events have a timelike separation. This makes sense. The entry of the particle into the detector and the decay should be separated in time. Otherwise, there would be a reference frame in which the two events were simultaneous (the particle decayed exactly as it entered the detector, and yet left a 1.05 mm long track), which would not make sense.

14.8 Particle production

1. Getting Started If all kinetic energy of the pion (along with rest energy of the proton) could be converted to rest mass energy of the kaon and the lambda particle, that would be most efficient. That would minimize our required initial kinetic energy in the pion. However, we are not free to impose such a condition, because momentum must still be conserved. Even if K and Λ are at rest relative to each other after the collision, they cannot be at rest relative to the proton's initial reference frame R. We must first find the zero momentum reference frame.

We will need to consider reference frames R and Z (zero momentum reference frame). While we do need information about the motion of the pion, no calculations have to be done in the reference frame of the pion. We illustrate the two relevant reference frames here:



We define our system to consist initially of the pion and proton (and finally of the kaon and lambda particle). After the collision, the center of mass of the system must still be moving at the speed of the zero-momentum reference frame Z. In addition to that, the kaon and lambda particle could have motion relative to each other, meaning left over kinetic energy. To minimize the required energy, we will assume that the kaon and lambda particles are at rest in the zero momentum reference frame after the collision.

2. Devise a Plan The invariant momentum of any particle can be written as $\vec{p} = \gamma m \vec{v}$, and its energy can be written $E = \gamma m c_0^2$. We can write down statements of the conservation of energy and momentum in reference frame R:

$$\frac{K_{\mathrm{R}\pi}}{c_0^2} + m_\pi + m_p = \gamma_{\mathrm{R}K} m_K + \gamma_{\mathrm{R}\Lambda} m_\Lambda \tag{1}$$

and

$$\gamma_{\mathrm{R}\pi} m_\pi u = \gamma_{\mathrm{R}K} m_K v_K + \gamma_{\mathrm{R}\Lambda} m_\Lambda v_\Lambda$$

Because we want the minimum possible energy required to produce these paricles, we want to consider the case where the Kaon and lambda particle have as little energy as possible after their creation. We do need to conserve momentum, such that they must have some non-zero kinetic energy after their production. However, they do not need to be moving relative to each other. The Kaon and lambda particle will thus be moving with the same speed after their formation, and we can write

$$\gamma_{\mathrm{R}\pi} m_\pi u = \gamma_{\mathrm{Rf}} (m_K + m_\Lambda) v_{\mathrm{Rf}} \tag{2}$$

In the zero momentum reference frame, the speeds of the pion and proton are found using relativistic velocity addition:

$$u_{\mathrm{Z}\pi} = \frac{m_p u}{m_p + m_\pi \sqrt{1 - u^2/c_0^2}} \tag{3}$$

$$u_{\mathrm{Z}p} = -\frac{m_\pi u}{m_\pi + m_p \sqrt{1 - u^2/c_0^2}} \tag{4}$$

Applying conservation of energy in this reference frame yields

$$\gamma_{\mathrm{Z}\pi} m_\pi + \gamma_{\mathrm{Z}p} m_p = m_K + m_\Lambda \tag{5}$$

Equations (1-5) relate the variables $u_{\mathrm{Z}\pi}$, $u_{\mathrm{Z}p}$, u, v_{Rf}, and $K_{\mathrm{R}\pi}$. It may appear that we don't have enough equations, but all variables (mostly Lorentz factors) can be written in terms of these five variables. We can use these to solve for $K_{\mathrm{R}\pi}$.

3. Execute Plan Rewriting explicitly, equations (1-5) in terms of the five variables discussed above, we find:

$$\frac{K_{\mathrm{R}\pi}}{c_0^2} + m_\pi + m_p = \frac{(m_K + m_\Lambda)}{\sqrt{1 - v_{\mathrm{Rf}}^2/c_0^2}} \tag{6}$$

$$\frac{m_\pi u}{\sqrt{1 - u^2/c_0^2}} = \frac{(m_K + m_\Lambda)}{\sqrt{1 - v_{\mathrm{Rf}}^2/c_0^2}} v_{\mathrm{Rf}} \tag{7}$$

$$u_{\mathrm{Z}\pi} = \frac{m_p u}{m_p + m_\pi \sqrt{1 - u^2/c_0^2}} \tag{8}$$

$$u_{\mathrm{Z}p} = -\frac{m_\pi u}{m_\pi + m_p \sqrt{1 - u^2/c_0^2}} \tag{9}$$

$$\frac{m_\pi}{\sqrt{1 - u_\pi^2/c_0^2}} + \frac{m_p}{\sqrt{1 - u_p^2/c_0^2}} = m_K + m_\Lambda \tag{10}$$

We can combine equations (8-10) to obtain u in terms of other variables. One can then use equation (7) to eliminate v_{Rf}. We then write the kinetic energy in terms of the masses of the particles:

$$K_{\mathrm{R}\pi} = \frac{(m_K + m_\Lambda)^2 c_0^4 - m_\pi^2 c_0^4 - m_p^2 c_0^4}{2 m_p c_0^2} - m_\pi c_0^2$$

4. Evaluate Result This answer makes sense in a few respects. First, if the mass of the kaon or the lambda particle is very large, there would be more energy associated with their rest energy, and we would expect a larger required energy input from the pion. Indeed, we see that the pion's initial kinetic energy increases if either of those masses increases. We also expect that if the pion itself were more massive, then it could carry more energy in its mass, and would not need to have quite as much kinetic energy. We see from the subtracted terms involve the pion mass, that the kinetic energy would decrease in this case.

The kaon and lambda particle must be moving after they are created, so their final total energy should be more than just their rest energies. Since it is the total energy that is conserved, this means that the initial energy of the pion and proton should be greater than the rest energy of the kaon and lambda particle, as is the case in our answer.

Questions and Problems

14.1. An event is something that occurs at a specific location at a specific instant. Option (*a*) occurs over a 30 minute interval. Option (*c*) occurs over some considerable time interval and over a range of geographic locations, as does option (*d*). Options (*b*), (*e*), and (*f*) are events.

14.2. You record your own proper time interval.

14.3. Consider clock A first. It will take light a period of time $\Delta t_{\text{A}} = \dfrac{\Delta d_{\text{OA}}}{c} = \dfrac{(1.00\times10^4\ \text{m})}{(3.00\times10^8\ \text{m/s})} = 3.33\times10^{-5}\ \text{s}$. This means that by the time light reaches clock A and starts it, clock O will already read $33.3\ \mu\text{s}$ after noon. In order for the clocks to be synchronized, clock A should be set to $33.3\ \mu\text{s}$ after noon. In exactly the same way, we see that it will take $66.7\ \mu\text{s}$ for light from clock O to reach clock B. Thus for clocks O and B to be synchronized, clock B should be set to $66.7\ \mu\text{s}$ after noon. Clock C is a distance $d_{\text{OC}} = \sqrt{(20.0\ \text{km})^2 + (10.0\ \text{km})^2} = 22.36\ \text{km}$ from clock O. The time required for light to travel from clock O to clock C is thus $\Delta t_{\text{C}} = \dfrac{\Delta d_{\text{OC}}}{c} = \dfrac{(2.236\times10^4\ \text{m})}{(3.00\times10^8\ \text{m/s})} =$ $7.45\times10^{-5}\ \text{s}$. Thus clock C must be set to $74.5\ \mu\text{s}$ after noon. Finally, clock D is a distance $d_{\text{OD}} =$ $\sqrt{(20.0\ \text{km})^2 + (20.0\ \text{km})^2} = 28.28\ \text{km}$ from clock O. It will take light a period of time $\Delta t_{\text{D}} = \dfrac{\Delta d_{\text{OD}}}{c} =$ $\dfrac{(2.828\times10^4\ \text{m})}{(3.00\times10^8\ \text{m/s})} = 9.43\times10^{-5}\ \text{s}$ to travel from clock O to clock D. Thus clock D must be set to $94.3\ \mu\text{s}$ after noon.

14.4. Let us call the time of the two explosions in the reference frame of the detector $t_{\text{detector explosions}} = 0$. Light from event 1 must travel 3.40 m before reaching the detector. Thus $\Delta t_{\text{detector 1}} = \dfrac{\Delta d_{\text{detector 1}}}{c_0} = \dfrac{(3.40\ \text{m})}{(3.00\times10^8\ \text{m/s})} =$ $1.13\times10^{-8}\ \text{s}$. Similarly, light from event 2 must travel 2.10 m to the detector, such that $\Delta t_{\text{detector 2}} = \dfrac{\Delta d_{\text{detector 2}}}{c_0} =$ $\dfrac{(2.10\ \text{m})}{(3.00\times10^8\ \text{m/s})} = 7.00\times10^{-9}\ \text{s}$. Event 1 will be detected at $t_1 = 11.3\ \text{ns}$ and event 2 will be detected at $t_2 = 7.00\ \text{ns}$.

14.5. (*a*) The clocks read a later time. The distant clocks will not start until light reaches them from the source of the pulse, which takes time. They must be set later so that even after this lag, they are synchronized with the reference clock. (*b*) All clocks will read the same time once the pulse is sent. That was the point of the synchronizing process. (*c*) You will read an earlier time. It takes time for light to travel from the distant clock to your eyes. So your eyes will not be seeing light from the clock at that instant, but rather light that left the clock some time in the past. Thus the light reaching your eyes brings an image of the clock at an earlier time.

14.6. (*a*) Upon seeing the first firework. A and B could be different distances from the fireworks location. If so, both light and sound from the explosions must travel different distances to A and B. Because sound travels at a relatively low speed, any significant difference in how far A and B are from the fireworks would cause a difference of several seconds in the time interval needed for the sound to reach them. Light travels so quickly that the time interval difference is likely to be only microseconds. Thus, observer A's plan of starting the clocks upon seeing the first firework will result in better synchronization. (*b*) Yes, if the two observers are equidistant from the first explosion. In that case, light will take the same amount of time to reach each observer and their synchronization could theoretically be perfect. Even though sound moves much more slowly, it would still take sound the same amount of time to reach each of the two observers. So their synchronization could theoretically be perfect using sound also.

14.7. The observer measures the speed of light to be c_0. All observers in all reference frames always measure the speed of light to be c_0.

14.8. For the purposes of constructing a concrete picture, assume that observer A is in the reference frame of the alarm clocks, and is at the origin. This is not a necessary condition, just a useful choice for discussing simultaneity. Clearly, observer A will say that both alarms went off simultaneously if light from each alarm clock reaches her at the same moment (since they are equidistant from her). Assume observer B is moving toward observer A, but we do not know if observe B is moving in the $+x$ or $-x$ direction. Consider each case:

B moving in the $+x$ direction: Observer B must agree that light strikes both sides of observer A's face at the same time (observer B might even be able to see both sides of observer's face illuminated, so there can be no disagreement on that point). But according to observer B, both clocks and observer A are moving in the $-x$ direction. Specifically, observer B sees observer A moving away from the initial position of clock 2 and toward the initial position of clock 1. So observer B would say that light from clock 2 has to "catch up" to observer A, whereas observer A is moving toward a light pulse emitted by alarm 1. Since light moves at c_0 (for everyone) the only way for light from clock 2 to each observer A at the same moment as light from clock 1 is for light from clock 2 to have been emitted earlier. Thus, observer B would say that alarm 2 went off first.

B moving in the $-x$ direction: A similar argument shows that observer B would see observer A moving away from the point at which alarm 1 emitted its pulse, and toward the position at which alarm 2 emitted its pulse. Thus the only way for light from alarm 1 to "catch up" to observer A and reach A at the same moment as light from alarm 2 is if the light from alarm 1 was emitted earlier. Thus, in this direction of motion, observer B would claim that alarm 1 went off first.

Clearly, B is moving in the $+x$ direction relative to A.

14.9. The light pulse must travel the distance d to the mirror, but it only travels part of the way back. You cover part of the distance to the mirror before the light reaches you. What you can say is that the distance travelled by the light and the distance travelled by you together must make up a distance of $2d$: $v\Delta t + c_0\Delta t = 2d \Rightarrow d = \dfrac{(1.20)c_0\Delta t}{2} =$

$$\frac{(1.20)(3.00\times 10^{8}\ \text{m/s})(0.80\times 10^{-6}\ \text{s})}{2} = 1.4\times 10^{2}\ \text{m}.$$

14.10. (*a*) No, you only know that the light reached you eyes at the same instant. The two strikes could have happed at very different distances from you. (*b*) The answer would not change; we still cannot say they were simultaneous. (*c*) If we know that the strikes occurred equal distances from the helicopter and we knew that the light reached us from both strikes at the same instant, then we could say that the strikes were simultaneous in our reference frame. (*d*) An observer would agree that neither we nor the observer could say for certain that the strikes were simultaneous in parts (*a*) and (*b*). In part (*c*), an observer on the ground would agree that we are able to make the claim (if the observer understands relativity), but the observer on the ground would disagree with our claim that the strikes were simultaneous.

14.11. The ends of the rod would move at a speed $v = \frac{d}{t} = \frac{N2\pi r}{t} = \frac{(100)2\pi(0.5\times10^6 \text{ m})}{(1.0 \text{ s})} = 3.1\times10^8 \text{ m/s}$, which is greater than c_0. This cannot happen because the concept of a rigid object is an approximation that breaks down under such extreme conditions: the rod would bend into a spiral shape so that no portion of it moves faster than the speed of light.

14.12. (*a*) $\Delta t_{\text{BB}} < \Delta t_{\text{A}}$ because moving clocks lag behind stationary clocks. (*b*) The two times are equal: $\Delta t_{\text{BC}} = \Delta t_{\text{CB}}$. The two observers are moving relative to A at the same speed, and the same time interval was measured for each in frame A. Put another way: nothing physically changes if we flip our axes, which amounts to changing the roles of observers B and C.

14.13. The description of time dilation tells us that a stationary observer will claim that a moving clock is running slow. Thus an observer on Earth would say that the nap was actually longer than 10 minutes.

14.14. Yes, you could live to see a later calendar year on Earth. Checkpoint 14.8 asks if you could live a longer life. While you are moving at nearly the speed of light, the rate at which you see time pass is lower than the rate at which time passes according to clocks on Earth. This means you do not age as much as people on Earth, but you also don't experience things as quickly as they do. Thus you would not be aware of living a longer life because your brain synapses, heartbeat, and physical surroundings would all be moving more slowly than on Earth. However, time would continue to pass on Earth, and a date 200 years in the future could arrive before you come out of your near-light-speed travel and continue living your life.

14.15. The distance between the two mirrors can be written a the sum of the distances from you to the left mirror and from you to the right mirror: $d = d_{\text{right}} + d_{\text{left}}$. But note that in order for you to observe the light, it actually had to travel to the mirror and back in the times quoted. Thus $d = c_0 \frac{\Delta t_{\text{right}}}{2} + c_0 \frac{\Delta t_{\text{left}}}{2} = \left(\frac{1}{2}\right)(3.00\times10^8 \text{ m/s})$ $\left((2.5\times10^{-6} \text{ s}) + (6.5\times10^{-6} \text{ s})\right) = 1.4\times10^3 \text{ m}$.

14.16. Call your stationary reference frame A. Since the two ships are travelling at the same speed toward each other, each one will cover 25,000 km before they pass each other. Thus $\Delta t_{\text{A pass}} = \frac{d_{\text{A pass}}}{v_{\text{A}}} = \frac{(2.50\times10^7 \text{ m})}{(0.800)(3.00\times10^8 \text{ m/s})} =$ $1.04\times10^{-1} \text{ s}$.

14.17. The time measured by the engineers will be the time required for the signal to reach Earth added to the time required for the reply to reach the ship: $\Delta t_{\text{E}} = \Delta t_{\text{E message}} + \Delta t_{\text{E reply}}$. Note that the ship will have continued moving away from Earth throughout the problem, such that the distance travelled by the reply is much greater than 65,000,000 km. Call this initial distance to the ship in the Earth reference frame $d_{\text{E ship,1}}$, and call the distance to the ship once the message is received by Earth $d_{\text{E ship,2}}$. We know $\Delta t_{\text{E message}} = \frac{d_{\text{E ship,1}}}{c_0} = \frac{(6.50\times10^{10} \text{ m})}{(3.00\times10^8 \text{ m/s})} = 216.7 \text{ s}$. In this time interval, the ship will have moved an additional distance away from Earth, such that when the reply is sent, the ship will be a distance $d_{\text{E ship,2}} = d_{\text{E ship,1}} + v_{\text{ship}} \Delta t_{\text{E message}} = (6.50\times10^{10} \text{ m}) + (0.850)(3.00\times10^8 \text{ m/s})(216.7 \text{ s}) =$ $1.203\times10^{11} \text{ m}$. The reply message moves toward the ship at speed c_0, but the ship moves away from the message (in Earth's reference frame) at a speed of $0.850c_0$. So the message gains on the ship at a speed of $0.150c_0$. It is critical at this point that we note this is only true in the Earth reference frame. An observer on the ship would say the message is gaining on it at exactly c_0. Thus we require that the message travel the distance $0.150c_0\Delta t_{\text{E reply}} =$ $d_{\text{E ship,2}} \Rightarrow \Delta t_{\text{E reply}} = \frac{d_{\text{E ship,2}}}{(0.150c_0)} = \frac{(1.203\times10^{11} \text{ m})}{(0.150)(3.00\times10^8 \text{ m/s})} = 2672.2 \text{ s}$. Adding the times for the initial message and for the reply, we obtain $\Delta t_{\text{E}} = 2.94\times10^3$ s or 49.0 minutes.

14.18. Observe A will measure the proper length of spaceship A (which is the greatest length an observer can measure). Spaceship C is moving relative to A and so observer A will observe spaceship C to be Lorentz contracted. However, spaceship B is moving even faster relative to A and will therefore be seen by observer A as being even more contracted than spaceship C. Thus $\mathrm{A} > \mathrm{C} > \mathrm{B}$.

14.19. Since crew B is travelling at the same speed as crew A, crew B will see the diagonal parallel to its path contracted by the same factor of 0.50. The diagonal line perpendicular to the path of crew B will not be contracted as seen by crew B. Thus, in the reference frame of crew B, the base will be diamond-shaped with the diagonals having lengths $\sqrt{2}$ km and $\dfrac{\sqrt{2}}{2}$ km. The area of such a kite is $0.5\ \mathrm{km}^2$.

14.20. [**NOTE: Problem statement in text will be revised to read:** What is the kinetic energy of a pin in your reference frame if you measure its inertia to be three times its mass?] In the rest frame of the pin, the mass and inertia are identical. Mass and internal energy are also invariant, which means only the kinetic energy of the pin will change with a change of reference frame. Hence if it has inertia equal to three times its mass in your reference frame, then it must have kinetic energy equal to two times its mass. The conversion from mass changes to energy changes is given in *Principles* Exercise 14.6: $\Delta E/\Delta m = 8.98\times10^{16}$ J/kg. The mass of a pin is about $40\ \mathrm{mg} = 4.0\times10^{-5}$ kg. This means the pin must have kinetic energy in your reference frame equivalent to double this mass, or $K = 2m_{\mathrm{pin}}(\Delta E/\Delta m) = 2(4.0\times10^{-5}\ \mathrm{kg})(8.98\times10^{16}\ \mathrm{J/kg}) = 7.2\times10^{12}\ \mathrm{J}$.

14.21. Zero. Mass is unchanged by motion, but may change slightly if the golf ball's internal energy changed when it was hit. However, such changes in internal energy are likely to be dissipated fairly quickly, so that the mass is ultimately unchanged. The inertia changes by an amount $m_{v,\mathrm{f}} - m_{v,\mathrm{i}} = (\gamma - 1)m = \left(\dfrac{1}{\sqrt{1-v^2/c_0^2}} - 1\right)m = \left(\dfrac{1}{\sqrt{1-(40.0\ \mathrm{m/s})^2/(3.00\times10^8\ \mathrm{m/s})^2}} - 1\right)m = (8.9\times10^{-15})m$ or 8.9×10^{-13} % of the mass while the ball is in motion.

14.22. (*a*) In Exercise 14.5b we found the ratio of energy decrease to mass loss is 8.98×10^{16} J/kg. The spring looses energy as it expands, and amount of the loss is $\Delta U^{\mathrm{spring}} = \dfrac{1}{2}k(x_\mathrm{f}^2 - x_\mathrm{i}^2) = \dfrac{1}{2}(1500\ \mathrm{N/m})\left(0-(2.40\ \mathrm{m})^2\right) = -4.32\times10^3$ J. Thus, the change is mass of the spring is $\Delta m = \dfrac{\Delta E}{(8.98\times10^{16}\ \mathrm{J/kg})} = \dfrac{(-4.32\times10^3\ \mathrm{J})}{(8.98\times10^{16}\ \mathrm{J/kg})} = -4.81\times10^{-14}$ kg. (*b*) The mass of the system does not change. The internal energy of the system also does not change, only the form of the internal energy (gravitational potential energy, spring potential energy, etc.) changes.

14.23. Energy is added to the system as you move the book, and it goes into internal energy (gravitational potential energy). This means energy, mass and therefore inertia all change (slightly). Kinetic energy is the same at the beginning and end, but does change while the book is in motion.

14.24. Ignoring spin, the observers disagree on kinetic energy and inertia, because they are in two different inertial frames. Each sees the other object in the system moving, and their own reference frame as stationary. However, they agree on the mass and energy of the system.

14.25. We know from Exercise 14.5 that $\dfrac{\Delta E}{\Delta m} = 8.98\times10^{16}$ J/kg. The change in energy of the system will be $\Delta E = \Delta U^G = -\dfrac{Gm_\mathrm{E}m_\mathrm{b}}{r_\mathrm{f}} + \dfrac{Gm_\mathrm{E}m_\mathrm{b}}{R_\mathrm{E}}$. Thus

$$\Delta E = \Delta m(8.98\times10^{16}\ \text{J/kg}) = -\frac{Gm_{\text{E}}m_{\text{b}}}{r_{\text{f}}} + \frac{Gm_{\text{E}}m_{\text{b}}}{R_{\text{E}}} \Rightarrow r_{\text{f}} = Gm_{\text{E}}m_{\text{b}}\left[\frac{Gm_{\text{E}}m_{\text{b}}}{R_{\text{E}}} - \Delta m(8.98\times10^{16}\ \text{J/kg})\right]^{-1}$$

so $r_{\text{f}} = \left[\frac{1}{(6.378\times10^{6}\ \text{m})} - \frac{(2.5\times10^{-6}\ \text{kg})(8.98\times10^{16}\ \text{J/kg})}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(1.00\times10^{4}\ \text{kg})}\right]^{-1} = 9.959\times10^{6}\ \text{m}$ from the center of Earth. Thus the height above Earth's surface is $h = r_{\text{f}} - R_{\text{E}} = (9.959\times10^{6}\ \text{m}) - (6.378\times10^{6}\ \text{m}) = 3.58\times10^{6}\ \text{m}.$

14.26. No, there is not an inertial frame in which the soda can explodes before the rifle is fired. Such an observation would violate causality.

14.27. In each case we calculate the square of the space-time interval and determine the type of separation from this interval. (*a*) $s^2 = (c_0\Delta t)^2 - (\Delta x)^2 = ((3.00\times10^{8}\ \text{m/s})(500\ \text{s}))^2 - (1.50\times10^{11}\ \text{m})^2 = 0$ so this separation is lightlike. (*b*) $s^2 = (c_0\Delta t)^2 - (\Delta x)^2 = ((3.00\times10^{8}\ \text{m/s})(8.45\times10^{4}\ \text{years})(365.25\ \text{days/year})(24\ \text{h/day})(3600\ \text{s/h}))^2 - (8.00\times10^{20}\ \text{m})^2$ $= 0,$ so this separation is lightlike. (*c*) The two events are the two observations of a signal by you. Thus both events take place at the same point in space, and are separated only in time. No calculation is necessary. The separation is timelike. (*d*) The simplest way to solve this is to work in the reference frame of the pilot in which both events occur at the same position. Thus the separation is clearly timelike. Of course, the answer does not change in the reference frame of the planets. We demonstrate this by calculating the square of the space-time interval in the reference frame of the planets. Note that the spatial separation in this frame is $\Delta x = v_{\text{ship}}\Delta t = 0.75c_0\Delta t.$ So the space-time interval squared is $s^2 = (c_0\Delta t)^2 - (0.75c_0\Delta t)^2 > 0,$ so the separation is indeed timelike.

14.28. In each part we simply calculate $\gamma = \frac{1}{\sqrt{1 - v^2/c_0^2}},$ or (to obtain the correction) $\gamma - 1 = \frac{1}{\sqrt{1 - v^2/c_0^2}} - 1.$ Some methods of computation (such as numerical solvers on computers) will have no trouble with this. But if you are restricted to the use of a pocket calculator, you may need to use the Taylor expansion to obtain the correction: $\gamma - 1 = \frac{1}{\sqrt{1 - v^2/c_0^2}} - 1 \approx \frac{1}{2}\left(\frac{v}{c_0}\right)^2 + O\left[\left(\frac{v}{c_0}\right)^4\right].$ (*a*) Note $60\ \text{mi/h} = 26.8\ \text{m/s}.$ So $\gamma = \frac{1}{\sqrt{1 - v^2/c_0^2}} =$ $\frac{1}{\sqrt{1 - (26.8\ \text{m/s})^2/(3.00\times10^{8}\ \text{m/s})^2}} = 1 + 4.0\times10^{-15}.$ (*b*) $\gamma = \frac{1}{\sqrt{1 - v^2/c_0^2}} = \frac{1}{\sqrt{1 - (340\ \text{m/s})^2/(3.00\times10^{8}\ \text{m/s})^2}} = 1 + 6.4\times10^{-13}.$ (*c*) Note that $28\times10^{3}\ \text{km/h} = 7.78\times10^{3}\ \text{m/s}.$ So $\gamma = \frac{1}{\sqrt{1 - v^2/c_0^2}} = \frac{1}{\sqrt{1 - (7.78\times10^{3}\ \text{m/s})^2/(3.00\times10^{8}\ \text{m/s})^2}} =$ $1 + 3.4\times10^{-10}.$ (*d*) $\gamma = \frac{1}{\sqrt{1 - (0.1c_0)^2/c_0^2}} = \frac{1}{\sqrt{1 - (0.1)^2}} = 1 + 5\times10^{-3}.$ (*e*) $\gamma = \frac{1}{\sqrt{1 - (0.3c_0)^2/c_0^2}} = \frac{1}{\sqrt{1 - (0.3)^2}} = 1 +$ 5×10^{-2} (*f*) $\gamma = \frac{1}{\sqrt{1 - (0.995c_0)^2/c_0^2}} = \frac{1}{\sqrt{1 - (0.995)^2}} = 1 + 9.00$ or $10.0.$

14.29. We know

$$T = \frac{\Delta t_{\text{proper}}}{\sqrt{1 - v_{\text{A}}^2/c_0^2}} \tag{1}$$

and

$$3T = \frac{\Delta t_{\text{proper}}}{\sqrt{1 - v_{\text{B}}^2/c_0^2}} = \frac{\Delta t_{\text{proper}}}{\sqrt{1 - 4v_{\text{A}}^2/c_0^2}} \tag{2}$$

Diving equation (2) by equation (1) we find $3 = \frac{\sqrt{1 - v_{\text{A}}^2/c_0^2}}{\sqrt{1 - 4v_{\text{A}}^2/c_0^2}} \Rightarrow 9(1 - 4v_{\text{A}}^2/c_0^2) = 1 - v_{\text{A}}^2/c_0^2 \Rightarrow v_{\text{A}} = c_0\sqrt{\frac{8}{35}} = 0.48c_0.$ We are told $v_{\text{B}} = 2v_{\text{A}},$ so $v_{\text{A}} = 0.48c$ and $v_{\text{A}} = 0.96c.$

14.30. (*a*) $\Delta t_{\text{A}} = \dfrac{\Delta t_{\text{proper}}}{\sqrt{1 - v_{\text{A}}^2/c_0^2}} = \dfrac{(1.00\text{ h})}{\sqrt{1-(0.400)^2}} = 1.09\text{ h}$. (*b*) This is identical to part (*a*): $\Delta t_{\text{B}} = \dfrac{\Delta t_{\text{proper}}}{\sqrt{1 - v_{\text{B}}^2/c_0^2}} = \dfrac{(1.00\text{ h})}{\sqrt{1-(0.400)^2}} = 1.09\text{ h}$. (*c*) Now we require $\Delta t_{\text{A}} = 2.00\text{ h} = \dfrac{\Delta t_{\text{proper}}}{\sqrt{1 - v_{\text{A}}^2/c_0^2}} \Rightarrow 1 - v_{\text{A}}^2/c_0^2 = \dfrac{1}{4} \Rightarrow v_{\text{A}} = c_0\sqrt{\dfrac{3}{4}} = 0.866c_0$.

14.31. From the definition of a half-life, we know $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{\Delta t_{\text{proper}}/\Delta t_{\text{half}}}$. Rearranging, we can write

$$\frac{\ln\left(\dfrac{N}{N_0}\right)}{\ln\left(\dfrac{1}{2}\right)}\Delta t_{\text{half}} = \Delta t_{\text{proper}} = \Delta t_{\text{Earth}}\sqrt{1 - v^2/c_0^2} = \frac{h}{v}\sqrt{1 - v^2/c_0^2}$$

where h is the height at which the first measurement is made. Solving for the speed yields

$$v = \left(\left[\frac{\ln\left(\dfrac{N}{N_0}\right)}{\ln\left(\dfrac{1}{2}\right)}\Delta t_{\text{half}}\right]^2 + \frac{h^2}{c_0^2}\right)^{-1/2} h = \left(\left[\frac{\ln\left(\dfrac{380}{600}\right)}{\ln\left(\dfrac{1}{2}\right)}(1.5\times10^{-6}\text{ s})\right]^2 + \frac{(1900\text{ m})^2}{(3.00\times10^8\text{ m/s})^2}\right)^{-1/2} (1900\text{ m}) = 2.96\times10^8\text{ m/s}$$

or $0.99c_0$.

14.32. (*a*) Yes. The space-time interval squared between these events is $s^2 = (c_0\Delta t)^2 - (\Delta x)^2 = \left(c_0(5\text{ y})\right)^2 - (10c_0\text{ y})^2 = -75(c_0\text{ y})^2$, meaning these events have a spacelike separation. We have not reduced the light year to base SI units in this case, because only the sign of the square of the space-time interval is relevant. A space-like separation means there is a reference frame in which the two events are simultaneous. (*b*) No, as we found in part (*a*), these events have a spacelike separation. (*c*) There is no proper time interval. A proper time interval is the time interval between events measured in a reference frame in which the two events happen at the same position. As we saw from parts (*a*) and (*b*), there is no reference frame in which the events happen at the same position; the events have spacelike separation. (*d*) In this case the square of the space-time interval is $s^2 = (c_0\Delta t)^2 - (\Delta x)^2 = \left(c_0(10\text{ y})\right)^2 - (5c_0\text{ y})^2 = 75(c_0\text{ y})^2$, such that the events have a timelike separation. Now there is not a reference frame in which the two events are simultaneous, but there is a frame in which they occur at the same position. Hence there is a proper time. Given the invariant space-time interval, it is trivial to calculate the proper time: we simply set the spatial separation to zero and require that the invariant space-time interval be unchanged. $s^2 = (c_0\Delta t)^2 - (\Delta x)^2 = \left(c_0(\Delta t_{\text{proper}})\right)^2 - (0)^2 = 75(c_0\text{ y})^2$ $\Rightarrow \Delta t_{\text{proper}} = \sqrt{75}\text{ y} = 8.7\text{ y}$.

14.33. From the definition of a half-life, we know $\dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{\Delta t_{\text{proper}}/\Delta t_{\text{half}}}$. Rearranging and using the relationship between proper and dilated times, we can write

$$\frac{\ln\left(\dfrac{N}{N_0}\right)}{\ln\left(\dfrac{1}{2}\right)}\Delta t_{\text{half}} = \Delta t_{\text{proper}} = \Delta t_{\text{Earth}}\sqrt{1 - v^2/c_0^2} = \frac{2R_{\text{E}}}{v}\sqrt{1 - v^2/c_0^2}$$

Solving for the speed, we find

$$v = \left(\left[\frac{\ln\left(\frac{N}{N_0}\right)}{\ln\left(\frac{1}{2}\right)} \Delta t_{\text{half}} \right]^2 + \frac{4R_{\text{E}}^2}{c_0^2} \right)^{-1/2} 2R_{\text{E}} = \left(\left[\frac{\ln\left(\frac{1}{10^6}\right)}{\ln\left(\frac{1}{2}\right)} (1.5\times10^{-6}\ \text{s}) \right]^2 + \frac{4(6.378\times10^6\ \text{m})^2}{(3.00\times10^8\ \text{m/s})^2} \right)^{-1/2} 2(6.378\times10^6\ \text{m})$$

$$= (1 - 2.47\times10^{-6})c_0$$

14.34. Equation 14.13 is valid only for inertial reference frames, but the cosmonaut was in roughly circular orbit, meaning his reference frame was constantly accelerating. An additional complication is that clocks on Earth are also in an accelerating reference frame, due to Earth's rotation. It would be important to know whether the cosmonaut's travel direction in orbit was the same as the direction of Earth's rotation, opposite the direction of that rotation, perpendicular to it, or anything in between. Using Eq. 14.13 yields a disagreement between the clocks of only $\Delta t_{\text{proper}} - \Delta t = \Delta t_{\text{proper}}\left(1 - \sqrt{1 - v^2/c_0^2}\right) = (437\ \text{days})(24\ \text{h/day})(3600\ \text{s/h})\left(1 - \sqrt{1 - (7700\ \text{m/s})^2/(3.00\times10^8\ \text{m/s})^2}\right) = 0.012\ \text{s}.$

14.35. (*a*) The proper length is the distance between two points in a reference frame in which the two points are at rest. Thus the proper length between the two events is the 8.00 km measured in the Earth reference frame. (*b*) The proper time between events is the time between events as measured by an observer who sees the two events happen in the same place. In this case, the an observer moving along with the cosmic ray would measure the proper time: $\Delta t_{\text{proper}} = \Delta t_{\text{E}}\sqrt{1 - v^2/c_0^2} = \frac{\Delta x_{\text{E}}}{v_{\text{muon}}}\sqrt{1 - v^2/c_0^2} = \frac{(8.00\times10^3\ \text{m})}{(0.400)(3.00\times10^8\ \text{m/s})}\sqrt{1 - (0.400)^2} = 6.11\times10^{-5}\ \text{s}.$

14.36. This problem can be solved either using length contraction or time dilation. In the reference frame of the particle, the distance to the Regulus system will be length-contracted. Then we have $v_{\text{particle}} = \frac{d_{\text{particle Earth-Regulus}}}{\Delta t_{\text{particle}}} = \frac{d_{\text{E Earth-Regulus}}\sqrt{1 - v_{\text{particle}}^2/c_0^2}}{\Delta t_{\text{particle}}}$. Solving for the speed of the particle yields $v_{\text{particle}} = \left(\frac{\Delta t_{\text{particle}}^2}{d_{\text{E Earth-Regulus}}^2} + \frac{1}{c_0^2}\right)^{-1/2} = c_0\left(\frac{(10\ \text{y})^2}{(77.5\ \text{y})^2} + 1\right)^{-1/2} = 0.99c_0.$

14.37. Rearranging equation (14.33) we find

$$v_{\text{AB}\,x} = \frac{(v_{\text{Bo}\,x} - v_{\text{Ao}\,x})}{\left(\frac{v_{\text{Ao}\,x}v_{\text{Bo}\,x}}{c_0^2} - 1\right)} = \frac{\left((0.600c_0) - (-0.600c_0)\right)}{\left(\frac{(0.600c_0)(-0.600c_0)}{c_0^2} - 1\right)} = -0.8823c_0$$

The Lorentz factor associated with this speed is $\gamma = \frac{1}{\sqrt{1 - (v_{\text{AB}\,x}^2/c_0^2)}} = \frac{1}{\sqrt{1 - (0.8823)}} = 2.13.$

14.38. Call the two particles A and B. Rearranging equation (14.33) we find

$$v_{\text{AB}\,x} = \frac{(v_{\text{Bo}\,x} - v_{\text{Ao}\,x})}{\left(\frac{v_{\text{Ao}\,x}v_{\text{Bo}\,x}}{c_0^2} - 1\right)} = \frac{\left((0.800c_0) - (-0.800c_0)\right)}{\left(\frac{(0.800c_0)(-0.800c_0)}{c_0^2} - 1\right)} = -0.976c_0$$

So the relative speed of one particle as measured by an observer moving along with the other particle is $0.976c_0$.

14.39. (*a*) The apparent "length" of Earth will be contracted along this direction, such that the diameter of Earth in the cosmic ray's reference frame is $D_{\text{E}} = 2R_{\text{E}} = 2R_{\text{E0}}\sqrt{1 - v_{\text{ray}}^2/c_0^2} = 2(6370\ \text{km})\sqrt{1 - (0.800)^2} = 7.64\times10^6\ \text{m}.$ (*b*)

Lengths will not appear contracted along that direction. The diameter of Earth is simply $D_E = 2R_E = 2R_{E0} = 2(6.370\times10^6 \text{ m}) = 1.27\times10^7 \text{ m}$.

14.40. Equations. 14.31 and 14.32 have no dependence on v_{AB} and are already identical to the Galilean transformations. In the limit $v_{AB} \ll c_0$, the Lorentz factor, $\gamma = 1/\sqrt{1-v_{AB}^2/c_0^2}$, approaches 1. With $\gamma = 1$, Eq. 14.30 to reduce to $x_{Be} = x_{Ae} - v_{AB}t_{Ae}$, which parallels Eq. 6.5. In Eq. 14.29, the limit $v_{AB} << c_0$ means $v_{AB}/c_0^2 \approx 0$, the second term inside the parentheses approaches zero, and the equation reduces to $t_{Be} = t_{Ae}$, which is Eq. 6.4.

14.41. (*a*) $\Delta x = v_{\text{particle}}\Delta t_{\text{proper}} = (0.9990)(3.00\times10^8 \text{ m/s})(14 \text{ days})(24 \text{ h/day})(3600 \text{ s/h}) = 3.63\times10^{14} \text{ m}$. (*b*) $\Delta x = v_{\text{particle}}\Delta t_{\text{Earth}} = \dfrac{(0.9990)c_0\Delta t_{\text{proper}}}{\sqrt{1-v_{\text{particle}}^2/c_0^2}} = \dfrac{(0.9990)(3.00\times10^8 \text{ m/s})(14 \text{ days})(24 \text{ h/day})(3600 \text{ s/h})}{\sqrt{1-(0.9990)^2}} = 8.11\times10^{15} \text{ m}$.

14.42. (*a*) It would be possible to detect that the moon is slightly contracted along the direction of your motion, but the difference would be very small. The Moon would appear to be spherical to a very good approximation. (*b*) The moon would appear ellipsoidal with the semimajor axis equal to the Moon's radius measured at rest relative to the moon (the proper length of its radius) which is 1.738×10^6 m. The semiminor axis would be along the direction of your motion, and it would have a contracted length of $R_M = R_{M0}\sqrt{1-v_{\text{observer}}^2/c_0^2} = (1.738\times10^6 \text{ m})\sqrt{1-(0.50)^2} = 1.5\times10^6 \text{ m}$. (*c*) The moon would appear ellipsoidal with the semimajor axis equal to the Moon's radius measured at rest relative to the moon (the proper length of its radius) which is 1.738×10^6 m. The semiminor axis would be along the direction of your motion, and it would have a contracted length of $R_M = R_{M0}\sqrt{1-v_{\text{observer}}^2/c_0^2} = (1.738\times10^6 \text{ m})\sqrt{1-(0.95)^2} = 5.4\times10^5 \text{ m}$.

14.43. (*a*) Call the width in the rest reference frame w, and call the length in the rest reference frame ℓ_0. From simple trigonometry, we have $\alpha = 2\tan^{-1}\left(\dfrac{w}{2\ell_0}\right) = 2\tan^{-1}\left(\dfrac{8.00 \text{ m}}{2(7.00 \text{ m})}\right) = 59.5°$. (*b*) The length is now $\ell = \ell_0\sqrt{1-v^2/c_0^2} = (7.00 \text{ m})\sqrt{1-(0.700)^2} = 5.00 \text{ m}$, the width remains 8.00 m, and $\alpha = 2\tan^{-1}\left(\dfrac{w}{2\ell}\right) = 2\tan^{-1}\left(\dfrac{8.00 \text{ m}}{2(5.00 \text{ m})}\right) = 77.3°$.

14.44. Let us call you observer A, and your friend observer B. The y and z coordinates of both events are the same, so we need not consider motion in those directions. We use the Lorentz transformation, specifically equation (14.29), to write the requirement that $t_{B\,e2} - t_{B\,e1} = \gamma\left(t_{A\,e2} - \dfrac{v_{AB\,x}}{c_0^2}x_{A\,e2} - t_{A\,e1} + \dfrac{v_{AB\,x}}{c_0^2}x_{A\,e1}\right) = 0$. Rearranging, we find

$$v_{AB\,x} = \frac{c_0^2(t_{A\,e1} - t_{A\,e2})}{(x_{A\,e1} - x_{A\,e2})} = \frac{(3.00\times10^8 \text{ m/s})^2\left((20\times10^{-6} \text{ s}) - (30\times10^{-6} \text{ s})\right)}{\left((10\times10^3 \text{ m}) - (30\times10^3 \text{ m})\right)} = 4.5\times10^7 \text{ m/s or } 0.15c_0.$$

So your friend must travel at $0.15c_0$ in the $+x$ direction.

14.45. Start with the expression for the Lorentz invariant space-time interval $s^2 = (c_0\Delta t_B)^2 - (\Delta x_B)^2$, initially using measurements from reference frame B. Use the Lorentz transformation equations to substitute in the expressions for time and space as measured from reference frame A, yielding

$$s^2 = \left[c_0\gamma\left(t_A - \frac{v_{AB}}{c_0^2}x_A\right)\right]^2 - \left[\gamma(x_A - v_{AB}t_A)\right]^2$$
$$= \frac{1}{1-v_{AB}^2/c_0^2}(c_0^2t_A^2 - 2c_0^2t_A^2v_{AB}x_A/c_0^2 + v_{AB}^2x_A^2/c_0^2 - x_A^2 + 2x_Av_{AB}t_A - v_{AB}^2t_A^2)$$
$$= \frac{1}{1-v_{AB}^2/c_0^2}\left[-(1-v_{AB}^2/c_0^2)x_A^2 + (1-v_{AB}^2/c_0^2)c_0^2t_A^2\right]$$
$$= \frac{\dfrac{1}{1-v_{AB}^2/c_0^2}}{\dfrac{1}{1-v_{AB}^2/c_0^2}}\left[c_0^2t_A^2 - x_A^2\right] = \left[(c_0t_A)^2 - x_A^2\right]$$

This last line is exactly the definition of the Lorentz invariant space-time interval.

14.46. (*a*) $\Delta t_S = \dfrac{\Delta x_S}{v_{\text{particle}}} = \dfrac{(8.14\times10^{12}\text{ m})}{(0.840)(3.00\times10^8\text{ m/s})} = 3.23\times10^4\text{ s (or 8.97 h)}$ (*b*) $\Delta t_P = \Delta t_S\sqrt{1-v_{\text{particle}}^2/c_0^2} =$ $(3.23\times10^4\text{ s})\sqrt{1-(0.840)^2} = 1.75\times10^4\text{ s (or 4.87 h)}$ (*c*) The proper length is given as 8.14×10^{12} m. (*d*) The proper time interval is that measured by an observer moving along with the particle. Thus the proper time interval is 1.75×10^4 s (or 4.87 h). (*e*) $\ell_P = \ell_S\sqrt{1-v_{\text{particle}}^2/c_0^2} = (8.14\times10^{12}\text{ m})\sqrt{1-(0.840)^2} = 4.42\times10^{12}\text{ m}.$

14.47. (*a*) From the ship, observers see the pulse originate at $x = 0$. This is because the pulse is sent at the moment the ship passes the origin. (*b*) From the ship, observers see the pulse originate at $t = 0$. This is because observers in S agreed to start their clocks at the moment they pass the origin. (*c*) $x_{\text{S reply}} = \dfrac{1}{\sqrt{1-v_{\text{CS}\,x}^2/c_0^2}}(x_{\text{C reply}} - v_{\text{CS}\,x}t_{\text{C reply}}) =$ $\dfrac{1}{\sqrt{1-(0.482)^2}}\left((3.000\times10^8\text{ m}) - (0.482)(3.00\times10^8\text{ m/s})(1.00\text{ s})\right) = 1.77\times10^8\text{ m.}$ (*d*) $t_{\text{S reply}} = \dfrac{1}{\sqrt{1-v_{\text{CS}\,x}^2/c_0^2}}$ $\left(t_{\text{C reply}} - \dfrac{v_{\text{CS}\,x}}{c_0^2}x_{\text{C reply}}\right) = \dfrac{1}{\sqrt{1-(0.482)^2}}\left(\dfrac{(3.000\times10^8\text{ m})}{(3.00\times10^8\text{ m/s})} - \dfrac{(0.482)}{(3.00\times10^8\text{ m/s})}(3.000\times10^8\text{ m})\right) = 0.591\text{ s.}$

14.48. In either case, we simply use the Lorentz transformation equations to compare the time interval and space interval as seen by the moving observes to the values measured by an observe in O.

(*a*)

$$x_{C\,2} - x_{C\,1} = \frac{1}{\sqrt{1-v_{\text{OC}\,x}^2/c_0^2}}[(x_{O\,2} - v_{\text{OC}\,x}t_{O\,2}) - (x_{O\,1} - v_{\text{OC}\,x}t_{O\,1})]$$
$$= \frac{1}{\sqrt{1-(0.600)^2}}\left[\left((-3.000\times10^9\text{ m}) - (0.600)(3.00\times10^8\text{ m/s})(0)\right) - \left((3.000\times10^9\text{ m}) - (0.600)(3.00\times10^8\text{ m/s})(0)\right)\right]$$
$$= -7.50\times10^9\text{ m}$$
$$t_{C\,2} - t_{C\,1} = \frac{1}{\sqrt{1-v_{\text{OC}\,x}^2/c_0^2}}\left[\left(t_{O\,2} - \frac{v_{\text{OC}\,x}}{c_0^2}x_{O\,2}\right) - \left(t_{O\,1} - \frac{v_{\text{OC}\,x}}{c_0^2}x_{O\,1}\right)\right]$$
$$= \frac{1}{\sqrt{1-(0.600)^2}}\left[\left(0 - \frac{(0.600)}{(3.00\times10^8\text{ m/s})}(-3.000\times10^9\text{ m})\right) - \left(0 - \frac{(0.600)}{(3.00\times10^8\text{ m/s})}(3.000\times10^9\text{ m})\right)\right]$$
$$= 15\text{ s}$$

Thus $\Delta x_C = -7.5\times10^9$ m and $\Delta t_C = 15$ s.

(*b*)

$$x_{D\,2}-x_{D\,1}=\frac{1}{\sqrt{1-v_{OD\,x}^2/c_0^2}}\left[\left(x_{O\,2}-v_{OD\,x}t_{O\,2}\right)-\left(x_{O\,1}-v_{OD\,x}t_{O\,1}\right)\right]$$

$$=\frac{1}{\sqrt{1-(-0.600)^2}}\left[\left((-3.000\times10^9\text{ m})-(-0.600)(3.00\times10^8\text{ m/s})(0)\right)-((3.000\times10^9\text{ m})-(-0.600)(3.00\times10^8\text{ m/s})(0))\right]$$

$$=-7.50\times10^9\text{ m}$$

$$t_{D\,2}-t_{D\,1}=\frac{1}{\sqrt{1-v_{OD\,x}^2/c_0^2}}\left[\left(t_{O\,2}-\frac{v_{OD\,x}}{c_0^2}x_{O\,2}\right)-\left(t_{O\,1}-\frac{v_{OD\,x}}{c_0^2}x_{O\,1}\right)\right]$$

$$=\frac{1}{\sqrt{1-(-0.600)^2}}\left[\left(0-\frac{(-0.600)}{(3.00\times10^8\text{ m/s})}(-3.000\times10^9\text{ m})\right)-\left(0-\frac{(-0.600)}{(3.00\times10^8\text{ m/s})}(3.000\times10^9\text{ m})\right)\right]$$

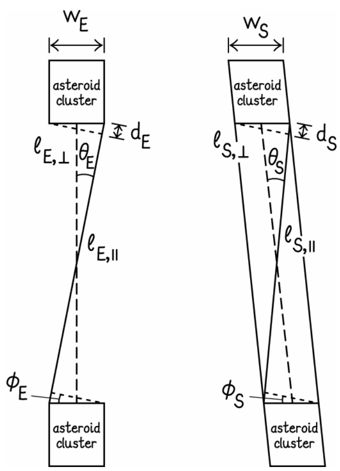
$$=-15\text{ s}$$

Thus $\Delta x_D=-7.5\times10^9\text{ m}$ and $\Delta t_D=-15$ s.

14.49. (*a*) Call the distance from Earth to the ship in Earth's reference frame at $t=0$ d_{E0}, and call the additional distance the ship travels while the message is being sent to Earth (also in Earth's reference frame) $d_{E\text{ message}}$. The ship also continues moving away from Earth as the reply is being sent. Call the additional distance travelled by the ship (in Earth's reference frame) while the reply is approaching it $d_{E\text{ reply}}$. Then the distance that the reply must travel in order to catch up with the ship is $d_{E\text{ total}}=d_{E\,0}+d_{E\text{ message}}+d_{E\text{ reply}}=d_{E\,0}+\dfrac{d_{E\,0}}{c_0}v_{\text{ship}}+v_{\text{ship}}\Delta t_{E\text{ reply}}$. Since the signal itself moves at the speed of light in a vacuum, we can write $c_0\Delta t_{E\text{ reply}}=d_{E\,0}+\dfrac{d_{E\,0}}{c_0}v_{\text{ship}}+v_{\text{ship}}\Delta t_{E\text{ reply}}$. Rearranging, we find $\Delta t_{E\text{ reply}}=\dfrac{d_{E\,0}\left(1+\dfrac{v_{\text{ship}}}{c_0}\right)}{(c_0-v_{\text{ship}})}=\dfrac{(3.47\times10^{11}\text{ m})(1+0.610)}{(0.390)(3.00\times10^8\text{ m/s})}=4.775\times10^3$ s. This is the time required for the reply to travel from Earth to the spaceship. However, the ground crew set their clocks in such a way that $t=0$ corresponds to the moment the initial message was sent by the ship. That message required $\Delta t_{E\text{ message}}=\dfrac{d_{E\,0}}{c_0}=\dfrac{(3.47\times10^{11}\text{ m})}{(3.00\times10^8\text{ m/s})}=$ 1.157×10^3 s to reach the station. Thus the reading on the ground crew's clock when the reply reaches the spaceship is $t=\Delta t_{E\text{ message}}+\Delta t_{E\text{ reply}}=(4.775\times10^3\text{ s})+(1.157\times10^3\text{ s})=5.93\times10^3$ s (or 98.9 minutes). (*b*) As outlined in part (*a*) the total distance is $d_{E\text{ total}}=d_{E\,0}+d_{E\text{ message}}+d_{E\text{ reply}}=d_{E\,0}\left(1+\dfrac{v_{\text{ship}}}{c_0}\right)+v_{\text{ship}}\Delta t_{E\text{ reply}}=(3.47\times10^{11}\text{ m})(1+0.610)+$ $(0.610)(3.00\times10^8\text{ m/s})(4.775\times10^3\text{ s})=1.43\times10^{12}$ m. (*c*) Since the ship has been moving at the same velocity relative to Earth throughout the communication, this is a straightforward application of time dilation: $\Delta t_{\text{ship total}}=\Delta t_{E\text{ total}}\sqrt{1-v_{\text{ship}}^2/c_0^2}=(5.93\times10^3\text{ s})\sqrt{1-(0.610)^2}=4.70\times10^3$ s (or 78.3 minutes).

14.50. (*a*) We know the relative speed of reference frame M as seen by observers on A, so we can calculate the effect of time dilation: $\Delta t_A=\dfrac{\Delta t_M}{\sqrt{1-v_{AM}^2/c_0^2}}=\dfrac{(1.00\text{ s})}{\sqrt{1-(0.525)^2}}=1.17$ s. (*b*) We know the speeds of both ships A and B relative to Earth only, so we choose to work in the Earth reference frame. We first determine the speed of M relative to Earth using equation (14.33): $v_{EM\,x}=\dfrac{v_{AM\,x}-v_{AE\,x}}{1-\dfrac{v_{AE\,x}}{c_0^2}v_{AM\,x}}=\dfrac{(0.525c_0)-(-0.862c_0)}{1-\dfrac{(-0.862c_0)}{c_0^2}(0.525c_0)}=0.9549c_0$. Now that we know the speed of M and the speed of B in the same reference frame (Earth's) we can determine their relative velocity by a second application of equation (14.33): $v_{\text{BM}\,x} = \dfrac{v_{\text{EM}\,x} - v_{\text{EB}\,x}}{1 - \dfrac{v_{\text{EB}\,x}}{c_0^2} v_{\text{EM}\,x}} = \dfrac{(0.9549c_0) - (-0.717c_0)}{1 - \dfrac{(-0.717c_0)}{c_0^2}(0.9549c_0)} = 0.9924c_0$. Finally, now that we know the relative speed of the reference frames M and B, we can use time dilation to write $\Delta t_{\text{B}} = \dfrac{\Delta t_{\text{M}}}{\sqrt{1 - v_{\text{BM}}^2/c_0^2}} = \dfrac{(1.00\text{ s})}{\sqrt{1-(0.9924)^2}} = 8.15\text{ s}$. (*c*) In part (*b*), we calculated the speed of M relative to Earth. We use this speed and write the dilated time: $\Delta t_{\text{E}} = \dfrac{\Delta t_{\text{M}}}{\sqrt{1 - v_{\text{EM}}^2/c_0^2}} = \dfrac{(1.00\text{ s})}{\sqrt{1-(0.9549)^2}} = 3.37\text{ s}$.

14.51. (*a*) Call the width of an asteroid cluster w, and call the distance between them y. Simple trigonometry tells us $\theta = \tan^{-1}\left(\dfrac{w}{y}\right) = \tan^{-1}\left(\dfrac{(6.00\times10^8\text{ m})}{(3.00\times10^9\text{ m})}\right) = 11.3°$. (*b*) Call the reference frame of the spaceship S, and call the reference frame of a person at rest relative to the clusters E. Consider the figure below.



Note that the length of the diagonal path taken by the spaceship is $\ell_{\text{E},\|} = 3.0594\times10^9$ m. We use the "$\|$" notation because this distance is parallel to the motion of reference frame S (the spaceship reference frame) and is therefore length contracted. The line perpendicular to this path that points from the path to the far corner of the asteroid cluster is labeled $\ell_{\text{E},\perp}$ in the figure. This distance is important because it is perpendicular to the direction of motion of S, and will therefore not be length contracted. These two paths $\ell_{\text{E},\|}$ and $\ell_{\text{E},\perp}$ will be orthogonal to each other even in reference frame S. The angles ϕ_{E} and θ_{E} happen to be the same, but this is only true in E. The angles ϕ_{S} and θ_{S} will not, in general, be equal. Simple geometry shows us that $\ell_{\text{E},\perp} = w_{\text{E}}\cos(\phi_{\text{E}}) = w_{\text{E}}\cos(\theta_{\text{E}}) = (6.00\times10^8\text{ m})\cos(11.3°) = 5.884\times10^8$ m. So $\ell_{\text{S},\perp} = 5.884\times10^8$ m also, whereas $\ell_{\text{S},\|} = \ell_{\text{E},\|}\sqrt{1 - v_{\text{S}}^2/c_0^2} = (3.0594\times10^9\text{ m})\sqrt{1-(0.9)^2} = 1.334\times10^9$ m. The resulting geometry is shown in the right half of the figure. Geometry also shows us that $d_{\text{E}} = w_{\text{E}}\sin(\phi_{\text{E}}) = (6.00\times10^8\text{ m})\sin(11.3°) = 1.176\times10^8$ m, which tells us that $d_{\text{S}} = d_{\text{E}}\sqrt{1 - v_{\text{S}}^2/c_0^2} = 1.176\times10^8\text{ m} = 5.125\times10^7$ m. With these distances, we can now calculate $\theta_{\text{S}} = \tan^{-1}\left(\dfrac{\ell_{\text{S},\perp}}{\ell_{\text{S},\|} - d_{\text{S}}}\right) = \tan^{-1}\left(\dfrac{(5.884\times10^8\text{ m})}{(1.334\times10^9\text{ m}) - (5.125\times10^7\text{ m})}\right) = 24.6°$.

14.52. Using equation (14.42) we write $p = \gamma m v = \dfrac{1}{\sqrt{1 - v^2/c_0^2}}(207)m_{\text{e}}v = \dfrac{1}{\sqrt{1-(0.500)^2}}(207)(9.11\times10^{-31}\text{ kg})(0.500)(3.00\times10^8\text{ m/s}) = 3.27\times10^{-20}\text{ kg}\cdot\text{m/s}$.

14.53. (*a*) The mass does not increase. The factor is 1.00. (*b*) The inertia can be calculated at each speed to determine the factor by which the inertia increases. We calculate $\dfrac{\dfrac{m_e}{\sqrt{1-v_f^2/c_0^2}}}{\dfrac{m_e}{\sqrt{1-v_i^2/c_0^2}}}=\dfrac{\sqrt{1-v_i^2/c_0^2}}{\sqrt{1-v_f^2/c_0^2}}=\dfrac{\sqrt{1-(0.700)^2}}{\sqrt{1-(0.900)^2}}=1.64.$

14.54. (*a*) Using equation (14.42) we write $p=\gamma mv=\dfrac{m_e v}{\sqrt{1-v^2/c_0^2}}=\dfrac{1}{\sqrt{1-(0.500)^2}}(9.11\times10^{-31}\text{ kg})(0.500)$ $(3.00\times10^8\text{ m/s})=1.58\times10^{-22}\text{ kg}\cdot\text{m/s}$. (*b*) We require that $\dfrac{1}{\sqrt{1-v^2/c_0^2}}=\dfrac{2}{\sqrt{1-(0.500)^2}}\Rightarrow v=c_0\sqrt{1-\dfrac{3}{16}}=0.901c_0$.

14.55. (*a*) We know the relative speeds of frames A an B. Let us call the direction of B as measured by A the $+\hat{x}$ direction. Using equation (14.42) we write

$$p_{AB}=\gamma_{AB}m_B v_{AB}=\frac{m_B v_{AB}}{\sqrt{1-v_{AB}^2/c_0^2}}=\frac{1}{\sqrt{1-(0.875)^2}}(7.71\times10^4\text{ kg})(0.875)(3.00\times10^8\text{ m/s})=4.18\times10^{13}\text{ kg}\cdot\text{m/s}$$

so $\vec{p}_{AB}=4.18\times10^{13}\text{ kg}\cdot\text{m/s }\hat{\imath}$. (*b*) This is nearly identical to (*a*), except we use the mass of object A, and the sign of the relative velocity changes. Thus

$$p_{BA}=\gamma_{BA}m_A v_{BA}=\frac{m_A v_{BA}}{\sqrt{1-v_{BA}^2/c_0^2}}=\frac{1}{\sqrt{1-(0.875)^2}}(4.24\times10^5\text{ kg})(0.875)(3.00\times10^8\text{ m/s})=2.30\times10^{14}\text{ kg}\cdot\text{m/s}$$

so $\vec{p}_{BA}=-2.30\times10^{14}\text{ kg}\cdot\text{m/s }\hat{\imath}$. (*c*) We are given the speed of reference frame C relative to reference frame A, so calculating the momentum of object A is done exactly as in part (*b*):

$$p_{CA}=\gamma_{CA}m_A v_{CA}=\frac{m_A v_{CA}}{\sqrt{1-v_{CA}^2/c_0^2}}=\frac{1}{\sqrt{1-(0.300)^2}}(4.24\times10^5\text{ kg})(0.300)(3.00\times10^8\text{ m/s})=4.00\times10^{13}\text{ kg}\cdot\text{m/s}$$

so $\vec{p}_{CA}=-2.30\times10^{14}\text{ kg}\cdot\text{m/s }\hat{\imath}$. Now, to determine the relative speed of reference frame B compared to reference frame C, we must use equation (14.33): $v_{CBx}=\dfrac{v_{ABx}-v_{ACx}}{1-\dfrac{v_{ACx}}{c_0^2}v_{ABx}}=\dfrac{(0.875c_0)-(0.300c_0)}{1-\dfrac{(0.300c_0)(0.875c_0)}{c_0^2}}=0.780c_0$. Now that we have the relative speed between the two reference frame, we can calculate the momentum of object B in reference frame C:

$$p_{CB}=\gamma_{CB}m_B v_{CB}=\frac{m_B v_{CB}}{\sqrt{1-v_{CB}^2/c_0^2}}=\frac{1}{\sqrt{1-(0.780)^2}}(7.71\times10^4\text{ kg})(0.780)(3.00\times10^8\text{ m/s})=2.88\times10^{13}\text{ kg}\cdot\text{m/s}$$

so $\vec{p}_{CB}=2.88\times10^{13}\text{ kg}\cdot\text{m/s }\hat{\imath}$.

14.56. (*a*) Note that the side length must be 2.00 m in the rest reference frame, for the given volume of the cube. The cube will only be contracting along the direction of motion. Call the direction of motion and call the length of a side parallel to the x axis ℓ_x. Then $\ell_x=\ell_{0x}\sqrt{1-v_x^2/c_0^2}=(2.00\text{ m})\sqrt{1-(0.672)^2}=1.48\text{ m}$. Thus the new mass density is $\rho=\dfrac{m}{V}=\dfrac{m}{\ell_x\ell_{0y}\ell_{0z}}=\dfrac{(200\text{ kg})}{(1.48\text{ m})(2.00\text{ m})(2.00\text{ m})}=33.8\text{ kg/m}^3$. (*b*) In order for this new object to stop the cube, it must have momentum that is equal in magnitude and opposite in direction compared to the cube. Equating the two momentum magnitudes we find

$$\frac{m_{\text{cube}} v_{\text{E cube}}}{\sqrt{1-v_{\text{E cube}}^2/c_0^2}} = \frac{m_{\text{object}} v_{\text{E object}}}{\sqrt{1-v_{\text{E object}}^2/c_0^2}} \Rightarrow \frac{(m_{\text{cube}} v_{\text{E cube}})^2}{(1-v_{\text{E cube}}^2/c_0^2)}(1-v_{\text{E object}}^2/c_0^2) = (m_{\text{object}} v_{\text{E object}})^2 \Rightarrow v_{\text{object}}$$

$$= \sqrt{\frac{(m_{\text{cube}} v_{\text{E cube}})^2}{\left[\frac{(m_{\text{cube}} v_{\text{E cube}})^2}{(1-v_{\text{E cube}}^2/c_0^2)c_0^2} + m_{\text{object}}^2\right](1-v_{\text{E cube}}^2/c_0^2)}}$$

$$= \sqrt{\frac{((200 \text{ kg})(0.672c_0))^2}{\left[\frac{((200 \text{ kg})(0.672c_0))^2}{(1-(0.672c_0)^2)c_0^2} + (100 \text{ kg})^2\right](1-(0.672)^2)}} = 0.876c_0$$

14.57. The half-life of a muon is 1.5×10^{-6} s. From the definition of a half-life, we know $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\Delta t_{\text{proper}}/\Delta t_{\text{half}}}$.

Rearranging, we can write

$$\frac{\ln\left(\frac{N}{N_0}\right)}{\ln\left(\frac{1}{2}\right)}\Delta t_{\text{half}} = \Delta t_{\text{proper}} = \Delta t_{\text{Earth}}\sqrt{1-v^2/c_0^2} = \frac{h}{v}\sqrt{1-v^2/c_0^2}$$

where h is the height at which the muons are created. Solving for the speed yields

$$v = \left(\left[\frac{\ln\left(\frac{N}{N_0}\right)}{\ln\left(\frac{1}{2}\right)}\Delta t_{\text{half}}\right]^2 + \frac{h^2}{c_0^2}\right)^{-1/2} h = \left(\left[\frac{\ln\left(\frac{1}{10{,}000}\right)}{\ln\left(\frac{1}{2}\right)}(1.5\times10^{-6} \text{ s})\right]^2 + \frac{(1.00\times10^4 \text{ m})^2}{(3.00\times10^8 \text{ m/s})^2}\right)^{-1/2} (1.00\times10^4 \text{ m})$$

$$= 2.57\times10^8 \text{ m/s or } 0.858c_0$$

This yields a momentum of $p = \frac{m_\mu v_\mu}{\sqrt{1-v_\mu^2/c_0^2}} = \frac{(207)(9.11\times10^{-31} \text{ kg})(2.57\times10^8 \text{ m/s})}{\sqrt{1-(0.858)^2}} = 9.45\times10^{-20} \text{ kg}\cdot\text{m/s}.$

14.58. Call the direction toward the star $+x$. Conservation of momentum allows us to write $p_{\text{SA}x,\text{i}} + p_{\text{SB}x,\text{i}} = p_{\text{SA}x,\text{f}} + p_{\text{SB}x,\text{f}}$, or $\frac{m_{\text{A}} v_{\text{SA}x,\text{i}}}{\sqrt{1-v_{\text{SA}x,\text{i}}^2/c_0^2}} + \frac{m_{\text{B}} v_{\text{SB}x,\text{i}}}{\sqrt{1-v_{\text{SB}x,\text{i}}^2/c_0^2}} = \frac{(m_{\text{A}}+m_{\text{B}})v_{(\text{A+B})x,\text{f}}}{\sqrt{1-v_{(\text{A+B})x,\text{f}}^2/c_0^2}}$. Solving for the final x component of the velocity yields

$$v_{(\text{A+B})x,\text{f}} = \sqrt{\frac{\left(\frac{m_{\text{A}} v_{\text{SA}x,\text{i}}}{\sqrt{1-v_{\text{SA}x,\text{i}}^2/c_0^2}} + \frac{m_{\text{B}} v_{\text{SB}x,\text{i}}}{\sqrt{1-v_{\text{SB}x,\text{i}}^2/c_0^2}}\right)^2}{\left[\left(\frac{m_{\text{A}} v_{\text{SA}x,\text{i}}}{\sqrt{1-v_{\text{SA}x,\text{i}}^2/c_0^2}} + \frac{m_{\text{B}} v_{\text{SB}x,\text{i}}}{\sqrt{1-v_{\text{SB}x,\text{i}}^2/c_0^2}}\right)^2 \frac{1}{c_0^2} + (m_{\text{A}}+m_{\text{B}})^2\right]}}$$

$$= \sqrt{\frac{\left(\frac{(1.00 \text{ kg})(0.500c_0)}{\sqrt{1-(0.500)}} + \frac{(1.00 \text{ kg})(-0.300c_0)}{\sqrt{1-(0.300)}}\right)^2}{\left[\left(\frac{(1.00 \text{ kg})(0.500c_0)}{\sqrt{1-(0.500)}} + \frac{(1.00 \text{ kg})(-0.300c_0)}{\sqrt{1-(0.300)}}\right)^2 \frac{1}{c_0^2} + (2.00 \text{ kg})^2\right]}} = 0.130c_0$$

Thus the final velocity of the chunks of rock is $0.130c_0$ toward the star.

14.59. Call the 150 kg probe probe A and call the 250 kg probe probe B. Call the direction of motion of probe A the $+x$ direction. We require that the momentum in some new frame (call it reference frame 0) be zero. Thus $p_{0\mathrm{A}x} + p_{0\mathrm{B}x} = 0$ or

$$\frac{m_\mathrm{A} v_{0\mathrm{A}x}}{\sqrt{1 - v_{0\mathrm{A}x}^2/c_0^2}} + \frac{m_\mathrm{B} v_{0\mathrm{B}x}}{\sqrt{1 - v_{0\mathrm{B}x}^2/c_0^2}} = 0 \tag{1}$$

Now we use equation (14.33) to relate the above expression to the speeds in the Earth reference frame. We find that

$$v_{0\mathrm{A}x} = \frac{v_{\mathrm{EA}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EA}x}} \tag{2}$$

and

$$v_{0\mathrm{B}x} = \frac{v_{\mathrm{EB}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EB}x}} \tag{3}$$

Inserting equations (2) and (3) into equation (1), we find

$$\frac{m_\mathrm{A}\left(\dfrac{v_{\mathrm{EA}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EA}x}}\right)}{\sqrt{1 - \dfrac{1}{c_0^2}\left(\dfrac{v_{\mathrm{EA}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EA}x}}\right)}} + \frac{m_\mathrm{B}\left(\dfrac{v_{\mathrm{EB}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EB}x}}\right)}{\sqrt{1 - \dfrac{1}{c_0^2}\left(\dfrac{v_{\mathrm{EB}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EB}x}}\right)}} = 0 \tag{4}$$

Equation (4) is algebraically mess, but the only unknown in the entire expression is $v_{\mathrm{E}0x}$. Solving for $v_{\mathrm{E}0x}$, we find $v_{\mathrm{E}0x} = 8.44 \times 10^7$ m/s or $v_{\mathrm{E}0x} = 0.2812 c_0$. Inserting this back into equations (2) and (3) respectively shows us

$$v_{0\mathrm{A}x} = \frac{v_{\mathrm{EA}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EA}x}} = \frac{(0.860c_0) - (0.2812c_0)}{1 - \frac{(0.281c_0)}{c_0^2}(0.860c_0)} = 0.763c_0$$

and

$$v_{0\mathrm{B}x} = \frac{v_{\mathrm{EB}x} - v_{\mathrm{E}0x}}{1 - \frac{v_{\mathrm{E}0x}}{c_0^2} v_{\mathrm{EB}x}} = \frac{(-0.355c_0) - (0.2812c_0)}{1 - \frac{(0.281c_0)}{c_0^2}(-0.355c_0)} = -0.578c_0$$

Quickly re-inserting these values into the expression for the relativistic momentum should verify that the total momentum in this frame is indeed zero, with $p_{0\mathrm{A}x} = 5.32 \times 10^{10}\ \mathrm{kg \cdot m/s}$ and $p_{0\mathrm{B}x} = -5.32 \times 10^{10}\ \mathrm{kg \cdot m/s}$.

14.60. We require $(\gamma - 1)mc_0^2 = mc_0^2 \Rightarrow \dfrac{1}{\sqrt{1 - v^2/c_0^2}} - 1 = 1 \Rightarrow v = c_0\sqrt{\dfrac{3}{4}} = 0.866c_0.$

14.61. (*a*) Because we know the total energy $E = \gamma m c_0^2$, we can say by inspection that $\gamma_\mathrm{C} > \gamma_\mathrm{B} > \gamma_\mathrm{A}$. (*b*) Because the kinetic energy is $K = (\gamma - 1)mc_0^2$, the order will be the same as in part (*a*): $K_\mathrm{C} > K_\mathrm{B} > K_\mathrm{A}$. (*c*) Because $\gamma = \dfrac{1}{\sqrt{1 - v^2/c_0^2}}$, a larger Lorentz factor always corresponds to a larger speed. Thus the order is the same as in parts (*a*) and (*b*): $|\vec{v}_\mathrm{C}| > |\vec{v}_\mathrm{B}| > |\vec{v}_\mathrm{A}|$. (*d*) Since $p = \gamma m v$, and particle C has the largest speed and the largest Lorentz factor, it must also have the greatest magnitude of momentum. Particle A has the smallest Lorentz factor and the smallest speed, so particle A must have the smallest magnitude of momentum. Thus $|\vec{p}_\mathrm{C}| > |\vec{p}_\mathrm{B}| > |\vec{p}_\mathrm{A}|$.

14.62. We know the internal energy is independent of speed: $E_{\text{internal}} = m_0c_0^2 = (9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2 = 8.20\times10^{-14}\ \text{J}$.

14.63. Note that the difference in mass when one atom of uranium is reacted is $\Delta m = m_{\text{f}} - m_{\text{i}} = m_{\text{Th}} + m_{\text{He}} - m_{\text{U}} = (388.638509\times10^{-27}\ \text{kg}) + (6.646478\times10^{-27}\ \text{kg}) - (395.292599\times10^{-27}\ \text{kg}) = -7.612\times10^{-30}\ \text{kg}$. When 1.00 kg reacts, the change in mass of the system will be $\Delta m_{1.00\ \text{kg}} = \dfrac{1.00\ \text{kg}}{395.292599\times10^{-27}\ \text{kg}}(-7.612\times10^{-30}\ \text{kg}) = -1.9257\times10^{-5}\ \text{kg}$. This corresponds to a change of internal energy equal to $\Delta E_{\text{internal}} = \Delta m c_0^2 = (-1.9257\times10^{-5}\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2 = -1.73\times10^{12}\ \text{J}$. Thus the uranium releases $1.73\times10^{12}\ \text{J}$ per kilogram, which is almost 58,000 times the energy released by burning a kilogram of coal.

14.64. The speed that the rocket must attain to achieve the trip described can be written $v = \dfrac{d_{\text{E}}}{\Delta t_{\text{E}}} = \dfrac{d_{\text{E}}\sqrt{1-v^2/c_0^2}}{\Delta t_{\text{A}}}$, or

$$v = \frac{d_{\text{E}}}{\sqrt{(\Delta t_{\text{A}})^2 + \dfrac{d_{\text{E}}^2}{c_0^2}}} = \frac{(4.24c_0\cdot\text{y})}{\sqrt{\left((437\ \text{days})\times\dfrac{(1\ \text{y})}{(365.25\ \text{days})}\right)^2 + \dfrac{(4.24c_0\cdot\text{y})^2}{c_0^2}}} = 0.9624c_0$$

Ignoring losses of any kind, we simply find the final kinetic energy of the rocket, and equate that to the energy required to get the rocket up to that speed:

$$K = (\gamma - 1)mc_0^2 = \left(\frac{1}{\sqrt{1-v^2/c_0^2}} - 1\right)mc_0^2 = \left(\frac{1}{\sqrt{1-(0.9624)^2}} - 1\right)(2.00\times10^{6}\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2 = 4.83\times10^{23}\ \text{J}$$

The rocket would require $4.83\times10^{23}\ \text{J}$ of energy, which is about 8×10^{2} times the predicted global energy consumption for 2015.

14.65. Conservation of momentum tells us that the magnitudes of the two final momenta must be equal:

$$p_{\text{E1}} = p_{\text{E2}} \Rightarrow \frac{m_1^2v_1^2}{1-\dfrac{v_1^2}{c_0^2}} = \frac{m_2^2v_2^2}{1-\dfrac{v_2^2}{c_0^2}}$$

Which we solve for v_2 to obtain

$$v_2^2 = \frac{m_1^2v_1^2\left(1-\dfrac{v_1^2}{c_0^2}\right)^{-1}}{m_2^2 + \dfrac{m_1^2v_1^2}{c_0^2 - v_1^2}} \tag{1}$$

Conservation of energy tells us that $E_0 = E_1 + E_2$ or

$$m_0c_0^2 = \gamma_1 m_1c_0^2 + \gamma_2 m_2c_0^2 = \frac{m_1c_0^2}{\sqrt{1-v_1^2/c_0^2}} + \frac{m_2c_0^2}{\sqrt{1-v_2^2/c_0^2}} \tag{2}$$

Inserting equation (1) into equation (2) gives us an algebraically difficult expression that can be rearranged to yield and conservation of energy tells us $v_1 = \dfrac{c_0}{m_0}\sqrt{m_0^2 - (m_1+m_2)^2}$. Inserting this expression into the equations describing relativistic energy and momentum yields

$$E_1 = \frac{m_1c_0^2}{\sqrt{1 - \dfrac{(m_1 - m_2 - m_{\text{orig}})(m_1 + m_2 - m_{\text{orig}})(m_1 - m_2 + m_{\text{orig}})(m_1 + m_2 + m_{\text{orig}})}{(m_1^2 - m_2^2 + m_{\text{orig}}^2)^2}}}$$

$$p_1 = \frac{m_1c_0\sqrt{(m_1 - m_2 - m_{\text{orig}})(m_1 + m_2 - m_{\text{orig}})(m_1 - m_2 + m_{\text{orig}})(m_1 + m_2 + m_{\text{orig}})}}{\sqrt{(m_1^2 - m_2^2 + m_{\text{orig}}^2)^2 - (m_1 - m_2 - m_{\text{orig}})(m_1 + m_2 - m_{\text{orig}})(m_1 - m_2 + m_{\text{orig}})(m_1 + m_2 + m_{\text{orig}})}}$$

14.66. Only energy stored internally (as potential energy) can contribute to the mass. Thus only (*b*) contributes to the mass.

14.67. (*a*) The kinetic energy is given by $K = (\gamma - 1)mc_0^2$, such that $v = c_0\sqrt{1 - \left(\dfrac{K}{mc_0^2} + 1\right)^{-2}} =$ $c_0\sqrt{1 - \left(\dfrac{(7.000\times10^{12}\text{ eV})}{(938\times10^{6}\text{ eV})} + 1\right)^{-2}} = (1 - 8.98\times10^{-9})c_0$. (*b*) The maximum possible mass would be achieved if all energy from both protons were converted to mass. In this case $m_{\text{particle}}c_0^2 = 2(E_{\text{proton}}) \Rightarrow m_{\text{particle}} = 1.400\times10^{13}\text{ eV}/c_0^2$, or alternatively $m_{\text{particle}} = \dfrac{2(7.000\times10^{12}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{(3.00\times10^{8}\text{ m/s})^2} = 2.49\times10^{-23}\text{ kg}$.

14.68. The minimum ratio is achieved when all energy in the electron and positron go into the internal energy of the proton and the antiproton (with no kinetic energy left over). In that case, we can write $2E_{\text{electron}} = 2\gamma m_{\text{electron}}c_0^2 = 2m_{\text{proton}}c_0^2$, which tells us that $\gamma = \dfrac{m_{\text{proton}}}{m_{\text{electron}}}$. The ratio of the kinetic energy to the rest mass energy is $\dfrac{K_{\text{electron}}}{E_{0,\text{electron}}} = \dfrac{(\gamma - 1)m_{\text{electron}}c_0^2}{m_{\text{electron}}c_0^2} = \gamma - 1 = \left(\dfrac{m_{\text{proton}}}{m_{\text{electron}}}\right) - 1 = 1836.15 - 1 = 1835.15.$

14.69. (*a*) The minimum amount of energy released would correspond to a case where the hydrogen and antihydrogen were both essentially at rest, such that only internal energy is released. In that case, $\Delta E_{\text{internal}} = \Delta mc_0^2 = -2m_{\text{H}}c_0^2 = -2(1.6737\times10^{-27}\text{ kg})(3.00\times10^{8}\text{ m/s})^2 = -3.01\times10^{-10}\text{ J}$. This loss of internal energy is equal in magnitude to the energy released, meaning that 3.01×10^{-10} J of energy is released. (*b*) The energy in 10 mg of each would be $E = E_{\text{per atom}}N_{\text{atoms}} = E_{\text{per atom}}\left(\dfrac{m}{m_{\text{atomic}}}\right) = (3.013\times10^{-10}\text{ J})\left(\dfrac{1.0\times10^{-5}\text{ kg}}{1.6737\times10^{-27}\text{ kg}}\right) = 1.80\times10^{12}\text{ J}$. Using the given ratio of energy used per meter travelled at highway speeds we find $d = \dfrac{E_{\text{fuel}}}{E_{\text{per meter}}} = \dfrac{(1.80\times10^{12}\text{ J})}{(2.5\times10^{3}\text{ J/m})} = 7.2\times10^{8}\text{ m}$.

14.70. We apply equation (14.33) and find $v_{\text{BS}x} = \dfrac{v_{\text{AS}x} - v_{\text{AB}x}}{1 - \dfrac{v_{\text{AB}x}}{c_0^2}v_{\text{AS}x}} = \dfrac{(0.500c_0) - (-0.600c_0)}{1 - \dfrac{(-0.600c_0)}{c_0^2}(0.500c_0)} = 0.846c.$

14.71. Since the muon is moving parallel to the tree, the tree's height will be contracted: $h = h_0\sqrt{1 - v^2/c_0^2} =$ $(32.6\text{ m})\sqrt{1 - (0.382)^2} = 30.1\text{ m}$. So the height of the tree has been contracted, but the distance perpendicular to the tree to the vertical line has not contracted. To me more concrete, we might imagine a line from the top of our angled tree and perpendicular to it over to a specific point on a vertical tree. That distance cannot change. Using the geometry in the rest frame of the tree, we see that distance $d_\perp = h_0\tan(\phi) = (32.0\text{ m})\tan(26.0°) = 15.9\text{ m}$. After the height of the tree is contracted, the new angle between the tree and vertical is given by $\phi' = \tan^{-1}\left(\dfrac{d_\perp}{h}\right) =$ $\tan^{-1}\left(\dfrac{(15.9\text{ m})}{(30.1\text{ m})}\right) = 27.8°$. Thus the tree is 30.1 m tall and is at an angle of $27.8°$ from vertical.

14.72. The spot speed varies, depending on the angle at any given instant, but it appears to move at an average speed of 3.81×10^{8} m/s, or $1.27c_0$. This is not a violation of special relativity because nothing actually moves along the path of the moving spot. No information is transmitted from one region of the cloud to another, and no matter moves from one region of the cloud to another. Photons leaving the searchlight take $3.3\ \mu\text{s}$ to reach the nearest region of the cloud and $4.85\ \mu\text{s}$ to reach the farthest region (after the light has been rotated). Those are the actual paths taken by information and/or particles, and neither path violates special relativity.

14.73. Yes, any reference frame moving along the perpendicular bisector of a line drawn between the two events (moving at any speed that is physically possible) would see the two events as being simultaneous.

14.74. (*a*) Pilot A sees ship B contracted to a length of $\ell_{\text{AB}} = \ell_{\text{BB}}\sqrt{1-v_{\text{BA}}^2/c_0^2} = (250\text{ m})\sqrt{1-(0.580)^2} = 203.7\text{ m}$. The time required for ship B to travel that distance, allowing the entire ship to pass pilot A is $\Delta t_{\text{A}} = \dfrac{\ell_{\text{AB}}}{v_{\text{BA}}} = \dfrac{(203.7\text{ m})}{(0.580)(3.00\times10^8\text{ m/s})} = 1.17\times10^{-6}\text{ s}$. (*b*) The easiest way to find this time is to use time dilation in comparison to part (*a*). $\Delta t_{\text{B}} = \dfrac{\Delta t_{\text{A}}}{\sqrt{1-v_{\text{BA}}^2/c_0^2}} = \dfrac{(1.17\times10^{-6}\text{ s})}{\sqrt{1-(0.580)^2}} = 1.44\times10^{-6}\text{ s}$. (*c*) Pilot A does not see his own ship contracted, because it is not moving relative to pilot A. Thus pilot A will see a distance of 250 m between the two events: (1) the front of ship A passing pilot B, and (2) the rear of ship A passing pilot B. Thus Pilot A will measure a time difference of $\Delta t_{\text{A}} = \dfrac{\ell_{\text{AA}}}{v_{\text{BA}}} = \dfrac{(250\text{ m})}{(0.580)(3.00\times10^8\text{ m/s})} = 1.44\times10^{-6}\text{ s}$. (*d*) We can use time dilation as before, only now it is pilot B who measures the proper time. Thus $\Delta t_{\text{B}} = \Delta t_{\text{A}}\sqrt{1-v_{\text{BA}}^2/c_0^2} = (1.44\times10^{-6}\text{ s})\sqrt{1-(0.580)^2} = 1.17\times10^{-6}\text{ s}$.

14.75. (*a*) We can simply use equation (14.33) to write $v_{\text{BA}x} = \dfrac{v_{\text{EA}x}-v_{\text{EB}x}}{1-\dfrac{v_{\text{EB}x}}{c_0^2}v_{\text{EA}x}} = \dfrac{(0.732c_0)-(-0.914c_0)}{1-\dfrac{(-0.914c_0)}{c_0^2}(0.732c_0)} = 0.986c_0$.

(*b*) There is a subtlety here. It is not possible for any object to move at a speed greater than c_0 in any reference frame. It is possible, however, for two objects to cover a distance between them at a speed greater than c_0 as measured by a third reference frame. As we just showed in part (*a*), even with one ship moving to the right at $0.732c_0$ and the other moving to the left at $0.914c_0$, they still do see each other moving at a speed greater than c_0. But to observers on Earth, the two ships work together to cover the distance between them at a rate that is greater than c_0. It is very important to note that there is still no object (or information) that is moving faster than c_0. In Earth's reference frame, we can write $\Delta x = |\Delta x_{\text{EA}}| + |\Delta x_{\text{EB}}| = |v_{\text{EA}}|\Delta t + |v_{\text{EB}}|\Delta t \Rightarrow \Delta t = \dfrac{\Delta x}{|v_{\text{EA}}|+|v_{\text{EB}}|} =$ $\dfrac{(4.5\times10^{10}\text{ m})}{\big((0.732)+(0.914)\big)(3.00\times10^8\text{ m/s})} = 91.1\text{ s}$.

14.76. From the conservation of energy and momentum, we obtain the two equations:

$$\gamma_1 m_{\text{p1}}c_0^2 + m_{\text{p2}}c_0^2 = \gamma_{\text{f}}(40)m_{\text{p}}c_0^2$$
$$\gamma_1 m_{\text{p1}}v_1 = \gamma_{\text{f}}(40)m_{\text{p}}v_{\text{f}}$$

or

$$\frac{1}{\sqrt{1-v_1^2/c_0^2}} + 1 = \frac{(40)}{\sqrt{1-v_f^2/c_0^2}}$$
$$\frac{v_1}{\sqrt{1-v_1^2/c_0^2}} = \frac{(40)v_{\text{f}}}{\sqrt{1-v_f^2/c_0^2}}$$

Solving this simple system of two equations and two unknowns yields $v_1 = (1 - 7.83\times10^{-7})c_0$ and $v_1 = (0.9987)c_0$. Then the ratio of the energy of p_1 to its internal energy is

$$\gamma_1 = \frac{1}{\sqrt{1 - v_1^2/c_0^2}} = \frac{1}{\sqrt{1-(1-7.83\times10^{-7})^2}} = 799$$

14.77. At the moment the Orion detects the hostile cruiser it is a distance $d_{\text{O,SO}} = 1.800\times10^{11}\ \text{m}$ from the space station as seen from Orion's reference frame. Since we must ultimately determine how much time the station has, let us relate known quantities in all reference frames to the corresponding quantities in the space station reference frame. The distance the signal must cross from Orion to the space station is $d_{\text{S,SO}} = \dfrac{d_{\text{O,SO}}}{\sqrt{1 - v_{\text{OS}}^2/c_0^2}} = \dfrac{(1.800\times10^{11}\ \text{m})}{\sqrt{1-(0.6000)^2}} =$ 2.25×10^{11}. Thus the warning signal would require at time $\Delta t_{\text{S,warning}} = \dfrac{d_{\text{S,SO}}}{c_0} = \dfrac{(2.25\times10^{11}\ \text{m})}{(3.00\times10^{8}\ \text{m/s})} = 750\ \text{s}.$ Now there is some ambiguity in the script. Saying that Orion detects a hostile cruiser at $8.00\times10^{10}\ \text{m}$ dead ahead could mean that they see an image of a ship at that distance (meaning they are seeing light that left the ship several minutes ago, or it could mean that they see the ship coming and are able to calculate where the ship actually is. Let us initially assume that the crew of the Orion is careful and that they are giving the actual position of the hostile cruiser. In that case the Orion would see a distance between the cruiser and the space station of $d_{\text{O,CS}} = 2.60\times10^{11}\ \text{m},$ such that the space station would see a distance to the cruiser of $d_{\text{S,CS}} = \dfrac{d_{\text{O,CS}}}{\sqrt{1 - v_{\text{SO}}^2/c_0^2}} = \dfrac{(2.60\times10^{11}\ \text{m})}{\sqrt{1-(0.6000)^2}} = 3.25\times10^{11}\ \text{m}.$ We can use equation 14.33 to determine the speed at which the cruiser is approaching the space station in the reference frame of the space station:

$$v_{\text{SC}x} = \frac{v_{\text{OC}x} - v_{\text{OS}x}}{1 - \dfrac{v_{\text{OS}x}}{c_0^2}v_{\text{OC}x}} = \frac{(0.8000c_0) - (0.6000c_0)}{1 - \dfrac{(0.6000c_0)}{c_0^2}(0.8000c_0)} = 0.3846c_0$$

So from the moment Orion sends the signal, in the reference frame of the space station, it will take the hostile cruiser a time $\Delta t_{\text{S,CS}} = \dfrac{d_{\text{S,CS}}}{v_{\text{S,C}}} = \dfrac{(3.25\times10^{11}\ \text{m})}{(0.3846)(3.00\times10^{8}\ \text{m/s})} = 2816.7\ \text{s}$ or 46.9 minutes. This is just over the time required for an evacuation, but this time is measured from the moment Orion sends the warning. The warning will not reach the space station for 750 s, leaving the station with only 2066.7 s or 34.4 minutes to evacuate. There is not sufficient time for an evacuation. Going back to our original assumption, if the crew of the Orion had been seeing light that left the battle cruiser several minutes earlier, that would only make the situation worse, since the cruiser would have advanced its position as the light travelled to the Orion. Thus, there is not sufficient time for an evacuation regardless of assumptions. Whether or not a re-write is required depends on where the story is going.

14.78. Rearranging the equations for spring potential energy and the relationship between mass and energy, you determine that this spring must satisfy the relationship

$$\Delta x = \sqrt{\frac{m_{\text{spring}}c^2}{k\cdot10^6}}$$

and you immediately note the difficulty of making the compression distance plausible. If a 1.0-kg spring had had an amazing spring constant of $10^6\ \text{N/m},$ you would need to compress it 300 m. This would require placing a mass of $3\times10^7\ \text{kg}$ on the spring. No material allows Hooke's law to apply at these scales, nor is there any material from which you could make a spring that is 300 m long and has a mass of only 1.0 kg. These implausible dimensions suggest that the only way to construct such a spring is to go to the microscopic scale. If, for example, a system of graphene nanotubes or sheets could be used as a spring, it might be possible to satisfy this constraint on a very small scale. Suppose a spring system is fashioned from 10^8 carbon atoms and has a spring constant of $5\ \text{N/m}.$ This spring needs to be compressed $55\ \mu\text{m},$ which require the spring to support a mass of 0.27 g. This suggestion is pushing the limits of what is scientifically possible and is probably not practical for your boss, but at least you have found a plausible solution.

14.79. The time measured in Earth's reference frame is $\Delta t_{\text{E}} = \dfrac{\Delta x_{\text{E}}}{v} = \dfrac{\Delta x_{\text{S}}}{v\sqrt{1-v^2/c_0^2}}$. In order to find the extremum of this function of the ship's speed v, we take the derivative and set it equal to zero:

$$\frac{d\Delta t_{\text{E}}}{dv} = \frac{d}{dv}\left(\frac{\Delta x_{\text{S}}}{v\sqrt{1-v^2/c_0^2}}\right) = \frac{d}{c_0^2(1-v^2/c_0^2)^{3/2}} - \frac{d}{v^2(1-v^2/c_0^2)^{1/2}} = 0$$

$$\Rightarrow v = \frac{c}{\sqrt{2}}$$

You can complete this race in the minimum possible amount of time (as measured in Earth's frame) by flying at a speed of $c/\sqrt{2}$. The winning time would be $\Delta t_{\text{E,min}} = \dfrac{\sqrt{2}(1.00\times 10^9\ \text{m})}{c_0\sqrt{1-(1/2)}} = 6.67\ \text{s}$.

15

PERIODIC MOTION

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

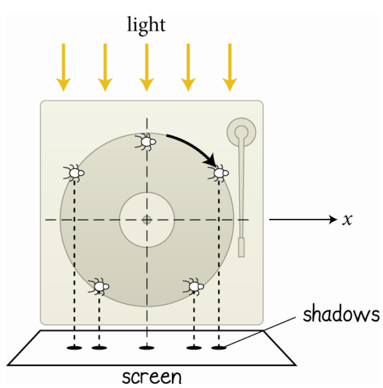
1. 10^2 J **2.** 10^2 J **3.** 10^0 Hz **4.** 10^0 m **5.** 10^1 s^{-1} **6.** 10^2 s^{-1} and 10^1 **7.** 10^3 N·m **8.** 10^4 kg/s

Guided Problems

15.2 Archaic music medium

1. Getting Started Let us call the direction in which the light is shining the y direction. Since the light is shining in the y direction, it gives us no information about where the bug is along the y axis. The position of the shadow along the x axis should be the same as the bug's position along the x axis. So in this way, we get information only about the motion of the bug in the x direction.

In the figure below we have shown the bug in several positions along its circular path. At each point the light will cause a shadow to appear on the distant screen. The shadows have been marked by black dots. Thus illustrates how the shadow will move back and forth across the screen, but we will have no information about the motion of the bug in the y direction.



For any position of the bug, we can always draw a vector pointing from the turntable's center to the bug's position. Thus vector can always be decomposed into components in the x and y directions. If ϑ is the angle from the positive x axis to the position vector $\vec{r}$, we can write the x and y components of the position vector as $r\cos(\vartheta)$ and $r\sin(\vartheta)$, respectively. We expect to find that the shadow moves across the screen in simple harmonic motion, and that simple harmonic motion is a projection or profile view of circular motion. We also know that the angle from the x axis will change as the turntable rotates, obeying $\vartheta = \omega t + \phi_i$. So we expect the motion along the x axis to be of the form $r\cos(\omega t + \phi_i)$.

2. Devise Plan We already expect the shadow to move according to $x(t) = x_{\max}\sin(\omega t + \phi_i)$. We need to determine the amplitude of the oscillation, the initial phase, and the rotational frequency. We can see that the maximum position along the x direction is just the radius of the bug's path, and we can determine the rotational frequency from the given number of revolutions per minute:

$$\omega = \frac{\Delta\vartheta}{\Delta t} = \frac{33.33(2\pi)}{1\text{min}}\left(\frac{1\text{ min}}{60\text{ s}}\right) = 3.49\text{ s}^{-1}$$

We find the initial phase by looking at the position when $t = 0$.

We can obtain the speed by taking the derivative of position with respect to time. And we can obtain the acceleration by taking another derivative.

The two lengths of the ship are different because of length contraction. Length contraction depends on the relative speed between two objects according to

3. Execute Plan (a) The position as a function of time will be

$$x(t) = x_{\max}\sin(\omega t + \phi_i) \tag{1}$$

Where $x_{\max} = 0.153\text{ m}$ $\omega = 3.49\text{ s}^{-1}$. The initial phase is chosen such that we satisfy $x(t=0) = (0.153\text{ m})\sin(\phi_i) = 0$, which means $\phi_i = 0$. Thus $x(t) = (0.153)\sin((3.49\text{ s}^{-1})t)$. ($b$) We take the time derivative of equation (1) and find

$$\begin{aligned} v_x(t) &= \frac{d}{dt}x(t) \\ v_x(t) &= x_{\max}\omega\cos(\omega t + \phi_i) \end{aligned} \tag{2}$$

The maximum value of the x component of the velocity in equation (2) is simply the amplitude $x_{\max}\omega$. One can think of this as inserting the maximum value of the time dependent cosine function (which is one). Thus

$$v_{\max} = x_{\max}\omega = (0.153\text{ m})(3.49\text{ s}^{-1}) = 0.532\text{ m/s}$$

(c) We now take another derivative of equation (2) to obtain the x component of acceleration as a function of time.

$$\begin{aligned} a_x(t) &= \frac{d}{dt}v(t) \\ a_x(t) &= -x_{\max}\omega^2\sin(\omega t + \phi_i) \end{aligned} \tag{3}$$

The maximum value that the x component of acceleration can reach is simply the amplitude $x_{\max}\omega^2$. One can think of this as maximizing the function $-\sin(\omega t + \phi_i)$ to find one.

Thus the maximum magnitude of the acceleration is

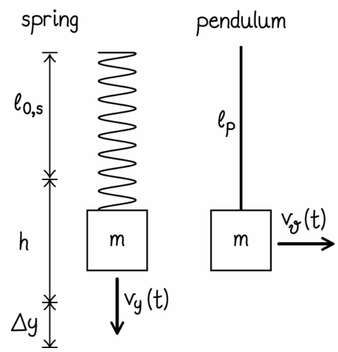
$$a_{\max} = x_{\max}\omega^2 = (0.153\text{ m})(3.49\text{ s}^{-1})^2 = 1.85\text{ m/s}^2$$

4. Evaluate Result The orders of magnitude we obtained are reasonable. But we can calculate the speed and acceleration of the bug in the plane of the record and compare to be sure. The speed of the bug around its circular path is $v = \frac{d}{\Delta t} = \frac{(33.33)2\pi(0.153\text{ m})}{60\text{ s}} = 0.532\text{ m/s}$, which is exactly the maximum speed we found for the shadow. This makes sense, because the fastest the shadow can ever move is when all the velocity of the bug is in the x direction. The radial component of the bug's acceleration is given by $a = \frac{v^2}{r} = \frac{(0.5323\text{ m/s})^2}{(0.153\text{ m})} = 1.85\text{ m/s}^2$. This acceleration always points toward the center of the circle, and sometimes that inward direction corresponds to the x axis. So it makes perfect sense that the maximum magnitude of the acceleration of the shadow would equal the radial acceleration of the bug.

15.4 In sync

1. Getting Started The spring motion and the pendulum motion are both periodic, and under the right conditions both are simple harmonic motion.

We sketch the two types of motion below.



We will assume that the oscillations of the pendulum are small enough that the period of the oscillation remains constant.

2. Devise Plan The general expression for the period of a pendulum is

$$T_{\mathrm{p}} = 2\pi\sqrt{\frac{\ell_{\mathrm{p}}}{g}} \tag{1}$$

And the period of a spring-mass system is

$$T_{\mathrm{s}} = 2\pi\sqrt{\frac{m}{k}} \tag{2}$$

We are given m as a variable. It is not immediately obvious that it will cancel out of our calculation, but it may. We know g, and we are trying to find ℓ_{p}. That leaves k to be determined. When we added the mass to the spring, it caused a stretch of the spring equal to h. We can use this to determine the spring constant by writing the sum of all forces on the block in the vertical direction. Let us choose $+y$ to point upward. Then

$$\sum F_y = F^{\mathrm{c}}_{\mathrm{sb}} - F^{G}_{\mathrm{Eb}} = ma_y = 0$$
$$kh - mg = 0$$
$$k = \frac{mg}{h} \tag{3}$$

Now we have enough information to equate the two expressions for the periods.

3. Execute Plan: Inserting equation (3) into equation (2), we find

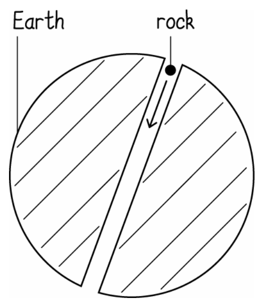
$$T_{\mathrm{s}} = 2\pi\sqrt{\frac{h}{g}}$$

If we now equate this expression to (1), we trivially see that the length of the pendulum is $\ell_{\mathrm{p}} = h.$

4. Evaluate Result It is surprising to many that the answer depends only on the stretch of the spring upon hanging the block. One might expect the answer to depend on the spring constant. But both the period and the additional stretch of the spring depend on the spring constant, so it is plausible that one could eliminate the dependence on the spring constant. It does make sense that the length of the pendulum depends on the additional stretch. If the block caused the spring to stretch a great deal, that would indicate a relatively weak spring and therefore a longer period of oscillation. In order for the pendulum to have a long period, the string would have to be long.

15.6 One deep hole

1. Getting Started We dig a hole completely through the center of the Earth and out the other side. We drop a rock into the hole and watch its position as it falls. We measure the amount of time required for the rock to return.



We expect the rock will accelerate downward (toward the center of Earth) upon release. Initially, the rock will have an acceleration equal to g. But as the rock moves in toward the center of the Earth, this acceleration will decrease.

2. Devise Plan When the rock is a distance $r < R_E$ from the center of Earth, only the fraction of Earth that is within that radius r will contribute to the net gravitational force on the rock. The density of Earth is not actually uniform, but we will assume it is for the purposes of this problem. In that case, the mass within the radius r is

$$m_{<r} = \frac{m_E r^3}{R_E^3}$$

Thus the gravitational force exerted on the rock at such a position is

$$F^G_{\text{Er},r} = -\frac{Gm_E m_r r}{R_E^3} = m_r a_{r,r} \tag{1}$$

This means

$$a = \frac{Gm_E r}{R_E^3} = g\left(\frac{r}{R_E}\right) \tag{2}$$

From equation (2) we see that the acceleration of the rock at the center of Earth is zero, which makes sense.

There is no reason to change variables at this point from $r \to x$, except to illustrate that the motion is one-dimensional, and can be described using our tools from one-dimensional periodic motion. With that change of variables, we write

$$a = g\left(\frac{x}{R_E}\right)$$

It is not obvious at this point that the position-dependent force described by equation (1) must yield simple harmonic motion. It is clear that the motion must be periodic. But not all periodic motion is harmonic. To see that the position will obey simple harmonic motion, let us re-write equation (1) by collecting all the constants together into one variable, which we will call $k = \frac{Gm_E m_r}{R_E^3}$:

$$F^G_{\text{Er},x} = -kx = m_r a_r$$

This looks exactly like the equation that describes the motion of a spring-mass system. The source of the restoring force is not a spring, it is gravity. But the force of gravity depends linearly on the distance from equilibrium (the center of Earth) just as the spring force does. The mathematical steps to determine the resulting motion are exactly the same. With our clever choice of the constant k, it is now clear that the motion will be harmonic motion, and we even have an equation that describes the period of the oscillation:

$$T = 2\pi\sqrt{\frac{m_r}{k}} \tag{3}$$

3. Execute Plan Since we know the rock will undergo simple harmonic motion, we an write its position as a function of time as:

$$x(t) = x_{\max}\sin(\omega t + \phi_i)$$

Here, the maximum distance from the center of Earth is $x_{\max} = R_E$. If we choose the moment of release to be $t = 0$, then we find $x(t) = R_E\sin(\phi_i) = R_E$, meaning $\phi_i = \frac{\pi}{2}$. Finally, the angular frequency is given (by extension of equation 3) by

$$\omega = \sqrt{\frac{k}{m_r}} = \sqrt{\frac{Gm_E}{R_E^3}}$$

$$\omega = \sqrt{\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.97\times10^{24}\ \mathrm{kg})}{(6.378\times10^{6}\ \mathrm{m})^3}}$$

$$\omega = 1.24\times10^{-3}\ \mathrm{s^{-1}}$$

It is now trivial to write the period of one oscillation as

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(1.24\times10^{-3}\ \mathrm{s^{-1}})} = 5.07\times10^{3}\ \mathrm{s}$$

or

$$T = 84.5\ \mathrm{min}$$

4. Evaluate Result We obtain the same period as in Worked Problem 15.5. This is reasonable. Note that Worked Problem 15.4 demonstrated that a spring that stretches a distance h due to a mass m being hung on it will have the same period as a pendulum with a length h with a bob of the same mass m. Here, we have treated the gravitational force as though it was being provided by a spring, and the amplitude of the oscillation was R_E. The pendulum described in Worked Problem 15.5 did not have the pendulum hung at the center of Earth, but as Figure WG15.5 shows, the geometry is that same as if the pendulum were swinging about a point at Earth's center. Thus, it is no surprise that we arrived at the same period for the two systems.

15.8 Prevent oscillation

1. Getting Started The damping coefficient characterizes the velocity-dependent forces that might convert energy into incoherent forces. An example of such a force is the viscous drag that a fluid exerts on an object moving through the fluid. In that case the damping coefficient would contain information about the viscosity of the fluid.

In Worked Problem 15.7, the pendulum went through a full oscillation before stopping. We want the oscillator to return to equilibrium without completing a full oscillation. This means the oscillator must be slowed at a greater rate, meaning we need larger damping than in Worked Problem 15.7.

2. Devise Plan Since we require that $\omega_d = 0$, we can find the damping coefficient in terms of other variables by using equation (2) from Worked Problem 15.5:

$$\omega_d = \sqrt{\frac{g}{\ell} - \left(\frac{b}{2m}\right)^2} \quad (1)$$

To determine when the amplitude is reduced by some fraction, we can look at the expression for the position of the damped pendulum as a function of time:

$$\vartheta(t) = Ae^{-bt/2m}\sin(\theta_d t + \phi_i) \quad (2)$$

We can simply use the prefactor in front of the sine function $Ae^{-bt/2m}$ as the time-dependent amplitude.

3. Execute Plan (*a*) We determine the damping constant by simply setting equation (1) equal to zero. We find

$$b = 2m\sqrt{\frac{g}{\ell}}$$

$$b = 2(2.0\text{ kg})\sqrt{\frac{(9.8\text{ m/s}^2)}{(9.0\text{ m})}} = 4.2\text{ kg/s}$$

(*b*) We require that the time-dependent amplitude in equation (2) be reduced to 1% of its original value, meaning $Ae^{-bt/2m} = (0.01)A.$ Rearranging and inserting our value for the damping coefficient, we find

$$t = -\frac{2m}{b}\ln(0.01)$$

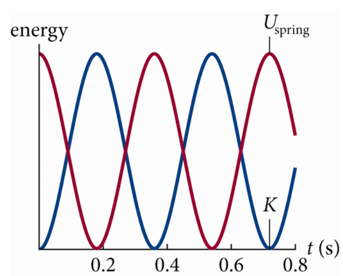
$$t = -\frac{1(2.0\text{ kg})}{(4.17\text{ kg/s})} = 0.50\text{ s}$$

4. Evaluate Result If we increase the damping coefficient to twice the critical value, the time required for the amplitude to drop to 1% of its initial value would be reduced by half. If we decrease the damping coefficient to half the critical value, the time required would double. This makes perfect sense; this basically says if you push harder against the motion of the pendulum, it will take less time to reduce its amplitude.

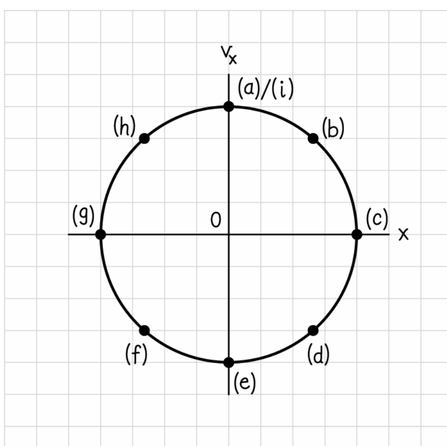
Questions and Problems

15.1. The wings are least likely to be blurry when they are in the highest or lowest positions that the bird's wings ever reach. At these moments the vertical motion of the wing is momentarily zero.

15.2.



15.3.



Points on the plot have been labeled to show which cell in *Principles* Figure 15.2 each point corresponds to. The units of x and v_x are arbitrary.

15.4. (*a*) We know that in simple harmonic motion, the energy is transformed from kinetic to potential and back again. In the absence of dissipation, the maximum kinetic energy should be the same as the maximum spring potential energy, which can of course be written as $U_{\text{max}} = \frac{1}{2}k(\Delta x)^2_{\text{max}} = \frac{1}{2}kA^2.$ Obviously, half the maximum kinetic energy is half the maximum potential energy: $\frac{K_{\text{max}}}{2} = \frac{U_{\text{max}}}{2} = \frac{1}{4}kA^2 = \frac{1}{2}k(\Delta x_{\text{half}})^2 \Rightarrow \Delta x_{\text{half}} = \frac{A}{\sqrt{2}}.$ (*b*) We can write half the maximum kinetic energy in two different ways, and equate them: $\frac{K_{\text{max}}}{2} = \frac{1}{2}mv^2_{\text{half}} = \frac{1}{4}mv^2_{\text{max}} \Rightarrow v_{\text{half}} = \frac{v_{\text{max}}}{\sqrt{2}}.$

15.5. We see that if a moving object moves to the left of $x = 0.5,$ it will continue moving to the left indefinitely. On the right hand side, if an oscillation were to carry the object past the point $x = 3.5,$ it would pass the $x = 0.5$ point upon swinging back to the left. Thus $0.5 < x < 3.5.$

15.6. We can start counting oscillations at any initial position of the oscillator; we choose one point of maximum displacement. Each oscillation consists of a displacement of A toward equilibrium, then a displacement of A to the other side away from equilibrium, and then back to the initial position. This is a distance of $4A.$ Thus 2.5 such oscillations would involve the oscillating object covering a distance of $10A.$

15.7. The period is 5 s. Although the shape of the wave is complicated, we can see that the shape repeats itself every 5 s.

15.8. In order for the piston to have perfectly simple harmonic motion, its motion in the *y* direction would have to exactly match the *y* component of the pin on the motor that is moving in a circle. As it is, the rod connecting the pin on the motor to the piston makes various angles as a circle is completed. It is the length of this rod that is held constant, rather than the vertical distance between the pin and the piston. In order to better approximate simple harmonic motion, a longer rod should be used to join the pin and the piston.

15.9. It is periodic, in that it repeats after some time. It is not simple harmonic motion, because its position and velocity cannot be described by a simple sine or cosine function. As it rises and falls, not all of its potential energy is converted into translational kinetic energy. Some of it becomes rotational kinetic energy.

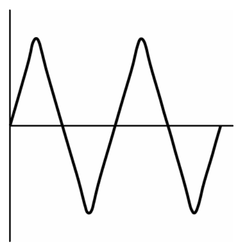
15.10. (*a*) If no energy is lost, then the ball with return up to its initial height. This is required by conservation of energy applied between the first time the ball is at its peak height and the second. But there is nothing special about the first and second peak positions; as long as no energy is lost in the collision or to air, this should continue indefinitely. (*b*) We can determine how much time is required for the ball to fall by using kinematics. We use $\Delta y = v_{y,\text{i}}\Delta t + \frac{1}{2}a_y(\Delta t)^2,$ noting that the ball is dropped, not thrown downward. Thus $\Delta t = \sqrt{\frac{2\Delta y}{a_y}}.$ A complete oscillation consists of the ball falling to the ground and rising back to its peak height. Since the path is symmetric in time, we can write $T = 2\Delta t = 2\sqrt{\frac{2\Delta y}{a_y}} = 2\sqrt{\frac{2(-2.0\text{ m})}{(-9.8\text{ m/s}^2)}} = 1.3\text{ s}.$ (*c*) No, this is not simple harmonic motion. The velocity goes from its peak negative value instantly to its peak positive value. This cannot be described by a sinusoidal function.

15.11. We know from the text that all higher harmonics must be integer multiples of the fundamental frequency. From this is follows that the difference between any two harmonics must also be an integer multiple of the fundamental frequency. This means the smallest possible difference between two harmonics would be $f_{n+1} - f_n = (n+1)f_1 - nf_1 = f_1.$ Note that the second and third frequencies have a difference of $(1270\text{ Hz}) - (1143\text{ Hz}) = 127\text{ Hz}.$ Since we are told $f_1 > 100\text{ Hz},$ this difference of 127 Hz cannot be only be $1 \times f_1.$ Thus $f_1 = 127\text{ Hz}.$ It is worth nothing that the difference between the first and second frequencies listed is $(1143\text{ Hz}) - (889\text{ Hz}) = 254\text{ Hz} = 2f_1.$

15.12. Sound difference is due to difference in higher harmonics emitted, which consist of superposition of many sinusoidal waves; different shape of piano and trumpet cause differences in form of the superposed waves, yielding different sounds for a given frequency.

15.13. There appear to be three harmonics. There is a low frequency (the fundamental frequency) that is peaked at 0 s, 5 s, 10 s, etc. The second harmonic adds to the peaks of the fundamental frequency, but also produces a peak at 2.5 s, and 7.5 s. The third harmonic produces the small peaks between the times already noted, such as near 1.5 s, and near 4.0 s.

15.14. (*a*) Because the amplitudes of the higher-frequency components decrease rapidly, one could visualize this function as a basic sine function with smaller and smaller "ripples" added to it. Superposing the ripples and the fundamental sine curve will increase the magnitude in some locations and decrease it at other locations. Only odd multiples of the fundamental frequency are used, which share all the zeroes of the fundamental frequency but alternate maxima and minima. Subtracting every other term will therefore tend to reinforce the fundamental maxima as well. The result will be a somewhat triangular wave shape (if all odd terms are included, the shape will be exactly triangular). The period will be $T = 1/f$, so the function will have zeroes and maxima at all the same locations as $\sin(2\pi ft)$. (*b*) The general shape between each pair of zeroes is triangular, so the function is alternating triangles of positive and negative displacement, as shown here:



15.15. The square wave looks like a sine function with the smooth curves replaced by flat regions. The zeroes and maxima of a sine curve of fundamental frequency f equal to that of the desired square wave, thus make a good starting point. In order to beef up the low-amplitude portions of the fundamental sine wave without increasing the maxima as much, we will need to add terms with zeroes in the same locations but with alternating maxima and minima. This suggests an odd sequence of frequencies is needed, similar to that in Problem 15.14 above. This time we will need to add each term rather than subtracting every other term, so that the alternating maxima and minima can produce cancellation rather than reinforcement of maxima and minima. To get the specific series that produces a square wave, we might Google "square wave", or derive the series as follows:
Any given a periodic function $f(t)$ can be written as a (possibly infinite) sum of sine and cosine terms. Specifically:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{N}\left(a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right) \tag{1}$$

Where the coefficients are

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos\left(\frac{2\pi nt}{T}\right)dt \tag{2}$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin\left(\frac{2\pi nt}{T}\right)dt \tag{3}$$

In this case the function we wish to treat is the square wave function in Figure P15.15, which can be expressed mathematically as

$$f(t) = A\theta\left(\frac{T}{2} - t\right) - A\theta\left(t - \frac{T}{2}\right) \tag{4}$$

on the interval $0 \le t \le T$, where T is the period of one full square well "oscillation", and

$$\theta(x) = \begin{cases} 1 & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases}$$

is called the Heavyside function. Using the functional form (4) in the equations for the coefficients (2) and (3) yields

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} A\left[\theta\left(\frac{T}{2}-t\right)-\theta\left(t-\frac{T}{2}\right)\right]\cos\left(\frac{2\pi nt}{T}\right)dt = -\frac{2A(\cos(n\pi)-1)\sin(n\pi)}{n\pi} = 0 \quad (5)$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} A\left[\theta\left(\frac{T}{2}-t\right)-\theta\left(t-\frac{T}{2}\right)\right]\sin\left(\frac{2\pi nt}{T}\right)dt = -\frac{4A(\cos(n\pi)-1)\sin^2\left(\frac{n\pi}{2}\right)}{n\pi} = \begin{cases} -\dfrac{4A(-1)^n}{n\pi} & \text{n odd} \\ 0 & \text{n even} \end{cases} \quad (6)$$

Inserting these results into equation (1) yields

$$f(t) = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{2n-1}\sin\left[(2n-1)\frac{2\pi t}{T}\right]$$

15.16. In order for a force to produce simple harmonic motion, the force must be in the opposite direction of the displacement (to return an oscillator to equilibrium), and the force must be linear in displacement. Options (*b*) and (*e*) satisfy this constraints, and therefore forces (*b*) and (*e*) could produce simple harmonic motion.

15.17. The restoring force is the stiffness or rigidity of the board itself. This comes from bonds on the microscopic scale behaving similarly to springs. When the board is pulled downward in one direction, some of these bonds are stretched and they pull the board back into its equilibrium position. When you have left the diving board, the relevant mass is the mass of the board itself. However, this is slightly more complicated than a single spring and a single point-like object that has mass. The mass of the diving board is distributed along its length, as are the bonds that give rise to the restoring force.

15.18. The spring-cart system will have exactly the same period as on Earth. The oscillation period of a spring-mass system depends on the mass of the object, but this is because the spring has to accelerate it back and forth; it has nothing to do with gravitational acceleration. The pendulum, however, uses gravity as its restoring force. In the absence of gravity (or in freefall) the pendulum will not oscillate at all. One could refer to the absence of oscillatory motion in the pendulum as an infinite period.

15.19. (*a*) In the plot of the force, the slope of the force as a function of position is different in each case. In the picture of the ground, the concavity of the ground is different in each case. (*b*) $\frac{\partial F_x}{\partial x} < 0$ describes stable equilibrium, $\frac{\partial F_x}{\partial x} > 0$ describes unstable equilibrium, and $\frac{\partial F_x}{\partial x} = 0$ describes neutral equilibrium. (*c*) These criteria have been established only in one dimension. But in that one dimension, yes, the criteria are general.

15.20. (*a*) The pendulum clock will run slower at higher altitude. The pendulum uses gravity as its restoring force. As the altitude is increased, gravity becomes weaker and the pendulum cannot be restored as quickly. (*b*) It is not possible to say. The restoring force due to gravity is weaker at higher altitude, but air resistance decreases also. Which effect dominates cannot be determined without data on atmospheric pressure at different altitudes, as well as the shape of the pendulum.

15.21. We simply insert $\omega = \frac{2\pi}{T}$ into each equation to obtain

$$x(t) = A\sin\left(\frac{2\pi t}{T}+\phi_1\right)$$

$$v_x(t) = \frac{2\pi A}{T}\cos\left(\frac{2\pi t}{T}+\phi_1\right)$$

$$a_x(t) = -\left(\frac{2\pi}{T}\right)^2 A\sin\left(\frac{2\pi t}{T}+\phi_1\right)$$

15.22. (*a*) Here the initial phase is zero, because the function appears to be exactly $A\sin(\omega t)$. (*b*) If the argument of a sine function is slightly negative, the value of the sine function will be negative as is the case here. The initial phase is negative.

15.23. (*a*) Yes (*b*) The horizontal component is increasing, meaning the vertical component must be decreasing. In order for the vertical component to be approaching zero as the phasors rotate counterclockwise, the vertical component must be lagging behind the horizontal component. Clearly the two phasors are orthogonal. Thus $\phi_v - \phi_h = -\pi/2$.

15.24. The spring constant is given by

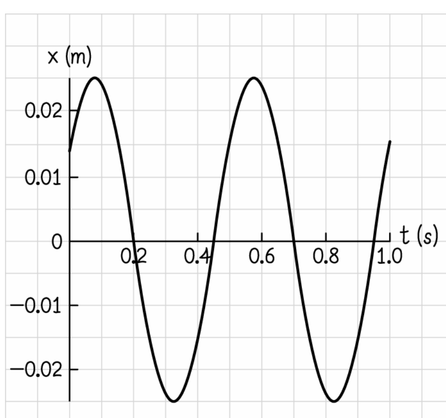
$$k = \omega_E^2 m_E = \left(\frac{2\pi}{T_E}\right)^2 m_E = \left(\frac{2\pi}{(365.25 \text{ days})} \times \left(\frac{1 \text{ day}}{24 \text{ h}}\right) \times \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\right)^2 (5.97 \times 10^{24} \text{ kg}) = 2.37 \times 10^{11} \text{ N/m}$$

15.25. (*a*) The velocity of the phasor tip is $A\vec{\omega}$, and the vertical component of this is $A\vec{\omega}\sin(\vartheta)$, where ϑ is measured counterclockwise from the positive x axis. The vertical component of the velocity of the tip is the same as the speed of an object undergoing simple harmonic motion in the vertical direction with the same rotational speed. (*b*) The acceleration of the phasor tip is $A\omega^2$ radially inward. The vertical projection of this acceleration is $A\omega^2\cos(\vartheta)$. This also has exactly the form of the acceleration of an object undergoing simple harmonic motion.

15.26. (*a*) We simply read off from the function that the amplitude is 20 mm. (*b*) We compare the given function to the general equation for an object undergoing simple harmonic motion: $A\cos(\omega t) = A\cos(2\pi f t) = (20 \text{ mm})\cos(8\pi t) \Rightarrow 2\pi f = 8\pi \text{ s}^{-1}$ or $f = 4$ Hz. (*c*) $T = \frac{1}{f} = \frac{1}{(4 \text{ Hz})} = 0.25$ s. (*d*) The cosine function is zero when the argument is $\left(n + \frac{1}{2}\right)\pi$. Thus we set $t = \frac{1}{8}\left(n + \frac{1}{2}\right) \Rightarrow t_0 = 0.063 \text{ s}, t_1 = 0.19 \text{ s}, t_2 = 0.31 \text{ s}$. (*e*) $x(t = 0.75 \text{ s}) = (20 \text{ mm})\cos(8\pi(0.75 \text{ s})) = 20 \text{ mm}$, $v_x(t = 0.75 \text{ s}) = -(8\pi)(20 \text{ mm})\sin(8\pi t) = 0$, $a_x(t = 0.75 \text{ s}) = -(8\pi)^2(20 \text{ mm})\cos(8\pi t) = -1.3 \times 10^4 \text{ mm/s}^2$.

15.27. (*a*) There are several correct ways of writing this. One might use a sine function or a cosine function. There are also two correct (non-trivial) initial phases that fit the description we are given. This is because we do not know if the position is increasing or decreasing at $t = 0$. Correct answers include $x(t) = A\cos(\omega t + \phi)$, $A = 25$ mm, $\omega = \frac{2\pi}{T} = 4\pi$ rad/s, and $\phi = 2.5$ or 0.59.

(*b*)



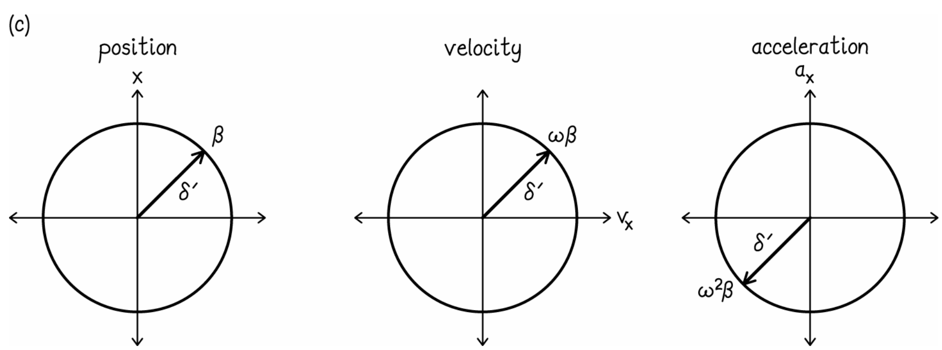
15.28. (*a*) We know that the rotational speed is given by $\omega = 2\pi f = 2\pi\left(\frac{33.3}{60\text{ s}}\right) = 3.5\text{ s}^{-1}$. (*b*) We found the rotational speed in part (*a*), and the amplitude is clearly the radius of the record (0.15 m). All that remains is to determine the initial phase. We require that $y(t=0) = A$ or $\sin(\omega(0)+\phi) = 1 \Rightarrow \phi = \sin^{-1}(1) = \frac{\pi}{2} = 1.6$. Thus $y(t) = A\sin(\omega t + \phi)$, where $A = 0.15$ m, $\omega = 3.5\text{ s}^{-1}$, and $\phi = 1.6$.

15.29. The maximum amplitude of the acceleration of an object undergoing simple harmonic motion is $a_{\text{max}} = A\omega^2$. We can write the total energy as the maximum of the kinetic energy, and insert the given values: $E = K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}m(\omega A)^2 = \frac{1}{2}ma_{\text{max}}A = \frac{1}{2}(1.0\text{ kg})(5.0\text{ m/s}^2)(0.12\text{ m}) = 0.30\text{ J}$.

15.30. (*a*) 8.00 m (*b*) $b = \omega = 2\pi f \Rightarrow f = \frac{b}{2\pi} = \frac{(2.00\text{ s}^{-1})}{2\pi} = 0.318\text{ Hz}$ (*c*) $T = \frac{1}{f} = \pi\text{ s} = 3.14\text{ s}$ (*d*) The speed is $v_x(t) = -ab\sin(bt + \pi/3)$, so that $v_x(t = \pi/2\text{ s}) = -(8.00\text{ m})(2.00\text{ s}^{-1})\sin((2.00\text{ s}^{-1})(\pi/2\text{s}) + \pi/3) = 13.9\text{m/s}$. The acceleration is $a_x(t) = -ab^2\cos(bt + \pi/3)$, so that $a_x(t = \pi/2\text{ s}) = -(8.00\text{ m})(2.00\text{ s}^{-1})^2\cos((2.00\text{ s}^{-1})(\pi/2\text{s}) + \pi/3) =$ 16.0m/s^2. (*e*) The maximum magnitude of the x component of acceleration is $\left|a_{x,\text{max}}\right| = A\omega^2 = ab^2 =$ $(8.00\text{ m})(2.00\text{ s}^{-1})^2 = 32.0\text{ m/s}^2$. This occurs when $\cos(bt + \pi/3) = -1 \Rightarrow bt + \pi/3 = \pi$ or $t = \frac{2\pi}{3b} = \frac{2\pi}{3(2.00\text{ s}^{-1})} =$ 1.05 s. (*f*) The maximum magnitude of the x component of the velocity is $\left|v_{x,\text{max}}\right| = A\omega = ab = (8.00\text{ m})(2.00\text{ s}^{-1}) =$ 16.0 m/s^2. This occurs when $\sin(bt + \pi/3) = 1 \Rightarrow bt + \pi/3 = \frac{\pi}{2}$ or $t = \frac{\pi}{6b} = \frac{\pi}{6(2.00\text{ s}^{-1})} = 0.262$ s.

15.31. (*a*) Simple harmonic motion is used in the text to refer to any system the motion of which can be described using a sinusoidal function. This can be done using sine or cosine functions. But we can turn $x(t) = \beta\cos(\omega t + \delta)$ into a sine function by shifting the argument by $\pi/2$. If we call $\delta' \equiv \delta + \pi/2$ then $x(t) = \beta\sin(\omega t + \delta')$. (*b*) Simply differentiating the expression for position with respect to time yields $v_x(t) = \beta\omega\cos(\omega t + \delta')$ and $a_x(t) = -\beta\omega^2\sin(\omega t + \delta')$.

(*c*)



15.32. The path from Boston to Paris constitutes a fraction of a great circle equal to $\frac{d_{\text{Boston-Paris}}}{2\pi R_{\text{E}}} = \frac{(5.850\times10^6\text{ m})}{2\pi(6.378\times10^6\text{ m})} = 0.146$ or $52.55°$. Simply geometry tells us that the angle between the path of the train and radially inward toward the center of the Earth is then $\theta_{\text{max}} = 63.72°$. The component of the force of gravity along the direction of motion will be $F_x^G = mg\cos(\theta)$, but of course the angle θ will change as the train moves.

Thus the maximum force that gravity exerts in that direction is $F^G_{x,\max} = mg\cos(\theta_{\max})$, meaning that the maximum acceleration in that direction will be $a_{x,\max} = g\cos(\theta_{\max})$. We are assuming that the train will move in simple harmonic motion, such that $a_{\max} = A\omega^2$, also. Thus $T = \sqrt{\dfrac{A4\pi^2}{g\cos(\theta_{\max})}} = \sqrt{\dfrac{R_E \sin((52.55°)/2)4\pi^2}{g\cos(\theta_{\max})}} = \sqrt{\dfrac{(6.378\times10^6\text{ m})\sin((52.55°)/2)4\pi^2}{(9.8\text{m/s}^2)\cos(63.72°)}} = 5068\text{ s}$ or 84.5 minutes. This is the entire period for an oscillation from Boston to Paris, and back to Boston again. The one-way trip would take only 42.2 minutes.

15.33. Call the axis along which the shaker oscillates the x axis. Then $\sum F_{x,\text{ on penny}} = F^s_x = ma_x \Rightarrow (a_x)_{\max} = \dfrac{1}{m}(F^s_x)_{\max} = g\mu_s$. But we also know the maximum acceleration in simple harmonic motion is given by $a_{x,\max} = A\omega^2$. Thus $\mu_s = \dfrac{A}{g}(2\pi f)^2 = \dfrac{(0.050\text{ m})}{(9.8\text{ m/s}^2)}(2\pi(1.85\text{ Hz}))^2 = 0.69.$

15.34. Because the spring has inertia; the inertia of the material at the bottom of a vertically hanging spring requires a nonzero time interval to be accelerated up and down by the rest of the spring.

15.35. From the initial condition we have $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_1}}$, and from the second condition (after the second block is added) we have $\dfrac{f}{2} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_1+m_2}}$. Dividing the first expression by the second, we obtain:

$$2 = \sqrt{\frac{m_1+m_2}{m_1}} \Rightarrow \frac{m_1}{m_2} = \frac{1}{3}$$

15.36. They are equal; the ratio is 1:1. The effect of the additional inertia on the second spring is to lower its equilibrium position, and to decrease the maximum speed.

15.37. (*a*) We know that the frequency increases if the spring constant increases. In this arrangement, a displacement Δx now requires that both springs be compressed (or stretched) by Δx. So clearly, the effective spring constant of the arrangement is doubled by adding the second spring in parallel. Thus the frequency increases. (*b*) Now a displacement of Δx requires that each spring be compressed (or stretched) by only $\Delta x/2$. So causing the same displacement as before will now lead to a restoring force that is half as large. Clearly the effective spring constant for this arrangement is decreased. Thus the frequency decreases.

15.38. We refer to the balls as A and B. The two balls stretch their respective springs by the same amount as they reach equilibrium. In order for the spring restoring force to counteract gravity, we know

$$\Delta y_A = -\frac{m_A g}{k_A}$$

$$\Delta y_B = -\frac{m_B g}{k_B}$$

Thus we can equate $-\dfrac{m_A g}{k_A} = -\dfrac{m_B g}{k_B} \Rightarrow \sqrt{\dfrac{m_A}{k_A}} = \sqrt{\dfrac{m_B}{k_B}} \Rightarrow \omega_A = \omega_B$. Thus the two balls oscillate with the same frequency.

15.39. The cup will lose contact with the table only if the table accelerates downward away from the cup faster than gravity can accelerate the cup downward.

(*a*) Yes. Acceleration increases with amplitude. So an increase in amplitude could cause the maximum acceleration to exceed the acceleration due to gravity. (*b*) No. Increasing the period decreases the angular frequency. Since $a \sim \omega^2$, this will decrease the acceleration. (*c*) Yes. Decreasing the period increases the angular frequency. Since $a \sim \omega^2$, this will increase the acceleration. (*d*) Yes. Since $a \sim \omega^2 \sim \dfrac{k}{m}$, decreasing the mass of the table would increase the acceleration. (*e*) No. As discussed in (*d*), this would decrease the acceleration. (*f*) No. This has no effect on the maximum acceleration. (*g*) If the cup is glued to the table, the answers do not change. If the cup is free to move, then the coffee would not fly out of the cup even if the cup flew into the air in free fall (at least until it hit the table again).

15.40. (*a*) We set $U_{\text{max}}^{\text{spring}} = \frac{1}{2}kA^2 = E_{\text{total}} = K_{\text{i}} + U_{\text{i}}^{\text{spring}}$, such that

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{i}}^2 + \frac{1}{2}k(\Delta x)_{\text{i}}^2 \Rightarrow A = \sqrt{\frac{m}{k}v_{\text{i}}^2 + (\Delta x)_{\text{i}}^2} = \sqrt{\frac{(2.0\ \text{kg})}{(50\ \text{N/m})}(2.0\ \text{m/s})^2 + (0.24\ \text{m})^2} = 0.47\ \text{m}$$

(*b*) At the equilibrium position, all energy is kinetic energy. Thus we can write

$$\frac{1}{2}mv_{\text{eq}}^2 = \frac{1}{2}kA^2 \Rightarrow v_{\text{eq}} = A\sqrt{\frac{k}{m}} = (0.466\ \text{m})\sqrt{\frac{(50\ \text{N/m})}{(2.0\ \text{kg})}} = 2.3\ \text{m/s}$$

(*c*) The answers would not change at all.

15.41. (*a*) We set $U_{\text{max}}^{\text{spring}} = \frac{1}{2}kA^2 = E_{\text{total}} = K_{\text{i}} + U_{\text{i}}^{\text{spring}}$, such that

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{i}}^2 + \frac{1}{2}k(\Delta x)_{\text{i}}^2$$

$$\Rightarrow A = \sqrt{\frac{m}{k}v_{\text{i}}^2 + (\Delta x)_{\text{i}}^2} = \sqrt{\left(\frac{T}{2\pi}\right)^2 v_{\text{i}}^2 + (\Delta x)_{\text{i}}^2} = \sqrt{\left(\frac{(2.3\ \text{s})}{2\pi}\right)^2 (0.12\ \text{m/s})^2 + (0.040\ \text{m})^2} = 0.059\ \text{m}$$

(*b*) $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{(2.3\ \text{s})} = 2.7\ \text{s}^{-1}$ (*c*) $E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}(0.35\ \text{kg})(2.73\ \text{s}^{-1})^2(0.0594\ \text{m})^2 = 4.6\ \text{mJ}$ (*d*) $k = m\omega^2 = (0.35)(2.73\ \text{s}^{-1})^2 = 2.6\ \text{N/m}$ (*e*) The initial vertical position is $y(t=0) = -0.040\ \text{m}$. We require that $y(t=0) = A\sin(\phi) \Rightarrow \phi = \sin^{-1}\left(\dfrac{y(t=0)}{A}\right) = \sin^{-1}\left(\dfrac{-0.040\ \text{m}}{(0.0594\ \text{m})}\right) = 3.9\ \text{rad or}\ 5.5\ \text{rad}.$ Both are valid solutions to the constraints we put on the position. However, we also require that the block is moving up toward equilibrium at time $t = 0$. This additional information limits our solution to $\phi = 5.5\ \text{rad}$. (*f*) Using the results of parts (*a*), (*b*), and (*e*), we can write $y(t) = A\sin(\omega t + \phi)$ where $A = 0.059\ \text{m}$, $\omega = 2.7\ \text{s}^{-1}$, and $\phi = 5.5\ \text{rad}$.

15.42. The amplitude is exactly 0.050 m; the magnitude of the maximum displacement from equilibrium is the definition of the amplitude. We can find the period by using the sum of all forces in the vertical direction at equilibrium. Call vertically upward the $+y$ direction. Then $\sum F_y = -k\Delta y_{\text{eq}} - mg = 0 \Rightarrow \dfrac{m}{k} = \dfrac{-\Delta y_{\text{eq}}}{g}$. We can insert this result into the know expression for the period: $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{-\Delta y_{\text{eq}}}{g}} = 2\pi\sqrt{\dfrac{-(-0.10\ \text{m})}{(9.8\ \text{m/s}^2)}} = 0.63\ \text{s}.$

15.43. Let us begin by writing the sum of all forces in the vertical direction at the moment the block is brought to rest. Call the upward direction $+y$. Then $\sum F_y = -k\Delta y_{\text{eq}} - mg = 0 \Rightarrow \dfrac{m}{k} = \dfrac{-\Delta y_{\text{eq}}}{g}$. This allows us to write the period of the oscillation as $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{-\Delta y_{\text{eq}}}{g}}$ or $\Delta y_{\text{eq}} = -\dfrac{T^2 g}{4\pi^2} = -\dfrac{(0.50\ \text{s})^2(9.8\ \text{m/s}^2)}{4\pi^2} = -0.062\ \text{m}.$ So the spring length is reduced by 0.062 m upon removing the block.

15.44. (*a*) The system is not specified, so there could be many answers. However, looking at later parts of the problem it is clear that at some point we must include Earth in the system, so let's just do that from the start. (Without Earth as a part of the system, there will be no gravitational potential energy. This means that as the spring-object system oscillates vertically, the force of gravity will do work on it, changing the energy of the system.) However, Earth has other forms of energy, such as kinetic energy, that are hard to compute and have no bearing on the motion of the suspended object (because these other forms of energy remain constant during the oscillatory motion—Earth does not expend some of its kinetic energy to increase the motion of the object, for example). Thus we include Earth in our system, but ignore any forms of energy unavailable to influence the motion of the object. This leaves us with the kinetic energy of the oscillating object, the gravitational potential energy of the Earth-object-spring system, and the potential energy stored in the spring. The sum of these energies is the energy of the system, and this sum remains constant during the motion (in the absence of dissipation). We are free to choose the location of zero gravitational potential energy, but must choose the zero of spring potential energy at its unstretched position (in this case, the highest point in the motion). Choosing the gravitational potential energy to be zero at the lowest point in the motion, we can compute the system energy at the highest point by summing the three terms. Because the kinetic energy of the object is zero at the highest point, two of those terms are zero:

$$E = K^{\text{object}} + U^{\text{spring}} + U^{G} = 0 + 0 + mgh = (4.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) = 39\text{ J}$$

We made use of the fact that the height measured from lowest to highest positions is double the amplitude, or one full meter. (b) As mentioned in part (a), we are told that at the object's highest point the spring returns to its equilibrium length and therefore stores no potential energy. Now consider the equilibrium position. We know the sum of all forces in the vertical direction is zero at that point. Calling the upward direction $+y$, we can write $\sum F_y = -k\Delta y_{\text{eq}} - mg = 0 \Rightarrow k = \dfrac{mg}{-\Delta y_{\text{eq}}} = \dfrac{(4.0\text{ kg})(9.8\text{ m/s}^2)}{-(-0.50\text{ m})} = 78.4\text{ N/m}.$ Thus, as the object passes through its equilibrium position, the spring potential energy is $U^{\text{spring}}_{\text{eq}} = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(78.4\text{ N/m})(0.50\text{ m})^2 = 9.8\text{ J}.$ Finally, at the object's lowest point, the spring potential energy of the system is $U^{\text{spring}}_{\text{lowest}} = \dfrac{1}{2}k(2A)^2 = \dfrac{1}{2}(78.4\text{ N/m})(2(0.50\text{ m}))^2 =$ 39 J. Thus the spring potential energies are $U^{\text{spring}}_{\text{lowest}} = 39\text{ J},$ $U^{\text{spring}}_{\text{eq}} = 9.8\text{ J},$ and $U^{\text{spring}}_{\text{highest}} = 0.0.$ (c) Clearly, the kinetic energy at the top and at the bottom of the object's path is zero. The system energy must be conserved between, for example, the highest point and the equilibrium position. Thus we can equate:

$$K_{\text{highest}} + U^{\text{spring}}_{\text{highest}} + U^{G}_{\text{highest}} = K_{\text{eq}} + U^{\text{spring}}_{\text{eq}} + U^{G}_{\text{eq}}$$

$$K_{\text{eq}} = U^{G}_{\text{highest}} - U^{G}_{\text{eq}} - U^{\text{spring}}_{\text{eq}}$$

$$K_{\text{eq}} = mgA - \frac{1}{2}kA^2 = (4.0\text{ kg})(9.8\text{ m/s}^2)(0.50\text{ m}) - \frac{1}{2}(78.4\text{ N/m})(0.50\text{m})^2$$

$$K_{\text{eq}} = 9.8\text{ J}$$

Thus, the kinetic energies are $K_{\text{lowest}} = 0.0,$ $K_{\text{eq}} = 9.8\text{ J},$ and $K_{\text{highest}} = 0.0.$ (*d*) One can choose to put the zero point of gravitational potential energy at any point, but we have previously selected the lowest point in the motion. At any position *y*, then, the gravitational potential energy will be given by $U^G = mg(y - y_{\text{lowest}}).$ Thus, the gravitational potential energies are $U^G_{\text{lowest}} = 0,$ $U^G_{\text{eq}} = mg(y_{\text{eq}} - y_{\text{lowest}}) = (4.0\text{ kg})(9.8\text{ m/s}^2)(0.50\text{ m}) = 20\text{ J},$ and $U^G_{\text{highest}} =$ $mg(y_{\text{highest}} - y_{\text{lowest}}) = (4.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) = 39\text{ J}.$

15.45. (*a*) The effective spring constant is 300 N/m. If this is not clear by inspection, consider displacing the block to the right by 1.00 m. Spring 1 would be stretched 1.00 m and would therefore exert a restoring force to the left with a magnitude of 100 N. Spring 2 would be compressed 1.00 m and would therefore exert a restoring force to the right with a magnitude of 200 N. Thus the springs exert 300 N of restoring force for every meter of displacement, which makes the spring constant 300 N/m. (*b*) We are told the amplitude explicitly. The angular frequency can be written $\omega = \sqrt{\dfrac{k_{\text{eff}}}{m}} = \sqrt{\dfrac{(300\text{ N/m})}{(6.0\text{ kg})}} = 7.1\text{ s}^{-1}.$ Finally, we need to choose an initial phase to satisfy the initial condition: $x(t=0)=A\sin(\phi)=A \Rightarrow \phi=\sin^{-1}(1)=\dfrac{\pi}{2}=1.6.$ Thus $x(t)=A\sin(\omega t+\phi)$ where $A=0.020$ m, $\omega=7.1\ \text{s}^{-1}$, and $\phi=1.6.$

15.46. (*a*) Call the time at which block B has a speed of 0.24 m/s and a displacement of 0.060 m t_1. Let us simply equate the energy at t_1 to the maximum potential energy, and rearrange terms:

$$\frac{1}{2}kA^2=\frac{1}{2}mv_1^2+\frac{1}{2}k(\Delta x)_1^2$$
$$\Rightarrow \sqrt{\frac{A^2-(\Delta x)_1^2}{v_1^2}}=\sqrt{\frac{m}{k}}=\frac{1}{\omega}=\frac{T}{2\pi}$$
$$\Rightarrow T=2\pi\sqrt{\frac{A^2-(\Delta x)_1^2}{v_1^2}}=2\pi\sqrt{\frac{(0.10\text{ m})^2-(0.060\text{ m})^2}{(0.24\text{ m/s})^2}}=2.1\text{ s}$$

(*b*) We know that the magnitude of the maximum acceleration that block C will undergo is $a_{\max}=A\omega^2$. The only force acting on block C to accelerate it is friction. Thus, calling the horizontal direction of acceleration $+x$, we can write $\sum F_{\text{C}x}=F^{\text{s}}_{\text{BC}x}=m_{\text{C}}a_x$ or $(F^{\text{s}}_{\text{BC}x})_{\max}=m_{\text{C}}(a_x)_{\max}\Rightarrow m_{\text{C}}g\mu_{\text{s,max}}=m_{\text{C}}A\omega^2$. Using steps from part (*a*) allows us to rewrite this as

$$\mu_{\text{s,max}}=\frac{A\omega^2}{g}=\left(\frac{A}{g}\right)\left(\frac{v_1^2}{A^2-(\Delta x)_1^2}\right)=\left(\frac{(0.10\text{ m})}{(9.8\text{ m/s}^2)}\right)\left(\frac{(0.24\text{ m/s})^2}{(0.10\text{ m})^2-(0.060\text{ m})^2}\right)=0.092$$

15.47. Call the lower block A, and the upper block B. Block A will fall off when a force greater than or equal to 20 N is required to accelerate block A along with block B. The acceleration of the blocks is greatest when their displacement from equilibrium is greatest, so we look at the lowest point in their oscillation. Call the vertically upward direction the $+y$ direction. Then the sum of all forces acting on block A is $\sum F_{\text{A}y}=F^{\text{c}}_{\text{glue A}}-m_{\text{A}}g=m_{\text{A}}a_y$. We know that the acceleration at this point is the maximum acceleration $a_y=a_{y,\max}=A\omega^2=\dfrac{Ak}{m}$. If this maximizes the force exerted by the glue, then it corresponds to the maximum amplitude. Thus

$$(F^{\text{c}}_{\text{glue A}})_{\max}-m_{\text{A}}g=m_{\text{A}}\left(\frac{A_{\max}k}{m_{\text{A}}+m_{\text{B}}}\right)$$
$$\Rightarrow A_{\max}=\left(\frac{(F^{\text{c}}_{\text{glue A}})_{\max}}{m_{\text{A}}}-g\right)\frac{(m_{\text{A}}+m_{\text{B}})}{k}=\left(\frac{(20\text{ N})}{(0.50\text{ kg})}-(9.8\text{m/s}^2)\right)\frac{((0.50\text{ kg})+(0.50\text{ kg}))}{(500\text{ N/m})}=0.060\text{ m}$$

15.48. (*a*) The center of mass of the system does not move. No external force is applied to the system. (*b*) Note first that the fact that the center of mass does not move implies

$$m_1x_1(t)=-m_2x_2(t) \tag{1}$$

The same magnitude force must be exerted on each block. Let the $+x$ direction point to the right in Figure P15.48. Then $F^{\text{spring}}_{\text{spring},1x}(t)=m_1\dfrac{d^2x_1(t)}{dt^2}$ and $F^{\text{spring}}_{\text{spring},2x}(t)=m_2\dfrac{d^2x_2(t)}{dt^2}$. Since each block undergoes simple harmonic motion, we can write $x_1(t)=A_1\sin(\omega t+\phi)$ and $x_2(t)=A_2\sin(\omega t+\phi)$. Note that the angular frequencies must be the same. Otherwise the blocks could get out of phase and the center of mass would move. Thus

$$F^{\text{spring}}_{\text{spring},1x}(t)=-m_1\omega^2x_1(t) \tag{2}$$
$$F^{\text{spring}}_{\text{spring},2x}(t)=-m_2\omega^2x_2(t) \tag{3}$$

and of course, the spring force is given by

$$F^{\text{spring}}(t)=k\Delta x_{\text{total}}(t)=k(x_1(t)-x_2(t)) \tag{4}$$

Setting the magnitudes of the expressions in equations (3) and (4) equal to each other and inserting equation (1) yields

$$m_1\omega^2 x_1(t) = kx_1(t)\left(1+\frac{m_1}{m_2}\right)$$
$$\Rightarrow \omega = \sqrt{\frac{k(m_1+m_2)}{m_1m_2}} \tag{5}$$

(c) When $m_2 \gg m_1$, then $\sqrt{\frac{k(m_1+m_2)}{m_1m_2}} \to \sqrt{\frac{k(m_2)}{m_1m_2}}$ such that $\omega = \sqrt{\frac{k}{m_1}}$. ($d$) When $m_2 = m_1$ then $\omega = \sqrt{\frac{2k}{m_1}}$.

15.49. The period is 1.0 s. We know $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \ell = g\left(\frac{T}{2\pi}\right)^2 = (9.8\,\text{m/s}^2)\left(\frac{(1.0\text{ s})}{2\pi}\right)^2 = 0.25\text{ m}.$

15.50. The period increases. A pendulum uses gravity as a restoring force, and only the component of gravity in a direction back toward equilibrium can contribute. When the stand is tilted, the component of gravity in the entire plane of the pendulum's motion is decreased. With a smaller restoring force, a pendulum will take longer to swing.

15.51. The simple pendulum (the pendulum on the left) has a greater period. The period of either pendulum can be written as $T = 2\pi\sqrt{\frac{I\ell_{\text{cm}}g}{m}}$. The rotational inertia per kilogram is larger for the simple pendulum because all its inertia is concentrated far from the pivot. For the same reason, the center of mass distance is larger for the simple pendulum. Thus the period is larger for the pendulum.

15.52. (a) *Small* in context of Eq. 15.31 means that the approximation $\sin(\vartheta) \approx \vartheta$ holds for the entire range of motion, which happens when ℓ_{cm} is much greater than the arc length over which system center of mass is displaced. (b) If we expand the sine function in a Taylor series around $\vartheta \approx 0$, we obtain $\sin(\vartheta) \approx \vartheta - \frac{\vartheta^3}{6} + O[\vartheta^5]$. Thus the percent different can be approximately written as

$$\frac{\vartheta - \sin(\vartheta)}{\sin(\vartheta)} \approx \frac{\vartheta^3}{(6\vartheta - \vartheta^3)} = 0.01$$
$$\Rightarrow \vartheta = \pm\sqrt{\frac{6(0.01)}{1+(0.01)}} = \pm 0.24\text{ rad}$$

The two solutions (positive and negative) simply indicate that the angular displacement could be to either side of equilibrium. The magnitude at which 1% error is exceeded is $0.24\text{ rad},$ which is equivalent to $14°$.

15.53. We use conservation of energy. Let use choose the zero of our gravitational potential energy to be at the lowest point in the pendulum's swing (when the bob is at the equilibrium position). Initially, all energy is gravitational potential energy, and this initial energy is equal to $U_{\text{i}}^G = mgh_{\text{i}} = mg\ell(1-\cos\vartheta_{\max})$. At any other angular position, the energy will be a combination of kinetic and potential energy. Thus

$$U_{\text{i}}^G = U_{\text{f}}^G + K_{\text{f}}$$
$$mg\ell(1-\cos\vartheta_{\max}) = mg\ell(1-\cos\vartheta) + \frac{1}{2}mv_{\text{f}}^2$$
$$\Rightarrow v_{\text{f}} = \sqrt{2g\ell(\cos(\vartheta) - \cos(\vartheta_{\max}))}$$

15.54. (*a*) $I = mR^2 = (8.00\times10^{-4}\ \text{kg})(0.015\ \text{m})^2 = 1.8\times10^{-7}\ \text{kg}\cdot\text{m}^2$ (*b*) From equation (15.29) we know $\kappa = \omega^2 I = \left(\frac{2\pi}{T}\right)^2 I,$ and we know from the description that the period is 0.50 s. Thus $\kappa = \left(\frac{2\pi}{(0.50\ \text{s})}\right)^2 (1.8\times10^{-7}\ \text{kg}\cdot\text{m}^2) = 2.8\times10^{-5}\ \text{kg}\cdot\text{m}^2/\text{s}^2.$

15.55. (*a*) We use the parallel axis theorem to write $I = I_{\text{cm}} + m(\Delta x)^2,$ where Δx is the distance from the axis of rotation to the center of mass of the rod (in this case $\Delta x = (\ell/2) - (0.0625\ \text{m}) = 0.0625\ \text{m}.$ Thus $I = \frac{1}{12}m\ell^2 + m(\Delta x)^2 = (0.100\ \text{kg})\left(\frac{(0.250\ \text{m})^2}{12} + (0.0625\ \text{m})^2\right) = 9.11\times10^{-4}\ \text{kg}\cdot\text{m}^2.$ (*b*) Equation (15.33) tells us $\omega = \sqrt{\frac{m\ell_{\text{cm}}g}{I}} = \sqrt{\frac{(0.100\ \text{kg})(0.0625\ \text{m})(9.8\ \text{m/s}^2)}{(9.115\times10^{-4}\ \text{kg}\cdot\text{m}^2/\text{s}^2)}} = 8.198\ \text{s}^{-1}.$ We also know $T = \frac{2\pi}{\omega} = \frac{2\pi}{(8.198\ \text{s}^{-1})} = 0.77\text{s}.$

15.56. We will determine the period by modifying equation (15.33) to give us $T = 2\pi\sqrt{\frac{I}{m_{\text{total}}\ell_{\text{cm}}g}},$ where $I = I_{\text{rod}} + I_{\text{cube}} = I_{\text{rod,end}} + I_{\text{cube,cm}} + m_{\text{cube}}(\Delta x)^2.$ Here we have use the parallel axis theorem to write the rotational inertia of the cube in terms of Δx the distance between the cube's center of mass and the new axis of rotation. In this case $\Delta x = \frac{3\ell}{2},$ and we know $I_{\text{rod,end}} = \frac{1}{3}m\ell^2$ and $I_{\text{cube,cm}} = \frac{1}{6}m\ell^2.$ Thus

$$T = 2\pi\sqrt{\frac{I_{\text{rod,end}} + I_{\text{cube,cm}} + m_{\text{cube}}(\Delta x)^2}{m_{\text{total}}\ell_{\text{cm}}g}} = \sqrt{\frac{\left(\frac{1}{3} + \frac{1}{6} + \left(\frac{3}{2}\right)^2\right)m\ell^2}{2m\ell_{\text{cm}}g}} = \sqrt{\frac{11\ell}{8g}}$$

15.57. (*a*) We choose our gravitational potential energy to be zero at the lowest point in the oscillation. We can use conservation of energy to write

$$K_{\text{i}} = U_{\text{f}}^G$$
$$\frac{1}{2}mv_{\text{i}}^2 = mg\ell(1-\cos\vartheta_{\text{max}})$$
$$\Rightarrow \vartheta_{\text{max}} = \cos^{-1}\left(1 - \frac{v_{\text{i}}^2}{2g\ell}\right) = \cos^{-1}\left(1 - \frac{(0.25\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)(0.30\ \text{m})}\right) = 0.15\ \text{radians}$$

(*b*) Using conservation of energy as in part (*a*) we can write

$$K_{\text{i}} = U_{\text{f}}^G + K_{\text{f}}$$
$$\frac{1}{2}mv_{\text{i}}^2 = mg\ell(1-\cos(\vartheta_{\text{max}}/2)) + \frac{1}{2}mv_{\text{f}}^2$$
$$\Rightarrow v_{\text{f}} = \sqrt{v_{\text{i}}^2 - 2g\ell(1-\cos(\vartheta_{\text{max}}/2))} = \sqrt{(0.25\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(0.30\ \text{m})(1-\cos(0.0730))} = 0.22\ \text{m/s}$$

15.58. Starting from equation (15.33), we can use the relationship between angular frequency and period to write $T = 2\pi\sqrt{\frac{I}{m\ell_{\text{cm}}g}}.$ Here, the rotational of inertia is that around a point other than the center of mass, so we use the parallel axis theorem: $I = I_{\text{cm}} + m(\Delta x)^2$ where Δx is the distance from the center of mass to the pivot. In this problem we are told $\Delta x = \ell_{\text{cm}}.$ Since the rotational of inertia of a disk about its center of mass is $I_{\text{cm}} = \frac{1}{2}mR^2,$ it follows that $T = 2\pi\sqrt{\frac{\frac{1}{2}mR^2 + m\ell_{\text{cm}}^2}{m\ell_{\text{cm}}g}} = 2\pi\sqrt{\frac{\frac{1}{2}R^2 + \ell_{\text{cm}}^2}{\ell_{\text{cm}}g}}.$

15.59. Starting from equation (15.33), we can use the relationship between angular frequency and period to write $T = 2\pi\sqrt{\dfrac{I}{m\ell_{\text{cm}}g}}$. Here, the rotational of inertia is that around a point other than the center of mass, so we use the parallel axis theorem: $I = I_{\text{cm}} + m(\Delta x)^2$ where Δx is the distance from the center of mass to the pivot. In this problem $\Delta x = R$. Since the rotational of inertia of a ring about its center of mass is $I_{\text{cm}} = mR^2$, it follows that $T = 2\pi\sqrt{\dfrac{mR^2 + mR^2}{mRg}} = 2\pi\sqrt{\dfrac{2R}{g}} = 2\pi\sqrt{\dfrac{2(0.10\text{ m})}{(9.8\text{ m/s}^2)}} = 0.90\text{ s}.$

15.60. We can write the frequency of any pendulum as $f = \dfrac{1}{2\pi}\sqrt{\dfrac{m\ell_{\text{cm}}g}{I}}$. To reasonable accuracy, we can model the swinging leg as a rod, meaning $I = \dfrac{1}{3}m\ell^2$. Let us assume that the length of the leg ℓ, the distance from the hip to the leg's center of mass ℓ_{cm} and the inertia of the leg m all scale linearly with the height of the person. This would mean:

$$\frac{f_{\text{short}}}{f_{\text{average}}} = \frac{\dfrac{1}{2\pi}\sqrt{\dfrac{(0.8m)(0.8\ell_{\text{cm}})g}{\frac{1}{3}(0.8m)(0.8\ell)^2}}}{\dfrac{1}{2\pi}\sqrt{\dfrac{m\ell_{\text{cm}}g}{\frac{1}{3}m\ell^2}}} = \sqrt{\frac{1}{0.8}} = 1.12$$

So $f_{\text{short}} : f_{\text{average}} = 1.12 : 1.$

15.61. Let us first write the period of the pendulum when we ignore the mass of the rod. In this approximation, the pendulum is a simple pendulum and we can apply the results of example 15.6:

$$T_0 = 2\pi\sqrt{\frac{\ell}{g}} \tag{1}$$

If we do not ignore the mass of the rod, we obtain a slightly more complicated expression using equation (15.33): $T = 2\pi\sqrt{\dfrac{I}{m\ell_{\text{cm}}g}}$. Here the rotational inertia is $I = I_{\text{rod}} + I_{\text{bob}} = \dfrac{1}{3}m_{\text{rod}}\ell^2 + m_{\text{b0b}}\ell^2$. The center of mass in this case is $\ell_{\text{cm}} = \left(\left(\dfrac{\ell}{2}\right)m_{\text{rod}} + \ell m_{\text{bob}}\right) / (m_{\text{rod}} + m_{\text{bob}})$. Thus

$$T = 2\pi\sqrt{\frac{\left(\dfrac{m_{\text{rod}}}{3} + m_{\text{bob}}\right)\ell^2}{(m_{\text{rod}} + m_{\text{bob}})\ell_{\text{cm}}g}} = 2\pi\sqrt{\frac{\left(\dfrac{m_{\text{rod}}}{3} + m_{\text{bob}}\right)\ell}{((m_{\text{rod}}/2) + m_{\text{bob}})g}} = 2\pi\sqrt{\frac{\left(\dfrac{1}{3}\left(\dfrac{m_{\text{rod}}}{m_{\text{bob}}}\right) + 1\right)\ell}{\left(\dfrac{1}{2}\left(\dfrac{m_{\text{rod}}}{m_{\text{bob}}}\right) + 1\right)g}}$$

Since we are told that the ratio $\dfrac{m_{\text{rod}}}{m_{\text{bob}}} \ll 1$, we can expand this expression in a Taylor series around $\dfrac{m_{\text{rod}}}{m_{\text{bob}}} \approx 0$. This yields

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(1 - \frac{1}{12}\left(\frac{m_{\text{rod}}}{m_{\text{bob}}}\right) + \frac{11}{288}\left(\frac{m_{\text{rod}}}{m_{\text{bob}}}\right)^2 + O\left[\left(\frac{m_{\text{rod}}}{m_{\text{bob}}}\right)^3\right]\right) \tag{2}$$

To find the difference between the two calculated periods, we subtract equation (1) from equation (2):

$\Delta T = T - T_0 = 2\pi\sqrt{\dfrac{\ell}{g}}\left(-\dfrac{1}{12}\dfrac{m_{\text{rod}}}{m_{\text{bob}}} + \dfrac{11}{288}\left(\dfrac{m_{\text{rod}}}{m_{\text{bob}}}\right)^2\right)$ plus higher order terms.

15.62. The damping constant b has units of kg/s. Hence the quantity m/b has units of $\text{kg}/(\text{kg/s}) = \text{s}$.

15.63. The quality factor is related to the undamped frequency through $Q = \omega\tau$. In order for Q to increase without affecting ω (and therefore the undamped period), the time constant τ must increase. The time constant is $\tau = m/b$, which allows us to re-write equation (15.38) as $\omega_\text{d} = \sqrt{\omega^2 - \left(\frac{1}{2\tau}\right)^2}$. Since τ increases, the quantity in the square root increases. Thus the angular frequency of the damped motion increases.

15.64. The sinusoidal factor in equation (15.37) describes the oscillatory behavior, whereas the prefactor describes the damped amplitude. Thus, we look only at the prefactor and require $Ae^{-bt/2m} = A/2$. Since $\tau = m/b$, we can also write this as $Ae^{-t/2\tau} = A/2 \Rightarrow t = -\tau\ln\left(\frac{1}{2}\right)$.

15.65. (*a*) The damping constant b has units of kg/s. Hence the quantity m/b has units of $\text{kg}/(\text{kg/s}) = \text{s}$. (*b*) Equation (15.38) tells us the damped angular frequency. So $f_\text{d} = \frac{1}{2\pi}\left[\omega^2 - \left(\frac{b}{2m}\right)^2\right]^{1/2} = \frac{1}{2\pi}\left[\frac{k}{m} - \left(\frac{b}{2m}\right)^2\right]^{1/2} =$ $\frac{1}{2\pi}\left[\frac{(300\text{ N/m})}{(0.400\text{ kg})} - \left(\frac{(5.00\text{ kg/s})}{2(0.400\text{ kg})}\right)^2\right]^{1/2} = 4.24\text{ Hz}.$ (*c*) $\tau = \frac{m}{b} = \frac{(0.400\text{ kg})}{(5.00\text{ kg/s})} = 0.080\text{ s}$ (*d*) $Q = \omega\tau = \sqrt{\frac{k}{m}}\tau =$ $\sqrt{\frac{(300\text{ N/m})}{(0.400\text{ kg})}}(0.080\text{ s}) = 2.19\text{ radians}$

15.66. We can determine the time constant by writing $\vartheta_\text{max,i}e^{-t/2\tau} = \vartheta_\text{max,f}$. Thus

$$\tau = \frac{-t}{2\ln\left(\frac{\vartheta_\text{max,f}}{\vartheta_\text{max,i}}\right)} = \frac{-(27.0\text{ s})}{2\ln\left(\frac{5.40°}{6.00°}\right)} = 128\text{ s}.$$

From this it follows immediately that $b = \frac{m}{\tau} = \frac{(1.00\text{ kg})}{(128.1\text{ s})} = 7.80\times10^{-3}\text{ kg/s}$.

15.67. (*a*) We can determine the time constant by writing $\vartheta_\text{max,i}e^{-t/2\tau} = \vartheta_\text{max,f}$. Thus

$$\tau = \frac{-t}{2\ln\left(\frac{\vartheta_\text{max,f}}{\vartheta_\text{max,i}}\right)} = \frac{-(35\text{ s})}{2\ln\left(\frac{1}{2}\right)} = 25\text{ s}$$

(*b*) Equation (15.40) shows us $E_0e^{-t/\tau} = E$, so $t = -\tau\ln\left(\frac{E}{E_0}\right) = -(25.2\text{ s})\ln\left(\frac{1}{2}\right) = 18\text{ s}.$

15.68. (*a*) We know that $Ae^{-T/\tau} = A(0.98)$. Raising each side of this equation to the 25th power yields $e^{-25T/\tau} =$ $(0.98)^{25} = 0.603$. Thus the amplitude after 25 cycles is $Ae^{-25T/\tau} = (0.10\text{ m})(0.60) = 0.060\text{ m}$. (*b*) Equation (15.40) shows us $E_0e^{-t/\tau} = E$. So $\frac{E}{E_0} = e^{-t/\tau} = (e^{-T/\tau})^{2t/T} = (0.98)^{2(6.3\text{ s})/(0.50\text{ s})} = 0.60\text{ or }60\%$.

15.69. (*a*) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{(12.5\ \text{N/m})}{(0.500\ \text{kg})}} = 5.00\ \text{s}^{-1}$ (*b*) Equation (15.38) tells us $\omega_\text{d} = \left[\omega^2 - \left(\dfrac{b}{2m}\right)^2\right]^{1/2}$. Rearranging, we find $b = 2m\sqrt{\omega^2 - \omega_\text{d}^2} = 2(0.500\ \text{kg})\sqrt{(5.00\ \text{s}^{-1})^2 - (4.58\ \text{s}^{-1})^2} = 2.01\ \text{kg/s}$. (*c*) $y(t) = Ae^{-t/2\tau}\sin(\omega t + \phi)$ where $A = 0.100\ \text{m}$, $\tau = \dfrac{m}{b} = \dfrac{(0.500\ \text{kg})}{(2.01\ \text{kg/s})} = 0.249\ \text{s}$, and $\omega = 4.58\ \text{s}^{-1}$. Or, in terms of the damping constant: $y(t) = Ae^{-bt/2m}\sin(\omega t + \phi)$ where $b = 2.01\ \text{kg/s}$.

15.70. (*a*) From equation (15.41) we can write $\dfrac{\tau}{T} = \dfrac{Q}{2\pi}$. The factor by which the energy is decreased each cycle is $\dfrac{E}{E_0} = e^{-T/\tau} = e^{-(2\pi/Q)} = e^{-(2\pi/20)} = 0.73$. (*b*) Equation (15.38) tells us $\omega_\text{d} = \left[\omega^2 - \left(\dfrac{b}{2m}\right)^2\right]^{1/2}$, such that the percent difference between the damped and undamped angular frequencies is

$$\frac{\omega - \omega_\text{d}}{\omega} = 1 - \sqrt{1 - \left(\frac{b}{2m\omega}\right)^2} = 1 - \sqrt{1 - \left(\frac{T}{4\pi\tau}\right)^2} = 1 - \sqrt{1 - \left(\frac{1}{4\pi}\right)^2\left(\frac{2\pi}{Q}\right)^2} = 1 - \sqrt{1 - \left(\frac{1}{4\pi}\right)^2\left(\frac{2\pi}{20}\right)^2} = 3.1\times10^{-4}$$

or 0.031%.

15.71. (*a*) From equation (15.41) we can write $\dfrac{\tau}{T} = \dfrac{Q}{2\pi}$. The factor by which the energy is decreased each cycle is $\dfrac{E}{E_0} = e^{-T/\tau} = e^{-(2\pi/Q)} = e^{-(2\pi/400)} = 0.9844$. This means that 1.6% of the energy is lost in each cycle. (*b*) $\dfrac{E}{E_0} = e^{-t/\tau} = (e^{-T/\tau})^{t/T} = (e^{-(2\pi/Q)})^{t/T} = (e^{-(2\pi/400)})^{(2880\ \text{min})/(54\ \text{min})} = 0.43$ or 43%.

15.72. (*a*) Equation (15.40) tells us $\dfrac{E}{E_0} = e^{-t/\tau}$ so $\tau = -\dfrac{t}{\ln\left(\dfrac{E}{E_0}\right)} = -\dfrac{(4.0\ \text{s})}{\ln(0.85)} = 25\ \text{s}$. (*b*) Rearranging equation (15.40) we find $t = -\tau\ln\left(\dfrac{E}{E_0}\right) = -(24.6\ \text{s})\ln(0.25) = 34\ \text{s}$. (*c*) $\dfrac{E}{E_0} = e^{-t/\tau} = e^{-(60\ \text{s})/(24.6\ \text{s})} = 0.087$ or 8.7%.

15.73. (*a*) In critical damping, the system does not even complete one oscillation, meaning that the damped frequency should be zero. Thus we set equation (15.38) to zero when $b = b_\text{crit}$: $\omega_\text{d} = \sqrt{\omega^2 - \left(\dfrac{b_\text{crit}}{2m}\right)^2} = \sqrt{\dfrac{k}{m} - \left(\dfrac{b_\text{crit}}{2m}\right)^2} = 0$. Rearranging, we find $b_\text{crit} = 2\sqrt{mk}$. (*b*) The Damping occurs so quickly that the spring returns to the equilibrium position before being fully compressed. In a car, this means the system returns quickly to the normal level rather than oscillating for a long time interval; the negative aspect is that the springs do not absorb shock from road-bumps as well as a spring system having $b \le b_\text{crit}$.

15.74. (*a*) We are told to assume that the drag force is of the form: $\vec{F}_\text{drag} = -b\vec{v}$. In the case of the falling object, we can say $\sum F_y = -mg - bv_y = ma_y = 0$, where we have set the acceleration equal to zero because the object reaches a terminal speed. Thus

$$b = -\frac{mg}{v_y} \qquad (1)$$

When the object is set into oscillatory motion, the quality factor is given by equation (15.40): $Q = \omega\tau \Rightarrow$ $Q = \sqrt{\dfrac{k}{m}}\left(\dfrac{m}{b}\right)$. Inserting equation (1) yields

$$Q = \sqrt{\frac{k}{m}}\left(\frac{-v_y}{g}\right) = \sqrt{\frac{(230 \text{ N/m})}{(3.0 \text{ kg})}}\left(\frac{-(-25 \text{ m/s})}{(9.8 \text{ m/s}^2)}\right) = 22$$

(*b*) We use equation (15.39) to write $x_{\max}(t) = Ae^{-t/2\tau} \Rightarrow t = -2\tau \ln\left(\dfrac{x_{\max}(t)}{A}\right) = -\left(\dfrac{2m}{b}\right)\ln\left(\dfrac{x_{\max}(t)}{A}\right)$. Inserting equation (1) from part (*a*) yields $t = \left(\dfrac{2v_y}{g}\right)\ln\left(\dfrac{x_{\max}(t)}{A}\right) = \left(\dfrac{2(-25 \text{ m/s})}{(9.8 \text{ m/s}^2)}\right)\ln\left(\dfrac{1}{2}\right) = 3.5 \text{ s}$. (*c*) Equation (15.40) tells us $\dfrac{E}{E_0} = e^{-t/\tau} = e^{-bt/m} = e^{gt/v_y} = e^{(9.8 \text{ m/s}^2)(3.54 \text{ s})/(-25 \text{ m/s})} = 0.25$. Only 25% of the original energy remains. So the amount of energy lost is $\Delta E = -(0.75)E_{\text{i}} = -(0.75)\dfrac{1}{2}kA^2 = -(0.75)\dfrac{1}{2}(230 \text{ N/m})(0.20 \text{ m})^2 = -3.5 \text{ J}$. So 3.5 J of energy is lost.

15.75. We know that damped oscillations have the general form of equation (15.37): $x(t) = Ae^{-t/\tau}\sin(\omega t + \phi)$. We determine the numerical values of all quantities. First, we know the period of one damped oscillation is $T = 1.0$ s, which means $\omega_{\text{d}} = 2\pi \text{ s}^{-1}$. Now let us determine the time constant from the damped energy:

$$\frac{E}{E_0} = e^{-\Delta t/\tau} \Rightarrow \tau = \frac{-\Delta t}{\ln\left(\dfrac{E}{E_0}\right)} = \frac{-(t_{\text{f}} - t_{\text{i}})}{\ln\left(\dfrac{\left[x_{\max}(t_{\text{f}})\right]^2}{\left[x_{\max}(t_{\text{i}})\right]^2}\right)} = \frac{-((2.5 \text{ s}) - (1.5 \text{ s}))}{\ln\left(\dfrac{(0.056 \text{ m})^2}{(0.060 \text{ m})^2}\right)} = 7.2 \text{ s}.$$

This time constant also allows us to determine the initial amplitude (at $t = 0$): $A = \dfrac{x_{\max}(t)}{e^{-t/2\tau}} = \dfrac{(0.060 \text{ m})}{e^{-(1.5 \text{ s})/2(7.25 \text{ s})}}$ $= 0.067$ m. Finally, we determine the initial phase constant by using the condition given at $t = 1.5$ s. We can say $x(t) = x_{\max}(t)\sin(\omega t + \phi) = x_{\max}(t) \Rightarrow \sin(\omega t + \phi) = 1$ at $t = 1.5$ s, then $\phi = \sin^{-1}(1) - \omega t = \dfrac{\pi}{2} - (2\pi \text{ s}^{-1})(1.5 \text{ s}) = -2.5\pi$ or $\dfrac{3\pi}{2}$. Thus $x(t) = Ae^{-t/\tau}\sin(\omega t + \phi)$ where $A = 67$ mm, $\tau = 7.2$ s, $\omega = 2\pi \text{ s}^{-1}$, and $\phi = \dfrac{3\pi}{2}$.

15.76. Yes. Since the spring has the same stiffness regardless of which direction the object moves, it will exert the same force magnitude and do the same amount of work as the object moves from equilibrium to either stopping point. Thus the force must be applied over the same distance from equilibrium to either stopping point.

15.77. (*a*) The energy would increase by a factor of 4. Consider an instant at which all energy has been converted to potential energy. In the case of a spring-object system, the potential energy is $U^{\text{spring}} = \dfrac{1}{2}kA^2$, which clearly increases with the square of the amplitude. In the case of a pendulum, the gravitational potential energy is $E = U^G_{\max} = mgh_{\max} = mg\ell(1 - \cos(\vartheta_{\max}))$. The motion of a pendulum is not simple harmonic motion unless the amplitude of the oscillations are kept small. With that assumption, we can Taylor expand the cosine around small angle to obtain $U^G = mgh = mg\ell\left(1 - \left(1 - \dfrac{\vartheta^2_{\max}}{2} + O(\vartheta^4_{\max})\right)\right) \approx \dfrac{mg\ell}{2}\vartheta^2_{\max}$, which again increases with the square of the amplitude. (*b*) We have established that the maximum energy increases by a factor of 4. Since the maximum speed occurs when all energy is kinetic energy, we can write $\dfrac{1}{2}mv^2_{\max} = E \Rightarrow v_{\max} = \sqrt{\dfrac{2E}{m}}$. So if the energy increases by a factor of 4, the maximum speed increases by a factor of 2. (*c*) The period would not change.

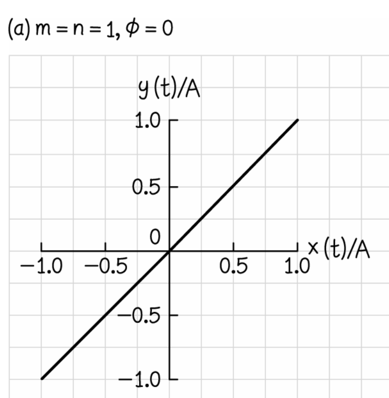
15.78. (*a*) You know that in an elevator, your apparent weight is greater if you are accelerating upward, than it is when the elevator is stationary or moving with a constant speed. Similarly, the period of a pendulum decreases, as though gravity had become stronger. Of course, the strength of the gravitational force and the acceleration due to gravity have not actually changed. What is actually happening is that you are in a reference frame that is being accelerated upward. But the effect is the same: the period of the pendulum decreases. (*b*) The period does not change. (*c*) For the same reasons described in part (*a*), the period decreases.

15.79. The molecules in the material of the spring resist compression and/or shear in a way that cannot change when you cut the spring. So let us consider the initial spring (length ℓ) as being made of two springs, each of length $\ell/2$. In order for the hanging object to move a certain distance Δy, each of the two springs would only need to stretch by a distance $\Delta y/2$. So the force exerted by the lower spring on the hanging object would be $F_{\text{whole spring}} = -k_{\text{whole spring}}\Delta y/2$. When the spring is cut in half, and the hanging object stretches the spring by Δy, the spring stretches a distance Δy, and the force it exerts is $F_{\text{half spring}} = -k_{\text{half spring}}\Delta y$. Comparing these two forces demonstrates that $k_{\text{half spring}} = 2k_{\text{whole spring}}$. Now, since $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$, we can write $\dfrac{f_{\text{half}}}{f_{\text{whole}}} = \sqrt{\dfrac{k_{\text{half}}}{k_{\text{whole}}}} = \sqrt{2}$.

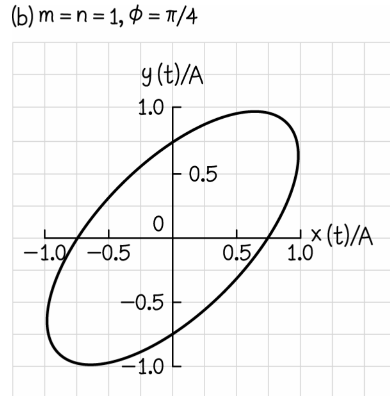
15.80. Call the initial compress of the spring Δy_{i}, and call the spring constant k. Including Earth in our system, and choosing the initial position of the block to correspond to zero gravitational potential energy, we can use conservation of energy to write $\frac{1}{2}k(\Delta y_{\text{i}})^2 = \frac{1}{2}k(d - \Delta y)^2 + mgd$. This equation is quadratic in d and has solutions $d = 0$, and $d = \dfrac{2gm}{k} + 2h$. Now, if we double the mass of the block being released and repeat the experiment, we will find it falls a new distance $d' = \dfrac{4gm}{k} + 2h = d + \dfrac{2gm}{k}$. This can be rewritten in terms of given variables by noting that the fall described is one half of a full oscillation. So we can write $k = m\omega^2 = \dfrac{m(2\pi)^2}{T^2} = \dfrac{m\pi^2}{\Delta t^2}$. Inserting this into our expression above yields $d' = d + \dfrac{2g\Delta t^2}{\pi^2}$.

15.81. The trajectories will be closed paths as long as there exists some time T after which $x(T) = x(0)$ and $y(T) = y(0)$ (meaning the trajectory returns to its original point). This is satisfied if $m\omega T = 2\pi p$ and $n\omega T = 2\pi p'$, where p and p' are integers. Dividing one condition by the other yields $\dfrac{m}{n} = \dfrac{p}{p'}$. Thus they always be closed paths if the ratio m/n is rational. This is obviously satisfied by choosing m and n to be positive integers.

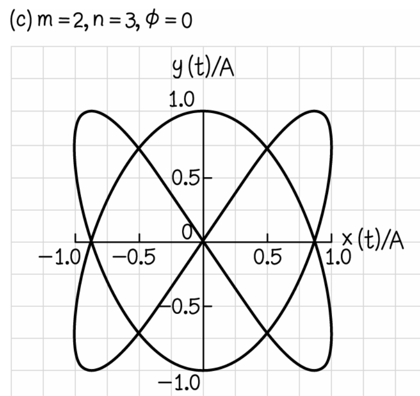
(*a*)



(*b*)



(*c*)



15.82. Different printings may vary slightly. Let us assume the book is 0.30 m tall, 0.22 m wide, and has a mass of 2.0 kg. The distance from one corner to the center of mass will be

$$\ell_{\text{cm}} = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{w}{2}\right)^2} = \sqrt{\left(\frac{(0.15\text{ m})}{2}\right)^2 + \left(\frac{(0.11\text{ m})}{2}\right)^2} = 0.186\text{ m} \tag{1}$$

The book will not be rotating around its center of mass, so we need to use the parallel axis theorem to determine the rotational inertia of the book around one corner.

$$\begin{aligned} I &= I_{\text{cm}} + m\ell_{\text{cm}}^2 = \frac{1}{12}m(h^2 + w^2) + m(\ell_{\text{cm}}^2) = \frac{1}{12}(2.0\text{ kg})((0.30\text{ m})^2 + (0.22\text{ m})^2) + (2.0\text{ kg})(0.186\text{ m})^2 \\ &= 0.0923\text{ kg}\cdot\text{m}^2 \end{aligned} \tag{2}$$

Rearranging equation (15.33) and inserting these results yields

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{m\ell_{\text{cm}}g}} = 2\pi\sqrt{\frac{(0.0923\text{ kg}\cdot\text{m}^2)}{(2.0\text{ kg})(0.186\text{ m})(9.8\text{m/s}^2)}} = 1.0\text{ s}$$

Certainly, there can be many correct answers, but they should be close to 1.0 s.

15.83. Rearranging equation (15.33) allows us to write

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{m\ell_{\text{cm}}g}} \quad\text{or}\quad I = \frac{m\ell_{\text{cm}}gT^2}{(2\pi)^2} = \frac{(2.00\text{ kg})(0.20\text{ m})(9.8\text{ m/s}^2)((11\text{ s})/(10))^2}{(2\pi)^2} = 0.12\text{ kg}\cdot\text{m}^2$$

15.84. (*a*) The initial energy can be written as $E = U_{\text{i}}^G + K_{\text{i}} = \frac{1}{2}k(\Delta x_{\text{i}})^2 + \frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}(200\text{ N/m})(0.0500\text{ m})^2 + \frac{1}{2}(2.00\text{ kg})(2.00\text{ m/s})^2 = 4.25\text{ J}$. (*b*) We know that the general form of this simple harmonic motion is

$x(t) = A\sin(\omega t + \phi)$. We determine the numerical value of the amplitude, angular frequency and initial phase constant. When the object reaches its maximum displacement from equilibrium, all the energy will be spring potential energy. At that instant, we can write $E_{\text{i}} = \frac{1}{2}k(\Delta x_{\max})^2 = \frac{1}{2}kA^2 \Rightarrow A = \sqrt{\frac{2E_{\text{i}}}{k}} = \sqrt{\frac{2(4.25\text{ J})}{(200\text{ N/m})}} = 0.206\text{ m}$. The angular frequency is given by $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{(200\text{ N/m})}{(2.00\text{ kg})}} = 10.0\text{ s}^{-1}$. Finally, to determine the initial phase, we look at the displacement at the moment the initial push is given. We know $x(t=0) = A\sin(\phi) \Rightarrow \phi = \sin^{-1}\left(\frac{x(t=0)}{A}\right) = \sin^{-1}\left(\frac{(0.0500\text{ m})}{(0.2062\text{ m})}\right) = 0.245$. Thus $x(t) = A\sin(\omega t + \phi)$ where $A = 0.206$ m, $\omega = 10.0\text{ s}^{-1}$, and $\phi = 0.245$.

15.85. (*a*) Rearranging equation (15.33) allows us to write $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{m\ell_{\text{cm}}g}} \Rightarrow I = m\ell_{\text{cm}}g\left(\frac{T}{2\pi}\right)^2 = (0.960\text{ kg})((0.670\text{ m}) - (0.030\text{ m}))(9.8\text{ m/s}^2)\left(\frac{(1.85\text{ s})}{2\pi}\right)^2 = 0.522\text{ kg}\cdot\text{m}^2$. (*b*) Using the parallel axis theorem, we can write $I = I_{\text{cm}} + m(\Delta x)^2$, where Δx is the distance from the pivot to the center of mass. Thus $I_{\text{cm}} = I - m(\Delta x)^2 = (0.5220\text{ kg}\cdot\text{m}^2) - (0.960\text{ kg})((0.670\text{ m}) - (0.030\text{ m}))^2 = 0.129\text{ kg}\cdot\text{m}^2$. (*c*) In this case, the distance from the center of mass to the pivot would be $\Delta x = (0.670\text{ m}) - (0.200\text{ m}) = 0.470\text{ m}$. We can apply the parallel axis theorem to this new pivot position to obtain $I = I_{\text{cm}} + m(\Delta x)^2 = (0.1288\text{ kg}\cdot\text{m}^2) + (0.960\text{ kg})(0.470\text{ m})^2 = 0.3408\text{ kg}\cdot\text{m}^2$. Inserting this rotational inertia into our rearrangement of equation (15.33) above, we find

$$T = 2\pi\sqrt{\frac{I}{m\ell_{\text{cm}}g}} = 2\pi\sqrt{\frac{(0.3408\text{ kg}\cdot\text{m}^2)}{(0.960\text{ kg})(0.470\text{ m})(9.8\text{ m/s}^2)}} = 1.74\text{ s}$$

15.86. (*a*) From equation (15.33) we know $f = \frac{1}{2\pi}\sqrt{\frac{m\ell_{\text{cm}}g}{I}}$. The distance from the pivot to the center of mass $\ell_{\text{cm}} = (0.50\text{m}) - x$. We know the rotational inertia of a rod around its center of mass, but here it rotates around a different point. So we need to use the parallel axis theorem. This yields $I = I_{\text{cm}} + m\ell_{\text{cm}}^2 = \frac{1}{12}(m\ell^2) + m((0.50\text{ m}) - x)^2$. Combining these results, we have

$$f = \frac{1}{2\pi}\left[\frac{m((0.50\text{ m}) - x)g}{\frac{1}{12}m(1.0\text{ m})^2 + m((0.50\text{ m}) - x)^2}\right]^{1/2} = \frac{1}{2\pi}\left[\frac{(0.50\text{ m} - x)g}{\frac{1}{12}(1.0\text{ m})^2 + (0.50\text{ m} - x)^2}\right]^{1/2}$$

(*b*) The period is

$$T = 2\pi\left[\frac{(0.50\text{ m} - x)g}{\frac{1}{12}(1.0\text{ m})^2 + (0.50\text{ m} - x)^2}\right]^{-1/2}$$

We can find the extrema of this as a function of x by differentiating and setting the first derivative to zero:

$$\frac{dT}{dx}=(2\pi)\frac{d}{dx}\left[\frac{(0.50\text{ m}-x)g}{\frac{1}{12}(1.0\text{ m})^2+(0.50\text{ m}-x)^2}\right]^{-1/2}=\frac{\pi\left[-\frac{g}{\frac{1}{12}+((0.50\text{ m})-x)^2}+\frac{2g((0.50\text{ m})-x)^2}{\left(\frac{1}{12}+((0.50\text{ m})-x)^2\right)^2}\right]}{\sqrt{\frac{g((0.50\text{m})-x)}{\frac{1}{12}+((0.50\text{m})-x)^2}}}=0$$

This yields two solutions: $x=0.21$ m and $x=0.79$ m, but they correspond to the same physical position, just from different ends. Because we are told $0<x<0.50$ m, we choose $x=0.21$ m.

15.87. Consider a tiny segment of the spring at a position x_{seg} and of length dx_{seg} with mass $dm=\frac{m}{x}dx_{\text{seg}}$. If we assume that the spring is uniform and stretches in a uniform way, we can write $v_{\text{seg}}=\frac{x_{\text{seg}}}{x}v.$ We can write the kinetic energy of this tiny segment as $K_{\text{seg}}=\frac{1}{2}dmv_{\text{seg}}^2=\left(\frac{m}{2x}\right)\left(\frac{x_{\text{seg}}}{x}v\right)^2dx_{\text{seg}}$. If we integrate the small contributions of such segments to the kinetic energy over the entire length of the spring, we obtain:

$$K=\int_0^x K_{\text{seg}}=\int_0^x\left(\frac{m}{2x}\right)\left(\frac{x_{\text{seg}}}{x}v\right)^2dx_{\text{seg}}=\frac{mv^2}{2x^3}\int_0^x x_{\text{seg}}^2dx_{\text{seg}}=\frac{mv^2}{6}\quad mv^2/6$$

15.88. (*a*) We start from equation (15.7) and find $v_x(x)=A\omega\cos(\omega t)=A\omega\sqrt{1-\sin^2(\omega t)}=\omega\sqrt{A^2-A^2\sin^2(\omega t)}=\omega\sqrt{A^2-(x(t))^2}$. (*b*) Since $v_x=\frac{dx}{dt}$ we know $dt=\frac{dx}{v_x}$ and $\int_0^t dt'=\int_0^x\frac{dx'}{v_x}=\int_0^x\frac{dx'}{\omega\sqrt{A^2-(x')^2}}$. Integrating, we find $t=\frac{1}{\omega}\left[\tan^{-1}\left(\frac{x}{\sqrt{A^2-x^2}}\right)-\phi\right]$. Taking the difference suggested using equation (15.6) yields $t=\frac{1}{\omega}\left[\sin^{-1}\left(\frac{x}{A}\right)-\phi\right]$. It is a trivial geometry problem to show that these two expressions are equivalent. It may be helpful to draw a right triangle with legs of lengths x and $\sqrt{A^2-x^2}$, such that the hypotenuse is A. Call the angle between the hypotenuse and the leg of length $\sqrt{A^2-x^2}$ angle θ. Then clearly $\theta=\tan^{-1}\left(\frac{x}{\sqrt{A^2-x^2}}\right)$ and $\theta=\sin^{-1}\left(\frac{x}{A}\right)$. Thus the two expressions are equivalent.

15.89. You wish to construct the new pendulum such that it matches the frequency of the spring-object system. The angular frequencies can be written as $\omega_{\text{spring}}=\sqrt{\frac{k}{m}}$ and $\omega_{\text{pendulum}}=\sqrt{\frac{g}{\ell}}$. So we require string of length $\ell=\frac{mg}{k}$. Now, when you hold the spring vertically, and attach the ball, it will eventually come to rest after stretching the spring some through some displacement Δy. At this new equilibrium position, we can write $\sum F_y=-mg-k\Delta y=0\Rightarrow\Delta y=-\frac{mg}{k}$. Thus the length of the pendulum wire must be the same as the distance by which the ball stretched the spring.

15.90. Total energy of spring-mass-Earth system is

$$E=\tfrac{1}{2}mv^2+\tfrac{1}{2}I\omega^2+\tfrac{1}{2}kx^2+\underbrace{(kx_{eq}-mg)}_{=0}x+\tfrac{1}{2}kx_{eq}-mgx_{eq}$$

where x is the displacement of the spring and the ball from their equilibrium position x_{eq}. Note that the last two terms are constants. Using $\omega = \frac{v}{R}$ gives

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\tfrac{I}{R^2}v^2 + \tfrac{1}{2}kx^2 + \tfrac{1}{2}kx_{eq} - mgx_{eq}$$

Since the total energy is constant, its time derivative must be zero:

$$\begin{aligned}\tfrac{dE}{dt} &= 0 = mva + \tfrac{I}{R^2}va + kxv \\ &= ma + \tfrac{I}{R^2}a + kx \\ &= (m + \tfrac{I}{R^2})a + kx\end{aligned}$$

$$\underbrace{\left(m + \tfrac{I}{R^2}\right)}_{m_{\text{eff}}} a = -kx$$

This is the equation of motion for a simple harmonic oscillator with an effective mass $m_{\text{eff}} = m + \frac{I}{R^2}$, so the frequency of oscillation of the spring-ball system in the horizontal configuration is

$$\omega_{hor} = \sqrt{\frac{k}{m_{\text{eff}}}} = \sqrt{\frac{k}{m + \frac{I}{R^2}}}$$

We know the spring constant because we measured the period of the spring-ball system in the vertical configuration:

$$\sqrt{\frac{k}{m}} = \omega_{vert} = \frac{2\pi}{T_{vert}} \quad \Rightarrow \quad k = \left(\frac{2\pi}{T_{vert}}\right)^2 m$$

We also know the oscillation frequency ω_{hor} of the spring-ball system in the horizontal configuration:

$$\omega_{hor} = \frac{2\pi}{T_{hor}} = \frac{2\pi}{1.1T_{vert}}$$

Combining these last three equations gives

$$\omega_{hor} = \sqrt{\frac{k}{m_{\text{eff}}}} \quad \Rightarrow \quad I = \left(\frac{k}{\omega_{hor}^2} - m\right)R^2 = (0.50\text{ kg})(1.1^2 - 1)(0.035\text{ m})^2 = 1.3\times 10^{-4}\text{ kg}\cdot\text{m}^2.$$

15.91. We wish to relate periods of oscillation for different systems of different oscillating masses. Let us start by determining the period for a general system of masses m_1 and m_2 connected by a spring with spring constant k. Clearly, since nothing exerts an external force on the system of the two massive objects and spring, the center of mass cannot accelerate, and for simplicity we assume it is stationary.

Note first that the fact that the center of mass does not move implies

$$m_1x_1(t) = -m_2x_2(t) \tag{1}$$

The same magnitude force must be exerted on each object. Let the x be parallel to the spring. Then $F^{\text{spring}}_{\text{spring},1x}(t) = m_1\dfrac{d^2x_1(t)}{dt^2}$ and $F^{\text{spring}}_{\text{spring},2x}(t) = m_2\dfrac{d^2x_2(t)}{dt^2}$. Since each object undergoes simple harmonic motion, we can write $x_1(t) = A_1\sin(\omega t + \phi)$ and $x_2(t) = A_2\sin(\omega t + \phi)$. Note that the angular frequencies must be the same. Otherwise the objects could get out of phase and the center of mass would move. Thus

$$F^{\text{spring}}_{\text{spring},1x}(t) = -m_1\omega^2x_1(t) \tag{2}$$

$$F^{\text{spring}}_{\text{spring},2x}(t) = -m_2\omega^2x_2(t) \tag{3}$$

and of course, the spring force is given by

$$F^{\text{spring}}(t) = k\Delta x_{\text{total}}(t) = k(x_1v - x_2(t)) \tag{4}$$

Setting the magnitudes of the expressions in equations (3) and (4) equal to each other and inserting equation (1) yields

$$m_1\omega^2x_1(t) = kx_1(t)\left(1 + \frac{m_1}{m_2}\right)$$

$$\Rightarrow \omega = \sqrt{\frac{k(m_1 + m_2)}{m_1m_2}} \tag{5}$$

Equivalently,

$$T = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1+m_2)}} \tag{6}$$

We apply equation (6) to the two experiments and find the ratios of the two periods:

$$\frac{T_{\text{you box}}}{T_{\text{box box}}} = \frac{2\pi\sqrt{\dfrac{m_{\text{you}}m_{\text{box}}}{k(m_{\text{you}}+m_{\text{box}})}}}{2\pi\sqrt{\dfrac{m_{\text{box}}^2}{k(2m_{\text{box}})}}} = \sqrt{\frac{2m_{\text{you}}}{(m_{\text{you}}+m_{\text{box}})}}$$

Thus

$$m_{\text{you}} = \frac{\left(\dfrac{T_{\text{you box}}}{T_{\text{box box}}}\right)^2 m_{\text{box}}}{\left(2-\left(\dfrac{T_{\text{you box}}}{T_{\text{box box}}}\right)^2\right)} = \frac{\left(\dfrac{(2.21\text{ s})}{(3.13\text{ s})}\right)^2(215\text{ kg})}{\left(2-\left(\dfrac{(2.21\text{ s})}{(3.13\text{ s})}\right)^2\right)} = 71.4\text{ kg}$$

Thus your mass is 71.4 kg.

16

WAVES IN ONE DIMENSION

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^0 m **2.** 10^2 m/s **3.** 10^2 m/s **4.** 10^8 Hz **5.** 10^{-9} m **6.** 10^{-11} m^{-1} **7.** 10^1 W **8.** 10^1

Guided Problems

16.2 Human wave

1. Getting Started A "human wave" travels around a stadium. When one person rises, the person next to them rises slightly later. When the first person sits, the next person sits slightly later. If they continue standing and sitting, the wave will continue to move around the stadium.

If the crowd is able to maintain a sinusoidal waveform, the vertical height of a person's head in the crowd should be described by

$$D_y = f(x,t) = A\sin(kx - \omega t) \tag{1}$$

We need to estimate A, k, and ω, in order to write the complete wave function.

2. Devise Plan We can confidently estimate the amplitude of the wave. The difference between the height of a standing person and a sitting person will vary from person to person. A reasonable difference is around 0.6 m. That corresponds to an amplitude of $A = 0.3$ m. The angular frequency is related to the period of one oscillation through

$$\omega = \frac{2\pi}{T} \tag{2}$$

The period depends on how quickly a person stands up and sits down. If one is continually standing and sitting, the amount of time require to go from a seated position to standing and back, might be on the order of 2 seconds. Of course, with planning, this time could be tuned to almost anything greater than about 1 second.

It is more difficult to estimate the wavelength. Indeed, this is entirely up to the discretion of the crowd. When the person to my right begins to stand, I could match that person's motion almost exactly, so that there is very little variation between my neighbor's height and my own. In that case, for the wave to drop from a peak to a trough would require many people. But that near-perfect matching of motion is not likely for short periods, as people have a non-zero response time. Human response time is often estimated to be around 0.5 seconds, but since we are anticipating the person's motion (not surprised when our neighbor stands), we might do somewhat better. If we estimate that our lag behind our neighbor will correspond to about 0.25 s, that means we lag behind our neighbor by 1/8 of a period. This means that a full waveform would require 8 people. Stadium seats may have a separation a little smaller than 1.0 m in some cases, but 1 m is the right order of magnitude. This means the wavelength could be around 8 m. We have assumed very small response time, so it is reasonable to guess that the wavelength could be anywhere from 8-20 m depending on the crowd. We will use the correct order of magnitude $(\lambda = 10\ \text{m})$ for our remaining calculations.

Equation (1) describes the vertical motion of people (not the lateral movement of the waveform). So we can obtain an expression for the speed of a person by simply taking a time derivative of equation (1).

3. Execute Plan (*a*) Using equation (2) above, and our estimate of the time required to stand and sit again, we find

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(2\ \text{s})} = 3\ \text{s}^{-1}$$

Anything close to this is acceptable.

We have already estimated the amplitude to be $A = 0.3$ m. Again, any numbers close to this could be valid, as this number would vary from person to person.

We have estimated the wavelength to be $\lambda = 1\times10^1$ m, and this value depends on the reaction time between neighbors.

Finally, the wave speed can be calculated from our other estimates according to

$$c = \lambda f = \frac{\lambda}{T} = \frac{(1\times10^1\ \text{m})}{(2\ \text{s})} = 5\ \text{m/s}$$

This number could be very different, depending on the estimates used. For example, if the crowd reacted very quickly to neighbors such that the wavelength was long, and each (nearly synced) neighbor moved very quickly, the wave speed might be as high as 20 m/s.

Using these estimates, and using $k = 2\pi/\lambda = 2\pi/(1\times10^1\ \text{m}) = 0.6\ \text{m}^{-1}$, the equation describing the wave motion is $y(x,t) = (0.3\ \text{m})\sin((0.6\ \text{m}^{-1})x - (0.3\ \text{s}^{-1})t)$. (*b*) Taking the time derivative of equation (1) yields

$$v_y(x,t) = -A\omega\cos(kx - \omega t) \tag{3}$$

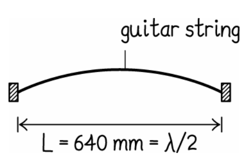
The maximum value that this vertical component of velocity can take is $A\omega$ (which simply comes from maximizing the cosine oscillation to obtain one). And inserting the values from part (*a*) yields

$$v_y(x,t) = A\omega = (0.3\ \text{m})(3\ \text{s}^{-1}) = 0.9\ \text{m/s}$$

4. Evaluate Result All estimates and assumptions are plausible for human beings. Our results are also plausible. A person sitting or standing rapidly could easily move vertically at a rate of 0.9 m/s. The amplitude of the oscillation (and by extension, the maximum vertical speed of a human) does depend on the person's height. But this variation is predictable (given a person's standing height, we could determine a good estimate of the difference between their height standing and sitting). The uncertainty in the wavelength of the wave and the period of the oscillation is much greater, since humans could choose to time their motions to make these values almost anything, above certain minima set by reaction times and power output of muscles.

16.4 Pitch-perfect guitar

1. Getting Started We start by drawing a diagram of a string oscillating in the first harmonic, and fixed at both ends:



We note that the curve above only shows a node, a peak, and a second node. It does not show a trough. A full wavelength must also include a trough and then return back to a node. Thus the length shown is only half of a full wavelength.

It is always the case that $f = c/\lambda$. So the frequency obviously depends on the wavelength, and thus on the length of the guitar string (oscillating portion, not the portion wrapped around a peg). The frequency also depends on the wave speed, which in turn depends on the linear mass density of the string and the tension applied.

2. Devise Plan We know the wave speed is given by $c = \sqrt{\frac{\mathcal{T}}{\mu}}$. This can be set equal to $c = \lambda f$ to yield

$$\lambda f = \sqrt{\frac{\mathcal{T}}{\mu}} \tag{1}$$

As mentioned above, the single antinode means that only half of a full wavelength fits between the fixed ends of the guitar string. Thus $\lambda = 2L$. The frequency is given, and the tension is what we are asked to find. The only other quantity in equation (1) that we need to determine is the linear mass density $\mu = m/L$. We can write this in terms of the density and radius as

$$\begin{aligned} \mu &= \frac{m}{L} = \frac{\rho V}{L} \\ \mu &= \rho \pi r^2 \end{aligned} \tag{2}$$

We can combine $\lambda = 2L$ with equations (1) and (2) to determine the tension.

3. Execute Plan Inserting $\lambda = 2L$ and (2) into equation (1) we find

$$2Lf = \sqrt{\frac{\mathcal{T}}{\rho \pi r^2}}$$

Rearranging, we obtain our expression for the tension:

$$\begin{aligned} \mathcal{T} &= \rho \pi r^2 (2Lf)^2 \\ \mathcal{T} &= (7.8\times 10^3 \text{ kg/m}^3)\pi(1.20\times 10^{-4} \text{ m})^2(2(0.640\text{ m})(330 \text{ s}^{-1}))^2 \\ \mathcal{T} &= 63 \text{ N} \end{aligned}$$

4. Evaluate Result 63 N is equal to the weight of about 6.4 kg on Earth, or 14 lbs. This is a reasonable value for the tension in a guitar string. If the tension were orders of magnitude higher, we would need machines to tighten our guitar strings. If it were orders of magnitude lower, we would not feel the resistance as we tighten guitar strings. So this is the right order of magnitude.

16.6 At the junction

1. Getting Started The boundary between ropes is free to oscillate up and down. We are given the wavefunction for the incident wave:

$$y_{\text{I}}(x,t) = A_{\text{I}} \sin(k_{\text{A}} x - \omega t) \tag{1}$$

The reflected wave is still on rope A which still has the same tension and linear mass density, so the speed of the reflected wave must be the same as the speed of the incident wave. Also, the reflected wave is produced by the upward and downward oscillation of the boundary. This happens at one particular rate, whether I consider it as being part of the incident or reflected wave. So the frequency must be the same in the reflected and incident waves. If the speed and frequency are the same, then the wavelengths (and therefore the wave numbers) are the same. The only things that change are the direction of propagation and the amplitude of the wave. Thus:

$$y_{\text{R}}(x,t) = A_{\text{R}} \sin(k_{\text{A}} x + \omega t) \tag{2}$$

The transmitted wave enters rope B, a medium with a different linear mass density, but the same tension as rope A. Thus the wave speed will change. The same arguments above about the frequency coming from oscillations of the boundary still hold. So the frequency will be the same, but the wavelength (and thus the wave number) will be different. The amplitude may be different than in either the incident or reflected waves. Thus

$$y_\mathrm{T}(x,t) = A_\mathrm{T}\sin(k_\mathrm{B}x - \omega t) \tag{3}$$

2. Devise Plan Since string B is twice as heavy as string A, the wave will move more slowly in string B. The frequencies and tensions are the same in both ropes. We know from Worked Problem 16.5 that

$$\frac{\lambda_\mathrm{A}}{\lambda_\mathrm{B}} = \sqrt{\frac{\mu_\mathrm{B}}{\mu_\mathrm{A}}}$$

Or equivalently

$$\frac{k_\mathrm{B}}{k_\mathrm{A}} = \sqrt{\frac{\mu_\mathrm{B}}{\mu_\mathrm{A}}} \tag{4}$$

And since the frequencies are the same in the two ropes, equation (4) tells us

$$\frac{c_\mathrm{A}}{c_\mathrm{B}} = \frac{\lambda_\mathrm{A} f_\mathrm{A}}{\lambda_\mathrm{B} f_\mathrm{B}} = \sqrt{\frac{\mu_\mathrm{B}}{\mu_\mathrm{A}}} \tag{5}$$

We will require that the wavefunction and its derivative be continuous at the junction, and use the above relationships (4,5) to relate the amplitudes.

3. Execute Plan The continuity of the wavefunction reads:

$$y_\mathrm{I}(x_\mathrm{J},t) + y_\mathrm{R}(x_\mathrm{J},t) = y_\mathrm{T}(x_\mathrm{J},t)$$

$$A_\mathrm{I}\sin(k_\mathrm{A}x_\mathrm{J} - \omega t) + A_\mathrm{R}\sin(k_\mathrm{A}x_\mathrm{J} + \omega t) = A_\mathrm{T}\sin(k_\mathrm{B}x_\mathrm{J} - \omega t)$$

The above equation must hold for all choices of axes, including $x_\mathrm{J} = 0$. We simplify our work by considering that set of axes. This is not physically necessary; it is simply a way of reducing our workload. We obtain

$$A_\mathrm{I}\sin(\omega t) + A_\mathrm{R}\sin(-\omega t) = A_\mathrm{T}\sin(\omega t)$$

$$A_\mathrm{I} - A_\mathrm{R} = A_\mathrm{T} \tag{6}$$

The first (spatial) derivative of the wavefunctions must also be continuous at the junction, yielding

$$\frac{\partial}{\partial x}[y_\mathrm{I}(x_\mathrm{J},t) + y_\mathrm{R}(x_\mathrm{J},t)] = \frac{\partial}{\partial x} y_\mathrm{T}(x_\mathrm{J},t)$$

$$A_\mathrm{I}k_\mathrm{A}\cos(k_\mathrm{A}x_\mathrm{J} - \omega t) + A_\mathrm{R}k_\mathrm{A}\cos(k_\mathrm{A}x_\mathrm{J} + \omega t) = A_\mathrm{T}k_\mathrm{B}\cos(k_\mathrm{B}x_\mathrm{J} - \omega t)$$

Again, we choose $x_\mathrm{J} = 0$. This time, we might also set our clocks to $t = 0$ and find

$$A_\mathrm{I}k_\mathrm{A} + A_\mathrm{R}k_\mathrm{A} = A_\mathrm{T}k_\mathrm{B} \tag{7}$$

We can rearrange equation (6) to write A_R in terms of other variables

$$A_\mathrm{R} = A_\mathrm{I} - A_\mathrm{T} \tag{8}$$

Inserting equation (8) into equation (7), and using equation (4) to determine that $k_\mathrm{B} = \sqrt{2}k_\mathrm{A}$ we have

$$\frac{A_\mathrm{T}}{A_\mathrm{I}} = \frac{2k_\mathrm{A}}{(k_\mathrm{A} + k_\mathrm{B})}$$

$$\frac{A_\mathrm{T}}{A_\mathrm{I}} = \frac{2}{(1+\sqrt{2})} \approx 0.828 \tag{9}$$

Plugging this back into equation (8), we find

$$\frac{A_\mathrm{R}}{A_\mathrm{I}} = 1 - \frac{2}{(1+\sqrt{2})}$$

$$\frac{A_\mathrm{R}}{A_\mathrm{I}} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \approx 0.172 \tag{10}$$

Now that we know the fractions of the initial amplitude that are reflected and transmitted, we can determine what fraction of the power is reflected and what fraction of the power is transmitted.

$$\frac{P_{\text{av,R}}}{P_{\text{av,I}}} = \frac{\frac{1}{2}\mu_{\text{A}} A_{\text{R}}^2 \omega^2 c_{\text{A}}}{\frac{1}{2}\mu_{\text{A}} A_{\text{I}}^2 \omega^2 c_{\text{A}}} = \left(\frac{A_{\text{R}}}{A_{\text{I}}}\right)^2 = \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)^2 = 0.0294$$

$$\frac{P_{\text{av,T}}}{P_{\text{av,I}}} = \frac{\frac{1}{2}\mu_{\text{B}} A_{\text{T}}^2 \omega^2 c_{\text{B}}}{\frac{1}{2}\mu_{\text{A}} A_{\text{I}}^2 \omega^2 c_{\text{A}}} = \frac{\mu_{\text{B}} A_{\text{T}}^2 c_{\text{B}}}{\mu_{\text{A}} A_{\text{I}}^2 c_{\text{A}}} = \sqrt{\frac{\mu_{\text{B}}}{\mu_{\text{A}}}}\left(\frac{A_{\text{T}}}{A_{\text{I}}}\right)^2 = \sqrt{2}\left(\frac{2}{(1+\sqrt{2})}\right)^2 = 0.971$$

Thus, 97.1% of the power is transmitted and 2.9% is reflected.

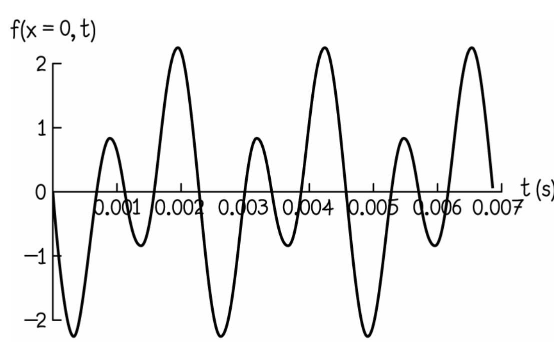
4. Evaluate Result The fractions of power transmitted and reflected do sum to one. This is what we expect from conservation of energy.

16.8 Superposed

1. Getting Started We are explicitly given the wave number and the amplitudes. We are told that the entire pattern repeats at a rate of 440 Hz. But we must consider how to write this in terms of the angular frequency $\omega_{\text{superpostion}} = 2\pi f$. Also, since the two sine waves have different frequencies (ω and 2ω), we need to determine how these are related to $\omega_{\text{superposition}}$ the angular frequency of the superposition of both sine waves. The term containing 2ω oscillates faster than the term containing ω. So if the second term (containing 2ω) undergoes one full oscillation, we would still have to wait for the first term to finish its oscillation. Thus it is the first term with angular frequency ω that determines the angular frequency of the overall wave. Thus we can set $\omega_{\text{superposition}} = \omega = 2\pi f = 2\pi(440\text{ Hz}) = 2.76\times 10^3\text{ s}^{-1}$. The period of one full oscillation is $T = \frac{1}{f} = \frac{1}{440\text{ Hz}} =$ 2.27 ms. This is the same as the time interval required for the first term to repeat itself, but the second term repeats twice as fast and would require only 1.14 ms for a full oscillation.

2. Devise Plan (*a*) We want to be able to clearly resolve all features of the oscillations of both terms. The faster of the two takes only 1.14 ms to repeat. If we want to see peaks, troughs, and nodes in the oscillation, we should calculate the value of the function on time intervals significantly less than 1.14 ms. Using a time interval of $\Delta t = 1.0\times 10^{-4}\text{ s}$ is probably sufficient, but one can use as small an interval as one likes. This choice of time interval means we would need about 60 data points to plot the whole range. Thus, we will not reproduce a data table here. (*b*) To show that this function satisfies the wave equation, we simply take two time derivatives and two spatial derivatives and see if they can be related by a simple multiplicative factor. If so, then we know the wave speed.

3. Execute Plan (*a*) The function is plotted below:



Here we have used a high number of data points, so that the curve appears smooth. One could still obtain a rougher figure showing all key peaks and troughs by using as few as 60 data points for the full three periods. (*b*) We start from the symbolic expression for the wavefunction. We introduce symbols for the amplitude of each term: $A_1 = 1.0$ and $A_2 = 1.5$, such that

$$f(x,t) = A_1 \sin(kx - \omega t) + A_2 \sin(2kx - 2\omega t) \tag{1}$$

We first take two spatial derivatives of equation (1) to obtain

$$\begin{aligned} \frac{\partial^2}{\partial x^2} f(x,t) &= \frac{\partial}{\partial x} A_1 k \cos(kx - \omega t) + 2A_2 k \cos(2kx - 2\omega t) \\ \frac{\partial^2}{\partial x^2} f(x,t) &= -A_1 k^2 \sin(kx - \omega t) - 4A_2 k^2 \sin(2kx - 2\omega t) \end{aligned} \tag{2}$$

Then we take two time derivatives:

$$\begin{aligned} \frac{\partial^2}{\partial t^2} f(x,t) &= -\frac{\partial}{\partial t} A_1 \omega \cos(kx - \omega t) - 2A_2 \omega \cos(2kx - 2\omega t) \\ \frac{\partial^2}{\partial t^2} f(x,t) &= -A_1 \omega^2 \sin(kx - \omega t) - 4A_2 \omega^2 \sin(2kx - 2\omega t) \end{aligned} \tag{3}$$

If we plug these derivatives into the wave equation, we can obtain

$$-A_1 k^2 \sin(kx - \omega t) - 4A_2 k^2 \sin(2kx - 2\omega t) = \frac{1}{c^2}(-A_1 \omega^2 \sin(kx - \omega t) - 4A_2 \omega^2 \sin(2kx - 2\omega t))$$

This equality clearly holds if

$$-A_1 k^2 \sin(kx - \omega t) = -\frac{A_1 \omega^2}{c^2} \sin(kx - \omega t) \Rightarrow c = \frac{\omega}{k}$$

and

$$-4A_2 k^2 \sin(2kx - 2\omega t) = \frac{1}{c^2}(-4A_2 \omega^2 \sin(2kx - 2\omega t)) \Rightarrow c = \frac{\omega}{k}$$

So, the wave has one consistent speed, even though it is made up of two oscillatory terms, and it is a solution to the wave equation with $c = \frac{\omega}{k}$. (*c*) Inserting the numerical values of ω and k, we find

$$c = \frac{\omega}{k} = \frac{(2.76 \times 10^3 \text{ s}^{-1})}{(8.00 \text{ m}^{-1})} = 346 \text{ m/s}$$

4. Evaluate Result We know that a superposition of waves is still a wave. So we expect that the sum of two wavefunctions that obey the wave equation should still obey the wave equation. We found that this is the case, so our result is consistent with our expectation.

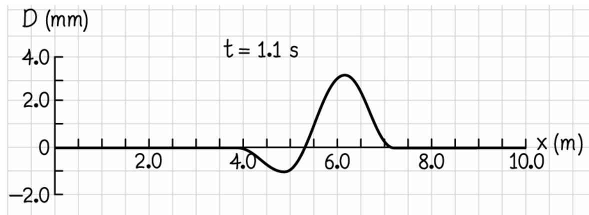
We can check the numerical value of the wave speed we found, using the more basic equation $c = \lambda f$. Since $k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{(8.00 \text{ m}^{-1})} = 0.785 \text{ m}$, we can write $c = \lambda f = (0.785 \text{ m})(440 \text{ Hz}) = 346 \text{ m/s}$, validating our answer to part (*c*).

Questions and Problems

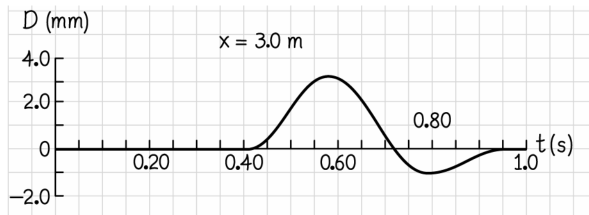
16.1. Yes, water can carry both transverse and longitudinal waves.

16.2. Motion in the *z* direction would correspond to a longitudinal wave, and motion in the *x* or *y* directions would correspond to transverse waves.

16.3. (*a*)

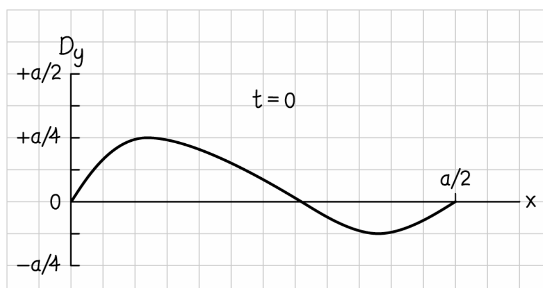


(*b*)



16.4. The piece of string moves from the position $y = 0$ to $y = a$ in the time interval $t_2 - t_1$. In the same time interval, the peak of the wave moves from $x = a/2$ to $x = a$. Thus $v_{\text{av}} = \dfrac{a}{t_2 - t_1}$, and $c = \dfrac{a}{2(t_2 - t_1)}$. So $v_{\text{av}}/c = 2$.

16.5.



16.6. Rope B has greater tension. We know this because the wave on rope B has already reached the tree, meaning it is moving faster than the wave on rope A. Man B shakes with a higher frequency than man A. We know this because man B has already made 7 peaks in the waveform on his rope at the time the image in Figure P16.6 is captured. Man A has made only 6 peaks.

16.7. The definition is the same as for a harmonic periodic wave: the number of cycles per second executed by each medium particle.

16.8. (*a*) Yes. This would be the case if her segment of rope were heavier than your segment. (*b*) Yes. This would be the case if her segment of rope and yours had the same linear mass density. (*c*) Yes. This would be the case if her segment of rope were lighter than your segment.

16.9. The first crate must have struck when the cable was under minimal tension, resulting in a low wave speed; the second crate must have struck when the cable was under significant tension, resulting in a high wave speed. The tension change is probably the result of ship's drifting away from dock in the time interval between the two incidents.

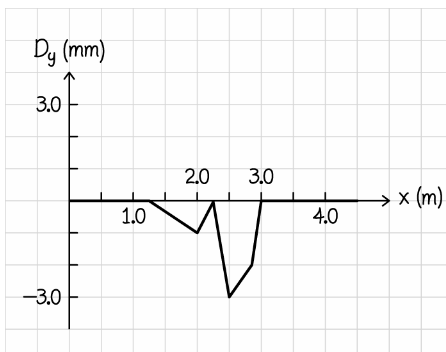
16.10. (*a*) Your hand is the source of the wave, so if your hand completes an oscillation in half the time, the wave completes an oscillation in half the time and the period is halved. (*b*) The wave speed has not changed, because nothing has changed about the medium through which the wave propagates. The only way for a wave to move at the same speed, even though twice as many oscillations pass a point each second, is if the length of each oscillation is also halved. Thus the wavelength is halved. (*c*) There is no change in the amplitude. This is determined by the amplitude of your hand's motion. (*d*) There is no change in the speed. Nothing has changed about the medium through which the wave propagates. (*e*) The transverse speed doubles. In order for a piece of medium to move from equilibrium to maximum displacement in half the time, it must move at twice the speed on average.

16.11. Waves generated by the machine propagate out into the pool and reflect off of the walls. This allows waves to interfere with each other. At some positions, the superposition of the waves may produce wild, large-amplitude oscillations. In other positions, the waves may tend to cancel each other, producing very little oscillation.

16.12. We know that the total energy of each wave is given by $E = P\Delta t = (10\ \text{W})(1.5\ \text{s}) = 15\ \text{J}$. We also know that a wave carries equal amounts of kinetic and potential energy. So half of this total energy is kinetic and $K = 7.5\ \text{J}$.

16.13. (*a*) The wave initially on the left will move away from the origin and will not perturb the rope at the position $x = 0$ at any time $t > 0$. The only displacement will come from the wave initially on the right. So the largest displacement from equilibrium at $x = 0$ will be 0.10 m. (*b*) The wave initially on the right will move away from the point $x = 0.90$ m and will not perturb the rope at that position at any time $t > 0$. The only displacement will come from the wave initially on the left. So the largest displacement from equilibrium at $x = 0.90$ m will be 0.20 m. (*c*) As the waves pass each other, the displacements of the string due to each wave will add, and the maximum displacement of the rope from equilibrium will be 0.30 m. This will happen at the point $x = 0.50$ m.

16.14.



16.15. The energy in wave 1 can be written $E_1 = U_1 + K_1 = 2U_1$. Similarly, $E_2 = U_2 + K_2 = 2U_2$. This means the total energy in both waves together is $E = E_1 + E_2 = 4U_1 = 4U_2$. As the two waves overlap, the displacement of the medium from zero will double (the displacements due to the two waves will add). Since the potential energy depends on the amplitude squared, doubling the amplitude quadruples the potential energy. Thus the potential energy at this moment is $U_{\text{superposition}} = 4U_1 = 4U_2$. Thus all energy is in potential energy at this instant. This means the kinetic energy at this instant is zero.

16.16. The wavelength changes, but the frequency does not.

16.17. The reflected wave is not inverted. A mechanical reflected wave is only inverted when the medium is not free to move at the boundary.

16.18. (*a*) No. Here the reflected wave is inverted correctly, but the pulse with the larger amplitude should be leading in the reflected pulse. (*b*) No. The reflected wave should be inverted, and the pulse with the larger amplitude should be leading in the reflected pulse. (*c*) Yes. (*d*) No. The reflected wave should be inverted.

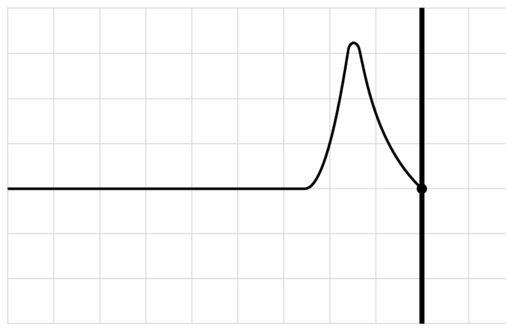
16.19. (*a*) The thinner string carried the initial pulse. When a pulse moves from a lighter medium to a heavier/denser medium, the reflected wave will be inverted. This ensures that the reflected and transmitted waves will be on opposite sides of the string as is the case in Figure P16.19. If the wave had originated on the thicker string, the reflected and transmitted wave would both be on the same side of the string (either both above or both below).

(*b*)

16.20. (*a*)

The transmitted wave remains upright, and the wavelength must be reduced by a factor of two, since the speed is reduced by a factor of two. The reflected wave is inverted and has its original wavelength. The amplitude of each wave must be smaller than the original amplitude by energy conservation. (*b*) The periods are the same.

16.21.

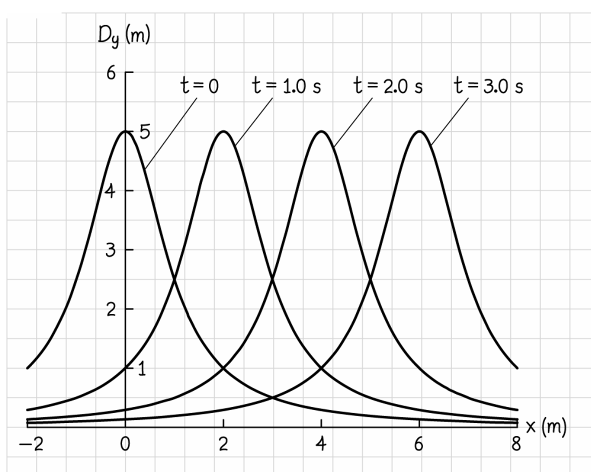


16.22. $c = \lambda f = \dfrac{\lambda}{T} = \dfrac{(2.5\ \text{m})}{(4.0\ \text{s})} = 0.63\ \text{m/s}.$

16.23. $c = \lambda f \Rightarrow \lambda = \dfrac{c}{f} = \dfrac{(6.0\ \text{m/s})}{(12\ \text{waves/min})} \times \left(\dfrac{60\ \text{s}}{1\ \text{min}}\right) = 30\ \text{m}.$

16.24. (*a*) We know $f = \dfrac{1}{T} = \dfrac{1}{(15\ \text{s})} = 0.067\ \text{s}^{-1}$. (*b*) The wave has to make it entirely around the stadium in the 15 seconds between standing up. Thus $c = \dfrac{\Delta x}{T} = \dfrac{(560\ \text{m})}{(15\ \text{s})} = 37\ \text{m/s}.$

16.25. (*a*)

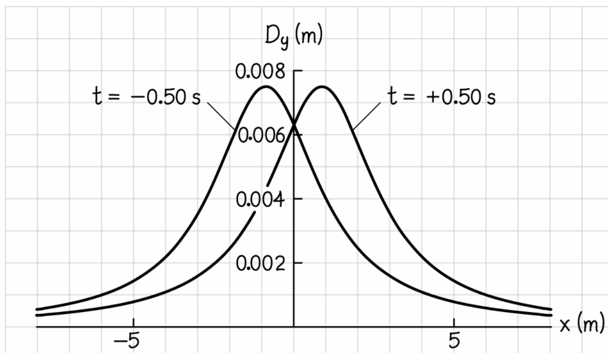


(*b*) 2.0 m/s

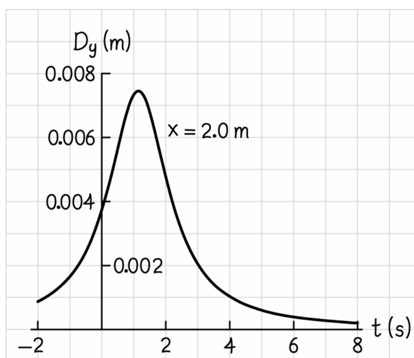
16.26. (*a*) The function has no well-defined first derivative at the discontinuity. (*b*) This is not a function. There are multiple y values that correspond to a single t value. (*c*) The first derivative of the curve is discontinuous at the point, and this is not a function, for the same reason as in (*b*).

16.27. (*a*) As time passes, you wish the wavefunction to move in the positive x direction. In other words, as time increases you want a given feature of your wavefunction to occur at a larger (or more positive) value of x. This is accomplished by shifting the argument from x to $x-ct$. Thus $f(x)=\dfrac{a}{b^2+(x-ct)^2}$ where $c=1.75$ m/s.

(*b*)



(*c*)



16.28. (*a*) The amplitude is just the prefactor in front of the sine function, and is given explicitly as $A=0.030$ m. The wavelength is the distance after which the wavefunction repeats itself, meaning $\pi b\lambda=2\pi\Rightarrow$ $\lambda=\dfrac{2}{b}=\dfrac{2}{(0.33\ \text{m}^{-1})}=6.1$ m. Similarly, the period is the time after which the wavefunction repeats itself, meaning $qT=2\pi\Rightarrow T=\dfrac{2\pi}{q}=\dfrac{2\pi}{(10.47\ \text{s}^{-1})}=0.60$ s. Finally, the speed of the wave is given by $c=\dfrac{\lambda}{T}=\dfrac{(6.06\ \text{m})}{(0.600\ \text{s})}=10$ m/s.

(*b*) $f(x,t)=a\sin(\pi bx+qt)=(0.030\ \text{m})\sin(\pi(0.33\ \text{m}^{-1})(0.50\ \text{m})+(10.47\ \text{s}^{-1})(1.60\ \text{s}))=-0.030$ m.

16.29. (*a*) The wavelength is the distance after which the wave has completed a full oscillation, thus $b\lambda=2\pi\Rightarrow\lambda=\dfrac{2\pi}{b}=\dfrac{2\pi}{(2.25\ \text{m}^{-1})}=2.79$ m. (*b*) We know $c=\lambda f\Rightarrow f=\dfrac{c}{\lambda}=\dfrac{(17.0\ \text{m/s})}{(2.793\ \text{m})}=6.09$ Hz. (*c*) The angular frequency is related to the frequency by $\omega=2\pi f=2\pi(6.088\ \text{Hz})=38.3\ \text{s}^{-1}$.

16.30. The wave speed is given by $c=\dfrac{\lambda}{T}.$ We are explicitly told the period, and the wavelength can be determined by $\lambda=\dfrac{2\pi}{b}.$ Thus $c=\dfrac{2\pi}{Tb}.$ The wave must cross a distance $d=1.50\times10^5\text{ m}$ before it reaches you, so $\Delta t=\dfrac{d}{c}=\dfrac{dTb}{2\pi}=\dfrac{(1.50\times10^5\text{ m})(0.50\text{ s})(0.0157\text{ m}^{-1})}{2\pi}=187\text{ s}.$

16.31. As time passes, the argument inside the sine function becomes larger. Any given feature of the wave (say the peak) exists for a specific argument of the sine function. If the passage of time has increased the argument, the spatial part of the argument must decrease in order to find the same wave feature. Thus, as time passes, the peak (or any other feature) will be found at a more negative (or less positive) position. Thus the wave moves in the $-x$ direction. The wave speed is given by $c=\lambda f=\left(\dfrac{2\pi}{b}\right)\left(\dfrac{q}{2\pi}\right)=\dfrac{q}{b}=\dfrac{(12.0\text{ s}^{-1})}{(\pi\times10^{-2}\text{ m}^{-1})}=382\text{ m/s}.$ Thus the wave moves at 382 m/s in the $-x$ direction.

16.32. (*a*) We require $f(0,0)=a\sin(\phi_i)=0\Rightarrow\phi_i=0$ or $\pi.$ (*b*) As time T elapses, the argument of the sine function changes by $-bcT.$ For a full period, this must correspond to a change in the argument of magnitude $2\pi.$ So we set $T=\dfrac{2\pi}{bc}=\dfrac{2\pi}{(33.05\text{ m}^{-1})(245\text{ m/s})}=7.76\times10^{-4}\text{ s}.$

16.33. (*a*) The wavelength is the distance after which the wave has gone through a full oscillation at fixed time, meaning $\lambda=\dfrac{2\pi}{b}=\dfrac{2\pi}{(4\pi\text{ m}^{-1})}=0.50\text{ m}.$ (*b*) The angular frequency is the coefficient of time inside the sine function, meaning $\omega=bc=(4\pi\text{ m}^{-1})(45\text{ m/s})=5.7\times10^2\text{ s}^{-1}.$ (*c*) The frequency is related to the angular frequency by $f=\dfrac{\omega}{2\pi}=\dfrac{(565\text{ s}^{-1})}{2\pi}=90\text{ Hz}.$ (*d*) The period is $T=\dfrac{1}{f}=\dfrac{1}{(90\text{ Hz})}=0.011\text{ s}.$

16.34. (*a*) We assume this wave is a sinusoidal wave (or equivalently, one arising from harmonic motion). This assumption is not necessary for calculating the wavelength, but it certainly is necessary for the functional form of the wave. The wavelength is given by $\lambda=\dfrac{c}{f}=\dfrac{(17.5\text{ m/s})}{(80.0\text{ Hz})}=0.219\text{ m}.$ Note that this makes $b=\dfrac{2\pi}{\lambda}=\dfrac{2\pi}{(0.2188\text{ m})}=28.7\text{ m}^{-1}.$ Thus the time-independent wavefunction is $f(x)=a\sin(bx)$ where $a=0.250\text{ m}$ and $b=28.7\text{ m}^{-1}.$ (*b*) Continuing with our assumption that the wave is sinusoidal, we know the displacement as a function of position and time will be $f(x,t)=a\sin(bx-qt),$ where $b=\dfrac{2\pi f}{c}$ and $q=cb=2\pi f.$ Also, the velocity in the transverse direction, is just the first time derivative of this displacement: $v_{\text{trans}}(x,t)=\dfrac{d}{dt}\left[f(x,t)\right]=-qa\cos(bx-qt).$ Finally, we can calculate

$$f(x=1.25\text{ m},t=3.00\text{ s})=(0.250\text{ m})\sin\left(\frac{2\pi(80.0\text{ Hz})}{(17.5\text{ m/s})}(1.25\text{ m})-2\pi(80.0\text{ Hz})(3.00\text{ s})\right)=-.244\text{ m}$$

$$v_{\text{trans}}(x,t)=-2\pi(80.0\text{ Hz})(0.250\text{ m})\cos\left(\frac{2\pi(80.0\text{ Hz})}{(17.5\text{ m/s})}(1.25\text{ m})-2\pi(80.0\text{ Hz})(3.00\text{ s})\right)=28.0\text{ m/s}$$

(*c*) Let us no longer assume that the initial phase is zero, to obtain the most general answer possible. Using the values determined in previous parts, we have $f(x,t)=a\sin\left(b(x-ct)+\phi_i\right)$ where $a=0.250\text{ m},$ $b=28.7\text{ m}^{-1},$ and $c=17.5\text{ m/s}.$ Once can also write this as $f(x,t)=a\sin(bx-qt+\phi_i)$ with $q=503\text{ s}^{-1}.$ (*d*) In this part, we will keep an unusual number of significant digits, because we initially obtain a phase that corresponds to many full oscillations. We require $f(x,t)=a\sin(b(x-ct)+\phi_1)=0.210\text{ m},$ such that $\phi_1=\sin^{-1}\left(\frac{0.210\text{ m}}{a}\right)-b(x-ct)=$ $\sin^{-1}\left(\frac{0.210\text{ m}}{0.250\text{ m}}\right)-\frac{2\pi(80\text{ Hz})}{(17.5\text{ m/s})}((1.25\text{ m})-(17.5\text{ m/s})(3.00\text{ s}))=-1471.063274.$ Clearly, this is not the smallest possible initial phase. To find the smallest possible phase, we determine how many full oscillations are in our answer above: $\frac{-1471.063274}{2\pi}=-234.12699,$ meaning 234 full oscillations. Thus, this initial phase is equivalent to an initial phase of -0.12699 oscillations, or -0.798 rad. This is equivalent to 5.49 rad.

16.35. Let $x=0$ correspond to the edge of the tank. There will be a node whenever $\sin(bx)=0\Rightarrow x=\frac{n\pi}{b}$ where $n=0,1,2....$ Thus there are nodes at $x_0=\frac{(0)\pi}{(19.6\text{ m}^{-1})}=0.00,$ $x_1=\frac{(1)\pi}{(19.6\text{ m}^{-1})}=0.160\text{ m},$ and $x_1=\frac{(2)\pi}{(19.6\text{ m}^{-1})}=0.321\text{ m}.$

16.36. We use the general functional form: $f(x,t)=a\sin(b(x-ct)).$ The amplitude is given as $a=0.50$ m. The wave number b is given by $b=\frac{2\pi}{\lambda}=\frac{2\pi}{(1.33\text{ m})}=4.72\text{ m}^{-1}$ and c is the wave speed.

16.37. Note that the material and tension are fixed, meaning that the wave speed is fixed. We can write the initial wavelength as $\lambda_i=\frac{c}{f},$ and we note that $\lambda_i=\frac{4}{5}\ell,$ where ℓ is the entire length of rope shown. Let us rearrange these equations to write:

$$c=\lambda_i f=\frac{4}{5}\ell f \tag{1}$$

Apparently, the type of standing wave being created can accommodate an antinode at one end and anode at the other end. Thus the longest possible wavelength would correspond to one in which just a quarter of a full oscillation (one node to the nearest antinode) occupies the entire length of string: $\lambda_{\text{max}}=4\ell.$ This the longest wavelength would correspond to the smallest frequency. This standing wave would still have the same speed so we can write

$$c=\lambda_{\text{max}}f_{\text{min}}=4\ell f_{\text{min}} \tag{2}$$

Comparing equations (1) and (2) yields $f_{\text{min}}=f/5.$

16.38. The frequency of the wave will be the same in either rope segment. Note that $\sqrt{\frac{T}{\mu_1}}=c_1=\lambda_1 f,$ and since there is a standing wave, we know that the wavelength must obey $\frac{n_1\lambda_1}{2}=\ell,$ where ℓ is the length of either rope segment, and n_1 is the number of half-wavelengths on the segment. Rearranging, we can write $\mu_1=\frac{T}{(\lambda_1 f)^2}=\frac{Tn_1^2}{4\ell^2 f^2}.$

Exactly the same logic applies to segment 2: $\mu_1=\frac{Tn_2^2}{4\ell^2 f^2}.$ It is now clear, that the segment with the greater linear mass density must also have the greater number of nodes, and therefore the greater value of $n.$ Since $\mu_2>\mu_1,$ we know $n_2=4$ and $n_1=3.$ Thus we can finally write

$$\frac{\mu_1}{\mu_2}=\frac{\frac{Tn_1^2}{4\ell^2 f^2}}{\frac{Tn_2^2}{4\ell^2 f^2}}=\frac{n_1^2}{n_2^2}=\left(\frac{3}{4}\right)^2=\frac{9}{16}.$$

16.39. (*a*) A guitar string has two fixed ends, such that there must always be nodes at the ends. This corresponds to allowed wavelengths that obey $\frac{n\lambda}{2}=\ell,$ meaning that the longest possible wavelength (which corresponds to the smallest possible frequency) is $\lambda_{\text{max}}=2\ell.$ We also know that $c=\lambda f,$ such that $f_{\text{min}}=\frac{c}{\lambda_{\text{max}}}=\frac{c}{2\ell}=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}=\frac{1}{2}\sqrt{\frac{T}{\ell m}}=\frac{1}{2}\sqrt{\frac{(126\text{ N})}{(0.650\text{ m})(4.00\times10^{-3}\text{ kg})}}=110\text{ Hz}.$ While it is appropriate to refer to this smallest allowed frequency as $f_{\text{min}},$ it is most often denoted $f_1=110\text{ Hz},$ because it is the first harmonic. (*b*) The string could vibrate at any positive integer multiple of this fundamental frequency $nf_1.$

16.40. (*a*) The net displacement will be the superposition of the two waves:

$$f(x,t)=f_1(x,t)+f_2(x,t)=a\left(\sin(bx-qt)+\sin\left(bx+qt+\frac{\pi}{3}\right)\right)$$

Using the trigonometric identity $\sin(\alpha)+\sin(\beta)=2\sin\left(\frac{1}{2}(\alpha+\beta)\right)\cos\left(\frac{1}{2}(\alpha-\beta)\right),$ we can write this as:

$$f(x,t)=2a\left(\sin\left(bx+\frac{\pi}{6}\right)\cos\left(qt+\frac{\pi}{6}\right)\right)$$

We are asked for the amplitude at a specific time, which is equivalent to the prefactor in front of the spatial sinusoidal function, or $2a\cos\left(qt+\frac{\pi}{3}\right)=2(3.00\times10^{-2}\text{ m})\cos\left((500\text{ s}^{-1})(3.00\text{ s})+\frac{\pi}{6}\right)=1.20\times10^{-2}\text{ m}.$ The wavelength is the distance after which the wave has gone through a full oscillation, meaning $b\lambda=2\pi\Rightarrow\lambda=\frac{2\pi}{b}=\frac{2\pi}{(4\pi\text{ m}^{-1})}=0.500\text{ m}.$ (*b*) We use $f(x,t)=2a\left(\sin\left(bx+\frac{\pi}{6}\right)\cos\left(qt+\frac{\pi}{6}\right)\right)$ to calculate $f(x=2.00\text{ m},t=1.70\text{ s})=2(3.00\times10^{-2}\text{ m})\left(\sin\left((4\pi\text{ m}^{-1})(2.00\text{ m})+\frac{\pi}{6}\right)\cos\left((500\text{ s}^{-1})(1.70\text{ s})+\frac{\pi}{6}\right)\right)=-1.98\times10^{-2}\text{ m}.$

16.41. The amplitude of a standing wave is twice the amplitude of either constituent travelling wave. Thus $a=0.030\text{ m}.$ The speed of either travelling wave is given by $c=\lambda f=\left(\frac{2\pi}{b}\right)\left(\frac{q}{2\pi}\right)=\frac{q}{b}=\frac{\left(40\pi\text{ s}^{-1}\right)}{\left(\frac{\pi}{3}\times10^{-2}\text{ m}^{-1}\right)}=1.2\text{ m/s}.$

Clearly, in order for the travelling waves to form a standing wave, they must be moving in opposite directions.

16.42. In order for the amplitude to be constant, the waves must have the same frequency. Because the resulting wave is to have a fixed wavelength (the same wavelength as the initial travelling wave) the wavelength of the second wave must be the same as the wavelength of the first. These facts mean $b=5\pi\text{ m}^{-1}$ and $q=314\text{ s}^{-1},$ identical to the first wave. However, we wish to reduce the amplitude by 0.020 m. This means the second wave must have an amplitude of 0.020 m and must be out of phase with the first wave by $\pi.$ Thus for the second wave $f(x,t)=a\sin(bx-ct+\phi_1)$ where $a=0.020\text{ m},$ $b=5\pi\text{ m}^{-1},$ $q=314\text{ s}^{-1},$ and $\phi_1=\pi.$

16.43. (*a*) The distance between nodes is only half a wavelength. Thus the wavelength is 0.60 m. (*b*) The leaf is lifted up on the surface of the water, and its displacement is therefore equal to the displacement of the wave at that point. Thus the displacement is 10 mm in the $+y$ (or vertical) direction. (*c*) We start with the most general form of a standing wave, allowing for an initial phase: $f(x,t)=a(\sin(bx+\phi_1)\cos(qt+\phi_2)).$ We know that the position of the leaf is one wavelength from the edge of the trough, so we know that the edge of the trough does not correspond to a node. Upon further consideration, we see that there is nothing to constrain the water from moving at the edges, and we surmise that a standing water wave in such a trough must have antinodes at the edges. This tells us two things: first, the amplitude is the same as the displacement of the leaf $a = 10$ mm, and secondly there must be an initial phase of $\phi_1 = \dfrac{\pi}{2}$ in order for the sine function to be maximal at $x = 0$. We can use the wavelength to determine $b = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{(0.60\text{ m})} = 10\text{ m}^{-1}$, and we use the period to determine $q = \dfrac{2\pi}{T} = \dfrac{2\pi}{(0.60\text{ s})} = 10\text{ s}^{-1}$. Finally, we know that the leaf has to attain its highest point at the time $t_{\text{peak}} = 3.2$ s, such that $\cos(qt_{\text{peak}} + \phi_2) = 1$, or $\phi_2 = -qt_{\text{peak}} = -(10.47\text{ s}^{-1})(3.2\text{ s}) = 33.51$ rad, which is clearly equivalent to $\phi_2 = \dfrac{2\pi}{3}$. Collecting these results, we have $f(x,t) = a(\sin(bx + \phi_1)\cos(qt + \phi_2))$ where $a = 10$ mm, $b = 10\text{ m}^{-1}$, $q = 10\text{ s}^{-1}$, $\phi_1 = \dfrac{\pi}{2}$, and $\phi_2 = \dfrac{2\pi}{3}$.

16.44. (*a*) The phase difference is just the difference between the arguments of the sine functions: $\left(bx - qt - \dfrac{\pi}{4}\right) - \left(bx - qt - \dfrac{\pi}{3}\right) = \dfrac{\pi}{12}$. (*b*) We simply add the two constituent waves using the trigonometric identity $\sin(\alpha) + \sin(\beta) = 2\sin\left(\dfrac{\alpha+\beta}{2}\right)\cos\left(\dfrac{\alpha-\beta}{2}\right)$:

$$f(x,t) = f_1(x,t) + f_2(x,t)$$
$$f(x,t) = a\left[\sin\left(bx - qt - \frac{1}{4}\pi\right) + \sin\left(bx - qt - \frac{1}{3}\pi\right)\right]$$
$$f(x,t) = 2a\sin\left(bx - qt - \frac{7\pi}{24}\right)\cos\left(\frac{\pi}{24}\right)$$

The amplitude of the resultant wave is the entire prefactor of the sine function: $2a\cos\left(\dfrac{\pi}{24}\right) = 2(5.00\times10^{-2}\text{ m})\cos\left(\dfrac{\pi}{24}\right) = 9.91\times10^{-2}$ m. The phase shift is now $\phi_1 = 7\pi/24$. Other quantities have not changed. So this wave function can be written as $f(x,t) = a'\sin\left(bx - qt - \phi_1\right)$ where $a' = 9.91\times10^{-2}$ m, $b = 0.120\text{ m}^{-1}$, $c = 180\text{ s}^{-1}$, and $\phi_1 = 7\pi/24$. (*c*) Plugging in the given values, we have $f(x,t) = a'\sin(bx - qt - \phi_1) = (9.91\times10^{-2}\text{ m})\sin\left((0.120\text{ m}^{-1})(2.00\text{ m}) - (180\text{ s}^{-1})(1.70\text{ s}) - \dfrac{7\pi}{24}\right) = 9.24\times10^{-2}$ m in the y direction.

16.45. These waves are moving past each other, travelling in opposite directions. Regardless of an initial phase difference, sometimes their peaks will line up with each other, and other times the peak of one wave will line up with the trough of another wave. The maximum displacement of the wave created by the superposition is the same as the amplitude of the standing wave that is formed. This maximum displacement is obviously the displacement caused when the peaks of one wave line up with the peaks of the other wave, causing an amplitude that is twice the amplitude of either constituent travelling wave. Thus the maximum displacement of the resultant superposition is 0.0578 m.

16.46. From equation (16.30) we know $c = \sqrt{\dfrac{\mathcal{T}}{\mu}} = \sqrt{\dfrac{(1.35\times10^{3}\text{ N})}{(5.00\times10^{-3}\text{ kg/m})}} = 520$ m/s.

16.47. Rearranging equation (16.30) we find $\mu = \dfrac{\mathcal{T}}{c^2}$ or $m = \mu\ell = \dfrac{\mathcal{T}\ell}{c^2} = \dfrac{(100\text{ N})(10\text{ m})}{(15\text{ m/s})^2} = 4.4$ kg.

16.48. If we assume that the storage unit is not accelerating, but is being lowered at a constant speed, then the tension in the cable is equal to the weight of the storage unit. In that case, equation (16.30) tells us $c = \sqrt{\frac{\mathcal{T}}{\mu}} = \sqrt{\frac{mg}{\mu}}$. This means that the time required for a wave to traverse the cable is $\Delta t = \frac{\ell}{c} = \ell\sqrt{\frac{\mu}{mg}} = (12\text{ m})\sqrt{\frac{(1.78\text{ kg/m})}{(10{,}000\text{ kg})(9.8\text{ m/s}^2)}} = 0.051\text{ s}.$

16.49. The violinist could shorten the length of the string that is free to oscillate (by pinching off part of the string). The standing wave on the string has a maximum wavelength equal to the length of the string itself, and it is this wavelength (corresponding to the fundamental frequency) that is most prominent. Decreasing the string length decreases the wavelength of sound and raises the frequency. Alternatively the violinist could use a string that has a lower linear mass density. Equation (16.30) tells us $c = \sqrt{\frac{\mathcal{T}}{\mu}}$ meaning $f = \frac{1}{\lambda}\sqrt{\frac{\mathcal{T}}{\mu}}$, such that the frequency will increase if the linear mass density of the string decrease. By looking at the same equation, we can see that the violinist could also increase the tension in the strings to raise the frequency of the sound.

16.50. The force exerted on the back of the propeller would be carried through the body of the ship at an extremely high rate. The motion would appear instantaneous to an observer, but in reality it does take a nonzero time interval for the force (and compression of atoms/molecules in the ship) to travel from the rear of the ship to the front of the ship. If you use the speed of sound in steel $(\text{about } 4.5\times10^3\text{ m/s})$ and a ship length of $4\times10^2\text{ m}$, then it would take about 0.1 s for the front of the ship to start moving.

16.51. Using equation (16.30) we can write $c_\text{i} = \sqrt{\frac{\mathcal{T}_\text{i}}{\mu}}$ and $c_\text{f} = \sqrt{\frac{\mathcal{T}_\text{f}}{\mu}}$, such that $c_\text{f} = c_\text{i}\sqrt{\frac{\mathcal{T}_\text{f}}{\mathcal{T}_\text{i}}} = (24\text{ m/s})\sqrt{\frac{(100\text{ N})}{(120\text{ N})}} = 22\text{ m/s}.$

16.52. We know from equation (16.30) that a wave will travel faster through the lighter rope (because $c = \sqrt{\mathcal{T}/\mu}$). So we calculate $c_\text{light} - c_\text{heavy} = \sqrt{\frac{\mathcal{T}}{\mu_\text{light}}} - \sqrt{\frac{\mathcal{T}}{\mu_\text{heavy}}} = \sqrt{\frac{(300\text{ N})}{(0.29\text{ kg/m})}} - \sqrt{\frac{(300\text{ N})}{(0.45\text{ kg/m})}} = 6\text{ m/s}.$

16.53. We know $c = f\lambda$, and using equation (16.30) we can write $f = \frac{1}{\lambda}\sqrt{\frac{\mathcal{T}_\text{i}}{\mu}}$. Thus

$$\frac{f_\text{1f}}{f_\text{1i}} = \frac{\frac{1}{\lambda_1}\sqrt{\frac{\mathcal{T}}{\mu_\text{f}}}}{\frac{1}{\lambda_1}\sqrt{\frac{\mathcal{T}}{\mu_\text{i}}}} = \sqrt{\frac{\mu_\text{i}}{\mu_\text{f}}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

meaning the fundamental frequency decreases by a factor of four.

16.54. We can write the total time required for a wave to travel through both sections of wire as $\Delta t = \Delta t_\text{copper} + \Delta t_\text{steel} = \frac{\Delta x_\text{copper}}{c_\text{copper}} + \frac{\Delta x_\text{steel}}{c_\text{steel}}$. Inserting equation (16.30) here, we find

$$\Delta t = \Delta x_{\text{copper}}\sqrt{\frac{\mu_{\text{copper}}}{\mathcal{T}}} + \Delta x_{\text{steel}}\sqrt{\frac{\mu_{\text{steel}}}{\mathcal{T}}} = \Delta x_{\text{copper}}\sqrt{\frac{\rho_{\text{copper}}\pi R_{\text{copper}}^2}{\mathcal{T}}} + \Delta x_{\text{steel}}\sqrt{\frac{\rho_{\text{steel}}\pi R_{\text{steel}}^2}{\mathcal{T}}}$$

$$= (50.0\text{ m})\sqrt{\frac{(8.92\times10^3\text{ kg/m}^3)\pi(0.450\times10^{-3}\text{ m})^2}{(145\text{ N})}} + (25.0\text{ m})\sqrt{\frac{(7.86\times10^3\text{ kg/m}^3)\pi(0.450\times10^{-3}\text{ m})^2}{(145\text{ N})}}$$

$$= 0.460\text{ s}$$

16.55. Given the volume mass density, we can determine the linear mass density: $\mu = \rho A = \rho\pi\left(\frac{d}{2}\right)^2 =$ $(7.86\times10^3\text{ kg/m}^3)\pi\left(\frac{(0.0720\text{ m})}{2}\right)^2 = 32.0\text{ kg/m}$. Now we can simply rearrange equation (16.30) to yield $\mathcal{T} = c^2\mu = (380\text{ m/s})^2(32.0\text{ kg/m}) = 4.62\times10^6\text{ N}$.

16.56. The only vertical forces being applied on the rod in the vertical direction are the component of the tensile force in the cable, and the downward gravitational force. This means $\mathcal{T}\sin(\theta) = mg$. Inserting the tension into equation (16.30) yields $c = \sqrt{\frac{\mathcal{T}}{\mu}} = \sqrt{\frac{mg}{\mu\sin(\theta)}} = \sqrt{\frac{(75\text{ kg})(9.8\text{ m/s}^2)}{(0.067\text{ kg/m})\sin(27°)}} = 1.6\times10^2\text{ m/s}$.

16.57. We simply insert numbers into equation (16.42), rewriting the angular frequency in terms of the wave speed and wavelength: $P_{\text{av}} = \frac{1}{2}\mu A^2\omega^2 c = \frac{2mA^2\pi^2c^3}{\ell\lambda^2} = \frac{2(0.128\text{ kg})(0.200\text{ m})^2\pi^2(25.0\text{ m/s})^3}{(3.60\text{ m})(0.600\text{ m})^2} = 1.22\text{ kW}$.

16.58. Let us re-write the expression for average power given by equation (16.42) in terms of the wavelength: $P_{\text{av}} = \frac{1}{2}\mu A^2\omega^2 c = \frac{2\mu A^2\pi^2c^3}{\lambda^2}$. Because the waves travel through the same medium, the linear mass density and wave speed must be the same. Thus we can write $\frac{P_{\text{av,Y}}}{P_{\text{av,X}}} = \frac{\frac{2\mu A_{\text{Y}}^2\pi^2c^3}{\lambda_{\text{Y}}^2}}{\frac{2\mu A_{\text{X}}^2\pi^2c^3}{\lambda_{\text{X}}^2}} = \frac{A_{\text{Y}}^2\lambda_{\text{X}}^2}{A_{\text{X}}^2\lambda_{\text{Y}}^2} = 1 \Rightarrow \frac{\lambda_{\text{Y}}}{\lambda_{\text{X}}} = \frac{A_{\text{Y}}}{A_{\text{X}}} = 2$. So, $\lambda_{\text{Y}}/\lambda_{\text{X}} = 2$, meaning of course that λ_{Y} is larger.

16.59. (*a*) There is no change in power required. (*b*) The average power has the form given by equation (16.42): $P_{\text{av}} = \frac{1}{2}\mu A^2\omega^2 c$. Doubling the amplitude will increase the average power by a factor of four, as will doubling the angular frequency. Thus these two changes together increase the power by a factor of 16. (*c*) We can re-write equation (16.42) as $P_{\text{av}} = \frac{1}{2}\mu A^2\omega^2 c = \frac{2\mu A^2\pi^2c^3}{\lambda^2}$. Now we see clearly that there is no dependence of the average power on the rope length, but doubling the wavelength decreases the average power by a factor of 0.25.

16.60. (*a*) The work done by your hand as you lift the string is given by equation (16.33): $W = \mu cv^2\Delta t =$ $(0.067\text{ kg/m})(31\text{ m/s})(12\text{ m/s})^2(0.016\text{ s}) = 4.8\text{ J}$. (*b*) Clearly, if you did 4.8 J of work on the string, the energy of the resulting wave is 4.8 J. (*c*) Exactly half of the energy in a travelling wave is in the form of kinetic energy, and half is in potential energy. Thus the potential energy in the wave is 2.4 J. (*d*) As described in part (*c*), the kinetic energy in the wave is 2.4 J.

16.61. The work done by your hand as you lift the string (and therefore the energy of) is given by equation (16.33): $W = \mu cv^2\Delta t = (3.5\text{ kg/m})(78\text{ m/s})(45\text{ m/s})^2(6.7\times10^{-3}\text{ s}) = 3.7\text{ kJ}$. Half of this is kinetic energy, so $K = 1.9\text{ kJ}$.

16.62. (*a*) The speed of the wave is given by $c=\lambda f=\left(\frac{2\pi}{b}\right)\left(\frac{q}{2\pi}\right)=\frac{q}{b}=\frac{(76.3\text{ s}^{-1})}{(5.85\text{ m}^{-1})}=13.0\text{ m/s}.$ (*b*) The frequency is given by $f=\frac{q}{2\pi}=\frac{(76.3\text{ s}^{-1})}{2\pi}=12.1\text{ Hz}.$ (*c*) The average power supplied by the wave is given by equation (16.42) as $P_{\text{av}}=\frac{1}{2}\mu A^2\omega^2 c.$ In terms of the variables given, this can be written as $P_{\text{av}}=\frac{\mu a^2 q^3}{2b}=$ $\frac{(0.0456\text{ kg/m})(0.0268\text{ m})^2(76.3\text{ s}^{-1})^3}{2(5.85\text{ m}^{-1})}=1.24\text{ W}.$

16.63. We can use equation (16.42) to write the average power as $P_{\text{av}}=\frac{1}{2}\mu A^2\omega^2 c=\frac{2\mu A^2\pi^2 c^3}{\lambda^2}.$ We solve this for the linear mass density to obtain $\mu=\frac{P_{\text{av}}\lambda^2}{2A^2\pi^2 c^3}.$ We also know from equation (16.30) that $c=\sqrt{\frac{\mathcal{T}}{\mu}}\Rightarrow\mu=\frac{\mathcal{T}}{c^2}.$ Equating the two expressions for the linear mass density and solving for the wave speed, we find $c=$ $\frac{P_{\text{av}}\lambda^2}{2A^2\pi^2\mathcal{T}}=\frac{(0.350\text{ W})(1.80\text{ m})^2}{2(0.0165\text{ m})^2\pi^2(14.8\text{ N})}=14.3\text{ m/s}.$

16.64. (*a*) We can use equation (16.42) to write the average power as $P_{\text{av}}=\frac{1}{2}\mu A^2\omega^2 c=\frac{2\mu A^2\pi^2 c^3}{\lambda^2}.$ Inserting equation (16.30): $c=\sqrt{\frac{\mathcal{T}}{\mu}}$ and rearranging yields

$$\lambda=\sqrt{\frac{2\mu A^2\pi^2}{P_{\text{av}}}\left(\frac{\mathcal{T}}{\mu}\right)^{3/2}}=\sqrt{\frac{2(0.360\text{ kg/m})(8.00\times10^{-3}\text{ m})^2\pi^2}{(200\text{ W})}\left(\frac{(30.0\text{ N})}{(0.360\text{ kg/m})}\right)^{3/2}}=4.16\times10^{-2}\text{ m}.$$

(*b*) We use equation (16.42) to write $P_{\text{av}}=\frac{1}{2}\mu A^2\omega^2 c=\frac{2\mu A^2\pi^2 c^3}{\lambda^2},$ and again we insert $c=\sqrt{\frac{\mathcal{T}}{\mu}}$ to obtain

$$P_{\text{av}}=\frac{2\mu A^2\pi^2}{\lambda^2}\left(\frac{\mathcal{T}}{\mu}\right)^{3/2}=\frac{2(0.360\text{ kg/m})(8.00\times10^{-3}\text{ m})^2\pi^2}{(0.0832\text{ m})^2}\left(\frac{(15.0\text{ N})}{(0.360\text{ kg/m})}\right)^{3/2}=17.7\text{ W}.$$

16.65. (*a*) We will assume that the initial phase of the wave is zero, such that the wavefunction will be of the form $f(x,t)=a\sin(bx-qt).$ We are told that $a=0.0725$ m. We require that the wavefunction go through a full oscillation when x changes by one wavelength, meaning $b\lambda=2\pi\Rightarrow b=\frac{2\pi}{\lambda}=\frac{2\pi}{(3.00\text{ m})}=2.09\text{ m}^{-1}.$ We also require that the wavefunction go through a full oscillation when the time changes by a full period, meaning $qT=\frac{q}{f}=2\pi\Rightarrow q=2\pi f=2\pi(60.0\text{ Hz})=377\text{ s}^{-1}.$ (*b*) Using equation (16.42) we can write the power of the wave as $P_{\text{av}}=\frac{1}{2}\mu A^2\omega^2 c=2\left(\frac{m}{\ell}\right)A^2\pi^2\lambda f^3=2\left(\frac{(1.20\times10^{-2}\text{ kg})}{(9.00\text{ m})}\right)(0.0725\text{ m})^2\pi^2(3.00\text{ m})(60.0\text{ Hz})^3=89.6\text{ W}.$

16.66. We start by writing the wave equation, and then we will plug in values to see if it is indeed satisfied:

$$\frac{\partial^2 f(x,t)}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 f(x,t)}{\partial t^2}$$

where c refers to the wave speed. Note that in the equation given, the variable v has been used to refer to the wave speed $(v=c)$. Taking the derivatives of the exponential function yields:

$$b^2e^{b(x-vt)}=\frac{1}{c^2}(-bv)^2e^{b(x-vt)}$$

Dividing both sides by $e^{b(x-vt)}$ yields

$$1=\frac{v^2}{c^2}=\frac{c^2}{c^2}$$

This is obviously true, and shows that the wave equation is satisfied.

16.67. The fact that speed at which surfer moves forward toward wave base equals speed at which wave moves forward.

16.68. (*a*) Let us assume that the shape the wave takes on is sinusoidal. The general form of the wavefunction will be $f(x,t)=a\sin(bx-qt+\phi)$. If we start our clocks at the moment the hand first begins to pull on the rope, then the initial phase $\phi_i=0$. We are told that the duration of a single pulse is 0.050 s, meaning $qT=2\pi \Rightarrow q=\frac{2\pi}{T}=\frac{2\pi}{(0.050\text{ s})}=1.3\times 10^2\text{ s}^{-1}$. We are told the wave speed, which is equal to $c=\lambda f=\left(\frac{2\pi}{b}\right)\left(\frac{q}{2\pi}\right)=\frac{q}{b}\Rightarrow b=\frac{q}{c}=\frac{(126\text{ s}^{-1})}{(67\text{ m/s})}=1.9\text{ m}^{-1}$. Finally, the string is displaced at a speed of 25 m/s for 0.040 s, making the amplitude equal to 1.0 m. Collecting these results, we have $f(x,t)=a\sin(bx-qt+\phi)$ where $a=1.0$ m, $q=1.3\times 10^2\text{ s}^{-1}$, $b=1.9\text{ m}^{-1}$, and $\phi_i=0$. (*b*) We re-write our wavefunction from part (*a*) in terms of more physically transparent quantities (like wavelengths and frequencies) to facilitate our discussion:

$$f(x,t)=a\sin(bx-qt+\phi)=a\sin\left(\frac{2\pi}{\lambda}x-2\pi ft+\phi_i\right)$$

where $a=1.0$ m and $\phi_i=0$ as before. Now $\lambda=3.35$ m and $f=20$ Hz. Applying the left hand side of equation (16.51) we have

$$\frac{\partial^2 f(x,t)}{\partial x^2}=-\left(\frac{2\pi}{\lambda}\right)^2 A\sin\left(\frac{2\pi}{\lambda}x-2\pi ft+\phi\right)$$

And applying the right hand side of equation (16.51) to our wavefunction yields

$$\frac{\partial^2 f(x,t)}{\partial t^2}=-(2\pi f)^2 A\sin\left(\frac{2\pi}{\lambda}x-2\pi ft+\phi\right)$$

So the wave equation (16.51)

$$\frac{\partial^2 f(x,t)}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 f(x,t)}{\partial x^2}$$

becomes

$$-\left(\frac{2\pi}{\lambda}\right)^2 A\sin\left(\frac{2\pi}{\lambda}x-2\pi ft+\phi\right)=-\frac{1}{c^2}(2\pi f)^2 A\sin\left(\frac{2\pi}{\lambda}x-2\pi ft+\phi\right)$$

$$\left(\frac{2\pi}{\lambda}\right)=\frac{1}{c^2}(2\pi f)$$

$$\frac{1}{(\lambda f)^2}=\frac{1}{c^2}$$

which is clearly true. Hence the equation from part (*a*) satisfies the wave equation.

16.69. We start by writing the wave equation for the superposition of the two waves $f(x,t)=f_1(x,t)+f_2(x,t)$:

$$\frac{\partial^2 f(x,t)}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 f(x,t)}{\partial t^2}$$

or

$$\frac{\partial^2 f(x,t)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 f(x,t)}{\partial t^2}=0$$

Now we write the function explicitly as the sum of the two original waves

$$\frac{\partial^2(f_1(x,t)+f_2(x,t))}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2(f_1(x,t)+f_2(x,t))}{\partial t^2}=0$$
$$\frac{\partial^2 f_1(x,t)}{\partial x^2}+\frac{\partial^2 f_2(x,t)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 f_1(x,t)}{\partial t^2}-\frac{1}{c^2}\frac{\partial^2 f_2(x,t)}{\partial t^2}=0$$
$$\left(\frac{\partial^2 f_1(x,t)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 f_1(x,t)}{\partial t^2}\right)+\left(\frac{\partial^2 f_2(x,t)}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2 f_2(x,t)}{\partial t^2}\right)=0$$

Because $f_1(x,t)$ satisfies the wave equation (given) we know that the first term in parentheses on the left must be zero. Similarly, because $f_2(x,t)$ satisfies the wave equation, the second term in parentheses must also be zero. The entire expression on the left reduces to zero, and the wave equation is satisfied for the superposition of the two waves.

16.70. (*a*) We start by writing the wave equation in general, and we will fill in values for each wavefunction:

$$\frac{\partial^2 f(x,t)}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 f(x,t)}{\partial t^2}$$

Wavefunction 1:

$$-Ak^2\cos(kx+\omega t)=\frac{1}{c^2}A\omega^2\cos(kx+\omega t)$$
$$\frac{k^2}{\omega^2}=\frac{1}{c^2}$$

and note

$$\frac{k^2}{\omega^2}=\frac{(2\pi/\lambda)^2}{(2\pi f)^2}=\frac{1}{(\lambda f)^2}$$

Wavefunction 2:

$$(-2b+4b^2(x-qt)^2)e^{-b(x-qt)^2}=\frac{1}{c^2}(-2bq^2+4b^2q^2(x-qt)^2)e^{-b(x-qt)^2}$$

Which is satisfied if $c=q$.

Wavefunction 3:

$$-2=\frac{1}{c^2}-2b^4$$

It is not true that this must always satisfy the wave equation. In order for this to satisfy the wave equation, the speed of the wave must be $c=b^2$. (*b*) In wavefunction 1 $c=\omega/k$, in wavefunction 2: the wave speed $c=q$, and in wavefunction 3: $c=b^2$.

16.71. (*a*) No, because pulse needs nonzero time interval to travel length of rope. (*b*) No, again because pulse needs nonzero time interval to travel length of bar, though pulse speed is much greater in bar than in rope.

16.72. The pulse has a greater speed at the top. We know that wave speed increases with the tension in a rope according to $c=\sqrt{\frac{\mathcal{T}}{\mu}}$. Introductory physics courses often say that the tension throughout a rope is the same, to a very good approximation. However, logic tells us that a massive rope must have greater tension near the top, because a segment of rope at the top is supporting not only the weight of the block, but also the weight of the length of rope below that segment. Thus the tension is greater near the top, and therefore the wave speed is also greater near the top.

16.73. We write the general expression for the time interval: $\Delta t=\frac{\ell}{c}=\ell\sqrt{\frac{\mu}{\mathcal{T}}}=\sqrt{\frac{m\ell}{k\Delta x}}$. Here ℓ is the total length of the spring and Δx is the distance the spring is stretched beyond its equilibrium length. Now we compare pulses A and B:

$$\frac{\Delta t_A}{\Delta t_B} = \frac{\sqrt{\dfrac{m\ell_A}{k\Delta x_A}}}{\sqrt{\dfrac{m\ell_B}{k\Delta x_B}}} = \sqrt{\frac{\ell_A \Delta x_B}{\ell_B \Delta x_A}} = \sqrt{\frac{(2\ell_0)(2\ell_0)}{(3\ell_0)(\ell_0)}} = 1.15$$

So $\Delta t_A = (1.15)\Delta t_B$, or equivalently $\Delta t_B = (0.87)\Delta t_A$.

16.74. Equation (16.20) describes a standing wave as $f(x,t) = 2a\sin(kx)\cos(\omega t)$. We know $k = \dfrac{2\pi}{\lambda}$, and we are given the amplitude of $2a = 0.75$ mm. The largest displacement (at any position) occurs at a time when $\cos(\omega t) = 1$. So, for our particular position, we can write $f_{max}(x,t) = 2a\sin(kx) = 2a\sin\left(\dfrac{2\pi x}{\lambda}\right) = (0.75\text{ mm})\sin\left(\dfrac{2\pi(0.500\text{ m})}{(1.3\text{ m})}\right) =$ 0.50 mm.

16.75. (*a*) We know $c = \lambda f = (0.170\text{ m})\left(\dfrac{15}{5.0\text{ s}}\right) = 0.51\text{ m/s}$. (*b*) $\Delta t = \dfrac{\Delta x}{c} = \dfrac{(6.0\text{ m})}{(0.51\text{ m/s})} = 12\text{ s}$.

16.76. Shower stall walls reflect sound waves. Depending on the stall geometry, sound waves of certain wavelengths are harmonics of the stall cavity, and these waves are amplified as they resonate, making them louder than sound waves that are not harmonics of the stall.

16.77. (*a*) Because there are eight nodes, there must be seven peaks and troughs (antinodes) on the string. This means there are $7/2$ wavelengths on the string, or $\dfrac{7\lambda}{2} = \ell \Rightarrow \lambda = \dfrac{2\ell}{7} = \dfrac{2(1.75\text{ m})}{7} = 0.500\text{ m}$. (*b*) $c = \lambda f \Rightarrow f = \dfrac{c}{\lambda} = \dfrac{(130\text{ m/s})}{(0.50\text{ m})} = 260\text{ Hz}$. (*c*) $f(x,t) = a\sin(kx)\cos(\omega t)$ where $a = 0.0200$ m, $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{(0.500\text{ m})} = 12.6\text{ m}^{-1}$, and $\omega = 2\pi f = 2\pi(260\text{ Hz}) = 1.63\times 10^3\text{ s}^{-1}$.

16.78. The water wave would travel more slowly because the restoring force (gravity) is weaker on Mars than on Earth.

16.79. We can use equation (16.33) to write $v = \sqrt{\dfrac{W}{\mu c \Delta t}} = \sqrt{\dfrac{(1.2\text{ J})}{(0.0282\text{ kg/m})(67\text{ m/s})(0.18\text{ s})}} = 1.9\text{ m/s}$.

16.80. As the wave pulse travels down the rope, it will be reflected two times. It will first be reflected from the interface between the two ropes, and then the portion of the wave that makes it through that interface (the transmitted wave) will reach the end of the second rope and will be reflected back. We can tell which rope we are holding by looking at the first reflected pulse. If we make a pulse upward, and the returning pulse pulls our hand upward, then it was reflected off of the interface without being inverted and we must be holding the heavier rope. If it returns to us inverted, we are holding the lighter rope. Regardless of the order of the ropes, the transmitted pulse that reaches the end of the second rope will be upright. If the second rope is tied in place, then the wave will be inverted and the second pulse that returns to us will be inverted. If the end of the second rope is free, the second pulse that returns to us will be upright. We can estimate the relative densities of the ropes by estimating the time required for the reflected waves to return to our hands. Suppose we give the initial pulse at time $t = 0$ and the first pulse that returns to us (reflected off the two-rope interface) arrives to us at time t_1 and the second pulse arrives at time t_2. We can write these times as $t_1 = \dfrac{2\ell}{c_1} = 2\ell\sqrt{\dfrac{\mu_1}{\mathcal{T}}}$ and $t_2 = \dfrac{2\ell}{c_1} + \dfrac{2\ell}{c_2} = 2\ell\left(\sqrt{\dfrac{\mu_1}{\mathcal{T}}} + \sqrt{\dfrac{\mu_2}{\mathcal{T}}}\right)$. Thus

$$\frac{t_2}{t_1} = \frac{\left(\sqrt{\frac{\mu_1}{\mathcal{T}}} + \sqrt{\frac{\mu_2}{\mathcal{T}}}\right)}{\sqrt{\frac{\mu_1}{\mathcal{T}}}} = 1 + \sqrt{\frac{\mu_2}{\mu_1}} \Rightarrow \frac{\mu_2}{\mu_1} = \left(\frac{t_2}{t_1} - 1\right)^2.$$

16.81. Assume that you grab the rope exactly at the bottom to start your pulse. We can write the speed of the pulse in two different ways: $\sqrt{\frac{\mathcal{T}}{\mu}} = v = \frac{dy}{dt}$, where y is the vertical position along the rope. The tension at any given segment of the rope depends on how high up the rope that segment is (because the tension depends on how much of the rope is being supported by that segment). This means $\mathcal{T} = m_{\text{below}}g = y\mu g = ymg/\ell$. Inserting this into our expressions for the pulse speed, we can write $\frac{dy}{dt} = \sqrt{\frac{ymg/\ell}{m/\ell}} = \sqrt{yg}$. So the amount of time require for the pulse to traverse a small differential segment dy is $dt = \frac{dy}{\sqrt{yg}}$. We can find the total amount of time required for the pulse to reach the ceiling by integrating: $T = \int_0^\ell \frac{dy}{\sqrt{yg}} = 2\sqrt{\frac{\ell}{g}}$. This is the general expression. For the specific case of a 10 meter rope, the pulse takes $T = 2\sqrt{\frac{\ell}{g}} = 2\sqrt{\frac{(10\text{ m})}{(9.8\text{ m/s}^2)}} = 2.0\text{ s}$.

17

WAVES IN TWO AND THREE DIMENSIONS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{-1} m **2.** 10^{2} **3.** 10^{-2} m **4.** 10^{5} m **5.** 10^{1} W **6.** 10^{5} m **7.** 10^{-2}

Guided Problems

17.2 Dripping disturbance

1. Getting Started: As the waves spread out, the power that they carry is distributed over larger and larger wavefronts. If we are only considering the wave that travels across the surface of the water (and not the compression wave that travels downward toward the bottom of the sink), the wavefronts are spreading in only two dimensions. In three dimensions, waves spread out such that the energy in a single wavefront is distributed over the surface of a sphere. In that case, one uses $I = \dfrac{P}{4\pi r^2}$. But in this two-dimensional case, the energy in a single wavefront (and also the power) is distributed over the radius of a circle. So here we will use

$$I = \frac{P}{2\pi r} \tag{1}$$

2. Devise Plan For part (*b*), we only need to use conservation of energy. Because energy is conserved, the power delivered in one period of the wave must be constant. If the power is constant, we can relate the intensities through equation (1) above. For example, we can find an expression for the power from the information given about the wave 10 mm from P, and then use that expression for the power to find the intensity 150 mm from point P.

To answer part (*a*), we must think back to Chapter 16. A general property of waves is that the energy delivered by a small section of the wave depends on the square of the amplitude in that section. This can be seen (for example) from equation 16.42. So to find the amplitude of the waves, we first must determine how much energy is being delivered by a small section of the wave. We can write for example, $dE = P(dt) = Id\ell dt,$ to relate the energy delivered by a small section of the wave to the intensity. This will allow us to relate the energies at the two points (10 mm from P and 150 mm from P) through the intensities. The ratio of energies will be related to the ratio of amplitudes according to

$$\frac{dE_{10\text{ mm}}}{dE_{150\text{ mm}}} = \frac{A^2_{10\text{ mm}}}{A^2_{150\text{ mm}}} \tag{2}$$

3. Execute Plan (*a*) Rewriting equation (2) using intensity yields

$$\frac{I_{10\text{ mm}}d\ell dt}{I_{150\text{ mm}}d\ell dt} = \frac{A^2_{10\text{ mm}}}{A^2_{150\text{ mm}}}$$

And inserting equation (1) for the intensity yields

$$\frac{P2\pi r_{150\text{ mm}}d\ell dt}{P2\pi r_{10\text{ mm}}d\ell dt}=\frac{A_{10\text{ mm}}^2}{A_{150\text{ mm}}^2}$$

The power and differential factors are the same whether we choose to look at a position 10 mm away from the source or 150 mm away. Thus

$$A_{150\text{ mm}}=A_{10\text{ mm}}\sqrt{\frac{r_{10\text{ mm}}}{r_{150\text{ mm}}}}=(7.0\text{ mm})\sqrt{\frac{(10\text{ mm})}{(150\text{ mm})}}=1.8\text{ mm}$$

So the wave will have an amplitude of 1.8 mm when it reaches a distance of 150 mm from the point P.

(*b*) Here, we simply apply equation (1) to each of the positions specified, and take the ratio:

$$\frac{I_{150\text{ mm}}}{I_{10\text{ mm}}}=\left(\frac{P}{2\pi r_{150\text{ mm}}}\right)\left(\frac{2\pi r_{10\text{ mm}}}{P}\right)=\frac{r_{10\text{ mm}}}{r_{150\text{ mm}}}=\frac{(10\text{ mm})}{(150\text{ mm})}=6.7\times10^{-2}$$

When the wave reaches a distance 150 mm from point P, the intensity is about 6.7% what it was at a position only 10 mm from point P.

4. Evaluate Result We know from everyday experience that the amplitude of such a wave decreases noticeably as it propagates outward. We know that the ripples become difficult to see after they propagate some distance (less than a meter, perhaps 20-30 cm depending on the drops and the waves they produce). So it is reasonable to find that the amplitude is reduced to about 26% in the interval described. Note that if we square this ratio of amplitudes

$$\left(\frac{A_{150\text{ mm}}}{A_{10\text{ mm}}}\right)^2=\left(\frac{1.81\text{ mm}}{7.0\text{ mm}}\right)^2=0.67$$

we obtain the ratio of intensities. This should be the case, since we know the energy transported in a section of the wave depends on the amplitude squared.

17.4 Bottle orchestra

1. Getting Started If the two bottles produce notes whose frequencies differ by just a few Hz, the only effect the ear will be able to detect is the warbling of the sounds moving out of phase with each other and back into phase with each other (the beats). Note that if the frequency, timbre or volume of the two sounds were drastically different, one could discern two different sounds from the two sources. But with the volumes and timbre the same, you and your friend won't be able to hear a second musical note until the beat frequency itself corresponds to a musical note audible to the human ear. The lowest frequency the human ear can detect is about 20 Hz, though this may vary slightly from person to person, and may change with age. So when the two fundamental frequencies differ by 20 Hz or more, you and your friend will hear a second musical note.

The bottles are open-closed tubes. So the standing waves that can be sustained in the bottles will have wavelengths given by

$$\lambda=\frac{4\ell}{n}\qquad n=1,3,5\ldots \tag{1}$$

Here ℓ is the length of the air cavity in the bottles. We know how to relate the wavelength to frequency: $\lambda=c/f$. So inserting this into equation (1), we have

$$f_n=\frac{nc}{4\ell}\qquad n=1,3,5\ldots \tag{2}$$

We use this to relate the frequency difference required to the length of the air cavity.

2. Devise Plan The fundamental frequency is the lowest-frequency standing wave. Thus, we want to choose $n=1$ in equation (2). Also, given the height of the bottle h, and the depth of fluid d, the length of the air cavity is $\ell=h-d$. So equation (2) becomes

$$f_1=\frac{c}{4(h-d)} \tag{3}$$

In Equation 17.11, we see that the sine term contains a difference in frequencies and the cosine term contains the sum of frequencies, effectively the average frequency. If the frequency from each bottle separately is audible, then the average frequency will be audible. The second audible tone we are seeking comes from the difference of frequencies. We set this difference in frequencies equal to the minimum frequency audible to humans $f_{\min}$.

Let us refer to the bottles as A and B. Then we have

$$f_{\min} = f_{1,\mathrm{A}} - f_{1,\mathrm{B}} = \frac{c}{4}\left(\frac{1}{(h-d_{\mathrm{A}})} - \frac{1}{(h-d_{\mathrm{B}})}\right) \tag{4}$$

3. Execute Plan Rearranging equation (4), we find

$$d_{\mathrm{A}} - d_{\mathrm{B}} = \frac{4f_{\min}}{c}(h-d_{\mathrm{A}})(h-d_{\mathrm{B}}) \tag{5}$$

We are told the height of the bottles and the approximate depth of the bottles. If the depths were exactly the same, there would be no frequency difference. But if the difference in depths is small, we can obtain an answer using the approximate values. Thus

$$d_{\mathrm{A}} - d_{\mathrm{B}} = \frac{4(20\ \mathrm{Hz})}{(343\ \mathrm{m/s})}((0.23\ \mathrm{m}) - (0.10\ \mathrm{m}))^2 = 3.9\ \mathrm{mm}$$

So you and your friend will hear a second sound when the depths of the fluids in the bottles differs by 3.9 mm.

4. Evaluate Result It is plausible for the depths of fluid in two beverage bottles to differ by 3.9 mm. It is suggested in the problem statement that the depths are very similar, so this small difference is expected. The frequencies you and your friend would hear are the beat frequency of 20 Hz (used in the solving process) and the average of the two frequencies. We can find the average of the two frequencies by writing

$$f_{\mathrm{av}} = \frac{1}{2}(f_{1,\mathrm{A}} + f_{1,\mathrm{B}}) = \frac{c}{8}\left(\frac{1}{(h-d_{\mathrm{A}})} + \frac{1}{(h-d_{\mathrm{B}})}\right) = \frac{c}{8}\left(\frac{2h-d_{\mathrm{A}}-d_{\mathrm{B}}}{(h-d_{\mathrm{A}})(h-d_{\mathrm{B}})}\right)$$

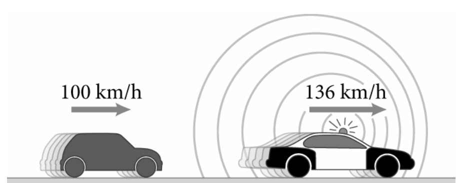
Again inserting approximate values given, we find

$$f_{\mathrm{av}} = \frac{c}{8}\left(\frac{2(0.23\ \mathrm{m}) - 2(0.10\ \mathrm{m})}{((0.23\ \mathrm{m}) - (0.10\ \mathrm{m}))^2}\right) = 6.6\times10^2\ \mathrm{Hz}$$

This is well within the audible range, and it is a perfectly reasonable sound to produce using bottles.

17.6 Cut to the chase

1. Getting Started We begin by making a diagram of the situation once the police car has passed the motorist. This is practically identical to Figure WG17.2, except that the police car is on the other side of the car. Note that the wavecrests approaching the car in Figure WG17.2 were very close together, and those approaching the car in this figure are farther apart.



We can solve parts (*a*) and (*b*) separately.

2. Devise Plan In part (*a*), the motion of the source is toward the observer, but the motion of the observer is away from the source. In part (*b*), the situation is reversed. The motion of the observer is toward the source, but the motion of the source is away from the observer. Since both parts (*a*) and (*b*) involve motion of both the source and the observer. We must use equation 17.21 from Principles to determine the Doppler shifted frequency heard by the motorist. We choose our signs carefully in each case.

3. Execute Plan (*a*) Equation 17.21 tells us

$$f_{\rm o} = \frac{c \pm v_{\rm o}}{c \pm v_{\rm s}} f_{\rm s}$$

Before the police car has passed the motorist, the motion of the police car serves to increase the frequency the motorist hears, and the motion of the motorist serves to decrease the frequency. Thus, the appropriate sign choices are

$$f_{\rm o} = \left(\frac{c - v_{\rm o}}{c - v_{\rm s}} \right) f_{\rm s} \tag{1}$$

Inserting given values into equation (1) yields

$$f_{\rm o} = \left(\frac{(343\ \text{m/s}) - (27.8\ \text{m/s})}{(343\ \text{m/s}) - (37.8\ \text{m/s})} \right) (1526\ \text{Hz}) = 1.58\ \text{kHz}$$

(*b*) We again use equation 17.21, but now the choices of signs are

$$f_{\rm o} = \left(\frac{c + v_{\rm o}}{c + v_{\rm s}} \right) f_{\rm s} \tag{2}$$

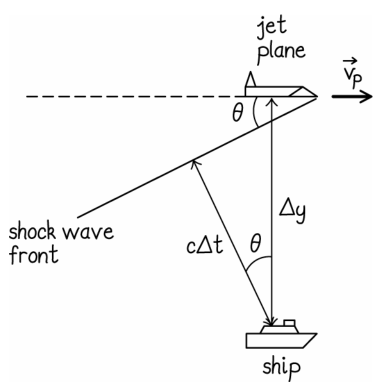
Inserting the given values yields

$$f_{\rm o} = \left(\frac{(343\ \text{m/s}) + (27.8\ \text{m/s})}{(343\ \text{m/s}) + (37.8\ \text{m/s})} \right) (1526\ \text{Hz}) = 1.49\ \text{kHz}$$

4. Evaluate Result Our answers are reasonable. As the police car approaches from behind, the distance between source and observer is decreasing. We know that the effect of the motion of the source is to increase the frequency and the effect of the motion of the observer in this case is to decrease the frequency, but because the two are getting closer, we expect the effects due to the motion of the source to dominate. Thus it fits our expectation that the frequency observed is larger than the frequency emitted by the source. The same intuition fits in part (*b*). Because the distance between the source and observer is increasing, we expect the frequency observed to be smaller than the frequency emitted, and this is the case.

17.8 Shocked, I say—shocked

1. Getting Started: We start by drawing a diagram of the situation:



Let us assume that the plane is flying horizontally throughout the problem. The figure shows the dashed line of flight of the plane, and the dashed shock wave front propagating outward from the path. The sound is moving perpendicular to this dashed wavefront at the speed of sound. This means that that the distance it covers is equal to the speed of sound times the amount of time it takes for the sound to reach us on the boat. We call the altitude of the plane Δy.

2. Devise Plan From the figure, we can see that geometry allows us to relate the altitude and the distance covered by the shock wave:

$$\cos(\theta) = \frac{c\Delta t}{\Delta y} \tag{1}$$

We also know that the speed of the jet is related to the Mach angle through equation 17.22:

$$\sin(\theta) = \frac{c}{v_{\text{p}}} \tag{2}$$

We can combine the two expressions to solve for the speed of the plane in terms of given quantities.

3. Execute Plan Dividing equation (2) by equation (1), we find

$$\cos(\theta) = \sqrt{1-\sin^2(\theta)} = \sqrt{1-\left(\frac{c}{v_{\text{p}}}\right)^2} = \frac{c\Delta t}{\Delta y}$$

$$v_{\text{p}} = c\left(1-\left(\frac{c\Delta t}{\Delta y}\right)^2\right)^{-1} \tag{3}$$

Inserting given values, we arrive at

$$v_{\text{p}} = (343\text{ m/s})\left(1-\left(\frac{(343\text{ m/s})(34\text{ s})}{(1.5\times10^4\text{ m})}\right)^2\right)^{-1/2} = 5.5\times10^2\text{ m/s}$$

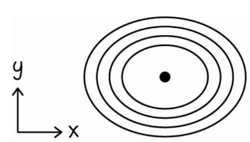
So the plane is travelling about 1.6 times the speed of sound.

4. Evaluate Result This is a very high speed, but supersonic jets certainly exist. A speed of 1.6 times the speed of sound is very reasonable for a jet producing a sonic boom.

Questions and Problems

17.1. An approaching train emits waves that travel through air and the track; those traveling through air spread out spherically, while those traveling in track remain (largely) confined to the track, making the vibrations they cause detectable at greater distance. Also, the speed of vibrations through steel is greater than the speed of sound through air.

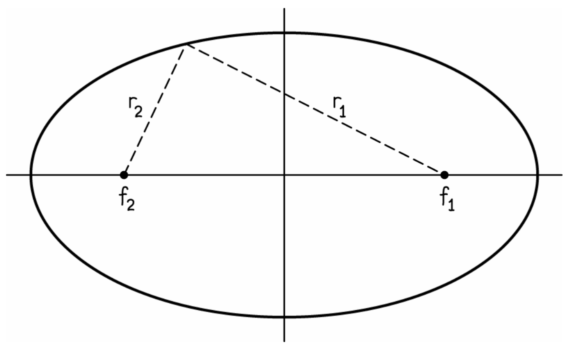
17.2. Because the material is stiffer in the x direction, waves will propagate more quickly in the x direction than in the y direction. The result would be ellipsoidal wavefronts as shown below.



17.3. As long as the sound is free to spread over a sphere centered at the professor's mouth, yes, the amplitude would roughly drop off like $1/r$. But large lecture halls (like concert halls) are often designed to make use of the reflection of sound off of walls. Chances are that the amplitude won't drop off quite that quickly.

17.4. (*a*) The sound energy that reaches your ear drops off like $1/r^2$ where r is the distance from you to the cannon. (*b*) The first estimate is higher than the second. If you assume there is no dissipation, then the only reason for the sound energy to become small is for the cannon to be very far away. If some of the decrease in sound energy can be explained by dissipation, then the cannon need not be quite so far away.

17.5. When a person whispers at one focus, the sound will travel some distance r_1 before hitting the wall. A wave that bounces off the wall and reaches the other focus will travel a distance r_2 from the wall to the second focus. Note that the definition of an ellipse is the set of all points such that the sum of the distances from any point on the ellipse to the two foci is constant. That means that $r_1 + r_2$ is a constant no matter what path the sound takes to go from the first focus, reflect off a wall, and reach the second focus (see figure next page). If two sound waves travel different paths, they may end up interfering with one another in a constructive or destructive way. But if they travel exactly the same distance, then peaks in the wavefunction of one sound wave will line up with peaks in the wavefunction of the other sound wave and they will interfere constructively. In an elliptical room, any sound waves that travel from one focus to a wall and then to the other focus will all travel the same distance and all such waves will thus interfere constructively. This will increase the amplitude and make the sound quite audible.

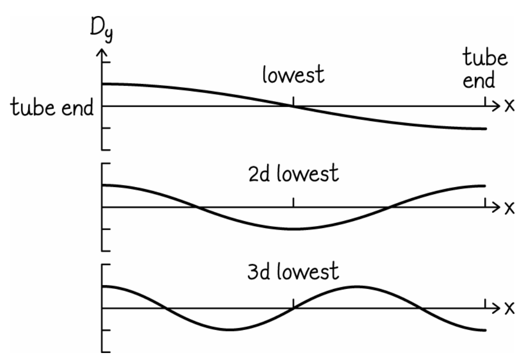


17.6. We know from PRIN Chapter 17.2 that humans can hear sounds in the range 20 Hz to 20 kHz. We also know that sounds that differ by one octave have frequencies that differ by a factor of two. Thus we write $20\text{ Hz}(2^N) = 20\text{ kHz},$ where N is the number of octaves. Rearranging, we find $N = \ln(1000)/\ln(2) = 10.$ So there are 10 octaves in the human audible-frequency range.

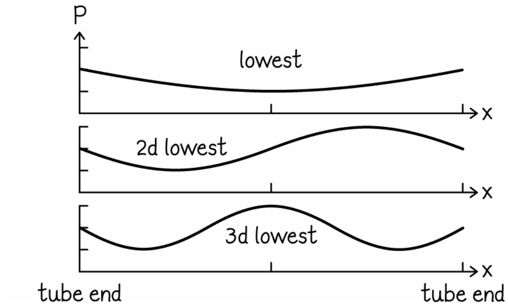
17.7. Bats can still feel compression of the air around them, just as we can feel the rumbling from a passing truck or from a small tremor. Bats and humans can still feel vibrations that are outside their range of hearing.

17.8. In a tube that is open at both ends, each end will be analogous to a free end of a string in the context of standing waves on strings. That is, we expect an antinode at each end. There could be any integer number of nodes in the tube, between these two antinodes. This means there could be $\dfrac{n\lambda}{2}$ wavelengths in the tube of length ℓ, or equivalently $\lambda = \dfrac{2\ell}{n}.$ Thus $\lambda_1 = 2L$ (fundamental, or first harmonic), $\lambda_2 = \ell$ (second harmonic), $\lambda_3 = 2\ell/3$ (third harmonic).

17.9. (*a*)



(*b*)



17.10. Increased frequency creates more nodal lines, so the lines get closer together.

17.11. Nodal lines become antinodal lines, and vice-versa.

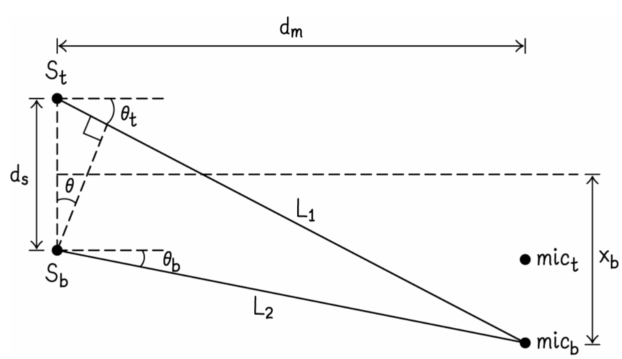
17.12. No stationary nodal lines are formed. Since one wave undergoes more oscillations each second than the other, the phase difference between the two waves at any given point is always changing.

17.13. A wall behind the speaker will allow a good deal of sound to be reflected back into the room. If the geometry of the speaker or room is such that a dead zone forms, it is very unlikely that sound coming from a different location (the wall behind the speaker) will also form a dead zone in exactly the same spot.

17.14. Choose the $+x$ axis to point from the left source to the right source. Half-way between the two sources along the *x* axis, the phase difference between the two sources would be zero if the sources themselves were oscillating in phase. This is the minimum phase difference. The phase difference would also be zero at any point along the perpendicular bisector of the line connected the left and right sources, forming a line of antinodes along what we will call the $+y$ axis. At a point on the $-x$ axis to the left of the left source, waves from the right source must travel 2.5 additional wavelengths compared to waves coming from the left source. So the waves are out of phase by 2.5 full oscillations. Thus, there will be a nodal line along the $-x$ axis. Note that the phase difference must change smoothly from zero (along the $+y$ axis) to 2.5λ (along the $-x$ axis), meaning at some angle the phase difference must pass through 0.5λ and 1.5λ, yielding two more nodal lines. The same must be true between the $-x$ axis and the $-y$ axis. Thus, to the left of the y axis there are five nodal lines. By symmetry, there must be the same number of nodal lines to the right of the y axis. Thus there are 10 nodal lines.

17.15. The ripples are two-dimensional surface waves. Vibration causes the cup interior to act like a chain of sources. Because waves propagate away from sources, wavefronts are parallel to the cup interior and therefore circular.

17.16. The figure shows the geometry of the problem.

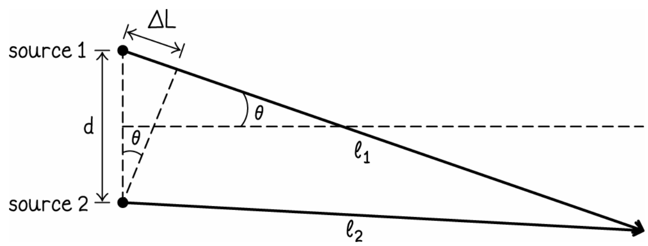


Using the small angle approximation makes $\theta_t \approx \theta_b \approx \theta$. If the distances travelled by the two interfering sound waves (L_1 and L_2) differ by a full number of wavelengths, then the sound heard by the microphone will be loud. This means $d_s \sin(\theta) = n\lambda$. Since the angle can be assumed to be small, $\sin(\theta) \approx \tan(\theta) = x_b/d_m$. This means that one could write the position of two adjacent microphones that each hear a loud sound as:

$$x_b = d_m n\lambda/d_s$$
$$x_t = d_m(n-1)\lambda/d_s$$

Taking the difference of the two equations yields: $\Delta x = x_b - x_t = d_m\lambda/d_s$.

17.17. (*a*) Consider the geometry in the figure below.



Clearly the path length difference $\ell_1 - \ell_2 = d\sin(\theta)$. If this path length difference is to produce a point on a nodal line to the perpendicular bisector, then we can set the path length difference equal to $\left(m+\frac{1}{2}\right)\lambda$. Thus we write $d\sin(\theta) = \left(m+\frac{1}{2}\right)\lambda = \frac{c}{2f} \Rightarrow \theta = \sin^{-1}\left[\frac{(m+\frac{1}{2})c}{df}\right]$, $m = 0,1,2,\ldots$. (*b*) The process is exactly the same as in part (*a*), except that now the path length difference has to make up for the phase difference. The phase difference corresponds to a certain fraction of one wave being emitted before the other wave, and the fraction is $\phi/2\pi$. Thus $\ell_1 - \ell_2 = d\sin(\theta) = \left(m+\frac{1}{2}\right)\lambda - \frac{\phi}{2\pi}\lambda$ in order for destructive interference to occur. Thus $\theta = \sin^{-1}\left[\left(m+\frac{1}{2}-\frac{\phi}{2\pi}\right)\frac{c}{df}\right]$, $m = 0,1,2,\ldots$

17.18. Let us consider the waves from the three sources at a particular point x_0, which we require must be on a nodal line. Let the three sources to behave like:

$$y_1(x = x_0, t) = A\sin(\omega t)$$
$$y_2(x = x_0, t) = A\sin(\omega t + \delta)$$
$$y_3(x = x_0, t) = A\sin(\omega t + 2\delta)$$

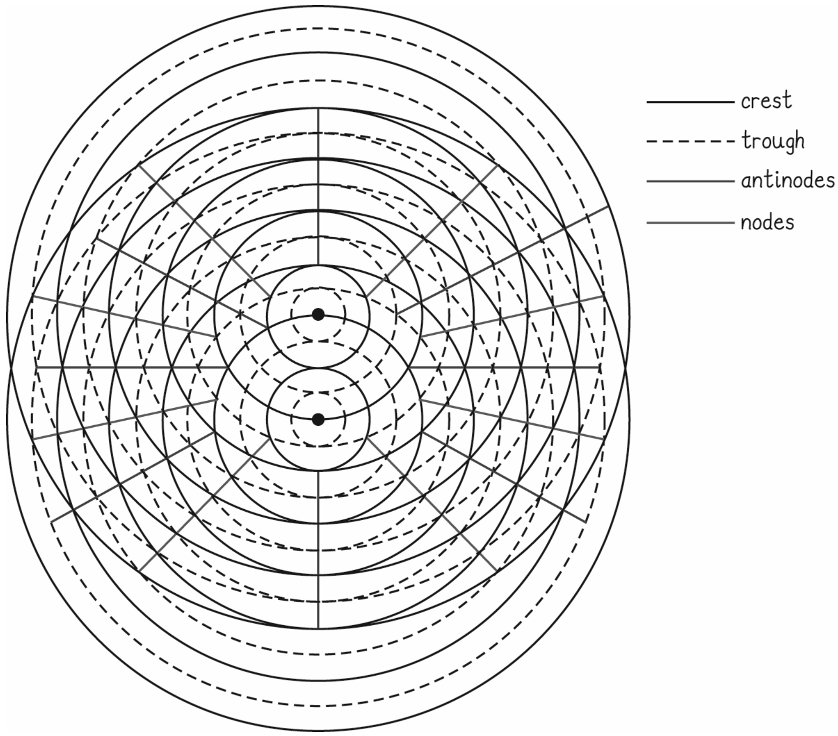
Here A is the amplitude of the oscillation and δ is the phase shift inside the sine functions that results from the additional distance each subsequent source must travel. Call the angle θ the angle between this nodal line and the perpendicular bisector of a line connecting the three sources. It is easy to see geometrically that the path length difference between two adjacent sources is $\Delta\ell = d\sin(\theta)$. This corresponds to a phase of $\delta = d\sin(\theta)2\pi/\lambda$.

If we picture adding phasors, it is easy to see that the only way for these displacements to sum to zero is if the phase shift $\delta = 2\pi m/3$. This corresponds to $2\pi m/3 = d\sin(\theta)2\pi/\lambda$ or $\theta = \sin^{-1}\left(\frac{mc}{3df}\right)$, where m is any integer.

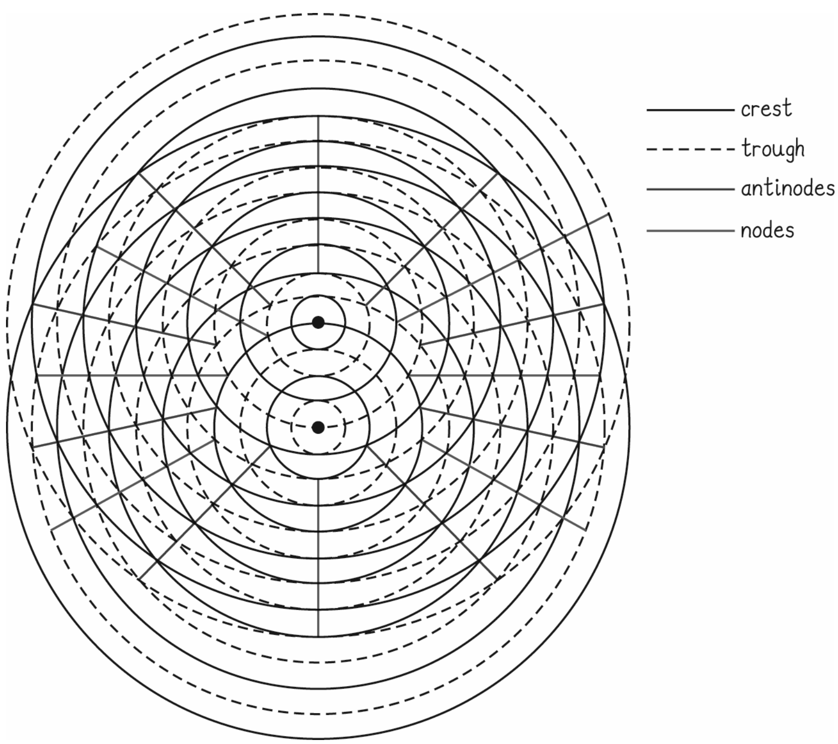
17.19. Sound waves she emitted diffracted around corner and reached your ears.

17.20. Yes, because sound waves diffract around panel edges and reach microphone.

17.21. (*a*)



(*b*)



(*c*) The positions of nodes and antinodes have switched. One easy position to spot this change is along the perpendicular bisector of the line joining the two sources. In part (*a*), since all points on this line are equidistant from the two sources, antinodes occurred along this line. In part (*b*), the sources started out of phase, so that they remained out of phase after travelling equal distances from the sources to the perpendicular bisector. Hence, in part (*b*) nodes occur on this line.

17.22. Adding phasors for the two constituent waves helps us see that the magnitude of the resultant phasor (and amplitude of the combined waves) is $A = \sqrt{(A_1 + A_2\cos(\phi_2))^2 + (A_2\sin(\phi_2))^2} = \sqrt{((1.00\times10^{-8}\text{ m}) + (0.600\times10^{-8}\text{ m})\cos(\pi/3))^2 + ((0.600\times10^{-8}\text{ m})\sin(\pi/3))^2} = 1.40\times10^{-8}\text{ m}.$

Similarly, we see that the phase is given by

$$\phi = \tan^{-1}\left(\frac{A_2\sin(\phi_2)}{A_1 + A_2\cos(\phi_2)}\right) = \tan^{-1}\left(\frac{(0.600\times10^{-8}\text{ m})\sin(\pi/3)}{(1.00\times10^{-8}\text{ m}) + (0.600\times10^{-8}\text{ m})\cos(\pi/3)}\right) = 0.380$$

17.23. Using equation (17.2) we can write $I = \dfrac{P}{A} = \dfrac{P}{4\pi r^2} = \dfrac{(140\text{ W})}{4\pi(1.5\times10^3\text{ m})^2} = 5.0\times10^{-6}\text{ W/m}^2.$

17.24. Using equation (17.2) we can write $I = \dfrac{P}{A} = \dfrac{P}{4\pi r^2} \Rightarrow r = \sqrt{\dfrac{P}{4\pi I}} = \sqrt{\dfrac{(90\text{ W})}{4\pi(5.70\text{ W/m}^2)}} = 1.1\text{ m}.$

17.25. We can determine the intensity of the sound from one mosquito from the sound level given using equation (17.5): $I_{1\text{ mosquito}} = I_{\text{th}}10^{\left(\frac{\beta_{1\text{ mosquito}}}{10\text{ dB}}\right)}$. Clearly the intensity of sound due to 100 such mosquitoes is $I_{100\text{ mosquitoes}} = (100I_{\text{th}})10^{\left(\frac{\beta_{1\text{ mosquito}}}{10\text{ dB}}\right)} = I_{\text{th}}10^{\left(\frac{\beta_{100\text{ mosquitoes}}}{10\text{ dB}}\right)} \Rightarrow 10^{\left(\frac{\beta_{100\text{ mosquitoes}} - \beta_{1\text{ mosquito}}}{10\text{ dB}}\right)} = 100.$ Taking the decadic logarithm of both sides yields $\beta_{100\text{ mosquitoes}} = 2(10\text{ dB}) + \beta_{1\text{ mosquito}} = (20\text{ dB}) + (15\text{ dB}) = 35\text{ dB}.$

17.26. Using equation (17.5) we can write the intensity of the sound from 10 children as $I_{10\text{ children}} = I_{\text{th}}10^{\left(\frac{\beta_{10\text{ children}}}{10\text{ dB}}\right)}$. Similarly, we require an unknown number of children producing a sound level of 82 dB, such that $I_{N\text{ children}} = I_{\text{th}}10^{\left(\frac{\beta_{N\text{ children}}}{10\text{ dB}}\right)}$. Clearly the intensity of sound will scale with the number of children such that we can write $\dfrac{I_{N\text{ children}}}{I_{10\text{ children}}} = \dfrac{N}{10}$. Taking the ratio of the two intensity expressions above, we find

$$\frac{I_{N\text{ children}}}{I_{10\text{ children}}} = \frac{N}{10} = \frac{I_{\text{th}}10^{\left(\frac{\beta_{N\text{ children}}}{10\text{ dB}}\right)}}{I_{\text{th}}10^{\left(\frac{\beta_{10\text{ children}}}{10\text{ dB}}\right)}} \Rightarrow N = 10\times10^{\left(\frac{\beta_{N\text{ children}} - \beta_{10\text{ children}}}{10\text{ dB}}\right)} = 10\times10^{\left(\frac{(82\text{ dB})-(80\text{ dB})}{10\text{ dB}}\right)} = 16\text{ children}$$

17.27. Call the side length of the cubical room ℓ. The distance to the nearest wall is $\ell/2$ and the distance to the corner of the room is $\ell\sqrt{3}/2$. We can write the ratio of the intensities using equation (17.2)

$$\frac{I_{\text{wall}}}{I_{\text{corner}}} = \frac{\left(\dfrac{P}{4\pi r_{\text{wall}}^2}\right)}{\left(\dfrac{P}{4\pi r_{\text{corner}}^2}\right)} = \frac{r_{\text{corner}}^2}{r_{\text{wall}}^2} = \frac{3(\ell^2/4)}{(\ell^2/4)} = 3$$

Similarly, we can write the ratio of the intensities in terms of sound levels using equation (17.5) to obtain

$$\frac{I_{\text{wall}}}{I_{\text{corner}}} = 3 = 10^{\left(\frac{\beta_{\text{wall}} - \beta_{\text{corner}}}{10\text{ dB}}\right)} \Rightarrow \beta_{\text{wall}} - \beta_{\text{corner}} = (10\text{ dB})\log_{10}(3) = 4.8\text{ dB}$$

17.28. Using equations (17.2) and (17.5) we can write the power as

$$P = IA = I(4\pi r^2) = (4\pi r^2)I_{\text{th}}(10^{\beta/10\text{dB}}) = (4\pi(3.0\text{ m})^2)(10^{-12}\text{ W/m}^2)(10^{(80\text{ dB})/10\text{ dB}}) = 11\text{ mW}$$

17.29. The intensity is fixed, meaning

$$I = \frac{P_{\text{total}}}{A_{\text{total}}} = \frac{P_{\text{received}}}{A_{\text{receiver}}} \Rightarrow P_{\text{received}} = \frac{P_{\text{total}}A_{\text{receiver}}}{A_{\text{total}}} = \frac{(12\text{ W})(400\text{ mm}^2)}{(8.0\text{ m}^2)} \times \frac{(1\text{ m}^2)}{(10^6\text{ mm}^2)} = 0.60\text{ mW}$$

17.30. (*a*) We determine the intensity of each source of sound rearranging equation (17.5) and then add the intensities.

$$I = I_{\text{baby}} + I_{\text{music}} = I_{\text{th}}10^{(\beta_{\text{baby}}/10\text{ dB})} + I_{\text{th}}10^{(\beta_{\text{music}}/10\text{ dB})} = I_{\text{th}}(10^{((75\text{ dB})/(10\text{ dB}))} + 10^{((80\text{ dB})/(10\text{ dB}))}) = 1.3\times10^{-4}\text{ W/m}^2$$

(*b*) We use the result of part (*a*) to determine the sound level using equation (17.5)

$$\beta = (10\text{ dB})\log_{10}(I/I_{\text{th}}) = (10\text{ dB})\log_{10}((1.32\times10^{-4}\text{ W/m}^2)/(10^{-12}\text{ W/m}^2)) = 81\text{ dB}$$

17.31. (*a*) The intensity is related to the intensity level through equation (17.5). $I = I_{\text{th}} \times 10^{(\beta/(10\text{ dB}))} = (10^{-12}\text{ W/m}^2)10^{((95\text{ dB})/(10\text{ dB}))} = 3.2\times10^{-3}\text{ W/m}^2$. (*b*) The intensity found in part (*a*) is related to the power of the source through equation (17.2): $P = IA = I(2\pi r^2) = (3.16\times10^{-3}\text{ W/m}^2)(2\pi(20\text{ m})^2) = 7.9\text{ W}$.

17.32. (*a*) We first determine the power of the source (the sound production of the jet engines) using equation (17.5) to determine the intensity and then equation (17.2) to relate this intensity to the power of the source.

$$P = I(4\pi r^2) = I_{\text{th}}(10^{\beta/(10\text{ dB})})(4\pi r_{\text{i}}^2) = (10^{-12}\text{ W/m}^2)(10^{(140\text{ dB})/(10\text{ dB})})(4\pi(50\text{ m})^2) = 3.14\times10^6\text{ W}$$

Now that we know the power, let us write the sound intensity level at the hotel parking lot in terms of the power and unknown distance from the hotel to the airport.

$$\beta = (10\text{ dB})\log_{10}(I/I_{\text{th}}) = (10\text{ dB})\log_{10}\left(\frac{P}{4\pi r_{\text{hotel}}^2 I_{\text{th}}}\right)$$

Or, rearranging, we find $r_{\text{hotel}} = \sqrt{\frac{P}{4\pi I_{\text{th}}}10^{-(\beta_{\text{park}}/(10\text{ dB}))}} = \sqrt{\frac{(3.14\times10^6\text{ W/m}^2)}{4\pi(10^{-12}\text{ W/m}^2)}10^{-((125\text{ dB})/(10\text{ dB}))}} = 2.8\times10^2\text{ m}$. (*b*) The intensity is related to the sound intensity level through equation (17.5):

$$I_{\text{room}} = I_{\text{th}}(10^{(\beta_{\text{room}}/(10\text{ dB}))}) = (10^{-12}\text{ W/m}^2)(10^{((50\text{ dB})/(10\text{ dB}))}) = 1.0\times10^{-7}\text{ W/m}^2$$

17.33. We first determine the intensity of sound in the empty library, and then compare this intensity to the intensity in a full library (with 120 students):

$$I_{\text{empty}} = I_{\text{th}} \times 10^{(\beta_{\text{empty}}/(10\text{ dB}))} = (10^{-12}\text{ W/m}^2)10^{((20\text{ dB})/(10\text{ dB}))} = 1.0\times10^{-10}\text{ W/m}^2$$

$$I_{\text{full}} = I_{\text{th}} \times 10^{(\beta_{\text{full}}/(10\text{ dB}))} = (10^{-12}\text{ W/m}^2)10^{((70\text{ dB})/(10\text{ dB}))} = 1.0\times10^{-5}\text{ W/m}^2$$

Comparing the two intensities, we see that each student contributes an intensity of $I_{\text{student}} = \frac{I_{\text{full}} - I_{\text{empty}}}{120} = \frac{(1.0\times10^{-5}\text{ W/m}^2) - (1.0\times10^{-10}\text{ W/m}^2)}{120} = 8.33\times10^{-8}\text{ W/m}^2$. Thus we expect an intensity with 60 students equal to $I_{60\text{ students}} = 60I_{\text{student}} = 5.00\times10^{-6}\text{ W/m}^2$. Finally, we calculate the intensity level with 60 students:

$$\beta_{60\text{ students}} = (10\text{ dB})\log_{10}(I/I_{\text{th}}) = (10\text{ dB})\log_{10}((5.00\times10^{-6}\text{ W/m}^2)/(10^{-12}\text{ W/m}^2)) = 67\text{ dB}$$

17.34. (*a*) The intensities of the 16 cars will add. So first we find the intensity of a single car at that distance by rearranging equation (17.5): $I_{1\text{ car}} = I_{\text{th}}(10^{(\beta_{1\text{ car}}/(10\text{ dB}))}) = (10^{-12}\text{ W/m}^2)(10^{((90\text{ dB})/(10\text{ dB}))}) = 1.0\times10^{-3}\text{ W/m}^2$.

Thus the intensity of 16 such cars a distance 1.0 m away is $I_{16\text{ cars}} = 16I_{1\text{ car}} = 1.6\times10^{-2}\text{ W/m}^2$. Finally, we find the intensity level corresponding to this intensity using equation (17.5) again. $\beta_{16\text{ cars}} = (10\text{ dB})\log_{10}(I/I_{\text{th}}) = (10\text{ dB})\log_{10}((1.6\times10^{-2}\text{ W/m}^2)/(10^{-12}\text{ W/m}^2)) = 1.0\times10^2\text{ dB}$. (*b*) The power output is independent of the distance from the cars. We can find the power output from the intensity we found in part (*a*). Applying equation (17.2), we can write $P = IA = I(4\pi r^2) = (1.6\times10^{-2}\ \text{W/m}^2)(4\pi(1.0\ \text{m})^2) = 0.20\ \text{W}$. (*c*) Using the power from part (*b*), we first determine the intensity 300 m away: $I = \dfrac{P}{4\pi r^2} = \dfrac{(0.201\ \text{W})}{4\pi(300\ \text{m})^2} = (1.78\times10^{-7}\ \text{W/m}^2)$. Now we apply equation (17.5) to determine $\beta = (10\ \text{dB})\log_{10}(I/I_{\text{th}}) = (10\ \text{dB})\log_{10}((1.78\times10^{-7}\ \text{W/m}^2)/(10^{-12}\ \text{W/m}^2)) = 52\ \text{dB}$.

17.35. (*a*) Applying equation (17.5) we find $\beta = (10\ \text{dB})\log_{10}(I/I_{\text{th}}) = (10\ \text{dB})\log_{10}((9.0\times10^{-6}\ \text{W/m}^2)/(10^{-12}\ \text{W/m}^2)) = 70\ \text{dB}$. (*b*) We rearrange equation (17.2) to find $P = IA = I(4\pi r^2) = (9.0\times10^{-6}\ \text{W/m}^2)(4\pi(2.3\times10^3)^2) = 6.0\times10^2\text{W}$. (*c*) We first use equation (17.5) to determine the intensity of sound required to produce an intensity level of 100 dB: $I = I_{\text{th}}(10^{(\beta/(10\ \text{dB}))}) = (10^{-12}\ \text{W/m}^2)(10^{((100\ \text{dB})/(10\ \text{dB}))}) = 1.0\times10^{-2}\ \text{W/m}^2$. Dividing this intensity by the intensity of a single whale's song, we find $N = \dfrac{I_{N\ \text{whales}}}{I_{1\ \text{whale}}} = \dfrac{(1.0\times10^{-2}\ \text{W/m}^2)}{(9.0\times10^{-6}\ \text{W/m}^2)} =$ 1.1×10^3 whales.

17.36. (*a*) We determine the power output by rearranging equation (17.2): $P = IA = I(4\pi r^2) = (0.050\ \text{W/m}^2)(4\pi(12\ \text{m})^2) = 90\ \text{W}$. (*b*) We assume the distance remains 12 m. Then the intensity is given by applying equation (17.2): $I = \dfrac{P}{4\pi r^2} = \dfrac{(92\ \text{W})}{4\pi(12\ \text{m})^2} = 0.051\ \text{W/m}^2$.

17.37. Let us first consider how the locations of these points affects the intensity observed at a given point. For some arbitrary point P (not necessarily R or Q) call the distance from P to source 1 d_{P1}, and call the distance from P to source 2 d_{P2}. From the discussion of travelling waves in Chapter 16 of PRIN, we know that the energy carried in any wave is proportional to its amplitude squared. The intensity is proportional to the power of the source, so we write $I \sim a^2$ or $I = \kappa a^2$, where κ is an unknown constant with units W/m^4. Relating this to equation (17.2) lets us write

$$I = \frac{P}{4\pi r^2} = \kappa a^2 \qquad (1)$$

Equation (1) should hold for any source. We also know the principle of superposition, which says we can add the displacements due to the waves from sources 1 and 2. Thus, we write the total amplitude of the resulting wave as the sum (in the case of constructive interference) or the difference (in the case of destructive interference) of the two amplitudes due to each source:

$$a_{\text{total}} = a_1 \pm a_2 = \sqrt{\frac{P_1}{4\pi\kappa d_{\text{P1}}^2}} \pm \sqrt{\frac{P_2}{4\pi\kappa d_{\text{P2}}^2}} = \sqrt{\frac{P}{4\pi\kappa}}\left(\frac{1}{d_{\text{P1}}} \pm \frac{1}{d_{\text{P2}}}\right)$$

Now we apply equation (1) to this sum or difference of amplitudes to obtain an expression for the intensity of the superposition of the two waves:

$$I_{\text{total}} = \kappa a_{\text{total}}^2 = \frac{P}{4\pi}\left(\frac{1}{d_{\text{P1}}} \pm \frac{1}{d_{\text{P2}}}\right)^2 \qquad (2)$$

We will use equation (2) to determine all required intensities, but we must first determine the possible values of d_{P1} and d_{P2} for the two points in question (P = R and P = Q). Let us begin with point R. Because point R is on a line of maximum destructive interference, we know that the waves from the two sources must be out of phase by half an oscillation at point R. Since the waves are produced in phase, the only way for them to be half an oscillation out of phase is for the distance d_{R2} to differ from d_{R1} by $\left(n + \dfrac{1}{2}\right)\lambda$. Thus, the possible values of d_{R2} are 7.5λ, 8.5λ, 9.5λ, or 10.5λ. Note that because the sources are separated by a distance of 2λ, the minimum distance that is geometrically possible for d_{R2} is 7λ and the maximum possible value of d_{R2} is $11\lambda,$ thus giving us only the above mentioned possible values. Thus, the possible intensities at the point R are:

$$I_R = \frac{P}{4\pi}\left(\frac{1}{(9.0\lambda)} - \frac{1}{d_{R2}}\right)^2 \text{ where } d_{R2} = 7.5\lambda,\ 8.5\lambda,\ 9.5\lambda, \text{ or } 10.5\lambda$$

The process is similar for point Q. Because it is along a line of maximum constructive interference, d_{Q2} must differ from d_{Q1} by and integer number of wavelengths $n\lambda.$ Again, considering the separation between sources, this gives us possible values of d_{Q2} equal to $4\lambda,\ 5\lambda,\ 6\lambda,\ 7\lambda,$ or $8\lambda.$ The only other difference for point Q is that here the amplitudes due to sources 1 and 2 are added in equation (2), rather than subtracted. Thus the possible intensities at the point Q are:

$$I_Q = \frac{P}{4\pi}\left(\frac{1}{(6.0\lambda)} + \frac{1}{d_{Q2}}\right)^2 \text{ where } d_{Q2} = 4\lambda,\ 5\lambda,\ 6\lambda,\ 7\lambda, \text{ or } 8\lambda$$

17.38. We can use equation (17.2) to write the power of your stereo in terms of the intensity of sound in your living room: $P = IA = I_{\text{living room}}(4\pi r^2_{\text{living room}}).$ We can also write the intensity of sound in the living room in terms of the intensity levels given using equation (17.5): $I_{\text{living room}} = I_{\text{th}}(10^{(\beta_{\text{living room}}/(10\text{ dB}))}),$ such that the power emitted by your stereo can be written $P = I_{\text{th}}(10^{(\beta_{\text{living room}}/(10\text{ dB}))})(4\pi r^2_{\text{living room}}).$ Applying this expression to the instants before and after you turn down your stereo, we can calculate the difference in the power output of your stereo:

$$\Delta P = P_f - P_i = I_{\text{th}}(4\pi r^2_{\text{living room}})(10^{(\beta_{\text{living room,f}}/(10\text{ dB}))} - 10^{(\beta_{\text{living room,i}}/(10\text{ dB}))})$$
$$= (10^{-12}\text{ W/m}^2)(4\pi(8.0\text{ m})^2)(10^{((45\text{ dB})/(10\text{ dB}))} - 10^{((50\text{ dB})/(10\text{ dB}))}) = -5.5\times10^{-5}\text{ W}$$

Thus, you reduced the power output of your stereo by a magnitude of 5.5×10^{-5} W.

17.39. You hear both frequencies because a frequency difference of $\Delta f = 20$ Hz should be easily discernable. You would also hear 20 beats of louder and quieter sound each second. That is much too fast for a person to count the beats, or even recognize them as beats. In fact, any oscillation with a frequency greater than 20 Hz is recognized by the human ear as a separate sound. Thus you would hear a very low tone in addition to the two frequencies mentioned previously.

17.40. Equation (17.8) tell us $f_{\text{beat}} = |f_1 - f_2| = |(355\text{ Hz}) - (350\text{ Hz})| = 5$ Hz.

17.41. You are hearing the average of the two frequencies, so $f_{\text{av}} = \frac{1}{2}(f_1 + f_2),$ and we know from equation (17.8) that $f_{\text{beat}} = |f_1 - f_2|.$ Solving, we find $f_1 = 198$ Hz and $f_2 = 194$ Hz.

17.42. Because you hear four beats per second, you know the unknown frequency must differ from 762 Hz by 4 Hz. If the frequency were 758 Hz, it would not be detectable by the detector. This leaves us with the answer of 766 Hz.

17.43. Increasing frequency A causes beat frequencies to increase, no matter which other frequency is played. This mean that the difference $f_A - f_{\text{other}}$ increases for any other frequency, meaning frequency A must be greater than any other frequency. Similarly, increasing the frequency of B increases the beat frequency when frequency C is also played. This tells us frequency B is greater than frequency C. But increasing frequency B causes the beat frequencies to decrease when either frequency D or A is also played. Thus, frequency B is less than frequency D (and also less than frequency A, which we already knew). Collecting these results, we find $C < B < D < A$.

17.44. When you place modeling clay on a tuning fork, you increase the amount of mass that oscillates back and forth. You do nothing to the rigidity of the fork that drives the oscillation (the restoring force). Thus, you must decrease the frequency of the oscillation. When you did this, it caused the beat frequency to increase, meaning you must have lowered the frequency of the tuning fork that already had the lower of the two frequencies. Otherwise, if the clay had been placed on the higher frequency fork, lowering the frequency would have brought its frequency closer to that of the other fork and the beat frequency would have decreased. Thus, you placed the clay on the 524 Hz fork.

17.45. The possible beat frequencies are equal to the pairwise differences between the three frequencies being emitted. These are $|f_1 - f_2| = |(262\text{ Hz}) - (264.3\text{ Hz})| = 2\text{ Hz}$, $|f_2 - f_3| = |(264.3\text{ Hz}) - (258\text{ Hz})| = 6\text{ Hz}$, and $|f_3 - f_1| = |(258\text{ Hz}) - (262\text{ Hz})| = 4\text{ Hz}$. So the possible beat frequencies are 2 Hz, 4 Hz, and 6 Hz.

17.46. We are not told which is larger: the beat frequency or the average frequency. Let us first question whether it is possible that the beat frequency might be twice the average frequency. Then we would write $2f_{\text{av}} = f_{\text{beat}} \Rightarrow (f_1 + f_2) = (f_1 - f_2)$, which has no solution other than $f_2 = 0$. We thus proceed with the calculation noting that the average frequency is the larger of the two. We have $f_{\text{av}} = 2f_{\text{beat}} \Rightarrow \frac{1}{2}(f_1 + f_2) = 2(f_1 - f_2)$, where we have arbitrarily assigned f_1 to be the higher of the two frequencies. Then $(f_1 + f_2) = 4(f_1 - f_2) \Rightarrow \frac{f_1}{f_2} = \frac{5}{3}$. It is not specified that we must give the ratio of the higher frequency to the lower, so the answer $\frac{f_2}{f_1} = \frac{3}{5}$ is also perfectly valid.

17.47. We can write the frequency of either violin string as $f = \frac{c}{\lambda} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$. Nothing changes about the linear mass density or the length of the string as the tension is increased. Thus, we can write

$$\frac{f_\text{f}}{f_\text{i}} = \frac{\frac{1}{2\ell}\sqrt{\frac{T_\text{f}}{\mu}}}{\frac{1}{2\ell}\sqrt{\frac{T_\text{i}}{\mu}}} = \sqrt{\frac{T_\text{f}}{T_\text{i}}} \Rightarrow f_\text{f} = f_\text{i}\sqrt{\frac{T_\text{f}}{T_\text{i}}}$$

Thus, the difference between the two frequencies can be written

$$f_{\text{beats}} = f_\text{f} - f_\text{i} = f_\text{i}\left(\sqrt{\frac{T_\text{f}}{T_\text{i}}} - 1\right) = (660\text{ Hz})\left(\sqrt{\frac{(102\text{ N})}{(100\text{ N})}} - 1\right) = 6.6\text{ Hz}$$

17.48. (*a*) The beat frequency is 2.5 Hz, so $f_2 - f_1 = 2.5\text{ Hz}$. By definition, the average frequency must differ from each of the two combining frequencies by the same amount. Clearly, the two frequencies are given by $(3500\text{ Hz}) \pm (1.25\text{ Hz})$. This gives us frequencies of 3499 Hz and 3501 Hz. (*b*) We can add the intensities from the two sources to obtain $I = I_1 + I_2 = \frac{2P}{4\pi r^2} = \frac{2(60\text{ W})}{4\pi(15\text{ m})^2} = 0.042\text{ W/m}^2$.

17.49. The shift magnitude decreases. The discussion of the Doppler effect involves the speed at which a source approaches a listener or detector. Unless you are standing directly on the tracks (not recommended) the velocity of the train is not directed toward you. Rather there is some component of the velocity directed toward you. The speed with which the train “approaches” you refers only to this component. If you stand very near the tracks, the velocity of the train will be almost directly toward you. As you move away, the component of the velocity that is directed toward you becomes smaller and smaller, reducing the magnitude of the Doppler effect.

17.50. (*a*) We apply equation (17.14) because the source (your car horn) is approaching the observer (your friend). Thus

$$f_\text{o} = \left(\frac{c}{c - v_\text{s}}\right)f_\text{s} = \left(\frac{(343\text{ m/s})}{(343\text{ m/s}) - (19.4\text{ m/s})}\right)(360\text{ Hz}) = 382\text{ Hz}$$

(*b*) Once we pass the friend, the source will be receding away from the listener. In this case, we use equation (17.15) to obtain

$$f_{\text{o}} = \left(\frac{c}{c+v_{\text{s}}}\right) f_{\text{s}} = \left(\frac{(343\text{ m/s})}{(343\text{ m/s})+(19.4\text{ m/s})}\right)(360\text{ Hz}) = 341\text{ Hz}$$

17.51. We use equation (17.17), because the observer (you) is approaching the source (the bells). Thus

$$f_{\text{o}} = \left(\frac{c+v_{\text{o}}}{c}\right) f_{\text{s}} = \left(\frac{(343\text{ m/s})+(4.47\text{ m/s})}{(343\text{ m/s})}\right)(400\text{ Hz}) = 405\text{ Hz}$$

17.52. We start with equations (17.13) and (17.14) to obtain expressions for the frequencies you observe when the car is approaching you, and receding from you. Since the frequency must be higher as the car approaches you than when it recedes, and since one octave difference corresponds to a factor of two, we can write

$$f_{\text{o,approaching}} = 2 f_{\text{o,receding}} \Rightarrow \left(\frac{c}{c-v_{\text{s}}}\right) f_{\text{s}} = 2\left(\frac{c}{c+v_{\text{s}}}\right) f_{\text{s}} \text{ or } v_{\text{s}} = c/3 = (343\text{ m/s})/3 = 114\text{ m/s}$$

We are asked to estimate. So we know the magnitude of the speed of the car is approximately 1×10^{2} m/s.

17.53. (*a*) Let the percentage by which we wish to change the observed frequency by *p*. Since the observer is moving away from the source, we use equation (17.19) to write $f_{\text{s}} - f_{\text{o}} = \left(1 - \left(\frac{c-v_{\text{o}}}{c}\right)\right) f_{\text{s}} = pf_{\text{s}}$, or $v_{\text{o}} = pc$. So for the case $p = 0.01$, we have $v_{\text{o}} = pc = (0.01)(343\text{ m/s}) = 3\text{ m/s}$. (*b*) In part (*a*) we showed that for a reduction in observed frequency of percent *p*, the speed required is $v_{\text{o}} = pc$. In this case $v_{\text{o}} = pc = (0.10)(343\text{ m/s}) = 34\text{ m/s}$. (*c*) In part (*a*) we showed that for a reduction in observed frequency of percent *p*, the speed required is $v_{\text{o}} = pc$. In this case $v_{\text{o}} = pc = \frac{1}{2}(343\text{ m/s}) = 172\text{ m/s}$.

17.54. We have two unknowns: the speed of the float and the frequency being played by the flute. We can write equations describing the Doppler shift in the frequency as the float approaches and recedes from the observer using equations (17.13) and (17.14):

$$f_{\text{o,approaching}} = \left(\frac{c}{c-v_{\text{s}}}\right) f_{\text{s}} \tag{1}$$

$$f_{\text{o,receding}} = \left(\frac{c}{c+v_{\text{s}}}\right) f_{\text{s}} \tag{2}$$

Solving equation (1) for the frequency of the source, and inserting the expression into equation (2), we obtain

$$f_{\text{o,receding}} = \left(\frac{c-v_{\text{s}}}{c+v_{\text{s}}}\right) f_{\text{o,approaching}}$$

Or equivalently

$$v_{\text{s}} = \frac{(f_{\text{o,approaching}} - f_{\text{o,receding}})}{(f_{\text{o,approaching}} + f_{\text{o,receding}})} c = \frac{((352\text{ Hz})-(347\text{ Hz}))}{((352\text{ Hz})+(347\text{ Hz}))}(343\text{ m/s}) = 2\text{ m/s}$$

17.55. (*a*) Because both the source and the observer are in motion, we use equation (17.21):

$$f_{\text{o}} = \left(\frac{c-v_{\text{o}}}{c-v_{\text{s}}}\right) f_{\text{s}} = \left(\frac{(343\text{ m/s})-(2.70\text{ m/s})}{(343\text{ m/s})-(2.00\text{ m/s})}\right)(300\text{ Hz}) = 299\text{ Hz}$$

(*b*) We can continue using equation (17.21) in this case, but we insert zero for the speed of the observer. This yields

$$f_{\text{o}} = \left(\frac{c-v_{\text{o}}}{c+v_{\text{s}}}\right) f_{\text{s}} = \left(\frac{(343\text{ m/s})-(0)}{(343\text{ m/s})+(2.00\text{ m/s})}\right)(300\text{ Hz}) = 298\text{ Hz}$$

17.56. Call the direction of your motion the $+x$ direction. Since the cars cover the 200 m between them in 3.0 s, the one component of the relative velocity of the two cars is $v_{\text{you},x} - v_{\text{police},x} = \dfrac{(200 \text{ m})}{(3.00 \text{ s})} = 66.67 \text{ m/s}.$ We know the x component of your velocity is $v_{\text{you},x} = 26.94 \text{ m/s},$ which means the x component of the police car's velocity is $v_{\text{police},x} = v_{\text{you},x} - (66.67 \text{ m/s}) = (26.94 \text{ m/s}) - (66.67 \text{ m/s}) = -39.72 \text{ m/s}.$ Now we use equation (17.21) to relate the observed frequency to the emitted frequency. We note that the signs in equation (17.21) are not necessarily the same as the signs of the x components of the velocities. The signs in equation (17.21) are determined either by reviewed the description in Principles, or by noting whether each motion should serve to increase or decrease the frequency. In this case, we have

$$f_\text{o} = \left(\frac{c+v_\text{o}}{c-v_\text{s}}\right) f_\text{s} \Rightarrow f_\text{s} = \left(\frac{c-v_\text{s}}{c+v_\text{o}}\right) f_\text{o} = \left(\frac{(343 \text{ m/s}) - (39.72 \text{ m/s})}{(343 \text{ m/s}) + (26.94 \text{ m/s})}\right)(310 \text{ Hz}) = 254 \text{ Hz}$$

17.57. (*a*) We apply equation (17.21), because both the source and the observer are in motion relative to the medium (air). The signs in equation (17.21) are determined either by reviewed the description in Principles, or by noting whether each motion should serve to increase or decrease the frequency. In this case, we have

$$f_\text{o} = \left(\frac{c+v_\text{o}}{c-v_\text{s}}\right) f_\text{s} = \left(\frac{(343 \text{ m/s}) + (25 \text{ m/s})}{(343 \text{ m/s}) - (18.1 \text{ m/s})}\right)(400 \text{ Hz}) = 4.5\times10^2 \text{ Hz}$$

(*b*) The solving process is identical to that in part (*a*), but now the signs in equation (17.21) will change:

$$f_\text{o} = \left(\frac{c-v_\text{o}}{c+v_\text{s}}\right) f_\text{s} = \left(\frac{(343 \text{ m/s}) - (25 \text{ m/s})}{(343 \text{ m/s}) + (18.1 \text{ m/s})}\right)(400 \text{ Hz}) = 3.5\times10^2 \text{ Hz}$$

17.58. (*a*) We apply equation (17.17) because here the observer is approaching a stationary source. Thus

$$f_\text{o} = \left(\frac{c+v_\text{o}}{c}\right) f_\text{s} = \left(\frac{(343 \text{ m/s}) + (5.89 \text{ m/s})}{(343 \text{ m/s})}\right)(150 \text{ Hz}) = 153 \text{ Hz}$$

(*b*) We apply equation (17.19) because here the observer is receding away from a stationary source. Thus

$$f_\text{o} = \left(\frac{c-v_\text{o}}{c}\right) f_\text{s} \Rightarrow v_\text{o} = \left(1 - \frac{f_\text{o}}{f_\text{s}}\right) c = \left(1 - \frac{(149 \text{ Hz})}{(150 \text{ Hz})}\right)(343 \text{ m/s}) = 2.29 \text{ m/s}$$

17.59. (*a*) The electronic machine gun on A emits 300 rounds per minute. So a burst that lasts 10 seconds according to pilot A will consist of 50 rounds. That is the total number of pulses emitted, and that number cannot be different for different observers. Pilot B agrees that there are a total of 50 sound pulses in that burst. (*b*) We use equation (17.21) to relate the frequencies of sound pulses heard by the two pilots, which can trivially be rewritten in terms of the period of one sound pulse:

$$f_\text{o} = \left(\frac{c+v_\text{o}}{c-v_\text{s}}\right) f_\text{s} \Rightarrow T_\text{o} = \left(\frac{c-v_\text{s}}{c+v_\text{o}}\right) T_\text{s} = \left(\frac{(343 \text{ m/s}) - (79.17 \text{ m/s})}{(343 \text{ m/s}) + (81.94 \text{ m/s})}\right)\left(\frac{10 \text{ s}}{50}\right) = 0.124 \text{ s}$$

So the duration of 50 such pulses is $50T_\text{o} = 50(0.124 \text{ s}) = 6.2 \text{ s}.$

17.60. (*a*) This is a simple case of a source receding from the observer, so we use equation (17.15):

$$f_\text{o} = \left(\frac{c}{c+v_\text{s}}\right) f_\text{s} = \left(\frac{(343 \text{ m/s})}{(343 \text{ m/s}) + (18.0 \text{ m/s})}\right)(560 \text{ Hz}) = 532 \text{ Hz}$$

(*b*) It may seem like this situation is physically the same as in part (*a*). But sound waves propagate through the air, so the relevant speeds are those relative to the air. So we use $v_\text{o} = 12.0 \text{ m/s}$ and $v_\text{s} = 6.0 \text{ m/s},$ and equation (17.21) to write

$$f_\text{o} = \left(\frac{c-v_\text{o}}{c+v_\text{s}}\right) f_\text{s} = \left(\frac{(343 \text{ m/s}) - (12.0 \text{ m/s})}{(343 \text{ m/s}) + (6.0 \text{ m/s})}\right)(560 \text{ Hz}) = 531 \text{ Hz}$$

The difference between (*a*) and (*b*) is small, but they are not the same. (*c*) The speed of the buzzer relative to the surrounding air is 6.0 m/s. But we do not know where the person by the tracks is standing (ahead of you or behind you). We do not know if the source is approaching the observer or receding from the observer. Thus we find two possible answers. If the person is standing such that the buzzer approaches him or her, we have

$$f_o = \left(\frac{c}{c+v_s}\right) f_s = \left(\frac{(343\text{ m/s})}{(343\text{ m/s}) - (6.0\text{ m/s})}\right)(560\text{ Hz}) = 570\text{ Hz}$$

If the buzzer is thrown such that it recedes away from the person, then the frequency heard by the person is

$$f_o = \left(\frac{c}{c+v_s}\right) f_s = \left(\frac{(343\text{ m/s})}{(343\text{ m/s}) + (6.0\text{ m/s})}\right)(560\text{ Hz}) = 550\text{ Hz}$$

17.61. The highest frequency that is audible to humans is approximately 20 kHz. Clearly, we can only lower the frequency heard from a stationary source by moving the observer away from the source. We can apply equation (17.21), inserting zero for the speed of the source. We find

$$f_o = \left(\frac{c - v_o}{c}\right) f_s \Rightarrow v_o = \left(1 - \frac{f_o}{f_s}\right) c = \left(1 - \frac{(20\text{ kHz})}{(21\text{ kHz})}\right)(343\text{ m/s}) = 16\text{ m/s}$$

So, you must move at a speed of 16 m/s away from the source in order for the whistle to become audible.

17.62. This problem involves reflection, which can be tricky. Initially, waves are approaching the submarine, and we are interested in how many pulses of sound strike the hull each second. From this perspective, the submarine plays the role of an observer (in that it is receiving a sound signal) that is receding away from the source. Let us call this intermediate frequency with which the sound reaches the submarine f_{int}. Clearly, through equation (17.19) $f_{int} = \left(\frac{c - v_o}{c}\right) f_s = \left(\frac{c - v}{c}\right) f$. But as waves are absorbed and re-emitted by the hull in the reflection process, the submarine then acts as the source of waves. Now the source is receding and the frequency detected by a stationary observer will be $f' = \left(\frac{c}{c + v_s}\right) f_{int} = \left(\frac{c}{c + v}\right) f_{int}$ or, equivalently $f' = \left(\frac{c - v}{c + v}\right) f$.

17.63. This problem involves reflection, which can be tricky. Initially, waves are emitted by the device and approach the stationary flat surface. From this perspective, the device plays the role of a source that is approaching the observer (in the sense that the flat surface can be considered an observer, as it receives the sound signal). Let us call this intermediate frequency with which the sound reaches the flat surface f_{int}. Clearly, through equation (17.13) $f_{int} = \left(\frac{c}{c - v_s}\right) f_s$. But as waves are absorbed and re-emitted by the surface in the reflection process, the surface then acts as the source of waves, and the moving device acts as an observer approaching a stationary source. Using equation (17.17), we see the device will detect a frequency $f' = \left(\frac{c + v_o}{c}\right) f_{int} = \left(\frac{c + v_o}{c - v_s}\right) f_s = \left(\frac{(343\text{ m/s}) + (22.2\text{ m/s})}{(343\text{ m/s}) - (22.2\text{ m/s})}\right)(700\text{ Hz}) =$ 8.0×10^2 Hz.

17.64. (*a*) This is a case of a moving source approaching a stationary observer, so we apply equation (17.13) and find $f_{int} = \left(\frac{c}{c - v_s}\right) f_s = \left(\frac{(343\text{ m/s})}{(343\text{ m/s}) - (16.0\text{ m/s})}\right)(680\text{ Hz}) = 713\text{ Hz}$. (*b*) This is also a case of a moving source approaching a stationary observer, so we again apply equation (17.13): $f_{int} = \left(\frac{c}{c - v_s}\right) f_s = \left(\frac{(343\text{ m/s})}{(343\text{ m/s}) - (15.0\text{ m/s})}\right)(670\text{ Hz}) = 701\text{ Hz}$. (*c*) We are not told which beat frequency we are to find: the beat frequency detected by student A or the beat frequency detected by student B. We calculate both, and we start with student A. Student A hears the sound emitted by the football that he just threw (call this football A), as an observer hearing a sound from a receding source. So we can apply equation (17.15) to write

$$f_{\mathrm{AA}} = \left(\frac{c}{c+v_{\mathrm{A}}}\right) f_{\mathrm{A}} = \left(\frac{(343\ \mathrm{m/s})}{(343\ \mathrm{m/s})+(16.0\ \mathrm{m/s})}\right)(680\ \mathrm{Hz}) = 649.7\ \mathrm{Hz}$$

Student A similarly hears the sound from the ball thrown by student B (call this football B) as an observer hearing a sound from an approaching source. So we can apply equation (17.14) to write

$$f_{\mathrm{AB}} = \left(\frac{c}{c-v_{\mathrm{B}}}\right) f_{\mathrm{B}} = \left(\frac{(343\ \mathrm{m/s})}{(343\ \mathrm{m/s})-(15.0\ \mathrm{m/s})}\right)(670\ \mathrm{Hz}) = 700.6\ \mathrm{Hz}$$

Thus, the beat frequency detected by student A is $f_{\mathrm{beat,A}} = |f_{\mathrm{AA}} - f_{\mathrm{AB}}| = |(649.7\ \mathrm{Hz}) - (700.6\ \mathrm{Hz})| = 51\ \mathrm{Hz}.$

Performing exactly the same processes, we find the frequencies detected by B due to footballs A and B respectively are:

$$f_{\mathrm{BA}} = \left(\frac{c}{c-v_{\mathrm{A}}}\right) f_{\mathrm{A}} = \left(\frac{(343\ \mathrm{m/s})}{(343\ \mathrm{m/s})-(16.0\ \mathrm{m/s})}\right)(680\ \mathrm{Hz}) = 713.3\ \mathrm{Hz}$$

$$f_{\mathrm{BB}} = \left(\frac{c}{c+v_{\mathrm{B}}}\right) f_{\mathrm{B}} = \left(\frac{(343\ \mathrm{m/s})}{(343\ \mathrm{m/s})+(15.0\ \mathrm{m/s})}\right)(670\ \mathrm{Hz}) = 641.9\ \mathrm{Hz}$$

This means the beat frequency detected by student B is $f_{\mathrm{beat,B}} = |f_{\mathrm{BA}} - f_{\mathrm{BB}}| = |(713.3\ \mathrm{Hz}) - (641.9\ \mathrm{Hz})| = 71\ \mathrm{Hz}.$

So the beat frequencies are 51 Hz (detected by student A) and 71 Hz (detected by student B). Note that neither student actually hears beats, because the beat frequencies are in the range of normal human hearing as separate tones.

17.65. Using equation (17.22), we can write $v_{\mathrm{s}} = \dfrac{c}{\sin(\theta)} = \dfrac{(343\ \mathrm{m/s})}{\sin(47°)} = 4.7\times10^{2}\ \mathrm{m/s}.$

17.66. Using equation (17.22), we can write $\theta = \sin^{-1}\left(\dfrac{c}{v_{\mathrm{s}}}\right) = \sin^{-1}\left(\dfrac{(20.1\ \mathrm{km/h})}{(48.0\ \mathrm{km/h})}\right) = 24.8°.$

17.67. Using equation (17.22), we can write $c = v_{\mathrm{s}}\sin(\theta) = (73.0\ \mathrm{km/h})\sin(14.3°) = 18.0\ \mathrm{km/h}$ or, equivalently 5.00 m/s.

17.68. The plane travelling at mach 1.5 is heard first. Because the plane travelling at mach 1.5 is the slower of the two, it will have a wider angle to its Mach cone. It is easy to see geometrically that this wave will reach you first. Another way to understand this is to think about the sound that will reach your ears first. It is not sound that was produced by the jets as they flew over your head (those sounds will reach you at the same time, because sound propagates at the same speed from each source). The sound that reaches your ears first is sound that was emitted a short time before the jets passed over your head. For any time interval before the jets passed overhead, the faster jet was farther away.

17.69. Drawing a diagram of this setup verifies that the angle between your line of sight and the horizontal is an opposite interior angle to the angle of the Mach cone at the front of the jet. Thus, those two angles are the same, and we can write $\theta = \sin^{-1}\left(\dfrac{c}{v_{\mathrm{s}}}\right) = \sin^{-1}\left(\dfrac{1}{1.30}\right) = 50.3°.$

17.70. As the plane emits sound, this sound disturbs the surrounding air, causing turbulence, and even the condensation of water. The region of disturbed air and water droplets moves out away from the plane at the speed of sound. As the plane passes the speed of sound, it catches up to this region of turbulence. Once the jet passes the speed of sound, the jet moves forward faster than the region of turbulence and water droplets, such that this region no longer has any adverse effect on the ride.

17.71. The reasoning is invalid because it is based on the sonic boom being created at only one instant, the instant the plane attains Mach 1 speed. A sonic boom is continuous at all speeds above Mach 1.

17.72. Imagine a line perpendicular to the shock wave of the plane and passing through the woman's position on the ground. This can also be thought of as a line from the woman to the part of the shock wave that will strike her. Simple geometry shows that the angle between this line and vertical is also $30°$. If we call the height of the plane *h*, then the length of this line is $\ell = h\cos(30°) = (20{,}000 \text{ m})\cos(30°) = 17.3 \text{ km}$. This line is the path that sound will travel to the woman, so we can determine the time as $\Delta t = \dfrac{\ell}{c} = \dfrac{1.73\times10^4 \text{ m}}{343 \text{ m/s}} = 50 \text{ s}$.

17.73. (*a*) We determine the angle by simply applying equation (17.22): $\theta = \sin^{-1}\left(\dfrac{c}{v_s}\right) = \sin^{-1}\left(\dfrac{1}{2.1}\right) = 28°$. (*b*) Imagine a line perpendicular to the shock wave of the bullet and passing through the position of the microphone. This can also be thought of as a line from the microphone to the part of the shock wave that will strike it. Simple geometry shows that the angle between this line and vertical is also $28.4°$. If we call the vertical height of the bullet above the microphone *h*, then the length of this line is $\ell = h\cos(28.4°)$. This line is the path that sound will travel to the microphone, so we can equate $c = \dfrac{\ell}{\Delta t} = \dfrac{h\cos(28.4°)}{\Delta t}$, such that $h = \dfrac{c\Delta t}{\cos(28.4°)} = \dfrac{(343 \text{ m/s})(0.0021 \text{ s})}{\cos(28.4°)} = 0.82 \text{ m}$.

17.74. (*a*) Imagine a line perpendicular to the shock wave of the plane and passing through your position on the ground. This can also be thought of as a line from you to the part of the shock wave that will strike you. Simple geometry shows that the angle between this line and vertical is also $42°$. If we call the height of the plane *h*, then the length of this line is $\ell = h\cos(42°) = (3{,}000 \text{ m})\cos(42°) = 2.23 \text{ km}$. This line is the path that sound will travel to you, so we can determine the time as $\Delta t = \dfrac{\ell}{c} = \dfrac{2.23\times10^3 \text{ m}}{343 \text{ m/s}} = 6.5 \text{ s}$. (*b*) We use equation (17.22) to determine the speed of the jet: $v_s = \dfrac{c}{\sin(\theta)} = \dfrac{(343 \text{ m/s})}{\sin(42°)} = 513 \text{ m/s}$. Thus $d = v\Delta t = (513 \text{ m/s})(6.50 \text{ s}) = 3.3\times10^3 \text{ m}$.

17.75. Let us choose axes such that the car move along the $+y$ direction. Call the perpendicular distance from you to the car's path Δx. Imagine a line perpendicular to the shock wave of the plane and passing through your position on the ground. This can also be thought of as a line from you to the part of the shock wave that will strike you. Simple geometry shows that the angle between this line and the *y* axis is also $37.0°$. The length of this line is $\ell = \Delta x\cos(37.0°)$. This line is the path that sound will travel to you, so we can relate this distance to the time required using the speed of sound: $\Delta t = \dfrac{\ell}{c} = \dfrac{\Delta x\cos(37.0°)}{c} \Rightarrow \Delta x = \dfrac{c\Delta t}{\cos(37.0°)} = \dfrac{(343 \text{ m/s})(0.045 \text{ s})}{\cos(37.0°)} = 19 \text{ m}$.

17.76. One hears no sound from the jet before the sonic boom. After the sonic boom one hears basically normal sound from the rear of the plane, although the frequency of the sound would be Doppler shifted.

17.77. No. The boom arises because the object moves faster than sound. This means the source emits sound from many positions, and the sound waves from all these positions combine into a high-amplitude (very loud) sound. This effect exists for all speeds greater than Mach 1. But exceeding Mach 2 does nothing to the sonic boom, except narrow the angle of the Mach cone.

17.78. Yes, the pilots can hear each other. The speed of sound is given relative to the motion of the medium. Since the air inside the cockpit is at rest relative to the pilots, they have no problem hearing each other. If one tried to fly at supersonic speeds with an open cockpit (which would present many other problems) the pilot in front would not be able to hear the pilot in the rear.

17.79. (*a*) $f_1 = \dfrac{b}{2\pi} = \dfrac{(2512 \text{ s}^{-1})}{2\pi} = 399.8 \text{ Hz}$ and $f_2 = \dfrac{d}{2\pi} = \dfrac{(2575 \text{ s}^{-1})}{2\pi} = 409.8 \text{ Hz}$ (*b*) Taking the difference of the two frequencies in part (*a*) yields $f_{\text{beat}} = |f_1 - f_2| = |(399.8 \text{ Hz}) - (409.8 \text{ Hz})| = 10.0 \text{ Hz}$.

17.80. The highest frequency the driver will hear corresponds to the moment when the car is moving directly toward the spectator (the air horn). At this point, we can apply equation (17.17) to find

$$f_{\text{o,max}} = \left(\frac{c+v_{\text{o}}}{c}\right)f_{\text{s}} = \left(\frac{(343\text{ m/s})+(97.2\text{ m/s})}{(343\text{ m/s})}\right)(400\text{ Hz}) = 513\text{ Hz}$$

The lowest frequency will be head by the driver as the car recedes away from the air horn. In this case we can use equation (17.19) to find

$$f_{\text{o,min}} = \left(\frac{c-v_{\text{o}}}{c}\right)f_{\text{s}} = \left(\frac{(343\text{ m/s})-(97.2\text{ m/s})}{(343\text{ m/s})}\right)(400\text{ Hz}) = 287\text{ Hz}$$

As the driver passes the spectator, the velocity of the car is no longer directed exactly toward or away from the air horn. Rather the velocity has some component toward or away from the air horn, and this component changes continuously. So the frequency heard by the driver will sweep continuously from the maximum to the minimum. Thus the driver will hear all frequencies between 287 Hz and 513 Hz.

17.81. Hyperbola. The hyperbola can be defined using two points (foci) f_1 and f_2, as the set of all points P such that the difference between the distances from P to f_1 and from P to f_2 is constant.

17.82. The edges of any aperture act as sources for the sound. If the edges are separated by a distance that is close to the wavelength of the sound being emitted, there may be many dead zones in front of the speaker. Higher frequency sounds have shorter wavelengths. The large aperture of the woofer would put sources of high-frequency sound so far apart that they could interfere with each other destructively in several places in front of the speaker. Using a "tweeter" ensures that the sources of the sound are never so far apart as to produce many dead spots.

17.83. The speed of the children around the Ferris wheel is $v = \dfrac{2\pi R}{T} = \dfrac{\pi(27.0\text{ m})}{(24.0\text{ s})} = 3.534\text{ m/s}.$ The highest frequency will be heard from a child moving directly toward you, and the lowest will be heard from a child moving directly away from you. Because the children are on opposite sides of the Ferris where, you can hear the maximum and minimum frequencies at the same time. We calculate these Doppler shifted frequencies using equation (17.21), inserting zero for the speed of the observer:

$$f_{\text{o,max}} = \left(\frac{c}{c-v_{\text{s}}}\right)f_{\text{s}} = \left(\frac{(343\text{ m/s})}{(343\text{ m/s})-(3.534\text{ m/s})}\right)(600\text{ Hz}) = 606.2\text{ Hz}$$

$$f_{\text{o,min}} = \left(\frac{c}{c+v_{\text{s}}}\right)f_{\text{s}} = \left(\frac{(343\text{ m/s})}{(343\text{ m/s})+(3.534\text{ m/s})}\right)(600\text{ Hz}) = 593.9\text{ Hz}$$

These frequencies will produce a beat frequency of $f_{\text{beats}} = \left|f_{\text{o,max}} - f_{\text{o,min}}\right| = \left|(606.2\text{ Hz})-(593.9\text{ Hz})\right| = 12\text{ Hz},$ or 12.4 beats per second.

17.84. Call the distance from one row to the next row d. For a given position, we can write $\dfrac{P}{4\pi r^2} = I = I_{\text{th}}(10^{(\beta/(10\text{ dB}))})$. We apply this to the two relevant positions: a distance $4d$ from the stage, and a distance Nd from the stage, and we take the ratio of the two expressions:

$$\frac{I_{4\text{th row}}}{I_{N\text{th row}}} = \frac{\left(\dfrac{P}{4\pi(4d)^2}\right)}{\left(\dfrac{P}{4\pi(Nd)^2}\right)} = \frac{I_{\text{th}}(10^{(\beta_{4\text{th row}}/(10\text{ dB}))})}{I_{\text{th}}(10^{(\beta_{N\text{th row}}/(10\text{ dB}))})}$$

Or

$$N = 4\sqrt{10^{((\beta_{4\text{th row}}-\beta_{N\text{th row}})/(10\text{ dB}))}} = 4\sqrt{10^{(((100\text{ dB})-(80\text{ dB}))/(10\text{ dB}))}} = 40$$

You want to sit in the 40th row.

17.85. Let us use the information given to determine the maximum distance from a radio station at which the signal would still be sufficient for you to detect it with your dish. The power you must pick up in your dish can be written in terms of the intensity at the location of your dish and the dish's surface area: $P_{\text{dish,min}} = I_{\text{min}} A_{\text{dish}} = I_{\text{min}} \pi \left(\frac{d}{2} \right)^2$. We can write this minimum intensity in terms of the power of the radio station, such that $P_{\text{dish,min}} = \left(\frac{P_{\text{station}}}{4\pi r_{\text{station,max}}^2} \right) \pi \left(\frac{d}{2} \right)^2$. Rearranging, we find $r_{\text{station,max}} = d\sqrt{\frac{P_{\text{station}}}{16 P_{\text{dish,min}}}} = (0.60 \text{ m})\sqrt{\frac{(50 \text{ W})}{16(1.0 \times 10^{-7} \text{ W})}} = 3.4 \text{ km}$. Your current setup would only allow you to pick up radio stations 3.4 km down the road, or 2.1 miles. You might try using a larger dish to detect more distant signals.

17.86. Call the frequency heard by the engineer f_{eng} and the frequency you hear *f*. Call the angle away from the line perpendicular to the track to the train θ. If we draw the velocity vector of the train, it is easy to decompose this into components: one along a line directly toward you and one perpendicular to that. Then simple geometry shows that the component of the train's velocity directed toward you (as it approaches) is $v_{\text{train}} \sin(|\theta|)$, where the absolute value is there only to avoid confusion between negative signs in the Doppler effect, and possible negative values of an angle. Then, as simple application of equation (17.21) shows us that the frequency you hear is related to the frequency that the engineer hears according to $f = f_{\text{eng}} \left(\frac{c}{c - v\sin(|\theta|)} \right)$, where $c = 343 \text{ m/s}$ is the speed of sound in air. Once the whistle has passed you and is receding away, the expression would become $f = f_{\text{eng}} \left(\frac{c}{c + v\sin(|\theta|)} \right)$.

18

FLUIDS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{19} kg **2.** 10^2 kg/m^3 **3.** 10^8 Pa **4.** 10^3 Pa **5.** 10^4 N **6.** 10^5 N **7.** 10^5 N **8.** 10^3 N **9.** 10^{-3} m^3/s
10. 10^{-3} kg

Guided Problems

18.2 Trial balloon

1. Getting Started In order for the balloon to hover, the gravitational force exerted downward by Earth on the balloon must be equal and opposite to the buoyant force acting on the balloon due to the surrounding atmosphere. We can determine the weight of the balloon by considering the force of gravity being exerted on all mass that makes up the thin outer film of the balloon and the gas that fills the balloon. The buoyant force is given by the weight of the mass of air displaced by the balloon.

2. Devise Plan Essentially the entire solving process used in Worked Problem 18.1 is still applicable here. The differences are reflected in the values used for the density of the atmosphere, and the density of the gas used to fill the balloon.

3. Execute Plan The force of gravity that Earth exerts downward on the balloon is given by

$$F^G_{\text{E,bal}} = m_{\text{bal}}g + m_{\text{hy}}g = (\rho_{\text{film}}V_{\text{film}} + \rho_{\text{hy}}V_{\text{bal}})g$$

This is exactly the same as in Worked Problem 18.1. Equating the gravitational and buoyant force magnitudes, we find

$$F^{\text{b}}_{\text{atm,bal}} = F^G_{\text{Earth,bal}}$$

$$\rho_{\text{atm}}V_{\text{bal}}g = (\rho_{\text{film}}V_{\text{film}} + \rho_{\text{hy}}V_{\text{bal}})g$$

$$\rho_{\text{atm}}\left(\frac{4}{3}\pi r^3_{\text{bal}}\right) = \rho_{\text{film}}(4\pi r^2_{\text{bal}}t) + \rho_{\text{hy}}\left(\frac{4}{3}\pi r^3_{\text{bal}}\right)$$

$$\left(\frac{r_{\text{bal}}}{3}(\rho_{\text{atm}} - \rho_{\text{hy}}) - \rho_{\text{film}}t\right)r^2_{\text{bal}} = 0$$

This has solutions $r_{\text{bal}} = 0$ or $r_{\text{bal}} = \dfrac{3\rho_{\text{film}}t}{(\rho_{\text{atm}} - \rho_{\text{hy}})}$. Clearly the former corresponds to having no balloon at all, and we want to work with the second expression. Inserting the appropriate constants, we have

$$r_{\text{bal}} = \frac{3\rho_{\text{film}}t}{(\rho_{\text{atm}} - \rho_{\text{hy}})} = \frac{3(1.2\times10^3\ \text{kg/m}^3)(3.0\times10^{-4}\ \text{m})}{((1.286\times10^0\ \text{kg/m}^3) - (9.0\times10^{-2}\ \text{kg/m}^3))} = 0.90\ \text{m}$$

In order to use helium, we simply change the density of gas filling the balloon. All other quantities remain the same.

$$r_{\text{bal}} = \frac{3\rho_{\text{film}}t}{(\rho_{\text{atm}} - \rho_{\text{he}})} = \frac{3(1.2\times10^3 \text{ kg/m}^3)(3.0\times10^{-4} \text{ m})}{((1.286\times10^0 \text{ kg/m}^3) - (1.79\times10^{-1} \text{ kg/m}^3))} = 0.98 \text{ m}$$

4. Evaluate Result Our answer is of the order 1 m independent of the choice of hydrogen or helium. This is drastically different from the radius required on Mars. This makes sense because the densities of the Earth and Martian atmospheres are so drastically different. This is also consistent with our day to day experience. Most of us have seen a helium balloon supported by the buoyant force on Earth's surface. Most have a radius smaller than 1 m. This is likely because balloons for children's parties do not need to be made of a material as strong as that used in a research balloon. The density of the material and the thickness of the film used on these common balloons are likely smaller than what we have considered here. Even with the given density and thickness, on Earth, helium is a perfectly reasonable gas to use for such a balloon.

18.4 Under pressure

1. Getting Started Let us assume that the substance responsible for the pressures is uniform in all relevant properties. For example, we assume that the temperature, density, molecule type, etc are all uniform across the inner surface and across the outer surface separately. We allow for differences only between the two sides. In that case the pressure difference is uniform all across the surface.

Note that the problem has rotational symmetry about the x axis. This means there can be no net force component perpendicular to the x axis, and we need only consider force components along the x axis.

2. Devise Plan Let us call the angle from the x axis θ. Let us consider a differential section of area made up of a thin vertical ring, centered on the x axis, at an angle θ and with a width of $Rd\theta$. The radius of such a ring r can be described as the vertical distance from the x axis to the ring, or $r = R\sin(\theta)$, where R is the radius of the hemisphere. The surface area of such a ring is $dA = 2\pi rRd\theta = 2\pi R^2\sin(\theta)d\theta$. The pressure is made up of forces being exerted perpendicular to the surface of the hemisphere. These forces have components parallel to the x axis and perpendicular to it. We already mentioned that only the components parallel to the x axis will be of importance. We can see this from the differential rings: a force at the top of the ring will have a vertical component that will be canceled out by a similar component to a force at the bottom of the ring.

A small contribution to the force can be written $dF = (P_{\text{out}} - P_{\text{in}})dA = (P_{\text{out}} - P_{\text{in}})(2\pi R^2\sin(\theta)d\theta)$, and the component of this force along the x axis is

$$dF_x = dF\cos(\theta) = (P_{\text{out}} - P_{\text{in}})(2\pi R^2\sin(\theta)\cos(\theta)d\theta) \tag{1}$$

We can integrate this to find the total force along the x axis.

3. Execute Plan The variable θ runs from 0 to $\pi/2$, because we defined it to run from the x axis to the edge of the hemisphere. Integrating equation (1), we find

$$F_x = \int_{\text{hemisphere}} dF_x = 2\pi R^2(P_{\text{out}} - P_{\text{in}})\int_0^{\pi/2}\sin(\theta)\cos(\theta)d\theta$$

$$F_x = \pi R^2(P_{\text{out}} - P_{\text{in}})$$

This is the same as the equatorial area times the pressure difference, so this is the equivalence we were asked to show.

4. Evaluate Result The result we obtained is consistent with Laplace's law, in that a fixed force implies an inverse relationship between radius of curvature and pressure difference:

$$(P_{\text{out}} - P_{\text{in}}) = \frac{F_x}{2\pi R}$$

It is also consistent with simple logic. Consider closing off the hemisphere shown in Figure WG 18.3 with a flat circular surface on the left side. Obviously, a fluid would not retain sharp points at the circle's edges in equilibrium, but one might constrain a fluid to have that form for an instant. In that instant, if the force from the left and right were not the same, the entire droplet would accelerate one direction or the other. The momentum of the closed, isolated system would suddenly change. That cannot happen. So the force due to the pressure difference on the left side (the flat equatorial cross-section) must be the same in magnitude as the force due to the pressure difference on the hemispherical right hand side.

18.6 Devastating flood

1. Getting Started The mass flow rate is related to the volume flow rate trivially through the density. The maximum speed is related to maximum volume flow rate through the area. These relations require that the fluid be incompressible. In a realistic dam break, there is enough turbulence, circulation, and whitewater that the entire volume of water cannot be said to be incompressible. But the bulk of the water, deep inside the flow still has relatively little dissolved oxygen, such that the incompressible fluid model should work fairly well. The same is true of viscosity. The viscosity of water is not zero. But a breaking dam is also not described by laminar flow, and we have no tools at this point to treat the viscosity of highly turbulent flow. Also, the relatively small fraction of the water that is in contact with the dam, and the middling to lo

2. Devise Plan The volume flow rate is related to the speed and mass flow rates according to the following:

$$v = Q_{\text{vol}}/A \tag{1}$$

$$Q_{\text{mass}} = \rho Q_{\text{vol}} \tag{2}$$

To relate this volume flow rate to the energy flow rate, we write the flow of kinetic energy as

$$Q_{\text{energy}} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$Q_{\text{energy}} = \frac{1}{2}\rho Q_{\text{vol}}\left(\frac{Q_{\text{vol}}}{A}\right)^2$$

$$Q_{\text{energy}} = \frac{1}{2}\frac{\rho}{A^2}Q_{\text{vol}}^3 \tag{3}$$

We can use equations (1-3) to determine the required quantities.

3. Execute Plan Inserting the given numbers into equation (1), we find that the maximum speed is

$$v = \frac{Q_{vol}}{A} = \frac{(8500\text{ m}^3/\text{s})}{(13\text{ m})(90\text{ m})} = 7.3\text{ m/s}$$

The mass flow rate is $Q_{\text{mass}} = \rho Q_{\text{vol}} = (1000\text{ kg/m}^3)(8500\text{ m}^3/\text{s}) = 8.5\times10^6\text{ kg/s}$.

Finally, the energy flow rate is

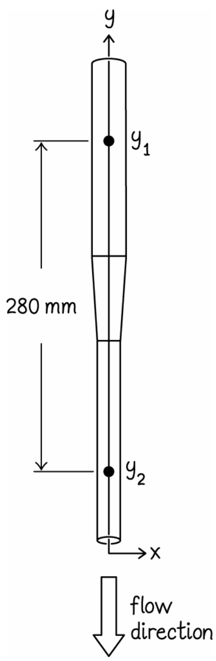
$$Q_{\text{energy}} = \frac{1}{2}\frac{\rho}{A^2}Q_{\text{vol}}^3 = \frac{1}{2}\frac{(1000\text{ kg/m}^3)}{((13\text{ m})(90\text{ m}))^2}(8500\text{ m}^3/\text{s})^3 = 2.2\times10^8\text{ J/s}$$

4. Evaluate Result The speed may seem low at first glance, but recall that pressure depends only on the depth of a fluid, not on the amount of water. This means that the pressure driving the water out of the dam was not so drastically different than the pressure in a garden hose. It is reasonable that water might have been moving at only 7.3 m/s.

A single car on the freeway might have a mass of 10^3 kg and might drive at 3×10^1 m/s. If the car is about 3 m long, it would take only a tenth of a second for the entire length of the car to move past a fixed point. A single such car would correspond to a mass flow rate of 10^3 kg/0.1 s or 10^4 kg/s. So the mass flow rate during the breach of the dam was similar to a stream of cars with a cross-section of 850 cars driving side-by-side out of the breach at a speed 30 m/s. The ratio of energies is not exactly the same, because of the higher speed of the cars on a freeway. The energy flow rate would correspond to a stream of cars with a cross-section of about 50 cars driving side-by-side at 30 m/s.

18.8 Reoriented flow

1. Getting Started We start by drawing a diagram of the situation:



The general approach of using the continuity equation and Bernoulli's equation will still work. The difference is that with the pipe oriented vertically, there will also now be a pressure difference due to hydrostatic forces.

2. Devise Plan This problem is essentially the same as Worked Problem 18.7, except $y_1 \neq y_2$. As in that problem, the continuity equation immediately yields

$$v_1 d_1^2 = v_2 d_2^2 \tag{1}$$

Bernoulli's equation reads

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \tag{2}$$

Assuming the same fluid (water) as in Worked Problem, we have sufficient information to solve for the pressure difference.

3. Execute Plan Rearranging equation (2), we have $P_2 - P_1 = \rho g(y_1 - y_2) + \frac{1}{2}\rho(v_1^2 - v_2^2)$. And inserting equation (1) yields

$$P_2 - P_1 = \rho g(y_1 - y_2) + \frac{1}{2}\rho v_1^2\left(1 - \left(\frac{d_1}{d_2}\right)^4\right)$$

Inserting numerical values, we obtain

$$P_2 - P_1 = (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.280\ \text{m}) + \frac{1}{2}(1000\ \text{kg/m}^3)(0.750\ \text{m/s})^2\left(1 - \left(\frac{15\ \text{mm}}{10\ \text{mm}}\right)^4\right) = 1.6\ \text{kPa}$$

This means that position 2 is at the higher pressure, by 1.6 kPa.

4. Evaluate Result It is certainly reasonable that the lower position in the pipe has a higher pressure. If the water were not moving, the pressure difference would be entirely due to the depth, and would be 2.7 kPa. It makes sense that the movement of the water reduces this since the water moves fastest in the lower section of pipe. The pressure difference is small compared to atmospheric pressure, which is also reasonable.

Questions and Problems

18.1. When the right piston is pushed leftward, it causes the pressure in the fluid to increase, and this pressure is transmitted to the left piston. If the left piston is not to move, then we must exert a rightward force on it equal in magnitude to the force exerted on it by the water. Because pressure equals the ratio of force to area, we have

$$P = F^c_{\text{rw}}/A_\text{r} = F^c_{\text{lw}}/A_\text{l}$$

$$F^c_{\text{lw}} = F^c_{\text{rw}}A_\text{l}/A_\text{r}$$

$$F^c_{\text{lw}} = \frac{(30.0\ \text{N})(\pi(10.0\ \text{mm})^2)}{\pi(30.0\ \text{mm})^2} = 3.33\ \text{N}$$

18.2. Atmospheric pressure in the International Space Station is the same as that on Earth, so the pressure at the top surface is the same. Because the International Space Station and everything on board is in free fall, there is no need for the bottom of the container to support the water against the force of gravity, so the pressure at the bottom of the container is the same as the pressure at the top. That is, the pressure at the bottom of the container is smaller on the International Space Station than it is on Earth.

18.3. (*a*) Water pressure increases with depth, so the pressure is greatest on the bottom surface and least on the top surface. Because they are all at the same depth, the pressure on each of the vertical sides is the same and is intermediate between the pressure on the top and bottom surfaces. That is, $P_{\text{top}} < P_A = P_B < P_{\text{bottom}}$. (*b*) The force exerted by the water on a side of the box equals the product of the pressure in the water times the area of the side. Because the pressure is greatest at the bottom and that side has the greatest area, it also has the greatest force exerted on it. Because the pressure is the same on the vertical sides, the sides with the greater areas, namely *B*, have greater forces exerted on them than on the smaller sides *A*. We need more information to determine the force exerted on the top side, as the pressure is smallest there but its area is large. If we assume the box is not very tall, so that the difference in pressure between the top and bottom sides is small, then the force exerted on the top will be almost as large as the force exerted on the bottom, and we have $F_A < F_B < F_{\text{top}} < F_{\text{bottom}}$.

18.4. (*a*) The ratio of the magnitude of the force exerted by the fluid on a side to the area of that side is just the average pressure of the fluid, which is the same for all three sides. That is, the ratios will be equal for any angle θ. (*b*) The magnitude of the force exerted by the fluid on a side equals the product of the pressure in the fluid times the area of the side, so the forces will have the same magnitudes if the sides have the same areas, that is, when $\theta = 60°$.

18.5. (*a*) If the wall at A were not present, the liquid would move to the right, so the wall must exert a force on the liquid directed toward the left. Because they form an interaction pair, the liquid must exert a force on the wall directed toward the right. (*b*) Pressure is a scalar, so it has no direction. (*c*) At location B, the wall supports the liquid against the force of gravity, so it must exert an upward-directed force on the liquid. (*d*) If the piston is pushed upward, it exerts an upward force on the liquid. Because they form an interaction pair, the liquid must exert a downward force on the piston. (*e*) Because the container is open at the top, pulling downward on the piston merely allows more liquid to flow down into its cylinder. As long as the piston's acceleration is smaller than the acceleration of gravity, it must still support the liquid against the force of gravity, so it exerts an upward force on the liquid. Because they form an interaction pair, the liquid must exert a downward force on the piston. (*f*) If the container were filled with gas, the magnitudes of the forces would be much smaller, but the directions of the forces would not change. Note, too, that if the mass density of the gas is less than the mass density of air, it will float out of the open top of the container, leaving the container filled with air.

18.6. [**NOTE:** In the problem statement, 57 kPa is changed to 27 kN/m^2.] Pressure is the ratio of force to area, so to determine the area given the pressure, we need to calculate the force. There are two forces exerted on me, the downward force of gravity and the upward contact force of the half-pipe's surface, and the vector sum of these provides the centripetal acceleration that makes me follow the circular path of the half-pipe. Equation 11.15 relates the magnitude of the centripetal acceleration to my speed and the radius of my circular path, $a_c = v^2/r$, so

$$F^c_{sm} - F^G_{Em} = ma_c = \frac{mv^2}{r}$$

$$F^c_{sm} = \frac{mv^2}{r} + mg$$

The pressure on the half-pipe equals the magnitude of the contact force exerted by me on the surface, which is the same as the magnitude of the contact force exerted by the surface on me, divided by the area of the snowboard.

$$P = F^c_{sm}/A = F^c_{sm}/(\ell w)$$

$$w = \frac{F^c_{sm}}{\ell P} = \frac{m\left(\dfrac{v^2}{r} + g\right)}{\ell P} = \frac{(70\text{ kg})\left(\dfrac{(8.2\text{ m/s})^2}{5.0\text{ m}} + (9.8\text{ m/s}^2)\right)}{(1.3\text{ m})(27\times 10^3\text{ N/m}^2)} = 4.6\times 10^{-2}\text{ m}$$

18.7. [**NOTE:** In the problem statement, 3 kPa and 60 kPa are changed to 3 kN/m^2 and 60 kN/m^2.] Suppose the gravitational force exerted by Earth on all the objects resting on the floor equals the maximum that the floor can support. Then we have $F^G_{Eo} = L_{floor}A_{floor}$. Suppose that, for each object, the point loading is also at its maximum value, so $F^G_{Eo} = L_{point}A_{objects}$. Setting these expressions for the gravitational force equal to each other gives

$$L_{floor}A_{floor} = L_{point}A_{objects}$$

$$\frac{A_{objects}}{A_{floor}} = \frac{L_{floor}}{L_{point}} = \frac{3\times 10^3\text{ N/m}^2}{60\times 10^3\text{ N/m}^2} = 0.05$$

That is, five percent of the floor area can be subjected to the point loading without exceeding the floor loading.

18.8. (*a*) We know that the pressure in a fluid increases linearly with depth, regardless of the shape of the container. Because the water in all three jars has the same depth, the pressures are all equal, $P_A = P_B = P_C$. (*b*) When only half the volume of water is added to the jars, they will each be filled to different heights. Because jar A is wider at the bottom, it will be filled to a smaller height than the others, and similarly, jar C will be filled to a greater height than the others. Because the pressure is proportional to the depth of the water, we have $P_A < P_B < P_C$.

18.9. At the outer surface of the planet, the pressure is zero. As we move toward the center of the planet, the pressure increases so that it is sufficient to support the outer portions against the force of gravity.
Consider a spherical shell of thickness dr a distance $r < R$ from the center of the planet. It contributes an increment of pressure dP to the interior of the planet equal to the gravitational force exerted on the shell divided by the shell's area, $dP = dF^G/A$. The gravitational force exerted on the shell equals its mass times the acceleration of gravity at that distance from the planet's center, $dF^G = dM\, g(r)$. The mass of the shell equals the mass density of the fluid times the volume of the shell, and the volume of the shell equals its thickness times its area, so $dM = \rho dV = \rho A dr$. The acceleration of gravity at a distance r from the planet's center is just that due to the mass enclosed by the shell (see *Principles* Section 13.8), $g(r) = \dfrac{GM_{enclosed}}{r^2} = \dfrac{G\frac{4}{3}\pi r^3\rho}{r^2} = \frac{4}{3}\pi G\rho r$. Combining these we have $dP = \dfrac{(\rho A dr)(\frac{4}{3}\pi G\rho r)}{A} = \frac{4}{3}\pi G\rho^2 r\, dr$, and the pressure at a distance r' from the center of the planet is the integral of these pressure increments from all the shells further from the center: $P(r') = \int_{r'}^{R} dP = \int_{r'}^{R} \frac{4}{3}\pi G\rho^2 r\, dr = \frac{2}{3}\pi G\rho^2(R^2 - r'^2)$. So, midway between the center and the surface we have $P = \frac{2}{3}\pi G\rho^2(R^2 - (R/2)^2) = \frac{1}{2}\pi G\rho^2 R^2$.

18.10. When the cube floats, the downward force of gravity exerted by Earth on the cube balances the buoyant force exerted by the water on the cube. The magnitude of the buoyant force equals the magnitude of the force of gravity on the water displaced by the submerged portion of the cube. The volume of water displaced by the submerged portion of the cube is $(100\ \text{mm})(100\ \text{mm})(70.0\ \text{mm}) = 7.00\times10^{-4}\ \text{m}^3$, and the mass density of water is $1000\ \text{kg/m}^3$, so the mass of the cube is $(7.00\times10^{-4}\ \text{m}^3)(1000\ \text{kg/m}^3) = 0.700\ \text{kg}$.

18.11. When the weight is submerged, there are two forces exerted on it, the downward force of gravity and the upward buoyant force of the water. The magnitude of the buoyant force equals the magnitude of the force of gravity on the water displaced by the weight. We could calculate the volume of the weight from its mass and mass density, or we could just notice that its mass density is twice that of water, so the magnitude of the buoyant force exerted by the water on the weight equals one half of the magnitude of the force of gravity exerted by Earth on the weight. So, the magnitude of the vector sum of the forces exerted on the submerged weight is $\frac{1}{2}mg = \frac{1}{2}(5.0\ \text{kg})(9.8\ \text{m/s}^2) = 25\ \text{N}$, and this is the magnitude of the force required to lift it.

18.12. (*a*) The magnitude of the buoyant force exerted on a submerged object equals the magnitude of the force of gravity on the water displaced by the object. Because all three bricks are the same size, they all displace the same volume of water, so the same buoyant force is exerted on each of them. (*b*) An object will float if its mass is less than the mass of the fluid it displaces when fully submerged. So the brick most likely to float is the one with the smallest mass density, which is most likely the plastic one.

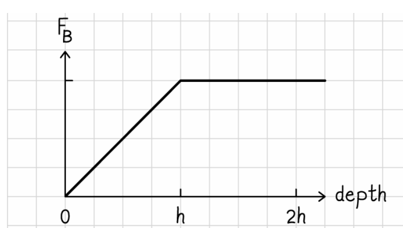
18.13. (*a*) The mass density of the ship as a whole equals its mass divided by its volume, $\rho = \dfrac{m}{V} = \dfrac{2.50\times10^6\ \text{kg}}{(50.0\ \text{m})(20.0\ \text{m})(20.0\ \text{m})} = 125\ \text{kg/m}^3$. (*b*) An object will float if its mass is less than the mass of the fluid it displaces when fully submerged, that is, if its mass density is less than that of the fluid. The mass density of the ship is less than the mass density of water, so the ship floats. (*c*) The loaded ship will just barely float if its mass equals the mass of the water it displaces. In that case we have

$$m_{\text{ship}} + m_{\text{cargo}} = \rho_{\text{water}} V_{\text{ship}}$$

$$m_{\text{cargo}} = \rho_{\text{water}} V_{\text{ship}} - m_{\text{ship}}$$

$$m_{\text{cargo}} = (1000\ \text{kg/m}^3)(50.0\ \text{m})(20.0\ \text{m})(20.0\ \text{m}) - (2.50\times10^6\ \text{kg}) = 1.75\times10^7\ \text{kg}$$

18.14. The magnitude of the buoyant force on the block equals the magnitude of the force of gravity on the water displaced by the submerged portion of the block. As the block is pushed into the water, the volume of water displaced by the submerged portion of the block increases linearly until the block is completely submerged, after which it remains constant. So, the magnitude of the buoyant force on the block increases linearly from zero until the block is completely submerged, after which it remains constant.



18.15. The water levels in the two glasses are the same. One way to see this is to consider what happens to the water level in glass B when the ice cube melts. The submerged portion of the ice cube displaces a volume of water with the same mass as the ice cube. But this is exactly the volume that the water from the melted ice cube will occupy, so the water level remains the same when the ice cube melts. After the ice cube has melted, the two glasses contain the same amount of liquid water, so their levels are the same, and as we just saw, this is the same level as was in glass B before the ice cube melted.

18.16. When the block is in the rowboat, the rowboat displaces a volume of water, the mass of which equals the combined mass of the rowboat and the block. When the block is in the pool, it displaces a volume of water equal to its volume, while the empty rowboat displaces a volume of water, the mass of which equals the mass of the rowboat.

Because the block sinks, we know that its mass density is greater than the mass density of water, so the volume of water it displaces when it's in the pool is smaller than the volume of water it displaces when it's in the rowboat. So, because a smaller volume of water is displaced when the block is thrown overboard, the water level in the pool falls.

18.17. The raft will just barely float when the combined mass of the raft and its passengers equals the mass of the water it can displace, which equals the mass density of water times the volume of the raft. So we have

$$m_{\text{raft}} + m_{\text{me}} + Nm_{\text{friend}} \le \rho_{\text{water}} V_{\text{raft}}$$

$$N \le \frac{\rho_{\text{water}} V_{\text{raft}} - m_{\text{raft}} - m_{\text{me}}}{m_{\text{friend}}} = \frac{(1000\ \text{kg/m}^3)(2.00\ \text{m})(2.00\ \text{m})(0.100\ \text{m}) - (40.0\ \text{kg}) - (55.0\ \text{kg})}{65.0\ \text{kg}} = 4.69$$

That is, four friends can ride with me.

18.18. (*a*) The balloon will barely float if the combined mass of the balloon, the gas it contains, and its payload equals the mass of the air it displaces. (That is, we're assuming we can ignore the mass of the air displaced by the payload.) In this case we have

$$m_{\text{balloon}} + m_{\text{gas}} + m_{\text{payload}} = m_{\text{displaced air}}$$

$$m_{\text{balloon}} + \rho_{\text{gas}} V_{\text{balloon}} + m_{\text{payload}} = \rho_{\text{air}} V_{\text{balloon}}$$

$$V_{\text{balloon}} = \frac{m_{\text{balloon}} + m_{\text{payload}}}{\rho_{\text{air}} - \rho_{\text{gas}}} = \tfrac{4}{3}\pi (d/2)^3$$

$$d = 2\left(\frac{3}{4\pi}\frac{m_{\text{balloon}} + m_{\text{payload}}}{\rho_{\text{air}} - \rho_{\text{gas}}}\right)^{1/3}$$

$$d = 2\left(\frac{3}{4\pi}\frac{(0.500\ \text{kg}) + (1.50\ \text{kg})}{(1.286\ \text{kg/m}^3) - (0.179\ \text{kg/m}^3)}\right)^{1/3} = 1.51\ \text{m}$$

(*b*) If the balloon is filled with nitrogen instead of helium, we have the same equation for the balloon's diameter, but we must use the mass density of nitrogen instead of helium.

$$d = 2\left(\frac{3}{4\pi}\frac{(0.500\ \text{kg}) + (1.50\ \text{kg})}{(1.286\ \text{kg/m}^3) - (1.251\ \text{kg/m}^3)}\right)^{1/3} = 4.78\ \text{m}$$

18.19. Because the blocks are floating, they each displace a volume of water, the mass of which equals the mass of the block. Block 1 displaces twice the volume of water as block 2, so the mass of block 1 must be twice the mass of block 2. Because the blocks have the same size, and mass density equals mass divided by volume, the ratio $\rho_1/\rho_2 = 2$.

18.20. (*a*) There are three forces exerted on the float, the downward force of gravity, the upward buoyant force of the water, and the downward contact force of the rope. Choosing our system to be the float, the water, and Earth, only the contact force of the rope does work on the system. If the float descends at constant speed, its acceleration is zero, and so the vector sum of the three forces must be zero, $F^G_{\text{Ef}} - F^{\text{b}}_{\text{wf}} + F^{\text{c}}_{\text{rf}} = 0$. The magnitude of the buoyant force is equal to the magnitude of the force of gravity on the water displaced by the float, so we have

$$\begin{aligned} F^{\text{c}}_{\text{rf}} &= F^{\text{b}}_{\text{wf}} - F^G_{\text{Ef}} \\ &= m_{\text{displaced water}} g - m_{\text{f}} g \\ &= V_{\text{f}} \rho_{\text{water}} g - m_{\text{f}} g \\ &= (V_{\text{f}} \rho_{\text{water}} - m_{\text{f}}) g \end{aligned}$$

and the work done by the rope is

$$\begin{aligned} W &= F^{\text{c}}_{\text{rf}} \Delta x_F \\ &= (V_{\text{f}} \rho_{\text{water}} - m_{\text{f}}) g \Delta x_F \\ &= ((1.00\ \text{m}^3)(1000\ \text{kg/m}^3) - (5.00\ \text{kg}))(9.8\ \text{m/s}^2)(50.4\ \text{m}) \\ &= 4.91 \times 10^5\ \text{J} \end{aligned}$$

(*b*) Average power is the rate at which work is done, that is, the work done divided by the time interval during which the work was done: $P = \dfrac{W}{\Delta t} = \dfrac{4.91\times10^{5}\ \text{J}}{10\ \text{min}}\dfrac{1\ \text{min}}{60\ \text{s}} = 819\ \text{W}.$

18.21. As the balloon floats, the combined mass of the balloon, the gas it contains, and the cord it supports equals the mass of the air it displaces:

$$m_{\text{balloon}} + m_{\text{gas}} + m_{\text{cord}} = m_{\text{displaced air}}$$

$$m_{\text{balloon}} + \rho_{\text{gas}} V_{\text{balloon}} + \lambda_{\text{cord}} \ell_{\text{cord}} = \rho_{\text{air}} V_{\text{balloon}}$$

$$V_{\text{balloon}} = \frac{m_{\text{balloon}} + \lambda_{\text{cord}} \ell_{\text{cord}}}{\rho_{\text{air}} - \rho_{\text{gas}}}$$

$$V_{\text{balloon}} = \frac{(2300\times10^{-6}\ \text{kg}) + (0.0160\ \text{kg/m})(63.2\times10^{-3}\ \text{m})}{(1.286\ \text{kg/m}^3) - (0.179\ \text{kg/m}^3)} = 2.99\times10^{-3}\ \text{m}^3$$

18.22. When the object is placed in the water, the water level in the tank rises. Because the water is now deeper, and pressure increases with depth, the pressure at the bottom of the tank is greater.

When the object floats, it displaces a volume of water, the mass of which equals the mass of the object. That is $m_{\text{object}} = m_{\text{displaced water}} = \rho_{\text{water}} V_{\text{displaced water}}$, or $V_{\text{displaced water}} = m_{\text{object}}/\rho_{\text{water}}$. The depth of the water increases by this volume divided by the horizontal cross-sectional area of the tank, $\Delta h = V_{\text{displaced water}}/A$. As we saw in *Principles* Example 18.1, pressure increases linearly with depth, $\Delta P = \Delta h \rho g$. Combining these we have

$$\begin{aligned}
\Delta P &= \frac{V_{\text{displaced water}}}{A}\rho g \\
&= \frac{m_{\text{object}}}{\rho A}\rho g \\
&= \frac{m_{\text{object}} g}{A} \\
&= \frac{m_{\text{object}} g}{\pi R^2} \\
&= \frac{(40.0\ \text{kg})(9.8\ \text{m/s}^2)}{\pi\left[(2.50\ \text{m})/2\right]^2} = 79.9\ \text{N/m}^2
\end{aligned}$$

18.23. (*a*) As the balloon floats, the combined mass of the balloon, the gas it contains, and the cord it supports equals the mass of the air it displaces.

$$m_{\text{balloon}} + m_{\text{gas}} + m_{\text{cord}} = m_{\text{displaced air}}$$

$$m_{\text{balloon}} + \rho_{\text{gas}} V_{\text{balloon}} + \lambda_{\text{cord}} \ell_{\text{cord}} = \rho_{\text{air}} V_{\text{balloon}}$$

$$\rho_{\text{gas}} = \frac{\rho_{\text{air}} V_{\text{balloon}} - m_{\text{balloon}} - \lambda_{\text{cord}} \ell_{\text{cord}}}{V_{\text{balloon}}}$$

$$\rho_{\text{gas}} = \frac{(1.286\ \text{kg/m}^3)(6.90\times10^{-2}\ \text{m}^3) - (6.00\times10^{-3}\ \text{kg}) - (0.260\ \text{kg/m})(283\times10^{-3}\ \text{m})}{6.90\times10^{-2}\ \text{m}^3}$$

$$\rho_{\text{gas}} = 0.133\ \text{kg/m}^3$$

(*b*) We know the mass of one cubic meter of gas. It contains some volume V_{He} of helium and a volume V_{H} of hydrogen, with $V_{\text{He}} + V_{\text{H}} = 1\ \text{m}^3$. So,

$$V_{\text{He}}\rho_{\text{He}} + V_{\text{H}}\rho_{\text{H}} = (1\ \text{m}^3)\rho_{\text{gas}}$$

$$V_{\text{He}} = \frac{(1\ \text{m}^3)\rho_{\text{gas}} - V_{\text{H}}\rho_{\text{H}}}{\rho_{\text{He}}} = (1\ \text{m}^3) - V_{\text{H}}$$

$$(1\ \text{m}^3)\rho_{\text{gas}} - V_{\text{H}}\rho_{\text{H}} = (1\ \text{m}^3)\rho_{\text{He}} - V_{\text{H}}\rho_{\text{He}}$$

$$V_{\text{H}} = \frac{(1\ \text{m}^3)\rho_{\text{He}} - (1\ \text{m}^3)\rho_{\text{gas}}}{\rho_{\text{He}} - \rho_{\text{H}}} = \frac{\rho_{\text{He}} - \rho_{\text{gas}}}{\rho_{\text{He}} - \rho_{\text{H}}}(1\ \text{m}^3)$$

$$\frac{V_{\text{He}}}{V_{\text{H}}} = \frac{(1\ \text{m}^3) - V_{\text{H}}}{V_{\text{H}}}$$

$$= \frac{(1\ \text{m}^3) - \dfrac{\rho_{\text{He}} - \rho_{\text{gas}}}{\rho_{\text{He}} - \rho_{\text{H}}}(1\ \text{m}^3)}{\dfrac{\rho_{\text{He}} - \rho_{\text{gas}}}{\rho_{\text{He}} - \rho_{\text{H}}}(1\ \text{m}^3)}$$

$$= \frac{\dfrac{\rho_{\text{He}} - \rho_{\text{H}}}{\rho_{\text{He}} - \rho_{\text{H}}} - \dfrac{\rho_{\text{He}} - \rho_{\text{gas}}}{\rho_{\text{He}} - \rho_{\text{H}}}}{\dfrac{\rho_{\text{He}} - \rho_{\text{gas}}}{\rho_{\text{He}} - \rho_{\text{H}}}}$$

$$= \frac{\rho_{\text{gas}} - \rho_{\text{H}}}{\rho_{\text{He}} - \rho_{\text{gas}}}$$

$$= \frac{(0.133\ \text{kg/m}^3) - (0.090\ \text{kg/m}^3)}{(0.179\ \text{kg/m}^3) - (0.133\ \text{kg/m}^3)} = 0.921$$

We could equally well express the ratio in terms of the masses of helium and hydrogen:

$$\frac{m_{\text{He}}}{m_{\text{H}}} = \frac{V_{\text{He}}\rho_{\text{He}}}{V_{\text{H}}\rho_{\text{H}}}$$

$$= 0.921\frac{0.179\ \text{kg/m}^3}{0.090\ \text{kg/m}^3} = 1.83$$

18.24. In a time interval Δt, the volume of water that passes through a plane perpendicular to the axis of the pipe through point A is $\Delta V = A_{\text{A}}v_{\text{A}}\Delta t$, where A_{A} is the pipe's cross-sectional area at A and v_{A} is the speed of the water at A. In the same time interval, the same volume of water must pass through a plane perpendicular to the axis of the pipe through point B, so we have $A_{\text{B}}v_{\text{B}}\Delta t = A_{\text{A}}v_{\text{A}}\Delta t$, or $v_{\text{B}} = v_{\text{A}}A_{\text{A}}/A_{\text{B}}$. So, $v_{\text{B}} = v_{\text{A}}A_{\text{A}}/A_{\text{B}} = v_{\text{A}}d_{\text{A}}^2/d_{\text{B}}^2 =$ $(100\ \text{mm/s})(100\ \text{mm})^2/(10.0\ \text{mm})^2 = 1.00\times10^4\ \text{mm/s}$.

18.25. Suppose the duct has cross-sectional area A. In a time interval Δt, the mass of air that passes through a plane perpendicular to the axis of the duct equals $\Delta m = \rho Av\Delta t$, which holds for all locations within the duct because air would have to accumulate somewhere in the duct if it did not. So, we have $\rho_{\text{outside}}Av_{\text{outside}}\Delta t = \rho_{\text{inside}}Av_{\text{inside}}\Delta t$, or $v_{\text{inside}} = v_{\text{outside}}\rho_{\text{outside}}/\rho_{\text{inside}}$. So, $v_{\text{inside}} = (5.38\ \text{m/s})(1.20\ \text{kg/m}^3)/(1.24\ \text{kg/m}^3) = 5.21\ \text{m/s}$.

18.26. Because the pipe is narrower at point 4 than at point 1, the speed of the liquid must be greater at point 4 than at point 1. Because the liquid is viscous, its speed at point 3 is nearly zero and its speed at point 2 is intermediate between its speeds at points 1 and 3. So, in order of increasing speed we have 3 < 2 < 1 < 4.

18.27. The streamlines are less densely spaced in region A than in region B, so the flow speed is smaller in region A than in region B. Because the flow speed is smaller in region A than in region B, the pressure is greater in region A than in region B. In order for the same quantity of water to flow through both regions of the pipe in a given time interval, the pipe diameter must be greater in region A than in region B.

18.28. In order for the water's speed to decrease while still maintaining the same rate of flow, the cross-sectional area of the river must have increased. We are told that the river has uniform width, so the only way for its cross-sectional area to have increased is by its depth having increased.

18.29. In the reference frame of the car, the air outside is moving rapidly while the air inside is nearly at rest. That is, the passenger compartment is not air-tight, and any air that enters the car's interior slows down, so the pressure inside the car is greater than the pressure outside, and the smoky air blows out through the small opening in the window.

18.30. We can relate the speed of the water in the pipe to the speed of the water as it flows through the hole by noting that the same volume of water passes through each in a given time interval. We can determine the speed of the water as it flows through the hole by considering each small quantity of water as a projectile launched horizontally from the hole. In the time interval Δt it takes for the drop to fall a distance Δy to the floor, it travels a horizontal distance Δx, and we have

$$\Delta y = \tfrac{1}{2} g(\Delta t)^2$$
$$\Delta t = \sqrt{2\Delta y/g}$$
$$\Delta x = v_{\text{hole}} \Delta t$$
$$v_{\text{hole}} = \Delta x/\Delta t = \Delta x/\sqrt{2\Delta y/g}$$

The volume of water that flows per unit time through either the pipe or the hole is given by the product of the water's speed and the cross-sectional area of the region it flows through:

$$v_{\text{hole}} A_{\text{hole}} = v_{\text{pipe}} A_{\text{pipe}}$$
$$v_{\text{pipe}} = v_{\text{hole}} A_{\text{hole}}/A_{\text{pipe}} = v_{\text{hole}} d_{\text{hole}}^2/d_{\text{pipe}}^2$$
$$v_{\text{pipe}} = \frac{d_{\text{hole}}^2 \Delta x}{d_{\text{pipe}}^2 \sqrt{2\Delta y/g}}$$
$$v_{\text{pipe}} = \frac{(3.00\text{ mm})^2(6.00\text{ m})}{(50.0\text{ mm})^2\sqrt{2(2.00\text{ m})/(9.8\text{ m/s}^2)}} = 3.38\times10^{-2}\text{ m/s}$$

18.31. (*a*) The volume of water that flows per unit time through any part of the hose is given by the product of the water's speed and the cross-sectional area of the region it flows through:

$$v_1 A_1 = v_2 A_2$$
$$v_2 = v_1 A_1/A_2 = v_1 d_1^2/d_2^2$$
$$v_{\text{middle}} = v_{\text{end}} d_{\text{end}}^2/d_{\text{middle}}^2 = (2.00\text{ m/s})(40.0\text{ mm})^2/(30.0\text{ mm})^2 = 3.56\text{ m/s}$$

(*b*) The time it takes water to flow through the hose equals the sum of the times it takes to flow through each portion:

$$\Delta t = \sum \Delta x/v$$
$$\Delta t = \frac{15.0\text{ m}}{2.00\text{ m/s}} + \frac{5.00\text{ m}}{3.56\text{ m/s}} + \frac{15.0\text{ m}}{2.00\text{ m/s}} = 16.4\text{ s}$$

18.32. When we discussed the Bernoulli effect in *Principles*, we considered the flow of fluid in a narrow, closed pipe. There we saw that when the cross-sectional area of the pipe changes, the fluid's speed changes. This change in speed must be due to a force exerted on the fluid, and the only candidate for that force is the difference in pressure between the regions of different flow speed.

The situation is different in an open channel. As noted in the problem statement, the pressure at the surface is atmospheric pressure. But the height of the surface above some arbitrarily chosen reference point is not fixed, so the force of gravity can do work on the fluid, changing its speed, as the height of the surface changes. It is also the case that the pressure in the fluid increases with depth. We need to account for these differences to give a complete description of the flow in an open channel.

18.33. The vector sum of the cohesive and adhesive forces on the particle results in a force directed perpendicular to the liquid surface because, if the force had a component parallel to the surface, it would cause the liquid to flow. So, the vector sum of the forces is directed upward at an angle of $104° - 90° = 14°$ above the solid surface, and in terms of the components of the vector sum of the forces perpendicular and parallel to the solid surface, we have $(\sum \vec{F})_\perp / (\sum \vec{F})_\| = \tan 14°$. We also know that the adhesive force is directed perpendicular to the solid surface, so $(\sum \vec{F})_\perp = F_{\text{cohesive}\,\perp} - F_{\text{adhesive}} = F_{\text{cohesive}} \sin 52.0° - F_{\text{adhesive}}$ and $(\sum \vec{F})_\| = F_{\text{cohesive}\,\|} = F_{\text{cohesive}} \cos 52.0°$. Combining these we have

$$\frac{F_{\text{cohesive}} \sin 52.0° - F_{\text{adhesive}}}{F_{\text{cohesive}} \cos 52.0°} = \tan 14°$$

$$F_{\text{adhesive}} = F_{\text{cohesive}}(\sin 52.0° - (\cos 52.0°)(\tan 14°))$$

$$F_{\text{adhesive}} = (4.00 \text{ N})(0.788 - (0.616)(0.249)) = 2.54 \text{ N}$$

18.34. The pressure difference across a liquid's surface increases as the radius of curvature of the surface decreases, and all three droplets have the same atmospheric pressure on their outsides, so the pressure is greatest in droplet C because its radius of curvature is smallest.

18.35. (*a*) The tension in the balloon membrane is greatest where its radius of curvature is greatest, so C < B < A. (*b*) The pressure is the same at all points inside the balloon, so A = B = C.

18.36. If the liquid's behavior is exactly between wetting and not wetting the solid surface, that means that the contact angle between liquid and solid is exactly $90°$, which means that the cohesive force on a particle of fluid at the solid-liquid-air boundary is directed at an angle of $45°$ above the solid surface. The vector sum of the cohesive and adhesive forces on such a particle results in a force directed perpendicular to the liquid surface because, if the force had a component parallel to the surface, it would cause the liquid to flow. So, the vector sum of the forces is directed parallel to the solid surface. The adhesive force on the liquid particle is directed perpendicular to the solid surface, so the magnitude of the adhesive force must equal the component of the cohesive force that is perpendicular to the solid surface. That is, $F_{\text{adhesive}} = F_{\text{cohesive}} \sin 45° = (\sqrt{2}/2) F_{\text{cohesive}}$.

18.37. [**NOTE:** The problem statement is changed: $35.0°$ is changed to $17.0°$; 1.00 N is changed to 6.84 N.] Because the cohesive force is directed at an angle of $17.0°$ to the wall, the contact angle between the liquid and the wall is $34.0°$. The line that passes through the point at the edge of the meniscus and the center of curvature of the meniscus is perpendicular to the liquid surface, so it forms an angle of $180° - (34.0° + 90°) = 56.0°$ with the wall. The radius of curvature times the sine of this angle equals the radius of the tube, so

$$R \sin 56.0° = d/2$$

$$R = \frac{d}{2 \sin 56.0°} = \frac{2.00 \text{ mm}}{2(0.829)} = 1.21 \text{ mm}$$

18.38. Within the suds, there is no difference in pressure between one bubble and the next, provided that both are of the same size; if there were, one bubble would expand and the other contract until the pressures on both sides of their shared surface were the same. Another way to see this is to notice that the shared surface between same-sized bubbles in the suds is flat, that is, its radius of curvature is infinite, so there is no pressure difference across the surface. For a free-floating bubble, the tension in the soap film causes the pressure inside the bubble to be greater than atmospheric pressure.

18.39. The line that passes through the point at the edge of the meniscus where it meets the wall and the center of curvature of the meniscus is perpendicular to the liquid surface. The angle θ between this line and the wall is determined by the contact angle θ_c. In the case of a wetting liquid, $\theta = 180° - (\theta_c + 90°) = 90° - \theta_c$; in the case of a nonwetting liquid, $\theta = \theta_c - 90°$. In either case, we have $R_{\text{tube}} = R_{\text{men}} \sin\theta$, or substituting the preceding, $R_{\text{tube}} = R_{\text{men}} |\cos\theta_c|$.

18.40. The pressure difference across a curved liquid-gas surface is proportional to the surface tension of the liquid and inversely proportional to the radius of curvature of the surface. The capillary rise is proportional to the pressure difference and inversely proportional to the mass density of the fluid. Because liquid B has twice the surface tension of liquid A and it forms a meniscus with half the radius of curvature of that formed by liquid A, the pressure difference across the surface of liquid B is four times that across the surface of liquid A. Because liquid B has twice the mass density of liquid A, the capillary rise of liquid B is twice that of liquid A, $h_B = 2h_A$.

18.41. Let's add a coordinate system to *Principles* Figure 18.32 with the origin at the solid-liquid-gas junction, the positive x axis pointing to the left, the positive y axis pointing upward, and let's measure angles clockwise from the positive x axis. Then the cohesive force on a particle at the junction is directed at an angle $\theta_c/2$ above the x axis, the adhesive force is directed downward in the negative y direction, and the vector sum of these forces is directed at an angle $\theta_c - 90°$, which will be negative or positive when the liquid wets or does not wet the surface, respectively. Expressing the vector sum of the forces in terms of their x and y components, we have $(\sum \vec{F})_x = F_{\text{cohesive}} \cos(\theta_c/2)$, $(\sum \vec{F})_y = F_{\text{cohesive}} \sin(\theta_c/2) - F_{\text{adhesive}}$, and $(\sum \vec{F})_y/(\sum \vec{F})_x = \tan(\theta_c - 90°)$.
This is the physics of the problem; all that remains is to apply some trigonometry and algebra:

$$\tan(\theta_c - 90°) = \frac{\sin(\theta_c - 90°)}{\cos(\theta_c - 90°)} = \frac{-\cos\theta_c}{\sin\theta_c}$$

$$\cos\theta_c = \cos(2(\theta_c/2)) = \cos^2(\theta_c/2) - \sin^2(\theta_c/2)$$

$$\sin\theta_c = \sin(2(\theta_c/2)) = 2\sin(\theta_c/2)\cos(\theta_c/2)$$

$$\frac{(\sum \vec{F})_y}{(\sum \vec{F})_x} = \frac{\sin^2(\theta_c/2) - \cos^2(\theta_c/2)}{2\sin(\theta_c/2)\cos(\theta_c/2)} = \frac{F_{\text{cohesive}} \sin(\theta_c/2) - F_{\text{adhesive}}}{F_{\text{cohesive}} \cos(\theta_c/2)}$$

$$F_{\text{cohesive}} \cos(\theta_c/2)\left[\sin^2(\theta_c/2) - \cos^2(\theta_c/2)\right] = \left[F_{\text{cohesive}} \sin(\theta_c/2) - F_{\text{adhesive}}\right] 2\sin(\theta_c/2)\cos(\theta_c/2)$$

$$F_{\text{adhesive}} 2\sin(\theta_c/2)\cos(\theta_c/2) = F_{\text{cohesive}}\left[2\sin^2(\theta_c/2)\cos(\theta_c/2) - \cos(\theta_c/2)\sin^2(\theta_c/2) + \cos^3(\theta_c/2)\right]$$

$$F_{\text{adhesive}} 2\sin(\theta_c/2) = F_{\text{cohesive}}\left[\sin^2(\theta_c/2) + \cos^2(\theta_c/2)\right] = F_{\text{cohesive}}$$

$$\frac{F_{\text{cohesive}}}{F_{\text{adhesive}}} = 2\sin(\theta_c/2)$$

18.42. (*a*) Water does not wet paraffin wax, so the meniscuses are convex. (*b*) If we add soap or detergent to the water, it will then wet the wax, making the meniscuses concave. (*c*) Provided that the tubes are narrow enough that the meniscuses are spherical, the vertical separation distance between the edge of the meniscus where it meets the wall and its peak or trough is proportional to the radius of the tube, so it is greatest for the largest tube, that is, the 3.00 mm-diameter tube.

To see this, consider the angle θ subtended by half of the meniscus from its center of curvature. The vertical separation distance equals the difference between the radius of curvature R and $R\cos\theta$, that is, $h = R(1 - \cos\theta)$. It is also the case that the radius of curvature is related to the radius of the tube by $R_{\text{tube}} = R\sin\theta$. But because the line from the center of curvature to the edge of the meniscus is perpendicular to the liquid surface where it meets the wall, θ is $90°$ less than the contact angle, that is, $\theta = \theta_c - 90°$. Combining these, we have

$$h = \frac{R_{\text{tube}}}{\sin\theta}(1 - \cos\theta) = R_{\text{tube}} \frac{1 - \sin\theta_c}{|\cos\theta_c|}$$

Because the contact angle does not depend on the radius of the capillary tube, and provided that the tubes are narrow enough that the meniscuses are spherical, the vertical separation distance is proportional to the radius of the tube.

18.43. I expect the liquid to rise to the top of the tube. Clearly, the liquid can rise no higher than the top of the tube because the adhesive force between the liquid and the tube is an essential part of capillary rise. Also, if the liquid were to rise higher than the top of the tube, it would spill over in a continuing flow, so we would have a device that continually pumps liquid upward, continually doing work in violation of conservation of energy. On the other hand, there is no reason to suppose that the liquid would stop short of the top of the tube.

In our analysis of capillary rise in *Principles*, we assumed that the meniscus was spherical, so that the liquid would rise higher in a narrower tube, but this need not be the case. For example, in a wide bowl, the surface of the water is flat over most of its surface, and it only rises in the region immediately adjacent to the sides of the bowl. Something similar must happen here. The pressure difference across the surface of the liquid in the tube, which is proportional to its radius of curvature, corresponds to the pressure difference between the top and bottom of the column of liquid in the tube due to its height. So, I expect the meniscus to be somewhat flattened in the short tube, compared to the meniscus in a tall tube, creating a smaller pressure difference across its surface.

Another way to look at the problem is to consider a tube that is initially submerged in the liquid and is then pushed upward. As the tube just reaches the surface, the liquid surface remains flat, and there is no pressure difference across the liquid surface inside the tube. As the tube rises above the surface, the liquid inside it rises, too, and the surface of the liquid inside the tube becomes more sharply curved as it rises higher, creating a pressure difference corresponding to that due to the height of the column of liquid in the tube. This continues until the meniscus finally becomes spherical. As the tube is raised even higher, the liquid level remains constant at its maximal capillary rise for a tube of that diameter.

18.44. We have in Eq. 18.8 a relationship between pressure and depth,

$$\begin{aligned}P &= P_{\text{surface}} + \rho g d \\ &= (1.0\times10^5\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(730\ \text{m}) = 7.3\times10^6\ \text{Pa}\end{aligned}$$

18.45. Pressure equals the magnitude of force exerted on an area divided by the area, so the magnitude of the force exerted on the eye equals the pressure at this depth times the surface area of the eye. We can use Eq. 18.8 to find the pressure,

$$\begin{aligned}P &= P_{\text{surface}} + \rho g d \\ &= (1.0\times10^5\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(400\ \text{m}) = 4.0\times10^6\ \text{Pa}\end{aligned}$$

so

$$\begin{aligned}F &= PA \\ &= P(\pi R^2) \\ &= \pi(4.0\times10^6\ \text{Pa})(0.050\ \text{m})^2 = 1.3\times10^5\ \text{N}\end{aligned}$$

18.46. The magnitude of the force exerted on the lowest timber in the pile equals the magnitude of the gravitational force exerted on all the timbers above it plus the force exerted on the top of the pile by atmospheric pressure. The pressure on the lowest timber equals this force magnitude divided by the area of the pile. The gravitational force is determined by the pile's mass, which equals its volume times the mass density of wood, and its volume equals its area times its height. Combining these we have

$$\begin{aligned}P &= \frac{F}{A} \\ &= \frac{P_{\text{atm}}A + \rho A h g}{A} \\ &= P_{\text{atm}} + \rho h g\end{aligned}$$

so

$$\begin{aligned}h &= \frac{P - P_{\text{atm}}}{\rho g} \\ &= \frac{(1.4\times10^7\ \text{Pa}) - (1.0\times10^5\ \text{Pa})}{(960\ \text{kg/m}^3)(9.8\ \text{m/s})} = 1.5\times10^3\ \text{m}\end{aligned}$$

18.47. (*a*) The magnitude of the buoyant force exerted on the capsule equals the magnitude of the gravitational force exerted on the volume of water it displaces.

$$\begin{aligned} F_{\text{wc}}^{\text{b}} &= \rho_{\text{w}} V_{\text{disp}} g \\ &= \rho_{\text{w}} \left[\pi (d/2)^2 (\ell - h) \right] g \\ &= \pi (1025 \text{ kg/m}^3) \left[(1.10 \text{ m})/2 \right]^2 \left[(2.34 \text{ m}) - (0.820 \text{ m}) \right] (9.8 \text{ m/s}^2) \\ &= 1.45 \times 10^4 \text{ N} \end{aligned}$$

(*b*) The magnitude of the buoyant force exerted on the capsule still equals the magnitude of the gravitational force exerted on the volume of water it displaces, but because it is now entirely submerged, the displaced volume is larger.

$$\begin{aligned} F_{\text{wc}}^{\text{b}} &= \rho_{\text{w}} V_{\text{disp}} g \\ &= \rho_{\text{w}} \left[\pi (d/2)^2 \ell \right] g \\ &= \pi (1025 \text{ kg/m}^3) \left[(1.10 \text{ m})/2 \right]^2 (2.34 \text{ m})(9.8 \text{ m/s}^2) \\ &= 2.23 \times 10^4 \text{ N} \end{aligned}$$

(*c*) It sinks because its average mass density has become greater than the mass density of water. This is because the interior spaces, which had been filled with air, are now filled with water.

18.48. There are two forces exerted on the window, an outward force caused by the pressure of the air inside the bell and an inward force caused by the pressure of the water outside. The pressure inside is constant, but the pressure outside increases with depth in accordance with Eq. 18.8, and the magnitude of the forces caused by the pressure equals the pressure times the area of the window. Taking the inward direction to be positive, we have

$$\begin{aligned} F &= P_{\text{w}} A - P_{\text{a}} A \\ &= (P_{\text{surface}} + \rho g d - P_{\text{a}}) A \\ d &= \frac{(F/A) - P_{\text{surface}} + P_{\text{a}}}{\rho g} \\ &= \frac{\left[(195 \times 10^3 \text{ N})/(0.250 \text{ m})^2 \right] - (1.01 \times 10^5 \text{ Pa}) + 2.00(1.01 \times 10^5 \text{ Pa})}{(1025 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} \\ &= 321 \text{ m} \end{aligned}$$

18.49. Pascal's demonstration worked because the pressure increase at the bottom of the tube compared to the top is proportional to the height of the water in the tube, regardless of the shape or size of the tube. If the tube is narrow, only a small amount of water is needed to fill the tube to a large height, creating a large increase in pressure at the bottom, which equals the pressure at the top of the barrel.

There are two forces exerted on the barrel's top, that due to the pressure in the water and that due to atmospheric pressure, and the magnitudes of these forces equals the pressure times the area of the barrel's top. The pressure in the water at the level of the barrel's top is given by Eq. 18.8, $P = P_{\text{surface}} + \rho g h,$ where P_{surface} is atmospheric pressure, so the vector sum of the forces on the barrel's top is $F = \rho g h A_{\text{barrel}}.$ The height of the water in the tube is its volume divided by the cross-sectional area of the tube,

$$\begin{aligned} h &= \frac{V}{\pi (d_{\text{tube}}/2)^2} \\ &= \frac{(3.39 \times 10^{-4} \text{ m}^3)}{\pi \left[(6.0 \times 10^{-3} \text{ m})/2 \right]^2} = 12 \text{ m} \end{aligned}$$

The force on the barrel's top is

$$\begin{aligned}F &= \rho g \frac{V}{A_{\text{tube}}} A_{\text{barrel}} \\ &= \rho g \frac{V}{\pi (d_{\text{tube}}/2)^2} \left[\pi (d_{\text{barrel}}/2)^2 \right] \\ &= \rho g V d_{\text{barrel}}^2 / d_{\text{tube}}^2 \\ &= (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(3.39 \times 10^{-4} \text{ m}^3)(0.42 \text{ m})^2/(6.0 \times 10^{-3} \text{ m})^2 \\ &= 1.6 \times 10^4 \text{ N}\end{aligned}$$

18.50. When the water exits the hose, it has some amount of kinetic energy, which allows it to rise until all of its kinetic energy has been converted into gravitational potential energy in the Earth-water system. Per unit volume of water, we have $\frac{1}{2}\rho v^2 = \rho g h,$ or $h = v^2/(2g)$ and $v^2 = 2gh.$

I know from experience that when I partially cover the end of the hose with my thumb, the water exits with greater speed. As a first approximation, it seems reasonable to suppose that the volume of water that exits the hose per unit time remains constant as I partially cover the opening. Because the volume of water that exits the hose per unit time equals the product of the opening's area times the exit speed, the exit speed increases as the opening's area decreases, $A_1 v_1 = A_2 v_2$ or $v_2 = v_1 A_1/A_2.$ Combining these we have

$$\begin{aligned}h_2 &= v_2^2/(2g) \\ &= (v_1 A_1/A_2)^2/(2g) \\ &= \frac{v_1^2 \left[\pi (d_1/2)^2 \right]^2}{\left[\pi (d_2/2)^2 \right]^2 2g} \\ &= \frac{2gh_1 \left[\pi (d_1/2)^2 \right]^2}{\left[\pi (d_2/2)^2 \right]^2 2g} \\ &= \frac{h_1 d_1^4}{d_2^4} \\ &= \frac{(0.15 \text{ m})(20 \text{ mm})^4}{(10 \text{ mm})^4} = 2.4 \text{ m}\end{aligned}$$

18.51. (*a*) If we make the same assumptions as we made in Problem 50, we arrive at the same equation for the height of the stream,

$$\begin{aligned}h_2 &= \frac{h_1 d_1^4}{d_2^4} \\ &= \frac{(0.15 \text{ m})(20 \text{ mm})^4}{(1.0 \text{ mm})^4} = 2.4 \times 10^4 \text{ m}\end{aligned}$$

(*b*) This result is very much contrary to experience. Our main assumption in solving Problem 50 was that the volume of water that exits the hose per unit time remains constant as the opening's area decreases, but this must not be the case when the opening's area decreases by a large factor. Other experience confirms this: For example, the further we open a faucet valve, the greater the volume of water that flows from the tap per unit time. In this example, the area of the opening in the valve is on the order of square millimeters, compared to the cross-sectional area of the pipes, which are on the order of square centimeters.

The first physical effect that comes to mind is viscosity. If the diameter of the opening is large in comparison to the thickness of the boundary layer (see *Principles* Figure 18.19), viscosity will have little effect on the flow's speed. If, on the other hand, the diameter of the opening is comparable to the thickness of the boundary layer, the flow's speed will be greatly reduced compared to that of a nonviscous fluid's flow.

18.52. (*a*) We have that $P(y)+\rho(y)gy=\text{constant}$ for all y. Differentiating with respect to y gives $(dP/dy)+\rho g+(d\rho/dy)gy=0$, or $dP=-\left[\rho g+(d\rho/dy)gy\right]dy$. (*b*) Using a subscript zero to denote values at sea level, our simplified model says that $\rho(y)=\rho_0 P(y)/P_0$, so our differential equation becomes $(dP/dy)+(\rho_0/P_0)Pg+(\rho_0/P_0)(dP/dy)gy=0$, or $\dfrac{1}{P}dP=\dfrac{-(\rho_0/P_0)g}{1+(\rho_0/P_0)gy}dy$. Integration gives $\ln P-\ln P_0=-\ln\left[(\rho_0/P_0)gy+1\right]$, or $\ln(P/P_0)=\ln\left\{1/\left[(\rho_0/P_0)gy+1\right]\right\}$. Finally, exponentiation and rearrangement gives $P=P_0/\left[(\rho_0/P_0)gy+1\right]$.

18.53. The force exerted by any portion of the water on the dam equals the pressure in the water times the area of the dam that it presses against. The pressure in the water increases with depth, $P=P_{\text{surface}}+\rho gd$, so one approach is to conceptually divide the face of the dam into thin, horizontal strips of height dy, along which the pressure is constant, and integrate the contributions to the force of each strip. The width of each strip is proportional to its height above the valley floor, $w(y)=wy/h$. Combining these, we have

$$\begin{aligned}
F &= \int_{y=0}^{h} P(y)w(y)\,dy \\
&= \int_{y=0}^{h}\left[P_{\text{surface}}+\rho g(h-y)\right](wy/h)\,dy \\
&= \int_{y=0}^{h} \frac{P_{\text{surface}}wy}{h}+\rho gwy-\frac{\rho gwy^2}{h}\,dy \\
&= \tfrac{1}{2}\frac{P_{\text{surface}}w}{h}y^2+\tfrac{1}{2}\rho gwy^2-\tfrac{1}{3}\frac{\rho gw}{h}y^3\Big|_{y=0}^{h} \\
&= \tfrac{1}{2}P_{\text{surface}}wh+\tfrac{1}{2}\rho gwh^2-\tfrac{1}{3}\rho gwh^2 \\
&= \tfrac{1}{2}P_{\text{surface}}wh+\tfrac{1}{6}\rho gwh^2=(\tfrac{1}{2}P_{\text{surface}}+\tfrac{1}{6}\rho gh)wh
\end{aligned}$$

18.54. In a barometer, we have $P=\rho gh$, so if the pressure changes, the height of the mercury does, too, $\Delta P=\rho g\Delta h$. So the change in height is

$$\begin{aligned}
\Delta h &= \frac{\Delta P}{\rho g} \\
&= \frac{0.10\text{ atm}}{(13{,}534\text{ kg/m}^3)(9.8\text{ m/s}^2)}\frac{1.01\times10^5\text{ Pa}}{1\text{ atm}}=0.076\text{ m}
\end{aligned}$$

18.55. The pressure in the flask supports the piston against the combined force of atmospheric pressure on the piston and the gravitational force on the block. The force on the piston caused by pressure equals the product of the pressure and the area of the piston. Combining these, we have

$$\begin{aligned}
P_{\text{flask}}A_{\text{piston}} &= P_{\text{atm}}A_{\text{piston}}+mg \\
P_{\text{flask}} &= P_{\text{atm}}+\frac{mg}{A_{\text{piston}}} \\
P_{\text{flask}} &= P_{\text{atm}}+\frac{mg}{\pi R^2}
\end{aligned}$$

18.56. Because the pistons are at the same height, the pressure in the water just beneath them is the same. The gauge pressure in the water supports each piston against the gravitational force on the objects resting on them, and pressure equals force divided by area, so

$$\begin{aligned}P_1 &= P_2\\ F_1/A_1 &= F_2/A_2\\ (m_1g)/(\pi R_1^2) &= (m_2g)/(\pi R_2^2)\\ R_1 &= R_2\sqrt{m_1/m_2}\\ R_1 &= (1.6\text{ m})\sqrt{(65\text{ kg})/(1000\text{ kg})} = 0.41\text{ m}\end{aligned}$$

18.57. Gauge pressure equals absolute pressure minus atmospheric pressure, so the gauge pressures at given depths are just $P_{\text{gauge}} = \rho g d$. This expression does not depend on the cross-sectional area of the container, only the depth below the surface. So we have for the ratio,

$$\begin{aligned}(P_{\text{A}}/P_{\text{B}})/(P_{\text{A}'}/P_{\text{B}'}) &= \left[(\rho_{\text{A}} g d_{\text{A}})/(\rho_{\text{B}} g d_{\text{B}})\right]/\left[(\rho_{\text{A}'} g d_{\text{A}'})/(\rho_{\text{B}'} g d_{\text{B}'})\right]\\ &= (d_{\text{A}}/d_{\text{B}})/(d_{\text{A}'}/d_{\text{B}'})\\ &= \left[(h/2)/h\right]/\left[(h/2)/h\right]\\ &= 1\end{aligned}$$

That is, because the fluids are incompressible, their mass densities do not depend on depth, so the gauge pressures at depth h are twice the gauge pressures at depth $h/2$, regardless of the fluid in the container or the container's shape, so the ratio is exactly one.

18.58. The pressure in the water in the two legs of the tube is the same at any horizontal level. In particular, at the level of the bottom of the oil in the left leg, the pressures are the same and they are sufficient to support the fluids above them against the force of gravity. That is,

$$\begin{aligned}\rho_1 V_1 g &= \rho_2 V_2 g + \rho_{\text{water}} V_{\text{water}} g\\ \rho_1 h_1 A g &= \rho_2 (h_1 - h_2) A g + \rho_{\text{water}} h_2 A g\\ h_1(\rho_1 - \rho_2) &= h_2(\rho_{\text{water}} - \rho_2)\\ h_1 &= \frac{h_2(\rho_{\text{water}} - \rho_2)}{\rho_1 - \rho_2}\\ h_1 &= \frac{(20\text{ mm})\left[(1000\text{ kg/m}^3) - (700\text{ kg/m}^3)\right]}{(800\text{ kg/m}^3) - (700\text{ kg/m}^3)} = 60\text{ mm}\end{aligned}$$

18.59. The inward force exerted by the balloon's elasticity on the air in the balloon equals the product of the gauge pressure of the air in the balloon times the surface area of the balloon. That pressure equals the pressure at the bottom of a column of mercury $(65.0\text{ mm}) - (40.0\text{ mm}) = 25.0\text{ mm}$ high. That is,

$$\begin{aligned}F &= PA\\ &= \rho g h A\\ &= (13.5\times10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(25.0\times10^{-3}\text{ m})(0.300\text{ m}^2) = 992\text{ N}\end{aligned}$$

18.60. (*a*) The magnitude of the contact force exerted by the road surface on the car's tires equals the magnitude of the force of gravity on the car. It also equals the product of the gauge pressure of the air in the tires times the area of the tires in contact with the road surface. Because the magnitude of the force of gravity on the car is unchanged, if the gauge pressure of the air in the tires increases by 10%, the area of the tires in contact with the road surface must decrease by 9.1% to keep their product the same:

$$\begin{aligned}F &= P_1 A_1 = P_2 A_2\\ A_2 &= P_1 A_1/P_2 = P_1 A_1/(1.10 P_1) = 0.909 A_1\end{aligned}$$

(*b*) The gauge pressure increased from 2.0 atm to 2.2 atm, so the absolute pressure increased from 3.0 atm to 3.2 atm, an increase of 6.7%.

18.61. (*a*) The air from the compressor is at the same gauge pressure as the hydraulic fluid in the lift because it presses on the other side of the small piston, and that gauge pressure is equal to the pressure caused by the gravitational force on the car pressing down on the large piston:

$$\begin{aligned} P &= F/A \\ &= mg/\pi R^2 \\ &= (1500\ \text{kg})(9.8\ \text{m/s}^2)/\pi(0.100\ \text{m})^2 = 4.68\times 10^5\ \text{Pa} \end{aligned}$$

(*b*) The same volume of fluid that raises the car must be pushed out of the small piston.

$$h_1A_1 = h_2A_2$$

$$h_2 = h_1A_1/A_2 = h_1(R_1/R_2)^2 = (1.8\ \text{m})\left[(100\ \text{mm})/(15\ \text{mm})\right]^2 = 80\ \text{m}$$

18.62. The buoyant force that lifts the stone and basket has the same magnitude as the gravitational force exerted on the air the balloon displaces, which means that the displaced air has the same mass as the stone and basket combined. The mass of the displaced air equals its mass density times its volume, so

$$\begin{aligned} m_{\text{stone}} + m_{\text{basket}} &= m_{\text{displaced air}} \\ &= \rho_{\text{air}} V_{\text{balloon}} \\ &= \rho_{\text{air}} \tfrac{4}{3}\pi R^3 \\ R &= \left(\frac{3(m_{\text{stone}} + m_{\text{basket}})}{4\pi\rho_{\text{air}}}\right)^{1/3} \\ &= \left(\frac{3\left[(125\ \text{kg}) + (15.0\ \text{kg})\right]}{4\pi(1.29\ \text{kg/m}^3)}\right)^{1/3} = 2.96\ \text{m} \end{aligned}$$

18.63. Initially, there are two forces on the cylinder, the upward contact force exerted by the spring and the downward force of gravity, and their vector sum is zero. That is, $k\Delta x_1 - m_{\text{cyl}}g = 0,$ which we can write as $k\Delta x_1 - \rho_{\text{Al}}V_{\text{cyl}}g = 0,$ or $k = \rho_{\text{Al}}V_{\text{cyl}}g/\Delta x_1.$

After the vat is filled, there are three forces on the cylinder, the upward contact force exerted by the spring, the upward buoyant force exerted by the liquid, and the downward force of gravity, and their vector sum is also zero. Because the magnitude of the buoyant force equals the magnitude of the gravitational force on the displaced liquid, and the cylinder is completely submerged, we have $k\Delta x_2 + \rho_{\text{liquid}}V_{\text{cyl}}g - \rho_{\text{Al}}V_{\text{cyl}}g = 0.$ Substituting our earlier expression for the spring constant $k,$ we can solve this equation for the mass density of the liquid:

$$(\rho_{\text{Al}}V_{\text{cyl}}g/\Delta x_1)\Delta x_2 + \rho_{\text{liquid}}V_{\text{cyl}}g - \rho_{\text{Al}}V_{\text{cyl}}g = 0$$

$$\begin{aligned} \rho_{\text{liquid}} &= \rho_{\text{Al}} - (\rho_{\text{Al}}/\Delta x_1)\Delta x_2 \\ &= \rho_{\text{Al}}(1 - \Delta x_2/\Delta x_1) \\ &= (2700\ \text{kg/m}^3)\left[1 - (42.0\ \text{mm} - 11.8\ \text{mm})/(42.0\ \text{mm})\right] \\ &= 759\ \text{kg/m}^3 \end{aligned}$$

18.64. There are three forces on the block, the upward contact force exerted by the wire, the upward buoyant force exerted by the water, and the downward force of gravity, and their vector sum is zero. So, the magnitude of the contact force exerted by the wire, which equals the tension in the wire, is equal to the difference between the magnitude of the downward force of gravity on the block and that of the upward buoyant force exerted on it by the water, $\mathcal{T} = F^G_{\text{Eb}} - F^{\text{b}}_{\text{wb}}.$

The magnitude of the buoyant force equals the magnitude of the force of gravity on the displaced volume of water, which is proportional to the length of the block that is submerged, $F^{\text{b}}_{\text{wb}} = \rho_{\text{water}}gV_{\text{submerged}} = \rho_{\text{water}}g\ell_{\text{submerged}}A,$ where A is the cross-sectional area of the block. Combining the preceding, we have $\mathcal{T} = mg - \rho_{\text{water}}g\ell_{\text{submerged}}A,$ or $\ell_{\text{submerged}} =$

$\dfrac{mg-\mathcal{T}}{\rho_{\text{water}}gA}$. The distance between the top of the block and the surface of the water equals the length of the block minus the length that is submerged, $h=\ell-\ell_{\text{submerged}}$, so we have

$$\begin{aligned}h&=\ell-\frac{mg-\mathcal{T}}{\rho_{\text{water}}gA}\\&=(0.750\text{ m})-\frac{(15.0\text{ kg})(9.8\text{ m/s}^2)-(135\text{ N})}{(1000\text{ kg/m}^3)(9.8\text{ m/s}^2)(0.00125\text{ m}^2)}=0.652\text{ m}\end{aligned}$$

18.65. When we described a barometer in *Principles*, we took the pressure at the top of the mercury column to be zero because it is in contact with vacuum. But surface tension causes the top of the mercury column to form a convex meniscus, the curvature of which results in a nonzero pressure at the top of the column, and this pressure corresponds to the decrease in the level of the mercury, $P=\rho gh$.

The pressure at the top of the mercury column depends of the radius of curvature of the meniscus, $P=2\gamma/R$. If the tube is narrow enough that the meniscus is spherical, then because the line from the center of curvature to the edge of the meniscus is perpendicular to the mercury surface where it meets the wall, the angle between this line and the vertical is 90° less than the contact angle, and we have for the radius of curvature $R\sin(\theta_{\text{c}}-90°)=R_{\text{tube}}$, or $R=-R_{\text{tube}}/\cos\theta_{\text{c}}$.

Combining the preceding, we have $P=\rho gh=2\gamma/R=-2\gamma\cos\theta_{\text{c}}/R_{\text{tube}}$, or

$$\begin{aligned}h&=\frac{-2\gamma\cos\theta_{\text{c}}}{\rho gR_{\text{tube}}}\\&=\frac{-2(0.465\text{ N/m})\cos 137°}{(13.5\times10^3\text{ kg/m}^3)(9.8\text{ m/s}^2)(3.5\times10^{-3}\text{ m})}=1.5\times10^{-3}\text{ m}\end{aligned}$$

18.66. (*a*) When the cube is released, it gains kinetic energy as it falls due to the work done on it by gravity. Once the cube is in the water, the upward buoyant force does negative work on the cube, slowing it down, while gravity continues to do positive work on it as it sinks to its lowest level. At that point, its kinetic energy is again zero, so the sum of the positive work done by gravity and the negative work done by the buoyant force equals zero. That is,

$$\begin{aligned}m_{\text{b}}g(h+d)-F^{\text{b}}_{\text{wb}}d&=0\\V_{\text{b}}\rho_{\text{b}}g(h+d)-V_{\text{b}}\rho_{\text{w}}gd&=0\\V_{\text{b}}\rho_{\text{b}}gh&=V_{\text{b}}\rho_{\text{w}}gd-V_{\text{b}}\rho_{\text{b}}gd\end{aligned}$$

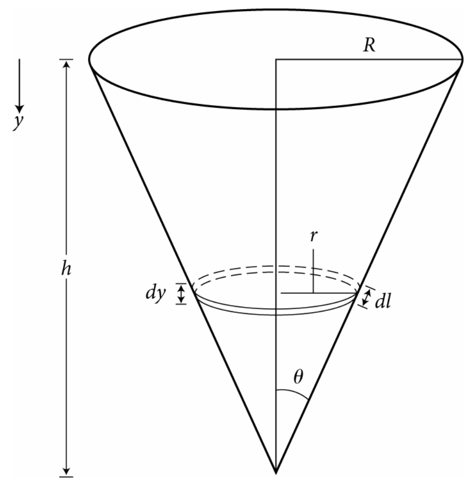
or

$$\begin{aligned}d&=\frac{V_{\text{b}}\rho_{\text{b}}gh}{V_{\text{b}}\rho_{\text{w}}g-V_{\text{b}}\rho_{\text{b}}g}\\&=\frac{\rho_{\text{b}}}{\rho_{\text{w}}-\rho_{\text{b}}}h\\&=\frac{850\text{ kg/m}^3}{(1000\text{ kg/m}^3)-(850\text{ kg/m}^3)}(5.00\text{ m})=28.3\text{ m}\end{aligned}$$

(*b*) While the block is under water, there are two forces exerted on it, the upward buoyant force and the downward gravitational force, the vector sum of which gives the block a constant upward acceleration. When the block is at its deepest position in the water, its speed is zero, so we can easily calculate the time it takes to return to the surface from the kinematic equation $\Delta x=\frac{1}{2}a(\Delta t)^2$. Because of symmetry, it takes the same amount of time to reach that position as it descends from the surface. So, we can find the time interval it spends in the water as

$$\begin{aligned}\Delta t &= 2\sqrt{\frac{2d}{a}}\\ &= 2\sqrt{\frac{2d}{F/m_b}}\\ &= 2\sqrt{\frac{2d}{(\rho_w-\rho_b)V_b g/(\rho_b V_b)}}\\ &= 2\sqrt{\frac{2d\rho_b}{(\rho_w-\rho_b)g}}\\ &= 2\sqrt{\frac{2(28.3\text{ m})(850\text{ kg/m}^3)}{\left[(1000\text{ kg/m}^3)-(850\text{ kg/m}^3)\right](9.8\text{ m/s}^2)}} = 11.4\text{ s}\end{aligned}$$

18.67. (*a*)



The pressure in the liquid causes it to exert a downward force on the flat side of the cone and an upward force on its curved surface. Let's call the pressure at the level of the flat side P_0, and measure depth *y* downward from there.

Because the pressure is uniform over the flat side, the downward force exerted on it by the liquid equals the pressure times its area, $F_{\text{down}} = P_0 A = P_0 \pi R^2$. The pressure increases with depth, $P = P_0 + \rho g y$, so to calculate the force on the curved surface of the cone, we mentally divide it into thin strips of height dy, along which the pressure does not change. The length of the diagonal side of the strip is $d\ell = dy/\cos\varphi$, where φ is the angle between the cone's side and the vertical, and so $\tan\varphi = R/h$, and the radius of the strip is $r = R(1-y/h)$. If we divide each strip into surface elements of width $dw = r d\theta$, then the magnitude of the force exerted on any element is $dF = P d\ell dw = (Pr/\cos\varphi) d\theta dy$, and this force is directed perpendicular to the cone's surface. If we take the vector sum of the forces exerted on two diametrically opposed surface elements, the horizontal components cancel, leaving only the vertical components, $dF_{\text{up}} = 2Pr(\sin\varphi/\cos\varphi) d\theta dy$. Integrating this over half the surface of the cone (because it includes contributions from diametrically opposed surface elements) gives the upward force exerted by the liquid on the cone,

$$\begin{aligned}
F_{\text{up}} &= \int_{y=0}^{h}\int_{\theta=0}^{\pi} 2(P_0+\rho g y)R(1-y/h)\tan\varphi\, d\theta dy \\
&= \int_{y=0}^{h}\int_{\theta=0}^{\pi} (2R^2/h)\left[P_0-(P_0y/h)+\rho g y-(\rho g y^2/h)\right]d\theta dy \\
&= (2\pi R^2/h)\int_{y=0}^{h} P_0-(P_0y/h)+\rho g y-(\rho g y^2/h)\,dy \\
&= (2\pi R^2/h)(P_0h-\tfrac{1}{2}P_0h+\tfrac{1}{2}\rho g h^2-\tfrac{1}{3}\rho g h^2) \\
&= \pi R^2(P_0+\tfrac{1}{3}\rho g h)
\end{aligned}$$

The total force exerted by the liquid on the cone is then

$$\begin{aligned}
F &= F_{\text{up}}-F_{\text{down}} \\
&= \pi R^2(P_0+\tfrac{1}{3}\rho g h)-P_0\pi R^2 \\
&= \tfrac{1}{3}\pi R^2\rho g h \\
&= \rho g V
\end{aligned}$$

(*b*) Because the cone is completely submerged, the volume of liquid it displaces is equal to its volume, so the force we calculated in part *a* is just the magnitude of the gravitational force exerted on the liquid displaced by the cone, in accord with Archimedes' principle.

18.68. The volume flow rate in the pipe equals the volume flow rate into the bucket. It also equals the product of the speed of the water in the pipe times the cross-sectional area of the pipe.

$$\begin{aligned}
\frac{V}{\Delta t} &= Av \\
v &= \frac{V}{A\Delta t} \\
&= \frac{4.00\text{ L}}{\pi\left[(12.5\text{ mm})/2\right]^2(1\text{ min})}\frac{1\text{ m}^3}{1000\text{ L}}\left(\frac{1000\text{ mm}}{1\text{ m}}\right)^2\frac{1\text{ min}}{60\text{ s}} = 0.543\text{ m/s}
\end{aligned}$$

18.69. As in *Principles* Example 18.9, we can consider the lake and dam to form a tube running from the lake's surface to the hole in the dam. The pressure at both surfaces is atmospheric and the speed of the water in the lake is approximately zero, so Bernoulli's equation reduces to $\rho g d = \frac{1}{2}\rho v^2$ or $v=\sqrt{2gd}$. Substituting the numerical values gives $v=\sqrt{2(9.8\text{ m/s}^2)\left[(25.2\text{ m})-(1.40\text{ m})\right]} = 21.6\text{ m/s}$.

18.70. Because the air flows more quickly between the tractor trailers than along their other sides, the pressure is less than atmospheric between them, so the air exerts a force on each equal to the product of its area times the pressure difference. Because the air speeds are not too large, we can assume the mass density of the air is constant, so we can use Bernoulli's equation to calculate the pressure difference. The air does not change height, so Bernoulli's equation reduces to $P_1+\frac{1}{2}\rho v_1^2 = P_2+\frac{1}{2}\rho v_2^2$ or $P_1-P_2=\frac{1}{2}\rho(v_2^2-v_1^2)$, and the magnitude of the force exerted on either of the tractor trailers is $F=(P_1-P_2)A=\frac{1}{2}\rho(v_2^2-v_1^2)A$. Substituting numerical values gives $F = \frac{1}{2}(1.0\text{ kg/m}^3)\left[(20\text{ m/s})^2-(5.0\text{ m/s})^2\right](16\text{ m})(4.0\text{ m}) = 1.2\times10^4\text{ N}$.

18.71. As in *Principles* Example 18.9, we can consider the bucket to form a tube running from the surface to the hole in the side. The pressure at both locations is atmospheric, so Bernoulli's equation reduces to $\rho g d+\frac{1}{2}\rho v_1^2=\frac{1}{2}\rho v_2^2$ or $v_2=\sqrt{2gd+v_1^2}$.

Note that, because $v_1\neq 0$, the height y of the water surface above the hole is not constant; in particular, its time rate of change is $dy/dt=-v_1$. So, our expression is only valid at the instant the hole is opened. At later times, we would have to replace d and v_1 with their then-current values.

If we knew the cross-sectional area of the bucket and the area of the hole, we could use the continuity equation (Eq. 18.24) to eliminate one of the speeds:

$$\rho g d + \tfrac{1}{2}\rho v_1^2 = \tfrac{1}{2}\rho v_2^2$$
$$\rho g d + \tfrac{1}{2}\rho (v_2 A_2/A_1)^2 = \tfrac{1}{2}\rho v_2^2$$
$$2gd = v_2^2\left[1-(A_2/A_1)^2\right]$$
$$v_2 = \sqrt{\frac{2gd}{1-(A_2/A_1)^2}}$$

18.72. From the position that the water strikes the roof, we can determine its speed as it exits the hose. Each bit of water behaves like a horizontally fired projectile, traveling $7.0\ \text{m}$ in the time it takes to fall $1.0\ \text{m}$. That is,

$$v = \Delta x/\Delta t$$
$$\Delta t = \sqrt{2h/g}$$
$$v = \Delta x/\sqrt{2h/g}$$
$$= (7.0\ \text{m})/\sqrt{2(1.0\ \text{m})/(9.8\ \text{m/s}^2)} = 15\ \text{m/s}$$

We can now use Bernoulli's equation to determine the pressure in the chamber. Because the chamber is described as "large," we can assume that the speed of the water in the chamber is approximately zero. We also know that the pressure in the water is atmospheric pressure as it exits the hose.

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$
$$P_1 - P_2 = \rho\left[g(y_2 - y_1) + \tfrac{1}{2}(v_2^2 - v_2^2)\right]$$
$$P_1 - P_2 = (1000\ \text{kg/m}^3)\left[(9.8\ \text{m/s}^2)(9.5\ \text{m}) + \tfrac{1}{2}(15\ \text{m/s})^2\right] = 2.1\times10^5\ \text{Pa}$$

This is the gauge pressure in the chamber; the absolute pressure is $3.1\times10^5\ \text{Pa}$.

18.73. The transmural pressure is the difference in pressure inside and outside the blood vessel. It seems reasonable to suppose that the pressure outside is approximately atmospheric pressure, but so long as it is the same at both points along the aorta, we can use the transmural pressure in Bernoulli's equation. That is, we can add the same pressure to both sides of the equation and have it still hold.

Combining Bernoulli's equation (where we omit the height-dependent term because it is the same on both sides) with the continuity equation we have

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$
$$A_1 v_1 = A_2 v_2$$
$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho (v_1 A_1/A_2)^2$$

Setting $P_2 = 0$ and solving for A_2/A_1 gives

$$P_1 + \tfrac{1}{2}\rho v_1^2 = \tfrac{1}{2}\rho (v_1 A_1/A_2)^2$$
$$\frac{P_1 + \tfrac{1}{2}\rho v_1^2}{\tfrac{1}{2}\rho v_1^2} = (A_1/A_2)^2$$
$$A_2/A_1 = \sqrt{\frac{1}{1+2P_1/(\rho v_1^2)}}$$
$$= \sqrt{\frac{1}{1+2(12\times10^3\ \text{Pa})/\left[(1060\ \text{kg/m}^3)(0.350\ \text{m/s})^2\right]}} = 0.073$$

That is, $100\% - 7.3\% = 93\%$ of the cross-sectional area of the aorta must be blocked in order for the transmural pressure to equal zero. If any more of the blood vessel were blocked, the transmural pressure would become negative, which is to say that the blood vessel would collapse under the external pressure.

18.74. The laminar flow of a viscous fluid should exhibit the Bernoulli effect. Regardless of the fluid's viscosity, the continuity equation requires the fluid's average speed to increase as it passes from a region of wider to narrower flow, and this increase in speed must be due to a force exerted on the fluid. The only candidate for that force is the difference in pressure between the two regions.

Bernoulli's equation in the form of Eq. 18.36 does not apply to a viscous fluid. Our derivation of Bernoulli's equation was based on conservation of energy, but viscous effects cause dissipation of a fluid's mechanical energy. In order to obtain an equation that would apply to a viscous fluid, we would need to include terms to account for the dissipation of mechanical energy, that is, its conversion into thermal energy.

18.75. We can solve this problem using the continuity equation and Bernoulli's equation. The former relates the speed of the water in the tube to the speed of the water flowing through the hole, while the latter relates these speeds to the depth of the water in the tube (the pressures are the same, namely, atmospheric pressure, at the upper surface of the water in the tube and at the water as it exits the hole). (*a*) From the continuity equation we have $v_1A_1 = v_2A_2$ or $v_1 = v_2A_2/A_1 = v_2(R_2/R_1)^2$. From Bernoulli's equation we have $\rho gd + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$. Combining these we have

$$\rho gd + \tfrac{1}{2}\rho\left[v_2(R_2/R_1)^2\right]^2 = \tfrac{1}{2}\rho v_2^2$$
$$2gd = v_2^2\left[1-(R_2/R_1)^4\right]$$
$$v_2 = \sqrt{\frac{2gd}{1-(R_2/R_1)^4}}$$
$$= \sqrt{\frac{2(9.8\text{ m/s}^2)(1.10\text{ m})}{1-(80.0\text{ mm}/150\text{ mm})^4}} = 4.84\text{ m/s}$$

(*b*) From the definition of volume flow rate we have

$$Q = V/\Delta t = Av$$
$$= (\pi R^2)v$$
$$= \pi(0.0800\text{ m})^2(4.84\text{ m/s}) = 9.74\times10^{-2}\text{ m}^3/\text{s}$$

(*c*) From the continuity equation we have $v_1A_1 = v_2A_2$ or $v_1 = v_2A_2/A_1 = v_2(R_2/R_1)^2$. That is, $v_1 = v_2(R_2/R_1)^2 = (4.84\text{ m/s})\left[(80.0\text{ mm})/(150\text{ mm})\right]^2 = 1.38\text{ m/s}$.

18.76. We have from Problem 18.71 that $\rho gy + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$, or $2gy + v_1^2 = v_2^2$, and we can use the continuity equation to relate the two speeds, $Q = v_1A_1 = v_2A_2$. We also know that the height of the water above the hole decreases as the water flows out, $dy/dt = -v_1$, which in turn decreases the flow rate. Combining these, we have

$$2gy + v_1^2 = (v_1A_1/A_2)^2$$
$$v_1 = \sqrt{\frac{2gy}{(A_1/A_2)^2-1}}$$
$$\frac{dy}{dt} = -\sqrt{\frac{2gy}{(A_1/A_2)^2-1}}$$

or

$$-\sqrt{\frac{(A_1/A_2)^2-1}{2gy}}\,dy = dt$$

If we integrate the right side to obtain the time interval over which the water drains from the bucket, the corresponding limits of integration on the left side will be as the height of the water decreases from its initial height above the hole to zero.

$$\int_{y=d}^{0} -\sqrt{\frac{(A_1/A_2)^2 - 1}{2gy}}\,dy = \int_{t=0}^{\Delta t} dt$$

$$2\sqrt{\frac{\left[(A_1/A_2)^2 - 1\right]d}{2g}} = \Delta t$$

$$\Delta t = \sqrt{\frac{2\left[(A_1/A_2)^2 - 1\right]d}{g}}$$

18.77. If we assume that the water wets the material perfectly, then the surface tension at the edge of the water exerts an upwardly directed force on the water, the magnitude of which is equal to the product of the surface tension times the circumference of the tube, $F = 2\pi R\gamma$. (If the water does not wet the material perfectly, we need to multiply our expression by $\cos\theta_c$, the cosine of the contact angle, to obtain the vertical component of the force.) At equilibrium, the magnitude of this force equals the magnitude of the force of gravity exerted on the water in the tube, so

$$\begin{aligned} 2\pi R\gamma &= mg \\ m &= 2\pi R\gamma/g \\ &= 2\pi(0.50\times10^{-3}\ \text{m})(7.28\times10^{-2}\ \text{N/m})/(9.8\ \text{m/s}^2) = 2.3\times10^{-5}\ \text{kg} \end{aligned}$$

This corresponds to a height of

$$\begin{aligned} h &= \frac{m}{\pi R^2 \rho} \\ &= \frac{2.3\times10^{-5}\ \text{kg}}{\pi(0.50\times10^{-3}\ \text{m})^2(1000\ \text{kg/m}^3)} = 3.0\times10^{-2}\ \text{m} \end{aligned}$$

18.78. If we assume that the liquid wets the material perfectly, then the surface tension at the edge of the liquid exerts an upwardly directed force on the liquid, the magnitude of which is equal to the product of the surface tension times the circumference of the tube, $F = 2\pi R\gamma$. (If the liquid does not wet the material perfectly, we need to multiply our expression by $\cos\theta_c$, the cosine of the contact angle, to obtain the vertical component of the force.) At equilibrium, the magnitude of this force equals the magnitude of the force of gravity exerted on the liquid in the tube, so

$$\begin{aligned} 2\pi R\gamma &= mg \\ &= (\rho V)g \\ &= \rho(\pi R^2 h)g \end{aligned}$$

or

$$\begin{aligned} \gamma &= \tfrac{1}{2}\rho g R h \\ &= \tfrac{1}{2}(1261\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.350\times10^{-3}\ \text{m})(29.0\times10^{-3}\ \text{m}) = 6.27\times10^{-2}\ \text{N/m} \end{aligned}$$

18.79. We know from Poiseuille's law that the flow rate of a fluid in a tube is inversely proportional to the viscosity of the fluid, so

$$\begin{aligned} Q_{\text{water}}\eta_{\text{water}} &= Q_{\text{castor oil}}\eta_{\text{castor oil}} \\ Q_{\text{castor oil}} &= Q_{\text{water}}\eta_{\text{water}}/\eta_{\text{castor oil}} \\ &= (2.00\times10^{-4}\ \text{m}^3/\text{s})(1.00\times10^{-3}\ \text{Pa}\cdot\text{s})/(0.986\ \text{Pa}\cdot\text{s}) = 2.03\times10^{-7}\ \text{m}^3/\text{s} \end{aligned}$$

18.80. The volume flow rate equals the product of the water's average flow speed and the cross-sectional area of the outlet. Because the showerhead is described as "low-flow," we should assume that we cannot ignore the viscosity of the water, so the water's average flow speed is one-half of its maximum flow speed.

$$
\begin{aligned}
Q &= \tfrac{1}{2} v_{\text{max}} A \\
&= \tfrac{1}{2} v_{\text{max}} \left[18\pi (d/2)^2 \right] \\
d &= 2\sqrt{\frac{2Q}{18\pi v_{\text{max}}}} \\
&= 2\sqrt{\frac{4.00 \text{ L/min}}{9\pi (4.30 \text{ m/s})} \frac{1 \text{ m}^3}{1000 \text{ L}} \frac{1 \text{ min}}{60 \text{ s}}} = 1.48\times 10^{-3} \text{ m}
\end{aligned}
$$

18.81. In either case, the work done equals the product of the force I exert on the wire times the displacement of the wire. The magnitude of the force I exert on the wire equals twice the product of the surface tension of the fluid times the length of the wire. So, we have

$$
\begin{aligned}
W_{\text{water}} &= 2\gamma_{\text{water}} \ell \Delta x \\
W_{\text{Hg}} &= 2\gamma_{\text{Hg}} \ell \Delta x/2 \\
\ell \Delta x &= \frac{W_{\text{water}}}{2\gamma_{\text{water}}} = \frac{W_{\text{Hg}}}{\gamma_{\text{Hg}}} \\
W_{\text{Hg}} &= \frac{W_{\text{water}} \gamma_{\text{Hg}}}{2\gamma_{\text{water}}} \\
&= \frac{(1.46\times 10^{-6} \text{ J})(0.465 \text{ N/m})}{2(7.28\times 10^{-2} \text{ N/m})} = 4.66\times 10^{-6} \text{ J}
\end{aligned}
$$

18.82. We know from Poiseuille's law that the pressure drop in the pipeline is proportional to its length:

$$
\begin{aligned}
Q &= \frac{\pi R^4}{8\eta \ell}(P_1 - P_2) \\
\ell &= \frac{\pi R^4}{8\eta Q}(P_1 - P_2) \\
&= \frac{\pi (0.250 \text{ m})^4}{8(0.224 \text{ Pa}\cdot\text{s})\left[(150 \text{ kg/s})/(850 \text{ kg/m}^3)\right]}\left[(3.75 \text{ atm}) - (2.50 \text{ atm})\right]\frac{1.01\times 10^5 \text{ Pa}}{1 \text{ atm}} = 4.90\times 10^3 \text{ m}
\end{aligned}
$$

18.83. The volume flow rate equals the product of the water's average flow speed and the cross-sectional area of the pipe. Assuming that we cannot ignore the viscosity of the water, the water's average flow speed is one-half of its maximum flow speed:

$$
\begin{aligned}
Q &= \tfrac{1}{2} v_{\text{max}} A \\
&= \tfrac{1}{2} v_{\text{max}} \pi (d/2)^2 \\
v_{\text{max}} &= \frac{2Q}{\pi (d/2)^2} \\
&= \frac{2(100 \text{ L})/(5.00 \text{ min})}{\pi \left[(12.5\times 10^{-3} \text{ m})/2\right]^2} \frac{1 \text{ m}^3}{1000 \text{ L}} \frac{1 \text{ min}}{60 \text{ s}} = 5.43 \text{ m/s}
\end{aligned}
$$

18.84. The flow rate must be great enough that the water's maximum flow speed is sufficient for it to rise to the given height against the acceleration of gravity,

$$
\begin{aligned}
\tfrac{1}{2} v_{\text{max}}^2 &= gh \\
v_{\text{max}} &= \sqrt{2gh}
\end{aligned}
$$

Assuming that we cannot ignore the viscosity of the water, the water's average flow speed is one-half of its maximum flow speed. So, we have for the minimum required volume flow rate

$$
\begin{aligned}
Q &= \tfrac{1}{2} v_{\max} A \\
&= \tfrac{1}{2}\sqrt{2gh}\,\pi(d/2)^2 \\
&= \tfrac{1}{2}\pi\sqrt{2(9.8\ \text{m/s}^2)(1.86\ \text{m})}\left[(1.60\times10^{-3}\ \text{m})/2\right]^2 = 6.07\times10^{-6}\ \text{m}^3\text{/s}
\end{aligned}
$$

18.85. By continuity, the flow rates through both pipes must be the same, so we have

$$\frac{\pi R_1^4}{8\eta\ell_1}(P_1 - P) = \frac{\pi R_2^4}{8\eta(2\ell_1)}(P - P_2)$$

We are also told that $P = (P_1 + P_2)/2,$ so

$$\frac{\pi R_1^4}{8\eta\ell_1}(P_1 - P_2)/2 = \frac{\pi R_2^4}{8\eta(2\ell_1)}(P_1 - P_2)/2$$

and dividing out common factors from both sides gives $R_2^4 = 2R_1^4$ or $R_2 = \sqrt[4]{2}R_1.$

18.86. (*a*) We know from Poiseuille's law that the volume flow rate through the pipe is inversely proportional to the viscosity. So, we must decrease the viscosity by the same proportion as we wish to increase the volume flow rate:

$$
\begin{aligned}
Q_1\eta_1 &= Q_2\eta_2 \\
\eta_2 &= \eta_1 Q_1/Q_2 \\
&= (0.456\ \text{Pa}\cdot\text{s})(2.00\ \text{m}^3\text{/s})/(8.50\ \text{m}^3\text{/s}) = 0.107\ \text{Pa}\cdot\text{s}
\end{aligned}
$$

(*b*) The mass flow rate equals the product of the mass density and the volume flow rate. With the original viscosity it equals $\rho Q_1 = (882\ \text{kg/m}^3)(2.00\ \text{m}^3\text{/s}) = 1.76\times10^3\ \text{kg/s}.$ (*c*) With the new, lower viscosity it equals $\rho Q_2 = (882\ \text{kg/m}^3)(8.50\ \text{m}^3\text{/s}) = 7.50\times10^3\ \text{kg/s}.$

18.87. (*a*) There are three forces exerted on the drops, the upward buoyant force of the water, the downward force of gravity, and the downward drag force of the water. Following the hint and assuming that the drops quickly reach their terminal speed, their upward speed corresponds to that at which the vector sum of the forces exerted on them is zero:

$$F = F_{\text{wd}}^{\text{b}} - F_{\text{Ed}}^{G} - F_{\text{wd}}^{\text{d}} = 0$$

$$\rho_{\text{w}} V_{\text{d}} g - \rho_{\text{d}} V_{\text{d}} g - (0.0200\ \text{Pa}\cdot\text{s})R v_{\text{term}} = 0$$

$$
\begin{aligned}
v_{\text{term}} &= \frac{(\rho_{\text{w}} - \rho_{\text{d}})V_{\text{d}} g}{(0.0200\ \text{Pa}\cdot\text{s})R} \\
&= \frac{(\rho_{\text{w}} - \rho_{\text{d}})(\tfrac{4}{3}\pi R^3)g}{(0.0200\ \text{Pa}\cdot\text{s})R} \\
&= \tfrac{4}{3}\pi\frac{(\rho_{\text{w}} - \rho_{\text{d}})R^2 g}{0.0200\ \text{Pa}\cdot\text{s}} \\
&= \tfrac{4}{3}\pi\frac{\left[(1000\ \text{kg/m}^3) - (700\ \text{kg/m}^3)\right](50.0\times10^{-6}\ \text{m})^2(9.8\ \text{m/s}^2)}{0.0200\ \text{Pa}\cdot\text{s}} = 1.54\times10^{-3}\ \text{m/s}
\end{aligned}
$$

At this speed, the time interval it takes them to rise $1000\ \text{m}$ to the surface is

$$
\begin{aligned}
\Delta t &= \Delta x/v_{\text{term}} \\
&= (1000\ \text{m})/(1.54\times10^{-3}\ \text{m/s}) = 6.50\times10^5\ \text{s}
\end{aligned}
$$

or more than seven days. (*b*) When the diameter of the drops is reduced, so is their terminal velocity,

$$v_{\text{term}} = \tfrac{4}{3}\pi\frac{\left[(1000\ \text{kg/m}^3) - (700\ \text{kg/m}^3)\right](1.0\times10^{-6}\ \text{m})^2(9.8\ \text{m/s}^2)}{0.0200\ \text{Pa}\cdot\text{s}} = 6.16\times10^{-7}\ \text{m/s}$$

and the time it takes for them to rise to the surface is correspondingly increased, $\Delta t = (1000\ \text{m})/(6.16\times10^{-7}\ \text{m/s}) = 1.62\times10^9\ \text{s},$ or more than 51 years. This means that, instead of forming an oil slick above the leak, the oil will likely be dispersed around the world by ocean currents over the course of many years.

Regarding our assumption that the drops spend essentially all of their upward journey traveling at their terminal speed, the equation of motion $F = ma$ can be solved to give a drop's speed as a function of time,

$v(t) = v_{\text{term}}(1 - e^{-[(0.0200\text{ Pa}\cdot\text{s})R/m]t})$. So, for example, we can calculate the time it takes for a drop to reach 95% of its terminal speed:

$$\begin{aligned}
1 - e^{-[(0.0200\text{ Pa}\cdot\text{s})R/m]t} &= 0.95 \\
e^{-[(0.0200\text{ Pa}\cdot\text{s})R/m]t} &= 0.05 \\
\left[(0.0200\text{ Pa}\cdot\text{s})R/m\right]t &= -\ln(0.05) \\
t &= \frac{-\ln(0.05)m}{(0.0200\text{ Pa}\cdot\text{s})R} \\
&= \frac{-\ln(0.05)(\frac{4}{3}\pi R^3\rho)}{(0.0200\text{ Pa}\cdot\text{s})R} \\
&= \frac{-\ln(0.05)\frac{4}{3}\pi R^2\rho}{0.0200\text{ Pa}\cdot\text{s}} \\
&= \frac{-\ln(0.05)\frac{4}{3}\pi(50.0\times10^{-6}\text{ m})^2(700\text{ kg/m}^3)}{0.0200\text{ Pa}\cdot\text{s}} = 1.10\times10^{-3}\text{ s}
\end{aligned}$$

which is an insignificant fraction of the travel time. For the smaller drops of part *b*, the time it takes to reach terminal speed is even smaller, and an even smaller fraction of the travel time.

18.88. Let's choose a coordinate system in which the cylinder's central axis is along the positive x axis, and the cylinder is divided by the xy plane. We can conceptually divide the tube into thin strips of thickness dx. At each angle θ from the y axis, we can define a surface element of area $Rd\theta dx$, and the force on this element has magnitude $(P_{\text{in}} - P_{\text{out}})Rd\theta dx$ and is directed perpendicular to the surface element, that is, radially outward. In particular, it has no x component. Considering the y and z components of the force separately we have

$$\begin{aligned}
F_y &= \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}(P_{\text{in}} - P_{\text{out}})R\cos\theta\, d\theta dx \\
&= (P_{\text{in}} - P_{\text{out}})R\int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\cos\theta\, d\theta dx \\
&= (P_{\text{in}} - P_{\text{out}})R\int_{x=0}^{\ell}(\sin\pi - \sin 0)\, dx \\
&= 0
\end{aligned}$$

and

$$\begin{aligned}
F_z &= \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}(P_{\text{in}} - P_{\text{out}})R\sin\theta\, d\theta dx \\
&= (P_{\text{in}} - P_{\text{out}})R\int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\sin\theta\, d\theta dx \\
&= (P_{\text{in}} - P_{\text{out}})R\int_{x=0}^{\ell}-(\cos\pi - \cos 0)\, dx \\
&= 2(P_{\text{in}} - P_{\text{out}})R\ell
\end{aligned}$$

(*a*) Because the magnitude of the force exerted by surface tension at the boundary of the half tube, that is, along the two sides of length ℓ, is $2\ell\gamma$, we see that Eq. 18.52 is valid. (*b*) Because only the z component of the force is nonzero, its direction is perpendicular to the plane that divides the cylinder.

18.89. In order for the pressure in the pipe to remain the same, the liquid's speed must also be the same. The volume flow rate is proportional to the cross-sectional area of the pipe, so $Q_1/A_1 = Q_2/A_2$ and

$$\begin{aligned}
Q_2 &= Q_1 A_2/A_1 \\
&= Q_1(d/4)^2/d^2 \\
&= Q_1/16
\end{aligned}$$

18.90. Given the assumptions of laminar flow of nonviscous blood, the volume flow rate is proportional to the cross-sectional area of the blood vessel. So, to double the volume flow rate, we must double the cross-sectional area of the blood vessel, which means that the new vessel diameter must be greater than the old vessel diameter by a factor of the square root of two:

$$\begin{aligned} A_2 &= 2A_1 \\ d_2^2 &= 2d_1^2 \\ d_2 &= \sqrt{2}d_1 \\ &= \sqrt{2}(3.20\ \text{mm}) = 4.53\ \text{mm} \end{aligned}$$

18.91. The diameter of Europa is more than $3000\ \text{km}$, so the acceleration due to gravity does not change much over the $125\ \text{m}$ of the probe's depth. The pressure at that depth is

$$\begin{aligned} P &= \rho g_{\text{Europa}} d_{\text{Europa}} \\ &= (1000\ \text{kg/m}^3)(1.3\ \text{m/s}^2)(125\ \text{m}) = 1.625\times10^5\ \text{Pa} \end{aligned}$$

where we have retained a couple of extra digits in this intermediate result. We might note that this is less than twice the atmospheric pressure here on Earth.

The depth in Earth's ocean at which the pressure is the same can be found as

$$\begin{aligned} P &= P_{\text{atm}} + \rho g_{\text{Earth}} d_{\text{Earth}} \\ d_{\text{Earth}} &= \frac{P - P_{\text{atm}}}{\rho g_{\text{Earth}}} \\ &= \frac{(1.625\times10^5\ \text{Pa}) - (1.01\times10^5\ \text{Pa})}{(1025\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 6.1\ \text{m} \end{aligned}$$

18.92. The air exerts a small buoyant force on my body. If my mass is about $70\ \text{kg}$, and my mass density is about that of water, I displace about $(70\ \text{kg})/(1000\ \text{kg/m}^3) = 0.07\ \text{m}^3$ of air, so the buoyant force exerted on me by the air is about

$$\begin{aligned} F^{\text{b}}_{\text{am}} &= \rho_{\text{air}} V_{\text{disp}} g \\ &= (1.29\ \text{kg/m}^3)(0.07\ \text{m}^3)(9.8\ \text{m/s}^2) = 0.9\ \text{N} \end{aligned}$$

That is, the spring scale reading is reduced by this amount, compared to what it would read in the absence of the air's buoyant force.

18.93. The increase in pressure in the brake fluid caused by my pressing on the brake pedal is transmitted to the piston that exerts a force on the brake shoe. (*a*) Pressure equals force divided by area, so

$$\begin{aligned} P &= F_{\text{pedal}}/A_{\text{pedal}} = F_{\text{piston}}/A_{\text{piston}} \\ F_{\text{piston}} &= F_{\text{pedal}} A_{\text{piston}}/A_{\text{pedal}} \\ &= F_{\text{pedal}}(d_{\text{piston}})^2/(d_{\text{pedal}})^2 \\ &= (125\ \text{N})(44.0\ \text{mm})^2/(12.0\ \text{mm})^2 = 1.68\times10^3\ \text{N} \end{aligned}$$

(*b*) The magnitude of the frictional force equals the product of the normal force and the coefficient of friction. If the car is moving, the coefficient of kinetic friction should be used and the magnitude of the frictional force is

$$\begin{aligned} F^k &= \mu_k F^n \\ &= \mu_k F_{\text{piston}} \\ &= (0.650)(1.68\times10^3\ \text{N}) = 1.09\times10^3\ \text{N} \end{aligned}$$

If the car is stopped, the coefficient of static friction should be used and the magnitude of the frictional force is

$$\begin{aligned} F^s &\le \mu_s F^n \\ &\le \mu_s F_{\text{piston}} \\ &\le (0.880)(1.68\times10^3\ \text{N}) = 1.48\times10^3\ \text{N} \end{aligned}$$

18.94. The oil and water both exert upward buoyant forces on the sphere, while Earth's gravity exerts a downward force, and the vector sum of the forces equals zero. So, we have

$$F_{\rm os}^{\rm b} + F_{\rm ws}^{\rm b} - F_{\rm Es}^{G} = 0$$

$$\rho_{\rm o}V_{\rm o\ disp}g + \rho_{\rm w}V_{\rm w\ disp}g - \rho_{\rm s}V_{\rm s}g = 0$$

$$\begin{aligned}\rho_{\rm s} &= (\rho_{\rm o}V_{\rm o\ disp}g + \rho_{\rm w}V_{\rm w\ disp}g)/V_{\rm s}g \\ &= (\rho_{\rm o}V_{\rm o\ disp} + \rho_{\rm w}V_{\rm w\ disp})/V_{\rm s} \\ &= \left[(800\ \text{kg/m}^3)(0.350V_{\rm s}) + (1000\ \text{kg/m}^3)(0.650V_{\rm s})\right]/V_{\rm s} \\ &= \left[0.350(800\ \text{kg/m}^3) + 0.650(1000\ \text{kg/m}^3)\right] = 930\ \text{kg/m}^3\end{aligned}$$

18.95. (*a*) If the ball neither rises nor falls, it must be neutrally buoyant, that is, its mass density must be equal to the mass density of the liquid, so its mass is

$$\begin{aligned}m &= \rho V \\ &= \rho\left[\tfrac{4}{3}\pi(d/2)^3\right] \\ &= \tfrac{4}{3}\pi(1350\ \text{kg/m}^3)\left[(10\times10^{-3}\ \text{m})/2\right]^3 = 7.1\times10^{-4}\ \text{kg}\end{aligned}$$

(*b*) The mass density of a substance changes with temperature, and the rate of change is different for different substances, so it seems reasonable to expect that the ball will no longer be neutrally buoyant if the temperature of the mixture changes.

18.96. (*a*) At equilibrium, the box displaces a volume of water, the mass of which equals the mass of the box. The depth to which it is submerged equals the volume of the water it displaces divided by its cross-sectional area. That is,

$$m_{\rm box} = m_{\rm water\ disp} = V_{\rm water\ disp}\rho$$

$$V_{\rm water\ disp} = m_{\rm box}/\rho$$

$$\begin{aligned}d &= V_{\rm water\ disp}/A \\ &= m_{\rm box}/\rho A \\ &= \frac{3000\ \text{kg}}{(1000\ \text{kg/m}^3)(1.50\ \text{m})^2} = 1.33\ \text{m}\end{aligned}$$

(*b*) The box sinks when the combined mass of the box and the water it contains equals the mass of water it displaces when fully submerged. That is,

$$m_{\rm box} + \rho Ah = \rho V$$

$$\begin{aligned}h &= \frac{\rho V - m_{\rm box}}{\rho A} \\ &= \frac{(1000\ \text{kg/m}^3)(1.50\ \text{m})^3 - (3000\ \text{kg})}{(1000\ \text{kg/m}^3)(1.50\ \text{m})^2} = 0.167\ \text{m}\end{aligned}$$

18.97. At the given depth, the water pressure is $P = P_{\rm atm} + \rho gd$, which means that the acceleration of gravity is $g = (P - P_{\rm atm})/\rho d$. The acceleration of gravity is also proportional to the planet's mass (see Section 13.5), so

$$g = G\frac{m_{\rm planet}}{R^2} = \frac{P - P_{\rm atm}}{\rho d}$$

$$\begin{aligned}m_{\rm planet} &= \frac{(P - P_{\rm atm})R^2}{G\rho d} \\ &= \frac{\left[(14.4\times10^5\ \text{Pa}) - (2.40\times10^5\ \text{Pa})\right](6.378\times10^6\ \text{m})^2}{(6.6738\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1000\ \text{kg/m}^3)(150\ \text{m})} = 4.88\times10^{24}\ \text{kg}\end{aligned}$$

18.98. We can determine the water's speed as it exits the hose by treating each bit of water as a projectile fired at $60.0°$ above the horizontal, that rises to a height of 25.0 m. So, we have for the vertical component of the water's initial velocity $v_y = \sqrt{2gh} = v\sin(60.0°)$, or $v = \sqrt{2gh}/\sin(60.0°)$. (*a*) The volume flow rate equals the product of the water's speed times the cross-sectional area of the hose,

$$\begin{aligned} Q &= vA \\ &= \frac{\sqrt{2gh}}{\sin(60.0°)}\pi R^2 \\ &= \frac{\sqrt{2(9.8\ \text{m/s}^2)(25.0\ \text{m})}}{\sin(60.0°)}\pi\left[(0.120\ \text{m})/2\right]^2 = 0.289\ \text{m}^3/\text{s} \end{aligned}$$

(*b*) The mass flow rate equals the product of the volume flow rate times the mass density,

$$\begin{aligned} Q_m &= Q\rho \\ &= (0.289\ \text{m}^3/\text{s})(1000\ \text{kg/m}^3) = 289\ \text{kg/s} \end{aligned}$$

18.99. There are four forces exerted on the sphere, the downward force of gravity, the upward buoyant force of the water, and the contact forces of the wires, and their vector sum is zero. In terms of their horizontal *x* and vertical *y* components, we have

$$\begin{aligned} F_x &= -\mathcal{T}_\text{A} + \mathcal{T}_\text{B}\cos(50.0°) = 0 \\ F_y &= -V_\text{sphere}\rho_\text{sphere}g + V_\text{sphere}\rho_\text{water}g + \mathcal{T}_\text{B}\sin(50.0°) = 0 \\ \mathcal{T}_\text{B} &= V_\text{sphere}(\rho_\text{sphere} - \rho_\text{water})g/\sin(50.0°) \\ &= \tfrac{4}{3}\pi(0.250\ \text{m})^3\left[(1500\ \text{kg/m}^3) - (1000\ \text{kg/m}^3)\right](9.8\ \text{m/s}^2)/(0.766) = 419\ \text{N} \\ \mathcal{T}_\text{A} &= \mathcal{T}_\text{B}\cos(50.0°) \\ &= (419\ \text{N})(0.643) = 269\ \text{N} \end{aligned}$$

18.100. The liquid rises to a height sufficient to make the pressure at the bottom of the column of liquid equal to atmospheric pressure. So,

$$\begin{aligned} \rho g h &= P_\text{atm} \\ \rho &= \frac{P_\text{atm}}{gh} \\ &= \frac{1.01\times10^5\ \text{Pa}}{(9.8\ \text{m/s}^2)(1.28\ \text{m})} = 8.05\times10^3\ \text{kg/m}^3 \end{aligned}$$

18.101. (*a*) If the ice is displaced vertically from its equilibrium position, the buoyant force exerted on it by the water changes as the submerged portion displaces more or less water. Because the ice has uniform cross-sectional area, the volume of water it displaces is proportional to the depth of its lower edge below the surface of the water, so the restoring force is linear, and the ice bobs in simple harmonic motion. The change in the buoyant force for a change in depth is

$$\begin{aligned} \Delta F^\text{b}_\text{wi} &= \rho_\text{w} g \Delta V_\text{submerged} \\ &= \rho_\text{w} g A \Delta h_\text{submerged} \end{aligned}$$

so the constant of proportionality between the change in force and change in depth is

$$\frac{\Delta F^\text{b}_\text{wi}}{\Delta h_\text{submerged}} = \rho_\text{w} g A$$

and the frequency of the oscillation is (see *Principles* Section 15.6)

$$
\begin{aligned}
f &= \frac{1}{2\pi}\sqrt{\frac{\rho_\mathrm{w} g A}{m}} \\
&= \frac{1}{2\pi}\sqrt{\frac{\rho_\mathrm{w} g A}{\rho_\mathrm{i} V}} \\
&= \frac{1}{2\pi}\sqrt{\frac{\rho_\mathrm{w} g}{\rho_\mathrm{i} \ell}} \\
&= \frac{1}{2\pi}\sqrt{\frac{(1024\ \mathrm{kg/m^3})(9.8\ \mathrm{m/s^2})}{(917\ \mathrm{kg/m^3})(75.0\ \mathrm{m})}} = 6.08\times 10^{-2}\ \mathrm{Hz}
\end{aligned}
$$

(*b*) If the iceberg had twice the mass, its sides would be longer by a factor of the cube root of two. Because the frequency is inversely proportional to the square root of the length of the cube's side, it would decrease by a factor of the sixth root of two,

$$
\begin{aligned}
f_2 &= f/(2)^{1/6} \\
&= (6.08\times 10^{-2}\ \mathrm{Hz})/1.122 = 5.42\times 10^{-2}\ \mathrm{Hz}
\end{aligned}
$$

18.102. (*a*) The pressure in the water exerts a force on the surface of the log, which we can calculate by dividing the surface into many small surface elements, the pressure on each of which is uniform, and integrating over the surface of the log. Let's choose a coordinate system in which the central axis of the log is on the positive *x* axis, the surface of the water is the *xy* plane, and the *z* axis points upward. The magnitudes of the forces exerted on the two ends of the log are equal and their directions are opposite, so they contribute nothing. The magnitude of the force exerted on a surface element $Rd\theta dx$, where θ is the angle below the surface of the water measured from the log's central axis, is $dF = P(z)Rd\theta dx$, where $P(z)$ is the pressure at depth *z*, $P(z) = P_\mathrm{atm} + \rho g R\sin\theta$. The direction of the force on each surface element is perpendicular to the surface, so it has no *x* component and we can integrate the *y* and *z* components of the force separately,

$$
\begin{aligned}
F_y &= \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}(P_\mathrm{atm} + \rho g R\sin\theta)R\cos\theta\, d\theta dx \\
&= \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}P_\mathrm{atm}R\cos\theta\, d\theta dx + \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\rho g R^2\sin\theta\cos\theta\, d\theta dx \\
&= P_\mathrm{atm}R\int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\cos\theta\, d\theta dx + \rho g R^2\int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\sin\theta\cos\theta\, d\theta dx \\
&= P_\mathrm{atm}R\int_{x=0}^{\ell}(\sin\pi - \sin 0)\, dx + \rho g R^2\int_{x=0}^{\ell}(\tfrac{1}{2}\sin^2\pi - \tfrac{1}{2}\sin^2 0)\, dx \\
&= 0
\end{aligned}
$$

and

$$
\begin{aligned}
F_z &= \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}(P_\mathrm{atm} + \rho g R\sin\theta)R\sin\theta\, d\theta dx \\
&= \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}P_\mathrm{atm}R\sin\theta\, d\theta dx + \int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\rho g R^2\sin^2\theta\, d\theta dx \\
&= P_\mathrm{atm}R\int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\cos\theta\, d\theta dx + \rho g R^2\int_{x=0}^{\ell}\int_{\theta=0}^{\pi}\sin\theta\cos\theta\, d\theta dx \\
&= P_\mathrm{atm}R\int_{x=0}^{\ell}-(\cos\pi - \cos 0)\, dx + \rho g R^2\int_{x=0}^{\ell}\tfrac{1}{2}(\pi - \sin\pi\cos\pi) - \tfrac{1}{2}(0 - \sin 0\cos 0)\, dx \\
&= 2\ell P_\mathrm{atm}R + \tfrac{1}{2}\pi\ell\rho g R^2
\end{aligned}
$$

(*b*) The first term is equal in magnitude to the pressure of the atmosphere on the top of the log, while the second is equal in magnitude to the gravitational force exerted on the volume of water displaced by the submerged portion of the log, in accordance with Archimedes' principle.

18.103. A ship will just barely float if the combined mass of the ship and its cargo equals the mass of the water it displaces. That is, for a ship of given mass, the greater the volume of water it displaces, the greater the mass of cargo it can carry.

As a very simple model, let's consider the ship to be an open-topped box of length, width, and height ℓ, w, and h, with walls of uniform thickness d. Then the mass of the ship is $m = (\ell w + 2\ell h + 2hw)d\rho_{\text{steel}}$, so our design is constrained to obey $\ell w + 2\ell h + 2hw = m/(d\rho_{\text{steel}})$, and the volume of water it can displace is $V = \ell wh$. If we choose the width of the ship, which we might do because the ship's width determines how narrow a channel it can sail through, we can vary the height and see how the volume changes.

Writing our constraint equation as $\ell w + 2\ell h + 2hw = C$, we have $\ell(w + 2h) = C - 2hw$ or $\ell = (C - 2hw)/(w + 2h)$. Substituting into our equation for the volume we have

$$\begin{aligned} V &= \ell wh \\ &= \frac{C - 2hw}{w + 2h} wh \\ &= \frac{Cwh - 2w^2h^2}{w + 2h} \end{aligned}$$

The volume will be maximal when its derivative with respect to h is zero:

$$\begin{aligned} \frac{\partial V}{\partial h} &= \frac{\partial}{\partial h} \frac{Cwh - 2w^2h^2}{w + 2h} \\ &= \frac{(Cw - 4w^2h)(w + 2h) - 2(Cwh - 2w^2h^2)}{(w + 2h)^2} \\ &= \frac{h^2(-4w^2) + h(-4w^3) + Cw^2}{(w + 2h)^2} = 0 \end{aligned}$$

or $h^2(-4w^2) + h(-4w^3) + Cw^2 = 0$. We can solve this quadratic equation for $h = \frac{1}{2}(\sqrt{w^2 + C} - w)$, where we have chosen the positive root. Substituting this into the constraint equation we find

$$\begin{aligned} \ell &= \frac{C - (\sqrt{w^2 + C} - w)w}{w + (\sqrt{w^2 + C} - w)} \\ &= \frac{w^2 + C - w\sqrt{w^2 + C}}{\sqrt{w^2 + C}} \\ &= \sqrt{w^2 + C} - w \\ &= 2h \end{aligned}$$

That is, for a given width, the ship will displace the greatest volume of water if its length is twice its height.

18.104. The air exerts an upward buoyant force on each person of magnitude equal to that of the force exerted by gravity on the mass of the air the person displaces. If each person has the same average mass density, then the volume of air they displace, which equals their volume, is proportional to their mass, so the magnitude of the buoyant force is proportional to their mass. The overall effect is that the downward contact force that each person exerts on the seesaw is reduced by the same factor, so their distances from the pivot to achieve balance are not affected at all.

18.105. From the values given, we can calculate the maximum height that water could be raised by capillary rise in the tree's xylem:

$$\begin{aligned} h &= (0.137 \text{ J/m}^2)/(\rho_{\text{water}} gR) \\ &= (0.137 \text{ J/m}^2)/\left[(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(2.5\times10^{-6} \text{ m})\right] = 6 \text{ m} \end{aligned}$$

This is much less than the height of the tree, so some other mechanism must be responsible. Maybe the pressure in the water in the xylem tubes near the base of the tree is greater than atmospheric pressure? Maybe the pressure in the xylem tubes near the top of the tree is less than atmospheric pressure?

19

ENTROPY

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{8000} **2.** 10^{14} s **3.** 10^{3} **4.** -10^{27} **5.** 10^{24} **6.** 10^{23} **7.** 10^{3} m/s **8.** 10^{2} m/s **9.** 10^{24} J **10.** -10^{41}

Guided Problems

19.2 Shared energy

1. Getting Started The particles have been labeled, meaning we can distinguish particle 1 from particle 2 and so on. Thus the particles are distinguishable in this case. The units of energy are indistinguishable. That is, we can know how much energy each particle has, but not which units make up that energy. The basic state refers to detailed information about the particles that make up the system. We might choose to describe basic states by specifying how much energy each particle has. For example, particles 1, 2, and 3 each having 2 units of energy would be a basic state. A macrostate refers to a system's large-scale properties. In the strictest sense, there is no macroscopic description of a system consisting of only three particles. However, one might still describe average properties of the system, leaving out particle-specific information. For example, we might describe the average energy per particle, or the average speed per particle, the total energy in the system, etc. If we describe macrostates in terms of the average energy per particle, the conditions set out in the problem statement constrain us to only one macrostate: that in which six units of energy are divided up between the three particles. Thus the one macrostate we consider here is that in which the total energy is six units, or equivalently, each particle has an average of 2 units of energy.

2. Devise Plan For the macrostate described above, we must count the number of basic states. We must associate a given number of units of energy to particle 1, then particle 2, then particle 3, such that the total number of energy units is six. Note that if we choose the number of energy units associated with particles 1 and 2, then the number associated with particle 3 is uniquely determined by the constraint of six total units of energy. Thus, we are only free to choose the number of energy units on particles 1 and 2. Note that if there are n units of energy on particle 1, particle 2 can have anywhere from 0 to $6-n$ units of energy. Thus when particle 1 has $n=0$ units, there are seven possible energies for particle 2: 0, 1, 2, 3, 4, 5, or 6 units. When particle 1 has $n=1$ unit, there are six possible energies for particle 2, and so on. Thus the total number of basic states is $N=7+6+5+4+3+2+1=28$. Note that particle 3 is included in this calculation. We simply did not need to consider what the number of units of energy could be on particle 3 because once we choose the number on particles 1 and 2, there is only one possible number of units left for particle 3.

It is more tedious, but it might be illuminating to compare the above argument to the table of possible values below. Convince yourself that the argument is correct.

Particle	1	2	3
Units:	0	0	6
	0	1	5
	0	2	4
	0	3	3
	0	4	2
	0	5	1
	0	6	0
	1	0	5
	1	1	4
	1	2	3
	1	3	2
	1	4	1
	1	5	0
	2	0	4
	2	1	3
	2	2	2
	2	3	1
	2	4	0
	3	0	3
	3	1	2
	3	2	1
	3	3	0
	4	0	2
	4	1	1
	4	2	0
	5	0	1
	5	1	0
	6	0	0

Now we must determine how many of the basic states we found assign two units of energy to particle 3. Again, we can argue that assigning two units to particle 3 leaves four units to be divided between particles 1 and 2. Particle 1 can have 0, 1, 2, 3, or 4 units, and that choice will determine how many units are left for particle 2. Thus there are five basic states that satisfy the condition that particle 3 have two units of energy. Alternatively, one could simply look at the table above and select those with two units of energy on particle 3.

3. Execute Plan The basic states that assign two units of energy to particle 3 are

Particle	1	2	3
Units	0	4	2
	1	3	2
	2	2	2
	3	4	2
	4	0	2

So we have five basic states that satisfy the criterion, out of a total of 28 basic states. Thus the probability of finding two units of energy on particle 3 is

$$P = \frac{\Omega(2)}{\Omega_{\text{total}}} = \frac{5}{28} = 17.9\%$$

4. Evaluate Result We expect each particle to have two units of energy on average, because there are six units being divided up between three particles. That certainly doesn't mean that each particle will always have two units of energy. We expect thermal fluctuations. So 17.9% is a plausible answer. We can check this numerically by finding the probability that particle 3 has n units of energy and using that to calculate the average energy. We expect to find a final answer of two units.

As discussed above, choosing n units for particle 3 leaves $7-n$ choices for the energy units assigned to particle 2. Once energy is assigned to those two particles, those assigned to particle 1 are determined by the condition that there be six units total. Thus the number of basic states in which particle 3 has n units is $7-n$, and the probability of being in that basic state is $P_n = \frac{7-n}{28}$. Then the average energy found on particle 3 is the sum over all possible energies times the probability of having that energy:

$$n_{\text{av}} = \sum_{n=0}^{6} nP_n = \sum_{n=0}^{6} n\frac{\Omega(n)}{\Omega_{\text{total}}} = \sum_{n=0}^{6} n\frac{(7-n)}{28} = 2$$

The last step can be carried out trivially numerically, or by brute force calculation. But we see that our process does, indeed yield an average of two energy units on particle 3. Since there is nothing special about particle 3 that would cause energy to be drawn to it or kept from it, we conclude that on average each particle will have two units of energy. This is what we expected.

19.4 Thermal equilibrium

1. Getting Started As the system evolves toward equilibrium, the temperature of each side changes, but the volume of each side stays the same. The total entropy of the system is made up of the entropy of the left and right sides. Thus we can write $S = S_{\text{X}} + S_{\text{Y}}$ or equivalently $\Delta S = \Delta S_{\text{X}} + \Delta S_{\text{Y}}$.

2. Devise Plan The temperature of the gas is related to the average kinetic energy of the gas. Since the gas is monatomic and ideal, all its energy is kinetic. Thus we can relate temperature and thermal energy using equation (19.50):

$$E_{th} = \frac{3}{2}Nk_{\text{B}}T$$

At equilibrium, we require that $T_{\text{X}} = T_{\text{Y}}$. Using equation (19.50) we obtain a condition for the total thermal energy in each compartment:

$$\frac{E_{th,\text{X,f}}}{N_{\text{X}}} = \frac{E_{th,\text{Y,f}}}{N_{\text{Y}}} \tag{1}$$

Initially, we have

$$T_{\text{Y,i}} = \frac{2E_{th,\text{Y,i}}}{3N_{\text{Y}}k_{\text{B}}}$$

$$T_{\text{X,i}} = \frac{2E_{th,\text{X,i}}}{3N_{\text{X}}k_{\text{B}}} = 3T_{\text{Y,i}} = \frac{2E_{th,\text{Y,i}}}{N_{\text{Y}}k_{\text{B}}} \tag{2}$$

Rearranging equation (2) allows us to write

$$E_{th,\text{X,i}} = 3\left(\frac{N_{\text{X}}}{N_{\text{Y}}}\right)E_{th,\text{Y,i}} \tag{3}$$

We can also write the total initial thermal energy of the system in terms of the energy in just one chamber using equation (3). We find

$$E_{th,\text{total,i}} = E_{th,\text{total,f}} = \left(3\left(\frac{N_\text{X}}{N_\text{Y}}\right)+1\right)E_{th,\text{Y,i}} = \left(1+\frac{1}{3}\left(\frac{N_\text{Y}}{N_\text{X}}\right)\right)E_{th,\text{X,i}} \tag{4}$$

The change in entropy for a given chamber depends on thermal energy according to equation (19.33):

$$\Delta S_\text{one chamber} = \frac{3}{2}N\ln\left(\frac{E_{th,\text{f}}}{E_{th,\text{i}}}\right)$$

We will apply this to each chamber separately to determine the change in entropy for the entire system.

3. Execute Plan We first write out the total change in entropy for the system using equation (19.33):

$$\Delta S_\text{total} = \Delta S_\text{X} + \Delta S_\text{Y} = \frac{3}{2}N_\text{X}\ln\left(\frac{E_{th,\text{X,f}}}{E_{th,\text{X,i}}}\right)+\frac{3}{2}N_\text{Y}\ln\left(\frac{E_{th,\text{Y,f}}}{E_{th,\text{Y,i}}}\right) \tag{5}$$

Clearly, it would be useful if we could write the final energy in either chamber in terms of the initial energy in that same chamber. Using equation (1), we can write the total energy in terms of the final energy in either chamber:

$$E_{th,\text{total,f}} = E_{th,\text{X,f}} + E_{th,\text{Y,f}} = \left(1+\frac{N_\text{X}}{N_\text{Y}}\right)E_{th,\text{Y,f}} = \left(1+\frac{N_\text{Y}}{N_\text{X}}\right)E_{th,\text{X,f}} \tag{6}$$

Using equation (4), we can write the initial total energy (which is the same as the final total energy) in terms of the thermal energy initially in either chamber. Thus we obtain:

$$E_{th,\text{X,f}} = \frac{\left(N_\text{X}+\frac{1}{3}N_\text{Y}\right)}{(N_\text{X}+N_\text{Y})}E_{th,\text{X,i}} \tag{7}$$

$$E_{th,\text{Y,f}} = \frac{(3N_\text{X}+N_\text{Y})}{(N_\text{Y}+N_\text{X})}E_{th,\text{Y,i}} \tag{8}$$

Inserting equations (7) and (8) into equation (5), we obtain

$$\Delta S_\text{total} = \frac{3}{2}N_\text{X}\ln\left(\frac{N_\text{X}+\frac{1}{3}N_\text{Y}}{N_\text{X}+N_\text{Y}}\right)+\frac{3}{2}N_\text{Y}\ln\left(\frac{3N_\text{X}+N_\text{Y}}{N_\text{Y}+N_\text{X}}\right) \tag{9}$$

We note that factors of 10^{23} will cancel inside the natural logarithms, such that we can write

$$\Delta S_\text{total} = \frac{3}{2}(6.0\times10^{23})\ln\left(\frac{(6.0)+\frac{1}{3}(15)}{(6.0)+(15)}\right)+\frac{3}{2}(1.5\times10^{24})\ln\left(\frac{3(6.0)+(15)}{(15)+(6.0)}\right)$$

$$\Delta S_\text{total} = 4.4\times10^{23}$$

4. Evaluate Result Chamber X experiences a decrease in temperature and chamber Y experiences an increase in temperature. We certainly expect the entropy to increase, since this system is isolated and closed and the second law of thermodynamics tells us that the entropy of such a system cannot decrease. Thus, our positive change in entropy makes sense. It is worth noting that the first term in equation (9) is negative. This corresponds to the entropy of chamber X being decreased as it cools. This is perfectly acceptable, provided that the increase in entropy of chamber Y is larger in magnitude than the decrease in chamber X. Clearly, this is the case.

19.6 Heating an ideal gas

1. Getting Started We can determine $v_\text{rms,f}$ using equation (19.47)

$$P = \frac{2}{3}\frac{N\left(\frac{1}{2}mv_\text{rms}^2\right)}{V}$$

We know the number of particles, and the volume; both of these are fixed throughout the process. We do not know the final pressure. We could determine the final pressure from the initial pressure and the initial and final temperatures, using the ideal gas equation. One finds

$$P_{\rm f} = \frac{P_{\rm i} T_{\rm f}}{T_{\rm i}} \tag{1}$$

2. Devise Plan We can determine the initial temperature of the gas by using the ideal gas equation:

$$T_{\rm i} = \frac{P_{\rm i} V_{\rm i}}{N k_{\rm B}} \tag{2}$$

The final temperature can be determine from the change in the entropy. Since the volume does not change, the change in entropy can be attributed entirely to the temperature difference, and we have from equation (19.56)

$$\Delta S = \frac{3}{2} N \ln\left(\frac{T_{\rm f}}{T_{\rm i}}\right)$$

We will use equations (2) and (19.56) to determine the initial and final temperatures. Inserting these into equation (1) will yield the final pressure. Then we can determine the final $v_{\rm rms,f}$ from equation (19.47).

3. Execute Plan Equation (2) gives us

$$T_{\rm i} = \frac{P_{\rm i} V_{\rm i}}{N k_{\rm B}} = \frac{(1.01\times10^5 \text{ Pa})(4.48\times10^{-2} \text{ m}^3)}{(9.03\times10^{23})(1.380\times10^{-23} \text{ J/K})} = 363.1 \text{ K}$$

Rearranging equation (19.56) we obtain the final temperature:

$$T_{\rm f} = T_{\rm i} e^{\frac{2\Delta S}{3N}} = (363.1 \text{ K}) e^{\frac{2(1.415\times10^{23})}{3(9.03\times10^{23})}} = 403.1 \text{ K}$$

Inserting both temperatures and the initial pressure into equation (1) we find

$$P_{\rm f} = \frac{P_{\rm i} T_{\rm f}}{T_{\rm i}} = (1.01\times10^5 \text{ Pa})\frac{(403.1 \text{ K})}{(363.1 \text{ K})} = 1.121\times10^5 \text{ Pa}$$

Finally, rearranging equation (19.47) allows us to use the pressure we obtained to find our answer:

$$v_{\rm rms,f} = \sqrt{\frac{3V_{\rm f} P_{\rm f}}{Nm}} = \sqrt{\frac{3(4.48\times10^{-2} \text{ m}^3)(1.121\times10^5 \text{ Pa})}{(9.03\times10^{23})(6.646\times10^{-27} \text{ kg})}} = 1.58\times10^3 \text{ m/s}$$

4. Evaluate Result This is about one tenth of the escape speed of Earth. It is also of the same order of magnitude as the speed of sound in many light gases. This answer is quite reasonable. We expect the number of basic states in the container to increase. We expect this for two reasons. First, with an increase in temperature come more units of energy that can be distributed the between the particles, and therefore there will be more ways of distributing that energy. Second, we are told that the entropy increases. This means the number of basic states must have increased.

19.8 Cooling and entropy change

1. Getting Started In order to determine the change in entropy from equation (19.61), we would need the initial and final temperatures and volumes, as well as the number of particles. We are given the initial and final temperatures and the number of particles. We are told nothing about the volumes, but we know the pressure is fixed.

2. Devise Plan The temperature decreases, and the pressure remains constant. This can only be true if the volume decreases as well. We can use the ideal gas law to determine by what fraction the volume must be reduced. Using the fact that the pressure is fixed, we can write:

$$\frac{N_{\rm i} k_{\rm B} T_{\rm i}}{V_{\rm i}} = P_{\rm i} = P_{\rm f} = \frac{N_{\rm f} k_{\rm B} T_{\rm f}}{V_{\rm f}}$$

From this we immediately see that

$$\frac{T_{\rm i}}{T_{\rm f}} = \frac{V_{\rm i}}{V_{\rm f}} \tag{1}$$

Note that in equation (19.61), we do not actually need each volume separately. All we need is the ratio of the final and initial volumes. Since we know the ratio of temperatures, we also know the ratio of volumes. Thus we have sufficient information to apply equation (19.61).

3. Execute Plan The temperature and volume change by the same factor. Equation (19.61) can be written

$$\Delta S = \frac{3}{2}N\ln\left(\frac{T_f}{T_i}\right) + N\ln\left(\frac{V_f}{V_i}\right) = \frac{5}{2}N\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{5}{2}(1.20\times10^{24})\ln\left(\frac{300\text{ K}}{400\text{ K}}\right) = -8.63\times10^{23}$$

4. Evaluate Result A negative entropy change is perfectly acceptable as long as the system is in contact with some other system or device, such that the total entropy increases or stays the same. In this case, the system has been cooled, and has decreased in volume. This means that energy has gone into some other system (increasing the entropy of that other system) and the volume of the surrounding area has increased (also increasing the entropy of the surrounding area/system). Thus it is reasonable to expect a negative entropy change when a system is cooled, and it is perfectly reasonable to assume that the negative change in entropy of this sample will be offset by a positive change in entropy elsewhere.

Questions and Problems

19.1. (*a*) There are four 8 cards in the deck, so the probability of picking any one of them at random is $4/52 = 0.0769$. (*b*) There are four of each card in the deck, for a total of 16 cards, so the probability of picking any one of them at random is $16/52 = 0.308$.

19.2. There are 6 ways for the two dice to show 7 dots ($1+6,\ 2+5,\ 3+4,\ 4+3,\ 5+2,$ and $6+1$). There are 36 ways for the dice to be thrown (6 possibilities for the first die times 6 possibilities for the second), so the probability is $6/36 = 0.167$.

19.3. It take 365 days, on average, for the system to cycle through the 5.00×10^{13} possible basic states, so the average time between collisions is

$$5.00\times10^{13}\Delta t = 365\text{ d}$$

$$\Delta t = \frac{365\text{ d}}{5.00\times10^{13}}\frac{24\text{ h}}{1\text{ d}}\frac{3600\text{ s}}{1\text{ h}} = 6.31\times10^{-7}\text{ s}$$

19.4. [**NOTE:** The last sentence of the problem statement will be changed to read: "If you reach in and grab one randomly, what is the probability of picking, **sequentially**, (*a*) a black one, (*b*) an orange one, and (c) a blue one?"] (*a*) The bag contains 45 jelly beans, 10 of which are black, so the probability of picking a black one is $10/45 = 0.222$. (*b*) There are 3 orange jelly beans in the bag, which now contains only 44 jelly beans, so the probability of picking one is $3/44 = 0.0682$. (*c*) There are 20 blue jelly beans in the bag, out of a total of 43, so the probability of picking one is $20/43 = 0.465$.

19.5. (*a*) There are 3 types, each available in 4 colors, and each with 2 kinds of motor, so the number of possible configurations is $3\times4\times2 = 24$. (*b*) If, in addition, each is available with 26 different option packages, the number of possible configurations is $3\times4\times2\times26 = 624$.

19.6. There are 3 ways to get a total of four ($1+1+2,\ 1+2+1,$ and $2+1+1$) out of 216 ways to throw three dice ($6\times6\times6$), so the probability is $3/216 = 0.0139$.

19.7. Six energy units can be divided among the three objects in 28 ways, and in 7 of those ways the pendulum has none, which we can demonstrate by counting:

P	A	B	P	A	B	P	A	B	P	A	B	P	A	B	P	A	B	P	A	B
6	0	0	5	1	0	4	2	0	3	3	0	2	4	0	1	5	0	0	6	0
			5	0	1	4	1	1	3	2	1	2	3	1	1	4	1	0	5	1
						4	0	2	3	1	2	2	2	2	1	3	2	0	4	2
									3	0	3	2	1	3	1	2	3	0	3	3
												2	0	4	1	1	4	0	2	4
															1	0	5	0	1	5
																		0	0	6

So, the probability of finding the pendulum with none of the energy units is $7/28 = 0.25$.

19.8. When the pendulum is released, all of its energy is in the form of gravitational potential energy in the Earth-pendulum system. Choosing the low point of the pendulum as our zero, we have $E_\text{p} = m_\text{p}gh$ or $m_\text{p} = E_\text{p}/(gh)$.

The kinetic energy of a typical nitrogen molecule is $\frac{1}{2}m_{\text{N}_2}v_{\text{N}_2}^2,$ and the kinetic energy of all the nitrogen molecules in the room is $\frac{1}{2}m_{\text{N}_2}v_{\text{N}_2}^2 N,$ so we have for the pendulum's mass

$$m_\text{p} = \frac{m_{\text{N}_2}v_{\text{N}_2}^2 N}{2gh}$$

$$= \frac{(4.65\times10^{-26}\text{ kg})(550\text{ m/s})^2(1.50\times10^{21})}{2(9.8\text{ m/s}^2)(1.15\text{ m})} = 0.936\text{ kg}$$

19.9. (*a*) When a coin is tossed randomly, any particular sequence is as likely as any other, so neither sequence is more likely than the other. (*b*) Landing heads up three times is more likely. There are $2^5 = 32$ possible outcomes, 10 of which have three heads, but only one of which has five heads.

We can find that number, 10, either by counting all the possible outcomes or by noticing that the number of heads *k* gotten in *n* tosses equals the number of ways to choose *k* things (the tosses that result in heads) from a set of *n* things (all the tosses), so it is equal to the binomial coefficient $\begin{pmatrix} n \\ k \end{pmatrix} = \frac{n!}{k!(n-k)!}.$ That is, the number of ways to get three heads in five tosses is $\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \frac{5!}{3!2!} = 10.$

19.10. (*a*) Because any valve can be in one of two positions, the number of tones that can be produced is $2^2 = 4,$ (*b*) $2^3 = 8,$ and (*c*) $2^6 = 64.$ (*d*) Because now any valve can be in one of three positions, the number of tones that can be produced is $3^3 = 27.$

19.11. [**NOTE:** The problem statement will be changed from "such that all six collide and change direction" to read "such that each collides and changes direction"] Each particle can move in any one of six directions, so if the particles are distinguishable, there are $6^6 = 46{,}656$ possible ways for the particles to be moving. That is, the first particle could be moving in any of the six directions, as could the second, and so on. Of these, there are 6 in which all six particles are moving in the same direction, so the probability of finding all six particles moving in the same direction is $6/46{,}656 = 1.286\times10^{-4}.$ Because the particles collide so frequently, they will on average divide their time evenly between all possible basic states over the course of a day, so the time they spend in these 6 basic states is $(1.286\times10^{-4})(24\text{ h})(3600\text{ s})/(1\text{ h}) = 11\text{ s}.$

If the particles are indistinguishable, there are only 462 basic states, so the probability of all six moving in the same direction is $6/462 = 1.299\times10^{-2}$, and the time they spend in these six basic states over the course of a day is $(1.299\times10^{-2})(24\text{ h})(3600\text{ s})/(1\text{ h}) = 1122\text{ s}$, or almost 19 minutes.

To see this, we could count the basic states by writing them all down, or we could calculate the number of basic states like so: Let's represent a basic state schematically as $\bullet||\bullet|\bullet\bullet|\bullet\bullet|$, where each dot stands for a particle, and the bars divide them among the six possible directions. So, for example, this basic state represents the first direction having one particle moving in that direction, the second direction having none, the third having one, the fourth and fifth each having two, and the sixth having none. Our string is made up of 11 symbols, and there are 11! possible strings made up of 11 symbols, but the 6 dots are identical and so are the 5 bars, so the number of different strings is $11!/(6!5!) = 462$.

19.12. To fill the first location, we can choose any of the 100 adjacent gas particles. To fill the second, we can choose any of the remaining 99, and so on, until to fill the sixth, we can choose any of the remaining 95 particles. So there are $100\times99\times98\times97\times96\times95 = 8.58\times10^{11}$ ways for the particles to be adsorbed onto the six locations.

In our calculation, above, we assumed that the locations were distinguishable. If they are not, then the first particle could have filled any of the 6 locations, the second could have filled any of the remaining 5, and so on, until the sixth filled the 1 remaining location, giving a total of $\dfrac{100\times99\times98\times97\times96\times95}{6\times5\times4\times3\times2\times1} = 1.19\times10^{9}$ ways for the particles to be adsorbed.

So far, we have also assumed that, when a gas particle is adsorbed, another does not immediately come in from the surroundings to take its place. If one did, there would always be 100 gas particles to choose from, so our results would be $100^6 = 10^{12}$ and 1.39×10^{9} ways for the cases of distinguishable and indistinguishable locations, respectively. Whether or not this happens depends on the rate at which particles are adsorbed compared to the rate at which particles change locations within the gas.

19.13. The energy can be divided among the particles in 10 ways, and one particle has all of it in 3 cases, so the probability is $3/10 = 0.3$. We can see this by counting the basic states:

A	B	C
3	0	0

A	B	C
2	1	0
2	0	1

A	B	C
1	2	0
1	1	1
1	0	2

A	B	C
0	3	0
0	2	1
0	1	2
0	0	3

19.14. The fewer units of energy particle A has, the greater the number of corresponding basic states the system can be in, because there are more ways to divide the greater number of remaining units among the other two particles. So, at any instant, it is more likely for particle A to have two units of energy than for it to have three.

19.15. At equilibrium, each molecule has the same kinetic energy, so we have

$$\tfrac{1}{2}m_{N_2}v_{N_2}^2 = \tfrac{1}{2}m_{O_2}v_{O_2}^2$$

$$\frac{v_{N_2}}{v_{O_2}} = \sqrt{\frac{m_{O_2}}{m_{N_2}}}$$

$$= \sqrt{\frac{5.3135\times10^{-26}\text{ kg}}{4.652\times10^{-26}\text{ kg}}} = 1.069$$

19.16. At equilibrium, the energy is evenly divided among all the particles. The total energy is 120 + 180 = 300 units, and there are 50 + 100 = 150 particles, so each particle has 2 units. That is, (*a*) each A particle has 2 units and (*b*) each B particle has 2 units.

19.17. There are 56 ways to distribute five units of energy among four distinguishable particles, and in four of these cases one particle has all the energy, so the probability is $4/56 = 0.0714$.

To see this, we could count the basic states by writing them all down, or we could calculate the number of basic states the same way as we did in the solution to Problem 19.11. Let's represent a basic state schematically as $\bullet||\bullet|\bullet\bullet\bullet,$ where each dot stands for an energy unit, and the bars divide them among the particles. So, for example, this basic state represents particle A having 1 unit of energy, particle B having none, particle C having 1 and particle D having 3. Our string is made up of 8 symbols, and there are 8! possible strings made up of 8 symbols, but the 5 dots are identical and so are the 3 bars, so the number of different strings is $8!/(5!3!) = 56$. In general, the number of ways to divide *n* indistinguishable objects among *k* containers is $\dfrac{(n+k-1)!}{n!(k-1)!}$.

19.18. As the pendulum swings, it collides with the nitrogen molecules, and eventually stops after all of its energy has been transferred to the gas. If we assume that each collision transfers the average amount of energy transferred per collision, which equals the initial energy of the pendulum divided by the number of collisions, we can solve the problem by proportional reasoning.

The number of collisions required to stop the pendulum remains the same. This number of collisions equals the product of the collision rate and the time interval Δt over which they take place, and the collision rate is proportional to the number density of the gas molecules in the box, which in turn is proportional to the number N of molecules in the box. So, we have

$$\begin{aligned}\Delta t_1 N_1 &= \Delta t_2 N_2\\ \Delta t_2 &= \Delta t_1 N_1/N_2\\ &= (5.0\text{ min})(1.0\times10^{23})/(2.5\times10^{23}) = 2.0\text{ min}\end{aligned}$$

19.19. The energy is evenly divided among the atoms, so a typical atom has speed satisfying $\frac{1}{2}mv^2 = E/N,$ or

$$\begin{aligned}v &= \sqrt{\frac{2E}{mN}}\\ &= \sqrt{\frac{2(498\text{ J})}{(6.63\times10^{-26}\text{ kg})(6.02\times10^{23})}} = 158\text{ m/s}\end{aligned}$$

19.20. The energy is evenly divided among all the atoms, so a typical A atom has the same amount of kinetic energy as a typical B atom:

$$\begin{aligned}\tfrac{1}{2}m_\mathrm{A}v_\mathrm{A}^2 &= \tfrac{1}{2}m_\mathrm{B}v_\mathrm{B}^2\\ v_\mathrm{A} &= \sqrt{\frac{m_\mathrm{B}v_\mathrm{B}^2}{m_\mathrm{A}}}\\ &= \sqrt{\frac{(1.39\times10^{-25}\text{ kg})(123\text{ m/s})^2}{3.35\times10^{-26}\text{ kg}}} = 251\text{ m/s}\end{aligned}$$

19.21. (*a*) The average argon atom has kinetic energy $E_\text{atom,i} = \frac{1}{2}m_\text{atom}v_\mathrm{i}^2 = \frac{1}{2}(6.63\times10^{-26}\text{ kg})(100\text{ m/s})^2 = 3.32\times10^{-22}\text{ J}$. (*b*) After the ball has stopped bouncing, its initial energy has been transferred to the argon atoms, and is evenly divided among them, so the kinetic energy of each argon atom has increased by $\Delta E_\text{atom} = E_\text{ball}/N = m_\text{ball}gh/N = (1.00\text{ kg})(9.8\text{ m/s}^2)(1.00\text{ m})/(6.02\times10^{23}) = 1.63\times10^{-23}\text{ J}$.
So, the new speed is given by

$$\tfrac{1}{2}m_{\text{atom}}v_{\text{f}}^2 = E_{\text{atom,i}} + \Delta E_{\text{atom}}$$

$$v_{\text{f}} = \sqrt{\frac{2(E_{\text{atom,i}} + \Delta E_{\text{atom}})}{m_{\text{atom}}}}$$

$$= \sqrt{\frac{2\left[(3.315\times10^{-22}\text{ J}) + (1.628\times10^{-23}\text{ J})\right]}{6.63\times10^{-26}\text{ kg}}} = 102\text{ m/s}$$

19.22. When the Sun warms the balloon, the energy it adds is evenly distributed among the helium atoms, and this additional energy increases the speed of each atom so that

$$\tfrac{1}{2}m_{\text{atom}}v_{\text{atom,f}}^2 = E_{\text{atom,i}} + \Delta E_{\text{atom}}$$

$$v_{\text{atom,f}} = \sqrt{\frac{2(\tfrac{1}{2}m_{\text{atom}}v_{\text{atom,i}}^2 + \Delta E_{\text{atom}})}{m_{\text{atom}}}}$$

The energy absorbed from the sunlight equals the product of the intensity, the cross-sectional area of the balloon, and the time interval, so the change in each atom's energy is

$$\begin{aligned}\Delta E_{\text{atom}} &= \Delta E_{\text{balloon}}/N \\ &= IA\Delta t/N \\ &= I(\pi R^2)\Delta t/N\end{aligned}$$

Combining these we have

$$\begin{aligned}v_{\text{atom,f}} &= \sqrt{\frac{2(\tfrac{1}{2}m_{\text{atom}}v_{\text{atom,i}}^2 + \pi IR^2\Delta t/N)}{m_{\text{atom}}}} \\ &= \sqrt{\frac{2\left[\tfrac{1}{2}(6.646\times10^{-27}\text{ kg})(367\text{ m/s})^2 + \pi(1120\text{ W/m}^2)(0.250\text{ m})^2(10.0\text{ min})(60\text{ s/1 min})/(6.02\times10^{23})\right]}{6.646\times10^{-27}\text{ kg}}} \\ &= 8.13\times10^3\text{ m/s}\end{aligned}$$

19.23. (*a*) There is only one basic state in which all three particles are in octant 1. (*b*) If two particles are in octant 1, then the third particle can be in any one of the seven other octants, giving seven basic states.

19.24. (*a*) We can imagine dividing the room into 64 same-sized volumes (let's call them "compartments" for convenience), the box being one of them, and the probability of finding either particle in any one of them is the same. There are 2080 ways to place 2 indistinguishable particles into 64 compartments, only one of which has both particles in the box, so the probability of finding both particles in the box is $1/2080 = 4.81\times10^{-4}$. To see this, we can enumerate the possibilities. It is either the case that both particles are in the same compartment, or that one particle is in one of them and the other particle is in another. There are 64 basic states with both particles in one of the 64 compartments. There are 63 basic states with one particle in compartment 1 and the other particle in one of the other 63 compartments. There are also 63 basic states with one particle in compartment 2 and the other particle in one of the other 63 compartments, and so on for all 64 compartments, giving 64×63 basic states. But we have counted each of these different-compartment basic states twice. For example, the basic state with one particle in compartment 1 and one particle in compartment 2 was counted both when we considered the basic states with one particle in compartment 1 and again when we considered the basic states with one particle in compartment 2. So, the total number of basic states is $64 + (64\times63)/2 = 2080$. (*b*) At first glance, one might suppose that this probability is just the probability of finding both particles in any of the 64 compartments, that is, $64/2080 = 3.08\times10^{-2}$, but this is not correct. For example, the particles might be in adjacent compartments, both of them near enough to the (notional) dividing wall that they are within the same $1/64$ of the room. The first particle can be anywhere in the room. The probability of the second particle being within the same $1/64$ of the room is $1/64$. But because the particles are indistinguishable, we also have to include the case where the two particles are exchanged. So, the probability of finding both particles in the same $1/64$ of the room is $2/64 = 3.125\times10^{-2}$.

19.25. (*a*) If the particles are indistinguishable, there are 20 basic states. We have three possibilities to consider: either all three particles are in the same quadrant, two particles are in the same quadrant and one is in another, or each of the three particles is in a different quadrant. There are four basic states corresponding to the first possibility (all three can be in any one of the four quadrants), and four corresponding to the third possibility (any one of the four quadrants can be empty). For the second possibility, the two particles can be in any one of the four quadrants, and in each of these cases the third particle can be in any one of the remaining three quadrants, for a total of $4\times 3=12$ basic states. So the total number of basic states is $4+12+4=20$. (*b*) If the particles are distinguishable, there are $4^3=64$ basic states. The first particle can go into any of the four quadrants, the second can go into any of the four, and likewise for the third.

19.26. (*a*) We are free to choose what we consider a macrostate based on the property of the system that interests us. For example, if the particles have different masses and we are interested in mass density, we might choose to define a macrostate as the mass of the particles in octant 1. If we are interested in number density, we might choose to define a macrostate as the number of particles in octant 1. If we are interested in each of the particles individually, we might choose to define a macrostate as the list of the individual particles in octant 1. (*b*) Each of the four particles can be in any one of the eight octants, so there are $8^4=4096$ basic states. That is, particle 1 can go into any of the eight octants, particle 2 can go into any of the eight, etc. (*c*) There are now 16 octants in which to distribute the particles, so there are now $16^4=65{,}536$ basic states. The number of basic states has increased by a factor of $2^4=16$ because $16^4=(2\times 8)^4=2^4\times 8^4$.

19.27. Assuming that the recurrence time is much less than a day, the probability of finding this number of particles in the upper left quadrant is $\dfrac{51.43\text{ min}}{1\text{ day}}\dfrac{1\text{ day}}{24\text{ h}}\dfrac{1\text{ h}}{60\text{ min}}=3.572\times 10^{-2}$. *Principles* Figure 19.12 enumerates the ways to distribute six identical particles among four quadrants. Because the total number of basic states is 84, the macrostate we need must correspond to $84\times 3.572\times 10^{-2}=3$ basic states, which means we need five particles in the upper left quadrant.

19.28. Because a particle is equally likely to be found anywhere in the cube, the probability of finding any given particle in a smaller volume V of a cube with volume 1.00 m^3 equals $V/(1.00\text{ m}^3)$. The probability of finding all N distinguishable particles in this smaller volume is $\left[V/(1.00\text{ m}^3)\right]^N$, that is, the probability of finding the first one there, times the probability of finding the second one there, and so on. So, we have (*a*) $\left[(0.500\text{ m}^3)/(1.00\text{ m}^3)\right]^1=0.500$, (*b*) $\left[(0.250\text{ m}^3)/(1.00\text{ m}^3)\right]^1=0.250$, (*c*) $\left[(0.0100\text{ m}^3)/(1.00\text{ m}^3)\right]^1=0.0100$, (*d*) $\left[(0.500\text{ m}^3)/(1.00\text{ m}^3)\right]^2=0.250$, and (*e*) $\left[(0.500\text{ m}^3)/(1.00\text{ m}^3)\right]^4=0.0625$.

19.29. The volume of the detector is $\pi R^2h=\pi(7.50\times 10^{-3}\text{ m})^2(5.00\times 10^{-3}\text{ m})=8.84\times 10^{-7}\text{ m}^3$. If this is attached to a container of volume 1.00 m^3, the probability at any instant of finding a given particle from the container in the detector equals the ratio of the detector's volume to their combined volumes, because the particle is equally likely to be found anywhere in this volume. Because the detector is so much smaller than the container, this equals the ratio of the detector's volume to the container's, or 8.84×10^{-7}.

If the number density of particles is assumed to be $1/(1.00\times 10^{-6}\text{ m}^3)$, there should be $(1.00\text{ m}^3)/(1.00\times 10^{-6}\text{ m}^3)=1.00\times 10^6$ particles in the container, and the probability of finding one or more of them in the detector at any instant is $(1.00\times 10^6)(8.84\times 10^{-7})=0.884$. So, provided that the duration of each test is much, much less than the recurrence time of the system of particles, out of 100 tests we expect that one or more particles will be found in the detector $(100)(0.884)=88$ times.

But note that we have played fast and loose with the probabilities in the preceding. (There must be something wrong with our calculation because, if there were twice as many particles, it would give a probability greater than one.) It is not strictly correct to say that the probability of finding one or more of the 1.00×10^6 particles in the detector equals 1.00×10^6 times the probability of finding any one of them there; that would only be the case if the possibilities of finding one or another of the particles in the detector were mutually exclusive. So, it is a reasonable approximation only when both the probability of finding a given particle in the detector and the number of particles are small enough that their product is much, much less than 1, that is, when the probability of finding more than one particle in the detector is small enough to be ignored, which is not the case here.

If the probability of finding any given particle in the detector is p, then the probability of finding one or more of the particles in the detector equals one minus the probability of finding none of them in the detector, which latter equals $(1-p)^N$, that is, the product of the probability of not finding the first particle in the detector, times the probability of not finding the second particle there, and so on. So, the probability of finding one or more of the particles in the detector equals $1-(1-p)^N = 1-(1-8.84\times10^{-7})^{1{,}000{,}000} = 0.587$. We can also see that our first method for calculating the probability amounts to using the approximation $(1-p)^N \approx 1-Np$.

The preceding applies to distinguishable particles. If the particles are indistinguishable, the answer is different. There are $1/(8.84\times10^{-7})$ = 1,131,768 detector-sized compartments in the cubic meter, plus one more for the detector. The number of ways to distribute a million indistinguishable particles among these 1,131,769 compartments is about $1.268\times10^{639{,}953}$. As before, the probability of finding one or more in the detector equals one minus the probability of finding none there. The probability of finding none in the detector equals the ratio of the number of ways of distributing them among the 1,131,768 detector-sized compartments in the cubic meter (i.e., the number of basic states for which there are no particles in the detector, which is to say that all one million particles are distributed among the other compartments) to the number of ways of distributing them among the 1,131,769 detector-sized compartments in the cubic meter plus the detector (i.e., all the basic states), which we can find using the equation we developed in the solution to Problem 19.17:

$$\frac{2{,}131{,}767!}{(1{,}000{,}000!)(1{,}131{,}767!)}\frac{(1{,}000{,}000!)(1{,}131{,}768!)}{2{,}131{,}768!} = \frac{1{,}131{,}768}{2{,}131{,}768}$$

or about 0.531. So, the probability of finding one or more in the detector is about $1-0.531 = 0.469$.

19.30. Following the hint, we see that the maximum number of particles that the box could contain is $4\times30 = 120$ particles. That is, the box contains $120-114 = 6$ possible locations for particles that have no particle in them. We can solve this problem by considering how these empty locations are distributed among the quadrants. (*a*) If a macrostate is defined as the number of particles in the upper left quadrant, it is equally well defined as 30 minus the number of unfilled locations in it. There are 7 possible macrostates, corresponding to zero through six unfilled locations in the upper left quadrant. The number of basic states equals the number of ways that the six unfilled locations can be distributed among the four quadrants, which is 84 (see *Principles* Figure 19.12). (*b*) The most probable number of unfilled locations in the upper left quadrant is zero (because there are then the greatest number of ways of distributing them among the other three quadrants), so the most probable number of particles in the upper left quadrant is 30. (*c*) The average number of particles in the upper left quadrant is 28.5. Because each particle is equally likely to be found in any of the quadrants, the average number found in the upper left one is $114/4 = 28.5$. Or we could, as in *Principles* Example 19.5, add up the probabilities of finding each number of unfilled locations, giving on average 1.5 unfilled locations out of 30, or 28.5 particles.

19.31. The number of basic states for the macrostate $E_A = 7$ is $77{,}520\times56$, while the number of basic states for the macrostate $E_A = 3$ is 560×792, so the macrostate $E_A = 7$ is $(77{,}520\times56)/(560\times792) = 9.79$ times more likely than the macrostate $E_A = 3$.

19.32. The number of ways that 10 indistinguishable energy units can be distributed among 14 distinguishable particles is the value of Ω_A for the macrostate $E_A = 10$ in Table P19.31, that is, $1{,}144{,}066$. We can check this using the formula we derived in the solution to Problem 19.17: $\dfrac{(10+14-1)!}{10!(14-1)!} = 1{,}144{,}066.$

19.33. (*a*) The system is in equilibrium in the macrostate with the greatest number of associated basic states, which is macrostate C. (*b*) Macrostates A and E have the smallest number of associated basic states, so they are least likely. (*c*) The system has 16 possible basic states, the sum of the basic states associated with all the macrostates. (*d*) The probability of finding the system in macrostate B is the ratio of the number of basic states associated with macrostate B to the number of possible basic states, $4/16 = 0.25$.

19.34. When the system is in equilibrium, the energy units will be evenly divided among the particles, so the average number of energy units per particle will be $9/(25+20) = 0.2$. That means that compartment B will contain $20 \times 0.2 = 4$ energy units. Because one energy unit can be exchanged per collision, it takes at least 5 collisions to reduce the number of energy units in compartment B from its initial value of 9 to its equilibrium value of 4.

19.35. The number of basic states equals the product of Ω_A, the number of basic states in compartment A, times Ω_B, the number of basic states in compartment B. (*a*) In the initial configuration, there are five energy units distributed among the two particles in compartment A and none distributed among the three particles in compartment B. So, $\Omega_A = 6$ (particle 1 can have zero while particle 2 has 5, or particle 1 can have 1 while particle 2 has 4, etc.) and $\Omega_B = 1$ (there is only one way to distribute nothing) and the number of basic states equals $\Omega_A \Omega_B = 6$. (*b*) When the system has reached equilibrium, the energy units will be distributed evenly among the particles, so there will be 2 energy units distributed among the 2 particles in compartment A and 3 energy units distributed among the 3 particles in compartment B. So, $\Omega_A = 3$ (particle 1 can have zero while particle 2 has 2, or particle 1 can have 1 while particle 2 has 1, or particle 1 can have 2 while particle 2 has zero) and $\Omega_B = 10$ (particle 1 can have zero while 3 are distributed among the other two particles in 4 possible ways, particle 1 can have 1 while 2 are distributed among the other two in 3 possible ways, etc.) and the number of basic states equals $\Omega_A \Omega_B = 30$.

19.36. (*a*) The number of ways to distribute *N* energy units among 2 distinguishable particles equals $N+1$ (particle 1 can have zero while particle 2 has *N*, or particle 1 can have 1 while particle 2 has $N-1$, or ... particle 1 can have *N* while particle 2 has zero), so the table is as follows:

E_A	E_B	Ω_A	Ω_B	Ω	$\ln\Omega$
0	4	1	5	5	1.61
1	3	2	4	8	2.08
2	2	3	3	9	2.20
3	1	4	2	8	2.08
4	0	5	1	5	1.61
Total		15	15	35	

(*b*) Yes, it does. The macrostate $E_A = 2$, in which the energy units are evenly divided among all the particles, is the one with the greatest number of corresponding basic states, which is to say, the most likely macrostate in which to find the system.

19.37. If the probability of finding the systems in their equilibrium states is p, the number of basic states in system 1 is N_1, and the number of basic states corresponding to its equilibrium state is $\Omega_{1,\mathrm{eq}}$, then we have $p = \Omega_{1,\mathrm{eq}}/N_1$, and similarly for system 2. So,

$$\frac{\Omega_{1,\text{eq}}}{N_1} = \frac{\Omega_{2,\text{eq}}}{N_2}$$

$$\begin{aligned}\Omega_{1,\text{eq}} &= \frac{\Omega_{2,\text{eq}} N_1}{N_2} \\ &= \frac{\Omega_{2,\text{eq}}(3422N_2)}{N_2} \\ &= 3422\Omega_{2,\text{eq}} \\ &= (3422)(489) = 1,673,358\end{aligned}$$

19.38. (*a*) Because macrostate 2 is six times more likely to occur than macrostate 1, it must have six times as many corresponding basic states, which is to say six. So the total number of basic states for the system is eight, and the probability of finding the system in macrostate 2 is $6/8 = 0.75$. (*b*) The probability that macrostate 1 occurs is $1/8 = 0.125$, and the probability that macrostate 3 occurs is $1/8 = 0.125$. (*c*) The sum of the probabilities for all macrostates is 1, as it must be because the system is certain to be in one of the three possible macrostates.

19.39. The system's most probable state is the one in which the energy units are evenly divided among all the particles, which is to say that each particle has $39/(3\times 13) = 1$ energy unit. To reach this state, region A must lose $39 - 13 = 26$ energy units, which means at least 26 collisions must occur. (*a*) 18 is less than 26, so the answer is no. (*b*) Yes. Fourteen of the collisions could transfer energy units from region A to region B, 12 from region A to region C, and one from region B to region C. (*c*) Yes. For example, we could have the same 27 collisions as in part *b*, followed by one that transfers an energy unit from region A to region B and another that transfers an energy unit from region B to region A.

19.40. Each particle can be placed in any of the 1000 compartments, so the number of basic states for the system is $\Omega = 1000^{25} = 10^{75}$, and its entropy is $S = \ln\Omega = 173$.

19.41. Let's call the number of beads *N*. The entropy of either system equals the natural logarithm of the number of basic states, and the number of basic states equals the number of compartments *M* raised to the power *N*, so $S = N\ln M$. The number of compartments is inversely proportional to the compartments' volumes, $M = (1.00\text{ m}^3)/(2R)^3$. The system with the smaller beads has the greater entropy because there are more containers to distribute them over, and we have

$$\begin{aligned}S_{15} &= N\ln\left(\frac{1.00\text{ m}^3}{(0.0300\text{ m})^3}\right) \\ &= N\ln(0.0300^{-3}) \\ &= -3N\ln(0.0300) \\ &= 10.5N\end{aligned}$$

$$\begin{aligned}S_{20} &= N\ln\left(\frac{1.00\text{ m}^3}{(0.0400\text{ m})^3}\right) \\ &= N\ln(0.0400^{-3}) \\ &= -3N\ln(0.0400) \\ &= 9.66N\end{aligned}$$

and

$$\begin{aligned}\frac{S_{15}}{S_{20}} &= \frac{-3N\ln(0.0300)}{-3N\ln(0.0400)} \\ &= \frac{\ln(0.0300)}{\ln(0.0400)} = 1.09\end{aligned}$$

We also have

$$\begin{aligned}S_{15}-S_{20}&=-3N\left[\ln(0.0300)-\ln(0.0400)\right]\\&=-3N\ln(3/4)\\&=0.863N\end{aligned}$$

That is, the difference in the systems' entropies is proportional to the logarithm of the ratio of the spheres' radii.

19.42. There are 80 basic states corresponding to one of the particles having all 80 energy units, but only one basic state corresponding to each of the particles having exactly one energy unit, so the latter is less likely. (Compare the solution to Problem 19.13, where the possible basic states are listed for the case of three energy units divided among three particles.)

19.43. The change in entropy when a gas expands is given by Eq. 19.8, so we have

$$\begin{aligned}\Delta S&=N\ln(V_\mathrm{f}/V_\mathrm{i})\\N&=\frac{\Delta S}{\ln(V_\mathrm{f}/V_\mathrm{i})}\\&=\frac{6.91\times10^{18}}{\ln((0.100\text{ m}^3)/(0.0100\text{ m}^3))}=3.00\times10^{18}\end{aligned}$$

19.44. (*a*) At equilibrium, the number densities of the particles on both sides of the partition are equal, and the number of particles is the same on each side of the partition, so the partition divides the box in half, at position $x=0.500$ m. (*b*) No, provided that the particles occupy a negligible fraction of the volume of the box, which was one of the assumptions we made at the beginning of *Principl*es Section 19.5. When we derived the equipartition of space by finding the volume ratio that maximized the entropy of the box as a whole, we found that the size of the compartments into which we divided the volume did not matter. That is, $\dfrac{dS}{dV}$ does not depend on δV because δV does not depend on V, so it makes no difference whether or not the same value is used for δV on each side of the partition.

19.45. We can calculate the entropy, that is, the natural logarithm of the number of basic states available to the system, for each system.

For system A, we can assign any of a million colors to the first star, any of a million colors to the second star, and so on, so

$$\begin{aligned}S_\mathrm{A}&=\ln(1{,}000{,}000^{70\times10^{21}})\\&=(70\times10^{21})\ln(1{,}000{,}000)\\&=9.7\times10^{23}\end{aligned}$$

For system B, we can assign any of N_A particles to the first site, any of $N_A-1\approx N_A$ particles to the second site, and so on, so

$$\begin{aligned}S_\mathrm{B}&=\ln(N_\mathrm{A}{}^{83})\\&=(83)\ln(6.02\times10^{23})\\&=4.5\times10^3\end{aligned}$$

For system C, we have

$$\begin{aligned}S_\mathrm{C}&=\ln((0.8N_\mathrm{A})^{106})\\&=(106)\ln((0.8)(6.02\times10^{23}))\\&=5.78\times10^3\end{aligned}$$

For system D, we can assign any of three health levels to the first cell, any of three to the second cell, and so on, so

$$\begin{aligned} S_{\mathrm{D}} &= \ln(3^{(10^{12})}) \\ &= (10^{12})\ln(3) \\ &= 1.1\times 10^{12} \end{aligned}$$

For system E, we can assign either of two conditions to the first microbe, either of two to the second, and so on, so

$$\begin{aligned} S_{\mathrm{E}} &= \ln(2^{1\times 10^{14}}) \\ &= (1\times 10^{14})\ln(2) \\ &= 7\times 10^{13} \end{aligned}$$

So, the order is $S_{\mathrm{B}} < S_{\mathrm{C}} < S_{\mathrm{D}} < S_{\mathrm{E}} < S_{\mathrm{A}}$.

19.46. The entropy of the two-unit system equals the sum of the entropy of each unit, $S = S_{\text{unit A}} + S_{\text{unit B}}$, so $\Delta S = \Delta S_{\text{unit A}} + \Delta S_{\text{unit B}}$, and we have from Eq. 19.8

$$\begin{aligned} \Delta S &= \Delta S_{\text{unit A}} + \Delta S_{\text{unit B}} \\ &= N_{\text{unit A}} \ln\left(\frac{V_{\text{unit A,f}}}{V_{\text{unit A,i}}}\right) + N_{\text{unit B}} \ln\left(\frac{V_{\text{unit B,f}}}{V_{\text{unit B,i}}}\right) \\ &= 3N_{\mathrm{A}} \ln(3) + 2N_{\mathrm{A}} \ln(3) \\ &= 5(6.02\times 10^{23})(1.10) = 3\times 10^{24} \end{aligned}$$

19.47. [**NOTE:** The problem statement will be changed so that "Suppose 50 distinguishable objects have a volume" will read "Suppose 50 distinguishable objects each has a volume"] (*a*) If we divide the box into object-sized compartments, there are initially $(1.00\times 10^{-6}\ \text{m}^3)/(1.00\times 10^{-9}\ \text{m}^3) = 1000$ compartments, so the initial number of basic states is $\Omega_{\mathrm{i}} = M_{\mathrm{i}}^N = 1000^{50} = 10^{150}$. If the volume is doubled, there are twice as many compartments and the new number of basic states is $\Omega_{\mathrm{f}} = M_{\mathrm{f}}^N = 2000^{50} = 2^{50}\times 10^{150}$. So, the number of basic states increased by a factor of $\Omega_{\mathrm{f}}/\Omega_{\mathrm{i}} = 2^{50} = 1.13\times 10^{15}$. (*b*) The entropy is the natural logarithm of the number of basic states, so $S_{\mathrm{i}} = \ln(10^{150}) = 150\ln(10) = 345$, $S_{\mathrm{f}} = \ln(2^{50}\times 10^{150}) = 50\ln(2) + 150\ln(10) = 380$, and $S_{\mathrm{f}}/S_{\mathrm{i}} = 1.10$.

19.48. (*a*) The left compartment contains three times as many particles as the right compartment, so in the new equilibrium state the left compartment will have three times the volume of the right compartment. That is, $V = V_{\mathrm{L,f}} + V_{\mathrm{R,f}} = 3V_{\mathrm{R,f}} + V_{\mathrm{R,f}} = 4V_{\mathrm{R,f}}$, or $V_{\mathrm{R,f}} = V/4$ and $V_{\mathrm{L,f}} = 3V_{\mathrm{R,f}} = 3V/4$. (*b*) The change in the entropy of the system equals the sum of the changes of the entropy of each compartment. From Eq. 19.8 we have

$$\begin{aligned} \Delta S &= \Delta S_{\mathrm{L}} + \Delta S_{\mathrm{R}} \\ &= N_{\mathrm{L}} \ln\left(\frac{V_{\mathrm{L,f}}}{V_{\mathrm{L,i}}}\right) + N_{\mathrm{R}} \ln\left(\frac{V_{\mathrm{R,f}}}{V_{\mathrm{R,i}}}\right) \\ &= N_{\mathrm{L}} \ln\left(\frac{3V/4}{V/2}\right) + N_{\mathrm{R}} \ln\left(\frac{V/4}{V/2}\right) \\ &= N_{\mathrm{L}} \ln(3/2) + N_{\mathrm{R}} \ln(1/2) \\ &= (3000)(0.4055) + (1000)(-0.6931) = 523.2 \end{aligned}$$

19.49. [**NOTE:** In the problem statement "increases the entropy by a factor of 1.10" will be changed to "increases the entropy per particle by 1.10".] The number of basic states is $\Omega = M^N$ and the entropy is $S = \ln\Omega = N\ln M$. We are given that $\Omega_2 = 6633\Omega_1$, so

$$M_2^N = 6633 M_1^N$$
$$(M_2/M_1)^N = 6633$$
$$N \ln(M_2/M_1) = \ln 6633$$

We are also given that $\dfrac{S_2}{N} = \dfrac{S_1}{N} + 1.10,$ so

$$\frac{N \ln M_2}{N} = \frac{N \ln M_1}{N} + 1.10$$
$$\ln M_2 - \ln M_1 = 1.10$$
$$\ln(M_2/M_1) = 1.10$$

Combining these we have

$$N(1.10) = \ln 6633$$
$$N = \frac{\ln 6633}{1.10} = 8.00$$

19.50. At equilibrium, the number density of the particles will be the same on both sides of the partition. Because the right compartment has six times as many particles as the left compartment, it will have six times the volume at equilibrium, so

$$V = V_{\text{L,f}} + V_{\text{R,f}} = V_{\text{L,f}} + 6V_{\text{L,f}} = 7V_{\text{L,f}}$$
$$V_{\text{L,f}} = V/7$$
$$V_{\text{R,f}} = 6V/7$$

Initially, we have $V_{\text{L,i}} = 3V/4$ and $V_{\text{R,i}} = V/4$. The change in the entropy of the system equals the sum of the changes of the entropy of each compartment. From Eq. 19.8 we have

$$\begin{aligned}
\Delta S &= \Delta S_{\text{L}} + \Delta S_{\text{R}} \\
&= N_{\text{L}} \ln\left(\frac{V_{\text{L,f}}}{V_{\text{L,i}}}\right) + N_{\text{R}} \ln\left(\frac{V_{\text{R,f}}}{V_{\text{R,i}}}\right) \\
&= N_{\text{L}} \ln\left(\frac{V/7}{3V/4}\right) + N_{\text{R}} \ln\left(\frac{6V/7}{V/4}\right) \\
&= N_{\text{L}} \ln(4/21) + N_{\text{R}} \ln(24/7) \\
&= (10)(-1.658) + (60)(1.232) = 57.3
\end{aligned}$$

19.51. At equilibrium we have equipartition of the energy over all the gas molecules. From Eq. 19.38 we have $\dfrac{1}{k_{\text{B}}T} = \dfrac{dS}{dE_{\text{th}}},$ and from Eq. 19.41 we have $\dfrac{dS}{dE_{\text{th}}} = \frac{3}{2}\dfrac{N}{E_{\text{th}}}.$ Combining these we have $\dfrac{1}{k_{\text{B}}T} = \frac{3}{2}\dfrac{N}{E_{\text{th}}},$ or $E_{\text{th}} = \frac{3}{2}Nk_{\text{B}}T.$ So, the first gas had thermal energy $E_{1,\text{th}} = \frac{3}{2}N_1 k_{\text{B}} T_1,$ and the second gas added its thermal energy $E_{2,\text{th}} = \frac{3}{2}N_2 k_{\text{B}} T_2,$ for a total $E_{\text{th}} = \frac{3}{2}k_{\text{B}}(N_1T_1 + N_2T_2).$ Substituting into the last equation above and solving for the absolute temperature we find

$$\begin{aligned}
\tfrac{3}{2}Nk_{\text{B}}T &= \tfrac{3}{2}k_{\text{B}}(N_1T_1 + N_2T_2) \\
T &= \frac{N_1T_1 + N_2T_2}{N} \\
&= \frac{N_1T_1 + N_2T_2}{N_1 + N_2} \\
&= \frac{(1.0\text{ mol})(72\text{ K}) + (0.50\text{ mol})(126\text{ K})}{(1.0\text{ mol}) + (0.50\text{ mol})} = 90\text{ K}
\end{aligned}$$

19.52. Eq. 19.33 relates the change in entropy of a system to the change in its thermal energy:

$$\Delta S = \tfrac{3}{2} N \ln\left(\frac{E_{\text{th,f}}}{E_{\text{th,i}}}\right)$$

$$= \tfrac{3}{2}(6.02\times 10^{23})\ln\left(\frac{(75.0\text{ J})+(25.0\text{ J})}{75.0\text{ J}}\right) = 2.60\times 10^{23}$$

19.53. (*a*) Their average speed equals the sum of their speeds divided by their number.

$$v_{\text{av}} = \tfrac{1}{8}\left[(6.2\text{ m/s})+(7.4\text{ m/s})+(7.4\text{ m/s})+(7.8\text{ m/s})+(8.3\text{ m/s})+(12.6\text{ m/s})+(20.1\text{ m/s})+(20.1\text{ m/s})\right]$$
$$= 11.2\text{ m/s}$$

(*b*) Their root-mean-square speed equals the square root of the mean of the squares of their speeds:

$$v_{\text{rms}} = \sqrt{\tfrac{1}{8}\left[(6.2\text{ m/s})^2+(7.4\text{ m/s})^2+(7.4\text{ m/s})^2+(7.8\text{ m/s})^2+(8.3\text{ m/s})^2+(12.6\text{ m/s})^2+(20.1\text{ m/s})^2+(20.1\text{ m/s})^2\right]}$$
$$= 12.5\text{ m/s}$$

19.54. Their root-mean-square speed equals the square root of the mean of the squares of their speeds, and the squares of their speeds equals the sum of the squares of the Cartesian coordinates of their speeds:

$$v_{\text{rms}} = (\tfrac{1}{4}[(4.0\text{ m/s})^2+(6.0\text{ m/s})^2+(2.0\text{ m/s})^2+(8.0\text{ m/s})^2+(-3.0\text{ m/s})^2+(8.0\text{ m/s})^2+$$
$$(7.0\text{ m/s})^2+(1.0\text{ m/s})^2+(6.0\text{ m/s})^2+(8.0\text{ m/s})^2+(9.0\text{ m/s})^2+(5.0\text{ m/s})^2])^{1/2}$$
$$= 11\text{ m/s}$$

19.55. In the solution to Problem 19.51 we found a relationship between thermal energy and absolute temperature, $E_{\text{th}} = \tfrac{3}{2}Nk_{\text{B}}T.$ We also have a relationship between thermal energy and root-mean-square speed in Eq. 19.28, $v_{\text{rms}} = \left(\dfrac{2E_{\text{th}}}{mN}\right)^{1/2}$. So, in order for the root-mean-square speed to triple, the thermal energy must increase by a factor of nine, which means that the absolute temperature must increase by a factor of nine, too. The absolute temperature in kelvins equals 273.15 plus the temperature in degrees Celsius, so the new absolute temperature is $T_{\text{new}} = 9(125+273.15)\text{ K} = 3.58\times 10^3\text{ K},$ which corresponds to $3.31\times 10^3\ ^\circ\text{C}.$

19.56. The rate of change of entropy with respect to thermal energy is inversely proportional to absolute temperature, $\dfrac{dS}{dE_{\text{th}}} = \dfrac{1}{k_{\text{B}}T},$ so if the rate of change doubles, the absolute temperature decreases to half its value, that is, to $200\text{ K}.$

19.57. By definition, $\dfrac{dS}{dE_{\text{th}}} = \dfrac{1}{k_{\text{B}}T},$ so

$$\frac{1}{k_{\text{B}}T} = \frac{d}{dE_{\text{th}}}(\tfrac{7}{2}N\ln E_{\text{th}} + NE_{\text{th}}^{2/15})$$
$$= \tfrac{7}{2}\frac{N}{E_{\text{th}}} + \tfrac{2}{15}NE_{\text{th}}^{(2/15)-1} = \frac{1}{E_{\text{th}}}(\tfrac{7}{2}N + \tfrac{2}{15}NE_{\text{th}}^{2/15})$$

19.58. (*a*) The average velocity of the particles equals the vector sum of their velocities divided by their number,

$$\vec{v}_{\text{av}} = \left(\frac{-6.0+4.0+7.0-4.0}{4}, \frac{5.0+5.0+0-9.0}{4}, \frac{1.0-2.0+8.0-6.0}{4}\right)\text{ m/s}$$
$$= (0.25, 0.25, 0.25)\text{ m/s}$$

(*b*) Their root-mean-square speed equals the square root of the mean of the squares of their speeds, and the squares of their speeds equals the sum of the squares of the Cartesian coordinates of their speeds,

$$v_{\text{rms}} = \left(\tfrac{1}{4}[(-6.0\text{ m/s})^2 + (4.0\text{ m/s})^2 + (7.0\text{ m/s})^2 + (-4.0\text{ m/s})^2 + (5.0\text{ m/s})^2 + (5.0\text{ m/s})^2 + (0\text{ m/s})^2 + (9.0\text{ m/s})^2 + (1.0\text{ m/s})^2 + (-2.0\text{ m/s})^2 + (8.0\text{ m/s})^2 + (-6.0\text{ m/s})^2]\right)^{1/2} = 9.4\text{ m/s}$$

(*c*) It is unlikely to be correct because the number of particles is so small.

19.59. After the system sits for a long time, the gas in the four quadrants will have come to thermal equilibrium, that is, it will all have the same absolute temperature. Because quadrants 1 and 2 can exchange gas particles, the number density of the particles in these two quadrants will be equal, too, but this does not affect the temperature or the root-mean-square speed.

We have a relationship between thermal energy and root-mean-square speed in Eq. 19.27, $\tfrac{1}{2}mv_{\text{rms}}^2 = \dfrac{E_{\text{th}}}{N}$, or $E_{\text{th}} = N\tfrac{1}{2}mv_{\text{rms}}^2$. We don't know the values of N or m, but their product equals the mass of the gas in each quadrant, $N_i m = m_i$. The thermal energy in the box is then

$$\begin{aligned}E_{\text{th}} &= E_{1\text{th}} + E_{2\text{th}} + E_{3\text{th}} + E_{4\text{th}}\\ &= N_1\tfrac{1}{2}mv_{1\text{rms}}^2 + N_2\tfrac{1}{2}mv_{2\text{rms}}^2 + N_3\tfrac{1}{2}mv_{3\text{rms}}^2 + N_4\tfrac{1}{2}mv_{4\text{rms}}^2\\ &= \tfrac{1}{2}(m_1v_{1\text{rms}}^2 + m_2v_{2\text{rms}}^2 + m_3v_{3\text{rms}}^2 + m_4v_{4\text{rms}}^2)\end{aligned}$$

and at equilibrium it is equipartioned among all the gas particles. In that case, the average kinetic energy of a gas particle is $K_{\text{av}} = E_{\text{th}}/N$, so

$$\begin{aligned}\tfrac{1}{2}mv_{\text{rms}}^2 &= \frac{E_{\text{th}}}{N}\\ v_{\text{rms}} &= \left(\frac{2E_{\text{th}}}{mN}\right)^{1/2}\\ &= \left(\frac{m_1v_{1\text{rms}}^2 + m_2v_{2\text{rms}}^2 + m_3v_{3\text{rms}}^2 + m_4v_{4\text{rms}}^2}{m_1 + m_2 + m_3 + m_4}\right)^{1/2}\\ &= \left(\frac{(3.00\text{ g})(400\text{ m/s})^2 + (5.50\text{ g})(500\text{ m/s})^2 + (2.00\text{ g})(420\text{ m/s})^2 + (6.75\text{ g})(445\text{ m/s})^2}{(3.00\text{ g}) + (5.50\text{ g}) + (2.00\text{ g}) + (6.75\text{ g})}\right)^{1/2}\\ &= 453\text{ m/s}\end{aligned}$$

19.60. We have in Eq. 19.53 a relationship between root-mean-square speed and absolute temperature. A temperature of $24.0\ ^\circ\text{C}$ corresponds to an absolute temperature of $(24.0 + 273.15)\text{ K} = 297.2\text{ K}$. So the root-mean-square speed is

$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3k_{\text{B}}T}{m}}\\ &= \sqrt{\frac{3(1.381\times10^{-23}\text{ J/K})(297.2\text{ K})}{6.646\times10^{-27}\text{ kg}}} = 1.36\times10^{3}\text{ m/s}\end{aligned}$$

19.61. We have in Eq. 19.53 a relationship between root-mean-square speed and absolute temperature, so

$$\begin{aligned}v_{\text{rms}} &= \sqrt{\frac{3k_{\text{B}}T}{m}}\\ &= \sqrt{\frac{3(1.381\times10^{-23}\text{ J/K})(1.8\times10^{4}\text{ K})}{6.646\times10^{-27}\text{ kg}}} = 1.1\times10^{4}\text{ m/s}\end{aligned}$$

19.62. (*a*) Eq.19.48 gives us a relationship between pressure and thermal energy, $P = \tfrac{2}{3}\dfrac{E_{\text{th}}}{V}$, or $E_{\text{th}} = \tfrac{3}{2}PV$. So, the thermal energy of the gas in the box is $E_{\text{th}} = \tfrac{3}{2}PV = \tfrac{3}{2}(2.0\text{ atm})(1.01\times10^{5}\text{ Pa/atm})(1.25\text{ m})^3 = 5.9\times10^{5}\text{ J}$. (*b*) If half of this energy were converted to the kinetic energy of the cat, its speed would be

$$\begin{aligned} K &= \tfrac{1}{2}mv^2 \\ v &= \sqrt{\frac{2K}{m}} \\ &= \sqrt{\frac{E_{\text{th}}}{m}} \\ &= \sqrt{\frac{5.9\times10^5\ \text{J}}{3.0\ \text{kg}}} = 4.4\times10^2\ \text{m/s} \end{aligned}$$

19.63. (*a*) The average kinetic energy of a gas particle equals the thermal energy of the gas divided by the number of gas particles. We have a relationship between pressure and thermal energy in Eq. 19.48. Combining these we have $P = \tfrac{2}{3}\dfrac{E_{\text{th}}}{V}$, or $E_{\text{th}} = \tfrac{3}{2}PV$.

$$\begin{aligned} K_{\text{av}} &= E_{\text{th}}/N \\ &= \tfrac{3}{2}PV/N \\ &= \tfrac{3}{2}\frac{(1.01\times10^5\ \text{Pa})(2.00\ \text{L})}{(0.0100\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})}\frac{1\ \text{m}^3}{1000\ \text{L}} = 5.03\times10^{-20}\ \text{J} \end{aligned}$$

(*b*) The kinetic energy of the bacterium is $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.0\times10^{-14}\ \text{kg})(1.0\times10^{-7}\ \text{m/s})^2 = 5.0\times10^{-29}\ \text{J}$, or 1.0×10^9 times smaller than the average kinetic energy of a gas particle. The kinetic energy of the slug is $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.0010\ \text{kg})(0.0100\ \text{m/s})^2 = 5.0\times10^{-8}\ \text{J}$, or 1.0×10^{12} times larger than the average kinetic energy of a gas particle.

19.64. After the accelerator has been running for a long time, the average kinetic energy of a gas particle will equal the kinetic energy of the accelerated particles. We can calculate the kinetic energy of the accelerated particles from kinematics. For constant acceleration from rest we have

$$\begin{aligned} v_{\text{av}} &= \frac{\Delta x}{\Delta t} = \tfrac{1}{2}v_{\text{f}} \\ v_{\text{f}} &= \frac{2\Delta x}{\Delta t} \\ K_{\text{f}} &= \tfrac{1}{2}mv_{\text{f}}^2 \\ &= \tfrac{1}{2}m\left(\frac{2\Delta x}{\Delta t}\right)^2 \\ &= 2m\left(\frac{\Delta x}{\Delta t}\right)^2 \end{aligned}$$

Combining Eqs. 19.27 and 19.50 we have

$$\begin{aligned} K_{\text{av}} &= \frac{E_{\text{th}}}{N} \\ E_{\text{th}} &= \tfrac{3}{2}Nk_{\text{B}}T \\ T &= \frac{2K_{\text{av}}}{3k_{\text{B}}} \\ &= \frac{2(2m)}{3k_{\text{B}}}\left(\frac{\Delta x}{\Delta t}\right)^2 \\ &= \frac{4(1.6726\times10^{-27}\ \text{kg})}{3(1.381\times10^{-23}\ \text{J/K})}\left(\frac{100\ \text{m}}{0.0179\ \text{s}}\right)^2 = 5.04\times10^3\ \text{K} \end{aligned}$$

19.65. After the system has come to equilibrium, the average kinetic energies of gas A particles and gas B particles will be equal. So, we have

$$\begin{aligned}\tfrac{1}{2}m_{\text{A}}v_{\text{A,rms}}^2 &= \tfrac{1}{2}m_{\text{B}}v_{\text{B,rms}}^2\\ m_{\text{A}} &= \frac{m_{\text{B}}v_{\text{B,rms}}^2}{v_{\text{A,rms}}^2}\\ &= \frac{m_{\text{B}}v_{\text{B,rms}}^2}{(6v_{\text{B,rms}})^2}\\ &= \tfrac{1}{36}m_{\text{B}}\end{aligned}$$

19.66. (*a*) We have from the ideal gas law

$$\begin{aligned}P &= \frac{Nk_{\text{B}}T}{V}\\ N &= \frac{PV}{k_{\text{B}}T}\\ &= \frac{(2.50\ \text{atm})(3.00\ \text{L})}{(1.381\times10^{-23}\ \text{J/K})\left[(50.0+273.15)\ \text{K}\right]}\frac{1\ \text{m}^3}{1000\ \text{L}}\frac{1.01\times10^5\ \text{Pa}}{1\ \text{atm}} = 1.70\times10^{23}\end{aligned}$$

(*b*) We doubled the thermal energy in the gas, so the amount of energy we added equals the amount that was initially there, which we can determine using Eq. 19.48,

$$\begin{aligned}P &= \tfrac{2}{3}\frac{E_{\text{th}}}{V}\\ E_{\text{th}} &= \tfrac{3}{2}PV\\ &= \tfrac{3}{2}(2.50\ \text{atm})(3.00\ \text{L})\frac{1\ \text{m}^3}{1000\ \text{L}}\frac{1.01\times10^5\ \text{Pa}}{1\ \text{atm}} = 1.14\times10^3\ \text{J}\end{aligned}$$

(*c*) The thermal energy of the atoms is proportional to the square of their root-mean-square speed, so doubling the thermal energy increased their root-mean-square speed by a factor of $\sqrt{2}$.

19.67. The time rate of change of momentum equals the force exerted on the atoms by the wall, the magnitude of which equals the product of the pressure in the gas and the area of the wall, $\Delta p/\Delta t = PA.$ The average kinetic energy of the atoms is related to the pressure by Eqs. 19.48 and 19.27,

$$\begin{aligned}P &= \tfrac{2}{3}\frac{E_{\text{th}}}{V} = \tfrac{2}{3}\frac{NK_{\text{av}}}{V}\\ K_{\text{av}} &= \tfrac{3}{2}\frac{PV}{N}\end{aligned}$$

Combining this with our expression for the time rate of change of momentum we have

$$\begin{aligned}K_{\text{av}} &= \tfrac{3}{2}\frac{\Delta p}{A\Delta t}\frac{V}{N}\\ \Delta p &= \tfrac{2}{3}\frac{K_{\text{av}}NA\Delta t}{V}\\ &= \tfrac{2}{3}K_{\text{av}}nA\Delta t\\ &= \tfrac{2}{3}(7.50\times10^{-21}\ \text{J})(3.61\times10^{24}\ \text{m}^{-3})(1.00\times10^{-2}\ \text{m}^2)(1\ \text{s}) = 181\ \text{kg}\cdot\text{m/s}\end{aligned}$$

19.68. Because chamber Y contains twice the number of gas particles as chamber X but the mass is the same, the mass of each particle of the gas in chamber Y must be one-half that of the particles of the gas in chamber X. Eq. 19.53 relates the root-mean-square speed of the particles to the absolute temperature of the gas,

$$\begin{aligned}
v_{\mathrm{Y,rms}} &= \sqrt{\frac{3k_{\mathrm{B}}T_{\mathrm{Y}}}{m_{\mathrm{Y}}}} \\
&= \sqrt{\frac{3k_{\mathrm{B}}(1.2T_{\mathrm{X}})}{\frac{1}{2}m_{\mathrm{X}}}} \\
&= \sqrt{2.4}\sqrt{\frac{3k_{\mathrm{B}}T_{\mathrm{X}}}{m_{\mathrm{X}}}} \\
&= \sqrt{2.4}v_{\mathrm{X,rms}} \\
&= \sqrt{2.4}(42\ \mathrm{m/s}) = 65\ \mathrm{m/s}
\end{aligned}$$

19.69. The pressure due to each gas is proportional to the number density of that gas. We can see this by examining Eq. 19.47, $P = \frac{2}{3}\frac{N(\frac{1}{2}mv_{\mathrm{rms}}^2)}{V}$. The term in parentheses is the average kinetic energy of a gas atom, which at equilibrium will be the same for both gasses. The number of gas atoms equals the mass of the gas divided by the mass of an atom, so because the jar contains the same mass of each gas, the ratio of the number densities is the inverse of the ratio of the atoms' masses. So, the ratio of the pressures due to each gas is $\frac{P_{\mathrm{He}}}{P_{\mathrm{Kr}}} = \frac{m_{\mathrm{Kr}}}{m_{\mathrm{He}}} = \frac{1.391\times10^{-25}\ \mathrm{kg}}{6.646\times10^{-27}\ \mathrm{kg}} = 20.9.$ The sum of the pressures equals 100% of the pressure in the jar, so

$$\begin{aligned}
P_{\mathrm{He}} + P_{\mathrm{Kr}} &= 100\%P \\
P_{\mathrm{He}} &= 20.9P_{\mathrm{Kr}} \\
21.9P_{\mathrm{Kr}} &= 100\%P \\
P_{\mathrm{Kr}} &= 4.56\%P \\
P_{\mathrm{He}} &= 95.4\%P
\end{aligned}$$

19.70. (*a*) Immediately after coming through the fan, the temperature is still $260\ \mathrm{K}$. Temperature is related to incoherent thermal energy, the average kinetic energy of randomly moving atoms, but the fan has given coherent kinetic energy to the neon on a macroscopic scale so that, in addition to each atom's random motion, they all participate in a collective motion, namely, the breeze. This coherent kinetic energy could, for example, be transferred to a macroscopic object by means of a sail, which could not be done with incoherent thermal energy. Eventually, as the moving neon interacts with its surroundings, its coherent kinetic energy will be dissipated into thermal energy as the gas becomes still, increasing its temperature. (*b*) In still air, the air adjacent to a person approaches thermal equilibrium with the person, after which it is unable to cool the person any more. The breeze from the fan moves the warmed air away from the person, replacing it with fresh air, which is better able to cool the person. Something similar happens with perspiration. As we will learn in the next chapter, when sweat evaporates, it removes thermal energy from a person. In still air, the air adjacent to the person becomes saturated with water vapor, so no more sweat can evaporate. The breeze from the fan moves the moist air away from the person, replacing it with drier air, so more sweat can evaporate.

19.71. (*a*) A temperature of $936\ ^\circ\mathrm{C}$ corresponds to an absolute temperature of $(936 + 273.15)\ \mathrm{K} = 1209\ \mathrm{K}.$ Given the mass density and diameter of the particles, we can calculate their mass: $m = \rho V = \rho(\frac{4}{3}\pi R^3) = \frac{4}{3}\pi(5600\ \mathrm{kg/m^3})(5.0\times10^{-9}\ \mathrm{m})^3 = 2.93\times10^{-21}\ \mathrm{kg}.$ So from Eq. 19.53, the particle's root-mean-square speed is

$$v_{\mathrm{rms}} = \sqrt{\frac{3k_{\mathrm{B}}T}{m}} = \sqrt{\frac{3(1.381\times10^{-23}\ \mathrm{J/K})(1209\ \mathrm{K})}{2.93\times10^{-21}\ \mathrm{kg}}} = 4.1\ \mathrm{m/s}$$

(*b*) A temperature of $190\ ^\circ\mathrm{C}$ corresponds to an absolute temperature of $(190 + 273.15)\ \mathrm{K} = 463\ \mathrm{K},$ and the mass of the larger particles is $m = \frac{4}{3}\pi(5600\ \mathrm{kg/m^3})(0.50\times10^{-6}\ \mathrm{m})^3 = 2.93\times10^{-15}\ \mathrm{kg},$ so their root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{3(1.381\times10^{-23}\ \text{J/K})(463\ \text{K})}{2.93\times10^{-15}\ \text{kg}}} = 2.6\times10^{-3}\ \text{m/s}$$

19.72. Radon atoms are much heavier than air molecules (order of magnitude 10^2), so at a given temperature, when both types of particle have the same average kinetic energy, air molecules move more quickly than radon atoms (order of magnitude 10). That is, at a given temperature, it takes a radon atom on the order of 10 times longer to traverse a given distance than an air molecule, so we might be able to use this difference to detect radon in the air. (*a*) The mass of a radon atom is 3.69×10^{-25} kg, so at 255 K radon gas has root-mean-square speed

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{3k_{\text{B}}T}{m}} \\ &= \sqrt{\frac{3(1.381\times10^{-23}\ \text{J/K})(255\ \text{K})}{3.69\times10^{-25}\ \text{kg}}} = 169\ \text{m/s} \end{aligned}$$

At that speed, the time it takes to traverse 50 mm is $\Delta t = \frac{\Delta x}{v} = \frac{50\times10^{-3}\ \text{m}}{169\ \text{m/s}} = 3.0\times10^{-4}\ \text{s}$. (*b*) If we had a longer tube, or the temperature of the gas were lower, the traversal time would be longer, but it is fairly easy to measure fractions of a millisecond with readily available electronic components, so this isn't really necessary. One source of error is different atoms having different path lengths. For example, a radon atom that travels straight through the tube might traverse it in a shorter amount of time than it takes for an air molecule to do so if the air molecule is traveling at a steep angle and bounces off the sides of the tube many times as it makes its way from one end of the tube to the other. We could eliminate this source of error by *collimating* the beam of atoms, that is, only accepting atoms that are moving parallel (or nearly parallel) to the axis of the tube. We could do this, for example, by making the atoms pass through a series of disks with small holes in their centers. (*c*) We might need to calibrate our system to account for collisions between radon atoms and air molecules. Considering only the particles that are moving down the tube, because the air molecules are moving faster, on average, than the radon atoms, a radon atom will most likely be struck by an air molecule that is overtaking it from behind, and such a collision will increase the speed of the radon atom and decrease that of the air molecule. We will also want to calibrate our system because we do not know the range of speeds the particles have. That is, we know that the root-mean-square speed of the air molecules is on the order of ten times that of the radon atoms, but not every particle moves at exactly the root-mean-square speed. It would be useful to know how the speeds of the fastest radon atoms compares to that of the slowest air molecules.

19.73. The change in entropy of a monatomic ideal gas as its temperature changes and its volume remains constant is given by Eq. 19.56:

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right)$$

So, the change in entropy per particle is

$$\frac{\Delta S}{N} = \tfrac{3}{2}\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right) = \tfrac{3}{2}\ln\left(\frac{400\ \text{K}}{300\ \text{K}}\right) = 0.432$$

19.74. The change in entropy of a monatomic ideal gas as its temperature changes and its volume remains constant is given by Eq. 19.56:

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right) = \tfrac{3}{2}(1.85\times10^5)\ln\left(\frac{500\ \text{K}}{300\ \text{K}}\right) = 1.42\times10^5$$

19.75. If we double the root-mean-square speed of the atoms, we increase their temperature by a factor of four (Eq. 19.52), so the change in entropy (Eq. 19.56) is

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right) = \tfrac{3}{2}(4.20\times10^{24})\ln(4) = 8.73\times10^{24}$$

19.76. A temperature of $25.0\ ^\circ\text{C}$ corresponds to an absolute temperature of $(25.0+273.15)\ \text{K}=298.2\ \text{K},$ and a temperature of $-135\ ^\circ\text{C}$ corresponds to an absolute temperature of $(-135+273.15)\ \text{K}=138\ \text{K}.$ (*a*) The change in the thermal energy of the gas can be calculated using Eq. 19.50:

$$\begin{aligned}\Delta E_{\text{th}} &= E_{\text{th,f}}-E_{\text{th,i}}\\ &=\tfrac{3}{2}Nk_{\text{B}}T_{\text{f}}-\tfrac{3}{2}Nk_{\text{B}}T_{\text{i}}\\ &=\tfrac{3}{2}Nk_{\text{B}}(T_{\text{f}}-T_{\text{i}})\\ &=\tfrac{3}{2}(8.72\times10^{23})(1.381\times10^{-23}\ \text{J/K})\big[(138\ \text{K})-(298.2\ \text{K})\big]=-2.89\times10^{3}\ \text{J}\end{aligned}$$

(*b*) The change in the entropy of the gas is given by Eq. 19.56,

$$\Delta S=\tfrac{3}{2}N\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right)=\tfrac{3}{2}(8.72\times10^{23})\ln\left(\frac{138\ \text{K}}{298.2\ \text{K}}\right)=-1.01\times10^{24}$$

19.77. A temperature of $30.0\ ^\circ\text{C}$ corresponds to an absolute temperature of $(30.0+273.15)\ \text{K}=303.2\ \text{K}.$ From the ideal gas law (Eq. 19.51), we can find the number of neon atoms in the chamber,

$$\begin{aligned}P&=\frac{N}{V}k_{\text{B}}T\\ N&=\frac{PV}{k_{\text{B}}T}\\ &=\frac{(2.00\ \text{atm})(3.50\ \text{L})}{(1.381\times10^{-23}\ \text{J/K})(303.2\ \text{K})}\frac{1\ \text{m}^3}{1000\ \text{L}}\frac{1.01\times10^{5}\ \text{Pa}}{1\ \text{atm}}=1.69\times10^{23}\end{aligned}$$

We can also find the change in the final absolute temperature of the gas from the ideal gas law,

$$\begin{aligned}\frac{N}{V_{\text{f}}}k_{\text{B}}T_{\text{f}}&=\frac{N}{V_{\text{i}}}k_{\text{B}}T_{\text{i}}\\ \frac{T_{\text{f}}}{T_{\text{i}}}&=\frac{V_{\text{f}}}{V_{\text{i}}}\end{aligned}$$

and the change in entropy from Eq. 19.61,

$$\begin{aligned}\Delta S&=\tfrac{3}{2}N\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right)+N\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)\\ &=\tfrac{3}{2}N\ln\left(\frac{8.00\ \text{L}}{3.50\ \text{L}}\right)+N\ln\left(\frac{8.00\ \text{L}}{3.50\ \text{L}}\right)\\ &=\tfrac{5}{2}(1.69\times10^{23})\ln\left(\frac{8.00\ \text{L}}{3.50\ \text{L}}\right)=3.50\times10^{23}\end{aligned}$$

19.78. If a monatomic ideal gas has its volume halved while its pressure is held constant, then its temperature must change in accordance with the ideal gas law (Eq. 19.51),

$$\begin{aligned}\frac{N}{V_{\text{f}}}k_{\text{B}}T_{\text{f}}&=\frac{N}{V_{\text{i}}}k_{\text{B}}T_{\text{i}}\\ \frac{T_{\text{f}}}{T_{\text{i}}}&=\frac{V_{\text{f}}}{V_{\text{i}}}\\ \frac{T_{\text{f}}}{T_{\text{i}}}&=\frac{\frac{1}{2}V_{\text{i}}}{V_{\text{i}}}\\ T_{\text{f}}&=\tfrac{1}{2}T_{\text{i}}\end{aligned}$$

So, the change in entropy per atom is given by Eq. 19.61,

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_f}{T_i}\right) + N\ln\left(\frac{V_f}{V_i}\right)$$

$$\begin{aligned}\frac{\Delta S}{N} &= \tfrac{3}{2}\ln\left(\frac{T_f}{T_i}\right) + \ln\left(\frac{V_f}{V_i}\right)\\ &= \tfrac{3}{2}\ln\left(\tfrac{1}{2}\right) + \ln\left(\tfrac{1}{2}\right)\\ &= \tfrac{5}{2}\ln\left(\tfrac{1}{2}\right) = -1.73\end{aligned}$$

19.79. We can use Eq. 19.52 to get the temperatures from the root-mean-square speeds, and then Eq.19.56 to find the change in entropy per atom:

$$\tfrac{1}{2}mv_{rms}^2 = \tfrac{3}{2}k_B T$$

$$T = \frac{mv_{rms}^2}{3k_B}$$

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_f}{T_i}\right)$$

$$\begin{aligned}\frac{\Delta S}{N} &= \tfrac{3}{2}\ln\left(\frac{T_f}{T_i}\right)\\ &= \tfrac{3}{2}\ln\left(\frac{v_{rms,f}^2}{v_{rms,i}^2}\right)\\ &= \tfrac{3}{2}\ln\left(\frac{(540\text{ m/s})^2}{(350\text{ m/s})^2}\right) = 1.30\end{aligned}$$

19.80. (*a*) To find the change in entropy we need to know both the change in volume and the change in temperature, and we can find the change in temperature from the ideal gas law. Because we know the number of atoms in the box does not change, we have

$$\frac{P_f V_f}{T_f} = \frac{P_i V_i}{T_i}$$

$$\frac{T_f}{T_i} = \frac{P_f V_f}{P_i V_i} = \frac{(3P_i)\left[\left(\tfrac{1}{2}\right)^3 V_i\right]}{P_i V_i} = \tfrac{3}{8}$$

and the change in entropy per particle is

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_f}{T_i}\right) + N\ln\left(\frac{V_f}{V_i}\right)$$

$$\begin{aligned}\frac{\Delta S}{N} &= \tfrac{3}{2}\ln\left(\frac{T_f}{T_i}\right) + \ln\left(\frac{V_f}{V_i}\right)\\ &= \tfrac{3}{2}\ln\left(\tfrac{3}{8}\right) + \ln\left(\tfrac{1}{8}\right) = -3.55\end{aligned}$$

(*b*) Because the entropy decreases, the system must not be closed.

19.81. If the gas is heated but its entropy remains constant, then it must be compressed into a smaller volume, so that the increase in entropy caused by the increase in temperature is compensated by the decrease in entropy caused by the decrease in volume. We can use Eq. 19.56 to calculate the final volume of the gas, and then the ideal gas law to calculate the change in pressure:

$$\Delta S = \tfrac{3}{2} N \ln\left(\frac{T_f}{T_i}\right) + N \ln\left(\frac{V_f}{V_i}\right) = 0$$

$$\ln\left(\frac{V_f}{V_i}\right) = -\tfrac{3}{2}\ln\left(\frac{T_f}{T_i}\right) = \ln\left[\left(\frac{T_f}{T_i}\right)^{-3/2}\right]$$

$$\frac{V_f}{V_i} = \left(\frac{T_f}{T_i}\right)^{-3/2}$$

$$V_f = V_i\left(\frac{T_f}{T_i}\right)^{-3/2}$$

$$P_f = \frac{Nk_B T_f}{V_f}$$

$$= \frac{Nk_B T_f}{V_i}\left(\frac{T_f}{T_i}\right)^{3/2}$$

$$= \frac{Nk_B T_i}{V_i}\left(\frac{T_f^{2/3}}{T_i^{2/3}}\frac{T_f}{T_i}\right)^{3/2}$$

$$= P_i\left(\frac{T_f^{5/3}}{T_i^{5/3}}\right)^{3/2}$$

$$= P_i\left(\frac{T_f}{T_i}\right)^{5/2}$$

$$\frac{P_f}{P_i} = \left(\frac{T_f}{T_i}\right)^{5/2}$$

$$= \left(\frac{458\text{ K}}{289\text{ K}}\right)^{5/2} = 3.16$$

19.82. [**NOTE:** The problem statement will be changed: (1) In the 3rd line, "This closed system" will read "This system" (2) In the 4th-5th lines "boils away" will read "evaporates"] (*a*) As the temperature of the ideal gas increases, its entropy increases according to Eq. 19.56,

$$\Delta S = \tfrac{3}{2} N \ln\left(\frac{T_f}{T_i}\right) = \tfrac{3}{2}(6.02\times10^{23})\ln\left(\frac{297\text{ K}}{77.2\text{ K}}\right) = 1.22\times10^{24}$$

(*b*) As the temperature of the ideal gas decreases, its entropy decreases again according to Eq. 19.56,

$$\Delta S = \tfrac{3}{2} N \ln\left(\frac{T_f}{T_i}\right) = \tfrac{3}{2}(6.02\times10^{23})\ln\left(\frac{77.2\text{ K}}{297\text{ K}}\right) = -1.22\times10^{24}$$

(*c*) The entropy of the ideal gas increased in the first part of the process, when thermal energy from the room-temperature environment flowed into the nitrogen and the ideal gas. This spontaneous flow of thermal energy from warmer to cooler regions is irreversible in accordance with the second law of thermodynamics. In the second part of the process, when the nitrogen gas was liquefied, some outside agent (such as a refrigerator) acted on the system to decrease its entropy.

19.83. The entropy change at constant volume is given by Eq. 19.56: $\Delta S = \tfrac{3}{2} N \ln\left(\frac{T_f}{T_i}\right)$. The entropy at any temperature is given by Eq. 19.55: $S = \tfrac{3}{2} N \ln T + C$. Because the entropy doubles, we know that $\Delta S = S_i$. There is no unique solution for the initial temperature because of the unknown constant *C*. We do not have enough information about the gas to compute this constant directly, though we can see from Eq. 19.54 that it involves two terms that will take opposite signs. In the absence of additional information, we assume that the constant C is zero. This allows us to compute an initial temperature:

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_\text{f}}{T_\text{i}}\right) = \tfrac{3}{2}N(\ln T_\text{f} - \ln T_\text{i}) = S_\text{i} = \tfrac{3}{2}N\ln T_\text{i}$$

$$\ln T_\text{f} = 2\ln T_\text{i}$$

$$T_\text{i} = e^{(\frac{1}{2}\ln T_\text{f})} = e^{(\frac{1}{2}\ln 634\ \text{K})} = 25.2\ \text{K}$$

19.84. A temperature of $85.0\ °\text{C}$ corresponds to an absolute temperature of $(85.0 + 273.15)\ \text{K} = 358.2\ \text{K}$. To determine the change in entropy, we need to know both the change in volume and the change in absolute temperature. We are given the change in volume and can use the ideal gas law to determine the change in absolute temperature:

$$\frac{P_\text{f}V_\text{f}}{T_\text{f}} = \frac{P_\text{i}V_\text{i}}{T_\text{i}}$$

$$T_\text{f} = \frac{P_\text{f}V_\text{f}}{P_\text{i}V_\text{i}}T_\text{i}$$

We also need to know the number of number of atoms in the gas, which we can also determine from the ideal gas law, $N = \dfrac{PV}{k_\text{B}T}$. So, we have

$$\begin{aligned}
\Delta S &= \tfrac{3}{2}N\ln\left(\frac{T_\text{f}}{T_\text{i}}\right) + N\ln\left(\frac{V_\text{f}}{V_\text{i}}\right) \\
&= \frac{P_\text{i}V_\text{i}}{k_\text{B}T_\text{i}}\left[\tfrac{3}{2}\ln\left(\frac{P_\text{f}V_\text{f}}{P_\text{i}V_\text{i}}\right) + \ln\left(\frac{V_\text{f}}{V_\text{i}}\right)\right] \\
&= \frac{(3.20\ \text{atm})(2.25\ \text{L})}{(1.381\times10^{-23}\ \text{J/K})(358.2\ \text{K})}\frac{1.01\times10^{5}\ \text{Pa}}{1\ \text{atm}}\frac{1\ \text{m}^3}{1000\ \text{L}}\left[\tfrac{3}{2}\ln\left(\frac{(4.40\ \text{atm})(1.10\ \text{L})}{(3.20\ \text{atm})(2.25\ \text{L})}\right) + \ln\left(\frac{1.10\ \text{L}}{2.25\ \text{L}}\right)\right] \\
&= 1.93\times10^{23}
\end{aligned}$$

19.85. (*a*) In the new equilibrium state, both gases are evenly distributed in both compartments. The entropy of the system equals the sum of the entropies of each gas, so

$$\begin{aligned}
\Delta S &= \Delta S_\text{He} + \Delta S_\text{Ar} \\
&= N_\text{He}\ln\left(\frac{V_\text{He,f}}{V_\text{He,i}}\right) + N_\text{Ar}\ln\left(\frac{V_\text{Ar,f}}{V_\text{Ar,i}}\right) \\
&= (9.00\times10^{23})\ln(2) + (2.70\times10^{24})\ln(2) = 2.50\times10^{24}
\end{aligned}$$

The final state has the greater entropy, because each gas is distributed over a greater volume. (*b*) Because the chamber is thermally insulated, there is no change in the thermal energy of the gases.

19.86. As we add thermal energy to the gas, its temperature increases and so does its entropy. To use Eq. 19.50, $E_\text{th} = \tfrac{3}{2}Nk_\text{B}T$, to find the change in temperature, we need to determine N, which we can find from the ideal gas law, Eq. 19.51,

$$P = \frac{N}{V}k_\text{B}T$$

$$N = \frac{P_\text{i}V}{k_\text{B}T_\text{i}}$$

The initial thermal energy of the gas is $E_{\text{th,i}} = \frac{3}{2}Nk_BT_i = \frac{3}{2}\frac{P_iV}{k_BT_i}k_BT_i = \frac{3}{2}P_iV$ and the final temperature is $T_f = \dfrac{E_{\text{th,f}}}{\frac{3}{2}Nk_B} = \dfrac{E_{\text{th,i}} + \Delta E_{\text{th}}}{\frac{3}{2}Nk_B} = \dfrac{\frac{3}{2}P_iV + \Delta E_{\text{th}}}{\frac{3}{2}Nk_B}$, so from Eq. 19.56 the change in entropy is

$$\begin{aligned}\Delta S &= \tfrac{3}{2}N\ln\left(\frac{T_f}{T_i}\right)\\ &= \tfrac{3}{2}N\ln\left(\frac{\frac{3}{2}P_iV + \Delta E_{\text{th}}}{\frac{3}{2}Nk_BT_i}\right)\\ &= \tfrac{3}{2}\frac{P_iV}{k_BT_i}\ln\left(\frac{\frac{3}{2}P_iV + \Delta E_{\text{th}}}{\frac{3}{2}P_iV}\right)\\ &= \tfrac{3}{2}\frac{P_iV}{k_BT_i}\ln\left(1 + \tfrac{2}{3}\frac{\Delta E_{\text{th}}}{P_iV}\right)\end{aligned}$$

A temperature of $150\ ^\circ\text{C}$ corresponds to an absolute temperature of $(150 + 273.15)\ \text{K} = 423\ \text{K}$, and converting to SI units we have $P_iV = (2.75\ \text{atm})(1.20\ \text{L})\dfrac{1.01\times10^5\ \text{Pa}}{1\ \text{atm}}\dfrac{1\ \text{m}^3}{1000\ \text{L}} = 333.3\ \text{J}$, so

$$\Delta S = \tfrac{3}{2}\frac{333.3\ \text{J}}{(1.381\times10^{-23}\ \text{J/K})(423\ \text{K})}\ln\left(1 + \tfrac{2}{3}\frac{755\ \text{J}}{333.3\ \text{J}}\right) = 7.87\times10^{22}$$

19.87. The first piece can land on any of the 64 squares, as can the second, and so on, for a total of $64^5 = 1{,}073{,}741{,}824$ possible basic states.

19.88. For the one-dozen carton, the first egg can be placed in any of 12 spaces, the second egg can be placed in any of the remaining 11 spaces, and so on, for a total of $12\times11\times10\times9\times8\times7 = 665{,}280$ possible basic states. For the half-dozen carton, the first egg can be placed in any of 6 spaces, the second egg can be placed in any of the remaining 5 spaces, and so on, for a total of $6\times5\times4\times3\times2\times1 = 720$ possible basic states, and the ratio is 924.

19.89. The change in entropy of the system equals the sum of the changes of entropy of the two gases, and during the process each gas is redistributed over twice its initial volume, so

$$\begin{aligned}\Delta S &= N_{\text{He}}\ln\left(\frac{V_f}{V_i}\right) + N_{\text{Ne}}\ln\left(\frac{V_f}{V_i}\right)\\ &= (1000 + 2000)\ln(2) = 2079\end{aligned}$$

19.90. (*a*) The average kinetic energy of an atom is proportional to the absolute temperature. A temperature of $100\ ^\circ\text{C}$ corresponds to an absolute temperature of $(100 + 273.15)\ \text{K} = 373\ \text{K}$, so $K_{\text{av}} = \frac{3}{2}k_BT = \frac{3}{2}(1.381\times10^{-23}\ \text{J/K})(373\ \text{K}) = 7.73\times10^{-21}\ \text{J}$. (*b*) The thermal energy of the gas is the sum of the kinetic energies of the atoms, so

$$\begin{aligned}E_{\text{th}} &= NK_{\text{av}}\\ N &= \frac{E_{\text{th}}}{K_{\text{av}}} = \frac{175\ \text{J}}{7.73\times10^{-21}\ \text{J}} = 2.26\times10^{22}\end{aligned}$$

19.91. (*a*) We have from Eq. 19.53 $v_{\text{rms}} = \sqrt{\dfrac{3k_BT}{m}}$. A temperature of $35\ ^\circ\text{C}$ corresponds to an absolute temperature of $(35 + 273.15)\ \text{K} = 308\ \text{K}$, so $m = \dfrac{3k_BT}{v_{\text{rms}}^2} = \dfrac{3(1.381\times10^{-23}\ \text{J/K})(308\ \text{K})}{(186\ \text{m/s})^2} = 3.69\times10^{-25}\ \text{kg}$. (*b*) The average kinetic energy is $K_{\text{av}} = \frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}k_BT = \frac{3}{2}(1.381\times10^{-23}\ \text{J/K})(308\ \text{K}) = 6.38\times10^{-21}\ \text{J}$.

19.92. [**NOTE:** Problem statement will be changed: "A 0.0423-kg sample" will read "A 0.00126-kg sample"] The mass of one gas particle equals the mass of the gas divided by the number of particles in the gas, $m = \frac{m_{\text{gas}}}{N}$. From the ideal gas law we have $N = \frac{PV}{k_{\text{B}}T}$. A temperature of $50.0\ ^\circ\text{C}$ corresponds to an absolute temperature of $(50.0 + 273.15)\ \text{K} = 323.2\ \text{K}$, so we have

$$m = m_{\text{gas}} \frac{k_{\text{B}}T}{PV} = (0.00126\ \text{kg}) \frac{(1.381 \times 10^{-23}\ \text{J/K})(323.2\ \text{K})}{(2.24\ \text{atm})(0.750\ \text{L})} \frac{1\ \text{atm}}{1.01 \times 10^5\ \text{Pa}} \frac{1000\ \text{L}}{1\ \text{m}^3} = 3.31 \times 10^{-26}\ \text{kg}$$

19.93. The root-mean-square speed of the atoms is inversely proportional to the square root of their masses, so the less massive isotope has the greater root-mean-square speed and

$$\frac{v_{235}}{v_{238}} = \sqrt{\frac{m_{238}}{m_{235}}}$$

$$v_{235} = v_{238}\sqrt{\frac{m_{238}}{m_{235}}}$$

$$v_{235} - v_{238} = v_{238}\left(\sqrt{\frac{m_{238}}{m_{235}}} - 1\right)$$

$$\frac{v_{235} - v_{238}}{v_{238}} = \sqrt{\frac{m_{238}}{m_{235}}} - 1$$

$$= \sqrt{\frac{3.95}{3.90}} - 1 = 0.00639$$

or 0.639%.

19.94. (*a*) Thermal energy is related to absolute temperature by Eq. 19.50, $E_{\text{th}} = \frac{3}{2}Nk_{\text{B}}T$. A temperature of $20\ ^\circ\text{C}$ corresponds to an absolute temperature of $(20 + 273.15)\ \text{K} = 293\ \text{K}$, and a temperature of $232\ ^\circ\text{C}$ corresponds to an absolute temperature of $(232 + 273.15)\ \text{K} = 505\ \text{K}$. We can find the number of helium atoms in the chamber from the ideal gas law, $N = \frac{PV}{k_{\text{B}}T}$. So, the thermal energy that was added is

$$\Delta E_{\text{th}} = \tfrac{3}{2}Nk_{\text{B}}\Delta T = \tfrac{3}{2}\frac{P_{\text{i}}V}{k_{\text{B}}T_{\text{i}}}k_{\text{B}}(T_{\text{f}} - T_{\text{i}})$$

$$= \tfrac{3}{2}\left(1.00\ \text{atm}\frac{1.01 \times 10^5\ \text{Pa}}{1\ \text{atm}}\right)\left(1.50\ \text{L}\frac{1\ \text{m}^3}{1000\ \text{L}}\right)\frac{(505\ \text{K}) - (293\ \text{K})}{293\ \text{K}} = 164\ \text{J}$$

(*b*) The change in entropy is

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right) = \tfrac{3}{2}\frac{P_{\text{i}}V}{k_{\text{B}}T_{\text{i}}}\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right)$$

$$= \tfrac{3}{2}\frac{(1.00\ \text{atm})(1.50\ \text{L})}{(1.381 \times 10^{-23}\ \text{J/K})(293\ \text{K})}\ln\left(\frac{505\ \text{K}}{293\ \text{K}}\right)\frac{1.01 \times 10^5\ \text{Pa}}{1\ \text{atm}}\frac{1\ \text{m}^3}{1000\ \text{L}} = 3.06 \times 10^{22}$$

(*c*) The final pressure is given by the ideal gas law,

$$P_{\text{f}} = \frac{Nk_{\text{B}}T_{\text{f}}}{V} = \frac{P_{\text{i}}V}{k_{\text{B}}T_{\text{i}}}\frac{k_{\text{B}}T_{\text{f}}}{V}$$

$$= \frac{P_{\text{i}}T_{\text{f}}}{T_{\text{i}}} = (1.00\ \text{atm})\frac{505\ \text{K}}{293\ \text{K}} = (1.72\ \text{atm})\frac{1.01 \times 10^5\ \text{Pa}}{1\ \text{atm}} = 1.74 \times 10^5\ \text{Pa}$$

19.95. (*a*) The absolute temperature of an ideal gas is proportional to the average kinetic energy of the atoms in the gas. From Eq. 19.52 we have

$$\tfrac{1}{2}mv_{\rm rms}^2 = \tfrac{3}{2}k_{\rm B}T$$

$$T = \frac{mv_{\rm rms}^2}{3k_{\rm B}} = \frac{(3.35\times10^{-26}\ \text{kg})(344\ \text{m/s})^2}{3(1.381\times10^{-23}\ \text{J/K})} = 95.7\ \text{K}$$

(*b*) The thermal energy of the gas equals the average kinetic energy of the atoms times the number of atoms in the gas,

$$E_{\rm th} = \tfrac{1}{2}mv_{\rm rms}^2 N = \tfrac{1}{2}(3.35\times10^{-26}\ \text{kg})(344\ \text{m/s})^2(6.02\times10^{23}) = 1.19\times10^3\ \text{J}$$

19.96. (*a*) Increasing the root-mean-square speed of the gas particles increases the thermal energy of the gas, increasing its temperature. To double the pressure, we must double the absolute temperature. Because the absolute temperature is proportional to the square of the root-mean-square speed, we must increase the root-mean-square speed by a factor of $\sqrt{2}$. (*b*) Doubling the absolute temperature doubles the thermal energy of the gas.

19.97. (*a*) The change in entropy is given by Eq. 19.61,

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_{\rm f}}{T_{\rm i}}\right) + N\ln\left(\frac{V_{\rm f}}{V_{\rm i}}\right) = 3N$$

$$\ln\left(\frac{T_{\rm f}}{T_{\rm i}}\right) = \tfrac{2}{3}\left[3 - \ln\left(\frac{V_{\rm f}}{V_{\rm i}}\right)\right]$$

$$T_{\rm f} = T_{\rm i}e^{\frac{2}{3}\left[3+\ln(4)\right]} = 18.6T_{\rm i}$$

(*b*) The root-mean-square speed of the gas particles is proportional to the square root of the absolute temperature, so it increases by a factor of $\sqrt{e^{\frac{2}{3}\left[3+\ln(4)\right]}} = 4.32$.

19.98. The root-mean-square speed of the helium is proportional the square root of the absolute temperature, as given by Eq. 19.53

$$v_{\rm rms} = \sqrt{\frac{3k_{\rm B}T}{m}} = \sqrt{\frac{3(1.381\times10^{-23}\ \text{J/K})(1.0\times10^6\ \text{K})}{6.646\times10^{-27}\ \text{kg}}} = 7.9\times10^4\ \text{m/s}$$

The atoms can escape the Sun's gravitational pull if their kinetic energy is greater than or equal to the potential energy due to their gravitational interaction with the Sun (see *Principles* Section 13.7), that is, when

$$\tfrac{1}{2}mv^2 \geq \frac{Gmm_{\rm Sun}}{r}$$

$$v \geq \sqrt{\frac{2Gm_{\rm Sun}}{r}} \geq \sqrt{\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{6.96\times10^8\ \text{m}}} = 6.18\times10^5\ \text{m/s}$$

So, no, the typical helium atom in the Sun's corona cannot escape the Sun's gravitational pull.

19.99. The altitude to which it can rise is that at which its initial kinetic energy is entirely converted into potential energy due to its gravitational interaction with Earth. That is, when

$$\tfrac{1}{2}mv^2 = Gmm_{\rm E}\left(\frac{1}{r_{\rm i}} - \frac{1}{r_{\rm f}}\right)$$

$$\frac{v^2}{2Gm_{\rm E}} = \left(\frac{1}{r_{\rm i}} - \frac{1}{r_{\rm f}}\right)$$

$$r_{\rm f} = \left(\frac{1}{r_{\rm i}} - \frac{v^2}{2Gm_{\rm E}}\right)^{-1}$$

A temperature of $20\ ^\circ\text{C}$ corresponds to an absolute temperature of $(20 + 273.15)\ \text{K} = 293\ \text{K},$ so the root-mean-square speed of the helium is $v_{\text{rms}} = \sqrt{\dfrac{3k_{\text{B}}T}{m}},$ and we have

$$r_{\text{f}} = \left(\frac{1}{r_{\text{i}}} - \frac{3k_{\text{B}}T}{2Gmm_{\text{E}}}\right)^{-1}$$

$$= \left(\frac{1}{6.38\times 10^{6}\ \text{m}} - \frac{3(1.381\times 10^{-23}\ \text{J/K})(293\ \text{K})}{2(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.646\times 10^{-27}\ \text{kg})(5.97\times 10^{24}\ \text{kg})}\right)^{-1} = 6.47\times 10^{6}\ \text{m}$$

So, the altitude is $h = r_{\text{f}} - R_{\text{E}} = (6.47\times 10^{6}\ \text{m}) - (6.38\times 10^{6}\ \text{m}) = 9\times 10^{4}\ \text{m}.$

19.100. (*a*) If the particles are indistinguishable, there are three basic states (both particles on the left, both on the right, and one on each side), each of which has equal probability, so the probability of both particles being on the left is $1/3 = 0.333,$ and this happens on average every $3(2.50\ \text{s}) = 7.50\ \text{s}.$ If the particles are distinguishable, there are four basic states, so the probability of both particles being on the left is $1/4 = 0.25,$ and this happens on average every $4(2.50\ \text{s}) = 10.0\ \text{s}.$ (*b*) If the particles are indistinguishable, there are 11 basic states (zero particles on the left and 10 on the right, one on the left and 9 on the right, etc.), each of which has equal probability, so the probability of all 10 particles being on the left is $1/11 = 0.0909,$ and this happens on average every $11(2.50\ \text{s}) = 27.5\ \text{s}.$ If the particles are distinguishable, there are $2^{10} = 1024$ basic states, so the probability of both particles being on the left is $1/1024 = 9.77\times 10^{-4},$ and this happens on average every $1024(2.50\ \text{s}) = 2.56\times 10^{3}\ \text{s}.$

19.101. The piston is either at rest or moving with constant velocity, so the vector sum of the forces exerted on it is zero. The force exerted on the piston by the gas is proportional to the pressure of the gas, and there is no reason to assume the other forces exerted on the piston change, so the pressure of the gas must be constant throughout the process. So from the ideal gas law, we have $\dfrac{Nk_{\text{B}}T_{\text{f}}}{V_{\text{f}}} = \dfrac{Nk_{\text{B}}T_{\text{i}}}{V_{\text{i}}},$ or $\dfrac{T_{\text{f}}}{T_{\text{i}}} = \dfrac{V_{\text{f}}}{V_{\text{i}}} = \dfrac{\frac{1}{5}V_{\text{i}}}{V_{\text{i}}} = \dfrac{1}{5}.$ (*a*) The root-mean-square speed of the gas particles is proportional to the square root of the absolute temperature, so because the absolute temperature decrease to one-fifth of its initial value, the root-mean-square speed of the gas particles decreases to $\sqrt{\frac{1}{5}} = 0.447$ of its initial value. (*b*) The thermal energy of the gas is proportional to its temperature, so it decreases to $1/5 = 0.2$ of its initial value.

19.102. At a depth of $12.0\ \text{m},$ the pressure in the water is (see *Principles* Section 18.5) $P = P_{\text{atm}} + \rho g d =$ $(101{,}325\ \text{Pa}) + (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(12.0\ \text{m}) = 2.19\times 10^{5}\ \text{Pa}.$ (*a*) From the ideal gas law we have $\dfrac{P_{\text{i}}V_{\text{i}}}{T_{\text{i}}} = \dfrac{P_{\text{f}}V_{\text{f}}}{T_{\text{f}}},$ or

$$V_{\text{f}} = \frac{P_{\text{i}}V_{\text{i}}}{T_{\text{i}}}\frac{T_{\text{f}}}{P_{\text{f}}} = \frac{(101{,}325\ \text{Pa})(3.64\ \text{L})(280\ \text{K})}{(300\ \text{K})(218{,}925\ \text{Pa})} = 1.57\ \text{L}$$

(*b*) The change in entropy is given by Eq. 19.61,

$$\Delta S = \tfrac{3}{2}N\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right) + N\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$$

We can determine *N* from the ideal gas law, $N = \dfrac{P_{\text{i}}V_{\text{i}}}{k_{\text{B}}T_{\text{i}}},$ so we have

$$\Delta S = N\left[\tfrac{3}{2}\ln\left(\frac{T_f}{T_i}\right)+\ln\left(\frac{V_f}{V_i}\right)\right]$$
$$=\frac{P_iV_i}{k_BT_i}\left[\tfrac{3}{2}\ln\left(\frac{T_f}{T_i}\right)+\ln\left(\frac{V_f}{V_i}\right)\right]$$
$$=\frac{P_iV_i}{k_BT_i}\left[\tfrac{3}{2}\ln\left(\frac{T_f}{T_i}\right)+\ln\left(\frac{P_iT_f}{P_fT_i}\right)\right]$$
$$=\frac{(101{,}325\text{ Pa})(3.64\text{ L})}{(1.381\times10^{-23}\text{ J/K})(300\text{ K})}\frac{1\text{ m}^3}{1000\text{ L}}\left[\tfrac{3}{2}\ln\left(\frac{280\text{ K}}{300\text{ K}}\right)+\ln\left(\frac{(101{,}325\text{ Pa})(280\text{ K})}{(218{,}925\text{ Pa})(300\text{ K})}\right)\right]=-8.39\times10^{22}$$

It is not unreasonable for the entropy to decrease because the system is not closed; the canister was initially floating at the surface, so work must have been done on the canister (and the gas it contains) to pull it beneath the surface.

19.103. We know the probability of a corrosive reaction taking place each time a gas particle hits the container wall. In the derivation of the pressure of an ideal gas in *Principles* Section 19.7, we came up with an expression for the number of collisions the gas particles make with the container, $N_{\text{col}}=\tfrac{1}{2}\frac{N}{V}|v_x|A\Delta t,$ and we can approximate $|v_x|$ from $v_{\text{rms}}^2=3(v_x^2)_{\text{av}}$ (Eq. 19.25) to get $\frac{N_{\text{col}}}{A\Delta t}=\frac{1}{2\sqrt{3}}\frac{N}{V}v_{\text{rms}}=\frac{1}{2\sqrt{3}}\frac{N}{V}\sqrt{\frac{3k_BT}{m}}=\tfrac{1}{2}\frac{N}{V}\sqrt{\frac{k_BT}{m}}.$ (The exact result is about 20% smaller; instead of one-half, the leading factor should be one over the square root of two pi, about 0.399. But this approximate result is close enough for our purposes.) If the inter-atomic spacing in the container is about $d_{\text{atom}}=0.3$ nm, which is typical for a metal, each atom in the container wall occupies an area of about $A=9\times10^{-20}\text{ m}^2,$ and the rate at which it is struck by a gas particle is $\frac{N_{\text{col}}}{\Delta t}=\tfrac{1}{2}\frac{N}{V}\sqrt{\frac{k_BT}{m}}A.$ If it takes on average 3 million collisions for the atom to enter into a corrosive reaction, $N_{\text{col}}=3\times10^6,$ the time it takes for the container wall to decrease in thickness by one atom is $\Delta t=\frac{2N_{\text{col}}V}{NA}\sqrt{\frac{m}{k_BT}},$ and the time it takes for the half the thickness d_{wall} of the container wall to corrode away is $\Delta t=\frac{2N_{\text{col}}V}{NA}\sqrt{\frac{m}{k_BT}}\frac{\tfrac{1}{2}d_{\text{wall}}}{d_{\text{atom}}}.$ The number of gas particles in the container equals the mass of the gas divided by the mass of each particle, $N=m_{\text{gas}}/m,$ and a temperature of 30 °C corresponds to an absolute temperature of $(30+273.15)\text{ K}=303\text{ K},$ so half the thickness of the container wall will corrode away in a time interval of about

$$\Delta t=\frac{(3\times10^6)(6.45\text{ L})}{\left(\frac{4.00\times10^{-3}\text{ kg}}{6.054\times10^{-26}\text{ kg}}\right)(9\times10^{-20}\text{ m}^2)}\frac{1\text{ m}^3}{1000\text{ L}}\sqrt{\frac{6.054\times10^{-26}\text{ kg}}{(1.381\times10^{-23}\text{ J/K})(303\text{ K})}}\frac{0.0100\text{ m}}{0.3\times10^{-9}\text{ m}}=4\times10^5\text{ s}$$

which corresponds to less than five days. So, the container is not safe to use for three weeks.

There is one other thing we should check, namely, how the number of gas particles compares to the number of atoms in the container. That is, it seems reasonable to suppose that each gas particle can only react with one atom from the container wall (or some small number, depending on the chemical reaction), so the question arises as to whether there are enough gas particles to corrode a significant portion of the wall.

We already have an expression for the number of gas particles, $N=\frac{m_{\text{gas}}}{m}=\frac{4.00\times10^{-3}\text{ kg}}{6.054\times10^{-26}\text{ kg}}=6.61\times10^{22}.$ The number of atoms in the container wall is the volume occupied by the wall divided by the volume occupied by an atom. We can calculate the inner radius of the container from its volume,

$$V = \tfrac{4}{3}\pi R^3$$

$$R = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(\frac{3(6.45\text{ L})}{4\pi}\frac{1\text{ m}^3}{1000\text{ L}}\right)^{1/3} = 0.115\text{ m}$$

so the volume occupied by the container wall is $V_{\text{wall}} = \tfrac{4}{3}\pi\left[(R+d_{\text{wall}})^3 - R^3\right] = \tfrac{4}{3}\pi\left[(0.125\text{ m})^3 - (0.115\text{ m})^3\right] =$ $1.82\times10^{-3}\text{ m}^3$, and the number of atoms in the container wall is about $N_{\text{atom}} = \dfrac{V_{\text{wall}}}{V_{\text{atom}}} = \dfrac{V_{\text{wall}}}{d_{\text{atom}}^3} = \dfrac{1.82\times10^{-3}\text{ m}^3}{(0.3\times10^{-9}\text{ m})^3} =$ 7×10^{25}. This is about 1000 times the number of gas particles in the container, so it seems that there is not enough gas to corrode away a significant portion of the container wall. On the other hand, all of the gas will have reacted well before the three weeks are up, so we still need to find a better container.

19.104. Because the balloons have shrunk to one-half their initial radius, the gauge pressure has decreased to one-quarter of the initial gauge pressure, and the volume to one-eighth of the initial volume. If this was due solely to the helium in the balloon cooling from the temperature of the hot car to air temperature, we would have

$$\frac{P_{\text{i}}V_{\text{i}}}{T_{\text{i}}} = \frac{P_{\text{f}}V_{\text{f}}}{T_{\text{f}}}$$

$$T_{\text{i}} = \frac{P_{\text{i}}V_{\text{i}}}{P_{\text{f}}V_{\text{f}}}T_{\text{f}}$$

A temperature of 35.0 °C corresponds to an absolute temperature of $(35.0+273.15)\text{ K} = 308.15\text{ K}$, and the initial and final absolute pressures are $P_{\text{i}} = P_{\text{atm}} + (50.0\text{ kPa}) = 151.325\text{ kPa}$ and $P_{\text{f}} = P_{\text{atm}} + \tfrac{1}{4}(50.0\text{ kPa}) = 113.825\text{ kPa}$, so the initial absolute temperature would have to have been $T_{\text{i}} = \dfrac{151.325\text{ kPa}}{113.825\text{ kPa}}(8)(308.15\text{ K}) = 3.28\times10^3\text{ K}$, which is comparable to the temperature of the glowing filament in an incandescent light bulb. So, the balloons' shrinking cannot be due solely to the helium having cooled. They must also be losing helium, and will eventually deflate completely.

20

ENERGY TRANSFERRED THERMALLY

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

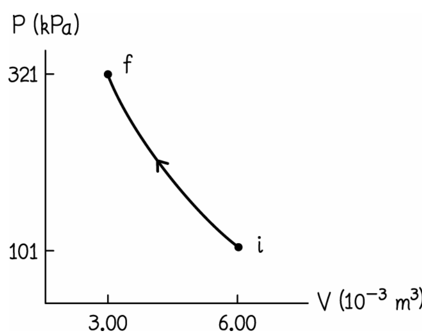
1. 10^4 J **2.** 10^2 **3.** 10^2 s **4.** 10^{-2} **5.** 10^3 K **6.** 10^{-1} m^3 **7.** 10^1 J **8.** 10^3 K **9.** 10^{22} **10.** -10^{22} **11.** 10^8 J

Guided Problems

20.2 Work during compression

1. Getting Started This is an isentropic process. We know this because it is adiabatic and quasistatic. This means there is no change in entropy, such that the process can be reversed without violating the second law of thermodynamics.

We gain some intuition by making a PV diagram:



Initially, we know everything about the ideal gas except the number of particles. We can use the ideal gas equation to determine this. Finally, we know only the pressure and volume. We do not know the final temperature.

2. Devise Plan For an isentropic process, we can find the work done using equation (20.19):

$$W = \Delta E_{\text{th}} = NC_V \Delta T$$

To find the work done, we need to determine the number of particles, the specific heat at constant volume, and the temperature change. The specific heat at constant volume is given by equation (20.14): $C_V = \frac{d}{2}k_B$. Because the gas is monatomic, we know it has only three degrees of (translational) freedom, such that $C_V = \frac{3}{2}k_B$. We can use the ideal gas law at the initial point on the PV diagram to determine the number of particles: $N = \frac{P_i V_i}{k_B T_i}$. Once the number of particles is known, we can use the ideal gas law at the final point on the PV diagram to determine the final temperature: $T_f = \dfrac{P_f V_f}{k_B N}$. This will give us all information necessary to determine the work from equation (20.19).

3. Execute Plan From the ideal gas law we have

$$N = \frac{P_i V_i}{k_B T_i} = \frac{(1.01\times10^5\ \text{Pa})(6.00\times10^{-3})}{(1.380\times10^{-23}\ \text{J/K})(250\ \text{K})} = 1.757\times10^{23}$$

Using this in the ideal gas law applied to the final position, we find

$$T_f = \frac{P_f V_f}{k_B N} = \frac{(3.21\times10^5\ \text{Pa})(3.00\times10^{-3})}{(1.380\times10^{-23}\ \text{J/K})(1.757\times10^{23})} = 396.9\ \text{K}$$

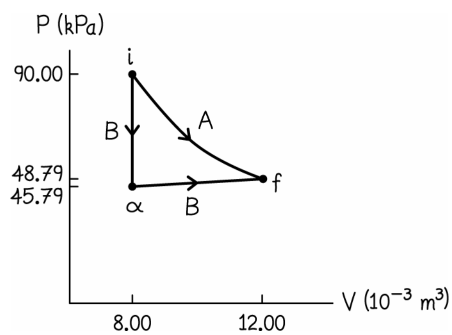
Inserting the number of particles, temperatures, and specific heat at constant volume, we see that the work is given by

$$W = NC_V\Delta T = (1.762\times10^{23})\left(\frac{3}{2}\right)(1.380\times10^{-23}\ \text{J/K})((396.9\ \text{K}) - (250\ \text{K})) = 534\ \text{J}$$

4. Evaluate Result The sign is certainly correct. We have compressed the gas, so we have done positive work on it. To determine whether or not the size is reasonable, we might compare the work done in this process to work done in different processes between the same two points in the PV diagram. For example, going from the same initial point to the same final point via isobaric compression at 101 kPa followed by isochoric increase in pressure would do 303 J of work. We see that not all the compression takes place at this lowest pressure, so we expect the work done in the isentropic process to be greater, and it is. We expect the work done by the isentropic process to be less than the work done by an initial isochoric increase in pressure up to 321 kPa, following by an isobaric compression, because here all compression takes place at the maximum pressure. That isochoric/isobaric process would do 963 J of work on the gas. Thus, the magnitude of our answer is also very reasonable.

20.4 Two-step versus one-step compression

1. Getting Started We begin by making an approximate PV diagram of the two paths described:



We break process B up into two parts: an isochoric drop in pressure, then an isobaric expansion. We can the point in between these two steps, point α.

We wish to determine the work done by processes A and B, so we can find the difference. We know that no work is done by an isochoric process, and we know expressions for work done by isobaric and isentropic processes. But we will need to determine several quantities to use these expressions. For example, we will need the heat capacity ratio for a monatomic ideal gas to determine the initial and final temperatures for the isentropic process. The ratio in this case is $\gamma = 5/3$. For work done by the isobaric expansion, we will need to know the final pressure. We are given all other quantities, including initial and final volumes, initial pressure, and the number of particles.

2. Devise Plan For process A, we determine the work using equation (20.19)

$$W = \Delta E_{th} = NC_V\Delta T$$

For which we need the initial and final temperatures. We can determine the initial temperature using the ideal gas equation:

$$T_\text{i} = \frac{P_\text{i}V_\text{i}}{k_\text{B}N}$$

For the final temperature, we make use of equation (20.44) which is valid for isentropes, to relate the temperatures to the volumes: $T_\text{i}V_\text{i}^{\gamma-1} = T_\text{f}V_\text{f}^{\gamma-1}$. For process B we need only the final pressure. Once we determine the final temperature as described above, we can determine the final pressure by, again, using the ideal gas law:

$$P_\text{f} = \frac{k_\text{B}NT_\text{f}}{V_\text{i}}$$

Once this pressure is determined, the work done by process B is simply

$$W_\text{B} = P_\text{f}(V_\text{f} - V_\text{i}) \tag{1}$$

3. Execute Plan The initial temperature is

$$T_\text{i} = \frac{P_\text{i}V_\text{i}}{k_\text{B}N} = \frac{(9.00\times10^{4}\ \text{Pa})(8.00\times10^{-3}\ \text{m}^3)}{(1.380\times10^{-23}\ \text{J/K})(0.200)(6.02\times10^{23})} = 433.3\ \text{K}$$

Rearranging equation (20.44) yields

$$T_\text{f} = T_\text{i}\left(\frac{V_\text{i}}{V_\text{f}}\right)^{\gamma-1} = (433.3\ \text{K})\left(\frac{(8.00\times10^{-3}\ \text{m}^3)}{(1.20\times10^{-2}\ \text{m}^3)}\right)^{(5/3)-1} = 330.7\ \text{K}$$

Using this final temperature, the ideal gas law yields the final pressure:

$$P_\text{f} = \frac{(1.380\times10^{-23}\ \text{J/K})(0.200)(6.02\times10^{23})(330.7\ \text{K})}{(1.20\times10^{-2}\ \text{m}^3)} = 4.579\times10^{4}\ \text{Pa}$$

Now equation (20.19) gives us the work done by process A:

$$W_\text{A} = N\left(\frac{3}{2}k_\text{B}\right)(T_\text{f} - T_\text{i}) = \left(\frac{3}{2}\right)(1.380\times10^{-23}\ \text{J/K})(0.200)(6.02\times10^{23})((330.7\ \text{K}) - (433.3\ \text{K})) = -256\ \text{J}$$

Equation (1) above gives us the work done by process B:

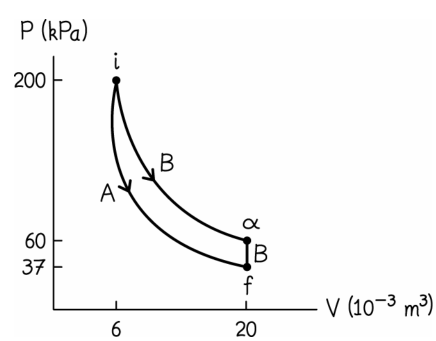
$$W_\text{B} = -P_\text{f}(V_\text{f} - V_\text{i}) = -(4.579\times10^{4}\ \text{Pa})((1.20\times10^{-2}\ \text{m}^3) - (8.00\times10^{-3}\ \text{m}^3)) = -183\ \text{J}$$

So, both processes do negative work on the gas. The magnitude of the work is greater for process A by 73 J.

4. Evaluate Result Based on the PV diagram, we see that in either process, the gas is allowed to expand. This corresponds to the gas doing positive work on the environment, or equivalently to the environment doing negative work on the gas. So we expected the work done on the gas for both processes to be negative. Since the area under curve A is greater in magnitude than the area under curve B, we expected process A to correspond to a larger magnitude work. This matches what we obtained algebraically.

20.6 Two-step versus one-step entropy change

1. Getting Started We begin by making an approximate PV diagram:



We label the point in process B between the isotherm and the isochor α. The heat capacity ratio is given, in general, by equation (20.26):

$$\gamma = 1 + \frac{2}{d}$$

where d is the number of degrees of freedom in the gas. For a diatomic molecule there are three degrees of freedom in translational motion, and one degree of freedom for each axis of rotation. But a diatomic molecule has all mass concentrated along the line joining the two atomic nuclei. Thus rotation around that axis would not cost any energy (the moment of inertia is effectively zero). So only two axes of rotation contribute, bringing the total degrees of freedom to five. If the gas is well above 10^3 K, we would need to consider vibrational degrees of freedom as well. For now, we assume the temperature is below that value, and we take $d = 5$, and therefore $\gamma = \frac{7}{5}$. But we must keep in mind that if we discover later that the temperature is too high, we may need to revisit this assumption.

2. Devise Plan A change in entropy in an isotherm is given by equation (20.35):

$$\Delta S_{\text{isotherm}} = N \ln\left(\frac{V_\text{f}}{V_\text{i}}\right)$$

And the change in entropy in an isochor is given by equation (20.38)

$$\Delta S_{\text{isochor}} = \frac{NC_V}{k_\text{B}} \ln\left(\frac{T_\text{f}}{T_\text{i}}\right)$$

For the first (isothermal) leg of process B, we have all required information. For the second (isochoric) leg of process B, we need to calculate both the initial and the final temperature, as well as the specific heat at constant volume. The initial temperature can be calculated from given information using the ideal gas law. The final temperature is most easily calculated using process A, since we know from equation (20.44) that in an isentrope: $T_\text{i} V_\text{i}^{\gamma-1} = T_\text{f} V_\text{f}^{\gamma-1}$. The specific heat at constant volume is easy to determine, since

$$\gamma = \frac{C_P}{C_V} = \frac{C_V + k_\text{B}}{C_V} \Rightarrow C_V = \frac{k_\text{B}}{\gamma - 1} = \frac{5}{2} k_\text{B}$$

Thus, we can obtain all required quantities to determine the change in entropy for process B. The change in entropy for process A is trivial. By definition, the change in entropy of an isentrope is zero.

3. Execute Plan The number of particles is obtained from the number of moles of gas given, and Avogadro's number: $N = (0.600 \text{ mol})(6.022 \times 10^{23} \text{ H}_2/\text{mol}) = 3.613 \times 10^{23} \text{ H}_2$. As already stated in part (2), the entropy is constant throughout process A, by definition: $\Delta S_\text{A} = 0$. Using the ideal gas law and the initial conditions given, we find

$$T_\text{i} = \frac{P_\text{i} V_\text{i}}{N k_\text{B}} = \frac{(2.00 \times 10^5 \text{ Pa})(6.00 \times 10^{-3} \text{ m}^3)}{(3.613 \times 10^{23})(1.380 \times 10^{-23} \text{ J/K})} = 240.7 \text{ K}$$

This is well below the regime that would require consideration of vibrational modes, so our initial assumption of five degrees of freedom is justified.

We determine the final temperature by rearranging equation (20.44)

$$T_\text{f} = T_\text{i} \left(\frac{V_\text{i}}{V_\text{f}}\right)^{\gamma-1} = (240.7 \text{ K}) \left(\frac{(6.00 \times 10^{-3} \text{ m}^3)}{(2.00 \times 10^{-2} \text{ m}^3)}\right)^{\left(\frac{7}{5}\right)-1} = 148.7 \text{ K}$$

We now have sufficient information to use equation (20.35):

$$\Delta S_{\text{isotherm}} = N \ln\left(\frac{V_\text{f}}{V_\text{i}}\right) = (3.613 \times 10^{23}) \ln\left(\frac{2.00 \times 10^{-2} \text{ m}^3}{6.00 \times 10^{-3} \text{ m}^3}\right) = 4.35 \times 10^{23}$$

And we also have enough to use equation (20.38):

$$\Delta S_{\text{isochor}} = \frac{NC_V}{k_\text{B}} \ln\left(\frac{T_\text{f}}{T_\text{i}}\right) = \frac{N}{k_\text{B}} \left(\frac{5}{2} k_\text{B}\right) \ln\left(\frac{T_\text{f}}{T_\text{i}}\right) = (3.613 \times 10^{23}) \left(\frac{5}{2}\right) \ln\left(\frac{148.7 \text{ K}}{240.7 \text{ K}}\right) = -4.35 \times 10^{23}$$

Combining the entropy changes from the isothermal and isochoric steps, we find that $\Delta S_\text{B} = 0$, also.

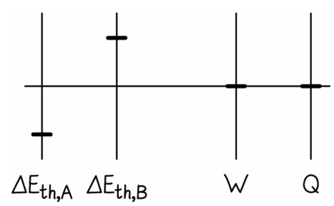
4. Evaluate Result The isothermal step of process B cannot have any entropy change associated with temperature change, so we need only consider the change in volume. This step is an expansion, so the volume occupied by the gas increases, meaning the number of ways of occupying space increases. Thus we expect the entropy to increase. Indeed, this is what we found. In the isochoric step, we need only consider temperature changes. Since the temperature decreases, we expect the entropy to decrease also. Again, our result agrees with this expectation.

The changes in entropy for process A and B do match. This is what we expect, since entropy is a state variable (it depends only on the state of the system, not how one got to that state).

20.8 Hot and cool water

1. Getting Started Let us define our closed system to consist of sample A and sample B. We know the initial temperature of each sample, but we do not know the final temperature of the mixture. We need to find this final temperature in order to find the temperature change of each sample. We know that these temperature changes are related to the transfer of energy through the specific heat capacity c_V. We can get the right order of magnitude by considering degrees of freedom. But we look up the exact value of the specific heat capacity for water, and find $c_V = 4.186 \text{ kJ/kg} \cdot \text{K}$.

2. Devise Plan We begin by making an energy diagram showing that sample A will lose thermal energy and sample B will gain exactly as much thermal energy:



Since we have the specific heat capacity at constant volume, we can obtain the entropy change from equation (20.53):

$$\Delta S = \frac{mc_V}{k_B} \ln\left(\frac{T_f}{T_i}\right)$$

We just need to find the final temperature of the mixture. We can do this by employing conservation of energy. All heat energy transferred from sample A must be transferred to sample B (because we have assumed that combination to be isolated). Noting, that one heat transfer is negative and the other positive, we can express this as $Q_A = -Q_B$. This heat transfer can be related to temperature changes through equation (20.49), yielding $m_A c_V (T_{A,f} - T_{A,i}) = -m_B c_V (T_{B,f} - T_{B,i})$.

In thermal equilibrium, the final temperature of each sample must be the same. Using that fact, and rearranging, we find

$$(m_A + m_B) c_V T_f = c_V (m_A T_{A,i} + m_B T_{B,i})$$

$$T_f = \frac{(m_A T_{A,i} + m_B T_{B,i})}{(m_A + m_B)} \qquad (1)$$

3. Execute Plan Inserting numbers into equation (1) above, we find

$$T_f = \frac{((1.50 \text{ kg})(373 \text{ K}) + (2.00 \text{ kg})(293 \text{ K}))}{((1.50 \text{ kg}) + (2.00 \text{ kg}))} = 327 \text{ K}$$

Now we can directly apply equation (20.53) to each of the two samples:

$$\Delta S_{\text{A}} = \frac{m_{\text{A}} c_V}{k_{\text{B}}} \ln\left(\frac{T_{\text{f}}}{T_{\text{A,i}}}\right) = \frac{(1.50\text{ kg})(4.186\times10^3\text{ J/kg}\cdot\text{K})}{(1.380\times10^{-23}\text{ J/K})} \ln\left(\frac{327.3\text{ K}}{373\text{ K}}\right) = -5.95\times10^{25}$$

$$\Delta S_{\text{B}} = \frac{m_{\text{B}} c_V}{k_{\text{B}}} \ln\left(\frac{T_{\text{f}}}{T_{\text{B,i}}}\right) = \frac{(2.00\text{ kg})(4.186\times10^3\text{ J/kg}\cdot\text{K})}{(1.380\times10^{-23}\text{ J/K})} \ln\left(\frac{327.3\text{ K}}{293\text{ K}}\right) = 6.71\times10^{25}$$

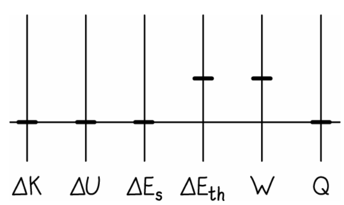
Thus the total entropy change is $\Delta S_{\text{A}} + \Delta S_{\text{B}} = -5.95\times10^{25} + 6.71\times10^{25} = 7.6\times10^{24}$.

4. Evaluate Result Since sample A decreases in temperature, we expect a negative change in entropy for A. Since sample B increases in temperature, we expect a positive change in entropy for B. One might see mathematically that the increase in temperature of B will result in a greater magnitude entropy change for B than for A. This can be seen from the fact that the heat transferred for each sample is the same in magnitude, but the temperature is always lower for sample B. But one could also argue that the magnitude of the entropy change for B must be larger than that of A, because the total entropy of the isolated system can only increase or remain the same; it cannot decrease. All of these expectations match the results of our calculation above.

Questions and Problems

20.1. When the volume of a gas increases, the force on its surroundings due to the pressure of the gas and the force displacement are both directed outward, in the direction of its expansion, so the gas does positive work on its surroundings.

20.2.



20.3. (*a*) If the water is boiling vigorously, its mass density is not uniform throughout the pot (that is, some regions are filled with steam bubbles while others are filled with liquid water), so the process is not quasistatic. Because thermal energy must be added to the water to make it boil, the process is not adiabatic. (*b*) Because the temperature, pressure, and so on are uniform throughout the air in the balloon at all times, the process is quasistatic. Because the balloon is cooled, the process is not adiabatic. (*c*) Because the temperature, pressure, and so on are uniform throughout the air at all times, the process is quasistatic. Because no energy is transferred thermally, the process is adiabatic.

20.4. As the gas expands quasistatically and adiabatically, its entropy does not change. So, from Eq. 19.61 we have

$$\Delta S = \tfrac{3}{2} N \ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right) + N \ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right) = 0$$

$$\ln\left(\frac{T_{\text{f}}}{T_{\text{i}}}\right) = -\tfrac{2}{3} \ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$$

$$T_{\text{f}} = T_{\text{i}}\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)^{-2/3}$$

We can determine the initial absolute temperature from the ideal gas law: $T_{\text{i}} = \dfrac{P_{\text{i}} V_{\text{i}}}{N k_{\text{B}}}$. Combining these we have

$$T_{\text{f}} = \frac{P_{\text{i}} V_{\text{i}}}{N k_{\text{B}}}\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)^{-2/3} = \frac{(30.5\times10^3\text{ Pa})(1.00\text{ L})}{(5.60\times10^{21})(1.381\times10^{-23}\text{ J/K})} \frac{1\text{ m}^3}{1000\text{ L}} \left(\frac{2.00\text{ L}}{1.00\text{ L}}\right)^{-2/3} = 248\text{ K}$$

20.5. Irreversible processes are those in which a system evolves toward a more probably macrostate, so a process can only be reversible if, at all times, the system is in equilibrium, that is, if the process is quasistatic. If the pistons move more quickly than the gas particles, the process will not be quasistatic (see *Principles* Figure 20.7), which places an upper limit on the number of cycles they can go through each second.

The root-mean-square speed of the gas particles is given by Eq. 19.53 $v_{\text{rms}} = \sqrt{\dfrac{3k_{\text{B}}T}{m}}$. The average speed of the piston is the stroke length divided by the time interval of each stroke, and there are two strokes per cycle, so the frequency of the piston is related to its average speed by $v_{\text{p,av}} = 2\ell f$, or $f = v_{\text{p,av}}/(2\ell)$. A temperature of $300\ °\text{F}$ corresponds to a temperature of $\left[(300-32)(5/9)\right]\ °\text{C} = 148.9\ °\text{C}$, or an absolute temperature of $(148.9+273.15)\ \text{K} = 422.0\ \text{K}$, so our upper limit on f is

$$f_{\text{max}} = \frac{1}{2\ell}\sqrt{\frac{3k_{\text{B}}T}{m}} = \frac{1}{2(0.150\ \text{m})}\sqrt{\frac{3(1.381\times10^{-23}\ \text{J/K})(422.0\ \text{K})}{7.548\times10^{-26}\ \text{kg}}} = 1.60\times10^{3}\ \text{Hz}$$

Note that, because we have used the average speed of the pistons and the maximum gas temperature, we have likely overestimated the maximum frequency. Our use of v_{rms} also results in an overestimate: because the gas particles move in random directions, only a fraction of them are moving parallel to the piston's motion at any given instant, so to maintain equilibrium at all times the piston would have to move more slowly.

Looking at our last equation, we see that the frequency could be increased by decreasing the stroke length, increasing the temperature, or decreasing the mass of the gas particles.

For comparison, a gasoline engine in a typical car operates at a few thousand RPM (revolutions per minute), which corresponds to a frequency of about $(3000\ \text{min}^{-1})(1\ \text{min}/60\ \text{s}) = 500\ \text{Hz}$. It also has a shorter stroke length, and the burning fuel has a higher temperature, so our reversibility constraint seems to give too high a frequency, even considering that it's likely an overestimate. There may well be other, more stringent constraints on the design, such as the maximum acceleration the pistons and other engine parts can withstand.

20.6. If the temperature has the same numerical value on both the Fahrenheit and Celsius scales, we have

$$\begin{aligned} T_{\text{C}} &= \tfrac{5}{9}(T_{\text{F}}-32) = \tfrac{5}{9}(T_{\text{C}}-32) \\ 9T_{\text{C}} &= 5T_{\text{C}} - 160 \\ T_{\text{C}} &= -40 \end{aligned}$$

so $T = (T_{\text{C}} + 273.15)\ \text{K} = 233.15\ \text{K}$.

20.7. The boiling and freezing points of water are given for all three temperature scales in *Principles* Table 20.1. (*a*) $\Delta T = (373.12\ \text{K}) - (273.15\ \text{K}) = 99.97\ \text{K}$ (*b*) $\Delta T = (100\ °\text{C}) - (0\ °\text{C}) = 100\ °\text{C}$ (*c*) $\Delta T = (212\ °\text{F}) - (32\ °\text{F}) = 180\ °\text{F}$

20.8. The temperature is $T = (T_{\text{C}} + 273.15)\ \text{K} = 77\ \text{K}$.

20.9. An ideal gas thermometer is constructed by attaching a gas-filled bulb to a thin, flexible tube filled with mercury, the other end of which is sealed and evacuated. It measures temperature by measuring the pressure in the gas. According to the ideal gas law, if the volume of the gas is held constant, its pressure is proportional to its temperature. The volume can be held constant by raising or lowering the tube as necessary to keep the mercury level on the bulb's side of the tube at some reference level, while the pressure can be measured by the height of the mercury above the reference level in the other side of the tube.

20.10. The height of the mercury above the reference point is proportional to the temperature. The temperature of the triple point of water is 273.16 K, and water boils at 373.12 K at atmospheric pressure, so the height is

$$h = h_{\text{tp}}\frac{T}{T_{\text{tp}}} = (986\text{ mm})\frac{373.12\text{ K}}{273.16\text{ K}} = 1.35\times 10^{3}\text{ mm}$$

20.11. (*a*) The height of the mercury above the reference point is proportional to the temperature. Water boils at 373.12 K at atmospheric pressure, so the temperature of the room is

$$T = T_{\text{boil}}\frac{h}{h_{\text{boil}}} = (373.12\text{ K})\frac{(1279\text{ mm})-(267\text{ mm})}{1279\text{ mm}} = 295\text{ K}$$

which corresponds to $T_{\text{C}} = (295-273)\ °\text{C} = 22\ °\text{C}$. (*b*) Yes, this is a comfortable room temperature, corresponding to $(32+\frac{9}{5}22)\ °\text{F} = 72\ °\text{F}$.

20.12. (*a*) The height of the mercury above the reference point is proportional to the temperature, so the temperature of the liquid is

$$T = T_{\text{freeze}}\frac{h}{h_{\text{freeze}}} = (273.15\text{ K})\frac{29\text{ mm}}{102\text{ mm}} = 78\text{ K}$$

(*b*) Depending on the gas in the thermometer, it might not exhibit ideal behavior at such a low temperature. Also, this temperature is below the freezing point of mercury (about 234 K), so if the mercury is liquid, the gas in contact with it is at a higher temperature than that of the liquid being measured.

20.13. If the specifications are in kelvins, the minimum required temperature is $\left[(0.95)(1960)-273\right]\ °\text{C} = 1589\ °\text{C}$. If the specifications are in degrees Celsius, the minimum required temperature is $(0.95)(1960\ °\text{C}) = 1862\ °\text{C}$. Because the weld can be performed safely at higher temperatures, she should specify $1862\ °\text{C}$.

20.14. (*a*) A given temperature interval is divided into more degrees Fahrenheit than degrees Celsius, in the ratio of 9 to 5, so the Fahrenheit scale will give us greater precision. (*b*) The temperature range of 255 K to 320 K corresponds to a range of $(255-273)\ °\text{C} = -18\ °\text{C}$ to $(320-273)\ °\text{C} = 47\ °\text{C}$, or a span of $10\left[47-(-18)\right] = 650$ tenths of a degree Celsius. It also corresponds to a range of $\left[\frac{9}{5}(-18)+32\right]\ °\text{F} = -0.4\ °\text{F}$ to $\left[\frac{9}{5}(47)+32\right]\ °\text{F} = 116.6\ °\text{F}$, or a span of $10\left[116.6-(-0.4)\right] = 1170$ tenths of a degree Fahrenheit. So, the ratio of possible readings on the Fahrenheit scale to those on the Celsius scale is $1170/650 = 1.8$.

20.15. The energy required equals the product of the specific heat capacity of the water, the mass of the water, and the change in temperature. We can determine the mass of the water from its volume and mass density, and the change in temperature in kelvins equals the change in degrees Celsius, so we have

$$\begin{aligned} E &= cm\Delta T = c\rho V\Delta T \\ &= (4181\text{ J/K}\cdot\text{kg})(1000\text{ kg/m}^3)\frac{1\text{ m}^3}{1000\text{ L}}(100\text{ L})(35\text{ K}) = 1.5\times 10^{7}\text{ J} \end{aligned}$$

20.16. The energy required equals the product of the specific heat capacity of the water, the mass of the water, and the change in temperature. The change in temperature in kelvins equals the change in degrees Celsius, so we have

$$E = cm\Delta T = (4181\text{ J/K}\cdot\text{kg})(0.236\text{ kg})(80\text{ K}) = 7.9\times 10^{4}\text{ J}$$

20.17. The energy required equals the product of the specific heat capacity of the aluminum, the mass of the aluminum, and the change in temperature. We can determine the mass of the aluminum from its weight, and the change in temperature in kelvins equals the change in degrees Celsius, so we have

$$\begin{aligned} E &= cm\Delta T = c(F^{G}_{\text{EAl}}/g)\Delta T \\ &= (897\text{ J/K}\cdot\text{kg})(1.7\times 10^{9}\text{ lb})\frac{4.448\text{ N}}{1\text{ lb}}\frac{1}{9.8\text{ N/kg}}(640\text{ K}) = 4.4\times 10^{14}\text{ J} \end{aligned}$$

20.18. (*a*) Thermal energy is equipartitioned over the degrees of freedom of the molecule, but only those for which the equipartition share of the energy is greater than the quantum of energy associated with it. Diatomic molecules have up to seven degrees of freedom, three translational, two rotational, and two vibrational, but the latter only contribute to the specific heat at high temperatures. So, the sample at 1000 K has the highest heat capacity per molecule, the sample at 298 K has the next highest, and the sample at 3 K has the lowest. That is, $C_{1000\ \mathrm{K}} > C_{298\ \mathrm{K}} > C_{3\ \mathrm{K}}$. (*b*) Each contributing degree of freedom carries its equipartition share of the thermal energy, $\frac{1}{2}k_\mathrm{B}T$. For the sample at 3 K, only the three translational degrees of freedom contribute, so the thermal energy per molecule is $\frac{3}{2}k_\mathrm{B}T$. For the sample at 298 K, the two rotational degrees of freedom also contribute, so the thermal energy per molecule is $\frac{5}{2}k_\mathrm{B}T$. For the sample at 1000 K, the two vibrational degrees of freedom also contribute, so the thermal energy per molecule is $\frac{7}{2}k_\mathrm{B}T$.

20.19. (*a*) The energy required equals the product of the specific heat capacity of the water, the mass of the water, and the change in temperature. We can determine the mass of the water from its volume and mass density, and the change in temperature in kelvins equals the change in degrees Celsius, so we have

$$\begin{aligned} E &= cm\Delta T = c\rho V\Delta T \\ &= (4181\ \mathrm{J/K\cdot kg})(1000\ \mathrm{kg/m^3})(50.0\ \mathrm{m})(35.0\ \mathrm{m})(2.00\ \mathrm{m})(1.00\ \mathrm{K}) = 1.46\times10^{10}\ \mathrm{J} \end{aligned}$$

(*b*) If this energy were used to lift a truck, it would be converted into gravitational potential energy, and we would have for the height

$$U^G = mgh$$

$$h = \frac{U^G}{mg} = \frac{1.46\times10^{10}\ \mathrm{J}}{(1.0\times10^4\ \mathrm{kg})(9.8\ \mathrm{N/kg})} = 1.5\times10^5\ \mathrm{m}$$

20.20. When the blocks come to thermal equilibrium, they will have the same temperature. The thermal energy lost by the iron block as it cools is gained by the copper block as it warms. Equating these we have

$$\begin{aligned} c_\mathrm{Fe}m_\mathrm{Fe}\Delta T_\mathrm{Fe} &= -c_\mathrm{Cu}m_\mathrm{Cu}\Delta T_\mathrm{Cu} \\ c_\mathrm{Fe}m_\mathrm{Fe}(T_\mathrm{f} - T_\mathrm{Fe,i}) &= -c_\mathrm{Cu}m_\mathrm{Cu}(T_\mathrm{f} - T_\mathrm{Cu,i}) \\ T_\mathrm{f}(c_\mathrm{Fe}m_\mathrm{Fe} + c_\mathrm{Cu}m_\mathrm{Cu}) &= c_\mathrm{Fe}m_\mathrm{Fe}T_\mathrm{Fe,i} + c_\mathrm{Cu}m_\mathrm{Cu}T_\mathrm{Cu,i} \\ T_\mathrm{f} &= \frac{c_\mathrm{Fe}m_\mathrm{Fe}T_\mathrm{Fe,i} + c_\mathrm{Cu}m_\mathrm{Cu}T_\mathrm{Cu,i}}{c_\mathrm{Fe}m_\mathrm{Fe} + c_\mathrm{Cu}m_\mathrm{Cu}} \end{aligned}$$

and the amount of energy transferred thermally is

$$\begin{aligned} E_\mathrm{th} &= c_\mathrm{Cu}m_\mathrm{Cu}(T_\mathrm{f} - T_\mathrm{Cu,i}) \\ &= c_\mathrm{Cu}m_\mathrm{Cu}\left(\frac{c_\mathrm{Fe}m_\mathrm{Fe}T_\mathrm{Fe,i} + c_\mathrm{Cu}m_\mathrm{Cu}T_\mathrm{Cu,i}}{c_\mathrm{Fe}m_\mathrm{Fe} + c_\mathrm{Cu}m_\mathrm{Cu}} - T_\mathrm{Cu,i}\right) \\ &= c_\mathrm{Cu}m_\mathrm{Cu}\left(\frac{c_\mathrm{Fe}m_\mathrm{Fe}(T_\mathrm{Fe,i} - T_\mathrm{Cu,i})}{c_\mathrm{Fe}m_\mathrm{Fe} + c_\mathrm{Cu}m_\mathrm{Cu}}\right) \\ &= (385\ \mathrm{J/K\cdot kg})(6.25\ \mathrm{kg})\left(\frac{(449\ \mathrm{J/K\cdot kg})(3.50\ \mathrm{kg})\left[(800\ \mathrm{K}) - (400\ \mathrm{K})\right]}{(449\ \mathrm{J/K\cdot kg})(3.50\ \mathrm{kg}) + (385\ \mathrm{J/K\cdot kg})(6.25\ \mathrm{kg})}\right) = 3.80\times10^5\ \mathrm{J} \end{aligned}$$

20.21. As the water falls, the force of gravity does work on it, increasing its kinetic energy. The change in thermal energy equals the product of the specific heat capacity of the water, the mass of the water, and the change in temperature. Combining these we have

$$\Delta E_\mathrm{th} = cm\Delta T = mgh$$

$$\Delta T = \frac{gh}{c} = \frac{(9.8\ \mathrm{N/kg})(82\ \mathrm{m})}{4181\ \mathrm{J/K\cdot kg}} = 0.19\ \mathrm{K}$$

20.22. The energy required equals the product of the specific heat capacity of the water, the mass of the water, and the change in temperature, and the energy delivered by the stove equals the product of the power and the time interval. To bring the water to the boiling point, we must raise its temperature to $100\ ^\circ\text{C},$ and the change in temperature in kelvins equals the change in degrees Celsius. We can determine the mass of the water from its volume and mass density. Combining these we have

$$P\Delta t = cm\Delta T$$

$$\Delta t = \frac{c\rho V\Delta T}{P} = \frac{(4181\ \text{J/K}\cdot\text{kg})(1000\ \text{kg/m}^3)(5.0\ \text{L})(80\ \text{K})}{1250\ \text{W}}\frac{1\ \text{m}^3}{1000\ \text{L}}\frac{1\ \text{min}}{60\ \text{s}} = 22\ \text{min}$$

20.23. (*a*) Because the atom can move in only one dimension, it has only one translational degree of freedom in which to store thermal energy, so the atom's thermal energy is $E_{\text{th}} = \frac{1}{2}k_{\text{B}}T = \frac{1}{2}(1.381\times10^{-23}\ \text{J/K})(77\ \text{K}) = 5.3\times10^{-22}\ \text{J}.$ (*b*) Now the atom has three degrees of freedom, so its thermal energy is $E_{\text{th}} = \frac{3}{2}k_{\text{B}}T = \frac{3}{2}(1.381\times10^{-23}\ \text{J/K})(295\ \text{K}) = 6.11\times10^{-21}\ \text{J}.$

20.24. (*a*) At this low a temperature, only the molecule's translational degrees of freedom contribute to its heat capacity. Because the molecule can move in only two dimensions, it has only two translational degrees of freedom in which to store thermal energy, so the molecule's thermal energy is $E_{\text{th}} = \frac{2}{2}k_{\text{B}}T = (1.381\times10^{-23}\ \text{J/K})(120\ \text{K}) = 1.66\times10^{-21}\ \text{J}.$ (*b*) At room temperature, the molecule's two rotational degrees of freedom also contribute to its heat capacity, so the molecule's thermal energy is $E_{\text{th}} = \frac{5}{2}k_{\text{B}}T = \frac{5}{2}(1.381\times10^{-23}\ \text{J/K})(298\ \text{K}) = 1.03\times10^{-20}\ \text{J}.$ (*c*) At this high a temperature, the molecule's two vibrational degrees of freedom also contribute to its heat capacity, so the molecule's thermal energy is $E_{\text{th}} = \frac{7}{2}k_{\text{B}}T = \frac{7}{2}(1.381\times10^{-23}\ \text{J/K})(3000\ \text{K}) = 1.450\times10^{-19}\ \text{J}.$

20.25. We know the relationship between thermal energy, specific heat capacity, mass, and temperature change, $\Delta E_{\text{th}} = cm\Delta T.$ Because the specific heat capacity depends on the temperature, we have to divide the process into many infinitesimal steps, during each of which the specific heat capacity does not change appreciably, and integrate them to get the total change in thermal energy, $\Delta E_{\text{th}} = \int_{T_\text{i}}^{T_\text{f}} c(T)m\,dT.$ Substituting the given formula for the specific heat capacity, we can solve for β,

$$\Delta E_{\text{th}} = \int_{T_\text{i}}^{T_\text{f}} \beta T^2 m\,dT = \tfrac{1}{3}\beta m(T_\text{f}^3 - T_\text{i}^3)$$

$$\beta = \frac{3\Delta E_{\text{th}}}{m(T_\text{f}^3 - T_\text{i}^3)} = \frac{3(231\ \text{J})}{(8000\times10^{-6}\ \text{kg})\left[(6.00\ \text{K})^3 - (1.00\ \text{K})^3\right]} = 403\ \text{J/K}^3\cdot\text{kg}$$

20.26. A quasistatic adiabatic process is isentropic. Because the gas expands, the process is not isochoric. Because isentropes cross both isotherms and lines of constant pressure, the process is not isothermal or isobaric, either.

20.27. Because the final volume is greater than the initial volume, the gas does positive work on its surroundings. The amount of work done by the gas in each process equals the area under the path in the *PV* diagram, which appears to be greater for path B. That is, the area to the left, where path B is above path A, appears to be larger than the area to the right, where path B is below path A.

20.28. (*a*) The process from state 1 to 2 takes place at constant pressure of $100\ \text{kPa},$ so it is isobaric. The process from state 2 to 3 takes place at constant volume of $0.3\ \text{m}^3,$ so it is isochoric. The process from state 3 to 4 takes place at constant pressure of $50\ \text{kPa},$ so it is isobaric. The process from state 4 to 1 takes place at constant volume of $0.1\ \text{m}^3,$ so it is isochoric. (*b*) We can find the temperatures from the ideal gas law.

$$T_1 = \frac{P_1V_1}{Nk_B} = \frac{(100\times10^3\ \text{Pa})(0.10\ \text{m}^3)}{(10\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})(1.381\times10^{-23}\ \text{J/K})} = 1.2\times10^2\ \text{K}$$

$$T_3 = \frac{P_3V_3}{Nk_B} = \frac{(50\times10^3\ \text{Pa})(0.30\ \text{m}^3)}{(10\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})(1.381\times10^{-23}\ \text{J/K})} = 1.8\times10^2\ \text{K}$$

20.29. None of the curves are straight lines, so they cannot be isochors or isobars. The isentrope passing through a given point on a PV diagram is steeper than the isotherm passing through the point, so the solid curves are isotherms and the dashed curves are isentropes. Because both the initial and final states are on the same isentrope, the change in the entropy of the system is zero.

20.30. The state of the gas does not depend on the process by which it reached that state. We can find the temperature from the ideal gas law,

$$T = \frac{PV}{Nk_B} = \frac{PV}{(m_{gas}/m_{N_2})k_B} = \frac{(4.652\times10^{-26}\ \text{kg})(150\times10^3\ \text{Pa})(0.150\ \text{m}^3)}{(0.300\ \text{kg})(1.381\times10^{-23}\ \text{J/K})} = 253\ \text{K}$$

20.31. From the ideal gas law, we know that for a given sample of gas, PV/T is constant. So, we have for A $\frac{P_AV_A}{T_A} = \frac{P_{400}V_{400}}{400\ \text{K}}$, or

$$T_A = \frac{P_AV_A}{P_{400}V_{400}}(400\ \text{K}) = \frac{(5.0\ \text{kPa})(0.160\ \text{m}^3)}{(6.0\ \text{kPa})(0.170\ \text{m}^3)}(400\ \text{K}) = 3.1\times10^2\ \text{K}$$

Similarly, we have for B

$$T_B = \frac{(3.0\ \text{kPa})(0.150\ \text{m}^3)}{(6.0\ \text{kPa})(0.170\ \text{m}^3)}(400\ \text{K}) = 1.8\times10^2\ \text{K}$$

and for C

$$T_C = \frac{(2.0\ \text{kPa})(0.130\ \text{m}^3)}{(6.0\ \text{kPa})(0.170\ \text{m}^3)}(400\ \text{K}) = 1.0\times10^2\ \text{K}$$

20.32. (*a*) The work done by the gas has the same magnitude and opposite sign as the work done on the gas, which is given by Eqs. 20.8 and 20.9. As the gas expands from state 1 to 2, it does positive work and $W_{1\to2} = P\Delta V = (1000\ \text{Pa})\left[(0.800\ \text{m}^3) - (0.100\ \text{m}^3)\right] = 700\ \text{J}$. As the pressure decreases from state 2 to 3, the volume does not change, so the gas does no work, $W_{2\to3} = 0$. As the gas is compressed from state 3 to 4, it does negative work and $W_{3\to4} = P\Delta V = (500\ \text{Pa})\left[(0.100\ \text{m}^3) - (0.800\ \text{m}^3)\right] = -350\ \text{J}$. As the pressure increases from state 4 to 1, the volume does not change, so the gas does no work, $W_{4\to1} = 0$. (*b*) The work done by the gas over the whole process is the sum of the work done over each leg, or $(700\ \text{J}) + 0 + (-350\ \text{J}) + 0 = 350\ \text{J}$.

20.33. The magnitude of the work done on the gas in each step of the process equals the area under the PV curve for that step, and the work is positive when the gas is compressed and negative when it expands. So, negative work is done on the gas as it expands from state 1 to state 2, positive work is done on it as it is compressed from state 2 to state 3, and no work is done on it as the pressure increases from state 3 to state 1. The total work done on the gas is negative, and its magnitude is the area of the triangle enclosed by the PV curves, $W = -\frac{1}{2}\Delta P\Delta V = -\frac{1}{2}(P_i - \frac{1}{3}P_i)(4V_i - V_i) = -\frac{1}{2}(\frac{2}{3}P_i)(3V_i) = -P_iV_i$.

20.34. If the volume of the gas does not change, it does no work, so the change in its thermal energy equals the energy it absorbed. The change in its thermal energy is proportional to the change in its temperature (Eq. 20.5), so

$$\Delta E_{\text{th}} = \tfrac{3}{2} N k_{\text{B}} \Delta T$$
$$\Delta T = \frac{\Delta E_{\text{th}}}{\tfrac{3}{2} N k_{\text{B}}}$$
$$T_{\text{f}} = \frac{\Delta E_{\text{th}}}{\tfrac{3}{2} N k_{\text{B}}} + T_{\text{i}}$$
$$= \frac{100\ \text{J}}{\tfrac{3}{2}(6.02\times 10^{23})(1.381\times 10^{-23}\ \text{J/K})} + (72\ \text{K}) = 80\ \text{K}$$

20.35. Because the thermal energy is transferred slowly, the process is quasistatic. If the expansion is isobaric, the work done on the gas is given by Eq. 20.9, $W = -P\Delta V = -P(A\Delta x) = -(5.0\times 10^4\ \text{Pa})(0.10\ \text{m}^2)(0.10\ \text{m}) = -5.0\times 10^2\ \text{J}$.

20.36. A temperature of $100\ °\text{F}$ corresponds to an absolute temperature of $\left[\tfrac{5}{9}(T_{\text{F}} - 32) + 273\right]\ \text{K} = 311\ \text{K}$. At this temperature, diatomic nitrogen molecules have five degrees of freedom contributing to their heat capacity, three translational and two rotational, so the thermal energy is (Eq. 20.4) $E_{\text{th}} = \dfrac{d}{2} N k_{\text{B}} T = \tfrac{5}{2}(5.6\times 10^{18})(1.381\times 10^{-23}\ \text{J/K})(311\ \text{K}) = 6.0\times 10^{-2}\ \text{J}$.

20.37. We know the quantity of gas in the chamber, its volume, and its temperature, so we can use the ideal gas law to determine its pressure. The pressure exerts a force on the piston, which must have the same magnitude as the force exerted on the piston by the block plus that exerted by atmospheric pressure. The magnitude of the force exerted by the piston on the block equals that of the force of gravity on the piston. Combining these we have

$$P = \frac{N k_{\text{B}} T}{V} = \frac{m_{\text{block}} g}{A_{\text{piston}}} + P_{\text{atm}}$$
$$A_{\text{piston}} = \frac{m_{\text{block}} g}{\dfrac{N k_{\text{B}} T}{V} - P_{\text{atm}}}$$
$$\pi R_{\text{piston}}^2 = \frac{m_{\text{block}} g}{\dfrac{N k_{\text{B}} T}{V} - P_{\text{atm}}}$$
$$R_{\text{piston}} = \sqrt{\frac{m_{\text{block}} g}{\pi\left(\dfrac{N k_{\text{B}} T}{V} - P_{\text{atm}}\right)}}$$
$$= \sqrt{\frac{(100\ \text{kg})(9.8\ \text{N/kg})}{\pi\left(\dfrac{(10.0\ \text{mol})(6.02\times 10^{23}\ \text{mol}^{-1})(1.381\times 10^{-23}\ \text{J/K})(300\ \text{K})}{0.100\ \text{m}^3} - (1.01\times 10^5\ \text{Pa})\right)}} = 4.58\times 10^{-2}\ \text{m}$$

20.38. (*a*) The vertical legs of both processes take place at constant volume, so they do no work on the gas. For the horizontal, isobaric legs, the work done on the gas is positive (because the gas is compressed) and proportional to the pressure. Because the isobaric leg of process A is at lower pressure than that of process B, less work is done on the gas in process A. (*b*) The product of the pressure times the volume of the gas is proportional to its temperature. Comparing these for the initial and final states we see that the final temperature of the gas is lower than the initial temperature:

$$P_{\text{i}} V_{\text{i}} = (9.68\ \text{kPa})(1.75\ \text{m}^3) = 16.9\ \text{kPa}\cdot\text{m}^3$$
$$P_{\text{f}} V_{\text{f}} = (22.4\ \text{kPa})(0.45\ \text{m}^3) = 10.0\ \text{kPa}\cdot\text{m}^3$$

Because the thermal energy of the gas is proportional to its temperature, its thermal energy must have decreased, so by Eq. 20.2, a negative amount of energy was transferred thermally to the gas. Because less work was done on the gas in process A, less energy must be removed by thermal transfer. That is, $|Q_A| < |Q_B|$. (*c*) As we saw in part *a*, work is only done on the gas during the horizontal, isobaric legs of the processes, and that work is given by Eq. 20.9, so

$$\begin{aligned} W_B - W_A &= (-P_B \Delta V_B) - (-P_A \Delta V_A) \\ &= (P_B - P_A)(-\Delta V) \\ &= \left[(22.4\times10^3 \text{ Pa}) - (9.68\times10^3 \text{ Pa})\right]\left[(1.75 \text{ m}^3) - (0.45 \text{ m}^3)\right] = 1.65\times10^4 \text{ J} \end{aligned}$$

20.39. In each temperature range, the change in thermal energy is given by Eq. 20.5. For the first stage, the gas has five contributing degrees of freedom, and $\Delta E_{th} = \frac{5}{2} N k_B \Delta T = \frac{5}{2}(6.02\times10^{23})(1.381\times10^{-23} \text{ J/K})(400 \text{ K}) = 8.31\times10^3 \text{ J}$. For the second stage, the gas has seven contributing degrees of freedom, and $\Delta E_{th} = \frac{7}{2} N k_B \Delta T = \frac{7}{2}(6.02\times10^{23})(1.381\times10^{-23} \text{ J/K})(200 \text{ K}) = 5.82\times10^3 \text{ J}$, giving a total of $(8.31\times10^3 \text{ J}) + (5.82\times10^3 \text{ J}) = 1.41\times10^4 \text{ J}$.

20.40. (*a*) We know the number of particles, the volume of the gas, and its pressure, so we can determine its temperature from the ideal gas law:

$$T_i = \frac{P_i V}{N k_B} = \frac{(1.84\times10^4 \text{ Pa})(0.0100 \text{ m}^3)}{(9.70\times10^{21})(1.381\times10^{-23} \text{ J/K})} = 1.37\times10^3 \text{ K}$$

and

$$T_f = \frac{P_f V}{N k_B} = \frac{(2.68\times10^4 \text{ Pa})(0.0100 \text{ m}^3)}{(9.70\times10^{21})(1.381\times10^{-23} \text{ J/K})} = 2.00\times10^3 \text{ K}$$

The chamber's volume does not change, so no work is done on the gas, and all the added energy increases its thermal energy. We can find the number of contributing degrees of freedom from Eq. 20.5,

$$\Delta E_{th} = \frac{d}{2} N k_B \Delta T$$

$$d = \frac{2\Delta E_{th}}{N k_B \Delta T} = \frac{2(378 \text{ J})}{(9.70\times10^{21})(1.381\times10^{-23} \text{ J/K})\left[(2.00\times10^3 \text{ K}) - (1.37\times10^3 \text{ K})\right]} = 9.0$$

If the gas were monatomic, it would have only three translational degrees of freedom, so the gas must be more complex. If the gas were diatomic, it would have three translational degrees of freedom, two rotational degrees of freedom, and maybe two vibrational degrees of freedom ("maybe" because the vibrational degrees of freedom might not contribute much at these temperatures for a light gas; see *Principles* Figure 20.15 for hydrogen), for a total of seven, so our gas particles must be more complex still. (*b*) Given these choices, ammonia is the only reasonable one because it is the only molecule complex enough to have nine contributing degrees of freedom.

20.41. No work is done on the gas in the isochoric leg of the process because the volume of the gas does not change.

20.42. (*a*) The mass of the gas is the product of its mass density and its volume, $m = \rho V = (1.13 \text{ kg/m}^3)(1.5 \text{ m})(2.0 \text{ m})(1.0 \text{ m}) = 3.4 \text{ kg}$. (*b*) If we assume the heating of the air was an isochoric process, then all of the energy went toward raising the temperature of the air, and we have $E_{Sun} = \Delta E_{th} = cm\Delta T = (720 \text{ J/K}\cdot\text{kg})(3.4 \text{ kg})(13 \text{ K}) = 3.2\times10^4 \text{ J}$. This assumption is not completely valid because the car is not completely air-tight. So rather than the air pressure increasing as it is heated, some of the air escapes from the car. But the relative change in absolute temperature is small (a few percent), so our answer should not be too far off.

20.43. (*a*) Using the values for the mass density and specific heat capacity for air from Problem 20.42, and assuming an isochoric process as we did there, the change in temperature is

$$E_{Sun} = \Delta E_{th} = cm\Delta T$$

$$E_{Sun} = \Delta E_{th} = cm\Delta T$$

$$\Delta T = \frac{E_{Sun}}{cm} = \frac{E_{Sun}}{c\rho V} = \frac{3.2\times10^4 \text{ J}}{(720 \text{ J/K}\cdot\text{kg})(1.13 \text{ kg/m}^3)(2.0\times10^2 \text{ m}^2)(3.0 \text{ m})} = 6.6\times10^{-2} \text{ K}$$

(*b*) Using the same values and assumptions, we have

$$\begin{aligned} E_{\text{Sun}} &= \Delta E_{\text{th}} = cm\Delta T = c\rho V\Delta T \\ &= (720\ \text{J/K}\cdot\text{kg})(1.13\ \text{kg/m}^3)(2.0\times10^2\ \text{m}^2)(3.0\ \text{m})(1\ \text{K}) = 5\times10^5\ \text{J} \end{aligned}$$

20.44. [**NOTE:** Part (c) of problem statement before the Hint is changed to read "Once the steam cools sufficiently, it begins to condense into liquid water and the can collapses inward. Why does the can retain its shape for much, but not all, of this process?"] (a) We can determine the pressure from the ideal gas law,

$$P_{\text{f}} = \frac{Nk_{\text{B}}T_{\text{f}}}{V} = \frac{P_{\text{i}}T_{\text{f}}}{T_{\text{i}}} = \frac{(101.3\ \text{kPa})\left[(101+273)\ \text{K}\right]}{\left[(120+273)\ \text{K}\right]} = 96.4\ \text{kPa}$$

(*b*) The force exerted by the atmosphere on the can equals the product of atmospheric pressure times the surface area of the can. For the curved portion, $F = P_{\text{atm}}A = P_{\text{atm}}(2\pi Rh) = (101.3\ \text{kPa})2\pi(0.050\ \text{m})(0.20\ \text{m}) = 6.4\times10^3\ \text{N}$. For each of the flat portions, $F = P_{\text{atm}}A = P_{\text{atm}}(\pi R^2) = (101.3\ \text{kPa})\pi(0.050\ \text{m})^2 = 8.0\times10^2\ \text{N}$. (*c*) At $101\ ^\circ\text{C}$, the vector sum of the forces exerted on the can's surfaces equals the pressure difference across the can times the surface area. For the curved portion,

$$\begin{aligned} F &= (P_{\text{atm}} - P_{\text{can}})A = (P_{\text{atm}} - P_{\text{can}})(2\pi Rh) \\ &= \left[(101.3\ \text{kPa}) - (96.4\ \text{kPa})\right]2\pi(0.050\ \text{m})(0.20\ \text{m}) = 3.1\times10^2\ \text{N} \end{aligned}$$

For each of the flat portions,

$$\begin{aligned} F &= (P_{\text{atm}} - P_{\text{can}})A = (P_{\text{atm}} - P_{\text{can}})(\pi R^2) \\ &= \left[(101.3\ \text{kPa}) - (96.4\ \text{kPa})\right]\pi(0.050\ \text{m})^2 = 38\ \text{N} \end{aligned}$$

The steel can can withstand these forces.

As the steam condenses into liquid water, the number of steam particles in the can decreases, further reducing the pressure until the can can no longer support the pressure difference.

20.45. As the gas expands adiabatically and quasistatically, it does so isentropically, so the work done on it equals the change in its thermal energy. The work it does equals minus the work done on it, so

$$\begin{aligned} W_{\text{by gas}} &= -W = -\Delta E_{\text{th}} \\ &= -\tfrac{3}{2}Nk_{\text{B}}\Delta T \\ N &= -\frac{2W_{\text{by gas}}}{3k_{\text{B}}\Delta T} \\ &= -\frac{2(9.05\ \text{J})}{3(1.381\times10^{-23}\ \text{J/K})(-4\ \text{K})} = 1\times10^{23} \end{aligned}$$

20.46. (*a*) Because the process is isobaric, the final pressure is the same as the initial pressure, namely, $(1.00\ \text{atm})\left[(101.3\ \text{kPa})/(1\ \text{atm})\right] = 101\ \text{kPa}$. (*b*) From the ideal gas law, we see that the change in volume is proportional to the change in temperature, so

$$V_{\text{f}} = \frac{V_{\text{i}}T_{\text{f}}}{T_{\text{i}}} = \frac{(2.00\ \text{L})(265\ \text{K})}{(273\ \text{K})}\frac{1\ \text{m}^3}{1000\ \text{L}} = 1.94\times10^{-3}\ \text{m}^3$$

20.47. For an isothermal process, the work done on the gas is given by Eq. 20.30,

$$\begin{aligned} W &= -Nk_{\text{B}}T\ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right) \\ &= -(1.00\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})(1.381\times10^{-23}\ \text{J/K})(293.15\ \text{K})\ln(\tfrac{1}{3}) = 2.68\times10^3\ \text{J} \end{aligned}$$

20.48. If the pressure remains constant, the work done on the gas is given by Eq. 20.22,

$$W = -P\Delta V$$
$$= -(1.00\ \text{atm})\frac{101.3\ \text{kPa}}{1\ \text{atm}}\left[(0.333\ \text{L}) - (1.00\ \text{L})\right]\frac{1\ \text{m}^3}{1000\ \text{L}} = 67.6\ \text{J}$$

20.49. Because the gas does work in an isobaric process that raises its temperature, the heat capacity per particle at constant pressure is greater than the heat capacity per particle at constant volume, so the heat capacity ratio should be greater than one. For a diatomic gas at room temperature, we expect to have five degrees of freedom, three translational and two rotational, contributing to its heat capacity, so we expect the heat capacity ratio to be about $7/5$ (Eq. 20.26).

20.50. Because the bubble rises slowly, it is a quasistatic process. At any point, the pressure in the gas equals that of its surroundings and so does its temperature. (*a*) If the number if gas particles in the bubble remains constant, the ideal gas law gives

$$\frac{P_\text{i}V_\text{i}}{T_\text{i}} = \frac{P_\text{f}V_\text{f}}{T_\text{f}}$$
$$\frac{V_\text{f}}{V_\text{i}} = \frac{P_\text{i}T_\text{f}}{T_\text{i}P_\text{f}}$$
$$= \frac{(2.00\ \text{atm})(293.15\ \text{K})}{(283.15\ \text{K})(1.00\ \text{atm})} = 2.07$$

(*b*) The change in the thermal energy of the gas is proportional to the change in temperature (Eq. 20.15), $\Delta E_\text{th} = NC_V\Delta T.$ The heat capacity per particle at constant volume for air is about $2.50k_\text{B}$, and we can determine the number of particles from the ideal gas law, $N = \dfrac{PV}{k_\text{B}T}.$ Combining these we have

$$\Delta E_\text{th} = \frac{P_\text{i}V_\text{i}}{k_\text{B}T_\text{i}}(2.50k_\text{B})\Delta T = \frac{P_\text{i}(\tfrac{4}{3}\pi R_\text{i}^3)}{T_\text{i}}(2.50)\Delta T$$
$$= \tfrac{4}{3}\pi\frac{(2.00\ \text{atm})(5.00\times10^{-3}\ \text{m})^3}{283.15\ \text{K}}\frac{101.3\ \text{kPa}}{1\ \text{atm}}(2.50)(10.0\ \text{K}) = 9.37\times10^{-3}\ \text{J}$$

20.51. (*a*) We know the formula for the work done on the gas in an isothermal process, Eq. 20.30. The work done by the gas is the negative of the work done on it, so

$$W_\text{by gas} = -W = Nk_\text{B}T\ln\left(\frac{V_\text{f}}{V_\text{i}}\right)$$
$$= (1.00\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})(1.381\times10^{-23}\ \text{J/K})(296\ \text{K})\ln(\tfrac{1}{3}) = -2.70\times10^3\ \text{J}$$

(*b*) In this case we have

$$W_\text{by gas} = -W = Nk_\text{B}T\ln\left(\frac{V_\text{f}}{V_\text{i}}\right)$$
$$= (1.00\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})(1.381\times10^{-23}\ \text{J/K})(296\ \text{K})\ln(3) = 2.70\times10^3\ \text{J}$$

20.52. The work done on a gas when its volume is changed isothermally is given by Eq. 20.30, so we have $W_1 = Nk_\text{B}T\ln(f_1)$ and $W_2 = Nk_\text{B}T\ln(f_2)$. We can combine these and rearrange to find

$$W_1 = Nk_\text{B}T\ln(f_1)$$
$$Nk_\text{B}T = \frac{W_1}{\ln(f_1)}$$
$$W_2 = Nk_\text{B}T\ln(f_2) = \frac{W_1}{\ln(f_1)}\ln(f_2)$$

20.53. We can solve this problem using the ideal gas law. The pressure in state 3 is $P_3 = \dfrac{Nk_B T_3}{V_3}$. Because the second process is isothermal, $T_3 = T_2$, and we can determine the remaining variables from those of state 1, $Nk_B = \dfrac{P_1 V_1}{T_1}$. Combining these we have $P_3 = \dfrac{P_1 V_1 T_2}{T_1 V_3}$.

20.54. The work done by the gas is minus the work done on the gas. For an isothermal expansion we have from Eq. 20.30 $W = -Nk_B T \ln\left(\dfrac{V_f}{V_i}\right)$. From the ideal gas law we know that $Nk_B T = PV$, so we have $W_{\text{by gas}} = -W = Nk_B T \ln\left(\dfrac{V_f}{V_i}\right) = P_f V_f \ln\left(\dfrac{V_f}{V_i}\right)$, which we can solve for the final pressure

$$P_f = \frac{W_{\text{by gas}}}{V_f \ln\left(\dfrac{V_f}{V_i}\right)} = \frac{26\text{ J}}{(2.0\text{ L})\ln(2)} \frac{1000\text{ L}}{1\text{ m}^3} = 1.9\times10^4\text{ Pa}$$

20.55. (*a*) The work done by the gas is minus the work done on the gas. For an isothermal expansion we have from Eq. 20.30 $W = -Nk_B T \ln\left(\dfrac{V_f}{V_i}\right)$. When the gas expands isothermally, the change in its entropy is given by Eq. 19.8, $\Delta S = N \ln\left(\dfrac{V_f}{V_i}\right)$. We are told that the entropy of the gas doubles, so $\Delta S = S_f - \frac{1}{2}S_f = \frac{1}{2}S_f$, and the work done by the gas is

$$W_{\text{by gas}} = -W = Nk_B T \ln\left(\frac{V_f}{V_i}\right) = k_B T \Delta S$$
$$= \tfrac{1}{2}k_B T S_f = \tfrac{1}{2}(1.381\times10^{-23}\text{ J/K})(100\text{ K})(1.345\times10^{24}) = 929\text{ J}.$$

(*b*) For an isothermal process, $Q = -W$, so the energy transferred thermally to the gas is also 929 J.

20.56. [NOTE: The problem statement is changed so that the number of helium atoms is specified as 7.55×10^{24}.]
(*a*) When the piston is released, the difference in pressure on its two surfaces causes it to accelerate upward, increasing both its kinetic and gravitational potential energy. The change in the piston's energy equals the work done on the piston, which in turn equals the integral of the force exerted on the piston with respect to its displacement. That is, choosing an upward-pointing *x* axis,

$$\Delta E_{\text{piston}} = \int_{x_i}^{x_f} F_{\text{piston}}\, dx$$

The force exerted on the piston equals the product of the difference in pressure on its surfaces times its area, so

$$\Delta E_{\text{piston}} = \int_{x_i}^{x_f} (P - P_{\text{atm}}) A_{\text{piston}}\, dx = \int_{x_i}^{x_f} P A_{\text{piston}}\, dx - \int_{x_i}^{x_f} P_{\text{atm}} A_{\text{piston}}\, dx$$

The first term on the right equals the work done by the gas, so we have for the work done on the gas $W = -\Delta E_{\text{piston}} - P_{\text{atm}} A_{\text{piston}} \Delta x$. For an isentropic expansion, $W = \Delta E_{\text{th}} = NC_V \Delta T$, or in the case of helium, $W = \frac{3}{2}Nk_B \Delta T$. Combining this with our previous result we have

$$\tfrac{3}{2}Nk_B \Delta T = -\Delta E_{\text{piston}} - P_{\text{atm}} A_{\text{piston}} \Delta x$$

$$\Delta T = \frac{-2(\Delta E_{\text{piston}} + P_{\text{atm}} A_{\text{piston}} \Delta x)}{3Nk_{\text{B}}} = \frac{-2\left[(\frac{1}{2}mv^2 + mg\Delta x) + P_{\text{atm}}(\pi R_{\text{piston}}^2)\Delta x\right]}{3Nk_{\text{B}}}$$

$$= \frac{-2}{3(7.55\times 10^{24})(1.381\times 10^{-23}\ \text{J/K})}\left[\tfrac{1}{2}(0.320\ \text{kg})(43.5\times 10^{-3}\ \text{m/s})^2 + (0.320\ \text{kg})(9.8\ \text{N/kg})(0.320\ \text{m}) + (101.3\ \text{kPa})\pi(0.200\ \text{m})^2(0.320\ \text{m})\right]$$

$$= -26.05\ \text{K}$$

$$T_{\text{f}} = T_{\text{i}} + \Delta T = (275.15\ \text{K}) + (-26.05\ \text{K}) = 249.1\ \text{K}$$

(*b*) For an isothermal expansion, the temperature of the gas does not change, so its final temperature equals its initial temperature, $275.15\ \text{K}$.

20.57. [**NOTE:** Problem statement is changed. In first line after the equation, "and n is the number density of the gas particles" changes to "and n is the number of moles of the gas particles."] We can determine the work done on a van der Waals gas in an isothermal process from Eq. 20.8, $W = -\int_{V_{\text{i}}}^{V_{\text{f}}} P\,dV$. Rearranging the van der Waals equation yields

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = Nk_{\text{B}}T$$

$$P + \frac{an^2}{V^2} = \frac{Nk_{\text{B}}T}{V - nb}$$

$$P = \frac{Nk_{\text{B}}T}{V - nb} - \frac{an^2}{V^2}$$

so the work done on the gas is

$$W = -\int_{V_{\text{i}}}^{V_{\text{f}}} \left(\frac{Nk_{\text{B}}T}{V - nb} - \frac{an^2}{V^2}\right) dV$$

$$= -Nk_{\text{B}}T \ln(V - nb) - \frac{an^2}{V}\Bigg|_{V_{\text{i}}}^{V_{\text{f}}}$$

$$= -Nk_{\text{B}}T \ln\left(\frac{V_{\text{f}} - nb}{V_{\text{i}} - nb}\right) - an^2\left(\frac{1}{V_{\text{f}}} - \frac{1}{V_{\text{i}}}\right)$$

When $a = b = 0$, this reduces to $W = -Nk_{\text{B}}T \ln\left(\frac{V_{\text{f}}}{V_{\text{i}}}\right)$, as expected for an ideal gas.

20.58. (*a*) During the isothermal expansion, the temperature of the gas does not change, so from the ideal gas law the product of the pressure times the volume remains constant, and we have $V_2 = \frac{P_1 V_1}{P_2} = \frac{(5.00\ \text{atm})(20.0\ \text{L})}{2.00\ \text{atm}} \frac{1\ \text{m}^3}{1000\ \text{L}} = 0.0500\ \text{m}^3$. The work done by the gas during the isothermal expansion equals minus the work done on the gas, and we have from Eq. 20.30

$$W_{\text{by gas } 1\to 2} = -W_{1\to 2} = Nk_{\text{B}}T \ln\left(\frac{V_2}{V_1}\right) = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$= (5.00\ \text{atm})\frac{101.3\ \text{kPa}}{1\ \text{atm}}(0.0200\ \text{m}^3)\ln\left(\frac{0.0500\ \text{m}^3}{0.0200\ \text{m}^3}\right) = 9.28\times 10^3\ \text{J}$$

where we have used the ideal gas law to substitute for *N*. (*b*) Eq. 20.46 gives a relation between initial and final states for an isentropic process that we can use to determine the volume,

$$P_3V_3^\gamma = P_2V_2^\gamma$$
$$V_3 = \left(\frac{P_2}{P_3}\right)^{1/\gamma} V_2$$
$$= \left(\frac{2.00 \text{ atm}}{5.00 \text{ atm}}\right)^{5/7} (0.0500 \text{ m}^3) = 0.0260 \text{ m}^3$$

and then we can use the ideal gas law to determine the temperature,

$$T_3 = \frac{P_3V_3}{Nk_\text{B}} = \frac{P_3V_3T_2}{P_2V_2} = \frac{(5.00 \text{ atm})(0.0260 \text{ m}^3)(320 \text{ K})}{(2.00 \text{ atm})(0.0500 \text{ m}^3)} = 416 \text{ K}$$

The work done on the gas during the isentropic compression equals the change in its thermal energy, which is given by Eq. 20.15,

$$W_{2\to3} = NC_V\Delta T = N(\tfrac{5}{2}k_\text{B})\Delta T = \tfrac{5}{2}\frac{P_2V_2}{T_2}\left(\frac{P_3V_3}{P_2V_2} - 1\right)T_2$$
$$= \tfrac{5}{2}(P_3V_3 - P_2V_2) = \tfrac{5}{2}\left[(5.00 \text{ atm})(0.0260 \text{ m}^3) - (2.00 \text{ atm})(0.0500 \text{ m}^3)\right]\frac{101.3 \text{ kPa}}{1 \text{ atm}} = 7.58\times10^3 \text{ J}$$

(*c*) For the isothermal expansion, the change in entropy is given by Eq. 19.8,

$$\Delta S_{1\to2} = N\ln\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1}{k_\text{B}T_1}\ln\left(\frac{V_2}{V_1}\right)$$
$$= \frac{(5.00 \text{ atm})(0.0200 \text{ m}^3)}{(1.381\times10^{-23} \text{ J/K})(320 \text{ K})}\frac{101.3 \text{ kPa}}{1 \text{ atm}}\ln\left(\frac{0.0500 \text{ m}^3}{0.0200 \text{ m}^3}\right) = 2.10\times10^{24}$$

For the isentropic compression, the change in entropy is zero, $\Delta S_{2\to3} = 0$, so the sum of the changes in entropy is $\Delta S_{1\to2} + \Delta S_{2\to3} = 2.10\times10^{24}$. (*d*) The change in entropy for an isobaric process is given by Eq. 20.40,

$$\Delta S_{1\to3} = N\left(1 + \frac{d}{2}\right)\ln\left(\frac{T_3}{T_1}\right) = \frac{P_1V_1}{k_\text{B}T_1}(1 + \tfrac{5}{2})\ln\left(\frac{T_3}{T_1}\right)$$
$$= \frac{(5.00 \text{ atm})(0.0200 \text{ m}^3)}{(1.381\times10^{-23} \text{ J/K})(320 \text{ K})}\frac{101.3 \text{ kPa}}{1 \text{ atm}}\tfrac{7}{2}\ln\left(\frac{416 \text{ K}}{320 \text{ K}}\right) = 2.10\times10^{24}$$

as expected, because the change in entropy depends only on the initial and final states, not on the process that links them.

20.59 We have a relation between initial and final states for an isentropic process that we can use to determine the final pressure (Eq. 20.46),

$$P_\text{f}V_\text{f}^\gamma = P_\text{i}V_\text{i}^\gamma$$
$$P_\text{f} = \left(\frac{V_\text{i}}{V_\text{f}}\right)^\gamma P_\text{i} = \left(\frac{0.61 \text{ m}^3}{1.00 \text{ m}^3}\right)^{1.4}(1.00 \text{ atm}) = (0.50 \text{ atm})\frac{101.3 \text{ kPa}}{1 \text{ atm}} = 51 \text{ kPa}$$

20.60. We have a relation between initial and final states for an isentropic process that we can use to determine the final pressure (Eq. 20.46),

$$P_\text{f}V_\text{f}^\gamma = P_\text{i}V_\text{i}^\gamma$$
$$P_\text{f} = \left(\frac{V_\text{i}}{V_\text{f}}\right)^\gamma P_\text{i} = \left(\frac{1.00 \text{ L}}{1.20 \text{ L}}\right)^{7/5}(100 \text{ Pa}) = 77.5 \text{ Pa}$$

20.61. The change in entropy depends only on the initial and final states, not on the process that transforms one into the other. For a monatomic ideal gas we have (Eq. 20.34)

$$\Delta S = N\ln\left(\frac{V_\text{f}}{V_\text{i}}\right) + \tfrac{3}{2}N\ln\left(\frac{T_\text{f}}{T_\text{i}}\right)$$

and we can use the ideal gas law to express the temperatures in terms of the desired variables, $T = \dfrac{PV}{Nk_B}$, combining which gives us $\Delta S = N\ln\left(\dfrac{V_f}{V_i}\right) + \frac{3}{2}N\ln\left(\dfrac{P_f V_f}{P_i V_i}\right)$. But we can simplify this still further using the values given in Figure P20.61. $\Delta S = N\ln(3) + \frac{3}{2}N\ln(9) = N\ln(3) + \frac{3}{2}N\left[2\ln(3)\right] = 4N\ln(3)$.

20.62. (*a*) We have a relation between initial and final states for an isentropic process that we can use to determine the final pressure (Eq. 20.46),

$$P_f V_f^\gamma = P_i V_i^\gamma$$

$$P_f = \left(\frac{V_i}{V_f}\right)^\gamma P_i = \left(\frac{1.5\ \text{m}^3}{0.50\ \text{m}^3}\right)^{1.4} (15\ \text{MPa}) = 70\ \text{MPa}$$

(*b*) Because the compression was isentropic, the entropy did not change. That is, $\Delta S = 0$.

20.63. The gas is heated at constant volume, and we know how to find the change in entropy for an isochoric process (Eq. 20.38), $\Delta S = \dfrac{NC_V}{k_B}\ln\left(\dfrac{T_f}{T_i}\right)$. We also know that, for an ideal gas, the heat capacities are related by Eq. 20.24, $C_P = C_V + k_B$. Combining these we have

$$\frac{C_P}{k_B} = \frac{C_V}{k_B} + 1 = \frac{\Delta S}{N\ln\left(\dfrac{T_f}{T_i}\right)} + 1$$

$$= \frac{1.8215\times10^{23}}{(\frac{1}{3}\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})\ln\left(\dfrac{500\ \text{K}}{273\ \text{K}}\right)} + 1 = 2.50$$

20.64. The change in entropy for an isothermal expansion is given by Eq. 19.8, $\Delta S = N\ln\left(\dfrac{V_2}{V_1}\right) =$ $(10\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})\ln(2) = 4.2\times10^{24}$.

20.65. As the air expands isentropically, it cools. We have a relation between initial and final states for an isentropic process that we can use to determine the final temperature (Eq. 20.47),

$$P_i^{(1/\gamma)-1} T_i = P_f^{(1/\gamma)-1} T_f$$

$$T_f = \left(\frac{P_i}{P_f}\right)^{(1/\gamma)-1} T_i = \left(\frac{5.0\ \text{atm}}{1.0\ \text{atm}}\right)^{-2/7} (298\ \text{K}) = 1.9\times10^2\ \text{K}$$

where we have used $\gamma = \frac{7}{5}$ for air, because it is mostly diatomic nitrogen, and have converted the temperature in degrees Celsius to the absolute temperature in kelvins, as we must for such ideal-gas equations.

20.66. For an isothermal process, the work done on the gas has the same magnitude but the opposite sign as the energy transferred thermally to the gas, so the change in entropy is (Eq. 20.36)

$$\Delta S = \frac{Q}{k_B T} = \frac{-W}{k_B T} = \frac{-2.0\times10^3\ \text{J}}{(1.381\times10^{-23}\ \text{J/K})(293\ \text{K})} = -4.9\times10^{23}$$

20.67. (*a*) We can rearrange Eq. 20.46 to find the heat capacity ratio, and then use Eq. 20.26 to find the number of degrees of freedom contributing to the heat capacity:

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$\frac{P_i}{P_f} = \left(\frac{V_f}{V_i}\right)^\gamma$$

$$\ln\left(\frac{P_i}{P_f}\right) = \gamma \ln\left(\frac{V_f}{V_i}\right)$$

$$\gamma = \frac{\ln(P_i/P_f)}{\ln(V_f/V_i)}$$

$$1 + \frac{2}{d} = \frac{\ln(P_i/P_f)}{\ln(V_f/V_i)}$$

$$\frac{2}{d} = \frac{\ln(P_i/P_f)}{\ln(V_f/V_i)} - 1$$

$$d = \frac{2}{\dfrac{\ln(P_i/P_f)}{\ln(V_f/V_i)} - 1} = \frac{2}{\dfrac{\ln(1.00\ \text{atm}/0.0215\ \text{atm})}{\ln(10.0\ \text{mL}/1.00\ \text{mL})} - 1} = 3.00$$

(*b*) We do not know the temperatures involved; if the temperature is sufficiently low, any ideal gas will have only three contributing degrees of freedom. But if the temperatures are not too low (say, greater than about 100 K), the gas is monatomic.

20.68. The change in entropy of the gas depends only on the initial and final states of the gas, and not on the processes that transform one into the other. From the ideal gas law, we have $P_1 = \dfrac{Nk_B T_1}{V_1} = \dfrac{Nk_B T_1}{V_i}$, and $V_2 = \dfrac{Nk_B T_2}{P_2} = \dfrac{Nk_B T_2}{P_1} = Nk_B T_2 \dfrac{V_i}{Nk_B T_1} = \dfrac{T_2}{T_1} V_i$. So, the change in entropy is (Eq. 20.34)

$$\Delta S = N\ln\left(\frac{V_f}{V_i}\right) + \tfrac{3}{2}N\ln\left(\frac{T_f}{T_i}\right) = N\ln\left(\frac{2V_2}{V_i}\right) + \tfrac{3}{2}N\ln\left(\frac{T_2}{T_i}\right)$$

$$= N\ln\left(\frac{2T_2}{T_1}\right) + \tfrac{3}{2}N\ln\left(\frac{T_2}{T_i}\right) = N\left[\ln\left(\frac{2T_2}{T_1}\right) + \tfrac{3}{2}\ln\left(\frac{T_2}{T_i}\right)\right]$$

$$= (6.02\times10^{23})\left[\ln\left(\frac{2(746\ \text{K})}{646\ \text{K}}\right) + \tfrac{3}{2}\ln\left(\frac{746\ \text{K}}{546\ \text{K}}\right)\right] = 7.86\times10^{23}$$

20.69. Following the hint, the work done by the gas is minus the work done on the gas, so

$$W_{\text{by gas}} = -W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{P_i V_i^\gamma}{V^\gamma}\,dV = P_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma}\,dV$$

$$= P_i V_i^\gamma \left.\frac{V^{1-\gamma}}{1-\gamma}\right|_{V_i}^{V_f} = \frac{P_i V_i^\gamma}{1-\gamma}\left[(6V_i)^{1-\gamma} - V_i^{1-\gamma}\right]$$

$$= \frac{P_i V_i}{1-\gamma}(6^{1-\gamma} - 1) = \tfrac{3}{2}P_i V_i(1 - 6^{-2/3})$$

20.70. The change in thermal energy equals the product of the mass of the water, its specific heat capacity, and the change in temperature (Eq. 20.50). We can determine the mass of the water from the number of moles by looking up its molar mass, which is 18.0 g/mol. So, the change in thermal energy is $\Delta E_{\text{th}} = mc_V \Delta T = (1.00\ \text{mol})$ $(0.0180\ \text{kg/mol})(4181\ \text{J/K}\cdot\text{kg})(2.0\ \text{K}) = 1.5\times10^2\ \text{J}$. We could also have calculated it from the heat capacity per particle (Eq. 20.15) and the data in Table 20.3 $\Delta E_{\text{th}} = NC_V\Delta T = (6.02\times10^{23})(8.97)(1.381\times10^{-23}\ \text{J/K})(2.0\ \text{K}) =$ $1.5\times10^2\ \text{J}$.

20.71. (*a*) The change in thermal energy equals the product of the mass of the water, its specific heat capacity, and the change in temperature (Eq. 20.50), $\Delta E_{\text{th}} = mc_V \Delta T = (0.500\text{ kg})(2900\text{ J/K}\cdot\text{kg})(-35.0\text{ K}) = -5.08\times10^4\text{ J}.$ (*b*) If the change in gravitational potential energy equals the magnitude of the change in thermal energy we have

$$mgh = -\Delta E_{\text{th}}$$

$$h = \frac{-\Delta E_{\text{th}}}{mg} = \frac{5.08\times10^4\text{ J}}{(3.80\text{ kg})(9.8\text{ N/kg})} = 1.36\times10^3\text{ m}$$

20.72. The change in entropy for an isothermal process is given by Eq. 20.54,

$$\Delta S = \frac{Q}{k_{\text{B}}T} = \frac{2.00\times10^4\text{ J}}{(1.381\times10^{-23}\text{ J/K})(283.15\text{ K})} = 5.11\times10^{24}$$

20.73. The change in entropy of the system equals the sum of the changes in entropy of the iceberg and the island. Because the process is approximately isothermal, we can use Eq. 20.54,

$$\Delta S = \Delta S_{\text{iceberg}} + \Delta S_{\text{island}} = \frac{Q_{\text{iceberg}}}{k_{\text{B}}T_{\text{iceberg}}} + \frac{Q_{\text{island}}}{k_{\text{B}}T_{\text{island}}}$$

$$= \frac{10\times10^3\text{ J}}{(1.381\times10^{-23}\text{ J/K})(273\text{ K})} + \frac{-10\times10^3\text{ J}}{(1.381\times10^{-23}\text{ J/K})(300\text{ K})} = 2.4\times10^{23}$$

20.74. To convert from Fahrenheit to Celsius, we subtract 32 and multiply by 5/9, $T_{\text{C}} = \frac{5}{9}(T_{\text{F}} - 32) = \frac{5}{9}(98.6 - 32) = 37.0.$ That is, normal body temperature is $37.0\text{ °C}.$

20.75. (*a*) Under normal circumstances, there are no negative temperatures on the kelvin scale, and the lowest possible temperature on the Celsius scale is $-273.15\text{ °C},$ so it is most likely a temperature in degrees Fahrenheit. (*b*) While temperatures of a fraction of a kelvin are achieved in laboratories, it is not possible to reach absolute zero, so a measured temperature of 0 K is not likely correct. Temperatures of 0 °C and 0 °F are both fairly common as winter outdoor air temperatures, depending on location. A glass of ice water has a temperature of $0\text{ °C}.$ A household freezer might have a temperature as low as $0\text{ °F}.$

20.76. We have an expression for the work done on an ideal gas in an isentropic process in terms of the change in its temperature (Eq. 20.19), and we can express the temperatures in terms of the pressures and volumes by using the ideal gas law. So, the work done on the gas is

$$W = NC_V\Delta T = NC_V(T_{\text{f}} - T_{\text{i}}) = NC_V\left(\frac{P_{\text{f}}V_{\text{f}}}{Nk_{\text{B}}} - \frac{P_{\text{i}}V_{\text{i}}}{Nk_{\text{B}}}\right) = \frac{C_V}{k_{\text{B}}}(P_{\text{f}}V_{\text{f}} - P_{\text{i}}V_{\text{i}})$$

20.77. From the ideal gas law we know $P = \frac{Nk_{\text{B}}T}{V},$ so $\Delta P = \frac{Nk_{\text{B}}}{V}\Delta T.$ We can determine *N* by looking up the molar mass of oxygen, which is $16.0\text{ g/mol},$ or 32.0 g/mol for diatomic oxygen molecules. So, we have

$$\Delta P = \frac{(22.7\text{ g})(6.02\times10^{23}\text{ mol}^{-1})(1.381\times10^{-23}\text{ J/K})}{(32.0\text{ g/mol})(0.0240\text{ m}^3)}(-21.50\text{ K}) = -5.28\times10^3\text{ Pa}$$

20.78. The change in entropy of the system equals the sum of the changes of the entropies of its parts. For isothermal processes, the change in entropy is given by Eq. 20.54, so we have $\Delta S = \Delta S_{\text{b}} + \Delta S_{\text{e}} = \frac{Q_{\text{b}}}{k_{\text{B}}T_{\text{b}}} + \frac{Q_{\text{e}}}{k_{\text{B}}T_{\text{e}}} = \frac{-Q_{\text{e}}}{k_{\text{B}}T_{\text{b}}} + \frac{Q_{\text{e}}}{k_{\text{B}}T_{\text{e}}}.$ Because the body's temperature is greater than the environment's, the change in the system's entropy is greater than zero, that is, the entropy of the system increases over time, in agreement with the second law of thermodynamics.

The rate at which the system's entropy increases is

$$\frac{\Delta S}{\Delta t}=\frac{Q_e}{\Delta t}\left(\frac{-1}{k_B T_b}+\frac{1}{k_B T_e}\right)=\frac{Q_e}{\Delta t}\frac{1}{k_B}\left(\frac{1}{T_e}-\frac{1}{T_b}\right)$$
$$=\frac{100\ \text{W}}{1.381\times10^{-23}\ \text{J/K}}\left(\frac{1}{293\ \text{K}}-\frac{1}{310\ \text{K}}\right)=1.35\times10^{21}\ \text{s}^{-1}$$

20.79. Because no energy is transferred thermally in an isentropic process, a greater quantity is transferred for sample A. For an isochoric process involving a monatomic ideal gas, the amount of energy transferred thermally is (Eq. 20.12)

$$Q_A=\tfrac{3}{2}Nk_B\Delta T=\tfrac{3}{2}Nk_B(5T_{tp}-T_{tp})=6Nk_BT_{tp}=6N(1.381\times10^{-23}\ \text{J/K})(273.16\ \text{K})=N(2.26\times10^{20}\ \text{J})$$

20.80. [**NOTE:** In the problem statement, 85.0 kg of liquid water is changed to 108 kg of liquid water.] There are at least two reasons not to touch the plate. First, the plate is in contact with the steam and so is hot. Second, the pressure of the steam in the reservoir is greater than atmospheric pressure, so removing the plate will release a jet of hot steam.

Because the steam was just able to raise the plate, the difference between the pressure of the steam in the reservoir and atmospheric pressure equals the force of gravity on the plate divided by the area of the vent, that is,

$$P_{steam}-P_{atm}=\frac{m_{plate}g}{A_{vent}}$$
$$P_{steam}=P_{atm}+\frac{m_{plate}g}{\pi R_{vent}^2}=(101.3\ \text{kPa})+\frac{(70.0\ \text{kg})(9.8\ \text{N/kg})}{\pi(0.0500\ \text{m})^2}=189\ \text{kPa}$$

Assuming that the portal through which our coworker saw a few drops of water condensing is thermally insulated, the steam is at its boiling point. (If not, the bulk of the steam may be at a higher temperature, in the same way that, in a warm room on a cold night, there might be a cold draft coming off an uninsulated window.) The boiling point of water varies with pressure. At this pressure, the boiling point is about 392 K, that is, about 19 K above the boiling point at atmospheric pressure.

At this temperature and pressure, the 100-m^3 reservoir contains $N=\frac{PV}{k_BT}=\frac{(189\ \text{kPa})(100\ \text{m}^3)}{(1.381\times10^{-23}\ \text{J/K})(392\ \text{K})}=3.49\times10^{27}$ steam particles, or $\frac{3.49\times10^{27}}{6.02\times10^{23}\ \text{mol}^{-1}}(18.0\times10^{-3}\ \text{kg/mol})=104\ \text{kg}$ of steam.

When the vent is uncovered and the pressure in the reservoir slowly falls to atmospheric pressure ("slowly" because the vent is small, which suggests that the temperature in the reservoir will not change much), it will contain $N=\frac{PV}{k_BT}=\frac{(101\ \text{kPa})(100\ \text{m}^3)}{(1.381\times10^{-23}\ \text{J/K})(392\ \text{K})}=1.87\times10^{27}$ steam particles, or $\frac{1.87\times10^{27}}{6.02\times10^{23}\ \text{mol}^{-1}}(18.0\times10^{-3}\ \text{kg/mol})=$ $55.8\ \text{kg}$ of steam.

As the 48.2 kg of steam that leaves the reservoir condenses, it will release $Q=L_vm=(2256\times10^3\ \text{J/kg})$ $(48.2\ \text{kg})=1.09\times10^8$ J of thermal energy. We and our coworker will not want to be nearby when this happens.

20.81. It is difficult to say how we should model the work done by the waves on the air in a tube. We might suppose that, as the volume of the air in the tube decreases, its pressure increases because the generator resists its flow. If this compression is quick enough, it will be an approximately isentropic process; if it is slow enough, it will be an approximately isothermal process. But to use either of these models, we would require more information, such as the change in pressure, the change in temperature, or the total volume of the air in the tube. All we know is the change in volume of the air in the tube, which suggests that we should model the process as isobaric. This is not unreasonable, provided that the generator does not restrict the flow of air through the tube very much.

In this case, the work done by the waves on the air in a tube is (Eq. 20.9)

$$W=-P\Delta V=-PA(-\Delta h)=PA\Delta h=(101.3\ \text{kPa})\left[\pi(1.00\ \text{m})^2\right](2.00\ \text{m})=6.36\times10^5\ \text{J}$$

With a wave arriving every 5.00 s, and a conversion efficiency of 68.0%, this corresponds to a power output per tube of $P = (68.0\%)\dfrac{W}{\Delta t} = (0.680)\dfrac{6.36\times10^5\ \text{J}}{5.00\ \text{s}} = 8.66\times10^4\ \text{W}$. So, in order to supply 0.500 MW, we will need $\dfrac{0.500\times10^6\ \text{W}}{8.66\times10^4\ \text{W}} = 5.78$ 2-m-diameter tubes, or at least 6 such tubes. Because we do not know how accurately our model describes a real tube, and also because the wave power fluctuates above and below the average, we will want to use more, say 10 or 12, tubes for the town's generator. As we can produce a few tubes per day, it should take no more than a single five-day work week to produce the required tubes.

21

DEGRADATION OF ENERGY

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{28} **2.** -10^{23} s^{-1} **3.** 10^{-1} **4.** 10^{5} kg/s **5.** 10^{1} **6.** 10^{5} m^{3} **7.** 10^{3} m^{3} **8.** 10^{-2} **9.** 10^{6} Pa

Guided Problems

21.2 Geothermal power

1. Getting Started In order for us to apply the laws of thermodynamics with confidence, we must assume that the system is isolated, and steady. That is, energy is not escaping the power plant halfway up the shaft, and energy input is not being used to raise portions of the power plant up to the steady-state temperature. We assume the steady state has already been achieved, and that all energy input occurs at the maximum depth in the form of heat input, and all energy output occurs at the surface in the form of work and heat output.

In order for our discussion of minimum thermal energy output to make sense, we must assume that the power plant is reversible. If the power plant were not operating using reversible steps, then the theoretical maximum efficiency for any thermodynamic process described in the chapter would not apply to this power plant.

2. Devise Plan The maximum possible efficiency, which is only valid for reversible processes, is given by equation (21.23):

$$\eta_{\max} = 1 - \frac{T_{\text{out}}}{T_{\text{in}}}$$

Equation (21.21) can be taken as the definition of the efficiency. Assuming maximum efficiency, we have:

$$\eta_{\max} = \frac{W}{Q_{\text{in}}}$$

Equating the two expressions for efficiency, and rearranging, we have

$$Q_{\text{in}} = \frac{W}{1 - \dfrac{T_{\text{out}}}{T_{\text{in}}}} \tag{1}$$

Because expression (1) was obtained by assuming the maximum possible efficiency, it corresponds to the minimum possible thermal energy input. Taking the time derivative of both sides of equation (1), and assuming no time dependence of the temperatures, we find

$$\left(\frac{dQ_{\text{in}}}{dt}\right)_{\min} = \frac{P}{1 - \dfrac{T_{\text{out}}}{T_{\text{in}}}} \tag{2}$$

Using conservation of energy, we can write $Q_{\text{in}} = W + Q_{\text{out}}$. Rearranging, and taking a time derivative of both sides yields

$$\frac{dQ_{\text{out}}}{dt} = \frac{dQ_{\text{in}}}{dt} - P$$

And substituting in equation (2), we find

$$\left(\frac{dQ_{\text{out}}}{dt}\right)_{\text{min}} = \left(\frac{T_{\text{out}}}{T_{\text{in}} - T_{\text{out}}}\right)P \tag{3}$$

3. Execute Plan Inserting numbers into equation (21.23), we find

$$\eta_{\text{max}} = 1 - \frac{T_{\text{out}}}{T_{\text{in}}} = 1 - \frac{(268\text{ K})}{(873\text{ K})} = 0.693$$

Inserting numbers into equations (2) and (3) above yields

$$\left(\frac{dQ_{\text{in}}}{dt}\right)_{\text{min}} = \frac{P}{1 - \dfrac{T_{\text{out}}}{T_{\text{in}}}} = \frac{(1.0\times10^7\text{ W})}{1 - \dfrac{(268\text{ K})}{(873\text{ K})}} = 1.44\times10^7\text{ W}$$

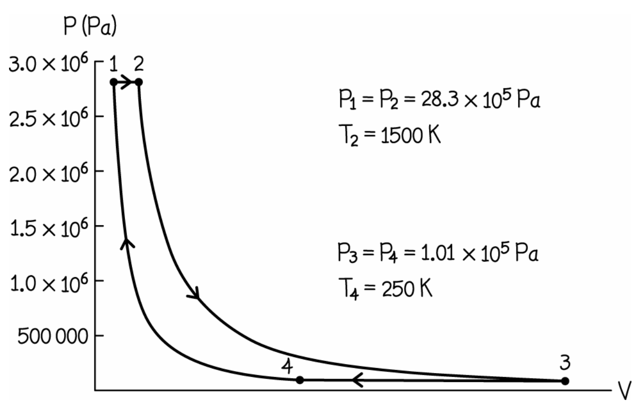
and

$$\left(\frac{dQ_{\text{out}}}{dt}\right)_{\text{min}} = \left(\frac{T_{\text{out}}}{T_{\text{in}} - T_{\text{out}}}\right)P = \left(\frac{(268\text{ K})}{(873\text{ K}) - (268\text{ K})}\right)(1.0\times10^7\text{ W}) = 4.4\times10^6\text{ W}$$

4. Evaluate Result The thermal output of energy is lower than the thermal input of energy, as it should be. This must be the case if any of the thermal input energy is to be turned into useful work.

21.4 Brayton cycle

1. Getting Started We begin by making a PV diagram:



Here we have labeled the highest and lowest pressures along the two isobars, as well as the two given temperatures. Note that air in taken in at point 4, just prior to the isentropic compression. The compressed working substance is then heated isobarically, meaning the maximum temperature is reached at the end of the top isobar.

2. Devise Plan One way of calculating the total work done by one cycle of a Brayton cycle is by using equation (21.37): $W = -Q = -NC_P(T_2 - T_1 + T_4 - T_3)$. This would require the determination of the remaining two temperatures, as well as the specific heat at constant pressure.

Another way to calculate the work done, is to determine the efficiency of the engine, and the heat taken in. This determines the work done on the gas because

$$\eta = \frac{W}{-Q_{\text{in}}}$$

The heat taken in is given by equation (21.38): $Q_{\text{in}} = NC_P(T_2 - T_1)$. And the efficiency is most compactly given by equation (21.47):

$$\eta = 1 - \left(\frac{P_{\text{H}}}{P_{\text{L}}}\right)^{\frac{1}{\gamma}-1}$$

This approach also requires the determination of temperature T_1. We will follow this approach, but we will go ahead and determine T_3 also and use equation (21.37) as a check of our work. We can determine T_1 by looking at the isentropic compression and applying equation (20.47): $T_1 P_1^{\frac{1}{\gamma}-1} = T_4 P_4^{\frac{1}{\gamma}-1}$. All that remains is to determine the specific heat ratio: $\gamma = 1 + \frac{2}{d}$.

And for diatomic nitrogen gas, there should be five degrees of freedom. In reality, the value of γ is slightly different from this ideal value of 1.4. But we will use this approximate value.

3. Execute Plan We first insert the given pressure ratio and the specific heat ratio into equation (21.38) to determine the efficiency:

$$\eta = 1 - \left(\frac{P_{\text{H}}}{P_{\text{L}}}\right)^{\frac{1}{\gamma}-1} = 1 - \left(\frac{28.3\times10^5 \text{ Pa}}{1.01\times10^5 \text{ Pa}}\right)^{\frac{1}{1.4}-1} = 0.6141$$

Next we determine T_1 by rearranging equation (20.47) and inserting the given values:

$$T_1 = T_4\left(\frac{P_4}{P_1}\right)^{\frac{1}{\gamma}-1} = (250 \text{ K})\left(\frac{1.01\times10^5 \text{ Pa}}{28.3\times10^5 \text{ Pa}}\right)^{\frac{1}{1.4}-1} = 647.9 \text{ K}$$

We can now write $W = -\eta Q_{\text{in}} = -\eta N C_P(T_2 - T_1) = -\eta N \frac{7}{2} k_{\text{B}}(T_2 - T_1)$.

Recalling that the total number of molecules is the number of moles times Avogadro's number, we rewrite the above expression as

$$\frac{W}{n} = \frac{7}{2}\eta N_{\text{A}} k_{\text{B}}(T_2 - T_1)$$

$$\frac{W}{n} = -\frac{7}{2}(0.6141)(6.022\times10^{23} \text{ mol}^{-1})(1.380\times10^{-23}\text{J/K})((1500 \text{ K}) - (647.9 \text{ K})) = -1.52\times10^4 \text{ J/mol}$$

This negative number is the work done on the gas. We want the work done by the gas on the environment, such that

$$\frac{W_{\text{on env}}}{n} = 1.52\times10^4 \text{ J/mol}$$

4. Evaluate Result It is easy to calculate T_3 since we have three of the four temperatures for the Brayton cycle, and equation (21.42) tells us

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\Rightarrow T_3 = \frac{T_2 T_4}{T_1} = \frac{(1500 \text{ K})(250 \text{ K})}{(647.9 \text{ K})} = 578.8 \text{ K}$$

We insert this into equation (21.37) to obtain the work in a different way:

$$W = -NC_P(T_2 - T_1 + T_4 - T_3)$$

$$\frac{W}{n} = -N_A \frac{7}{2} k_B (T_2 - T_1 + T_4 - T_3)$$

$$\frac{W}{n} = -\frac{7}{2}(6.022\times10^{23}\ \text{mol}^{-1})(1.380\times10^{-23}\ \text{J/K})((1500\ \text{K}) - (647.9\ \text{K}) + (250\ \text{K}) - (578.8\ \text{K}))$$

$$\frac{W}{n} = -1.52\times10^{4}\ \text{J/mol}$$

Again, this is the work done on the gas, such that

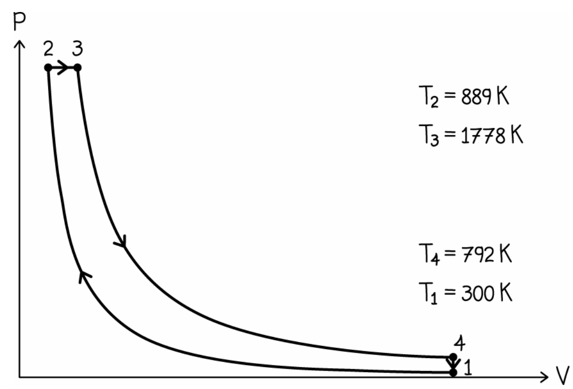
$$\frac{W_{\text{on env}}}{n} = 1.52\times10^{4}\ \text{J/mol}$$

This is exactly the answer we obtained before.

21.6 Diesel cycle

[NOTE: One printing of this problem statement incorrectly gave $T_4 = 884$ K. The problem should give $T_4 = 792$ K. We use this value in the solution below.]

1. Getting Started We begin by making a rough PV diagram:



No energy can be transferred thermally in the two isentropes. Energy is transferred thermally into the gas in the isobaric expansion $(2 \to 3)$, and energy is transferred thermally out of the gas into the environment in the isochoric reduction in pressure $(4 \to 1)$.

2. Devise Plan We can consult Table 20.6 to find expressions for the energy transferred thermally in various thermodynamic steps. We find

$$Q_{\text{in,isobar}} = Q_{23} = NC_P\Delta T = NC_P(T_3 - T_2) \tag{1}$$

$$Q_{\text{out,isochor}} = Q_{41} = NC_V\Delta T = NC_V(T_1 - T_4) \tag{2}$$

And of course $Q_{\text{isoentrope}} = 0$.

Since, air consists largely of diatomic molecules, we expect air to have three translational and two rotational degrees of freedom. At very high temperatures, we would also need to consider vibrational modes, but we will ignore these in this problem. This means there are five total degrees of freedom, which tells us

$$C_V = \frac{5}{2}k_B \tag{3}$$

$$C_P = \frac{7}{2}k_B \tag{4}$$

Throughout Principles, the term "work" is used to describe work on the gas (working substance) of the cycle. This is the sum of the work done on the gas in all steps of the thermodynamic process. In this case, work is done on the gas in the isobaric expansion $(2 \to 3)$, the isentropic expansion $(3 \to 4)$, and the isentropic compression $(1 \to 2)$. No work is done during the isochoric process. Clearly some of the work done by the above mentioned steps will be positive and some negative. We can get expressions for the work done during each step from Tables 20.6 and 21.1. We can write

$$W_{23} = W_{\text{isobar}} = -NC_P \Delta T = -NC_P (T_3 - T_2) \tag{5}$$

$$W_{34} = W_{\text{isentrope}} = NC_V \Delta T = NC_V (T_4 - T_3) \tag{6}$$

$$W_{12} = W_{\text{isentrope}} = NC_V \Delta T = NC_V (T_2 - T_1) \tag{7}$$

Efficiency is a measure of how much work we can get out of a process (on the environment) per energy we supply thermally to the process. Thus, the efficiency is

$$\eta = \frac{-W_{\text{on gas}}}{Q_{\text{in}}} \tag{8}$$

Equation (8) is sufficient to proceed, but we note also that all of Q_{in} that does not turn into mechanical work must leave the system as exhausted heat Q_{out}. Thus, the efficiency can also be written as

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} \tag{9}$$

Equation (9) is slightly more compact, so we will use it. But equation (8) could also be used to determine the efficiency in a slightly different way using different expressions from Tables 20.6 and 21.1. This would be one way of checking the final answer, although we use a different logical check below.

3. Execute Plan The heat transferred thermally into the system is given by equation (1). We insert equation (4) to obtain

$$Q_{\text{in,isobar}} = N\left(\frac{7}{2}\right)k_{\text{B}}(T_3 - T_2)$$

The heat transferred thermally out of the system is given by equation (2). We insert equation (3) to obtain

$$Q_{\text{out,isochor}} = N\left(\frac{5}{2}\right)k_{\text{B}}(T_1 - T_4)$$

This makes the efficiency

$$\eta = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{5(T_1 - T_4)}{7(T_3 - T_2)}$$

$$\eta = 1 - \left(\frac{5}{7}\right)\left(\frac{(300\text{ K}) - (792\text{ K})}{(1778\text{ K}) - (889\text{ K})}\right)$$

$$\eta = 0.605$$

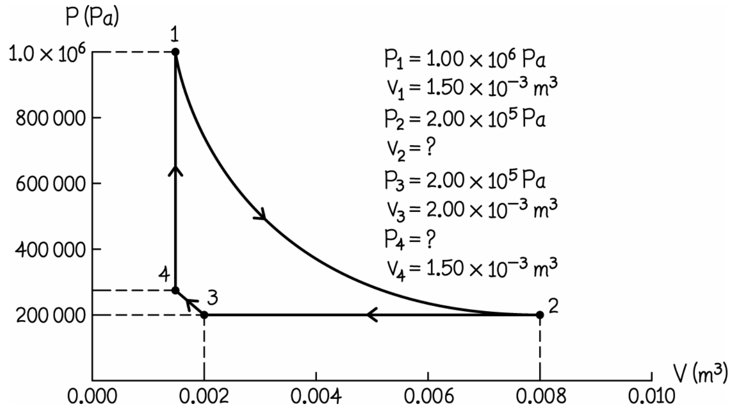
4. Evaluate Result A Carnot cycle running between a minimum temperature of 300 K and a maximum temperature of 1778 K would have an efficiency of

$$\eta_{\text{Carnot}} = 1 - \frac{T_{\text{out}}}{T_{\text{in}}} = 1 - \frac{(300\text{ K})}{(1778\text{ K})} = 0.831$$

This is somewhat higher than we found for our Diesel cycle. This is as it should be, since the Carnot cycle is the most efficient possible cycle. If we had obtained a higher efficiency for our Diesel cycle, we would know there was a mistake.

21.8 Cyclic process II

1. Getting Started Helium is a monatomic ideal gas, so it has only the three translational degrees of freedom. We can gain some intuition about the problem by drawing a PV diagram:



Here we have already used the fact that $P_2 = P_3$ (two ends of an isobaric compression) and $V_1 = V_4$ (isochoric pressure increase). We can determine P_4 and V_2 using the ideal gas law. This should be fairly straightforward because the points lie along isotherms ($T_1 = T_2$ and $T_3 = T_4$).

2. Devise Plan We know the molar specific heat for helium is given by $C_V = \frac{3}{2}k_\mathrm{B}$ and $C_V = \frac{5}{2}k_\mathrm{B}$.

Table 20.6 can be used to determine many of the quantities desired (W, Q, and ΔE_th). Only the work done in an isentrope is missing from Table 20.6. But that work is given in a variety of other places, including Table 21.1. The changes in entropy in various processes are given by Table 20.7. Some quantities can be conveniently written in terms of pressures and volumes, so we might want to complete our set of pressure and volume information by finding P_4 and V_2. Some quantities are more conveniently found using temperatures, which can be easily determined using the ideal gas law.

To guide us in our work, we keep in mind that the total change in entropy of the gas must be zero for the cycle, because the working substance returns to its initial state. Similarly, thermal energy is a state variable; the change in thermal energy of the working substance must also be zero for the same reason. The heat transfer and work done need not be zero, and in fact will not be zero.

3. Execute Plan We have the specific heats from part 2. We can determine temperature T_1 from the ideal gas law:

$$T_1 = \frac{P_1 V_1}{N k_\mathrm{B}} = \frac{(1.00\times10^{6}\ \mathrm{Pa})(1.50\times10^{-3}\ \mathrm{m}^3)}{(3.00\times10^{23})(1.380\times10^{-23}\ \mathrm{J/K})} = 362.3\ \mathrm{K}$$

We know that $T_2 = 362.3\ \mathrm{K}$ also, because points 1 and 2 are along an isotherm. Similarly, we determine T_3:

$$T_3 = \frac{P_3 V_3}{N k_\mathrm{B}} = \frac{(2.00\times10^{5}\ \mathrm{Pa})(2.00\times10^{-3}\ \mathrm{m}^3)}{(3.00\times10^{23})(1.380\times10^{-23}\ \mathrm{J/K})} = 96.62\ \mathrm{K}$$

From this, we know also that $T_4 = 96.62$.

It is trivial to apply the ideal gas law a second time to points 2 and 4 to obtain $P_4 = 2.67\times10^{5}\ \mathrm{Pa}$ and $V_2 = 7.50\times10^{-3}\ \mathrm{m}^3$.

We now insert expressions for each quantity required for each step in the process.

For the isothermal expansion $(1 \to 2)$:

$$W_{1\to2} = -Nk_\mathrm{B}T_{12}\ln\left(\frac{V_2}{V_1}\right) = -(3.00\times10^{23})(1.380\times10^{-23}\ \mathrm{J/K})(362.3\ \mathrm{K})\ln\left(\frac{7.50\times10^{-3}\ \mathrm{m}^3}{1.50\times10^{-3}\ \mathrm{m}^3}\right) = -2.41\ \mathrm{kJ}$$

$$\Delta E_{\mathrm{th},1\to2} = 0$$

$$Q_{1\to2} = -W_{1\to2} = 2.41\ \mathrm{kJ}$$

$$\Delta \mathrm{S}_{1\to2} = \frac{Q_{1\to2}}{k_\mathrm{B}T_{12}} = \frac{(2.41\times10^3\ \mathrm{J})}{(1.380\times10^{-23}\ \mathrm{J/K})(362.3\ \mathrm{K})} = 4.82\times10^{23}$$

For the isobaric compression $(2 \to 3)$

$$W_{2\to3} = -Nk_\mathrm{B}(T_3 - T_2) = -(3.00\times10^{23})(1.380\times10^{-23}\ \mathrm{J/K})((96.62\ \mathrm{K}) - (362.3\ \mathrm{K})) = 1.10\ \mathrm{kJ}$$

$$\Delta E_{\mathrm{th},2\to3} = NC_V(T_3 - T_2) = -(3.00\times10^{23})\left(\frac{3}{2}\right)(1.380\times10^{-23}\ \mathrm{J/K})((96.62\ \mathrm{K}) - (362.3\ \mathrm{K})) = -1.65\ \mathrm{kJ}$$

$$Q_{2\to3} = NC_P(T_3 - T_2) = -(3.00\times10^{23})\left(\frac{5}{2}\right)(1.380\times10^{-23}\ \mathrm{J/K})((96.62\ \mathrm{K}) - (362.3\ \mathrm{K})) = -2.75\ \mathrm{kJ}$$

$$\Delta \mathrm{S}_{2\to3} = \frac{NC_P}{k_\mathrm{B}}\ln\left(\frac{T_3}{T_2}\right) = (3.00\times10^{23})\left(\frac{5}{2}\right)\ln\left(\frac{96.62}{362.3}\right) = -9.91\times10^{23}$$

For the isothermal compression $(3 \to 4)$:

$$W_{3\to4} = -Nk_\mathrm{B}T_{34}\ln\left(\frac{V_4}{V_3}\right) = -(3.00\times10^{23})(1.380\times10^{-23}\ \mathrm{J/K})(96.62\ \mathrm{K})\ln\left(\frac{1.50\times10^{-3}\ \mathrm{m}^3}{2.00\times10^{-3}\ \mathrm{m}^3}\right) = 115\ \mathrm{J}$$

$$\Delta E_{\mathrm{th},3\to4} = 0$$

$$Q_{3\to4} = -W_{3\to4} = -115\ \mathrm{J}$$

$$\Delta \mathrm{S}_{3\to4} = \frac{Q_{3\to4}}{k_\mathrm{B}T_{34}} = \frac{(-115\ \mathrm{J})}{(1.380\times10^{-23}\ \mathrm{J/K})(96.62\ \mathrm{K})} = -8.63\times10^{22}$$

Finally, for the isochoric pressure increase $(4 \to 1)$:

$$W_{4\to1} = 0$$

$$\Delta E_{\mathrm{th},4\to1} = NC_V(T_1 - T_4) = (3.00\times10^{23})\left(\frac{3}{2}\right)(1.380\times10^{-23}\ \mathrm{J/K})((362.3\ \mathrm{K}) - (96.62\ \mathrm{K})) = 1.65\ \mathrm{kJ}$$

$$Q_{4\to1} = \Delta E_{\mathrm{th},3\to4} = 1.65\ \mathrm{kJ}$$

$$\Delta \mathrm{S}_{4\to1} = \frac{NC_V}{k_\mathrm{B}}\ln\left(\frac{T_1}{T_4}\right) = (3.00\times10^{23})\left(\frac{3}{2}\right)\ln\left(\frac{(362.3\ \mathrm{K})}{(96.62\ \mathrm{K})}\right) = 5.95\times10^{23}$$

Combining these results for the entire cycle, we find

$$W_\mathrm{cycle} = W_{1\to2} + W_{2\to3} + W_{3\to4} + W_{4\to1} = (-2.41\ \mathrm{kJ}) + (1.10\ \mathrm{kJ}) + (115\ \mathrm{J}) + 0 = -1.20\ \mathrm{kJ}$$

$$\Delta E_\mathrm{th,cycle} = \Delta E_{\mathrm{th},1\to2} + \Delta E_{\mathrm{th},2\to3} + \Delta E_{\mathrm{th},3\to4} + \Delta E_{\mathrm{th},4\to1} = 0 + (-1.65\ \mathrm{kJ}) + 0 + (1.65\ \mathrm{kJ}) = 0$$

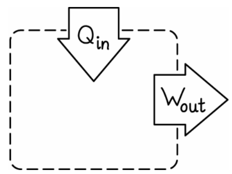
$$Q_\mathrm{cycle} = Q_{1\to2} + Q_{2\to3} + Q_{3\to4} + Q_{4\to1} = (2.41\ \mathrm{kJ}) + (-2.75\ \mathrm{kJ}) + (-115\ \mathrm{J}) + (1.65\ \mathrm{kJ}) = 1.20\ \mathrm{kJ}$$

$$\Delta S_\mathrm{cycle} = \Delta S_{1\to2} + \Delta S_{2\to3} + \Delta S_{3\to4} + \Delta S_{4\to1} = (4.82\times10^{23}) + (-9.91\times10^{23}) + (-8.63\times10^{22}) + (5.95\times10^{23}) = 0$$

4. Evaluate Result We expect the work done on the working substance in a complete cycle to be negative. The area under the curve is clearly positive, meaning the work done by the gas on the environment is positive. This means the work done by the environment on the gas must be negative. This fits what we found in our calculation. Our calculation also fits our expectation that the change in internal thermal energy of the gas and the change in entropy should both be zero.

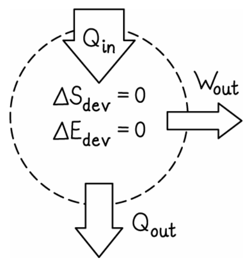
Questions and Problems

21.1. (*a*)



(*b*) Because the temperature of the gas does not change, neither does its thermal energy. Because the volume of the gas increases, its entropy increases. (*c*) The final state is different than the initial state, so it is not a steady device.

21.2.



21.3. In each case, some amount of mechanical energy is converted to thermal energy by friction, which increases the entropy of the snowboarder and her environment. Assuming that there is less friction on the ice than on the snow, the least amount of mechanical energy is converted to thermal energy in the third case, and the greatest amount of mechanical energy is converted to thermal energy in the second case, so the ranking is $\Delta S_3 < \Delta S_1 < \Delta S_2$.

21.4. There are two constraints on the operation of a steady device, because together with its environment, it must comply with both the energy law and the entropy law. To comply with the energy law, the sum of the energy inputs must equal the sum of the energy outputs. To comply with the entropy law, the entropy of the environment must not decrease. Thermal input of energy would cause a decrease in the entropy of the environment, and there is no thermal output of energy to compensate, so the we must have $Q_{\text{in}} = 0$, in which case the energy law gives $W_{\text{in}} = 320$ J.

21.5. We know that the energy in must equal the energy out, so $Q_{\text{out}} + W_{\text{out}} = 425$ J. Further, we know that Q_{out} cannot be exactly zero, since some heat energy must be delivered to the environment to allow the entropy law to be satisfied, that is, to make the net entropy greater than or equal to zero. But we have specified nothing about how much energy is required to make a certain entropy change to the environment. Therefore, the environment could gain energy with an arbitrarily large entropy increase, meaning that the energy needed in the form of Q_{out} could be arbitrarily small. So the largest amount of work one could get out of such a device is arbitrarily close to (but not quite equal to) 425 J.

21.6. (*a*) As the ball fell, the force of gravity did work on it, increasing its kinetic energy. When it hit the water, its kinetic energy was $K = mgh = (5.3\text{ kg})(9.8\text{ N/kg})(2.0\text{ m}) = 103.88\text{ J}$, where we have retained extra precision in this intermediate result.

All of the ball's kinetic energy was converted to other forms in the collision, and we have accounted for the amount converted to sound energy, thermal energy in the ball, and thermal energy that escaped the water, leaving the remainder as thermal energy in the water,

$$\Delta E_{\text{th,water}} = K - E_{\text{sound}} - \Delta E_{\text{th,ball}} - \Delta E_{\text{th,escaped}} = (103.88\text{ J}) - (0.80\text{ J}) - (4.50\text{ J}) - (21\text{ J}) = 78\text{ J}$$

(*b*) Assuming the other conversions are unchanged, the thermal energy absorbed by the water is $\Delta E_{\text{th,water}} = K - E_{\text{sound}} - \Delta E_{\text{th,ball}} = (103.88\text{ J}) - (0.80\text{ J}) - (4.50\text{ J}) = 99\text{ J}.$

21.7. (*a*) The device converts 68% of its input energy to work output, so it must output the remaining 32% of its input energy thermally. In one cycle, the thermal energy output is $Q_{\text{out}} = 32\%(600\text{ W})/(20\text{ Hz}) = 9.6\text{ J}.$ (*b*) In 8.0 h the thermal energy output is $Q_{\text{out}} = 32\%(600\text{ W})(8.0\text{ hr})(3600\text{ s})/(1\text{ hr}) = 5.5\times10^6\text{ J}.$ (*c*) If 50% of the output thermal energy were used to lift water, we would have

$$\tfrac{1}{2}Q_{\text{out}} = mgh$$

$$m = \frac{\frac{1}{2}Q_{\text{out}}}{gh} = \frac{\frac{1}{2}(5.5\times10^6\text{ J})}{(9.8\text{ N/kg})(10\text{ m})} = 2.8\times10^4\text{ kg}$$

21.8. (*a*) The entropy gradient is inversely proportional to the absolute temperature, so the reservoir at the lowest temperature is the one at the highest entropy gradient, namely, reservoir 3. (*b*) The reservoir with the smallest entropy gradient is most useful for converting thermal energy to mechanical energy, namely, reservoir 1.

21.9. The material with the lower entropy gradient is more useful for converting thermal energy to mechanical energy. The entropy gradients are $\dfrac{dS_1}{dE} = \dfrac{d}{dE}2\gamma E^2 = 4\gamma E$ and $\dfrac{dS_2}{dE} = \dfrac{d}{dE}\tfrac{1}{9}\gamma E^3 = \tfrac{1}{3}\gamma E^2,$ which for 4 units of energy give $\dfrac{dS_1}{dE} = 16\gamma$ and $\dfrac{dS_2}{dE} = \tfrac{16}{3}\gamma.$ So, material 2 is more useful for converting thermal energy to mechanical energy.

21.10. A reversible process is one in which the entropy does not change, so the decrease in the entropy of the high-temperature reservoir, as it transfers energy thermally to the device, must equal the increase in the entropy of the low-temperature reservoir, as the device transfers energy thermally to it. From the definition of absolute temperature (Eq. 19.38), we have

$$\begin{aligned}\Delta S_{\text{H}} + \Delta S_{\text{L}} &= \frac{dS_{\text{H}}}{dE}(-Q_{\text{in}}) + \frac{dS_{\text{L}}}{dE}Q_{\text{out}}\\ &= \frac{Q_{\text{out}}}{k_{\text{B}}T_{\text{L}}} - \frac{Q_{\text{in}}}{k_{\text{B}}T_{\text{H}}} = 0\end{aligned}$$

or $Q_{\text{out}} = Q_{\text{in}}\dfrac{T_{\text{L}}}{T_{\text{H}}}.$ From the energy law, we have $W_{\text{in}} + Q_{\text{in}} = W_{\text{out}} + Q_{\text{out}},$ so with no input of work, $W_{\text{out}} = Q_{\text{in}} - Q_{\text{out}},$ and

$$\begin{aligned}\frac{W_{\text{out}}}{Q_{\text{in}}} &= \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{in}}\dfrac{T_{\text{L}}}{T_{\text{H}}}}{Q_{\text{in}}}\\ &= 1 - \frac{T_{\text{L}}}{T_{\text{H}}} = 1 - \frac{250\text{ K}}{390\text{ K}} = 0.359\end{aligned}$$

That is, at most 35.9% of the input thermal energy can be output as mechanical energy.

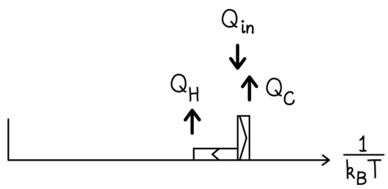
21.11. From the definition of absolute temperature (Eq. 19.38), we have

$$\frac{dS}{dE} = \frac{1}{k_{\text{B}}T}$$

$$T = \frac{1}{k_{\text{B}}\dfrac{dS}{dE}} = \frac{1}{(1.381\times10^{-23}\text{ J/K})(2.00\text{ J}^{-4})(3.00\text{ MJ})^3} = 1.34\times10^3\text{ K}$$

21.12. [**NOTE:** In the problem statement in part (*a*), J^{-3} should be J^{-2}.] (*a*) The entropy gradient of reservoir 1 is $\dfrac{dS_1}{dE} = \dfrac{d}{dE} aE^2 = 2aE,$ so when $E = 1.00\ \text{MJ},$ $\dfrac{dS_1}{dE} = 2(1.00\ \text{J}^{-2})(1.00\ \text{MJ}) = 2\times10^6\ \text{J}^{-1}.$ (*b*) The entropy gradient of reservoir 2 is $\dfrac{dS_2}{dE} = \dfrac{d}{dE} bEe^{-cE} = bE(-ce^{-cE}) + be^{-cE} = b(1-cE)e^{-cE}.$ (*c*) When $E = 1.00\ \text{J},$ $\dfrac{dS_2}{dE} = (3.00\ \text{J}^{-1})\left[1-(1.00\ \text{J}^{-1})(1.00\ \text{J})\right]e^{-(1.00\ \text{J}^{-1})(1.00\ \text{J})} = 0.00\ \text{J}^{-1}.$

21.13.



21.14. In an entropy diagram, the area of each rectangle equals the magnitude of the change in entropy of the environment, with leftward pointing arrows indicating a decrease in entropy and rightward pointing arrows indicating an increase. In this composite process, we have three rectangles. From left to right, the first represents degradation of high-quality energy with a corresponding increase in entropy, the second represents an upgrade of energy with a corresponding decrease in entropy, and the third represents another degradation of energy with a corresponding increase in entropy. The change in entropy for the composite process equals the sum of the changes represented by each of the three rectangles,

$$\begin{aligned}\Delta S &= \Delta S_1 + \Delta S_2 + \Delta S_3 = Q_1\Delta\left(\frac{1}{k_\text{B}T}\right)_1 - Q_2\Delta\left(\frac{1}{k_\text{B}T}\right)_2 + Q_3\Delta\left(\frac{1}{k_\text{B}T}\right)_3 \\ &= (4.0\ \text{MJ})(6.0\times10^{-19}\ \text{J}^{-1}) - (2.0\ \text{MJ})(8.0\times10^{-19}\ \text{J}^{-1}) + (2.0\ \text{MJ})(3.0\times10^{-19}\ \text{J}^{-1}) = 1.4\times10^{-12}\end{aligned}$$

21.15. For a reversible steady device, the entropy change of its environment is zero, so the entropy decrease caused by the upgrade of input thermal energy to mechanical work equals the entropy increase caused by the degradation of energy associated with the thermal transfer of energy out of the device. We can calculate the entropy changes with Eq. 20.54, $\Delta S = \dfrac{-Q_\text{in}}{k_\text{B}T_\text{in}} + \dfrac{Q_\text{out}}{k_\text{B}T_\text{out}} = 0,$ and from the energy law for a steady device we have $Q_\text{in} = Q_\text{out} + W_\text{out}.$ Combining these we have

$$\frac{-(Q_\text{out}+W_\text{out})}{k_\text{B}T_\text{in}} + \frac{Q_\text{out}}{k_\text{B}T_\text{out}} = 0$$

$$Q_\text{out}\left(\frac{1}{k_\text{B}T_\text{out}} - \frac{1}{k_\text{B}T_\text{in}}\right) = \frac{W_\text{out}}{k_\text{B}T_\text{in}}$$

$$\begin{aligned}Q_\text{out} &= \frac{W_\text{out}}{k_\text{B}T_\text{in}}\left(\frac{1}{k_\text{B}T_\text{out}} - \frac{1}{k_\text{B}T_\text{in}}\right)^{-1} \\ &= \frac{W_\text{out}}{T_\text{in}}\left(\frac{1}{T_\text{out}} - \frac{1}{T_\text{in}}\right)^{-1} \\ &= \frac{750\ \text{J}}{355\ \text{K}}\left(\frac{1}{300\ \text{K}} - \frac{1}{355\ \text{K}}\right)^{-1} = 4.09\times10^3\ \text{J}\end{aligned}$$

21.16. (*a*) From the energy law for a steady device we have

$$Q_{\text{in}} + W_{\text{in}} = Q_{\text{out}}$$
$$Q_{\text{in}} = Q_{\text{out}} - W_{\text{in}} = (12\ \text{J}) - (4.0\ \text{J}) = 8\ \text{J}$$

(*b*) The coefficient of performance of heating is the ratio of the thermal output of the heat pump to the work done on it, $\text{COP}_{\text{heating}} = \dfrac{Q_{\text{out}}}{W_{\text{in}}} = \dfrac{12\ \text{J}}{4.0\ \text{J}} = 3.0.$ (*c*) The coefficient of performance of cooling is the ratio of the thermal input of the heat pump to the work done on it, $\text{COP}_{\text{cooling}} = \dfrac{Q_{\text{in}}}{W_{\text{in}}} = \dfrac{8\ \text{J}}{4.0\ \text{J}} = 2.$

21.17. (*a*) The highest-quality energy is that associated with the smallest entropy cost $dS/dE = 1/k_{\text{B}}T,$ which is to say, the energy available at the greatest temperature, so reservoir 1 has the highest-quality energy. (*b*) A steady device upgrades the maximum amount of thermal energy input to mechanical work when it degrades as little energy as possible to thermal output, in which case the entropy change is zero. If the devices operate independently, we can calculate the entropy change of each device with Eq. 20.54, for example, for device A, $\Delta S_{\text{A}} = \dfrac{-Q_{\text{A,in}}}{k_{\text{B}}T_1} + \dfrac{Q_{\text{A,out}}}{k_{\text{B}}T_2} = 0.$ Combining this with the energy law we have

$$Q_{\text{A,in}} = Q_{\text{A,out}} + W_{\text{A,out}}$$
$$Q_{\text{A,out}} = Q_{\text{A,in}} - W_{\text{A,out}}$$
$$\frac{-Q_{\text{A,in}}}{k_{\text{B}}T_1} + \frac{(Q_{\text{A,in}} - W_{\text{A,out}})}{k_{\text{B}}T_2} = 0$$
$$\frac{W_{\text{A,out}}}{T_2} = Q_{\text{A,in}}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$
$$\frac{W_{\text{A,out}}}{Q_{\text{A,in}}} = T_2\left(\frac{1}{T_2} - \frac{1}{T_1}\right) = \frac{T_1 - T_2}{T_1}$$

and similarly for the other devices. So, the efficiencies of the three devices are

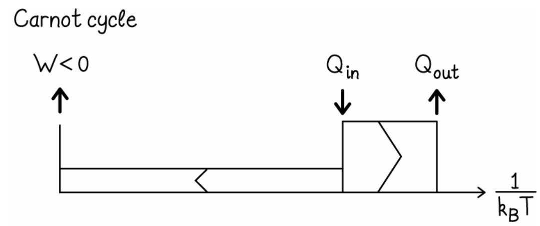
$$\frac{W_{\text{A,out}}}{Q_{\text{A,in}}} = \frac{T_1 - T_2}{T_1} = \frac{(800\ \text{K}) - (150\ \text{K})}{800\ \text{K}} = 0.813$$
$$\frac{W_{\text{B,out}}}{Q_{\text{B,in}}} = \frac{T_1 - T_2}{T_1} = \frac{(400\ \text{K}) - (150\ \text{K})}{400\ \text{K}} = 0.625$$
$$\frac{W_{\text{C,out}}}{Q_{\text{C,in}}} = \frac{T_1 - T_2}{T_1} = \frac{(800\ \text{K}) - (400\ \text{K})}{800\ \text{K}} = 0.500$$

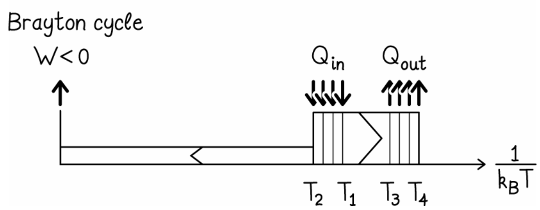
That is, device A is the most efficient of the three.

21.18. The magnitude of the work done on the gas in a thermodynamic cycle is equal to the area enclosed by the cycle on a *PV* diagram, and is positive for a counterclockwise cycle and negative for a clockwise cycle. So, the greatest magnitude of work is done on the gas in cycle (*a*), because it encloses the greatest area. But these cycles are clockwise, so negative work is done on the gas. The greatest amount of work is done on the gas in the cycle that does the least amount of negative work on it, which is cycle (*c*).

21.19. A thermodynamic cycle is more efficient when thermal energy is input at higher temperature and output at lower temperature. Only cycle (*a*) has all of its thermal energy input at the highest isotherm shown and all of its thermal energy output at the lowest one, so it must have the greatest efficiency.

21.20. (*a*)





(*b*) Because both cycles are reversible, the change in entropy of the environment is zero for both of them.

21.21. The work done on the gas during the cycle equals the sum of the work done on the gas during each part of the cycle, $W = (-6.4\ \text{J}) + (8.2\ \text{J}) = 1.8\ \text{J}$.

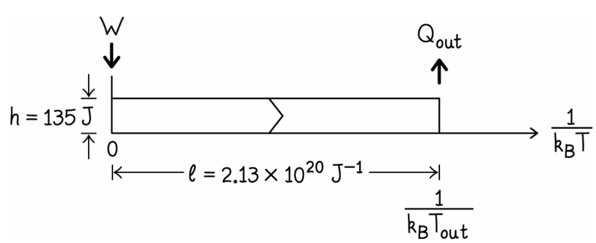
21.22. The entropy change in the environment equals the sum of the entropy changes in the reservoirs, for which Eq. 21.10 gives

$$\Delta S_{\text{env}} = Q_{\text{out}}\left(\frac{1}{k_{\text{B}}T_{\text{out}}} - \frac{1}{k_{\text{B}}T_{\text{in}}}\right) = \frac{Q_{\text{out}}}{k_{\text{B}}}\left(\frac{1}{T_{\text{out}}} - \frac{1}{T_{\text{in}}}\right) = \frac{5.0\ \text{MJ}}{1.381\times 10^{-23}\ \text{J/K}}\left(\frac{1}{500\ \text{K}} - \frac{1}{1000\ \text{K}}\right) = 3.6\times 10^{26}$$

21.23. For a steady device, the entropy change over a cycle is zero, $\Delta S_{\text{dev}} = 0$. The entropy change in the environment is given by Eq. 21.10,

$$\Delta S_{\text{env}} = Q_{\text{out}}\left(\frac{1}{k_{\text{B}}T_{\text{out}}} - \frac{1}{k_{\text{B}}T_{\text{in}}}\right) = \frac{Q_{\text{out}}}{k_{\text{B}}}\left(\frac{1}{T_{\text{out}}} - \frac{1}{T_{\text{in}}}\right) = \frac{1.55\times 10^{6}\ \text{J}}{1.381\times 10^{-23}\ \text{J/K}}\left(\frac{1}{300\ \text{K}} - \frac{1}{450\ \text{K}}\right) = 1.25\times 10^{26}$$

21.24.



The length of the entropy bar is the difference between the output and input points on the $1/k_{\text{B}}T$ scale, where mechanical energy is placed at the zero point on the scale. So for this device, the length is $\dfrac{1}{k_{\text{B}}T_{\text{out}}} - 0 = \dfrac{1}{(1.381\times 10^{-23}\ \text{J/K})(340\ \text{K})} = 2.13\times 10^{20}\ \text{J}^{-1}$. The height of the entropy bar is the amount of energy transferred, in this case 135 J. The resulting change in entropy equals the product of the length times the height, $\Delta S = (2.13\times 10^{20}\ \text{J}^{-1})(135\ \text{J}) = 2.88\times 10^{22}$.

21.25. Because the gas is held at constant temperature, its thermal energy remains constant, which means that $Q = -W$ (Eq. 20.28). That is, a quantity of thermal energy equal to the work done on the gas is transferred thermally to the environment, resulting in an entropy change (Eq. 20.54)

$$\Delta S_{\text{env}} = \frac{Q_{\text{out}}}{k_{\text{B}} T_{\text{out}}} = \frac{W}{k_{\text{B}} T_{\text{out}}} = \frac{mgh}{k_{\text{B}} T_{\text{out}}} = \frac{(5.0\text{ kg})(9.8)(0.200\text{ m})}{(1.381\times10^{-23}\text{ J/K})(70\text{ K})} = 1.0\times10^{22}$$

21.26. The change in entropy equals the product of the length times the height of the bar in the entropy diagram. We are told that the heights are related by $Q_{\text{A}} = \sqrt{2}Q_{\text{B}}$, and we can read the lengths off the graphs, so we have

$$\begin{aligned}
\Delta S_{\text{A}} &= (2.0\text{ J}^{-1})Q_{\text{A}} = 2.6\times10^{24}\\
Q_{\text{A}} &= 1.3\times10^{24}\text{ J}\\
Q_{\text{B}} &= Q_{\text{A}}/\sqrt{2} = 0.9192\times10^{24}\text{ J}\\
\Delta S_{\text{B}} &= (4.0\text{ J}^{-1})Q_{\text{B}} = 3.7\times10^{24}
\end{aligned}$$

21.27. The maximum efficiency of a reversible heat engine operating between two reservoirs is given by Eq. 21.23, $\eta_{\text{max}} = 1 - \dfrac{T_{\text{out}}}{T_{\text{in}}} = 1 - \dfrac{273\text{ K}}{373\text{ K}} = 0.268.$

21.28. The maximum possible value for the temperature of the low-temperature reservoir is that which corresponds to the engine operating at maximum efficiency, so from Eq. 21.23 we have

$$\begin{aligned}
\eta_{\text{max}} &= 1 - \frac{T_{\text{L,max}}}{T_{\text{in}}}\\
\frac{T_{\text{L,max}}}{T_{\text{in}}} &= 1 - \eta_{\text{max}}\\
T_{\text{L,max}} &= T_{\text{in}}(1-\eta_{\text{max}}) = (500\text{ K})(1-0.450) = 275\text{ K}
\end{aligned}$$

21.29. The quantities of thermal energy taken in and thermal energy expelled by a heat pump are related by Eq. 21.25, which we can solve for the thermal energy expelled,

$$\begin{aligned}
\text{COP}_{\text{heating}} &= \frac{1}{1 - Q_{\text{in}}/Q_{\text{out}}}\\
1 - Q_{\text{in}}/Q_{\text{out}} &= \frac{1}{\text{COP}_{\text{heating}}}\\
1 - \frac{1}{\text{COP}_{\text{heating}}} &= \frac{Q_{\text{in}}}{Q_{\text{out}}}\\
Q_{\text{out}} &= \frac{Q_{\text{in}}}{1 - \dfrac{1}{\text{COP}_{\text{heating}}}}
\end{aligned}$$

We can calculate the coefficient of performance of heating for a reversible heat pump from Eq. 21.26, $\text{COP}_{\text{heating,max}} = \dfrac{1}{1 - T_{\text{in}}/T_{\text{out}}}$. Combing these we have

$$Q_{\text{out}} = \frac{Q_{\text{in}}}{1 - (1 - T_{\text{in}}/T_{\text{out}})} = \frac{T_{\text{out}} Q_{\text{in}}}{T_{\text{in}}} = \frac{(320\text{ K})(16.0\text{ MJ})}{273\text{ K}} = 18.8\text{ MJ}$$

21.30. The coefficient of performance of heating is defined as the ratio of thermal energy output to work done on the heat pump (Eq. 21.24), and power equals energy divided by time interval, so we have $P = \dfrac{W}{\Delta t} = \dfrac{Q_{\text{out}}}{\text{COP}_{\text{heating}}\Delta t} = \dfrac{2.4\times10^9 \text{ J}}{(5.4)(24 \text{ h})}\dfrac{1 \text{ h}}{3600 \text{ s}} = 5.1\times10^3 \text{ W}.$

21.31. The energy law applied to a steady device says that the energy input equals the energy output (Eq. 21.2), so $Q_{\text{in}} = Q_{\text{out}} - W = Q_{\text{out}} + W_{\text{out}} = (110 \text{ J}) + (85 \text{ J}) = 195 \text{ J}.$ The efficiency of the engine is the ratio of work output to thermal energy input (Eq. 21.21), so $\eta = \dfrac{W_{\text{out}}}{Q_{\text{in}}} = \dfrac{85 \text{ J}}{195 \text{ J}} = 0.44.$

21.32. The energy law applied to a steady device says that the energy input equals the energy output (Eq. 21.2). (*a*) For the first heat pump we have $Q_{\text{out}} = W + Q_{\text{in}} = (2.0 \text{ J}) + (4.0 \text{ J}) = 6.0 \text{ J}.$ (*b*) For the second heat pump we have $W = Q_{\text{out}} - Q_{\text{in}} = (9.0 \text{ J}) + (6.0 \text{ J}) = 3.0 \text{ J}.$

21.33. The work done by the heat engine per cycle equals the area enclosed by the cycle on the *PV* diagram, $W = (2.25 \text{ atm})\left[(101.3 \text{ kPa})/(1 \text{ atm})\right](0.125 \text{ m}^3) = 2.85\times10^4 \text{ J}.$ The efficiency of the engine equals the ratio of the work done to the thermal energy input, and the thermal energy input equals the sum of the work done plus the thermal energy output, so $\eta = \dfrac{W_{\text{out}}}{Q_{\text{in}}} = \dfrac{W_{\text{out}}}{W_{\text{out}} + Q_{\text{out}}} = \dfrac{28.5 \text{ kJ}}{(28.5 \text{ kJ}) + (43.5 \text{ kJ})} = 0.396.$

21.34. The work done by the engine per cycle equals the difference between the thermal energy input and the thermal energy output, $W_{\text{out}} = Q_{\text{in}} - Q_{\text{out}} = (115 \text{ J}) - (75 \text{ J}) = 40 \text{ J}.$ The amount of energy required to lift the load is $E = mgh = (375 \text{ kg})(9.8 \text{ N/kg})(27.0 \text{ m}) = 9.92\times10^4 \text{ J}.$ So, to accomplish this task we need $(9.92\times10^4 \text{ J})/(40 \text{ J}) = 2.5\times10^3 \text{ cycles}$ of the engine.

21.35. The heating system takes thermal energy from the warm geothermal reservoir, converts some of it to mechanical energy, and uses the remainder to heat the house. If it really is reversible, it should operate at the maximum possible efficiency for a heat engine operating between the two reservoirs (Eq. 21.23), $\eta_{\max} = 1 - \dfrac{T_{\text{out}}}{T_{\text{in}}} = 1 - \dfrac{294.15 \text{ K}}{347 \text{ K}} = 0.152.$ The actual efficiency of the system is the ratio of the work done by the system to the thermal energy input (Eq. 21.21), which equals the ratio of the power output to the rate at which thermal energy is input. The rate at which thermal energy is input equals the sum of the power output and the rate at which thermal energy is output. Combining these we have

$$\begin{aligned}\eta &= \frac{W_{\text{out}}}{Q_{\text{in}}} = \frac{P_{\text{out}}}{dQ_{\text{in}}/dt} \\ &= \frac{P_{\text{out}}}{P_{\text{out}} + dQ_{\text{out}}/dt} = \frac{180 \text{ W}}{(180 \text{ W}) + (1000 \text{ W})} = 0.153\end{aligned}$$

With 5% uncertainty in the rate at which thermal energy is output, the actual efficiency lies between 0.159 and 0.146, which includes the maximum possible value, so the system could be reversible.

21.36. The efficiency of the engine is the ratio of the work done on the environment to the thermal energy input (Eq. 21.21), so we have $\eta_{\text{new}} = \dfrac{W_{\text{out,new}}}{Q_{\text{in}}} = \dfrac{1.14W_{\text{out,old}}}{Q_{\text{in}}} = 1.14\eta_{\text{old}} = (1.14)(0.22) = 0.25.$

21.37. (*a*) A Carnot cycle is reversible and all thermal transfers of energy take place at constant temperature, so the coefficient of performance is maximal, and Eq. 21.29 applies. The relevant temperatures are

$T_{\text{in}} = \left[\frac{5}{9}(72-32)+273.15\right]\text{ K} = 295.37\text{ K}$ and $T_{\text{out}} = \left[\frac{5}{9}(105-32)+273.15\right]\text{ K} = 313.71\text{ K}$, so the coefficient of performance of cooling is $\text{COP}_{\text{cooling}} = \dfrac{T_{\text{in}}}{T_{\text{out}}-T_{\text{in}}} = \dfrac{295.37\text{ K}}{(313.71\text{ K})-(295.37\text{ K})} = 16$. (*b*) The maximal coefficient of performance for heating is given by Eq. 21.26, and $T_{\text{in}} = \left[\frac{5}{9}(-15-32)+273.15\right]\text{ K} = 247.04\text{ K}$, so $\text{COP}_{\text{heating}} = \dfrac{T_{\text{out}}}{T_{\text{out}}-T_{\text{in}}} = \dfrac{295.37\text{ K}}{(295.37\text{ K})-(247.04\text{ K})} = 6.1$.

21.38. For a reversible heat engine operating between two thermal reservoirs, the ratio of the thermal energy output to the thermal energy input equals the ratio of the output temperature to the input temperature (Eq. 21.20). So we have for the two engines

$$\begin{aligned}\frac{Q_{2\text{out}}}{Q_{2\text{in}}} &= \frac{T_{2\text{out}}}{T_{2\text{in}}}\\ Q_{2\text{out}} &= Q_{2\text{in}}\frac{T_{2\text{out}}}{T_{2\text{in}}} = Q_{1\text{out}}\frac{T_{2\text{out}}}{T_{1\text{out}}}\\ &= \left(Q_{1\text{in}}\frac{T_{1\text{out}}}{T_{1\text{in}}}\right)\frac{T_{2\text{out}}}{T_{1\text{out}}} = Q_{1\text{in}}\frac{T_{2\text{out}}}{T_{1\text{in}}}\\ &= (3.70\times10^{11}\text{ J})\frac{422\text{ K}}{270\text{ K}} = 5.78\times10^{11}\text{ J}\end{aligned}$$

We might notice that the combined engines act like a single engine operating between the two reservoirs, regardless of the intermediate temperature.

21.39. (*a*) The maximum coefficient of performance of cooling is (Eq. 21.29) $\text{COP}_{\text{cooling,max}} = \dfrac{T_{\text{in}}}{T_{\text{out}}-T_{\text{in}}} = \dfrac{(-4+273.15)\text{ K}}{\left[35-(-4)\right]\text{ K}} = 6.9$. (*b*) In the cooler basement, the maximum coefficient of performance of cooling would be $\text{COP}_{\text{cooling,max}} = \dfrac{T_{\text{in}}}{T_{\text{out}}-T_{\text{in}}} = \dfrac{(-4+273.15)\text{ K}}{\left[19-(-4)\right]\text{ K}} = 12$. (*c*) The coefficient of performance of cooling equals the ratio of the quantity of thermal energy removed to the work done on the heat pump, so the minimum amount of work that must be done is $W_{\text{min}} = \dfrac{Q_{\text{in}}}{\text{COP}_{\text{cooling,max}}}$.

Assuming the heat pump would have to remove thermal energy from the freezer at the same rate in both locations, we have for the energy used in a day in the warm shop $E_{\text{shop}} = \dfrac{(3000\text{ J/s})}{6.90}\dfrac{3600\text{ s}}{1\text{ h}}\dfrac{24\text{ h}}{1\text{ day}} = 37.6\text{ MJ}$ and in the cool basement $E_{\text{basement}} = \dfrac{(3000\text{ J/s})}{11.7}\dfrac{3600\text{ s}}{1\text{ h}}\dfrac{24\text{ h}}{1\text{ day}} = 22.1\text{ MJ}$ for a potential savings of 15 MJ per day.

21.40. (*a*) The maximum coefficient of performance of cooling for the freezer is (Eq. 21.29) $\text{COP}_{\text{cooling,max}} = \dfrac{T_{\text{in}}}{T_{\text{out}}-T_{\text{in}}} = \dfrac{269\text{ K}}{(325\text{ K})-(269\text{ K})} = 4.8$. (*b*) The coefficient of performance for cooling equals the ratio of the thermal energy absorbed to the work done on the freezer (Eq. 21.27), and the thermal energy released to the environment equals the sum of the thermal energy absorbed and the work done by the freezer, so

$$\begin{aligned}Q_{\text{out}} &= Q_{\text{in}} + W = Q_{\text{in}} + \frac{Q_{\text{in}}}{\text{COP}_{\text{cooling}}}\\ &= Q_{\text{in}}\left(1+\frac{1}{\text{COP}_{\text{cooling}}}\right) = (118\text{ J})\left(1+\frac{1}{4.80}\right) = 143\text{ J}\end{aligned}$$

(*c*) The work done on the freezer per cycle is $W = \dfrac{Q_{\text{in}}}{\text{COP}_{\text{cooling}}} = \dfrac{118\ \text{J}}{4.80} = 24.6\ \text{J}$. (*d*) Considered as a heater, the coefficient of performance of heating of the freezer is one plus its coefficient of performance of cooling (Eq. 21.28), that is, $\text{COP}_{\text{heating}} = \text{COP}_{\text{cooling}} + 1 = 5.80$. An electric space heater converts the input electrical energy to thermal energy, so its coefficient of performance of heating is one, the lowest possible value.

Of course, unless the thermal energy inside the freezer is replenished, it will shut itself off when it reaches the temperature it is set for. It will then turn itself back on when some amount of thermal energy has leaked into the freezer compartment from the room, only to pump that thermal energy back out into the room. So, in the long term, there is no thermal energy being pumped from the freezer into the room, and the coefficient of performance of heating is one for the freezer, too.

21.41. For a Carnot cycle we have (Eq. 21.34)

$$\begin{aligned} W_{\text{out}} &= -W = Nk_{\text{B}}(T_{\text{in}} - T_{\text{out}})\ln\left(\frac{V_2}{V_1}\right) = Nk_{\text{B}}(3T_{\text{out}} - T_{\text{out}})\ln\left(\frac{2V_1}{V_1}\right) \\ &= Nk_{\text{B}}(2T_{\text{out}})\ln(2) = (6.02\times10^{23})(1.381\times10^{-23}\ \text{J/K})\big[2(273.15\ \text{K})\big]\ln(2) = 3.15\times10^{3}\ \text{J} \end{aligned}$$

21.42. The maximum efficiency of a heat engine that operates between two thermal reservoirs is (Eq. 21.23) $\eta_{\max} = 1 - \dfrac{T_{\text{out}}}{T_{\text{in}}} = 1 - \dfrac{311\ \text{K}}{339\ \text{K}} = 0.0826.$

21.43. The efficiency of a Carnot-cycle heat engine is (Eq. 21.36) $\eta = \dfrac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}} = 1 - \dfrac{T_{\text{out}}}{T_{\text{in}}}$, which we can rearrange to give,

$$\begin{aligned} \frac{T_{\text{out}}}{T_{\text{in}}} &= 1 - \eta \\ T_{\text{in}} &= \frac{T_{\text{out}}}{1-\eta} \end{aligned}$$

If we raise the temperature of the high-temperature reservoir while keeping the temperature of the low-temperature reservoir the same we have

$$\begin{aligned} T_{2\text{in}} - T_{1\text{in}} &= \frac{T_{\text{out}}}{1-\eta_2} - \frac{T_{\text{out}}}{1-\eta_1} \\ &= T_{\text{out}}\left(\frac{1}{1-\eta_2} - \frac{1}{1-\eta_1}\right) \\ &= (283.15\ \text{K})\left(\frac{1}{1-0.600} - \frac{1}{1-0.480}\right) = 163\ \text{K} \end{aligned}$$

Because the size of a degree Celsius is the same as the size of a kelvin, we must raise the temperature of the high-temperature reservoir by 163 °C.

21.44. We have a relationship between the work done by the engine and the thermal energy input, namely, the efficiency (Eq. 21.21). Combining this with Eq. 21.36 for the efficiency of a Carnot-cycle engine, and the definition of specific heat capacity (see *Principles* Section 20.3), we have

$$\eta = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}} = 1 - \frac{T_{\text{out}}}{T_{\text{in}}} = \frac{W_{\text{out}}}{Q_{\text{in}}}$$

$$W_{\text{out}} = Q_{\text{in}}\left(1 - \frac{T_{\text{out}}}{T_{\text{in}}}\right) = c_V m \Delta T\left(1 - \frac{T_{\text{out}}}{T_{\text{in}}}\right)$$

$$= (4181 \text{ J/K} \cdot \text{kg})(1.0 \text{ kg})(1.0 \text{ K})\left(1 - \frac{280 \text{ K}}{340 \text{ K}}\right) = 7.4 \times 10^2 \text{ J}$$

21.45. We can determine the values from those given using the ideal gas law (Eq. 19.51) and the equation for an isentrope (Eq. 20.44):

$$P_1 = \frac{Nk_{\text{B}}T_1}{V_1} = \frac{(6.02\times10^{23})(1.381\times10^{-23} \text{ J/K})(700 \text{ K})}{0.10 \text{ m}^3} = 5.8\times10^4 \text{ Pa}$$

$$P_2 = \frac{Nk_{\text{B}}T_2}{V_2} = \frac{(6.02\times10^{23})(1.381\times10^{-23} \text{ J/K})(700 \text{ K})}{0.30 \text{ m}^3} = 1.9\times10^4 \text{ Pa}$$

$$V_3 = \left(\frac{T_2}{T_3}\right)^{1/(\gamma-1)} V_2 = \left(\frac{700 \text{ K}}{300 \text{ K}}\right)^{3/2}(0.30 \text{ m}^3) = 1.1 \text{ m}^3$$

$$P_3 = \frac{Nk_{\text{B}}T_3}{V_3} = \frac{(6.02\times10^{23})(1.381\times10^{-23} \text{ J/K})(300 \text{ K})}{1.069 \text{ m}^3} = 2.3\times10^3 \text{ Pa}$$

$$V_4 = \left(\frac{T_1}{T_4}\right)^{1/(\gamma-1)} V_1 = \left(\frac{700 \text{ K}}{300 \text{ K}}\right)^{3/2}(0.10 \text{ m}^3) = 0.36 \text{ m}^3$$

$$P_4 = \frac{Nk_{\text{B}}T_4}{V_4} = \frac{(6.02\times10^{23})(1.381\times10^{-23} \text{ J/K})(300 \text{ K})}{0.3564 \text{ m}^3} = 7.0\times10^3 \text{ Pa}$$

21.46. Qualitatively, we can see that cycle A is more efficient. The work done equals the area enclosed by the cycle on a *PV* diagram, and we are told that cycle A does more work than cycle B, so cycle A encloses a greater area than cycle B. We are also told that the isothermal expansion of cycle B is greater than that of cycle A, so on a *PV* diagram cycle B is wider than cycle A, which means that cycle A must be taller, which in turn means that the low-temperature reservoir of cycle A must be cooler than that of cycle B. For the same input temperature, the efficiency will be greater for the cooler output temperature, so cycle A is more efficient.

Quantitatively, we can use Eq. 21.34 to relate the work done on a Carnot-cycle engine to the volume and temperature changes over the cycle, $W = -Nk_{\text{B}}(T_{\text{in}} - T_{\text{out}})\ln\left(\frac{V_2}{V_1}\right)$. We are told that N and T_{in} are the same for both cycles, and we are given the volume and work ratios, so we have

$$W_{\text{A}} = -Nk_{\text{B}}(T_{\text{in}} - T_{\text{A,out}})\ln(2) = 2W_{\text{B}}$$

$$W_{\text{B}} = -Nk_{\text{B}}(T_{\text{in}} - T_{\text{B,out}})\ln(3)$$

$$(T_{\text{in}} - T_{\text{A,out}})\ln(2) = 2(T_{\text{in}} - T_{\text{B,out}})\ln(3)$$

$$T_{\text{in}} - T_{\text{A,out}} = \frac{2\ln(3)}{\ln(2)}(T_{\text{in}} - T_{\text{B,out}})$$

$$\frac{T_{\text{in}} - T_{\text{A,out}}}{T_{\text{in}}} = \frac{2\ln(3)}{\ln(2)}\frac{T_{\text{in}} - T_{\text{B,out}}}{T_{\text{in}}}$$

$$\eta_{\text{A}} = \frac{2\ln(3)}{\ln(2)}\eta_{\text{B}} = 3.17\eta_{\text{B}}$$

21.47. Because the two engines have the same efficiency, we can equate our expressions for their efficiencies (Eq. 21.36) and solve for the unknown temperature,

$$\frac{T_{1\text{in}} - T_{1\text{out}}}{T_{1\text{in}}} = \frac{T_{2\text{in}} - T_{2\text{out}}}{T_{2\text{in}}}$$

$$T_{2\text{in}}(T_{1\text{in}} - T_{1\text{out}}) = T_{1\text{in}}(T_{2\text{in}} - T_{2\text{out}})$$

$$T_{2\text{in}} = \frac{T_{1\text{in}}T_{2\text{out}}}{T_{1\text{out}}} = \frac{T_{1\text{in}}T_{2\text{out}}}{2T_{2\text{out}}}$$

$$= \tfrac{1}{2}T_{1\text{in}} = \tfrac{1}{2}(373\text{ K}) = 187\text{ K}$$

21.48. (*a*) A Carnot cycle has maximal efficiency, so the coefficient of performance for cooling is (Eq. 21.29) $\text{COP}_{\text{cooling}} = \dfrac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} = \dfrac{(20 + 273.15)\text{ K}}{25\text{ K}} = 12.$ (*b*) The coefficient of performance for cooling is the ratio of the thermal energy taken in to the work done on the heat pump, so

$$W = \frac{Q_{\text{in}}}{\text{COP}_{\text{cooling}}}$$

$$\frac{dW}{dt} = \frac{dQ_{\text{in}}/dt}{\text{COP}_{\text{cooling}}} = \frac{dQ_{\text{in}}/dt}{\text{COP}_{\text{cooling}}}$$

$$= \frac{1.50\text{ kW}}{11.73} = 0.13\text{ kW}$$

21.49. (*a*) The ratio of the thermal energy added to the work done is the coefficient of performance for heating, so $W = \dfrac{Q_{\text{out}}}{\text{COP}_{\text{heating}}}$ or $\dfrac{dW}{dt} = \dfrac{dQ_{\text{out}}/dt}{\text{COP}_{\text{heating}}}$. A Carnot cycle has maximal efficiency, so the coefficient of performance for heating is given by Eq. 21.26 and we have $\dfrac{dW}{dt} = \dfrac{dQ_{\text{out}}}{dt}\dfrac{T_{\text{out}} - T_{\text{in}}}{T_{\text{out}}} = (1.35\text{ kW})\dfrac{47\text{ K}}{(22 + 273.15)\text{ K}} = 0.21\text{ kW}.$ (*b*) The pump's coefficient of performance for heating is $\text{COP}_{\text{heating}} = \dfrac{T_{\text{out}}}{T_{\text{out}} - T_{\text{in}}} = \dfrac{(22 + 273.15)\text{ K}}{47\text{ K}} = 6.3.$

21.50. The coefficient of performance for cooling is the ratio of the thermal energy taken in to the work done on the heat pump, and the thermal energy expelled equals the sum of the thermal energy taken in and the work done, so we have

$$\text{COP}_{\text{cooling}} = \frac{Q_{\text{in}}}{W}$$

$$Q_{\text{in}} = \text{COP}_{\text{cooling}}W$$

$$Q_{\text{out}} = Q_{\text{in}} + W = W(1 + \text{COP}_{\text{cooling}})$$

$$\frac{Q_{\text{out}}}{dt} = \frac{dW}{dt}(1 + \text{COP}_{\text{cooling}}) = (225\text{ W})(1 + 4.00) = 1.13 \times 10^3\text{ W}$$

21.51. The change in thermal energy is proportional to the change in temperature (Eq. 20.15), and we can determine the change in temperature from Eq. 21.36 and the efficiency and temperature given,

$$\eta = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}}$$

$$T_{\text{in}}\eta = T_{\text{in}} - T_{\text{out}}$$

$$T_{\text{in}} = \frac{T_{\text{out}}}{1 - \eta}$$

$$\Delta T = T_{\text{out}}\left(\frac{1}{1 - \eta} - 1\right) = T_{\text{out}}\left(\frac{\eta}{1 - \eta}\right)$$

In this temperature range, the heat capacity per molecule for diatomic nitrogen is $2.50k_B$, so the change in thermal energy is

$$\Delta E_{th} = NC_V\Delta T = N(2.50k_B)T_{out}\left(\frac{\eta}{1-\eta}\right)$$
$$= (7.50\times10^{23})(2.50)(1.381\times10^{-23}\text{ J/K})\left[(25+273.15)\text{ K}\right]\left(\frac{0.350}{1-0.350}\right) = 4.16\times10^3\text{ J}$$

21.52. (*a*) The coefficient of performance for heating is the ratio of the thermal energy output to the work done on the heat pump, and we know the COP for a Carnot cycle, so

$$\text{COP}_{heating} = \frac{Q_{out}}{W}$$
$$Q_{out} = \text{COP}_{heating}W = \frac{T_{out}}{T_{out}-T_{in}}W$$
$$= \frac{(20+273.15)\text{ K}}{10\text{ K}}(1.0\text{ J}) = 29\text{ J}$$

(*b*) With the colder input temperature we have $Q_{out} = \frac{(20+273.15)\text{ K}}{43\text{ K}}(1.0\text{ J}) = 6.8\text{ J}$. It takes more than four times as much electrical energy to transfer the same amount of thermal energy, so depending on the cost of electricity and the cost of drilling, it might well be worth it. (*c*) The coefficients of performance of heating are 29 in part (*a*) and 6.8 in part (*b*).

21.53. We know the change in entropy for an isothermal process (Eq. 20.54), $\Delta S = \frac{Q}{k_B T}$, or $\frac{dS}{dt} = \frac{dQ/dt}{k_B T}$, and we know the temperatures, so we need to determine the rate at which thermal energy is removed from the water and added to the air in the cabin. We can find these from the coefficient of performance for heating and the temperatures (Eqs. 21.24 and 21.26). For the air, we have

$$\text{COP}_{heating} = \frac{T_{out}}{T_{out}-T_{in}}$$
$$Q_{out} = W\text{COP}_{heating}$$
$$\frac{Q_{out}}{dt} = \frac{W}{dt}\text{COP}_{heating} = \frac{W}{dt}\frac{T_{out}}{T_{out}-T_{in}}$$
$$= (75\text{ W})\frac{(20+273.15)\text{ K}}{16\text{ K}} = 1.374\times10^3\text{ W}$$

and the rate of change of the entropy of the air is $\frac{dS}{dt} = \frac{1.374\times10^3\text{ W}}{(1.381\times10^{-23}\text{ J/K})\left[(20+273.15)\text{ K}\right]} = 3.4\times10^{23}\text{ s}^{-1}$. The rate at which thermal energy is removed from the water equals the rate at which it is added to the air minus the rate at which work is done on the pump, so

$$\frac{Q}{dt} = -\frac{W}{dt}(\text{COP}_{heating}-1) = -\frac{W}{dt}\frac{T_{in}}{T_{out}-T_{in}}$$
$$= -(75\text{ W})\frac{(4+273.15)\text{ K}}{16\text{ K}} = -1.299\times10^3\text{ W}$$

and the rate of change of the entropy of the water is $\frac{dS}{dt} = \frac{-1.299\times10^3\text{ W}}{(1.381\times10^{-23}\text{ J/K})\left[(4+273.15)\text{ K}\right]} = -3.4\times10^{23}\text{ s}^{-1}$. As expected, the rates have the same magnitude but opposite signs.

21.54. The time interval required is that in which the engine can do sufficient work to increase the car's kinetic energy the desired amount. We can use Eq. 21.34 to determine the work done by each cylinder per cycle, and use that to determine the number of cycles required. For the six-cylinder engine, the work done per cycle is

$$W_{\text{out}} = 6Nk_{\text{B}}(T_{\text{in}} - T_{\text{out}})\ln\left(\frac{V_2}{V_1}\right) = 6(6.02\times10^{23})(1.381\times10^{-23}\text{ J/K})(300\text{ K})\ln(2) = 1.0373\times10^4\text{ J}$$

where we have retained a few extra digits of precision in our intermediate result. The energy required is $K = \frac{1}{2}mv^2 = \frac{1}{2}(820\text{ kg})\left[(50.0\text{ km/h})\frac{1000\text{ m}}{1\text{ km}}\frac{1\text{ h}}{3600\text{ s}}\right]^2 = 7.9090\times10^4\text{ J}$, so the engine must complete $N_{\text{cycles}} = \frac{K}{W_{\text{out}}} = \frac{7.9090\times10^4\text{ J}}{1.0373\times10^4\text{ J}} = 7.625$ cycles. Each cycle lasts $2(9.00\text{ s}) + 2(2.00\text{ s}) = 22.00\text{ s}$, so the car takes $\Delta t = (7.625)(22.00\text{ s}) = 168\text{ s}$ to accelerate.

21.55. (*a*) In order to freeze the water, we must remove a quantity of thermal energy given by Eq. 20.56, $Q = mL$. We can determine the rate at which the freezer removes thermal energy from the rate at which it draws electrical energy, the definition of the coefficient of performance of cooling (Eq. 21.27), and the relationship between the coefficient of performance of cooling and the temperatures for a Carnot-cycle heat pump (Eq. 21.29). Combining these we have

$$Q = \frac{dQ}{dt}\Delta t$$

$$\Delta t = \frac{Q}{dQ/dt}$$

$$\frac{dQ}{dt} = \frac{dW}{dt}\text{COP}_{\text{cooling}} = \frac{dW}{dt}\frac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}}$$

$$\Delta t = \frac{mL}{dW/dt}\frac{T_{\text{out}} - T_{\text{in}}}{T_{\text{in}}} = \frac{(0.500\text{ kg})(334\text{ kJ/kg})}{115\text{ W}}\frac{40\text{ K}}{(-18+273.15)\text{ K}} = 2.3\times10^2\text{ s}$$

(*b*) The coefficient of performance of cooling is $\text{COP}_{\text{cooling}} = \frac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} = \frac{(-18+273.15)\text{ K}}{40\text{ K}} = 6.4$. (*c*) The thermal energy exhausted into the kitchen equals the sum of the thermal energy removed from the freezer plus the work done on the engine,

$$Q_{\text{out}} = Q_{\text{in}} + W = mL + \frac{dW}{dt}\Delta t = (0.500\text{ kg})(334\text{ kJ/kg}) + (115\text{ W})(228\text{ s}) = 1.93\times10^5\text{ J}$$

(*d*) The change in entropy of the water as it freezes is given by Eq. 20.58,

$$\Delta S_{\text{water}} = -\frac{mL}{k_{\text{B}}T} = -\frac{(0.500\text{ kg})(334\text{ kJ/kg})}{(1.381\times10^{-23}\text{ J/K})(273.15\text{ K})} = -4.43\times10^{25}$$

(*e*) The change in entropy of the kitchen is (Eq. 20.54)

$$\Delta S_{\text{kitchen}} = \frac{Q_{\text{out}}}{k_{\text{B}}T} = \frac{1.932\times10^5}{(1.381\times10^{-23}\text{ J/K})\left[(22+273.15)\text{ K}\right]} = 4.74\times10^{25}$$

so the change in entropy of the system made up of the water and the kitchen is $\Delta S = \Delta S_{\text{water}} + \Delta S_{\text{kitchen}} = -4.43\times10^{25} + 4.74\times10^{25} = 3.1\times10^{24}$. This is greater than zero, but that is to be expected as there is an irreversible transfer of thermal energy from the freezing water to the colder freezer.

21.56. For a Brayton cycle, the ratios of the temperatures at the endpoints of the two isobaric processes are equal (Eq. 21.42), so we have

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$T_4 = \frac{T_3T_1}{T_2} = \frac{(550\text{ K})(400\text{ K})}{750\text{ K}} = 293\text{ K}$$

21.57. The maximum efficiency for an engine operating on the Brayton cycle is related to the ratio of the temperature at which the isobaric expansion ends to the temperature at which the isobaric compression begins

(Eq. 21.44). We also know that the ratios of the temperatures at the endpoints of the two isobaric processes are equal (Eq. 21.42), so we have

$$\frac{T_2}{T_1}=\frac{T_3}{T_4}$$

$$\frac{T_1+(100\text{ K})}{T_1}=\frac{T_4+(85\text{ K})}{T_4}$$

$$T_4T_1+T_4(100\text{ K})=T_1T_4+T_1(85\text{ K})$$

$$\frac{T_4}{T_1}=\frac{85\text{ K}}{100\text{ K}}$$

$$\eta=1-\frac{T_4}{T_1}=1-\frac{85\text{ K}}{100\text{ K}}=0.15$$

21.58. Using Eq. 21.44,

$$\eta=1-\frac{T_4}{T_1}$$

$$T_1=\frac{T_4}{(1-\eta)}=\frac{267\text{K}}{(1-0.22)}\simeq 342.3\text{ K}$$

21.59. The maximum possible efficiency for an engine operating on the Brayton cycle is related to the pressure ratio by Eq. 21.47. Because the working substance is a monatomic ideal gas, the heat capacity ratio is $\frac{5}{3}$, and we have

$$\eta=1-\left(\frac{P_\text{H}}{P_\text{L}}\right)^{(1/\gamma)-1}=1-\left(\frac{103{,}500\text{ Pa}}{99{,}700\text{ Pa}}\right)^{-2/5}=1.4851\times10^{-2}$$

21.60. (*a*) The pressure ratio is the ratio of the higher pressure to the lower pressure, that is, $(6.0\text{ atm})/(1.0\text{ atm})=6.0$. (*b*) The engine's efficiency is (Eq. 21.47) $\eta=1-\left(\frac{P_\text{H}}{P_\text{L}}\right)^{(1/\gamma)-1}=1-\left(\frac{6.0\text{ atm}}{1.0\text{ atm}}\right)^{(1/1.40)-1}=$ 0.40. (*c*) The efficiency is now $\eta=1-\left(\frac{P_\text{H}}{P_\text{L}}\right)^{(1/\gamma)-1}=1-\left(\frac{4.8\text{ atm}}{0.40\text{ atm}}\right)^{(1/1.44)-1}=0.53$.

21.61. We can equate the formula for the efficiency of the Brayton-cycle engine (Eq. 21.47) with that of the Carnot-cycle engine (Eq. 21.36) and solve for the input temperature,

$$1-\left(\frac{P_\text{H}}{P_\text{L}}\right)^{(1/\gamma)-1}=1-\frac{T_\text{out}}{T_\text{in}}$$

$$T_\text{in}=T_\text{out}\left(\frac{P_\text{H}}{P_\text{L}}\right)^{1-(1/\gamma)}=\left[(15+273.15)\text{ K}\right](5)^{1-(1/1.333)}=431\text{ K}$$

21.62. We can equate the formula for the efficiency of the Brayton-cycle engine (Eq. 21.47) with that of the Carnot-cycle engine (Eq. 21.36) and solve for the pressure ratio,

$$1-\left(\frac{P_\text{H}}{P_\text{L}}\right)^{(1/\gamma)-1}=1-\frac{T_\text{out}}{T_\text{in}}$$

$$\frac{P_\text{H}}{P_\text{L}}=\left(\frac{T_\text{out}}{T_\text{in}}\right)^{\gamma/(1-\gamma)}=\left(\frac{(-10+273.15)\text{ K}}{(225+273.15)\text{ K}}\right)^{1.667/(-0.667)}=4.93$$

21.63. The rate at which the engine can perform work equals the product of the rate at which thermal energy is input times the efficiency, $\frac{dW_{\text{out}}}{dt} = \eta \frac{dQ_{\text{in}}}{dt} = \left[1 - \left(\frac{P_{\text{H}}}{P_{\text{L}}}\right)^{(1/\gamma)-1}\right]\frac{dQ_{\text{in}}}{dt}$. The rate at which it performs work equals the rate of change of the bricks' gravitational potential energy, $\frac{dW_{\text{out}}}{dt} = \frac{d}{dt}(mgh) = mg\frac{dh}{dt}$. Combining these, and noting that the heat capacity ratio for diatomic nitrogen is about 1.40, we have

$$\frac{dQ_{\text{in}}}{dt} = mg\frac{dh}{dt}\left[1 - \left(\frac{P_{\text{H}}}{P_{\text{L}}}\right)^{(1/\gamma)-1}\right]^{-1} = (535\text{ kg})(9.8\text{ N/kg})(0.100\text{ m/s})\left[1 - 10^{(1/1.40)-1}\right]^{-1} = 1.09\times10^{3}\text{ J}$$

21.64. (*a*) If the engine does 175 J of work while exhausting 65 J of thermal energy, the thermal energy input must be $Q_{\text{in}} = W_{\text{out}} + Q_{\text{out}} = (175\text{ J}) + (65\text{ J}) = 240\text{ J}$. We can determine the required pressure ratio from the efficiency (Eq. 21.47), where the heat capacity ratio is $\frac{5}{3}$ for helium,

$$\eta = \frac{W_{\text{out}}}{Q_{\text{in}}} = 1 - \left(\frac{P_{\text{H}}}{P_{\text{L}}}\right)^{(1/\gamma)-1}$$

$$\left(\frac{P_{\text{H}}}{P_{\text{L}}}\right)^{(1/\gamma)-1} = 1 - \frac{W_{\text{out}}}{Q_{\text{in}}}$$

$$\frac{P_{\text{H}}}{P_{\text{L}}} = \left(1 - \frac{W_{\text{out}}}{Q_{\text{in}}}\right)^{\gamma/(1-\gamma)} = \left(1 - \frac{175\text{ J}}{240\text{ J}}\right)^{-5/2} = 26.2$$

(*b*) No, it is not because the heat capacity ratio of carbon dioxide is smaller than that of helium. The heat capacity ratio of carbon dioxide varies with temperature, but at $100\ ^\circ\text{C}$ it is about 1.28, so the required pressure ratio would be $\frac{P_{\text{H}}}{P_{\text{L}}} = \left(1 - \frac{175\text{ J}}{240\text{ J}}\right)^{1.28/(-0.28)} = 392.$

21.65. For an ideal gas, the quantity PV^{γ} is constant along an isentrope (Eq. 20.46), so we have

$$P_2V_2^{\gamma} = P_3V_3^{\gamma}$$

$$\left(\frac{V_2}{V_3}\right)^{\gamma} = \frac{P_3}{P_2}$$

$$\gamma\ln(V_2/V_3) = \ln(P_3/P_2)$$

$$\gamma = \frac{\ln(P_3/P_2)}{\ln(V_2/V_3)}$$

The heat capacity ratio is related to the contributing degrees of freedom by Eq. 20.26,

$$\gamma = 1 + \frac{2}{d}$$

$$\frac{2}{d} = \gamma - 1$$

$$d = \frac{2}{\gamma - 1}$$

Combining these we have

$$d = \frac{2}{\frac{\ln(P_3/P_2)}{\ln(V_2/V_3)} - 1} = \frac{2}{\frac{\ln\left[(50\text{ kPa})/(250\text{ kPa})\right]}{\ln\left[(1.00\text{ m}^3)/(3.50\text{ m}^3)\right]} - 1} = 7.0$$

21.66. (*a*) We can express the ideal engine's efficiency in terms of the pressure ratio (Eq. 21.47) and from this determine the real engine's efficiency. If we use R to represent the pressure ratio, we can find the maximum efficiency by setting the derivative with respect to R equal to zero,

$$\eta_{\text{ideal}} = 1 - R^{(1/\gamma)-1}$$

$$\eta_{\text{real}} = (1.45)(1-\eta_{\text{ideal}})^{0.8}\eta_{\text{ideal}} = (1.45)\left[1-(1-R^{(1/\gamma)-1})\right]^{0.8}(1-R^{(1/\gamma)-1})$$

$$= (1.45)R^{0.8[(1/\gamma)-1]}(1-R^{(1/\gamma)-1}) = (1.45)(R^{0.8[(1/\gamma)-1]} - R^{1.8[(1/\gamma)-1]})$$

$$\frac{d\eta_{\text{real}}}{dR} = (1.45)\left\{0.8\left[(1/\gamma)-1\right]R^{-0.2[(1/\gamma)-1]} - 1.8\left[(1/\gamma)-1\right]R^{0.8[(1/\gamma)-1]}\right\} = 0$$

$$0.8\left[(1/\gamma)-1\right]R^{-0.2[(1/\gamma)-1]} = 1.8\left[(1/\gamma)-1\right]R^{0.8[(1/\gamma)-1]}$$

$$\frac{R^{-0.2[(1/\gamma)-1]}}{R^{0.8[(1/\gamma)-1]}} = \frac{1.8}{0.8}$$

$$R^{1-(1/\gamma)} = 2.25$$

$$R = (2.25)^{\gamma/(\gamma-1)} = (2.25)^{1.4/0.4} = 17.1$$

That is, we choose a pressure ratio of 17. (*b*) For a given input pressure and temperature, a greater pressure ratio means a greater maximum pressure, and input of thermal energy at greater temperatures. Limits on these will be set by the materials used to construct the engine, and also by the source of thermal energy (for example, if the source of thermal energy is burning fuel, thermal energy cannot be input at a temperature greater than the combustion temperature of the fuel). Another consideration is that at higher pressures the working fluid will likely deviate more from ideal-gas behavior.

21.67. For a transfer of thermal energy at constant temperature, the change in entropy is (Eq. 20.54) $\Delta S = \dfrac{Q}{k_{\text{B}}T} = \dfrac{4180\text{ J}}{(1.381\times10^{-23}\text{ J/K})(265\text{ K})} = 1.14\times10^{24}.$

21.68. For a transfer of thermal energy at constant temperature, the change in entropy is (Eq. 20.54) $\Delta S = \dfrac{Q}{k_{\text{B}}T}$, so the temperature is $T = \dfrac{Q}{k_{\text{B}}\Delta S} = \dfrac{10\text{ J}}{(1.381\times10^{-23}\text{ J/K})(3.8\times10^{21})} = 1.9\times10^{2}\text{ K}.$

21.69. The COP of cooling is the ratio of thermal energy input to work done on the heat engine, so if the work required has decreased by a factor of four, the COP of cooling has increased by a factor of four.

21.70. The change in entropy (Eq. 20.54) is proportional to the quantity of energy transferred thermally, which we can determine from the definition of the specific transformation energy, Eq. 20.55. That is, $Q = Lm,$ so the change in entropy of the ice is $\Delta S_{\text{ice}} = \dfrac{Lm}{k_{\text{B}}T_{\text{ice}}} = \dfrac{(334\text{ kJ/kg})(1000\text{ kg})}{(1.381\times10^{-23}\text{ J/K})(273.15\text{ K})} = 8.854\times10^{28},$ and the change in entropy of the water is $\Delta S_{\text{water}} = \dfrac{-Lm}{k_{\text{B}}T_{\text{water}}} = \dfrac{-(334\text{ kJ/kg})(1000\text{ kg})}{(1.381\times10^{-23}\text{ J/K})(275.15\text{ K})} = -8.790\times10^{28},$ so the change in entropy of the ice-water system is $\Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{water}} = 8.854\times10^{28} - 8.790\times10^{28} = 6.4\times10^{26}.$

21.71. (*a*) As the boulder is pulled up the ramp, it rises a vertical distance $h = \ell\sin\theta,$ increasing the gravitational potential energy of the Earth-boulder system by $\Delta E = mgh,$ so this amount of mechanical energy must be supplied by the engine. The amount of work the engine can do for a given amount of thermal energy input is determined by its efficiency, which for a Carnot-cycle engine depends on the ratio of the temperatures of the high- and low-temperature reservoirs (Eq. 21.36). Combining these, we can solve for the minimum required temperature of the high-temperature reservoir,

$$\frac{W_{\text{out}}}{Q_{\text{in}}} = 1 - \frac{T_{\text{out}}}{T_{\text{in}}}$$

$$T_{\text{in}} = \frac{T_{\text{out}}}{1 - \dfrac{W_{\text{out}}}{Q_{\text{in}}}} = \frac{T_{\text{out}}}{1 - \dfrac{mg\ell\sin\theta}{Q_{\text{in}}}}$$

$$= \frac{(-5 + 273.15)\text{ K}}{1 - \dfrac{(1200\text{ kg})(9.8\text{ N/kg})(65.0\text{ m})\sin(35.0°)}{1.00\text{ MJ}}} = 478\text{ K}$$

(*b*) From the energy law, we know that the thermal energy discarded equals the difference between the thermal energy input and the work done by the engine,

$$Q_{\text{out}} = Q_{\text{in}} - W_{\text{out}} = Q_{\text{in}} - mg\ell\sin\theta$$
$$= (1.00\text{ MJ}) - (1200\text{ kg})(9.8\text{ N/kg})(65.0\text{ m})\sin(35.0°) = 0.56\text{ MJ}$$

21.72. (*a*) The coefficient of performance of cooling of the heat pump is the ratio of the thermal energy removed from the house to the mechanical energy input to the pump, and is maximal for a Carnot-cycle heat pump, so we have (Eq. 21.29)

$$\frac{Q_{\text{in}}}{W_{\text{in}}} = \frac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}}$$

$$Q_{\text{in}} = W_{\text{in}}\frac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} = (1.0\text{ J})\frac{(20 + 273.15)\text{ K}}{18\text{ K}} = 16\text{ J}$$

(*b*) The coefficient of performance of cooling is the factor we just calculated, 16.

21.73. The efficiency of a Carnot-cycle heat engine is related to the temperatures of the high- and low-temperature reservoirs by Eq. 21.36. Given the efficiency and the temperature of the low-temperature reservoir, we can solve for the temperature of the high-temperature reservoir, and use this to find the new efficiency:

$$\eta_1 = 1 - \frac{T_{1\text{out}}}{T_{\text{in}}}$$

$$T_{\text{in}} = \frac{T_{1\text{out}}}{1 - \eta_1}$$

$$\eta_2 = 1 - \frac{T_{2\text{out}}}{T_{\text{in}}} = 1 - \frac{T_{2\text{out}}}{T_{1\text{out}}}(1 - \eta_1) = 1 - \frac{(18 + 273.15)\text{ K}}{(28 + 273.15)\text{ K}}(1 - 0.33) = 0.35$$

That is, the efficiency could be improved by about 6.7%.

21.74. The rate at which thermal energy must be supplied to the engine equals the rate at which the engine does work divided by its efficiency, and we have an expression for the efficiency of a Carnot-cycle engine, Eq. 21.36,

$$\eta = \frac{W_{\text{out}}}{Q_{\text{in}}}$$

$$Q_{\text{in}} = \frac{W_{\text{out}}}{\eta}$$

$$\frac{dQ_{\text{in}}}{dt} = \frac{dW_{\text{out}}/dt}{\eta} = \frac{dW_{\text{out}}/dt}{1 - \dfrac{T_{\text{out}}}{T_{\text{in}}}}$$

The rate at which work must be done is the power needed to lift the water at the required volume flow rate (see *Principles* Section 18.7), $P = Q_{\text{water}}\rho_{\text{water}}gh$, so the rate at which thermal energy must be supplied to the engine is

$$\frac{dQ_{\text{in}}}{dt} = \frac{Q_{\text{water}}\rho_{\text{water}}gh}{1-\dfrac{T_{\text{out}}}{T_{\text{in}}}}$$

$$= \frac{(6600\ \text{L/min})\dfrac{1\ \text{m}^3}{1000\ \text{L}}\dfrac{1\ \text{min}}{60\ \text{s}}(1000\ \text{kg/m}^3)(9.8\ \text{N/kg})(45.0\ \text{m})}{1-\dfrac{(-10+273.15)\ \text{K}}{(215+273.15)\ \text{K}}} = 1.05\times10^5\ \text{W}$$

21.75. (*a*) From the energy law, the rate at which thermal energy is output equals the sum of the rate at which thermal energy is input plus the rate at which work is done on the pump, so

$$Q_{\text{out}} = Q_{\text{in}} + W_{\text{in}}$$
$$\frac{dQ_{\text{out}}}{dt} = \frac{dQ_{\text{in}}}{dt} + \frac{dW_{\text{in}}}{dt}$$
$$\frac{dQ_{\text{in}}}{dt} = \frac{dQ_{\text{out}}}{dt} - \frac{dW_{\text{in}}}{dt}$$

The rate at which work is done on the pump equals the rate at which thermal energy is output divided by the coefficient of performance of heating (Eq. 21.24),

$$\text{COP}_{\text{heating}} = \frac{Q_{\text{out}}}{W_{\text{in}}}$$
$$W_{\text{in}} = \frac{Q_{\text{out}}}{\text{COP}_{\text{heating}}}$$
$$\frac{dW_{\text{in}}}{dt} = \frac{dQ_{\text{out}}/dt}{\text{COP}_{\text{heating}}}$$

and the coefficient of heating for a Carnot-cycle heat pump is maximal (Eq. 21.26), so $\dfrac{dW_{\text{in}}}{dt} = \dfrac{dQ_{\text{out}}}{dt}\dfrac{T_{\text{out}}-T_{\text{in}}}{T_{\text{out}}}$.

Combining these we have

$$\frac{dQ_{\text{in}}}{dt} = \frac{dQ_{\text{out}}}{dt}\left(1-\frac{T_{\text{out}}-T_{\text{in}}}{T_{\text{out}}}\right)$$
$$= \frac{dQ_{\text{out}}}{dt}\frac{T_{\text{in}}}{T_{\text{out}}} = (12.0\ \text{W})\frac{(5+273.15)\ \text{K}}{(35+273.15)\ \text{K}} = 10.8\ \text{W}$$

(*b*) We developed an expression for the rate at which mechanical energy is input, $\dfrac{dW_{\text{in}}}{dt} = \dfrac{dQ_{\text{out}}}{dt}\dfrac{T_{\text{out}}-T_{\text{in}}}{T_{\text{out}}} =$

$(12.0\ \text{W})\dfrac{30\ \text{K}}{(35+273.15)\ \text{K}} = 1.2\ \text{W}.$

21.76. We can determine the electric power required from the rate of thermal energy output and the coefficient of performance for heating (Eq. 21.24),

$$\text{COP}_{\text{heating}} = \frac{Q_{\text{out}}}{W_{\text{in}}}$$
$$W_{\text{in}} = \frac{Q_{\text{out}}}{\text{COP}_{\text{heating}}}$$
$$\frac{dW_{\text{in}}}{dt} = \frac{dQ_{\text{out}}/dt}{\text{COP}_{\text{heating}}}$$

and the coefficient of heating for a Carnot-cycle heat pump is maximal (Eq. 21.26), so $\dfrac{dW_{\text{in}}}{dt} = \dfrac{dQ_{\text{out}}}{dt}\dfrac{T_{\text{out}}-T_{\text{in}}}{T_{\text{out}}}$. The amount of electrical energy used is the product of the rate at which it is used and the time interval over which it is used, so the cost is

$$\text{cost} = \frac{dQ_{\text{out}}}{dt}\frac{T_{\text{out}} - T_{\text{in}}}{T_{\text{out}}}\Delta t(\text{price})$$
$$= (12.5\text{ kW})\frac{32\text{ K}}{(22 + 273.15)\text{ K}}(14\text{ h})\frac{3600\text{ s}}{1\text{ h}}(\$0.0269/\text{MJ}) = \$1.84$$

21.77. (*a*) The temperature of the high-temperature reservoir is the surface temperature plus the product of the temperature gradient and the depth, so the maximum efficiency is (Eq. 21.36)

$$\eta = 1 - \frac{T_{\text{out}}}{T_{\text{in}}} = 1 - \frac{T_{\text{out}}}{T_{\text{out}} + (dT/dx)\Delta x}$$
$$= 1 - \frac{(28 + 273.15)\text{ K}}{\left[(28 + 273.15)\text{ K}\right] + (25\text{ K/km})(30\text{ km})} = 0.71$$

(*b*) We can rearrange our equation above to find the depth,

$$\eta = 1 - \frac{T_{\text{out}}}{T_{\text{out}} + (dT/dx)\Delta x}$$
$$\frac{T_{\text{out}}}{T_{\text{out}} + (dT/dx)\Delta x} = 1 - \eta$$
$$T_{\text{out}} = (1-\eta)\left[T_{\text{out}} + (dT/dx)\Delta x\right]$$
$$\Delta x = \frac{\eta T_{\text{out}}}{dT/dx} = \frac{0.50\left[(28 + 273.15)\text{ K}\right]}{25\text{ K/km}} = 6.0\text{ km}$$

21.78. (*a*) The efficiency relates the work output to the thermal energy input (Eq. 21.21), and we know the efficiency of a Carnot-cycle engine (Eq. 21.36), so the rate at which the engine extracts thermal energy is

$$\eta = \frac{W_{\text{out}}}{Q_{\text{in}}}$$
$$Q_{\text{in}} = \frac{W_{\text{out}}}{\eta}$$
$$\frac{dQ_{\text{in}}}{dt} = \frac{dW_{\text{out}}/dt}{\eta} = \frac{W_{\text{out}}}{dt}\frac{T_{\text{in}}}{T_{\text{in}} - T_{\text{out}}}$$
$$= (2.50\text{ MW})\frac{1000\text{ K}}{(1000\text{ K}) - (380\text{ K})} = 4.03\text{ MW}$$

(*b*) The rate of change of the entropy of the core is (Eq. 20.54)

$$\Delta S = \frac{Q}{k_{\text{B}}T}$$
$$\frac{dS}{dt} = \frac{dQ/dt}{k_{\text{B}}T} = \frac{-4.03\text{ MW}}{(1.381\times10^{-23}\text{ J/K})(1000\text{ K})} = -2.92\times10^{26}\text{ s}^{-1}$$

(*c*) The thermodynamic efficiency is, as we saw above,

$$\eta = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}} = \frac{(1000\text{ K}) - (380\text{ K})}{1000\text{ K}} = 0.620$$

21.79. (*a*) For a device operating on a Carnot cycle, the efficiency is (Eq. 21.36) $\eta = \frac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}} = \frac{T_{\text{H}} - T_{\text{L}}}{T_{\text{H}}}$, and the coefficient of performance of cooling is (Eq. 21.29) $\text{COP}_{\text{cooling}} = \frac{T_{\text{in}}}{T_{\text{out}} - T_{\text{in}}} = \frac{T_{\text{L}}}{T_{\text{H}} - T_{\text{L}}}$, so $\text{COP}_{\text{cooling}} = \frac{1}{\eta}\frac{T_{\text{L}}}{T_{\text{H}}} = \frac{1}{\eta}(1-\eta) = \frac{1}{\eta} - 1$. (*b*) The coefficient of performance of heating is (Eq. 21.26) $\text{COP}_{\text{heating}} = \frac{T_{\text{out}}}{T_{\text{out}} - T_{\text{in}}} = \frac{T_{\text{H}}}{T_{\text{H}} - T_{\text{L}}}$, so

$\text{COP}_{\text{heating}} = \dfrac{1}{\eta}.$ (*c*) The efficiency is between zero and one, so the coefficient of performance of heating is greater than one, as it should be, and the coefficient of performance of cooling is greater than zero, as it should be.

21.80. We can determine the rate at which thermal energy must be supplied to the engine from its efficiency and the rate at which it performs work (Eq. 21.21), and we know the efficiency of a Carnot-cycle engine (Eq. 21.36), so

$$\eta = \frac{W_{\text{out}}}{Q_{\text{in}}}$$

$$Q_{\text{in}} = \frac{W_{\text{out}}}{\eta}$$

$$\frac{dQ_{\text{in}}}{dt} = \frac{dW_{\text{out}}/dt}{\eta} = \frac{dW_{\text{out}}}{dt}\frac{T_{\text{in}}}{T_{\text{in}} - T_{\text{out}}}$$

The rate at which it performs work equals the power needed to lift the oil at the required volume flow rate (see *Principles* Section 18.7), $P = Q_{\text{oil}}\rho_{\text{oil}}gh,$ so the rate at which thermal energy must be supplied is

$$\frac{dQ_{\text{in}}}{dt} = \frac{Q_{\text{oil}}\rho_{\text{oil}}ghT_{\text{in}}}{T_{\text{in}} - T_{\text{out}}}$$

$$= \frac{(25{,}000 \text{ bbl/day})\dfrac{0.159 \text{ m}^3}{1 \text{ bbl}}\dfrac{1 \text{ day}}{24 \text{ h}}\dfrac{1 \text{ h}}{3600 \text{ s}}(850 \text{ kg/m}^3)(9.8 \text{ N/kg})(1.70 \text{ km})\left\{\left[\tfrac{5}{9}(250-32)+273.15\right] \text{ K}\right\}}{(\tfrac{5}{9}200) \text{ K}} = 2.31\times10^6 \text{ W}$$

21.81. The kinetic energy of the rotating flywheel (see *Principles* Sections 11.5 and 11.6) is $K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{4}mR^2\omega^2,$ and this is the amount of work the engine must perform. The work is related to the thermal energy that must be transferred to the engine by the efficiency (Eq. 21.21),

$$\eta = \frac{W_{\text{out}}}{Q_{\text{in}}}$$

$$Q_{\text{in}} = \frac{W_{\text{out}}}{\eta}$$

and we know the efficiency of a Carnot-cycle engine (Eq. 21.36), $\eta = \dfrac{T_{\text{in}} - T_{\text{out}}}{T_{\text{in}}},$ so the thermal energy that must be transferred to the engine is

$$Q_{\text{in}} = \tfrac{1}{4}mR^2\omega^2\frac{T_{\text{in}}}{T_{\text{in}} - T_{\text{out}}}$$

$$= \tfrac{1}{4}(1500 \text{ kg})(0.500 \text{ m})^2(8.50 \text{ s}^{-1})^2\frac{(150+273.15) \text{ K}}{140 \text{ K}} = 2.05\times10^4 \text{ J}$$

The amount of thermal energy discarded is, by the energy law,

$$Q_{\text{out}} = Q_{\text{in}} - W_{\text{out}} = Q_{\text{in}} - \tfrac{1}{4}mR^2\omega^2$$

$$= (2.05\times10^4 \text{ J}) - \tfrac{1}{4}(1500 \text{ kg})(0.500 \text{ m})^2(8.50 \text{ s}^{-1})^2 = 1.37\times10^4 \text{ J}$$

21.82. The energy that must be transferred thermally to the engine is related to the work it does by the efficiency (Eq. 21.21),

$$\eta = \frac{W_{\text{out}}}{Q_{\text{in}}}$$

$$Q_{\text{in}} = \frac{W_{\text{out}}}{\eta}$$

and we know the efficiency of a Brayton-cycle engine (Eq. 21.47), $\eta = 1 - \left(\dfrac{P_{\text{H}}}{P_{\text{L}}}\right)^{(1/\gamma)-1},$ where $\gamma = \tfrac{5}{3}$ for helium. The work the engine must do equals the kinetic energy of the moving car, plus that lost to friction,

$$75\% W_{\text{out}} = \tfrac{1}{2}mv^2$$
$$W_{\text{out}} = \tfrac{2}{3}mv^2$$

Combining these we have

$$Q_{\text{in}} = \frac{\tfrac{2}{3}mv^2}{1-\left(\dfrac{P_{\text{H}}}{P_{\text{L}}}\right)^{(1/\gamma)-1}} = \frac{\tfrac{2}{3}(950\ \text{kg})(30\ \text{m/s})^2}{1-8^{-2/5}} = 1.0\times10^6\ \text{J}$$

21.83. We know the relationship between initial and final pressures and volumes for an isentropic process (Eq. 20.46), which we can use to find the heat capacity ratio of the working substance,

$$P_4V_4^\gamma = P_1V_1^\gamma$$
$$\frac{P_4}{P_1} = \left(\frac{V_1}{V_4}\right)^\gamma$$
$$\ln(P_4/P_1) = \gamma\ln(V_1/V_4)$$
$$\gamma = \frac{\ln(P_4/P_1)}{\ln(V_1/V_4)}$$

Assuming the working substance is an ideal gas, we can determine the number of contributing degrees of freedom from Eq. 20.26

$$\gamma = 1+\frac{2}{d}$$
$$d = \frac{2}{\gamma-1}$$

We are not given V_4, but we can determine it from the ideal gas law, $V_4 = \dfrac{Nk_{\text{B}}T_4}{P_4}$. Combining these we have

$$d = \frac{2}{\dfrac{\ln(P_4/P_1)}{\ln(V_1/V_4)}-1} = \frac{2}{\dfrac{\ln(P_4/P_1)}{\ln(V_1P_4/Nk_{\text{B}}T_4)}-1}$$
$$= \frac{2}{\dfrac{\ln\left[(100\ \text{kPa})/(149\ \text{kPa})\right]}{\ln\left[(0.10\ \text{m}^3)(100\ \text{kPa})/(2.00\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1})(1.381\times10^{-23}\ \text{J/K})(800\ \text{K})\right]}-1} = 5.0$$

21.84. If the device loses thermal energy at a constant rate, it will take

$$\Delta t = \frac{E_{\text{i}}}{-dE/dt} = \frac{1000\ \text{J}}{200\ \mu\text{J/s}} = (5.00\times10^6\ \text{s})\frac{1\ \text{h}}{3600\ \text{s}}\frac{1\ \text{day}}{24\ \text{h}} = 57.9\ \text{days}$$

for the device to dissipate its initial energy. We will likely want to check the rate at which it loses thermal energy at various time intervals, to see whether or not the rate changes and revise our estimate of when it will stop as necessary. But it looks, at first glance, as if we have plenty of time.

Assuming a room temperature of about 300 K, the device increases the entropy of the environment at a rate of

$$\frac{dS}{dt} = \frac{dQ/dt}{k_{\text{B}}T} = \frac{200\ \mu\text{J/s}}{(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})} = 4.8\times10^{16}\ \text{s}^{-1}$$

which is very small. Over the time it takes to run down, it will increase the entropy of the environment by $\Delta S = \dfrac{Q}{k_{\text{B}}T} = \dfrac{1000\ \text{J}}{(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})} = 2.4\times10^{23}.$

For comparison, 1000 J is the order of magnitude of the thermal energy lost by a cup of coffee as it cools one degree Celsius. It would have to be stored in a very well insulated container if it took nearly two months for it to cool by such a small amount, and its effect on the entropy of the environment would be negligible.

21.85. We would like to compare the power output of each of the engines, because that will let us compare their accelerations and top speeds. We would also like to compare their efficiencies, because that will determine their fuel requirements.

For a Brayton cycle, the amount of work provided per cycle is (Eq. 21.37) $W_{\text{out}} = NC_P(T_2 - T_1 + T_4 - T_3)$. We are given T_4 and T_2, and can calculate the other temperatures from the isentropic relationship between temperature and pressure (Eq. 20.47), the pressure ratio, and the degrees of freedom of the working substance (through Eq. 20.26, $\gamma = 1 + 2/d$), $T_1 = T_4\left(\dfrac{P_4}{P_1}\right)^{(1/\gamma)-1} = (350\text{ K})(\tfrac{1}{5.00})^{-1/3} = 598.5\text{ K}$ and $T_3 = T_2\left(\dfrac{P_2}{P_3}\right)^{(1/\gamma)-1} = (700\text{ K})5.00^{-1/3} = 409.4\text{ K}$. So, the work per cycle is

$$W_{1\text{out}} = (6.02\times10^{23})\left[3(1.381\times10^{-23})\text{ J/K}\right]\left[(700\text{ K}) - (598.5\text{ K}) + (350\text{ K}) - (409.4\text{ K})\right] = 1051\text{ J}$$

(where we have used Eq. 20.25 $C_P = (1 + d/2)k_B$, and retained extra digits in the intermediate results), and the power output of the engine is $P_1 = \dfrac{W_{1\text{out}}}{\Delta t} = (1051\text{ J})(8.00\text{ Hz}) = 8.41\text{ kW}$. We can use this to determine the mass of the car from its acceleration,

$$P = \frac{\Delta K}{\Delta t}$$
$$P\Delta t = \tfrac{1}{2}mv^2$$
$$m = \frac{2P\Delta t}{v^2} = \frac{2(8.41\text{ kW})(11.5\text{ s})}{\left[(100\text{ km/h})\dfrac{1000\text{ m}}{1\text{km}}\dfrac{1\text{ h}}{3600\text{ s}}\right]^2} = 251\text{ kg}$$

which is very light for a car.

The efficiency of the engine is (Eq. 21.46) $\eta_1 = 1 - \left(\dfrac{P_H}{P_L}\right)^{(1/\gamma)-1} = 1 - 5.00^{-1/3} = 0.415$, from which we can determine the thermal energy input (Eq. 21.21) per cycle,

$$\eta = \frac{W_{\text{out}}}{Q_{\text{in}}}$$
$$Q_{1\text{in}} = \frac{W_{1\text{out}}}{\eta_1} = \frac{1051\text{ J}}{0.415} = 2532\text{ J}$$

For comparison, gasoline provides about 3×10^7 J of thermal energy per liter when burned. Gasoline burns at about 2700 K, so it is likely not the fuel used by this engine, but the thermal energy provided by burning one liter of gasoline could power this engine for about 1.2×10^4 cycles, or about 25 min at 8 cycles per second. At its top speed, the car could travel about 43 km in this time interval.

For a Carnot cycle, the amount of work provided per cycle is (Eq. 21.34) $W_{\text{out}} = Nk_B(T_{\text{in}} - T_{\text{out}})\ln\left(\dfrac{V_2}{V_1}\right)$, but we do not have enough information to determine the volume ratio. We are told, however, that car 2 has the same fuel intake as car 1, so we can determine the work output from the thermal energy input and the efficiency. The efficiency of a Carnot-cycle engine is (Eq. 21.36) is $\eta_2 = 1 - \dfrac{T_{\text{out}}}{T_{\text{in}}} = 1 - \dfrac{350\text{ K}}{700\text{ K}} = 0.500$. So, for the same thermal energy input per cycle, the work output by this engine in each cycle is $W_{2\text{out}} = \eta_2 Q_{1\text{in}} = (0.500)(2532\text{ J}) = 1266\text{ J}$. This engine only runs at 1.00 cycles per second, so its power output is 1.27 kW. That is, $\dfrac{P_1}{P_2} = \dfrac{\eta_1 Q_{\text{in}} f_1}{\eta_2 Q_{\text{in}} f_2} = \dfrac{\eta_1 f_1}{\eta_2 f_2} = \dfrac{(0.415)(8.00\text{ Hz})}{(0.500)(1.00\text{ Hz})} =$ 6.64. This is also the ratio of the time intervals needed to accelerate the cars from zero to 100 km/h, so car 2 requires 76.4 s to reach that speed.

When the car reaches its top speed, all the power of the engine is being dissipated through air resistance. The drag force of the air is proportional to the square of the car's speed, and the power dissipated equals the product of the force times the speed, and so is proportional to the cube of the car's speed. So, the top speed of car 2 is

$$\frac{P_1}{P_2} = \frac{v_{1\text{top}}^3}{v_{2\text{top}}^3}$$

$$v_{2\text{top}} = v_{1\text{top}}\left(\frac{P_1}{P_2}\right)^{-1/3} = (105\ \text{km/h})(6.64)^{-1/3} = 55.9\ \text{km/h}$$

and our preceding result is not valid; car 2 cannot accelerate to 100 km/h. Instead, in the 11.5 s it took car 1 to reach 100 km/h, car 2 can accelerate to 38.8 km/h.

Because car 2 uses fuel at one-eighth the rate of car 1, it can run for about 200 minutes on the thermal energy produced by burning one liter of gasoline, in which time it could travel about 180 km at its top speed.

21.86. The temperatures given for our thermal reservoirs correspond to $\left[\frac{5}{9}(100-32)+273.15\right]\ \text{K} = 311\ \text{K}$ and $\left[\frac{5}{9}(70-32)+273.15\right]\ \text{K} = 294\ \text{K}$. An ideal Carnot-cycle engine operating between these temperatures would have an efficiency of (Eq. 21.36) $\eta = 1-\frac{T_\text{out}}{T_\text{in}} = 1-\frac{294\ \text{K}}{311\ \text{K}} = 0.0547,$ which more than meets our engineer's requirement. The efficiency of the prototype, $40\%(0.547) = 0.022,$ does not. We need to improve the engine so that its efficiency is at least $0.045/0.0547 = 82\%$ that of an ideal Carnot-cycle engine operating between these temperatures.

If a heat engine does not achieve maximal efficiency, it means that more energy is being degraded than is necessary to satisfy the entropy law, which means that irreversible processes are taking place. The most obvious candidate is the dissipation of mechanical energy to thermal energy through friction. If we have not already done so in our prototype, we could try using low-friction material, such as Teflon or Delrin, for the moving parts. Another possible way to increase the efficiency is to make the isothermal and isentropic processes more nearly ideal. So, for example, we might add fins (such as those on the condenser of an air conditioner) to the parts of the engine where thermal energy is transferred isothermally, and we might add better thermal insulation to the parts where the isentropic processes occur.

Another possibility is to change the temperature of the thermal reservoirs. We cannot change the temperature of the bath water or the air, and it might not be safe to use a hotter high-temperature reservoir, but maybe we could add a compartment filled with cold water or even an ice cube to use as the low-temperature reservoir. If the engine still operates at 40% of the ideal efficiency, the temperature required is

$$40\%\eta = 40\%\left(1-\frac{T_\text{out}}{T_\text{in}}\right) \geq 0.045$$

$$1-\frac{T_\text{out}}{T_\text{in}} \geq \frac{0.045}{0.40}$$

$$1-\frac{0.045}{0.40} \geq \frac{T_\text{out}}{T_\text{in}}$$

$$T_\text{out} \leq T_\text{in}\left(1-\frac{0.045}{0.40}\right) \leq (311\ \text{K})\left(1-\frac{0.045}{0.40}\right) = 276\ \text{K}$$

so an ice cube should work.

22

ELECTRIC INTERACTIONS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{-4} N **2.** 10^{-7} N **3.** 10^{-8} N **4.** 10^{27} **5.** 10^{51} **6.** 10^{23} N **7.** 10^{-17}% **8.** 10^{-8} C **9.** 10^{-6} C **10.** 10^{-10} C/kg

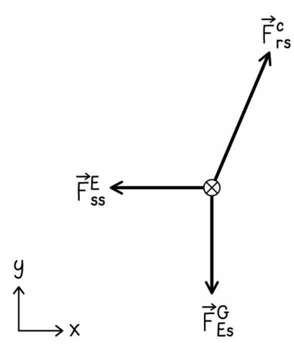
Guided Problems

22.2 Electroscope

1. Getting Started In Worked Problem 22.1, there were two objects carrying equal magnitudes of excess charge exerting forces on each other. But in that problem, the excess charge was of opposite sign, such that the two charged objects attracted each other. In this problem, the two objects carry charge of equal sign as well as magnitude, such that they repel each other. Also, in Worked Problem 22.1 we were not told about any other type of force that might be counteracting the electrostatic force. In this problem we are asked to relate the electrostatic force to others, such as tension and gravity.

When the spheres are very close to each other, their electrostatic repulsion is very strong, and pushes them apart. As they spread apart, the electrostatic force becomes weaker and is cancelled out exactly by tension and gravity. At that point the spheres are in equilibrium.

2. Devise Plan We begin by making a free body diagram:



The diagram above is for the left-most sphere.

The electrostatic repulsion is entirely along the negative x axis, and the gravitational force that Earth exerts on the sphere is in the negative y. Only the tension needs to be decomposed into components. Let us call the angle between the tension force and the positive x axis θ. Simple geometry tells us that

$$\cos(\theta) = \frac{(d/2)}{\ell} \Rightarrow \theta = \cos^{-1}\left(\frac{(d/2)}{\ell}\right) = \cos^{-1}\left(\frac{(93\,\text{mm})}{2(120\text{ mm})}\right) = 67.2°$$

Using that angle, we can write

$$\sum F_x = F^E_{\text{ss}\,x} + F^c_{\text{rs}\,x} = -F^E_{\text{ss}} + F^c_{\text{rs}}\cos(\theta) \tag{1}$$

$$\sum F_y = F^G_{\text{Es}\,y} + F^c_{\text{rs}\,y} = -F^G_{\text{Es}} + F^c_{\text{rs}}\sin(\theta) \tag{2}$$

3. Execute Plan We first rearrange equations (1) and (2) and insert formulas to describe the gravitational force and the electrostatic force:

$$F^c_{\text{rs}}\cos(\theta) = \frac{kq^2}{d^2} \tag{3}$$

$$F^c_{\text{rs}}\sin(\theta) = mg \tag{4}$$

Dividing equation (4) by equation (3), we find

$$\tan(\theta) = \frac{d^2 mg}{kq^2}$$

or

$$q = \pm\sqrt{\frac{d^2 mg}{k\tan(\theta)}} \tag{5}$$

Inserting the given numbers, we find

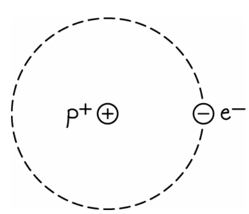
$$q = \pm\sqrt{\frac{(0.093\text{ m})^2(0.017\text{ kg})(9.8\text{ m/s}^2)}{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\tan(67.2°)}} = \pm 2.6\times10^{-7}\text{ C}$$

It is important that we understand the "$\pm$" symbol does not mean that the two spheres have opposite signs. If that were true, they would attract each other. The meaning of the symbol is that both spheres could carry a positive charge, or both spheres could carry excess negative charge. Either would result in the repulsion described in the problem.

4. Evaluate Result We know that adding more charge to each sphere should cause them to repel each other with a greater force, causing them to move farther apart. Thus, we expect that increasing d should correspond to increasing the magnitude of the charge on each sphere. Indeed, equation (5) shows that this is the case.

22.4 Electron orbit

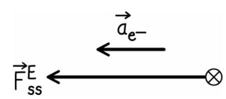
1. Getting Started We start by drawing a picture of the electron encircling the proton.



Clearly, in order for the electron to be moving in uniform circular motion, there must be a force acting on the electron directed toward the center of the circular orbit, and we know this force is the electrostatic attraction. That means the electron must also exert a force on the proton, which means the proton must not be completely stationary. However, because proton is about 2×10^3 times more massive than the electron, this force will have little effect on the proton. To a very good approximation the proton will be stationary.

There is no other force acting on the electron. The electrostatic force that the proton exerts on the electron acts toward the center of the electron's circular orbit, and thus plays the role of the centripetal force.

2. Devise Plan The free body diagram for this setup consists of just a single force:



This situation is similar to the planetary motion described in Chapter 13. In that chapter, the gravitational force exerted by the Sun on a planet (or by a planet on a moon) acted as the centripetal force to maintain uniform circular motion. In this model, gravity has been replaced by an electrostatic attraction.

We will determine the speed of the electron by equating the electrostatic attraction to the required centripetal force:

$$\frac{kq^2}{r^2} = \frac{mv^2}{r} \tag{1}$$

Once the speed is determined for part (a), we can determine the period by noting

$$v = \frac{d}{t} = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} \tag{2}$$

3. Execute Plan Rearranging equation (1), we find

$$v = \sqrt{\frac{kq^2}{mr}} \tag{3}$$

Inserting the given values, we obtain

$$v = \sqrt{\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(0.053\times10^{-9}\ \text{m})}} = 2.2\times10^{6}\ \text{m/s}$$

Inserting this speed into equation (2), we find

$$T = \frac{2\pi(0.053\times10^{-9}\ \text{m})}{(2.2\times10^{6}\ \text{m/s})} = 1.5\times10^{-16}$$

4. Evaluate Result In this model the electron's speed is about 0.7% the speed of light, meaning it is safe to ignore relativity in our calculation.

22.6 Charge pyramid

1. Getting Started Since each ion is missing one electron, they are all slightly positively charged. Thus the ions all repel each other. The lower five ions exert forces on the top ion at various angles.

Note that the four lower ions at the vertices of the pyramid will each exert exactly the same magnitude force on the top ion. This is clear because they are all equidistant from the top ion, and all carry the same excess charge. This means we need only calculate one of these four forces. Additionally, it is clear that any horizontal components of the forces will cancel pairwise. For example, the horizontal components of the forces exerted by the front right ion and the rear left ion will cancel each other. Thus, we need only consider the vertical component of one of these four forces. The force exerted by the central (green) ion on the top ion will be different, though, because the separation between those two ions is not a.

2. Devise Plan Let θ be the angle between the horizontal base of the pyramid and any one of the diagonal edges. Simple geometry tells us $\cos(\theta) = \frac{\sqrt{2}}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = 45°$. If one of the four ions at the lower vertices of the pyramid exerts a total force $\vec{F}^E_{\text{vt}}$, the upward component of this force is $F^E_{\text{vt}}\sin(\theta)$. The central (green) ion exerts a force on the top ion that is completely vertical, and so no component needs to be found. In either case, the magnitude of the repulsive force will be found using the standard formula for the electrostatic force exerted by one charged particle on another: $F^E_{12} = \frac{kq_1q_2}{r^2_{12}}$.

Because of the pairwise cancellation of horizontal components, we really need only choose one axis to be vertical. We will call this the z axis.

3. Execute Plan The four ions at the lower vertices of the pyramid each exerts a force with a vertical component $F^E_{\text{vt}\,z} = \frac{kq^2}{a^2}\sin(\theta)$, and the central (green) ion exerts an upward force of $F^E_{\text{ct}\,z} = \frac{kq^2}{(\sqrt{2}a/2)^2} = \frac{2kq^2}{a^2}$. This means the total vertical component of the force exerted on the top ion is

$$\sum F_z = 4\left(\frac{kq^2}{a^2}\sin(\theta)\right) + \frac{2kq^2}{a^2} = \frac{2kq^2}{a^2}\left(1 + 2\sin(\theta)\right)$$

$$\sum F_z = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(0.13\times10^{-9}\ \text{m})^2}(1+2\sin(45°))$$

$$\sum F_z = 6.6\times10^{-8}\ \text{N}$$

And since this vertical component is the only component, we can write that the sum of all forces on the top ion is

$$\sum \vec{F} = (7.7\times10^{-8}\ \text{N})\ \hat{z}$$

4. Evaluate Result The force between two protons a distance a apart is

$$F^E_{\text{pp}} = \frac{kq_\text{p}^2}{a^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(0.13\times10^{-9}\ \text{m})^2} = 1.4\times10^{-8}\ \text{N}$$

Since we have five interactions with the top ion that are comparable to this force magnitude, we expect our final answer to be several times the force between two protons separated by a distance a, which it is.

Questions and Problems

22.1. At that distance the electric force has the same magnitude as the gravitational force Earth exerts on the scrap. Thus $F^\text{E}_{\text{tp}} = F^G_{\text{Ep}} = mg = (100\times10^{-6}\ \text{kg})(9.8\ \text{m/s}^2) = 9.8\times10^{-4}\ \text{N}$.

22.2. Many answers possible, including (1) increase distance between the strips such as by taping them to opposite sides of bat at position where bat is wider), (2) touch one tape strip to remove some of the surplus charge, (3) fly the setup to Jupiter, where the gravitational force is much stronger than on Earth.

22.3. Choose the $+y$ direction to be vertically upward. We write the sum of all y components of forces: $\sum F_y = F^G_{\text{Es}y} + F^{\text{sp}}_{\text{sp}\,\text{s}y} + F^\text{E}_{\text{os}y} = 0 \Rightarrow F^\text{E}_{\text{os}y} = -F^{\text{sp}}_{\text{sp}\,\text{s}y} - F^G_{\text{Es}y} = k\Delta y + mg = (450\ \text{N/m})(-0.0050\ \text{m}) + (1.00\ \text{kg})(9.8\ \text{m/s}^2) = 7.55\ \text{N}$. Thus $\vec{F}^\text{E}_{\text{os}} = 7.55\ \text{N}$ upward.

22.4. Opposite. If they carried the same charge, they would repel. As they move farther apart, the strength of their interaction would decrease. But oppositely-charged objects will attract, and their interaction strength will increase.

22.5. There would be some type of charge such that an object carrying that charge would be attracted to both types of tape (top and bottom), or that would be repelled by both types of tape.

22.6. Negative. If the balloon were positively charged, the two objects would attract each other. In that case you would not have to do positive work to bring them together.

22.7. (*a*) Stick C to a flat surface, then press A down on top of C such that the two pieces overlap along only half their lengths. Next press B down on top of the half of C that is not covered by A, so that B and C also overlap along only half their lengths. Holding A and B by the ends not stuck to C, remove A and B from C. The ends that were stuck to C have the same charge and repel each other. (*b*) No. Charge is a conserved quantity. This means that as A and B acquire a surplus charge of one type from C, C is left with a surplus of the opposite type of charge and so is attracted to A and B.

22.8. (*a*) If q_1 and q_2 have the same sign, then regions I and III can be immediately rejected as possibilities, because both q_1 and q_2 would attract (or repel) charge q_3. The only possibility is region II. We can see that this is possible if we place q_3 closer to q_1 than to q_2. We can find a position where the two attractive or two repulsive forces acting on q_3 are equal in magnitude, and they will be in opposite direction. Thus this is possible in region II, and only in region II. (*b*) If q_1 and q_2 have opposite signs, then region II can be immediately rejected as a possibility, because q_1 and q_2 would exert forces on q_3 in the same direction. The two interactions could not possibly cancel each other out. We can further exclude region I, because everywhere in region I, q_3 would always be closer to q_2 than to q_1. Since q_2 is also larger in magnitude than q_1, this means q_3 would always feel a stronger force from q_2 than from q_1. We are left only with region III. It is possible to find a position in region III that is closer to q_1 than to q_2 such that the two interactions with q_3 could be of equal magnitude (and opposite in direction). Thus this is possible in region III, and only in region III. (*c*) Regardless of the sign of the charge on each particle, the strength of the interaction with q_2 will always be greater than the strength of the interaction with q_1 if the third particle is placed closest to q_2 (as is the case in region I). Thus, the only way the force can be zero is if the third particle has zero excess charge. There is no other way.

22.9. (*a*) The area is approximately equal to $10^{-4}\ \text{m}^2$ which is closest to the area of a fingernail. (*b*) To calculate the area we set the force from the electrical interaction equal to the gravitational force that Earth exerts on a 2000 kg object: $F^{\text{E}}_{\text{all charges}} = NF^{\text{E}}_{\text{single pair}} = mg \Rightarrow N = \dfrac{mg}{F^{\text{E}}_{\text{single pair}}}$. Now the total area needed for the plates is given by N times the area needed by a single pair: $A_{\text{total}} = NA_{\text{pair}} = \dfrac{mgA_{\text{pair}}}{F^{\text{E}}_{\text{single pair}}} = \dfrac{(2000\ \text{kg})(9.8\ \text{m/s}^2)(3\times10^{-21}\ \text{m}^2)}{(2.3\times10^{-12}\ \text{N})} = 2.6\times10^{-5}\ \text{m}^2$. Our estimate was off by one order of magnitude.

22.10. a, b, and e are all due to electric interactions.

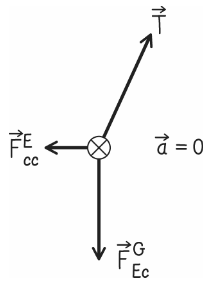
22.11. The wriststrap allows any small amount of surplus charge that might build up on the technician to immediately discharge to ground. This prevents a buildup of static charge and so eliminates any possibility of a spark.

22.12. Zero. Because the rods were made of nonconducting material, no charge moved during this process. The two rods remain neutral.

22.13. Lightning occurs when there is an enormous surplus of one type of charge built up in clouds. The clouds then exert an attractive force on oppositely charged particles at Earth's surface. Once a large amount of charge collects on the ground where a person is standing, the charge moves into the person's body and travels to the head and along each strand of hair. Because each strand carries the same type of charge, the strands repel each other and stand out away from the person's head.

22.14. The most likely particles are the positron (charge $+e$) and the products of the neutron decay: electron (charge $-e$), proton (charge $+e$), and antineutrino (charge 0), meaning the charge in the system after decay is what it was before decay: $+e$. The positron and electron might annihilate each other, but even if this happens the system charge is still $+e$, in this case carried by the proton.

22.15. (*a*)



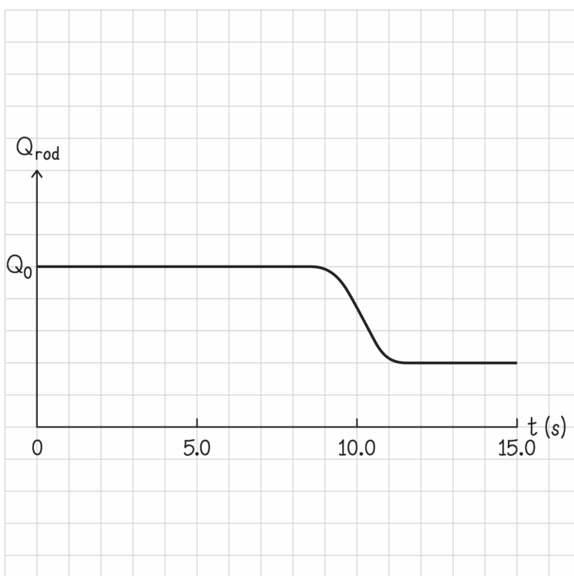
(*b*) Let us focus on the forces exert on paperclip 2. Let the $+y$ direction point vertically upward and let the $+x$ direction point from paperclip 1 to paperclip 2. Call the angle between the rope and the vertical direction $\theta = 16°$. We write the sum of the x and y components of forces acting on paperclip 2.

$$\sum F_x = F^{c}_{r2x} + F^{E}_{12x} = 0 \Rightarrow -F^{c}_{r2}\cos(\theta) + F^{E}_{12} = 0 \tag{1}$$

$$\sum F_y = F^{c}_{r2y} + F^{G}_{E2y} = 0 \Rightarrow F^{c}_{r2}\sin(\theta) - mg = 0 \tag{2}$$

Combining equations (1) and (2) we obtain $F^{E}_{12} = mg\tan(\theta) = (500 \times 10^{-6}\ \text{kg})(9.8\ \text{m/s}^2)\tan(16°) = 1.40\ \text{mN}$. Thus $\vec{F}^{E}_{12} = 1.40\ \text{mN}$ from paperclip 1 to paperclip 2. (*c*) No, they have the same charge type.

22.16. There is no change in the amount of charge on the rod as it is turned around. The charge on the rod decreases somewhat as the rod comes in contact with the pith ball, as some charge escapes onto the pith ball.



22.17. (*a*) Since A is repelled by both balls, we simply place balls B and C on the same side of A. The forces that B and C exert on A will add constructively. (*b*) Place balls B and C on opposite sides of A. This way the forces exerted on A by B and C will be directed opposite each other.

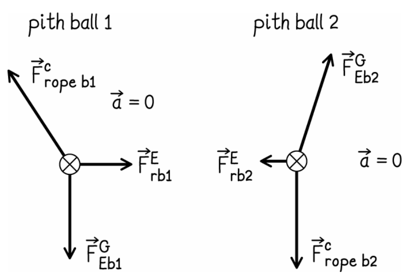
22.18. (*a*) Yes, the sphere acquires a negative charge via induction because the sphere's positive charge carriers, repelled by rod, move as far from rod as possible, exiting sphere via ground wire. The result is an attractive electric force between sphere and rod. (*b*) Yes, the sphere again acquires negative charge via induction as rod comes near.

Once rod touches sphere, positive carriers move from rod to sphere. These positive charges are free to escape from the system through the ground wire. However, because plastic is electrical insulator, not all surplus positive charge initially on rod moves to sphere. Thus, even while the rod and sphere are in contact, positive charge remains on the rod and attracts negative charge through the ground wire and onto the sphere. In addition to this net negative charge on the sphere, there is also polarization to consider. When rod and sphere separate, positive charge on rod causes polarization of positive and negative charge in sphere, with positive carriers as far from rod as possible and negative carriers as near rod as possible. Thus rod exerted attractive electric force on side of sphere nearest rod.

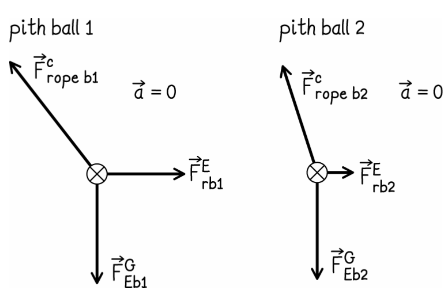
22.19. Scraps of paper are routinely taken to be electrically neutral, meaning that the attractive interaction with the rubber rod is due to polarization in the paper. If that is the case, then the fur-paper force is also attractive. But note that the paper could also have initially carried a large charge, and simply did not interact with the rubber because the rubber was uncharged. In that case, the fur could repel the paper.

22.20. In the *Principles* text it is explained that we refer to the charge that ends up on a comb when it is run through hair as negative. The scrap of paper most strongly attracted to the comb must be positive. The scrap that is only weakly attracted to the comb is neutral, and is attracted because of polarization. The scrap that is repelled by the comb is negative. Thus A is positive, B is neutral, and C is negative.

22.21. The diagrams could look different depending on the magnitude of the negative charge added to the rod. If a huge negative charge is placed on the rod, then the rod may repel the negatively charged ball despite polarization of the rod.



If only a very small negative charge is placed on the rod, then that negative charge will be attracted to the positive ball and move to the left end of the rod. In addition, more negative charge carriers from the rod will be attracted to the left side and repelled from the right. This will leave the right end of the rod slightly positive, and the negatively charged ball will be attracted to it.



22.22. Because the rod was initially neutral, it must still be neutral. Thus the far half of the rod carries -1×10^{4} units of charge.

22.23. (*a*) The sphere will roll toward the rod. (*b*) The answer depends on how much positive charge is on the sphere. If it is only a very small amount (say a single electron is removed), then the polarization of the sphere will be the dominant effect, and the sphere will still roll toward the rod. If a very large amount of positive charge is added to the sphere then it could roll away from the rod.

22.24. (*a*) While the positively charged sphere is near rod A, negative charge will be attracted to the sphere. Some of this negative charge will flow from the ends of rod A toward the center, and some will flow into rod A from rod B. This will leave rods A and B will opposite charges. Thus the rods will attract each other. (*b*) As described in part (*a*), excess negative charge flows onto rod A from rod B. Thus, rod A is negative; rod B is positive.

22.25. (*a*) If positive charge is added to both spheres, the charge carriers will be repelled away from each other. In the nonconducting pair of spheres, that repulsion doesn't matter; the carriers can't move. In the case of a single conducting sphere, the carriers on that sphere can move and will flow to the side of the conducting sphere farthest from the nondconducting sphere. This increases the distance between charge carriers and therefore decreases the repulsive force between the spheres. In the case of two conducting spheres, carriers on both spheres can move away from each other. This increases the distance between charge carriers even more than in the case of only one conducting sphere. Thus $F_{\mathrm{A}}^{\mathrm{E}} < F_{\mathrm{C}}^{\mathrm{E}} < F_{\mathrm{B}}^{\mathrm{E}}$. (*b*) If negative charge is added to both spheres, the charge carriers will again be repelled away from each other. Nothing at all about the arguments in part (*a*) changes now that the charge has a different sign. $F_{\mathrm{A}}^{\mathrm{E}} < F_{\mathrm{C}}^{\mathrm{E}} < F_{\mathrm{B}}^{\mathrm{E}}$. (*c*) Now that charges of opposite sign are on each sphere, the charge carriers will attract. Now, when charge is allowed to move on a conducting sphere it will move closer to the opposite charge and the force will increase. So now $F_{\mathrm{B}}^{\mathrm{E}} < F_{\mathrm{C}}^{\mathrm{E}} < F_{\mathrm{A}}^{\mathrm{E}}$.

22.26. The distance decreases. The charge carriers in C separate and move closer to the two spheres (A and B). This strengthens the interaction.

22.27. The larger chunks have some excess charge on the outside. The smaller chunks are electrically neutral throughout, but could become polarized due to close proximity with a larger chunk. If the large chunks have excess positive charge carriers on the outside, then negative charge on the small chunks could move to the side of the small chunk closest to the large chunk, such that attractive forces dominate.

22.28. When A and B are brought into contact, the charge divides evenly between the identical spheres and each carries $\frac{1}{2}q$. Then when spheres B and C are brought together, each ends up with $\frac{1}{4}q$. Sphere B will keep this charge. But now spheres A and C are brought into contact. They have a total charge of $\frac{3}{4}q$ between them, so each will end up with $\frac{3}{8}q$. Thus $q_{\mathrm{A}} = \frac{3}{8}q$, $q_{\mathrm{B}} = \frac{1}{4}q$, and $q_{\mathrm{C}} = \frac{3}{8}q$.

22.29. The magnitude of the electric force is given by $F_{ee}^{\mathrm{E}} = \frac{k|q_e||q_e|}{r^2} = \frac{(9.0\times10^{9}\ \mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\ \mathrm{C})^2}{(1.50\times10^{-9}\ \mathrm{m})^2} = 1.02\times10^{-10}\ \mathrm{N}$, and this force is repulsive.

22.30. The only other variables that can be changed are the charge q_2, and the distance between the particles r. So, either q_2 could be halved, or r could be changed to $\sqrt{2}r$.

22.31. (*a*) The force between the two particles is attractive. Since they are separated along the x axis, the force will also be along this line (the y component will be zero). $F_x^{\mathrm{E}} = \frac{k|q_1||q_2|}{r_{12}^2} = -\frac{(9.0\times10^{9}\ \mathrm{N\cdot m^2/C^2})(4.0\times10^{-6}\ \mathrm{C})(1.0\times10^{-6}\ \mathrm{C})}{(10\ \mathrm{m})^2} = -3.6\times10^{-4}\ \mathrm{N}$ and $F_y^{\mathrm{E}} = 0$. (*b*) This time the force between the particles will lie along the y axis, such that the x component will be zero. $F_x^{\mathrm{E}}=0$ and $F_y=-\dfrac{k|q_1||q_2|}{r_{12}^2}=$ $-\dfrac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(4.0\times10^{-6}\ \mathrm{C})(1.0\times10^{-6}\ \mathrm{C})}{(6.0\ \mathrm{m})^2}=1.0\times10^{-3}\ \mathrm{N}.$

22.32. (*a*) $F_{12}^{\mathrm{E}}=\dfrac{k|q_1||q_2|}{r_{12}^2}\Rightarrow r_{12}=\sqrt{\dfrac{kq^2}{F_{12}^{\mathrm{E}}}}=\sqrt{\dfrac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(1.0\ \mathrm{C})^2}{(1.0\ \mathrm{N})}}=95\ \mathrm{km}.$ (*b*) We can find the charge magnitude by writing $F_{12}^{\mathrm{E}}=\dfrac{k|q_1||q_2|}{r_{12}^2}=\dfrac{kq^2}{r_{12}^2}\Rightarrow q=\sqrt{\dfrac{r_{12}^2F_{12}^{\mathrm{E}}}{k}}=\sqrt{\dfrac{(1.0\ \mathrm{m})^2(1.0\ \mathrm{N})}{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})}}=1.1\times10^{-5}\ \mathrm{C}.$ Either a positive or negative charge would produce the same magnitude force at this distance.

22.33. Call vertically upward the $+y$ direction. We write the sum of all y components of forces: $\sum F_y=F_{\mathrm{E}1y}^{G}+F_{21y}^{\mathrm{E}}+F_{\mathrm{sp}\,1y}^{\mathrm{sp}}=ma_y=0$. Inserting know expressions for these forces, we find $-mg+\dfrac{kq_1q_2}{r_{12}^2}+-k\Delta y=0$ or $q_2=\dfrac{r_{12}^2(mg+k\Delta y)}{kq_1}=\dfrac{(0.20\ \mathrm{m})^2\left((5.0\times10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})+(30.0\ \mathrm{N/m})(-0.030\ \mathrm{m})\right)}{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(1.00\times10^{-6}\ \mathrm{C})}=-3.78\ \mu\mathrm{C}.$

22.34. Because the spheres are identical, when they touch the same amount of charge will remain on each sphere. We can examine just one sphere; call it sphere 1. Let the $+y$ direction point vertically upward, and let the $+x$ axis point from sphere 2 to sphere 1. We write the sum of all force components in x and y separately.

$$\sum F_x=F_{\mathrm{r}1x}^{\mathrm{c}}+F_{21x}^{\mathrm{E}}=m_1a_{1x}=0\Rightarrow -F_{\mathrm{r}1}^{\mathrm{c}}\cos(\theta)+F_{21}^{\mathrm{E}}=0 \quad (1)$$
$$\sum F_y=F_{\mathrm{r}1y}^{\mathrm{c}}+F_{\mathrm{E}1y}^{G}=m_1a_{1y}=0\Rightarrow F_{\mathrm{r}1}^{\mathrm{c}}\sin(\theta)-m_1g=0 \quad (2)$$

Combining equations (1) and (2) yields $F_{21}^{\mathrm{E}}=m_1g\tan(\theta)$. Inserting the formula for Coulomb's law, we find $\dfrac{kq_1q_2}{r_{12}^2}=m_1g\tan(\theta)$, where $r_{12}=2\ell\sin(\theta)$ from basic geometry. Solving for the charge on either sphere yields $-\sqrt{\dfrac{m_1g(2\ell\sin(\theta))^2\tan(\theta)}{k}}=-\sqrt{\dfrac{(9.60\times10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(2(0.300\ \mathrm{m})\sin(13.0°))^2\tan(13.0°)}{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})}}=-2.097\times10^{-7}\ \mathrm{C}.$

Note that either positive or negative solutions were mathematically possible. We have chosen the negative solution, simply because we are told that excess electrons are added to the spheres. To determine how many electrons this is, we simply divide the charge by the charge on each electron. Thus $N=\dfrac{q}{q_e}=\dfrac{(-2.097\times10^{-7}\ \mathrm{C})}{(-1.60\times10^{-19}\ \mathrm{C})}=1.31\times10^{12}$ electrons.

22.35. We take the ratio of the two interaction strengths: $\dfrac{F_{12}^{G}}{F_{12}^{\mathrm{E}}}=\dfrac{\frac{Gm_1m_2}{r_{12}^2}}{\frac{kq_1q_2}{r_{12}^2}}=\left(\dfrac{G}{k}\right)\left(\dfrac{m_1}{q_1}\right)\left(\dfrac{m_2}{q_2}\right)=$ $\left(\dfrac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})}{(8.99\times10^9\ \mathrm{N\cdot m^2/kC^2})}\right)(1.00\ \mathrm{kg/C})^2=7.42\times10^{-21}$. Equivalently, the electric force is 1.35×10^{20} times stronger than the gravitational force.

22.36. When identical spheres are brought into electrical contact, the surplus charge on the two spheres combined will distribute evenly over the two spheres. In this case, that means each sphere will carry a surplus charge of $\frac{1}{2}\left((6.0\ \mu\mathrm{C})+(-24\ \mu\mathrm{C})\right)=-9.0\ \mu\mathrm{C}$. Now write the ratio of the initial to final force the two spheres exert on each other.

$$\frac{F_{\mathrm{i}}^{\mathrm{E}}}{F_{\mathrm{f}}^{\mathrm{E}}}=\frac{\dfrac{k|q_{1,\mathrm{i}}||q_{2,\mathrm{i}}|}{r_{12}^2}}{\dfrac{k|q_{1,\mathrm{f}}||q_{2,\mathrm{f}}|}{r_{12}^2}}=\frac{|q_{1,\mathrm{i}}||q_{2,\mathrm{i}}|}{|q_{1,\mathrm{f}}||q_{2,\mathrm{f}}|}=\frac{(6.0\ \mu\mathrm{C})(24\ \mu\mathrm{C})}{(9.0\ \mu\mathrm{C})^2}=1.8$$

22.37. (*a*) When sphere 1 rises the first time it acquires 10 units of charge. This charge is distributed throughout all five spheres when they come into electrical contact. When sphere 5 rises it also acquires 10 units of charge, and that charge is distributed evenly between all five spheres. Thus, each sphere carries 4 units of charge just before sphere 1 rises a second time. Specifically, sphere 1 carries 4 units of charge at this time. (*b*) After one more round of charge transfer, sphere 1 will have risen and added another 10 units of charge to the system of spheres, and sphere 5 will have risen and added another 10 units. Thus, the total surplus charge on the system of five spheres is now 40 units. Charge will be distributed evenly among the identical spheres, so sphere 1 will carry 8 units of charge.

22.38. (*a*) We require $F_{12}^{G}=F_{12}^{\mathrm{E}}\Rightarrow\frac{Gm^2}{d^2}=\frac{kq^2}{d^2}\Rightarrow q=\sqrt{\frac{G}{k}}m=\sqrt{\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})}{(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})}}(1.00\ \mathrm{kg})=8.61\times10^{-11}\ \mathrm{C}.$ (*b*) To find the smallest mass that can balance in this way, we let it be balance by the smallest increment of charge that could be added to a particle (the magnitude of the charge on an electron). Thus $m_{\min}=\sqrt{\frac{k}{G}}q_{\min}=\sqrt{\frac{(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})}{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})}}(1.60\times10^{-19}\ \mathrm{C})=1.86\times10^{-9}\ \mathrm{kg}.$ (*c*) Both forces have $1/r^2$ behavior.

22.39. We can treat charge on the sphere like it is concentrated at the center of the sphere. Thus the distance from the charge on the sphere to the drop will be $r_{\mathrm{sd}}=50$ mm. Now, we simply need the electric force between the sphere and drop to balance the gravitational force that Earth exerts on the drop. Calling vertically upward the $+y$ direction, we write the sum of y components of forces and find $\sum F_y=F_{\mathrm{sd}y}^{\mathrm{E}}+F_{\mathrm{Ed}y}^{G}=ma_y=0\Rightarrow\frac{kq_{\mathrm{s}}q_{\mathrm{d}}}{r_{\mathrm{sd}}^2}=mg.$ Solving for the charge on the sphere, we obtain $q_{\mathrm{s}}=\frac{r_{\mathrm{sd}}^2mg}{kq_{\mathrm{d}}}=\frac{(0.050\ \mathrm{m})^2(5.00\times10^{-12}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})}{(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(10)(1.60\times10^{-19}\ \mathrm{C})}=8.51\times10^{-6}\ \mathrm{C}.$

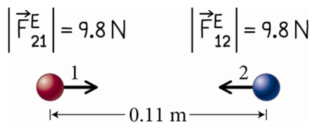
22.40. (*a*) The proton is much more massive than the electron, so we will treat the proton as though it is stationary. In order for the electron to move in a circle around the stationary proton, we require $\sum F_r=m_{\mathrm{e}}a_{\mathrm{centripetal}}=\frac{m_{\mathrm{e}}v^2}{R_{\mathrm{orbit}}}.$ The gravitational force is negligible, so the only force we have to make to the electron move in circular motion is the electric force. Note that the attractive force does point radially inward toward the proton. Thus we can write $\frac{kq_e^2}{R_{\mathrm{orbit}}^2}=\frac{m_{\mathrm{e}}v^2}{R_{\mathrm{orbit}}}\Rightarrow v=\sqrt{\frac{kq_e^2}{m_{\mathrm{e}}R_{\mathrm{orbit}}}}.$ (*b*) The period is obviously given by $T=\frac{2\pi R_{\mathrm{orbit}}}{v}.$ Inserting the result from part (*a*), we find $T=2\pi\sqrt{\frac{m_{\mathrm{e}}}{kq^2}}R_{\mathrm{orbit}}^{3/2}.$

22.41. (*a*) Symbolically, we can write the initial force $F_{12}^{\mathrm{E}}=\frac{kq^2}{d^2},$ where d is an arm's length. We note that at this time the force is repulsive, because each ball has the same charge type. When ball 3 is brought into electrical contact with ball 1, the total surplus charge $(-q)$ distributes evenly over the two identical balls such that balls 1 and 3 each carry a surplus charge of $-\frac{q}{2}.$ Now when balls 3 and 2 are brought into contact, the total surplus charge $\left(+\frac{q}{2}\right)$ divides evenly such that balls 2 and 3 each carry $\left(+\frac{q}{4}\right).$ Now the magnitude of the force exerted by 1 on 2 is

$F_{12,\mathrm{f}}^{\mathrm{E}} = \dfrac{k\left(\dfrac{q}{2}\right)\left(\dfrac{q}{4}\right)}{d^2} = \left(\dfrac{kq^2}{d^2}\right)\left(\dfrac{1}{8}\right) = \dfrac{F_{12}^{\mathrm{E}}}{8}$. (*b*) Yes, in the final case ball 1 carries a negative charge and ball 2 carries a positive charge. So the force between them is now attractive.

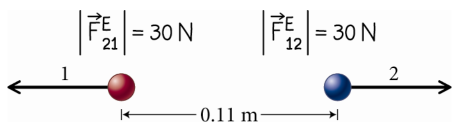
22.42. (*a*) The magnitude of the attractive force is

$$F_{12}^{\mathrm{E}} = \frac{k|q_1||q_2|}{r_{12}^2} = \frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(2.10\times10^{-6}\ \mathrm{C})(6.30\times10^{-6}\ \mathrm{C})}{(0.11\ \mathrm{m})^2} = 9.8\ \mathrm{N}$$

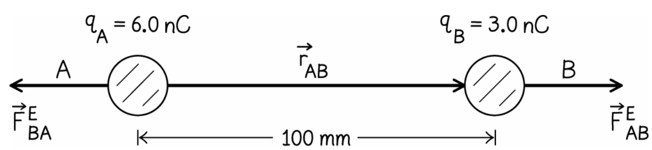


(*b*) The magnitude of the repulsive force is

$$F_{12}^{\mathrm{E}} = \frac{k|q_1||q_2|}{r_{12}^2} = \frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(6.30\times10^{-6}\ \mathrm{C})(6.30\times10^{-6}\ \mathrm{C})}{(0.11\ \mathrm{m})^2} = 3.0\times10^1\ \mathrm{N}.$$



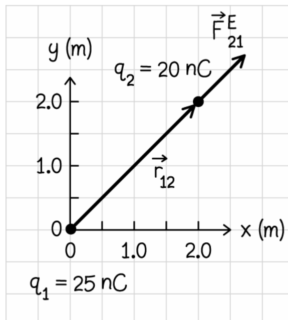
22.43. (*a*) $F_{\mathrm{BA}}^{\mathrm{E}} = \dfrac{k|q_{\mathrm{B}}||q_{\mathrm{A}}|}{r_{\mathrm{BA}}^2} = \dfrac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(3.0\times10^{-9}\ \mathrm{C})(6.0\times10^{-9}\ \mathrm{C})}{(0.100\ \mathrm{m})^2} = 1.6\times10^{-5}\ \mathrm{N}$. (*b* and *c*)



22.44. (*a*) $F_{\mathrm{pn}}^{\mathrm{E}} = \dfrac{k|q_{\mathrm{p}}||q_{\mathrm{n}}|}{r_{\mathrm{pn}}^2} = \dfrac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(1.0\times10^{-6}\ \mathrm{C})(0.50\times10^{-6}\ \mathrm{C})}{(0.100\ \mathrm{m})^2} = 0.45\ \mathrm{N}$. (*b*) They attract and move toward each other with increasing speed. (*c*) The total surplus charge between the two marbles is $+0.50\ \mu\mathrm{C}$. This will divide evenly, such that each marble carries a surplus charge of $+0.25\ \mu\mathrm{C}$. So the new force between the marbles will be $F_{\mathrm{pn,f}}^{\mathrm{E}} = \dfrac{k|q_{\mathrm{p,f}}||q_{\mathrm{n,f}}|}{r_{\mathrm{pn}}^2} = \dfrac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(0.25\times10^{-6}\ \mathrm{C})^2}{(0.100\ \mathrm{m})^2} = 5.6\times10^{-2}\ \mathrm{N}$.

22.45. From the force magnitude we can write $F_{12}^{\mathrm{E}} = \dfrac{k|q_1||q_2|}{r_{12}^2} = 8.0\times10^{-3}\ \mathrm{N}$. Because the force is attractive, we know that one of the two charges must be negative. Thus the magnitude of the electric force can be written $F_{12}^{\mathrm{E}} = -\dfrac{kq_1q_2}{r_{12}^2} = -\dfrac{kq_1(q_{\mathrm{both}} - q_1)}{r_{12}^2}$. This quadratic equation has solutions $q_1 = \dfrac{q_{\mathrm{tot}} \pm \sqrt{q_{\mathrm{tot}}^2 + 4F_{12}^{\mathrm{E}}r_{12}^2/k}}{2} =$ $\dfrac{(6.0\times10^{-6}\ \mathrm{C}) \pm \sqrt{(6.0\times10^{-6}\ \mathrm{C})^2 + 4(8.0\times10^{-3}\ \mathrm{N})(3.0\ \mathrm{m})^2/(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})}}{2}$, so $q_1 = -1.1\times10^{-6}\ \mathrm{C}$ or $q_1 =$ $7.1\times10^{-6}\ \mathrm{C}$. Of course, it does not matter which particle is called 1 and which is called 2. The two particles carry charges $-1.1\ \mu\mathrm{C}$ and $7.1\ \mu\mathrm{C}$.

22.46. Call the particle at the origin particle 1, and the other particle 2. Because the particles both carry a positive charge, the force exerted on particle 2 will be directed away from the origin. In this case, that corresponds to a direction $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2.0\text{ m}}{2.0\text{ m}}\right) = 45°$ counterclockwise from the $+x$ axis. The magnitude of the force is given by $F_{12}^{\text{E}} = \frac{k|q_1||q_2|}{r_{12}^2} = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(25\times10^{-9}\text{ C})(20\times10^{-9}\text{ C})}{(2.0\text{ m})^2+(2.0\text{ m})^2} = 5.6\times10^{-7}\text{ N}$. So $\vec{F}_{12}^{\text{E}} = 5.6\times10^{-7}\text{ N}$ at 45° ccw from $+x$.



22.47. We label the ions 1 and 2. $F_{12}^{\text{E}} = \frac{k|q_1||q_2|}{r_{12}^2} \Rightarrow q = \sqrt{\frac{F_{12}^{\text{E}} r_{12}^2}{k}} = \sqrt{\frac{(3.7\times10^{-9}\text{ N})(0.50\times10^{-9}\text{ m})^2}{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}} = 3.2\times10^{-19}\text{ C}.$ This is exactly twice the magnitude of the charge on an electron. Thus there are 2 electrons missing from each.

22.48. (*a*) Because $F_{12}^{\text{E}} = F_{21}^{\text{E}}$ we know $m_1 = \frac{m_2 a_2}{a_1} = \frac{(0.49\times10^{-6}\text{ kg})(9.0\text{ m/s}^2)}{(7.0\text{ m/s}^2)} = 0.63\times10^{-6}$. So $m_1 = 0.63\text{mg}$.
(*b*) We can get the magnitude of the electric force being exerted on either particle from the mass and acceleration of either particle. Assuming the electric force is the only force responsible for the acceleration of either particle, we can write $F_{12}^{E} = m_2 a_2$ or $\frac{kq^2}{r_{12}^2} = m_2 a_2 \Rightarrow r_{12} = \sqrt{\frac{kq^2}{m_2 a_2}} = \sqrt{\frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(71\times10^{-12}\text{ C})^2}{(0.49\times10^{-6}\text{ kg})(9.0\text{ m/s}^2)}} = 3.2\text{ mm}.$

22.49. (*a*) The balls are initially attracted to each other due to polarization of the uncharged ball. Once they touch, they share charge and repel each other. When they finally come to rest, each string will make the same angle with vertical. (*b*) Because of symmetry, it is sufficient to consider the forces acting only on one ball; call this ball 1, and the other ball will be ball 2. Call vertically upward the $+y$ direction, and call the direction from ball 2 toward ball 1 $+x$. We can write down the sum of all x and y components of forces:

$$\sum F_x = F_{\text{r}1x}^{\text{c}} + F_{21x}^{\text{E}} = ma_{1x} = 0 \Rightarrow -F_{\text{r}1}^{\text{c}}\cos(\theta) + F_{21}^{\text{E}} = 0 \quad (1)$$

$$\sum F_y = F_{\text{r}1y}^{\text{c}} + F_{\text{E}1y}^{G} = ma_{1y} = 0 \Rightarrow F_{\text{r}1}^{\text{c}}\sin(\theta) - mg = 0 \quad (2)$$

Combining equations (1) and (2) and inserting the expression for Coulomb's law yields $\frac{kq^2}{d^2} = mg\tan(\theta)$. Basic geometry allows this to be written as

$$q = \sqrt{\frac{mgd^3}{2k\sqrt{\ell^2 - \left(\frac{d}{2}\right)^2}}}$$

22.50. (*a*) Using this average separation of charges, we can write an average electric force $F_{\text{lr,av}}^{\text{E}} = \frac{kq^2}{r_{\text{lr,av}}^2}$. This average force has to do enough work over 100 mm to give the rocket its final desired kinetic energy. It will also have to lift the rocket by 100 mm against gravity, but this is a negligible correction. We write

$W_{\mathrm{lr}}^{\mathrm{E}} = \Delta K_{\mathrm{r}} \Rightarrow F_{\mathrm{lr,av}}^{\mathrm{E}} \Delta y = \frac{1}{2} m_{\mathrm{r}} v_{\mathrm{r,f}}^2$. Equivalently $\frac{kq^2}{r_{\mathrm{lr,av}}^2} \Delta y = \frac{1}{2} m_{\mathrm{r}} v_{\mathrm{r,f}}^2 \Rightarrow q = \sqrt{\frac{m_{\mathrm{r}} v_{\mathrm{r,f}}^2 r_{\mathrm{lr,av}}^2}{2k\Delta y}} = \sqrt{\frac{(100{,}000 \text{ kg})}{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(0.100 \text{ m})}}$ $(1.12\times10^4 \text{ m/s})(1.0 \text{ m}) = 84 \text{ C}$. (*b*) The number of elementary charges that must be added is $N = \frac{q_{\text{total}}}{q_{\text{elementary}}} = \frac{(83.5 \text{ C})}{(1.60\times10^{-19} \text{ C})} = 5.22\times10^{20}$. If each addition of an elementary charge requires adding mass equal to $m_{\mathrm{H}} = 1.67\times10^{-27}$ kg, then the total additional mass is $m = Nm_{\mathrm{H}} = (5.22\times10^{20})(1.67\times10^{-27} \text{ kg}) = 8.7\times10^{-7}$ kg.

22.51. The original electric force between the spheres can be written $F_{\mathrm{AB,i}}^{\mathrm{E}} = \frac{kq(q+2n)}{r_{\mathrm{AB}}^2} = \frac{k(q^2+2qn)}{r_{\mathrm{AB}}^2}$. The final electric force can be written $F_{\mathrm{AB,f}}^{\mathrm{E}} = \frac{k(q+n)^2}{r_{\mathrm{AB}}^2} = \frac{k(q^2+2qn+n^2)}{r_{\mathrm{AB}}^2}$. The final force has an extra term $\frac{kn^2}{r_{\mathrm{AB}}^2}$, which is positive. Thus the force has increased.

22.52. (*a*) For the given condition $q_1 + q_2 = q$, the electric force magnitude is $F_{12}^{\mathrm{E}} = \frac{kq_1q_2}{r^2} = \frac{kq_1(q-q_1)}{r^2}$. To determine the maximum value of F_{12}^{E}, we set its first derivative with respect to charge q_1 equal to zero,

$$\frac{dF_{12}^{\mathrm{E}}}{dq_1} = \frac{d}{dq_1}\left(\frac{kq_1(q-q_1)}{r^2}\right) = 0$$

which yields $\frac{k}{r^2}(q-2q_2) = 0$.

Because neither k nor r is zero, it must be true that $q - 2q_2 = 0$ and $q_2 = q/2$, which implies $q_1 = q/2$. (*b*) With an attractive electric force, the sum of the charges reduces the magnitude of the charge on either particle. Also, you could not maximize an attractive force in this manner because the surplus charge (either positive or negative) exists on both particles and so the force is repulsive.

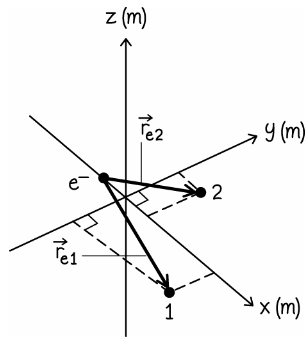
22.53. The two charged particles you add must each carry a charge $-q$. One of them goes 2.00 m from the origin with its position vector making an angle of 105° measured counterclockwise from the positive *x* axis and the other goes 2.00 m from the origin with its position vector making an angle of 195° measured counterclockwise from the positive *x* axis.

22.54. Because all the particles lie on the x axis, we need only consider the x components of forces. $\sum F_x = F_{13}^{\mathrm{E}} + F_{23}^{\mathrm{E}}$. Particle 1 attracts particle 3, and therefore exerts a force on 3 in the $+x$ direction. Particle 2 repels particle 3, and therefore exerts a force on 3 in the $-x$ direction. Thus $\sum F_x = \frac{k|q_1||q_3|}{r_{13}^2} - \frac{k|q_2||q_3|}{r_{23}^2} =$ $\frac{(9.0\times10^9 \text{ N}\cdot\text{m}^2\text{C}^2)(4.0\times10^{-6} \text{ C})(6.0\times10^{-6} \text{ C})}{(3.0 \text{ m})^2} - \frac{(9.0\times10^9 \text{ N}\cdot\text{m}^2\text{C}^2)(6.0\times10^{-6} \text{ C})(6.0\times10^{-6} \text{ C})}{(6.0 \text{ m})^2} = 0.015 \text{ N}$. So $\sum \vec{F}_3^{\mathrm{E}} = 0.015 \text{ N } \hat{i}$.

22.55. Let the +*x* direction point to the right, the $+y$ direction point into the page, and the +*z* direction point up toward the top of the page. The Cs^+ ions along the bottom front and across the rear top will exert forces on the Cl^- ion that will cancel pair-wise. We are left only with the electric forces exerted on the Cl^- ion by the Cs^+ ion at the rear bottom of the cube. Let us calculate the magnitude of one such electric force. $F_{\mathrm{Cs^+Cl^-}}^{\mathrm{E}} = \frac{kq_{\mathrm{e}}^2}{3\left(\frac{d}{2}\right)^2} = \frac{(9.0\times10^9 \text{ N}\cdot\text{m}^2\text{C}^2)(1.60\times10^{-19} \text{ C})^2}{3\left(\frac{412\times10^{-12} \text{ m}}{2}\right)^2} = 1.8098\times10^{-9} \text{ N}$.

Draw a perpendicular bisector to the line connecting the two rear bottom Cs^+ ions, that also passes through the Cl^- ion. The components of the forces from the two rear bottom Cs^+ ions that are parallel to this line will add, whereas the components perpendicular to this line will cancel. The force from a single Cs^+ ion makes an angle with this bisector equal to $\theta = \sin^{-1}\left(\dfrac{(d/2)}{\sqrt{3}(d/2)}\right) = 35.26°$. Adding the components parallel to the bisector yields 2.952×10^{-9} N directed parallel to the perpendicular bisector. Because that vector points midway between the $-y$ axis and the $-z$ axis we can write $\sum \vec{F}^{\text{E}}_{\text{Cl}^-} = -F^{\text{E}}_{\text{Cl}^-}\cos(45°)\,\hat{j} - F^{\text{E}}_{\text{Cl}^-}\sin(45°)\,\hat{k} = (-2.09\times10^{-9}\text{ N})\,\hat{j} + (-2.09\times10^{-9}\text{ N})\,\hat{k}.$

22.56. (*a*)



(*b*) Call the angle between $\vec{r}_{\text{e1}}$ and the $+x$ axis θ_1. Call the angle between $\vec{r}_{\text{e2}}$ and the $+x$ axis θ_2. Simple trigonometry shows us that $\theta_1 = \tan^{-1}\left(\dfrac{2.0\text{m}}{5.0\text{ m}}\right) = 21.8°$ and $\theta_2 = \tan^{-1}\left(\dfrac{2.0\text{m}}{2.0\text{ m}}\right) = 45°$. We write the sum of all components of forces on the electron:

$$\sum F_x = F^{\text{E}}_{1\text{e}x} + F^{\text{E}}_{2\text{e}x} = -\frac{k|q_1||q_\text{e}|}{r_{\text{e1}}^2}\cos(\theta_1) + \frac{k|q_2||q_\text{e}|}{r_{\text{e2}}^2}\cos(\theta_2)$$

$$= -\frac{(9.0\times10^9\text{ N}\cdot\text{m}^2\text{C}^2)(5.0\times10^{-6}\text{ C})(1.60\times10^{-19}\text{ C})}{(5.0\text{ m})^2 + (2.0\text{ m})^2}\cos(21.80°) + \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2\text{C}^2)(12\times10^{-6}\text{ C})(1.60\times10^{-19}\text{ C})}{(2.0\text{ m})^2 + (2.0\text{ m})^2}\cos(45°)$$

$$= 1.297\times10^{-15}\text{ N}$$

$$\sum F_y = F^{\text{E}}_{1\text{e}y} + F^{\text{E}}_{2\text{e}y} = \frac{k|q_1||q_\text{e}|}{r_{\text{e1}}^2}\sin(\theta_1) + \frac{k|q_2||q_\text{e}|}{r_{\text{e2}}^2}\sin(\theta_2)$$

$$= \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2\text{C}^2)(5.0\times10^{-6}\text{ C})(1.60\times10^{-19}\text{ C})}{(5.0\text{ m})^2 + (2.0\text{ m})^2}\sin(21.80°) + \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2\text{C}^2)(12\times10^{-6}\text{ C})(1.60\times10^{-19}\text{ C})}{(2.0\text{ m})^2 + (2.0\text{ m})^2}\sin(45°)$$

$$= 1.620\times10^{-15}\text{ N}$$

Thus in component form $\sum \vec{F} = 1.3\times10^{-15}\text{ N}\,\hat{i} + 1.6\times10^{-15}\text{ N}\,\hat{j}$. Or, using the Pythagorean Theorem, we can also write this as $\sum \vec{F} = 2.1\times10^{-15}$ N at 51° ccw from the $+x$ axis as seen from the $+z$ axis.

22.57. We write the sum of all components of forces on particle 4:

$$\sum F_x = F^{\text{E}}_{14x} + F^{\text{E}}_{24x} + F^{\text{E}}_{34x} = \frac{k|q_1||q_4|}{r_{14}^2}\cos(\theta_1) - \frac{k|q_2||q_4|}{r_{24}^2}\cos(\theta_2) + \frac{k|q_3||q_4|}{r_{34}^2}\cos(\theta_3) = \frac{kq^2}{4}(1) - \frac{2kq^2}{1}(1) + \frac{3q^2}{5}\left(\frac{2}{\sqrt{5}}\right) = (-1.21)kq^2$$

$$\sum F_y = F^{\text{E}}_{14y} + F^{\text{E}}_{24y} + F^{\text{E}}_{34y} = \frac{k|q_1||q_4|}{r_{14}^2}\sin(\theta_1) - \frac{k|q_2||q_4|}{r_{24}^2}\sin(\theta_2) - \frac{k|q_3||q_4|}{r_{34}^2}\sin(\theta_3) = \frac{kq^2}{4}(0) - \frac{2kq^2}{1}(0) + \frac{3q^2}{5}\frac{1}{\sqrt{5}} = (-0.27)kq^2$$

We calculate the angle from the $+x$ at which the vector sum of all forces acts using $\theta_{\text{sum}} = \tan^{-1}\left(\frac{\sum F_y}{\sum F_x}\right) =$ $\tan^{-1}\left(\frac{-0.27}{-1.21}\right) = (2\times 10^2)°$ clockwise from $+\hat{x}$. Thus the $190°$ counterclockwise from the $+x$ axis.

22.58. We write the sum of all components of forces on particle 3:

$$\sum F_x = F_{13x}^{\text{E}} + F_{23x}^{\text{E}} = -\frac{k|q_1||q_3|}{r_{13}^2}\cos(\theta_1) - \frac{k|q_2||q_3|}{r_{23}^2}\cos(\theta_2)$$

$$= -\frac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2\text{C}^2)(10\times 10^{-6}\ \text{C})(3\times 10^{-6}\ \text{C})}{(1.0\ \text{m})^2}\cos(60°) - \frac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2\text{C}^2)(5\times 10^{-6}\ \text{C})(3\times 10^{-6}\ \text{C})}{(1.0\ \text{m})^2}\cos(60°)$$

$$= -0.2025\ \text{N}$$

$$\sum F_y = F_{13y}^{\text{E}} + F_{23y}^{\text{E}} = -\frac{k|q_1||q_3|}{r_{13}^2}\sin(\theta_1) + \frac{k|q_2||q_3|}{r_{23}^2}\sin(\theta_2)$$

$$= -\frac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2\text{C}^2)(10\times 10^{-6}\ \text{C})(3\times 10^{-6}\ \text{C})}{(1.0\ \text{m})^2}\sin(60°) + \frac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2\text{C}^2)(5\times 10^{-6}\ \text{C})(3\times 10^{-6}\ \text{C})}{(1.0\ \text{m})^2}\sin(60°)$$

$$= -0.1169\ \text{N}$$

Using the Pythagorean Theorem and simple trigonometry, we can also write this as $\vec{F} = 0.234\ \text{N}$ at $210°$ counterclockwise from the $+x$ axis.

22.59. Note first that since the positively charged particle is at the center of the square, the vector sum of all electric forces on it must be zero. Thus, we only have to show that the vector sum of forces on all the negative particles is zero. Since each negatively charged particle is equivalent, we only need to show that the vector sum of all forces on any one negatively charged particle is zero. To further simplify the process, the entire setup is symmetry about a diagonal of the square. If we choose the x and y axis to lie parallel to edges of the square, this diagonal symmetry means the sum of all x components of forces will be the same as the sum of all y components. Thus we need only calculate one component of the sum of all forces on a single negatively charged particle. Call this particle 1 and let particle 1 be on the lower right corner of the square. Number the remaining negatively charged particles 2-4 going counterclockwise around the square. Choose the $+x$ axis to point to the right. We calculate the sum of all x components of the forces.

$$\sum F_x = F_{21x}^{\text{E}} + F_{31x}^{\text{E}} + F_{41x}^{\text{E}} + F_{\text{p}1x}^{\text{E}} = \frac{k|q_2||q_1|}{r_{21}^2}\cos(\theta_2) + \frac{k|q_3||q_1|}{r_{31}^2}\cos(\theta_3) + \frac{k|q_4||q_1|}{r_{41}^2}\cos(\theta_4) + \frac{k|q_{\text{p}}||q_1|}{r_{\text{p}1}^2}\cos(\theta_{\text{p}})$$

$$= \frac{kq_{\text{n}}^2}{d^2}\cos(90°) + \frac{kq_{\text{n}}^2}{2d^2}\cos(45°) + \frac{kq_{\text{n}}^2}{d^2}\cos(0) + \frac{kq_{\text{p}}q_{\text{n}}}{d^2/2}\cos(45°) = 0$$

$$q_{\text{n}}\left(\frac{\sqrt{2}}{4} + 1\right) + q_{\text{p}}\sqrt{2} = 0 \Rightarrow \frac{q_{\text{p}}}{q_{\text{n}}} = -\left(\frac{1}{4} + \frac{\sqrt{2}}{2}\right) \approx -0.957$$

22.60. Let the horizontal axis be the x axis of a standard xy coordinate system. Clearly, by symmetry the y components of the forces that particles 2 and 3 exert on 1 will cancel. We need only consider forces in the x direction, which means we will be placing particle 4 on the x axis somewhere. We write the sum of all x components of the forces on particle 1:

$$\sum F_x = F^{\mathrm{E}}_{21x} + F^{\mathrm{E}}_{31x} + F^{\mathrm{E}}_{41x} = 0$$

$$\frac{k|q_2||q_1|}{r_{21}^2}\cos(\theta_2) + \frac{k|q_3||q_1|}{r_{31}^2}\cos(\theta_3) - \frac{k|q_4||q_1|}{x^2}\cos(\theta_4) = 0$$

$$x = \pm\sqrt{\frac{|q_4||q_1|\cos(\theta_4)}{\frac{|q_2||q_1|}{r_{21}^2}\cos(\theta_2) + \frac{|q_3||q_1|}{r_{31}^2}\cos(\theta_3)}}$$

$$x = \pm\sqrt{\frac{(2.0\times10^{-6}\ \mathrm{C})^2\cos(0)}{2\frac{(1.0\times10^{-6}\ \mathrm{C})(2.0\times10^{-6}\ \mathrm{C})}{(2.0\times10^{-3}\ \mathrm{m})^2}\cos(45°)}}$$

$$x = -2.4\ \mathrm{mm}$$

So we should place particle 4 on the horizontal axis, 2.4 mm to the left of the origin.

22.61. Because particle 2 is twice as far from particle 1 as particle 3 is from 1, and because the magnitude of the charge on particle 2 is half that on particle 3, we can see by inspection that the ratio is 1/8. More explicitly, we can write $F^{\mathrm{E}}_{21} = \frac{k|q_2||q_1|}{r_{21}^2}$, and $F^{\mathrm{E}}_{31} = \frac{k|q_3||q_1|}{r_{31}^2} = \frac{k|2q_2||q_1|}{(r_{21}/2)^2} = 8\frac{k|q_2||q_1|}{r_{21}^2} = 8F^{\mathrm{E}}_{21}$. So, $F^{\mathrm{E}}_{21}/F^{\mathrm{E}}_{31} = 1/8$.

22.62. We write the sum of all x and y components of forces on particle 3:

$$\sum F_x = F^{\mathrm{E}}_{13x} + F^{\mathrm{E}}_{23x} = \frac{k|q_1||q_3|}{r_{13}^2}\cos(\theta_1) + \frac{k|q_2||q_3|}{r_{23}^2}\cos(\theta_2)$$

$$= \frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(1.00\times10^{-6}\ \mathrm{C})(3.00\times10^{-6}\ \mathrm{C})}{(5.0\ \mathrm{m})^2}\cos(36.9°)$$

$$+\frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(2.00\times10^{-6}\ \mathrm{C})(3.00\times10^{-6}\ \mathrm{C})}{(5.0\ \mathrm{m})^2}\cos(53.1°) = 2.1606\times10^{-3}\ \mathrm{N}$$

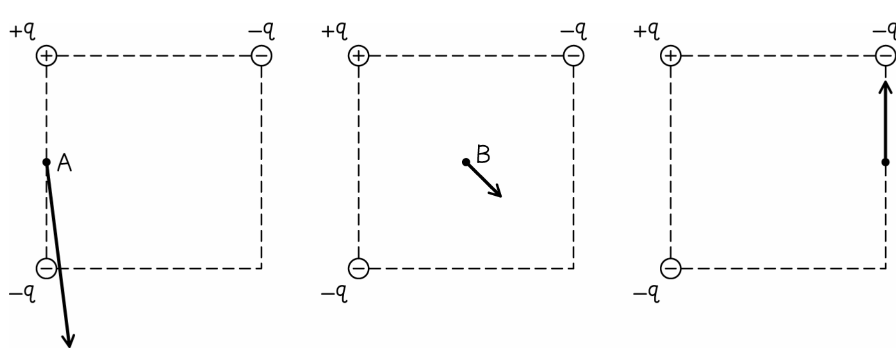
$$\sum F_y = F^{\mathrm{E}}_{13y} + F^{\mathrm{E}}_{23y} = \frac{k|q_1||q_3|}{r_{13}^2}\sin(\theta_1) - \frac{k|q_2||q_3|}{r_{23}^2}\sin(\theta_2)$$

$$= \frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(1.00\times10^{-6}\ \mathrm{C})(3.00\times10^{-6}\ \mathrm{C})}{(5.0\ \mathrm{m})^2}\sin(36.9°)$$

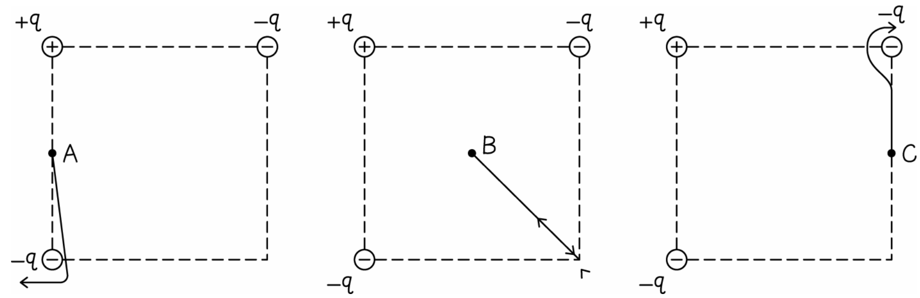
$$-\frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(2.00\times10^{-6}\ \mathrm{C})(3.00\times10^{-6}\ \mathrm{C})}{(5.0\ \mathrm{m})^2}\sin(53.1°) = -1.0789\times10^{-3}\ \mathrm{N}$$

Using the Pythagorean Theorem and basic trigonometry, this can also be expressed as $\sum \vec{F}^{\mathrm{E}}_3 = 2.41\ \mathrm{mN}$ at an angle of $333°$ counterclockwise from the $+x$ axis.

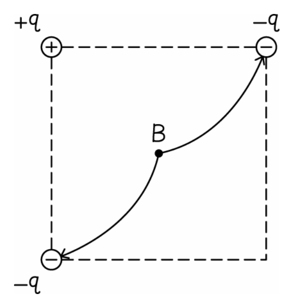
22.63. (*a*)



(*b*) In position B, the sum of all forces is initially down and to the right, not toward either negatively charged particles. The object will slowly move down and to the right. This path is shown below.



But if it shifted ever so slightly to one side, it would rapidly approach one of the negatively charged particles. Two possible paths are shown taking into account this instability. Which path the particle follows depends on whether the object moves some infinitesimal distance above or below the diagonal path.



22.64. Call the angle between $\vec{F}_{13}^{\text{E}}$ and $+\hat{x}$ θ_1, and call the angle between $\vec{F}_{23}^{\text{E}}$ and $+\hat{x}$ θ_2. Simple geometry shows us that the magnitude of each angle is the same, equal to $\theta_1 = \tan^{-1}\left(\frac{0.080\text{ m}}{0.030\text{ m}}\right) = 69.4°$. We write the sum of all x and y components of forces on particle 3:

$$\sum F_x = F_{13x}^{\text{E}} + F_{23x}^{\text{E}} = \frac{k|q_1||q_3|}{r_{13}^2}\cos(\theta_1) + \frac{k|q_2||q_3|}{r_{23}^2}\cos(\theta_2)$$

$$= \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-9}\text{ C})(5.0\times10^{-6}\text{ C})}{(3.0\times10^{-2}\text{ m})^2 + (8.0\times10^{-2}\text{ m})^2}\cos(69.4°)$$

$$+\frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-9}\text{ C})(5.0\times10^{-6}\text{ C})}{(3.0\times10^{-2}\text{ m})^2 + (8.0\times10^{-2}\text{ m})^2}\cos(69.4°) = 8.66\times10^{-3}\text{ N}$$

$$\sum F_y = F_{13y}^{\text{E}} + F_{23y}^{\text{E}} = \frac{k|q_1||q_3|}{r_{13}^2}\sin(\theta_1) - \frac{k|q_2||q_3|}{r_{23}^2}\sin(\theta_2)$$

$$= \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-9}\text{ C})(5.0\times10^{-6}\text{ C})}{(3.0\times10^{-2}\text{ m})^2 + (8.0\times10^{-2}\text{ m})^2}\sin(69.4°)$$

$$-\frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-9}\text{ C})(5.0\times10^{-6}\text{ C})}{(3.0\times10^{-2}\text{ m})^2 + (8.0\times10^{-2}\text{ m})^2}\sin(69.4°) = 0$$

Thus $\sum \vec{F}_3^{\text{E}} = 8.7\times10^{-3}\text{ N }\hat{i}$.

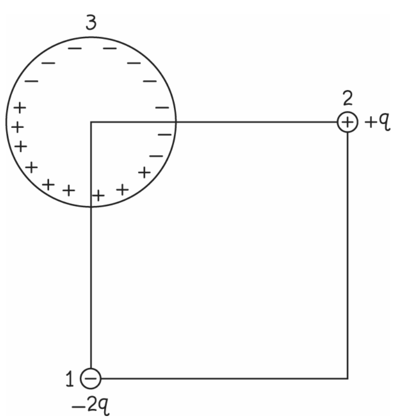
22.65. At the location $y = 0.080$ m, the force that particle 1 exerts on 3 and the force that particle 2 exerts on 3 cancel each other. This does not give us any information about the charge q. Because this position is twice as far from particle 1 as it is from particle 2 (and particle 1 has four times the charge of particle 2) these forces would cancel at this location regardless of the size of q. We write the sum of all y components of the forces acting on particle 3 when it is at the location $y = 0.040$ m. Note that the sum of all forces goes from zero to being directed in the $+y$ direction when particle 3 moves toward particle 1. This means that $\vec{F}^{\mathrm{E}}_{13}$ is in the $+y$ direction, which tells us that q is positive.

$$\sum F_y = F^{\mathrm{E}}_{13y} + F^{\mathrm{E}}_{23y} = \frac{k|q_1||q_3|}{r_{13}^2} - \frac{k|q_2||q_3|}{r_{23}^2} = \frac{k4q|q_3|}{r_{13}^2} - \frac{kq|q_3|}{4r_{13}^2}$$

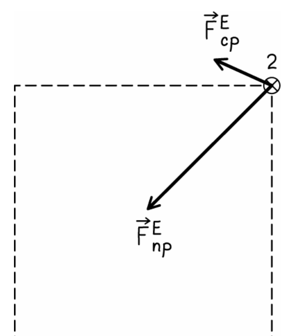
$$\Rightarrow q = \frac{\sum F_y}{\frac{k|q_3|}{r_{13}^2}\left(4 - \frac{1}{4}\right)} = \frac{(126.4\ \mathrm{N})}{\frac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(2.0\times10^{-6}\ \mathrm{C})}{(0.040\ \mathrm{m})^2}\left(4 - \frac{1}{4}\right)} = 3.0\times10^{-6}\ \mathrm{C}$$

So $q = 3.0\ \mu\mathrm{C}$.

22.66. (*a*) Sphere 1 will have the effect of polarizing sphere 3 in such a way as to repel negative charge away from sphere 1, or equivalently: drawing positive charge toward sphere 1. Sphere 2 will have the effect of polarizing sphere 3 in such a way as to attract negative charge toward sphere 2. If the charges on spheres 1 and 2 had equal magnitude, we might expect the charge distribution on 3 to have surplus positive charge on one side of a line through the square diagonal, and surplus negative charge on the other side. Since the effect due to sphere 1 is stronger (because it has a larger magnitude of charge) the division between positive and negative surplus charge is shifted somewhat away from this diagonal.



(*b*)



22.67. (*a*) The horizontal component of the force exerted by a given particle A in the ring cancels the horizontal component of the force exerted by particle opposite particle A in ring. Therefore only vertical components need to be summed. Because all 100 forces have same magnitude and the same distance from the ink droplet, you need to calculate only one force. Call the direction vertically upward the $+z$ axis. (*b*) We start by calculating the z component of the force exerted on the ink drop by a single charged particle A.

$$F^{\mathrm{E}}_{\mathrm{A\ ink}\ z} = \frac{k|q_{\mathrm{A}}||q_{\mathrm{ink}}|}{r_{\mathrm{A\ ink}}^2}\cos(\theta) = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.00\times10^{-9}\ \mathrm{C})(8.00\times10^{-9}\ \mathrm{C})}{(6.00\times10^{-3}\ \mathrm{m})^2 + (6.00\times10^{-3}\ \mathrm{m})^2}\cos(45°) = 7.063\times10^{-4}\ \mathrm{N}$$

We have 100 such force components, such that $\sum \vec{F}^{\mathrm{E}}_{\mathrm{ink}} = 7.06\times10^{-2}\ \mathrm{N}\ \hat{k}$. (*c*) We know that $\sum F^{\mathrm{E}}_{\mathrm{ink}} = N\frac{k|q_{\mathrm{A}}||q_{\mathrm{ink}}|}{r_{\mathrm{A\ ink}}^2}\cos(\theta) = m_{\mathrm{ink}}a_{\mathrm{ink}}$ so

$$a_{\mathrm{ink}} = N\frac{k|q_{\mathrm{A}}|}{r_{\mathrm{A\ ink}}^2}\left(\frac{|q_{\mathrm{ink}}|}{m_{\mathrm{ink}}}\right)\cos(\theta) = (100)\frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.00\times10^{-9}\ \mathrm{C})}{(6.00\times10^{-3}\ \mathrm{m})^2 + (6.00\times10^{-3}\ \mathrm{m})^2}\left(\frac{1.00\ \mathrm{C}}{1.00\ \mathrm{kg}}\right)\cos(45°) = 8.83\times10^4\ \mathrm{m/s^2}$$

So $\vec{a} = 8.83\times10^6\ \mathrm{m/s^2}\ \hat{k}$.

22.68. (*a*) Treating the distance between the puck and generator as constant, we can treat the acceleration of the puck during this 100 ms time interval as constant.

$$a_{\mathrm{p}} = \frac{F^{\mathrm{E}}_{\mathrm{gp}}}{m_{\mathrm{p}}} = \frac{k|q_{\mathrm{g}}||q_{\mathrm{p}}|}{m_{\mathrm{p}} r^2_{\mathrm{gp}}} = \frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})(0.10\ \mathrm{C})(5.0\times 10^{-5}\ \mathrm{C})}{(0.160\ \mathrm{kg})(61\ \mathrm{m})^2} = 75.6\ \mathrm{m/s^2}$$

We can use kinematics to determine the speed of the puck once the generator is shut off. $v_{\mathrm{p,f}} = v_{\mathrm{p,i}} + a_{\mathrm{p}}\Delta t = 0 + (75.6\,\mathrm{m/s^2})(1.00\times 10^{-1}\ \mathrm{s}) = 7.56\ \mathrm{m/s}$. We note as an aside that during this initial $100\ \mathrm{ms}$ period of acceleration, then puck moves only 0.378 m (also from kinematics). Now the generator is shut off, and the only force doing work on the puck is the force of kinetic friction that the ice exerts on it. Let us determine how far the puck can move before this force of friction does sufficient work on the puck to reduce its kinetic energy to zero. Let us call the direction that the puck moves the $+x$ direction. $W = \Delta K \Rightarrow -F^{\mathrm{E}}_{\mathrm{ice\ p}\,x}\Delta x_{\max} = K_{\mathrm{p,f}} - K_{\mathrm{p,i}} = -\frac{1}{2}mv^2_{\mathrm{p,i}}$. So $\Delta x_{\max} = \dfrac{v^2_{\mathrm{p,i}}}{2\mu_{\mathrm{k}} g} = \dfrac{(7.56\,\mathrm{m/s})^2}{2(0.015)(9.8\,\mathrm{m/s^2})} = 1.9\times 10^2\ \mathrm{m}$. This is much greater than the required 61 m. So, yes the puck will make it to the generator. (*b*) As in part (*a*), we set the work done by friction equal to the change in kinetic energy. This time we insert the known distance to the generator and solve for the final speed of the puck.

$$-F^{\mathrm{E}}_{\mathrm{ice\ p}\,x}\Delta x_{\mathrm{generator}} = \frac{1}{2}mv^2_{\mathrm{p,f}} - \frac{1}{2}mv^2_{\mathrm{p,i}} \Rightarrow v_{\mathrm{p,f}} = \sqrt{2\left(-\mu_{\mathrm{k}} g\Delta x_{\mathrm{generator}} + \frac{1}{2}v^2_{\mathrm{p,i}}\right)} = \sqrt{2(-(0.015)(9.8\ \mathrm{m/s^2})(61\ \mathrm{m}) + \frac{1}{2}(7.56\,\mathrm{m/s})^2)} = 6.3\ \mathrm{m/s}$$

22.69. We can see from symmetry that the x components of the forces on particle 3 will cancel, regardless of where particle 3 is located on the y axis. Symmetry also shows us that the magnitudes of F^{E}_{13} and F^{E}_{23} must be the same. Note also that in order for the force to be in the $+y$ direction, particle 3 must be on the $+y$ axis. Call the angle between $\vec{F}^{\mathrm{E}}_{13}$ and $+\hat{x}$ θ, and call the horizontal distance of particle 1 from the origin d. The sum of all y components of forces can be simply written as $\sum F_{3y} = F^{\mathrm{E}}_{13y} + F^{\mathrm{E}}_{23y} = 2F^{\mathrm{E}}_{13}\sin(\theta) = \dfrac{2k|q_1||q_3|}{(d^2+y^2)}\dfrac{y}{(d^2+y^2)^{1/2}}$. This equation is cubic in y, and can be solved graphically, or numerically to find $y = 10\ \mathrm{mm}$ or $y = 40\ \mathrm{mm}$.

22.70. A range of answers is possible. There are many simplifications and approximations that can be made. One may choose to treat the tape like a continuum of charge (which it is) or one may approximate the charge as being localized at a certain point on the tape. One may treat A charged section of tape as though it interacts with the entire strip of tap opposite it (which it does), or one may treat it as though the dominant interaction is between nearest-neighbor sections of tape. One must come up with an approximate treatment of the shape of the curved strands of tape, most likely a straight line. Here we present only one simple solution.

Ignore the curved top regions of the tap, and consider only the two straight sections of tape near the bottom. Assume these sections are approximately 10 cm long, and separated by about 7 cm. The forces acting on one of these sections are the electric force exerted on it by the opposite straight section of tape, and the tension in the top (curved) section of tape. We treat the charged particles as though they are localized at the center of the straight sections, and we treat the tensile force as acting at a $20°$ angle from vertical. We ignore the mass of the tape itself, and use the mass of a paper clip. The sum of all forces on one straight section of tape (section 1), broken into components is

$$\sum F_x = F^{\mathrm{c}}_{\mathrm{top}\ 1x} + F^{\mathrm{E}}_{21x} = m_{\mathrm{paperclip}}a_{1x} = 0 \Rightarrow -F^{\mathrm{c}}_{\mathrm{top}\ 1}\cos(\theta) + F^{\mathrm{E}}_{21} = 0 \quad (1)$$

$$\sum F_y = F^{\mathrm{c}}_{\mathrm{top}\ 1y} + F^{G}_{\mathrm{E}1y} = m_{\mathrm{paperclip}}a_{1y} = 0 \Rightarrow F^{\mathrm{c}}_{\mathrm{top}\ 1}\sin(\theta) - m_{\mathrm{paperclip}}g = 0 \quad (2)$$

Combining equations (1) and (2), we find $F^{\mathrm{E}}_{21} = m_{\mathrm{paperclip}}g\tan(\theta)$ or $q = \sqrt{\dfrac{d^2 m_{\mathrm{paperclip}}g\tan(\theta)}{k}}$. We will assume the mass of the paperclip is 1 g. Then $q = \sqrt{\dfrac{(0.07\ \mathrm{m})^2(0.001\ \mathrm{kg})(9.8\ \mathrm{m/s^2})\tan(20°)}{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})}} = 4.3\times 10^{-8}\ \mathrm{C}$ or 2.7×10^{11} electrons.

Different types of tape will differ. One roll of tape has 90 wrapping of tape in a thickness of 1 cm, meaning the thickness of a strip of tape is approximately 1×10^{-4} m. The tape was 2 cm wide. These numbers, along with our assumptions about the tape give us a volume of about $2\times10^{-7}\ \text{m}^3$. A table of hydrocarbon densities indicates that a common density is about $8\times10^{2}\ \text{kg/m}^3$, meaning our straight sections of tape have a mass of about $m_{\text{tape}} = 0.2$ g. The mass of a set of one C atom and two H atoms is $m_{\text{C+2H}} = 2.3\times10^{-26}$ kg, and each such set would have 14 electrons. Thus in one straight section of tape, we have $N_{\text{e}} = \dfrac{m_{\text{tape}}}{m_{\text{C+2H}}}\times N_{\text{per (C+2H)}} = \dfrac{(2\times10^{-4}\ \text{kg})}{(2.3\times10^{-26}\ \text{kg})}(14) = 1.2\times10^{23}$ electrons. Above, we calculated that we need approximately 2.7×10^{11} of these 1.2×10^{23} electrons, which is roughly one in 10^{12}.
Other methods may yield answers an order of magnitude smaller or larger than this. Approximately one electron in every 10^{11} to 10^{13} electrons must be removed from one of the strips.

22.71. Electrons in piece of metal do not settle because gravitational forces exerted on them are negligible relative to electric forces exerted on them. Thus difference between gravitational potential energy of electron at top of piece and gravitational potential energy of electron at bottom of piece is infinitesimally small.

22.72. When electrons move through metal conducting electricity, they carry kinetic energy with them. This kinetic energy does not remain on individual electrons but is transferred from one electron to another in collisions, becoming thermal energy. The relationship stated in the problem is often true, but not true for all materials. For example silver conducts electricity much better than diamond, but diamond conducts heat better than silver.

22.73. $\sum F_{sy} = F^{\text{E}}_{\text{rs}y} + F^{G}_{\text{Es}y} = m_{\text{s}}a_{\text{s},y}$, so $F^{\text{E}}_{\text{rs}y} = m_{\text{s}}a_{\text{s},y} + m_{\text{s}}g = (0.20\times10^{-3}\ \text{kg})((0.14\ \text{m/s}^2) + (9.8\ \text{m/s}^2)) = 2.0\times10^{-3}$ N. Thus $\vec{F}^{\text{E}}_{\text{rs}} = 2.0\times10^{-3}$ N upward.

22.74. When spheres A and B are brought into contact, their collective 16 units of negative charge divides evenly between them. Spheres A and B each carry 8 units of negative charge at this point. When Spheres B and C are brought into contact, the charge initially on B divides evenly between B and C, leaving each with 4 units of negative charge. Since A carries 8 units of negative charge, and C carries 4 units of negative charge, the ratio on the charge A:C is 2:1.

22.75. (*a*) The force between the two particles is repulsive, such that $\vec{F}^{\text{E}}_{\text{BA}}$ is in the $+y$ direction. The magnitude of the force is given by

$$F^{\text{E}}_{\text{BA}} = \frac{k|q_{\text{A}}||q_{\text{B}}|}{r^2_{\text{AB}}} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.0\times10^{-6}\ \text{C})(6.0\times10^{-6}\ \text{C})}{(3.0\ \text{m})^2} = 24\ \text{mN}$$

Thus $\vec{F}^{E}_{\text{BA}} = 24\ \text{mN}\,\hat{j}$. (*b*) By Newton's third law $\vec{F}^{E}_{\text{AB}} = -24\ \text{mN}\,\hat{j}$.

22.76. Proton: up, up, down. Neutron: up, down, down.

22.77. Clearly the charge must be negative, in order to attract the positive charge. We write the sum of all x components of forces acting on particle 2.

$$\sum F_{2x} = F^{\text{E}}_{12x} + F^{\text{E}}_{32x} = -\frac{k|q_1||q_2|}{r^2_{12}} + \frac{k|q_3||q_2|}{r^2_{32}} = 0 \Rightarrow |q_3| = \frac{r^2_{32}}{r^2_{12}}|q_1| = \frac{(30\ \text{mm})^2}{(10\ \text{mm})^2}(4.0\ \text{nC}) = 36\ \text{nC}.\ \text{So}\ q_3 = -36\ \text{nC}$$

22.78. There are many correct answers. Let particle 1 be at the origin. Place particle 2 a distance d to the left of particle 1. Place particle 3 a distance $2d$ to the right of particle 1. At this point there is a net force on particle 1 to the right, meaning we must place our final particle on the right side of the origin. We can determine exactly how far from the origin particle 4 has to be by writing the sum of all forces in the x direction.

$$\sum F_{1x} = F^{\text{E}}_{21x} + F^{\text{E}}_{31x} + F^{\text{E}}_{41x} = \frac{kq^2}{d^2} + -\frac{kq^2}{4d^2} - \frac{kq^2}{x^2} = 0 \Rightarrow \frac{1}{x^2} = \frac{1}{d^2}\left(\frac{3}{4}\right) \Rightarrow x = \frac{2}{\sqrt{3}}d$$

So we place our final particle a distance $\dfrac{2}{\sqrt{3}}d$ to the right of charge 1.

22.79. (*a*) Place the positively charged particles in corners opposite each other along a diagonal of the square. Do the same for the negatively charged particles. (*b*) Call the charge in the middle of an edge q_{m}, and let this edge be along the $+x$ axis. Call the positively charged particle at the origin particle 1, and number the other particles in counterclockwise order around the square. We write the sum of all force components in the x and y directions.

$$\sum F_{\text{m}x} = F^{\text{E}}_{1\text{m}x} + F^{\text{E}}_{2\text{m}x} + F^{\text{E}}_{3\text{m}x} + F^{\text{E}}_{4\text{m}x} = F^{\text{E}}_{1\text{m}} + F^{\text{E}}_{2\text{m}} - F^{\text{E}}_{3\text{m}}\cos(\theta) - F^{\text{E}}_{4\text{m}}\cos(\theta) = \frac{kq^2}{\ell^2}\left(4 + 4 - \frac{4}{5}\frac{1}{\sqrt{5}} - \frac{4}{5}\frac{1}{\sqrt{5}}\right) = 7.3\frac{kq^2}{\ell^2} \quad (1)$$

$$\sum F_y = F^{\text{E}}_{1\text{m}y} + F^{\text{E}}_{2\text{m}y} + F^{\text{E}}_{3\text{m}y} + F^{\text{E}}_{4\text{m}y} = (0) + (0) - F^{\text{E}}_{3\text{m}}\sin(\theta) + F^{\text{E}}_{4\text{m}}\sin(\theta) = \frac{kq^2}{\ell^2}\left(\frac{4}{5}\frac{1}{\sqrt{5}} - \frac{4}{5}\frac{1}{\sqrt{5}}\right) = 0 \quad (2)$$

From equations (1) and (2) we see $\sum \vec{F}_{\text{m}} = 7.3\frac{kq^2}{\ell^2}\,\hat{i}$ where $\hat{i}$ points along the edge of the square on which the charge was placed, pointing from positive to negative.

22.80. We can write down the sum of all x and y components of forces:

$$\sum F_x = F^{\text{c}}_{\text{rb}x} + F^{\text{E}}_{\text{sb}x} = ma_x = 0 \Rightarrow -F^{\text{c}}_{\text{rb}}\cos(\theta) + F^{\text{E}}_{\text{sb}} = 0 \quad (1)$$

$$\sum F_y = F^{\text{c}}_{\text{rb}y} + F^{G}_{\text{E}1y} = ma_y = 0 \Rightarrow F^{\text{c}}_{\text{rb}}\sin(\theta) - mg = 0 \quad (2)$$

Combining equations (1) and (2) yields $m = \dfrac{F^{\text{E}}_{\text{sb}}}{g\tan(\theta)} = \dfrac{(2.3\text{ N})}{(9.8\text{ m/s}^2)\tan(3.6°)} = 3.7\text{ kg}.$

22.81. Call the sliding block block 1, and call the fixed block block 2. We write the sum of all x components of forces on block 1.

$$\sum F_{1x} = F^{\text{sp}}_{\text{sp }1\,x} + F^{\text{E}}_{21x} = -k\Delta\ell\cos(\theta) + F^{\text{E}}_{21x} = 0 \Rightarrow k = \frac{F^{\text{E}}_{21x}}{\Delta\ell\cos(\theta)} = \frac{(0.100\text{ N})}{(3.00\times10^{-3}\text{ m})\cos(45°)} = 47.1\text{ N/m}$$

22.82. 0 (neutral). Regardless of what the decay products of the neutron are, or how they react with the anti-hydrogen atom, charge is still conserved.

22.83. Let us assume that by "balanced" we mean the proteins in the center will not experience a net electric force from the nucleic acids surrounding them. Then to complete the symmetry of the ring of nucleic acids, we need to insert the eighth one at $5\pi/4$.

22.84. (*a*) In uniform circular motion, we know $\sum F_r = \dfrac{mv^2}{R} = \dfrac{m}{R}\left(\dfrac{2\pi R}{T}\right)^2 = \dfrac{4m\pi^2 R}{T^2}$. Choose $+r$ to point radially inward toward the proton. If the gravitational force is responsible for holding the atom together, and it is the only force acting, we can write $\dfrac{Gm_{\text{e}}m_{\text{p}}}{R^2} = \dfrac{4m_{\text{e}}\pi^2 R}{T^2} \Rightarrow T = \sqrt{\dfrac{4\pi^2 R^3}{Gm_{\text{p}}}} = \sqrt{\dfrac{4\pi^2(53\times10^{-12}\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.67\times10^{-27}\text{ kg})}} = 7.3\times10^3\text{ s}.$ (*b*) If we now insert the electric force as that responsible for holding the atom together, we obtain $\dfrac{kq_{\text{e}}^2}{R^2} = \dfrac{4m_{\text{e}}\pi^2 R}{T^2} \Rightarrow T = \sqrt{\dfrac{4m_{\text{e}}\pi^2 R^3}{kq_{\text{e}}^2}} = \sqrt{\dfrac{4(9.11\times10^{-31}\text{ kg})\pi^2(53\times10^{-12}\text{ m})^3}{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ kg})}} = 1.5\times10^{-16}\text{ s}.$

22.85. (*a*) Let us assume the penny was minted prior to 1982, such that it is made entirely of copper. The number of copper atoms in a penny can be found by writing $N_{\text{Cu}} = \dfrac{m_{\text{penny}}}{M_{\text{Cu}}}N_{\text{A}}$. Since each Cu atom has 29 electrons.

The number of electrons is $N_{\text{e}} = N_{\text{e per Cu}}\left(\dfrac{m_{\text{penny}}}{M_{\text{Cu}}}\right)N_{\text{A}} = (29\text{ e/atom})\dfrac{(0.003\text{ kg})}{(.06355\text{ kg/mol})}(6.02\times10^{23}\text{ atoms/mol}) =$

8×10^{23} electrons. (*b*) This would yield a charge equal to $q = N_e q_e = (8.2\times10^{23}\text{ electrons})(1.60\times10^{-19}\text{ C/electron}) =$ 1×10^{5} C. (*c*) The repulsive force exerted by that charge on a single additional electron would be $F^{\text{E}}_{\text{ce}} = \dfrac{k|q_c||q_e|}{r^2_{\text{ce}}} =$ $\dfrac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.3\times10^{5}\text{ C})(1.60\times10^{-19}\text{ C})}{(1.0\times10^{-9}\text{ m})^2} = 2\times10^{14}\text{ N}$. In order to bring the additional electron within that distance of the rest of the charge, you would need to push with a force of at least 2×10^{14} N. (*d*) No.

22.86. (*a*) Because the electron could reasonably be at the pole or on the equator, we could be justified using either the mean radius, or the mean equatorial radius of Earth. Using the mean equatorial radius of 6.38×10^{6} m, the magnitude of the force is given by $F^{\text{E}}_{\text{Ee}} = \dfrac{k|q_{\text{E}}||q_e|}{r^2_{\text{Ee}}} = \dfrac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(6.76\times10^{5}\text{ C})(1.60\times10^{-19}\text{ C})}{(6.38\times10^{6}\text{ m})^2} =$ 2.39×10^{-17} N. Because both charges involved are negative, $\vec{F}^{\text{E}}_{\text{ce}} = 2.39\times10^{-17}$ N away from the center of Earth.

(*b*) $$\frac{F^{\text{E}}_{\text{Ee}}}{F^{G}_{\text{Ee}}} = \frac{\dfrac{k|q_{\text{E}}||q_e|}{r^2_{\text{Ee}}}}{\dfrac{Gm_{\text{E}}m_e}{r^2_{\text{Ee}}}} = \frac{k|q_{\text{E}}||q_e|}{Gm_{\text{E}}m_e} = \frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(6.76\times10^{5}\text{ C})(1.60\times10^{-19}\text{ C})}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})(9.11\times10^{-31}\text{ kg})} = 2.68\times10^{12}$$

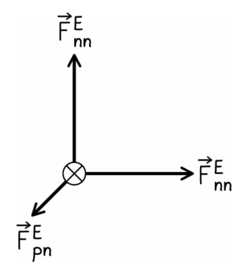
(*c*) We write the sum of all forces radially outward from the center of Earth, and require that this sum be zero so that the penny just hovers. The mass of a penny can vary based on wear, but the U.S. Mint lists the mass as 2.5 g. Thus

$$\sum F_r = F^{\text{E}}_{\text{E}pr} + F^{G}_{\text{E}pr} = 0 \Rightarrow \frac{k|q_{\text{E}}||q_p|}{r^2_{\text{Ee}}} = mg \Rightarrow |q_p| = \frac{mgr^2_{\text{Ee}}}{k|q_{\text{E}}|} = \frac{(0.0025\text{ kg})(9.8\text{ m/s}^2)(6.38\times10^{6}\text{ m})^2}{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(6.76\times10^{5}\text{ C})} = 1.6\times10^{-4}\text{ C}$$

Note that if you use the very rough estimate of a penny's mass from problem 22.86 (3 g), you obtain a charge magnitude of 2×10^{-4} C. The most accurate answer is $q_p = -1.6\times10^{-4}$ C, and we will use this below for part (*d*).

(*d*) $$N_e = \frac{q}{q_e} = \frac{(1.64\times10^{-4}\text{ C})}{(1.60\times10^{-19}\text{ C/e}^-)} = 1.0\times10^{15}\text{ e}^-.$$

22.87. (*a*)



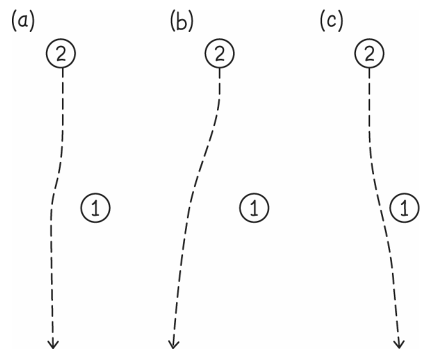
(*b*) Call the negatively charged particle in the lower right corner A, the particle in the upper right B, in the upper left C, and call the positively charged particle in the lower left corner D. Choose the $+x$ axis to lie along the bottom edge of the square and point from D to A; let the $+y$ axis lie along the left edge and point from D to C. We write the sum of all x and y components of forces on particle B.

$$\sum F_{\text{B}x} = F^{\text{E}}_{\text{AB}x} + F^{\text{E}}_{\text{CB}x} + F^{\text{E}}_{\text{DB}x} = (0) + F^{\text{E}}_{\text{CB}} - F^{\text{E}}_{\text{DB}}\cos(\theta) = \frac{kq^2}{\ell^2}\left(1 - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\right) = \frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-9}\text{ C})^2}{(0.050\text{ m})^2}\left(1 - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\right)$$
$$= 2.09\times10^{-5}\text{ N}$$

$$\sum F_{\text{B}y} = F^{\text{E}}_{\text{AB}y} + F^{\text{E}}_{\text{CB}y} + F^{\text{E}}_{\text{DB}y} = F^{\text{E}}_{\text{AB}} + (0) - F^{\text{E}}_{\text{DB}}\cos(\theta) = \frac{kq^2}{\ell^2}\left(1 - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\right) = \frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-9}\text{ C})^2}{(0.050\text{ m})^2}\left(1 - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\right)$$
$$= 2.09\times10^{-5}\text{ N}$$

Using the Pythagorean Theorem and basic trigonometry, we can write this as $\sum \vec{F}_{\mathrm{B}}^{\mathrm{E}} = 3.0 \times 10^{-5}\ \mathrm{N}$ away from the positive charge (45° counterclockwise from the $+x$ axis).

22.88. Because we do not know the charge to mass ratio of the falling sphere, nor the absolute value of the charge, we cannot be quantitative about the paths taken by the falling sphere. What we can say is that in (*a*), the spheres should repel and the path of the sphere 2 should bend away from sphere 1. In part (*b*) this repulsion should be much more pronounced. In part (*c*), the spheres should attract. Depending on how close sphere 2 is to being directly above sphere 1, the spheres may collide in case (*c*). Or sphere 2 may simply veer toward sphere 1. Here we show one possible figure.



22.89. The two positively charged spheres will repel each other, so your only hope of maintaining equilibrium is to place a negatively charged sphere between spheres 1 and 2. Call this negatively charged sphere, sphere 4. In order to exactly cancel the repulsion of 1 and 2, sphere 4 will have to be closer to sphere 1 than to sphere 2. Note that this is only possible if the magnitude of the charge on sphere 4 is smaller than the magnitude of the charge on either sphere 1 or 2. Sphere 4 is always closer to spheres 1 and 2 than spheres 1 and 2 are to each other. If the charge on sphere 4 had the same magnitude (or greater) than the magnitude of charge on sphere 1, for example, the attraction of 1 and 2 to 4 would overwhelm the repulsion of 1 and 2.

22.90. Set the blocks on a nonconducting material. Rub your sweater over the rest of your clothing to accumulate static charge and then touch the sweater to the left face of each block. Tie a styrofoam peanut to a length of the string and dangle the peanut next to the right face of each block. The block that attracts the peanut more strongly is the conducting block. (If instead of charging the blocks, you use the sweater to charge the peanut,, the charged peanut, when you dangle it near the blocks, may polarize the conducting block or polarize molecules at the surface of the nonconducting block. If this happens, the peanut is attracted to both blocks. Therefore so it is better to charge the blocks.)

22.91. The field is only zero at the center of the ring. Consider a placing a small particle carrying a positive charge q_{test} at a point in the plane of the ring that is off-center. Call the distance from the point to the closest segment of the ring r_1 and call the distance to the farthest point on the ring r_2. Rather than picturing conical surfaces passing through the off-center point (as in Newton's and Priestley's arguments) you use a roughly triangular pie-slice off to either side of the point. If the charge is distributed evenly along the ring, then the relationship between the charge q_1 on the smaller pie crust and the charge q_2 on the larger pie crust is $\dfrac{q_1}{r_1} = \dfrac{q_2}{r_2}$. The sum of the electric forces on q_{test} due to the large and small pie crusts is $\sum \vec{F}_{q_{\text{test}}}^{\mathrm{E}} = \vec{F}_{1q_{\text{test}}}^{\mathrm{E}} + \vec{F}_{2q_{\text{test}}}^{\mathrm{E}} = kq_{\text{test}}\left(\dfrac{q_1}{r_1^2} - \dfrac{q_2}{r_2^2}\right)$ in the direction of the larger pie crust (crust 2). Using the charge ratio above, this simplifies to $\sum \vec{F}_{q_{\text{test}}}^{\mathrm{E}} = \dfrac{kq_2 q_{\text{test}}}{r_2}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$ in the direction of the larger pie crust. This is only zero if the two distances to the edges are equal (meaning the point is in the center of the ring).

23

THE ELECTRIC FIELD

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

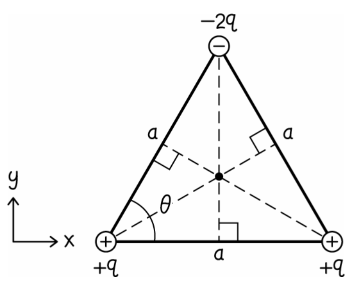
Developing a Feel

1. 10^{11} N/C **2.** 10^{-3} N/C **3.** 10^{7} C/kg **4.** 10^{-10} C/kg **5.** 10^{26} C/m^3 **6.** 10^{-6} C/m^2 **7.** 10^{-5} C/m **8.** 10^{7} N/C **9.** 10^{-7} N/C upward **10.** 10^{-29} C·m

Guided Problems

23.2 Charge triangle

1. Getting Started We begin by drawing the arrangement of charged particles. We choose to orient the triangle such that the two positive charges are on the horizontal base, and the negative charge is at the peak of the triangle.

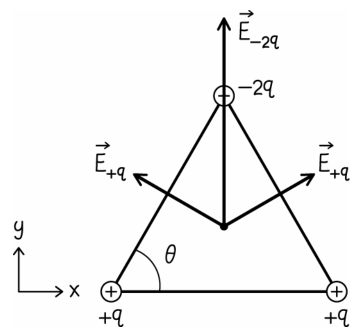


The center of the triangle must lie along all of the perpendicular bisectors of the edges (shown as thin lines across the interior of the triangle). For an equilateral triangle, the angle $\theta = 60°$. The position of the center along the x axis is clearly $a/2$ (with the origin chose to be at the lower left particle). Simple geometry gives us the vertical position:

$$\tan(\theta/2) = \frac{y_{\text{center}}}{x_{\text{center}}} \Rightarrow y_{\text{center}} = x_{\text{center}} \tan(\theta/2) = \frac{\sqrt{3}a}{6}$$

Each of the three charged particles causes an electric field at the center of the triangle. The total electric field is the vector sum of the fields due to each particle. The setup is reflection symmetric about a vertical line through its center. This tells us that there can be no electric field component along the x axis. To see this, imagine that there is an x component of the electric field, and flip the system over, rotating around that vertical line of symmetry. The charge arrangement would be exactly the same, but the electric field direction would change. The same charge distribution cannot produce both a leftward and a rightward total electric field. Thus the horizontal component must be zero. We need only consider the vertical components. Based on the same symmetry, we know the left positively charged particle will cause an electric field with the same vertical component as that from the right positively charged particle. So we only need to calculate that quantity once.

2. Devise Plan We draw a diagram of the relevant electric fields, to ensure that we have the correct signs for all terms.



Considering only the vertical components, we find

$$\begin{aligned} E_y &= 2E_{+q,y} + E_{-2q,y} \\ E_y &= 2E_{+q}\cos(\theta) + E_{-2q} \end{aligned} \tag{1}$$

3. Execute Plan We already found that the position of the center of the triangle is $(a/2, a\sqrt{3}/6)$. This means that the distance from the lower left particle to the center is

$$d_{\text{base}} = \sqrt{(a/2)^2 + (\sqrt{3}a/6)^2} = \sqrt{3}a/3$$

This is clearly also the distance from the lower right particle to the center. Because we are looking at the center of the triangle, we expect this to also be the same as the distance to the top particle, but this is easy to check. The height of the equilateral triangle is $h = \left(\frac{a}{2}\right)\tan(60°) = a\sqrt{3}/2,$ so the distance from the center to the particle at the top of the triangle is

$$d_{\text{top}} = h - y_{\text{center}} = a\frac{\sqrt{3}}{2} - a\frac{\sqrt{3}}{6} = a\frac{\sqrt{3}}{3}$$

So we drop the subscripts and simply write $d = a\frac{\sqrt{3}}{3}$.

Now inserting the form of the Coulomb force between two charged particles into equation (1) we have

$$\begin{aligned} E_y &= 2E_{+q}\cos(\theta) + E_{-2q} \\ E_y &= 2\left(\frac{kq}{d^2}\right)\cos(\theta) + \frac{k(2q)}{d^2} \\ E_y &= \frac{9kq}{a^2} \end{aligned}$$

So $\vec{E} = \frac{9kq}{a^2}$ toward the negatively charged particle.

4. Evaluate Result All terms in our answer are toward the negatively charged particle. We can see from our diagram that this should be the case. We certainly expect the electric field magnitude to increase if the charge on each particle increases, and this is reflection in our final answer. We also see that the electric field decreases when the side length of the triangle is increased.

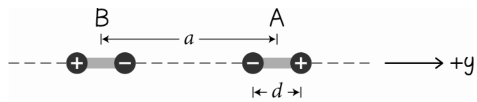
23.4 Interacting dipoles

1. Getting Started We will follow the hint and treat one dipole (dipole A) as a source of an electric field, and threat the other dipole (dipole B) as a set of two charged particles placed into the electric field produced by dipole A.

2. Devise Plan The only way to take advantage of symmetry in this problem is to choose an axis parallel/antiparallel to the two dipole axes. If we choose the y axis to point along the axis of dipole A, then we can use the given expression for the electric field along the dipole axis, verbatim:

$$E_y = kq_\mathrm{p}\left(\frac{1}{[y-(d/2)]^2} - \frac{1}{[y+(d/2)]^2}\right) \tag{1}$$

This corresponds to the following choice of axes and dipole labeling:



But we must be careful to note that the expression given, which is equation (23.11) is valid for $y > d/2$. That is, it is valid on the dipole axis for points closer to the positive charge than to the negative charge. In this situation, the dipole axes of dipoles A and B point away from each other. So regardless of our choice of dipole and axes labeling, the dipole feeling the electric field from the source will always be in a region where $y < -d/2$. If you look at equation (1), it is easy to see that this changes the overall sign. Thus, the equation we will use for the electric field due to A at the location of B is

$$E_y = kq_\mathrm{p}\left(\frac{1}{[y+(d/2)]^2} - \frac{1}{[y-(d/2)]^2}\right) \tag{2}$$

The force acting on a particle carrying charge q in an electric field $\vec{E}$ is given by $\vec{F} = q\vec{E}$. This can be used to describe the force on either of the charged particles in dipole B. But we have to be careful, because the positive and negative charges in B are at slightly different locations.

3. Execute Plan The negatively charge particle in dipole B is located at the point $y_{B,\mathrm{n}} = -a + \dfrac{d}{2}$, and the positively charge particle in dipole B is located at the point $y_{B,\mathrm{p}} = -a - \dfrac{d}{2}$. We use equation (2) to write a general equation for the force, and insert these positions to find

$$F_y = q_{B,\mathrm{n}}(E_y)_{y=y_{B,\mathrm{n}}} + q_{B,\mathrm{p}}(E_y)_{y=y_{B,\mathrm{p}}}$$

$$F_y = q_\mathrm{p}\left[-(E_y)_{y=-a+\frac{d}{2}} + (E_y)_{y=-a-\frac{d}{2}}\right]$$

$$F_y = kq_\mathrm{p}^2\left(-\frac{1}{[-a+d]^2} + \frac{1}{[-a]^2} + \frac{1}{[-a]^2} - \frac{1}{[-a-d]^2}\right)$$

So $\vec{F} = kq_\mathrm{p}^2\left(\dfrac{2}{a^2} - \dfrac{1}{(-a+d)^2} - \dfrac{1}{(a+d)^2}\right)$ along the axis of dipole A.

4. Evaluate Result It is not obvious whether our final answer is positive or negative. Experience tells us that dipoles with parallel axes tend to attract, and those with antiparallel axes tend to repel. So we expect the total force on B to be negative. If we let d be very small, we can expand our terms in a series to obtain

$$\frac{1}{(-a+d)^2} \approx \frac{1}{a^2} + \frac{2}{a^2}\left(\frac{d}{a}\right) + \frac{3}{a^2}\left(\frac{d}{a}\right)^2 + O\left[\left(\frac{d}{a}\right)^3\right]$$

$$\frac{1}{(a+d)^2} \approx \frac{1}{a^2} - \frac{2}{a^2}\left(\frac{d}{a}\right) + \frac{3}{a^2}\left(\frac{d}{a}\right)^2 + O\left[\left(\frac{d}{a}\right)^3\right]$$

This allows us to write the force as

$$F_y = kq_{\text{p}}^2\left(\frac{2}{a^2}-\left(\frac{1}{a^2}+\frac{2}{a^2}\left(\frac{d}{a}\right)+\frac{3}{a^2}\left(\frac{d}{a}\right)^2\right)-\left(\frac{1}{a^2}-\frac{2}{a^2}\left(\frac{d}{a}\right)+\frac{3}{a^2}\left(\frac{d}{a}\right)^2\right)\right)+O\left[\left(\frac{d}{a}\right)^3\right]$$

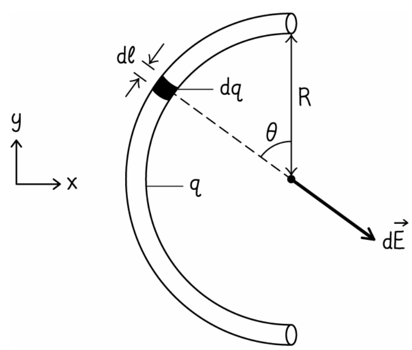
$$F_y = kq_{\text{p}}^2\left(-\frac{6}{a^2}\left(\frac{d}{a}\right)^2\right)+O\left[\left(\frac{d}{a}\right)^3\right]$$

This is clearly negative. One could also simply insert reasonable numbers to check the sign. We find that the direction of our answer agrees with our intuition.

In the limit as $a \gg d$, our expansion above should be a good approximation. Ignoring higher order terms (which are smaller by a factor of (d/a), we find $\vec{F} = \frac{6kq_{\text{p}}^2 d^2}{a^4}$ away from the source dipole.

23.6 Electric field created by a curved charged rod

1. Getting Started We begin by drawing a diagram showing the curved charged rod.



We consider a tiny length of the charged rod, denoted $d\ell$ and carrying a small quantity of charge dq. If we make this section of the rod very small, it will be the same as a charged particle. We can determine the small contribution to the electric field $d\vec{E}$ due to this small section of rod. To determine the total electric field at the center, we will need to add all such small contributions (integrate). But we must add these electric fields like the vectors they are. It is not correct to integrate the magnitudes of the $d\vec{E}$ s. Some of their components will cancel each other; integrating over the magnitudes would be like assuming that all the contributions $d\vec{E}$ from different sections of rod are exactly parallel, which they are not. Thus, we must first break down the small contributions $d\vec{E}$ into components dE_x and dE_y. These components are just numbers and can be integrated to yield the total E_x and E_y.

2. Devise Plan The magnitude of a small (differential) contribution to the electric field from a small segment of the rod is given by

$$dE = \frac{kdq}{r^2}$$

Which is simply equation (23.3) for a charge dq. The symbol r refers to the distance from the charged segment to the point at which we are finding the electric field (the center of the semicircle). However, in this case all segments are the same distance from the center: R. So we are free to use

$$dE = \frac{kdq}{R^2} \tag{1}$$

Because the charge density is uniform, we know $\frac{q}{\ell}=\frac{dq}{d\ell}$. That is, the charge to length ratio on a small segment is the same as the charge per length ratio on the whole rod. Equivalently $dq=\frac{q}{\ell}d\ell=\frac{q}{\pi R}d\ell$. Inserting this into equation (1), we have

$$dE=\frac{kqd\ell}{\pi R^3} \tag{2}$$

It may not be obvious at this point that θ is a useful integration variable. Either ℓ or θ would work. But remember, we have to take components of these differential contributions to the electric field:

$$dE_x=\frac{kqd\ell}{\pi R^3}\sin(\theta) \tag{3}$$

$$dE_y=\frac{kqd\ell}{\pi R^3}\cos(\theta) \tag{4}$$

We need to write this in terms of only one variable, and θ is a natural choice. Recall for an arc of a circle: $d\ell=Rd\theta$, such that equations (3) and (4) can be written

$$dE_x=\frac{kq}{\pi R^2}\sin(\theta)d\theta \tag{5}$$

$$dE_y=\frac{kq}{\pi R^2}\cos(\theta)d\theta \tag{6}$$

These expressions can be integrated to give us the components of the total electric field.

Because the semicircle is reflection symmetric across the x axis, we expect there to be no y component to the electric field. But we will go ahead and integrate equation (6) to verify this, as a check.

3. Execute Plan With the angle θ defined in the figure above, the range of this angle is $0<\theta<\pi$. Integrating equations (5) and (6) over this interval yields

$$E_x=\int dE_x=\frac{kq}{\pi R^2}\int_0^\pi \sin(\theta)d\theta=\frac{2kq}{\pi R^2}$$

$$E_y=\int dE_y=\frac{kq}{\pi R^2}\int_0^\pi \cos(\theta)d\theta=\frac{kq}{\pi R^2}(\sin(\pi)-\sin(0))=0$$

Thus, we have $\vec{E}=\frac{2kq}{\pi R^2}$ in the $+x$ direction.

4. Evaluate Result The direction of the electric field we obtained is very reasonable. It is exactly what we expected based on symmetry.

If the radius of the semicircle is increased, the electric field at the center drops off like $\frac{1}{R^2}$, which is very reasonable. Also, increasing the charge on the curved rod increases the electric field, as it should.

23.8 Cathode ray tube

1. Getting Started The upward electric field will exert a downward electric force on the electron. This will cause the electron to accelerate downward, changing its trajectory, as long as the electron is in the electric field. We are not told how the electric field drops off around the edge of the plate. Therefore, we will assume that there is an abrupt cutoff, such that we treat the electric field as constant between the plates and zero immediately outside the plates. Thus we expect a downward acceleration when the electron is between the plates, and no acceleration (but still a downward component to the velocity) once the electron leaves the plates and continues traveling toward the screen.

2. Devise Plan Since the electric field is uniform inside the plates, the electron will experience a constant force while in the plates, meaning the acceleration will be constant. This allows us to use kinematic equations. But the acceleration abruptly drops to zero as the electron leaves the area between the plates. So we must break up the motion into two regions: between the plates and outside the plates.

Let us call the instant the electron enters the area between plates t_0, call the instant the electron leaves the area between plates t_1, and the instant the electron strikes the screen t_2. Let us call the length of the plates ℓ and the horizontal distance from the plates to the screen d. Call the total height of the screen h.

We can use the kinematic equations to come up with an expression for the vertical component of the velocity when the electron leaves the plates v_{y1}, and for the vertical displacement of the electron while it is in between the plates $y_1 - y_0$. For v_{y1}, we write

$$v_{y1} = v_{y0} + a_y(t_1 - t_0) = 0 + \frac{F_y}{m}(t_1 - t_0)$$

Using the facts that $v_x = \dfrac{\ell}{(t_1 - t_0)}$ and that the force $F_y = qE$, we find

$$v_{y1} = \frac{qE}{m}\left(\frac{\ell}{v_x}\right) \tag{1}$$

For $y_1 - y_0$ we use

$$y_1 - y_0 = v_{y0} + \frac{1}{2}a_y(t_1 - t_0)^2 = 0 + \frac{qE}{2m}(t_1 - t_0)^2$$

Again, using $v_x = \dfrac{\ell}{(t_1 - t_0)}$, we find

$$y_1 - y_0 = \frac{qE}{2m}\left(\frac{\ell}{v_x}\right)^2 \tag{2}$$

Note that the charge on the electron q is a negative number, meaning both $y_1 - y_0$ and v_{y1} are negative quantities, as they should be.

We can use $y_1 - y_0$ to determine the remaining vertical distance the electron must cross once it leaves the plates: $(y_2 - y_1) = -\dfrac{h}{2} - (y_1 - y_0)$. We know the electron must cross this vertical distance in the same time $(t_2 - t_1)$ that it takes to cross the horizontal distance to the screen. Equivalently:

$$(t_2 - t_1) = \frac{(y_2 - y_0)}{v_{y1}} = \frac{d}{v_x}$$

$$\Rightarrow \frac{-\frac{h}{2} - (y_1 - y_0)}{v_{y1}} = \frac{d}{v_x} \tag{3}$$

This can be rearranged to yield an equation for v_{y1}. Equating that to (1), we can solve for the electric field.

3. Execute Plan Rearranging equation (3), we find

$$v_{y1} = \left(-\frac{h}{2} - (y_1 - y_0)\right)\left(\frac{v_x}{d}\right)$$

And inserting equation (2) yields

$$v_{y1} = -\left(\frac{h}{2} + \frac{qE}{2m}\left(\frac{\ell}{v_x}\right)^2\right)\left(\frac{v_x}{d}\right)$$

Setting this equal to equation (1), and rearranging we finally obtain

$$\frac{qE\ell d}{mv_x^2} = -\left(\frac{h}{2} + \frac{qE}{2m}\left(\frac{\ell}{v_x}\right)^2\right)$$

$$E\left(\frac{q\ell d}{mv_x^2} + \frac{qE\ell^2}{2mv_x^2}\right) = -\frac{h}{2}$$

$$E = -\frac{hmv_x^2}{q\ell(\ell + 2d)}$$

Recalling that the charge q is a negative number, we can use $e \equiv |q_e|$ to write

$$E = \frac{hmv_x^2}{e\ell(\ell + 2d)} \tag{4}$$

Inserting the numbers given, we find

$$E = \frac{(0.30\text{ m})(9.11\times10^{-31}\text{ kg})(3.0\times10^{5}\text{ m/s})^2}{(1.60\times10^{-19}\text{ C})(0.030\text{ m})((0.030\text{ m}) + 2(0.20\text{ m}))} = 12\text{ N/C}$$

4. Evaluate Result Equation (4) makes sense for several reasons. First, we see that if the length of the plates or the distance to the screen is increased, the electric field could be smaller. This is reasonable, since the electron would have longer to cover the vertical distance to the edge of the screen. If the screen is made higher, then the electron would need to travel a greater vertical distance and the electric field required would increase. The magnitude of our final numerical answer is an electric field that would be easy to produce, and so it is reasonable for electronic devices such as old cathode ray-based televisions.

Questions and Problems

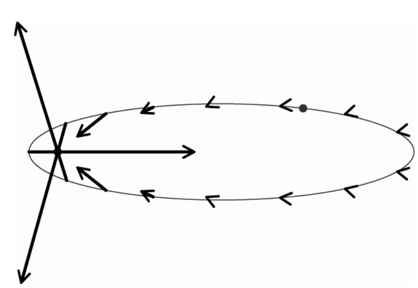
23.1. $g_{\text{SE}} = \dfrac{F_{\text{SE}}^G}{m_{\text{E}}} = \dfrac{Gm_{\text{S}}}{r_{\text{SE}}^2} = \dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\text{ kg})}{(1.496\times10^{11}\text{ m})^2} = 5.96\times10^{-3}\text{ N/kg}$

23.2. (*a*) $g_{\text{Ep}} = \dfrac{F_{\text{Ep}}^G}{m_{\text{p}}} = \dfrac{Gm_{\text{E}}}{r_{\text{Ep}}^2} = \dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{(6.378\times10^{6}\text{ m})^2} = 9.79\text{ m/s}^2$

(*b*) $g_{\text{Es}} = \dfrac{F_{\text{Es}}^G}{m_{\text{s}}} = \dfrac{Gm_{\text{E}}}{r_{\text{Es}}^2} = \dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{(6.528\times10^{6}\text{ m})^2} = 9.34\text{ m/s}^2$

(*c*) $g_{\text{EM}} = \dfrac{F_{\text{EM}}^G}{m_{\text{M}}} = \dfrac{Gm_{\text{E}}}{r_{\text{EM}}^2} = \dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\text{ kg})}{(3.844\times10^{8}\text{ m})^2} = 2.69\times10^{-3}\text{ m/s}^2$

23.3.

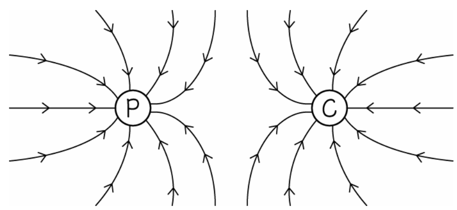


23.4. No, for two reasons. First, the two fields have different units (Newtons per kilogram and Newtons per Coulomb) and so cannot be added. Second, the magnitude of the gravitational force exerted on the proton is many orders of magnitude smaller than the magnitude of the electric force; for all practical purposes, the gravitational field magnitude can be ignored in such a problem.

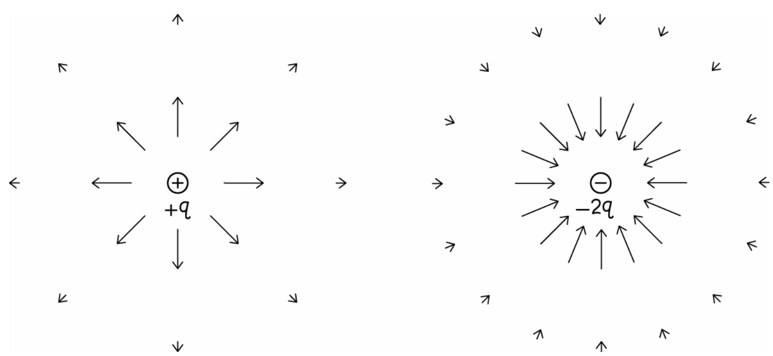
23.5. No, because the kilogram portion of the unit *newton* is cancelled in gravitational-field units but not in electric-field units. Gravitational: $\mathrm{N/kg} = (\mathrm{kg \cdot m/s^2})/\mathrm{kg} = \mathrm{m/s^2}$; electric: $\mathrm{N/C} = (\mathrm{kg \cdot m/s^2})/\mathrm{C}$.

23.6. $E_1 = \dfrac{F_{12}^{\mathrm{E}}}{q_2} = \dfrac{(2.5\times10^{-20}\ \mathrm{N})}{(1.60\times10^{-19}\ \mathrm{C})} = 0.16\ \mathrm{N/C}$

23.7.

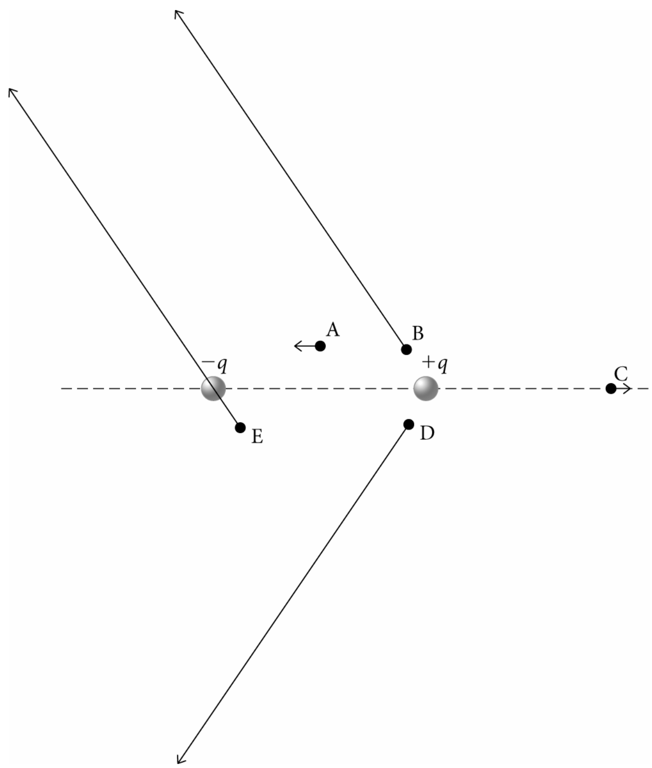


23.8.



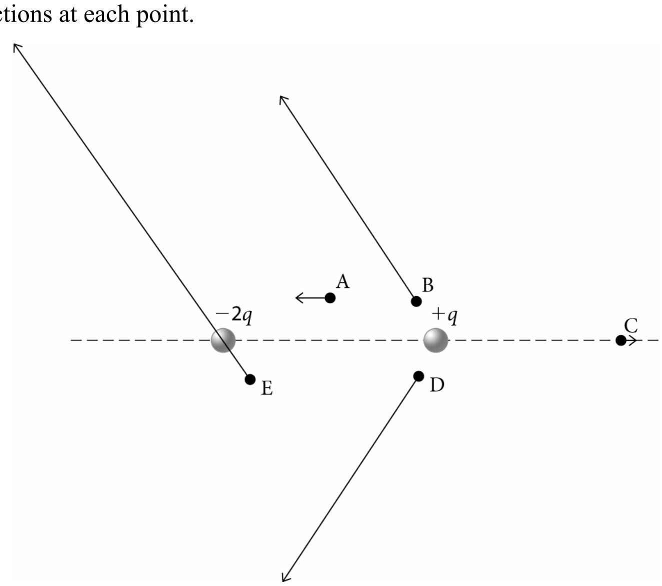
The principal differences are as follow: The field line density in the –2*q* diagram is twice that in the +*q* diagram; at comparable positions, vectors in the –2*q* diagram twice as long as those in the +*q* diagram; and vectors point away from the particle in the +*q* diagram but toward the particle in the –2*q* diagram.

23.9. At point A, the electric field due to the negative charge will point downward and to the left. The electric field due to the positive charge will point upward and to the left. Because A is roughly the same distance from the positive and negative charge, we expect the vertical components of the fields will cancel, leaving a total electric field that points to the left. We proceed in this way, finding approximate directions at each point.

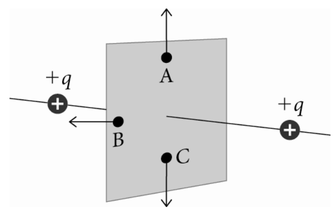


23.10. Each tiny charged particle on the rod will produce an electric field that points radially outward from that point. Particles near the top of the rod will create an electric field at the center of the circle that is downward and to the right. Particles near the bottom of the rod will create an electric field at the center of the circle that is upward and to the right. The vertical components of the electric field will cancel and the horizontal components will add. By symmetry, we can see the electric field will point away from the center of the arc, which is to the right in the figure.

23.11. At point A, the electric field due to the negative charge will point downward and to the left. The electric field due to the positive charge will point upward and to the left. A is roughly the same distance from the positive and negative charge, but the negative charge is twice as large in magnitude as the positive charge. Thus, we expect a downward vertical component of the fields and a large horizontal component to the left. We proceed in this way, finding approximate directions at each point.



23.12. At point A, the positively charge particle in front of the plane produces an electric field that has a component into the page, and a component toward the top of the page. The positively charged particle behind the plane produces a field at A that has a component out of the page, and a component toward the top of the page. The components into and out of the page cancel because the two charged objects carry the same charge and are the same distance from the plane. The components pointing toward the top of the page add. Thus the total electric field at point A points toward the top of the page. We proceed in this manner, finding the approximate direction of the field at each point.



23.13. All charges have the same magnitude, and all charges are the same distance from the center of the square. So the electric field magnitude at the center of the square due to each individual charge will always be the same; call this electric field magnitude E_q. We only need to consider how the fields add in each case. In (*a*), the total field is simply $E_\text{a} = E_q$. In (*b*), the vertical components of the fields due to each charge will cancel, and the horizontal components will add, such that the electric field magnitude will be $E_\text{b} = 2E_q \cos(45°) = \sqrt{2}E_q$. In (*c*), charges opposite each other along the square diagonal will cause electric fields at the center in opposite directions. The field will cancel completely. $E_\text{c} = 0$. In (*d*) The fields due to the charge in the lower left corner and the upper right corner will cancel, leaving only the field due to the charge in the lower right corner. Thus $E_\text{d} = E_q$. Finally, in (*e*) the electric fields due to the positive charge in the lower left and due to the negative charge in the upper right will add, giving a field of $2E_q$. Perpendicular to that, there will also be a field due to the charge in the lower right corner. We use the Pythagorean Theorem to find $E_\text{e} = \sqrt{5}E_q$. Combining these results, we can write $|E|_\text{c} < |E|_\text{a} = |E|_\text{d} < |E|_\text{b} < |E|_\text{e}$.

23.14. Let us express this linear dependence of charge on position around the ring as $q(\theta) = q_\text{i} + k\theta$, where $q(\theta)$ is the charge on a small section of the ring at position θ. The charge at two points opposite each other will produce electric fields at the ring's center that are directly opposite each other. Consider one particular section of the ring at angular position θ_1, and the point opposite to it, at angular position $\theta_1 + \pi$. The magnitude of the resulting electric field will be $E(\theta_1) - E(\theta_1 + \pi)$. Because the electric field strength is proportional to charge, we expect the electric field due to these two charged regions to be proportional to $q(\theta) - q(\theta + \pi) = -k\pi$. Note that this proportionality is totally independent of the location of θ_1. Thus, the electric field due to every opposite pair will yield the same magnitude field at the center (call it E_pair), and that field will always be directed toward the point in the pair of lower angular position. To get the entire field, we would need to add up all of these pair-fields. We would only integrate these fields from positions between 0 and π, because we have already taken into account the positions between π and 2π in the pairs. These fields $\vec{E}_\text{pair}$ will have vertical components that will cancel out the vertical components from other pairs, but the horizontal components will add. Thus the electric field at the center points to the right.

23.15. The gravitational field in this room is very uniform. The field due to Earth is very uniform because all points in the room are the same distance from Earth's center to a very good approximation. The fields due to other objects are extremely small, and cause negligible non-uniformity.

23.16. (*a*) If these particles are constrained to one dimension (such as sliding along a wire) then this equilibrium is stable. If the particles are in two or three dimensional space, then the equilibrium is unstable since the central particle will slip to one side and be pushed arbitrarily far away. (*b*) This is unstable equilibrium regardless of dimensionality. The slightest movement of the particle to either side would cause it to be more strongly attracted to the charge on that side.

23.17. The electric field due to other charged particles in the box does not change. So, inserting the new charge will reduce the force magnitude and cause it to switch directions. We can calculate the new component in each direction:

$$\frac{F^{E}_{\text{north, i}}}{q_{\text{i}}} = \frac{F^{E}_{\text{north, f}}}{q_{\text{f}}} \Rightarrow F^{E}_{\text{north, f}} = \frac{q_{\text{f}}}{q_{\text{i}}} F^{E}_{\text{north, i}} = \frac{(-50\text{ nC})}{(120\text{ nC})}(1.2\times10^{-3}\text{ N}) = -5.0\times10^{-4}\text{ N}$$

$$\frac{F^{E}_{\text{east, i}}}{q_{\text{i}}} = \frac{F^{E}_{\text{east, f}}}{q_{\text{f}}} \Rightarrow F^{E}_{\text{east, f}} = \frac{q_{\text{f}}}{q_{\text{i}}} F^{E}_{\text{east, i}} = \frac{(-50\text{ nC})}{(120\text{ nC})}(5.7\times10^{-4}\text{ N}) = -2.4\times10^{-4}\text{ N}$$

$$\frac{F^{E}_{\text{up, i}}}{q_{\text{i}}} = \frac{F^{E}_{\text{up, f}}}{q_{\text{f}}} \Rightarrow F^{E}_{\text{up, f}} = \frac{q_{\text{f}}}{q_{\text{i}}} F^{E}_{\text{up, i}} = \frac{(-50\text{ nC})}{(120\text{ nC})}(2.2\times10^{-4}\text{ N}) = -9.2\times10^{-5}\text{ N}$$

Thus $\vec{F} = 5.0\times10^{-4}\text{ N south}, 2.4\times10^{-4}\text{ N west}, 9.2\times10^{-5}\text{ N downward.}$

23.18. (*a*) Electric fields point in the direction in which a positive charge would feel a force, and opposite the direction that a negative charge would feel a force. In order for the electron to have been deflected downward, the electric field between the plates must have pointed upward. A proton, having a positive charge, should be deflected in the direction of the electric field, and should thus be deflected vertically upward. (*b*) The electric field still points upward, so the negatively-charged electron will still be deflected vertically downward. (*c*) The electric field still points upward, so the positively-charged proton will still be deflected vertically upward. (*d*) The electric field still points upward, so the positively-charged proton will still be deflected vertically upward.

23.19. Choose the $+y$ axis to point vertically upward. The sum of all forces acting on the oil drop in the y direction must be zero, so we write

$$\sum F_{\text{oil }y} = F^{G}_{\text{E oil }y} + F^{E}_{\text{field oil }y} = -mg + E_y q = 0 \Rightarrow E_y = \frac{mg}{q} = \frac{(30.0\times10^{-6}\text{ kg})(9.8\text{ m/s}^2)}{(3.5\times10^{-6}\text{ C})} = 84\text{ N/C}$$

So $\vec{E} = 84\text{ N/C}$ vertically upward.

23.20. It is most likely that the dipoles will align in the same direction along the axis joining their centers. Because the dipole on the left is more massive than the dipole on the right, it is most likely that the right dipole will flip over.

23.21. Call $\Delta t_{\text{p}} = \dfrac{\Delta x_{\text{p}}}{v_{\text{p}\,x,\text{i}}}$ the time required for the proton to strike the plate. From the geometric information, we can determine the acceleration of the proton: $\Delta z_{\text{p}} = v_{\text{p}\,z,\text{i}}\Delta t + \frac{1}{2}a_{\text{p}\,z}\Delta t_{\text{p}}^2 \Rightarrow a_{\text{p}\ z} = \dfrac{2(\Delta z_{\text{p}})v^2_{\text{p}\,x,\text{i}}}{(\Delta x_{\text{p}})^2}$. A totally equivalent statement is $m_{\text{p}}a_{\text{p}\,z} = \dfrac{2m_{\text{p}}(\Delta z_{\text{p}})v^2_{\text{p}\,x,\text{i}}}{(\Delta x_{\text{p}})^2}$. Exactly the same process allows us to write a similar statement for the electron $m_{\text{e}}a_{\text{e}\,z} = \dfrac{2m_{\text{e}}(\Delta z_{\text{e}})v^2_{\text{e}\,x,\text{i}}}{(\Delta x_{\text{e}})^2}$. Because the charge on a proton is equal in magnitude and opposite in sign to the charge on an electron, we know that the forces experienced by the two particles between the plates will be equal in magnitude and opposite in direction. Thus we can say $\vec{F}^{E}_{\text{on p}} = \vec{F}^{E}_{\text{on e}} \Rightarrow m_{\text{p}}a_{\text{p}\,z} = -m_{\text{e}}a_{\text{e}\,z}$. This allows us to write $\dfrac{2m_{\text{p}}(\Delta z_{\text{p}})v^2_{\text{p}\,x,\text{i}}}{(\Delta x_{\text{p}})^2} = -\dfrac{2m_{\text{e}}(\Delta z_{\text{e}})v^2_{\text{e}\,x,\text{i}}}{(\Delta x_{\text{e}})^2}$. We are told that the initial speed of the two particles is the same, and we know that if the proton struck the top plate, the electron will strike the bottom plate. So $\Delta z_{\text{e}} = -\Delta z_{\text{p}}$. This allows us to write $\Delta x_{\text{e}} = \sqrt{\dfrac{m_{\text{e}}}{m_{\text{p}}}}\Delta x_{\text{p}} = \sqrt{\dfrac{(9.11\times10^{-31}\text{ kg})}{(1.67\times10^{-27})}}(200\text{ mm}) = 4.7\text{ mm}$. Thus the electron strikes at the position. (4.7 mm, 0.0 mm, 0.0 mm).

23.22. Statement (*a*) is false. The electric field at the position of A has not doubled. Statement (*b*) is not exactly correct. The increase in electric force on A is not a direct result of the increase in field at position B. In that sense the statement is false. However, the field at B only changes because the magnitude of charge A is increased. Increasing the charge on A is directly responsible for the increase in electric force. In that indirect sense one could argue this statement is true. Statement (*c*) is the most correct.

23.23. $E = \dfrac{kq}{r^2} = \dfrac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0\times 10^{-6}\ \text{C})}{(0.200\ \text{m})^2} = 6.8\times 10^5\ \text{N/C}$

23.24. The magnitude of the electric field is given by $E = \dfrac{k\,|q|}{r^2} = \dfrac{kN_\text{e}\,|q_\text{e}|}{x^2+y^2} = \dfrac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.30\times 10^4)(1.60\times 10^{-19}\ \text{C})}{(2.00\times 10^{-3}\ \text{m})^2 + (1.00\times 10^{-3}\ \text{m})^2} = 9.5\ \text{N/C}.$ The direction of the electric field is toward the origin, meaning $\theta = \tan^{-1}\left(\dfrac{E_y}{E_x}\right).$ This angle can also be obtained from the spatial geometry. Mathematically, $\theta = \tan^{-1}\left(\dfrac{1.00\ \text{mm}}{2.00\ \text{mm}}\right) = 26.6^\circ$ or $207^\circ.$ But physically, we know the only valid angle is 207° counterclockwise from $+\hat{x}.$ Thus $\vec{E} = 9.5\ \text{N/C at } 207^\circ$ couterclockwise from $+\hat{x}.$

23.25. (*a*) The field from each proton is the same in magnitude and they are in opposite directions. Thus $\vec{E} = \vec{0}.$
(*b*) Let the $+x$ axis point from A to B. We write the sum of all *x* components of the electric field.

$$\sum E_x = E_{\text{A}x} + E_{\text{B}x} = E_\text{A} - E_\text{B} = \frac{kq}{r_\text{A}^2} - \frac{kq}{r_\text{B}^2} = kq\left(\frac{16}{d^2} - \frac{16}{9d^2}\right) = \frac{16(9.0\times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-9}\ \text{C})}{(9.00\times 10^{-6}\ \text{m})^2}\left(1 - \frac{1}{9}\right) = 2.5\times 10^2\ \text{N/C}$$

Thus $\vec{E} = 2.5\times 10^2\ \text{N/C}$ from A to B.

23.26. Choose the +*y* axis to point vertically upward. We write the sum of all force components acting on the particle in the *y* direction.

$$\sum F_{\text{p}y} = F^G_{\text{Ep}y} + F^\text{E}_{\text{field p}\,y} = -mg + qE_y = ma \Rightarrow E_y = \frac{m}{q}(a+g) = \frac{1\,\text{kg}}{0.100\ \text{C}}\left(\frac{(100\ \text{m/s})}{(2.00\ \text{s})} + (9.8\ \text{m/s}^2)\right) = 6.0\times 10^2\ \text{N/C}$$

$\vec{E} = 6.0\times 10^2\ \text{N/C}$ upward.

23.27. (*a*) We note first that these particles both carry positive charge, and will therefore repel each other. So the electric force exerted by the charge at the origin on the proton will be in the direction pointing from the origin toward the proton. We calculate the magnitude of this force through $F^E_{\text{op+}} = \dfrac{k\,|q_\text{origin}||q_\text{p+}|}{r^2_\text{origin p+}} = \dfrac{k\,|q_\text{origin}||q_\text{p+}|}{x^2+y^2} = \dfrac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.95\times 10^{-6}\ \text{C})(1.60\times 10^{-19}\ \text{C})}{(4.00\times 10^{-3}\ \text{m})^2 + (3.00\times 10^{-3}\ \text{m})^2} = 4.0\times 10^{-10}\ \text{N}.$ So $\vec{F}^\text{E}_\text{op} = 4.0\times 10^{-10}\ \text{N}$ from the origin to the proton. (*b*) $E_\text{p+} = \dfrac{F^\text{E}_\text{op}}{q_\text{p}} = \dfrac{(4.00\times 10^{-10}\ \text{N})}{(1.60\times 10^{-19}\ \text{C})} = 2.5\times 10^9\ \text{N/C}.$ So $\vec{E}_\text{p+} = 2.5\times 10^9\ \text{N/C}$ directed from the origin to the position of the proton.

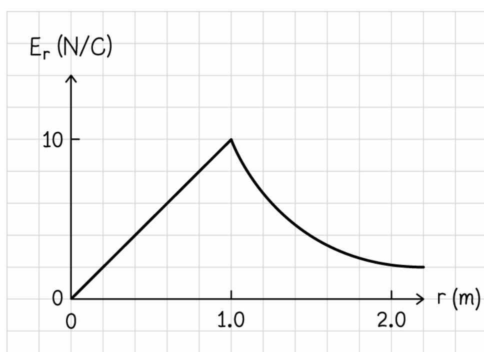
23.28. (*a*) The magnitude of the electric field is given by $E_\text{p} = \dfrac{kq}{r_\text{P}^2} = \dfrac{(9.0\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.89\times 10^{-9}\ \text{C})}{(4.00\times 10^{-3}\ \text{m})^2} = 2.2\times 10^6\ \text{N/C},$ and since this is a positive charge, the electric field points along a line from the origin toward the point specified. Thus $\vec{E}_\text{p} = 2.2\times 10^6\ \text{N/C}$ directed from origin to (4.00 mm, 0).

(*b*) The magnitude of the electric field is given by $E_{\mathrm{p}} = \dfrac{kq}{r_{\mathrm{p}}^2} = \dfrac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(3.89\times10^{-9}\ \mathrm{C})}{(4.00\times10^{-3}\ \mathrm{m})^2} = 2.2\times10^6\ \mathrm{N/C}$, and since this is a positive charge, the electric field points along a line from the origin toward the point specified. Thus $\vec{E}_{\mathrm{p}} = 2.2\times10^6\ \mathrm{N/C}$ directed from origin to (0, 4.00 mm). (*c*) The magnitude of the electric field is given by $E_{\mathrm{p}} = \dfrac{kq}{r_{\mathrm{p}}^2} = \dfrac{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(3.89\times10^{-9}\ \mathrm{C})}{(-2.829\times10^{-3}\ \mathrm{m})^2 + (2.829\times10^{-3}\ \mathrm{m})^2} = 2.2\times10^6\ \mathrm{N/C}$, and since this is a positive charge, the electric field points along a line from the origin toward the point specified. Thus $\vec{E}_{\mathrm{p}} = 2.2\times10^6\ \mathrm{N/C}$ directed from origin to (−2.829 mm, 2.829 mm).

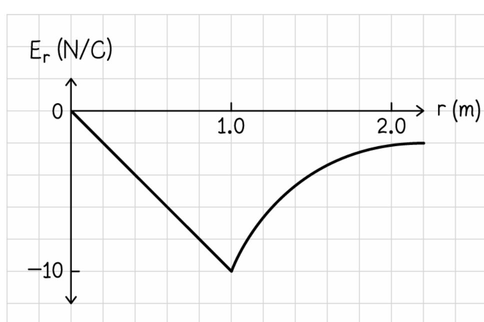
23.29. (*a*) Call the radius of the pellet *R*. For distances $r > R$, the electric field simply decreases like $1/r^2$. Inside the pellet, the behavior is not so obvious. Let us imagine that we place a charged test particle at a particular point $\vec{r}_1$ inside the pellet, and let us consider the forces on that test particle. Imagine now that the pellet is made up of an inner sphere where $r < r_1$, and an outer spherical shell with $r_1 < r < R$. You know from Newton's and Priestley's argument that the test particle will not experience any force due to the spherical shell, because it is inside this spherical shell. Further, if the test particle experiences no force due to the outer shell, then there must be no electric field due to the outer shell. Thus, once inside the shell, there will only be an electric field due to the charge at radial distances $r < r_1$.

At a particular r_1, the charge in the sphere with $r < r_1$ is given by $q_{r<r_1} = q_{\mathrm{total}}\left(\dfrac{V_{r<r_1}}{V_{\mathrm{total}}}\right) = q_{\mathrm{total}}\left(\dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi R^3}\right) = q_{\mathrm{total}}\left(\dfrac{r_1}{R}\right)^3$.

The electric field due to this charge, at position $\vec{r}_1$ is $E_{r_1} = \dfrac{kq_{r<r_1}}{r_1^2} = \dfrac{kq_{\mathrm{total}}}{R^3}r_1$. We used r_1 to refer to a particular point, but this is valid for any point inside the pellet. This means that inside the pellet, the electric field increases linearly with *r*. This will become more clear in Chapter 24.



(*b*)



23.30. We can use the magnitude of the acceleration to determine the magnitude of the charge on the sphere by considering the force exerted by the sphere on the electron. $F_{\text{se}}^{E}=\dfrac{k\,|q_{\text{s}}||q_{\text{e}}|}{r_{\text{se}}^{2}}=m_{\text{e}}a_{\text{e}}\Rightarrow |q_{\text{s}}|=\dfrac{m_{\text{e}}a_{\text{e}}r_{\text{se}}^{2}}{k\,|q_{\text{e}}|}=$ $\dfrac{(9.11\times10^{-31}\ \text{kg})(4.0\times10^{7}\ \text{m/s}^2)(0.10\ \text{m})^2}{(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})}=2.5\times10^{-16}\ \text{C}.$

23.31. We require that the electric fields due to the two charged particles cancel each other. Call the particle carrying 6.0 nC particle 1, and the particle carrying 3.0 nC of charge particle 2. Call the line connecting the two particles the x axis, and call the distance from particle 1 to the point where the fields cancel x_1. In order for the fields to cancel each other, they must be directed opposite each other, and this is only the case for points between the two particles. For those particles, the total electric field can be written as $E_{\text{p}}=\dfrac{k\,|q_1|}{x_1^2}-\dfrac{k\,|q_2|}{(d-x_1)^2}=0\Rightarrow x_1=$ $\pm\sqrt{\dfrac{q_1}{q_2}}(d-x_1)\Rightarrow x_1=\dfrac{d\sqrt{q_1}}{\sqrt{q_1}\pm\sqrt{q_2}}$. This appears as though there are two solutions. But plugging in numbers reveals that the two positions are $x_1=59\ \text{mm}$ and $341\ \text{mm}$. The latter position is not between the two charges as we required. Here we have found that the electric fields due to each particle have the same magnitude at the position $x_1=341\ \text{mm},$ but they also have the same direction at that point. They only cancel at the point 59 mm from the particle with 6.0 nC of charge (or equivalently 41 mm from the particle with 3.0 nC of charge).

23.32. (*a*) Call the particle that carries $+q$ particle 1, and call the particle that carries $+4q$ particle 2. Let us choose our origin to be on particle 1, and let the line pointing from paricle 1 to particle 2 be the $+x$ axis. In order for the electric fields from these two particles to cancel each other, the fields must be pointing in opposite directions. This is only the case for points between the two particles. With that restriction, we write the difference of the two fields a distance x_1 from particle 1.

$$E_{\text{p}}=\frac{k\,|q_1|}{x_1^2}-\frac{k\,|q_2|}{x_2^2}=\frac{kq}{x_1^2}-\frac{4kq}{(d-x_1)^2}=0\Rightarrow x_1=\pm\frac{1}{2}(d-x_1)\Rightarrow x_1=\frac{d}{3}\ \text{or}\ x_1=-d$$

The latter solution is not between the two particles, so the fields cannot actually cancel there. What we have found is that the fields from each particle have the same magnitude at the position $x_1=-d,$ but since they also have the same direction there, this point is not useful to us. The point where the fields can actually cancel is $d/3$ from the particle carrying $+q$, or equivalently $2d/3$ from the particle carrying $+4q$. (*b*) No, no other such point exists.

23.33. (*a*) Choose the $+y$ direction to point vertically upward. We are not told whether the field points upward or downward. If the electric force were the only force acting on the ink droplet, the direction would not matter in terms of deflection magnitudes. But since gravity may also act on the droplet, we are forced to make an assumption about the relative directions of the gravitational and electric forces. For now, assume that the electric field is directed vertically upward, and we return to the question of the gravitational force in part (*c*). While the charged ink droplet is between the plates, it experiences an electric force and a gravitational force such that $\sum F_y=F_{\text{plates ink }y}^{E}+F_{\text{E ink }y}^{G}=qE_{\text{plates}}-m_{\text{ink}}g=m_{\text{ink}}a_{\text{ink},y}$. Thus we can equate $a_{\text{ink},y}=\dfrac{qE_{\text{plates}}-m_{\text{ink}}g}{m_{\text{ink}}}$. Now we can use kinematics in the x and y directions to find

$$v_{\text{ink},x}=\frac{\ell}{\Delta t}\Rightarrow \Delta t=\frac{\ell}{v_{\text{ink},x}} \tag{1}$$

$$\Delta y=v_{\text{ink},y,\text{i}}\Delta t+\frac{1}{2}a_{\text{ink},y}\ \Delta t^2 \tag{2}$$

Inserting the results of equation (1) into equation (2), and the fact that the initial velocity of the ink is purely in the $\hat{x}$ direction, we find $\Delta y=\dfrac{1}{2}a_{\text{ink,y}}\left(\dfrac{\ell}{v_{\text{ink},x}}\right)^2$. Inserting our expression for the acceleration, we find $\Delta y=$

$\frac{1}{2}\left(\frac{qE_{\text{plates}}-m_{\text{ink}}g}{m_{\text{ink}}}\right)\left(\frac{\ell}{v_{\text{ink},x}}\right)^2$, or in terms of the variables given, $\Delta y=\frac{(Eq-mg)\ell^2}{2mv^2}$. (*b*) $q=\frac{1}{E}\left(\frac{2m\Delta yv^2}{\ell^2}+mg\right)=$ $\left(\frac{1}{(1.2\times10^{6}\text{ N/C})}\right)\left(\frac{2(1.5\times10^{-10}\text{ kg})(1.3\times10^{-3}\text{ m})(20\text{ m/s})^2}{(10\times10^{-3}\text{ m})^2}+(1.5\times10^{-10}\text{ kg})(9.8\text{ m/s}^2)\right)=1.3\times10^{-12}$. (*c*) From the expressions used in part (*a*), we can see that $|\vec{F}^G|=1.47\times10^{-9}$ N and $|\vec{F}^C|=1.56\times10^{-6}$ N. The gravitational force is three orders of magnitude smaller than the electric force and can be ignored for most purposes.

23.34. (*a*) Choose the +*y* direction to point vertically upward. We write the sum of all *y* components of forces, and require that sum to be zero.

$$\sum F_y=F^{\text{E}}_{\text{field e}\,y}+F^{G}_{\text{Ee}\,y}=E_yq_e-m_\text{e}g=0\Rightarrow E_y=\frac{m_\text{e}g}{q_\text{e}}=\frac{(9.11\times10^{-31}\text{ kg})(9.8\text{ m/s}^2)}{(-1.60\times10^{-19}\text{ C})}=-5.6\times10^{-11}\text{ N/C}$$

So the required magnitude of the electric field is 5.6×10^{-11} N/C. (*b*) We require that $E=\frac{kq_\text{p}}{r_\text{ep}^2}\Rightarrow r_\text{ep}=$ $\pm\sqrt{\frac{kq_\text{p}}{E}}=\pm\sqrt{\frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})}{(5.58\times10^{-11}\text{ N/C})}}=\pm5.1\text{ m}$. Of course, because the electron is attracted to the proton, we must place the proton 5.1 m above the electron.

23.35. The distance from any corner to the point in question is $r=\sqrt{a^2+x^2}$. The electric field magnitude due to any one of the charged particles is $|\vec{E}_1|=\frac{kq}{r^2}$ and this field will have a component in the *x* direction and a component along the body diagonal. Call the angle between the *x* axis and this electric field *θ*. Components along the diagonal will cancel pairwise, so only the component along *x* is required. $E_{1x}=E_1\cos(\theta)$ and the angle can be determined by the geometry of the setup: $\cos(\theta)=x/r$. So $E_{1x}=E_1x/r=kqx/r^3$. We have four such charged particle contributing to the field so we have as the total field $\vec{E}=\frac{4kqx}{(x^2+a^2)^{3/2}}\hat{x}$.

23.36. (*a*) Call the particle at the origin particle 1, and the other particle 2. Clearly the electric field at the point $(5.0\text{ m},\ 0)$, which we will call point *a*, due to particle 1 will be entirely along the +*x* axis and the electric field due to particle 2 will have a component in the +*x* and a component in the –*y* direction. Call the angle between the electric field at point *a* due to particle 2 and the +*x* axis θ_2. We write the sum of the *x* and *y* components of the electric field.

$$E_x=E_{1x}+E_{2x}=E_1+E_2\cos(\theta_2)=\frac{k|q_1|}{r_1^2}+\frac{k|q_2|}{r_2^2}\cos(\theta_2)$$
$$=\frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(6.0\times10^{-6}\text{ C})}{(5.0\text{ m})^2}+\frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(4.0\times10^{-6}\text{ C})}{(5.0\text{ m})^2+(5.0\text{ m})^2}\cos(45°)=2.67\times10^{3}\text{ N/C}$$
$$E_y=E_{1y}+E_{2y}=(0)-E_2\sin(\theta_2)=-\frac{k|q_2|}{r_2^2}\sin(\theta_2)=-\frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(4.0\times10^{-6}\text{ C})}{(5.0\text{ m})^2+(5.0\text{ m})^2}\sin(45°)=-5.09\times10^{2}\text{ N/C}$$

Combining these components using the Pythagorean theorem and simple trigonometry, we find $\vec{E}=2.7\times10^{3}$ N/C at 349° counterclockwise from $+\hat{x}$. (*b*) By symmetry, we can see that the electric field must be the mirror image (reflection across the *y* axis) of the electric field we found in part (*a*). Thus $\vec{E}=2.7\times10^{3}$ N/C at 191° counterclockwise from $+\hat{x}$. (*c*) Here it is clear that the electric fields due to each particle are both entirely along the –*y* axis. We need only carry out the sum of *y* components of electric fields.

$$E_y=E_{1y}+E_{2y}=-E_1-E_2=-\frac{k|q_1|}{r_1^2}-\frac{k|q_2|}{r_2^2}$$
$$=\frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(6.0\times10^{-6}\text{ C})}{(5.0\text{ m})^2}-\frac{(9.0\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(4.0\times10^{-6}\text{ C})}{(10.0\text{ m})^2}=2.5\times10^{3}\text{ N/C}$$

Thus $\vec{E}=-2.5\times10^{3}$ N/C directed along the –*y* axis.

23.37. (*a*) Start at the bottom of the half-ring and number the charged particles 1-5. By symmetry, it is clear that the *y* components of the electric fields due to charges opposite each other across the *x* axis will cancel at the origin (meaning $E_{1y} = -E_{5y}$ and so on). The *x* components, however, will add. Let θ_n be the magnitude of the angle between the +*x* axis and the electric field at the center of the half-ring due to particle *n*. Then the sum of all *x* components of electric fields is

$$E_x = E_{1x} + E_{2x} + E_{3x} + E_{4x} + E_{5x} = E_1\cos(\theta_1) + E_2\cos(\theta_2) + E_3\cos(\theta_3) + E_4\cos(\theta_4) + E_5\cos(\theta_5)$$

Since all the particles are the same distance from the origin, and since all particles carry the same charge, we can write the above sum as

$$E_x = \frac{kq}{R^2}\left(\cos(\theta_1) + \cos(\theta_2) + \cos(\theta_3) + \cos(\theta_4) + \cos(\theta_5)\right) = \frac{kq}{R^2}\left(\cos(90°) + \cos(45°) + \cos(0) + \cos(45°) + \cos(90°)\right)$$

$$E_x = \frac{kq}{R^2}\left(\sqrt{2}+1\right) = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.00\times10^{-9}\ \text{C})}{(0.100\ \text{m})^2}\left(\sqrt{2}+1\right) = 2.2\times10^3\ \text{N/C}$$

So $\vec{E} = 2.2\times10^3\ \text{N/C}$ along the +*x* axis. (*b*) Clearly, in order for the negative charge to cause an electric field in the $-\hat{x}$ direction, the negatively charged particle must be placed to the left of the origin, and in order for it to produce no *y* component to the electric field, it must lie on the –*x* axis. Using the result from part (*a*), we can write the sum of all *x* components of the electric field in terms of the unknown position of the negative charge, x_n.

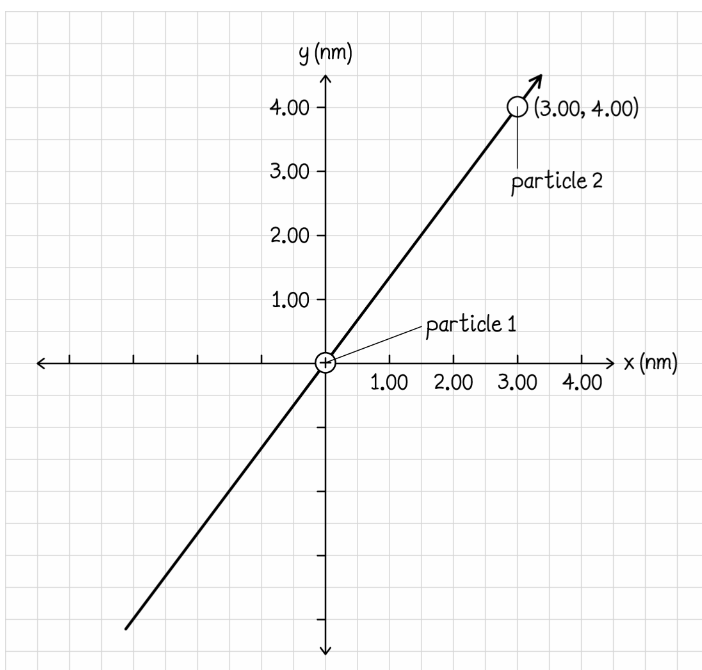
$$E_x = E_{\text{ring}\,x} + E_{\text{n}\,x} = E_{\text{ring}} - E_{\text{n}} = 0 \Rightarrow \frac{k\,|\,q_n\,|}{x_n^2} = E_{\text{ring}}$$

or equivalently

$$x_n = \pm\sqrt{\frac{k\,|\,q_{\text{n}}\,|}{E_{\text{ring}}}} = -\sqrt{\frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.0\times10^{-9}\ \text{C})}{(2.17\times10^3\ \text{N/C})}} = -0.14\ \text{m}$$

Thus we place the negatively charged particle at $(x, y) = (-0.144\ \text{m},\ 0)$.

23.38. (*a*)



$\vec{E}_1$. is directed along dashed line. Any particle 2 that cancels $\vec{E}_1$ must lie along that dashed line so that $\vec{E}_1$ and $\vec{E}_2$ can be directed opposite each other at the point in question. Charge on particle 2 must be negative if particle placed anywhere below (3.00 nm, 4.00 nm) on dashed line and positive if particle placed anywhere above (3.00 nm, 4.00 nm) on dashed line. The closer particle 2 is to t (3.00 nm, 4.00 nm), the smaller the charge magnitude q_2 required to have $E_{(3.00\text{ nm, }4.00\text{ nm})} = 0$. (*b*) We have established in part (*a*) that the second charged particle must be along the diagonal line. If the particle 2 is to hold the same charge as particle 1, then for the two fields to cancel each other at the point (3.00 nm, 4.00 nm), the two particles must clearly be equidistant from that point. Thus we place particle two at (6.00 nm, 8.00 nm).

23.39. (*a*) It is straightforward to write the sum of electric field components due to each charged particle and find that the electric field at the center is zero. However, we solve this problem using symmetry. Let the triangle be oriented such that one edge lies along the *x* axis. Call the particle at the top vertex particle 1, and proceed numbering the particles counterclockwise. Because this triangle is symmetric about a vertical line passing through particle 1, there cannot be any electric field component in the *x* direction. The only possibility for a non-zero electric field is $\vec{E} = E_y\hat{j}$. Now imagine that we rotate the triangle counterclockwise by 120 degrees. Nothing about the geometry of charge has changed, and therefore the electric field must still be $\vec{E} = E_y\hat{j}$. But this field pointed toward particle 1, meaning it now points $210°$ from the +*x* axis. The only way for the electric field to be both $\vec{E} = E_y\hat{j}$ and $\vec{E} = E_y\cos(210°)\hat{i} + E_y\sin(210°)\hat{j}$ is if $E_y = 0$. Thus at the center of the triangle $\vec{E} = \vec{0}$. (*b*) Let us specifically look at the center of the side connecting particles 2 and 3. Particles 2 and 3 are equidistant from this point and carry the same charge. Their fields at this midpoint with thus cancel, and we will be left only with the downward electric field from particle 1. Simple geometry shows that the distance from particle 1 to this midpoint is $r_{1\text{m}} = \frac{\sqrt{3}}{2}a$, such that the electric field magnitude due to particle 1 at this midpoint is $E_{1\text{m}} = \frac{kq_1}{r_{1\text{ m}}^2} = \frac{4kq}{3a^2}$. Because all midpoints of sides are identical in the equilateral triangle, we can say $\vec{E}_{\text{midpoint}} = \frac{4kq}{3a^2}$ perpendicular to the side, away from the center of the triangle. (*c*) The electric field components in the *x* direction due to particles 2 and 3 will cancel, and there is no *x* component to the electric field due to particle 1. So we need only consider the electric field components in the *y* direction. Note that the magnitude of the angle between the field due to particles 2 or 3 and the vertical is given by

$$\theta = \tan^{-1}\left(\frac{a/2}{a\left(1+\frac{\sqrt{3}}{2}\right)}\right) = 15°$$

And the distance from either particle 2 or 3 to this point above the vertex is

$$d = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\left(1+\frac{\sqrt{3}}{2}\right)a\right)^2} = a\sqrt{2+\sqrt{3}} \text{ or } 1.932a$$

We write the sum of all *y* components of the electric fields at the point a distance *a* above the vertex:

$$E_y = E_{1y} + E_{2y} + E_{3y} = \frac{kq}{a^2} + 2\left(\frac{kq}{\left(\sqrt{2+\sqrt{3}a}\right)^2}\right)\cos(15°) = \left(1+\frac{1}{\sqrt{2+\sqrt{3}}}\right)\frac{kq}{a^2}$$

Thus $\vec{E} = \left(1+\frac{1}{\sqrt{2+\sqrt{3}}}\right)\frac{kq}{a^2}$ away from the center of the triangle. One might also write the magnitude as $1.52\frac{kq}{a^2}$.

23.40. (*a*) The magnitude of the electric force is given by $F_{12}^{E} = \frac{kq^2}{r_{12}^2} \Rightarrow q = \sqrt{\frac{F_{12}^{\mathrm{E}} r_{12}^2}{k}} = \sqrt{\frac{(0.10\ \mathrm{N})(0.050\ \mathrm{m})^2}{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})}} = 1.7\times 10^{-7}\ \mathrm{C}$. (*b*) Choose the $+\hat{y}$ direction to point upward and the $+\hat{x}$ to point to the right. Let θ_2 be the angle between the electric field due to sphere 2, and the $+\hat{x}$ axis. We write the sum of all electric field components:

$$E_x = E_{1x} + E_{2x} = 0 + E_2\cos(\theta_2) = \frac{k|q_2|}{r_2^2}\cos(\theta_2) = \frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})(1.67\times 10^{-7}\ \mathrm{C})}{2(0.050\ \mathrm{m})^2}\cos(45°) = 2.1\times 10^5\ \mathrm{N/C}$$

$$E_y = E_{1y} + E_{2y} = E_1 + E_2\ \sin(\theta_2) = \frac{k|q_1|}{r_1^2} + \frac{k|q_2|}{r_2^2}\sin(\theta_2)$$

$$= \frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})(1.67\times 10^{-7}\ \mathrm{C})}{(0.050\ \mathrm{m})^2} + \frac{(9.0\times 10^9\ \mathrm{N\cdot m^2/C^2})(1.67\times 10^{-7}\ \mathrm{C})}{2(0.050\ \mathrm{m})^2}\sin(45°) = 8.1\times 10^5\ \mathrm{N/C}$$

Combining the above components using the Pythagorean, we find $\vec{E} = 8.4\times 10^5\ \mathrm{N/C}$. (*c*) Since the spheres are insulating, if the charge distribution is uniform, then the radius does not matter provided the radii do not become so large that the two spheres overlap.

23.41. Call the particle at the origin particle 1 and the other particle 2. Let us define a new coordinate system in which $+\hat{x}'$ points from particle 1 to particle 2, and $+\hat{y}'$ is perpendicular to this. Note for later use that this $+\hat{x}'$ axis makes a $26.6°$ with the original $+\hat{x}$ direction. Because there can be no electric field component along $+\hat{y}'$, we look only for points along the $+\hat{x}'$ axis. Further, we know that the electric field cannot be zero between the two particles, because their electric fields add in that region. Nor can the electric field be zero anywhere farther from the origin than particle 2 along the $\hat{x}'$ axis, because all such points are closer to the charge of greater magnitude; the electric fields could never be equal in size, as required for them to cancel each other. Hence the only possibility is for the electric field to be zero a distance (call it *d*) from the origin in the $-\hat{x}'$ direction. At such a point, we can write:

$$E_{x'} = E_{1x'} + E_{2x'} = E_1 - E_2 = \frac{k|q_1|}{r_1^2} - \frac{k|q_2|}{r_2^2} = \frac{k|q_1|}{d^2} - \frac{k|q_2|}{(d+\Delta x')^2} = 0$$

where $\Delta x'$ is the distance between the two particles along the x' axis. Thus we have

$$\sqrt{\frac{|q_1|}{|q_2|}}(d+\Delta x') = d \Rightarrow d = \frac{\Delta x'\sqrt{\frac{|q_1|}{|q_2|}}}{\left(1-\sqrt{\frac{|q_1|}{|q_2|}}\right)} = \frac{\sqrt{(1.0\ \mathrm{m})^2 + (0.50\ \mathrm{m})^2}\sqrt{\frac{5.0\ \mu\mathrm{C}}{12\ \mu\mathrm{C}}}}{\left(1-\sqrt{\frac{5.0\ \mu\mathrm{C}}{12\ \mu\mathrm{C}}}\right)} = 2.036\ \mathrm{m}$$

We use this distance along the $-\hat{x}'$ axis to determine the coordinates in the original frame. $x = -d\ \cos(\theta) = -(2.036)\cos(26.6°) = -1.8\ \mathrm{m}$, and $y = -d\ \sin(\theta) = -(2.036)\sin(26.6°) = -0.91\ \mathrm{m}$. Thus the position at which the electric field is zero is $(-1.8\ \mathrm{m}, -0.91\ \mathrm{m})$.

23.42. (*a*) Assume the eighth Na^+ ion is located at the origin, and let the $+\hat{x}$, $+\hat{y}$, and $+\hat{z}$ all point directly away from nearest neighbor Cl^- ions. Call the side length of this cubic arrangement $a = 280\ \mathrm{pm}$. Let us consider the x component of the electric field first. The nearest neighbor Cl^- ion in the $-\hat{x}$, direction will produce an electric field of $E_{1x} = -\frac{kq}{a^2}$ at the origin. The other nearest neighbor Cl^- ions will produce electric fields that have no x component. Two of the Na^+ ions will each produce an electric field with an x component of $E_{2x} = \frac{kq}{2a^2}\cos(45°)$ at the origin. Finally, the Cl^- across the body diagonal from the origin will produce an electric field with an x

component of $E_{3x} = -\frac{kq}{3a^2}\left(\frac{1}{\sqrt{3}}\right)$. Thus the total x component of the electric field at the origin is $E_x = E_{1x} + 2E_{2x} + E_{3x} = -\frac{kq}{a^2} + 2\left(\frac{kq}{2a^2}\right)\frac{\sqrt{2}}{2} - \frac{kq}{3a^2}\left(\frac{1}{\sqrt{3}}\right) = -\frac{kq}{a^2}\left(1 - \frac{\sqrt{2}}{2} + \frac{1}{3\sqrt{3}}\right)$.

By symmetry, we know we will obtain exactly the same expression for E_y and E_z. Thus the electric field must be directed along the body diagonal, and its magnitude is given by

$$E = \sqrt{3}E_x = \sqrt{3}\left(1 - \frac{\sqrt{2}}{2} + \frac{1}{3\sqrt{3}}\right)\frac{kq}{a^2} = \sqrt{3}\left(1 - \frac{\sqrt{2}}{2} + \frac{1}{3\sqrt{3}}\right)\frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})}{(280\times10^{-12}\ \text{m})^2} = 1.5\times10^{10}\ \text{N/C}$$

Thus $\vec{E} = 1.5\times10^{10}$ N/C toward the center of the cubic arrangement. (*b*) $F = Eq =$ $(1.54\times10^{10}\ \text{N/C})(1.60\times10^{-19}\ \text{C}) =$ 2.5×10^{-9} N. Thus $\vec{F} = 2.5\times10^{-19}$ N toward the center of the cubic arrangement.

23.43. Treating the bond as ionic yields a dipole moment with a magnitude of $p = q_p r_p =$ $2(1.60\times10^{-19}\ \text{C})(0.058\times10^{-9}\ \text{m}) = 1.86\times10^{-29}\ \text{C}\cdot\text{m}$. This is a factor of 3 times too large to fit experiment. Since the charge of the constituent atoms is known, the only way for the dipole moment to be smaller is for the charge separation to be smaller. Hence the bond is not ionic, but must be covalent.

23.44. $\vec{P} = q_p\vec{r}_p = (1.0\times10^{-10}\ \text{C})(0.600\ \text{m})\hat{r}_\text{p} = 6.0\times10^{-11}\ \text{C}\cdot\text{m}$ pointing from the negative to the positive ball.

23.45. Each dipole moment along a bond will be directed $52.25°$ away from the vertical direction in Figure 23.43 (call this direction $+\hat{y}$). Adding these dipole moments like vectors tells us $p_y = 2p_\text{bond}\cos(\theta) =$ $p_{\text{H}_2\text{O}} \Rightarrow p_\text{bond} = \frac{p_{\text{H}_2\text{O}}}{2\cos(\theta)} = \frac{(6.186\times10^{-30}\ \text{C}\cdot\text{m})}{2\cos(52.25°)} = 5.052\times10^{-30}\ \text{C}\cdot\text{m}$. Thus $\vec{p} = 5.052\times10^{-30}\ \text{C}\cdot\text{m}$ along the bond axis pointing from O to H.

23.46. The electric field due to a small charged object will decrease as the distance from the object increases, like $1/r^2$. So the fact that the electric field at the position of the proton decreased by a factor of 4, means that the distance from the proton to the charged object doubled. The electric field due to a dipole has a different behavior. With the orientation given, as long as the initial distance x_0 from the object to the proton is much larger than the charge separation in the dipole r_p, we can use equation (23.10) to see that the electric field will drop off like $1/r^3$. Thus when the distance doubles, the electric field magnitude will be reduced to 1/8 the original field.

23.47. Far from the origin, we can use equation (23.13) for the electric field due to the dipole. This electric field will be in the same direction as the electric field due to the single proton at the origin. Thus, at either separation, the total electric field at a distance z is $E_z = \frac{2kqd}{z^3} + \frac{kq}{z^2}$. Thus the ratio of the fields is

$$\frac{E(z=10d)}{E(z=20d)} = \frac{\frac{2kqd}{(10d)^3} + \frac{kq}{(10d)^2}}{\frac{2kqd}{(20d)^3} + \frac{kq}{(20d)^2}} = \frac{\frac{2}{1000} + \frac{1}{100}}{\frac{2}{8000} + \frac{1}{400}} = 4.36$$

23.48. From equation (23.12), y only enters in terms that are squared, so the sign of y does not matter. But if $|y| < \frac{d}{2}$, then the two quadratic terms in the second factor diverge, so $|y| > \frac{d}{2}$. More physically, the dipole has an orientation, so you would expect the electric field to mirror that orientation on the *y* axis, and the symmetry of the dipole precludes any difference in the positive or negative *y* direction.

23.49. Label the bottom right particle 1, and then number the particles in counterclockwise order. Clearly the x components of the electric fields due to particles 1 and 2 will cancel each other, as will the x components of the fields due to particles 3 and 4. Hence, we need only consider the y components of the fields. The sum of all y components is

$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y} = -2E_1\left(\frac{d}{2(r-d/2)}\right) + 2E_3\left(\frac{d}{2(r+d/2)}\right)$$

$$E_y = -2\left(\frac{kd}{(r-d/2)^2+(d/2)^2}\right)\left(\frac{d}{2(r-d/2)}\right) + 2\left(\frac{kd}{(r+d/2)^2+(d/2)^2}\right)\left(\frac{d}{2(r+d/2)}\right)$$

$$E_y = \frac{2kq}{r^2}\left[-\left(\frac{1}{(1-d/2r)^2+(d/2r)^2}\right)\left(\frac{d/r}{(2-d/r)}\right) + \left(\frac{1}{(1+d/2r)^2+(d/2r)^2}\right)\left(\frac{d/r}{(2+d/r)}\right)\right]$$

A Taylor expansion of this expression around small d/r yields

$$E_y = \frac{kq}{r^2}\left(-\frac{3d^2}{2r^2} + \text{higher order terms}\right)$$

Thus for $r \gg d,$ we keep only the lowest order and say $E = -\frac{3}{2}\frac{kqd^2}{r^4},$ such that $E \sim \frac{1}{r^4}.$

23.50. (*a*) If the negatively charged sphere experiences a force in the positive y direction, then the electric field at that position is in the negative y direction. This means the dipole must be pointing in the positive y direction. We use equation (23.10):

$$E_y \approx -k\frac{p}{|x^3|} = \frac{kdq_{\text{dipole}}}{|x^3|} \Rightarrow x = \left(\frac{kd|q_{\text{dipole}}|}{|E_y|}\right)^{1/3} = \left(\frac{kd|q_{\text{dipole}}||q_{\text{sphere}}|}{|F_y|}\right)^{1/3}$$

$$= \left(\frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(20\times10^{-3}\ \text{m})(10\times10^{-9}\ \text{C})(3.0\times10^{-9}\ \text{C})}{(200\times10^{-9}\ \text{N})}\right)^{1/3} = 0.30\ \text{m}$$

(*b*) As discussed in part (*a*), the dipole moment points in the $+y$ direction.

23.51. Because the linear charge density is uniform, we can write the magnitude of the total charge on either rod as $q = \lambda\ell.$ The dipole moment is defined as $\vec{p} = q_p\vec{r}_p$ where r_p is the magnitude of the charge separation. In this case the charge centers are the geometric centers of the two rods. Thus, the separation between charge centers is also $\ell.$ Thus $p = q_\text{p}r_\text{p} = (\lambda\ell)(\ell) = \lambda\ell^2.$

23.52. (*a*) Let a Cartesian coordinate system have its origin at the center of a dipole with the y axis oriented along the dipole moment. Call the electric field due to the positively and negatively charged ends $\vec{E}_+$ and $\vec{E}_-,$ respectively. At any position (x, y), Coulomb's law and simple geometry yield

$$\vec{E}_+ = \frac{kq}{x^2+(y-d/2)^2}\ \text{at}\ \theta = \tan^{-1}\left(\frac{y-d/2}{x}\right)\ \text{ccw from}\ +x\ \text{axis}$$

$$\vec{E}_- = \frac{kq}{x^2+(y+d/2)^2}\ \text{at}\ \phi = \tan^{-1}\left(\frac{y+d/2}{x}\right)\ \text{ccw from}\ -x\ \text{axis}$$

The x and y components of the electric field are the sum of the projections of $\vec{E}_+$ and $\vec{E}_-$ onto the x and y axes:

$$E_x = E_+\cos(\theta) - E_-\cos(\phi) = \frac{kq}{x^2+(y-d/2)^2}\frac{x}{\sqrt{x^2+(y-d/2)^2}} - \frac{kq}{x^2+(y+d/2)^2}\frac{x}{\sqrt{x^2+(y+d/2)^2}}$$

$$E_y = E_+\sin(\theta) - E_-\sin(\phi) = \frac{kq}{x^2+(y-d/2)^2}\frac{y-d/2}{\sqrt{x^2+(y-d/2)^2}} - \frac{kq}{x^2+(y+d/2)^2}\frac{y-d/2}{\sqrt{x^2+(y+d/2)^2}}$$

Expanding to first order in d/y yields

$$E_x = E_+ \cos(\theta) - E_- \cos(\phi) = \frac{3kqydx}{(x^2+y^2)^{5/2}} = \frac{3kypx}{(x^2+y^2)^{5/2}}$$

$$E_y = E_+ \sin(\theta) - E_- \sin(\phi) = \frac{kqd(2y^2-x^2)}{(x^2+y^2)^{5/2}} = \frac{kp(2y^2-x^2)}{(x^2+y^2)^{5/2}}$$

(*b*) for $x=0$ these give $E_y = \dfrac{2kp}{y^3}, E_x = 0;$ for $y = 0,$ these give $E_y = \dfrac{-kp}{x^3}, E_x = 0.$

23.53. Inside the sphere the electric field magnitude is zero. Outside the sphere, the electric field is the same as if the charge were located at the center of the spherical balloon. Clearly, the maximum electric field will be found right at the surface of the balloon. Thus we set $E_{\text{max}} = \dfrac{kq_{\text{max}}}{r_{\text{ballon}}^2} \Rightarrow q_{\text{max}} = \dfrac{E_{\text{max}} r_{\text{ballon}}^2}{k} = \dfrac{(100{,}000 \text{ N/C})(1.75 \text{ m})^2}{(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)} = 3.4\times10^{-5} \text{ C}.$ This charge could be negative or positive, since we are only concerned with the magnitude of the electric field here.

23.54. The particle will oscillate back and forth through the center of the ring. This motion will be periodic, but since the force is not exactly proportional to the distance from the particle to the ring, the motion will not be simple harmonic motion.

23.55. Because the point $(40 \text{ mm}, 30 \text{ mm}, 0)$ still lies in the $z = 0$ plane, this point still lies along a perpendicular bisector of the rod. This means we can use the results of Example 23.4 to determine the magnitude of the electric field: $E = \dfrac{kq}{r\sqrt{\ell^2/4 + r^2}}$ where ℓ is the entire length of the rod and r is the entire distance from the rod to the point along the perpendicular bisector. Thus

$$E = \frac{kq}{r\sqrt{\ell^2/4+r^2}} = \frac{(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-8} \text{ C})}{(0.050 \text{ m})\sqrt{(0.200 \text{ m})^2/4 + (0.030 \text{ m})^2 + (0.040 \text{ m})^2}} = 3.2\times10^4 \text{ N/C}$$

Because the rod is positively charged, the electric field points in the direction from the origin to the point $(40 \text{ mm}, 30 \text{ mm}, 0).$ This could be expressed as $\vec{E} = 3.2\times10^4$ N/C at $37°$ ccw from the $+x$ axis as seen from the $+z$ axis, or as $\vec{E} = (2.6\times10^4 \text{ N/C})\hat{\imath} + (1.9\times10^4 \text{ N/C})\hat{\jmath}.$

23.56. Let the origin be at the center of the circle, and let the $+x$ axis point to the right along the dashed line, and the $+y$ axis vertically upward toward the top of the page. By symmetry, the fields due to the two positively charged arcs will cancel each other. We need only consider the electric field due to the single negatively charged arc. The differential contribution to the electric field dE from a differential segment of charge on the ring dq is just given by $dE = \dfrac{k|dq|}{R^2} = \dfrac{k|\lambda|d\ell}{R^2} = \dfrac{k|\lambda|d\theta}{R},$ because this tiny segment can be treated like a charged particle. We wish to integrate over the entire arc, but in order to add the contributions of the many particle-like segments, we need to break the field into components. Let the angular position of a differential segment of charge relative to the $+x$ axis be θ. Then the x component of the electric field due to that segment will be $dE_x = dE\cos(\theta) = \dfrac{k|\lambda|d\theta}{R}\cos(\theta).$ The negatively charged arc extends from $\theta = 90°$ to $\theta = 180°.$ So the x component of the total electric field from the entire arc is

$$E_x = \int_{\text{arc}} dE_x = \frac{k|\lambda|}{R}\int_{90°}^{180°} \cos(\theta)d\theta = -\frac{k|\lambda|}{R}(1) = -\frac{k}{R}\left(\frac{q}{\pi R/2}\right) = -\frac{2kq}{\pi R^2}$$

By symmetry, we must obtain the same expression for E_y. Thus the electric field is $\vec{E} = \dfrac{2\sqrt{2}kq}{\pi R^2}$ from the origin toward the center of the negatively charged arc.

23.57. (*a*) First, we must obtain an expression for the linear charge density of the rod. We know it is of the form $\lambda(x) = Cx,$ where C is a constant with units of Cm^{-2}. We know that if we integrate over the entire length of the rod we must obtain q_{rod}. Thus

$$\int_0^\ell Cxdx = \frac{1}{2}Cx^2\Big|_0^\ell = \frac{C}{2}\ell^2 = q_{\text{rod}} \Rightarrow C = \frac{2q_{\text{rod}}}{\ell^2}$$

Thus the linear charge density of the rod is $\lambda(x) = \frac{2q_{\text{rod}}}{\ell^2}x.$ Every tiny segment of the charged rod carrying a differential charge dq will contribute an electric field $dE = \frac{kdq}{r^2}.$ In the case of point P along the axis of the rod (the x axis) all electric field components at P will point in the $+x$ direction. So the total electric field is simply

$$E = \int_{\text{rod}} dE = \int_0^\ell \frac{k\lambda dx}{r^2} = \int_0^\ell \frac{k\lambda dx}{(d+\ell-x)^2} = \frac{2kq_{\text{rod}}}{\ell^2}\int_0^\ell \frac{xdx}{(d+\ell-x)^2}$$

$$E = \frac{2kq_{\text{rod}}}{\ell^2}\left(\frac{(\ell+d)}{(\ell+d-x)} + \ln[\ell+d-x]\right)\Bigg|_0^\ell$$

$$E = \frac{2kq_{\text{rod}}}{\ell^2}\left(\frac{\ell}{d} + \ln\left[\frac{d}{\ell+d}\right]\right)$$

So $\vec{E} = \frac{2kq_{\text{rod}}}{\ell^2}\left(\frac{\ell}{d} + \ln\left[\frac{d}{\ell+d}\right]\right)\hat{i}.$

(*b*) From the answer to part (*a*), we can take the term in parentheses $\left(\frac{\ell}{d} + \ln\left[\frac{d}{\ell+d}\right]\right),$ and write it as $\left(\frac{\ell}{d} + \ln\left[\frac{1}{\ell/d+1}\right]\right).$ We now construct a Taylor series around small $\frac{\ell}{d}$ and find that this term is approximately equal to $\left(\frac{\ell}{d}\right)^2$ to lowest order. Inserting this in place of the term in parentheses, we find $E = \frac{kq_{\text{rod}}}{d^2}.$ Note that this must be correct, because very far away from the rod, it would look basically like a point.

23.58. In order for the fields due to the two disks to cancel each other, the charge on disk 2 must be negative. Given that, we can use the results of example 23.6 in PRIN to write the electric field due to each disk. We also see from this example, as well as from symmetry alone, that the electric field due to either disk will be exactly parallel or antiparallel to the z axis. Call the distance from disk 1 to $z_{E=0} \cdot \Delta z_1,$ and call the distance from disk 2 to $z_{E=0} \cdot \Delta z_2.$ We can write

$$E_z = E_{1z} + E_{2z} = \frac{2kq_1}{R^2}\left(1 - \frac{\Delta z_1}{\left(\Delta z_1^2 + R^2\right)^{1/2}}\right) + \frac{2kq_2}{R^2}\left(1 - \frac{\Delta z_2}{\left(\Delta z_2^2 + R^2\right)^{1/2}}\right) = 0$$

$$q_2 = -q_1\frac{\left(1 - \frac{\Delta z_1}{\left(\Delta z_1^2 + R^2\right)^{1/2}}\right)}{\left(1 - \frac{\Delta z_2}{\left(\Delta z_2^2 + R^2\right)^{1/2}}\right)} = -(1.50\times10^{-6}\text{ C})\frac{\left(1 - \frac{(88.0\times10^{-3}\text{ m})}{((88.0\times10^{-3}\text{ m})^2 + (25.0\times10^{-3}\text{ m})^2)^{1/2}}\right)}{\left(1 - \frac{(56.0\times10^{-3}\text{ m})}{((56.0\times10^{-3}\text{ m}) + (25.0\times10^{-3}\text{ m})^2)^{1/2}}\right)} = -6.57\times10^{-7}\text{ C}$$

or $q_2 = -0.657\ \mu\text{C}.$

23.59. In each case we want the percent difference between the exact expression for a disk and the approximation of the disk as an infinite sheet. Thus, in each case we wish to calculate

$$\frac{|E_{\text{disk}} - E_{\text{sheet}}|}{E_{\text{disk}}} = \frac{2k\pi\sigma\left(\dfrac{z}{\sqrt{z^2+R^2}}\right)}{2k\pi\sigma\left(1-\dfrac{z}{\sqrt{z^2+R^2}}\right)} = \frac{1}{\dfrac{\sqrt{z^2+R^2}}{z}-1} \tag{1}$$

(*a*) From equation (1) above, we can write

$$\frac{|E_{\text{disk}} - E_{\text{sheet}}|}{E_{\text{disk}}} = \frac{1}{\dfrac{\sqrt{z^2+R^2}}{z}-1} = \frac{1}{\dfrac{R\sqrt{(0.1)^2+1}}{(0.1)R}-1} = 0.11 \text{ or } 11.1\%$$

(*b*) From equation (1) above, we can write

$$\frac{|E_{\text{disk}} - E_{\text{sheet}}|}{E_{\text{disk}}} = \frac{1}{\dfrac{\sqrt{z^2+R^2}}{z}-1} = \frac{1}{\dfrac{R\sqrt{(0.5)^2+1}}{(0.5)R}-1} = 0.809 \text{ or } 80.9\%$$

(*c*) From equation (1) above, we can write

$$\frac{|E_{\text{disk}} - E_{\text{sheet}}|}{E_{\text{disk}}} = \frac{1}{\dfrac{\sqrt{z^2+R^2}}{z}-1} = \frac{1}{\dfrac{R\sqrt{(1)^2+1}}{(1)R}-1} = 2.41 \text{ or } 241\%$$

23.60. Principles Example 23.4 gives us the expression for the electric field along the perpendicular bisector of a charged wire or rod: $E = \dfrac{kq}{d\sqrt{\ell^2/4+d^2}}$ where ℓ is the entire length of the rod and d is the distance from the rod to the point along the perpendicular bisector. Note that because the denominator $d\sqrt{\ell^2/4+d^2}$ is always greater than d^2, treating the charge as though it is localized at a point will always overestimate the field. Thus we require:

$$E_{\text{point}} - E_{\text{rod}} \le (0.05)E_{\text{rod}}$$

$$\frac{kq}{d^2} - \frac{(1.05)kq}{d\sqrt{\ell^2/4+d^2}} \le 0$$

We set the difference equal to zero to find the limiting distance

$$\frac{1}{d} = \frac{(1.05)}{\sqrt{\ell^2/4+d^2}}$$

$$(1.05)^2 d^2 = \frac{\ell^2}{4} + d^2$$

$$d = \sqrt{\frac{\ell^2/4}{(1.05)^2-1}} = \sqrt{\frac{(0.250\text{ m})^2/4}{(1.05)^2-1}} = 3.9\times10^2 \text{ mm}$$

23.61. From Principles Example 23.6 , we have expressions for the electric fields due to a disk along its axis of symmetry and due to a sheet. We require that the percent difference between the exact expression for a disk and the approximation of the disk as an infinite sheet be less than 10%. Thus

$$\frac{|E_{\text{disk}} - E_{\text{sheet}}|}{E_{\text{disk}}} = \frac{2k\pi\sigma\left(\dfrac{z}{\sqrt{z^2+R^2}}\right)}{2k\pi\sigma\left(1-\dfrac{z}{\sqrt{z^2+R^2}}\right)} = \frac{1}{\dfrac{\sqrt{z^2+R^2}}{z}-1} \le 0.10$$

or equivalently

$$121z^2 \le z^2 + R^2$$

$$z \le \frac{R}{\sqrt{120}} = \frac{(9.0\times10^{-3}\text{ m})}{\sqrt{120}} = 8.2\text{ mm}$$

So the error exceeds 10% a distance 8.2 mm from the disk along its axis of symmetry.

23.62. (*a*) From Principles Example 23.5, we know the expression for the electric field due to a single charged ring, a distance y along its axis of symmetry is $E_y = \dfrac{kqy}{(y^2+R^2)^{3/2}}$. We combine three such terms to find the total electric field at the point $(0, 100\text{ mm}, 0)$.

$$E_y = E_{1y} + E_{2y} + E_{3y} = ky\left[\frac{q_1}{(y^2+R_1^2)^{3/2}} + \frac{q_2}{(y^2+R_2^2)^{3/2}} + \frac{q_3}{(y^2+R_3^2)^{3/2}}\right]$$

$$E_y = (9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(0.100\text{ m})\left[\frac{(1.0\times10^{-6}\text{ C})}{((0.100\text{ m})^2+(0.050\text{ m})^2)^{3/2}} + \frac{(-2.0\times10^{-6}\text{ C})}{((0.100\text{ m})^2+(0.070\text{ m})^2)^{3/2}} + \frac{(1.0\times10^{-6}\text{ C})}{((0.100\text{ m})^2+(0.090\text{ m})^2)^{3/2}}\right]$$

$$= 2.4\times10^4\text{ N/C}$$

Thus $E = 2.4\times10^4$ N/C. (*b*) We require that a charged particle produce an electric field that can cancel that found in part (*a*). In order for the field at $(0, 100\text{ mm}, 0)$ to be downward, the charge we place at $(0, -100\text{ mm}, 0)$ must be negative. We find its magnitude by requiring

$$E_y = E_{\text{rings},\,y} + E_{\text{particle},\,y} = 0$$

$$E_{\text{rings}} = \frac{k|q|}{r^2} \Rightarrow |q| = \frac{E_{\text{rings}}r^2}{k}$$

$$|q| = \frac{(2.39\times10^4\text{ N/C})(0.200\text{ m})^2}{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)} = 1.1\times10^{-7}\text{ C}$$

Thus $q = -1.1\times10^{-7}$ C.

23.63. (*a*) Since the electron is repelled by the bottom plate, the bottom plate must be negatively charged. (*b*) From Principles Example 23.6, we know the electric field due to a single large sheet is given by $E_{\text{sheet}} = 2k\pi\sigma$. Between the two oppositely-charged sheets, the electric fields due to each sheet will be in the same direction, such that $E_{2\text{ sheets}} = 4k\pi\sigma$. If the force due to this electric field is the only force exerted on the electron, then we can clearly relate this to the acceleration. Let us call up toward the positive plate the $+z$ direction. Then

$$\frac{E_{\text{sheet}}|q_{\text{e}}|}{m_{\text{e}}} = \frac{4k\pi|\sigma||q_e|}{m_e} = |a_z| \tag{1}$$

The acceleration can be determined from kinematics. Call the direction in which the electron is initially fired (parallel to the plates) the $+x$ direction. We can write the displacement in the z direction as $\Delta z = v_{z,\text{i}}\Delta t + \frac{1}{2}a_z\Delta t^2 \Rightarrow a_z = \dfrac{2\Delta z}{\Delta t^2}$. Because there is no acceleration in the *x* direction, we can insert $\Delta t = \dfrac{\Delta x}{\Delta_{x,\text{i}}}$ and obtain $a_z = \dfrac{2\Delta z v_{x,\text{i}}^2}{\Delta x^2}$. Inserting this into equation (1) and rearranging yields

$$|\sigma| = \frac{2\Delta z v_{x,\text{i}}^2 m_e}{4k\pi|q_{\text{e}}|\Delta x^2} = \frac{2(5.0\times10^{-3}\text{ m})(4.0\times10^6\text{ m/s})^2(9.11\times10^{-31}\text{ kg})}{4(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\pi(1.60\times10^{-19}\text{ C})(0.020\text{ m})^2} = 2.0\times10^{-8}\text{ C/m}^2$$

Clearly one plate is positive and one is negative, so depending on the plate $\sigma = \pm2.0\times10^{-8}\text{ C/m}^2$.

23.64. (*a*) If $\dfrac{q}{m} > \dfrac{g}{2\pi k\sigma}$, particle undergoes constant acceleration away from sheet. If $\dfrac{q}{m} = \dfrac{g}{2\pi k\sigma}$, particle does not move. If $\dfrac{q}{m} < \dfrac{g}{2\pi k\sigma}$, particle undergoes constant acceleration toward sheet. (*b*) Let us assume that the charged particle is above the charged sheet, where vertically upward is the $+z$ direction. The sum of all z components of forces yields

$$\sum F_z = F^G_{\text{Ep}z} + F^{\text{E}}_{\text{sp}z} = -mg + 2k\pi\sigma q_{\text{p}} = ma_z$$

$$a_z = \frac{2k\pi\sigma q_{\text{p}}}{m} - g$$

So $\vec{a} = \left(\dfrac{2k\pi\sigma q_{\text{p}}}{m} - g\right)\hat{k}$. ($c$) Since the force does not vary with distance (provided the particle is very close to the sheet such that the sheet appears approximately infinite), we can write the work done on the particle as $\Delta K = W = \sum_n \vec{F}_n \cdot \vec{r} = F^G_{\text{sp}z}\Delta z + F^{\text{E}}_{\text{sp}z}\Delta z = (2k\pi\sigma q_{\text{p}} - mg)\Delta z.$

Note that both the $\vec{a}$ expression and the ΔK expression simplify if gravity is negligible, which is true for fundamental particles.

23.65. (a) $E_x = \displaystyle\int_0^\infty \frac{-k\lambda x dx}{(x^2+y^2)^{3/2}} = -\frac{k\lambda}{y}$

$$E_y = \int_0^\infty \frac{k\lambda y dx}{(x^2+y^2)^{3/2}} = -\frac{k\lambda}{y}$$

(b) $\theta = \tan^{-1}\left(\dfrac{E_y}{E_x}\right) = \tan^{-1}(-1) = 135°$

23.66. (a) Let the axis of symmetry of the ring be the z axis, and let the $+x$ axis point from the center of half-ring 1 to the center of half-ring 2. For some charged segment of a ring carrying charge dq, let ϕ be the angle between the z axis and the electric field due to dq. Let the angular position of a charged segment of ring 1 be given by θ_1. Then

$$E_z + E_{1z} + E_{2z} = \int_{\text{ring 1}} dE_z + \int_{\text{ring 2}} dE_z = \int_{\text{ring 1}} \frac{kdq_1}{r^2}\cos(\phi) + \int_{\text{ring 2}} \frac{kdq_2}{r^2}\cos(\phi)$$

$$E_z = \int_{\text{ring 1}} \frac{k\lambda_1 R d\theta_1}{R^2+z^2}\cos(\phi) + \int_{\text{ring 2}} \frac{k\lambda_2 R d\theta_2}{R^2+z^2}\cos(\phi) = \frac{k\lambda_1 Rz}{(R^2+z^2)^{3/2}}\int_{\pi/2}^{3\pi/2} d\theta_1 + \frac{k\lambda_2 Rz}{(R^2+z^2)^{3/2}}\int_{-\pi/2}^{\pi/2} d\theta_2$$

$$Ez = \frac{k\pi Rz(\lambda_1+\lambda_2)}{(R^2+z^2)^{3/2}}$$

$$Ez = \frac{kz(q_1+q_2)}{(R^2+z^2)^{3/2}}$$

Note that in the special case where $q_1 = q_2 = q$ we recover the familiar old field due to a uniformly charged ring (with a total charge of $2q$). (b) We proceed as in part (a).

$$E_x + E_{1x} + E_{2x} = \int_{\text{ring 1}} dE_x + \int_{\text{ring 2}} dE_z = -\int_{\text{ring 1}} \frac{kdq_1}{r^2}\sin(\phi)\cos(\theta_1) - \int_{\text{ring 2}} \frac{kdq_2}{r^2}\sin(\phi)\cos(\theta_2)$$

$$E_x = \int_{\text{ring 1}} \frac{k\lambda_1 R d\theta_1}{R^2+z^2}\sin(\phi)\cos(\theta_1) - \int_{\text{ring 2}} \frac{k\lambda_2 R d\theta_2}{R^2+z^2}\sin(\phi)\cos(\theta_2) = -\frac{k\lambda_1 R^2}{(R^2+z^2)^{3/2}}\int_{\pi/2}^{3\pi/2}\cos(\theta_1)d\theta_1 - \frac{k\lambda_2 R^2}{(R^2+z^2)^{3/2}}\int_{-\pi/2}^{\pi/2}\cos(\theta_2)d\theta_2$$

$$E_x = -\frac{kR^2\lambda_1}{(R^2+z^2)^{3/2}}\sin(\theta_1)\Big|_{\pi/2}^{3\pi/2} - \frac{kR^2\lambda_2}{(R^2+z^2)^{3/2}}\sin(\theta_2)\Big|_{-\pi/2}^{\pi/2}$$

$$E_x = \frac{2kR^2(\lambda_1-\lambda_2)}{(R^2+z^2)^{3/2}}$$

$$E_x = \frac{2kR}{\pi}\frac{(q_1-q_2)}{(R^2+z^2)^{3/2}}$$

23.67. For some charged segment of the rod carrying charge dq, call the angle between the field due to this segment and the $+x$ axis θ. Then

$$E_x = \int_{\text{rod}} dE_x = \int_{\text{rod}} \frac{kdq}{r^2}\cos(\theta) = \int_{-\ell/2}^{\ell/2} \frac{kqdx}{\ell\left(d^2+\left(\frac{\ell}{2}-x\right)^2\right)^{3/2}}\left(\frac{\ell}{2}-x\right)$$

$$E_x = \frac{kq}{\ell}\int_{-\ell/2}^{\ell/2} \frac{dx}{\left(d^2+\left(\frac{\ell}{2}-x\right)^2\right)^{3/2}}\left(\frac{\ell}{2}-x\right) = \frac{kq}{\ell}\left(\frac{1}{d}-\frac{1}{\sqrt{\ell^2+d^2}}\right)$$

Similarly,

$$E_y = \int_{\text{rod}} dE_y = \int_{\text{rod}} \frac{kdq}{r^2}\sin(\theta) = \int_{-\ell/2}^{\ell/2} \frac{kqdx}{\ell\left(d^2+\left(\frac{\ell}{2}-x\right)^2\right)^{3/2}}(d)$$

$$E_y = \frac{kqd}{\ell}\int_{-\ell/2}^{\ell/2} \frac{dx}{\left(d^2+\left(\frac{\ell}{2}-x\right)^2\right)^{3/2}} = \frac{kq}{d}\left(\frac{1}{\sqrt{\ell^2+d^2}}\right)$$

23.68. (*a*) Let the $+x$ axis point from the rod center toward the point P, and let the origin be coincident with the center of the rod. The charge density as a function of position can be written as $\lambda(x) = -\frac{\lambda_0}{\ell}x.$ Due to symmetry about the x axis, we need only consider the x components of the electric field.

$$E_x = \int_{\text{rod}} dE_x = \int_{\text{rod}} \frac{kdq}{r^2} = \int_{-\ell}^{\ell} \frac{k\lambda(x)dx}{(d-x)^2} = -\frac{k\lambda_0}{\ell}\int_{-\ell}^{\ell}\frac{xdx}{(d-x)^2} = \frac{k\lambda}{\ell}\left(\frac{2d\ell}{d^2-\ell^2}+\text{Ln}\left(\frac{d-\ell}{d+\ell}\right)\right)$$

So $E = \frac{k\lambda}{\ell}\left(\frac{2d\ell}{d^2-\ell^2}+\text{Ln}\left(\frac{d-\ell}{d+\ell}\right)\right)$ (*b*) Expanding around $\ell/d \ll 1$ yields $E = \frac{2kp}{d^3}$ which is the field due to a dipole along the axis of the dipole.

23.69. Due to symmetry about the x axis, we need only consider the x component of the electric field. Call the distance from the near end of the rod to the point at which we measure the field d. We first determine the form of the charge density. Call the total charge placed on the rod Q. Then we require. $Q = \int_0^\infty q_0 e^{-x/\ell}dx = \ell q_0 \Rightarrow q_0 = \frac{Q}{\ell}.$ Now we can write the electric field

$$E_x = \int_{\text{rod}} dE_x = \int_{\text{rod}} \frac{kdq(x)}{r^2} = \int_0^\infty \frac{kq_0 e^{-x/\ell}dx}{(d+x)^2} = -\frac{kQ}{\ell}\int_0^\infty \frac{e^{-x/\ell}dx}{(d+x)^2}$$

This integral is very difficult. It can be done by hand in terms of gamma functions, but it is most efficiently solved numerically. One obtains

$$E_x = -\frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.2\times10^{-6}\ \text{C})}{(0.0286\ \text{m})}\int_0^\infty \frac{e^{-x/(0.0286\ \text{m})}dx}{\left((0.020\ \text{m})+x\right)^2} = 1.6\times10^7\ \text{N/C}$$

23.70. We find the magnitude of the torque using $\tau = 2r_\perp F_+^E = d\sin(\theta)q_\text{p}E = (6.186\times10^{-30}\ \text{C}\cdot\text{m})(8500\ \text{N/C})$ $\sin(48°) = 3.9\times10^{-26}\ \text{N}\cdot\text{m}.$ The right hand rule tells the direction, such that $\vec{\tau} = 3.9\times10^{-26}\ \text{N}\cdot\text{m}\,\hat{j}.$

23.71. Any substance heats up when the kinetic energy of its molecules increases. In liquid water, the polar water molecules tend to align with the positive region of each molecule next to the negative region of neighboring molecules, and the molecules have a natural frequency associated with their oscillation around that equilibrium position. The transmitter in a microwave oven produces an oscillating electric field that matches that natural frequency. The oscillating electric field causes the water molecules to flip over in one orientation and then reverse; this increases the rotational kinetic energy of the water molecules. Collisions turn this rotational kinetic energy into a mixture of rotational and translational kinetic energy in the water molecules in a food placed in the oven, and the food is heated. Being nonpolar, oil molecules have a natural oscillation frequency different from that of liquid water, so the frequency of the microwaves is not a good match to the natural frequency of the oil molecules. The oil molecules in a food sample therefore do not increase their rotational kinetic energy and the food does not heat up.

Because the water molecules in ice are constrained, their natural frequency differs from that of liquid water. Thus the frequency of the microwaves (that of liquid water) does not match that of water molecules in a sample of frozen food. Therefore the kinetic energy of the molecules does not increase, and the food does not heat up.

23.72. *a*) We know the torque on a dipole and the electric field are related by $\tau = d\sin(\theta)q_\mathrm{p}E \Rightarrow E = \dfrac{\tau}{d\sin(\theta)q_\mathrm{p}} = \dfrac{\tau}{p\sin(\theta)} = \dfrac{(10.0\times10^{-9}\ \mathrm{N\cdot m})}{(8.0\times10^{-12}\ \mathrm{C\cdot m})\sin(30°)} = 2.5\times10^{3}\ \mathrm{N/C}$. (*b*) $q_\mathrm{p} = \dfrac{p}{d} = \dfrac{(8.0\times10^{-12}\ \mathrm{C\cdot m})}{(0.0025\ \mathrm{m})} = 3.2\times10^{-9}\ \mathrm{C}$.

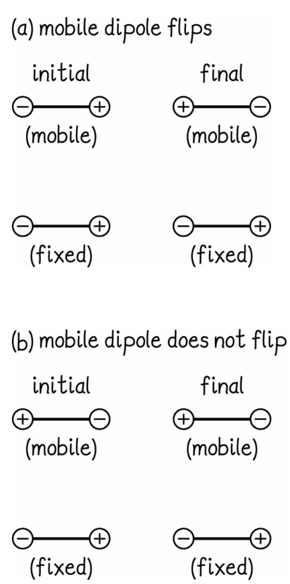
23.73. (*a*) There would be no motion; the dipole would remain stationary. (*b*) The dipole spins toward the equilibrium orientation (with the dipole moment parallel to the electric field). But in the absence of dissipation, the dipole's inertia carries it past this equilibrium orientation. The dipole oscillates around the equilibrium orientation with an angular amplitude of nearly 90°.

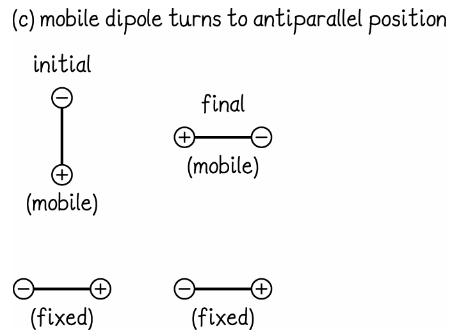
23.74. (*a*) The relationship between a restoring torque and an angular displacement from equilibrium is

$$\tau = \kappa\theta \tag{1}$$

We know that the torque on a dipole in an electric field is $\tau = pE\sin(\theta)$, which for very small angles can be rewritten $\tau \approx pE\theta$. Comparing this to equation (1) shows us that $\kappa = PE$. (*b*) $\omega = \sqrt{\dfrac{\kappa}{1}} = \sqrt{\dfrac{pE}{I}}$.

23.75.

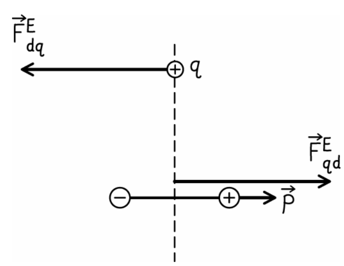




$F_{12} = F_{21} = 30\ \text{N}$

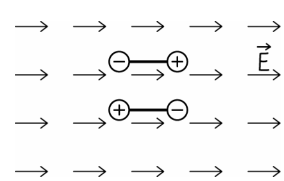
23.76. The answer depends somewhat on the size of the dipole separation d, relative to the length scale over which the electric field varies. If dipole separation d is small relative to the distance over which the electric field varies, the linear variation in the electric field does not significantly affect torque and the dipole rotates about its center of mass as if the field were uniform. Another way of saying this is that the positive and negative ends of the dipole experience approximately the same electric field. In this case the dipole will swing toward equilibrium (orientation in which the dipole moment and electric field are parallel). In the absence of dissipation, the dipole will swing past this equilibrium position and oscillate around it. In general, the oscillation will not be simple harmonic. If however there is an appreciable difference between the electric field strengths at the locations of the positive and negative charge, the effects of the linear increase in the electric field strength become important. Since the dipole rotates toward equilibrium, the positive end of the dipole will always be in a region where the electric field is stronger than (or equal to) the electric field at the position of the negative charge. This means the dipole will accelerate in the direction of the electric field. This acceleration may be large (when the dipole is momentarily parallel to the electric field) or zero (as the dipole is momentarily perpendicular to the field) or anything in between. As the dipole moves and the electric field becomes stronger, the oscillations around equilibrium will become smaller and faster. This can be understood by considering the potential energy of a dipole in an electric field. When the field becomes very strong the dipole no longer has sufficient energy to reach an orientation perpendicular to the field. If these oscillations become very small, the motion may be simple harmonic motion to a very good approximation.

23.77. (*a*)



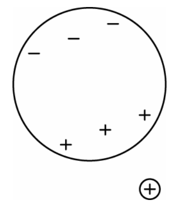
(*b*) Newton's third law tells us that the force that the particle exerts on the dipole must be equal in magnitude and opposite in direction to the force the dipole exerts on the particle. See the figure in part (*a*). (*c*) 1×10^{-6} N.

23.78. (*a*)



(*b*) No, when the dipole moment is parallel to the electric field the arrangement is stable. When dipole is oriented opposite the electric field the arrangement is unstable. (*c*) The stable arrangement should have less potential energy, meaning the arrangement with the dipole moment parallel to the electric field has less potential energy.

23.79. (*a*)



(*b*) The molecule would accelerate away from the positively charged particle.

23.80. Let the dipole moment $\vec{p}$ be parallel to the $+x$ axis, with the origin being at the center of the dipole. Call the charged particle on the $-x$ axis particle 1 and the particle on the $+x$ axis particle 2. Clearly by symmetry there will be no components of any forces in the y direction, so we consider only x components.

$$\sum F_{\text{d}x} = F^{\text{E}}_{1\text{d}x} + F^{\text{E}}_{2\text{d}x} = F^{\text{E}}_{1-x} + F^{\text{E}}_{1+x} + F^{\text{E}}_{2-x} + F^{\text{E}}_{2+x}$$

$$\sum F_{\text{d}x} = \frac{kq_1q_{\text{p}}}{\left(\frac{r-d}{2}\right)^2} + \frac{kq_1q_{\text{p}}}{\left(\frac{r+d}{2}\right)^2} + \frac{kq_2q_{\text{p}}}{\left(\frac{r+d}{2}\right)^2} - \frac{kq_2q_{\text{p}}}{\left(\frac{r-d}{2}\right)^2} \tag{1}$$

$$= \frac{kqq_{\text{p}}}{r^2}\left[-\frac{1}{\left(\frac{1}{2}-\frac{d}{2r}\right)^2} + \frac{1}{\left(\frac{1}{2}+\frac{d}{2r}\right)^2} + \frac{1}{\left(\frac{1}{2}+\frac{d}{2r}\right)^2} - \frac{1}{\left(\frac{1}{2}-\frac{d}{2r}\right)^2}\right]$$

We can now expand expression (1) in a Taylor series around small $\frac{d}{r}$. This yields

$$\sum F_{\text{d}x} = \frac{kqq_p}{r^2}\left(-32\frac{d}{r}\right) = \frac{-32kqp}{r^3}$$

Recalling the direction of the dipole moment, we can write $\sum \vec{F}_{\text{d}x} = \frac{-32kq\vec{p}}{r^3}$.

23.81. Let the imaginary line joining the two charged particles be the x axis, with the $+x$ direction pointing from the positively charged particle (particle 1) to the negatively charged particle (particle 2). All fields, forces, and polarizations will clearly be along the x axis, so we work with the x components. The x component of the electric field at the position of the dipole is that from the two charged particles: $E_x = \frac{k(q_1+q_2)}{r^2}$, where r is the distance from a charged particle to the center of the dipole. Note that $q_2 < 0$. From equation (23.26) we know the form of the force exerted on an induced dipole due to a charged particle. In this case we have two such particles, so the x component of the force is $\sum F_{\text{d}x} = -\frac{2kp_{\text{ind}}}{r^3}(q_1+q_2)$. We can use this to write $p_{\text{ind}} = -\frac{(\Sigma F_{\text{d}x})r^3}{2k(q_1+q_2)}$. We are not told the direction of the net force, but by looking at the expression for the total force (or just looking at the relative charge magnitude on the two charged particles) we can see that the force will be in the $-x$ direction. We can now use equation (23.24) to relate the above expressions for the induced dipole moment and the electric field

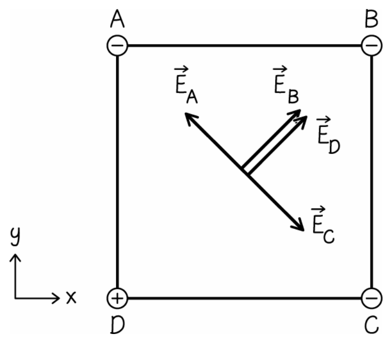
$$\vec{p}_{\text{ind}} = \alpha\vec{E} \Rightarrow p_{\text{ind},x} = \alpha E_x$$

$$-\frac{(\Sigma F_{\text{dx}})r^3}{2k(q_1+q_2)} = \alpha\left(\frac{k(q_1+q_2)}{r^2}\right)$$

$$\alpha = \frac{(\Sigma F_{\text{dx}})r^5}{2k^2(q_1+q_2)^2} = -\frac{(-45.0\times10^{-9}\text{ N})(2.57\times10^{-6}\text{ m})^5}{2(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)^2\left((3.56\times10^{-6}\text{ m})+(-1.05\times10^{-6}\text{ m})\right)^2} = 4.9\times10^{-45}\text{ C}^2\cdot\text{m/N}$$

23.82. J/(A·s·m)

23.83.



Clearly the electric fields from the charge on the upper left and lower right corners will cancel each other. The electric fields from the charge on the lower left and upper right corners will be in the same direction such that their magnitudes can simply be added. Thus the magnitude of the total electric field is $E = \frac{2kq}{r^2} = \frac{2kq}{(\ell/\sqrt{2})} = \frac{4(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-9}\text{ C})}{(0.050\text{ m})^2} = 4.0\times10^4\text{ N/C}$. Thus $\vec{E} = 4.3\times10^4\text{ N/C}$ from the center of the square to the upper right corner.

23.84. In all cases, the magnitude of the electric field is simply given by $E = \frac{kq}{r^2} = \frac{kq}{x^2+y^2}$. The direction from the origin to the point $(8.00\text{ nm}, 6.00\text{ nm})$ is always 37° ccw from the $+x$ axis.

(*a*) $E = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(8.00\times10^{-6}\text{ C})}{(8.00\times10^{-9}\text{ m})^2+(6.00\times10^{-9}\text{ m})^2} = 7.2\times10^{20}\text{ N/C}$ so that $\vec{E} = 7.2\times10^{20}\text{ N/C}$ at 37° ccw from the $+x$ axis.

(*b*) $E = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(23.0\times10^{-6}\text{ C})}{(8.00\times10^{-9}\text{ m})^2+(6.00\times10^{-9}\text{ m}^2)} = 2.1\times10^{21}\text{ N/C}$ so that $\vec{E} = 2.1\times10^{21}\text{ N/C}$ at 217° ccw from the $+x$ axis.

(*c*) $E = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10.0\times10^{-9}\text{ C})}{(8.00\times10^{-9}\text{ m})^2+(6.00\times10^{-9}\text{ m})^2} = 9.0\times10^{17}\text{ N/C}$ so that $\vec{E} = 9.0\times10^{17}\text{ N/C}$ at 37° ccw from the $+x$ axis.

23.85. (*a*) If the charges have the same sign, then the only place the fields from the two charges will be in opposite directions is between them on the x axis. In order for the fields to cancel, they must be the same in magnitude. This means the point at which they cancel must be equidistant from each of the two particles. Thus the electric field is zero at $x = d/2$. (*b*) There is no such point.

23.86. There is no acceleration in the x direction, such that the time required is just $\Delta t = \dfrac{\Delta x}{v_{x,\mathrm{i}}}$. We can use this along with the acceleration in the y direction $a_y = \dfrac{F^{\mathrm{E}}_{\mathrm{e},y}}{m} = \dfrac{qE_y}{m}$ to determine the distance travelled in the y direction from $\Delta y = v_{y,\mathrm{i}}\Delta t + \dfrac{1}{2}a_y \Delta t^2$. $\Delta y = 0 + \dfrac{1}{2}\left(\dfrac{qE_y}{m}\right)\left(\dfrac{\Delta x}{v_{x,\mathrm{i}}}\right)^2 = \dfrac{1}{2}\left(\dfrac{(-1.60\times10^{-19}\text{ C})(2\times10^4\text{ N/C})}{(9.11\times10^{-31}\text{ kg})}\right)\left(\dfrac{(0.040\text{ m})}{(2.1\times10^7\text{ m/s})}\right)^2 = -6\text{ mm}.$

23.87. (*a*) The greatest possible torque occurs when the electric field is perpendicular to the dipole moment. In this case

$$\tau = F_{\mathrm{p}}^{E} r_{\mathrm{p}} \sin(\theta_{\mathrm{p}}) + F_{\mathrm{n}}^{\mathrm{E}} r_{\mathrm{n}} \sin(\theta_{\mathrm{n}})$$
$$\tau = Eq_{\mathrm{p}} r_{\mathrm{p}} \sin(\theta_{\mathrm{p}}) + E|q_{\mathrm{n}}| r_{\mathrm{n}} \sin(\theta_{\mathrm{n}})$$
$$\tau = (100.0\text{ N/C})(1.60\times10^{-19}\text{ C})\sin(90^\circ)\left((0.30\times10^{-9}\text{ m}) + (0.10\times10^{-9}\text{ m})\right)$$
$$\tau = 6.4\times10^{-27}\text{ N}\cdot\text{m}$$

(*b*) The smallest torque the molecule could experience is zero. This occurs when the dipole moment is parallel to the electric field, or exactly opposite the electric field.

23.88. (*a*) For two charged particles, $F_{12}^{\mathrm{E}} \sim \dfrac{1}{r_{12}^2}$. So the new force magnitude is $F^{\mathrm{E}}/4$. (*b*) It was shown in problem 23.80 that the force between two dipoles separated in a direction parallel to their dipole moments depends on the separation according to $F_{12}^{\mathrm{E}} \sim \dfrac{1}{r_{12}^3}$. Thus the new force magnitude is $F^{\mathrm{E}}/8$. (*c*) Let the dipole on the left be dipole 1 and the dipole on the right be dipole 2. Let their dipole moments be oriented parallel and antiparallel to the $+y$ axis, respectively, and let them be separated along the x axis with the $+x$ axis pointing to the right. The total force on dipole 2 due to dipole 1 is given by

$$\sum F_{12x} = F^{\mathrm{E}}_{\mathrm{p_1p_2}x} + F^{\mathrm{E}}_{\mathrm{p_1n_2}x} + F^{\mathrm{E}}_{\mathrm{n_1p_2}x} + F^{\mathrm{E}}_{\mathrm{n_1n_2}x}$$

$$\sum F_{12x} = \frac{kq_1q_2}{x^2+d^2}\cos(\theta) - \frac{kq_1q_2}{x^2} - \frac{kq_1q_2}{x^2} + \frac{kq_1q_2}{x^2+d^2}\cos(\theta) = \frac{kq^2}{x^2}\left[-2 + \frac{2}{\left(1+\left(\frac{d}{x}\right)^2\right)}\frac{x}{\sqrt{x^2+d^2}}\right]$$

$$\sum F_{12x} = \frac{kq^2}{x^2}\left[-2 + \frac{2}{\left(1+\left(\frac{d}{x}\right)^2\right)^{3/2}}\right] \qquad (1)$$

We can simplify expression (1) by Taylor expanding this around small $\dfrac{d}{x}$. We obtain

$$\sum F_{12x} = \frac{kq^2}{x^2}\left(-\frac{3d^2}{2x^2}\right) = -\frac{3kd^2q^2}{2x^4}$$

Since the force between the two dipoles depends on separation according to $F_{12}^{\mathrm{E}} \sim \dfrac{1}{r_{12}^4}$, the new force magnitude is $F^{\mathrm{E}}/16$. (*d*) The field due to the dipole at the location of the charged particle depends on the separation according to $E_{\mathrm{d}} \sim \dfrac{1}{r^3}$. So the force between the two also depends on separation like $\sim \dfrac{1}{r^3}$. Thus the new force magnitude is

$F^{\text{E}}/8$. (*e*) From equation 23.26, we know that the force between a charged particle and an induced dipole depends on separation according to $F^{\text{E}} \sim \dfrac{1}{r^5}$. Thus the new force magnitude is $F^{\text{E}}/32$.

23.89. (*a*) We begin by calculating the electric field at the origin. Once that is done, we can obtain any of the forces required by simply multiplying this electric field by the charge. Call the particle carrying 32 nC of charge particle 1, and call the particle carrying 98 nC of charge particle 2. Call the angle between the $+x$ axis and the electric field at the origin due to particle 1 θ_1. We write the x and y components of the electric field at the origin.

$$E_x = E_{1x} + E_{2x} = \frac{kq_1}{x_1^2 + y_1^2}\cos(\theta_1) + \frac{kq_2}{x_2^2 + y_2^2}\cos(\theta_2) = -\frac{kq_1 x_1}{(x_1^2 + y_1^2)^{3/2}} - \frac{kq_2 x_2}{(x_2^2 + y_2^2)^{3/2}}$$

$$E_x = -\frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.0\times10^{-9}\ \text{C})(10.0\times10^{-9}\ \text{m})}{\left((10.0\times10^{-9}\ \text{m})^2 + (95.0\times10^{-9}\ \text{m})^2\right)^{3/2}} - \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(98.0\times10^{-9}\ \text{C})(45.0\times10^{-9}\ \text{m})}{\left((45.0\times10^{-9}\ \text{m})^2 + (56.0\times10^{-9}\ \text{m})^2\right)^{3/2}}$$

$$E_x = -1.10\times10^{17}\ \text{N/C}$$

$$E_y = E_{1y} + E_{2y} = \frac{kq_1}{x_1^2 + y_1^2}\sin(\theta_1) + \frac{kq_2}{x_2^2 + y_2^2}\sin(\theta_2) = -\frac{kq_1 y_1}{(x_1^2 + y_1^2)^{3/2}} - \frac{kq_2 x_2}{(x_2^2 + y_2^2)^{3/2}}$$

$$E_x = -\frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.0\times10^{-9}\ \text{C})(95.0\times10^{-9}\ \text{m})}{\left((10.0\times10^{-9}\ \text{m})^2 + (95.0\times10^{-9}\ \text{m})^2\right)^{3/2}} - \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(98.0\times10^{-9}\ \text{C})(56.0\times10^{-9}\ \text{m})}{\left((45.0\times10^{-9}\ \text{m})^2 + (56.0\times10^{-9}\ \text{m})^2\right)^{3/2}}$$

$$E_x = -1.646\times10^{17}\ \text{N/C}$$

Using the Pythagorean theorem and simple trigonometry we find $\vec{E} = 2.0\times10^{17}\ \text{N/C}$ at 266° ccw from the $+x$ axis. Thus, for the first charged particle we place at the origin $\vec{F} = q\vec{E} = (3.50\times10^{-6}\ \text{C})(2.0\times10^{17}\ \text{N/C})\hat{E} = 6.9\times10^{11}\ \text{N}$ at 266° ccw from the $+x$ axis.

(*b*) $\vec{F} = q\vec{E} = (7.22\times10^{-6}\ \text{C})(2.0\times10^{17}\ \text{N/C})\hat{E} = 1.4\times10^{12}$ N at 266° ccw from the $+x$ axis.

(*c*) $\vec{F} = q\vec{E} = (95.1\times10^{-9}\ \text{C})(2.0\times10^{17}\ \text{N/C})\hat{E} = 1.9\times10^{10}$ N at 266° ccw from the $+x$ axis.

(*d*) $\vec{F} = q\vec{E} = (-77.5\times10^{-9}\ \text{C})(2.0\times10^{17}\ \text{N/C})\hat{E} = 1.5\times10^{10}$ N at 86° ccw from the $+x$ axis.

(*e*) $\vec{F} = q\vec{E} = (1.00\times10^{-3}\ \text{C})(2.0\times10^{17}\ \text{N/C})\hat{E} = 2.0\times10^{14}$ N at 266° ccw from the $+x$ axis.

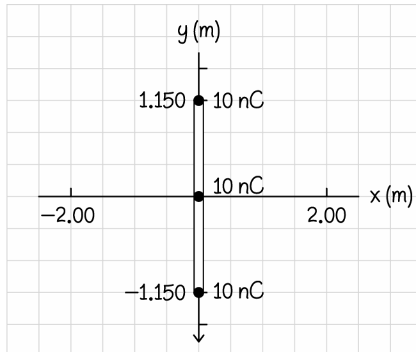
(*f*) $\vec{F} = q\vec{E} = (3.32\times10^{1}\ \text{C})(2.0\times10^{17}\ \text{N/C})\hat{E} = 6.6\times10^{18}$ N at 266° ccw from the $+x$ axis.

23.90. The electric fields from the two charged particles on the top of the square will be directed opposite each other and will be equal in magnitude. Thus they will cancel each other. The electric fields due to the two charged particles along the bottom of the square will have horizontal components that will cancel each other, but vertical components that will add. Call the angle between the electric field due to the bottom left particle and vertical θ. Then

$$E_y = \frac{2kq}{r^2}\cos(\theta) = \frac{2kq\ell}{\left(\left(\dfrac{\ell}{2}\right)^2 + \ell^2\right)^{3/2}} = \frac{2(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\ \text{C})(5.00\times10^{-3}\ \text{m})}{\left(\left(\dfrac{(5.00\times10^{-3}\ \text{m})}{2}\right)^2 + (5.00\times10^{-3}\ \text{m})^2\right)^{3/2}} = 1.5\times10^9\ \text{N/C}$$

So $\vec{E} = 1.5\times10^9\ \text{N/C}$ vertically upward from P.

23.91. (*a*)



(*b*) Along the x axis the y components of the electric field will cancel. We consider only the x components.

$$E_x = E_{\text{top},x} + E_{\text{mid},x} + E_{\text{bottom},x} = \frac{2kq}{r_{\text{top}}^2}\cos(\theta) + \frac{kq}{r_{\text{mid}}^2}$$

$$E_x = \frac{2kqx}{(x^2+y^2)^{3/2}} + \frac{kq}{x^2} = (9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10\times10^{-9}\ \text{C})\left(\frac{2(0.200\ \text{m})}{\left((0.200\ \text{m})^2+(0.150\ \text{m})^2\right)^{3/2}} + \frac{1}{(0.200\ \text{m})^2}\right) = 4.6\times10^3\ \text{N/C}$$

(*c*) The exact answer is given by the result of example 23.4:

$$E_x = \frac{kq}{x\sqrt{\ell^2/4+x^2}} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(30\times10^{-9}\ \text{C})}{(0.200\ \text{m})\sqrt{\dfrac{(0.300\ \text{m})^2}{4}+(0.200\ \text{m})^2}} = 5.4\times10^3\ \text{N/C}$$

So the percent error is $\dfrac{(E_x)_{\text{exact}} - (E_x)_{\text{approx}}}{(E_x)_{\text{exact}}} = \dfrac{(5.40\times10^3\ \text{N/C}) - (4.55\times10^3\ \text{N/C})}{(5.40\times10^3\ \text{N/C})} = 0.16 \text{ or } 16\%.$ (*d*) We could approximate the rod as five charged particles, evenly spaced along the position of the rod, each with 1/5 the overall charge.

23.92. (*a*) The equilibrium position is that at which the gravitational and electric forces cancel. Thus we require $\sum F_{1y} = F^G_{E1y} + F^E_{21y} = -mg + \dfrac{kq_1q_2}{h_2} = 0 \Rightarrow h = \sqrt{\dfrac{kq_1q_2}{mg}}.$ (*b*) The sum of all forces in the y direction on sphere 1 is $\sum F_1^E = \dfrac{kq_1q_2}{h^2\left(1+\dfrac{\Delta y}{h}\right)^2} = -mg,$ where Δy is the small displacement from equilibrium. If one expands this around $\Delta y/h \ll 1$ one obtains that the sum of all forces is $F_1^E = -\dfrac{2kq_1q_2}{h^3}\Delta y.$ This is a restoring force that is linear in the displacement. In other words, one could define $k_{\text{spring}} = \dfrac{2kq_1q_2}{h^3}\Delta y,$ and one would have exactly the equation describing a spring restoring force. This has been shown to produce simple harmonic motion. Recall that in simple harmonic motion $\omega = \sqrt{k/m},$ meaning $\omega = \sqrt{\dfrac{2kq_1q_2}{mh^3}}.$ Recall also that $g = \dfrac{kq_1q_2}{mh^2}$ from part (*a*). This yields $\omega = \sqrt{\dfrac{2g}{h}}.$

23.93. The exact electric field is given by equation (23.12). The approximation of the field in equation (23.13) came from Taylor expanding the exact expression around small $\dfrac{d}{z},$ and keeping only the lowest order. If the higher orders account for more than 1% of the total expression, then it is not sufficient to keep only the first order term. We compare the next term in the Taylor expansion to the exact expression. The Taylor expansion is

$$E_z = \frac{kq_p}{z^2}\left[\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right] \approx \frac{kq_\text{p}}{z^2}\left(2\left(\frac{d}{z}\right)+\left(\frac{d}{z}\right)^3+\frac{3}{8}\left(\frac{d}{z}\right)^5+\ldots\right)$$

We require $E_\text{exact} - E_\text{approx} \le (0.01)E_\text{exact}$ or $(0.99)E_\text{exact} \le E_\text{approx}$. Inserting equations (23.12) and (23.13) we write

$$\frac{(0.99)kq_\text{p}}{z^2}\left[\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right] \le \frac{2kq_\text{p}d}{z^3}$$

With some algebra, this reduces to the constraint $z > 2.83d$. This may be sufficient for your boss. If not, one solution would be to include the next lowest order term in the Taylor series and use

$$E_\text{approx,z} = \frac{kq_\text{p}}{z^2}\left(2\left(\frac{d}{z}\right)+\left(\frac{d}{z}\right)^3\right)$$

23.94. The electric field at the center of this arc is $kq\sin(\theta)/(\theta r^2)$ where θ is measured in radians. You can maximize the electric field in two ways. Since q is fixed, one way of maximizing the electric field is to pack that charge onto the smallest possible length (minimize the length of the arc). This could be accomplished by either minimizing θ or R. The second way is to maximize the field is to get the charge as close to the ink droplet as possible, meaning minimize R. Minimizing the radius of the arc accomplishes both of these and has the strongest effect. But choosing a smaller θ could also help.

23.95. Call the perpendicular bisector of the neutral segment the x axis with the $+x$ axis pointing out away from the center of the semicircle. Let angle θ be the angle between the electric field due to some small section of the rod and the $+x$ axis. In general, the x component of the electric field due to an arc length is

$$E_{\text{arc},x} = \int_{\theta_\text{min}}^{\theta_\text{max}} \frac{kdq}{R^2}\cos(\theta) = \int_{\theta_\text{min}}^{\theta_\text{max}} \frac{k\lambda d\theta}{R}\cos(\theta) = \frac{kq}{R^2(\theta_\text{max}-\theta_\text{min})}\left[\sin(\theta_\text{max}) - \sin(\theta_\text{max})\right] \qquad (1)$$

In our case, the central segment has no charge, so the expression in equation (1) is zero. One arc extends from $\pi/6$ to $\pi/2$ and the other extends from $3\pi/2$ to $11\pi/6$. Thus

$$E_x = \frac{3kq}{R^2\pi}\left(\left[\sin\left(\frac{\pi}{2}\right)-\sin\left(\frac{\pi}{6}\right)\right]+\left[\sin\left(\frac{11\pi}{6}\right)-\sin\left(\frac{3\pi}{2}\right)\right]\right) = \frac{3kq}{\pi R^2}$$

By symmetry, the y components of the electric fields should cancel. Thus $E = \dfrac{3kq}{\pi R^2}$.

24

GAUSS'S LAW

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

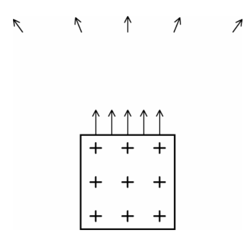
Developing a Feel

1. $10^3\ \text{N}\cdot\text{m}^2/\text{kg}$ **2.** $10^3\ \text{N}\cdot\text{m}^2/\text{C}$ **3.** $10^{-4}\ \text{C/m}^2$ **4.** $10^{-21}\ \text{C/m}^2$ **5.** $10^{-9}\ \text{C/m}^2$

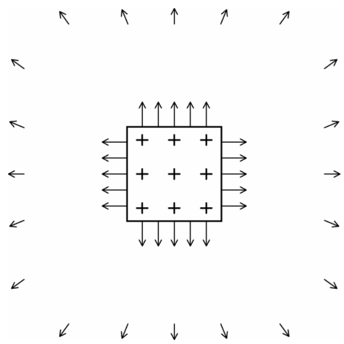
Guided Problems

24.2 Charged square sheet

1. Getting Started We begin by making a sketch of the charged sheet. We are not asked for any quantitative results, we only need to determine the electric field lines qualitatively. For example, we know that very near an edge of the square, the electric field will point directly away from that edge. We also know that very far from the square, it will appear very much like a charged particle (the exact geometry won't be as noticeable). In that case, the electric field lines should point radially away from the center. Here we draw an intermediate step in which we show these limiting cases for just one side of the square.



2. Devise Plan The square sheet of charge will look exactly the same if we rotate it by any integer multiple of $90°$ about an axis through the square's center and perpendicular to the plane of the square (for later reference, let us call this axis the z axis). This means that the electric field lines must also be unchanged by the same rotation. In other words, the electric field lines near the bottom of the square sheet should look the same as those drawn for the top of the sheet, just rotated by $180°$. Thus we can complete the sketch of the limiting cases based on symmetry.



Because electric field lines are continuous in empty space, and because they cannot cross, connecting the two limiting cases shown above (near the sheet and far from the sheet) is straightforward.

Before completing our sketch, we note that the same symmetry we used to complete the limiting cases above can be applied to the question of flux. One might try to use equation (24.4) directly:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

But finding the electric field as a function of position would already require a non-trivial two-dimensional integral over the charge distribution. The dot product with the area elements of the sheet would also be non-trivial, and the entire process would be quite difficult. It would be much better to use symmetry.

We are asked about the flux through a sheet that is perpendicular to the charged square sheet and passes through its center. Let us call the flux through the sheet $\Phi_{E,\text{sheet,i}}$. Note that such a sheet must necessarily contain the axes we called z. If we rotate the charged sheet by $180°$ about z, the charge distribution does not change (rotating a square by $180°$ about z still just produces a square). Now we make two arguments based on this symmetry. First, rotating any charge distribution by $180°$ flips the direction of all electric fields. That means the sign of the electric flux through every small segment of the perpendicular sheet will reverse sign. So we can write

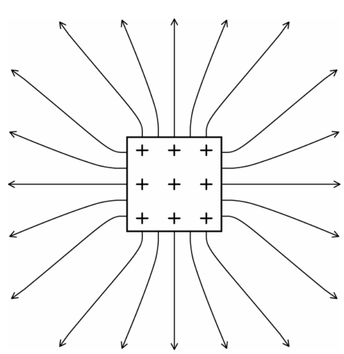
$$\Phi_{E,\text{sheet,i}} = -\Phi_{E,\text{sheet,f}} \quad (1)$$

Secondly, because of the special symmetry of our charged sheet, we know that rotating it by $180°$ about z changes nothing about the charge distribution. That means nothing about the electric fields can be different, and thus nothing about the flux can be different. Thus

$$\Phi_{E,\text{sheet,i}} = \Phi_{E,\text{sheet,f}} \quad (2)$$

Combining equations (1) and (2), we see $\Phi_{E,\text{sheet,i}} = -\Phi_{E,\text{sheet,i}}$ which can only be true if $\Phi_{E,\text{sheet,i}} = 0$. We might have guessed that the flux would be zero, just from our intuition about symmetry. But the above argument is more than just a guess; it constitutes proof that the flux is zero.

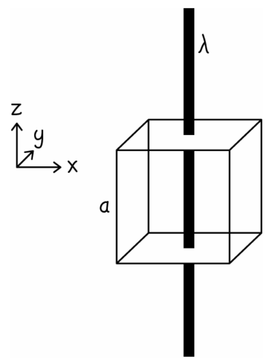
3. Execute Plan We now connect the limited cases in our previous sketch to obtain our final answer:



4. Evaluate Result If a charged sheet has reflection symmetry across a plane, then the electric flux through that plane must be zero. To state this terms of rotations, we would need to refer to the axis of rotation (which we continue to call z). If a charge distribution has $180°$ rotational symmetry about some axis z, then the flux through a plane containing z must be zero.

24.4 Charged rod, cubed

1. Getting Started We are given the dimensions of the cube and the linear charge density of the wire. We are asked for the flux through each face of the cube. We begin by making a sketch of the information we have:



This problem is similar to Worked Problem 24.3 in that we are required to determine the flux through direct integration. Another similarity is that there is symmetry in both problems that can be used to our advantage. This problem is different from Worked Problem 24.3 in that the symmetry of the problem is different, and here we are given a linear charge density rather than a charged particle.

We can use symmetry to argue that the flux through the top and bottom faces of the cube must be zero. One might simply recall that the electric field from an infinitely long wire points radially outward from the wire. That means electric field lines are running along the surface of the top and bottom faces, but never actually passing through the faces. Alternatively, one might appear directly to symmetry and say that if the charged wire is flipped upside-down, the charge distribution does not change, so the flux through the top face cannot change. But flipping any charge distribution flips the direction of the electric fields and must change the sign of the flux through a surface. Thus, the flux through the top face Φ_{top} is the same as $-\Phi_{\text{top}}$, such that $\Phi_{\text{top}} = 0$. The same can be said of Φ_{bottom}.

There is more symmetry that we can use. For example, rotating the cubic box by $90°$ around the wire itself changes nothing. Thus the flux through each of the vertical faces must be the same.

2. Devise Plan As we argued above, symmetry tells us that the flux through the top and bottom faces is zero.

We determine the flux through a given face by using equation (24.4)

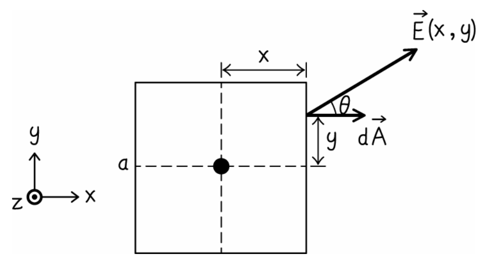
$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

To make use of this we need an expression for the electric field due to a long straight wire, which is worked out in *Principles* Example 23.4:

$$\vec{E}_{\text{rod}} = \frac{2k\lambda}{r}\hat{r} \tag{1}$$

Here $\hat{r}$ is the unit vector pointing to a point where we wish to determine the field (sometimes called a "field point") from the closest point on the wire to the field point. In the context of our charge distribution and axes, $r = \sqrt{x^2 + y^2}$. Note that there is no z dependence in r, since $\hat{r}$ must always be perpendicular to the wire. Since r has no z

dependence, and neither does $\vec{E}_{\text{rod}}$, we expect integration over z to be rather trivial, and we suspect we may get most salient details by looking at a two-dimensional slice of the setup, in the plane $z = z_0$. We do this only to simplify our sketches; it is not required. Consider the sketch below.



We know we can rewrite the dot produce in equation (24.4) to obtain

$$\Phi_E = \int E dA \cos(\theta)$$

where the integration is over one face. Note that for the right-most face, the plane of the face lies in the yz plane, so our integral should be over y and z. Note also that we have measured y from the center of the cube, such that it must run from $-a/2$ to $a/2$. We have no specified whether z is measured from the bottom of the cube or the center. Since the integrand is independent of z, it doesn't matter whether we integrate z from 0 to a or from $-a/2$ to $a/2$. Written in terms of spatial variables,

$$\Phi_E = \int_0^a \int_{-a/2}^{a/2} E \cos(\theta) dy dz \tag{2}$$

From the diagram, we can see that

$$E = \frac{2k\lambda}{\sqrt{x^2 + y^2}} \tag{3}$$

We know that the angle θ is required for relating the direction of the area element and the direction of the electric field. But we see that geometrically it is also given by

$$\cos(\theta) = \frac{x}{\sqrt{x^2 + y^2}} \tag{4}$$

Inserting equations (3) and (4) into equation (2), and noting that everywhere on the right-most face $x = a/2$, we obtain

$$\Phi_E = k\lambda a \int_0^a \int_{-a/2}^{a/2} \frac{1}{(a/2)^2 + y^2} dy dz \tag{5}$$

We need only evaluate equation (5) to obtain the flux through the right-most face. The flux through all other faces must be the same, by symmetry.

3. Execute Plan The integration over z in equation (5) is trivial. The integration over y can be carried out using appropriate software, or by using a trigonometric substitution $y = \frac{a}{2}\tan(\theta)$ such that $dy = \frac{a d\theta}{2\cos^2(\theta)}$.

$$\Phi_E = k\lambda a \int_0^a \int_{-a/2}^{a/2} \frac{1}{(a/2)^2 + y^2} dydz$$
$$\Phi_E = k\lambda a^2 \int_{-a/2}^{a/2} \frac{1}{(a/2)^2 + y^2} dy$$
$$\Phi_E = k\lambda a^2 \int_{-\pi/4}^{\pi/4} \frac{(a/2)}{(a/2)^2 \cos^2(\theta)(1+\tan^2(\theta))} d\theta$$
$$\Phi_E = 2k\lambda a \int_{-\pi/4}^{\pi/4} \frac{d\theta}{(\cos^2(\theta)+\sin^2(\theta))}$$
$$\Phi_E = 2k\lambda a \int_{-\pi/4}^{\pi/4} d\theta$$
$$\Phi_E = k\lambda a\pi = \frac{\lambda a}{4\varepsilon_0}$$

The flux through the top and bottom faces is zero, and the flux through all vertical faces is $\Phi_E = \dfrac{\lambda a}{4\varepsilon_0}$.

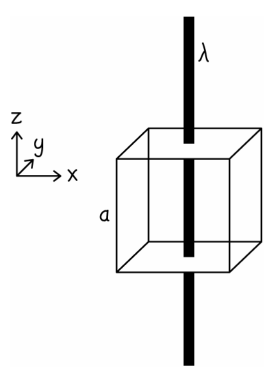
4. Evaluate Result Gauss's Law tells us that if we add up the flux through all sections of a closed surface, we should get $\dfrac{q_{\text{enclosed}}}{\varepsilon_0}$. In this case, we are not told an enclosed charge; we are given a linear charge density. So the charge enclosed in the cube is $q_{\text{enclosed}} = \lambda a$. Adding up the flux through all surfaces, we find

$$\Phi_{E,\text{total}} = \Phi_{E,\text{top}} + \Phi_{E,\text{bottom}} + 4\Phi_{E,\text{side}}$$
$$\Phi_{E,\text{total}} = 0 + 0 + 4\left(\frac{\lambda a}{4\varepsilon_0}\right) = \frac{q_{\text{enclosed}}}{\varepsilon_0}$$

Thus, our result is in agreement with Gauss's Law.

24.6 Revisiting charged rod cubed

1. Getting Started The same sketch we used in Worked Problem 24.4 should work here as well.



This problem is similar to Worked Problem 24.5, in that we are asked to use Gauss's Law to determine the flux through a surface. The alternative would be to calculate flux directly using equation (24.4). This problem involves a linear charge density rather than a charged particle.

There is a great deal of symmetry in this problem. Note first that there is reflection symmetry across a plane passing through the center of the cube and perpendicular to the wire. There is also rotational symmetry in that the entire setup can be rotated around the axis of the wire by integer multiples of $90°$ without changing anything about the setup. These two symmetries will be very important.

2. Devise Plan We can use symmetry to argue that the flux through the top and bottom faces of the cube must be zero. Consider the following argument, which we also used in Guided Problem 24.4. One might simply recall that the electric field from an infinitely long wire points radially outward from the wire. That means electric field lines are running along the surface of the top and bottom faces, but never actually passing through the faces. Alternatively, one might appear directly to symmetry and say that if the charged wire is flipped upside-down, the charge distribution does not change, so the flux through the top face cannot change. But flipping any charge distribution flips the direction of the electric fields and must change the sign of the flux through a surface. Thus, the flux through the top face Φ_{top} is the same as $-\Phi_{\text{top}}$, such that $\Phi_{\text{top}} = 0$. The same can be said of Φ_{bottom}.

The rotational symmetry of the setup means that the flux through the right-most face must equal the flux through the back face, and so on. The flux through all vertical faces must be the same. This allows us to right the total flux through the cube in terms of the flux through a single side.

In order to use Gauss's Law, we need an expression for the charge enclosed by the Gaussian surface. Since we are asked about flux through the cubic surface, it makes sense to choose that cubic surface as our Gaussian surface. We are given the linear charge density $\lambda = q/\ell$. In this case, the length of wire enclosed is the side length a. Thus $q_{\text{enclosed}} = \lambda a$.

We now have information about the enclosed charge, and we have used all possible symmetry to simplify our work in finding the flux. We are ready to relate flux and enclosed charge using Gauss's Law:

$$\Phi_{E,\text{total}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

3. Execute Plan Inserting our expression for the enclosed charge and our symmetry-based results for the flux into Gauss's Law, we obtain

$$\Phi_{E,\text{total}} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

$$\Phi_{E,\text{top}} + \Phi_{E,\text{bottom}} + 4\Phi_{E,\text{side}} = \frac{\lambda a}{\varepsilon_0}$$

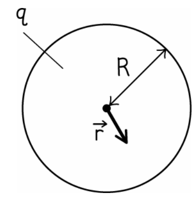
$$0 + 0 + 4\Phi_{E,\text{side}} = \frac{\lambda a}{\varepsilon_0}$$

$$\Phi_{E,\text{side}} = \frac{\lambda a}{4\varepsilon_0}$$

4. Evaluate Result Our answer should agree with the answer we obtained in Worked Problem 24.4, and it does.

24.8 Nonuniformly charged sphere

1. Getting Started We are given the radius of the charged sphere R, and the total charge contained in the sphere q. We are not given an explicit expression for the charge density, but we are told that it has to decrease near the outer surface of the sphere and increase near the center of the sphere, varying like $1/r$. This is shown below.



This is similar to Worked Problem 24.7, in that it involves a charge distribution that is not uniform, but which does have the same symmetry as the charged object. In this problem, we have spherical symmetry, whereas in Worked Problem 24.7 there was cylindrical symmetry. Another different between the two problem is that Worked Problem 24.7 involved a charge distribution that increased with r, whereas our distribution here decreases when r increases.

We do have enough information to solve the problem, although it may not seem like it at first. We need a charge distribution as a function of r, but we can find it just by knowing its dependence on r and by requiring that the total charge inside the sphere be q.

2. Devise Plan This problem has spherical symmetry both inside and outside the charged sphere. This means we need not consider angular changes in charge or in the electric field. The electric field (and the charge distribution itself) will be functions of r only. The electric field will point radially outward from the center of the sphere, regardless of whether the field is being calculated inside or outside the sphere.

We know that the volume charge density can be written in the form $\rho(r) = C/r$, where C is some unknown constant. We also know that if we integrate this charge density over the volume of the sphere, we are required to obtain q. Thus

$$q = \int_{\text{sphere}} \rho(r) dV$$

$$q = \int_{\text{sphere}} \rho(r) r^2 \sin(\theta) dr d\theta d\phi$$

$$q = \int_0^R \left(\frac{C}{r}\right) r^2 dr \int_0^\pi \sin(\theta) d\theta \int_0^{2\pi} d\phi$$

$$q = 4\pi C \int_0^R r dr$$

$$q = 4\pi C \left(\frac{R^2}{2}\right)$$

$$\Rightarrow C = \frac{q}{2\pi R^2}$$

Thus, we can write the volume charge density as $\rho(r) = \left(\dfrac{q}{2\pi R^2}\right)\left(\dfrac{1}{r}\right)$.

Because of the spherical symmetry of the problem, a spherical Gaussian surface is most useful. Note that Gauss's Law holds true regardless of what surface you choose. But if we choose a spherical surface centered on the same point that the charge distribution is centered, then symmetry enables us to claim that the electric field magnitude everywhere on the Gaussian sphere will be the same. If we chose a different shape, we could not make such a claim, and the math would be a great deal harder.

We will use a Gaussian sphere of radius r'. We can determine the charge enclosed by such a Gaussian surface by integrating the charge distribution up to that radius r'. In this context, r' is being treated as a constant (the radius of our Gaussian surface), whereas r is a variable that runs from 0 to r', so that we can integrate the charge distribution over the entire volume inside the Gaussian surface. Thus

$$q_{\text{encl}} = \int_{\substack{\text{Gaussian}\\\text{sphere}}} \rho(r)dV$$

$$q_{\text{encl}} = \int_{\text{sphere}} \rho(r)r^2\sin(\theta)drd\theta d\phi$$

$$q_{\text{encl}} = \int_0^{r'}\left(\frac{q}{2\pi R^2}\right)\left(\frac{1}{r}\right)r^2dr\int_0^{\pi}\sin(\theta)d\theta\int_0^{2\pi}d\phi$$

$$q_{\text{encl}} = \frac{2q}{R^2}\int_0^{r'}rdr$$

$$q_{\text{encl}} = q\left(\frac{r'}{R}\right)^2$$

The use of two variables (r and r') was critical to the process. But we can choose to call the distance from the origin whatever we like. So now that we have our result for the charge enclosed, we drop the prime and simply write

$$q_{\text{encl}} = q\left(\frac{r}{R}\right)^2 \tag{1}$$

3. Execute Plan Apart from dropping the prime symbol, we choose our Gaussian surface as before: a sphere of radius r and concentric with the spherical charge distribution. Gauss's Law tells us

$$\oint_{\substack{\text{Gaussian}\\\text{sphere}}} \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

Since we have chosen a Gaussian sphere that is concentric with the spherical charge distribution, both the area element and the electric field will always point radially outward (meaning they will always be parallel). Thus the scalar product yields $\cos(0)=1$ in all cases:

$$\oint_{\substack{\text{Gaussian}\\\text{sphere}}} EdA = \frac{q_{\text{encl}}}{\varepsilon_0}$$

Due to spherical symmetry, the electric field should have the same magnitude at all points along the spherical surface. Since E is a constant for all points over which we are integrating, it can be pulled out of the integral.

$$E\oint_{\substack{\text{Gaussian}\\\text{sphere}}} dA = \frac{q_{\text{encl}}}{\varepsilon_0} \tag{2}$$

Now the left hand side of equation (2) has an integral of differential area elements over the surface of a sphere, which is just the surface area of a sphere: $4\pi r^2$. Inside the spherical charge distribution, the charge enclosed by our Gaussian surface is given by equation (1). Inserting the surface area and equation (1) into equation (2), we obtain

$$E_{\text{in}}4\pi r^2 = \frac{q}{\varepsilon_0}\left(\frac{r}{R}\right)^2$$

$$E_{\text{in}} = \frac{q}{4\pi\varepsilon_0 R^2} \tag{3}$$

Recall that R is a constant. The radius of the Gaussian sphere (r) is what determined where we were evaluating the electric field. That distance has dropped out of our expression such that equation (3) tells us the electric field inside the spherical charge distribution is constant.

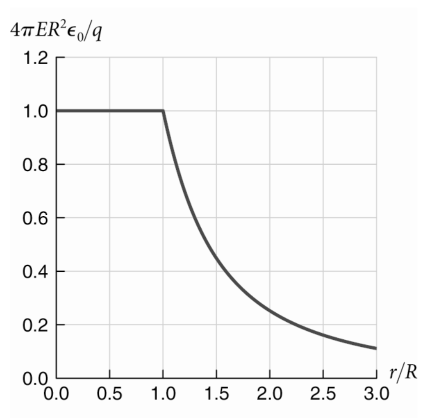
Outside the sphere, our assertion that the charge enclosed is given by equation (1) is not valid. Outside the spherical charge distribution, the charge enclosed by a Gaussian sphere of radius $r > R$ is just q. In that case, equation (2) becomes

$$E_{\text{out}} 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E_{\text{out}} = \frac{q}{4\pi\varepsilon_0 r^2} \tag{4}$$

Thus we find that the electric field outside of the spherical charge distribution just looks like the electric field due to a charged particle at the center of the sphere.

Note that at the surface of the sphere, when $r = R$, equations (3) and (4) yield the same result. Using these two expressions, we can plot the electric field as a function of radial distance from the center of the charge distribution:

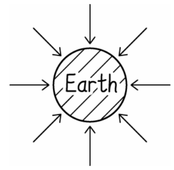


4. Evaluate Result We are accustomed to seeing expressions for the electric field at a given point that depend on the distance from that point to a charged object or two the center of a charge distributions. A constant result inside the charged sphere may have been unexpected. But note that the volume of a sphere increases with radius like r^3. Since the charge distribution varies like $1/r$, this means the charge contained in a sphere increases like r^2. Thus the total amount of charge inside of a spherical surface grows like r^2, while the electric field due to any point-like section of that distribution drops off like $1/r^2$. It is perfectly plausible that these two effects would cancel to make the electric field inside the charge distribution independent of r. So while it may have been unexpected, our answer does make logical sense.

Questions and Problems

24.1. (*a*) The electric field will point radially outward from the origin, meaning at the point $(0.60,\ 1.2)$ the direction will be from the origin to $(0.60,\ 1.2)$, or $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1.2}{0.6}\right) = 63°$ counterclockwise from the $+x$ axis. (*b*) Now that the charge is negative, the electric field lines will point radially inward, meaning the direction of the electric field at $(0.60,\ 1.2)$ will point from $(0.60,\ 1.2)$ to the origin, or $243°$ counterclockwise from the $+x$ axis.

24.2.



24.3. No. Although in some sources you may find an electric field with a magnitude of zero is sometimes indicated by a dot.

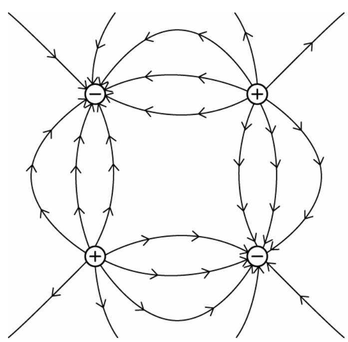
24.4. Both are correct. What is important is that twice as many field lines emanate from the first object as terminate on the second object. How many lines you choose to draw is up to the artist.

24.5. (*a*) The particle accelerates along the direction of $\vec{E}$. (*b*) The initial velocity would be different, but the path would not change. (*c*) The component of the initial velocity parallel to the field $(v_{\mathrm{i}}\cos(\theta))$ would just make the particle move more quickly. But the trajectory would be changed by the component of the initial velocity perpendicular to the field $(v_{\mathrm{i}}\sin(\theta))$. This would cause the trajectory to change from linear to parabolic. (*d*) In case (*a*) the particle would now accelerate in the direction opposite $\vec{E}$. In case (*b*), the particle would initially move in the direction of $\vec{E}$, and then reverse course and travel against $\vec{E}$. In case (*c*), the trajectory would again change from linear to parabolic.

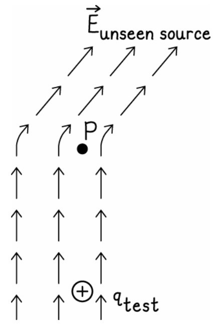
24.6. 12 lines terminate on the particle carrying $-3\ \mu\mathrm{C}$, 8 lines terminate on the particle carrying $-2\ \mu\mathrm{C}$.

24.7. (*a*) It would accelerate to the right. (*b*) It would accelerate to the left. (*c*) It would accelerate to the left. (*d*) It would accelerate to the right and then curve downward toward the negative charge. (*e*) Although the initial velocity would no longer be to the right, the acceleration would still be to the right. The trajectory would curve back toward the negative charge.

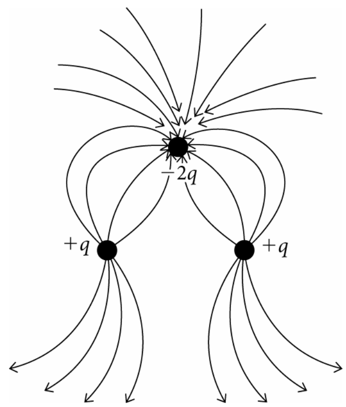
24.8.



24.9. No. The electric field points in the direction of the force that would be exerted on a positively charged particle. This means it also points in the direction of the acceleration, but not necessarily the position. For example in Figure_24.009_Solution the particle would accelerate upward until it reaches point P. At P, the acceleration would suddenly change to have a horizontal component. But the velocity of the particle would not change instantly. The path would deviate from the field line at P.



24.10.



24.11. The quarter covers one or more charged particles carrying a combined negative charge equal in magnitude to 7/17 of the charge on the positively charged particle shown to the right of the quarter.

24.12. We describe our method in two dimensions, simply because a three-dimensional picture of vectors would be extremely difficult to read and would probably not be useful. We could begin by calculating the electric field at a particular point (x, y).

Step1: Calculate the components of the electric field at (x, y) due to a single point source (the nth source) located at (x_{0n}, y_{0n}):

$$E_{n,x} = \frac{kq_n}{(x - x_{0n})^2 + (y - y_{0n})^2} \frac{(x - x_{0n})}{\sqrt{(x - x_{0n})^2 + (y - y_{0n})^2}}$$

$$E_{n,y} = \frac{kq_n}{(x - x_{0n})^2 + (y - y_{0n})^2} \frac{(y - y_{0n})}{\sqrt{(x - x_{0n})^2 + (y - y_{0n})^2}}$$

Step 2: Find the total electric field at (x, y) by summing over all contributions to each component separately:

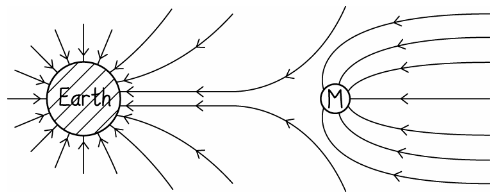
$$E_x(x, y) = \sum_n E_{n,x}$$

$$E_y(x, y) = \sum_n E_{n,y}$$

This allows us to draw the vector arrow describing the electric field at (x, y).

Step 3: Iterate in either of two ways. (1) Step through values of $x \to x + \Delta x$ and $y \to y + \Delta y$, and repeat above calculation to create array of regularly spaced arrows showing $\vec{E}$ direction at any location. (2) Use each electric field vector to step forward to new value of (x, y): $x \to x + E_x f$ and $y \to y + E_y f$, where f is constant having units of meters-Coulombs per Newton. This operation moves location of interest a small step in direction of initial electric field, such that next electric field is calculated at location that lies along initial electric field. This operation produces sequence of overlapping electric field vectors that form nearly continuous electric field lines.

24.13. (*a*) With gravitational field lines defined as pointing in direction of gravitational force exerted on any object having mass, Earth-moon gravitational field is as shown in Figure.



(*b*) Both objects must carry negative charge. (*c*) All gravitational forces are attractive, but electric forces can be attractive or repulsive. Gravitational field lines always terminate on all objects in the field; electric field lines emanate from positively charged objects and terminate on negatively charged objects.

24.14. (*a*) Yes. The two surfaces are at the same point in space. Since there is no polarization, the field at one surface must be the same as the other. (*b*) Yes. The electric field is proportional to the field line density.

24.15. The largest electric field is at the point where the field lines are most dense, in the lower right.

24.16. Let us say that the field lines are horizontal, and that the surface is initially vertical. If we tilt the surface by $60°$, a square meter of surface will now have a vertical height of only $(1 \text{ m})\cos(60°) = 0.50 \text{ m}$. Thus the square meter of surface will only have half as many field lines passing through it: 1000 field lines.

24.17. The electric field is proportional to the field line density. Thus $E_B > E_A > E_C$.

24.18. Whether pointing away from positively charged wire or toward negatively charged wire, field lines spread out radially from wire all along wire length. Choose arbitrary length ℓ along wire, and picture field lines forming infinite number of concentric imaginary cylinders centered on wire, each cylinder having length ℓ and radius r, where r is radial distance from wire. Surface area of curved portion of each cylinder is $2\pi r \ell$. Thus field line density through curved portion of any cylinder is $N/2\pi r \ell$, where N is constant number of field lines passing through curved portion of any given cylinder and r is radius of that cylinder. Because field line density is proportional to electric field magnitude, this expression for field line density as function of r shows that $E \propto 1/r$.

24.19. Treat plate as infinite and lying horizontally in xy plane. Because arrangement is reflection symmetric about xz and yz planes, electric field can have only E_z component. For finite-sized plate, this symmetry is broken as you move away from plate, but for infinite plate, symmetry argument holds for any distance from plate. This means field line density is constant around plate (the lines never bend toward or away from any region of space) and electric field is uniform.

24.20. (*a*) A; the greatest electric field occurs where the field lines are most dense. (*b*) C; the smallest electric field occurs where the field lines are least dense. (*c*) Because the electric field is proportional to the number of field lines we can write $\dfrac{E_A}{E_C} = \dfrac{\text{field lines at A}}{\text{field lines at C}} = 3.$

24.21. (*a*) The particle in the strongest electric field will experience the greatest acceleration and will thus require the smallest amount of time. Thus $\Delta t_{\text{C}\to\text{D}} < \Delta t_{\text{B}\to\text{C}} < \Delta t_{\text{A}\to\text{B}}$. (*b*) There is no change, as long as the pellets do not leave the area with the field.

24.22. (*a*) $1.5\ \text{C}\times\left(\dfrac{16\ \text{lines}}{1\ \text{C}}\right)=24\ \text{lines}.$ These 24 lines leave the particle because it is positively charged. (*b*) $0.375\ \text{C}\times\left(\dfrac{16\ \text{lines}}{1\ \text{C}}\right)=6\ \text{lines}.$ These 6 lines enter the particle because it is negatively charged. (*c*) No, not precisely. This charge would require 12.8 field lines leaving it.

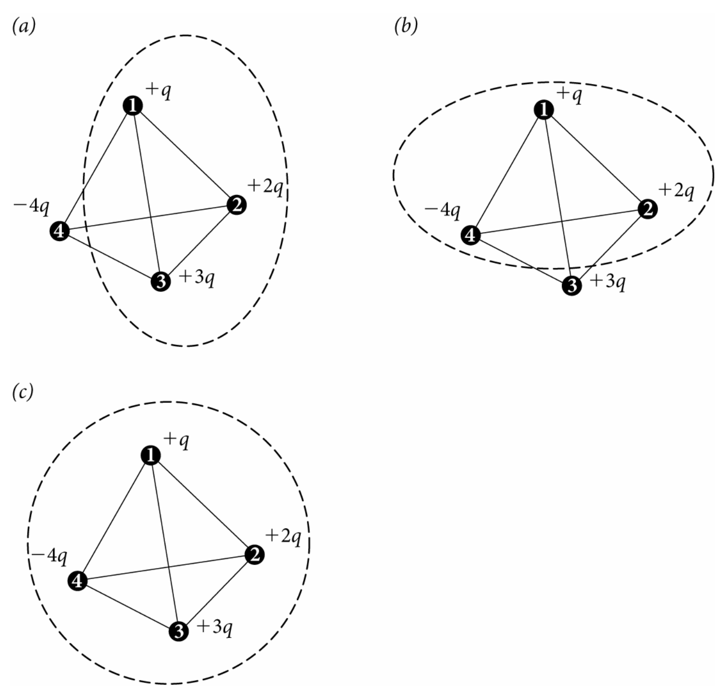
24.23. Let us assume the field lines are horizontal. Call the area of each surface $A=\ell\times h.$ The field line density at A is $\dfrac{N}{A}$. Surface B is tilted such that field lines make a $72°$ angle with the normal line. Starting from normal incidence, this could be accomplished by tilting surface B around an axis parallel to its length, height or anything in between. For simplicity, let us assume it has been rotated around an axis parallel to its length. Thus the vertical height of the surface is only $h\cos(72°)$. Thus the $N/2$ field lines could also pass through a vertical surface (at normal incidence) with dimensions $\ell\times h\cos(72°)=0.309A.$ So the field line density at this location is $\dfrac{N/2}{0.309A}=1.6\left(\dfrac{N}{A}\right).$ The field line density is greater at B by a factor of 1.6.

24.24. No. If the charged object is inside the surface the flux will be positive. If the charged object is outside the surface then all field lines that enter the surface must exit again, and the total flux will be zero.

24.25. $A=B=C.$ Because all surfaces enclose the same amount of charge, all surfaces have the same field line flux.

24.26. Surface (*c*) encloses a net charge of zero, and thus has no net flux. Surface (*b*) encloses a total charge q. Surface (*a*) encloses a total charge $2q$ and therefore has the greatest flux. Thus $(c)<(b)<(a)$.

24.27.



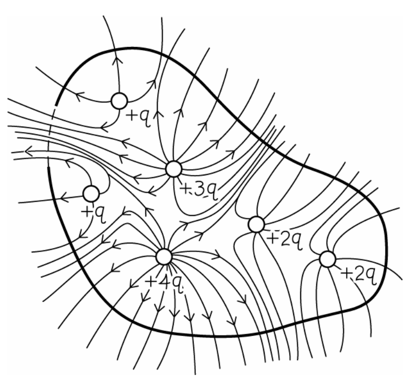
24.28. (*a*) From the information given, we know the diagram displays 12 field lines per charge 1 C. Surface A encloses $-5\ \text{C}$ of charge. Thus the diagram should display $5\ \text{C}\times\left(\dfrac{12\ \text{field lines}}{1\ \text{C}}\right)=60$ field lines. Because the charge is negative the diagram should show 60 field lines inward. (*b*) Surface B encloses -1 C of charge. Thus the diagram should display $1\text{ C}\times\left(\dfrac{12\text{ field lines}}{1\text{ C}}\right)=12$ field lines. Because the charge is negative the diagram should show 12 field lines inward. (*c*) Surface C encloses -3 C of charge. Thus the diagram should display $3\text{ C}\times\left(\dfrac{12\text{ field lines}}{1\text{ C}}\right)=36$ field lines. Because the charge is negative the diagram should show 36 field lines inward. (*d*) Surface A encloses $+1\text{ C}$ of charge. Thus the diagram should display $1\text{ C}\times\left(\dfrac{12\text{ field lines}}{1\text{ C}}\right)=$ 12 field lines. Because the charge is positive the diagram should show 12 field lines outward. (*e*) Surface A encloses -2 C of charge. Thus the diagram should display $2\text{ C}\times\left(\dfrac{12\text{ field lines}}{1\text{ C}}\right)=60$ field lines. Because the charge is negative the diagram should show 24 field lines inward.

24.29. The charge inside this closed surface must be equal to the charge on 4 protons, $+4e$. All that can be said about the number of fundamental particles present is that there can only be an even number, and there must be 4 or more fundamental particles.

24.30. (*a*) Yes. Such a surface must enclose both charged particles. (*b*) Yes. Such a surface must enclose the positively charged particle only. (*c*) Yes. Such a surface must enclose the negatively charged particle only. (*d*) No.

24.31. Let us consider our charged object to be at the origin. We want to consider forces being exerted on this charge, so all discussion of electric fields in the will refer to fields due to other particles that our charged object can interact with. In order for it to be held in stable equilibrium, the value of the electric field at the origin would have to be zero. Further, stable equilibrium means there must be a restoring force. If the charged object is moved slightly in the $+x$ direction, it must experience a force in the $-x$ direction, and vice versa. This applies for any axis. Hence, if the object is positively charged, there must be an electric field pointing radially inward toward the origin. If the object is negatively charged, there must be an electric field pointing radially outward away from the origin. In either case, surrounding the origin with an imaginary sphere and integrating over that surface would yield a non-zero value. According to Gauss' Law, that means there must be a charge enclosed by the sphere (remember we are only considering electric fields due to charges other than that on our charged object). So, this situation is not possible unless there is another charged particle in our volume of interest. But if we keep shrinking our imaginary sphere to a smaller and smaller radius around the origin, we see that there must be a net non-zero charge right at the origin. But equilibrium requires that the field at the origin is zero. Hence the situation is not possible even if we are allowed to put charge in our volume of interest. Charged objects cannot be held in stable equilibrium by electrostatic forces alone.

24.32. Consider the charge distribution below. The distribution is surrounded by a closed surface that is represented by a combination of a solid line and a dashed portion. Were the dashed portion to be removed, a significant number of field lines would not cross the surface, so the calculation of the charged contained within would be significantly altered.



24.33. (*a*) No. (*b*) The relationship still holds for dipoles. There is simply not a Gaussian surface that makes the calculation simple.

24.34. No. This is a dipole. The electric field is non-zero in magnitude at any arbitrary distance away.

24.35. By symmetry, the flux through each of the six faces must be the same. Since the flux through all six faces must be Φ, we have $6\Phi_{1\text{ face}} = \Phi$ or $\Phi_{1\text{ face}} = \Phi/6$.

24.36. (*a*) There is no dependence of the electric field strength on r inside the hole. The field is zero. (*b*) The field strength decreases like $1/r$. To see this consider surrounding the charged cylinder with a cylindrical Gaussian surface. The number of field lines leaving the charged cylinder N is fixed, but the surface area of the Gaussian surface increases with r since the area of the curved side of a cylinder is $2\pi\ell r$. Thus the field line density a given radial distance away is $N/(2\pi\ell r)$.

24.37. There are many other shapes that would be equally useful. A cubic box or rectangular box (similarly centered on the plate) would work equally well.

24.38. Yes, this is possible. It is simply not useful. The integration process would be very ugly, since the magnitude of the electric field and the angle it makes with the line normal to the surface would be different at different points on the sphere.

24.39. (*a*) The electric field is the same as if the charge were located at the center of the sphere. Thus the electric field is $E = \dfrac{kq}{(2R)^2} = \dfrac{1}{4}\left(\dfrac{kq}{R^2}\right) = \dfrac{E_0}{4}$. (*b*) $E = \left(\dfrac{kq}{R^2}\right) = E_0$. (*c*) The volume charge density is $\rho = \dfrac{q}{V} = \dfrac{3q}{4\pi R^3}$. Thus the charge enclosed by a Gaussian sphere of radius $r = R/4$ is $q_{\text{enc}} = \rho V = \left(\dfrac{3q}{4\pi R^3}\right)\left(\dfrac{4}{3}\pi\left(\dfrac{R}{4}\right)^3\right) = \dfrac{q}{64}$. Now, using Gauss's Law and the symmetry of the charged sphere as well as the Gaussian sphere, we can write

$$\oint_{\text{sphere } r=R/4} \vec{E}\cdot d\vec{A} = E \oint_{\text{sphere } r=R/4} dA = E\left(4\pi\left(\frac{R}{4}\right)^2\right) = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\Rightarrow E = \left(\frac{q}{64}\right)\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{16}{R^2}\right) = \frac{1}{4}\left(\frac{kq}{R^2}\right) = \frac{E_0}{4}$$

So now $E = E_0/4$.

24.40. Removing a small, disk-shaped region from the spherical shell means there will be no charge at the location of the disk. Electrically, this is no different than having the complete spherical shell with a negatively charged disk of the same charge density at that location. Inside a complete charged spherical shell the electric field is zero, so the only electric field at the center of the sphere must come from the negatively charged disk. We are told that the hole removes 0.01% of the material, meaning the surface area of the disk is $\pi R_{\text{disk}}^2 = (10^{-4})4\pi R^2 \Rightarrow R_{\text{disk}} = \left(\dfrac{1}{50}\right)R$. From example 23.6, we know the form of the electric field along the symmetry axis of a disk:

$$E = 2k\pi\rho\left[1 - \frac{z}{\sqrt{z^2 + R_{\text{disk}}^2}}\right]$$

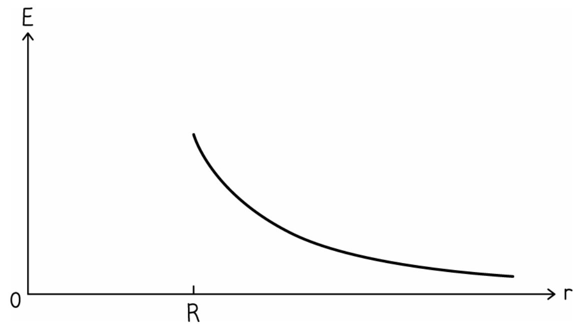
The variable z is the distance from the center of the disk along its axis of symmetry at which the electric field is being measured, which in this case is R. The surface charge density is the same as on the sphere $\sigma = \dfrac{q}{4\pi R^2}$. Thus

$$E=\frac{kq}{2R^2}\left[1-\frac{R}{\sqrt{R^2+R_{\text{disk}}^2}}\right]=\frac{kq}{2R^2}\left[1-\frac{1}{\sqrt{1+\frac{R_{\text{disk}}^2}{R^2}}}\right]=\frac{kq}{2R^2}\left[1-\frac{1}{\sqrt{1+\left(\frac{1}{50}\right)^2}}\right]=(10^{-4})\frac{kq}{R^2}$$

Because the disk we added to the complete spherical shell carried a negative charge, the electric field at the center of the sphere must be directed toward the hole. Thus $\vec{E}=10^{-4}\frac{kq}{R^2}$ toward the hole.

24.41. Yes, because both the charge enclosed by the surfaces and the flux through the two surfaces are proportional to the area of the slab that is enclosed. This assumes that the plate is large enough to be treated as approximately infinite. Her cube is more likely to be in the regime where it is valid to treat the plate as being infinite, because her cube is shorter than your cylinder.

24.42.



24.43. (*a*) Inside the material of the metal the electric field is zero. Thus if we enclose a cavity with a spherical (or other) Gaussian surface the electric field at all points on the surface must be zero. Thus by Gauss' Law we know the total charge enclosed by the surface must be zero. This means there must be a charge of $-q$ on the inner surface of cavity 1, and $+2q$ on the inner surface of cavity 2. (*b*) The metal object is neutral. If there is a total of $+q$ on the inner surface of all cavities, this leaves $-q$ on the outer surface.

24.44. (*a*) If we construct a Gaussian surface with the same shape as the metal object, such that all points on the Gaussian surface are in the metal material, then the electric field at all points on our Gaussian surface must be zero. From Gauss' Law we can see that the total charge enclosed must also be zero. Since there is a charge of $-2q$ on the inner surface of the metal, there must be $+2q$ in the cavity. (*b*) The total charge on the object is the sum of the charge on the inner and outer surfaces (there can't be any inside the material of the conductor). Thus the total charge is $-q$.

24.45. Because metal is electrical conductor, it remains in electrostatic equilibrium even when place in external electric field. Electric field inside metal box is therefore zero and any device inside box is not exposed to electric field.

24.46. (*a*) Consider a conductor that has one edge that is flat for some time interval, and then begins to curve. Call the flat region I, and the curved region II, and consider an electron at point P right between the two regions. Recall that all charge will accumulate on the surface of the conductor, so we only need to consider charged particles at the surface. All the electrons in region I exert a repulsive force on the electron at P, and these repulsive forces are all directed exactly parallel to the surface. All electrons in region II exert a repulsive force on the electron at P, but these forces are not parallel to the surface. Only some component of these forces are parallel to the surface. But the sum of all forces exerted on P from electrons in region I must be equal in magnitude to the sum of all forces exerted on the electron at point P by electrons in region II. Otherwise, there would be a nonzero vector sum of forces on the electron at P that is parallel to the surface, and this electron would accelerate (meaning the system was not in electrostatic equilibrium). So if the components parallel to the surface of the repulsive forces exerted by region II on the electron at P must be as large as all the forces exerted by region I, then there must be more charge in region II. The same argument can be repeated for two curved regions by noting that the region of higher curvature will exert forces that have a smaller component parallel to the surface. Hence the surface charge density will be greater on regions of the surface that have higher curvature.

(*b*)

24.47. (*a*) The electric field everywhere in the material of the inner shell must be zero. Choosing a spherical Gaussian surface entirely inside that material shows us that there can be no surplus charge inside the material or on the inner surface. Thus the entire surplus charge of $+2q$ must be on the outer surface of this inner sphere. (*b*) Similar to the case in part (*a*) the electric field in the material of the outer shell must also be zero. As in (*a*) we can use Gauss' Law to see that the surplus charge enclosed by a Gaussian surface entirely in the material of the outer sphere is zero. Thus, there must be a charge of $-2q$ on the inner surface of the outer sphere to balance the $+2q$ on the inner sphere. (*c*) Since the outer sphere carries a total surplus charge of $-q,$ and a charge of $-2q$ is on the inner surface, the outer surface must have $+q.$ (*d*) Zero charge on outer surface of inner shell, zero charge on inner surface of outer shell, charge $+q$ on outer surface of outer shell. The two sphere are now part of the same conducting body, so all surplus charge will flow to the outer surface.

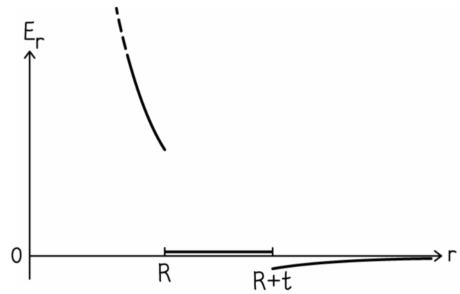
24.48. (*a*) The electric field everywhere in the material of the shell must be zero. Choosing a spherical Gaussian surface entirely inside that material shows us that there can be no surplus charge inside the Gaussian surface. This means the charge on the inner surface of the shell must be equal in magnitude and opposite in sign to the charge on the ball. Thus a charge of $+2q$ must be on the inner surface of this sphere. (*b*) The sphere carries a total surplus charge of $+q.$ A charge of $+2q$ is on the inner surface, meaning the outer surface must carry a charge $-q.$ (*c*) The charge remains $+2q$ on inner surface and $-q$ on outer surface, but now the charge distribution is nonuniform on each surface.

24.49. The conducting block is neutral, so the fact that it carries $-5q$ on its outer surface means that a total of $+5q$ must have accumulated on the inner surfaces of the cavities. Because the block is conducting, a Gaussian surface completely contained inside the material of the block cannot have any flux through it, and therefore cannot contain any surplus charge. Thus Gauss' Law tells us that the charge that accumulates on the inner surface of a cavity must be equal in magnitude and opposite in sign to the charge suspended inside the cavity. Thus the total charge on the inner surface of all cavities can be written: $\sum q_{\text{inner}} = -q - q - 2q + q_? = +5q \Rightarrow q_? = +9q.$ If the charge on the inner surface of the fourth chamber is $+9q,$ a second application of Gauss' Law tells us that there must be $-9q$ inside the cavity.

24.50. (*a*) The electric field everywhere in the conducting material must be zero. Choosing a spherical Gaussian surface entirely inside that material shows us that there can be no surplus charge inside the Gaussian surface. This means the charge on the inner surface of the cavity must be equal in magnitude and opposite in sign to the charge placed inside the cavity. Thus a charge of $-q$ must be on the inner surface of this sphere. Because the $+q$ charge is centered inside the cavity, the charge on the inner surface of the cavity will be distributed uniformly. (*b*) Since the sphere carries a surplus charge of $-2q$ and $-q$ is already on the inner surface, the outer surface must carry a charge $-q.$ Because the cavity is not centered in the sphere, this $-q$ charge will be distributed non-uniformly.

24.51. (*a*) The electric field everywhere in the conducting material must be zero. Choosing a spherical Gaussian surface entirely inside that material shows us that there can be no surplus charge inside the Gaussian surface. This means the charge on the inner surface of the spherical shell must be equal in magnitude and opposite in sign to the charge placed inside the shell at the origin. Thus a charge of $-2q$ must be on the inner surface of this spherical shell. (*b*) The spherical shell carries a surplus charge of $-3q,$ and $-2q$ is on the inner surface. Thus $-q$ must remain on the outer surface.

(*c*)



24.52. The setup is symmetric, such that the flux through any face of the cube should be the same. This means the total flux through the surface is $\Phi = 6\Phi_{\text{side}} = \dfrac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow q_{\text{enc}} = 6\varepsilon_0\Phi_{\text{side}} = 6(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.2\times10^2\ \text{N}\cdot\text{m}^2/\text{C}) =$ $2.8\times10^{-8}\ \text{C}$ or 28 nC.

24.53. (*a*) Yes. $\Phi = AE\cos(\theta) \Rightarrow \theta = \cos^{-1}\left(\dfrac{\Phi}{AE}\right) = \cos^{-1}\left(\dfrac{(6\ \text{N}\cdot\text{m}^2/\text{C})}{(3.0\ \text{m}^2)(10\ \text{N/C})}\right) = 78°.$ The required flux occurs when there is an angle of $78°$ between the electric field and the line normal to the surface. (*b*) No, at normal incidence the flux is only $30\ \text{N}\cdot\text{m}^2/\text{C}$. There is nothing we can do to make the flux larger than this, without increasing the area and/or the electric field.

24.54. (*a*) $\Phi_{\text{E, spherical}} = \Phi_{\text{E, cylindrical}}$. This follows from Gauss' Law, since by construction they enclose the same charge. (*b*) The cylindrical Gaussian surface is more appropriate. Because the electric field magnitude is constant over the cylindrical surface, and because electric field lines all pass through surface in the same orientation, either parallel or perpendicular to normal to the surface, the integration to obtain Φ_{E} is relatively simple. In contrast, the electric field magnitude varies over the spherical Gaussian surface and field lines pass through the surface at different angles, making the integration for Φ_{E} more complex with spherical surface.

24.55. (*a*) Using Gauss' Law, we can write $\oint_{\text{surface}} \vec{E}\cdot d\vec{A} = \dfrac{q_{\text{enc}}}{\varepsilon_0} = \dfrac{(6.0\times10^{-6}\ \text{C})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 6.8\times10^5\ \text{N}\cdot\text{m}^2/\text{C}.$ (*b*) Nothing has changed about the charge enclosed, so nothing has changed about the total flux through a surface surrounding that charge. The flux is still $6.8\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$.

24.56. Imagine a Gaussian surface enclosing both charged particles as being spherical and centered at the dipole midpoint. Split this Gaussian surface into two along plane that bisects the dipole axis and is perpendicular to the axis. You now have two Gaussian surfaces 1 and 2, each consisting of a curved hemispheric portion and a flat circular portion. These Gaussian surfaces are identical except that one of them—surface 1, say—encloses the positively charged particle and other encloses the negatively charged particle. Hence, electric fluxes through 1 and 2 have same magnitude but opposite signs:

$$\Phi_{\text{E1}} = \Phi_{\text{E1,hemispheric portion}} + \Phi_{\text{E1,circular portion}} = -\Phi_{\text{E2}} = -\Phi_{\text{E2,hemispheric portion}} - \Phi_{\text{E2, circular portion}}$$

Because $\Phi_{\text{E1,circular portion}} = -\Phi_{\text{E2, circular portion}}$ (electric field lines leaving 1 through the circular portion of 1 enter 2 through the circular portion of 2). Hence $\Phi_{\text{E1,hemispheric portion}} = -\Phi_{\text{E2,hemispheric portion}}$. Thus, to determine the electric flux through the original sphere, you add two terms that are equal in magnitude and opposite in sign, making the sum zero.
As long as the Gaussian surface you choose encloses both poles of the dipole, the shape you choose for the surface does not affect the value you calculate for Φ_{E}.

24.57. $\Phi_{\text{E, right side }(a)} = \Phi_{\text{E, right side }(b)} = \Phi_{\text{E, right side }(c)}$. Because all three surfaces enclose the same amount of charge, the flux through the entire surface is the same in all three cases. Further, the left side is identical in each case, so the flux through the left side must be identical in each case. Thus the flux through the right side must also be the same in all three cases.

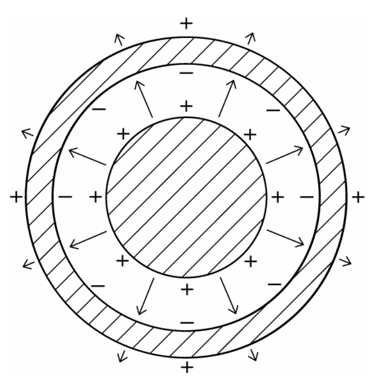
24.58. (*a*) All electric field lines that pass through the loop must also pass through the net. The flux through a closed boundary is independent of the surface you choose to consider. Thus, it suffices to find the flux through the loop itself: $\Phi = EA\cos(\theta) = (150\text{ N/C})\pi(0.200\text{ m})^2\cos(0) = 18.8\text{ N}\cdot\text{m}^2/\text{C}.$ (*b*) The flux decreases.

24.59. (*a*) The electric field everywhere inside the conducting shell must be zero. So if we imagine a Gaussian sphere in the material of the shell and concentric to it, the flux through this surface must be zero. Thus the total charge enclosed by this Gaussian sphere must also be zero. This means the charge on the inner surface of the spherical shell must be equal in magnitude and opposite in sign to the charge at the center. Thus the charge at the center is $+2q.$ (*b*) Outside the sphere, the electric field is the same as if all charge on or inside the sphere were located at the center. The electric field is $E = \dfrac{3kq}{r^2}$, which corresponds to a charge magnitude of $3q.$ Since the field is toward the sphere, we know the total charge on/in the sphere is $-3q.$ Since $+2q$ is insider the sphere, the sphere itself must hold $-5q.$

24.60. (*a*) The electric field everywhere inside the conducting shell must be zero. So if we imagine a Gaussian sphere in the material of the shell and concentric to it, the flux through this surface must be zero. Thus the total charge enclosed by this Gaussian sphere must also be zero. This means the charge on the inner surface of the spherical shell must be equal in magnitude and opposite in sign to the charge at the center. Thus the charge density on the inner surface is $\sigma_{\text{inner}} = \dfrac{q_{\text{inner}}}{4\pi R_{\text{inner}}^2}\dfrac{(-30.0\times10^{-9}\text{ C})}{4\pi(0.100\text{ m})^2} = 239\text{ nC/m}^2$ of negative charge. (*b*) The inner surface of the shell carries a total of $-30.0\times10^{-9}\text{ C}$ of charge. Since the shell is neutral, there must be $+30.0\times10^{-9}\text{ C}$ of charge on the outer surface. Thus $\sigma_{\text{outer}} = \dfrac{q_{\text{outer}}}{4\pi R_{\text{outer}}^2}\dfrac{(+30.0\times10^{-9}\text{ C})}{4\pi(0.150\text{ m})^2} = 106\text{ nC/m}^2$ of positive charge. (*c*) Let $q = 30.0\text{ nC}.$ Then

$$E(r < R_{\text{inner}}) = \frac{kq}{r^2}\text{ directed away from the sphere center}$$
$$E(R_{\text{inner}} < r < R_{\text{outer}}) = 0$$
$$E(r > R_{\text{outer}}) = \frac{kq}{r^2}\text{ directed away from the sphere center}$$

24.61. (*a*) The key features are that all charge is located on surfaces of the conductors, all electric fields are zero inside the conducting material and radially outward from the center, otherwise. The electric field is weaker outside the outer shell than in between the sphere and the shell, as indicated by denser electric field lines between the sphere and shell.



(*b*) The charge on the solid sphere accumulates on the outer surface, such that $\sigma_{\text{solid}} = \dfrac{q_{\text{solid}}}{4\pi R_{\text{solid}}^2} = \dfrac{(5.0\times10^{-9}\ \text{C})}{4\pi(0.060\ \text{m})^2} = 1.1\times10^2\ \text{nC/m}^2$ of positive charge. The charge on the inner surface of the spherical shell and the charge on the solid sphere must be equal in magnitude and opposite in sign. Thus $\sigma_{\text{inner}} = \dfrac{q_{\text{inner}}}{4\pi R_{\text{inner}}^2} = \dfrac{(-5.0\times10^{-9}\ \text{C})}{4\pi(0.100\ \text{m})^2} = 40\ \text{nC/m}^2$ of negative charge. Finally, since the spherical shell carries a charge of $-4.0\ \text{nC}$ and $-5.0\ \text{nC}$ is on the inner surface, there must be $+1.0\ \text{nC}$ of charge on the outer surface. Thus and $\sigma_{\text{outer}} = \dfrac{q_{\text{outer}}}{4\pi R_{\text{outer}}^2} = \dfrac{(1.0\times10^{-9}\ \text{C})}{4\pi(0.120\ \text{m})^2} = 5.5\ \text{nC/m}^2$ of positive charge.

(*c*) The electric field is directed away from the center and has magnitude:

$$E = \begin{cases} 0, & r \le 60\ \text{mm} \\ \dfrac{kq_1}{r^2}, & 60\ \text{mm} < r < 100\ \text{mm} \\ 0, & 100\ \text{mm} \le r \le 120\ \text{mm} \\ \dfrac{kq_2}{r^2}, & r > 120\ \text{mm} \end{cases}, \text{ where } q_1 = 5.0\ \text{nC and } q_2 = 1.0\ \text{nC}$$

24.62. (*a*) The charged particles at the vertices of the equilateral triangle are a distance $\dfrac{\sqrt{3}}{3}a \approx 0.58a$ from the origin. So the spherical Gaussian surface of radius $a/4$ does not enclose any charge. Thus the flux through this surface is zero. (*b*) Now the radius of the surface is large enough to enclose all three charged particles. Thus the flux is $+3q/\varepsilon_0$.

24.63. (*a*) In order for the total flux through the closed surface to be zero, the flux through the fourth side must be $-4.0\ \text{N}\cdot\text{m}^2/\text{C}$. (*b*) The total flux through the closed surface is now $\Phi = -4.0\ \text{N}\cdot\text{m}^2/\text{C} = \dfrac{q_{\text{enc}}}{\varepsilon_0} \Rightarrow q_{\text{enc}}$ $= \Phi\varepsilon_0 = (-4.0\ \text{N}\cdot\text{m}^2/\text{C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = -3.5\times10^{-11}\ \text{C}$.

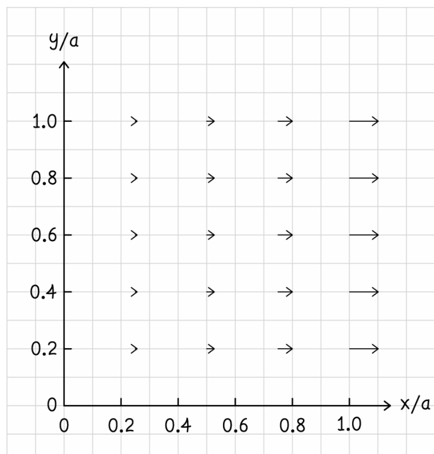
24.64. They emerge through the curved side of the cylinder.

24.65. (*a*) Yes; use a cylindrical Gaussian surface. (*b*) No. (*c*) Yes; use a spherical Gaussian surface. If $r \gg \ell$, then the rod can be roughly treated like a particle.

24.66. Let $\vec{E}_{\text{orig}}(\vec{r})$ be electric field due to original collection of *N* particles, where $\vec{r}$ points from origin of Cartesian coordinate system to some point in space along Gaussian surface. Let position of *N*th particle be given by $\vec{r}_N$. Electric flux through any Gaussian surface surrounding original collection is $\oint \vec{E}_{\text{orig}}(\vec{r})\cdot d\vec{a}$. To show that this expression is equal to sum of fluxes due to individual particles, let $\vec{E}_N(\vec{r})$ be electric field due to *N*th particle. Then electric flux due to *N*th particle is $\oint \vec{E}_N(\vec{r})\cdot d\vec{a}$, and sum of all such fluxes is $\sum_N \oint \vec{E}_N(\vec{r})\cdot d\vec{a}$. Because summation and integration can be interchanged (provided both yield finite results), this can be written as $\oint \sum_N (\vec{E}_N(\vec{r})\cdot d\vec{a})$. In this expression, the scalar product selects only component of $\vec{E}$ normal to Gaussian surface, at every point $\vec{r}$ along surface. Hence sum is sum of scalars at a point, $\oint\left(\sum_N \vec{E}_N(\vec{r})\right)\cdot d\vec{a} = \oint \vec{E}_{\text{orig}}(\vec{r})\cdot d\vec{a}$.

24.67 (*a*) This problem can be solved by using the expression for the electric field due to an infinitely long charged rod, and integrating $\oint_{\text{sheet}} \vec{E}\cdot d\vec{A}$. However, we could also use the symmetry of the setup to find an answer without integrating. Let us add three other such sheets to form a box (open at the ends) enclosing a portion of the wire. These four sheets are all equivalent, meaning we can rotate them around the wire without changing anything. Thus the flux through each one must be the same. We already know that the electric field due to a positively charged, long wire is radially outward away from the wire. Thus, we can cap the ends of our box, noting that there is no flux through the square ends. Now the total flux through our closed surface is $\Phi_{\text{closed}} = 4\Phi_{\text{side}} = \dfrac{q_{\text{enc}}}{\varepsilon_0}$. The charge enclosed is just $q_{\text{enc}} = \lambda\ell$. Thus $\Phi_{\text{side}} = \dfrac{\lambda\ell}{4\varepsilon_0}$. (*b*) The electric flux would increase. All points on the surface of the sheet would move closer to the wire, meaning the electric field at all points on the sheet would become stronger.

24.68. (*a*)



(*b*) We determine this by calculating the flux through the cubic shell. Because the electric field is entirely in the x direction, only the square faces at $x = 0$ and $x = a$ will have non-zero flux. The electric field magnitude at $x = 0$ is zero, such that the flux through that face is also zero. We are left with $\Phi_{\text{cube}} = \Phi_{x=a\ \text{side}} = E(x=a)A\cos(\theta) = ba^4$. Using Gauss' Law, we can write $q_{\text{enc}} = b\varepsilon_0 a^4$.

24.69. (*a*) 12 electric field lines directed out from the center of the cube (*b*) 2 electric field lines (*c*) The flux through the cube is given by Gauss' Law: $\Phi_{\text{cube}} = \dfrac{q_{\text{enc}}}{\varepsilon_0} = \dfrac{(3.0\times10^{-6}\ \text{C})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.4\times10^5\ \text{N}\cdot\text{m}^2/\text{C}$. (*d*) By symmetry, the flux through each face must be one sixth the electric flux through the closed surface: $\Phi_{\text{face}} = \dfrac{\Phi_{\text{cube}}}{6} = \dfrac{(3.39\times10^5\ \text{N}\cdot\text{m}^2/\text{C})}{6} = (5.6\times10^4\ \text{N}\cdot\text{m}^2/\text{C})$. (*e*) The answers to (*b*) and (*d*) would change. Those answers required that the system be symmetric such that all faces were equivalent.

24.70. (*a*) Mass (*b*) For a surface enclosing mass *m*, the gravitational flux Φ_G through the surface is $\Phi_G = \oint \vec{\gamma}\cdot d\vec{A} = -4\pi Gm$, where $\vec{\gamma} \equiv -\dfrac{Gm}{r^2}\hat{r}$ is the gravitational field (analogous to $\vec{E}$), G is the gravitational constant, r is the distance from object of mass m to location at which $\vec{\gamma}$ measured, $\hat{r}$ is the unit vector pointing toward the object, and $d\vec{A}$ is an area vector normal to surface enclosing mass m, and pointing outward. The above expression relates a flux through a closed surface to a mass enclosed, which is a gravitational analog to Gauss's law for electricity.

24.71. (*a*) Consider first a single positively charged particle located at $y = d/2,$ and two infinite planar Gaussian surfaces at $y = 0$ and $y = d$. Because the planes are infinite, we could connect them with a band of height d off at infinity and the flux through this band would be zero. Thus we can say that the electric flux through a set of two infinite planes is the same as the flux through a closed surface that contains all charge between the infinite planes. By symmetry, the flux through each plane must be the same. Using Gauss's law we know that the flux through each plane is $\dfrac{q}{2\varepsilon_0}$ out of the region enclosed. We can carry out exactly the same process for a negatively charged particle at $y = -d/2,$ with infinite planar Gaussian surfaces at $y = 0$ and $y = -d$ and by the same arguments we determine that the flux through each plane is $\dfrac{q}{2\varepsilon_0}$ into the region enclosed. With the positively charged particle and the negatively charged particle along the y axis as described, the flux due to each charge add along the $y = 0$ plane. Thus the flux across the xz axis is $\dfrac{q}{\varepsilon_0}$. (*b*) We know the electric field due to a dipole along the plane perpendicular to the dipole and bisecting it is $E_y = \dfrac{kqd}{\left[r^2 + \left(\dfrac{d}{2}\right)^2\right]^{3/2}},$ where r is the distance from the dipole center in the xz plane.

Thus

$$\Phi = \int_{xz\text{ plane}} \vec{E} \cdot d\vec{A} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(x,y)\,dxdy = kqd\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dxdy}{\left[r^2 + \left(\dfrac{d}{2}\right)^2\right]^{3/2}}$$

$$\Phi = 4\pi kq = \frac{q}{\varepsilon_0}$$

(*c*) Gauss's Law is easier.

24.72. (*a*) A spherical Gaussian surface with a radius $r = 20\text{ mm}$ does not enclose any charge. Thus the flux through the surface must be zero. By symmetry, the electric field at all points on the surface must be zero. Thus $\vec{E}(r = 20\text{ mm}) = \vec{0}$. (*b*) As in part (*a*), a spherical Gaussian surface with a radius of $r = 90\text{ mm}$ does not enclose any charge, and again it follows that $\vec{E}(r = 90\text{ mm}) = \vec{0}$. (*c*) Now, Gauss's Law yields $\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E} \cdot d\vec{A} = \dfrac{q_{\text{enc}}}{\varepsilon_0},$ which by symmetry can be written as

$$E \oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA = E4\pi r_{\text{Gaussian}}^2 = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q_{\text{enc}}}{4\pi\varepsilon_0 r_{\text{Gaussian}}^2} = \frac{\sigma 4\pi r_{\text{sphere}}^2}{4\pi\varepsilon_0 r_{\text{Gaussian}}^2} = \frac{(10\times10^{-9}\text{ C/m}^2)(0.100\text{ m})^2}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.110\text{ m})^2} = 9.3\times10^2\text{ N/C}$$

Because the spherical shell is positively charged we can say $\vec{E} = 9.3\times10^2\text{ N/C}$ radially outward away from the sphere.

24.73. (*a*) For all parts, the procedure we will us is to first determine an expression for the charge that is enclosed by a spherical Gaussian sphere with a radius r_{G}. Then we apply Gauss's Law. Let the radius of the charged sphere be R_{S}. As long as our Gaussian surface is inside the solid sphere, $q_{\text{enc}} = \rho V_{\text{enc}} = \rho\left(\dfrac{4}{3}\pi r_{\text{G}}^3\right)$. Applying Gauss's Law yields

$$\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\left(\frac{4}{3}\pi r_{\text{G}}^3\right)$$

By symmetry, the electric field can come out of the integral, because its magnitude is constant over the Gaussian surface and its direction is always normal to the surface. Thus

$$E \oint_{\substack{\text{Gaussian}\\ \text{surface}}} dA = E4\pi r_{\text{G}}^2 = \frac{q_{\text{enc}}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\left(\frac{4}{3}\pi r_{\text{G}}^3\right)$$

$$\Rightarrow E = \frac{\rho r_{\text{G}}}{3\varepsilon_0}$$

For part (*a*) we have $E = \dfrac{\rho r_{\text{G}}}{3\varepsilon_0} = \dfrac{(250\times 10^{-9}\ \text{C/m}^2)(0.020\ \text{m})}{3(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = (1.9\times 10^2\ \text{N/C})$. Because the sphere is positively charged, we can write $\vec{E} = 1.9\times 10^2\ \text{N/C}$ radially outward from the center of the sphere. (*b*) We can apply the expression obtained in part (*a*): $E = \dfrac{\rho r_{\text{G}}}{3\varepsilon_0} = \dfrac{(250\times 10^{-9}\ \text{C/m}^2)(0.090\ \text{m})}{3(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = (8.5\times 10^2\ \text{N/C})$. Since the sphere is positively charged, we can write $\vec{E} = 8.5\times 10^2\ \text{N/C}$ radially outward from the center of the sphere. (*c*) Now the expression for the charge enclosed no longer applies, because the charge density is zero outside the charged sphere. Now we must use $q_{\text{enc}} = \rho V_{\text{enc}} = \rho\left(\dfrac{4}{3}\pi R_{\text{S}}^3\right)$. Our expression for the electric field consequently becomes

$$E = \frac{\rho}{3\varepsilon_0}\frac{R_{\text{S}}^3}{r_{\text{G}}^2} = \frac{(250\times 10^{-9}\ \text{C/m}^2)}{3(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\frac{(0.100\ \text{m})^3}{(0.110\ \text{m})^2} = (7.8\times 10^2\ \text{N/C})$$

Since the sphere is positively charged, we write $\vec{E} = 7.8\times 10^2\ \text{N/C}$ radially outward from the center of the sphere.

24.74. (*a*) $q = \sigma A = 2\pi r\ell\sigma = 2\pi(0.050\ \text{m})(10\ \text{m})(9\times 10^{-9}\ \text{C/m}^2) = 3\times 10^{-8}\ \text{C}$. (*b*) A cylindrical Gaussian surface concentric with the cylindrical shell, and having a radius $r = 49\ \text{mm}$ will not enclose any charge. Gauss's Law shows us that the flux through the surface is thus zero, and the symmetry of the cylinder allows us to state that the electric field at this radius is zero. (*c*) Gauss's Law tells us $\displaystyle\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$. Since the electric field is the same at all points along the cylindrical Gaussian surface, and since the field is always normal to the surface, we can write $E = \dfrac{q_{\text{enc}}}{2\pi\varepsilon_0 r\ell} = \dfrac{(2.8\times 10^{-8}\ \text{C})}{2\pi(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.051\ \text{mm})(10\ \text{m})} = 1.0\times 10^3\ \text{N/C}$. Since the cylinder is positively charged, we can say $\vec{E} = 1.0\times 10^3\ \text{N/C}$ radially outward from the center of the cylinder.

24.75. (*a*) Choose a Gaussian cylinder inside the inner cylinder and concentric to it. Then $\displaystyle\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = E2\pi r\ell = \frac{q_{\text{enc}}}{\varepsilon_0} = 0 \Rightarrow \vec{E} = \vec{0}$. If a cylindrical Gaussian surface is constructed between the two charged cylinders, it will enclose a charge $q_{\text{enc}} = \lambda_a\ell$. Gauss's Law then yields $\displaystyle\oint_{\substack{\text{Gaussian}\\ \text{surface}}} \vec{E}\cdot d\vec{A} = E2\pi r\ell = \frac{q_{\text{enc}}}{\varepsilon_0} = 0 \Rightarrow E = \frac{\lambda_a}{2\pi r}$.

Finally, outside the outer cylinder, the charge enclosed is $q_{\text{enc}} = (\lambda_a + \lambda_b)\ell$, and the same process as before shows $E = \dfrac{(\lambda_a + \lambda_b)}{2\pi r}$.

Thus the fields are given by

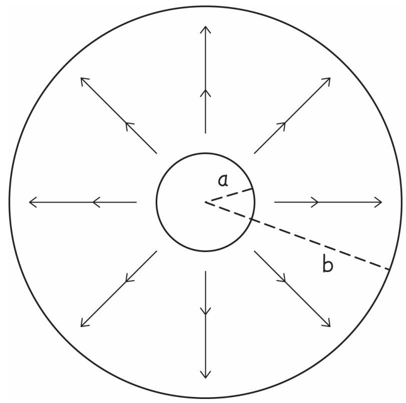
$$\vec{E}(r < a) = \vec{0}$$

$$\vec{E}(a < r < b) = \frac{2k\lambda_a}{r} \text{ radially outward from the center of the cylinder}$$

$$\vec{E}(r > b) = \frac{2k(\lambda_a + \lambda_b)}{r} \text{ radially outward from the center of teh cylinder}$$

(*b*) From part (*a*) we can see that the field outside the outer cylinder is only zero if $\lambda_b = -\lambda_a = -5.0\ \text{nC/m}$. So the linear charge density on the outer cylinder must be $5.0\ \text{nC/m}$ of negative charge.

(*c*)



24.76. (*a*) Let region I be to the left of the left-most sheet, region II be between the two sheets, and region III be to the right of the right-most sheet. We can determine the magnitude of the electric field due to a single sheet by using the results of example 24.8: $E_{\text{sheet}} = \dfrac{\sigma}{2\varepsilon_0}$. In region I, the electric fields due to each plate are in the same direction, as they are in region III. But in region II, the fields are in opposite directions. Thus

$$\vec{E}_{\text{I}} = \frac{\sigma}{\varepsilon_0}\ \text{left} = \frac{(3.0\times10^{-9}\ \text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\ \text{left} = 340\ \text{N/C left}$$

$$E_{\text{II}} = 0$$

$$\vec{E}_{\text{III}} = 340\ \text{N/C right}$$

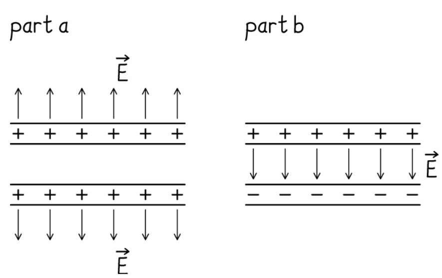
(*b*) With the regions labeled as in part (*a*), the fields from each plate are directed opposite each other in regions I and III, but are in the same direction in region II. Thus we have

$$E_{\text{I}} = 0$$

$$\vec{E}_{\text{II}} = 340\ \text{N/C right}$$

$$E_{\text{III}} = 0$$

(*c*)



24.77. (*a*) Because the electric field inside the material of the conducting cylindrical shell must be zero, Gauss's Law can be used to demonstrate that the total charge enclosed by any cylindrical Gaussian surface within the conducting material and centered on the wire must be zero. Thus the total linear charge density on the inner surface of the conducting shell must be $\lambda_{\text{inner}} = -1.5\ \mu\text{C/m}$. We are asked for a surface charge density. $\sigma = \dfrac{q}{2\pi r\ell}$ and we know the linear charge density $\lambda = \dfrac{q}{\ell}$ such that $\sigma_{\text{inner}} = \dfrac{\lambda_{\text{inner}}}{2\pi r_{\text{inner}}} = \dfrac{(-1.5\ \mu\text{C/m})}{2\pi(0.050\ \text{m})} = -4.8\times10^{-6}\ \text{C/m}^2$. If the conducting shell is neutral, then the same magnitude of linear charge density must exist on the outer surface, just spread over a larger area: $\sigma_{\text{outer}} = \dfrac{\lambda_{\text{outer}}}{2\pi r_{\text{outer}}} = \dfrac{(+1.5\ \mu\text{C/m})}{2\pi(0.070\text{ m})} = 3.4\times10^{-6}\text{ C/m}^2$. So $\sigma_{\text{inner}} = 4.8\ \mu\text{C/m}^2$ of negative charge and $\sigma_{\text{outer}} = 3.4\ \mu\text{C/m}^2$ of positive charge. (*b*) Using the results of exercise 24.7, we can write the electric field due to an infinite wire as $E = \dfrac{2k\lambda}{r} = \dfrac{2(9.0\times10^9\text{ Nm}^2/\text{C}^2)(1.5\times10^{-6}\text{ C/m})}{(0.049\text{ m})} = 5.5\times10^5\text{ N/C}$. By symmetry, the electric field is radial, and since the wire carries a positive charge, the electric field points radially outward. Thus the electric field is $\vec{E} = (+5.5\cdot10^5\text{ N/C})\,\hat{r}$. (*c*) A point 0.051 m from the central wire is inside the material of the conducting shell. Thus the electric field at this point is zero. (*d*) Outside the conducting shell, the electric field will be the same as if the total charge were located all at the central wire. Again, we can use the results of exercise 24.7. $E = \dfrac{2k\lambda}{r} = \dfrac{2(9.0\times10^9\text{ Nm}^2/\text{C}^2)(1.5\times10^{-6}\text{ C/m})}{(0.100\text{ m})} = 2.7\times10^5\text{ N/C}$. As before, symmetry and the sign of the charge tells us that the electric field must be directed radially outward such that $\vec{E} = 2.7\cdot10^5\text{ N/C}$ radially outward.

24.78. We consider three regions: inside the empty hole at the center, in the material of the shell, and outside the shell. Inside the hole at the center, a spherical Gaussian surface concentric with the shell would enclose no charge, such that the total electric flux through any such surface would be zero. Because of the spherical symmetry of the setup, all points on such a spherical surface would be equivalent, such that zero total flux implies the magnitude of the electric field anywhere on the surface must also be zero. Thus $E(r < R/2) = 0$.

To examine the region inside the material of the shell, let us choose a spherical Gaussian surface, concentric with the shell and having a radius r such that $R/2 < r < R$. The charge enclosed by such a spherical shell would be $q_{\text{enc}} = \rho V = \rho\left(\dfrac{4}{3}\pi\right)\left(r^3 - \left(\dfrac{R}{2}\right)^3\right)$. Using Gauss's Law and the spherical symmetry of the setup, we can write $E(4\pi r^2) = \left(\dfrac{\rho}{\varepsilon_0}\right)\left(\dfrac{4}{3}\pi\right)\left(r^3 - \left(\dfrac{R}{2}\right)^3\right)$.

Solving for the electric field, and using symmetry to determine the direction, we obtain $\vec{E}(R/2 < r < R) = \left(\dfrac{\rho}{3\varepsilon_0}\right)\left(r - \dfrac{R^3}{8r^2}\right)$ radially outward from the center of the shell.

As before, we apply Gauss's Law to a spherical Gaussian surface, concentric with the spherical shell, but this time with a radius $r > R$. Outside the shell, the total charge enclosed in independent of the size of the spherical Gaussian surface: $q_{\text{enc}} = \rho V = \rho\left(\dfrac{4}{3}\pi\right)\left(R^3 - \left(\dfrac{R}{2}\right)^3\right) = \dfrac{7\pi\rho R^3}{6}$. Gauss's Law tells us $E(4\pi r^2) = \dfrac{7\pi\rho R^3}{6\varepsilon_0}$. Solving for the electric field and using symmetry and the sign of the charge to determine the direction, we find $\vec{E}(r > R) = \dfrac{7\rho R^3}{24\varepsilon_0 r^2}$ radially outward from the center of the spherical shell.

Collecting the expressions from all regions, we have

$$\vec{E}(r < R/2) = \vec{0}$$

$$\vec{E}(R/2 < r < R) = \frac{\rho}{3\varepsilon_0}\left(r - \frac{R^3}{8r^2}\right)\text{ radially outward}$$

$$\vec{E}(r > R) = \frac{7\rho R^3}{24\varepsilon_0 r^2}\text{ radially outward}$$

24.79. We consider the regions $r \le a$, $a \le r \le b$, and $b \le r$ separately. In all cases, we form a spherical Gaussian surface with a radius in the specified regime, and use Gauss's Law to set the flux through that surface equal to $\dfrac{q_{\text{enc}}}{\varepsilon_0}$.

In the region $r \le a$, the charge enclosed by such Gaussian surface is $q_{\text{enc}} = \rho_0 V = \rho_0\left(\frac{4}{3}\pi r^3\right)$. Any such Gaussian surface with $a \le r \le b$ must necessarily enclose all charge on the inner sphere: $q_{\text{inner}} = \rho_0\left(\frac{4}{3}\pi a^3\right)$, but it will also enclose some of the charge on the shell. We determine this contribution by integrating over the charge density $q_{\text{shell}} = \int_0^{\pi}\int_0^{2\pi}\int_a^{r}\frac{\rho_0 r'}{a}(r')^2\sin(\theta)dr'd\phi d\theta = \frac{\pi\rho_0}{a}(r^4 - a^4)$. Thus, the total enclosed charge in this region is $q_{\text{enc}} = \rho_0\left(\frac{4}{3}\pi a^3\right) + \frac{\pi\rho_0}{a}(r^4 - a^4) = \frac{\pi\rho_0 r^4}{a} + \frac{\pi\rho_0 a^3}{3}$. Finally, in the region satisfying $r \ge b$, the total charge enclosed is $q_{\text{enc}} = \frac{\pi\rho_0 b^4}{a} + \frac{\pi\rho_0 a^3}{3}$. In all cases the flux through the Gaussian surface can be written as $E(4\pi r^2)$. Applying Gauss's Law in each region separately yields

$$\vec{E}(r \le a) = \frac{\rho r}{3\varepsilon_0} \text{ radially outward}$$

$$\vec{E}(a \le r \le b) = \frac{\rho a^3}{\varepsilon_0 r^2}\left(\frac{1}{12} + \frac{1}{4}\left(\frac{r}{a}\right)^3\right) \text{ radially outward}$$

$$\vec{E}(r \ge b) = \frac{\rho a^3}{\varepsilon_0 r^2}\left(\frac{1}{3} + \frac{1}{4}\left(\frac{b^4}{a^4} - 1\right)\right) \text{ radially outward}$$

24.80. (*a*) We integrate the charge density over the volume specified: $q = \int_{\text{cylinder}} \rho dV = \int_0^R\int_0^{2\pi}\int_0^{\ell}\left(\frac{c}{r}\right)dzd\phi rdr = 2\pi c\ell R.$ (*b*) Construct a cylindrical Gaussian cylinder concentric with the nonconducting solid cylinder, and having a radius $r < R$. Then the charge enclosed per unit length is $\lambda_{\text{enc}} = \frac{q}{\ell} = \int_{\text{cylinder}} \rho dV = \frac{1}{\ell}\int_0^r\int_0^{2\pi}\int_0^{\ell}\left(\frac{c}{r'}\right)dzd\phi r'dr' = 2\pi cr.$ Gauss's Law then lets us write $E(2\pi r\ell) = \frac{2\pi c\ell r}{\varepsilon_0}$. Solving for the electric field magnitude, we obtain $E = \frac{c}{\varepsilon_0}$. (*c*) Now we construct a cylindrical Gaussian cylinder concentric with the nonconducting solid cylinder, and having a radius $r > R$. Then the charge enclosed per unit length is $\lambda_{\text{enc}} = \frac{q}{\ell} = \int_{\text{cylinder}} \rho dV = \frac{1}{\ell}\int_0^R\int_0^{2\pi}\int_0^{\ell}\left(\frac{c}{r}\right)dzd\phi rdr = 2\pi cR.$ Gauss's Law then lets us write $E(2\pi r\ell) = \frac{2\pi c\ell R}{\varepsilon_0}$. Solving for the electric field magnitude, we obtain $E = \frac{cR}{\varepsilon_0 r}$.

24.81. The electric field due to the second charged wire could either cause an additional electric field of E_0 directed to the right, or it could cause an electric field of $3E_0$ to the left. We know from the first wire that $E_0 = \frac{2k\lambda}{d}$. We now require that $E_0 = \frac{2k|\lambda'|}{3d}$ such that $\lambda' = -3\lambda$ (where the negative sign came from considering the rightward direction of this contribution), or $3E_0 = \frac{2k|\lambda'|}{3d}$ such that $\lambda' = 9\lambda$.

24.82. (*a*) If we choose a cylindrical Gaussian surface inside the material of the conductor, the electric flux must be zero, meaning that the charge enclosed must also be zero. Since there is a charge of $-q$ on the inner surface of the shell, there must be a charge of $+q$ on the wire. (*b*) We are not told that the conducting shell is neutral. What we are given is the number of field lines entering the inner surface of the shell (12) and entering the outer shell (9). This means the electric flux into the outer surface is 9/12 the size of the flux into the inner surface. This tells us that the total charge enclosed by a cylindrical Gaussian surface coaxial with the shell and enclosing the entire shell $q_{\text{all}} = q_{\text{outer}} + q_{\text{inner}} + q_{\text{wire}}$ is −3/4 the total charge q_{wire} enclosed by a similar Gaussian surface that encloses only the central wire. Thus $q_{\text{outer}} + q_{\text{inner}} + q_{\text{wire}} = -\frac{3}{4}q_{\text{wire}} \Rightarrow q_{\text{outer}} = -\frac{3}{4}q - q + q.$ So $q_{\text{outer}} = -\frac{3}{4}q.$ (*c*) By combining the charge on the inner and outer surfaces, we see the total charge per length ℓ is $\frac{q}{4}.$ (*d*) $\frac{\sigma_{\text{in}}}{\sigma_{\text{out}}} = \frac{\left(\frac{-q}{2\pi R}\right)}{\left(\frac{-3q/4}{2\pi(2R)}\right)} = \frac{8}{3}$

24.83. (*a*) Since the volume charge density is uniform, we can simply multiply the charge density by the volume: $q = \rho V = \rho \ell \pi R^2 = (9.0\times10^{-9}\ \text{C/m}^3)(10\ \text{m})\pi(0.050\ \text{m})^2 = 7.1\times10^{-10}\ \text{C}.$ (*b*) Choose a cylindrical Gaussian surface that is coaxial with the charged cylinder and has a radius $r = 0.040$ m. Applying Gauss's Law yields

$$E(2\pi r\ell) = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{\rho(\pi r^2\ell)}{\varepsilon_0(2\pi r\ell)} = \frac{\rho r}{2\varepsilon_0} = \frac{(9.0\times10^{-9}\ \text{C/m}^3)(0.040\ \text{m})}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 20\ \text{N/C}$$

(*c*) We proceed exactly as in part (*b*), except that now the charge enclosed is determined by the radius of the charged cylinder, not the radius of the cylindrical Gaussian surface.

$$E(2\pi r\ell) = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{\rho(\pi R^2\ell)}{\varepsilon_0(2\pi r\ell)} = \frac{\rho R^2}{2\varepsilon_0 r} = \frac{(9.0\times10^{-9}\ \text{C/m}^3)(0.050\ \text{m})^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.060\ \text{m})} = 21\ \text{N/C}$$

24.84. (*a*) Exercise 24.8 tells us that the electric field due to an infinite sheet with surface charge density σ has a magnitude $\frac{\sigma}{2\varepsilon_0},$ and a direction toward or away from the sheet depending on the sign of the charge. We can determine the contribution of each sheet to the total electric field by considering the direction of the field each produces at the point in question. Thus the electric field due to the three sheets at the position $x = 1.5$ m is $-\frac{\sigma}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0} = 0 \Rightarrow \sigma_2 = 2\sigma.$ So the charge density on sheet 2 is $2\sigma.$ (*b*) This problem can be answered by inspection. Since the electric field due to an infinite sheet does not vary with distance, all that matters is to which side of each sheet the point in question is located. The origin is to the left of all sheets. The point $x = -2.0$ m is to the left of all sheets. Therefore the electric field at $x = -2.0$ m must also be $\vec{E}_0.$ This is a sufficient answer. But one may wish to see the contributions individually, in which case one obtains an expression for the electric field magnitude at $x = 0$: $E_0 = \frac{\sigma}{2\varepsilon_0} - \frac{2\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$ Now the electric field at $x = -2.0$ m is $\frac{\sigma}{2\varepsilon_0} - \frac{2\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0},$ which of course is the same as at the origin. (*c*) The x component of the electric field at $x = 3.0$ m is $-\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{3\sigma}{2\varepsilon_0} = \frac{2\sigma}{\varepsilon_0} = 2E_0.$ Thus the electric field at $x = 3.0$ m is $2\vec{E}_0.$ (*d*) The x component of the electric field at $x = 6.0$ m is $-\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} - \frac{3\sigma}{2\varepsilon_0} = -\frac{\sigma}{\varepsilon_0} = -E_0.$ Thus the electric field at $x = 6.0$ m is $-\vec{E}_0.$

24.85. (*a*) 2 (*b*) There is a thick, neutral, conducting spherical shell of inner radius R and outer radius $2R.$ We know this because the electric field drops to zero in this region. There is also a thin, insulating spherical shell of radius $3R$ that carries $-2q.$ We know this because the flux through a Gaussian surface outside this shell has the same magnitude but opposite sign as the flux through a Gaussian surface inside this shell. Thus the total charge enclosed by a sphere outside this shell must be equal in magnitude and opposite in sign to the charge enclosed by a Gaussian surface inside the shell.

24.86. Yes. This could be produced using four charged particles: $+q$ at origin, $-q$ at any point for which $|\vec{r}| = R$, $+q$ at any point for which $|\vec{r}| = 2R$, $-2q$ at any point for which $|\vec{r}| = 3R$.

24.87. Example 23.4 gives the expression for the electric field a distance r along the perpendicular bisector of a rod of length ℓ carrying a charge q: $E_{\text{rod}} = \dfrac{kq}{r\sqrt{\ell^2/4 + r^2}}$. Treating the rod as infinite and using Gauss's Law yields the expression from Exercise 24.7: $E_{\text{Gauss}} = \dfrac{2k\lambda}{r}$ or $E_{\text{Gauss}} = \dfrac{2kq}{r\ell}$. To find the distance r at which the error reaches 5%, we require $\dfrac{E_{\text{Gauss}} - E_{\text{rod}}}{E_{\text{rod}}} = f$, where f is the percent error. Thus we write

$$\frac{2kq}{r\ell} - \frac{kq}{r\sqrt{\ell^2/4 + r^2}} = f\left(\frac{kq}{r\sqrt{\ell^2/4 + r^2}}\right)$$

$$\frac{2}{r\ell} = \frac{(f+1)}{r\sqrt{\ell^2/4 + r^2}}$$

$$4(\ell^2/4 + r^2) = \ell^2(f+1)^2$$

$$r = \sqrt{\frac{\ell^2(f+1)^2}{4} - \frac{\ell^2}{4}}$$

$$r = \sqrt{\frac{(0.25\ \text{m})^2(1.05)^2}{4} - \frac{(0.25\ \text{m})^2}{4}} = 40\ \text{mm}$$

Thus the error first exceeds 5% a distance 0.040 m from the center of the rod, along its perpendicular bisector.

24.88. (*a*) Both charged objects must be carrying positive charge because of the directions of the electric fields given. We know at the point $x = d$, the field due to the point is twice as large as the field due to the wire. Thus we can write $\dfrac{kq}{d^2} = 2\left(\dfrac{2k\lambda}{d}\right) \Rightarrow q = 4\lambda d$. Now we require that the two fields cancel somewhere along the x axis. Because both objects carry positive charge, such a point can only exist between the two objects (as opposed to off to the left or off to the right). Thus we require $\dfrac{2k\lambda}{x} - \dfrac{kq}{(2d-x)^2} = 0$, or equivalently: $\dfrac{2k\lambda}{x} = \dfrac{4k\lambda d}{(2d-x)^2} \Rightarrow$ $x^2 - 6dx + 4d^2 = 0$. This quadratic equation leads to two solutions: $x = (3 \pm \sqrt{5})d$, but only one of those is physically valid. If our point is greater than $2d$, the fields due to each charged object would be in the same direction, and could not possibly cancel each other. Thus $x = (3 - \sqrt{5})d$ is the only correct answer. (*b*) $E_{\text{wire}} = \dfrac{2k\lambda}{x} = \dfrac{2k\lambda}{(3-\sqrt{5})d}$.

24.89. (*a*) Choose a spherical Gaussian surface concentric with the charged sphere, and having radius $r = R/4$. By symmetry, the electric flux through this Gaussian surface is $E4\pi(R/4)^2$. The charge enclosed by such a surface is

$$q_{\text{enc}} = q_{\text{sphere}} \frac{\frac{4}{3}\pi\left(\frac{R}{4}\right)^3}{\frac{4}{3}\pi R^3}$$

Thus Gauss's Law tells us that the electric field at $x=\dfrac{R}{4}$ is $E_{\text{sphere}}=\dfrac{q_{\text{sphere}}(R/4)^3}{4\pi\varepsilon_0(R/4)^2R^3}=\dfrac{kq_{\text{sphere}}}{4R^2}$. At this location, we require that the fields due to the sphere and to the particle cancel one another. Thus $\dfrac{kq_{\text{sphere}}}{4R^2}=\dfrac{kq_{\text{particle}}}{(7R/4)^2}\Rightarrow$ $q_{\text{particle}}=\dfrac{49}{64}q_{\text{sphere}}$. (*b*) For any point outside the sphere, and between the sphere and the particle, we require $E_{\text{sphere}}=E_{\text{part}}\Rightarrow\dfrac{kq_{\text{sphere}}}{x^2}=\dfrac{k\left(\dfrac{49}{64}\right)q_{\text{sphere}}}{(2R-x)^2}$. After some algebra, this reduces to t $x=\dfrac{2R}{\left(1\pm\sqrt{\left(\dfrac{49}{64}\right)}\right)}$. But since the point must be between the sphere and the particle, only and $x=\dfrac{16}{15}R$ is a valid solution.

24.90. (*a*) Since the objects carry the same sign, they can only cancel at a point on the x axis that is between them. Thus we require $E_{\text{part}}=E_{\text{sphere}}$, and assuming the point of interest is outside the sphere, we write $\dfrac{4kq}{x^2}=\dfrac{kq}{(6R-x)^2}\Rightarrow x=4R$. We must consider points inside and outside the sphere. A spherical Gaussian surface of radius $r<R$ and concentric with the charged sphere will enclose a charge $q_{\text{enc}}=q_{\text{sphere}}\dfrac{r^3}{R^3}$, such that the electric field at that radial distance from the center is $E=\dfrac{q_{\text{sphere}}r}{4\pi\varepsilon_0R^3}$ or $E=\dfrac{kq(6R-x)}{R^3}$. Between the center of the sphere and the particle, the electric fields from the two objects will be directed opposite each other. Thus we can require $E_{\text{sphere}}=E_{\text{particle}}$ or equivalently $\dfrac{kq(6R-x)}{R^3}=\dfrac{4kq}{x^2}$. The resulting cubic equation, $x^3-6Rx^2+4R^3=0$ is most readily solved using graphical or numerical methods. One finds that the electric field is also zero at $x=5.9R$. (*b*) The electric field would still be equal to zero at $x=4R$, but now it would also be zero for $5R<x<7R$.

24.91. (*a*) In all cases our process will be to integrate over the charge density to obtain the charge enclosed by a spherical Gaussian surface of radius r, and then use Gauss's Law and spherical symmetry to obtain the electric field. For our first charge distribution

$$q_{\text{enc}}=\int_0^{\pi}\int_0^{2\pi}\int_0^{r}\rho(r')(r')^2\sin(\theta)dr d\phi d\theta=\int_0^{\pi}\int_0^{2\pi}\int_0^{r}\frac{\rho_0}{R}(r')^3\sin(\theta)dr d\phi d\theta$$
$$q_{\text{enc}}=\frac{\rho_0\pi r^4}{R}$$

Writing the electric flux through the spherical Gaussian surface as $E(4\pi r^2)$, we obtain $\vec{E}(r<R)=\dfrac{\rho_0r^2}{4\varepsilon_0R}$ radially outward from the center of the sphere. Once outside the sphere, the charge enclosed no longer depends on the radius of the Gaussian surface, and we find $\vec{E}(r>R)=\dfrac{\rho_0R^3}{4\varepsilon_0r^2}$ radially outward from the center of the sphere.

(*b*) $q_{\text{enc}}=\displaystyle\int_0^{\pi}\int_0^{2\pi}\int_0^{r}\rho(r')(r')^2\sin(\theta)dr d\phi d\theta=\int_0^{\pi}\int_0^{2\pi}\int_0^{r}\rho_0\left(1-\frac{R}{2r'}\right)(r')^2\sin(\theta)dr d\phi d\theta$

$$q_{\text{enc}}=\frac{\rho_0\pi r^2}{3}(4r-3R)$$

Writing the electric flux through the spherical Gaussian surface as $E(4\pi r^2)$, we obtain $\vec{E}(r<R)=\frac{\rho_0}{4\varepsilon_0}\left(\frac{4}{3}r-R\right)$ radially outward from the center of the sphere. Once outside the sphere, the charge enclosed no longer depends on the radius of the Gaussian surface, and we find $\vec{E}(r>R)=\frac{\rho_0 R^3}{12\varepsilon_0 r^2}$ radially outward from the center of the sphere.

(*c*) $q_{\text{enc}}=\int_0^{\pi}\int_0^{2\pi}\int_0^{r}\rho(r')(r')^2\sin(\theta)drd\phi d\theta=\int_0^{\pi}\int_0^{2\pi}\int_0^{r}\rho_0\left(1-\frac{R}{r'}\right)(r')^2\sin(\theta)drd\phi d\theta$

$$q_{\text{enc}}=\frac{2\rho_0\pi r^2}{3}(2r-3R)$$

Writing the electric flux through the spherical Gaussian surface as $E(4\pi r^2)$, we obtain $\vec{E}(r<R)=\frac{\rho_0}{4\varepsilon_0}\left(\frac{4}{3}r-2R\right)$ radially outward from the center of the sphere. Once outside the sphere, the charge enclosed no longer depends on the radius of the Gaussian surface, and we find $\vec{E}(r>R)=\frac{\rho_0 R^3}{6\varepsilon_0 r^2}$ radially inward toward the center of the sphere.

24.92. (*a*) We integrate each charge density over the relevant volume:

$$q=\int_0^{\pi}\int_0^{2\pi}\int_0^{R/2}\rho_0(r')^2\sin(\theta)drd\phi d\theta+\int_0^{\pi}\int_0^{2\pi}\int_{R/2}^{R}2\rho_0\left(1-\frac{r'}{R}\right)(r')^2\sin(\theta)drd\phi d\theta$$

$$q=\frac{5}{8}\pi R^3\rho_0$$

(*b*) Consider first the region $r\le R/2$. Choose a spherical Gaussian surface concentric with the charged sphere, and with radius r. The total charge enclosed by this Gaussian surface is given by integrating the charge density over the volume enclosed, and one obtains $q_{\text{enc}}=\rho_0\left(\frac{4}{3}\pi r^3\right)$. We can write the electric flux through our Gaussian surface as $E(4\pi r^2)$. Using Gauss's Law, we obtain $\vec{E}(r<R/2)=\frac{\rho_0 r}{3\varepsilon_0}$ radially outward from the center of the sphere.

In the region $R/2\le r\le R$, the charge enclosed is given by

$$q_{\text{enc}}=q_{\text{enc, inner sphere}}+q_{\text{enc, shell}}=\rho_0\left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)+\int_0^{\pi}\int_0^{2\pi}\int_{R/2}^{r}2\rho_0\left(1-\frac{r'}{R}\right)(r')^2\sin(\theta)drd\phi d\theta$$

$$q_{\text{enc}}=\rho_0\pi\left(\frac{R^3}{6}-\frac{5R^3}{24}+\frac{8}{3}r^3-2\frac{r^4}{R}\right)$$

We can write the electric flux through our Gaussian surface as $E(4\pi r^2)$. Using Gauss's Law, we obtain $\vec{E}(R/2<r<R)=\frac{\rho_0}{4\varepsilon_0 r^2}\left(\frac{8}{3}r^3-\frac{2}{R}r^4-\frac{1}{24}R^3\right)$ radially outward from the center of the sphere.

Finally, outside the sphere the charge enclosed is a fixed $q_{\text{enc}}=\rho_0\pi\left(\frac{R^3}{6}-\frac{5R^3}{24}+\frac{8R^3}{3}-2R^3\right)=\frac{5\rho_0\pi R^3}{8}$. Again, we write the electric flux through our Gaussian surface as $E(4\pi r^2)$. Using Gauss's Law, we obtain $\vec{E}(r>R)=\frac{5\rho_0 R^3}{32\varepsilon_0 r^2}$ radially outward from the center of the sphere.

(c) Consider approaching the boundaries at $R/2$ and R from just inside and just outside and check that they match.

$$\vec{E}\left(r \to (R/2)_-\right) = \frac{\rho_0 R}{6\varepsilon_0}\hat{r}$$

$$\vec{E}\left(r \to (R/2)_+\right) = \frac{\rho_0}{4\varepsilon_0 \left(R/2\right)^2}\left(\frac{8}{3}\left(\frac{R}{2}\right)^3 - \frac{2}{R}\left(\frac{R}{2}\right)^4 - \frac{1}{24}R^3\right)\hat{r} = \frac{\rho_0 R}{\varepsilon_0}\left(\frac{1}{3} - \frac{2}{16} - \frac{1}{24}\right)\hat{r} = \frac{\rho_0 R}{6\varepsilon_0}\hat{r}$$

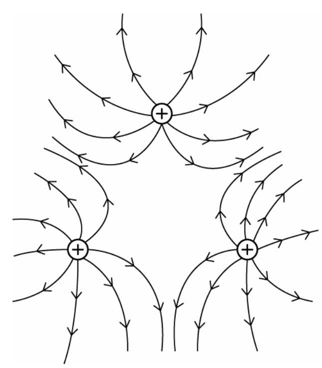
$$\vec{E}\left(r \to (R)_-\right) = \frac{\rho_0}{4\varepsilon_0 \left(R\right)^2}\left(\frac{8}{3}R^3 - \frac{2}{R}R^4 - \frac{1}{24}R^3\right)\hat{r} = \frac{\rho_0 R}{4\varepsilon_0}\left(\frac{8}{3} - \frac{6}{3} - \frac{1}{24}\right)\hat{r} = \frac{5\rho_0 R}{32\varepsilon_0}\hat{r}$$

$$\vec{E}(r \to (R)_+) = \frac{5\rho_0 R^3}{32\varepsilon_0 R^2}\hat{r} = \frac{5\rho_0 R}{32\varepsilon_0}\hat{r}$$

We see that they match.

24.93. No, in electrostatics (as long as the charged particles in the electric field are not suddenly accelerated) electric fields have no kinks.

24.94. (a)



(b) By simply applying Gauss's Law, we see that $\Phi_E = \dfrac{q_{\text{enc}}}{\varepsilon_0}$, meaning that in this case the electric flux through the surface is $\dfrac{q}{\varepsilon_0}$. (c) This is identical to part (b), except that now twice as much charge is enclosed, such that the flux through the surface is now $\dfrac{2q}{\varepsilon_0}$.

24.95. The force experienced by the particle is $\vec{F}^E_{\text{ps}} = q_{\text{p}}\vec{E}_{\text{s}}$, such that the acceleration is $\vec{a} = \dfrac{q\sigma}{2m\varepsilon_0}$ upward. The acceleration is constant and independent of d.

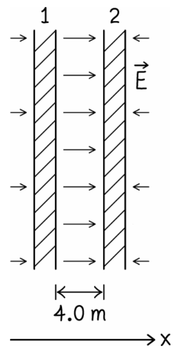
24.96. (a) Let region I be to the left of sheet 1, region II be between the two sheets, and region III be to the right of sheet 2. Then

$$\vec{E}_{\text{I}} = (-E_1 + E_2)\hat{x} = \left(-\frac{|\sigma_1|}{\varepsilon_0} + \frac{|\sigma_2|}{\varepsilon_0}\right)\hat{x} = \left(-\frac{(4.0\times10^{-9}\ \text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} + \frac{(8.0\times10^{-9}\ \text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\right)\hat{x} = 2.3\times10^2\ \text{N/C}\ \hat{x}$$

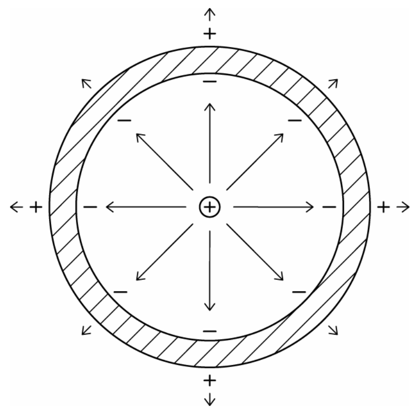
$$\vec{E}_{\text{II}} = (E_1 + E_2)\hat{x} = \left(\frac{|\sigma_1|}{\varepsilon_0} + \frac{|\sigma_2|}{\varepsilon_0}\right)\hat{x} = \left(\frac{(4.0\times10^{-9}\ \text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} + \frac{(8.0\times10^{-9}\ \text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\right)\hat{x} = 6.8\times10^2\ \text{N/C}\ \hat{x}$$

$$\vec{E}_{\text{III}} = (-E_1 - E_2)\hat{x} = \left(-\frac{|\sigma_1|}{\varepsilon_0} - \frac{|\sigma_2|}{\varepsilon_0}\right)\hat{x} = \left(-\frac{(4.0\times10^{-9}\ \text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} - \frac{(8.0\times10^{-9}\ \text{C/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\right)\hat{x} = -2.3\times10^2\ \text{N/C}\ \hat{x}$$

(*b*)



24.97. (*a*)



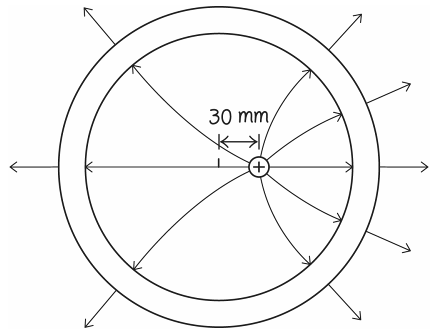
(*b*) Since the electric field in the conductor must be zero, it follows from Gauss's Law that the total charge enclosed by a spherical Gaussian surface in the wall of the conductor must also be zero. Thus the total charge on the inner surface must be -3.0 nC, leaving $+3.0$ nC on the outer surface. The surface charge density on the inner surface is given by $\sigma_{\text{inner shell surface}} = \dfrac{q_{\text{inner shell surface}}}{A} = \dfrac{q_{\text{inner shell surface}}}{4\pi r_{\text{inner shell surface}}^2} = \dfrac{(-3.0\times 10^{-9}\ \text{C})}{4\pi(0.100\ \text{m})^2} = 24\ \text{nC/m}^2$ of negative charge, and the surface charge density on the outer surface is $\sigma_{\text{outer shell surface}} = \dfrac{q_{\text{outer shell surface}}}{A} = \dfrac{q_{\text{outer shell surface}}}{4\pi r_{\text{outer shell surface}}^2} = \dfrac{(3.0\times 10^{-9}\ \text{C})}{4\pi(0.120\ \text{m})^2} = 17\ \text{nC/m}^2$ of positive charge. (*c*) Let $q = 3.0$ nC. Inside the cavity, there is only the electric field due to the charged particle at the center. In the conducting material the electric field is zero, and outside the shell the field is the same as if all charge were located at the center. Thus

$$\vec{E}(r < R_{\text{inner}}) = \frac{kq}{r^2} \text{ radially outward from the center}$$

$$\vec{E}(R_{\text{inner}} < r < R_{\text{outer}}) = \vec{0}$$

$$\vec{E}(r > R_{\text{outer}}) = \frac{kq}{r^2} \text{ radially outward from the center}$$

(*d*)



24.98. (*a*) Cylinder I is a solid, non-conducting cylinder of radius R. Cylinder O is a thick, conducting shell of inner radius $2R$ and outer radius $3R$. (*b*) The electric field a distance R from the center is four units on the graph in Figure P24.98. Let us call this electric field magnitude $4E_0$. From Exercise 24.7 we know the formula for the electric field outside a long charged cylinder is given by $4E_0 = \frac{2k\lambda_I}{R}$. Similarly, a distance $3R$ away, we have the same magnitude of electric field $4E_0 = \frac{2k(\lambda_I + \lambda_O)}{3R}$. Equating the two expressions, we obtain $q_I = \frac{(q_I + q_O)}{3} \Rightarrow \frac{q_I}{q_O} = \frac{1}{2}$.

24.99. $E = \frac{kq}{R_E^2} \Rightarrow \frac{q}{4\pi R_E^2} = \frac{E}{4\pi k} = E\varepsilon_0 = (150\text{ N/C})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 1.3\times10^{-9}\text{ C/m}^2$. In order for the direction to be radially inward, the surface charge density must be 1.3 nC/m^2 of negative charge.

24.100. (*a*) Call the direction in which the electron is fired the $+x$ direction. The x component of the acceleration of the electron is given by $a_x = \frac{F_x}{m} = \frac{E_x q}{m} = -\frac{\sigma q}{2m\varepsilon_0}$, which is constant as the electron moves. We can use kinematics to write $a_x = \frac{v_{x,f}^2 - v_{x,f}^2}{2\Delta x}$ or $\sigma = -\frac{m\varepsilon_0(v_{x,f}^2 - v_{x,f}^2)}{\Delta x q} = -\frac{(9.11\times10^{-31}\text{ kg})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0-(400\text{ m/s})^2)}{(2.00\text{ m})(1.60\times10^{-19}\text{ C})} =$ $4.03\times10^{-18}\text{ C/m}^2$ of negative charge.

(*b*) Moving the electron closer or farther from the sheet does not affect the acceleration. Thus the electron will still cross a distance of 2.00 m as it slows. If it is to just reach the sheet, it should be fired from 2.00 m away.

24.101. (*a*) Outside the spheres in B and C, the electric field is the same as if the charge were located at the center of the sphere. Thus, for the electric field at $x = R$ to be the same for A-C, the charge on particle A, and on spheres B and C must all be the same. The electric field will drop off like $1/x^2$ for all of these cases, whereas the electric field is constant outside the slab. Thus $E_D > E_A = E_B = E_C$. (*b*) Call the electric field at $x = R$ E_0. At $x = R/2$, the electric field in sphere B is zero because it is a conductor. The electric field at $x = R/2$ in case A is $4E_0$. In case D the electric field is still E_0. Finally in case C, the electric field is $E_0/2$. Thus $E_A > E_D > E_C > E_B$.

24.102. (*a*) By symmetry, the electric field at the center of the circle must be zero. (*b*) This charge arrangement is the same as having a complete sheet of positive charge with uniform charge density, with a disk having an equal and opposite surface charge density placed at the position of the vacancy. Thus the electric field is the same as $E = E_{\text{sheet}} - E_{\text{disk}} = \frac{\sigma}{2\varepsilon_0} - 2k\pi\sigma\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$. Inserting the values given, we obtain $E = \left(\frac{\sigma}{2\varepsilon_0}\right)\frac{4R}{\sqrt{16R^2+R^2}} = \left(\frac{2}{\sqrt{17}}\right)\frac{\sigma}{\varepsilon_0}$.

24.103. (*a*) Note first that the left half of the slab and the right half of the slab will have equal and opposite charges, such that the effective surface charge density of this slab is zero, outside the slab. Thus the electric field will be zero everywhere outside the slab. Now consider a pillbox-shaped Gaussian surface that extends from the left of the slab inside the slab to a position x, with its flat circular sides parallel to the slab. The charge enclosed by this box is $q_{\text{enc}} = \frac{A\rho_0}{a}\int_{-a}^{x} x'dx' = \frac{A\rho_0}{2a}(x^2 - a^2)$. The flux through the box is simply AE. Thus Gauss's Law tells us that $\vec{E}(-a < x < a) = \frac{\rho_0}{2\varepsilon_0 a}(a^2 - x^2)$ in the $-x$ direction. (*b*) Let region I be to the left slab 1, region II be inside slab 1, region III be between the two slabs, region IV be in the left half of slab 2, region V be in the right half of slab 2, and region VI be to the right of slab 2. The electric field from slab 2 will simply be added or subtracted to the electric field from slab 1 in the various regions. The electric field magnitude outside slab 2 due to slab 2 is $\frac{\rho_0 t}{\varepsilon_0}$. In order to find the electric field inside slab 2, consider a pillbox-shaped Gaussian surface that extends from the left of the slab inside the slab to a depth $x - d$ with its flat circular sides parallel to the slab. The charge enclosed by this box is $q_{\text{enc}} = A\rho_0(x - d)$. The flux through the box is simply AE. Thus Gauss's Law tells us that $E = \frac{\rho_0}{\varepsilon_0}(x - d)$. Adding and subtracting the fields as appropriate in the different regions, we obtain

$$\vec{E}_{\text{I}} = -\frac{\rho_0 t}{\varepsilon_0}\,\hat{i}$$

$$\vec{E}_{\text{II}}(x) = -\left(\frac{\rho_0}{2\varepsilon_0 a}(a^2 - x^2) + \frac{\rho_0 t}{\varepsilon_0}\right)\hat{i}$$

$$\vec{E}_{\text{III}} = -\frac{\rho_0 t}{\varepsilon_0}\,\hat{i}$$

$$\vec{E}_{\text{IV}}(x) = -\frac{\rho_0}{\varepsilon_0}(d - x)\,\hat{i}$$

$$\vec{E}_{\text{V}}(x) = \frac{\rho_0}{\varepsilon_0}(x - d)\,\hat{i}$$

$$\vec{E}_{\text{VI}} = \frac{\rho_0 t}{\varepsilon_0}\,\hat{i}$$

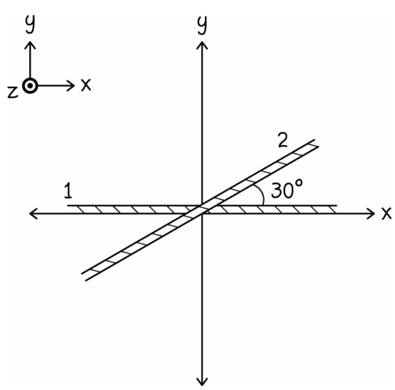
24.104. (*a*) Choose a cylindrical Gaussian surface concentric with the rod, having a radius r. If $r < R$ then the charge enclosed is given by $q_{\text{enc}} = \rho_0 \pi r^2 \ell$ and flux through the surface is $E(2\pi r\ell)$. Thus Gauss's Law yields $E = \frac{\rho_0 r}{2\varepsilon_0}$. So $\vec{E}_{\text{in}} = -\frac{\rho r}{2\varepsilon_0}$ radially outward from the center of the rod. Outside the rod, the charge enclosed no longer depends on the radius of the Gaussian surface $(q_{\text{enc}} = \rho_0 \pi R^2 \ell)$, so we obtain and $\vec{E}_{\text{out}} = \frac{\rho R^2}{2\varepsilon_0 r}$ radially outward from the center of the rod. (*b*) We proceed as in part (*a*), except that now the charge enclosed must be determined by integrating the charge density over the appropriate volume. For values $r < R$ $q_{\text{enc}} = \int_0^{\ell}\int_0^{2\pi}\int_0^{r} \rho_0\left(1 - \frac{2r'}{R}\right) r'dr'd\phi dz = 2\pi\ell\rho_0\left(\frac{r^2}{2} - \frac{2r^3}{3R}\right)$. The flux can still be written as $E(2\pi r\ell)$. So Gauss's Law tells us $\vec{E}_{\text{in}} = \frac{\rho}{2\varepsilon_0}\left(r - \frac{4r^2}{3R}\right)$ radially outward from the center of the rod. In the case of $r > R$, the charge enclosed is $q_{\text{enc}} = -\frac{\pi\ell\rho_0 R^2}{3}$ and we obtain $\vec{E}_{\text{out}} = \frac{\rho R^2}{6\varepsilon_0 r}$ radially inward toward the center of the rod.

24.105. If the hole drilled is very small, we can neglect the charged material that was removed. We have shown that the electric field inside such a uniformly charged nonconducting sphere is $E = \frac{kq_s r}{R^3}$. Let us orient the drilled hole such that it lies along the x axis. Since the charge on the sphere is negative, we can write $E_x = -\frac{k|q_s|x}{R^3}$. Clearly the pellet will experience a force due to this electric field, and if this is the only force exerted on the pellet, we can write $F_x = q_p E_x = -\frac{k|q_s|q_p x}{R^3} = ma_x$. Recall from simple harmonic motion that a restoring force has the form $F_x = -k_{\text{restoring}} x$, where $k_{\text{restoring}}$ may be a spring constant, or any other constant describing a restoring force. Recall also that for an object solely under the influence of a restoring force, the frequency of the oscillations will be $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{restoring}}}{m}}$. In this case, we obtain $f = \frac{1}{2\pi}\sqrt{\frac{k|q_s|q_p}{mR^3}}$.

24.106. Let $\vec{d}$ point from the center of the solid sphere to the center of the cavity. Let $\vec{r}_1$ point from the center of the solid sphere to some point P in the cavity, and let $\vec{r}_2$ point from the center of the cavity to the same point P. The electric field will be the superposition of the electric field due to the complete larger charged sphere, and an oppositely charged smaller sphere: $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho\vec{r}_1}{3\varepsilon_0} - \frac{\rho\vec{r}_2}{3\varepsilon_0}$. Note that $\vec{r}_1 = \vec{d} + \vec{r}_2$. Thus $\vec{E} = \frac{\rho}{3\varepsilon_0}\left[\vec{r}_1 - (\vec{r}_1 - \vec{d})\right] = \frac{\rho\vec{d}}{3\varepsilon_0}$. Thus the resulting electric field in the cavity is given by $\vec{E}_{\text{cavity}} = \frac{\rho\vec{d}}{3\varepsilon_0}$.

24.107. Call the electric field at which dissociation of air occurs $E_{\max}$. The electric field at the surface is $E = \frac{2k\lambda}{R_{\text{wire}}} < \frac{E_{\max}}{3} \Rightarrow \lambda < \frac{E_{\max}R_{\text{wire}}}{6k} = \frac{(3\times 10^6\text{ N/C})(0.0166\text{ m})/2}{6(9.0\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)} = 5\times 10^{-7}\text{ C/m}$. So the wire should not carry more than about 5×10^2 nC/m.

24.108. (*a*)



(*b*) $E_x = \left(\frac{\sigma_2}{2\varepsilon_0}\sin(\theta)\right)$ and $E_y = \left(\frac{\sigma_1 - \sigma_2\cos(\theta)}{2\varepsilon_0}\right)$ where $\theta = 30°$. Hence at (3 m, 1 m, 0)

$E_x = \left(\frac{(90.0\times 10^{-9}\text{ C/m}^2)}{2(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\sin(30°)\right) = 2.54\times 10^3\text{ N/C}$ and

$E_y = \left(\frac{(130.0\times 10^{-9}\text{ C/m}^2) - (90.0\times 10^{-9}\text{ C/m}^2)\cos(30°)}{2(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\right) = 2.94\times 10^3\text{ N/C}$, or equivalently

$\vec{E}(3\text{m},1\text{m},0) = 3.9\times 10^3\text{ N/C}$ at 49° ccw from the $+x$ axis.

25

WORK AND ENERGY IN ELECTROSTATICS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

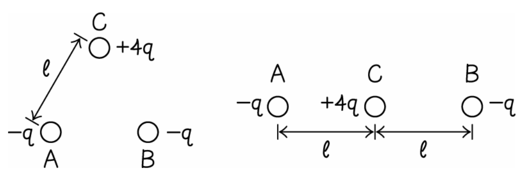
Developing a Feel

1. 10^{-17} J **2.** 10^{-13} J **3.** 10^{7} V **4.** 10^{6} V **5.** at least 10^{-9} C/m^2 **6.** 10^{-7} V **7.** 10^{0} V **8.** $10^{-1}c_0$ **9.** 10^{-24} J

Guided Problems

25.2 Forming and straightening a charged triangle

1. Getting Started We begin by making a sketch of the charge arrangements.

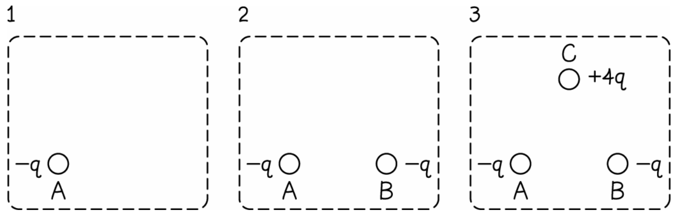


We need to determine the amount of work required to form these two arrangements, starting from a situation where all three charged particles are very far apart (so far that we can neglect their initial interactions). We know that each pair of charged particles will either attract or repel each other. Since we have a combination of like and unlike charges, it is not obvious whether the total amount of work will be positive or negative.

2. Devise Plan We know that work is the change in potential energy. In this case, where gravity and other forces can be ignored, we need only consider the electrostatic potential energy. Thus

$$W = \Delta U = U_{\mathrm{f}}$$

So we can simply find the final electrostatic potential energy. In order to do that, it is sometimes useful to avoid double-counting of interactions by making a diagram after the addition of each particle. For example, one might draw the images below.



Imagining building up the charge distribution one particle at a time makes it easier to count and label interactions. In this case, placing particle A in panel 1 comes with no energy cost. Adding particle B in panel 2 requires that we put enough energy into the system to account for the final potential energy in panel 2, U_{AB}. Finally, the addition of particle C in panel 3 yields two more potential energy terms, U_{AC} and U_{BC}. Make sure you understand that it is not correct to add other terms with the indices swapped. Our total potential energy in the arrangement above is

$$U_{\mathrm{f}} = U_{\mathrm{AB}} + U_{\mathrm{AC}} + U_{\mathrm{BC}} \tag{1}$$

Applying the same process to the linear charge arrangement yields the same expression as equation (1). We still have the same three pair-wise interactions. The only difference is that one of the terms (U_{AB}) will be different in the two cases. In either case it costs nothing to add particle A. In either case, particle A and C are a distance ℓ apart, and C and B are a distance ℓ apart. In the triangular case, particles A and B are also a distance ℓ apart, but in the linear case they are a distance 2ℓ apart. So we expect our calculation to differ only in that U_{AB} term.

3. Execute Plan To calculate the final potential energy of the triangle, we use equation (1) and obtain

$$U_{\mathrm{tri,f}} = U_{\mathrm{AB,tri}} + U_{\mathrm{AC,tri}} + U_{\mathrm{BC,tri}}$$

$$U_{\mathrm{tri,f}} = \frac{kq_{\mathrm{A}}q_{\mathrm{B}}}{\ell} + \frac{kq_{\mathrm{A}}q_{\mathrm{C}}}{\ell} + \frac{kq_{\mathrm{B}}q_{\mathrm{C}}}{\ell}$$

$$U_{\mathrm{tri,f}} = \frac{kq^2}{\ell} - \frac{4kq^2}{\ell} - \frac{4kq^2}{\ell}$$

$$U_{\mathrm{tri,f}} = \frac{-7kq^2}{\ell}$$

Since the particles are initially very far away, the initial potential energy is zero, and we can equate the necessary work to this final potential energy. Thus

$$W_{\mathrm{tri}} = \frac{-7kq^2}{\ell}$$

The process is the same for the linear arrangement.

$$U_{\mathrm{lin,f}} = U_{\mathrm{AB,lin}} + U_{\mathrm{AC,lin}} + U_{\mathrm{BC,lin}}$$

$$U_{\mathrm{lin,f}} = \frac{kq_{\mathrm{A}}q_{\mathrm{B}}}{\ell} + \frac{kq_{\mathrm{A}}q_{\mathrm{C}}}{\ell} + \frac{kq_{\mathrm{B}}q_{\mathrm{C}}}{\ell}$$

$$U_{\mathrm{lin,f}} = \frac{kq^2}{2\ell} - \frac{4kq^2}{\ell} - \frac{4kq^2}{\ell}$$

$$U_{\mathrm{lin,f}} = \frac{-15kq^2}{2\ell}$$

Thus $W_{\mathrm{lin,f}} = \dfrac{-15kq^2}{2\ell}$

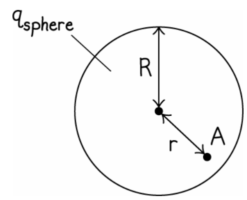
Once the potential energy of the triangular arrangement is known, the potential energy of the linear arrangement could be calculated without all the work above. One could also say that the potential energy of the triangular arrangement includes the $U_{\mathrm{AB,tri}} = \dfrac{kq^2}{\ell}$ term. The potential energy of the linear arrangement is exactly the same as that of the triangular arrangement, except for the increased spacing between particle A and B. The term describing the energy stored by the interaction between A and B in the linear arrangement is $U_{\text{lin,f}} = \dfrac{-kq^2}{2\ell}$. Thus, if we remove $U_{\text{AB,tri}}$ from $U_{\text{tri,f}}$ and replace $U_{\text{AB,tri}}$ with $U_{\text{AB,lin}}$, we should get $U_{\text{lin,f}}$. To check this:

$$U_{\text{tri,f}} - U_{\text{AB,tri}} + U_{\text{AB,lin}} = \frac{-7kq^2}{\ell} - \frac{kq^2}{\ell} + \frac{kq^2}{2\ell} = \frac{-15kq^2}{2\ell} = U_{\text{lin,f}}$$

4. Evaluate Result Particles A and B carry the same type of charge. Having them close together stores positive potential energy, and moving them apart lowers the potential energy. The only difference between the triangular and linear charge arrangements is that particles A and B are farther apart in the linear arrangement. Thus, we expect the triangular arrangement to have higher potential energy than the linear arrangement. This agrees with our answers. Note that the energy of the energy of the triangular arrangement is a negative number with a smaller magnitude than that of the linear arrangement. Thus the potential energy of the triangular arrangement is higher.

25.4 Charged plastic sphere

1. Getting Started We make a sketch of the sphere and use the sketch to clarify our understanding of the variables.



We are told that the sphere carries a total charge q_{sphere}, and has a radius R. We are asked to find the potential difference between the center of the sphere and the point A. In the figure above, we show the point A located inside the sphere, but we are asked to consider the possibilities of A being inside and outside the sphere. r is the distance from the center of the sphere to point A.

2. Devise Plan We determine the potential difference between two points (call them points a and b) using equation (25.25)

$$V_{ab} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$$

In words, this means we define a path from a to b and integrate the electric field's projection along that path. This gives us the potential difference between the two points. In order to apply this, of course, we must know the electric field both inside and outside the sphere. We can do this using Gauss's Law.

Consider a spherical Gaussian surface, concentric with the charged sphere and having radius r. Gauss's Law reads

$$\oint_{\substack{\text{Gaussian}\\\text{sphere}}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

Due to spherical symmetry of both the charge distribution and Gaussian surface, both the area element and the electric field will always point radially outward (meaning they will always be parallel). Thus the scalar product yields $\cos(0) = 1$ in all cases.

$$\oint_{\substack{\text{Gaussian}\\\text{sphere}}} E dA = \frac{q_{\text{encl}}}{\varepsilon_0}$$

Also due to spherical symmetry, the electric field should have the same magnitude at all points along the spherical surface. Since E is a constant for all points over which we are integrating, it can be pulled out of the integral.

$$E \oint_{\substack{\text{Gaussian}\\\text{sphere}}} dA = \frac{q_{\text{encl}}}{\varepsilon_0} \tag{1}$$

Note that since the charge is uniformly distributed, we can say $\dfrac{q_{\text{encl}}}{V_{\text{encl}}} = \dfrac{q_{\text{sphere}}}{V_{\text{sphere}}}$ or $q_{\text{encl}} = \dfrac{r^3}{R^3} q_{\text{sphere}}$. This is true for any Gaussian sphere that is smaller than the charged sphere $(r < R)$. Once we begin closing the Gaussian sphere outside the charged sphere, the expression above breaks down because the charge enclosed is simply q_{sphere}, regardless of the volume we enclose. First treating the case where the Gaussian surface is closed inside the charged sphere, we obtain an expression for the electric field inside the charged sphere.

$$E_{\text{in}} 4\pi r^2 = \frac{q_{\text{sphere}}}{\varepsilon_0}\left(\frac{r}{R}\right)^3$$

$$E_{\text{in}} = \frac{q_{\text{sphere}} r}{4\pi\varepsilon_0 R^3} \tag{2}$$

If $r > R$, the charge enclosed is simply $q_{\text{encl}} = q_{\text{sphere}}$, and we obtain

$$E_{\text{out}} 4\pi r^2 = \frac{q_{\text{sphere}}}{\varepsilon_0}$$

$$E_{\text{out}} = \frac{q_{\text{sphere}}}{4\pi\varepsilon_0 r^2} \tag{3}$$

Right between the two cases $(r = R)$, note that equations (2) and (3) give the same result.

We now have the electric fields required to determine potential differences between the center of the sphere and the point A. But before we do so, we note a subtlety. Equation (1) only gives us the difference in potential between two points, not the potential at either point. We can still shift the value of the potential by any arbitrary amount without changing the physics of the setup. Choosing the potential to go to zero as $r \to \infty$ is a common and useful convention, but it is only a convention.

We also note that the potential on the surface of the sphere must be the same whether we approach the surface from outside or inside. Indeed, finding two potentials at the same point would yield infinite electric fields. So $V(r = R)$ will have only one value, regardless of how we calculate it.

3. Execute Plan Let us first consider the case $r \le R$. In that case, we determine the potential difference from the center to point A by inserting equation (2) into equation (25.25). Let us call the distance from the sphere's center to the point A r_{A}. We know $\vec{E}_{\text{in}}$ points radially outward. If we choose our path of integration from the center toward point A, then $d\vec{\ell}$ also points radially outward. In fact, $d\vec{\ell}$ is exactly the same as $d\vec{r}$ in that case, even up to the limits of integration. Thus we can write:

$$V_{\text{A}} - V_0 = -\int_0^{r_{\text{A}}} \vec{E}_{\text{in}} \cdot d\vec{\ell}$$

$$V_{\text{A}} - V_0 = -\frac{q_{\text{sphere}}}{4\pi\varepsilon_0 R^3}\int_0^{r_{\text{A}}} r\,dr$$

$$V_{\text{A}} - V_0 = -\frac{q_{\text{sphere}}}{4\pi\varepsilon_0 R^3}\left(\frac{1}{2}r_{\text{A}}^2 - 0\right)$$

$$V_{\text{A}} - V_0 = -\frac{q_{\text{sphere}} r_{\text{A}}^2}{8\pi\varepsilon_0 R^3} \tag{4}$$

Now for the case $r \geq R,$ we first determine the potential difference between the origin and the outer edge of the sphere, and then determine the potential difference between the outer edge and the position A. Symbollically, $V_A - V_0 = V_A - V_R + V_R - V_0,$ where the potential difference $V_R - V_0$ comes from our result (4) above. Thus

$$V_A - V_0 = V_A - V_R + V_R - V_0$$

$$V_A - V_0 = -\int_R^{r_A} \vec{E}_{out} \cdot d\vec{\ell} - \frac{q_{sphere}}{8\pi\varepsilon_0 R}$$

We insert equation (3) into the expression above, and note that again the electric field and differential step along the path of integration are parallel, such that

$$V_A - V_0 = -\frac{q_{sphere}}{4\pi\varepsilon_0}\int_R^{r_A} \frac{dr}{r^2} - \frac{q_{sphere}}{8\pi\varepsilon_0 R}$$

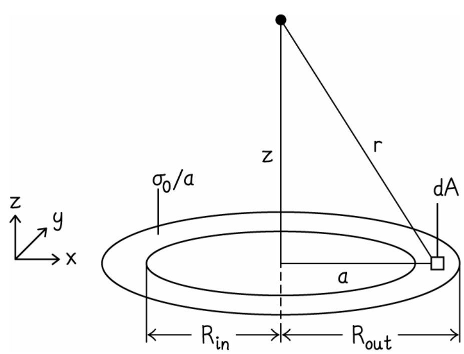
$$V_A - V_0 = -\frac{q_{sphere}}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{r_A}\right) - \frac{q_{sphere}}{8\pi\varepsilon_0 R}$$

$$V_A - V_0 = \frac{q_{sphere}}{8\pi\varepsilon_0}\left(\frac{2}{r_A} - \frac{3}{R}\right) \tag{5}$$

4. Evaluate Result Very far from the sphere's center, the potential should approach the potential at infinite distance. We cannot claim that this potential must be zero. We are always free to measure our potential relative to anything we like, and choosing the potential to be zero at infinity is merely a convention. What we can say without ambiguity is that the electric field must go to zero at infinite distance from a source. That is equivalent to saying that the potential must approach a constant. So the logical check we perform is whether or not the result (5) approaches a constant as $r_A \to \infty.$ We see that in that limit $V_A - V_0 = \dfrac{3q_{sphere}}{8\pi\varepsilon_0 R},$ which is indeed a constant, and which has the right units.

25.6 Charged ring

1. Getting Started We begin by making a sketch of the ring and the variables we plan to use to solve the problem.



We expect that the problem will be easier to solve for points along the axis, because of the symmetry of the charge distribution about that axis. For example, rotating the ring around that vertical (z) axis changes nothing about the charge distribution. This should mean that our integral around the azimuthal angle ϕ will be trivial (there is no ϕ dependence in the distribution). If we were asked about the potential off-center, we would expect a more complicated integral.

2. Devise Plan We know that the potential from a single charged particle is $V_{particle} = kq/r.$ Each small (differential) section of area of the ring is like a tiny charged particle with charge

$$dq = \sigma(a)dA = \frac{\sigma_0}{a} r dr d\phi \tag{1}$$

Each such section will contribute a small amount $dV_{\text{particle}} = kdq/r$ to the total potential at a given point. To find the total potential, we integrate these dV_{particle} over the entire ring.

Note that $r = \sqrt{a^2 + z^2}$. This means that r is constant if a is constant. In other words, all differential area sections that lie along a thin ring of radius a with contribute the same $V_{\text{particle}} = kq/\sqrt{a^2 + z^2}$ to the total potential. The contribution from such a thin ring comes from adding all such differential contributions around the ring, integrating $d\phi$ from 0 to 2π. We use this as a guide for setting up our integral.

$$V = \int_{\substack{\text{thick}\\\text{ring}}} dV_{\text{particle}}$$

$$V = \int_{\substack{\text{thick}\\\text{ring}}} \frac{kdq}{r}$$

$$V = \int_{r_{\min}}^{r_{\max}} \int_0^{2\pi} \frac{k\sigma_0}{ar} r dr d\phi$$

Note that r is the distance from the point where we want to know the potential to a charge element on the ring. Its maximum and minimum values are not $R_{\max}$ and $R_{\min}$. Those are the limits on a. r is a variable, as is a. To carry out the integral we write one in terms of the other: $r = \sqrt{a^2 + z^2}$, and note that $dr = \frac{a}{r} da$. Thus

$$V = \int_{R_{\text{in}}}^{R_{\text{out}}} \int_0^{2\pi} \frac{k\sigma_0}{a} \left(\frac{a}{r}\right) da d\phi$$

$$V = 2\pi k\sigma_0 \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{da}{\sqrt{a^2 + z^2}} \tag{2}$$

3. Execute Plan The integral in equation (2) can be solved by looking it up in an integral table, by using software. One finds that

$$\int \frac{da}{\sqrt{a^2 + z^2}} = \ln\left(a + \sqrt{a^2 + z^2}\right)$$

such that

$$V = 2\pi k\sigma_0 \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{da}{\sqrt{a^2 + z^2}} = 2\pi k\sigma_0 \ln\left(\frac{R_{\text{out}} + \sqrt{R_{\text{out}}^2 + z^2}}{R_{\text{in}} + \sqrt{R_{\text{in}}^2 + z^2}}\right)$$

Or equivalently

$$V = \frac{\sigma_0}{2\varepsilon_0} \ln\left(\frac{R_{\text{out}} + \sqrt{R_{\text{out}}^2 + z^2}}{R_{\text{in}} + \sqrt{R_{\text{in}}^2 + z^2}}\right) \tag{3}$$

4. Evaluate Result In the limit as $z \gg R_{\text{out}}$, both the numerator and denominator in the logarithm of equation (3) approach z. Thus the logarithm approaches zero, such that the electric potential goes to zero at very large distances. One can also check the result by considering units, although one must be careful in this case. Since the surface charge density was given by $\sigma(a) = \sigma_0/a$, the units of σ_0 are actually C/m, not C/m^2. This does yield the correct units for electric potential.

25.8 Electric field due to a charged disk

1. Getting Started We know the radius of the disk is R, and the surface charge density on the disk is σ. We know that we want to describe the potential and eventually the electric field on an axis that passes perpendicularly through the center of the disk. Thus we can use the expression from the chapter summary for the potential along this axis:

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - |z|\right) \tag{1}$$

Thus we have everything we need to determine the electric potential a given distance z from the disk, along the symmetry axis. We need to determine the electric field.

2. Devise Plan Equation (1) from the chapter summary already assumed that the point at which the potential was to be determined was on the symmetry axis (z axis). Thus

$$V(x = y = 0, z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - |z|\right)$$

The electric field can be determined by this electrostatic potential using equation (25.40):

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

We already know that the x and y components of the electric field should be zero, based on symmetry. That is, the disk can be rotated around its axis of symmetry without changing the charge distribution at all. Thus, rotating the system cannot affect the electric field. So the field cannot have any component that would change under rotation bout the z axis. But now we can also see mathematically that these components are zero, since equation (1) has no x or y dependence. Recall that R is the constant radius of the disk. Everything in equation (1) is held fixed, except z.

3. Execute Plan With the symmetry arguments above, we can write

$$\vec{E} = E\hat{k} = -\frac{\partial V}{\partial z}\hat{k}$$

$$\vec{E} = -\frac{\partial}{\partial z}\left(\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - |z|\right)\right)\hat{k}$$

In the case where $z > 0$, we find

$$\vec{E} = -\frac{\partial}{\partial z}\left(\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)\right)\hat{k}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)\hat{k} \tag{2}$$

In the case where $z < 0$, we find

$$\vec{E} = -\frac{\partial}{\partial z}\left(\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} + z\right)\right)\hat{k}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(-1 - \frac{z}{\sqrt{z^2 + R^2}}\right)\hat{k} \tag{3}$$

One way of compactly writing equations (2) and (3) together is

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2}} - \frac{z}{\sqrt{z^2 + R^2}}\right)\hat{k}$$

$$\vec{E} = \frac{\sigma z}{2\varepsilon_0}\left(\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{z^2 + R^2}}\right)\hat{k} \tag{4}$$

4. Evaluate Result This agrees exactly with the result of Example 23.6. That example sought merely one component. Here we are expressing the additional information that the z component is the only component. Other than that, the expressions are identical.

Questions and Problems

25.1. The greatest electrostatic potential energy is attained when the dipole moment and the electric field are antiparallel. The smallest electrostatic potential energy is attained when the dipole moment and the electric field are parallel. One way of seeing this is to consider what would happen to a dipole released from either position and see if it could gain kinetic energy by lowering its potential energy.

25.2. All particles will have the same final speed. The easiest way of seeing this is through energy conservation. For an arbitrary particle P, the initial energy in a system consisting of the plate and the particle can be written as $K_{P,\text{i}} + U_{P,\text{i}}$, and the final energy for the particle can be written $K_{P,\text{f}} + U_{P,\text{f}}$. Because the total energy of the system is constant (no outside forces do any work) we can write $K_{P,\text{i}} + U_{P,\text{i}} = K_{P,\text{f}} + U_{P,\text{f}}$ or equivalently

$$K_{P,\text{f}} = K_{P,\text{i}} + (U_{P,\text{i}} - U_{P,\text{f}}) \tag{1}$$

Since all particles start at the same point, and end on the plate, the change in potential energy must be the same. That coupled with the fact that the particles are launched with the same initial speed means that the right-hand side of equation (1) is the same for all three particles. Thus the final speed of all three particles must be the same.

25.3. (*a*) The 1 kg ball will have the greatest final speed. The force of gravity on each ball would give each ball an acceleration of $-9.8\text{ m/s}^2\,\hat{j}$. The electric force will also be exerted downward, increasing the acceleration of the balls further. The electric force is the same on all three balls, but it will cause the largest additional acceleration in the case of the 1 kg ball. Hence the 1 kg ball will experience the greatest acceleration as it falls and will have the greatest final speed. (*b*) The 3 kg ball will reach the ground first. In this case the electric force is exerted upward. This upward force will cause the smallest acceleration in the 3 kg ball, meaning it will counteract the acceleration due to gravity less for the 3 kg ball than for any other. So the 3 kg ball will reach the ground first.

25.4. (*a*) The change in kinetic energy will be given by the work done by the electric force. The force is exerted across the same distance for all three particles. But since the alpha particle has twice the charge of the proton or deuteron, the force exerted on the alpha particle will be twice as great as the force exerted on the proton or deuteron. Hence $K_\text{d} = K_\text{p} = K_\alpha/2$. (*b*) In order to determine relative momenta, we must have the relative final speeds. Given the relationship between the final kinetic energies in (*a*), and the relationship between the masses of the particles $(m_\text{d} = 2m_\text{p},\ m_\alpha = 4m_\text{p})$ we can write $\frac{1}{2}m_\text{p}v_{\text{p,f}}^2 = \frac{1}{2}m_\text{d}v_{\text{d,f}}^2 = \frac{1}{4}m_\alpha v_{\alpha,\text{f}}^2$ and solve for the relative speeds. Writing all masses in terms of the proton mass yields $\frac{1}{2}m_\text{p}v_{\text{p,f}}^2 = \frac{1}{2}2m_\text{p}v_{\text{d,f}}^2 = \frac{1}{4}4m_\text{p}v_{\alpha,\text{f}}^2$ or $\frac{1}{\sqrt{2}}v_{\text{p,f}} = v_{\text{d,f}} = v_{\alpha,\text{f}}$. From here we write $p = mv$ for each particle. For comparison purposes, we write the momenta in terms of the proton momentum:

$$p_{\text{p,f}} = m_\text{p}v_{\text{p,f}}$$

$$p_{\text{d,f}} = m_\text{d}v_{\text{d,f}} = 2m_\text{p}\frac{1}{\sqrt{2}}v_{\text{p,f}} = \frac{2}{\sqrt{2}}p_{\text{p,f}}$$

$$p_{\alpha,\text{f}} = m_\alpha v_{\alpha,\text{f}} = 4m_\text{p}\frac{1}{\sqrt{2}}v_{\text{p,f}} = \frac{4}{\sqrt{2}}p_{\text{p,f}}$$

which can be written as $(p_{\text{p,f}} : p_{\text{d,f}} : p_{\alpha,\text{f}}) = (\sqrt{2} : 2 : 4)$

Or, dropping the "final" subscript: $p_\text{p} = (\sqrt{2}p_\text{d})/2 = (\sqrt{2}p_\alpha)/4$. (*c*) As shown in part (*b*) $v_\text{p} = \sqrt{2}v_\text{d} = \sqrt{2}v_\alpha$. (*d*) Since the field is uniform, the acceleration will be constant ($a = F^E/m = qE/m$). So we can use the kinematic equation $\Delta x = v_{x,\text{i}}t + \frac{1}{2}a_x t^2$ to obtain the expression for time $t = \sqrt{\frac{2\Delta x}{E}\frac{m}{q}}$. The distance covered Δx, and the electric field strength are the same for all three particles. Again, we write each time in terms of the time required by the proton.

$$t_{\mathrm{p}} = \sqrt{\frac{2\Delta x}{E}\frac{m_{\mathrm{p}}}{q_{\mathrm{p}}}}$$

$$t_{\mathrm{d}} = \sqrt{\frac{2\Delta x}{E}\frac{m_{\mathrm{d}}}{q_{\mathrm{d}}}} = \sqrt{\frac{2\Delta x}{E}\frac{2m_{\mathrm{p}}}{q_{\mathrm{p}}}} = \sqrt{2}t_{\mathrm{p}}$$

$$t_{\alpha} = \sqrt{\frac{2\Delta x}{E}\frac{m_{\alpha}}{q_{\alpha}}} = \sqrt{\frac{2\Delta x}{E}\frac{4m_{\mathrm{p}}}{2q_{\mathrm{p}}}} = \sqrt{2}t_{\mathrm{p}}$$

Combining these results, we write $\Delta t_{\mathrm{p}} = (\sqrt{2}\Delta t_{\mathrm{d}})/2 = (\sqrt{2}\Delta t_{\alpha})/2$. (*e*) The change in electrostatic potential energy can be written as $\Delta U = q\Delta V$. All particles move through the same potential difference. So the ratios of potential energy change will be equal to the ratios of the charges. Hence $\Delta U_{\mathrm{p}}^{\mathrm{E}} = \Delta U_{\mathrm{d}}^{\mathrm{E}} = \Delta U_{\alpha}^{\mathrm{E}}/2$.

25.5. (*a*) A force exerted over a certain time interval causes a change in momentum, in this case $qE\Delta t = mv_{\mathrm{f}}$ or $v_{\mathrm{f}} = qE\Delta t/m$. Hence we can write the final kinetic energy of any particle as $\frac{1}{2}m\left(\frac{qE\Delta t}{m}\right)^2$. We will write each kinetic energy in terms of the kinetic energy of the proton for comparison purposes:

$$K_{\mathrm{p,f}} = \frac{(E\Delta t)^2}{2}\frac{q_{\mathrm{p}}^2}{m_{\mathrm{p}}}$$

$$K_{\mathrm{d,f}} = \frac{(E\Delta t)^2}{2}\frac{q_{\mathrm{d}}^2}{m_{\mathrm{d}}} = \frac{1}{2}K_{\mathrm{p,f}}$$

$$K_{\alpha,\mathrm{f}} = \frac{(E\Delta t)^2}{2}\frac{q_{\alpha}^2}{m_{\alpha}} = K_{\mathrm{p,f}}$$

Combining these results, we write $K_{\mathrm{p}} = 2K_{\mathrm{d}} = K_{\alpha}$. (*b*) The final momentum is just given by $p_{\mathrm{f}} = qE\Delta t$. So the ratios of final momentum will be equal to the ratios of charge on the particles. Hence $p_{\mathrm{p}} = p_{\mathrm{d}} = p_{\alpha}/2$. (*c*) The final speed is given by $v_{\mathrm{f}} = qE\Delta t/m$. So we write an expression for the final speed of each particle, and express this in terms of the proton's final momentum.

$$v_{\mathrm{p,f}} = q_{\mathrm{p}}E\Delta t/m_{\mathrm{p}}$$

$$v_{\mathrm{d,f}} = q_{\mathrm{d}}E\Delta t/m_{\mathrm{d}} = q_{\mathrm{p}}E\Delta t/2m_{\mathrm{p}} = \frac{1}{2}v_{\mathrm{p,f}}$$

$$v_{\alpha,\mathrm{f}} = q_{\alpha}E\Delta t/m_{\alpha} = 2q_{\mathrm{p}}E\Delta t/4m_{\mathrm{p}} = \frac{1}{2}v_{\mathrm{p,f}}$$

Combining these results, we write $v_{\mathrm{p}} = 2v_{\mathrm{d}} = 2v_{\alpha}$. (*d*) Because the acceleration is constant, we can use kinematic equations, and we can write $\Delta x = \frac{1}{2}(v_{x,\mathrm{i}} + v_{x,\mathrm{f}})\Delta t = \frac{v_{x,\mathrm{f}}\Delta t}{2}$. Since each particle is accelerated for the same amount of time, the ratio of the distances will just be the ratio of the final velocities. Hence $\Delta x_{\mathrm{p}} = 2\Delta x_{\mathrm{d}} = 2\Delta x_{\alpha}$. (*e*) The change in electrostatic potential energy can be written $\Delta U = q\Delta V = -qE\Delta x$. Since all particles move through the same electric field, the ratio of changes in potential energies will be equal to the product of the charge times the distance ratios. Hence $\Delta U_{\mathrm{p}}^{\mathrm{E}} = 2\Delta U_{\mathrm{d}}^{\mathrm{E}} = \Delta U_{\alpha}^{\mathrm{E}}$. This is also what one would expect by looking at the changes in kinetic energy.

25.6. (*a*) is lower in electrostatic potential energy than (*b*). This can be seen by simply drawing the dipole fields, or by noting the relative distances between positive and negative charges in the two cases. In (*a*) the positive regions of the two dipoles are adjacent to each other. So are the negative regions. In (*b*) the positive regions of the two dipoles are across from each other along a diagonal, meaning they are farther apart than in (*a*). In case (*a*) the positive regions repel each other and try to lower their potential energy. In (*b*) they have repelled each other and have lowered their potential energy.

25.7. The magnitude of the change in electrostatic potential energy of either charged end can be written as $\Delta U^E = qEd$. Since the positive charge moves to a higher potential and the negative charge moves to a lower potential, the changes in potential energy for both charged ends is positive going from (*a*) to (*b*). Hence $U_b - U_a = 2qEd$.

25.8. The work done along path 2 is also W_1. Electrostatic work depends only on the endpoints, not on the path.

25.9. Since the particle now carries a negative charge, that change alone could make the work negative. However, we are also changing the direction that our particle is moving. We are now moving from B to A. This means that we have switched the direction of the electrostatic forces and the direction of our motion. Hence there is no overall sign change. However, since the particle now carries twice the charge (in magnitude) as it did before, we will have to do twice as much work. The work done is now $2W$.

25.10. The work done is still *W*. Electrostatic work is independent of the path; it depends only on the endpoints of the path.

25.11. If the potential is higher at point B than at point A, then the field points from B to A. One way of seeing this is to consider the source of the electric field. If the potential is higher at B than at A, then point B must be closer to some positively charged object than A, or A must be closer to some negatively charged object than point B is. It is clear that either of these scenarios would produce a field from B to A.

25.12. The change in electrostatic potential energy of object 3 is positive, since the oppositely-charged objects move farther apart. The change in electrostatic potential energy is negative for objects 1 and 2. Since objects 1 and 2 carry the same charge, the negative change in potential energy is the same for objects 1 and 2. Hence $\Delta U_3 > \Delta U_2 = \Delta U_1$.

25.13. If electrostatic work W is done in moving from A to B, then $-W$ must be done by electrostatic forces in moving from B back to A. In order to overcome these forces, we have to do work W just to get the particle back to point A. We also provide $2W$ of work that goes to kinetic energy. Hence we must do work equal to $3W$, altogether.

25.14. Consider that moving the particle all around the triangle along the path $A \to B \to C \to A$ will result in no net change in electrostatic potential energy and therefore cannot require any net work. This means that $W_{\text{AB}} + W_{\text{BC}} + W_{\text{CA}} = 0$. We can write this as $W_{\text{AB}} + W_{\text{BC}} - W_{\text{AC}} = 0$ or $W_{\text{AB}} + W_{\text{BC}} - (-W_{\text{AB}}) = 0$. Hence $W_{\text{BC}} = -\frac{3}{2}W_{\text{AB}}$.

25.15. (*a*) The electrostatic potential energy would remain the same. If we only consider our system to be the electron, then there is no electrostatic potential energy. Rather, electrostatic forces are external forces that do work on our system. (*b*) The electrostatic potential energy must have increased. We had to do positive work on the system to make the change. Since this work did not go into kinetic energy, it must have increased potential energy. (*c*) $V_{\text{B}} - V_{\text{A}} < 0$. It was argued in part (*b*) that the electrostatic potential energy must have increased. Since an electron is negatively charged, this corresponds to moving it to a lower potential. Hence the potential at B must have been lower than the potential at A.

25.16. (*a*) The electrostatic potential energy would remain the same. If we only consider our system to be the proton, then there is no electrostatic potential energy. Rather, electrostatic forces are external forces that do work on our system. (*b*) The electrostatic potential energy must have increased. We had to do positive work on the system to make the change. Since this work did not go into kinetic energy, it must have increased potential energy. (*c*) $V_{\text{B}} - V_{\text{A}} > 0$. It was argued in part (*b*) that the electrostatic potential energy must have increased. Since a proton is positively charged, this corresponds to moving it to a higher potential. Hence the potential at B must have been higher than the potential at A.

25.17. (*a*) The electrostatic potential energy increases. The electron has a negative charge and moves to a region of lower potential. This corresponds to an increase in potential energy. (*b*) The electron's kinetic energy must decrease. Here, electrostatic interactions are being taken into account through changes in potential energy (all sources are included in our system). Hence there are no external forces. This means the total energy of the system must remain unchanged. The increase in potential energy described in part (*a*) must therefore be accompanied by a decrease in kinetic energy. (*c*) The answer to (*a*) would change. Rather than increasing, the electrostatic potential energy would remain the same. It would be zero because the sources of the electric field are not included in our system and electrostatic interactions are external forces that do work on our system. The answer to (*b*) would not change. The electron would still slow down, and kinetic energy is associated with the electron alone. It is not dependent on what we include in our system.

25.18. (*a*) The electrostatic potential energy decreases. The proton has a positive charge and moves to a region of lower potential. This corresponds to a decrease in potential energy. (*b*) The proton's kinetic energy must increase. Here, electrostatic interactions are being taken into account through changes in potential energy (all sources are included in our system). Hence there are no external forces. This means the total energy of the system must remain unchanged. The decrease in potential energy described in part (*a*) must therefore be accompanied by an increase in kinetic energy. (*c*) The answer to (*a*) would change. Rather than decreasing, the electrostatic potential energy would remain the same. It would be zero because the sources of the electric field are not included in our system and electrostatic interactions are external forces that do work on our system. The answer to (*b*) would not change. The proton would still speed up, and kinetic energy is associated with the proton alone. It is not dependent on what we include in our system.

25.19. $W_{(a)} = W_{(b)} < W_{(c)} < W_{(d)}$. Note that the electric field in this region points vertically downward, so that all forces are exerted in the vertical direction. Any horizontal motion or horizontal component of motion cannot result in any work being done. Since the motion described in (*a*) and (*b*) both move $+q$ the same distance along the vertical direction, the same amount of work is done in those cases. Since the charged particles are moved opposite their attraction to the negatively charge plate, the work done must be negative. The motion described in (*c*) is perpendicular to any electrostatic forces and results in no work being done. Finally, the motion in (*d*) results in a negative charge being moved parallel to the repulsive force exerted on it due to the negatively charged plate. This results in positive work being done. Hence $W_{(a)} = W_{(b)} < W_{(c)} < W_{(d)}$.

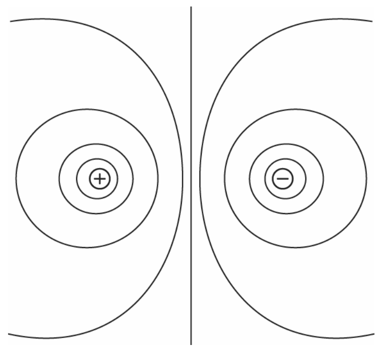
25.20. For all parts below we use the given relations $\Delta V_{AC} = -\Delta V_{AB}$ and $\Delta V_{AC} = 2\Delta V_{BD}$.
(*a*) Clearly the difference in potential between A and C can be broken down into steps and written as the potential difference between A and B added to the potential difference between B and C. This means $\Delta V_{BC} = \Delta V_{AC} - \Delta V_{AB}$. Using $\Delta V_{AC} = -\Delta V_{AB}$ yields $\Delta V_{BC} = -2\Delta V_{AB}$.
(*b*) Similar to our approach in part (*a*) we write $\Delta V_{CD} = \Delta V_{BD} - \Delta V_{BC}$. Inserting the given information that $\Delta V_{BD} = -\frac{1}{2}\Delta V_{AB}$ and the results from part (*a*): $\Delta V_{BC} = -2\Delta V_{AB}$, we obtain $\Delta V_{CD} = -\frac{1}{2}\Delta V_{AB} - (-2\Delta V_{AB})$. Hence $\Delta V_{CD} = \frac{3}{2}\Delta V_{AB}$. (*c*) We start with $\Delta V_{AD} = \Delta V_{AB} + \Delta V_{BD}$. We insert $\Delta V_{BD} = -\frac{1}{2}\Delta V_{AB}$ to obtain $\Delta V_{AD} = \Delta V_{AB} - \frac{1}{2}\Delta V_{AB}$ or $\Delta V_{AD} = \frac{1}{2}\Delta V_{AB}$.

25.21. Yes, there are some cases where an equipotential line may cross itself at a point where the electric field becomes zero. Note that this occurrence is only sensible at a point where the electric field is zero, because the electric field would otherwise have an ill-defined direction. As an example of this, consider the equipotential line around two equally-charged particles that includes a point half-way between them.

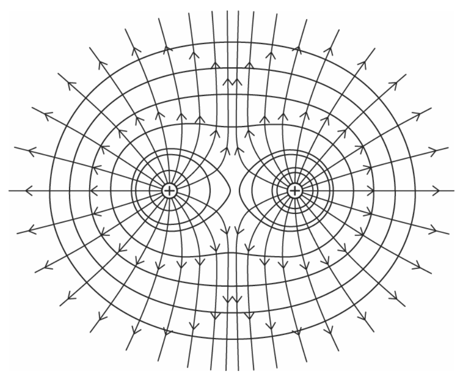
25.22. Yes, if the electric field vanishes only at a single point, the equipotential line passing through that point can still be drawn. As an example of this, consider the equipotential line around two equally-charged particles that includes a point half-way between them.

25.23. The equipotential surfaces associated with a long, uniformly-charged wire would be long cylindrical shells. The cylinders would have the same central axis as the wire, but could have any radius depending on what the value of the potential is along that surface. This is valid as long as the wire can be treated as being infinite. For any real wire, near the ends, the shape would deviate from a perfect cylinder; they would be more rounded.

25.24.



25.25.



25.26. (*a*) The electric field is highest to the right of the particle. (*b*) The direction of the electric field is perpendicular to the equipotential line, pointing roughly inward toward the negatively charged particle. In this case, the electric field points to the left. (*c*) The outer-most equipotential line has the highest potential. The potential decreases as we get closer to the negatively charged particle.

25.27. (*a*) No. A and B could lie to two disjoint equipotential surfaces. If no work is required to move a charged particle from A to B, then the potential difference between A and B must be zero. If both points are at the same potential, then they lie along equipotential surfaces that describe the same potential. However, they could in principle lie along two different surfaces that describe the same potential. Consider two equally and positively charged particles separated by some distance *d*. The potential surface corresponding to a very high potential will be almost spherical and very close to either particle. That is, there will be such a surface around each particle, and these surfaces will not be connected. Hence, the two points A and B must be on equipotential surfaces that describe the same potential, but these surfaces could be separate. (*b*) No. The path can travel to a higher or lower potential before returning to the same potential. In fact, if the two points are on different equipotential surfaces that are separated in space, then there would be no path from point A to B that is entirely along an equipotential surface.

25.28. (*a*) The potential would have to depend linearly on *r*, say according to $V(r) = V_0 + \kappa r$ where κ is any constant with units of V/m. (*b*) There would be no *r*-dependence of the electric field in this case. The electric field would be a constant (more specifically, it would be equal to $-\kappa\hat{r}$).

25.29. (*a*) The potential due to a charge particle is given by $V = \dfrac{kq}{r} = \dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-9}\ \text{C})}{(4.0\ \text{m})} = 6.7\,\text{V}.$

(*b*) If the particle is not sped up at all, then the work required will be equal to the change in potential energy of the particle. Hence $W = \Delta U = q\Delta V.$ In this case, since the potential at infinity is taken to be zero, the potential difference for particle B is just the final potential, 4 m from particle A. So $W = qV_\text{f} = (3.0\times10^{-9}\ \text{C})(6.7\ \text{V}) = 2.0\times10^{-8}\ \text{J}.$ (*c*) With particle A moved to $r = 4.0\ \text{m},$ the potential at the origin will now be 6.7 V. Particle B will then be moving through exactly the same potential difference as in part (*b*); it will move from a potential of zero at an infinite distance away to a potential of 6.7 V at the origin. Hence the work required to move particle B will be exactly the same as in part (*b*): $2.0\times10^{-8}\ \text{J}.$

25.30. (*a*) In the case of a uniform electric field, the work is given by $W = q\vec{E}\cdot\vec{d} = (6.0\times10^{-9}\ \text{C})(2.0\times10^{3}\ \text{N/C})(4.0\ \text{m}) = 4.8\times10^{-5}\ \text{J}.$ (*b*) Since our system is just the charged particle, no electrostatic potential energy is being stored as work is done on the particle. Hence all work done must be contributing to kinetic energy. Since the particle started from rest, the final kinetic energy is exactly the same as the amount of electrostatic work done on the particle: $4.8\times10^{-5}\ \text{J}.$

25.31. For all cases, the procedure is to add up the pairs of particles that can interact with each other and store electrostatic potential energy. In general, the potential energy stored will be given by $U = U_\text{AB} + U_\text{BC} + U_\text{AC} + U_\text{AD} + U_\text{BD} + U_\text{CD}.$ In terms of the charged particles (the signs of which may change) this can be written as $U = \dfrac{k}{a}\left(q_\text{A}q_\text{B} + q_\text{B}q_\text{C} + q_\text{C}q_\text{D} + q_\text{A}q_\text{D} + \dfrac{q_\text{A}q_\text{C}}{\sqrt{2}} + \dfrac{q_\text{B}q_\text{D}}{\sqrt{2}}\right).$ Here *a* is the side length of the square.

(*a*) If all particles carry a positive charge, then all the terms in the general expression are positive, and we can write $U = \dfrac{kq^2}{a}\left(4 + \dfrac{2}{\sqrt{2}}\right).$ So $U = \dfrac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-9}\ \text{C})^2}{(3.0\ \text{m})}\left(4 + \dfrac{2}{\sqrt{2}}\right) = 1.5\times10^{-7}\ \text{J}.$

(*b*) Now the general expression $U = \dfrac{k}{a}\left(q_\text{A}q_\text{B} + q_\text{B}q_\text{C} + q_\text{C}q_\text{D} + q_\text{A}q_\text{D} + \dfrac{q_\text{A}q_\text{C}}{\sqrt{2}} + \dfrac{q_\text{B}q_\text{D}}{\sqrt{2}}\right)$ simplifies to $U = \dfrac{kq^2}{a}\left(-1 + 1 + 1 - 1 + \dfrac{-1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}\right) = 0.$

(*c*) The general expression becomes

$$U = \frac{kq^2}{a}\left(-\frac{2}{\sqrt{2}}\right) = U = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-9}\ \text{C})^2}{(3.0\ \text{m})}\left(\frac{-2}{\sqrt{2}}\right) = -3.8\times10^{-8}\ \text{J}$$

(*d*) The general expression becomes

$$U = \frac{kq^2}{a}\left(-4 + \frac{2}{\sqrt{2}}\right) = U = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-9}\ \text{C})^2}{(3.0\ \text{m})}\left(-4 + \frac{2}{\sqrt{2}}\right) = -7.0\times10^{-8}\ \text{J}$$

25.32. (*a*) The potential at the apex of the triangle is given by $V = \dfrac{2kq}{r} = \dfrac{2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-9}\ \text{C})}{(2.0\ \text{m})} = 18\ \text{V}.$ (*b*) Assuming the particle does not speed up such that all the work goes to increasing the electrostatic potential energy of the particle, then $W = \Delta U = q\Delta V.$ Since the 5.0 nC of charge is being brought in from an infinite distance, the change in potential is just the final potential at the apex of the triangle. Hence $W = (5.0\times10^{-9}\ \text{C})(18\ \text{V}) = 9.0\times10^{-8}\ \text{J}.$ (*c*) The potential at the apex of the triangle would change to $V = \dfrac{k(q_\text{A} + q_\text{B})}{r} = \dfrac{2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.0\times10^{-9}\ \text{C})}{(2.0\ \text{m})} = -4.5\ \text{V}.$ The work required to move the 5.0 nC charge to the apex only changes because the potential at the apex changes. The new work required would be $W = (5.0\times10^{-9}\ \text{C})(-4.5\ \text{V}) = -2.2\times10^{-8}\ \text{J}.$ (*d*) Call the particle at the apex particle C, and call the side length of the triangle a. The general expression for the electrostatic potential energy is $U = \frac{k}{a}(q_A q_B + q_B q_C + q_C q_A)$. For the charge distribution presented in (a), we have

$$U = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})}{(2.0\ \mathrm{m})}\left((2.0\times10^{-9}\ \mathrm{C})^2 + 2(2.0\times10^{-9}\ \mathrm{C})(5.0\times10^{-9}\ \mathrm{C})\right)$$
$$= 1.1\times10^{-7}\ \mathrm{J}$$

For the distribution described in part (c), we have

$$U = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})}{(2.0\ \mathrm{m})}[(2.0\times10^{-9}\ \mathrm{C})(-3.0\times10^{-9}\ \mathrm{C}) + (2.0\times10^{-9}\ \mathrm{C})(5.0\times10^{-9}\ \mathrm{C}) + (5.0\times10^{-9}\ \mathrm{C})(-3.0\times10^{-9}\ \mathrm{C})]$$
$$U = -4.9\times10^{-8}\ \mathrm{J}$$

25.33. (a) All six charged particles are the same distance from the center of the sphere. So the total potential can just be written as $V = \frac{6kq}{R} = \frac{6(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.0\times10^{-9}\ \mathrm{C})}{(0.60\ \mathrm{m})} = 2.7\times10^2\ \mathrm{V}$.

(b) As in part (a), all charged particles are the same distance from the top of the sphere, the only difference here is that the distance is not simply the radius of the sphere R, but $\sqrt{2}R$. Hence $V = \frac{6kq}{\sqrt{2}R} = \frac{6(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.0\times10^{-9}\ \mathrm{C})}{\sqrt{2}(0.60\ \mathrm{m})} = 1.9\times10^2\ \mathrm{V}$.

25.34. (a) Zero. The potential at the origin is zero, just as it is at infinity. There is no difference in electrostatic potential energy of the electron, so no net work is required. (b) Using conservation of energy, we can say that the total initial energy is equal to the final energy at a position where the speed is momentarily zero. Let the q_1 refer to the stationary electron and q_2 refer to the proton. Let the initial and final positions of the moving electron be denoted x_i and x_f. Then we can write: $\frac{1}{2}mv_i^2 + \frac{kq_1q_{e^-}}{x_i - 1} + \frac{kq_2q_{e^-}}{x_i + 1} = \frac{kq_1q_{e^-}}{x_f - 1} + \frac{kq_2q_{e^-}}{x_f + 1}$ or $\frac{1}{2}\frac{mv_i^2}{kq_{e^-}^2} + \frac{1}{x_i - 1} + \frac{-1}{x_i + 1} = \frac{1}{x_f - 1} + \frac{-1}{x_f + 1}$.

Solving for the final position yields

$$x_f = \sqrt{\frac{mv^2(1 - x_i^2) + 4kq_{e^-}^2 x_i^2}{mv^2(1 - x_i^2) + 4kq_{e^-}^2}}$$

$$x_f = \sqrt{\frac{(9.11\times10^{-31}\ \mathrm{kg})(5.000\times10^2\ \mathrm{m/s})^2(1-(20.00\ \mathrm{m})^2) + 4(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\ \mathrm{C})^2(20.00\ \mathrm{m})^2}{(9.11\times10^{-31}\ \mathrm{kg})(5.000\times10^2\ \mathrm{m/s})(1-(20.00\ \mathrm{m})^2) + 4(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\ \mathrm{C})^2}}$$

$$x_f = 1.002\ \mathrm{m}$$

This answer contains more significant digits than we have in our textbooks for the physical constants such as the mass of the electron. But these numbers are well known out to many decimal places, and could be looked up to higher precision. Because our measurements were given to four significant digits, we keep our answer of 1.002 m, and simply note that we would need to obtain more precise values for physical constants if this calculation were for publication or construction. So the closest the electron will get to the origin is 1.002 m.

25.35. (a) Negative (b) Let the charged objects be lettered A through D. Let and A and C be positive, and let B and D be negative. The total electrostatic potential energy can be written as $U = \frac{k}{d}\left(q_A q_B + q_B q_C + q_C q_D + q_A q_D + \frac{q_A q_C}{\sqrt{2}} + \frac{q_B q_D}{\sqrt{2}}\right)$. Inserting positive and negative charges where appropriate, we obtain $U = \frac{kq^2}{d}(\sqrt{2} - 4)$.

25.36. $\frac{\mathrm{N}}{\mathrm{C}} = \frac{\mathrm{N\cdot m}}{\mathrm{C\cdot m}} = \frac{\mathrm{J}}{\mathrm{C\cdot m}} = \frac{\mathrm{V}}{\mathrm{m}}$

25.37. The electrostatic work done along all paths is the same: $W_A = W_B = W_C = W_D$. Electrostatic work is independent of the path, and only depends on the initial and final positions. Since the initial positions are at the same potential for all paths, and the final positions are at the same potential for all paths, all paths require the same amount of electrostatic work.

25.38. The magnitude of the electric field near a very large charged sheet is given by $E = \dfrac{\sigma}{2\varepsilon_0}$. Since the electric field is uniform, its magnitude can also be written as $E = \dfrac{\Delta V}{\Delta x}$, where we choose our x axis has been chosen to be along the electric field. Equating the two expressions gives us $\Delta x = \dfrac{2\varepsilon_0 \Delta V}{\sigma} = \dfrac{2(8.85\times10^{-12}\ \text{m}^2/\text{N}\cdot\text{C}^2)(100\ \text{V})}{(3.5\times10^{-9}\ \text{C/m}^2)} =$ 0.51 m.

25.39. In each of the cases described, the potential at the center of the square is given by $V = \sum_i \dfrac{kq_i}{r}$, where $r = \sqrt{2}(1.5\ \text{m})$ is the distance from one corner to the center.

(*a*) When all charges are positive, the potential at the center is given by

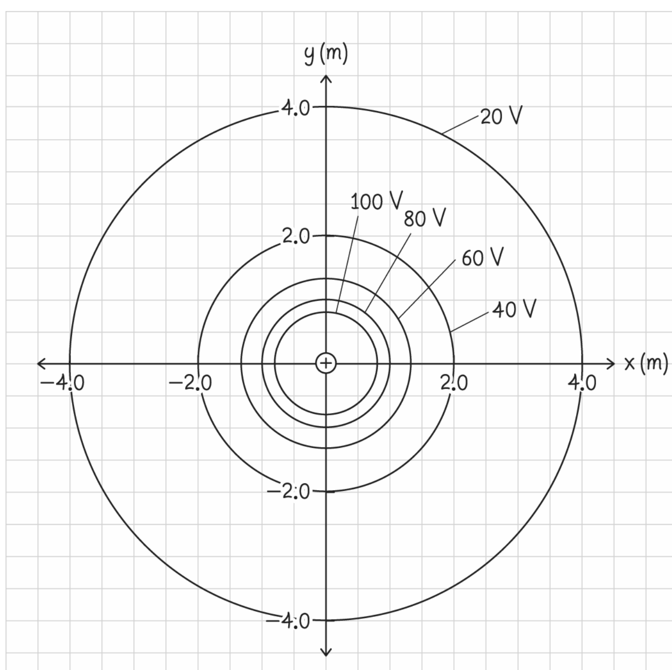
$$V = \frac{4kq}{r} = \frac{4(8.99\times10^9\ \text{Nm}^2/\text{C}^2)(3.0\times10^{-9}\ \text{C})}{\sqrt{2}(1.5\ \text{m})} = 51\ \text{J/C}$$

(*b*) With one negatively charged particle, the sum of all the potentials yields

$$V = \frac{2kq}{r} = \frac{2(8.99\times10^9\ \text{Nm}^2/\text{C}^2)(3.0\times10^{-9}\ \text{C})}{\sqrt{2}(1.5\ \text{m})} = 25\ \text{J/C}$$

(*c*) With two positively charged particles and two negatively charged particles, the potential in the center is zero.

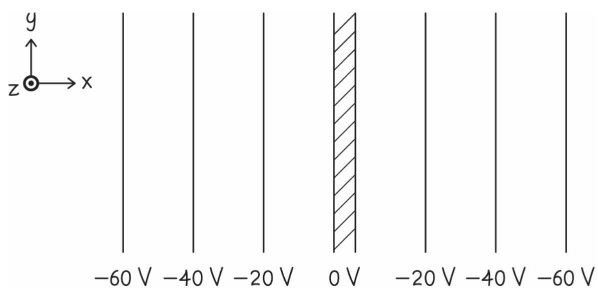
25.40.



No, the surfaces are not evenly spaced.

25.41. (*a*) Any point to the left of $x=-7d$ would be closer to the larger charge, so the potential due to the smaller, positive charge could not possible cancel out the potential of the larger charge. We need only consider the two cases when the point in question is between the two charged particles and when it is to the right of $x=+d$. Let the distance from the positively charged particle be r. If the point is between the two charged particles, we can write the potential as $V=\frac{kq}{r}+\frac{k(-3q)}{(8d-r)}=0$ which yields $r=2d$. This corresponds to the position $x=-d$. Now consider that the point could be to the right of the positively charged particle. Now the potential can be written $V=\frac{kq}{r}+\frac{k(-3q)}{(8d+r)}=0$, which yields $r=4d$. This corresponds to the position $x=+5d$. Hence the potential could be zero along the *x* axis at $x=-d$ or $x=+5d$. (*b*) Clearly the setup is reflection symmetric across the *x* axis. So, we solve for a point along the $+y$ axis at which the potential is zero, and we know by symmetry that reflection across the *x* axis will give a second point. The potential can be written as $V=\frac{kq}{\sqrt{d^2+y^2}}+\frac{k(-3q)}{\sqrt{49d^2+y^2}}=0$, which yields $y=\pm\sqrt{5}d$. (*c*) There is nowhere along either axis that the potential due to these two positively charged objects could be zero. A positive potential will exist due to each. Of course, in the limit as *x* or *y* become infinite, the potential would approach zero.

25.42. (*a*) The electric field is given by $\vec{E}=\frac{\sigma}{2\varepsilon_0}\hat{x}=\frac{(2.5\times10^{-6}\ \mathrm{C/m^2})}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})}\hat{x}=1.4\times10^5\ \mathrm{N/C}$ in the $+x$ direction.
(*b*) In a region where the electric field is uniform, we can write $E_x=\frac{-\Delta V}{\Delta x}$. Hence $E_x=-\frac{V(x=0.20\ \mathrm{m})-V(x=0.50\ \mathrm{m})}{(0.20\ \mathrm{m})-(0.50\ \mathrm{m})}$, such that $V(x=0.20\ \mathrm{m})-V(x=0.50\ \mathrm{m})=(0.30\ \mathrm{m})E_x=4.2\times10^4\ \mathrm{V}$.
(*c*) In this case all the work done by an external force will cause an increase in electrostatic potential energy (the object does not speed up). Hence we can write $W=\Delta U=q\Delta V=(1.5\times10^{-9}\ \mathrm{C})(4.2\times10^4\ \mathrm{V})=6.4\times10^{-5}\ \mathrm{J}$.
(*d*)

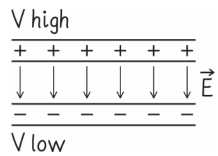


25.43. (*a*) The potential drops steadily as you move away from the sheet. Hence the potential at point B is lower than at A, and V_B-V_A is negative. (*b*) By symmetry, the electric field should be along the x axis. Since the field is uniform in this case, we can write $|E_x|=\frac{\Delta V}{\Delta x}=\frac{15\ \mathrm{V}}{5.0\ \mathrm{m}}=3.0\ \mathrm{V/m}$. Since electric field lines point away from positive charge, we know that $\vec{E}=3.0\ \mathrm{V/m}$ along the $+x$ direction. (*c*) The electric field is related to the charge density through $E=\frac{\sigma}{2\varepsilon_0}$ or $\sigma=2\varepsilon_0E=2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(3.0\ \mathrm{V/m})=5.3\times10^{-11}\ \mathrm{C/m^2}$.

25.44. (*a*) Configurations 1 and 2 consist of the same charged particles, placed the same distance from the origin, so they produce the same potential at the origin. Configuration 3 is nearly the same, but with the signs of the charges switched. Hence $V_1=V_2>V_3$. (*b*) If we label the charged particles as q_{top}, q_{left}, and q_{right}, respectively, then the total electrostatic potential energy in any arrangement can be written as $U=k\left(\frac{q_{\mathrm{top}}q_{\mathrm{right}}}{\sqrt{2}d}+\frac{q_{\mathrm{top}}q_{\mathrm{left}}}{\sqrt{2}d}+\frac{q_{\mathrm{left}}q_{\mathrm{right}}}{2d}\right)$.

Plugging in numbers from each configuration yields $U_1^E = -\frac{kq^2}{d}\left(6+\frac{2}{\sqrt{2}}\right)$, $U_2^E = -\frac{kq^2}{d}\left(4+\frac{6}{\sqrt{2}}\right)$, and $U_3^E = -\frac{kq^2}{d}\left(6+\frac{2}{\sqrt{2}}\right)$. Hence $U_1^{\mathrm{E}} = U_3^{\mathrm{E}} > U_2^{\mathrm{E}}$.

25.45. (*a*)



(*b*) $|E_x| = \frac{\Delta V}{\Delta x} = \frac{500\ \text{V}}{0.10\ \text{m}} = 5.0\times10^3\ \text{V/m}$ (*c*) Here we treat the system as consisting of the parallel plates and the electron. In that case $\Delta U = q\Delta V = (-1.60\times10^{-19}\ \text{C})(+500\ \text{V}) = -8.0\times10^{-17}\ \text{J}$. (*d*) Now we treat the electric field between the plates as being external, and we take the system to be the electron alone. In that case the work done by the electric force is equal to $W = \vec{F}^E \cdot \Delta\vec{x} = qE_x\Delta x = q\left(-\frac{\Delta V}{\Delta x}\right)\Delta x = -q\Delta V$. Clearly the answer is the same in magnitude as the answer to (*c*). The only difference is that now we are discussing a positive external work being done on the system. $W = -(-1.60\times10^{-19}\ \text{C})(500\ \text{V}) = 8.0\times10^{-17}\ \text{J}$. (*e*) Whether electrostatic potential energy is converted into kinetic energy, or external work speeds up the electron, either treatment tells us that the kinetic energy must be 8.0×10^{-17} J.

25.46. (*a*) For two parallel plates, we can write the electric field as $|E| = \frac{\sigma}{\varepsilon_0}$. So $\sigma = \varepsilon_0 E = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)$ $(50\ \text{V/m}) = 4.4\times10^{-10}\ \text{C/m}^2$. (*b*) Since the electric field is uniform, we can also write $|E_x| = \left|\frac{\Delta V}{\Delta x}\right|$, such that $|\Delta x| = \left|\frac{\Delta V}{E_x}\right| = \frac{0.25\ \text{V}}{50\ \text{V/m}} = 5.0\ \text{mm}$. (*c*) $W = \vec{F}^E \cdot \Delta\vec{x} = -qE_x\Delta x = -(-1.60\times10^{-19}\ \text{C})(50\ \text{V/m})(5.0\times10^{-3}\ \text{m}) = 4.0\times10^{-20}\ \text{J}$.

25.47. (*a*) Since the electrostatic potential energy is positive, the second charge must also be positive. (*b*) We have sufficient information to write down two equations: one for the electrostatic potential energy, and one for the electric potential at the origin. We can write:

$$U = \frac{kq_1q_2}{|x+d|} = \frac{2kq_1^2}{d} \tag{1}$$

and

$$V = V_1 + V_2 = \frac{kq_1}{d} + \frac{kq_2}{|x|} = \frac{4kq_1}{d} \tag{2}$$

Here x is the position of q_2 along the x axis. Because of the absolute value on x, there are three cases to consider: $x > 0$, $0 > x > -d$ and $x < -d$.

Case 1: If $x > 0$, then we can write the electrostatic potential energy of the arrangement as $\frac{kq_1q_2}{x+d} = \frac{2kq_1^2}{d}$ or

$$\frac{q_2}{x+d} = \frac{2q_1}{d} \tag{3}$$

The potential at the origin can be written as $V_1 + V_2 = \frac{kq_1}{d} + \frac{kq_2}{x} = \frac{4kq_1}{d}$ or

$$\frac{q_2}{x} = \frac{3q_1}{d} \tag{4}$$

Combining equations (3) and (4) yields $x = 2d$ and $q_2 = 6q_1$.

Case 2: If $0 > x > -d$, then we can write the electrostatic potential energy of the arrangement as $\frac{kq_1q_2}{x+d} = \frac{2kq_1^2}{d}$ or

$$\frac{q_2}{x+d} = \frac{2q_1}{d} \tag{5}$$

The potential at the origin can be written as $V_1 + V_2 = \frac{kq_1}{d} + \frac{kq_2}{-x} = \frac{4kq_1}{d}$, yielding

$$\frac{q_2}{x} = -\frac{3q_1}{d} \tag{6}$$

Now, combining equations (5) and (6) yields $x = -\frac{2}{5}d$ and $q_2 = \frac{6}{5}q_1$.

Case 3: If $x < -d$ then we can write the electrostatic potential energy of the arrangement as $-\frac{kq_1q_2}{x+d} = \frac{2kq_1^2}{d}$ or

$$\frac{q_2}{x+d} = -\frac{2q_1}{d} \tag{7}$$

The potential at the origin can be written as $V_1 + V_2 = \frac{kq_1}{d} + \frac{kq_2}{-x} = \frac{4kq_1}{d}$ or

$$\frac{q_2}{x} = -\frac{3q_1}{d} \tag{8}$$

Combining equations (7) and (8) yields $x = 2d$, which violates our assumption that $x < -d$. Hence there is no solution for $x < -d$.

The only possible values of the second charge and its position are $q_2 = 6q_1$, $x = 2d$ and $q_2 = \frac{6}{5}q_1$, $x = -\frac{2}{5}d$.

25.48. (*a*) Since the magnitude of the electric field at the origin is $E = \frac{2kq}{d^2}$ (which is larger than the field due to particle 1 alone), the electric fields of the two particles must be pointing in the same direction at $x = 0$. This allows for two possibilities: q_2 is positive and x is also positive, or q_2 and x are both negative. But the electric potential is given by $V = \frac{3kq}{d}$, which is larger than the potential of particle 1 alone. Hence particle 2 must carry a positive charge (and $x > 0$). Now we can write expressions for the electric field and electric potential at the origin:

$$E = \frac{kq}{d^2} + \frac{kq_2}{x^2} = \frac{2kq}{d^2}$$

or

$$\frac{q_2}{x^2} = \frac{q}{d^2} \tag{1}$$

and

$$V = \frac{kq}{d} + \frac{kq_2}{x} = \frac{3kq}{d}$$

or

$$\frac{q_2}{x} = \frac{2q}{d} \tag{2}$$

Combining equations (1) and (2) yields $x = 2d$ and $q_2 = 4q$. (*b*) Clearly, this setup is reflection symmetric across the *x* axis. We expect to find possible locations of q_2 both above and below the *x* axis, equidistant from the origin. Let us initially assume that $y > 0$.

We can write the magnitude of the electric field and the potential at the origin as

$$E = \frac{2kq}{d^2} = \sqrt{\left(\frac{kq}{d^2}\right)^2 + \left(\frac{kq_2}{y^2}\right)^2}$$

or

$$\frac{4q^2}{d^4} = \frac{q^2}{d^4} + \frac{q_2^2}{y^4} \tag{3}$$

and

$$V = \frac{kq}{d} + \frac{kq_2}{y} = \frac{3kq}{d}$$

or

$$\frac{q_2}{y} = \frac{2q}{d} \tag{4}$$

Combining equations (3) and (4) yields $y = \frac{2}{\sqrt{3}}d,\ q_2 = \frac{4\sqrt{3}}{3}q.$ Clearly, $y = -\frac{2}{\sqrt{3}}d,\ q_2 = \frac{4\sqrt{3}}{3}q$ is also a solution. Note that in equation (4) the absence of an absolute value sign on *y* is an artifact of our assumption that $y > 0$.

25.49. For all cases we use the result of Example 25.7: $V(z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - |z|\right)$.

(*a*) $V(z = 5.0\times10^{-3}\text{ m}) = \frac{(7.5\times10^{-9}\text{ C/m}^2)}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\sqrt{(5.0\times10^{-3}\text{ m})^2 + (62.5\times10^{-3}\text{ m})^2} - \left|5.0\times10^{-3}\text{ m}\right|\right) = 24\text{ V}$

(*b*) $V(z = 30\times10^{-3}\text{ m}) = \frac{(7.5\times10^{-9}\text{ C/m}^2)}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\sqrt{(30\times10^{-3}\text{ m})^2 + (62.5\times10^{-3}\text{ m})^2} - \left|30\times10^{-3}\text{ m}\right|\right) = 17\text{ V}$

(*c*) $V(z = 62.5\times10^{-3}\text{ m}) = \frac{(7.5\times10^{-9}\text{ C/m}^2)}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\sqrt{(62.5\times10^{-3}\text{ m})^2 + (62.5\times10^{-3}\text{ m})^2} - \left|62.5\times10^{-3}\text{ m}\right|\right) = 11\text{ V}$

25.50. (*a*) Just outside the shell, the electric field looks as if the charge were located at a single point at the shell's center. Hence $\vec{E}_{\text{out}}(r \to R_+) = \frac{kq}{R^2}\hat{r} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10\times10^{-9}\text{ C})}{(120\times10^{-3}\text{ m})^2}\hat{r} = (+6.2\times10^3\text{ N/C})$ radially outward.

Inside the shell, the electric field is zero. This is most easily seen using Gauss' Law. Since a concentric Gaussian sphere would enclose no charge, one could write $\oint \vec{E}_{\text{in}} \cdot d\vec{a} = 0.$ By symmetry, the electric field should be constant and radial, everywhere on the Gaussian surface. This allows us to write $E_{\text{in}}(4\pi r^2) = 0,$ meaning $\vec{E}_{\text{in}} = 0.$ (*b*) Just outside the shell, the potential will be the same as if the charge were located at a single point at the shell's center. Hence

$$V_{\text{out}}(r \to R_+) = \frac{kq}{R} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10\times10^{-9}\text{ C})}{(120\times10^{-3}\text{ m})} = (+7.5\times10^2\text{ J/C})$$

The potential inside is also $(+7.5\times10^2\text{ J/C})$. Since there is no electric field inside the shell, there can be no electric potential difference between two points inside the shell. (*c*) $\vec{E}(r = 0) = 0,$ since we showed in part (*a*) that the electric field vanishes at all points inside the shell, and $V(r = 0) = (+7.5\times10^2\text{ J/C})$.

25.51. In general, we can find the potential difference between two points using the electric field via $\Delta V_{21} = -\int_1^2 \vec{E}(\vec{r}) \cdot d\vec{r}$. We also know the electric field due to an infinitely long wire is $\vec{E} = \frac{2k\lambda}{r}\hat{r}$. Let point 2 be the reference potential of $V(r = 2.5) = 0$. As long as we integrate in the direction of the electric field, the potential difference becomes

$$\Delta V_{21} = 0 - V_1 = -2k\lambda\left[\mathrm{Ln}(r_2) - \mathrm{Ln}(r_1)\right]$$

Clearly, if one integrates against the field, one picks up a an additional minus sign from the dot product. The electric field points radially outward from the wire.

(*a*) Here, we integrate from $r_1 = 2.0$ m to $r_2 = 2.5$ m, so we integrate in the direction of the field. Hence

$$\begin{aligned} V(r = 2.0 \text{ m}) &= 2k\lambda\left[\mathrm{Ln}(r_2) - \mathrm{Ln}(r_1)\right] \\ &= 2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-9}\ \text{C/m})\left[\mathrm{Ln}(2.5\text{ m}) - \mathrm{Ln}(2.0\text{ m})\right] \\ & 6.0\times10^{2}\ \text{V} \end{aligned}$$

(*b*) Since we now integrate from $r_1 = 4.0$ m to $r_2 = 2.5$ m, we integrate against the direction of the field. Hence

$$\begin{aligned} V(r = 4.0 \text{ m}) &= -2k\lambda\left[\mathrm{Ln}(r_2) - \mathrm{Ln}(r_1)\right] \\ &= -2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-9}\ \text{C/m})\left[\mathrm{Ln}(2.5\text{ m}) - \mathrm{Ln}(4.0\text{ m})\right] \\ & -1.3\times10^{3}\ \text{V} \end{aligned}$$

(*c*) Again, integrating from $r_1 = 12$ m to $r_2 = 2.5$ m, we integrate against the direction of the field. Hence

$$\begin{aligned} V(r = 12 \text{ m}) &= -2k\lambda\left[\mathrm{Ln}(r_2) - \mathrm{Ln}(r_1)\right] \\ &= -2(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-9}\ \text{C/m})\left[\mathrm{Ln}(2.5\text{ m}) - \mathrm{Ln}(12\text{ m})\right] \\ & -4.2\times10^{3}\ \text{V} \end{aligned}$$

25.52. In order to find the potentials at all points, we integrate the electric fields in each region. The electric fields in each region are:

$$\vec{E}(r > 3R) = \frac{kq}{r^2}\hat{r}$$

$$\vec{E}(3R > r > 2R) = \frac{-3kq}{r^2}\hat{r}$$

$$\vec{E}(2R > r > R) = 0$$

$$\vec{E}(R > r) = \frac{kq}{r^2}\hat{r}$$

We first find the potential on the thin outer shell by integrating

$$\Delta V = V(r \to \infty) - V(r = 3R) = -\int_{3R}^{\infty} \vec{E}(\vec{r}) \cdot d\vec{r}$$

$$V(r = 3R) = -\int_{3R}^{\infty} \frac{kq}{r^2} dr = \frac{kq}{3R}$$

We now find the potential on the outer surface of the thick shell by looking at the potential difference

$$\Delta V = V(r = 3R) - V(r = 2R) = -\int_{2R}^{3R} \vec{E}(\vec{r}) \cdot d\vec{r}$$

$$V(r = 2R) = \frac{kq}{3R} + \int_{2R}^{3R} \frac{-3kq}{r^2} dr = \frac{kq}{3R} + \left.\frac{3kq}{r}\right|_{2R}^{3R} = -\frac{kq}{6R}$$

This potential is constant throughout the conducting shell. Clearly, between $r = 2R$ and $r = 3R$ the potential changed sign, meaning it must have passed through zero. We can find the exact position it passed through zero by requiring

$$V(r) = \frac{kq}{3R} + \int_r^{3R} \frac{-3kq}{(r')^2} dr' = \frac{kq}{3R} + \frac{3kq}{r'}\bigg|_r^{3R} = 0$$
$$\Rightarrow \frac{kq}{3R} + \frac{3kq}{3R} - \frac{3kq}{r} = 0 \Rightarrow r = \frac{9R}{4}$$

So the potential passes through zero at $r = \frac{9R}{4}$, but we have not yet checked the region $r < R$. We expect that as we approach the positively charged particle at the center the potential should increase and again pass through zero. Inside the thick shell, we can relate the potential at some point r_0 to the potential on the thick shell through $V(r = R) - V(r = r_0) = -\int_{r_0}^{R} \frac{-kq}{(r')^2} dr'$. We can find the location where the potential passes through zero by requiring

$$V(r_0) = -\frac{kq}{6R} + \int_{r_0}^{R} \frac{kq}{(r')^2} dr' = 0$$
$$\Rightarrow -\frac{kq}{6R} - \frac{kq}{R} + \frac{kq}{r_0} = 0$$
$$r_0 = \frac{6R}{7}$$

Thus the potential is zero at $r = \frac{6R}{7}$ and $r = \frac{9R}{4}$, and of course the potential is assumed to be zero at infinite distance from the arrangement.

25.53. (*a*) The electric field is zero. This can be seen by symmetry. (*b*) Example 25.6 gives us the equation for the electrostatic potential due to a rod of length ℓ' carrying a charge q at a point a distance d from the endpoint of the rod along a line perpendicular to the rod: $V_{\text{rod}} = \frac{kq}{\ell'} \ln\left(\frac{\ell' + \sqrt{(\ell')^2 + d^2}}{d}\right)$. In this case, the square is made up of 8 such rods (treating each side as two such rods pushed together), such that $\ell' = \ell/2$ and $d = \ell/2$. Note also that the charge on each of the 8 rods would only be $q/2$. Thus

$$V_{\text{square}} = 8V_{\text{rod}} = \frac{8k(q/2)}{(\ell/2)} \ln\left(\frac{(\ell/2) + \sqrt{(\ell/2)^2 + (\ell/2)^2}}{(\ell/2)}\right) = \frac{8kq}{\ell} \ln\left(1 + \sqrt{2}\right)$$

25.54. (*a*) In distribution C, electric fields due to differential elements of charge on opposite sides of the circle will cancel each other and the total electric field magnitude will be zero. In distribution B, *y* components will cancel, but *x* components will add. Hence the magnitude of the electric field at the origin will not be zero, but it will also not be as large as in distribution A. In distribution A, all charge causes an electric field in exactly the same direction. Hence $E_{\text{C}} < E_{\text{B}} < E_{\text{A}}$. (*b*) Because electrostatic potential is a scalar, only the distance from the charge to the origin matters; it does not matter on which side the charge lies. Hence all three distributions cause exactly the same electrostatic potential at the origin. $V_{\text{A}} = V_{\text{B}} = V_{\text{C}}$. (*c*) $V = \frac{kq}{R}$.

25.55. The electric potential will just be the superposition of the electric field from each charged object:

$$V = V_{\text{p}} + V_{\text{r}} = \frac{kq_{\text{p}}}{|y_{\text{p}} - y|} + \frac{kq_{\text{r}}}{\ell} \ln\left(\frac{\ell + \sqrt{(\ell)^2 + y^2}}{|y|}\right)$$

25.56. We treat the annulus like it is a complete disk of radius *R* carrying a uniform charge density $+\sigma$ and a disk of radius *a* carrying a uniform charge density $-\sigma$. Then the total potential will be

$$V = V_{\text{annulus}} + V_{\text{disk}} = V_{\text{large disk}} + 2V_{\text{small disk}}$$

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - |z|\right) - \frac{\sigma}{\varepsilon_0}\left(\sqrt{z^2+a^2} - |z|\right)$$

Setting this potential equal to zero at $z = R$ yields

$$\frac{1}{2}\left(\sqrt{2}R - R\right) = \left(\sqrt{R^2+a^2} - R\right) \Rightarrow a = \left[\left(\frac{\sqrt{2}+1}{2}\right)^2 - 1\right]^{1/2} R \text{ or } a \approx 0.676R$$

25.57. Following the general procedure used by Example 25.7, we consider a ring–shaped section of the disk with a radius r and a thickness dr. All points on this ring are equidistant from any point on the x axis, so the contribution of this ring to the electrostatic potential is $V_{\text{ring}} = \dfrac{k(2\pi r dr)\sigma(r)}{\sqrt{x^2+r^2}} = \dfrac{cr^2 dr}{2\varepsilon_0\sqrt{x^2+r^2}}$. Integrating this expression over many such differential rings so as to cover the entire disk, we obtain

$$V_{\text{disk}} = \int_0 \frac{cr^2 dr}{2\varepsilon_0\sqrt{x^2+r^2}}$$

$$V_{\text{disk}} = \frac{c}{8\epsilon_0}\left[2R\sqrt{R^2+x^2} + x^2 \ln\left(\frac{x^2}{\left(R+\sqrt{R^2+x^2}\right)^2}\right)\right]$$

25.58. First let us use Gauss's Law to determine the electric field inside the cylinder. Then we will integrate the electric field over the radius of the cylinder to determine the potential difference. We choose a cylindrical Gaussian surface of radius r that is concentric with the charged cylinder. The charge enclosed by this surface is

$$q_{\text{enc}} = \int_0^{\ell}\int_0^{2\pi}\int_0^{r} (ar')r'dr'd\phi dz = \frac{2\pi\ell a r^3}{3}$$

The flux through the Gaussian surface is $E(2\pi r\ell)$. Gauss's Law tells us that $\vec{E} = \dfrac{ar^2}{3\varepsilon_0}$ radially outward. Now that we have this electric field, we can write $V_R - V_0 = -\displaystyle\int_0^R \frac{ar^2}{3\varepsilon_0}dr = \frac{-aR^3}{9\varepsilon_0}$.

25.59. We know $E_x = -\dfrac{dV}{dx}$, so $\left|\vec{E}\right| = B$, and the electric field points in the $-x$ direction.

25.60. $E_x = -\dfrac{dV}{dx} = -3y$ and $E_y = -\dfrac{dV}{dy} = -3x + 10y$, so $\vec{E}(x,y) = (-3y)\hat{i} + (-3x+10y)\hat{j}$.

25.61. (*a*) Note that we are only given k to two significant digits in the text, and we are only given ε_0 to three significant digits. Due to the high number of significant digits given in the problem, we will not limit our answers to three significant digits in this case. The justification for this is that this problem is illustrating how changes in a quantity over very small distances can approach a derivative. The point is mathematical, and so we try to keep significant digits as long as possible. If the values were important, more precise values of physical constants could always be looked up when higher precision is needed.
$V(x = 3.0000 \text{ m}) = \dfrac{kq}{x} = \dfrac{q}{4\pi\varepsilon_0 x} = \dfrac{(3.00\times10^{-9} \text{ C})}{4\pi(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.0000 \text{ m})} = 8.9918 \text{ V}$ and $V(x = 3.0000 \text{ m}) = \dfrac{kq}{4\pi\varepsilon_0 x} = \dfrac{(3.00\times10^{-9} \text{ C})}{4\pi(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.0100 \text{ m})} = 8.9619 \text{ V}$.

(*b*) The electric potential decreases as x increases. $\dfrac{\Delta V}{\Delta x} = \dfrac{(8.9619 \text{ V}) - (8.9918 \text{ V})}{(0.0100 \text{ m})} = -2.9873 \text{ V/m}$.

(*c*) $E(x = 3.0000\text{ m}) = \dfrac{kq}{x^2} = \dfrac{q}{4\pi\varepsilon_0 x^2} = \dfrac{(3.00\times 10^{-9}\text{ C})}{4\pi(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0000\text{ m})^2} = 2.9973\text{ V}$

$\vec{E} = 2.9973\text{ V/m}$ to the right, which is very close to the value obtained in (*b*), but with opposite sign, as expected.

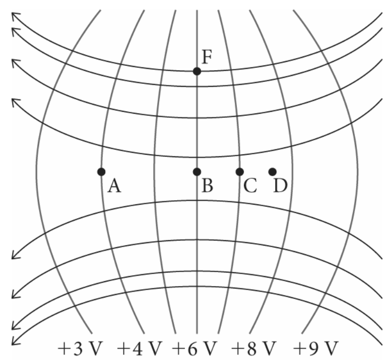
(*d*) $V = \dfrac{kq}{r} = \dfrac{q}{4\pi\varepsilon_0 r} = \dfrac{(3.00\times 10^{-9}\text{ C})}{4\pi(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)\sqrt{(3.0000\text{ m})^2 + (0.0100\text{ m})^2}} = 8.9912\text{ V}$

$V = 8.9912\text{ V},$ which is between the values obtained in part (*a*). This makes sense, since the distance from the origin to this point is between the two distances used in part (*a*).

25.62. (*a*) We simply add the potential due to each particle to obtain $V(x) = \dfrac{2kq}{\sqrt{x^2 + a^2}}$. (*b*) $\vec{E}(x) = -\dfrac{dV}{dx}\hat{i} = \dfrac{2kqx}{(x^2 + a^2)^{3/2}}\hat{i}.$

25.63. (*a*) B (*b*) F

(*c*)

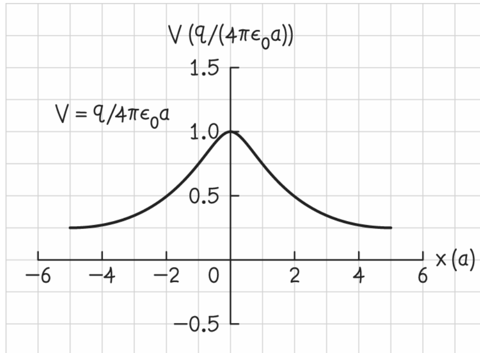


25.64. (*a*) The point $x = 3.00\text{ m}$ is equidistant from the two charged particles (which carry charges equal in magnitude and opposite in sign), so the potential at this point must be zero. (*b*) We just take the superposition of the electric fields due to each particle:

$$\vec{E} = \vec{E}_+ + \vec{E}_- = 2\frac{kq}{x^2}\hat{i} = 2\frac{q}{4\pi\varepsilon_0 x^2}\hat{i} = 2\frac{(3.00\times 10^{-9}\text{ C})}{4\pi(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\text{ m})^2}\hat{i} = 5.99\text{ V/m }\hat{i}$$

(*c*) $V = V_\text{p} + V_\text{n} = \dfrac{q_\text{p}}{4\pi\varepsilon_0 x_\text{p}} + \dfrac{q_\text{n}}{4\pi\varepsilon_0 x_\text{n}} = \dfrac{(3.00\times 10^{-9}\text{ C})}{4\pi(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left(\dfrac{1}{3.01\text{ m}} - \dfrac{1}{2.99\text{ m}}\right) = -0.0599\text{ V}$ (*d*) $\dfrac{\Delta V}{\Delta x} =$ $\dfrac{(-0.0599\text{ V}) - 0}{(0.010\text{ m})} = -5.99\text{ V/m},$ which has the same magnitude as our answer to part (*b*). This is what we expect since $E_x = -\dfrac{dV}{dx},$ which is well-approximated here as $E_x \approx -\dfrac{\Delta V}{\Delta x}$ to the precision given.

25.65. (*a*)The functional form of the potential is $V_\text{ring} = \dfrac{k(2\pi r dr)\sigma(r)}{\sqrt{x^2 + r^2}} = \dfrac{kq}{\sqrt{x^2 + r^2}}.$



(b) One can see by inspection that the potential is maximum at $x = 0$. (c) One can see by symmetry that the electric field must be zero at the center of the ring. More generally, since the magnitude of the electric field depends on a spatial derivative of the potential, any point where a potential reaches a maximum the electric field must be zero.

25.66. (a) We know $E_r = -\dfrac{dV}{dr}$, so we take the derivative in each region separately to obtain

$$\vec{E}(r > 2R) = \frac{q}{4\pi\varepsilon_0 r^2} \text{ radially outward from sphere center}$$

$$\vec{E}(R \le r \le 2R) = 0$$

$$E(r < R) = \frac{3q}{4\pi\varepsilon_0 r^2} \text{ radially outward from sphere center}$$

(b) A particle carrying a charge of $-3q$ at the origin would produce the required field inside the region $r < R$. Since the potential is constant from $R < r < 2R$, we need a thick, conducting spherical shell in this region. By looking at the potential outside of this thick shell, we can see that the total charge of the system must be $-q$, meaning the thick shell must carry a total charge $+2q$.

25.67. (a) Positive. The work done on the electron must be positive because the kinetic energy of the electron increased. (b) Lower. The electric field must point from B to A in order for the electron to be accelerated from A to B. Since the electric field always points from higher potential to lower potential, the potential at A must be lower than the potential at B.

25.68. (a) Positive. The work done on the proton must be positive because the kinetic energy of the proton increased. (b) Higher. The electric field must point from A to B in order for the proton to be accelerated from A to B. Since the electric field always points from higher potential to lower potential, the potential at A must be higher than the potential at B.

25.69. In each case we simply use $E_x = -\dfrac{dV}{dx}$. (a) $E_x = -b = -6000\ \text{V/m}$ (b) $E_x = \dfrac{a}{x^2} + b$ where $a = 2000\ \text{Vm}$ and $b = -1500\ \text{V/m}$ (c) $E_x = a + bx$ where $a = 2000\ \text{V/m}$ and $b = 6000\ \text{V/m}^2$ (d) $E_x = 0$.

25.70. For any point with $r \ge R$ the potential is simply given by $V = \dfrac{kq}{r}$. Inside the spherical shell, there is no electric field (because there is no charge enclosed), meaning there cannot be any change in the potential inside the sphere. Thus (a) $V_\text{A} - V_\text{B} = \dfrac{kq}{R} - \dfrac{kq}{R/2} > 0$ (b) $V_\text{A} - V_\text{C} = \dfrac{kq}{R} - \dfrac{kq}{R} = 0$ (c) $V_\text{A} - V_\text{D} = \dfrac{kq}{R} - \dfrac{kq}{R} = 0$.

25.71. (a) One can solve this problem by inspection, noting that to increase the potential by a factor of two, we must reduce the distance from the charged particle by a factor of two. Alternatively, one can work it out algebraically: we are told that $(5.0\text{ V}) = \dfrac{kq}{(0.050\text{ m})} \Rightarrow kq = (5.0\text{ V})(0.050\text{ m}) = 0.25\text{ V}\cdot\text{m}$. So we require $(10\text{ V}) = \dfrac{kq}{r} \Rightarrow r = \dfrac{kq}{(10\text{ V})} = \dfrac{(0.25\text{ V}\cdot\text{m})}{(10\text{ V})} = 0.025\text{ m}$ or 25 mm. (*b*) Exactly as in part (*a*), we require $(2\text{ V}) = \dfrac{kq}{r} \Rightarrow r = \dfrac{kq}{(2\text{ V})} = \dfrac{(0.25\text{ V}\cdot\text{m})}{(2\text{ V})} = 0.13\text{ m}$ or 1.3×10^2 mm.

25.72. The potential due to a tiny segment of the rod carrying a charge dq at the point specified is $dV = \dfrac{k(dq)}{r}$, where $r = d - x$. Because the charge density is uniform, we can also write this as $dV = \dfrac{kq}{(d-x)\ell}dx$. To find the total potential, we integrate these differential contributions over the length of the rod:

$$V = \int_{\text{rod}} dV = \int_{-\ell}^{0} \frac{kq}{(d-x)\ell}dx = \frac{kq}{\ell}\ln\left(\frac{d+\ell}{d}\right)$$

25.73. (*a*) The electric field outside the conducting shell is $\dfrac{k(q_{\text{shell}} + q_{\text{sphere}})}{r^2}$ radially outward from the center. We can chose the potential to be any constant we like as $r \to \infty$, and we make the common choice that $V(r\to\infty) = 0$, such that we require the potential at the center of the solid sphere also be zero. We can find the potential on the outer shell by integrating the electric field as follows:

$$V(r\to\infty) - V(r = 3R) = -\int_{3R}^{\infty} \vec{E}\cdot d\vec{r}$$

$$\Rightarrow V(r = 3R) = 0 + \int_{3R}^{\infty} \frac{k(q_{\text{shell}} + q_{\text{sphere}})}{r^2}dr = \frac{k(q_{\text{shell}} + q_{\text{sphere}})}{3R}$$

This will be the potential throughout the conducting spherical shell. In the region $R < r < 2R$ the electric field will be $\dfrac{kq_{\text{sphere}}}{r^2}$ radially outward. Again we calculate the potential difference

$$V(r = 2R) - V(r = R) = -\int_{R}^{2R} \vec{E}\cdot d\vec{r}$$

$$\Rightarrow V(r = R) = \frac{k(q_{\text{shell}} + q_{\text{sphere}})}{3R} + \int_{R}^{2R} \frac{kq_{\text{sphere}}}{r^2}dr = \frac{k(q_{\text{shell}} + q_{\text{sphere}})}{3R} + \frac{kq_{\text{sphere}}}{2R}$$

Because the solid sphere in the center is conducting, the potential at the surface is also the potential at the center. Thus we require $\dfrac{k(q_{\text{shell}} + q_{\text{sphere}})}{3R} + \dfrac{kq_{\text{sphere}}}{2R} = 0$ or equivalently $q_{\text{sphere}} = -\dfrac{2q_{\text{shell}}}{5}$. (*b*) All steps in part (*a*) still apply here, except the final assertion that the potential on the outside of the solid sphere is the same as the potential at the center. We still know that $V(r = R) = \dfrac{k(q_{\text{shell}} + q_{\text{sphere}})}{3R} + \dfrac{kq_{\text{sphere}}}{2R}$, but now we have to consider the region $r < R$ in which the electric field is $\dfrac{kqr}{R^3}$ radially outward. We find the difference between the potential at the center and outer edge as follows:

$$V(r = R) - V(r = 0) = -\int_{0}^{R} \vec{E}\cdot d\vec{r}$$

$$\Rightarrow V(r = 0) = \frac{k(q_{\text{shell}} + q_{\text{sphere}})}{3R} + \frac{kq_{\text{sphere}}}{2R} + \int_{0}^{R} \frac{kq_{\text{sphere}}r}{R^3}dr = \frac{k(q_{\text{shell}} + q_{\text{sphere}})}{3R} + \frac{kq_{\text{sphere}}}{R}$$

Requiring that this potential at the center be zero, we obtain $q_{\text{sphere}} = -\dfrac{q_{\text{shell}}}{4}$.

25.74. To be extremely precise, there are quantum mechanical considerations, such as different allowed angular momentum states even in a given energy level. However, classical physics can give a very good estimate. No additional information is needed. We simply apply conservation of energy so that we can state that the change in electrostatic potential energy must be equal in magnitude to the energy carried off by radiation. Thus

$$E_{\text{rad}} = U_{\text{i}} - U_{\text{f}} = \frac{kq_{\text{e}}(2q_{\text{p}})}{r_{\text{i}}} - \frac{kq_{\text{e}}(2q_{\text{p}})}{r_{\text{f}}} =$$

$$E_{\text{rad}} = -2(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2\left(\frac{1}{(0.42\times10^{-9}\ \text{m})} - \frac{1}{(0.24\times10^{-9}\ \text{m})}\right) = 8.2\times10^{-19}\ \text{J}$$

25.75. In order to hold, say, proton 92 in nucleus, strong interaction must be sufficient to overcome repulsive interactions exerted on this proton by other 91 protons in nucleus, interactions tending to expel proton 92 from nucleus. If each proton occupies volume $\delta V = \frac{4}{3}\pi r^3$, the radius R of $n = 91$ protons is approximately $R = rn^{1/3}$. Using this radius in Coulomb's law yields magnitude of force between proton 92 and 91 other protons: $F \approx (91e^2)/(4\pi\varepsilon_0 R^2) \approx 300$ N.

Alternatively, one could speak of the energy stored in this arrangement. In that case, the binding energy associated with the strong nuclear force must be greater (or equal) in magnitude to the positive potential energy associated with the electrical interaction: $\left|U^{\text{nuclear}}\right| \geq (91e^2)/(4\pi\varepsilon_0 R) \approx 2\times10^{-12}$ J.

25.76. (*a*) Conservation of momentum tells us that $m_1 v_{1x} = -m_2 v_{2x} \Rightarrow v_1 = \dfrac{m_2}{m_1} v_2$. Thus we can say $\dfrac{K_2}{K_1} = \dfrac{\frac{1}{2}m_2 v_2^2}{\frac{1}{2}m_1 v_1^2} = \dfrac{m_2}{m_1}\dfrac{v_2^2}{\left(-\dfrac{m_2}{m_1}v_2\right)^2} = \dfrac{m_1}{m_2} = \dfrac{1}{2}$. (*b*) We know that all electrostatic potential energy that has been lost has been converted to the kinetic energy of both objects. We can write this total kinetic energy as $K_1 + K_2 = \dfrac{3}{2}K_1 = \dfrac{3}{4}mv_1^2$. Equating this kinetic energy to the electrostatic potential energy lost yields $\dfrac{3}{4}mv_1^2 = 2kq^2\left(\dfrac{1}{r} - \dfrac{1}{4r}\right) \Rightarrow v_1 = \sqrt{\dfrac{2kq^2}{mr}}$. (*c*) From the momentum conservation comment in part (*a*), we know $v_2 = \dfrac{1}{2}v_1 = \sqrt{\dfrac{kq^2}{2mr}}$. (*d*) Here, the process is the same as in parts (*b*) and (*c*), expect that the final separation approaches infinity. Thus $\dfrac{3}{4}mv_1^2 = 2kq^2\left(\dfrac{1}{r} - \dfrac{1}{\infty}\right) \Rightarrow v_1 = \sqrt{\dfrac{8kq^2}{3mr}}$. (*e*) As before, we employ the conservation of momentum and write $v_2 = \dfrac{1}{2}v_1 = \sqrt{\dfrac{2kq^2}{3mr}}$.

26

CHARGE SEPARATION AND STORAGE

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^5 V **2.** 10^{-5} C **3.** 10^{10} V **4.** 10^4 m$\times 10^4$ m **5.** 10^{-12} F **6.** 10^{-11} m^2 **7.** 10^2 J/m^3 **8.** 10^4 J **9.** 10^{-10} F **10.** 10^{-11} F

Guided Problems

26.2 Home-made capacitor

1. Getting Started The approach of Worked Problem 26.1 is applicable here. That problem describes a similar geometry with similar dimensions and materials. The final answer will not be the same, because the geometry is slightly different. Here we have a solid pencil at the center of the cylinder, rather than having rolled capacitor all the way to the center. We are also using slightly different materials here.

2. Devise Plan The length of an unused number 2 pencil is approximately 0.19 m. Its radius is approximately 4.0 mm. Aluminum foil and waxed paper can be found is rolls approximately 30 m long. While it is certainly possible to connect two such rolls to obtain a longer effective sheet, handling 30 m of foil and paper seems difficult enough. We will use 30 m as the maximum tractable length for both foil and paper.

We know that if the layers of foil are very close together, the curvature will have a negligible effect and we can treat the system like a parallel plate capacitor. Thus we employ the results of Example 26.2 and write

$$C = \frac{\kappa \varepsilon_0 A}{d} \tag{1}$$

The area A in equation (1) is the area of the aluminum sheets: $A = \ell w.$ The width of the sheet (that we can use) is the width that will fit onto the pencil, or 0.20 m. The distance between the plates d is the width of the waxed paper sheet placed between the two sheets of aluminum. The thickness of a sheet of waxed paper is approximately $t_{\text{wax}} = 1.0\times 10^{-4}$ m, although anything of this order is reasonable. The thickness of aluminum foil is given in Worked Problem 26.1 as $t_{\text{Al}} = 5.0\times 10^{-5}$ m.

3. Execute Plan The only information still missing from equation (1) is the dielectric constant of waxed paper. This may vary somewhat from brand to brand. We estimate the dielectric constant to be $\kappa = 2.5$.

Inserting all numbers into equation (1), we find

$$C = \frac{\kappa\varepsilon_0 A}{d}$$
$$C = \frac{\kappa\varepsilon_0 \ell w}{t_\mathrm{w}}$$
$$C = \frac{(2.5)(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(30\ \mathrm{m})(0.20\ \mathrm{m})}{(1.0\times10^{-4}\ \mathrm{m})}$$
$$C = 1.3\times10^{-6}\ \mathrm{F}$$

Anything of order $1\ \mu\mathrm{F}$ is reasonable.

4. Evaluate Result A capacitance of $1\ \mu\mathrm{F}$ is reasonable, in general. But the capacitance obtain in Worked Problem 26.1 was an order of magnitude smaller. That is in spite of the fact that the Mylar in Worked Problem 26.1 has a slightly higher dielectric constant, and the spacing between plates in that problem was half what it is here. This can only be sensible if the area of the plates in Worked Problem 26.1 was much smaller. Since the height of the capacitor is 0.20 m in either case, we must check to see if the length of the sheets used in the capacitor of Worked Problem 26.1 was significantly smaller than in our case here (more than an order of magnitude smaller). Using the expression for length and the numerical values from Worked Problem 26.1, we have

$$\ell = \frac{\pi R^2}{4t} = \frac{\pi(6.00\times10^{-3}\ \mathrm{m})^2}{4(0.0500\times10^{-3}\ \mathrm{m})} = 0.57\ \mathrm{m}$$

So we see the length of our plates here is in fact more than an order of magnitude larger than the length of the plates in Worked Problem 26.1. So it makes perfect sense that our capacitance here is an order of magnitude larger.

26.4 Laptop power

1. Getting Started The energy required is equal to the required power times the amount of time you want to use the laptop before recharging:

$$U^E = P\Delta t = (1\ \mathrm{W})(4\ \mathrm{h})\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 1.4\times10^4\ \mathrm{J}$$

Here we have kept one additional significant digit, as this is an intermediate calculation.

Also, the capacitor must maintain a potential difference greater than or equal to 8 V. That means it is not sufficient for the capacitor to hold a total of $1.4\times10^4\ \mathrm{J}$. It must hold more, such that after converting $1.4\times10^4\ \mathrm{J}$ of energy, there is still enough charge left over on the capacitor plates for the potential difference to be 8 V.

2. Devise Plan We know the capacitance is related to the energy stored in the capacitor by

$$U_\mathrm{i}^E = \frac{1}{2}CV_\mathrm{i}^2$$
$$U_\mathrm{f}^E = \frac{1}{2}CV_\mathrm{f}^2$$

And we also know the limits on the potential differences initially and finally: $V_\mathrm{i} \le 48\ \mathrm{V}$ and $V_\mathrm{f} \ge 8\ \mathrm{V}$. We also know the difference between the initial and final energies must account for the energy used by the laptop:

$$E_\mathrm{laptop} = U_\mathrm{i}^E - U_\mathrm{f}^E = \frac{1}{2}C(V_\mathrm{i}^2 - V_\mathrm{f}^2)$$
$$C = \frac{2E_\mathrm{laptop}}{(V_\mathrm{i}^2 - V_\mathrm{f}^2)} \qquad (1)$$

This capacitance is related to the geometry and materials of the capacitor by

$$C = \frac{\kappa\varepsilon_0 A}{d} \qquad (2)$$

3. Execute Plan (a) We can certainly power the laptop if we use an arbitrarily large capacitor. The interesting question is how small we can make the capacitor. For that, we want to minimize equation (1). That corresponds to choosing the largest possible initial potential difference and the smallest possible final potential difference. Inserting these numbers, we find

$$C = \frac{2E_{\text{laptop}}}{(V_{\text{i}}^2 - V_{\text{f}}^2)}$$

$$C = \frac{2(1.4\times10^4 \text{ J})}{((48 \text{ V})^2 - (8 \text{ V})^2)}$$

$$C = 1\times10^1 \text{ F}$$

This is an enormous capacitance. We report 1×10^1 F as our final answer because we had only one significant digit. We will use a value of 13 F for further calculation so as to avoid accumulated errors from intermediate rounding. (b) Rearranging equation (2) and inserting values, we find

$$A = \frac{Cd}{\kappa\epsilon_0} = \frac{(13 \text{ F})(5\times10^{-5} \text{ m})}{(3.3)(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)} = 2\times10^7 \text{ m}^2$$

4. Evaluate Result The energy required to run the laptop was already calculated in part 1. Although we used two significant digits to avoid errors due to intermediate rounding, the actual answer to the accuracy we are give is 1×10^4 J. We can now use the capacitance to calculate the energy stored initially and finally on the plates.

$$U_{\text{i}}^E = \frac{1}{2}CV_{\text{i}}^2 = \frac{1}{2}(13 \text{ F})(48 \text{ V})^2 = 1.498\times10^4 \text{ J}$$

$$U_{\text{f}}^E = \frac{1}{2}CV_{\text{f}}^2 = \frac{1}{2}(13 \text{ F})(8 \text{ V})^2 = 416 \text{ J}$$

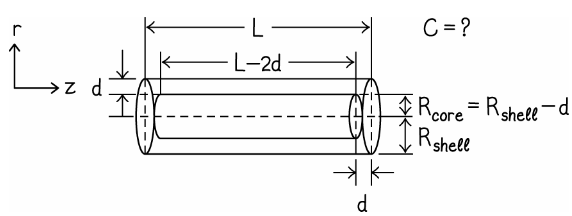
Such that the difference is $U_{\text{i}}^E - U_{\text{f}}^E = 1.46\times10^4$ J, which is 1×10^4 J to the accuracy given. So our work appears to be correct.

The plate area is completely absurd. If the plate were square, it would be over 4 km long on a side. Not even the cleverest methods of folding and rolling can make this a feasible alternative to batteries. But this result assumed the use of Mylar and assumed a thickness of 0.05 mm. If a material could be used that was much thinner than 0.05 mm, such a large plate could be folded more compactly. If a material could be used with a much higher dielectric constant (such as barium titanate), then this required plate area could be reduced by orders of magnitude.

Given that 1 F can be fit into a volume of 10^{-5} m^3, the required capacitance would correspond to a volume of order 10^{-4} m^3. If cubic, such a volume is just 50 mm on a side. The use of capacitors for laptop power sources now begins to seem more plausible.

26.6 Tin can capacitor

1. Getting Started We begin by making a sketch of the capacitor and the variables we plan to use to solve the problem. Let us call the radius of the solid core R_{core}:



The geometry of this capacitor is almost identical to the coaxial capacitor. The difference is that the ends are capped in this problem, and the inner rod is slightly shorter than the conducting shell.

The metal ends are held at the same potential as the rest of the shell, because they are part of a continuous conductor.

The surface area of the circular end caps is πR_{shell}^2 per cap, and the surface area of the curved length of the cylinder is $2\pi R_{\text{shell}} L$.

The distance from the rod to the shell is constant at all points along the length of the shell, except very near the corners between the curved side and the end caps.

2. Devise Plan The capacitance of a coaxial capacitor is (from Example 26.3):

$$C_{\text{coaxial}} = \frac{2\pi\varepsilon_0 \ell}{\ln(R_2/R_1)}$$

In this expression R_2 is the outer radius (in our case R_{shell}). The variable R_1 is the inner/core radius, which in our case is $R_{\text{shell}} - d$.

The end caps are parallel plate capacitors. This is a very good approximation because $d \ll R_{\text{shell}}$. The total capacitance will be the sum of the coaxial capacitance and the two parallel plate end cap capacitances.

3. Execute Plan Using the expressions for the coaxial and parallel plate capacitances, we have

$$C = C_{\text{coaxial}} + 2C_{\text{par}}$$

$$C = \frac{2\pi\varepsilon_0 L}{\ln(R_{\text{shell}}/(R_{\text{shell}} - d))} + \frac{2\varepsilon_0(\pi R_{\text{shell}}^2)}{d} \tag{1}$$

Note that in the limit $d/R_{\text{shell}} \to 0$, $\ln(R_{\text{shell}}/(R_{\text{shell}} - d)) \to R/d$. One can prove this with a simple Taylor series expansion. In that case

$$C = \frac{2\pi\varepsilon_0 L R_{\text{shell}}}{d} + \frac{2\varepsilon_0(\pi R_{\text{shell}}^2)}{d}$$

$$C = \frac{2\pi\varepsilon_0 R_{\text{shell}}}{d}(R_{\text{shell}} + L) \tag{2}$$

Note that this is equivalent to the capacitance of a single parallel plate capacitor with an area that includes both the curved side and the circular end caps.

4. Evaluate Result Our answer has the right units. It is also sensible that the final expression should be that of a parallel plate capacitor, since at all points the distance between the positive and negative surfaces is constant and much smaller than any other length scale in the problem.

Let us modify the capacitance trivially by filling the gap with Mylar. Then we can compare this geometry with the rolled capacitor of Worked Problem 26.1. Let us assume the length of the capacitor is 0.20 m, and the distance between plates is the thickness of the Mylar sheet: $d = 5.00 \times 10^{-5}$ m. The radius of the shell will be $R_{\text{shell}} = 6.00 \times 10^{-3}$ m. Then the geometry in this current problem yields a capacitance of

$$C = \frac{2\pi\kappa\varepsilon_0 R_{\text{shell}}}{d}(R_{\text{shell}} + L)$$

$$C = \frac{2\pi(3.3)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(6.00 \times 10^{-3}\ \text{m})}{(5.00 \times 10^{-5}\ \text{m})}((6.00 \times 10^{-3}\ \text{m}) + (2.000 \times 10^{-2}\ \text{m}))$$

$$C = 1.4 \times 10^{-9}\ \text{F}$$

This is two orders of magnitude smaller than the capacitor in Worked Problem 26.1. This also makes sense, since the rolled capacitor made use of the entire volume inside the shell for charge storage. This current model only stores charge near the outer shell.

26.8 Vacuum capacitors

1. Getting Started The electric field in any system is related to changes in the potential according to $E_s = -\dfrac{dV}{ds}$. But in a parallel plate capacitor the electric field is constant. So we can write $Ed = \Delta V$ or simply $Ed = V$. This way we can relate the maximum potential difference to the maximum electric field. The largest the electric field can be is 10 times the breakdown threshold for air. That value is given several times throughout Chapter 26, notably in Table 26.1: $E_{\text{max,air}} = 3.0\times10^{6}$ V/m.

2. Devise Plan We already know how to determine the potential difference that can be attained, by relating it to the electric field just before breakdown. We can relate that potential difference to the charge on the capacitor using $q = CV$.

We are explicitly given the capacitance of the vacuum capacitor, so we don't really even need to worry about what the dielectric constant of the vacuum is. But recall that the dielectric constant was a measure of how the capacitance (or electric field) changed compared to the vacuum when a material was put between capacitor plates. So obviously $\kappa_{\text{vacuum}} = 1$.

3. Execute Plan The maximum potential difference is given by

$$V_{\text{max,vacuum}} = E_{\text{max,vacuum}}d = d(10E_{\text{max,air}})$$
$$V_{\text{max,vacuum}} = (1.0\times10^{-3}\ \text{m})(10)(3.0\times10^{6}\ \text{V/m})$$
$$V_{\text{max,vacuum}} = 3.0\times10^{4}\ \text{V}$$

The maximum charge stored on the plates is then

$$q_{\text{max}} = CV_{\text{max}}$$
$$q_{\text{max}} = (1.0\times10^{-9}\ \text{F})(3.0\times10^{4}\ \text{V})$$
$$q_{\text{max}} = 3.0\times10^{-5}\ \text{C}$$

4. Evaluate Result The maximum potential difference is much larger than common household items, which typically operate at 120 V of potential difference. The amount of charge stored is also very large.

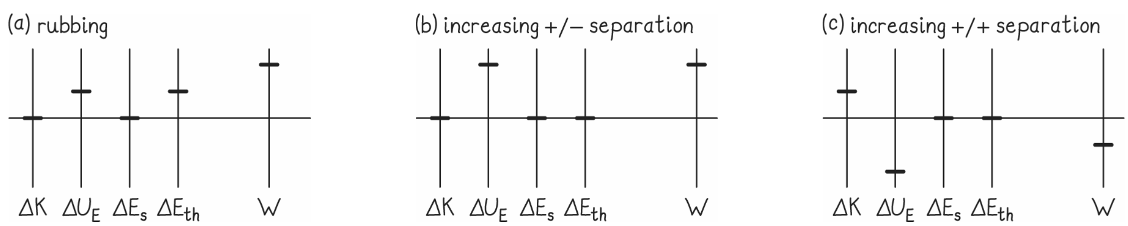
Questions and Problems

26.1. The magnitudes of the charges are equal, and they are opposite in sign.

26.2. The electric field magnitude decreases between the charged particles, and increases everywhere else.

26.3. The sphere should have a large radius. Once inside the sphere there is no electric field due to charge that has accumulated on the sphere. Thus charge on the belt only has to be pushed against electrostatic forces up to the edge of the sphere. The larger the sphere, the weaker the electrostatic force at the edge for a given quantity of charge.

26.4. Note that several answers may be possible depending on assumptions about initial and final kinetic energies. Two possible answers have been explicitly shown in (*c*).



26.5. The work done on the system is

$$W = \Delta E_{\text{th}} + \Delta U^E = \Delta E_{\text{th}} + \frac{2kq^2}{d} - \frac{2kq^2}{2d} - \frac{2kq^2}{\sqrt{5}d} = \Delta E_{\text{th}} + \frac{kq^2}{d}\left(1 - \frac{2}{\sqrt{5}}\right)$$

$$W = (34.0\text{ J}) + \frac{(2.00\times10^{-6}\text{ C})^2}{4\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(30.0\times10^{-3}\text{ m})}\left(1 - \frac{2}{\sqrt{5}}\right) = 34.1\text{ J}$$

26.6. Call the charges on the two objects q_1 and q_2. We know $F = \frac{kq_1q_2}{r^2}$ and $q_1 + q_2 = q_{\text{total}} = -23.5\ \mu\text{C}$. This leads us to the quadratic equation $\frac{kq_1(q_{\text{total}} - q_1)}{r^2} - F = 0$, which has solutions

$$q_1 = \frac{q_{\text{total}} \pm \sqrt{q_{\text{total}}^2 - \frac{4Fr^2}{k}}}{2} = \frac{(-23.5\times10^{-6}\text{ C}) \pm \sqrt{(-23.5\times10^{-6}\text{ C})^2 - \frac{4(7.00\text{ N})(0.200\text{ m})^2}{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}}{2}$$

Thus 1.41×10^{-6} C and 2.21×10^{-5} C.

26.7. The work you do causes the change in energy of the system of two spheres according to $W = \Delta U^E + \Delta K$. But we do not know if we are pushing the spheres apart or resisting their electrostatic repulsion by pushing them together. Thus the work could be positive or negative: $\pm 2Fd = kq^2\left(\frac{1}{r+2d} - \frac{1}{r}\right) + 2\left(\frac{1}{2}mv_{\text{f}}^2\right)$, where d is the distance you push each sphere, and r is their initial separation. Solving for the charge, we obtain $q = \pm\left(\frac{\pm 2Fd - mv_{\text{f}}^2}{k}\right)^{1/2}\left(\frac{1}{r+2d} - \frac{1}{r}\right)^{-1/2}$. Inserting the values given yields an imaginary result when the work is positive. Thus, the pushing force must have been resisting the electrostatic repulsion by pushing the two spheres toward each other:

$$q = \pm\left(\frac{-2(0.15\text{ N})(0.100\text{ m}) - (0.0450\text{ kg})(0.500\text{ m/s})^2}{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}\right)^{1/2}\left(\frac{1}{(0.200\text{ m}) + 2(0.100\text{ m})} - \frac{1}{(0.200\text{ m})}\right)^{-1/2}$$
$$= \pm 3.39\times10^{-6}\text{ C}$$

26.8. Assume (1) that surface charge density is uniform across belt and does not change as belt moves and (2) that all charge is removed from belt as belt passes through dome (meaning no force exerted on depleted half of belt). In order to put the last little bit of charge on the dome, the motor would have to overcome the repulsive force between the charged half of the belt and the nearly fully-charged dome. This repulsive force can be calculated by considering the electric field due to the dome at any point outside the dome: $E = \frac{kq_{\text{dome}}}{r^2}$. A differential segment of the belt carrying charge dq will feel a repulsive force $dF = \frac{kq_{\text{dome}}(dq)}{r^2}$. To find the total repulsive force we integrate this over the length of the belt outside the dome.

$$F = \int_{\text{belt}} dF = \int_{R_{\text{dome}}}^{\ell_{\text{belt}}} \frac{kq_{\text{dome}}\lambda dr}{r^2} = kq_{\text{dome}}\sigma w\left(\frac{1}{R_{\text{dome}}} - \frac{1}{\ell_{\text{belt}}}\right)$$

$$F = (9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(5.0\text{ C})(45\times10^{-6}\text{ C/m}^2)(0.100\text{ m})\left(\frac{1}{(1.3\text{ m})} - \frac{1}{(5.0\text{ m})}\right) = 1.4\times10^5\text{ N}$$

The motor must produce a maximum force magnitude of 1.4×10^5 N. This value could be reduced by increasing dome radius or decreasing belt length. These changes would alter amount of energy required to put given amount of charge on dome. The maximum force required could also be reduced by decreasing either belt surface charge density or belt width. These changes would cause dome to charge more slowly, but energy required to charge dome would be unchanged.

You could also make dome ellipsoidal instead of spherical. Orienting ellipsoid so that belt enters flatter region would move some of the charge on dome away from belt. All of these changes are physically reasonable to an extent.

26.9. If the battery stays connected, the potential difference between plates will remain constant when the plates move closer together. Since the same change in electric potential would have to occur over a shorter distance, the electric field would increase (doubling, since the plate separation is halved). If the plates are disconnected from the battery, then no additional charge can be placed on the plates. Since the charge density of the plates will not change, the electric field will not change. Thus, there is a greater electric field magnitude with battery connected: $E_{\text{batt connected}}/E_{\text{batt disconnected}} = 2$.

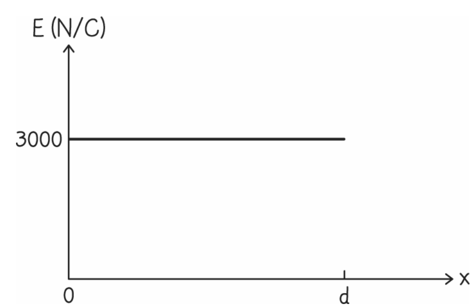
26.10. One can answer this question by inspection if one notes that doubling the charge doubles the strength of the electric field between plates, and doubling the distance between plates means this electric field persists through twice as large a change in potential, meaning $V_1 = 4V_2$. Algebraically, the electric field between capacitor plates is constant, meaning that $\Delta V = -E_x \Delta x$ if the plates are separated in the x direction. Further, we can insert the electric field due to a pair of large plates: $\Delta V = -\frac{\sigma \Delta x}{\varepsilon_0} = -\frac{q \Delta x}{A\varepsilon_0}$. It should now be clear that $V_1 = -\frac{(2q_2)(2\Delta x)}{A\varepsilon_0} = -4\frac{q\Delta x}{A\varepsilon_0} = 4V_2$.

26.11. Doubling the plate separation with a given electric field doubles the potential difference between plates. Thus, the fact that the plate separation in capacitor 1 is twice that in capacitor 2, means the potential difference would be twice as large between the plates of capacitor 1 if the electric field in both capacitors is the same. Since the electric fields are the same, the charge density and therefore the total charge on the plates must be the same: $q_1 = q_2$.

26.12. The plates are disconnected from the batter; they are isolated. No charge can leave or make its way onto either plate. The charge remains $-q$.

26.13. Because potential difference between plates maintained at some value by battery, $V_{\text{cap}} \neq 0$ always. This potential difference can be written $V_{\text{cap}} = \int_{+}^{-} \vec{E} \cdot d\vec{\ell}$. If electric field magnitude outside plates were zero, there would be a path from one plate to the other such that $\int_{+}^{-} \vec{E} \cdot d\vec{\ell} = 0 = V_{\text{cap}}$, which can never be true. Hence electric field magnitude outside plates can never be zero.

26.14. (*a*) The electric field points to the right, meaning the potential is lower on the right than on the left. Thus the left plate carries positive charge. (*b*) 2 (*c*) They are conductors; the field inside these slabs is zero. (*d*) Because there is no potential difference across the conducting slabs, initially the potential drops must occur over a total distance of $3d/5$. With the slabs removed, the electric potential can drop smoothly over a distance d. With the potential fixed on the plates at the same values, but dropping over a distance $5/3$ the previous distance, the electric field magnitude will be reduced to a distance 3000 N/C.



26.15. Nothing changes about the sign of the charge on either plate (battery leads are not swapped), so we concern ourselves only with the magnitude of the charge on each plate. We know that $\dfrac{|\Delta V|}{\Delta x} = |E_x| = \dfrac{|\sigma|}{\varepsilon_0} \Rightarrow |q| = \dfrac{\varepsilon_0 A|\Delta V|}{\Delta x}$, where we have assumed the electric field is parallel to the x axis. Thus if we double the potential difference and the separation, the charge on the plates remains unchanged: q.

26.16. The electric field due to one plate (that is felt by the other plate) is $E_{\text{plate}} = \dfrac{\sigma}{2\varepsilon_0}$. So the force per unit area felt by the opposite plate is attractive and has a magnitude $\dfrac{F_{\text{plates attractive}}}{A} = E\left(\dfrac{q}{A}\right) = \dfrac{\sigma^2}{2\varepsilon_0} \Rightarrow F_{\text{plates attractive}} = \dfrac{A\sigma^2}{2\varepsilon_0}$. In order to separate the plates, we must push them apart with a force equal in magnitude to this attractive force. We can write the work done by such a force as

$$W = F\Delta x = \frac{q^2\Delta x}{2A\varepsilon_0} = \frac{(3.30\times10^{-6}\ \text{C})^2(1.00\times10^{-3}\ \text{m})}{2(0.0100\ \text{m}^2)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 0.0615\ \text{J}$$

26.17. (*a*) We know that $\dfrac{V}{d} = |E_x| = \dfrac{|\sigma|}{\varepsilon_0} \Rightarrow q = \dfrac{\varepsilon_0 AV}{d}$, where we have assumed the electric field is parallel to the x axis. Thus, doubling the plate separation reduces the charge by half such that the positively charged plate now holds a charge $+q/2$. (*b*) If the plates are disconnected from the battery, they can be assumed to be electrically isolated. There is no way for charge to leave the plates or come to the plates. The charge on the positively charged plate is still $+q$.

26.18. (*a*) The maximum electric field can be written $|E_{\max}| = \dfrac{V_{\text{batt}}}{d/4}$, meaning that the potential must change over a distance of at least $d/4$, or else dielectric breakdown will occur. There will be no change in potential throughout the conducting slab. Thus, we can fill $3d/4$ of the gap without causing dielectric breakdown. The maximum thickness of the slab is $\dfrac{3d}{4}$. (*b*) Yes, it increases the amount of charge by a factor of 4. Note that initially $\dfrac{\sigma_{\text{i}}}{\varepsilon_0} = |E_{\text{i}}| = \dfrac{V_{\text{batt}}}{d}$ or $\sigma_{\text{i}} = \dfrac{V_{\text{batt}}\varepsilon_0}{d}$, and finally $\dfrac{\sigma_{\text{f}}}{\varepsilon_0} = |E_{\text{f}}| = \dfrac{V_{\text{batt}}}{d/4}$ or $\sigma_{\text{f}} = \dfrac{4V_{\text{batt}}\varepsilon_0}{d}$. Clearly $\sigma_{\text{f}} = 4\sigma_{\text{i}}$, and since the plate area has not changed, it follows that $q_{\text{f}} = 4q_{\text{i}}$.

26.19. Many answers possible. (*a*) In the case of either a dielectric or a conductor a surface charge accumulates on the slab. At constant potential difference, both slabs increase the charge on the capacitor plates. For an isolated capacitor, both slabs decrease V_{cap} and decrease E between plates. (*b*) With conductor, E between plates is zero; with dielectric, E between plates is lower than E outside plates but not zero. Charge that accumulates on slab is free on conductor but bound on dielectric. A dielectric slab increases the breakdown threshold but a conducting slab does not.

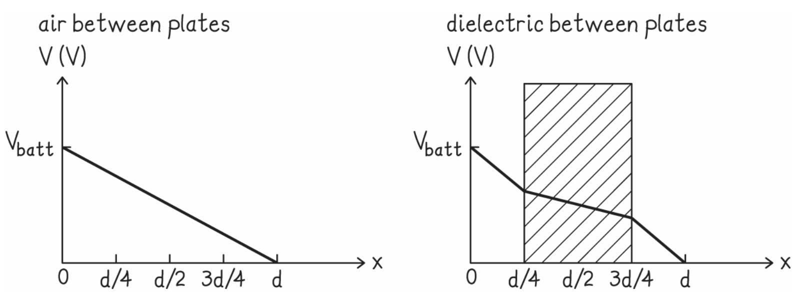
26.20. (*a*) The free charge is charge that has been placed on the capacitor plates, that is not bound to any dielectric. Because the plates are disconnected from the battery (and not connected to any other source or sink of free charge), the free charge on the plates is unchanged. (*b*) The electric field between capacitor plates causes molecules in the dielectrics to reorient themselves, placing as much bound positive charge close to the negative plates as possible, and as much negative bound charge close to the positive plate as possible. Thus, the magnitude of the total charge at either plate (bound charge and free charge together) is reduced.

26.21. (*a*) When capacitor 2 is filled with plastic, bound charge in the plastic will reduce the total charge on either plate. This lower charge density on either plate means there will be a smaller electric field in capacitor 2. $E_1 > E_2$. (*b*) Call the direction of the electric field the $+x$ direction. Then $E_x = -\dfrac{V_{\text{plates}}}{d}$. Since the capacitors have the same plate separation d, and $E_1 > E_2$, it follows that $\Delta V_1 > \Delta V_2$. (*c*) In capacitor 2, one plate feels a weaker potential due to the other plate. Thus $U_1^E > U_2^E$.

26.22. (*a*) Call the direction of the electric field the $+x$ direction. The potential difference is the same across either capacitor, and the distance over which that potential changes is also the same. Since $E_x = -\frac{V_{\text{plates}}}{d}$, we see $E_1 = E_2$. (*b*) The total charge at either plate (bound and free together) must be same, because the electric fields are the same and electric fields depend on the charge density at either plate according to $E = \frac{\sigma}{\varepsilon_0}$. In capacitor 1, this total charge is just the free charge $q_1 = q_{1,\text{free}}$. But in capacitor 2, the total charge is $q_2 = q_{2,\text{free}} - |q_{2,\text{bound}}|$. Thus $q_{2,\text{free}} > q_{1,\text{free}}$. (*c*) In capacitor 2, we have a greater amount of free charge on the plates, as well has having a large charge separation in the dielectric itself. Thus $U_2^E > U_1^E$.

26.23. The initial electric field is $E_\text{i} = \frac{\sigma_\text{i}}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$. The contribution to the electric field from the addition of the dielectric is $E_\text{d} = \frac{\sigma_\text{d}}{\varepsilon_0} = -0.3E_\text{i} = \frac{-0.3q}{A\varepsilon_0}$, which tells us that the polarization of the dielectric put a charge of $-0.3q$ up against the plate that initially held a positive charge $+q$. The battery was capable of maintaining a charge separation with $+q$ on one plate and $-q$ on the other. So the battery can now supply an additional $+0.3q$ to the plate to maintain the potential difference across the plates. Thus the free charge on the positive plate is $1.3q$.

26.24.



The electric field E in the region occupied by dielectric smaller than E in rest of region between plates. This corresponds to change in slope when $V_{\text{batt}}(x)$ curve enters dielectric portion of graph. The greater the quantity of charge bound on dielectric surface, the shallower the slope in this portion of graph. Also, $V_{\text{batt}}(x)$ curve for region outside dielectric is steeper than curve in air-between-plates graph because same potential difference must be maintained between plates.

26.25. (*a*) If the battery stays connected to the plates, then the same potential difference is maintained across the plates, regardless of the insertion of the dielectric. Since the plate separation is the same and the potential difference is the same, from $E_x = -\frac{V_{\text{plates}}}{\Delta x}$ (calling the direction of the electric field x), we see that the electric field between the plates is unchanged. The ratio of electric fields is 1:1. (*b*) The initial electric field in the empty capacitor can be written $E_{\text{empty}} = \frac{\sigma_{\text{empty}}}{\varepsilon_0} = \frac{q}{A\varepsilon_0}$. After the capacitor is filled, there is an additional contribution to the charge on the plates due to the bound charge on the dielectric, and we have $E_{\text{dielectric}} = \frac{\sigma_{\text{dielectric}}}{\varepsilon_0} = \frac{q_{\text{free}} - q_{\text{bound}}}{A\varepsilon_0} = \frac{q - q/3}{A\varepsilon_0} = \frac{2}{3}E_{\text{empty}}$. Thus the ratio $E_{\text{empty}} : E_{\text{dielectric}} = 3:2$.

26.26. Since we are asked for the smallest possible charge, we assume that the electric field upon removal of the dielectric is just exactly equal to the threshold electric field of air: $E_{\text{breakdown}} = 3.0\times10^{6}$ V/m. If the plates are isolated such that the free charge q is kept fixed, then we simply require that $E_{\text{breakdown}} = \dfrac{q}{A\varepsilon_0} \Rightarrow q = EA\varepsilon_0 E_{\text{breakdown}} =$ $(0.0045\text{ m}^2)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^{6}\text{ V/m}) = 1.2\times10^{-7}$ C. However, if the plates are still connected to a battery, then it is not the free charge but the potential difference (and total charge) that will remain constant. The total charge on the capacitors was initially $q_{\text{total}} = q_{\text{free}} + q_{\text{bound}} = q - \dfrac{3q}{4} = \dfrac{q}{4}$. When the dielectric material is removed, the battery will no longer be able to keep the same free charge on the plates as before; instead the free charge on the plates will be reduced to $\dfrac{q}{4}$. Now we require $E_{\text{breakdown}} = \dfrac{q}{4A\varepsilon_0} \Rightarrow q = 4A\varepsilon_0 E_{\text{breakdown}} = 4(0.0045\text{ m}^2)$ $(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^{6}\text{ V/m}) = 4.8\times10^{-7}$ C. So if the plates are disconnected from the battery and isolated such that the charge stays fixed $q \geq 1.2\times10^{-7}$ C, and if the plates remain connected to a battery $q \geq 4.8\times10^{-7}$ C.

26.27. Using a conducting slab does not help because its presence increases electric field magnitude in the space not filled by the slab, making the air in that region more likely to break down. Also, charge carriers can travel across conducting slab, a motion not possible (under normal conditions) in a dielectric.
Increasing plate separation does not help because this increases both the potential difference between the plates and the distance over which the potential changes. These two effects cancel when determining the electric field magnitude. (This argument is valid only when the plate areas are much greater than the separation distance. If plate separation is made very large, electric field magnitude in region between plates could decrease.)

26.28. The electromotive force is not a force, but rather the work done per unit charge. It has units of Volts, and behaves more like a potential difference than a force.

26.29. It remains the same. Charge carriers flow out of one terminal and into the other terminal at equal rates such that the total charge in the battery is constant.

26.30. The concentration of ions in electrolyte solution and the ability of electrodes to resist deterioration determine the lifetime of a battery.

26.31. The chemical reaction of one molecule of HSO_4^- provides two electrons to the negative terminal, meaning that 5.0×10^{7} HSO_4^- reactions take place each second. Thus the fraction of HSO_4^- reacting each second is $\dfrac{(5.0\times10^{7}\text{ molecules})}{(3.00\text{ mol})(6.02\times10^{23}\text{ molecules/mol})} = 2.8\times10^{-17}$.

26.32. (*a*) We simply balance the number of each atom and the charge on each side of the reaction to obtain: $Zn + 2OH^- \rightarrow ZnO + H_2O + 2e^-$. (*b*) We simply balance the number of each atom and the charge on each side of the reaction to obtain: $2MnO_2 + H_2O + 2e^- \rightarrow Mn_2O_3 + 2OH^-$. (*c*) Mn is the positive terminal and Zn is the negative terminal.

26.33. The potential difference is related to the charge density through the electric field magnitude according to $\dfrac{\sigma}{\varepsilon_0} = E = \dfrac{V_{\text{batt}}}{d} \Rightarrow \sigma = \dfrac{V_{\text{batt}}\varepsilon_0}{d} = \dfrac{(4.5\text{ V})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}{(1.00\times10^{-3}\text{ m})} = 4.0\times10^{-8}\text{ C/m}^2$.

26.34. The original capacitance of the parallel plate capacitor is $C = \dfrac{\varepsilon_0 A}{d}$, and the final capacitance is $C_\text{f} = \dfrac{\varepsilon_0 2A}{2d} = C.$ So the capacitance is still C.

26.35. From examples 26.3 and 26.4 we know $C_\text{cyl} = \dfrac{2\pi\varepsilon_0 \ell}{\ln(R_2/R_1)}$ and $C_\text{sphere} = \dfrac{4\pi\varepsilon_0 R_1 R_2}{(R_2 - R_1)}$. Equating the two, we have

$$\ell = \frac{2R_1R_2\ln(R_2/R_1)}{(R_2 - R_1)} = \frac{2(10.0\times10^{-3}\text{ m})(30.0\times10^{-3}\text{ m})\ln\big((30.0\times10^{-3}\text{ m})/(10.0\times10^{-3}\text{ m})\big)}{\big((30.0\times10^{-3}\text{ m}) - (10.0\times10^{-3}\text{ m})\big)} = 3.30\times10^{-2}\text{ m.}$$

26.36. For each part, we use the expression for the capacitance of a parallel plate capacitor: $C = \dfrac{\varepsilon_0 A}{d}$.

(a) $C_\text{f} = \dfrac{\varepsilon_0 A}{2d} = \dfrac{C}{2}$ (b) $C_\text{f} = \dfrac{\kappa\varepsilon_0 A}{d} = \kappa C$ (c) $C_\text{f} = \dfrac{\varepsilon_0 A}{d} = C$ (d) $C_\text{f} = \dfrac{\varepsilon_0 (A/2)}{d} = \dfrac{C}{2}$ (e) $C_\text{f} = \dfrac{\varepsilon_0 A}{d} = C$

26.37. From example 26.3, we know $C_\text{cyl} = \dfrac{2\pi\varepsilon_0 \ell}{\ln(R_\text{outer}/R_\text{wire})} = \dfrac{q}{V}$, such that $R_\text{wire} = R_\text{outer} e - \left(\dfrac{2\pi\varepsilon_0 \ell V}{q}\right) =$

$$(10.0\times10^{-3}\text{ m})e - \left(\frac{2\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(80.0\times10^{-3}\text{ m})\big(18.0\text{ V}\big)}{(1.1\times10^{-9}\text{ C})}\right) = 9.3\text{ mm.}$$

26.38. (a) The positive and negative charge distributions contribute equally to the electric field in the case of the parallel plates. In the case of the spherical capacitor, the inner negative charge distribution contributes the entire electric field inside, and the outer positive charge distribution contributes nothing. (b) The field magnitude is zero outside either capacitor. This is most easily seen by applying Gauss's Law. If we construct a pill-box shaped Gaussian cylinder enclosing sections of both plates of the parallel plate capacitor, we see that it would enclose no charge. It follows from Gauss's Law and symmetry that the electric field on either side of the parallel plate capacitor will be zero. A similar process using a spherical Gaussian surface around the spherical capacitor confirms that the field outside that capacitor is also zero.

26.39. (a) We require $C_\text{f} = \dfrac{4\pi\varepsilon_0 R_\text{inner} d}{(d - R_\text{inner})} = 2C_\text{i} = 2\dfrac{4\pi\varepsilon_0 R_\text{inner} R_\text{outer}}{(R_\text{outer} - R_\text{inner})}$. Rearranging, we obtain $\dfrac{d}{(d - R_\text{inner})} = 2\dfrac{R_\text{outer}}{(R_\text{outer} - R_\text{inner})}$ or $d = \dfrac{2R_\text{outer}R_\text{inner}}{(R_\text{outer} + R_\text{inner})}$. ($b$) We require $C_\text{f} = \dfrac{4\pi\varepsilon_0 R_\text{outer} d}{(R_\text{outer} - d)} = 2C_\text{i} = 2\dfrac{4\pi\varepsilon_0 R_\text{inner} R_\text{outer}}{(R_\text{outer} - R_\text{inner})}$. Rearranging, we obtain $d(R_\text{outer} - R_\text{inner}) = 2R_\text{inner}(R_\text{outer} - d)$ or $d = \dfrac{2R_\text{inner}R_\text{outer}}{(R_\text{outer} + R_\text{inner})}$. ($c$) Just as in part ($a$), we first require $C_\text{f} = \dfrac{2\pi\varepsilon_0 \ell}{\ln(d/R_\text{inner})} = 2C_\text{i} = \dfrac{4\pi\varepsilon_0 \ell}{\ln(R_2/R_1)}$. Rearranging, we obtain $\ln(d/R_\text{inner}) = \dfrac{1}{2}\ln(R_\text{outer}/R_\text{inner}) \Rightarrow d = \sqrt{R_\text{outer}R_\text{inner}}$. As in part ($b$), we require $C_\text{f} = \dfrac{2\pi\varepsilon_0 \ell}{\ln(R_\text{outer}/d)} = 2C_\text{i} = \dfrac{4\pi\varepsilon_0 \ell}{\ln(R_\text{outer}/R_\text{inner})}$. Rearranging, we obtain $\ln(R_\text{inner}/d) = \dfrac{1}{2}\ln(R_\text{outer}/R_\text{inner})$ $\Rightarrow d = \sqrt{R_\text{outer}R_\text{inner}}$. So $d = \sqrt{R_1 R_2}$ in either case.

26.40. (a) From examples 26.3 and 26.4, and inserting the inner and outer radii, we know $C_\text{cyl} = \dfrac{2\pi\varepsilon_0 \ell}{\ln(2)}$ and $C_\text{sphere} = 4\pi\varepsilon_0(2R)$. Equating the two, we have $\ell = 4R\ln(2)$. (b) We require $C_\text{sphere} = C_\text{par} \Rightarrow 4\pi\varepsilon_0(2R) = \dfrac{\varepsilon_0 \pi R^2}{d} \Rightarrow d = \dfrac{R}{8}$.

26.41. (*a*) In order to be added to x^2, $[b]=\text{m}^2$. Solving for a, we obtain $a=\dfrac{E(x)}{q(x^2+b)}$. Examining the units of the right hand side, we obtain $[a]=\dfrac{\text{N}}{C^2\text{m}^2}$. (*b*) The capacitance of any object is $C=\dfrac{q}{V}$. We can obtain the potential difference by integrating the electric field: $V=\int_{-4}^{4} aq(x^2+b)dx=\left(aqbx+\dfrac{aqx^3}{3}\right)\Bigg|_{-4}^{4}=(8\text{ m})aqb+\dfrac{(128\text{ m})aq}{3}$.
Dividing charge by this potential difference, we obtain $C=\dfrac{1}{a\left(\dfrac{(128\text{ m}^3)}{3}+(8\text{ m})b\right)}$.

26.42. (*a*) In the limit as $R_1\to R_2$, we can write R_1R_2 as approximately R^2. Then the expression for the capacitance of a spherical capacitor becomes $C_{\text{sphere}}=4\pi\varepsilon_0\left(\dfrac{R_1R_2}{R_2-R_1}\right)\to 4\pi R^2\varepsilon_0\left(\dfrac{1}{d}\right)$. Note that $4\pi R^2$ is the surface area of a sphere, so we can write $C_{\text{sphere}}\to\dfrac{A\varepsilon_0}{d}$. (*b*) The Taylor series of $\ln(1+x)$ for x near zero is given by $\ln(1+x)=x+2\text{nd order terms}$. We use this in the following expression for the capacitance of a cylindrical capacitor: $\dfrac{2\pi\varepsilon_0\ell}{\ln(R_2/R_1)}$. Writing $R_2=R_1+d$ gives us $C_{\text{cyl}}=\dfrac{2\pi\varepsilon_0\ell}{\ln\left(1+\dfrac{d}{R_1}\right)}$. Since d/R_1 is small by construction, we can use the Taylor series above to write $C_{\text{cyl}}\to\dfrac{2\pi\varepsilon_0\ell}{d/R_1}=\dfrac{(2\pi R_1\ell)\varepsilon_0}{d}$. Note that $(2\pi R_1\ell)$ is the surface area of a cylindrical shell. Hence we can write $C_{\text{cyl}}\to\dfrac{A\varepsilon_0}{d}$.

26.43. From equation 26.4, we have $U^E=\dfrac{1}{2}qV_{\text{cap}}=\dfrac{1}{2}(6.60\times10^{-6}\text{ C})(9.00\text{ V})=2.97\times10^{-5}\text{ J}$.

26.44. From equation 26.4, we have $U^E=\dfrac{1}{2}CV_{\text{cap}}^2=\dfrac{1}{2}(56.0\times10^{-6}\text{ F})\big((0.85)(9.00\text{ V})\big)^2=1.64\times10^{-3}\text{ J}$.

26.45. Initially, the electrostatic potential energy stored in the capacitor can be written $U_{\text{i}}^E=\dfrac{1}{2}CV_{\text{cap}}^2=\dfrac{1}{2}\left(\dfrac{\varepsilon_0A}{d}\right)V_{\text{batt,i}}^2$. The final electrostatic potential energy is $U_{\text{f}}^E=\dfrac{1}{2}C_{\text{f}}V_{\text{cap,f}}^2=\dfrac{1}{2}\left(\dfrac{\varepsilon_0(2A)}{d}\right)4V_{\text{batt,i}}^2=$ $8\left(\dfrac{1}{2}\left(\dfrac{\varepsilon_0A}{d}\right)V_{\text{batt,i}}^2\right)=8U_{\text{i}}^E$.

26.46. (*a*) The capacitance of a parallel plate capacitor is determine entirely by the structure of the capacitor (the geometry and dielectric material). If the two capacitors are structurally identical, then $C_1=C_2$. (*b*) We know that $V=q/C$. The capacitance is the same for each, but since capacitor 2 carries twice as much charge as capacitor 1, we have $V_2=2V_1$. (*c*) Capacitor 2 has twice the change in potential as capacitor 1, but across the same plate separation. Using $E=\dfrac{V}{d}$, we see $E_2=2E_1$. (*d*) Inserting the results of parts (*a*) and (*b*) into equation 26.4, we find $\dfrac{U_2^{\text{E}}}{U_1^{\text{E}}}=\dfrac{\frac{1}{2}C_2V_2^2}{\frac{1}{2}C_1V_1^2}=\left(\dfrac{C_2}{C_1}\right)\left(\dfrac{V_2}{V_1}\right)^2=(1)(2)^2=4$. So $U_2^{\text{E}}=4U_1^{\text{E}}$.

26.47. (*a*) Using the expression for the capacitance of a parallel place capacitor, we find $C_1 = \frac{\varepsilon_0 A}{d}$, $C_2 = \frac{2\varepsilon_0 A}{d}$, and $C_3 = \frac{\varepsilon_0 A}{2d}$. Thus $C_2 > C_1 > C_3$. (*b*) Since all capacitors are connected to the same battery, we can use $q = CV$ to determine $q_2 > q_1 > q_3$. (*c*) Capacitors 1 and 2 have the same plate separation. Since they are connected to the same battery and therefore have the same potential difference across that plate separation, they must have the same electric field magnitude between their plates. Capacitor 3 has a larger plate separation and therefore a smaller electric field. Thus $E_2 = E_1 > E_3$. (*d*) From equation 26.4 we know $U^E = \frac{1}{2}qV$. Since the potential difference is the same across all three capacitors, the ranking of energy stored is the same as the ranking of charge stored. Thus $U_2^{\mathrm{E}} > U_1^{\mathrm{E}} > U_3^{\mathrm{E}}$. (*e*) From equation 26.6, we know $u_E - \frac{1}{2}\varepsilon_0 E^2$. So the ranking for energy density is the same as the ranking of electric fields. Thus $u_{E,2} = u_{E,1} > u_{E,3}$.

26.48. (*a*) Using the expression for the capacitance of a parallel place capacitor, we find $C_1 = \frac{\varepsilon_0 A}{d}$, $C_2 = \frac{2\varepsilon_0 A}{d}$, and $C_3 = \frac{\varepsilon_0 A}{2d}$. Thus $C_2 > C_1 > C_3$. (*b*) Since all capacitors carry the same charge, we can use $V = q/C$ to determine $\Delta V_3 > \Delta V_1 > \Delta V_2$. (*c*) The electric field is given by $E = \frac{q}{A\varepsilon_0}$, and q is the same for all three capacitors. Since capacitors 1 and 3 have the same plate area, they must also have the same electric field between their plates. Capacitor 2, however, has a larger plate area and therefore a smaller electric field. Thus $E_1 = E_3 > E_2$. (*d*) From equation 26.4 we know $U^E = \frac{1}{2}qV$. Since the charge is the same on all three capacitors, the ranking of energy stored is the same as the ranking of potential difference. Thus $U_3^{\mathrm{E}} > U_1^{\mathrm{E}} > U_2^{\mathrm{E}}$. (*e*) From equation 26.6, we know $u_E - \frac{1}{2}\varepsilon_0 E^2$. So the ranking for energy density is the same as the ranking of electric fields. Thus $u_{E,1} = u_{E,3} > u_{E,2}$.

26.49. (*a*) Gauss's Law tells us that the electric field in this region is due entirely to the charge on the inner sphere. Thus $\vec{E} = \frac{kq}{r^2}\hat{r}$. To find the average energy density, we must integrate this over the volume of the capacitor, and then divide by that volume. Thus $U^E = \int_{\mathrm{cap}} u_E dV = \int_0^{2\pi}\int_0^{\pi}\int_R^{2R} \frac{1}{2}\varepsilon_0\left(\frac{kq}{r^2}\right)^2 r^2 dr \sin(\theta) d\theta d\phi = \frac{k^2\pi q^2\varepsilon_0}{R} = \frac{q^2}{16\pi\varepsilon_0 R}$. Dividing this total electrostatic potential energy by the volume of the capacitor yields $u_{E,\mathrm{av}} = \frac{U^E}{V} = \frac{\left(\frac{q^2}{16\pi\varepsilon_0 R}\right)}{\frac{4}{3}\pi(8R^3 - R^3)} = \frac{3q^2}{448\pi^2\varepsilon_0 R^4}$.

(*b*) As in part (*a*), we first determine that the electric field inside the capacitor is $\vec{E} = \frac{q}{2\pi\varepsilon_0 \ell r^2}\hat{r}$, and then integrate $u_E = \frac{1}{2}\varepsilon_0 E^2$ to determine the total electrostatic potential energy stored: $U^E = \int_{\mathrm{cap}} u_E dV = \int_0^{2\pi}\int_0^{\ell}\int_R^{2R} \frac{1}{2}\varepsilon_0\left(\frac{q}{2\pi\varepsilon_0 r\ell}\right)^2 r dr dz d\phi = \frac{q^2 \ln(2)}{4\pi\varepsilon_0 \ell}$. Dividing this total potential energy by the volume of the capacitor yields $u_{E,\mathrm{av}} = \frac{U^E}{V} = \frac{\left(\frac{q^2 \ln(2)}{4\pi\varepsilon_0 \ell}\right)}{\pi\ell(4R^2 - R^2)} = \frac{q^2 \mathrm{Ln}(2)}{12\pi^2\varepsilon_0 \ell^2 R^2}$.

26.50. (*a*) For a parallel plate capacitor, we know $C = \frac{\varepsilon_0 A}{d}$, so $4C = \frac{\varepsilon_0 A}{d/4}$, meaning the plate separation would be $\frac{d}{4}$. (*b*) With the capacitor connected to the same battery at all times, quadrupling the capacitance would also quadruple the charge on the capacitor. Thus this condition is no different than the condition in part (*a*), and the plate separation must be $\frac{d}{4}$. (*c*) From equation 26.4, we know $U^E = \frac{1}{2} q V_{\text{cap}}$. Again, with the potential difference kept fixed by the battery, quadrupling the charge on the plates would quadruple the potential energy stored. Thus, once again the plate separation is $\frac{d}{4}$. (*d*) The energy density is $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0 V_{\text{cap}}^2}{2d^2}$. So reducing the plate separation by $\frac{1}{2}$ would quadruple the energy density. Thus the new plate separation must be $\frac{d}{2}$.

26.51. (*a*) For a parallel plate capacitor, we know $C = \frac{\varepsilon_0 A}{d}$, so $4C = \frac{\varepsilon_0 A}{d/4}$, meaning the plate separation would be $\frac{d}{4}$. (*b*) This is not possible as long as the plates are isolated. Charge is conserved and the plates are not connected to anything. (*c*) From equation 26.4, we know $U^E = \frac{1}{2}\frac{q^2}{C}$. So with a fixed amount of charge, the capacitance must be reduced by a factor of 4 in order to increase the potential energy stored by a factor of 4. Since $C = \frac{\varepsilon_0 A}{d}$, we see the plate separation must be increased to $4d$. (*d*) One can write the electrostatic potential energy density as $u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left(\frac{\sigma}{\varepsilon_0}\right)^2 = \frac{q^2}{2A^2\varepsilon_0}$, which is completely independent of the plate separation. Thus this is not possible by only varying the plate separation.

26.52. (*a*) If the capacitor is isolated, then the charge is fixed which means the charge density and electric field is also fixed. Consider the electric field due to one plate that is felt by the second plate: $E_{1\text{ plate}} = \frac{\sigma}{2\varepsilon_0}$. This will cause an attractive force on the second plate of magnitude $F_{12}^E = qE_{1\text{ plate}} = \frac{q\sigma}{2\varepsilon_0} = \frac{q^2}{2A\varepsilon_0}$. In order to increase the separation from d to $3d$ you must do work equal to $W = F_{12}^E(2d) = \frac{dq^2}{A\varepsilon_0}$. (*b*) If the capacitor is connected to a battery, then the electric field is not constant. Also, the work we do by moving the plates is not the only work done on the system of the plates; the battery also does work. So we cannot equate the work we do on the plates to the change in potential energy. In this case, we write the electric field due to one plates that can be felt by the other plate as $E_{1\text{ plate}} = \frac{V_{\text{cap}}}{x}$, where x is the varying plate separation. Thus the force felt by plate 2 is $F_{12}^E = qE_{1\text{ plate}} = \frac{qV_{\text{cap}}}{x} = \frac{CV_{\text{cap}}^2}{x} = \frac{\varepsilon_0 A V_{\text{cap}}^2}{x^2}$. The work you must do to increase the plate separation is now $W = \int_d^{3d} F_{12}^E dx = \int_d^{3d} \frac{\varepsilon_0 A V_{\text{cap}}^2}{x^2} dx = \varepsilon_0 A V_{\text{cap}}^2 \left(\frac{1}{d} - \frac{1}{3d}\right) = \frac{2\varepsilon_0 A V_{\text{cap}}^2}{3d} = \frac{2}{3} C V_{\text{cap}}^2 = \frac{2}{3}\frac{q^2}{C} = \frac{2}{3}\frac{dq^2}{\varepsilon_0 A}$. (*c*) Increasing d to $3d$ in isolated capacitor increases U^E because greater separation distance means more positive work must be done to move charge carriers from one plate to the other. When d increased to $3d$ in capacitor connected to battery, battery does negative work to recombine charge carriers from plates to keep V_{cap} constant. Algebraic sum of this negative work and positive work you do results in decrease in U^E.

26.53 (*a*) It is not generally useful, but one could say that the dielectric constant of a conductor approaches infinity as the conductivity increases. (*b*) The breakdown threshold is zero. Any arbitrarily small field will separate charge.

26.54. (*a*) Charge carriers are ripped from material between plates (air or whatever dielectric is used) forming a spark that provides a conducting path from one plate to the other. The capacitor discharges at least partially (and often completely). (*b*) Use dielectric that has higher breakdown threshold (and therefore higher κ value) or increase plate separation distance.

26.55. (*a*) Increase plate area, decrease plate separation, insert dielectric with higher dielectric constant, change the geometry in some cases. (*b*) No. Capacitance is the ratio of charge stored per potential difference applied. For a given capacitance, connecting a stronger battery will simply increase the charge stored such that the ratio (the capacitance) is constant.

26.56. Stored energy increases by factor of κ (as you do positive work in pulling slab out). This happens because with only air between plates there is no bound surface charge on dielectric to reduce electric field magnitude or amount of energy stored.

26.57. (*a*) Since, for a parallel plate capacitor $C = \dfrac{\kappa\varepsilon_0 A}{d}$, it follows immediately that $C_2 = \kappa C_1$. (*b*) Since the capacitors are connected to identical batteries, we can equate the potential differences across them and we find $V_{\text{cap 1}} = \dfrac{q_1}{C_1} = V_{\text{cap 2}} = \dfrac{q_2}{C_2}$. Inserting the results of part (*a*), we find $q_2 = \dfrac{C_2}{C_1} q_1 = \kappa q_1$. (*c*) We know the electrostatic potential energy stored in a capacitor can be written $U^E = \dfrac{1}{2} q V_{\text{cap}}$. Since both capacitors are connected to identical batteries, we can say $\dfrac{U_2^E}{U_1^E} = \dfrac{\frac{1}{2} q_2 V_{\text{cap 2}}}{\frac{1}{2} q_1 V_{\text{cap 1}}} = \dfrac{q_2}{q_1} = \kappa$ such that $U_2^E = \kappa U_1^E$. (*d*) In each capacitor the same change in potential occurs over the same plate separation. Thus $E_1 = E_2$. (*e*) We know $U^E = \dfrac{1}{2} C V_{\text{cap}}^2$ so we can write $u_E = \dfrac{U^E}{V} = \dfrac{C V_{\text{cap}}^2}{2Ad} = \dfrac{\kappa\varepsilon_0 V_{\text{cap}}^2}{2d^2}$. Thus $u_2^E = \kappa u_1^E$.

26.58. (*a*) Since, for a parallel plate capacitor $C = \dfrac{\kappa\varepsilon_0 A}{d}$, it follows immediately that $C_2 = \kappa C_1$. (*b*) Since the capacitors carry identical charge, we can equate $q_1 = V_{\text{cap 1}} C_1 = q_2 = V_{\text{cap 2}} C_2$. Inserting the results of part (*a*), we find. $V_1 = \kappa V_2$. (*c*) We know the electrostatic potential energy stored in a capacitor can be written $U^E = \dfrac{1}{2} q V_{\text{cap}}$. Since both capacitors carry the same charge, we can say $\dfrac{U_2^E}{U_1^E} = \dfrac{\frac{1}{2} q_2 V_{\text{cap 2}}}{\frac{1}{2} q_1 V_{\text{cap 1}}} = \dfrac{V_{\text{cap 2}}}{V_{\text{cap 1}}} = \dfrac{1}{\kappa}$ such that $U_2^E = \dfrac{U_1^E}{\kappa}$. (*d*) The electric field between parallel plates can be written $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{q}{A\varepsilon_0}$. So $\dfrac{E_1}{E_2} = \dfrac{q_1}{q_2} = \kappa$ or $E_1 = \kappa E_2$. (*e*) We know $U^E = \dfrac{1}{2}\dfrac{q^2}{C}$ so we can write $u_E = \dfrac{U^E}{V} = \dfrac{q^2}{2CAd} = \dfrac{q^2}{2\kappa\varepsilon_0 A^2}$. Thus $u_2^E = \dfrac{u_1^E}{\kappa}$.

26.59. (*a*) From equation 26.17, we know $\dfrac{q_{\text{free}}}{\kappa} = q_{\text{free}} - q_{\text{bound}}$ or $\kappa = \dfrac{q_{\text{free}}}{q_{\text{free}} - q_{\text{bound}}} = \dfrac{q_{\text{free}}}{q_{\text{free}} - \frac{1}{4} q_{\text{free}}} = \dfrac{4}{3}$. (*b*) If the capacitor is isolated, then there is no change to the bound charge. If the capacitor remains connected to a battery, then the potential difference between plates is kept fixed but the charge can change. We can write the potential difference with dielectric filling the region between plates as $V_{\text{filled}} = Ed = \frac{E_{\text{free}}}{\kappa} d = \frac{q_{\text{filled}} d}{\kappa A \varepsilon_0}$. When the dielectric only fills have the region, the electric field will be difference in the dielectric and in the empty space, meaning $V_{\text{half}} = \frac{E_{\text{free}}}{\kappa}\left(\frac{d}{2}\right) + E_{\text{free}}\left(\frac{d}{2}\right) = \frac{qd}{2A\varepsilon_0}\left(\frac{1}{\kappa}+1\right)$. Equating the potential difference in each case yields $q = \frac{2q_{\text{filled}}}{(1+\kappa)} = \frac{2q_{\text{filled}}}{\left(1+\frac{4}{3}\right)} = \frac{6}{7} q_{\text{filled}}$. Thus, if the plates remain connected to a battery while the dielectric is changed to occupy only half the space, the bound charge is reduced to $6/7$ its initial value.

26.60. (*a*) We require that the electric field in the capacitor $E = \frac{V}{d}$ be less than the breakdown threshold of mica (which we obtain from Table 26.1): $\frac{V}{d} \le E_{\max} \Rightarrow d \ge \frac{V}{E_{\max}} = \frac{(6000\text{ V})}{(2\times 10^{8}\text{ V/m})} = 3\times 10^{-5}$ m. (*b*) We require that the electric field in the capacitor $E = \frac{V}{d}$ be less than the breakdown threshold of barium titanate (which we obtain from Table 26.1): $\frac{V}{d} \le E_{\max} \Rightarrow d \ge \frac{V}{E_{\max}} = \frac{(6000\text{ V})}{(8\times 10^{7}\text{ V/m})} = 8\times 10^{-5}$ m. (*c*) We know $q_{\max} = CV_{\max}$, and we already ensured that the maximum potential difference across the capacitors is 6000 V by making the plate separation as small as possible in parts (*a*) and (*b*). Thus $\frac{q_{\text{bt,max}}}{q_{\text{m,max}}} = \frac{C_{\text{bt}} V_{\max}}{C_{\text{m}} V_{\max}} = \frac{(\kappa_{\text{bt}} \varepsilon_0 A V_{\max}/d_{\text{bt,min}})}{(\kappa_{\text{m}} \varepsilon_0 A V_{\max}/d_{\text{m,min}})} = \frac{\kappa_{\text{bt}} d_{\text{m,min}}}{\kappa_{\text{m}} d_{\text{bt,min}}} = \frac{(1200)(3\times 10^{-5}\text{ m})}{(5)(7.5\times 10^{-5}\text{ m})} = 1\times 10^{2}.$

26.61. We know the electric field is $E = \frac{V}{d}$. From the data on the empty capacitor, we see $\frac{q}{V} = C = \frac{\varepsilon_0 A}{d} \Rightarrow d = \frac{\varepsilon_0 A V}{q} = \frac{(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(50\times 10^{-6}\text{ m}^2)(1\text{ V})}{(5.5\times 10^{-12}\text{ C})} = 8.05\times 10^{-5}$ m. The addition of barium titanate certainly affects capacitance and the charge stored on the plates, but it does not affect the potential difference across the plates (as the battery is still connected) and it does not change the plate separation. Thus $E = \frac{V}{d} = \frac{(9.0\text{ V})}{(8.05\times 10^{-5}\text{ m})} = 1.1\times 10^{5}\text{ V/m}.$

26.62. One can write the charge stored on a parallel plate capacitor as $Q = \Delta VC = \frac{\Delta V \kappa A \varepsilon_0}{d} = E\kappa A\varepsilon_0$. So for a given plate area, one maximizes the charge held by maximizing the product of the electric field times the dielectric constant. Since we can produce almost any potential difference, we are limited only by the breakdown threshold of each material. Thus, we rank materials according to the product $E_{\text{breakdown}}\kappa$. Since $E_{\text{breakdown, paper}}\kappa_{\text{paper}} = (4.0\times 10^{7}\text{ V/m})(3.0) = 1.2\times 10^{8}\text{ V/m}$, $E_{\text{breakdown, mylar}}\kappa_{\text{mylar}} = (4.3\times 10^{8}\text{ V/m})(3.3) = 1.4\times 10^{9}\text{ V/m}$, $E_{\text{breakdown, quartz}}\kappa_{\text{quartz}} = (8\times 10^{6}\text{ V/m})(4.3) = 3\times 10^{7}\text{ V/m}$, and $E_{\text{breakdown, mica}}\kappa_{\text{mica}} = (2\times 10^{8}\text{ V/m})(5) = 1\times 10^{9}\text{ V/m}$, this yields a ranking of Mylar, mica, paper, quartz.

26.63. The only thing doing work on the system is the battery, so we can equate the work done by the battery to the change in electrostatic potential energy of the system: $W_{\text{batt}} = U_{\text{f}}^{E} - U_{\text{i}}^{E} = \frac{1}{2}C_{\text{f}}V_{\text{batt}}^2 - \frac{1}{2}C_{\text{i}}V_{\text{batt}}^2 = \left(\frac{V_{\text{batt}}^2}{2}\right)(\kappa_{\text{oil}}-1)\left(\frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}\right)$. Solving for the potential difference provided by the battery, we find

$$V_{\text{batt}} = \sqrt{\frac{2W_{\text{batt}}}{(\kappa_{\text{oil}}-1)}\frac{(R_2-R_1)}{(4\pi\varepsilon_0 R_1 R_2)}} = \sqrt{\frac{2(8.90\times10^{-9}\text{ J})}{(4.5-1)}\frac{\left((8.50\times10^{-3}\text{ m})-(8.00\times10^{-3}\text{ m})\right)}{(4\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(8.00\times10^{-3}\text{ m})(8.50\times10^{-3}\text{ m})}} = 18\text{ V}.$$

26.64. (*a*) From equation 26.17 we know that $\frac{q_{\text{free}}}{\kappa} = q_{\text{free}} - q_{\text{bound}} \Rightarrow \kappa = \frac{q_{\text{free}}}{q_{\text{free}} - q_{\text{bound}}} = \frac{q_{\text{free}}}{q_{\text{free}} - 0.75q_{\text{free}}} = 4.$ (*b*) Because this is a parallel plate capacitor, we can write $\frac{\kappa\varepsilon_0 A}{d} = C = 480\varepsilon_0\ell \Rightarrow \frac{A}{d} = 120\ell.$ We also know that $q_{\max} = CV_{\max} = \frac{\kappa\varepsilon_0 A}{d}V_{\max} = \kappa\varepsilon_0 AE_{\max}$, and from the given information, we equate $\kappa\varepsilon_0 AE_{\max} = q_{\max} = 240\ell^2\varepsilon_0 E_{\max} \Rightarrow$ $A = 60\ell^2$. Combing this with the above result we find $\frac{A}{d} = \frac{60\ell^2}{d} = 120\ell \Rightarrow d = \frac{\ell}{2}.$ (*c*) As shown in part (*b*) $A = 60\ell^2$.

26.65. The electric field that one wire feels from the other wire in the distilled water will be $E = \frac{E_{\text{empty}}}{\kappa_{\text{distilled water}}}$. Thus the force felt by one wire due to the other will be $F = qE = \frac{qE_{\text{empty}}}{\kappa_{\text{distilled water}}} = \frac{F}{\kappa_{\text{distilled water}}} = \frac{F}{80.2}.$

26.66. (*a*) We know the electric field outside the shell is $\vec{E}_{\text{out}} = \frac{kq}{r^2}\hat{r}$. We integrate this to find the potential at the outer surface of the shell, and find $V_{\text{outer}} = \frac{kq}{2R}$. The electric field inside the spherical shell will be $\vec{E}_{\text{shell}} = \frac{kq}{\kappa r^2}\hat{r}$, and we integrate this to find the difference between the outer and inner surface of the shell: $V_{\text{in}} - V_{\text{out}} = \int_R^{2R}\frac{kq}{\kappa r^2}dr =$ $\frac{kq}{\kappa R} - \frac{kq}{2\kappa R} = \frac{kq}{2\kappa R}$. So, the potential at the inner surface of the shell is $V_{\text{in}} = \frac{kq}{2R}\left(1+\frac{1}{\kappa}\right) = \frac{q}{(6.0)\pi\varepsilon_0 R}$. Since there is no potential difference within the conducting sphere at the center, this is also the potential at the center of the sphere. (*b*) The potential would decrease.

26.67. (*a*) We know the electric field outside the shell is $\vec{E}_{\text{out}} = \frac{kq}{r^2}\hat{r}$. We integrate this to find the potential at the outer surface of the shell, and find $V_{\text{outer}} = \frac{kq}{2R}$. The electric field inside the spherical shell will be $\vec{E}_{\text{shell}} = \frac{kq}{\kappa r^2}\hat{r}$, and we integrate this to find the difference between the outer and inner surface of the shell: $V_{\text{in}} - V_{\text{out}} = \int_R^{2R}\frac{kq}{\kappa r^2}dr =$ $\frac{kq}{\kappa R} - \frac{kq}{2\kappa R} = \frac{kq}{2\kappa R}$. So, the potential at the inner surface of the shell is $V_{\text{in}} = \frac{kq}{2R}\left(1+\frac{1}{\kappa}\right) = \frac{q}{8\pi\varepsilon_0 R}\left(1+\frac{1}{\kappa}\right)$. We now equate our expressions for the electric field in the spherical shell and the potential on the inner shell surface to the expressions given in the problem:

$$\frac{q}{4\pi\varepsilon_0\kappa r^2} = \frac{3Q}{4\pi\varepsilon_0 r^2} \tag{1}$$

$$\frac{q}{8\pi\varepsilon_0 R}\left(1+\frac{1}{\kappa}\right) = \frac{15Q}{16\pi\varepsilon_0 R} \tag{2}$$

Solving equation (1) for the charge on the sphere we obtain

$$q = 3\kappa Q \tag{3}$$

Inserting this into equation (2), we find $q = \frac{9}{2}Q.$ (*b*) Inserting the charge found in part (*a*) into equation (3), we find $\kappa = \frac{3}{2}.$

26.68. The potential difference between the inner and outer surfaces can be written $\Delta V = \int_R^{R+d} \vec{E} \cdot dr = \int_R^{R+d} \frac{kq}{\kappa r^2} dr = \frac{kq}{\kappa}\left(\frac{1}{R} - \frac{1}{R+d}\right).$ Rearranging, we obtain

$$d = \left(\frac{1}{R} - \frac{\kappa \Delta V}{kq}\right)^{-1} - R = \left(\frac{1}{(2.25\text{ m})} - \frac{4\pi(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1200)(20{,}000\text{ V})}{(35.0\times10^{-3}\text{ C})}\right)^{-1} - (2.25\text{ m}) = 0.466\text{ m}$$

26.69. (*a*) This model shows that increasing the separation distance between charged objects increases the elastic potential energy stored in the electric field between the objects, just as stretching an elastic band increases the elastic potential energy stored in the band. (*b*) This analogy suggests that electric field lines run from a positively charged object to a negatively charged object along the shortest possible path (so as to minimize energy), which is not true.

26.70. In a capacitor made of two nonconducting objects, it is difficult to arrange the objects in such a way that charge carriers are uniformly distributed over the surfaces. In a capacitor made of two conducting objects, charge carriers easily spread out uniformly over the surfaces. Also, if positive charge is added to a positively charged object, charge carriers on the negatively charged object can shift closer to the positively charged object, lowering the energy and allowing more charge to remain on the capacitor. This interaction is not efficient when the capacitor is made of a nonconducting material.

26.71. (*a*) We know $V = \frac{q}{C} = \frac{qd}{\epsilon_0 A}.$ So for capacitors 1 and 2, $\frac{V_1}{V_2} = \frac{\frac{q_1 d_1}{\epsilon_0 A_1}}{\frac{q_2 d_2}{\epsilon_0 A_2}} = \left(\frac{q_1}{q_2}\right)\left(\frac{A_2}{A_1}\right) = (2)\left(\frac{1}{2}\right) = 1,$ so $V_1 = V_2.$

(*b*) Starting again from $V = \frac{q}{C} = \frac{qd}{\epsilon_0 A} \Rightarrow q = \frac{\epsilon_0 A V}{d},$ we can write $\frac{q_1}{q_2} = \frac{\left(\frac{\epsilon_0 A_1 V_1}{d_1}\right)}{\left(\frac{\epsilon_0 A_2 V_2}{d_2}\right)} = \left(\frac{A_1}{A_2}\right)\left(\frac{V_1}{V_2}\right) = (2)(2) = 4,$ so $q_1 = 4q_2.$

26.72. (*a*) The capacitance is $C = \frac{q}{V} = \frac{(10\text{ C})}{(3\times10^6\text{ V})} = 3\times10^{-8}\text{ F}$ which is of order 1×10^{-7} F. (*b*) The energy is given by $U^E = \frac{1}{2}qV = \frac{1}{2}(10\text{ C})(3\times10^8\text{ V}) = 2\times10^9\text{ J}$ which is of order 1×10^9 J. (*c*) We take $\frac{U^E_{\text{lightning}}}{U^{\text{chem}}_{\text{1 L gasoline}}} = \frac{(1.5\times10^9\text{ J})}{(36\times10^6\text{ J})} = 4\times10^1\text{ L},$ which is of order 1×10^2 L.

26.73. When positively and negatively charged particles are separated, they are subject to an attractive force and, so, are not in mechanical equilibrium. The only stable mechanical equilibrium occurs when the positively and negatively charged particles are as close to each other as possible.

26.74. (*a*) There will now be a charge on each plate with magnitude $2q$. One way to see this is to note that the capacitance doubled while the potential difference remained fixed, and $q = CV$. (*b*) More force is required after the plates are expanded. The plates now hold more charge, and there are more electric field lines between the plates. In the rubber band model, this corresponds to more rubber bands being stretched across the capacitor. Clearly, it takes more force to stretch more rubber bands.

26.75. We know the electric field due to a single large plate is $E_{1\text{ plate}} = \dfrac{\sigma}{2\varepsilon_0}$. The other plate will feel an attractive force equal to $qE_{1\text{ plate}} = \dfrac{q\sigma}{2\varepsilon_0} = \dfrac{q^2}{2A\varepsilon_0}$.

26.76. (*a*) With the battery connected, we have $q = CV = \kappa C_{\text{empty}}V = (80.2)(30.5\times10^{-6}\text{ F})(24.0\text{ V}) = 5.87\times10^{-2}\text{ C}$.
(*b*) With the battery disconnected, we have $q = CV = C_{\text{empty}}V = (30.5\times10^{-6}\text{ F})(24.0\text{ V}) = 7.32\times10^{-4}\text{ C}$.

26.77. If you insert the slabs of metal with thicknesses $d/2$, $d/4$, and $d/5$, then 95% of the gap between the plates will have been filled with a conductor. Since the electric field in the conducting plates is zero, the entire potential difference must now occur over the remaining 5% of the gap. Hence the electric field will increase by a factor of 20.

26.78. The highest power is achieved by maximizing the rate at which chemical reactions take place at the electrodes. This reaction rate is maximized by using a highly concentrated electrolyte solution and/or by increasing the electrode surface area. Given a fixed amount of metal, rods yield a relatively small surface area for reaction. Electrodes in the form of thin sheets have much larger surface area for a given amount of metal. This large-surface-area arrangement helps ensure that some large portion of each electrode is always available for chemical reaction.

26.79. Fold up a long thin sandwich of two 0.5-μm thick conducting sheets separated by a 0.5-μm thick dielectric into smaller and smaller squares to form cube of 10-mm edge length. With each capacitor—positive plate, negative plate, dielectric—$1.5\mu m$ thick, this folding forms 6,666 layers of capacitor (ignoring any compression of materials at the folds). This corresponds to a plate area of 0.6666 m^2 and a capacitance of $C = \dfrac{\kappa\varepsilon_0 A}{d} =$

$$\frac{(1000)(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.6666\text{ m}^2)}{(0.5\times10^{-6})} = 12\text{ mF.}$$

In this folded capacitor, each conducting surface (that is, capacitor plate) is folded over on itself and thus twice as thick as it needs to be. This doubling can be avoided by machining conducting "teeth" on the plates such that the teeth of adjacent plates interpenetrate, an architecture called an *interdigitated comb*. Such a design could increase capacitance to order of 20 mF.

27

MAGNETIC INTERACTIONS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

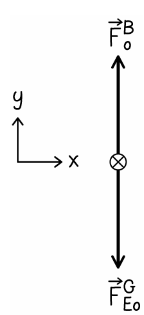
1. 10^2 A **2.** 10^{-4} Wb **3.** 10^{-18} N upward **4.** 10^{-26} T **5.** 10^{-1} T **6.** 10^{-4} T **7.** 10^{-1} A **8.** 10^2 m/s^2

Guided Problems

27.2 Magnetic scale

1. Getting Started A current flows horizontally in the rod. In the presence of a magnetic field, the current-carrying rod will feel a force. If the direction of the field is right, the magnetic force could be made to counteract the gravitational force that Earth exerts downward on the block and rod. We are given the direction of the current flow and the direction of the gravitational force. We must determine the direction of the magnetic field. We are also given the mass of the hanging object and rod together, and the strength of the magnetic field. We must determine the current required.

We begin by making a free-body diagram of the rod and hanging object together. Because the hanging object and the rod are joined together, we treat them as one extended object (denoted by the subscript "o").



Because we are tuning the current such that the rod and hanging object are suspended, we know the acceleration in the vertical direction is zero. Thus the sum of all forces in the y direction must be zero, meaning the magnetic force must exactly cancel out the gravitational force. We can write the magnetic force exerted on the rod in terms of the current I, the length of the rod ℓ, and the magnetic field B, using information from Chapter 27.

2. Devise Plan In order to determine the direction of the magnetic field, we employ the right hand rule introduced in Chapter 27. That is, we orient our right hand such that the outstretched fingers point in the direction of the current in the rod. We rotate the hand until the thumb points in the direction that the magnetic force must be acting. Then the fingers curl (or equivalently, the palm faces) the direction of the magnetic field. We know the direction of the current and the magnetic force, so applying this right hand rule should be straightforward.

We can relate the unknown current to other variables using equation (27.4). Equation (27.7) is a more general equation that allows for the possibility that the magnetic field and the current are not perpendicular to each other, but form an angle θ between their vectors. In this case, we are told to consider only the possibilities that the field is directed into the page or out of the page, which makes the field and current perpendicular. In this case we have $F_{\text{o}}^{B} = |I|\ell B$.

We also know from the sum of all forces in the vertical direction that

$$\sum F_y = F_{\text{o}}^{B} - F_{\text{Eo}}^{G} = ma_y = 0$$
$$F_{\text{o}}^{B} = F_{\text{Eo}}^{G} \tag{1}$$

We can clearly combine equations (27.4) and (1) to determine the current in terms of other variables.

3. Execute Plan Orienting our right hand as described above, our fingers point to the right (in the $+x$ direction), our thumb points upward to the top of the page (the $+y$ direction), such that the fingers curl to point into the page. Thus the magnetic field must be directed into the page in order for the magnetic force to point upward and counteract gravity.

Inserting equation (27.4) into equation (1) and inserting the known form of gravitational force on Earth, we obtain

$$|I|\ell B = mg$$
$$I = \frac{mg}{\ell B} = \frac{(0.157\text{ kg})(9.80\text{ m/s}^2)}{(0.100\text{ m})(0.150\text{ T})} = 103\text{ A}$$

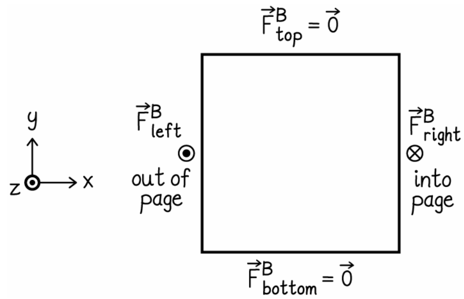
4. Evaluate Result It is often difficult for students to accept the fact that magnetic forces are perpendicular to the magnetic field and current, but it is explained throughout Chapter 27. So, yes, the direction of the magnetic field does makes sense.

The current required is very large. The maximum current that can pass through most household circuit breakers is 10 A to 20 A. Large appliances such as electric stoves may be put on a circuit breaker with a maximum current of 50 A. A common maximum current coming into an entire house in 100 A.

27.4 Torque on a current loop

1. Getting Started A wire that carries a current will experience a force when placed in a magnetic field. In this case we have four current-carrying wires, all carrying current in different directions. Thus, we expect magnetic forces might be exerted in as many as four different directions. We want to determine whether these forces result in overall force or torque on the system of four wires.

We begin by drawing a free-body diagram. Because we are interested in both torque and force, we draw the extended object, rather than treating all forces as acting at the center of mass. We apply the right hand rule to each wire to determine the direction of the force on each segment of wire. For the left and right vertical wires we find a force out of the page, and into the page, respectively. When attempting to apply the right hand rule to the top and bottom wire, you may find it difficult to maneuver your hand into the correct orientation. Indeed, the current and magnetic fields lie along the same axis, and there is no way to make your fingers and palm point in the same direction. The resolution of this contradiction is that the force on the top and bottom wires is zero. That can also be seen from equation (27.7). Thus we obtain



2. Devise Plan It does appear that the loop experiences a torque around an axis parallel to y and passing through the center of the loop. If we choose the axis of rotation be $+y$, the torque around this axis is also positive (counterclockwise when viewed from the a point along the $+y$ axis).

To find the vector sum of all forces and torques, we need the form of each force and each torque. Each of the two non-zero forces will have the form given by equation (27.7) with $\theta = 90°$: $F^B_w = |I|\ell B$. The direction of the above force has already been determined by the right hand rule.

Each torque is given by $\vec{\tau} = \vec{r} \times \vec{F}^B_w$. Note that the cross product yields a torque in the $+y$ direction for both of the non-zero forces. The magnitude of the torque is (in general) $\tau = rF^B_w \sin(\phi) = (\ell/2)|I|\ell B \sin(\phi)$ where ϕ is the angle between the force and the radial vector. However, in this case the radial vector points either left or right depending on the force and the forces are into and out of the page. So the angle ϕ is always $90°$, and the magnitude of either torque is

$$\tau = |I|\ell^2 B/2 \tag{1}$$

The vector sum of forces and torques are found in the usual way. In this case, all non-zero forces lie along the z axis, and all non-zero torques lie along the y axis. Thus we will calculate

$$\sum F_z = F^B_{\text{left},z} + F^B_{\text{right},z} \tag{2}$$

$$\sum \tau_y = \tau_{\text{left},y} + \tau_{\text{right},y} \tag{3}$$

3. Execute Plan Inserting equation (27.7) into equation (2), we note that the z components of the two forces have opposite signs, and we find

$$\sum F_z = |I|\ell B - |I|\ell B = 0$$

Since there are no forces in any other direction, the vector sum of all forces on the loop is zero.

Now inserting equation (1) into equation (3), and noting that the y components of the torques are both positive, we find

$$\sum \tau_y = |I|\ell^2 B/2 + |I|\ell^2 B/2 = |I|\ell^2 B$$

$$\sum \tau_y = (1.7\text{ A})(0.010\text{ m})^2(0.23\text{ T})$$

$$\sum \tau_y = 3.9\times 10^{-5}\text{ N}\cdot\text{m}$$

So the vector sum of all torques is $\sum \vec{\tau} = (3.9\times 10^{-5}\text{ N}\cdot\text{m})\,\hat{j}$.

4. Evaluate Result Since the loop size and the current are relatively small, it is reasonable to obtain a small answer for the torque, as we did. Because the loop has rotational symmetry about the z axis, we know there can be no force in the xy plane. Otherwise, we could rotate the system by $90°$ about the z axis and change the direction of the force without changing anything about the loop, which would be nonsense. So the only possibility is for forces to be the positive or negative z direction. We say that this was the case, but symmetry caused those two forces to cancel.

So it is very reasonable to find that the vector sum of all forces on the loop is zero. The torque direction is also what we concluded in making the free-body diagram.

27.6 Fusion energy

1. Getting Started When charged particles move in a uniform magnetic field, their trajectories will curve because of the magnetic force they experience. If no other force interferes, the charged particles will move in circular paths. The radius of these circular paths depends on many variables like the particle mass m, the magnetic field B, and the speed of the particles v. The radius also depends, in general, on the relative orientations of the magnetic field and the velocity. But here, we cannot consider all possible velocity orientations. A charge particle moving parallel to the magnetic field will experience no force and will not move in a circle. In order to achieve "containment" we must assume that the particles are not parallel to the magnetic field, and we will make the further simplification that the particles are moving perpendicular to the magnetic field.

Because protons are much more massive than electrons, we expect them to be harder to accelerate. Therefore we anticipate that the more difficult containment to achieve will be the containment of the protons.

2. Devise Plan The fact that we are looking for the minimum radius of the containment vessel implies a few things. First, if particles can move parallel to the magnetic field, then they will not move in a circle, and no finite radius would be sufficient for the particles to remain contained. If the particles have a component of their velocity parallel to the magnetic field then they will move in a helix, and eventually escape. The notion that we can contain the particles at all implies that we are considering only particles moving perpendicular to the magnetic field. Also, since we are considering the minimum radius, we set it equal to the larger of the two radii of the circular motion of the protons and electrons. We expect this larger radius to be the radius of the proton's motion, but we calculate both radii to be sure.

The radius of curvature of a charged particle in a magnetic field is given by equation (27.23):

$$R = \frac{mv}{|q|B}$$

All quantities on the right hand side of equation (27.23) are given in the problem statement or are known constants. Thus we can determine the radius of circular motion for either particle with relative ease. We note also that all quantities on the right hand side of equation (27.23) are the same for both particles, with the exception of the particle mass.

The orbital period is given by equation (27.24):

$$T = \frac{2\pi m}{|q|B}$$

Again, all quantities on the right hand side are known, so we can proceed inserting numbers.

3. Execute Plan Inserting the given numbers into equation (27.23) for both electrons and protons, we find

$$R_\mathrm{p} = \frac{m_\mathrm{p} v}{|q|B} = \frac{(1.67\times10^{-27}\ \mathrm{kg})(2.0\times10^{7}\ \mathrm{m/s})}{(1.60\times10^{-19}\ \mathrm{C})(8.0\ \mathrm{T})} = 0.026\ \mathrm{m}$$

$$R_\mathrm{p} = \frac{m_\mathrm{e} v}{|q|B} = \frac{(9.11\times10^{-31}\ \mathrm{kg})(2.0\times10^{7}\ \mathrm{m/s})}{(1.60\times10^{-19}\ \mathrm{C})(8.0\ \mathrm{T})} = 1.4\times10^{-5}\ \mathrm{m}$$

As we expected, the larger of the two radii is the radius required for the proton. Thus for all charge to remain contained, the chamber must have a radius of at least $26\ \mathrm{mm}$.

The period of the orbit will also be different for the two particle types, because equation (27.24) also depends on the mass of the particles. Inserting numerical values, we find

$$T_\mathrm{p} = \frac{2\pi m_\mathrm{p}}{|q|B} = \frac{2\pi(1.67\times10^{-27}\ \mathrm{kg})}{(1.60\times10^{-19}\ \mathrm{C})(8.0\ \mathrm{T})} = 8.2\times10^{-9}\ \mathrm{s}$$

$$T_\mathrm{e} = \frac{2\pi m_\mathrm{e}}{|q|B} = \frac{2\pi(9.11\times10^{-31}\ \mathrm{kg})}{(1.60\times10^{-19}\ \mathrm{C})(8.0\ \mathrm{T})} = 4.5\times10^{-12}\ \mathrm{s}$$

4. Evaluate Result The required radius of 26 mm would easily fit inside a laboratory device.

For a particle moving in a circular orbit, we know the speed should be given by the expression $v = d/t = 2\pi R/T$. We can insert the values we obtained for the radii and periods to check that the speed is consistent with the problem statement. We have

$$v_\mathrm{p} = \frac{2\pi R_\mathrm{p}}{T_\mathrm{p}} = \frac{2\pi(0.026\ \mathrm{m})}{(8.2\times10^{-9}\ \mathrm{s})} = 2.0\times10^{7}\ \mathrm{m/s}$$

$$v_\mathrm{e} = \frac{2\pi R_\mathrm{e}}{T_\mathrm{e}} = \frac{2\pi(1.4\times10^{-5}\ \mathrm{m})}{(4.5\times10^{-12}\ \mathrm{s})} = 2.0\times10^{7}\ \mathrm{m/s}$$

This the given velocity for all particles in the device, so we do not appear to have made any calculation errors.

The radii depend linear on the mass of the particle. Thus the ratio of the radii should be the same as the ratio of the masses of the two particles. In other words, we expect

$$\frac{R_\mathrm{p}}{R_\mathrm{e}} = \frac{m_\mathrm{p}}{m_\mathrm{e}}$$

$$\frac{(0.026\ \mathrm{m})}{(1.4\times10^{-5}\ \mathrm{m})} = \frac{(1.7\times10^{-27}\ \mathrm{kg})}{(9.1\times10^{-31}\ \mathrm{kg})}$$

$$1.9\times10^{3} = 1.9\times10^{3}$$

In the calculation, we have used only two significant digits for the masses of the elementary particles, simply so that we compare the two ratios to equal precision.

27.8 Half speed

1. Getting Started This problem is actually quite a bit simpler than Worked Problem 27.7. In this problem we are only asked for information in one reference frame: the Earth reference frame. The point is emphasized in Worked Problem 27.7, that the linear charge density of positive ions and negative electrons are equal in magnitude. This means there is no net linear charge density on the wire (the wire is neutral). We can also use the same expression and numerical result for the speed of electrons in the wire, if it is needed.

Many of the equations listed in section 27.8 are specific to the situation in which the proton and the electrons are moving with the same average speed. In order to answer the question for an observer stationary with respect to the wire, we must think back to previous sections/chapters.

The electric force the wire exerts on the proton is given by $F^{E}_{\mathrm{E,wp}} = qE_\mathrm{E}$. The magnetic force is given by $F^{B}_{\mathrm{E,wp}} = qvB_\mathrm{E}$.

2. Devise Plan The fields will actually be given by the same old familiar equations that came prior to section 27.8. Namely, the electric field is given by

$$E_\mathrm{E} = \frac{2k\lambda_\mathrm{E}}{r}$$

and the magnetic field is given by

$$B_{\mathrm{E}} = \frac{\mu_0 I}{2\pi r}$$

The reason is that we are asked for the electric and magnetic fields in the reference frame of someone at rest relative to the wire. Since the observer is not moving relative to the wire, the same equations we used before consider the relativistic motion of the observer must still apply. However, once we have determined the answers, we should ensure that they agree with results found in (27.8) in the special case where the speed of the observer $v \to 0$.

The important simplification that we are making is that the observer is stationary relative to the wire, which is equivalent to a power of v approaching zero in the equations of section 27.8.

3. Execute Plan We are told that the linear charge density is zero in the rest frame of the wire. Since that is the same reference frame as our observers, the linear charge density of the wire is zero. This means the electric field is zero, and the electric force is zero:

$$F_{\mathrm{E,wp}}^{E} = |q|E_{\mathrm{E}} = \frac{2|q|k\lambda_{\mathrm{E}}}{r} = \frac{2|q|k(0)}{r} = 0$$

This agrees with the expression from section 27.8. Note that in the derivation of the electric field seen by a relativistic observer, one obtains the linear charge density seen by the moving observer according to equation (27.31):

$$\lambda_{\mathrm{M}} = \gamma\lambda_{\text{proper}} - \frac{\lambda_{\text{proper}}}{\gamma}$$

In this case where the observer has speed $v_{\text{obs}} = 0,$ we have $\gamma = 1,$ such that $\lambda_{\mathrm{M}} = (1)\lambda_{\text{proper}} - \dfrac{\lambda_{\text{proper}}}{(1)} = 0.$ It is important to note that the speed that was used in the Lorentz factor above is the speed of the observer relative to the wire, and not (necessarily) the speed of the proton. Thus, using this charge density in equation (27.32)

$$E_{\mathrm{E}} = \frac{2k\lambda\gamma v^2}{rc_0^2}$$

shows us that this electric field is zero, and that the electric force is also zero.

The magnetic force is given by

$$F_{\mathrm{E,wp}}^{B} = |q|v_{\mathrm{p}}B = \frac{|q|v_{\mathrm{p}}\mu_0 I}{2\pi r} = 2k|q|v_{\mathrm{p}}I\mu_0\varepsilon_0 = \frac{2k|q|v_{\mathrm{p}}I}{c_0^2}$$

Note that this is the same expression as equation (27.37). Note also that the speed in the above expression is the speed of the proton, and not (necessarily) the speed of the observer. Inserting numerical values, we find

$$F_{\mathrm{E,wp}}^{B} = \frac{2k|q|v_{\mathrm{p}}I}{c_0^2} = \frac{2(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\ \mathrm{C})(1.56\times10^{-3}\ \mathrm{m/s})(5.0\ \mathrm{A})}{(3.0\times10^8\ \mathrm{m/s})^2} = 1.3\times10^{-18}\ \mathrm{N}$$

4. Evaluate Result The electric force we obtained is zero, even though the force found in Worked Problem 27.7 was non-zero. The reason is that the observer was moving relative to the wire in Worked Problem 27.7. Thus for that moving observer, the linear charge densities of the ions and electrons could be different than for an observer stationary relative to the wire. Without that motion of the observer relative to the wire, the wire is simply neutral and cannot exert any electric force. Thus our answer is quite reasonable.

The magnetic force we found is many orders of magnitude larger than the electric force found in Worked Problem 27.7. We must keep in mind that we are now comparing two different types of forces. The force we obtained here is the magnetic force that exists even when the observer is not moving relative to the wire. The force determined in Worked Problem 27.7 was the electric force which only exists due to differences in the length contraction applicable to the ions and electrons. Because the electrons' motion was so small (on the order of millimeters per second) the

Lorentz factor was also very small. It is perfectly reasonable that the electric force resulting front that would be much smaller than the magnetic force we calculated here.

Questions and Problems

27.1. The north pole of a compass needle points north, toward Earth's North Pole, even in the Southern Hemisphere.

27.2. By definition, the pole of a freely suspended magnet that points toward the north is a north magnetic pole. Because the north pole of a freely suspended magnet points away from Earth's geographic South Pole, it must be repelled by the magnetic pole located there. So, because like poles repel, a north magnetic pole must be located near Earth's geographic South Pole.

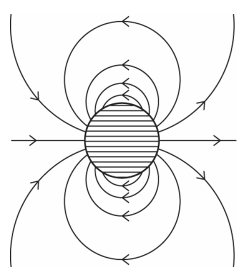
27.3. Because the paper clips are attracted to the flat faces of the magnet, those must be its poles. So, the elementary magnets are all oriented with one type of pole toward the flat face on the left and the other type of pole toward the flat face on the right.

27.4. Yes, it can. For example, an x-shaped piece of iron could have two north and two south poles.

27.5. Either end of the bar that is not magnetized will be attracted to either end of a magnet, and either end of a magnet will be attracted to either end of the unmagnetized bar. But one end of a magnet will be a attracted to one end of another magnet and repelled by the other end. So, choose two bars, and see if one end of the first bar is repelled by either end of the second bar. If so, these two are the magnets. If not, the third bar must be a magnet. Put down the first bar and repeat the experiment with the second and third bars. If one of the interactions is repulsive, these two are the magnets. If not, the first and third bars are the magnets.

27.6. If you place one end of the magnet near the midpoint of the other rod, there will be a strong attraction between them. If you place one end of the unmagnetized rod near the midpoint of the magnet, there will be very little attraction between them.

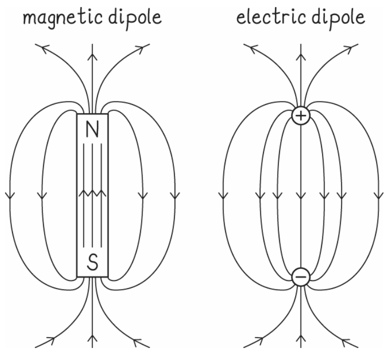
27.7. We can determine the poles of the magnetized sphere by sketching the elementary magnets it contains. One hemisphere will be a north pole and the other hemisphere a south pole, with the strength greatest at the points furthest from the dividing line between the two hemispheres.



27.8. Figures b and d could represent magnetic fields because the lines in both form closed loops, as magnetic field lines do. In Figure a, all the lines meet at a point, and in Figure c, the lines intersect, which magnetic field lines cannot do.

27.9. (*a*) The magnetic field line flux through a closed surface is always zero. (*b*) If the flux through one hemisphere is positive, then the flux through the other hemisphere must be negative. (*c*) The flux is still zero. The negative flux through the part of the spherical surface around the end of the magnet is offset by the positive flux through the part of the surface that intersects the magnet. (*d*) The flux is still zero.

27.10. They are similar far from the dipole. Near the source of the fields, however, the electric dipole field lines start and end on the charges making up the dipole, while the magnetic dipole field lines form closed loops.



27.11. Magnetic field lines point away from north poles and towards south poles, and the magnitude of the magnetic field is proportional to the field line density. So, the magnet will experience a counterclockwise torque that tends to align it with the field lines, with the north pole of the magnet being attracted toward the top right corner of the figure where the magnetic field is strongest.

27.12. Magnetic field lines point away from north poles and towards south poles, and the magnitude of the magnetic field is proportional to the field line density. So, the magnet will experience a clockwise torque that tends to align it with the field lines, with the north pole of the magnet being attracted toward the lower left corner of the figure where the magnetic field is strongest.

27.13. No, because electric field lines begin and end on the charges that create the field, while magnetic field lines always form closed loops.

27.14. The field lines are about twice as far apart at location 2 as they are at location 1, so about four times as many field lines would pass through an area near location 1 as would pass through an equal area near location 2. So the magnitude of the field at location 2 is about one-fourth the magnitude at location 1, or about 0.07 T.

27.15. There is an attractive force between the unlike poles of the two magnets and a repulsive force between their like poles, and the closer the poles the greater the force. The magnitude of the torques produced by these forces is proportional to the magnitude of the force and the lever arm distance from the axis of rotation, which will be the center of mass of the magnet. So, from smallest to largest magnitude of torque, we have $0 = a = b < e < c = d$.

27.16. (*a*) Magnetic field lines form loops around current-carrying wires, so if the field points out of the page at position P it must point into the page at position S. (*b*) Applying the right-hand current rule by curling the fingers of our right hand around the wire in the direction of the magnetic field, we find that the current is directed toward the right.

27.17. (*a*) Magnetic field lines form loops around current-carrying wires, so if the field points to the right at position P it must point to the left at position S. (*b*) Applying the right-hand current rule by curling the fingers of our right hand around the wire in the direction of the magnetic field, we find that the current is directed into the page.

27.18. (*a*) We can consider each moving particle to be a tiny current, with the direction of the current the same as the velocity for the positively charged protons and opposite the velocity for the negatively charged electron. Directly above the north pole of the magnet, the magnetic field points upward. Applying the right-hand force rule, by pointing the fingers of our right hand in the direction of the current so that they can curl towards the direction of the magnetic field, we find that the forces exerted on particles 1 and 3 are directed into the page, while there is no magnetic force exerted on particle 2. (*b*) The force exerted by the magnet on the particle is one half of an interaction pair, the other half being the force exerted by the particle on the magnet. So, the forces exerted by particles 1 and 3 on the magnet are directed out of the page, while there is no magnetic force exerted on the magnet by particle 2.

27.19. (*a*) If the wires are not connected to the battery, the rod carries no current, so there is no magnetic force exerted on it. If the wires are connected to the battery, the rod carries a current and there could be a magnetic force exerted on it. Applying the right-hand force rule, by pointing the fingers of our right hand in the direction of the current so that they can curl towards the direction of the magnetic field, we find that the magnetic force exerted on the current-carrying rod is directed upward when the direction of the current is from S to P and downward when the direction of the current is from P to S. Exerting a force on the rod in either of these directions changes the tension in the spring. (*b*) As we found in part a, the direction of the force depends on the direction of the current, which in turn depends on which wire is connected to which battery terminal. In order to increase the tension in the spring, we want the magnetic force on the rod to be directed downward, so the direction of the current should be from P to S, so we should connect the wire at P to the positive battery terminal.

27.20. (*a*) When current passes through the coil, the current in one turn is parallel to the current in the adjacent turns. As we saw in Checkpoint 27.12, that means there will be an attractive force between adjacent turns, so the coil will tend to shorten. (*b*) The current in one turn is parallel to the current in the adjacent turns regardless of the direction of the current, so the answer does not depend on the current direction.

27.21. (*a*) Applying the right-hand force rule to side 1, by pointing the thumb of our right hand in the direction of the force and seeing which direction the current must have so that our fingers can curl towards the direction of the magnetic field, we find the direction of the current is upward in side 1. That is, there is a counterclockwise current in the loop. (*b*) Applying the right-hand force rule to the other sides, we find that the magnetic force on side 2 is directed downward, that on side 3 is directed to the left, and that on side 4 is directed upward.

27.22. Each rod produces a magnetic field with the field lines encircling the rod. The current in the vertical rod is directed upward, and the current in the horizontal rod is directed toward the left. Applying the right-hand current rule, we find that the magnetic field produced by the current in the vertical rod is directed in counterclockwise circles, as seen from above, and the magnetic field produced by the current in the horizontal rod is directed in counterclockwise circles, as seen from the left. That is, at the center of either rod, the magnetic field produced by the current in the other rod is antiparallel to the current in the rod, and there is no magnetic force exerted on the central portion of the rod. Because the magnetic field lines curve, however, there is a magnetic force exerted on the ends of each rod. Applying the right-hand force rule to the left end of the horizontal rod, we see that the magnetic force on it is directed upward, while the magnetic force on the right end is directed downward. Because of the symmetry of the arrangement, the vector sum of the forces is zero, but they produce a torque that tends to make the rod rotate clockwise as seen from the front. That is, the torque on the horizontal rod tends to rotate it so that its current is parallel to the current in the vertical rod. Similarly, the magnetic forces exerted on the ends of the vertical rod produce a torque that tend to make the rod rotate counterclockwise as seen from the front. Here, too, the torque tends to rotate the rod so that its current is parallel to the current in the other rod. If the rods are free to move, they will begin to rotate, and as the currents become more nearly parallel, the torques will get smaller and an attractive force will develop between the rods.

27.23. Not in the Earth reference frame. In the Earth reference frame, particle M is moving, so it produces a magnetic field. But because particle S is at rest, there is no magnetic force exerted on S. The same is true in the reference frame moving along with particle M, because in this reference frame particle M is at rest and so does not produce a magnetic field. There are reference frames in which both particles are moving, and in these a magnetic force is exerted on S.

27.24. Observers in all of the reference frames measure an electric field because charge, the source of electric fields, is an invariant.

27.25. Suppose the particle has charge q and is located at position z in reference frame S. In reference frame S, the electric field is just the electric field of a particle that we learned about in Chapter 23: it is directed away from the particle and its magnitude decreases with the square of the distance from the particle. In particular, at the origin, the magnitude of the electric field is $E = \dfrac{kq}{z^2}$ and it is directed downward.

In reference frame S' the particle is seen to be moving in the negative x' direction. Similar to the way that length contraction makes the measured charge density of a moving charged rod greater than the measured charge density when the rod is at rest, we might reasonably (and, it turns out, correctly) guess that length contraction also causes the pattern of the electric field lines of the charged particle to be compressed along the direction of motion. In that case, the electric field would still be directed away from the particle (in particular, downward at the origin), but the magnitude of its components in the y' and z' directions would be greater that those measured in the y and z directions in reference frame S.
In reference frame S'' the particle is seen to be moving in the positive x'' direction. So, we expect to find that the electric field is still directed away from the particle (in particular, downward at the origin), but that the magnitude of its components in the y'' and z'' directions are greater that those measured in reference frame S. In reference frame S''' the particle moves in the negative y''' direction, and we expect to find that the electric field is still directed away from the particle (in particular, downward at the origin), but that the magnitude of its components in the x''' and z''' directions are greater that those measured in reference frame S.

27.26. In reference frame S the particle is at rest and we measure no magnetic field. In the other reference frames the particle is moving, and so should be surrounded by a circular magnetic field, like that surrounding a very short current-carrying wire. The direction of the magnetic field can be found from the right-hand current rule. In reference frame S' the particle is seen to be moving in the negative x' direction, so at the origin the direction of the magnetic field will be in the negative y' direction. In reference frame S'' the particle moves in the positive x'' direction, so at the origin the direction of the magnetic field will be in the positive y'' direction. In reference frame S''' the particle moves in the negative y''' direction, so at the origin the direction of the magnetic field will be in the positive x''' direction. Because the particle has the same speed in all three primed reference frames, the magnitude of the magnetic field is the same in each.

27.27. The magnitude of the magnetic force exerted on the wire is given by Eq. 27.4 $F_{\text{w}}^{B} = |I|\ell B\sin\theta.$ Rearranging to solve for B we have $B = \dfrac{F_{\text{w}}^{B}}{|I|\ell\sin\theta} = \dfrac{0.20\text{ N}}{(1.4\text{ A})(0.70\text{ m})\sin 53^\circ} = 0.26\text{ T}.$

27.28. The magnitude of the magnetic forces exerted on the wires is given by Eq. 27.4 $F_{\text{w}}^{B} = |I|\ell B\sin\theta.$ $F_{\text{w1}}^{B} = |I|\ell B\sin 90^\circ = |I|\ell B,$ $F_{\text{w2}}^{B} = |6I|(\ell/2)B\sin 0 = 0,$ $F_{\text{w3}}^{B} = |I/10|(15\ell)B\sin 90^\circ = 1.5|I|\ell B,$ and $F_{\text{w4}}^{B} = |3I/5|(2\ell)B\sin 45^\circ = 0.85|I|\ell B.$ So, the rank from smallest to largest is $F_{\text{w2}}^{B} < F_{\text{w4}}^{B} < F_{\text{w1}}^{B} < F_{\text{w3}}^{B}.$

27.29. The wire will not fall if the magnetic force is directed upward with magnitude equal to the force of gravity. By the right-hand force rule, the magnetic field will be directed upward if the current is directed from left to right. Equating the magnitudes of the magnetic and gravitational forces on the wire, we find

$$F_{\text{w}}^{B} = |I|\ell B\sin\theta$$

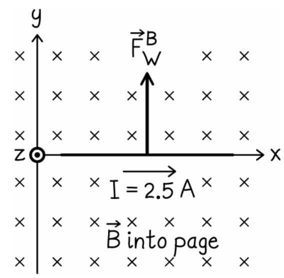
$$F_{\text{Ew}}^{G} = mg$$

$$|I|\ell B\sin\theta = mg$$

$$|I| = \frac{mg}{\ell B\sin\theta} = \frac{(4.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{(80.0\times10^{-3}\text{ m})(0.250\text{ T})} = 1.96\text{ A}$$

27.30. The magnetic force exerted on the loop equals the vector sum of the forces exerted on each portion of the loop. If the direction of the current is counterclockwise as seen from a point on the positive z axis, we can see by the right-hand force rule that at any point along the loop the magnetic force is directed radially outward. If the direction of the current is clockwise, the magnetic force is directed radially inward. Because the field is uniform and so is the current in the wire, the magnitude of the magnetic force exerted on any same-sized segment of the wire is the same at every point, and because of symmetry, the force on any segment of the wire has the opposite direction as the force on the diametrically opposed segment. So, the vector sum of the forces equals zero.

27.31. (*a*) From the definition of current (Eq. 27.1) and the ampere (Eq. 27.3) we know that 2.5 C of charge pass through a cross-sectional area of the wire each second. The magnitude of the charge on an electron is 1.60×10^{-19} C, so $(2.5\text{ C})/(1.60\times10^{-19}\text{ C/electron}) = 1.6\times10^{19}$ electrons pass through a cross-sectional area of the wire each second. (*b*) Because the current is in the positive *x* direction and the charge on the electron is negative, the electrons are moving in the negative *x* direction. (*c*) By the right-hand force rule, the magnetic force is directed in the positive *y* direction and its magnitude per meter of wire is (2.5 A)(0.20 T) = 0.50 N/m. (*d*)



27.32. (*a*) The magnetic force exerted on the wire equals the vector sum of the forces exerted on each segment of the wire. The magnitude of the force on each segment is given by Eq. 27.4, $F^B_\text{w} = |I|\ell B\sin\theta,$ and the direction by the right-hand force rule:

$$\vec{F}^B_\text{w} = (4.10\text{ A})(30.0\times10^{-3}\text{ m})(0.500\text{ T})(-\hat{j}) + (4.10\text{ A})(40.0\times10^{-3}\text{ m})(0.500\text{ T})\hat{i}$$
$$= (8.20\times10^{-2}\text{ N})\hat{i} + (-6.15\times10^{-2}\text{ N})\hat{j}$$

So, the magnitude of the force is $F^B_\text{w} = \sqrt{(8.20\times10^{-2}\text{ N})^2 + (-6.15\times10^{-2}\text{ N})^2} = 1.03\times10^{-1}\text{ N}$ and its direction is given by $\tan\theta = (-6.15\times10^{-2}\text{ N})/(8.20\times10^{-2}\text{ N})$ or $\theta = 36.9°$ below the positive *x* axis. (*b*) The magnitude of the force is $F^B_\text{w} = (4.10\text{ A})(50.0\times10^{-3}\text{ m})(0.500\text{ T}) = 1.03\times10^{-1}\text{ N}$ and its direction is perpendicular to the wire, namely, $36.9°$ below the positive *x* axis.

27.33. When we attach the piece of plastic to the bar, it sinks down a small distance until it reaches its new equilibrium position. To determine how much current will make it rise a distance of 1.5 mm above this new equilibrium position, we proceed exactly as in *Principles* Example 27.2: When the bar rises *d* = 1.5 mm, the magnitude of the force exerted on the bar by each spring decreases by *kd*, so the magnitude of the magnetic force is 2*kd*, the same as without the piece of plastic, and the current required is exactly the same, *I* = 0.45 A.

27.34. (*a*) By the right-hand force rule, the magnetic force on the crossbar is directed toward the right. (*b*) In order to make the crossbar move, the magnetic force must be greater than the maximum force of static friction between the rods and crossbar. So, $I_0\ell B > mg\mu_s$ or $I_0 > \dfrac{mg\mu_s}{\ell B}.$

27.35. We can use the right-hand force rule to determine what magnetic field directions could cause the changes. We orient the fingers of our right hand in the direction of the current, point our right thumb in the direction of the required force, and curl our fingers toward the direction of the magnetic field. (*a*) The change is caused by a force directed outward from the center of the loop. A magnetic field directed out of the page, in the negative *y* direction, could cause this change. (*b*) The change is caused by a force directed inward toward the center of the loop. A magnetic field directed into the page, in the positive *y* direction, could cause this change. (*c*) The loop was originally in the *xz* plane, but is now in the *xy* plane. The change is caused by a torque in the negative *x* direction, which could be caused by a force in the positive *y* direction on the top segment of the loop or a force in the negative *y* direction on the bottom segment of the loop. An upward magnetic field, in the positive *z* direction, could cause these forces. (*d*) The loop was originally in the *xz* plane, but is now in the *yz* plane. The change is caused by a torque in the positive *z* direction, which could be caused by a force in the negative *y* direction on the left segment of the loop or a force in the positive *y* direction on the right segment of the loop. A magnetic field in the positive *x* direction could cause these forces. (*e*) The loop has rotated in the *xz* plane, which could be caused by forces in the direction of the current. But the magnetic force on a current-carrying wire is always perpendicular to the current, so it is not possible for a magnetic field to have caused this change.

27.36. Imagine dividing the wire into many small segments of length $d\ell$. The magnetic force on the whole wire equals the vector sum of the magnetic forces on each segment of the wire, $\vec{F}_{\text{w}}^{B} = \sum I_0\, d\vec{\ell}_n \times \vec{B}_0$. In the limit as the length of each segment approaches zero, the sum becomes an integral and we have $\vec{F}_{\text{w}}^{B} = \int_{\vec{r}_1}^{\vec{r}_2} I_0\, d\vec{\ell} \times \vec{B}_0$. Because $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$, $\vec{F}_{\text{w}}^{B} = -\int_{\vec{r}_1}^{\vec{r}_2} I_0\vec{B}_0 \times d\vec{\ell}$. Both I_0 and $\vec{B}_0$ are constants, so we can bring them outside the integral giving $\vec{F}_{\text{w}}^{B} = -I_0\vec{B}_0 \times \int_{\vec{r}_1}^{\vec{r}_2} d\vec{\ell}$. The remaining integral is the vector sum of the infinitesimal segments of wire, that is, the displacement between the ends of the wire $\int_{\vec{r}_1}^{\vec{r}_2} d\vec{\ell} = \vec{r}_2 - \vec{r}_1$, so $\vec{F}_{\text{w}}^{B} = -I_0\vec{B}_0 \times (\vec{r}_2 - \vec{r}_1) = I_0(\vec{r}_2 - \vec{r}_1)\times\vec{B}_0$.

27.37. (*a*) Let's choose a coordinate system with the *y* axis pointing upward, parallel to the surface of the incline, and the *x* axis pointing down into the surface of the incline. The *z* axis then points from back to front horizontally across the incline. When there is no current in the bar, the only forces exerted on it are the downward force of gravity $\vec{F}_{\text{Eb}}^{G}$, the normal force of the surface of the incline $\vec{F}_{\text{sb}}^{n}$, and the force of static friction $\vec{F}_{\text{sb}}^{s}$. We know from experience that on such a steep incline with such a small coefficient of friction, the bar will slide down.

When there is current in the metal bar, the direction of the magnetic force exerted on the bar $\vec{F}_{\text{b}}^{B}$ can be determined from the right-hand force rule. If the direction of the current is from back to front, along the positive *z* axis, the magnetic force is directed to the left, away from the surface of the incline, and it cannot help keep the bar from moving. If the direction of the current is from front to back, along the negative *z* axis, the magnetic force is directed to the right, toward the surface of the incline. The *y* component of this force is directed up the incline and the *x* component is directed into the surface of the incline, where it contributes to the force of static friction holding the bar in place.

If the bar is not to move when it's released, the vector sum of the forces exerted on it must be zero. Writing this out in component form, we have for the *x* components

$$F_{\text{Eb}}^{G}\cos\theta - F_{\text{sb}}^{n} + F_{\text{b}}^{B}\sin\theta = 0 \qquad (1)$$

and for the *y* components

$$-F_{\text{Eb}}^{G}\sin\theta + F_{\text{sb}}^{s} + F_{\text{b}}^{B}\cos\theta = 0 \qquad (2)$$

where we have assumed the force of static friction is directed up the incline, helping to counter the force of gravity. The magnitude of the force of static friction can take on any value up to a maximum determined by the magnitude of the normal force and the coefficient of friction

$$F_{\text{sb}}^{s} \le \mu_s F_{\text{sb}}^{n} \qquad (3)$$

Rearranging Eqs. 1 and 2 and substituting into Eq. 3 gives

$$F_{\text{sb}}^{s} = F_{\text{Eb}}^{G}\sin\theta - F_{\text{b}}^{B}\cos\theta \le \mu_s F_{\text{sb}}^{n} = \mu_s(F_{\text{Eb}}^{G}\cos\theta + F_{\text{b}}^{B}\sin\theta) \qquad (4)$$

Collecting terms for the gravitational and magnetic forces gives

$$F_{\text{Eb}}^{G}\sin\theta - \mu_s F_{\text{Eb}}^{G}\cos\theta \le F_{\text{b}}^{B}\cos\theta + \mu_s F_{\text{b}}^{B}\sin\theta \text{ or}$$

$$F_{\text{b}}^{B} \ge \frac{F_{\text{Eb}}^{G}(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta} \text{ or, finally}$$

$$I \ge \frac{mg(\sin\theta - \mu_s\cos\theta)}{\ell B(\cos\theta + \mu_s\sin\theta)}$$

where in the last step we have substituted the formulas for the gravitational and magnetic forces and solved for the current. So, the minimum current required is

$$I = \frac{(0.900\text{ kg})(9.80\text{ m/s}^2)(\sin 65.0° - (0.200)\cos 65.0°)}{(1.00\text{ m})(0.850\text{ T})(\cos 65.0° + (0.200)\sin 65.0°)} = 14.1\text{ A}$$

(*b*) If the magnetic force it too great, it could begin to push the bar up the incline, overcoming the gravitational force and the force of friction. In this case, the force of friction exerted on the bar will be directed downward, in the negative *y* direction. We proceed as in part a. Eq. 1 is unchanged, but the sign of F^s_{sb} in Eq. 2 is now negative, so the new version of Eq. 4 is

$$F^s_{\text{sb}} = -F^G_{\text{Eb}}\sin\theta + F^B_{\text{b}}\cos\theta \le \mu_s F^n_{\text{sb}} = \mu_s(F^G_{\text{Eb}}\cos\theta + F^B_{\text{b}}\sin\theta)$$

$$-F^G_{\text{Eb}}\sin\theta - \mu_s F^G_{\text{Eb}}\cos\theta \le -F^B_{\text{b}}\cos\theta + \mu_s F^B_{\text{b}}\sin\theta$$

$$F^B_{\text{b}} \le \frac{F^G_{\text{Eb}}(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}$$

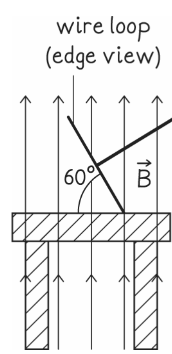
$I \le \dfrac{mg(\sin\theta + \mu_s\cos\theta)}{\ell B(\cos\theta - \mu_s\sin\theta)}$ and the maximum current that can be used is

$$I = \frac{(0.900\text{ kg})(9.80\text{ m/s}^2)(\sin 65.0° + (0.200)\cos 65.0°)}{(1.00\text{ m})(0.850\text{ T})(\cos 65.0° - (0.200)\sin 65.0°)} = 42.6\text{ A}$$

27.38. According Gauss's law for magnetism, the magnetic flux through any closed surface must always be zero. So, the fluxes in all four cases are the same, namely, zero.

27.39. Objects b, d, and e are all closed surfaces, so by Gauss's law for magnetism, the magnetic flux through each of them is zero. Objects a and c appear to be flat surfaces perpendicular to $\vec{B}$, so the magnetic flux through each of them is BA. Object c has twice the area of object a, so the ranking is 0 = b = d = e < a < c.

27.40. The greatest magnetic flux through the loop is when the loop is perpendicular to the magnetic field direction, so the magnetic field is perpendicular to the table. So, when the loop makes a $60°$ angle with the table, its area vector makes a 60° angle with the magnetic field and the flux is $\Phi_B = BA\cos\theta = (0.25\text{ T})(100\times10^{-3}\text{ m})^2\cos 60° =$ 1.3×10^{-3} Wb.



27.41. The flux through the loop is given by $\Phi_B = BA\cos\theta$, which we can rearrange to solve for the angle $\theta = \cos^{-1}(\Phi_B/BA)$. Substituting the given values, $\theta = \cos^{-1}((3.00\times10^{-4}\text{ T}\cdot\text{m}^2)/(0.030\text{ T})\pi(100\times10^{-3}\text{ m})^2) = 71.4°$. This is the angle between the normal to the loop and the magnetic field, so the angle between the plane of the loop and the field is $90° - \theta = 18.6°$.

27.42. (*a*) The sides of the loop make a $25.0°$ angle with the horizontal, so the area vector, which is perpendicular to the sides, makes a $65.0°$ angle with the horizontal. Because the perimeter of the square loop is 4.00 m, each side must have length 1.00 m, and its area is 1.00 m^2. So, the flux through the loop is $\Phi_B = BA\cos\theta = (0.100\text{ T})(1.00\text{ m}^2)\cos 65.0° = 4.23\times10^{-2}$ Wb. (*b*) Because the area of the irregular loop is the same as that of the square loop, and it is oriented in the same plane, the flux is the same, $\Phi_B = 4.23\times10^{-2}$ Wb.

27.43. The total flux through the loop is equal to the sum of the fluxes through each part of the loop. Taking the area vector to point out of the plane of the page, we get $\Phi_B = B_1A_1\cos\theta_1 + B_2A_2\cos\theta_2 = (2.0\text{ T})(2.0\text{ m})(2.0\text{ m})\cos 0 +$ $(1.0\text{ T})(1.0\text{ m})(2.0\text{ m})\cos 180° = 6.0\text{ Wb}$.

27.44. We can solve this problem easily using Gauss's law for magnetism. If we make the hemispherical bowl into a closed surface by adding a circular lid, we know that the flux through the closed surface is zero. Because the magnetic field is uniform, the magnitude of the flux through the hemispherical bowl equals the magnitude of the flux through the circular lid, which we can easily calculate, $\Phi_B = B_0(\pi R^2)$.

27.45. (*a*) The condition for a charged particle to move undeflected through a velocity selector is $v = E/B$ (Eq. 27.26), which we can solve for $B = E/v = (2.0\times10^5\text{ N/C})/(6.67\times10^5\text{ m/s}) = 0.30\text{ T}$. The electric field exerts a force on the proton in the positive *z* direction, so the magnetic field must exert a force in the negative *z* direction, which means that the direction of the magnetic field must be the negative *y* direction. (*b*) If the proton's speed doubles, the magnetic force exerted on it also doubles, while the electric force remains the same. So, the proton is initially deflected in the negative *z* direction.

27.46. A charged particle moving perpendicular to a uniform magnetic field orbits in a circle, and the period of the orbit depends only on the particle's charge and mass and the magnitude of the magnetic field (Eq. 27.24). Our particles traverse a semicircular path, so the time it takes is one-half of an orbital period, $\Delta t = \tfrac{1}{2}T = \tfrac{1}{2}\dfrac{2\pi m}{|q|B} = \dfrac{\pi(2.6\times10^{-26}\text{ kg})}{(2\cdot1.6\times10^{-19}\text{ C})(0.20\text{ T})} = 1.3\times10^{-6}\text{ s}$.

27.47. (*a*) A charged particle moving perpendicular to a uniform magnetic field orbits in a circle, and the angular frequency and period of the orbit depend only on the particle's charge and mass and the magnitude of the magnetic field (Eqs. 27.25 and 27.24).

$$\omega = \frac{|q|B}{m} = \frac{(2\cdot1.60\times10^{-19}\text{ C})(0.75\text{ T})}{6.64\times10^{-27}\text{ kg}} = 3.6\times10^7\text{ s}^{-1}$$

$$T = \frac{2\pi}{\omega} = 1.7\times10^{-7}\text{ s}$$

(*b*) The particle's speed equals the circumference of the orbit divided by the orbital period, $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(0.75\text{ m})}{1.7\times10^{-7}\text{ s}} =$ $2.7\times10^7\text{ m/s}$. (*c*) This speed is not relativistic, so the kinetic energy $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(6.64\times10^{-27}\text{ kg})(2.7\times10^7\text{ m/s})^2 =$ $2.4\times10^{-12}\text{ J}$.

27.48. (*a*) A charged particle moving perpendicular to a uniform magnetic field orbits in a circle, and the angular frequency and period of the orbit depend only on the particle's charge and mass and the magnitude of the magnetic field (Eqs. 27.25 and 27.24).

$$\omega = \frac{|q|B}{m} = \frac{(1.60\times10^{-19}\text{ C})(0.25\text{ T})}{1.67\times10^{-27}\text{ kg}} = 2.4\times10^7\text{ s}^{-1}$$

$$T = \frac{2\pi}{\omega} = 2.6\times10^{-7}\text{ s}$$

(*b*) The particle's speed equals the circumference of the orbit divided by the orbital period, $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(0.15\text{ m})}{2.6\times10^{-7}\text{ s}} = 3.6\times10^6\text{ m/s}$. (*c*) This speed is not relativistic, so the kinetic energy $K = \tfrac{1}{2}mv^2 =$ $\tfrac{1}{2}(1.67\times10^{-27}\text{ kg})(3.6\times10^6\text{ m/s})^2 = 1.1\times10^{-14}\text{ J}$.

27.49. (*a*) We have an expression for the radius of the trajectory of a charged particle moving perpendicular to a uniform magnetic field $R = \frac{mv}{|q|B}$ (Eq. 27.23). We can express the speed in terms of the kinetic energy $K = \frac{1}{2}mv^2$, $v = \sqrt{2K/m}$. Combining these we get $R = \frac{\sqrt{2Km}}{|q|B} = \frac{\sqrt{2(7.5\times10^{-17}\text{ J})(9.11\times10^{-31}\text{ kg})}}{(1.60\times10^{-19}\text{ C})(0.35\text{ T})} = 2.1\times10^{-4}$ m. (*b*) We have expressions for the angular frequency and period of the electron's orbit (Eqs. 27.25 and 27.24).

$$\omega = \frac{|q|B}{m} = \frac{(1.60\times10^{-19}\text{ C})(0.35\text{ T})}{9.11\times10^{-31}\text{ kg}} = 6.1\times10^{10}\text{ s}^{-1}$$

$$T = \frac{2\pi}{\omega} = 1.0\times10^{-10}\text{ s}$$

(*c*) We found an expression for its speed in terms of its kinetic energy in part a, $v = \sqrt{2K/m} = \sqrt{2(7.5\times10^{-17}\text{ J})/(9.11\times10^{-31}\text{ kg})} = 1.3\times10^{7}$ m/s. This speed is not relativistic, so our expression for the kinetic energy in part *a* was valid.

27.50. (*a*) We have an expression for the radius of the trajectory of a charged particle moving perpendicular to a uniform magnetic field $R = \frac{mv}{|q|B}$ (Eq. 27.23). We can rearrange this to get an expression for each particle's speed and compare them.

$$v_\text{p} = \frac{|q_\text{p}|BR}{m_\text{p}}$$

$$v_\text{d} = \frac{|q_\text{d}|BR}{m_\text{d}} = \frac{|q_\text{p}|BR}{2m_\text{p}} = \tfrac{1}{2}v_\text{p}$$

$$v_\alpha = \frac{|q_\alpha|BR}{m_\alpha} = \frac{|2q_\text{p}|BR}{4m_\text{p}} = \tfrac{1}{2}v_\text{p}$$

(*b*) We can write expressions for each particle's kinetic energy and compare them.

$$K_\text{p} = \tfrac{1}{2}m_\text{p}v_\text{p}^2$$

$$K_\text{d} = \tfrac{1}{2}m_\text{d}v_\text{d}^2 = \tfrac{1}{2}(2m_\text{p})(\tfrac{1}{2}v_\text{p})^2 = \tfrac{1}{2}K_\text{p}$$

$$K_\alpha = \tfrac{1}{2}m_\alpha v_\alpha^2 = \tfrac{1}{2}(4m_\text{p})(\tfrac{1}{2}v_\text{p})^2 = K_\text{p}$$

(*c*) We can write expressions for each particle's angular momentum and compare them.

$$L_\text{p} = m_\text{p}v_\text{p}R$$

$$L_\text{d} = m_\text{d}v_\text{d}R = (2m_\text{p})(\tfrac{1}{2}v_\text{p})R = L_\text{p}$$

$$L_\alpha = m_\alpha v_\alpha R = (4m_\text{p})(\tfrac{1}{2}v_\text{p})R = 2L_\text{p}$$

27.51. As an ion is accelerated from rest through the potential difference *V*, its kinetic energy increases to $K = qV$, which determines its speed $v = \sqrt{2K/m} = \sqrt{2qV/m}$. Then the ion enters the region of uniform magnetic field *B* and travels through a semicircular arc of radius $R = \frac{mv}{|q|B}$ (Eq. 27.23), striking the detector at a point a distance $2R = \frac{2mv}{|q|B}$ from the entrance. Combining this with our expression for the speed gives $2R = \frac{2m}{|q|B}\sqrt{\frac{2qV}{m}} = \frac{2}{B}\sqrt{\frac{2mV}{q}}$, and the separation distance between the two types of ions is $\Delta x = \frac{2}{B}\sqrt{\frac{2m_{26}V}{q}} - \frac{2}{B}\sqrt{\frac{2m_{24}V}{q}}$. Rearranging, we can solve for *V*,

$$\Delta x = \frac{2}{B}\sqrt{\frac{2(\frac{26}{24})m_{24}V}{q}} - \frac{2}{B}\sqrt{\frac{2m_{24}V}{q}}$$

$$\Delta x = \frac{2}{B}\sqrt{\frac{2m_{24}V}{q}}\left(\sqrt{\tfrac{26}{24}} - 1\right)$$

$$(\Delta x)^2 = \frac{4}{B^2}\frac{2m_{24}V}{q}\left(\sqrt{\tfrac{26}{24}} - 1\right)^2$$

$$V = \frac{(\Delta x)^2 qB^2}{8m_{24}\left(\sqrt{\frac{26}{24}} - 1\right)^2} = \frac{(2.60\times10^{-3}\text{ m})^2(1.60\times10^{-19}\text{ C})(0.577\text{ T})^2}{8(3.983\times10^{-26}\text{ kg})\left(\sqrt{\frac{26}{24}} - 1\right)^2} = 678\text{ V}$$

27.52. (*a*) Suppose the particle is traveling along the *x* axis. In this case, there will be no magnetic force on the particle and it will continue to travel in a straight line at constant speed. Suppose the particle is traveling along the *y* axis. Now, the particle experiences a magnetic force perpendicular to its velocity and orbits in a circle in the *yz* plane. For a particle that enters in a different direction, the *x* component of its velocity will remain unchanged, while what is originally the *y* component vector of its velocity will rotate parallel to the *yz* plane. We can see this by writing the formula for the magnetic force in terms of component vectors:

$$\begin{aligned}\vec{F}^B_{\text{p}} &= q\vec{v}\times\vec{B}\\ &= q\left(\vec{v}_x + \vec{v}_y + \vec{v}_z\right)\times\vec{B}_x\\ &= q\vec{v}_x\times\vec{B}_x + q\vec{v}_y\times\vec{B}_x + q\vec{v}_z\times\vec{B}_x\\ &= \vec{0} + q\vec{v}_y\times\vec{B}_x + q\vec{v}_z\times\vec{B}_x\\ &= -qv_yB_x\hat{k} + qv_zB_x\hat{j}\end{aligned}$$

This force, initially in the negative *z* direction, changes the direction of the particle's velocity but not its magnitude, and only has components in the *yz* plane. If we write $\vec{v}_\perp = \vec{v}_y + \vec{v}_z$ for the component of the velocity perpendicular to the magnetic field, that is, perpendicular to the *x* axis, we see that $\vec{v}_\perp$ rotates about the *x* axis and the particle travels along a helical path. By considering the perpendicular and parallel components of the particle's motion separately and following the derivation of Eq. 27.23, we can find a similar formula for the radius of the helix $R = \dfrac{mv_\perp}{qB}$. (*b*) Following the derivation of the particle's cyclotron frequency (Eq. 27.25), but using only the component of the particle's velocity perpendicular to the magnetic field, we arrive at the same formula. So, in a time interval $\Delta t = 2\pi m/qB$ the particle undergoes an angular displacement of one full turn or 2π.

27.53. (*a*) Suppose the current is made up of positive charge carriers. Then the magnetic force on the moving charge carries would be toward the right, and the right side would acquire a positive charge and be at the higher potential. This is the opposite of what has happened, so the mobile charge carriers must be negatively charged. (*b*) The potential difference across the width of the strip reaches equilibrium when the electric force it produces on the charge carriers cancels the magnetic force, that is, when $v = E/B$. So, $v = (\Delta V/w)/B = ((4.27\times10^{-6}\text{ V})/(20\times10^{-3}\text{ m}))/(2.0\text{ T}) = 1.1\times10^{-4}\text{ m/s}$. (*c*) The magnitude of the current is the amount of charge that passes through a cross-sectional area of the conductor each second, $I = \Delta Q/\Delta t$. The flow of charge is related to the number density of the charge carriers and their speed, so $I = \Delta Q/\Delta t = Av\Delta tnq/\Delta t = Avnq$, where A is the cross-sectional area of the conductor and q is the charge of an individual charge carrier. Here, the cross-sectional area of the conductor equals its width times its thickness, $A = wh$, the charge carriers are electrons, and the speed of the charge carriers is given by our answer to part *b*, so we can combine these and rearrange to get

$$I = wh\frac{\Delta V}{wB}ne$$

$$n = \frac{IB}{h\Delta Ve} = \frac{(20\text{ A})(2.0\text{ T})}{(1.0\times10^{-3}\text{ m})(4.27\times10^{-6}\text{ V})(1.60\times10^{-19}\text{ C})} = 5.9\times10^{28}\text{ m}^{-3}$$

27.54. (*a*) The magnitude of the current is the amount of charge that passes through a cross-sectional area of the conductor each second, $I = \Delta Q/\Delta t$. The flow of charge is related to the number density of the charge carriers and their speed, so $I = \Delta Q/\Delta t = Av\Delta tnq/\Delta t = Avnq$, where A is the cross-sectional area of the conductor and q is the charge of an individual charge carrier. Solving for the speed we get $v = \dfrac{I}{Anq} = \dfrac{10\text{ A}}{(1.0\times10^{-3}\text{ m})(20\times10^{-3}\text{ m})(8.47\times10^{19}\text{ mm}^{-3})(1000\text{ mm}/1\text{ m})^3(1.60\times10^{-19}\text{ C})} = 3.7\times10^{-5}\text{ m/s}$. (*b*) As the charge carriers move through the conductor, the magnetic field pushes them to one side creating a potential difference. At equilibrium, the electric and magnetic forces on the charge carriers cancel, and the condition for this to happen is $v = E/B$. So, $v = (\Delta V/w)/B$ or $\Delta V = vwB = (3.7\times10^{-5}\text{ m/s})(20\times10^{-3}\text{ m})(2.0\text{ T}) = 1.5\times10^{-6}\text{ V}$.

27.55. As the proton travels through the magnetic field, the component of the proton's velocity parallel to the magnetic field is unaffected because the magnetic force is perpendicular to both the field and the proton's velocity, while the component of its velocity perpendicular to the field rotates around the direction of the field at the cyclotron frequency. So, the proton will traverse a half turn through a helical path before striking the wall from the inside. The z component of the proton's position will equal the product of the z component of its velocity on entering the field times one-half the period corresponding to its cyclotron frequency, $z = v_z\dfrac{\pi m}{qB}$. The y component will be given by the diameter of the helical path the proton follows, $y = \dfrac{2m|v_x|}{qB}$. To find the proton's speed, we know its kinetic energy after being accelerated through the potential difference $K = \tfrac{1}{2}mv^2 = q\Delta V$, which we can solve for the speed $v = \sqrt{\dfrac{2q\Delta V}{m}}$. Combining these, we have

$$\begin{aligned}
z &= (v\sin 25^\circ)\frac{\pi m}{qB}\\
&= \sqrt{\frac{2q\Delta V}{m}}\frac{\pi m}{qB}\sin 25^\circ\\
&= \sqrt{\frac{2m\Delta V}{q}}\frac{\pi\sin 25^\circ}{B}\\
&= \sqrt{\frac{2(1.67\times10^{-27}\text{ kg})(120\text{ V})}{1.60\times10^{-19}\text{ C}}}\frac{\pi\sin 25^\circ}{0.15\text{ T}}\\
&= 1.4\times10^{-2}\text{ m}
\end{aligned}$$

and

$$\begin{aligned}
y &= \frac{2m(v\cos 25^\circ)}{qB}\\
&= \frac{2m\cos 25^\circ}{qB}\sqrt{\frac{2q\Delta V}{m}}\\
&= \frac{2\cos 25^\circ}{B}\sqrt{\frac{2m\Delta V}{q}}\\
&= \frac{2\cos 25^\circ}{0.15\text{ T}}\sqrt{\frac{2(1.67\times10^{-27}\text{ kg})(120\text{ V})}{1.60\times10^{-19}\text{ C}}}\\
&= 1.9\times10^{-2}\text{ m}
\end{aligned}$$

27.56. In chamber 1, a force is exerted on the protons that initially points in the negative z direction. A magnetic field in the negative y direction would cause the protons to move in such a circular trajectory with radius $R = mv/qB$. Solving for B, we have $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\text{ kg})(300\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.40\text{ m})} = 7.8\times10^{-6}\text{ T}$, or $B_y = -7.8\times10^{-6}\text{ T}$. Because the magnetic force in chamber 1 is perpendicular to the protons' velocity, they enter chamber 2 with the same 300 m/s speed. Here, they follow a circular trajectory in the opposite direction as in chamber 1, so the magnetic field must have the same magnitude but the opposite direction as that in chamber 1, or $B_y = +7.8\times10^{-6}$ T. In chamber 3 there is an electric field that accelerates the protons in the positive *x* direction, that is, in their direction of motion. If there were a magnetic field perpendicular to the path of the protons in this chamber, they would curve in some direction, but they do not, so the magnetic field here must be zero. As the protons pass through chamber 4, they follow a circular trajectory such as would be caused by a magnetic field in the positive *y* direction. To determine its magnitude, we first must find the protons' speed as they enter the chamber. As they passed through chamber 3, their kinetic energy increased by $\Delta K = qEd$. We can find their speed as they exit chamber 3 kinematically:

$$K_\mathrm{f} = K_\mathrm{i} + \Delta K$$

$$\tfrac{1}{2}mv_\mathrm{f}^2 = \tfrac{1}{2}mv_\mathrm{i}^2 + qEd$$

$$v_\mathrm{f} = \sqrt{v_\mathrm{i}^2 + \frac{2qEd}{m}}$$

$$v_\mathrm{f} = \sqrt{(300\ \text{m/s})^2 + \frac{2(1.60\times10^{-19}\ \text{C})(2.09\times10^{-3}\ \text{N/C})(1.0\ \text{m})}{1.67\times10^{-27}\ \text{kg}}} = 700\ \text{m/s}$$

So, the magnitude of the magnetic field in chamber 4 is $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\ \text{kg})(700\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.40\ \text{m})} = 1.8\times10^{-5}$ T, or $B_y = +1.8\times10^{-5}$ T. In chamber 5, there is an electric field perpendicular to the protons' path, but that path is a straight line, so there must be a magnetic force with the same magnitude and the opposite direction as the electric force. The electric force is in the negative *x* direction, so the magnetic force must be in the positive *x* direction, and the magnetic field must be in the negative *y* direction. Its magnitude is given by the condition $v = E/B$ or $B = E/v = (6.00\times10^{-6}\ \text{N/C})/(700\ \text{m/s}) = 8.6\times10^{-9}$ T, or $B_y = -8.6\times10^{-9}$ T.

27.57. As the electrons move through the conductor, the magnetic field pushes them to one side creating a potential difference. At equilibrium, the electric and magnetic forces on the charge carriers cancel, and the condition for this to happen is $v = E/B$, or $v = (\Delta V/w)/B$. We can determine the speed of the electrons from the current and other data given because the current is the quantity of charge that passes through a cross-sectional area of the conductor each second:

$$I = \frac{\Delta Q}{\Delta t} = \frac{whv\Delta tnq}{\Delta t} = whvnq$$

$$v = \frac{I}{whnq}$$

where *w* and *h* are the width and height of the conductor, *n* is the number density of electrons in the conductor, and *q* is the charge on each electron. Combining this with our previous result we have

$$v = \frac{I}{whnq} = \frac{\Delta V}{wB}$$

$$\Delta V = \frac{IB}{hnq} = \frac{(12.0\ \text{A})(2.00\ \text{T})}{(1.00\times10^{-3}\ \text{m})(8.46\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})} = 1.77\times10^{-6}\ \text{V}$$

27.58. (*a*) The linear charge density is the quantity of charge per unit length. The number density of electrons is the number of electrons per unit volume. In a given volume of wire, the quantity of charge equals the number density of electrons times the charge on an electron times the volume, and the volume equals the cross-sectional area times the length, so the linear charge density in the wire equals the number density of electrons times the charge on an electron times the cross-sectional area of the wire, $\lambda = nqA = (8.46\times10^{19}\ \text{mm}^{-3})(1000\ \text{mm}/1\ \text{m})^3(1.60\times10^{-19}\ \text{C})\pi(1.00\times10^{-3}\ \text{m})^2 = 4.25\times10^4$ C/m. (*b*) In a reference frame moving along with the electrons, the distance between positive ions is reduced by length contraction from that measured in the Earth reference frame, while the distance between electrons is increased because their separation is no longer length contracted. Because the linear charge density is inversely proportional to the distance between the charges, $\lambda' = \lambda'_{\text{ions}} + \lambda'_{\text{electrons}} = \gamma\lambda_{\text{ions}} + \dfrac{1}{\gamma}\lambda_{\text{electrons}}$. Because the wire is electrically neutral in the Earth reference frame, the magnitudes of λ_{ions} and $\lambda_{\text{electrons}}$ are the same, so $\lambda' = \left(\gamma - \dfrac{1}{\gamma}\right)\lambda_{\text{ions}}$. For $v = 4.70\times10^{-4}$ m/s, $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c_0^2}}} = \dfrac{1}{\sqrt{1-\dfrac{(4.70\times10^{-4}\text{ m/s})^2}{(3.00\times10^8\text{ m/s})^2}}} = 1 + 1.23\times10^{-24}$ and $\gamma - \dfrac{1}{\gamma} = 2.45\times10^{-24}$. Using the linear charge density we calculated in part *a* we have $\lambda' = (2.45\times10^{-24})(4.25\times10^4\text{ C/m}) = 1.04\times10^{-19}\text{ C/m}$.

27.59. Because there is a clockwise current in the loop, the electrons travel counterclockwise around the loop. So, on side +*x* they travel in the positive *y* direction, on side –*x* they travel in the negative *y* direction, on side +*y* they travel in the negative *x* direction, and on side –*y* they travel in the positive *x* direction. Observer M traveling with the same speed as the electrons in the positive *y* direction sees the spacing between the positive ions on all four sides contracted in the *y* direction. This increases the positive linear charge density he measures on sides +*x* and –*x*, but has no effect on the linear charge density on sides +*y* and –*y* (length contraction makes these wires thinner, increasing the volumetric charge density, but does not affect the spacing between the positive ions along the length of the wire). To observer M, the electrons on side +*x* are at rest, so the spacing between them is greater than that measured in the Earth reference frame. Similarly, to observer M, the electrons on side –*x* are moving at greater speed than that measured in the Earth reference frame, so the spacing between them is smaller than that measured in the Earth reference frame, and smaller than the spacing observer M measures between the positive ions on this side. As with the positive ions, length contraction has no effect on the spacing between electrons along the length of the wire on sides +*y* and –*y*. So, (*a*) observer M sees a surplus of positive ions on side +*x* and (*b*) a surplus of negative electrons on side –*x*.

27.60. To observer M, the spacing between positive ions is less than measured in the Earth reference frame, while the spacing between negative ions is greater, increasing the positive charge density and decreasing the negative charge density, each by a factor of γ. Combining these effects, we have (similar to Eq. 27.30) $\lambda_{\text{M}} = -\lambda_{\text{En}}\gamma\dfrac{v^2}{c_0^2}$. (*a*) If $v = 3.0\times10^5$ m/s, $\gamma = 1 + 5.0\times10^{-7}$, and $\lambda_{\text{M}} = (-1.0\times10^{-6})\lambda_{\text{En}}$. (*b*) If $v = 3.0\times10^{-3}$ m/s, $\gamma = 1 + 5.0\times10^{-23}$, and $\lambda_{\text{M}} = (-1.0\times10^{-22})\lambda_{\text{En}}$.

27.61. To observer M, the spacing between positive ions is length contracted, while the spacing between negative ions is not, so to observer M, the linear charge density in the wire is $\lambda_{\text{M}} = \gamma\lambda_{\text{Ep}} + \lambda_{\text{En}}$. The magnitude of the electric field measured by M is related to the linear charge density by $E_{\text{M}} = \dfrac{2k\lambda_{\text{M}}}{r}$, or $\lambda_{\text{M}} = \dfrac{E_{\text{M}}r}{2k} = \dfrac{(3.00\times10^7\text{ N/C})(0.0100\text{ m})}{2(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)} = 1.67\times10^{-5}\text{ C/m}$. Substituting into our first expression and solving for γ we get $\gamma = \dfrac{\lambda_{\text{M}} - \lambda_{\text{En}}}{\lambda_{\text{Ep}}} = \dfrac{(1.67\times10^{-5}\text{ N/m}) - (-4.500\times10^{-5}\text{ N/m})}{6.000\times10^{-5}\text{ N/m}} = 1.028$. From the definition of $\gamma = 1/\sqrt{1 - v^2/c_0^2}$ we get $v = c_0\sqrt{1 - 1/\gamma^2} = 0.231c_0 = 6.93\times10^7$ m/s.

27.62. We can use the right-hand force rule to see that the magnetic force on wire 1 is directed in the negative *z* direction, the magnetic force on wire 2 is directed in the negative *x* direction, there is no magnetic force on wire 3, the magnetic force on wire 4 is directed at a $30°$ angle to the left of the positive *z* direction, there is no magnetic force on wire 5, and there is no magnetic force on wire 6.

27.63. The period with which the particles revolve is $T = \dfrac{2\pi m}{|q|B}$ (Eq. 27.24). The period does not depend of the particle's speed, so the period of particle 1 remains the same $T_1 = T$. Particle 2 enters a region where the magnetic field is reduced by a factor of 2, so its period is now $T_2 = \dfrac{2\pi m}{|q|(B/2)} = 2T$. The mass of particle 3 doubles, so it's period also doubles (it doesn't matter that its speed is also reduced, as we saw with particle 1), that is, $T_3 = 2T$.

27.64. (*a*) Because the loop is parallel to the magnetic field, the magnetic flux through it is zero. (*b*) When the axis points northwest, the flux is $\Phi_B = BA\cos\theta = (3.5\times10^{-5}\ \text{T})\pi(0.10\ \text{m})^2\cos45° = 7.8\times10^{-7}\ \text{T}\cdot\text{m}^2$. (*c*) When the axis points due north, the flux is $\Phi_B = BA = (3.5\times10^{-5}\ \text{T})\pi(0.10\ \text{m})^2 = 1.1\times10^{-6}\ \text{T}\cdot\text{m}^2$. (*d*) When the axis points to the west, the loop is parallel to the magnetic field, so the flux is zero.

27.65. In its first orientation, the magnetic force on the wire tells us nothing about the *x* component of the magnetic field, because that component is parallel to the current. However, we can tell from the right-hand force rule that the *y* and *z* components are in the ratio of 2 to –3. We can find the actual components by using Eq. 27.8 and writing out the components of the vector product (omitting terms where the current is zero):

$$\begin{aligned}
\vec{F}^B_{\text{w}} &= I\vec{\ell}\times\vec{B}\\
F^B_{\text{w}y} &= -I\ell_x B_z\\
B_z &= -F^B_{\text{w}y}/I\ell_x = -(3.0\ \text{N})/(20\ \text{A})(1.0\ \text{m}) = -0.15\ \text{T}\\
F^B_{\text{w}z} &= I\ell_x B_y\\
B_y &= F^B_{\text{w}z}/I\ell_x = (2.0\ \text{N})/(20\ \text{A})(1.0\ \text{m}) = 0.10\ \text{T}
\end{aligned}$$

In the second orientation, the current is in the positive *y* direction. Proceeding as previously, we can see that the *z* and *x* components of the magnetic field are in the ratio of –3 to 2 and find the components to be

$$\begin{aligned}
F^B_{\text{w}x} &= I\ell_y B_z\\
B_z &= F^B_{\text{w}x}/I\ell_y = (-3.0\ \text{N})/(20\ \text{A})(1.0\ \text{m}) = -0.15\ \text{T}\\
F^B_{\text{w}z} &= -I\ell_y B_x\\
B_x &= -F^B_{\text{w}z}/I\ell_y = -(-2.0\ \text{N})/(20\ \text{A})(1.0\ \text{m}) = 0.10\ \text{T}
\end{aligned}$$

So, the magnitude of the magnetic field is $B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(0.10\ \text{T})^2 + (0.10\ \text{T})^2 + (-0.15\ \text{T})^2} = 0.21\ \text{T}$ and its direction is given by the components.

27.66. As the blood flows through the artery, the magnetic force on the positive ions is in the opposite direction of the magnetic force on the negative ions, which causes charge separation across the artery. This charge separation reaches equilibrium when the electric field it produces causes an electric force that cancels the magnetic force, which happens when $E = vB$ (assuming the magnetic field is perpendicular to the artery). The potential difference is related to the electric field magnitude by $E = \Delta V/d$, so $\Delta V/d = vB$ or $d = \Delta V/vB = (1.0\times10^{-3}\ \text{V})/(0.60\ \text{m/s})(0.20\ \text{T}) = 8.3\times10^{-3}\ \text{m}$.

27.67. (*a*) The electrons will be undeflected when the electric and magnetic forces on them cancel, that is, when $E = vB$ or $v = E/B = (2.0\times10^3\ \text{N/C})/(1.2\times10^{-5}\ \text{T}) = 1.7\times10^8\ \text{m/s}$. (*b*) As the electrons pass through the region with the electric field, they are accelerated in the direction opposite to the field direction. This does not change the *x* component of their velocity; it just adds a new, perpendicular component, in what we'll call the *y* direction. When the electrons reach the end of the electric-field region, the *y* component of their velocity is $v_y = a_y\Delta t = \dfrac{Eq}{m}\dfrac{\Delta x}{v_x} = \dfrac{(2.0\times10^3\ \text{N/C})(1.60\times10^{-19}\ \text{C})(40.0\times10^{-3}\ \text{m})}{(9.1\times10^{-31}\ \text{kg})(1.7\times10^8\ \text{m/s})} = 8.4\times10^4\ \text{m/s}$, and the *y* component of their displacement is $\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{1}{2} \frac{Eq}{m}\left(\frac{\Delta x}{v_x}\right)^2 = \frac{(2.0\times 10^3\text{ N/C})(1.60\times 10^{-19}\text{ C})(40.0\times 10^{-3}\text{ m})^2}{2(9.1\times 10^{-31}\text{ kg})(1.7\times 10^8\text{ m/s})^2} = 1.0\times 10^{-5}\text{ m}.$ As they travel an additional 300 mm to the screen, the y component of their displacement increases to $\Delta y_\text{f} = \Delta y_\text{i} + v_y \Delta t = \Delta y_\text{i} + v_y \frac{\Delta x}{v_x} = (1.0\times 10^{-5}\text{ m}) + \frac{(8.4\times 10^4\text{ m/s})(300\times 10^{-3}\text{ m})}{(1.7\times 10^8\text{ m/s})} = 1.6\times 10^{-4}\text{ m}.$

27.68. If the particle is moving in a circle of radius r in a uniform magnetic field of magnitude B, we know that $r = \frac{mv}{|q|B}$ (Eq. 27.23). So, its momentum $p = mv = r|q|B,$ and its kinetic energy $K = \frac{p^2}{2m} = \frac{(r|q|B)^2}{2m} = \frac{r^2 q^2 B^2}{2m}.$

27.69. Take two of the magnets and find two poles that attract. Label these poles, say, by wrapping a piece of tape around them. According to your colleague, these must be like poles. Set one of these magnets aside (let's call it the first one), pick up the third, and find which pole of the third magnet is attracted to the labeled pole of the second magnet. According to your colleague, this must be another like pole, so it should be attracted to the labeled pole of the first magnet. But it's not; it's repelled. If the labeled pole on the first magnet was a north pole, then the labeled pole on the second magnet was a south pole, which attracted the north pole of the third magnet, and the north poles of the first and third magnets repel each other.

27.70. When there is a current in the rod, it experiences an upward magnetic force of magnitude $F_\text{r}^B = I\ell B \sin 45°.$ Both with and without the current, when the rod is in equilibrium, the vector sum of the forces on the rod is zero. Aside from the magnetic force, the other forces exerted on the rod are the force of gravity and the forces due to the tension in the ropes. Without the current, we have for the z component of the forces

$$-m_\text{rod} g + 2\mathcal{T}\sin 30° = 0$$

$$\mathcal{T} = \frac{m_\text{rod} g}{2\sin 30°} = \frac{(0.350\text{ kg})(9.8\text{ m/s}^2)}{2\sin 30°} = 3.43\text{ N}$$

With the current we have

$$-m_\text{rod} g + I\ell B \sin 45° + 2\mathcal{T}\sin 5.0° = 0$$

$$\begin{aligned} I &= \frac{m_\text{rod} g - 2\mathcal{T}\sin 5.0°}{\ell B \sin 45°} \\ &= \frac{(0.350\text{ kg})(9.8\text{ m/s}^2) - 2(3.43\text{ N})\sin 5.0°}{(1\text{ m})(0.25\ \mathcal{T})\sin 45°} = 16\text{ A} \end{aligned}$$

27.71. As we saw in Problem 36, the solution is easier than it first appears. Imagine dividing the wire into many small segments of length $d\ell$. The magnetic force on the whole wire equals the vector sum of the magnetic forces on each segment of the wire, $\vec{F}_\text{w}^B = \sum I\, d\vec{\ell}_n \times \vec{B}.$ In the limit as the length of each segment approaches zero, the sum becomes an integral and we have $\vec{F}_\text{w}^B = \int_{\vec{r}_1}^{\vec{r}_2} I\, d\vec{\ell} \times \vec{B},$ where $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the beginning and end of the wire. Because $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a},$ $\vec{F}_\text{w}^B = -\int_{\vec{r}_1}^{\vec{r}_2} I\vec{B}\times d\vec{\ell}.$ Both I and $\vec{B}$ are constants, so we can bring them outside the integral giving $\vec{F}_\text{w}^B = -I\vec{B}\times\int_{\vec{r}_1}^{\vec{r}_2} d\vec{\ell}.$ The remaining integral is the vector sum of the infinitesimal segments of wire, that is, the displacement between the ends of the wire $\int_{\vec{r}_1}^{\vec{r}_2} d\vec{\ell} = \vec{r}_2 - \vec{r}_1,$ so $\vec{F}_\text{w}^B = -I\vec{B}\times(\vec{r}_2 - \vec{r}_1) = I(\vec{r}_2 - \vec{r}_1)\times\vec{B}.$ So, the magnitude of the magnetic force is $F_\text{w}^B = 2IRB$ and its direction is perpendicular to both the magnetic field and the diameter connecting the ends of the wire.

27.72. For convenience, let's choose a coordinate system. Let the x axis point to the right along the rotation axis of the loop, let the y axis point upward, and let the z axis point out of the plane of the page.

When there is current in the loop, there will be magnetic forces on the wires that make up the loop. If the magnetic field has a nonzero x component, there will be magnetic forces on the left and right sides of the loop directed into and out of the plane of the page, but these won't cause the loop to move because it is attached to the axle. No matter how the loop might turn, this remains the case, so we can ignore this component of the magnetic field.

If the magnetic field has only a nonzero y component, there will be magnetic forces on the top and bottom sides of the loop, directed into the plane of the page on one side and out of the plane of the page on the other, creating a torque about the rotation axis that causes the loop to rotate. As the loop rotates, the torque becomes smaller because the lever arm of the forces, which continue to point in the positive and negative z directions, becomes smaller, and when the plane of the loop is oriented parallel to the magnetic field, the torque becomes zero. If there is friction in the axle, the loop stops in this orientation.

If the magnetic field has only a nonzero z component, there will be magnetic forces on all four sides, directed either toward or away from the center of the loop depending on whether the current is clockwise or counterclockwise. These two cases, however, are very different should the loop turn by even the tiniest amount. If the magnetic forces are directed toward the center of the loop, as the loop turns they create a torque that tends to make the loop turn further, while if the magnetic forces are directed away from the center of the loop, they create a torque that tends to return the loop to its original position. That is, in the first case, the loop is in a state of unstable equilibrium, while in the second case, the loop is in a state of stable equilibrium. This state of stable equilibrium is where the loop winds up after it is released, so it was originally in the state of unstable equilibrium. That is, the magnetic field has only a z component (ignoring any x component, which, as described above, does not affect the loop's motion) and, if that component is positive, there is a clockwise current in the loop, or a counterclockwise current if the z component of the magnetic field is negative.

There are at least two ways to keep the loop rotating in the same direction. If we could get the magnetic field to rotate, the loop could continue rotating in the same direction. We might do this by moving the magnets that produce the field, or by using multiple magnets in different orientations and switching them on and off at the appropriate times. Or we could cause the current in the loop to change direction at the appropriate times. In fact, both of these methods—rotating magnetic fields and reversing current directions—are used in electric motors, more about which we will learn in the next chapter.

28

MAGNETIC FIELDS OF CHARGED PARTICLES IN MOTION

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{-5} T **2.** 10^{1} A **3.** 10^{-3} T **4.** 10^{-2} T **5.** 10^{-2} N/m **6.** 10^{9} A **7.** 10^{1} T **8.** 10^{-12} T

Guided Problems

28.2 Bell wire

1. Getting Started The student runs a wire to a bell and the wire carries current. We know that a current-carrying wire produces a magnetic field. We know the direction of the field is given by the right hand rule, and we have determined magnitudes for the magnetic fields produced by some geometries of wire. Let us assume that the long, straight wire segments are semi-infinite. That is, let us assume that they extend off in one direction so far that the endpoint can be assumed to be approximately infinitely far away. Of course, we can only assume that for the free ends of the straight wire segments.

2. Devise Plan Let us break the problem down into parts by considering each of three segments separately: the top semi-infinite wire, the curved segment, and the lower semi-infinite wire. One way of finding the magnetic field at the point P is to apply the following equations:

$$\vec{B} = \int_{\substack{\text{current}\\ \text{path}}} d\vec{B}_s \qquad (28.10)$$

$$d\vec{B}_s = \frac{\mu_0}{4\pi}\frac{I d\hat{\ell} \times \hat{r}_{sP}}{r_{sP}^2} \qquad (28.12)$$

One could write out the differential contribution to the magnetic field using (28.12), in terms of the given geometry. One could then integrate as in equation (28.10) to add up the differential contributions and obtain the entire field. This can be done using Example 28.6 as a guide. This is a fine approach. However, in this case, we can simplify the problem a great deal by modifying expressions we already have. Recall that the expressions for the magnetic field due to an infinitely long wire, and due to a circular loop of wire have already been determined in Examples 28.3 and 28.7 respectively:

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d}$$

$$B_{\text{arc}} = \frac{\mu_0 I \phi}{4\pi R} \qquad (1)$$

The formula for the wire was derived for a single infinite wire. But we can use symmetry to argue that each half of an infinitely long wire contributes the same amount to the magnetic field. Thus we know that the magnetic field produced by a semi-infinite wire is

$$B_{\text{semi}} = \frac{\mu_0 I}{4\pi d} \tag{2}$$

Thus we have expressions for each of the three segments of wire in the problem. We can add or subtract them once we determine their directions.

3. Execute Plan Using our right hands, we orient our thumbs along the direction in which the positive current is flowing. When we do this with the top semi-infinite wire, our fingers curl around and point into the page at the point P. Similarly, our fingers point into the page at point P when we orient our thumbs along the curved arc, and along the lower semi-infinite wire. Thus all wire segments contribute a magnetic field in the same direction: into the page. Now we can add the magnitudes of the fields due to each segment using equations (1) and (2) for the various segments:

$$\begin{aligned} B &= B_{\text{semi}} + B_{\text{loop}} + B_{\text{semi}} \\ B &= \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I\phi}{4\pi R} + \frac{\mu_0 I}{4\pi R} \\ B &= \frac{\mu_0 I}{4\pi R}(2+\phi) \qquad (3) \\ B &= \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}(0.540\ \text{A})}{4\pi(0.010\ \text{m})}\left(2+\frac{\pi}{2}\right) \\ B &= 1.9\times10^{-5}\ \text{T} \end{aligned}$$

Thus we find the magnetic field at point P is $1.9\times10^{-5}\ \text{T}$ into the page.

4. Evaluate Result Equation (3) shows the functional dependence of the magnetic field on the radius of curvature and on the angle subtended by the circular arc. Notice that the magnetic field decreases linearly with R, which fits our expectation. Note also that if the arc angle is decreased to zero, equation (3) becomes equal to the magnetic field due to a single infinite wire, as it should.

28.4 Force on a rectangular current loop

1. Getting Started We first determine the magnetic field caused by the long straight wire near the circuit element. Then we determine the force on each current-carrying segment of the rectangular loop as it sits in the magnetic field from the long straight wire.

We are not given dimensions of the long, straight wire, so we will treat it as though it is infinitely long. This is likely a good assumption because the other length scales involved are less than one millimeter (even a 1-m wire would be nearly infinite by comparison).

2. Devise Plan We know that the magnetic field produced by a long straight wire is given by (see Example 28.3)

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d} \tag{1}$$

Here d is the perpendicular distance away from the long straight wire.

The right hand current rule tells us that the magnetic field due to the long, straight wire points into the page below the wire where the circuit element sits. We can then determine the direction of the magnetic force using the right hand force rule. We find the force on the top segment is upward, on the right segment is rightward, on the bottom segment is downward, and on the left segment is leftward.

At first glance it may seem as though all the forces will cancel each other. But note that the magnetic field strength produced by the long straight wire decreases as we move away from it. So the force on the bottom segment of the circuit element is directed downward, but we expect it will be weaker than the upward force on the top segment. So we do expect a net upward force. The left and right segments do cancel, though. Because the left and right wires are not one fixed distance from the long straight wire, each differential piece of the left and right wires is sitting in a different magnetic field. But for every such segment on the left wire, there is an identical segment on the right wire at the same distance from the long straight wire. The forces on such segment on the left and right wires will cancel each other out, and there will be no net force to the left or right.

To determine the magnetic force acting on a segment of wire due to a long straight wire, we use equation (28.16):

$$F_{12}^{\mathrm{B}} = \frac{\mu_0 \ell I_1 I_2}{2\pi d}$$

3. Execute Plan We have already determined that only forces toward or away from the wire will contribute, because forces to the left and right cancel. Thus, the sum of all forces toward the wire is

$$F^B = F^B_{\text{wire top}} - F^B_{\text{wire bottom}}$$

$$F^B = \frac{\mu_0 \ell I_{\text{wire}} I_{\text{segment}}}{2\pi x} - \frac{\mu_0 \ell I_{\text{wire}} I_{\text{segment}}}{2\pi(x+w)}$$

$$F^B = \frac{\mu_0 \ell I_{\text{wire}} I_{\text{segment}}}{2\pi}\left(\frac{1}{x} - \frac{1}{x+w}\right)$$

$$F^B = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(5.7\times10^{-3}\ \mathrm{m})(0.039\ \mathrm{A})^2}{2\pi}\left(\frac{1}{(0.300\times10^{-3}\ \mathrm{m})} - \frac{1}{(0.300\times10^{-3}\ \mathrm{m})+(0.90\times10^{-3}\ \mathrm{m})}\right)$$

$$F^B = 4.3\times10^{-9}\ \mathrm{N}$$

So the total magnetic force on the rectangular loop is $4.3\times10^{-9}\ \mathrm{N}$ toward the long straight wire.

4. Evaluate Result This force is extremely weak. One would expect this to be the case, because one cannot have circuit elements being pulled apart inside a circuit board, simply because current is flowing. In the lines above we have the expression

$$F^B = \frac{\mu_0 \ell I_{\text{wire}} I_{\text{segment}}}{2\pi}\left(\frac{1}{x} - \frac{1}{x+w}\right)$$

From this we can see that as the width w becomes large, there is only an attractive term, and the force per unit length reduces to

$$\frac{F^B}{\ell} = \frac{\mu_0 I_{\text{wire}} I_{\text{segment}}}{2\pi x}$$

Which is practically identical to equation (28.16), except for slightly different variable names. Note that if w becomes very small, then it begins to look like two wires of equal and opposite current right on top of each other. In limit as the width approaches zero, we expect it to be as though there is no net current at the location of the two wires, and so we would except no force on the wires. We see that as $w \to 0$ the above expression reduces to

$$F^B \to \frac{\mu_0 \ell I_{\text{wire}} I_{\text{segment}}}{2\pi}\left(\frac{1}{x} - \frac{1}{x}\right) = 0$$

28.6 Magnetic field in a coaxial cable

1. Getting Started This problem is similar to Worked Problem 28.5 in that there is cylindrical symmetry which might allow us to use Ampere's Law. In both problems, we are trying to determine the magnetic field at different locations in and around cables by using Ampere's Law.

2. Devise Plan We are not asked for the magnetic field inside the metal of the inner conductor, only for the magnetic field between the inner conductor and outer shell, and for the magnetic field outside the outer shell. So we are not concerned with the exact radius of the inner conductor. For concreteness, we can take the limit as it becomes very small, so that for any reasonable distance r from the cylinder axis, that point will be outside the inner conductor. As in Worked Problem 28.5, we will call the outer radius of the wire (the radius of the outer shell) R. Also as in Worked Problem 28.5 we can make a circular Amperian path centered on the cylinder's axis and having radius r. We can apply Ampere's Law to relate the magnetic field to the enclosed current.

3. Execute Plan Ampère's Law tells us

$$\oint_{\substack{\text{Amperian}\\\text{path}}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

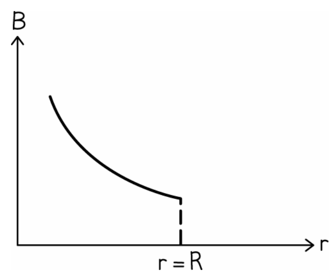
In this case, because our Amperian loop is centered on the cylinder's axis, the magnetic field should be parallel to the loop and equal in magnitude at all points. This allows us to reduce Ampere's law to $2\pi rB = \mu_0 I_{\text{enc}}$ or

$$B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} \tag{1}$$

In between the inner wire and the outer shell, our Amperian path has enclosed only the inner conducting wire. So we have

$$B_{\text{in}} = \frac{\mu_0 I}{2\pi r}$$

Once $r > R$, the Amperian loop encloses both the inner wire and the outer shell. Since the two conductors carry equal currents in opposite directions, the total enclosed current at this point is zero. Inserting this into equation (1), we find $B_{\text{out}} = 0$. Graphically, the magnetic field dependence is described by the following figure:



4. Evaluate Result The magnetic field in between the inner wire and the outer shell is very reasonable. It is the same $1/r$ dependence that we are accustomed to. In fact, equation (1) is simply the magnetic field due to a long straight wire. Outside the outer shell, the behavior is as described in the problem statement. One can understand this qualitatively by noting that the entire coaxial cable carries no net current in either direction. And the two current distributions (inner wire and outer shell) are centered on exactly the same axis. Thus, obtaining zero magnetic field outside the entire coaxial cable is quite reasonable.

28.8 Tubular current

1. Getting Started The problem setup has cylindrical symmetry. That is, the entire setup can be rotated around the axis at the center of the cylinder without changing any physical property. Because of this symmetry, the path integral in Ampere's Law is easy to calculate, making Ampere's Law a relatively simple way to approach the problem.

2. Devise Plan In order to use Ampere's law to answer the entire problem, we must consider an Amperian path at various positions. In all cases, the Amperian path should be a circle centered on the cylinder's axis of symmetry. We will need three such paths of radii $r < R_{\text{in}}$, $R_{\text{in}} < r < R_{\text{out}}$, and $r > R_{\text{out}}$ in order find the magnetic field in each of the three regions specified.

In the region (*a*), no current is enclosed, and this can be seen without calculation. In region (*c*), the entire current I is enclosed, because the entire wire is enclosed. Again this requires no calculation. However, in region (*b*), the amount of current enclosed by the loop depends on what fraction of the cross-sectional area of the loop is enclosed.

3. Execute Plan Ampère's law tells us

$$\oint_{\substack{\text{Amperian}\\ \text{path}}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

As long as we use the Amperian paths described above, the magnetic field should have the same magnitude at all points along a given Amperian path, and the field should always be parallel to the path. Thus Ampere's Law reduces to $2\pi r B = \mu_0 I_{\text{enc}}$ or

$$B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} \tag{1}$$

(a) Since no current is enclosed by an Amperian loop with $r < R_{\text{in}}$, it follows trivially from equation (1), that the magnetic field in region (a) is zero: $B(r < R_{\text{in}}) = 0$. (b) In region (b), the fraction of current enclosed is the fraction of the conducting cross-sectional area enclosed (because the current density is uniform). Note that the hollow region in the center does not carry current, so it cannot be counted in our total cross-sectional area of current-carrying wire, nor in the fraction of the cross section that is enclosed. Thus

$$\frac{I_{\text{enc}}}{I} = \frac{A_{\text{enc}}}{A}$$

$$\frac{I_{\text{enc}}}{I} = \frac{\pi(r^2 - R_{\text{in}}^2)}{\pi(R_{\text{out}}^2 - R_{\text{in}}^2)}$$

$$I_{\text{enc}} = I\frac{(r^2 - R_{\text{in}}^2)}{(R_{\text{out}}^2 - R_{\text{in}}^2)}$$

Inserting this into equation (1) yields

$$B(R_{\text{in}} < r < R_{\text{out}}) = \frac{\mu_0 I}{2\pi r}\frac{(r^2 - R_{\text{in}}^2)}{(R_{\text{out}}^2 - R_{\text{in}}^2)}$$

(c) In region (c), the entire current is enclosed, and equation (1) yields

$$B(r > R_{\text{out}}) = \frac{\mu_0 I}{2\pi r}$$

4. Evaluate Result Note that the expression in region (b) reduces to zero in the limit as $r \to R_{\text{in}}$, such that the expressions for region (a) and (b) match at their boundary. Note also that as $r \to R_{\text{out}}$, the expression in region (b) reduces to $\dfrac{\mu_0 I}{2\pi r}$, thus matching the field strength in region (c). The dependence on r in region (b) is rather complex: the difference between a term proportional to r and one proportional to $1/r$. But the matching at boundaries gives us confidence. The dependence on r in region (c) is clearly correct, since this is the same old expression for the magnetic field a distance r from a long straight wire.

Questions and Problems

28.1. Because the particle is at rest, there is no magnetic force exerted on it.

28.2. Each current-carrying wire produces a magnetic field that curls around the wire in the direction given by the right-hand current rule. At a point midway between the wires, the magnetic fields of the two wires point in opposite directions. Because the wires carry the same current, the magnitudes of the magnetic fields of the two wires are equal, so their vector sum is zero.

28.3. (a) Applying the right-hand current rule, we see that the magnetic field at the center of the loop due to the left side is directed downward. The magnetic fields due to (b) the top side, (c) the right side, and (d) the bottom side are all directed downward as well.

28.4. Because the particle is at rest, there is initially no magnetic force exerted on it. Because the particle is negatively charged, there is an electric force exerted on it in the negative y direction, opposite the direction of the electric field. So, the vector sum of the forces is initially in the negative *y* direction. If the particle is free to move, it will initially accelerate in the negative *y* direction, after which there will be a magnetic force exerted on it in a direction perpendicular to both the magnetic field and its velocity as determined by the right-hand force rule, that is, in the negative *z* direction at first.

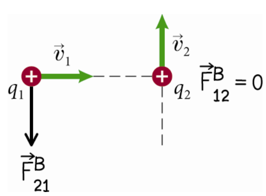
28.5. Considering only the electrons, there are three forces exerted on each of them by the other. Because of their like charge, there is a strong electric force pushing them away from each other. Because they are moving in the Earth reference frame, each creates a magnetic field and is subject to the magnetic field created by the other, so there is a weak magnetic force attracting them together. Because both electrons have mass, there is also an extremely weak gravitational force attracting them together.

28.6. The magnetic force between the protons is repulsive. To see this, consider each proton as a tiny length of current-carrying wire. The direction of each current is the same as the direction of the proton's velocity, that is, the direction of the two currents is opposed, and parallel wires carrying currents in opposite directions exert repulsive magnetic forces on each other.

28.7. The magnetic field of each wire forms loops around the wire and, in the plane defined by the wires, it is directed into the page on one side of the wire and out of the page on the other. The fields due to the horizontal wire are in one direction, either into or out of the page, at points 1 and 4, and in the opposite direction at points 2 and 3. The fields due to the vertical wire are in one direction at points 1 and 2, and the opposite direction at points 3 and 4. Because each wire carries the same current and the points are equidistant from the wires, the magnitudes of the fields produced by each wire are equal at each of the four points. So, there is (*a*) one point where the magnetic field is directed out of the page, (*b*) one point where it is directed into the page, and (*c*) two points where it is zero.

28.8. Each wire produces a magnetic field that loops around the wire. According to the right-hand current rule, the loops are all counterclockwise. The field at P caused by the top left wire is directed upward, that caused by the bottom left wire is directed upward and toward the left at a $45°$ angle from the vertical, and the field at P caused by the bottom right wire is directed toward the left. Because the top left and bottom right wires carry the same current and are equal distances from point P, the magnitude of the field each produces at P is the same, and their vector sum is directed upward and toward the left at a $45°$ angle from the vertical, the same as the field from the third wire. So, the vector sum of all three fields is directed upward and toward the left at a $45°$ angle from the vertical.

28.9. Each particle creates a magnetic field in the form of loops around its direction of motion. Because particle 2 is on the line of particle 1's velocity, particle 1 creates no magnetic field at particle 2's location and no magnetic force is exerted on particle 2. Because particle 1 is on a line perpendicular to particle 2's velocity, particle 2 creates a magnetic field at particle 1's location, which is directed out of the page, and so exerts a magnetic force on particle 1 in the downward direction.



28.10. The magnetic field produced by each side of the loop forms circles around the wire, with the direction given by the right-hand current rule. (*a*) At point P the magnetic field due to segment 1 points into the page, and that due to segment 3 points out of the page. If segments 2 and 4 produce any field at P, it should be small and directed into the page. (*b*) Side 3 is closest to P, so it produces the strongest field there. (*c*) We need to know how the strength of the field depends on the angle and distance from the wire.

28.11. Along the axis of the loop, we expect the magnitude to be greatest in the plane of the loop, because the contributions from all segments of the loop are in alignment there and the distance from a point on the axis to the loop is smallest there. Away from the axis, we would need to know how the field magnitude depends on distance, but we might expect it to be greatest right next to the inner surface of the wire.

28.12. We know that rectangular magnets have fields that are similar to those of circular magnets, so yes, it could, although the field would no longer be circularly symmetric around the long axis of the magnet.

28.13. We can find the field direction by applying the right-hand current rule for a few segments around the loop and considering how the components of the vector sum add together. At points 1 and 2, all segments of the loop produce a magnetic field in the positive z direction, that is, out of the page, so their vector sum is also in the positive z direction. At points 3 and 4, the nearer segments of the loop produce a magnetic field in the negative z direction, that is, into the page, while the more distant segments produce a magnetic field in the positive z direction. Because the magnitude of the field is greater near the wire than farther away, we expect the vector sum of the contributions from all the segments to be in the negative z direction.

28.14. The field of the spinning charged object is similar to that of a current loop in the xz plane. So, at points 1 and 3, the field is in the positive y direction, at points 2 and 5, the field is in the negative y direction, and at points 4 and 6, the field direction is mostly to the right, in the positive x direction, although it could also have a y component.

28.15. Far from the loops, we would not expect their shapes to have much effect on the magnetic fields they produce. Near the loop, the fields will be similar, but differ in detail. In particular, the field of the square loop will not have the circular symmetry that the field of the circular loop has.

28.16. A charged particle produces a magnetic field when it is in motion, either translating or spinning. Likewise, a charged particle interacts with a magnetic field when it is in motion, either translating or spinning. So, there can only be a magnetic interaction between the particles when both are in some sort of motion, either translating, spinning, or both, that is, under circumstances c, f, g, h, i, and j.

28.17. We can think of the disk as being made up of concentric circular rings, all spinning with the same angular velocity. The current in each ring, that is, the charge that passes through a cross-sectional area per unit time, is proportional to the charge it contains, which is proportional to its radius. The magnetic field of the disk equals the vector sum of the fields of these concentric circular rings. In the plane of the disk, at distances farther from the axis than the radius of the disk, the fields of all the concentric rings have the same direction, so they add together to give a field of greater magnitude than that of any of the individual rings. The same is true along the axis. In between, in the plane of the disk, the field due to the smaller rings has the opposite direction to that due to the larger rings, so the magnitude of the field will be smaller than that of a loop in this region.

28.18. The density of magnetic field lines is greatest in the body of the magnet, at point 2. We can see this by applying Gauss's law for magnetism to a spherical surface that encloses the north pole of the magnet. The outward flux through the large part of the surface outside the magnet is offset by the inward flux through the small part of the surface that intersects the magnet, so the field-line density must be greatest there.

28.19. The spinning charged particle produces a magnetic field similar to the one produced by a small current-carrying loop. Because the particle is negatively charged, the direction of the current in the loop is opposite to the direction the particle spins. So, (*a*) at points along the x axis, the magnetic field is in the positive x direction, (*b*) at points along the y axis, the magnetic field is in the negative x direction, and (*c*) at points along the z axis, the magnetic field is in the negative x direction.

28.20. The magnetic field lines form loops around the wire, the direction of which is given by the right-hand current rule. At point P, the magnetic field from any small segment of wire in the semicircular section points out of the page, so the vector sum of these contributions to the field at P must also point out of the page.

28.21. Because both particles are spinning, they create magnetic fields and are subject to magnetic interactions. So, each particle interacts magnetically with the other because of their spins. The electron is also in orbit around the proton, and its orbital motion gives it another way to interact with the magnetic field of the spinning proton. The orbiting electron also forms a current loop with an associated magnetic field, and the particles interact magnetically with this orbital field as well.

28.22. By definition, the north pole of a magnet is the one that is attracted to Earth's geographic North Pole. That is, outside of Earth, the magnetic field lines are directed toward the North Pole, so Earth's magnetic dipole moment points toward the South Pole.

28.23. By definition, the north pole of a magnet is the one that is attracted to Earth's geographic North Pole. That is, outside of Earth, the magnetic field lines are directed toward the North Pole, so Earth's magnetic dipole moment points toward the South Pole. So, by the right-hand dipole rule, the current must circulate clockwise as viewed by an observer above the North Pole.

28.24. Because the counterclockwise motion of the electrons corresponds to a clockwise current, by the right-hand dipole rule the disk's magnetic dipole moment points down into the page, in the negative z direction.

28.25. (*a*) Electrons move from lower to higher potential, so they move from end 2 to end 1, counterclockwise through the loop. (*b*) The direction of the current is opposite the direction of the negatively charged electrons, that is, clockwise through the loop. (*c*) By the right-hand dipole rule, the loop's magnetic dipole moment is directed into the page.

28.26. If the magnetic dipole moment of the loop had a component to the right, there would be a counterclockwise torque on the loop, turning it in the opposite direction. In order to keep the loop rotating in the clockwise direction, its magnetic dipole moment must have a left-pointing component, which is accomplished by periodically reversing the direction of the current through the loop. So, during no fraction of its rotation does the loop's magnetic dipole moment have a right-pointing component.

28.27. The torque on the loop tends to align its magnetic dipole moment with the external magnetic field, which points in the positive x direction. Because the torque caused the loop to rotate clockwise about the y axis, its magnetic dipole moment originally pointed in the negative z direction, so the current in the loop was clockwise as viewed from a point on the positive z axis.

28.28. According to the right-hand dipole rule, the loop's magnetic dipole moment initially points in the positive z direction. If the loop experiences a counterclockwise torque about the x axis, it tends to rotate so that its magnetic dipole moment points in the negative y direction. Because the torque on the loop tends to align its magnetic dipole moment with the external magnetic field, the field must point in the negative y direction.

28.29. In each case, we can apply the right-hand dipole rule to the current to find the direction of the magnetic dipole moment. (*a*) positive y direction (*b*) negative y direction (*c*) the current is directed from the higher potential end of the wire to the lower potential end, so the magnetic dipole moment points in the negative x direction (*d*) the counterclockwise (as viewed from a point on the positive z axis) motion of negative charge corresponds to a clockwise current, so the magnetic dipole moment points in the negative z direction (*e*) a long straight wire produces a magnetic field that loops around the wire and has no poles, so it has no magnetic dipole moment.

28.30. (*a*) The direction of current in the loop is from higher to lower potential. Applying the right-hand dipole rule, we see that when the black end of the loop is in contact with the positive half of the commutator, the magnetic dipole moment of the loop has a left-pointing component, and the same is true when the other end of the loop is in contact with the positive half of the commutator. Only arrows f and g have a left-pointing component, so only they could represent the magnetic dipole moment of the loop. (*b*) As we saw in part a, no matter what the orientation of the loop, its magnetic dipole moment is either zero (when the ends of the loop are not in contact with the commutator and the direction of the current in the loop is reversing), or it has a left-pointing component, which none of the other arrows have.

28.31. (*a*) The direction of the force is given by the right-hand force rule, and is perpendicular to both the direction of the current and the direction of the magnetic field. On side 1 the force is directed upward, on side 2 it's directed out of the page, on side 3 it's directed downward, and on side 4 it's directed into the page. (*b*) Only the forces exerted on sides 1 and 3 have a nonzero lever arm about the axis, and they produce a torque directed into the page. (*c*) Once the system reaches equilibrium, the magnetic dipole moment of the loop will be parallel to the magnetic field, so the plane of the loop will be perpendicular to the magnetic field with side 1 at the top.

28.32. (*a*) If the direction of the current is reversed, the direction of each of the forces is reversed, and so is the direction of the torque on the loop. (*b*) No, this situation is not any different—the magnetic dipole moment of the loop is again parallel to the external magnetic field—although the loop rotated in the opposite direction to reach this orientation.

28.33. (*a*) Yes. If the particle were not spinning, it would not be subject to a magnetic force because it is initially at rest. Because it is spinning, it has a magnetic dipole moment that points in the positive z direction, so the magnetic field causes a torque on the particle in the positive y direction. (*b*) If the particle were not spinning, its initial angular momentum would be zero. The electromagnetic field it produces would also be different.

28.34. According to Ampère's law, the line integral of a magnetic field along a closed path is proportional to the current encircled by the path. Path a encircles no current, so the line integral is zero. Path b also encircles no current, so the line integral is zero. Path c encircles current of magnitude I and has the same direction as the magnetic field, so the line integral is positive. Path d also encircles current of magnitude I, but its direction is opposite to the magnetic field, so the line integral is negative. Path e encircles the loop, but the current in the two sides of the loop are in opposite directions, so the net current encircled by the path is zero and so is the line integral.

28.35. The Ampèrian path does not encircle wires 2 and 4. Perhaps the easiest way to see this is to notice that, starting outside the path at the bottom of the figure, we can reach first wire 4 and then wire 2 without crossing the path. So, the path encircles a total current of magnitude $3I$ directed into the page. The path is counterclockwise, so it is in the same direction as the magnetic field produced by wire 1 and in the opposite direction to the fields produced by wires 3 and 5, so the path integral is negative.

28.36. The path encircles two wires, for a total encircled current of magnitude I directed out of the page. Because the direction of the path is the same as the direction of the magnetic field caused by the total encircled current, the line integral is positive.

28.37. According to Ampère's law, the line integral of a magnetic field along a closed path is proportional to the current encircled by the path. Paths a, b, and f each encircle a current of magnitude I. Path c encircles a current of magnitude $2I$, path d encircles a current of magnitude $3I$, and path e encircles a total current of zero. So, the ranking is e < a = b = f < c < d.

28.38. According to Ampère's law, the line integral of a magnetic field along a closed path is proportional to the current encircled by the path. (*a*) If the current is doubled but the path integral remains the same, it must be the case that the path integral equals zero, and so the total current encircled by the path must be zero. (*b*) If doubling the current doubles the path integral, we can't say anything about the current encircled by the path.

28.39. Because the line integrals around all three paths are positive, we know that the direction of the magnetic field produced by the enclosed current is the same as the direction of the path. Because the line integrals have the same magnitude, the enclosed currents must also have the same magnitude. So, wire 1 carries a current directed out of the page, wire 2 carries a current of the same magnitude directed into the page, and wire 3 carries a current of the same magnitude directed out of the page.

28.40. The spinning particle creates a magnetic dipole field and, by the right-hand dipole rule, its magnetic dipole moment is in the positive z direction. (*a*) The magnetic field is perpendicular to any path in the xy plane, so the line integral is zero. (*b*) The magnetic field is perpendicular to the path when it passes through the z axis. On the portion of the path where y is positive, the field is in the same direction as the path, while on the other half of the path, the field is in the opposite direction as the path. Because of the symmetry of the field, these two halves sum to zero. (*c*) The magnetic field is perpendicular to the path when it passes through the z axis. On the portion of the path where x

is negative, the field is in the same direction as the path, while on the other half of the path, the field is in the opposite direction as the path. Because of the symmetry of the field, these two halves sum to zero. Another way to see this is to consider the spinning particle as a tiny current loop. The path in part a does not encircle any current, while the paths in parts b and c encircle both sides of the loop, for a net current of zero.

28.41. The line integral of the magnetic field along the path is greater than zero. The path encircles only the central wire, and does so in a clockwise direction, which is the same direction as the magnetic field produced by the central wire.

28.42. By considering line integrals along Ampèrian paths around each of the numbered points, we see that they are nonzero only around points 3 and 7. Applying the right-hand current rule, we see that the current at 3 is directed out of the page, while the current at 7 is directed into the page.

28.43. (*a*) Both paths encircle the same current, so by Ampère's law, the line integral of the magnetic field along both paths is the same. (*b*) The average magnetic field is greater along path A because it is closer to the wire. (*c*) They are consistent because the line integral does not measure the average magnetic field; it measures the component of the magnetic field parallel to the path times the length of the path. Even though the magnetic field has greater magnitude along path A, path B is longer, so the integrals can have the same value.

28.44. Consider the closed Ampèrian path formed by first following path 2, then following the positive x axis until it intersects path 1, then following path 1 counterclockwise until it intersects the y axis, and finally following the y axis back to the start of path 2. This path encircles no current, so the line integral around it is zero. The line integral along the whole path is also equal to the sum of the line integrals along the four parts of the path. The magnetic field produced by the wire forms counterclockwise circles around the wire. This is perpendicular to the parts of the path that lie along the x and y axes, so the line integrals along these parts are zero. The magnetic field is parallel to the part of the path that lies along path 1, so the line integral along this part is positive. Because the magnetic field is symmetrical around the wire and this path is one quarter the length of path 1, the magnitude of the line integral equals one quarter of the magnitude of the line integral along path 1, that is, $1.38\ \text{T}\cdot\text{m}$. Because the line integral around our closed Ampèrian path is zero, the line integral along path 2 must be $-1.38\ \text{T}\cdot\text{m}$.

28.45. The magnetic field of the wire forms circles around the wire, so we can apply Ampère's law to a circular path of radius 25 mm centered on the wire.

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

$$\oint \vec{B}\cdot d\vec{\ell} = B\oint d\ell = 2\pi rB$$

$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi(25\times10^{-3}\ \text{m})(2.0\times10^{-5}\ \text{T})}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 2.5\ \text{A}$$

28.46. We used Ampère's law to find the magnetic field of a large, flat current sheet in Example 28.4. There we found that the magnetic field is uniform and directed both parallel to the surface and perpendicular to the direction of the current. We have an expression (Eq. 27.7) for the magnetic force on a straight wire in a uniform field. Combining these we have

$$\begin{aligned} F^B_{\text{sw}} &= I\ell B \\ &= I\ell(\tfrac{1}{2}\mu_0 K) \\ F^B_{\text{sw}}/\ell &= \tfrac{1}{2}I\mu_0 K \\ &= \tfrac{1}{2}(1.5\ \text{A})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.0\ \text{A/m}) = 2.8\times10^{-6}\ \text{N/m} \end{aligned}$$

Applying the right-hand force rule, we see that the magnetic force on the wire is attractive, that is, downward.

28.47. (*a*) The magnetic field of the wire forms circles around the wire, so we can apply Ampère's law to a circular path of radius 6.2 mm centered on the wire.

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

$$\oint \vec{B}\cdot d\vec{\ell} = B\oint d\ell = 2\pi rB$$

$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi(6.2\times10^{-3}\text{ m})(4.0\times10^{-3}\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 1.2\times10^{2}\text{ A}$$

(*b*) In Example 28.3, we found an expression for the magnetic field of a long, straight current-carrying wire.

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$B(77\text{ mm}) = B(6.2\text{ mm})\frac{6.2\text{ mm}}{77\text{ mm}} = (4.0\times10^{-3}\text{ T})\frac{6.2\text{ mm}}{77\text{ mm}} = 3.2\times10^{-4}\text{ T}$$

28.48. Far from the wires, the magnetic field will be approximately that of a single wire carrying the difference of the currents in the two wires, that is,

$$B \approx \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1.0\text{ A})}{2\pi r} = \frac{2.0\times10^{-7}\text{ T}\cdot\text{m}}{r}$$

28.49. We have an expression for the magnetic force on a moving charged particle, which we can solve for the magnitude of the field that causes it. We also have an expression for the magnetic field of a long, straight current-carrying wire, and can combine these expressions to solve for the magnitude of the current.

$$F_{\text{p}}^{B} = qvB$$

$$B = \frac{F_{\text{p}}^{B}}{qv} = \frac{\mu_0 I}{2\pi r}$$

$$I = \frac{2\pi rF_{\text{p}}^{B}}{\mu_0 qv} = \frac{2\pi(3.0\times10^{-6}\text{ m})(7.0\times10^{-13}\text{ N})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(1.60\times10^{-19}\text{ C})(2.5\times10^{7}\text{ m/s})} = 2.6\text{ A}$$

28.50. (*a*) We can apply Ampère's law to a circular path a distance *r* from the center of the wire. Because of the symmetry of the current, the magnetic field is parallel to the path, and the encircled current is the fraction of the total current that passes through the area encircled by the path.

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

$$\oint \vec{B}\cdot d\vec{\ell} = B\oint d\ell = 2\pi rB$$

$$I_{\text{enc}} = I\frac{\pi r^2}{\pi R^2}$$

$$2\pi rB = \mu_0 I\frac{r^2}{R^2}$$

$$B = \frac{\mu_0 Ir}{2\pi R^2} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(6.0\text{ A})(20\times10^{-3}\text{ m})}{2\pi(25\times10^{-3}\text{ m})^2} = 3.8\times10^{-5}\text{ T}$$

(*b*) Here, outside the wire, the encircled current is the total current in the wire and

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(6.0\text{ A})}{2\pi(50\times10^{-3}\text{ m})} = 2.4\times10^{-5}\text{ T}$$

(*c*) We can solve our equation from part b for *I* and find

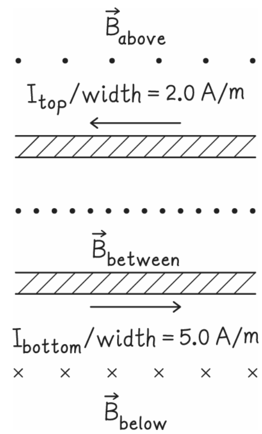
$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi(50\times10^{-3}\text{ m})(1.0\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = 2.5\times10^{5}\text{ A}$$

28.51. (*a*) The magnetic field is the vector sum of the magnetic fields produced by the two sheets. The fields of the two sheets are parallel between the sheets, and we have an expression for their magnitudes from Example 28.4.

$$B = \tfrac{1}{2}\mu_0 K_1 + \tfrac{1}{2}\mu_0 K_2 = \tfrac{1}{2}\mu_0 (K_1 + K_2)$$
$$= (2\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\big((2.0\ \text{A/m}) + (5.0\ \text{A/m})\big) = 4.4 \times 10^{-6}\ \text{T}$$

(*b*) Above the upper sheet, the fields of the two sheets are directed opposite to each other, so the magnitude is $B = (2\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\big((5.0\ \text{A/m}) - (2.0\ \text{A/m})\big) = 1.9 \times 10^{-6}\ \text{T}$. (*c*) Below the lower sheet, the fields are also directed opposite each other, so the magnitude is also $B = 1.9 \times 10^{-6}\ \text{T}$.

(*d*)



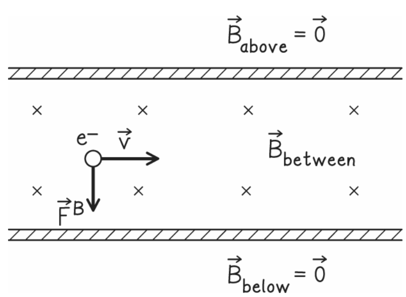
28.52. The magnitude of the magnetic force on the electron is given by Eq. 27.18. We have an expression for the magnetic field produced by a current sheet, and the total magnetic field is the vector sum of the field produced by each sheet. Between the sheets, the magnetic fields of the two sheets are parallel, so the magnitude of the total magnetic field is the sum of the magnitudes of the fields of each sheet, and they are directed perpendicular to the electron's velocity, so we have

$$B = \tfrac{1}{2}\mu_0 K_1 + \tfrac{1}{2}\mu_0 K_2 = \tfrac{1}{2}\mu_0 (K_1 + K_2)$$
$$= (2\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\big((8.0\ \text{A/m}) + (8.0\ \text{A/m})\big) = 1.0 \times 10^{-5}\ \text{T}$$

and

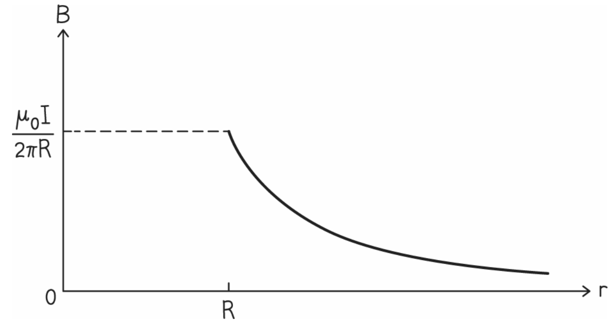
$$F_e^B = |q| vB$$
$$= (1.60 \times 10^{-19}\ \text{C})(3.0 \times 10^{6}\ \text{m/s})(1.0 \times 10^{-5}\ \text{T}) = 4.8 \times 10^{-18}\ \text{N}$$

The force on the electron is directed downward.



28.53. (*a*) Because of the cylindrical symmetry of the wire, we know that the magnetic field must form circular loops, so we can use Ampère's law on a circular path a distance r from the center of the wire. For the line integral, we get $\oint \vec{B}(r) \cdot d\vec{\ell} = B(r) \oint d\ell = 2\pi r B(r)$. For the encircled current, $I_{\text{enc}} = I$, for $r \geq R$, and $I_{\text{enc}} = 0$, for $r < R$. So, for $r \geq R$, $B(r) = \dfrac{\mu_0 I}{2\pi r}$, and for $r < R$, $B(r) = 0$.

(*b*)



28.54. Let's first assume that the sheet is very wide compared to the other distances in the problem, in which case we can approximate its field by that of an infinite sheet. Then, its field would be uniform and have magnitude $B_\text{s} = \frac{1}{2}\mu_0 K,$ where K is the current per unit width in the sheet, that is, $K = I_\text{s}/w.$ The field of the wire has magnitude $B_\text{w} = \dfrac{\mu_0 I_\text{w}}{2\pi r}.$ The fields have opposite directions at point P, so for the total field to have zero magnitude, the magnitudes of the individual fields must be equal. So we have $B_\text{s} = \frac{1}{2}\mu_0 K = \dfrac{\frac{1}{2}\mu_0 I_\text{s}}{w} = B_\text{w} = \dfrac{\mu_0 I_\text{w}}{2\pi d_2}.$ Solving for w gives $w = \frac{1}{2}\mu_0 I_\text{s}\dfrac{2\pi d_2}{\mu_0 I_\text{w}} = \dfrac{\pi d_2 I_\text{s}}{I_\text{w}},$ or $w = \dfrac{\pi(3.0\times 10^{-3}\text{ m})(40\text{ A})}{0.35\text{ A}} = 1.1\text{ m}.$ This is much greater than the other distances in the problem, so our initial assumption is justified.

28.55. Because the currents in both plates have the same direction, the magnetic force between them is attractive. We know the magnetic field produced by the lower plate is uniform and perpendicular to the current, with magnitude $B = \frac{1}{2}\mu_0 K.$ To calculate the force per unit area on the upper plate, let's consider an area of length ℓ and width w. We can imagine dividing this area into a number of thin wires of width dw, each carrying a current of Kdw. The magnetic force on one of these wires is $F_\text{w}^B = I\ell B = (Kdw)\ell B,$ and the force on the area is the sum of the forces on all the wires, that is, the integral with respect to w, $F_A^B = KBw\ell.$ The force per unit area is this force divided by the area, $F^B/A = F^B/w\ell = KB,$ and substituting our expression for the magnetic field, we have $F^B/A = \frac{1}{2}\mu_0 K^2 = \frac{1}{2}(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(100\text{ A/m})^2 = 6.28\times 10^{-3}\text{ N/m}^2$ directed downward.

28.56. We expect that the current is uniformly distributed within each conductor because the fixed metal ions are uniformly distributed within it, so if the electrons that make up the current were concentrated in one region, it would have a negative charge density and other regions would have a positive charge density, and electric forces would cause the electrons to move and restore the wire's neutral charge density. The arrangement has cylindrical symmetry, so the magnetic field should form loops around the center of the wire, and we can apply Ampère's law to circular paths around the center of the wire at different radii to determine the magnetic field magnitude there. Because the magnitude of the field is constant around the path, the path integral is just $\oint \vec{B}\cdot d\vec{\ell} = 2\pi r B.$

There are four cases to consider, $r \le R_\text{wire},$ $R_\text{wire} < r < R_\text{shell},$ $R_\text{shell} \le r \le 2R_\text{shell},$ and $r > 2R_\text{shell},$ that is, the region inside the wire, the region between the wire and the shell, the region inside the shell, and the region outside the shell. In the first case, the Ampèrian path does not encircle all the current in the wire, but only the portion inside the path, $I_\text{enc} = I_\text{wire}\dfrac{\pi r^2}{\pi R_\text{wire}^2},$ so we have $2\pi r B_1 = \mu_0 I_\text{wire}\dfrac{r^2}{R_\text{wire}^2},$ or $B_1 = \dfrac{\mu_0 I_\text{wire} r}{2\pi R_\text{wire}^2}.$ In the region between the wire and the shell, the Ampèrian path encircles all of the current in the wire and none of the current in the shell, so $B_2 = \dfrac{\mu_0 I_\text{wire}}{2\pi r}.$ In the region inside the shell, the Ampèrian path encircles all of the current in the wire and some of the current in the shell.

The portion of the shell's current that is encircled is $I_{\text{shell}}\dfrac{\pi r^2-\pi R_{\text{shell}}^2}{\pi(2R_{\text{shell}})^2-\pi R_{\text{shell}}^2}=I_{\text{shell}}\dfrac{(r/R_{\text{shell}})^2-1}{3}$, so $B_3=\dfrac{\mu_0}{2\pi r}\left(I_{\text{wire}}+I_{\text{shell}}\dfrac{(r/R_{\text{shell}})^2-1}{3}\right)$. In the region outside the shell, the Ampèrian path encircles all of the current in the wire and in the shell, so $B_4=\dfrac{\mu_0(I_{\text{wire}}+I_{\text{shell}})}{2\pi r}$.

(*a*) If $I_{\text{wire}}=I_{\text{shell}}$, the magnetic field is only zero at the very center of the wire. (*b*) If $I_{\text{wire}}=-I_{\text{shell}}$, the magnetic field is zero at the very center of the wire and also throughout all of region 4, outside the shell. (*c*) If $I_{\text{wire}}=-I_{\text{shell}}/2$, the magnetic field is zero at the very center of the wire and also in region 3, inside the shell, at a radius given by $I_{\text{wire}}+I_{\text{shell}}\dfrac{(r/R_{\text{shell}})^2-1}{3}=0$, which we can solve for $r=R_{\text{shell}}\sqrt{\tfrac{5}{2}}$.

28.57. The sheet produces a uniform magnetic field directed perpendicular to the current with magnitude $B=\tfrac{1}{2}\mu_0 K$. When the particle enters the field, it begins to follow a circular trajectory, the radius of which is $R=mv/|q|B$ (Eq. 27.23). The particle exits the field after having completed one half of a revolution, so the distance between where it enters and where it exits is $2R$. Combining these and solving for the magnitude of the charge, we have

$$d=2R=\frac{2mv}{|q|B}=\frac{2mv}{|q|\frac{1}{2}\mu_0K}$$

$$|q|=\frac{4mv}{d\mu_0K}=\frac{4(9.1\times10^{-31}\ \text{kg})(2.0\times10^4\ \text{m/s})}{(90\times10^{-3}\ \text{m})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.0\ \text{A/m})}=1.6\times10^{-19}\ \text{C}$$

28.58. Inside a long solenoid, the magnetic field is uniform with magnitude given by Eq. 28.6 $B=\mu_0 nI=(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(300\ \text{m}^{-1})(1.0\ \text{A})=3.8\times10^{-4}\ \text{T}$.

28.59. The magnetic field inside a solenoid is given by Eq. 28.6 $B=\mu_0 nI$, which we can rearrange to find the windings per unit length:

$$n=\frac{B}{\mu_0 I}=\frac{0.070\ \text{T}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20\ \text{A})}=2.8\times10^3\ \text{m}^{-1}$$

28.60. (*a*) The current is directed into the page at B and out of the page at A, so according to the right-hand dipole rule, the magnetic dipole moment is in the positive *z* direction. (*b*) The torque on the small solenoid will tend to align its magnetic dipole moment with the magnetic field of the large solenoid. Applying the right-hand dipole rule to the current in the large solenoid, we find that its magnetic field is in the positive *x* direction.

28.61. Let's choose a coordinate system with the *x* axis pointing right, the *y* axis pointing up, and the *z* axis pointing towards us. Because the current in the wire is toward the left, the magnetic field it produces is in the negative *z* direction at the center of the solenoid and its magnitude is $B_{\text{wire}}=\mu_0 I/2\pi r=(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.5\ \text{A})/2\pi(50\times10^{-3}\ \text{m})=1.0\times10^{-5}\ \text{T}$. We are not told the direction of the current in the solenoid, but the magnetic field it produces is directed along its length, in either the positive or negative *x* direction. Let's say it's in the positive *x* direction. The magnitude of the solenoid's magnetic field is $B_{\text{solenoid}}=\mu_0 nI=(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1000\ \text{m}^{-1})(45\times10^{-3}\ \text{A})=5.7\times10^{-5}\ \text{T}$. The magnitude of the combined field is $B=\sqrt{B_{\text{wire}}^2+B_{\text{solenoid}}^2}=\sqrt{(1.0\times10^{-5}\ \text{T})^2+(5.7\times10^{-5}\ \text{T})^2}=5.7\times10^{-5}\ \text{T}$ and it is directed at an angle to the *x* axis given by $\tan\theta=B_{\text{wire}}/B_{\text{solenoid}}$ or $\theta=10°$. It does not matter whether the wire is inside or outside the solenoid coils.

28.62. The magnetic field in the solenoid is directed along its axis and has magnitude $B = \mu_0 n I_{\text{solenoid}}$. The magnitude of the magnetic force on a current-carrying wire is $F_{\text{w}}^B = |I_{\text{wire}}|\ell B\sin\theta$. Combining these we get $F_{\text{w}}^B = |I_{\text{wire}}|\ell\mu_0 n I_{\text{solenoid}}\sin\theta = (4.0\text{ A})(20\times10^{-3}\text{ m})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(700\text{ m}^{-1})(3.0\text{ A})\sin45° = 1.5\times10^{-4}\text{ N}.$

28.63. (*a*) The magnetic field in a toroid depends on the distance from the axis through the center of the toroid. At the center of one of the square windings, this distance equals the sum of one half the length of the side of the winding plus the distance from the center to the inner surface of the windings, $r = \frac{1}{2}(50\text{ mm}) + (120\text{ mm}) = 145\text{ mm}$. The magnitude of the magnetic field is $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(250)(3.0\times10^{-3}\text{ A})}{2\pi(145\times10^{-3}\text{ m})} = 1.0\times10^{-6}\text{ T}$. (*b*) The magnetic field of the toroid is zero outside the cavity enclosed by the windings. (*c*) The magnetic field of the toroid is zero outside the cavity enclosed by the windings.

28.64. We have formulas for the magnitudes of the magnetic fields in a toroid and a solenoid, which we can set equal to each other and solve for *r*.

$$B_{\text{solenoid}} = \mu_0 nI = B_{\text{toroid}} = \frac{\mu_0 NI}{2\pi r}$$

$$r = \frac{\mu_0 NI}{2\pi\mu_0 nI} = \frac{N}{2\pi n} = \frac{200}{2\pi(500\text{ m}^{-1})} = 6.37\times10^{-2}\text{ m}$$

Note that this is only valid if *r* happens to be inside the cavity enclosed by the toroid's windings because the magnetic field is zero outside.

28.65. If the toroid has *n* windings per unit length along the inside edge of the windings, and the inner radius of the windings is R_{toroid}, then the number of windings is $N = 2\pi R_{\text{toroid}} n$. So, the magnetic field magnitude a distance *r* from the axis through the center of the toroid is $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0 R_{\text{toroid}} nI}{r}$.

28.66. (*a*) When the electron is in the solenoid, it follows a helical path. The *x* component of its velocity remains the same as outside the solenoid, while the perpendicular component rotates around the *x* axis. The radius of the helix is determined by the perpendicular component of the electron's velocity $R = \dfrac{mv_\perp}{|q|B}$, and the greater the electron's speed, the greater the radius. So, if the electron is not to strike the coils, we must have $v_\perp \le \dfrac{R_{\text{coil}}|q|B}{m}$.

Each turn of the solenoid uses a length of wire $\ell_{\text{coil}} = 2\pi R_{\text{coil}}$ and there are $N = 400$ turns in the solenoid using, in total, a length of wire $\ell_{\text{wire}} = N\ell_{\text{coil}} = 2\pi NR_{\text{coil}}$, so the radius of the coils is $R_{\text{coil}} = \dfrac{\ell_{\text{wire}}}{2\pi N}$.

The magnitude of the magnetic field inside the solenoid depends on the current and the windings per unit length. If the solenoid has length ℓ, then the windings per unit length is $n = N/\ell$, and the magnitude of the magnetic field is $B = \mu_0 nI = \mu_0 NI/\ell$.

Combining these results, we find the maximum possible perpendicular component of the electron's velocity $v_\perp \le \dfrac{R_{\text{coil}}|q|B}{m} = \dfrac{\ell_{\text{wire}}}{2\pi N}\dfrac{|q|}{m}\dfrac{\mu_0 NI}{\ell} = \dfrac{\ell_{\text{wire}}}{\ell}\dfrac{|q|\mu_0 I}{2\pi m}$. Because the angle θ of the electron's initial velocity is fixed, this also determines the maximum possible *x* component of the electron's velocity: $v_\perp = v\sin\theta$ and $v_x = v\cos\theta = (v_\perp/\sin\theta)\cos\theta = v_\perp/\tan\theta$. The amount of time it takes the electron to travel the length of the solenoid is just that length divided by the *x* component of the electron's velocity, $\Delta t = \ell/v_x$, and the shortest possible interval corresponds to the greatest possible speed:

$$\begin{aligned}\Delta t &= \frac{\ell}{v_x} \\ &= \frac{\ell \tan\theta}{v_\perp} \\ &\geq \frac{\ell^2 \tan\theta}{\ell_{\text{wire}}} \frac{2\pi m}{|q|\mu_0 I} \\ &= \frac{(0.200\ \text{m})^2 \tan 65°}{33.0\ \text{m}} \frac{2\pi(9.11\times10^{-31}\ \text{kg})}{(1.60\times10^{-19}\ \text{C})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})} = 7.40\times10^{-9}\ \text{s}\end{aligned}$$

(*b*) The number of revolutions M the electron makes in this time interval is the time interval divided by the period of revolution, $T = 2\pi R_{\text{coil}}/v_\perp$. That is,

$$M = \frac{\Delta t}{T} = \frac{\ell \tan\theta}{v_\perp} \frac{v_\perp}{2\pi R_{\text{coil}}} = \frac{\ell \tan\theta}{2\pi} \frac{2\pi N}{\ell_{\text{wire}}} = \frac{400(0.200\ \text{m})\tan 65°}{33.0\ \text{m}} = 5.20$$

28.67. We know that, in an ideal toroid, the magnitude of the magnetic field is inversely proportional to the distance from the central axis and its direction is in circles around that axis. We need to measure the field at several points in order to determine how accurately our real-world magnet reproduces these ideal properties. For example, an ideal toroid has perfect circular symmetry, which we used in deriving an expression for its magnetic field. If our real-world toroid is not perfectly circular or the windings are not perfectly uniform, the magnetic field will differ from what we calculate.

To explain to our partner why the magnetic field in a toroid is not uniform, while that of a solenoid is, we could apply Ampère's law to two similar configurations and note the differences. In both cases, we'll choose an Ampèrian path that is tangent to the magnetic field inside the coils, has "legs" passing through the coils that are perpendicular to the magnetic field, and is closed by a path outside the coils and parallel to the first one. Only the first segment of the path contributes to the line integral (because the legs are perpendicular to the field and there is no field outside), which is proportional to its length. Because of symmetry, the magnetic field in the solenoid must be parallel to its axis, while the magnetic field in the toroid must be circular. So, the first segment of the path in the solenoid is a straight line, while it's a circular arc in the toroid. Changing the length of the legs of our paths does not change the line integral because it does not change the current encircled by the path. In the case of the solenoid, this means that the magnetic field is uniform across the width of the solenoid. In the case of the toroid, however, changing the length of the legs changes the length of the circular arc, so in order for the line integral to remain unchanged, the magnitude of the magnetic field must change as we move across the width of the toroid.

28.68. We can use the Biot-Savart law to calculate the magnitude of the magnetic field. Each segment of length $d\ell$ of the wire contributes an amount $dB_{\text{s}} = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r_{\text{sP}}^2}$ to the magnetic field. For all the segments, the direction of the magnetic field is perpendicular to the plane of the arc, $r_{\text{sP}} = R$, $\theta = 90°$, and $d\ell = Rd\phi$, where ϕ is the angle from the end of the arc to the beginning of the segment, as measured from the center of the arc. So, the magnetic field is $B = \int_0^{\pi/2} \frac{\mu_0 I}{4\pi R} d\phi = \frac{\mu_0 I}{4\pi R}\frac{\pi}{2} = \frac{\mu_0 I}{8R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.0\ \text{A})}{8(25\times10^{-3}\ \text{m})} = 1.9\times10^{-5}\ \text{T}.$

28.69. We have an expression for the magnetic force between parallel, straight, current-carrying wires $F_{12}^B = \frac{\mu_0 \ell I_1 I_2}{2\pi d}$ (Eq. 28.16). We can rearrange this to find the current in wire 2: $I_2 = \frac{2\pi d F_{12}^B}{\mu_0 \ell I_1} =$ $\frac{2\pi(90\times10^{-3}\ \text{m})(4.0\times10^{-7}\ \text{N})}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.0\ \text{m})(3.0\ \text{A})} = 1.2\times10^{-2}\ \text{A}.$

28.70. Let's choose a coordinate system with the loop in the *xy* plane centered on the *z* axis, and our point of interest P located at position *z* on the positive *z* axis. Each segment of length $d\ell$ of the wire contributes an amount $dB_s = \dfrac{\mu_0}{4\pi}\dfrac{Id\ell\sin\theta}{r_{sP}^2}$ to the magnetic field. For all the segments, $r_{sP} = \sqrt{R^2 + z^2}$, $\theta = 90°$, and $d\ell = Rd\phi$, where ϕ is the polar angle from the positive *x* axis to the beginning of the segment. The direction of each magnetic-field contribution is different, however, the vector sum of the contributions from two diametrically opposed segments has only a component in the *z* direction of magnitude $2dB_s R/r_{sP}$. Integrating this around half the loop (because it already includes the contribution from the other half of the loop) gives $B = \int_0^\pi \dfrac{\mu_0}{4\pi}\dfrac{2IR^2}{r_{sP}^3}d\phi = \dfrac{\mu_0 IR^2}{2r_{sP}^3} = \dfrac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}}$, or substituting the given values, $B = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.0\ \text{A})(0.22\ \text{m})^2}{2\left((0.22\ \text{m})^2 + (70\times10^{-3}\ \text{m})^2\right)^{3/2}} = 7.4\times10^{-6}\ \text{T}.$

28.71. Because wire 1 stays in place when released, the vector sum of the forces exerted on it must be zero. There are two forces exerted on the wire, the downward gravitational force and the magnetic force due to wire 2, which must be directed upward and have equal magnitude to the gravitational force. Equating expressions for the magnitudes of these forces, we can solve for the magnitude of the current in wire 2:

$$\begin{aligned}
F_{E1}^G &= F_{21}^B \\
m_1 g &= \frac{\mu_0 \ell I_1 I_2}{2\pi d} \\
(\lambda\ell)g &= \frac{\mu_0 \ell I_1 I_2}{2\pi d} \\
I_2 &= \frac{2\pi d\lambda\ell g}{\mu_0 \ell I_1} = \frac{2\pi d\lambda g}{\mu_0 I_1} \\
I_2 &= \frac{2\pi(5.0\times10^{-3}\ \text{m})(0.010\ \text{kg/m})(9.8\ \text{m/s}^2)}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{A})} = 2.5\times10^2\ \text{A}
\end{aligned}$$

Because the magnetic force between the wires is attractive, their currents must have the same direction, that is, toward the right.

28.72. (*a*) Each segment of length $d\ell$ of the wire in the loop contributes an amount $dB_s = \dfrac{\mu_0}{4\pi}\dfrac{Id\ell\sin\theta}{r_{sP}^2}$ to the magnetic field. For all the segments, the direction of the magnetic field is perpendicular to the plane of the loop, $r_{sP} = R$, $\theta = 90°$, and $d\ell = Rd\phi$, where ϕ is the polar angle around the loop to the beginning of the segment. So, the magnetic field is $B = \int_0^{2\pi}\dfrac{\mu_0 I}{4\pi R}d\phi = \dfrac{\mu_0 I}{4\pi R}2\pi = \dfrac{\mu_0 I}{2R}$. (*b*) The magnitude of the magnetic field in the solenoid is $B_{\text{solenoid}} = \mu_0 nI$, which does not depend on its radius. If $n = 1/(10^{-3}\ \text{m}) = 1000\ \text{m}^{-1}$, then $B_{\text{solenoid}} = (4\pi\times10^{-4}\ \text{T/A})I.$ Using our result from part a, the magnitude of the magnetic field at the center of the 10-mm loop is $B_{\text{loop}} = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})I}{2(10\times10^{-3}\ \text{m})} = (2\pi\times10^{-5}\ \text{T/A})I.$ That is, the magnitude of the magnetic field at the center of the solenoid is 20 times the magnitude of the magnetic field at the center of the loop.

28.73. We can use the Biot-Savart law to calculate the magnetic field. Each segment of length $d\ell$ of the wire contributes an amount $dB_s = \dfrac{\mu_0}{4\pi}\dfrac{Id\ell\sin\theta}{r_{sP}^2}$ to the magnetic field. For the segments on the arcs, the direction of the magnetic field is perpendicular to the plane of the arc, $r_{sP} = R$, $\theta = 90°$, and $d\ell = Rd\phi$, where ϕ is the angle from the end of the arc to the beginning of the segment, as measured from the center of the arc. For the segments on the straight lines, $\theta = 0$, so these segments contribute nothing. Integrating around the arcs, we find the magnetic field is

$B=\int_0^{\pi}\frac{\mu_0 I}{4\pi R_1}d\phi+\int_{\pi}^{2\pi}\frac{\mu_0 I}{4\pi R_2}d\phi=\frac{\mu_0 I}{4\pi R_1}\pi+\frac{\mu_0 I}{4\pi R_2}\pi=\frac{\mu_0 I}{4}\left(\frac{1}{R_1}+\frac{1}{R_2}\right).$ Substituting the given values, we have $B=\frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(3.0\times 10^{-3}\ \text{A})}{4}\left(\frac{1}{70\times 10^{-3}\ \text{m}}+\frac{1}{20\times 10^{-3}\ \text{m}}\right)=6.1\times 10^{-8}\ \text{T},$ and by applying the right-hand current rule, we see that the magnetic field is directed out of the page.

28.74. The magnetic field at P equals the vector sum of the fields produced there by each of the segments of wire, and each of those can be calculated using the Biot-Savart law. Each segment of length $d\ell$ of the wire contributes an amount $dB_\text{s}=\frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r_\text{sP}^2}$ to the magnetic field. For the segments on the arcs, the direction of the magnetic field is perpendicular to the plane of the arc as given by the right-hand current rule, $r_\text{sP}=R,$ $\theta=90°,$ and $d\ell=Rd\phi,$ where ϕ is the polar angle from the end of the arc to the beginning of the segment. For the segments on the straight lines, $\theta=0,$ so these segments contribute nothing. Integrating around the arcs, we find the magnetic field is $\vec{B}=\int_0^{\pi}\frac{\mu_0 I}{4\pi R_1}d\phi\,\hat{z}+\int_0^{\pi}\frac{\mu_0 I}{4\pi R_2}d\phi\,\hat{x}=\frac{\mu_0 I}{4R_1}\hat{z}+\frac{\mu_0 I}{4R_2}\hat{x}.$ If $R_2=1.5R_1,$ then $\vec{B}=\frac{\mu_0 I}{4R_1}\hat{z}+\frac{\mu_0 I}{6R_1}\hat{x}=\frac{\mu_0 I}{2R_1}\left(\frac{\hat{z}}{2}+\frac{\hat{x}}{3}\right),$ and its direction is parallel to the *xz* plane at an angle θ above the *xy* plane, where $\tan\theta=3/2$ or $\theta=56°.$

28.75. We can follow the procedure used in Example 28.6 to get an expression for the magnitude of the field due to each infinitesimal segment of the wire, $dB_\text{s}=\frac{\mu_0 Ir}{4\pi}\frac{dx}{(x^2+r^2)^{3/2}}.$ Here, too, we integrate over the length of the wire to get $B=\frac{\mu_0 Ir}{4\pi}\int_0^{10\ \text{m}}\frac{dx}{(x^2+r^2)^{3/2}}=\frac{\mu_0 Ir}{4\pi}\left[\frac{1}{r^2}\frac{x}{(x^2+r^2)^{1/2}}\right]_{x=0}^{x=10\ \text{m}}.$ Simplifying, substituting the given values, and noting that $\lim_{x\to 0}\frac{x}{(x^2+r^2)^{1/2}}=0,$ we find $B=\frac{\mu_0 I}{4\pi r}\frac{10\ \text{m}}{\left((10\ \text{m})^2+r^2\right)^{1/2}}=\frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(3.0\ \text{A})}{4\pi(2.0\ \text{m})}\frac{10\ \text{m}}{\left((10\ \text{m})^2+(2.0\ \text{m})^2\right)^{1/2}}=$ $1.5\times 10^{-7}\ \text{T}.$

28.76. We can use the Biot-Savart law to calculate the magnitude of the magnetic field. The magnetic field at P equals the vector sum of the magnetic fields due to each portion of the wire. For the slanted portions, the angle between $d\vec{\ell}$ and $\vec{r}_\text{sP}$ is 0 or $180°,$ so these contribute nothing to the field at P, leaving us with only the horizontal portion to consider. We can follow the procedure used in Example 28.6, with the only difference being the limits of integration, which in this case are from $x=-\ell/2$ to $x=+\ell/2.$ So, $B=\frac{\mu_0 I}{4\pi r}\left[\frac{x}{(x^2+r^2)^{1/2}}\right]_{x=-\ell/2}^{x=+\ell/2}=$ $\frac{\mu_0 I}{4\pi r}\left[\frac{\ell}{((\ell/2)^2+r^2)^{1/2}}\right],$ and substituting the given values gives $B=\frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(45.0\ \text{A})}{4\pi(30.0\times 10^{-3}\ \text{m})}$ $\left[\frac{0.100\ \text{m}}{((0.100\ \text{m}/2)^2+(30.0\times 10^{-3}\ \text{m})^2)^{1/2}}\right]=2.57\times 10^{-4}\ \text{T}.$

28.77. We have an expression for the magnetic field of a moving charged particle in Eq. 28.21, $\vec{B}=\frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}_\text{pP}}{r_\text{pP}^2}.$ For our electron, $r_\text{pP}=\sqrt{(5.0\ \text{mm})^2+(15\ \text{mm})^2}=15.81\ \text{mm},$ and the sine of the angle between its velocity and the unit vector to our point of interest is $\sin\theta=(15\ \text{mm})/r_\text{pP}=0.9487,$ where we have included extra digits in these intermediate results. Substituting values into our formula, we have $B=\frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r_\text{pP}^2}=$

$\dfrac{(10^{-7}\ \text{T}\cdot\text{m/A})(1.60\times10^{-19}\ \text{C})(6.0\times10^{7}\ \text{m/s})(0.9487)}{(15.81\times10^{-3}\ \text{m})^2}=3.6\times10^{-15}\ \text{T},$ and its direction is perpendicular to the electron's velocity, as given by the right-hand current rule. That is, if the electron is moving to the right, the magnetic field is directed toward us.

28.78. We have an expression for the magnetic field of a moving charged particle in Eq. 28.21, $\vec{B}=\dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}_{\text{pP}}}{r_{\text{pP}}^2}.$ For our proton, $r_{\text{pP}}=\sqrt{(2.00\ \text{mm})^2+(1.00\ \text{mm})^2}=2.2361\ \text{mm},$ and the sine of the angle between its velocity and the unit vector to our point of interest is $\sin\theta=(1.00\ \text{mm})/r_{\text{pP}}=0.4472,$ where we have included extra digits in these intermediate results. Substituting values into our formula, we have $B=\dfrac{\mu_0}{4\pi}\dfrac{qv\sin\theta}{r_{\text{pP}}^2}=$ $\dfrac{(10^{-7}\ \text{T}\cdot\text{m/A})(1.60\times10^{-19}\ \text{C})(4.00\times10^{4}\ \text{m/s})(0.4472)}{(2.2361\times10^{-3}\ \text{m})^2}=5.72\times10^{-17}\ \text{T}.$

28.79. We have expressions for both the magnetic field and the electric field of a charged particle: $\vec{B}=\dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}_{\text{pP}}}{r_{\text{pP}}^2}$ (Eq. 28.21) and $\vec{E}=\dfrac{1}{4\pi\epsilon_0}\dfrac{q\hat{r}_{\text{pP}}}{r_{\text{pP}}^2}$ (Eqs. 23.3 and 24.7). Comparing them, we see that they both have the same dependence on the charge and the inverse square of the distance to the point of interest, but the preceding constants of each are different, and the magnetic field also depends on the particle's velocity. Consolidating these differences, we find $\vec{B}=\mu_0\epsilon_0\vec{v}\times\vec{E}=\dfrac{\vec{v}\times\vec{E}}{c_0^2}.$

28.80. Equation 28.24 gives the magnetic force exerted by one moving charged particle on another, $\vec{F}_{21}^B=\dfrac{\mu_0}{4\pi}\dfrac{q_1q_2}{r_{12}^2}\vec{v}_1\times(\vec{v}_2\times\hat{r}_{21}).$ Let's choose a coordinate system in which the electrons move in the $\pm x$ directions, and their paths are separated in the y direction by $\Delta y\,\hat{j},$ with $\Delta y=10$ nm. When the x component of the distance between the electrons is $\Delta x,$ $r_{12}^2=\Delta x^2+\Delta y^2,$ and the sine of the angle between $\vec{v}_2$ and $\hat{r}_{21}$ is $\sin\theta=\Delta y/r_{12}.$ So, the magnitude of the magnetic force exerted by electron 2 on electron 1 is $F_{21}^B=\dfrac{\mu_0}{4\pi}\dfrac{q_1q_2}{r_{12}^2}v_1v_2\sin\theta=\dfrac{\mu_0}{4\pi}\dfrac{e^2}{\Delta x^2+\Delta y^2}v_1v_2\dfrac{\Delta y}{\sqrt{\Delta x^2+\Delta y^2}}=\dfrac{\mu_0}{4\pi}\dfrac{e^2v_1v_2\Delta y}{(\Delta x^2+\Delta y^2)^{3/2}}.$ Substituting the given values gives $F_{21}^B=(10^{-7}\ \text{T}\cdot\text{m/A})\dfrac{(1.60\times10^{-19}\ \text{C})^2(4.0\times10^{7}\ \text{m/s})(7.0\times10^{6}\ \text{m/s})(10\times10^{-9}\ \text{m})}{\left(\Delta x^2+(10\times10^{-9}\ \text{m})^2\right)^{3/2}}=\dfrac{7.2\times10^{-39}\ \text{N}\cdot\text{m}^3}{\left(\Delta x^2+(10\times10^{-9}\ \text{m})^2\right)^{3/2}}.$ For the particular case $\Delta x=0,$ that is, just as the electrons pass each other, $F_{21}^B=7.2\times10^{-15}\ \text{N}.$ Of course, this assumes that the electrons continue to follow their straight-line paths.

28.81. Equations 28.23 and 28.24 gives the magnetic forces exerted by a pair of moving charged particle on each other, $\vec{F}_{12}^B=\dfrac{\mu_0}{4\pi}\dfrac{q_1q_2}{r_{12}^2}\vec{v}_2\times(\vec{v}_1\times\hat{r}_{12})$ and $\vec{F}_{21}^B=\dfrac{\mu_0}{4\pi}\dfrac{q_1q_2}{r_{12}^2}\vec{v}_1\times(\vec{v}_2\times\hat{r}_{21}).$ For our two protons, the magnitudes of these forces are $F_{12}^B=\dfrac{\mu_0}{4\pi}\dfrac{e^2}{r^2}v_1v_2$ and $F_{21}^B=\dfrac{\mu_0}{4\pi}\dfrac{e^2}{r^2}v_1v_2\sin45°.$

28.82. (*a*) In Section 27.7, we learned that a charged particle moving in a direction perpendicular to a uniform magnetic field travels along a circular trajectory of radius $R=\dfrac{mv}{|q|B_{\text{ext}}}$ (Eq 27.23). The magnetic field produced by this moving particle at the center of its trajectory is given by Eq. 28.21, $\vec{B}_{\text{p}} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times(-\hat{R})}{R^2}$, where $\hat{R}$ is the unit vector from the center of the particle's circular trajectory to the particle's position. Combining these, we can eliminate the particle's velocity and express the result in terms of the requested variables, $B_{\text{p}} = \dfrac{\mu_0}{4\pi}\dfrac{q}{R^2}\dfrac{RqB_{\text{ext}}}{m} = \dfrac{\mu_0}{4\pi}\dfrac{q^2B_{\text{ext}}}{Rm}$. (*b*) Multiplying numerator and denominator by the electric constant ε_0 we get $B_{\text{p}} = \dfrac{\mu_0\varepsilon_0}{4\pi\varepsilon_0}\dfrac{q^2B_{\text{ext}}}{Rm} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2B_{\text{ext}}}{Rmc_0^2}$.

28.83. From the acceleration of electron 1, we can get the electromagnetic force exerted on it by electron 2 $\vec{F}_{21}^{EB} = m_1\vec{a}_1$. Combining this with our expression for the electromagnetic force (Eq. 28.26) and rearranging, we can solve for the distance.

$$\vec{F}_{21}^{EB} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\left[\hat{r}_{21} + \frac{\vec{v}_1\times(\vec{v}_2\times\hat{r}_{21})}{c_0^2}\right] = m_1\vec{a}_1$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}^2}\left[1 + \frac{v_1v_2}{c_0^2}\right] = m_1a_1$$

$$r_{12}^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{m_1a_1}\left[1 + \frac{v_1v_2}{c_0^2}\right]$$

$$r_{12} = \sqrt{\frac{1}{4\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}\frac{(1.60\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(900\ \text{m/s}^2)}\left[1+\frac{(1.5\times10^6\ \text{m/s})(4.0\times10^6\ \text{m/s})}{(3.0\times10^8\ \text{m/s})^2}\right]} = 0.53\ \text{m}$$

28.84. (*a*) At this instant, the magnetic field produced by the proton at the electron's location is directed out of the page, so the magnetic force exerted by the proton on the electron is directed to the right, that is, toward the proton. Its magnitude is $F_{\text{pe}}^B = \dfrac{\mu_0}{4\pi}\dfrac{e^2v_{\text{e}}v_{\text{p}}}{r_{\text{pe}}^2} = \dfrac{(10^{-7}\ \text{T}\cdot\text{m/A})(1.60\times10^{-19}\ \text{C})^2(3.0\times10^4\ \text{m/s})^2}{(3.0\times10^{-6}\ \text{m})^2} = 2.6\times10^{-25}\ \text{N}$. (*b*) The electric force exerted by the proton on the electron is also directed toward the proton and its magnitude is $F_{\text{pe}}^E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{r_{\text{pe}}^2}$. So, the ratio of the magnetic to electric forces is $F_{\text{pe}}^B/F_{\text{pe}}^E = \mu_0\varepsilon_0v_{\text{e}}v_{\text{p}} = v^2/c_0^2$.

28.85. (*a*) The magnetic force exerted by electron 1 on electron 2 is $\vec{F}_{12}^B = \dfrac{\mu_0}{4\pi}\dfrac{q_1q_2}{r_{12}^2}\vec{v}_2\times(\vec{v}_1\times\hat{r}_{12})$. Because $\vec{v}_1$ is parallel to $\vec{r}_{12}$ the vector product in parentheses is zero and so is the magnetic force. (*b*) The magnetic force exerted by electron 2 on electron 1 is $\vec{F}_{21}^B = \dfrac{\mu_0}{4\pi}\dfrac{q_1q_2}{r_{12}^2}\vec{v}_1\times(\vec{v}_2\times\hat{r}_{21})$. The vector product in parentheses has magnitude v_2 and points in the negative y direction. The vector product of $\vec{v}_1$ with this has magnitude v_1v_2 and points in the negative x direction. So, the magnetic force has magnitude $F_{21}^B = \dfrac{\mu_0}{4\pi}\dfrac{e^2v_1v_2}{r_{12}^2} =$ $\dfrac{(10^{-7}\ \text{T}\cdot\text{m/A})(1.60\times10^{-19}\ \text{C})^2(300\ \text{m/s})(500\ \text{m/s})}{(2.0\times10^{-3}\ \text{m})^2} = 9.6\times10^{-35}\ \text{N}$ in the negative *x* direction.

28.86. (*a*) Let's choose a coordinate system with electron 1 traveling on the *x* axis and the *y* axis pointing upward. At $t = 3.0\ \mu\text{s}$, the locations of the electrons are $(x_1, y_1) = (6.0\ \text{mm}, 0)$ and $(x_2, y_2) = (10\ \text{mm}, -10\ \text{mm})$, assuming they followed straight-line paths. The point at which we want to calculate the magnetic field has coordinates

$(x, y) = (9.0\text{ mm}, -5.0\text{ mm})$. The magnetic field produced by electron 1 has magnitude $B_1 = \dfrac{\mu_0}{4\pi}\dfrac{qv_1 \sin\theta_1}{r_1^2}$, where $r_1^2 = (3.0\times10^{-3}\text{ m})^2 + (-5.0\times10^{-3}\text{ m})^2 = 3.4\times10^{-5}\text{ m}^2$ and $\sin\theta_1 = (5.0\times10^{-3}\text{ m})/r_1 = 0.8575$. So, $B_1 = \dfrac{(10^{-7}\text{ T}\cdot\text{m/A})(1.60\times10^{-19}\text{ C})(2.0\times10^{3}\text{ m/s})(0.8575)}{3.4\times10^{-5}\text{ m}^2} = 8.1\times10^{-19}\text{ T}$ and its direction is toward us, in the positive z direction. The magnetic field produced by electron 2 has magnitude $B_2 = \dfrac{\mu_0}{4\pi}\dfrac{qv_2 \sin\theta_2}{r_2^2}$, where $r_2^2 = (-1.0\times10^{-3}\text{ m})^2 + (5.0\times10^{-3}\text{ m})^2 = 2.6\times10^{-5}\text{ m}^2$ and $\sin\theta_2 = (5.0\times10^{-3}\text{ m})/r_2 = 0.9806$. So, $B_2 = \dfrac{(10^{-7}\text{ T}\cdot\text{m/A})(1.60\times10^{-19}\text{ C})(5.0\times10^{3}\text{ m/s})(0.9806)}{2.6\times10^{-5}\text{ m}^2} = 3.0\times10^{-18}\text{ T}$ and its direction is away from us, in the negative z direction. The total field is then $B = B_2 - B_1 = 2.2\times10^{-18}\text{ T}$, in the negative z direction. (*b*) The magnetic force exerted by one electron on the other is greatest when electron 2 passes electron 1. At that instant, its magnitude is $F^B = \dfrac{\mu_0}{4\pi}\dfrac{e^2 v_1 v_2}{r_{12}^2} = \dfrac{(10^{-7}\text{ T}\cdot\text{m/A})(1.60\times10^{-19}\text{ C})^2(2.0\times10^{3}\text{ m/s})(5.0\times10^{3}\text{ m/s})}{(10\times10^{-3}\text{ m})^2} = 2.6\times10^{-34}\text{ N}$ and would cause the electron to accelerate at $a = F/m = (2.6\times10^{-34}\text{ N})/(9.1\times10^{-31}\text{ kg}) = 2.8\times10^{-4}\text{ m/s}^2$. Over an interval of $2.0\ \mu\text{s}$, an acceleration of this magnitude would displace the electron by $\Delta y = \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}(2.8\times10^{-4}\text{ m/s}^2)(2.0\times10^{-6}\text{ s})^2 = 5.6\times10^{-16}\text{ m}$, which is too small to measure directly. The electric force between the electrons is greater than the magnetic force by a factor of $c_0^2/(v_1 v_2) = 9.0\times10^{9}$, but over an interval of $2.0\ \mu\text{s}$, it would displace the electron by only about $5.1\times10^{-6}\text{ m}$, which is still insignificant.

28.87. The magnetic dipole moment of the loop points in the positive z direction. An external magnetic field produces a torque in the loop that tends to align its magnetic dipole moment with the external field. (*a*) There would be a torque in the positive y direction. (*b*) There would be a torque in the negative x direction. (*c*) There would be no torque.

28.88. We have an expression for the magnetic field in a solenoid, which we can solve for the current.

$$B = \mu_0 n I$$
$$I = \frac{B}{\mu_0 n} = \frac{B\ell}{\mu_0 N} = \frac{(1.0\text{ T})(200\times10^{-3}\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(200)} = 8.0\times10^{2}\text{ A}$$

28.89. Because the currents in the wires are in opposite directions, the magnetic force between them is repulsive. The magnitude of the force exerted by wire 2 on wire 1 is $F_{21}^B = \dfrac{\mu_0 \ell I_1 I_2}{2\pi d} = \dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.3\text{ m})(2.2\text{ A})(3.0\text{ A})}{2\pi(0.25\text{ m})} = 1.2\times10^{-5}\text{ N}$.

28.90. (*a*) Applying the right-hand current rule, we can see that the magnetic field created by the right loop at the position of the left loop is downward. (*b*) Applying the right-hand dipole rule, we can see that the magnetic dipole moment of the left loop is initially directed upward. (*c*) The left loop rotates so as to align its magnetic dipole moment with the external magnetic field. (*d*) After the left loop rotates and reaches equilibrium, the magnetic dipole moments of the two loops are antiparallel.

28.91. Because wire 1 begins to slide, the current in wire 2 must be great enough that the magnetic force between the wires is greater than the maximum force of static friction between wire 1 and the table's surface. That force is proportional to the normal force exerted by the table's surface on the wire, and because the wire is not accelerating vertically, the magnitude of the normal force equals the gravitational force on the wire. Equating the magnetic force between the wires with the maximum force of static friction, we can rearrange to find the minimum current in wire 2.

$$F_{s1}^{s} \le \mu_s F_{s1}^{n} = \mu_s m_1 g$$

$$F_{21}^{B} = \frac{\mu_0 \ell I_1 I_2}{2\pi d}$$

$$\frac{\mu_0 \ell I_1 I_2}{2\pi d} = \mu_s m_1 g$$

$$I_2 = \frac{2\pi d \mu_s m_1 g}{\mu_0 \ell I_1} = \frac{(2.0\times 10^{-3}\ \text{m})(0.050)(0.010\ \text{kg})(9.8\ \text{m/s}^2)}{(2\times 10^{-7}\ \text{T}\cdot\text{m/A})(1.0\ \text{m})(1.5\ \text{A})} = 33\ \text{A}$$

28.92. Both of our friends have clearly made mistakes. Not only do their answers not agree, but they are not self-consistent. According to Ampère's law, the line integral of the magnetic field along any path is proportional to the encircled current. For concreteness, let's suppose that the wire carries current along the *z* axis, with the Ampèrian paths lying in the *xy* plane. Further, let's suppose the sides of the square and triangular paths have length *d*. Because of the cylindrical symmetry of the wire, we know that the magnetic-field lines form circular loops around the wire, so the easiest Ampèrian path to use would be a circle centered on the wire, as we did in Example 28.3, because the magnitude of the magnetic field is constant along the path and its direction is tangential to the path. In the Example, we found an expression for the magnetic field magnitude $B(r) = \frac{\mu_0 I}{2\pi r}$ by noting that the path integral is equal to the product of the magnetic field magnitude times the length of the path $\oint \vec{B}\cdot d\vec{\ell} = B2\pi r.$ Maybe one of our friends used this method to calculate the whole line integral, but made a mistake in calculating the more difficult integral over one of the straight sides?

It's easy to see the results they should have gotten without working through the more difficult integrals. Because of the symmetry of the square Ampèrian path, each side should contribute an equal quarter of the whole line integral. That is, if Andy's value of $64\ \text{T}\cdot\text{m}$ for the whole line integral is correct, then he should have gotten a value of $16\ \text{T}\cdot\text{m}$ for one side. In Beth's case, the common side should still contribute one quarter of the whole line integral, and the two remaining sides should contribute the remaining three quarters, divided evenly between them. So, if her value of $89\ \text{T}\cdot\text{m}$ for the whole line integral is correct, then she should have gotten a value of $22\ \text{T}\cdot\text{m}$ for the common side.

We can see this more formally by considering a circular Ampèrian path that intersects the corners of the square. It has radius $R = \frac{\sqrt{2}d}{2}$ and the line integral of the magnetic field along the path is just the product of the magnetic field at this distance from the wire times the circumference of the circle. The line integral around an Ampèrian path made up of one of the sides of the square plus three quarters of the circle is the same, so it must be that the integral along the square side is one quarter of the whole line integral.

28.93. Because the magnetic field magnitude at a point in a toroid is inversely proportional to the distance from the point to the axis of the toroid, the magnitude of the magnetic field is greatest at the inner radius of the toroid and smallest at the outer radius. So, if the ratio of these magnitudes must be four, the outer radius must be four times the inner radius, $R_{\text{outer}} = 4R_{\text{inner}}$. The field per ampere will be greatest when the number of turns is maximized, which for a given length of wire means choosing the smallest coils possible. Because the core radius must be at least 30 mm, and $R_{\text{outer}} = R_{\text{inner}} + 2R_{\text{core}}$, we have $R_{\text{inner}} = \frac{2}{3}R_{\text{core}} = 20\ \text{mm}$ and $R_{\text{outer}} = 4R_{\text{inner}} = 80\ \text{mm}$. Each turn has a circumference of $2\pi R_{\text{core}} = 188\ \text{mm},$ so with 100 m of wire we can make $N = \frac{100\ \text{m}}{2\pi R_{\text{core}}} = 530$ turns. The magnetic field per ampere of current ranges from $\frac{B}{I} = \frac{\mu_0 N}{2\pi R_{\text{inner}}} = \frac{(2\times 10^{-7}\ \text{T}\cdot\text{m/A})(530)}{0.020\ \text{m}} = 5.3\times 10^{-3}\ \text{T/A}$ at the toroid's inner radius to $\frac{B}{I} = \frac{\mu_0 N}{2\pi R_{\text{outer}}} = \frac{(2\times 10^{-7}\ \text{T}\cdot\text{m/A})(530)}{0.080\ \text{m}} = 1.3\times 10^{-3}\ \text{T/A}$ at the toroid's outer radius.

29

CHANGING MAGNETIC FIELDS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{-5} V **2.** 10^{-4} V **3.** 10^{-1} A **4.** 10^{-3} J **5.** 10^{-6} V **6.** 10^{-5} J **7.** 10^{-3} V/m **8.** 10^{2} T **9.** 10^{3} m **10.** 10^{-3} V

Guided Problems

29.2 Moving loops

1. Getting Started Worked Problem 29.1 delt with changing magnetic flux through a loop, as does this problem. The difference is that the flux in Worked Problem 29.1 changed because the current in the source (long straight wire) was changing. In this case, the magnetic flux is changing because the closed loop is moving away from the source. Another difference is that the field strength (and therefore the flux) was increasing in Worked Problem 29.1. In this case, the field being produced is constant. But since the loop is moving farther from the source, the field at the location of the loop will decrease in time. But there are many similarities between the two problems. Both problems involve the field due to a long straight wire. Since both problems require that we calculate the flux through a rectangular loop due to that field, we should be able to use the expression for the total flux from Worked Problem 29.1, verbatim. The rate of change of this flux will be different in this case, though.

2. Devise Plan From Example 28.3, we know the magnetic field due to a long straight wire a distance r away is given by

$$B_{\text{wire}} = \frac{\mu_0 I}{2\pi r}$$

A tiny contribution to the magnetic flux is given by $d\Phi_B = \vec{B} \cdot d\vec{A}.$ We obtain the total flux at a given time by integrating such differential contributions over the entire area of the loop. Because this situation is so similar to Worked Problem 29.1, we expect the same result for the flux. But we will check that assumption below.

Once we obtain the flux through the loop. We will take the time derivative of it to obtain the electromotive force:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

This is then trivially related to the current in the loop:

$$\frac{I_{\text{ind}}}{R} = \mathcal{E} \tag{1}$$

The electromotive force will involve the perpendicular distance r from the wire to one edge of the rectangular loop. It is therefore reasonable that the time derivative of something that depends on r would end up being written in terms of the rate of change of $r,$ meaning the speed $v.$ We will check shortly.

3. Execute Plan We first obtain the expression for the total flux through the rectangular loop.

$$\Phi_B = \int_{\text{loop}} d\Phi_B$$

$$\Phi_B = \int_0^{\ell} \int_r^{r+w} B(r')dr'd\ell'$$

$$\Phi_B = \ell \int_r^{r+w} \frac{\mu_0 I}{2\pi r'} dr'$$

$$\Phi_B = \frac{\mu_0 \ell I}{2\pi} \ln\left(\frac{r+w}{r}\right) \tag{2}$$

This is exactly the same expression obtain in Worked Problem 29.1, except for the use of slightly different variables. Now we determine the electromotive force by taking the time derivative of equation (2):

$$|\varepsilon| = \frac{d\Phi_B}{dt}$$

$$|\varepsilon| = \frac{d}{dt}\left[\frac{\mu_0 \ell I}{2\pi} \ln\left(\frac{r+w}{r}\right)\right]$$

$$|\varepsilon| = \frac{\mu_0 \ell I}{2\pi}\left(\frac{r}{r+w}\right)\frac{(vr-(r+w)v)}{r^2}$$

$$|\varepsilon| = \frac{\mu_0 \ell I}{2\pi}\left(\frac{wv}{r^2+wr}\right)$$

We now determine the induced current I_{ind} using equation (1):

$$I_{\text{ind}} = R\frac{\mu_0 \ell I}{2\pi}\left(\frac{wv}{r^2+wr}\right)$$

The flux through the loop is directed into the page, but it is decreasing. Lenz's Law tells us that the induced current produces a field that resists that change. Thus the induced field should be into the page, to counteract the decrease. Using the right hand current rule tells us that the induced current must flow clockwise in the loop. Thus

$$I_{\text{ind}} = R\frac{\mu_0 \ell I}{2\pi}\left(\frac{wv}{r^2+wr}\right) \text{ clockwise}$$

4. Evaluate Result The magnetic force on the left-most wire segment in the rectangular loop will be toward the long straight wire. The magnetic force on the right-most wire segment in theloop will be away from the long straight wire. But we know the field due to the long wire is stronger on the left side of the loop than on the right side. So the overall magnetic force on the rectangular loop will be back toward the long straight wire, opposite the motion of the loop. This makes sense.

The answer to Worked Problem 29.1 has slightly different behavior than our answer here, because the current was changing in the former problem, whereas the position of the loop was changing here. But we see that they have the same units and similar prefactors.

29.4 A falling loop

1. Getting Started The loop is being pulled down by gravity. But the change in flux with time means there will be an induced current in the loop. The direction of the induced current will be such that the loop experiences an upward magnetic force, opposing the change in flux. Thus, we do not expect that the speed will be determined by the flux, at least not directly. Rather, because the change in flux induces a current that yields a force and therefore an acceleration, we expect the rate of change of the flux to be related to the rate of change of the speed. However, we do not expect a simple proportionality between $\frac{d\Phi_B}{dt}$ and $a(t)$, because the change of flux through the loop also depends directly on how quickly the loop is falling, $v(t)$. Thus we expect a complex relationship between these variables, which we will determine by considering the sum of all forces on the loop in the vertical direction.

Let us call the length of the loop that is inside the region with the magnetic field h. The direction of the field is parallel to an area element of the loop (that is to say, normal to the surface). Thus the flux is given by $\Phi_B = whB$. Be sure you understand why it is not $w\ell B$.

2. Devise Plan Let the vertically upward direction be the $+y$ axis. Taking the time derivative of the above expression for flux, we have

$$\frac{d\Phi_B}{dt} = wB\frac{dh}{dt}$$

Note that the rate at which the height of the loop inside the field is changing is exactly the downward speed of the loop:

$$\frac{d\Phi_B}{dt} = wBv_y(t) \tag{1}$$

Note that when the loop falls, $v_y(t)$ is a negative number. Thus equation (1) correctly states that when the loop falls downward the flux is decreasing (the rate of change of the flux is negative).

As suggested, we use $I_{\text{ind}} = \mathcal{E}_{\text{ind}}/R$ and $\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -wBv_y(t)$ to relate the induced current to other variables in the problem. It is sometimes useful to employ these equations in finding only the magnitude of the induced current, since the direction can be obtained from Lenz's law. We proceed using only magnitudes:

$$I_{\text{ind}} = \frac{wB}{R}\left|v_y(t)\right|$$

Since the flux out of the page is decreasing, the induced current will flow in such a way as to increase the flux out of the page. The right-hand current rules shows us that this corresponds to a current flowing counterclockwise around the loop. This is consistent with the idea that the magnetic force should act to slow the descent of the loop, slowing the change of flux. Using the right-hand force rule on each side of the wire loop, we see there is no force on the bottom segment (it is not in a magnetic field). There is an upward force on the top wire segment. The rightward force on the right-most segment and the leftward force on the left-most segment cancel each other out, such that the total force on the loop is given only by the upward magnetic force on the top wire segment. Thus

$$\sum \vec{F}_{\text{loop}} = I_{\text{ind}}\vec{w}\times\vec{B} - mg\hat{y} = -\left(\frac{w^2B^2}{R}v_y(t) + mg\right)\hat{y} \tag{2}$$

In the last step of equation (2), we have noted that the right hand rule gives us a magnetic force that is always upward. Since $v_y(t)$ is always negative, we require a negative sign in front of the expression.

Equating the sum of all forces in the vertical direction to the mass times the acceleration in the vertical direction, we obtain

$$a_y(t) = -\left(\frac{w^2B^2}{mR}v_y(t) + g\right) \tag{2}$$

Or equivalently, using $a_y(t) = \frac{dv_y(t)}{dt}$, we have

$$\frac{dv_y(t)}{dt} = -\left(\frac{w^2B^2}{mR}v_y(t) + g\right) \tag{3}$$

In order to determine the speed as a function of time, we must solve this differential equation.

3. Execute Plan We see that we have a differential equation of the form $\frac{dv_y(t)}{dt} = \alpha - \beta v_y(t)$, where $\alpha = -g$, and $\beta = \frac{w^2 B^2}{mR}$. In keeping with the instructions, we try a solution of the form $v_y(t) = \frac{\alpha}{\beta}(1 - e^{-\beta t})$. First we check to see that the differential equation is satisfied:

$$\begin{aligned}\frac{dv_y(t)}{dt} &= \frac{\alpha}{\beta}\frac{d}{dt}(1 - e^{-\beta t}) \\ &= \alpha e^{-\beta t} \\ &= \alpha(e^{-\beta t} - 1) + \alpha \\ &= -\beta\frac{\alpha}{\beta}(1 - e^{-\beta t}) + \alpha \\ &= \alpha - \beta v_y(t)\end{aligned}$$

So this is a valid form for the vertical component of the velocity. Inserting the values of α and β in terms of variables given in the problem statement, we find

$$v_y(t) = \frac{-gmR}{w^2 B^2}\left(1 - e^{-\frac{w^2 B^2}{mR}t}\right) \tag{4}$$

Note that when $t = 0$, equation (4) reduces to $v_y(t = 0) = 0$. This is as it should be, since at the moment of release the loop has not had time to accelerate downward yet. Equation (4) is only valid until the top edge of the loop leaves the region with the magnetic field. After that instant, the loop will fall under the influence of gravity only.

4. Evaluate Result Differentiating equation (4) with respect to time, we find

$$\begin{aligned}\frac{dv_y(t)}{dt} &= \frac{-gmR}{w^2 B^2}\frac{d}{dt}\left(1 - e^{-\frac{w^2 B^2}{mR}t}\right) \\ a_y(t) &= -ge^{-\frac{w^2 B^2}{mR}t}\end{aligned}$$

Note that if the magnetic field is reduced to zero, the acceleration reduces to $a_y(t) = -g$, as must obviously be the case. It is interesting to note that as long as the top edge of the loop remains inside the region with the field, the acceleration of the loop decays exponentially. This means the loop would approach a constant (terminal) speed. Of course, once the loop leaves the field entirely, it would fall under the influence of gravity only.

29.6 Coaxial cable inductance and magnetic energy

1. Getting Started We first determine an expression for the magnetic field in every region of the setup. Then we use equation (29.29) to relate the magnetic energy density to the magnetic field strength. Finally, we integrate over that magnetic energy density to obtain the whole magnetic potential energy stored. Because of the cylindrical symmetry of the problem, the simplest way to determine the magnetic field in different regions is to use Ampere's Law.

Once the magnetic potential energy U^B has been determined, the inductance of the setup can simply be read off from equation (29.25): $U^B = \frac{1}{2}LI^2$.

2. Devise Plan Consider a circular Amperian path, centered on the central axis of rotational symmetry of the cylinder. Because of the cylindrical symmetry, the magnetic field must be parallel to the path at all points and the magnitude of the field must be constant along the path. Inserting this symmetry into Ampere's Law yields

$$\oint_{\substack{\text{Amperian}\\ \text{path}}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$$

$$2\pi r B = \mu_0 I_{\text{enc}}$$

$$B = \frac{\mu_0 I_{\text{enc}}}{2\pi r} \tag{1}$$

In the region $r > R_{\text{outer}}$, an Amperian loop encloses the leftward and rightward currents, meaning the total current enclosed is zero. Thus $B(r > R_{\text{outer}}) = 0$.

Between the two conductors, in the region $R_{\text{inner}} < r < R_{\text{outer}}$, only the leftward current in the inner conductor is enclosed. Thus $B(R_{\text{inner}} < r < R_{\text{outer}}) = \dfrac{\mu_0 I}{2\pi r}$.

Finally, inside the inner conductor $r < R_{\text{inner}}$, there is no conducting material, so the current enclosed is again zero. From the above results we can write down the magnetic energy density in the various regions using equation (29.25):

$$r > R_{\text{outer}} \qquad u_B = 0 \tag{2}$$

$$R_{\text{inner}} < r < R_{\text{outer}} \qquad u_B = \frac{\mu_0 I^2}{8\pi^2 r^2} \tag{3}$$

$$r < R_{\text{inner}} \qquad u_B = 0 \tag{4}$$

In order to determine the magnetic potential energy stored in the entire arrangement, we must integrate the magnetic energy density over the appropriate regions, according to equation (29.30):

$$U^B = \int_{\text{volume}} u_B dV$$

Equation (2) demonstrates that we need not carry out any integration of the magnetic energy density outside the outer conductor or inside the inner conductor. In the intermediate region $(R_{\text{inner}} < r < R_{\text{outer}})$, inserting the recommended differential volume, equation (29.30) becomes

$$U^B(R_{\text{inner}} < r < R_{\text{outer}}) = \int_{R_{\text{inner}}}^{R_{\text{outer}}} \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi r \ell dr \tag{5}$$

We need only carry out the integration to find the total magnetic potential energy stored.

3. Execute Plan Evaluating the integrals in (5) and (6), we obtain

$$U^B(R_{\text{inner}} < r < R_{\text{outer}}) = \frac{2\pi\ell\mu_0 I^2}{8\pi^2} \int_{R_{\text{inner}}}^{R_{\text{outer}}} \frac{dr}{r}$$

$$U^B(R_{\text{inner}} < r < R_{\text{outer}}) = \frac{\ell\mu_0 I^2}{4\pi} \ln\left(\frac{R_{\text{outer}}}{R_{\text{inner}}}\right)$$

So the total magnetic potential energy in the system is

$$U^B = \frac{\ell\mu_0 I^2}{4\pi} \ln\left(\frac{R_{\text{outer}}}{R_{\text{inner}}}\right)$$

Rearranging equation (29.25), we find

$$L = \frac{2U^B}{I^2} = \frac{\ell\mu_0}{2\pi} \ln\left(\frac{R_{\text{outer}}}{R_{\text{inner}}}\right)$$

4. Evaluate Result Note that the energy stored depends on the square of the current, which makes sense. Note also that both the energy stored and inductance scale with the length of the setup, which also makes sense. Finally, notice that if the radius of the inner and outer conductors become very close, the logarithmic factor in the inductance and total energy vanishes. This makes perfect sense, because in that limit there would be equal and opposite currents flowing along the same conducting surface. There would be no magnetic field anywhere because there would be no net current.

29.8 Betatron

1. Getting Started In order for the electron to move at a constant radius, the magnetic force must point inward toward the center of the circular path. For a given speed of the electron v, the force must have magnitude $\sum F_r = \frac{mv^2}{R}$. This radial force will depend on the magnetic field according to $\vec{F} = q\vec{v}\times\vec{B}(R)$. Thus if $B(R)$ increases in magnitude the force acting on the electron will increase. In order to stay at the same radial position, the electron must also move faster. We know that magnetic forces do no work, so this acceleration of the electron to a higher speed cannot be a direct result of a magnetic force. But when the magnetic flux through the electron's orbit changes, there is also an electromotive force. That can change the electron's speed.

2. Devise Plan In the radial direction, we can equate

$$q\vec{v}\times\vec{B}(R) = \frac{mv^2}{R}$$

And use the orthogonality of the path and the magnetic field to simplify this and obtain

$$v = \frac{qR}{m}B(R) \tag{1}$$

Differentiating equation (1) with respect to time (holding the radius fixed), we obtain

$$a = \frac{dv}{dt} = \frac{qR}{m}\frac{dB(R)}{dt} \tag{2}$$

The change in flux through the orbit of the electron is

$$\frac{d\Phi_B}{dt} = \pi R^2 \frac{dB_{\text{av}}}{dt} \tag{3}$$

We can relate this to the induced electric field using equation (29.17):

$$\oint \vec{E}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

In this case, the path of integration is the circular path of the electron. By symmetry, the electric field should be the same everywhere along that path. It also must be directed parallel to the path of integration. Thus we can write

$$|E| = \frac{1}{2\pi R}\left|\frac{d\Phi_B}{dt}\right| \tag{4}$$

Using the right hand current rule, we see that the electric field will be directed opposite the electron's direction of motion (which serves to speed up the electron). This electric field will cause an acceleration of the electron with a magnitude equal to

$$a = \frac{dv}{dt} = \frac{qE}{m} \tag{5}$$

Inserting equation (3) into equation (4), and inserting that result into equation (5) yields

$$a = \frac{qR}{2m}\frac{dB_{\text{av}}}{dt} \tag{6}$$

We can now compare equations (2) and (6), both of which describe the acceleration of the electron tangential to its path.

3. Execute Plan Setting equations (2) and (6) equal to each other and rearranging, we find

$$\frac{dB(R)}{dt} = \frac{1}{2}\frac{dB_{\text{av}}}{dt} \tag{7}$$

4. Evaluate Result If both fields start at zero, and increase linearly with time, equation (7) would reduce to $B(R) = \frac{1}{2}B_{\text{av}}$. Note also that if both fields are constant, equation (7) is satisfied. This would defeat the purpose of an accelerator, but it demonstrates that our result is consistent with a known case in which the electron's radius of orbit does not change.

Questions and Problems

29.1. (*a*) If the conducting rod moves along the *z* axis, the charge carriers in it also move in the *z* direction. Because this is parallel to the magnetic field, it exerts no force on the charge carriers and nothing happens. (*b*) If the conducting rod moves along the *y* axis, so do the charge carriers in it, and according to the right-hand force rule, the magnetic field exerts a force on them parallel to the *x* axis. So, some charge separation takes place between one side of the rod and the other in the *x* direction.

29.2. Charge separation occurs when a conductor moves perpendicular to a magnetic field. If the conductor moves parallel to the field, its velocity has no component perpendicular to the field, so no magnetic forces are exerted on the charge carriers in the conductor and no charge separation occurs. So, if the airplanes fly parallel to Earth's magnetic field, that is, parallel to Earth's surface and toward or away from Earth's magnetic poles, there is no charge separation on their metal surfaces.

29.3. The charge separation between the wing tips is maximized when the magnetic force exerted on the charge carriers in the wings is maximized in the direction from one wing tip to the other. This happens when both the airplane's velocity and the line from one wing tip to the other are perpendicular to Earth's magnetic field. The magnetic field is parallel to Earth's surface and points toward the North Pole, so the charge separation between the wing tips is maximized when both the airplane's velocity and the line between the wing tips are in the vertical east-west plane. For example, the airplane could fly straight up with its wings pointing east and west, or it could fly toward the east with its wings pointing up and down.

29.4. At the location of the loop, the magnetic field points away from the magnet's north pole. That is, it has a component in the positive *x* direction and a component directed away from the *x* axis. So, at point 1, by the right-hand force rule, the magnetic force on an electron in the conductor is in the positive *z* direction. At point 2, it is in the positive *y* direction; at point 3 in the negative *z* direction; and at point 4 in the negative *y* direction. These forces do not cause any charge separation, but they do induce a current in the loop, directed from point 1 to point 2, and so on around the loop. That is, the direction of the current is opposite the direction of the magnetic forces on the negatively charged electrons.

29.5. (*a*) If the rod rotates clockwise about the *z* axis, the top end of the rod initially moves to the right, in the positive *x* direction, while the bottom end of the rod initially moves to the left, in the negative *x* direction. So, according to the right-hand force rule, there is a downward magnetic force, in the negative *y* direction, exerted on positive charge carriers at the top end of the rod and an upward magnetic force, in the positive *y* direction, exerted on positive charge carriers at the bottom end of the rod. So, the magnetic forces cause a positive charge to build up toward the center of the rod, while a negative charge develops at the ends, and this remains the case as the rod rotates. (*b*) If the rod rotates clockwise about the *x* axis, the top end of the rod initially moves in the negative *z* direction, while the bottom end of the rod initially moves in the positive *z* direction. Because this motion is parallel to the magnetic field, there are no magnetic forces exerted on the charge carriers in the rod.

As the rod rotates, however, this changes. When the rod lies in the *xz* plane, one end moves in the negative *y* direction, while the other end moves in the positive *y* direction. In the former case, a magnetic force is exerted on the positive charge carriers in the rod in the negative *x* direction; in the latter case the magnetic force is in the positive *x* direction. In both cases, though, the magnetic forces are directed perpendicular to the long axis of the rod, so the charge separation that occurs will only be between one side of the rod and the other, not along the length of the rod.

As the rod continues through its rotation, it again becomes aligned with the *y* axis, and the magnetic force on the charge carriers in the rod vanishes. From there, the magnetic force continues to oscillate as the rod rotates. (*c*) As the rod rotates clockwise about the *y* axis, the back of the rod (that is, the part of the rod with the largest negative *z* component of its position) moves in the positive *x* direction, so a magnetic force is exerted on the positive charge carriers in this part of the rod directed downward, in the negative *y* direction. The left and right sides of the rod move in the negative and positive *z* directions, so there is no magnetic force on the charge carriers in these parts of the rod. The front of the rod moves in the negative *x* direction, so a magnetic force is exerted on positive charge carriers in this part of the rod directed upward, in the positive y direction. So there is a current down the back of the rod and up the front, clockwise as viewed from the positive x axis looking toward the origin.

29.6. (*a*) The magnitude of the induced current in the ring is greater when the rate of change of the magnetic flux through the ring is greater. As the current direction alternates, the magnetic field it produces also alternates and it does so in the same amount of time for all the locations, so the rate at which the magnetic flux through the ring changes is proportional to the maximum magnetic flux through the ring. The magnetic field forms circular loops around the wire and its magnitude decreases with distance away from the wire, so the maximum magnetic flux through the ring is greatest at position A, smaller at position D (because it is farther from the wire), and smallest at position C (because the magnetic field is in the plane of the ring, so the flux is zero). So, the induced current in the ring is greatest at position A and smallest at position C. (*b*) Because the wires carrying current in opposite directions are close together, the vector sum of their magnetic fields is small, even when relatively close to the pair of wires. So, because the magnetic fields are smaller, the rate of change of the magnetic flux through the ring is smaller, and so is the induced current in the ring.

29.7. (*a*) Yes. If we choose an area vector for the loop that points in the positive x direction, there is a positive magnetic flux through the loop at the instant shown in the Figure. When the magnet has rotated through a quarter turn, the magnetic flux through the loop becomes zero. Because the magnetic flux is changing, a current is induced in the loop. (*b*) The induced current varies with time. During one half of the magnet's rotation, the magnetic flux through the loop is decreasing from some maximum value, through zero, to some greatest negative value. During the other half of the magnet's rotation, the magnetic flux through the loop is increasing from its greatest negative value, through zero, and back to its greatest positive value. Because the rate of change of the magnetic flux through the loop changes sign, the induced current must change direction, which means it cannot be constant. (Compare *Principles* Figure 29.3, for example, where the current changes direction when the rate of change of magnetic flux through a loop changes sign.)

29.8. As in *Principles* Example 29.1, we would observe no current in the loop at locations (*a*), (*c*), and (*e*). At locations (*b*) and (*d*), we would observe currents in the same directions as in the Example because at one location the magnetic flux through the loop is increasing, while at the other it is decreasing. The magnitudes of the currents would be different from those in the Example. There, as the rectangular loop enters or leaves the magnetic field, the rate of change of magnetic flux through the loop is constant, so the magnitude of the induced current is constant during the transition. With a circular loop, the rate of change of magnetic flux increases and decreases smoothly, so the magnitude of the induced current increases and decreases smoothly as the loop enters or leaves the magnetic field.

29.9. The induced current is greatest when the rate of change of the magnetic flux through the coil is greatest. Because the magnetic field is uniform and we have no control over its rate of change, the only way we can increase the rate of change of magnetic flux through the coil is by increasing the area of the coil. For a single winding, a circular coil encloses the greatest area, $A_{\max} = \ell^2/4\pi.$ For two windings, each circular coil would enclose an area $A = \ell^2/16\pi,$ for a total area for both windings of $2A = \ell^2/8\pi,$ which is smaller than the area for one winding. This trend continues for more windings, so we should form the wire into a single circular loop.

29.10. It depends on the shape of the rectangle and the instant at which the comparison is made. Because the magnetic field is uniform, the rate of change of magnetic flux through the loop is proportional to the rate of change of the area of the loop that is within the region with the magnetic field. When the rectangle passes into or out of the magnetic field, assuming its velocity is parallel to one of its sides, the rate of change of magnetic flux through the loop is constant during the transition. (If the rectangle moves diagonally, there will be intervals at the beginning and end of the transition, while the corners are passing into or out of the field, when the rate of change will not be constant.) If the rectangle moves parallel to its longer sides, for a given speed, the rate of change of magnetic flux through the loop will be small; if it moves parallel to its shorter sides, it will be large. When the circle passes into or out of the magnetic field, the rate of change of the area of the loop that is in the field will vary smoothly from zero up to some maximum value and back down to zero.

29.11. The train from Paris to Avignon travels on average in a direction south of south-east. From the point of view of an observer in the Earth reference frame, our conducting rod, which is oriented perpendicular to the direction of the train's velocity, is moving through Earth's magnetic field, which is directed to the north and downward. So, there is a component of the rod's motion that is perpendicular to Earth's magnetic field, which causes magnetic forces to be exerted on the charge carriers in the rod, causing charge separation between its ends. For an observer in the moving reference frame of the train, the rod is at rest but the source of Earth's magnetic field is moving, and an electric field accompanies this changing magnetic field, causing charge separation between the ends of the rod.

29.12. [**NOTE:** The first line of the problem statement will be changed from "A flat conducting plate **lies in** the *xy* plane…." to "A flat conducting plate **is centered in** the *xy* plane…."] When the north pole of the magnet faces the plate above the origin, its magnetic field lines have a downward component and a radial component directed away from the origin where they intersect the plate. If the plate were moving toward the magnet, the magnetic force on the moving charge carriers in the plate would cause them to form current loops circulating counterclockwise around the origin as seen from the location of the magnet. Because the same current is induced when the plate is at rest and the magnet moves toward it, the electric field lines in the plate must form circular loops pointing counterclockwise around the origin. If the south pole faces the plate, the magnetic field lines have an upward component and a radial component directed toward the origin where they intersect the plate. In this case, the induced current forms clockwise loops around the origin as seen from the location of the magnet, so the electric field lines must form circular loops pointing clockwise around the origin.

29.13. Assuming that the bar magnet is oriented parallel to the pipe, its magnetic field lines have a component parallel to the pipe and a component perpendicular to the surface of the pipe, directed radially outward near the magnet's north pole and radially inward near the magnet's south pole. If the magnet were at rest and the pipe moving upward, these radial components would exert magnetic forces on the charge carriers in the pipe, inducing currents that circulate around the pipe's circumference in a counterclockwise direction near the magnet's north pole and in a clockwise direction near its south pole, as viewed from above. When the pipe is at rest and the magnet moves downward, the same currents are induced, so the electric field lines in the pipe form loops around the pipe's circumference in a counterclockwise direction near the magnet's north pole and in a clockwise direction near its south pole, as viewed from above.

29.14. The direction of the induced current reverses when the rate of change of magnetic flux through the loop changes sign. There are several ways to accomplish this. One way would be to suspend the bar magnet like a pendulum with one pole facing down over the center of the loop. When the pendulum is set swinging, the magnetic flux through the loop increases as the magnet approaches the center of the loop and decreases as it swings away, so the rate of change of the magnetic flux through the loop alternates from positive to negative and back again, and the induced current alternates from clockwise to counterclockwise and so on.

29.15. A positive current in the loop produces a magnetic field that points into the plane of the page at the center of the loop. To induce such a current, according to Lenz's law, we need a changing magnetic field in the loop that either points into the plane of the page and decreases in magnitude or points out of the plane of the page and increases in magnitude. There are many ways to do this. For example, we could point the south pole of a bar magnet at the loop and move it closer, or we could point the north pole of a bar magnet at the loop and move it farther away.

29.16. As the magnet moves through the pipe, the changing magnetic field in a given section of the pipe induces a current in that section of pipe, and the resulting induced magnetic field resists the change that created it, namely the motion of the magnet. That is, the induced magnetic field exerts an upward force on the falling magnet, decreasing its downward acceleration, so it takes longer to fall through the pipe than it would through air.

29.17. A clockwise current through the loop produces a magnetic field at the center of the loop that points down into the plane of the page. So, by Lenz's law, to induce a clockwise current in the loop, we must have a changing magnetic field in the loop that either points down into the page with its magnitude decreasing, or one that points up out of the page with its magnitude increasing. When the current is in the direction indicated in Figure P29.17, it produces a magnetic field that points down into the page at the location of the loop. If we take this as the positive current direction, then the field will be into the page with its magnitude decreasing when the current's phase is between one-half pi and pi, that is, when $\frac{1}{2}\pi < \omega t + 2n\pi < \pi$ or $(2n+\frac{1}{2})\pi/\omega < t < (2n+1)\pi/\omega,$ for integer $n = \ldots,-2,-1,0,1,2,\ldots.$ The field will be out of the page with its magnitude increasing when the current's phase is between pi and three-halves pi, that is, when $(2n+1)\pi/\omega < t < (2n+\frac{3}{2})\pi/\omega.$ So, the induced current will be clockwise when the phase of the current in the wire is between one-half pi and three-halves pi, that is, when $(2n+\frac{1}{2})\pi/\omega < t < (2n+\frac{3}{2})\pi/\omega.$

The induced current is zero when the magnetic flux through the loop is not changing, that is, at those instants when the current in the wire reaches its minimum and maximum values. These occur when its phase is an odd multiple of one-half pi, that is, when $t = (n+\frac{1}{2})\pi/\omega.$

The induced current is counterclockwise the remainder of the time, that is, when the phase of the current in the wire is between zero and one-half pi or between three-halves pi and two pi, that is, when $(2n-\frac{1}{2})\pi/\omega < t < (2n+\frac{1}{2})\pi/\omega.$

29.18. If there were no magnet, the pendulum would swing back and forth, converting gravitational potential energy into kinetic energy and back into gravitational potential energy. Ignoring friction, the pendulum would rise back to its initial $30°$ angle at the end of each swing.

When the pendulum swings between the poles of the magnet, it is subject to a changing magnetic field, which induces eddy currents in the disk. In accordance with Lenz's law, the eddy currents create induced magnetic fields that oppose the motion that caused them, namely, the motion of the pendulum through the magnetic field between the poles of the magnet. Work would have to be done on the disk for it to continue to move at constant velocity through the magnetic field, but there is nothing to supply this work, so the pendulum slows, reducing its kinetic energy. Because its kinetic energy is reduced, it can no longer rise back to its initial $30°$ angle at the end of the swing. This continues to happen with each swing of the pendulum, so the amplitude of the pendulum's swing continues to get smaller and smaller.

When the vertical slits are cut in the disk, the eddy currents are reduced. Instead of a disk, we now have, in effect, a series of thin conducting rods. As they pass through the magnetic field, some charge separation occurs along their lengths, but very little current flows, so the induced magnetic fields are small and offer little resistance to the pendulum's motion. That is, with the slits, the pendulum slows much less as it passes through the magnetic field, so it rises higher than it did without the slits. The amplitude of the pendulum's swing does decrease with each swing, but by a smaller amount than it did without the slits.

29.19. The induced emf is proportional to the rate of change of magnetic flux through the loop (Eq. 29.8), so the magnitude of the induced emf is

$$\left|\mathscr{E}_{\text{ind}}\right| = \left|-\frac{d\Phi_B}{dt}\right| = 3.0\ \text{T}\cdot\text{m}^2/\text{s} = 3.0\ \text{V}$$

29.20. The induced emf is proportional to the rate of change of magnetic flux through the loop (Eq. 29.8), so the average magnitude of the induced emf is

$$\left|\mathscr{E}_{\text{ind}}\right|_{\text{avg}} = \left|-\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{1.0\ \text{T}\cdot\text{m}^2}{5.0\ \text{s}} = 0.20\ \text{V}$$

29.21. [**NOTE:** In problem statement, "homogeneous" in problem statement should be "uniform".] The induced current in the loop is proportional to the induced emf and inversely proportional to the resistance (Eq. 29.4). The induced emf is proportional to the rate of change of magnetic flux through the loop (Eq. 29.8), and the magnetic flux through the loop is given by Eq. 27.9, $\Phi_B = BA\cos\theta.$ Combining these, we have

$$I_{\text{ind,avg}} = \frac{\mathcal{E}_{\text{ind,avg}}}{R} = \frac{-\Delta\Phi_B/\Delta t}{R} = \frac{\Phi_{B,\text{i}} - \Phi_{B,\text{f}}}{R\Delta t}$$
$$= \frac{BA\cos\theta_\text{i} - BA\cos\theta_\text{f}}{R\Delta t} = \frac{BA(\cos\theta_\text{i} - \cos\theta_\text{f})}{R\Delta t}$$
$$= \frac{(0.50\text{ T})(0.080\text{ m})(0.060\text{ m})(\cos 45° - \cos 135°)}{(20\text{ V/A})(0.40\text{ s})} = 4.2\times10^{-4}\text{ A}$$

29.22. The induced emf is proportional to the rate of change of magnetic flux through the coil (Eq. 29.8). As the coil rotates, the magnetic flux through it changes. The flux will be maximal when the plane of the coil is perpendicular to the magnetic field, and in this case as the coil rotates, the flux through it is given by Eq. 27.9, $\Phi_B = BA\cos\theta = BA\cos\omega t.$ So the emf is

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}BA\cos\omega t = BA\omega\sin\omega t$$

which attains its maximum value when $\sin\omega t = 1$ and

$$\mathcal{E}_{\text{ind,max}} = BA\omega = BA(2\pi f)$$
$$= (50\times10^{-6}\text{ T})(400\text{ mm}^2)\left(\frac{1\text{ m}}{10^3\text{ mm}}\right)^2 2\pi(10{,}000\text{ rotation/min})\frac{1\text{ min}}{60\text{ s}} = 2.1\times10^{-5}\text{ V}$$

29.23. The induced emf is proportional to the rate of change of magnetic flux through the coil (Eq. 29.8), and the flux through each winding of the coil is given by Eq. 27.9, $\Phi_B = BA\cos\theta.$ So, the flux through N windings is $\Phi_B = NBA\cos\theta = NBA\cos\omega t,$ and the induced emf is

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}NBA\cos\omega t = NBA\omega\sin\omega t$$

Given the required emf, we can solve for N,

$$NBA\omega\sin\omega t = V_{\text{max}}\sin\omega t$$
$$N = \frac{V_{\text{max}}}{BA\omega} = \frac{V_{\text{max}}}{B(\pi R^2)(2\pi f)} = \frac{155\text{ V}}{2\pi^2(0.5\text{ T})(0.050\text{ m})^2(60\text{ Hz})} = 1.0\times10^2$$

29.24. (*a*) The magnitude of the induced emf is proportional to the rate of change of magnetic flux through the loop (Eq. 29.8), and the magnetic flux through the loop is given by Eq. 27.9, $\Phi_B = BA\cos\theta,$ so

$$\left|\mathcal{E}_{\text{ind}}\right| = \left|-\frac{d\Phi_B}{dt}\right| = \left|\frac{dB}{dt}\right|A = (0.070\text{ T/s})(0.20\text{ m})^2 = 2.8\times10^{-3}\text{ V}$$

(*b*) The direction of the induced current is such that it opposes the changing magnetic field that caused it. The external magnetic field is in the positive *z* direction and decreasing, so the induced current produces a magnetic field in the loop that points in the positive *z* direction. By the right-hand dipole rule, the induced current is counterclockwise.

29.25. The induced emf is proportional to the rate of change of magnetic flux through the coil (Eq. 29.8), and the flux through each winding of the coil is given by Eq. 27.9, $\Phi_B = BA\cos\theta.$ If the coil is perpendicular to the magnetic field, the flux through N windings is $\Phi_B = NBA,$ and the induced emf is

$$\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}NBA = -NA\frac{dB}{dt} = -NAB_{\text{peak}}\omega\cos\omega t$$

The induced emf reaches it maximum value when $\cos\omega t = -1$ and $\mathcal{E}_{\text{ind,max}} = NAB_{\text{peak}}\omega,$ which we can solve for AN,

$$AN = \frac{\mathcal{E}_{\text{ind,max}}}{B_{\text{peak}}\omega} = \frac{4.0\text{ V}}{(5.0\times10^{-3}\text{ T})(8.52\times10^7\text{ s}^{-1})} = 9.4\times10^{-6}\text{ m}^2$$

29.26. We derived an expression (Eq. 29.13) for the potential difference between the ends of a conducting rod that moves through a magnetic field where both the magnetic field and the rod's motion are perpendicular to its length and to each other. Here, the rod and its motion are still perpendicular to the magnetic field, but the rod's length is not perpendicular to its motion, so the magnetic force exerted on the charge carriers in the rod is not parallel to the length of the rod. The electric force on the charge carriers is still parallel to the length of the rod, and at equilibrium it is balanced by the component of the magnetic force parallel to the length of the rod, that is, $E = vB\cos 30°$. Otherwise, the derivation is the same and we have

$$V = E\ell = vB\ell\cos 30° = (0.20\ \text{m/s})(0.40\ \text{T})(0.100\ \text{m})\cos 30° = 6.9\times 10^{-3}\ \text{V}$$

In the preceding, we assumed that the rod had infinitesimal thickness, so that there was no charge separation perpendicular to its length. More generally, we could have said that the electric force was perpendicular to the magnetic force (that is, perpendicular to the rod's velocity) and taken the projection of the length of the rod onto this direction in the line integral to calculate the potential difference between the ends of the rod. In that case, the potential difference would be expressed as $V = vB(\ell\cos 30°)$, which gives the same result as $V = (vB\cos 30°)\ell$.

29.27. The emf induced in the loop when the current in the solenoid is reduced to zero is proportional to the rate of change of magnetic flux through the loop. The magnetic field of a long solenoid is approximately uniform inside the solenoid and approximately zero outside of it, so the magnetic flux through the loop equals the product of the magnitude of the magnetic field inside the solenoid (Eq. 28.6, $B = \mu_0 nI$) times the cross-sectional area of the solenoid. Combining these, we can solve for the magnitude of the initial current:

$$\left|\mathscr{E}_{\text{ind}}\right|_{\text{avg}} = \left|-\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{|\Delta B| A_{\text{sol}}}{\Delta t} = \frac{B_{\text{i}} A_{\text{sol}}}{\Delta t} = \frac{\mu_0 n I A_{\text{sol}}}{\Delta t}$$

$$I = \frac{\left|\mathscr{E}_{\text{ind}}\right|_{\text{avg}}\Delta t}{\mu_0 n A_{\text{sol}}} = \frac{\left|\mathscr{E}_{\text{ind}}\right|_{\text{avg}}\Delta t}{\mu_0 n(\pi R_{\text{sol}}^2)} = \frac{(0.10\ \text{V})(0.100\ \text{s})}{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(1000\ \text{m}^{-1})\pi(0.50\ \text{m})^2} = 10\ \text{A}$$

29.28. (*a*) The emf induced in the loop is proportional to the rate of change of magnetic flux through the loop. Because the magnetic field is uniform to the right of the *y* axis and zero to the left, the magnetic flux through the loop equals the product of the magnetic field magnitude times the area of the loop that is to the right of the *y* axis. This area changes as the loop moves to the right, $A = hw = hv\Delta t$. Combining these, we can solve for the magnetic field magnitude,

$$\left|\mathscr{E}_{\text{ind}}\right| = \left|-\frac{d\Phi_B}{dt}\right| = B\frac{dA}{dt} = B\frac{hv\Delta t}{\Delta t} = Bhv$$

$$B = \frac{\left|\mathscr{E}_{\text{ind}}\right|}{hv} = \frac{0.24\ \text{V}}{(0.30\ \text{m})(2.0\ \text{m/s})} = 0.40\ \text{T}$$

(*b*) The induced current causes an induced magnetic field through the loop that opposes the change in magnetic flux through the loop. The magnetic field is in the positive *z* direction and the flux is increasing, so the induced magnetic field points in the negative *z* direction, which by the right-hand dipole rule means that the induced current is in the clockwise direction.

29.29. (*a*) The magnetic field is parallel to the area vector of the loop, so the magnetic flux through the loop is just the product of the magnetic field magnitude times the area of the loop, $\Phi_B = BA = \pi R^2 B_0 e^{-t/\tau}$. As time passes, the flux decreases. (*b*) Taking the area vector for the loop in the positive *z* direction, the induced emf in the loop is given by Eq. 29.8,

$$\mathscr{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\pi R^2 B_0 e^{-t/\tau} = \frac{\pi R^2 B_0}{\tau}e^{-t/\tau}$$

(*c*) Assuming that B_0 is positive, our expression for the induced emf is positive, so the current direction is counterclockwise as viewed from the positive *z* axis looking toward the origin. Lenz's law gives the same result: The magnetic field points in the positive *z* direction and is decreasing, so the induced magnetic field points in the positive *z* direction, which means the current direction is counterclockwise.

29.30. (*a*) As the coil turns, the magnetic flux through it varies with the cosine of the angle between the area vector and the magnetic field vector, $\Phi_B = NBA\cos\theta$. For constant rotational speed, we have $\Phi_B = NBA\cos\omega t$ and the induced emf is

$$\mathscr{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}NBA\cos\omega t = NBA\omega\sin\omega t$$

The induced emf reaches its maximum when $\sin\omega t = 1$, and we can solve for the frequency,

$$\mathscr{E}_{\text{ind,max}} = NBA\omega = NBA(2\pi f)$$

$$f = \frac{\mathscr{E}_{\text{ind,max}}}{2\pi NBA} = \frac{1.00\ \text{V}}{2\pi(100)(0.500\times10^{-4}\ \text{T})(0.0100\ \text{m}^2)} = 3.18\times10^5\ \text{Hz}$$

(*b*) This is a very high frequency, so it does not seem like a practical way to generate electricity.

29.31. The magnetic field inside the outer solenoid is approximately uniform and given by Eq. 28.6, $B = \mu_0 nI$. The induced emf in the inner solenoid is proportional to the rate of change of magnetic flux through it, and the magnetic flux through it equals the product of the number of windings, the cross-sectional area of each winding, and the magnetic field. Combining these we have

$$\mathscr{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}N_{\text{inner}}A_{\text{inner}}B = -\frac{d}{dt}N_{\text{inner}}A_{\text{inner}}\mu_0 n_{\text{outer}}I$$

$$= -\frac{d}{dt}N_{\text{inner}}A_{\text{inner}}\mu_0\frac{N_{\text{outer}}}{\ell_{\text{outer}}}I_0\sin\omega t = -N_{\text{inner}}A_{\text{inner}}\mu_0\frac{N_{\text{outer}}}{\ell_{\text{outer}}}I_0\omega\cos\omega t$$

The induced emf takes on its greatest value when $\cos\omega t = -1$, so we have

$$\mathscr{E}_{\text{ind,max}} = N_{\text{inner}}A_{\text{inner}}\mu_0\frac{N_{\text{outer}}}{\ell_{\text{outer}}}I_0\omega = N_{\text{inner}}(\pi R_{\text{inner}}^2)\mu_0\frac{N_{\text{outer}}}{\ell_{\text{outer}}}I_0\omega$$

$$= 150\pi(0.020\ \text{m})^2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})\frac{1000}{0.400\ \text{m}}(0.600\ \text{A})(100\ \text{s}^{-1}) = 3.6\times10^{-2}\ \text{V}$$

29.32. The induced emf in the loop is proportional to the rate of change of the magnetic flux through it. Because the magnetic field inside the solenoid is uniform and the area vector of the loop is parallel to the field direction, the flux equals the product of the area of the loop and the magnitude of the field, and its rate of change is proportional to the rate of change of the field. The magnetic field of the solenoid is proportional to the current in it (Eq. 28.6), so the rate of change of the field is proportional to the rate of change of the current. Combining these we have

$$\left|\mathscr{E}_{\text{ind}}\right| = \left|-\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}BA = A\frac{d}{dt}\mu_0 nI = A\mu_0 n\frac{dI}{dt}$$

From the graph in Figure P29.32 we can estimate the rate of change of the current at $t = 10\ \text{s}$ as

$$\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \frac{(0.56\ \text{A})-(0.24\ \text{A})}{(15\ \text{s})-(5\ \text{s})} = 0.032\ \text{A/s}$$

so the induced emf is

$$\left|\mathscr{E}_{\text{ind}}\right| = (0.40\ \text{m}^2)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2000\ \text{m}^{-1})(0.032\ \text{A/s}) = 3.2\times10^{-5}\ \text{V}$$

29.33. (*a*) As the disk rotates, the magnetic field exerts a force on the moving charge carriers in the disk, causing charge separation between the center of the disk and its circumference. In particular, the magnetic force drives positive charge carriers toward the circumference. At equilibrium, the magnetic force on the charge carriers is balanced by the electric force produced by the charge separation, so we have $|q|E = |q|vB$ or $E = vB$, with the electric field directed from the circumference to the center.

The potential difference between the center of the disk and the circumference equals minus the line integral of the electric field over that displacement, and the speed equals the product of the rotational speed and the radius, so the potential difference is

$$V = -\int_{\text{center}}^{\text{circumference}} \vec{E}\cdot d\vec{\ell} = \int_0^R E\,dr = \int_0^R vB\,dr = \int_0^R r\omega B\,dr$$
$$= \tfrac{1}{2}\omega BR^2 = \tfrac{1}{2}(300 \text{ rotation/min})\frac{2\pi}{\text{rotation}}\frac{1 \text{ min}}{60 \text{ s}}(1.5 \text{ T})(0.100 \text{ m})^2 = 0.24 \text{ V}$$

(*b*) As we have seen by considering the magnetic force on the charge carriers, the circumference is at the greater potential.

29.34. The induced emf in the loop is proportional to the rate of change of magnetic flux through the loop, so we need an expression for the magnetic flux through the loop. Because the magnetic field is not uniform, we must use Eq. 27.10 to integrate the field over the area of the loop. If we call the position of the left edge of the loop x_{L} and choose our area vector parallel to the magnetic field we have

$$\Phi_B = \int_{\text{loop}} \vec{B}\cdot d\vec{A} = \int_{x_{\text{L}}}^{x_{\text{L}}+\ell} B(x)w\,dx = \int_{x_{\text{L}}}^{x_{\text{L}}+\ell}\left[B(x_{\text{i}}) + \frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}(x-x_{\text{i}})\right]w\,dx$$
$$= B(x_{\text{i}})wx + \frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}\frac{w}{2}x^2 - \frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}wx\Bigg|_{x_{\text{L}}}^{x_{\text{L}}+\ell}$$
$$= B(x_{\text{i}})w\ell + \frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}\frac{w}{2}(2x_{\text{L}}\ell+\ell^2) - \frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}w\ell$$

Notice that the only non-constant term is the second, which depends on x_{L}. If we choose a coordinate system in which $x_{\text{L}} = x_{\text{i}}$ at $t = 0$, we have $x_{\text{L}} = x_{\text{i}} + vt$ and $dx_{\text{L}}/dt = v$, so the rate of change of flux through the loop is

$$\frac{d\Phi_B}{dt} = \frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}\frac{w}{2}2v\ell = \frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}w\ell v$$

and the induced emf is

$$\mathscr{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{B(x_{\text{f}})-B(x_{\text{i}})}{x_{\text{f}}-x_{\text{i}}}w\ell v$$
$$= -\frac{(0.060 \text{ T})-(0.010 \text{ T})}{(0.900 \text{ m})-(0.300 \text{ m})}(0.040 \text{ m})(0.050 \text{ m})(0.030 \text{ m/s}) = -5.0\times10^{-6} \text{ V}$$

where the minus sign indicates that the induced emf will be directed opposite the boundary of our chosen area vector, that is, it will cause an induced current in the counterclockwise direction.

It is interesting to note that the first term in our expression for the rate of change of magnetic flux through the loop is the gradient of the magnetic field, dB/dx, the next two terms give the area of the loop, $A = w\ell$, and in the reference frame of the loop v is the speed of the source of the magnetic field. That is, in the reference frame of the loop, we could have more simply found the rate of change of magnetic flux through the loop as

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}\int_{\text{loop}} B\,dA = \int_{\text{loop}}\frac{dB}{dt}dA = \int_{\text{loop}}\frac{dB}{dx}\frac{dx}{dt}dA = \frac{dB}{dx}\frac{dx}{dt}\int_{\text{loop}} dA = \frac{dB}{dx}vA$$

29.35. As the loop enters the magnetic field, an induced current arises in the loop (Eqs. 29.3 and 29.4), which is directed counterclockwise as seen in Figure P29.35. The magnetic field exerts a force on this induced current (Eq. 27.8), which is directed downward on the upper segment of the loop, upward on the lower segment of the loop, and towards the left on the leading segment of the loop; the vector sum of these forces is toward the left, reducing the loop's speed. Combining these we have

$$F = I_{\text{ind}}wB = \frac{\mathscr{E}_{\text{ind}}}{R}wB = \frac{Bwv}{R}wB = \frac{B^2w^2v}{R}$$

which gives the equation of motion $\dfrac{dv}{dt} = -\dfrac{B^2w^2}{mR}v.$ Because the derivative of the exponential function is proportional to itself, we have as a solution $v(t) = v_{\text{i}}e^{-(B^2w^2/mR)t}$, where the loop enters the magnetic field at $t = 0$.
The loop's speed is half its initial value when the exponential term equals one-half,

$$e^{-(B^2w^2/mR)t} = \tfrac{1}{2}$$
$$-(B^2w^2/mR)t = \ln\tfrac{1}{2}$$
$$(B^2w^2/mR)t = \ln 2$$
$$t = \frac{mR\ln 2}{B^2w^2} = \frac{(0.010\text{ kg})(5.0\text{ V/A})\ln 2}{(5.0\text{ T})^2(0.120\text{ m})^2} = 9.6\times10^{-2}\text{ s}$$

Note that this is only valid if the loop is long enough that the left side has not yet entered the magnetic field, that is, if

$$\ell > \int_0^{t_{1/2}} v\,dt > \int_0^{t_{1/2}} v_\text{i}e^{-(B^2w^2/mR)t}\,dt > -\frac{v_\text{i}mR}{B^2w^2}e^{-(B^2w^2/mR)t}\bigg|_0^{t_{1/2}} > \frac{v_\text{i}mR}{2B^2w^2}$$

If the loop is not this long, it will be entirely within the magnetic field, at which point the induced current, and so the retarding force, will be zero, before it slows to half its initial speed. That is, it will not slow to half its initial speed.

29.36. (*a*) The emf induced in the loop is proportional to the rate of change of magnetic flux through the loop, so to determine the induced emf we need an expression for the magnetic flux through the loop. There are at least two ways to get this expression. We could make the obvious choice for the surface enclosed by the loop, the half-cylindrical surface bounded by the loop, and integrate the magnetic flux through this surface. Another way to get this expression is to choose a surface for which the integration is easy, because by Gauss's law for magnetism, we are free to choose any surface bounded by the loop. If we choose a surface with a horizontal rectangular section between the two straight sides and two vertical semi-circular sections connecting the first section to the arcs of the loop, the magnetic field is either parallel or perpendicular to the surface. In both cases we find that, when the loop has moved a distance x into the magnetic field, the magnetic flux is $\Phi_B = Bwx$.

In the case of our three-sided surface, this is easy to see: The vertical sides are parallel to the magnetic field, so the flux through them is zero. The horizontal side is perpendicular to the magnetic field, so the flux through it equals the product of the magnetic field magnitude and the area of the surface in the magnetic field, $\Phi_B = Bwx$.

In the case of the curved surface, we can consider an area element dA of width w subtended by an angle $d\theta$ measured from the center of the circular arc. Writing R for the radius of the circular arc, the magnitude of the area element is $dA = wRd\theta$. When the loop has moved a distance x into the magnetic field, the surface in the magnetic field will be subtended by an angle θ_x from the leading edge, where $R\cos\theta_x + x = R$, or $\cos\theta_x = 1-(x/R)$. The angle between the magnetic field vector and that of the area element dA is $90° - \theta$, so the magnetic flux is

$$\Phi_B = \int_0^{\theta_x} BwR\cos(90°-\theta)\,d\theta = \int_0^{\theta_x} BwR\sin\theta\,d\theta = -BwR\cos\theta\Big|_0^{\theta_x} = BwR\big[1-(1-x/R)\big] = Bwx$$

So, the emf induced in the loop is

$$\mathscr{E}_\text{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}Bwx = -Bw\frac{dx}{dt} = -Bwv = -(0.600\text{ T})(0.500\text{ m})(0.800\text{ m/s}) = -0.240\text{ V}$$

where the minus sign indicates that the induced magnetic field will be in the opposite direction of B. (*b*) We can solve this in the same way, using either of the surfaces we used in the previous part. In both cases, we find that when the loop has moved a distance x into the magnetic field, the magnetic flux is $\Phi_B = B(2\ell/\pi)x$, where $2\ell/\pi$ is the distance between the two straight sides. So, the induced emf is

$$\mathscr{E}_\text{ind} = -\frac{d\Phi_B}{dt} = -B(2\ell/\pi)v = -(0.600\text{ T})\big[2(0.800\text{ m})/\pi\big](0.800\text{ m/s}) = -0.244\text{ V}$$

29.37. As the bar moves, the magnetic flux through the loop made of the bar and the rails increases, which induces a current in the loop (Eqs. 29.3 and 29.4) directed counterclockwise in the Figure. The magnetic field exerts a force on the current-carrying bar (Eq. 27.8), directed toward the left, which slows the bar. Combining these we have

$$F = I_\text{ind}\ell B = \frac{\mathscr{E}_\text{ind}}{R}\ell B = \frac{B\ell v}{R}\ell B = \frac{B^2\ell^2 v}{R}$$

which gives the equation of motion

$$\frac{dv}{dt} = -\frac{B^2\ell^2}{mR}v$$

Because the derivative of the exponential function is proportional to itself, we have as a solution $v(t) = v_i e^{-(B^2\ell^2/mR)t}$, where the bar has speed v_i at $t = 0$, and the time constant is $\frac{mR}{B^2\ell^2}$. This time constant is related by a factor of ½ to the time constant that describes the decrease of energy (*Principles* Section 15.8).

29.38. The induced current in the loop is proportional to the rate of change of the magnetic flux through the loop, so in order that no current be induced in the loop we must have constant magnetic flux through the loop. Because the area vector of the loop is aligned with the magnetic field, the magnetic flux is $\Phi_B = BA = B\ell w$. If the flux remains constant, we have

$$B(t)\ell(t)w_i = B_i\ell_i w_i$$

$$\ell(t) = \frac{B_i\ell_i}{B(t)} = \frac{B_i\ell_i}{B_i + \frac{\Delta B}{\Delta t}t}$$

So, at $t = 3.00$ s we have

$$\ell = \frac{(0.500\text{ T})(0.300\text{ m})}{(0.500\text{ T}) + \frac{0.200\text{ T}}{10\text{ s}}(3.00\text{ s})} = 0.268\text{ m}$$

and at $t = 10.0$ s we have

$$\ell = \frac{(0.500\text{ T})(0.300\text{ m})}{(0.500\text{ T}) + \frac{0.200\text{ T}}{10\text{ s}}(10.0\text{ s})} = 0.214\text{ m}$$

29.39. The force on the particle is exerted by the electric field that accompanies the changing magnetic field. Following *Principles* Example 29.7, we can find the magnitude of the electric field. Because of the cylindrical symmetry of the magnetic field, the electric field lines must form circles around the center of the magnetic field. Evaluating the line integral of the electric field around such a circular path and the rate of change of magnetic flux through the area enclosed by the path in Eq. 29.17, we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$2\pi rE = \pi r^2\left|\frac{dB}{dt}\right|$$

$$E = \tfrac{1}{2}r\left|\frac{dB}{dt}\right|$$

The electric force is given by Eq. 23.6, $F_q^E = |q|E$, so we have

$$F_q^E = |q|\tfrac{1}{2}r\left|\frac{dB}{dt}\right|$$

$$\left|\frac{dB}{dt}\right| = \frac{2F_q^E}{|q|r} = \frac{2(4.00\times10^{-6}\text{ N})}{(5.0\times10^{-3}\text{ C})(0.020\text{ m})} = 8.0\times10^{-2}\text{ T/s}$$

29.40. Following *Principles* Example 29.7, we can relate the magnitude of the electric field to the rate of change of the magnetic field. Because of the cylindrical symmetry of the magnetic field, the electric field lines must form circles around the center of the magnetic field. Evaluating the line integral of the electric field around such a circular path and the rate of change of magnetic flux through the area enclosed by the path in Eq. 29.17, we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$
$$2\pi r E = \pi r^2 \left|\frac{dB}{dt}\right|$$
$$E = \tfrac{1}{2} r \left|\frac{dB}{dt}\right|$$

Solving for the rate of change of the magnetic field, we have

$$\left|\frac{dB}{dt}\right| = \frac{2E}{r} = \frac{2(10\ \text{V/m})}{0.060\ \text{m}} = 3.3 \times 10^2\ \text{T/s}$$

29.41. (*a*) Following *Principles* Example 29.7, we can relate the magnitude of the electric field to the rate of change of the magnetic field. Because of the cylindrical symmetry of the magnetic field, the electric field lines must form circles around the center of the magnetic field. Evaluating the line integral of the electric field around such a circular path and the rate of change of magnetic flux through the area enclosed by the path in Eq. 29.17, we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$
$$2\pi r E = \pi r^2 \left|\frac{dB}{dt}\right|$$
$$E = \tfrac{1}{2} r \left|\frac{dB}{dt}\right| = \tfrac{1}{2}(0.20\ \text{m})(0.30\ \text{T/s}) = 0.030\ \text{V/m}$$

(*b*) In this case, where $r > R,$ we have to change the expression for the rate of change of magnetic flux through the area enclosed by the circular path. We have instead

$$2\pi r E = \pi R^2 \left|\frac{dB}{dt}\right|$$
$$E = \frac{R^2}{2r}\left|\frac{dB}{dt}\right| = \frac{(0.25\ \text{m})^2}{2(0.50\ \text{m})}(0.30\ \text{T/s}) = 0.019\ \text{V/m}$$

(*c*) Because the magnetic field is in the positive *z* direction and decreasing, Lenz's law says that the induced magnetic field is in the positive *z* direction, so the electric field lines encircle the *z* axis counterclockwise.

29.42. (*a*) Following *Principles* Example 29.7, we can relate the magnitude of the electric field to the rate of change of the magnetic field. Because of the cylindrical symmetry of the magnetic field, the electric field lines must form circles around the center of the magnetic field. Evaluating the line integral of the electric field around such a circular path and the rate of change of magnetic flux through the area enclosed by the path in Eq. 29.17, we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$
$$2\pi r E = \pi r^2 \left|\frac{dB}{dt}\right|$$
$$E = \tfrac{1}{2} r \left|\frac{dB}{dt}\right| = \tfrac{1}{2} r \left|\frac{d}{dt} B_{\max} \sin \omega t\right| = \tfrac{1}{2} r B_{\max}\, \omega \left|\cos \omega t\right|$$

(*b*) In the case where $r > R,$ we have to change the expression for the rate of change of magnetic flux through the area enclosed by the circular path. We have instead

$$2\pi r E = \pi R^2 \left|\frac{dB}{dt}\right|$$
$$E = \frac{R^2}{2r}\left|\frac{dB}{dt}\right| = \frac{R^2}{2r}\left|\frac{d}{dt} B_{\max} \sin \omega t\right| = \frac{R^2}{2r} B_{\max}\, \omega \left|\cos \omega t\right|$$

(*c*) At $t = 0,$ the magnetic field is zero and increasing in the positive *z* direction, so by Lenz's law the induced magnetic field is in the negative *z* direction, and the electric field lines encircle the *z* axis clockwise as seen from the positive *z* axis.

29.43. Following *Principles* Example 29.7, we can relate the magnitude of the electric field to the rate of change of the magnetic field. Because of the cylindrical symmetry of the magnetic field, the electric field lines must form circles around the center of the magnetic field. Evaluating the line integral of the electric field around such a circular path and the rate of change of magnetic flux through the area enclosed by the path in Eq. 29.17, we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$2\pi r E = \pi r^2 \left|\frac{dB}{dt}\right|$$

$$\left|\frac{dB}{dt}\right| = \frac{2E}{r} = \frac{2(3Crt^2)}{r} = 6Ct^2$$

which we can integrate to find the magnitude of the magnetic field. Because our expression for the rate of change of the magnetic field increases steadily from zero, and $B = 0$ at $t = 0,$ we have

$$B = \int_0^t \frac{dB}{dt}\,dt = \int_0^t 6Ct^2\,dt = 2Ct^3$$

29.44. (*a*) Following *Principles* Example 29.7, we can relate the magnitude of the electric field to the rate of change of the magnetic field. Because of the cylindrical symmetry of the magnetic field, the electric field lines must form circles around the center of the magnetic field. Evaluating the line integral of the electric field around such a circular path and the rate of change of magnetic flux through the area enclosed by the path in Eq. 29.17, we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$2\pi r E = \pi r^2 \left|\frac{dB}{dt}\right|$$

$$E = \tfrac{1}{2} r \left|\frac{dB}{dt}\right|$$

Because the electric field magnitude is proportional to the distance from the magnetic field center, $E = E_1/3$ when $r = r_1/3.$ (*b*) When $r > R,$ the expression for the magnetic flux changes and we have

$$2\pi r E = \pi R^2 \left|\frac{dB}{dt}\right|$$

$$E = \frac{R^2}{2r}\left|\frac{dB}{dt}\right|$$

So, when $E = E_1/3,$ we have

$$\frac{R^2}{2r}\left|\frac{dB}{dt}\right| = \tfrac{1}{3}\left(\tfrac{1}{2} r_1 \left|\frac{dB}{dt}\right|\right)$$

$$r = \frac{3R^2}{r_1}$$

29.45. The magnetic field inside a long solenoid is approximately uniform and directed along the solenoid's long axis and is approximately zero outside. Applying Eq. 29.17 to a circular path centered on and perpendicular to the solenoid's axis we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$2\pi r E = \pi r^2 \left|\frac{dB}{dt}\right|$$

$$E = \tfrac{1}{2} r \left|\frac{dB}{dt}\right|$$

That is, the magnitude of the electric field is proportional to the radial distance from the solenoid's axis and has its maximum value at the radius of the solenoid. By Lenz's law, the induced emf opposes the change in the magnetic field. Because the magnetic field in the solenoid is proportional to the current (Eq. 28.6, $B = \mu_0 nI$), this means that the induced emf opposes the change in the current. Even with no resistance in the wires, we would have to supply a potential difference of $2\pi rE$ per winding to increase the current sufficiently rapidly to create an electric field of magnitude E. For the solenoid radius and electric field magnitude given, we would have to supply a potential difference per winding of $V = 2\pi rE = 2\pi(0.030 \text{ m})(10^6 \text{ V/m}) = 2\times10^5 \text{ V}$.

Note that this is also the potential difference between a given point on one winding and the neighboring point on the adjacent winding. If the wires are thin and tightly wound, the magnitude of the electric field between one winding and the next would be very large. For example, if the windings are 1 mm apart, the magnitude of the electric field between one winding and the next would be on the order of 10^8 V/m, which exceeds the electrical breakdown threshold of most materials (see *Principles* Table 26.1).

There are also usually many windings in a long solenoid, so it seems unlikely that we could supply a great enough potential difference across the solenoid to cause a spark in the air inside of it. It seems more likely that the electrical breakdown threshold would be exceeded in some other part of the equipment (for example, the insulation on the wires) before we achieved the required potential difference unless extraordinary precautions were taken.

29.46. From the definition of inductance (Eq. 29.19) we have $\mathscr{E}_{\text{ind}} = -L\dfrac{dI}{dt} = -(2.0 \text{ H})(0.40 \text{ A/s}) = -0.80 \text{ V}$, and the minus sign indicates that the induced emf opposes the increasing flow of charge carriers.

29.47. From the definition of inductance (Eq. 29.19) we have

$$\mathscr{E}_{\text{ind}} = -L\frac{dI}{dt}$$
$$L = -\frac{\mathscr{E}_{\text{ind}}}{dI/dt} = -\frac{6.0 \text{ V}}{-2.0 \text{ A/s}} = 3.0 \text{ H}$$

29.48. We can calculate the inductance of the toroid using Eq. 29.21, which relates the rate of change of magnetic flux through the toroid to its inductance and the rate of change of the current through it. The magnetic field in a toroid is (Eq. 28.9)

$$B = \frac{\mu_0 NI}{2\pi r}$$

where r is the radial distance from the center of the toroid. The magnetic flux through one winding of the toroid is (Eq. 27.10) $\Phi_{B1} = \int \vec{B}\cdot d\vec{A}$. Because the radius of the toroid is much greater than the radius of the windings, the magnetic field varies little over the area of a winding and we can approximate the flux through one winding as

$$\Phi_{B1} = BA = \frac{\mu_0 NI}{2\pi R_{\text{t}}}\pi R_{\text{w}}^2 = \frac{\mu_0 NR_{\text{w}}^2}{2R_{\text{t}}} I$$

and the flux through all N windings is

$$\Phi_B = N\Phi_{B1} = \frac{\mu_0 N^2 R_{\text{w}}^2}{2R_{\text{t}}} I$$

So, Eq. 29.21 gives

$$\frac{d\Phi_B}{dt} = \frac{\mu_0 N^2 R_{\text{w}}^2}{2R_{\text{t}}}\frac{dI}{dt} = L\frac{dI}{dt}$$

or

$$L = \frac{\mu_0 N^2 R_{\text{w}}^2}{2R_{\text{t}}} = \frac{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(400)^2(0.010 \text{ m})^2}{2(0.10 \text{ m})} = 1.0\times10^{-4} \text{ H}$$

29.49. (*a*) From the definition of inductance (Eq. 29.19) we have

$$\mathscr{E}_{\text{ind}}(t) = -L\frac{dI}{dt} = -L\frac{d}{dt}I_{\max}\sin(\omega t) = -LI_{\max}\omega\cos(\omega t)$$

(*b*) At $t = 0$ the current in the inductor is increasing because $\sin(\omega t)$ is increasing. (*c*) Because the induced emf is negative at $t = 0,$ it is opposing the flow of the charge carriers. (*d*) The induced emf opposes the increase in the current, that is, it tends to reduce the change in the current.

29.50. (*a*) From the definition of inductance (Eq. 29.19) we have $\mathscr{E}_{\text{ind}}(t) = -L\frac{dI}{dt} = -2Ct,$ so, $\frac{dI}{dt} = \frac{2Ct}{L}$ and $\int_0^t \frac{dI}{dt}dt = \int_0^t \frac{2Ct}{L}dt,$ so because $I = 0$ at $t = 0$, $I(t) = \frac{Ct^2}{L}.$ (*b*) The current is increasing when $t > 0.$ (*c*) When $t > 0,$ the current is increasing and the induced emf is negative, which means that it opposes the increase in current. This is consistent with the behavior of inductors, which oppose changes to the current they carry.

29.51. (*a*) In *Principles* Example 29.8, we derived the inductance of a long solenoid, $L = \frac{\mu_0 N^2 A}{\ell}.$ The value of *A* is fixed by the diameter of the wooden rod, so we can maximize *L* by maximizing *N* and minimizing $\ell.$ Both of these are achieved by winding the wire as tightly as possible, in which case we have for the length of wire in each winding

$$\ell_{\text{winding}} = \sqrt{d_{\text{wire}}^2 + \left[\pi(d_{\text{rod}} + d_{\text{wire}})\right]^2} = \sqrt{(4.115\times10^{-3}\text{ m})^2 + \pi^2(0.0141\text{ m})^2} = 0.0445\text{ m}$$

where we have used the Pythagorean theorem to account for the windings being diagonal and taken the circumference at the wire's center.

The number of windings is $N = \frac{\ell_{\text{wire}}}{\ell_{\text{winding}}} = \frac{0.65\text{ m}}{0.0445\text{ m}} = 14.6.$ That is, our solenoid will have only 14 full windings, with a little wire left over for the leads, and its length will be $\ell = Nd_{\text{wire}} = 14(4.115\times10^{-3}\text{ m}) = 0.05761\text{ m}.$ So, the inductance is

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(14)^2\pi(5.0\times10^{-3}\text{ m})^2}{0.05761\text{ m}} = 3.4\times10^{-7}\text{ H}$$

(*b*) Our friend used the right equation, but the wrong values. It's not clear where we got the idea that we could wind the wire around the rod about 40 times, but that was not correct. The discrepancy is approximately a factor of pi, which is easily lost in computations. Our friend has also used the cross-sectional area of the wire instead of the area of the solenoid windings, and the length of the wire instead of the length of the solenoid.

29.52. We can calculate the inductance of the toroid using Eq. 29.21, which relates the rate of change of magnetic flux through the toroid to its inductance and the rate of change of the current through it. The magnetic field in a toroid is (Eq. 28.9) $B = \frac{\mu_0 NI}{2\pi r},$ where *r* is the radial distance from the center of the toroid. The magnetic flux through one winding of the toroid is (Eq. 27.10)

$$\Phi_{B1} = \int \vec{B}\cdot d\vec{A} = \int_{R_{\text{in}}}^{R_{\text{out}}} \frac{\mu_0 NI}{2\pi r}h\,dr = \frac{\mu_0 NIh}{2\pi}\int_{R_{\text{in}}}^{R_{\text{out}}} \frac{1}{r}dr$$

$$= \frac{\mu_0 NIh}{2\pi}(\ln R_{\text{out}} - \ln R_{\text{in}}) = \frac{\mu_0 NIh}{2\pi}\ln(R_{\text{out}}/R_{\text{in}})$$

and the magnetic flux through all *N* windings is $\Phi_B = N\Phi_{B1} = \frac{\mu_0 N^2 Ih}{2\pi}\ln(R_{\text{out}}/R_{\text{in}}).$ So, Eq. 29.21 gives $\frac{d\Phi_B}{dt} = \frac{\mu_0 N^2 h}{2\pi}\ln(R_{\text{out}}/R_{\text{in}})\frac{dI}{dt} = L\frac{dI}{dt}$ or

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln(R_{\text{out}}/R_{\text{in}}) = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(200)^2(0.020\text{ m})}{2\pi}\ln\left(\frac{240\text{ mm}}{160\text{ mm}}\right) = 6.5\times10^{-5}\text{ H}$$

29.53. The magnetic potential energy stored in the magnetic field of an inductor is given by Eq. 29.25, $U^B = \frac{1}{2}LI^2 = \frac{1}{2}(0.60\text{ H})(6.0\text{ A})^2 = 11\text{ J}.$

29.54. The magnetic potential energy stored in the magnetic field of an inductor is given by Eq. 29.25, $U^B = \frac{1}{2}LI^2,$ so

$$I = \sqrt{\frac{2U^B}{L}} = \sqrt{\frac{2(10\text{ J})}{5.0\text{ H}}} = 2.0\text{ A}$$

29.55. The magnetic potential energy stored in a magnetic field is given by Eq. 29.30, $U^B = \int u_B dV.$ Because the magnetic field is uniform, this reduces to $U^B = u_B V.$ The energy density of a magnetic field is given by Eq. 29.29, $u_B = \frac{1}{2}\frac{B^2}{\mu_0},$ so

$$U^B = \frac{1}{2}\frac{B^2}{\mu_0}\pi R^2 \ell = \frac{1}{2}\frac{(0.12\text{ T})^2}{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})}\pi(0.040\text{ m})^2(0.060\text{ m}) = 1.7\text{ J}$$

29.56. The heating element is a solenoid, which is to say, an inductor. When we unplug it, there is an induced emf that resists the change in current, and this causes the spark. The amount of energy dissipated in the spark equals the amount of magnetic potential energy stored in the solenoid.

From *Principles* Example 29.8, we have the inductance of a solenoid,

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(600)^2\pi(0.0010\text{ m})^2}{0.200\text{ m}} = 7.1\times 10^{-6}\text{ H}$$

so the magnetic potential energy stored in the solenoid is (Eq. 29.25)

$$U^B = \frac{1}{2}\frac{\mu_0 N^2 A}{\ell}I^2 = \frac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(600)^2\pi(0.0010\text{ m})^2}{2(0.200\text{ m})}(3.2\text{ A})^2 = 3.6\times 10^{-5}\text{ J}$$

The electrical breakdown threshold for air is about 10^6 V/m, so if the spark is about 1 mm long, the induced emf is about 10^3 V. From Eq. 29.19 we can determine the length of the time interval over which the current is reduced to zero,

$$\mathscr{E}_{\text{ind}} = -L\frac{dI}{dt}$$

$$\frac{\mathscr{E}_{\text{ind}}}{L} = -\frac{\Delta I}{\Delta t}$$

$$\Delta t = \frac{IL}{\mathscr{E}_{\text{ind}}} = \frac{I\mu_0 N^2 A}{\ell\mathscr{E}_{\text{ind}}} = \frac{(3.2\text{ A})(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(600)^2\pi(0.0010\text{ m})^2}{(0.200\text{ m})(10^3\text{ V})} = 2\times 10^{-8}\text{ s}$$

So, the average rate at which magnetic potential energy is dissipated by the spark is

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t} = \frac{3.6\times 10^{-5}\text{ J}}{2\times 10^{-8}\text{ s}} = 2\times 10^3\text{ W}$$

29.57. Assuming the magnetic field in the cylindrical region is uniform, we have from Eqs. 29.30 and 29.29

$$U^B = u_B V = \frac{1}{2}\frac{B^2}{\mu_0}\pi R^2 \ell = \frac{(3.0\text{ T})^2\pi(0.300\text{ m})^2(0.200\text{ m})}{2(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})} = 2.0\times 10^5\text{ J}$$

If this energy is dissipated in 20 s, the average rate is

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t} = \frac{2.0\times 10^5\text{ J}}{20\text{ s}} = 1.0\times 10^4\text{ W}$$

29.58. We can calculate the magnetic potential energy stored in the volume from Eqs. 29.30 and 29.29, $U^B = \int u_B\, dV = \int \frac{1}{2}\frac{B^2}{\mu_0}dV.$ We determined the magnetic field of a long current-carrying wire in *Principles* Example 28.3 and Checkpoint 28.12,

$$B = \begin{cases} \dfrac{\mu_0 I}{2\pi r} & r > R_{\text{wire}} \\ \dfrac{\mu_0 I r}{2\pi R_{\text{wire}}^2} & r < R_{\text{wire}} \end{cases}$$

Substituting into the preceding equation, and noting that the infinitesimal volume element for the cylindrical region is $dV = 2\pi r h_{\text{cylin}}\, dr$, we have

$$\begin{aligned} U^B &= \int_0^{R_{\text{wire}}} \frac{1}{2\mu_0}\left(\frac{\mu_0 I r}{2\pi R_{\text{wire}}^2}\right)^2 2\pi r h_{\text{cylin}}\, dr + \int_{R_{\text{wire}}}^{R_{\text{cylin}}} \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 2\pi r h_{\text{cylin}}\, dr \\ &= \frac{\mu_0 I^2 h_{\text{cylin}}}{4\pi}\left(\int_0^{R_{\text{wire}}} \frac{r^3}{R_{\text{wire}}^4} dr + \int_{R_{\text{wire}}}^{R_{\text{cylin}}} \frac{1}{r} dr\right) = \frac{\mu_0 I^2 h_{\text{cylin}}}{4\pi}\left[\tfrac{1}{4} + \ln(R_{\text{cylin}}/R_{\text{wire}})\right] \\ &= \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.3\ \text{A})^2(0.050\ \text{m})}{4\pi}\left[\tfrac{1}{4} + \ln\left(\frac{24\ \text{mm}}{2.1\ \text{mm}}\right)\right] = 7.1\times10^{-8}\ \text{J} \end{aligned}$$

29.59. (*a*) Substituting the expression for L into Eq. 29.25, and noting that $n = N/\ell$, we have

$$U^B = \tfrac{1}{2}\frac{\mu_0 N^2 A}{\ell} I^2 = \tfrac{1}{2}\frac{\mu_0 (n\ell)^2(\pi R^2)}{\ell} I^2 = \tfrac{1}{2}\mu_0 n^2 \ell \pi R^2 I^2$$

(*b*) Combining Eq. 29.30 with Eq. 29.29 we have $U^B = \int u_B\, dV = \int \tfrac{1}{2}\frac{B^2}{\mu_0} dV$. Substituting Eq. 28.6, $B = \mu_0 n I$, for the magnetic field in the solenoid we have

$$U^B = \int \tfrac{1}{2}\frac{(\mu_0 n I)^2}{\mu_0} dV = \tfrac{1}{2}\mu_0 n^2 I^2 \int dV = \tfrac{1}{2}\mu_0 n^2 I^2 V = \tfrac{1}{2}\mu_0 n^2 I^2 \ell \pi R^2$$

because the magnetic field is approximately zero outside the solenoid, and its volume equals its length times its area.

29.60. [**NOTE:** In the problem statement, 0.03 m is changed to 0.030 m.] Using the formula we developed in *Principles* Example 29.8 for the inductance of a solenoid, we have

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 N^2 \pi R^2}{\ell} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/a})(2000)^2 \pi (0.030\ \text{m})^2}{0.20\ \text{m}} = 7.1\times10^{-2}\ \text{H}$$

29.61. [**NOTE:** In the problem statement, 1.2 A is changed to 1.20 A.] The induced emf is given by Eq. 29.19, $\mathscr{E}_{\text{ind}} = -L\frac{dI}{dt}$, and we developed a formula for the inductance of a solenoid in *Principles* Example 29.8, $L = \frac{\mu_0 N^2 A}{\ell}$. Combining these we have

$$\begin{aligned} \mathscr{E}_{\text{ind}} &= -\frac{\mu_0 N^2 A}{\ell}\frac{dI}{dt} = -\frac{\mu_0 N^2 \pi R^2}{\ell}\frac{dI}{dt} \\ &= -\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/a})(400)^2 \pi (0.020\ \text{m})^2}{0.150\ \text{m}}\frac{0.80\ \text{A}}{2.0\ \text{s}} = -6.7\times10^{-4}\ \text{V} \end{aligned}$$

29.62. Combining Eq. 29.30 with Eq. 29.29 we have $U^B = \int u_B\, dV = \int \tfrac{1}{2}\frac{B^2}{\mu_0} dV$. If the magnetic field is uniform in the volume of interest, this reduces to $U^B = \frac{B^2 V}{2\mu_0}$, which we can solve for the magnetic field magnitude

$$B = \sqrt{\frac{2U^B \mu_0}{V}} = \sqrt{\frac{2(12\ \text{J})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{(0.050\ \text{m})^3}} = 0.49\ \text{T}$$

29.63. Combining Eq. 29.30 with Eq. 29.29 we have $U^B = \int u_B\, dV = \int \frac{1}{2}\frac{B^2}{\mu_0} dV$. If the magnetic field is uniform in the volume of interest, this reduces to

$$U^B = \frac{B^2 V}{2\mu_0} = \frac{B^2 \pi R^2 h}{2\mu_0} = \frac{(1.3\ \text{T})^2 \pi (0.050\ \text{m})^2 (0.025\ \text{m})}{2(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})} = 1.3\times 10^2\ \text{J}$$

29.64. As the magnetic flux changes, an emf is induced in the loop (Eq. 29.8) $\mathscr{E}_{\text{ind}} = -\frac{d\Phi_B}{dt}$, which causes an induced current (Eq. 29.4) $I_{\text{ind}} = \frac{\mathscr{E}_{\text{ind}}}{R}$. Combining these we have for the magnitude of the induced current

$$I_{\text{ind}} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{1}{R}\left|\frac{d}{dt}\Phi_{B,\text{i}} e^{-\beta t}\right| = \frac{1}{R}\left|-\beta \Phi_{B,\text{i}} e^{-\beta t}\right|$$
$$= \frac{1}{2.0\ \text{V/A}}(0.50\ \text{s}^{-1})(4.0\ \text{Wb})e^{-(0.50\ \text{s}^{-1})(10\ \text{s})} = 6.7\times 10^{-3}\ \text{A}$$

29.65. (*a*) The magnetic field in the solenoid is (Eq. 28.6)

$$B = \mu_0 n I = \mu_0 \frac{N}{\ell} I = (4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})\frac{400}{0.20\ \text{m}}(3.0\ \text{A}) = 7.5\times 10^{-3}\ \text{T}$$

(*b*) We developed a formula for the inductance of a solenoid in *Principles* Example 29.8,

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 N^2 \pi R^2}{\ell} = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(400)^2 \pi (0.025\ \text{m})^2}{0.20\ \text{m}} = 2.0\times 10^{-3}\ \text{H}$$

(*c*) Combining Eq. 29.30 with Eq. 29.29 we have $U^B = \int u_B\, dV = \int \frac{1}{2}\frac{B^2}{\mu_0} dV$. Because the magnetic field is uniform in the solenoid, this reduces to

$$U^B = \frac{B^2 V}{2\mu_0} = \frac{B^2 \pi R^2 \ell}{2\mu_0} = \frac{(7.54\times 10^{-3}\ \text{T})^2 \pi (0.025\ \text{m})^2 (0.20\ \text{m})}{2(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})} = 8.9\times 10^{-3}\ \text{J}$$

where we retained an extra digit of precision in our intermediate result, the magnetic field magnitude. (*d*) The magnetic potential energy stored in an inductor is (Eq. 29.25)

$$U^B = \tfrac{1}{2} L I^2 = \tfrac{1}{2}(1.97\times 10^{-3}\ \text{H})(3.0\ \text{A})^2 = 8.9\times 10^{-3}\ \text{J}$$

29.66. As the bar moves through the magnetic field, charge separation occurs between the ends creating a potential difference $V = vB\ell$ (Eq. 29.13). The force of gravity on the bar causes it to slide down the rails with constant acceleration $a = g\sin\theta$ (*Principles* Section 3.7), so its speed after a time interval Δt is $v_{\text{f}} = v_{\text{i}} + a\Delta t$. Combining these we have

$$V = (g\sin\theta)\Delta t B\ell = (9.8\ \text{N/kg})\sin(15°)(0.20\ \text{s})(0.60\ \text{T})(0.040\ \text{m}) = 1.2\times 10^{-2}\ \text{V}$$

29.67. Current in the large loop creates a magnetic field at the location of the small loop. As this current changes, so does the magnetic flux through the small loop, inducing a current in it (Eqs. 29.8 and 29.4). We determined the magnitude of the magnetic field at the center of a circular loop in *Principles* Checkpoint 28.16, $B = \frac{\mu_0 I}{2R}$, and the magnetic flux through the small loop is just the product of the magnitude of the magnetic field caused by the current in the large loop times the area of the small loop. Combining these we have

$$I_{\text{ind}} = \frac{\mathscr{E}_{\text{ind}}}{R} = -\frac{1}{R}\frac{d\Phi_B}{dt} = -\frac{1}{R}\frac{d}{dt}BA = -\frac{1}{R}\frac{d}{dt}\frac{\mu_0 I}{2a}\pi b^2 = -\frac{\mu_0 \pi b^2}{2aR}\frac{dI}{dt}$$
$$= -\frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})\pi(0.0020\ \text{m})^2}{2(0.50\ \text{m})(0.80\ \text{V/A})}\frac{4.5\ \text{A}}{0.30\ \text{s}} = -3.0\times 10^{-10}\ \text{A}$$

where the minus sign indicates that the induced current is in the opposite direction as the current in the large loop.

29.68. As the loop gets smaller, the magnetic flux through it decreases, inducing an emf given by Eq. 29.8, $\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}BA = -B\frac{dA}{dt}$. As we pull on the ends of the wire, we reduce the loop's circumference; after an interval Δt the circumference has been reduced by $2v\Delta t$, that is, $dC/dt = -2v$. This also reduces its area. The relationship between the area and circumference of a circle is

$$A = \pi R^2$$
$$C = 2\pi R$$
$$C^2 = 4\pi^2 R^2$$
$$A = \frac{1}{4\pi}C^2,$$

so

$$\frac{dA}{dt} = \frac{1}{2\pi}C\frac{dC}{dt} = \frac{1}{2\pi}C(-2v) = -\frac{v}{\pi}(C_{\text{i}} - 2v\Delta t) = -\frac{v}{\pi}(2\pi R_{\text{i}} - 2v\Delta t)$$

and

$$\mathcal{E}_{\text{ind}} = \frac{Bv}{\pi}(2\pi R_{\text{i}} - 2v\Delta t)$$
$$= \frac{(0.70\text{ T})(6.0\times10^{-3}\text{ m/s})}{\pi}\left[2\pi(0.030\text{ m}) - 2(6.0\times10^{-3}\text{ m/s})(2.0\text{ s})\right] = 2.2\times10^{-4}\text{ V}$$

29.69. (*a*) As the rod rotates in the magnetic field, charge separation occurs creating an electric field in the rod. At equilibrium, the magnitude of the magnetic force on the charge carriers in the rod equals the magnitude of the electric force, that is,

$$F_q^B = F_q^E$$
$$qvB = qE$$
$$E = vB$$

where the speed of each portion of the rod equals the product of its distance from the axis of rotation times the rotational speed, $v = r\omega$.

The potential difference between the ends of the rod equals the negative of the line integral of the electric field from one end of the rod to the other, $V = -\int \vec{E}\cdot d\vec{\ell}$. The magnetic force on a positive charge carrier in the rod is directed away from the axis of rotation, so the electric field points toward the axis and we have $\vec{E}\cdot d\vec{\ell} = -Edr$. Combining these we have $V = \int_0^\ell r\omega B\,dr = \frac{1}{2}\ell^2\omega B$. (*b*) The end of the rod not at the axis of rotation is at the higher potential. (*c*) Substituting the given values we have $V = \frac{1}{2}(0.15\text{ m})^2(377\text{ s}^{-1})(1.0\text{ T}) = 4.2\text{ V}$.

29.70. (*a*) The magnitude of the induced emf is proportional to the rate of change of magnetic flux through the loop (Eq. 29.8), $\mathcal{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}BA = -B\frac{dA}{dt}$. As the radius of the loop increases, so does its area

$$A = \pi r^2$$
$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} = 2\pi(vt)v = 2\pi v^2 t$$

so the magnitude of the induced emf is $|\mathcal{E}_{\text{ind}}| = 2\pi B v^2 t$. (*b*) The induced current is directed so as to oppose the change in the magnetic flux through the loop. Because the magnetic field is pointing up out of the page and the flux is increasing, the induced current will produce a magnetic field that points down into the page, so the direction of the induced current is clockwise.

29.71. The force of gravity causes the bar to accelerate down the incline, in the negative x direction. As the bar moves, the magnetic flux through the conducting path made up of the rod and rails changes, inducing a current in the conductors that opposes the change. The magnetic field then causes a magnetic force to be exerted on the current-carrying bar. When the bar is at its terminal speed, the x component of this force balances that of the force of gravity, and the bar moves at constant speed.

The x component of the gravitational force on the bar is $F^G_{\mathrm{b}\,x} = -mg\sin\theta,$ and the x component of the magnetic force is $F^B_{\mathrm{b}\,x} = B\left|I_{\mathrm{ind}}\right|w\cos\theta.$ The magnitude of the induced current is given by Eqs. 29.8 and 29.4,

$$\left|I_{\mathrm{ind}}\right| = \frac{\left|\mathscr{E}_{\mathrm{ind}}\right|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{1}{R}\left|\frac{d}{dt}BA\cos\theta\right| = \frac{1}{R}Bwv\cos\theta$$

so at the terminal speed we have

$$mg\sin\theta = B\left(\frac{1}{R}Bwv_{\mathrm{term}}\cos\theta\right)w\cos\theta = \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos^2\theta$$

which we can solve for the terminal speed

$$v_{\mathrm{term}} = \frac{Rmg\sin\theta}{B^2w^2\cos^2\theta} = \frac{(0.20\ \mathrm{V/A})(8.0\times10^{-3}\ \mathrm{kg})(9.8\ \mathrm{N/kg})\sin(15°)}{(0.50\ \mathrm{T})^2(0.12\ \mathrm{m})^2\cos^2(15°)} = 1.2\ \mathrm{m/s}$$

29.72. The force of friction exerted by the rails on the bar (see *Principles* Section 10.10) is $F^k_{\mathrm{rb}} = \mu_k F^n_{\mathrm{rb}},$ which changes our equation for the balance of forces at the bar's terminal speed in Problem 71 to $mg\sin\theta - \mu_k F^n_{\mathrm{rb}} = \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos^2\theta.$ The normal force exerted by the rails on the bar has two parts, one due to the vertical force of gravity and the other due to the horizontal magnetic force on the bar, $F^n_{\mathrm{rb}} = mg\cos\theta + \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos\theta\sin\theta.$ Substituting this expression into the previous equation and solving for the coefficient of friction gives

$$mg\sin\theta - \mu_k\left(mg\cos\theta + \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos\theta\sin\theta\right) = \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos^2\theta$$

$$\mu_k\left(mg\cos\theta + \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos\theta\sin\theta\right) = mg\sin\theta - \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos^2\theta$$

$$\begin{aligned}\mu_k &= \frac{mg\sin\theta - \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos^2\theta}{mg\cos\theta + \frac{1}{R}B^2w^2v_{\mathrm{term}}\cos\theta\sin\theta}\\ &= \frac{(8.0\times10^{-3}\ \mathrm{kg})(9.8\ \mathrm{N/kg})\sin(15°) - \frac{1}{0.20\ \mathrm{V/A}}(0.50\ \mathrm{T})^2(0.12\ \mathrm{m})^2(1.0\ \mathrm{m/s})\cos^2(15°)}{(8.0\times10^{-3}\ \mathrm{kg})(9.8\ \mathrm{N/kg})\cos(15°) + \frac{1}{0.20\ \mathrm{V/A}}(0.50\ \mathrm{T})^2(0.12\ \mathrm{m})^2(1.0\ \mathrm{m/s})\cos(15°)\sin(15°)}\\ &= 0.044\end{aligned}$$

29.73. Let's choose a coordinate system in which the wire is on the y axis with the upward direction positive, the square loop is located in the xy plane along the positive x axis, and the z axis points toward us. The current in the wire produces a magnetic field, with the magnetic field lines forming circles around the wire. We determined the magnetic field of a long current-carrying wire in *Principles* Example 28.3, $B = \frac{\mu_0 I}{2\pi r},$ and the direction of the field at the location of the loop is away from us, in the negative z direction. If we choose our area element to point in the positive z direction, which means that counterclockwise is the positive direction around the loop, the magnetic flux through the loop is (Eq. 27.10)

$$\Phi_B = \int \vec{B}\cdot d\vec{A} = \int_x^{x+a} -\frac{\mu_0 I}{2\pi x}a\,dx = -\frac{\mu_0 Ia}{2\pi}\left[\ln(x+a) - \ln(x)\right] = -\frac{\mu_0 Ia}{2\pi}\left[\ln(1+a/x)\right]$$

As the current in the wire changes, so does the magnetic flux through the loop, causing an induced emf in the loop (Eq. 29.8)

$$\mathscr{E}_{\text{ind}} = -\frac{d\Phi_B}{dt} = \frac{\mu_0 a}{2\pi}\left[\ln(1+a/x)\right]\frac{dI}{dt} = \frac{\mu_0 a}{2\pi}\left[\ln(1+a/x)\right]I_{\max}\omega\cos(\omega t)$$

29.74. When the coil rotates at constant angular speed ω in a magnetic field perpendicular to the axis of rotation, the magnetic flux through the coil changes sinusoidally because the scalar product of the magnetic field and the area vector is proportional to the cosine of the angle between them. For a coil with N windings of area A, the flux is $\Phi_B = N\vec{B}\cdot\vec{A} = NBA\cos\theta = NBA\cos(\omega t)$, and the induced emf in the coil is $\mathscr{E}_{\text{ind}} = -\dfrac{d\Phi_B}{dt} = NBA\omega\sin(\omega t)$, which reaches its maximum value $NBA\omega$ when $\sin(\omega t) = 1$.

The magnitude of Earth's magnetic field is about 5×10^{-5} T, so to give a useful signal we must have $1\times10^{-3}\ \text{V} = N(5\times10^{-5}\ \text{T})A\omega$ or $NA\omega = (1\times10^{-3}\ \text{V})/(5\times10^{-5}\ \text{T}) = 20\ \text{m}^2/\text{s}$. We might want to check that our units are correct:

$$\begin{aligned}
&1\ \text{V} = 1\ \text{J/C}\\
&1\ \text{T} = 1\ \text{N/A}\cdot\text{m}\\
&\frac{1\ \text{V}}{1\ \text{T}} = 1\frac{\text{N}\cdot\text{m}}{\text{A}\cdot\text{s}}\frac{\text{A}\cdot\text{m}}{\text{N}} = 1\ \text{m}^2/\text{s}
\end{aligned}$$

While it is possible to measure electrical signals of extremely short duration, we know that it is easy to measure household electricity with inexpensive equipment. Household electricity in North America oscillates at a frequency of 60 Hz, which corresponds to an angular speed of $377\ \text{s}^{-1}$, and this could easily be achieved in a hand-cranked device using appropriate gears, such as a hand-powered drill or eggbeater (if more precision is required, it could be driven by a clockwork mechanism of modest power). With this value for ω, we have $NA = (20\ \text{m}^2/\text{s})/(377\ \text{s}^{-1}) = 0.05\ \text{m}^2$. If we used circular windings of radius 25 mm, we would need about 30 of them in our coil.

29.75. A lightning bolt is more-or-less an enormous spark, so their should be a current associated with it, and currents produce magnetic fields. If we model the lighting bolt as a vertical wire in which a current is rapidly turned on and then off, it will produce a changing magnetic field, the lines of which form horizontal circles centered on the wire. If some of these field lines intercept our solenoid, the magnetic flux through the solenoid will increase rapidly, inducing an emf in one direction, and then it will decrease rapidly, inducing an emf in the other direction, and this is what we see in our data.

We can determine the magnitude of the current from the magnitude of the magnetic field using the formula for the magnetic field of a long current-carrying wire we derived in *Principles* Example 28.3, $B = \dfrac{\mu_0 I}{2\pi r}$, or $I = \dfrac{2\pi r B}{\mu_0}$. We can determine the magnetic field from the induced emf by integrating Eq. 29.8,

$$\begin{aligned}
\mathscr{E}_{\text{ind}} &= -\frac{d\Phi_B}{dt}\\
\int\mathscr{E}_{\text{ind}}\,dt &= -\int d\Phi_B\\
\Delta\Phi_B &= -\int\mathscr{E}_{\text{ind}}\,dt
\end{aligned}$$

The magnetic flux through our solenoid is $\Phi_B = NBA\cos\theta$, so

$$\begin{aligned}
N\Delta BA\cos\theta &= -\int\mathscr{E}_{\text{ind}}\,dt\\
\Delta B &= -\frac{1}{NA\cos\theta}\int\mathscr{E}_{\text{ind}}\,dt
\end{aligned}$$

The integral of the emf with respect to time is the area under the curve in Figure P29.75. It looks as if the lightning strike occurred during the interval $t = 0.120$ s to $t = 0.420$ s, where the first, upward deviation was caused by the lightning current increasing and the second, downward deviation was caused by the current decreasing. To calculate the integral, we can approximate the curve by a series of rectangles and sum their areas, $\int \mathscr{E}_{\text{ind}}\, dt \approx \sum \mathscr{E}_{\text{ind}} \Delta t,$ using either the positive or negative portion of the curve to find the maximum value of $\Delta B,$ which is also the maximum value of B because the magnetic field due to the lightning was initially zero.

Reading values off the graph, we have

t (s)	$\mathscr{E}$ (mV)
0.12	0.0
0.14	0.3
0.16	0.8
0.18	1.6
0.20	2.7
0.22	3.7
0.24	4.0
0.26	2.0
0.28	−0.7
0.30	−3.3
0.32	−3.7
0.34	−3.5
0.36	−2.0
0.38	−0.8
0.40	−0.3
0.42	0.0

The sum of the positive values of the emf is $15.1 \text{ mV},$ while the sum of the negative values is $-14.3 \text{ mV}.$ The difference of their absolute values is $0.8 \text{ mV},$ but this does not seem excessive because the error bars on the graph suggest that each emf value has an uncertainty of $\pm 0.2 \text{ mV}.$ For our calculation, we can use the average of the two values, $14.7 \text{ mV},$ where here and in what follows we will retain extra precision in our intermediate values. We will also ignore the sign because that depends on the orientation of the solenoid's windings, which we do not know. (It would be interesting if we did know it because then the sign would allow us to determine whether the current in the lightning is directed upward or downward.)

Because all of the time intervals are the same, we have for our approximate integral

$$\sum \mathscr{E}_{\text{ind}} \Delta t = \Delta t \sum \mathscr{E}_{\text{ind}} = (0.02 \text{ s})(14.7\times10^{-3} \text{ V}) = 2.94\times10^{-4} \text{ V}\cdot\text{s} = 2.94\times10^{-4} \text{ T}\cdot\text{m}^2$$

and

$$B = \frac{\Phi_B}{NA\cos\theta} = \frac{2.94 \text{ T}\cdot\text{m}^2}{(100)(0.10 \text{ m}^2)\cos(50°)} = 4.57\times10^{-5} \text{ T}$$

where we have used $\theta = 90° - 40° = 50°$ for the angle between the area vector of the solenoid's windings and the magnetic field because the magnetic field is directed perpendicular to our line of sight to the tree.

We heard the thunder 2.0 s after the emf pulse because it took that long for the sound to travel to us from its source, the lightning strike. The speed of sound in air is 343 m/s at $20 \text{ °C},$ so the distance to the lighting strike is about 686 m and the current is, from our first equation,

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(686 \text{ m})(4.57\times10^{-5} \text{ T})}{4\pi\times10^{-7} \text{ T}\cdot\text{m/A}} = 1.6\times10^5 \text{ A}$$

This is a very large current, and our calculation includes some fairly large uncertainties, but a little on-line research suggests that our result is reasonable for a large lightning bolt.

29.76. Induced emfs and currents are caused by changing magnetic fluxes. It is not, however, the linearity or non-linearity of the change that determines the magnitudes of the induced emfs and currents, but rather the instantaneous rate of change. Without knowing the details of the electronic equipment, we cannot calculate magnetic fluxes, but assuming the equipment does not include moving conductors, we can say that any magnetic flux is proportional to the magnetic field, and likewise for their rates of change, $I_{\text{ind}} \propto \mathscr{E}_{\text{ind}} \propto \dfrac{d\Phi_B}{dt} \propto \dfrac{dB}{dt}$, and we can estimate dB/dt from the graph. At its steepest, near the power surge, at around $t = 3$ s, the slope of the curve is about $\dfrac{\Delta B}{\Delta t} = \dfrac{0.20 \text{ T}}{0.67 \text{ s}} = 0.30$ T/s. At this rate, the time interval it would take for the magnetic field to increase from 0 to 0.90 T linearly is $\Delta t = \dfrac{\Delta B}{dB/dt} = \dfrac{0.90 \text{ T}}{0.30 \text{ T/s}} = 3.0$ s. So, unless the equipment was designed with the expectation that the field would increase over a much longer interval, it seems unlikely that the power supply is causing the damage.

29.77. [**NOTE:** In the problem statement, the word "cassette" is replaced by "tape."] The emf induced in the coil is proportional to the rate of change of magnetic flux through the coil. If 98% of the magnetic flux on the tape passes through the 250-turn coil, then the magnetic flux through the coil is $\Phi_{\text{coil}} = (250)(0.98)\Phi_{\text{tape}}$. The magnetic flux on the tape for a sine wave is $\Phi_{\text{tape}} = \Phi_{\text{tape,max}} \sin(2\pi ft)$, and we are given a value of $\Phi_{\text{tape,max}}$ for a particular tape and frequency, $\Phi_{\text{tape,max}} = (320 \text{ nWb/m})(6.3 \text{ mm})$ for $f = 1000$ Hz. The rate of change of magnetic flux through the coil is proportional to the rate of change of magnetic flux on the tape, and that is $\dfrac{d\Phi_{\text{tape}}}{dt} = \Phi_{\text{tape,max}} 2\pi f \cos(2\pi ft)$.
Combining these, we find the emf induced in the coil is

$$\mathscr{E}_{\text{ind}} = -\frac{d\Phi_{\text{coil}}}{dt} = -(250)(0.98)\frac{d\Phi_{\text{tape}}}{dt} = -(250)(0.98)\Phi_{\text{tape,max}} 2\pi f \cos(2\pi ft)$$

The induced emf has its maximum magnitude when the cosine term equals one, and for this particular tape and frequency it is

$$\left|\mathscr{E}_{\text{ind}}\right|_{\text{max}} = (250)(0.98)(320\times10^{-9} \text{ Wb/m})(6.3\times10^{-3} \text{ m})2\pi(1000 \text{ Hz}) = 3.1\times10^{-3} \text{ V}$$

Notice that the induced emf is proportional to the frequency of the signal. An interesting question is whether this effect is compensated for during recording (by recording high-frequency signals at reduced magnetic flux levels) or during playback (by reducing the amplification of high-frequency signals). We might be able to determine which by playing the tape at different speeds. If we double the speed of the tape, we will double the frequency of the signal, and so double the magnitude of the induced emf. We could measure the output of the tape player with an oscilloscope and see whether it, too, is doubled.

31

ELECTRIC CIRCUITS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

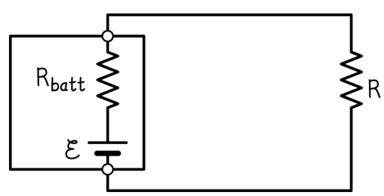
Developing a Feel

1. $10^2\ \Omega$ **2.** $10^3\ \Omega$ **3.** 10^3 m **4.** 10^{-1} m **5.** 10^{-3} m **6.** 10^3 V/m **7.** 10^5 J **8.** 10^1 dollars **9.** 10^3 W
10. 10^3 dollars

Guided Problems

31.2 Battery-powered lift

1. Getting Started We begin by drawing a circuit diagram. The winch is certain to be more complex than just a resistor. But the winch will have elements in it that resist the flow of current, and energy will be used by the winch in lifting a load. By focusing on the resistance of the winch, we can describe the energy used by the winch in terms of other circuit-related quantities.



When the winch is set to a slow speed, its internal resistance is much higher than the internal resistance of the battery. We see from the diagram that the internal resistance of the battery and that of the winch are in series, meaning the same current must pass through each resistance. Thus we can use the same current I to write $P_{R_{\text{batt}}} = I^2 R_{\text{batt}}$ and $P_{\text{winch}} = I^2 R$. Clearly, if $R \gg R_{\text{batt}}$, then $P_{\text{winch}} \gg P_{R_{\text{batt}}}$ and the total energy used in the circuit will be approximately equal to the power used by the winch. Note that P_{winch} is not the same thing as the rate of increase of the gravitational potential energy of the object being lifted (because the winch is only 50% efficient).

2. Devise Plan We are given an example of the current and potential difference that can be supplied by the battery for a certain length of time. Since the current and potential difference can be assumed to be constant over that lifetime, we can write $E_{\text{batt}} = P_{\text{batt}}\Delta t$, or

$$E_{\text{batt}} = IV\Delta t \qquad (1)$$

Also, the current is simply the charge delivered per unit time. So the total charge delivered is $q = I\Delta t$.

If the winch is lifting the load very slowly then a negligible amount of energy is being dissipated in the internal resistance of the battery. But we still must consider that the winch is only 50% efficient. Thus 50% of the total energy delivered by the battery will go into increasing the gravitational potential energy of the load object: $\frac{1}{2}E_{\text{batt}} = mgh,$ where h is the maximum height. Combining this with equation (1) yields

$$\frac{1}{2}IV\Delta t = mgh \tag{2}$$

The rate of energy transfer (to the gravitational potential energy of the load object) is half the energy delivered to the winch: $P_{\text{to } U^G} = \frac{1}{2}I^2R,$ or equivalently

$$P_{\text{to } U^G} = \frac{1}{2}\frac{\varepsilon^2}{R} \tag{3}$$

For a fixed resistance in the winch (much larger than R_{batt}) this power is a constant. However, if we wish to maximize it, we might reduce the resistance of the winch by changing the speed setting. If we reduce the resistance of the winch, it may no longer be negligible compare to the internal resistance of the battery, and our previous results for the maximum height may no longer hold. Let us assume that the battery is capable of maintaining the electromotive force of 1.5 V regardless of the current in the circuit. The maximum power output of the battery to the winch corresponds to maximizing the quantity $P_{\text{winch}} = I^2R.$ If the resistance of the winch is no longer negligible compared to that of the battery, we have $I = \frac{\varepsilon}{R_{\text{batt}} + R},$ and

$$P_{\text{winch}} = I^2R = \frac{\varepsilon^2 R}{(R_{\text{batt}} + R)^2} \tag{4}$$

It is not trivial to see where the maximum of this expression occurs. To determine this we take the derivative with respect to the variable resistance of the winch $R,$ and set that derivative equal to zero to obtain the resistance that results in the maximum possible power delivery to the winch.

$$\frac{dP_{\text{winch}}}{dR} = \varepsilon^2\left[\frac{(R_{\text{batt}} + R)^2 - 2R(R_{\text{batt}} + R)}{(R_{\text{batt}} + R)^4}\right] = 0$$

$$\Rightarrow \left(R_{\text{batt}} + R\right)^2 = 2R\left(R_{\text{batt}} + R\right)$$

$$\Rightarrow \left(R_{\text{batt}} + R\right) = 2R$$

$$\Rightarrow R_{\text{batt}} = R$$

So the maximum power delivery occurs when the resistance of the winch equals the internal resistance of the battery. Inserting this into equation (4) we have

$$(P_{\text{winch}})_{\max} = \frac{\varepsilon^2}{4R_{\text{batt}}} \tag{5}$$

It may appear that we have a logical inconsistency since the "maximum" power in equation (5) appears to be smaller than the power from equation (3). However, note that in equation (3) we had not yet set the condition $R = R_{\text{batt}}.$ In equation (3) the resistance of the wince R is still an undetermined (and large) quantity. Thus the numerical value of the power in equation (5) can still be much larger than that described by equation (3).

If we set half the maximum power output in (5) equal to the rate of increase of the load height, we find

$$\frac{1}{2}\left(\frac{\varepsilon^2}{4R_{\text{batt}}}\right) = mgv_{\max} \tag{4}$$

We can also see now that the current in the case of maximal power is $I = \frac{\varepsilon}{2R_{\text{batt}}}.$ Since the same amount of charge will be delivered by the battery regardless of the rate at which power is used, we can equate the charge delivered as fast as possible to the charge delivered in the test of the battery referred to at the beginning of the problem statement.

$$q_{\text{test}} = q_{\text{fast}}$$
$$I_{\text{test}}\Delta t_{\text{test}} = I_{\text{fast}}\Delta t_{\text{fast}}$$
$$\Delta t_{\text{fast}} = \frac{I_{\text{test}}\Delta t_{\text{test}}}{I_{\text{fast}}}$$
$$\Delta t_{\text{fast}} = \frac{2I_{\text{test}}\Delta t_{\text{test}}R_{\text{batt}}}{\varepsilon}$$
$$\Delta t_{\text{fast}} = \frac{2(0.025\text{ A})(300\text{ h})(1.0\ \Omega)}{(1.5\text{ V})}\left(\frac{3600\text{ s}}{1\text{ h}}\right)$$
$$\Delta \text{t}_{\text{fast}} = 3.6\times10^{4}\text{ s} \tag{6}$$

3. Execute Plan The total charge delivered by the battery is

$$q = I\Delta t = (25\times10^{-3}\text{ A})(300\text{ h})\frac{(3600\text{ s})}{(1\text{ h})} = 2.7\times10^{4}\text{ C}$$

Using equation (1), the total energy delivered by the battery is

$$E_{\text{batt}} = I\varepsilon\Delta t = (25\times10^{-3}\text{ A})(1.5\text{ V})(300\text{ h})\frac{(3600\text{ s})}{(1\text{ h})} = 4.1\times10^{4}\text{ J}$$

(*a*) At a very slow speed, we can use equation (2) to find the maximum height h that the object can be lifted:

$$h = \frac{IV_{\text{winch}}\Delta t}{2mg} = \frac{I\varepsilon\Delta t}{2mg} = \left(\frac{1}{2(60\text{ kg})(9.8\text{ m/s}^2)}\right)\left((25\times10^{-3}\text{ A})(1.5\text{ V})(300\text{ h})\frac{(3600\text{ s})}{(1\text{ h})}\right)$$
$$h = 34\text{ m}$$

Using the maximum power output, we can use $\Delta U^G = P_{\text{to }U^G}\Delta t = \frac{1}{2}P_{\text{winch}}\Delta t,$ and use the power from equation (5) and the time from equation (6):

$$h = \frac{\Delta U^G}{mg} = \frac{P_{\text{winch}}\Delta t_{\text{fast}}}{2mg} = \frac{\varepsilon^2\Delta t_{\text{fast}}}{8R_{\text{batt}}mg}$$
$$h = \frac{(1.5\text{ V})^2(3.6\times10^{4}\text{ s})}{8(1.0\ \Omega)(60\text{ kg})(9.8\text{ m/s}^2)}$$
$$h = 17\text{ m}$$

(*b*) The maximum speed is given by setting the maximum rate at which power is delivered to the winch equal to the maximum rate of increase of the gravitational potential energy mgv_{max}:

$$(P_{\text{to }U^G})_{\text{max}} = \frac{\varepsilon^2}{8R_{\text{batt}}}$$
$$(P_{\text{to }U^G})_{\text{max}} = \frac{\varepsilon^2}{8R_{\text{batt}}} = mgv_{\text{max}}$$
$$v_{\text{max}} = \frac{\varepsilon^2}{8R_{\text{batt}}mg}$$
$$v_{\text{max}} = \frac{(1.5\text{ V})^2}{8(1.0\ \Omega)(60\text{ kg})(9.8\text{ m/s}^2)}$$
$$v_{\text{max}} = 4.8\times10^{-4}\text{ m/s}$$

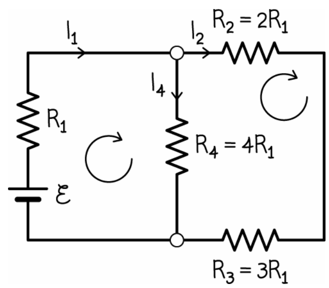
At this rate, it would take $\Delta t = \frac{h}{v_{\text{max}}} = \frac{(17.2\text{ m})}{(4.78\times10^{-4}\text{ m/s})} = 3.6\times10^{4}\text{ s}$ or $1.0\times10^{1}\text{ h}$ to lift the object to its maximum height.

4. Evaluate Result In terms of the expressions we obtained, it is reasonable that the maximum power would be delivered to the winch when the battery and load resistances are equal. Lowering the load resistance further would reduce the fraction of the power in the winch as compared to the power dissipated in the internal resistance of the battery. Increasing the load resistance further would lower the current in the circuit and reduce the overall power delivery.

Numerically, it is reasonable that a D cell battery would lift a 60 kg object very slowly. An answer of the order 1 mm/s is perfectly plausible. As a final numerical check, we note that when the battery delivers energy at its maximum rate, the current in the circuit is $\mathcal{E}/(R+R_{\text{batt}}) = (1.5\ \text{V})/(2.0\ \Omega) = 0.75\ \text{A}$. This is 30 times the current quoted in the description of the battery's total energy at the beginning of the problem statement (25 mA). Since the electromotive force in the battery is not changed, one would expect this to mean that the battery would be exhausted 30 times faster when delivering maximum power to the circuit. Indeed, we found that the battery would be exhausted after 10 h, compared to the 300 h given at the beginning of the problem statement.

31.4 Four resistors

1. Getting Started We begin by adding current directions and paths that we will take around the loops:



Since resistors R_2 and R_3 are in series, they must have the same current running through them. Thus, there is no reason to label or calculate I_3; I_2 will suffice.

2. Devise Plan Because we have three unknowns $(I_1, I_2,$ and $I_4)$ we will need three linearly independent equations to solve the problem. However, in this case, there may be a cognitively simpler way to determine the currents than simply by algebraically solving three coupled equations. In this case, we can find the equivalent resistance of the circuit to determine the total current supplied by the battery in terms of $\mathcal{E}$ and R_1. Then we can look at the relative resistances of the various paths to determine how that total current will split at the junction.

3. Execute Plan Resistors R_2 and R_3 are in series. Their combination (R_2+R_3) is in parallel with R_4 and that combination is in series with R_1. Thus the equivalent resistance of the entire circuit is

$$R_{\text{eq}} = R_1 + \left(\frac{1}{R_4} + \frac{1}{(R_2+R_3)}\right)^{-1}$$

$$R_{\text{eq}} = R_1 + \left(\frac{1}{4R_1} + \frac{1}{(5R_1)}\right)^{-1}$$

$$R_{\text{eq}} = \left(\frac{29}{9}\right)R_1$$

Thus the current delivered by the battery to the circuit is $I_{\text{circuit}} = \mathcal{E}_{\text{circuit}}/R_{\text{eq}} = \dfrac{9\mathcal{E}}{29R_1}$.

Because this entire current must pass through resistor R_1, it follows immediately that $I_1 = \left(\dfrac{9}{29}\right)\dfrac{\mathcal{E}}{R_1}$.

At this point we have several choices. One might use the junction rule to relate I_1 to I_2 and I_4, and compare relative resistances of the paths current might follow. Alternatively, we could use the loop rule on the left-most loop to determine the current through R_4. We follow this latter approach, though the former is just as valid. Beginning just below the battery and moving clockwise around the left-most loop, the sum of changes in potential is

$$\varepsilon - I_1R_1 - I_4R_4 = 0$$

$$\frac{\varepsilon}{R_1} = I_1 + 4I_4$$

$$I_4 = \frac{1}{4}\left(\frac{\varepsilon}{R_1} - \left(\frac{9}{29}\right)\frac{\varepsilon}{R_1}\right)$$

$$I_4 = \left(\frac{5}{29}\right)\frac{\varepsilon}{R_1}$$

Now beginning just below R_4 and moving clockwise around the right-most loop, the sum of changes in potential is given by

$$I_4R_4 - I_2(R_2 + R_3) = 0$$

$$I_2 = \frac{I_4R_4}{(R_2 + R_3)}$$

$$I_2 = \left[\left(\frac{5}{29}\right)\frac{\varepsilon}{R_1}\right]\frac{(4R_1)}{(5R_1)}$$

$$I_2 = \left(\frac{4}{29}\right)\frac{\varepsilon}{R_1}$$

Thus $I_1 = \left(\frac{9}{29}\right)\frac{\varepsilon}{R_1}$, $I_2 = \left(\frac{4}{29}\right)\frac{\varepsilon}{R_1}$ and $I_4 = \left(\frac{5}{29}\right)\frac{\varepsilon}{R_1}$.

4. Evaluate Result Because we have left our answers in terms of fractions, one can see that the junction rule is satisfied, by inspection:

$$I_1 = I_2 + I_4$$

$$\left(\frac{9}{29}\right)\frac{\varepsilon}{R_1} = \left(\frac{4}{29}\right)\frac{\varepsilon}{R_1} + \left(\frac{5}{29}\right)\frac{\varepsilon}{R_1}$$

Dividing both sides by $\left(\frac{1}{29}\right)\frac{\varepsilon}{R_1}$ yields $9 = 4 + 5$, meaning the junction rule is obviously satisfied.

31.6 Conductivity of seawater

1. Getting Started We are accustomed to thinking of resistors as linear components of circuits. We often think of them as being formed of a certain length L (in the direction of current flow) and a certain cross-sectional area A of a material with resistivity ρ or equivalently conductivity $\sigma = 1/\rho$. In this case, current is flowing from the inner cylinder to the outer shell, meaning that the relevant "length" of the resistor is actually $r_{\text{outer}} - r_{\text{inner}}$. The relevant cross-sectional area at a given distance r from the center of the inner cylinder is $2\pi r\ell$, but that obviously changes as one moves radially inward or outward. Thus, we must speak of the resistance of a thin (differential) cylindrical shell of seawater, which has a well-defined area $2\pi r\ell$. We will then add up the contributions of these differential shells. Because all current flowing through one such shell must then flow through the subsequent shell, these shells can be considered to be in series. Thus the contributions of the many shells will add directly (as opposed to inversely, as in parallel resistors). This allows us to add the contributions from the many shells by integrating.

2. Devise Plan In analog with the linear resistors described above, we write the differential contribution from an infinitely thin cylindrical shell of seawater as

$$dR = \frac{\rho L}{A} = \frac{\rho dr}{2\pi r\ell}$$

or equivalently

$$dR = \frac{dr}{2\pi r\ell\sigma} \tag{1}$$

We will integrate expression (1) from the inner to the outer radius to determine the total resistance of all seawater in the device. Once we have an expression for this resistance in terms of the conductivity, we can relate it to other known quantities (current and potential difference) using $\Delta V = IR$ (Ohm's Law).

3. Execute Plan The total resistance of all seawater in the device is

$$R = \int dR$$

$$R = \int_{r_{\text{inner}}}^{r_{\text{outer}}} \frac{dr}{2\pi r\ell\sigma}$$

$$R = \frac{1}{2\pi\ell\sigma}\ln\left(\frac{r_{\text{outer}}}{r_{\text{inner}}}\right) \tag{2}$$

Equating (2) to $\Delta V/I$ using equation (31.11) and rearranging, we have

$$\sigma = \frac{I}{2\pi\ell\Delta V}\ln\left(\frac{r_{\text{outer}}}{r_{\text{inner}}}\right) \tag{3}$$

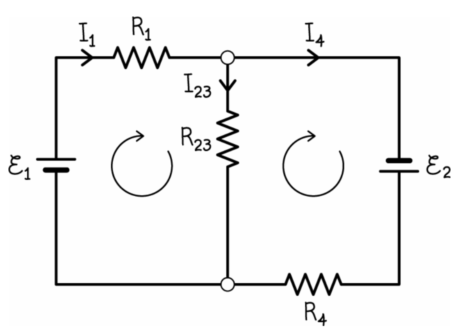
Inserting numerical values into equation (3), we find

$$\sigma = \frac{(2.93\text{ A})}{2\pi(0.400\text{ m})(0.500\text{ V})}\ln\left(\frac{(0.0400\text{ m})}{(0.0100\text{ m})}\right) = 3.23\text{ A/(V}\cdot\text{m)}$$

4. Evaluate Result Different sources quote different values for the conductivity of seawater. The reason for this variation is that the salinity in different regions of oceans varies based on other factors. However, practically all values will fall between $1-10$ A/(V · m). This is consistent with our answer.

31.8 Resistor network power

1. Getting Started We begin by noting that resistors R_2 and R_3 are wired in parallel, such that we can combine them into one equivalent resistor: $R_{23} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$. We then pick directions for the various currents and choose an arbitrary direction to move around the various loops. This is shown below.



2. Devise Plan We need to determine the three unknown currents in the figure above. To do so, we write down the junction rule:

$$I_1 = I_{23} + I_4 \tag{1}$$

We now start just below the battery on the left hand side and move clockwise around the left-most loop, writing down changes in potential according to the loop rule:

$$\mathcal{E}_1 - I_1 R_1 - I_{23} R_{23} = 0 \tag{2}$$

We do the same again, this time beginning just below resistor R_{23} and moving clockwise around the right-most loop:

$$I_{23} R_{23} + \mathcal{E}_2 - I_4 R_4 = 0 \tag{3}$$

This gives us three equations for our three unknown currents, so we can solve for the currents. Note that current I_{23} is really the total current flowing through resistors R_2 and R_3, wired in parallel. But it is particularly easy here to determine how much of I_{23} flows through each of those two resistors, because R_2 and R_3 are equal. Thus the current through each (I_2 and I_3) will each be half of I_{23}.
Once all four currents are know, we calculate the power dissipated in the n^{th} resistor using $P_n = I_n^2 R_n$.

3. Execute Plan First we note the numerical value of

$$R_{23} = \left(\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1} = \left(\frac{1}{(4.0\ \Omega)}+\frac{1}{(4.0\ \Omega)}\right)^{-1} = (2.0\ \Omega)$$

We will first use equation (1) to eliminate current I_1. Inserting equation (1) into equation (2) we have

$$\varepsilon_1 - (I_{23}+I_4)R_1 - I_{23}R_{23} = 0$$

$$I_{23} = \frac{\varepsilon_1 - I_4 R_1}{(R_1 + R_{23})} \tag{4}$$

Now we use equation (4) to eliminate current I_{23}. Inserting equations (1) and (4) into equation (3) yields an expression entirely in terms of one unknown: I_4

$$\frac{\varepsilon_1 - I_4 R_1}{(R_1+R_{23})}R_{23} + \varepsilon_2 - I_4 R_4 = 0$$

$$I_4 = \frac{\varepsilon_2(R_1+R_{23}) + \varepsilon_1 R_{23}}{R_4(R_1+R_{23}) + R_1 R_{23}}$$

$$I_4 = \frac{(20\ \text{V})((2.0\ \Omega)+(2.0\ \Omega)) + (50\ \text{V})(2.0\ \Omega)}{(2.0\ \Omega)((2.0\ \Omega)+(2.0\ \Omega)) + (2.0\ \Omega)(2.0\ \Omega)}$$

$$I_4 = 15\ \text{A}$$

Inserting this result back into equation (4) yields

$$I_{23} = \frac{(50\ \text{V}) - (15\ \text{A})(2.0\ \Omega)}{((2.0\ \Omega)+(2.0\ \Omega))}$$

$$I_{23} = 5.0\ \text{A}$$

Finally inserting the two known currents into equation (1), we find $I_1 = (15\ \text{A}) + (5.0\ \text{A}) = 20\ \text{A}$. Recalling our argument about current I_{23} splitting evenly between resistors R_2 and R_3, we see $I_2 = I_3 = 2.5\ \text{A}$. Inserting each of these currents into the expression for power, we have

$$P_1 = I_1^2 R_1 = (20\ \text{A})^2(2.0\ \Omega) = 8.0\times10^2\ \text{W}$$

$$P_2 = I_2^2 R_1 = (2.5\ \text{A})^2(4.0\ \Omega) = 2.5\times10^1\ \text{W}$$

$$P_3 = I_3^2 R_1 = (2.5\ \text{A})^2(4.0\ \Omega) = 2.5\times10^1\ \text{W}$$

$$P_4 = I_4^2 R_1 = (15\ \text{A})^2(2.0\ \Omega) = 4.5\times10^2\ \text{W}$$

4. Evaluate Result The currents do satisfy the junction rule since $(20\ \text{A}) = (2.5\ \text{A}) + (2.5\ \text{A}) + (15\ \text{A})$. The power delivered by a battery is $P = I\varepsilon$. So the total power delivered to the circuit by both batteries is

$$P_{\text{batt}} = \varepsilon_1 I_1 + \varepsilon_2 I_4 = (50\ \text{V})(20\ \text{A}) + (20\ \text{V})(15\ \text{A})$$

$$P_{\text{batt}} = 1.3\times10^3\ \text{W}$$

Summing over the powers dissipated in the various resistors, we find

$$P_{\text{res}} = P_1 + P_2 + P_3 + P_4$$

$$P_{\text{res}} = (8.0\times10^2\ \text{W}) + (2.5\times10^1\ \text{W}) + (2.5\times10^1\ \text{W}) + (4.5\times10^2\ \text{W})$$

$$P_{\text{res}} = 1.3\times10^3\ \text{W}$$

So the power supplied by the battery equals the power dissipated by the resistors, as it should.

Questions and Problems

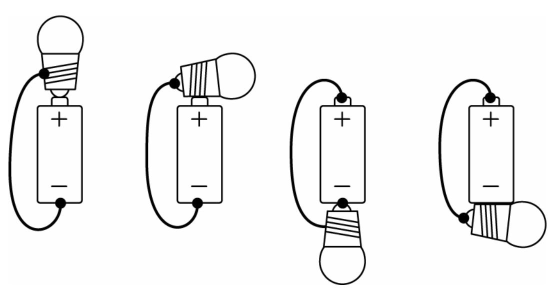
31.1. 3.0×10^{24} electrons remain in the battery. Whenever electrons leave one terminal of a battery, an equal number of electrons enter the other terminal.

31.2. A battery is drawn to represent the source of energy (chemical potential energy), and a resistor or several resistors can be drawn to represent elements that dissipate energy in the form of heat. For more complex circuits one may include other symbols to represent elements that store energy in electric or magnetic fields.

31.3. Current can only flow from one terminal of the battery to the other terminal in a closed loop. This means that current will only flow through wire making up the path $a \rightarrow b \rightarrow e \rightarrow f \rightarrow j \rightarrow a.$ None of the other wire segments will carry any current. Further, any charge that flows into point a must flow out again. There is nowhere for the flowing charge to accumulate or escape, so the current at all points along the above mentioned closed path must be the same. Thus $a=b=e=f=j>g=h=c=d=0.$

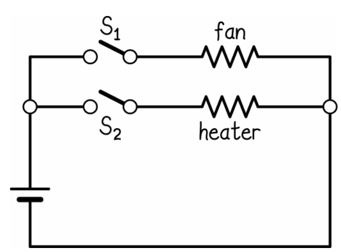
31.4. Current can only flow from one terminal of the battery to the other terminal in a closed loop. There is no such closed loop connecting the two ends of the left-most battery, so no current flows through bulbs A and B. If any current could flow through bulbs F and G, the charge on the right side would have nowhere to go. Thus, current flows through bulbs C, D, and E only.

31.5. We must complete a circuit. This can be done by connecting the light bulb directly to one terminal, and running the wire from the opposite terminal of the battery to the other contact on the lightbulb. But it does not matter which terminal on the battery is connected to which side of the lightbulb, and it does not matter whether electrons go first through the bulb and then through the wire, or first through the wire and then through the bulb. The four resulting setups are shown below.



31.6. There are many possible answers. Some examples include capacitors, inductors (quickly switched into the circuit in question), thermoelectric materials, and wall outlets.

31.7.



It is possible for a hair dryer to contain several resistive elements, but our circuit diagram groups all such elements into two categories: heating elements, and the fan. We draw two separate sections of the circuit with switches, because most hair dryers have settings for simply blowing room temperature air (fan only), and for blowing hot air (fan and heating element). Electrical energy is converted largely to thermal energy, as the hair dryer heats up. There is also kinetic energy of the fan inside.

31.8. (*a*) Before the cut is made, there is a complete loop of wire allowing charge to flow out of a battery, through bulb A, and then back to the battery. But once the cut is made, this path is interrupted. Thus, bulb A will go out. No other bulbs are affected by this cut. (*b*) One might initially think that bulb C would go out because there is no longer a closed path from the battery through C and back to the battery. However, bulb C was never lit. The ideal wire next to bulb C has short-circuited bulb C, meaning all the flowing charge moves through the ideal wire and bypasses bulb C entirely. Thus, no bulbs are affected by this cut. (*c*) As in part (*b*), we might expect that bulbs D and E would go out. However, these bulbs are short-circuited by the straight wire just above them. They were never lit. Thus, no bulbs are affected by this cut. (*d*) All bulbs that were initially lit would go out. Because C, D, and E were always short-circuited, bulbs A and B would go out if a cut were made at point d.

31.9. Your friend is correct that charge does not flow across the gap, meaning there is no current. There is however a displacement current. The electric field between the plates allows positive and negative charges to influence each other across the gap. Also, we know from our detailed study of capacitors that they can certainly discharge. If the air gap prevented charge from ever flowing from the plates, then no capacitor could ever discharge.

31.10. A, B, and K are all in series. Any current that flows through K must flow through the battery, and then through A and B. Also C and D are in series, and E and F are in series.

31.11. (*a*) No, only B, C, and D light up. There can be no current flowing through A because the current would have nowhere to go on the other side.(*b*) The power dissipated by a bulb increases with the resistance and with the current flowing through the bulb. Bulbs B through D have the same current flowing through them, but bulb C has a greater resistance than the others. Thus $\text{C} > \text{B} = \text{D} > \text{A}$.

31.12. The circuits shown in parts (*a*) through (*c*) are not closed. There is a gap in each that prevents charge from flowing in a complete loop. Thus the current in all these circuits in zero. In diagram (*f*), the loop is closed, but the capacitor has been discharging for hours. For any realizeable system, the capacitor would be fully discharged after a time much shorter than 10 hours. It is safe to say that the current through this circuit is also zero. Similarly, there is no commercially available battery that lasts for 20 years, making it safe to say that the current in circuit (*g*) is also zero. Figures (*d*) and (*e*) show closed circuits, and the potential differences in each should still be driving current. The source of potential difference driving the current in each system starts at 9.0 V, and the load attached to the circuit is the same. After a short time, the potential difference across the capacitor will decrease, whereas that across the battery will remain approximately constant. But here it is specified that the capacitor has been discharging for a very short amount of time. This means that the potential difference across the capacitor will still be close to 9.0 V, even equal to 9.0 V to our two significant digits of precision. Thus we say that the current in circuits (*d*) and (*e*) are the same. Thus $0 = a = b = c = f = g < d = e.$

31.13. (*a*) No. The power can be written as $P = IV$. All bulbs have the same current flowing through them, so the only way for bulb A to be brighter than bulb B is for bulb A to have a greater potential difference across it than B. If the potential difference across bulb C were 3 V, then the potential differences across A and B would each be greater than 3 V, resulting in a total potential drop over bulbs A-C of more than 9 V. The battery only supplies 9 V of potential difference. So, even if bulb D were completely off, there would be a larger potential drop than potential increase across the battery terminals. This is not possible. (*b*) Yes. The potential differences across bulbs A and B could be larger than 2 V without the circuit having a potential drop that exceeds the potential difference maintained by the battery. As an example, the potential differences of bulbs A, B, C, and D could be 3.0 V, 2.5 V, 2.0 V, and 1.5 V respectively.

31.14. No. With the two bulbs in series in the original circuit, the same current would pass through each bulb. The only way for A to be brighter than B is if A has a higher resistance than B. The resistance is a property of a bulb. Placing a bulb onto a different battery doesn't change its resistance. Placing the two different bulbs in identical circuits will not make their resistances the same, and so they will not glow with the same brightness.

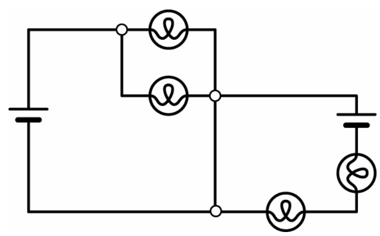
31.15. With the three bulbs in series, the same current must flow through each bulb. Since $P = I^2R,$ and we know $P_\text{B} = 2P_\text{A}$ and $P_\text{C} = 3P_\text{A},$ we can write $R_\text{B} = 2R_\text{A}$ and $R_\text{C} = 3R_\text{A}.$ This allows us to write the entire resistance of the set of three bulbs as $R = R_\text{A} + R_\text{B} + R_\text{C} = 6R_\text{A}.$ Applying Ohm's Law to the entire circuit allows us to write $V_\text{battery} = IR = 6IR_\text{A} \Rightarrow R_\text{A} = \dfrac{V_\text{battery}}{6I} = \dfrac{(9.0\text{ V})}{6(1.0\text{ A})} = 1.5\ \Omega.$ This allows us to calculate the power dissipated by bulb A in the form of light and heat: $P_\text{A} = I^2R_\text{A} = (1.0\text{ A})^2(1.5\ \Omega) = 1.5\text{ W}.$ Since $\Delta E = P\Delta t,$ the energy dissipated in bulb A is 1.5 J. Using the proportionality given in the problems statement, we know twice as much energy is dissipated in B, and three times as much energy is dissipated in C. Thus $\Delta E_\text{A} = 1.5\text{ J},$ $\Delta E_\text{B} = 3.0\text{ J},$ and $\Delta E_\text{C} = 4.5\text{ J}.$

31.16. All the charge flowing into the junction must flow out again. Let us choose to call a flow of charge into a junction positive and the flow out of a junction negative. Then, we require $I_\text{A} + I_\text{B} + I_\text{C} = 0 \Rightarrow$ $I_\text{C} = -I_\text{A} - I_\text{B} = -(3.2\text{ mA}) - (-4.3\text{ mA}) = 1.1\text{ mA},$ where the positive sign indicates that the current in wire C is 1.1 mA into the junction.

31.17. A, B, and C are in parallel. They are connected by ideal wires on each side. E and F are also in parallel.

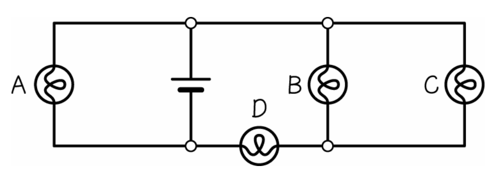
31.18. A and B are in series. This is clear because all current passing through one must pass through the other. C and D are in parallel. They are connected on both sides by ideal wire.

31.19.



31.20. (*a*) These bulbs are connected in parallel, which means the potential difference across each must be the same: $\Delta V_\text{A} = \Delta V_\text{B}.$ (*b*) We can write the power as $P = IV.$ Since the two bulbs have the same potential difference across them, but bulb B is dimmer, this means $I_\text{A} > I_\text{B}.$ (*c*) One can either use Ohm's Law directly, or simply write the power as $P = \dfrac{V^2}{R}.$ Since the two bulbs have the same potential difference across them, and B is dimmer, B must have the greater resistance. Thus $R_\text{A} < R_\text{B}.$

31.21. (*a*)



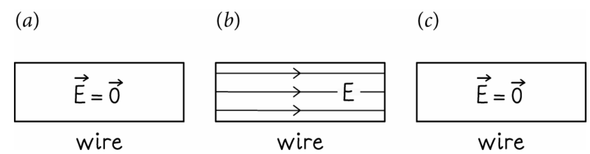
(*b*) Yes, all bulbs light up. (*c*) Call the resistance of each bulb R and the potential difference across the battery terminals $V_\text{batt}.$ The brightest bulb will be the one that has the most current flowing through it. We can write the current flowing through A as $I_\text{A} = \dfrac{V_\text{A}}{R} = \dfrac{V_\text{batt}}{R}.$ Electrons flowing through bulb D must then flow either through B or C on their way back to the battery. The resistance of this path (through D and the parallel arrangement of B and C) is $R_\text{eq} = R + \left(\dfrac{1}{R} + \dfrac{1}{R}\right)^{-1} = \dfrac{3}{2}R.$ Thus the current flowing through that path (and therefore also through bulb D) can be written $I_D = \frac{V_{batt}}{R_{eq}} = \frac{2V_{batt}}{3R}$. This current splits evenly between bulbs B and C (because they are identical bulbs). Thus $I_B = I_C = \frac{I_D}{2} = \frac{V_{batt}}{3R}$. Thus A is the brightest, B and C are tied for the dimmest. (*d*) B and C are in parallel, and none are wired in series.

31.22. (*a*) One bulb burning out would not cause any others to shut off. However, if two bulbs wired in parallel burned out, the entire string would go out. (*b*) These are wired completely in parallel such that any number of bulbs could burn out and the others would remain lit. (*c*) If one bulb in this strand goes out, then the bulb wired in series to it will also go out. But the rest of the strand will remain lit.

31.23. Lights in a house are wired in parallel with very few exceptions. If lights were wired in series, then neither would light up on unless both were switched on.

31.24. We know that in electrostatic equilibrium, the electric field inside an ideal conductor is exactly zero. Because charge is moving in the wires and because real materials are not ideal (there are collision between charge carriers and metal atoms that prevent true electrostatic equilibrium), the electric field there is likely not exactly zero. It is, however, very small compared to the electric field in the resistor. We claim only that to a very good approximation $E_a = E_c = 0$. The electric field in the resistor is given by $E_b = \left|\frac{\Delta V}{\Delta x}\right| = \left|\frac{(9.0 \text{ V})}{(0.010 \text{ m})}\right| = 9.0\times10^2 \text{ N/C}$.

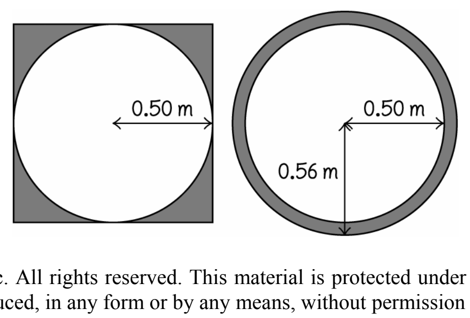
31.25.



31.26. Let us first express the initial resistance of the wire in terms of the length and cross-sectional area: $R = \frac{\rho\ell}{\pi r^2}$, where ρ is the resistivity of the material from which the wire is made, and r is the radius of the cylindrical wire. We can write the new resistance of the wire R' as $R' = R_{left} + R_{right} = \frac{\rho(\ell/2)}{\pi r^2} + \frac{\rho\ell}{\pi(r/\sqrt{2})^2} = \frac{R}{2} + 2R$. Here we have used that the cross-sectional area of the wire must decrease by a factor of two, because the length doubled and the volume of metal present is constant. Thus, $R' = 5R/2$.

31.27. **[NOTE:** In the second sentence of the problem statement "The area *A* of each plate…." is changed to "The area *A* of each square plate…."] We are required to use a straight length of rod, but we can choose any length of the rod. We note that the length of the rod is related to the maximum electric field through $E_{max} = \left|\frac{\Delta V_{max}}{\ell}\right| = \frac{Q_{max}}{\ell C_{\text{parallel plate}}} = \frac{Q_{max} d}{\ell \epsilon_0 A}$. Thus $\ell = \frac{Q_{max} d}{E_{max}\epsilon_0 A} = \frac{(4.51\times10^{-8} \text{ C})(0.100 \text{ m})}{(1000 \text{ N/C})(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(1.00 \text{ m}^2)} 0.510 \text{ m}.$

This means the rod must be attached to the opposite plate a radial distance of 0.50 m or more from the center. The region of acceptable attachment points could be different depending on the shape of the capacitor plates. The figure below shows the acceptable attachment regions (shaded areas) for square plates and circular plates.



31.28. Ohm's Law tells us $I = \dfrac{V}{R} = \dfrac{V\pi r^2}{\rho \ell} = \dfrac{V\pi r^2 \sigma}{\ell} = \dfrac{(9.0\text{ v})\pi(1.0\times10^{-3}\text{ m})^2(5.9\times10^7\text{ A/(V}\cdot\text{m))}}{(0.600\text{ m})} = 2.8\times10^3\text{ A}.$

31.29. We can write the resistance as $R = \dfrac{\rho\ell}{A} = \dfrac{\ell}{\sigma\pi r^2} \Rightarrow \ell = R\sigma\pi r^2$. For the nichrome, we have $\ell_{\text{nichrome}} = R\sigma_{\text{nichrome}}\pi r^2 = (12\ \Omega)(6.7\times10^5\text{ A/(V}\cdot\text{m))}\pi(0.20\times10^{-3}\text{ m})^2 = 1.0\text{ m}$. In the case of copper, we have $\ell_{\text{copper}} = R\sigma_{\text{copper}}\pi r^2 = (12\ \Omega)(5.9\times10^7\text{ A/(V}\cdot\text{m))}\pi(0.20\times10^{-3}\text{ m})^2 = 89\text{ m}$. Thus the wire would be 1.0 m when made of nichrome, and 89 m when made from copper.

31.30. Equation 31.5 defines the current density to be $J = \dfrac{|I|}{A} = \dfrac{|I|}{\pi r^2} = \dfrac{(1.20\text{ A})}{\pi(2.0575\times10^{-3}\text{ m})^2} = 9.02\times10^4\text{ A/m}^2.$

31.31. Equations 31.6 and 31.7 both give expressions equal to the current density. Equating the right hand sides of these two equations yields

$$nev_{\text{d}} = \frac{ne^2\tau}{m_{\text{e}}}E \Rightarrow E = \frac{v_{\text{d}}m_{\text{e}}}{e\tau} = \frac{(1.0\times10^{-2}\text{ m/s})(9.11\times10^{-31}\text{ kg})}{(1.60\times10^{-19}\text{ C})(1.0\times10^{-14}\text{ s})} = 5.7\text{ N/C}$$

31.32. The resistance is $R = \dfrac{V}{I} = \dfrac{3.0\text{ V}}{0.08\text{ A}} = 4\times10^1\ \Omega.$

31.33. We can write the resistance as $R = \dfrac{\rho\ell}{A} = \dfrac{\ell}{\sigma\pi r^2} \Rightarrow \ell = R\sigma\pi r^2$. For the 4-gauge wire, we have $\ell_4 = R\sigma\pi r_4^2 = (1.0\ \Omega)(5.9\times10^7\text{ A/(V}\cdot\text{m))}\pi(2.595\times10^{-3}\text{ m})^2 = 1.2\text{ km}$. In the case of 22-gauge wire, we have $\ell_{22} = R\sigma\pi r_{22}^2 = (1.0\ \Omega)(5.9\times10^7\text{ A/(V}\cdot\text{m))}\pi(0.32\times10^{-3}\text{ m})^2 = 19\text{ m}.$

31.34. Call the thinner, shorter wire 1, and the thicker longer wire 2. We calculate the ratio of the resistances as $\dfrac{R_1}{R_2} = \dfrac{\left(\dfrac{\ell_1}{\pi r_1^2\sigma}\right)}{\left(\dfrac{\ell_2}{\pi r_2^2\sigma}\right)} = \dfrac{\ell_1 r_2^2}{\ell_2 r_2^2} = \dfrac{\ell_1(2r_1)^2}{3\ell_1 r_2^2} = \dfrac{4}{3}$. Thus, wire 1 (the thinner, shorter wire) has the larger resistance, which is $\dfrac{4}{3}$ the resistance of the longer, thicker wire.

31.35. Equations 31.5 and 31.7 both give expressions for the current density. Equating the right hand sides of these equations yields $\dfrac{I}{A} = \dfrac{ne^2\tau}{m_{\text{e}}}E$ or

$$\tau = \frac{Im_{\text{e}}}{Ane^2E} = \frac{Im_{\text{e}}}{\ell^2ne^2(\Delta V/\ell)} = \frac{(6.10\times10^5\text{ A})(9.11\times10^{-31}\text{ kg})}{(1.00\times10^{-2}\text{ m})(6.60\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})^2(1.0\text{ V})} = 3.29\times10^{-14}\text{ s}$$

31.36. Using Equations 31.5 and 31.8 we can write $\dfrac{I}{A} = J = \sigma E \Rightarrow E = \dfrac{I}{A\sigma} = \dfrac{I}{\pi R^2\sigma} = \dfrac{(4.0\text{ A})}{\pi(1.0\times10^{-3}\text{ m})^2(3.6\times10^7\text{ A/(V}\cdot\text{m))}} = 3.5\times10^{-2}\text{ N/C}.$

31.37. (*a*) If the lattice ions are closer together, we expect electrons to collide with the lattice ions more often. Thus the average time between collisions would decrease. (*b*) If the size of the lattice ion decreases, then there will be more empty space between lattice ions through which electrons can move. We expect less frequent collisions in this case, meaning the average time between collisions would increase. (*c*) If the charge of the lattice ion is increased, it should interact more strongly with the electrons. We expect the time between collisions to decrease.

31.38. (*a*) In order to make use of Equation 31.5, we need the density of electrons in copper. We are told to assume it is the same as the density of copper atoms in copper, such that $n = \dfrac{\rho_{\text{Cu}}}{m_{\text{atom Cu}}} = \dfrac{(8.96\times10^3\ \text{kg/m}^3)}{(1.055\times10^{-25}\ \text{kg})} =$ 8.49×10^{28} electrons/m^3. Now we can write $I = nqv_{\text{d}}A = (8.49\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(7.08\times10^{-4}\ \text{m/s})\pi$ $((1.63\times10^{-3}\ \text{m})/2)^2 = 20.1\ \text{A}$. (*b*) Using our result from part (*a*), we can write $J = \dfrac{I}{A} = nqv_{\text{d}} =$ $(8.49\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(7.08\times10^{-4}\ \text{m/s}) = 9.62\times10^6\ \text{A/m}^2$. (*c*) We know $\Delta t = \dfrac{\Delta x}{v_{\text{d}}} = \dfrac{(3.00\ \text{m})}{(7.08\times10^{-4}\ \text{m/s})} =$ 4.24×10^3 s or 70.6 minutes.

31.39. The current already tells you how much charge is delivered each second. We determine the number of protons by writing $n = \dfrac{I}{q}\Delta t = \dfrac{(2.00\times10^{-9}\ \text{A})}{(1.60\times10^{-19}\ \text{C})}(1.00\ \text{s}) = 1.25\times10^{10}$ protons.

31.40. We can use Equation 31.5 to determine the drift speed of the electrons, but we must make an estimate of the length of the wire in order to find the time it takes an electron to move that distance. If the wiring is fairly direct, a distance of about 3 meters is a reasonable estimate. Then we can write

$$v_{\text{d}} = \frac{I}{nqA} = \frac{(2.0\ \text{A})}{(8.4\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})\pi((1.1\times10^{-3}\ \text{m})/2)^2} = 1.57\times10^{-4}\ \text{m/s}$$

Using our estimated length of wire, we have $\Delta t = \dfrac{\ell}{v_{\text{d}}} = \dfrac{3\ \text{m}}{(1.57\times10^{-4}\ \text{m/s})} = 2\times10^4$ s or 5 hours.

31.41. We first require that the resistances be the same to obtain information about the cross-sectional area of the wires. This also gives us a relationship between the volumes of metal required, since the wire must have a fixed length in either case. $R_{\text{silver}} = R_{\text{copper}}$ or

$$\frac{\ell}{\sigma_{\text{silver}}A_{\text{silver}}} = \frac{\ell}{\sigma_{\text{copper}}A_{\text{copper}}} \Rightarrow \frac{V_{\text{copper}}}{V_{\text{silver}}} = \frac{\sigma_{\text{silver}}}{\sigma_{\text{copper}}}$$

We can write this in terms of the mass of each metal required using the densities given:

$$\frac{m_{\text{copper}}}{m_{\text{silver}}} = \frac{V_{\text{copper}}\rho_{\text{copper}}}{V_{\text{silver}}\rho_{\text{silver}}} = \frac{\sigma_{\text{silver}}\rho_{\text{copper}}}{\sigma_{\text{copper}}\rho_{\text{silver}}}$$

Finally, we write the relative cost:

$$\frac{\text{cost silver}}{\text{cost copper}} = \frac{m_{\text{silver}}(100\ \text{c})}{m_{\text{copper}}\text{c}} = \frac{(100)\sigma_{\text{copper}}\rho_{\text{silver}}}{\sigma_{\text{silver}}\rho_{\text{copper}}} = \frac{(100)(5.9\times10^7\ \text{A/(V}\cdot\text{m}))(1.0490\times10^4\ \text{kg/m}^3)}{(6.3\times10^7\ \text{A/(V}\cdot\text{m}))(8.960\times10^3\ \text{kg/m}^3)} = 1.10\times10^2$$

Thus, the silver wire would cost 110 times as much as the copper wire.

31.42. Using Equation 31.10 we can express the current as $I = \dfrac{V}{R} = \dfrac{VA\sigma}{\ell}$. Equating the two currents yields $\dfrac{V_{\text{copper}}A\sigma_{\text{copper}}}{\ell} = \dfrac{V_{\text{carbon}}A\sigma_{\text{carbon}}}{\ell} \Rightarrow V_{\text{carbon}} = \dfrac{V_{\text{copper}}\sigma_{\text{copper}}}{\sigma_{\text{carbon}}} = \dfrac{(9.0\ \text{V})(5.9\times10^7\ \text{A/(V}\cdot\text{m}))}{(7.3\times10^4\ \text{A/(V}\cdot\text{m}))} = 7.3\times10^3\ \text{V}$.

31.43. The energy transferred to the lattice is $\Delta E = P\Delta t = IV\ell/v_{\text{d}} = IE\ell^2/v_{\text{d}}$. Using Equation 31.5, we can write this as

$$\begin{aligned}\Delta E &= (nqv_{\text{d}}A)E\ell^2/v_{\text{d}} = nqAE\ell^2\\ &= (1.20\times10^{27}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})\pi(1.00\times10^{-3}\ \text{m})^2(4.50\times10^2\ \text{N/C})(0.300\ \text{m})^2 = 2.44\times10^4\ \text{J}\end{aligned}$$

31.44. (*a*) The batteries in the diagram are both oriented so as to drive a clockwise current, such that there is a 21 V potential drop across the resistors. The total resistance of the circuit is $7.000\ \text{k}\Omega$. Thus, using Equation 31.10 we can say the current is $I=\dfrac{V}{R}=\dfrac{(21.0\ \text{V})}{(7.000\times10^{3}\ \Omega)}=3.00\times10^{-3}\ \text{A}$ clockwise. (*b*) Here the 12.0-V battery drives current in the clockwise direction, while the 3.0-V battery would (if it were the only battery) drive current in the counterclockwise direction. The result of the two batteries is that a 9.0-V potential difference is maintained across the $900\ \Omega$ resistor. Using Equation 31.10 we can write $I=\dfrac{V}{R}=\dfrac{(9.0\ \text{V})}{(9.00\times10^{2}\ \Omega)}=1.0\times10^{-2}\ \text{A}$ clockwise.

31.45. The 18.0-V battery maintains a potential difference that would drive a current clockwise around the circuit. The 9.0-V battery would (if it were the only battery in the circuit) drive a current counterclockwise around the circuit. If we add up the changes in potential across circuit elements, moving clockwise around the circuit, we obtain $\mathscr{E}_{\text{left}}+\mathscr{E}_{\text{right}}+V_{\text{resistor}}=0$ or $I=-\dfrac{(\mathscr{E}_{\text{left}}+\mathscr{E}_{\text{right}})}{R}=-\dfrac{((18.0\ \text{V})+(-9.0\ \text{V}))}{(10\ \Omega)}=0.90\ \text{A}$ clockwise.

31.46. (*a*) With the resistors connected in series, the same current must pass through each resistor. Thus, we can use Equation 31.21 to write $\mathscr{E}_{\text{battery}}+V_1+V_2+V_3=0$ or $\mathscr{E}_{\text{battery}}-I(R_1+R_2+R_3)=0\Rightarrow\mathscr{E}_{\text{battery}}=(2.3\ \text{A})((15\ \Omega)+(20\ \Omega)+(25\ \Omega))=1.4\times10^{2}\ \text{V}$. Because there is no internal resistance, $\mathscr{E}_{\text{battery}}=V_{\text{battery}}$, so $V_{\text{battery}}=1.4\times10^{2}\ \text{V}$. (*b*) One can solve this using the results of Exercise 31.8 in *Principles*. But it may be more instructive to simply consider that the resistance of the circuit outside the battery and the current through that circuit have not changed. Thus the potential difference driving current through that circuit cannot have changed. It should be clear that $V_{\text{battery}}=1.4\times10^{2}\ \text{V}$. The difference between this case and part (*a*) is that in part (*a*) this potential difference across the battery terminals was the same as $\mathscr{E}_{\text{battery}}$. But in this part, $\mathscr{E}_{\text{battery}}$ would have to be greater than $V_{\text{battery}}=1.4\times10^{2}\ \text{V}$, such that $V_{\text{battery}}=\mathscr{E}_{\text{battery}}-IR_{\text{battery}}$. But since we are asked only about the potential difference across the battery terminals, the answer is the same as in part (*a*) $V_{\text{battery}}=1.4\times10^{2}\ \text{V}$.

31.47. (*a*) We use Equation 31.21 to write $\mathscr{E}_{\text{battery}}+V_{\text{resistors}}=0$ or $I=\dfrac{\mathscr{E}_{\text{battery}}}{R}=\dfrac{(9.0\ \text{V})}{(100\ \Omega)}=9.0\times10^{-2}\ \text{A}$ clockwise. (*b*) Again we use Equation 31.21 to write $\mathscr{E}_{\text{battery, left}}+\mathscr{E}_{\text{battery, right}}+V_{\text{resistors}}=0$ or $I=\dfrac{\mathscr{E}_{\text{battery, left}}+\mathscr{E}_{\text{battery, right}}}{R_{\text{top}}+R_{\text{bottom}}}$. We plug in values by considering the potential differences as we step clockwise around the circuit, thus obtaining $I=\dfrac{(+9.0\ \text{V})+(-11.0\ \text{V})}{(150\ \Omega)+(180\ \Omega)}=-6.1\times10^{-3}\ \text{A}$. Here, the negative sign indicates that the direction of the current is opposite the direction in which we took steps across circuit elements. Thus the current is $6.1\times10^{-3}\ \text{A}$ counterclockwise. (*c*) Note first that all resistors are in series, so we obtain the equivalent resistance of the entire circuit by simply adding the individual resistances. Using Equation 31.21 we write $\mathscr{E}_{\text{battery, left}}+\mathscr{E}_{\text{battery, right}}+V_{\text{resistors}}=0$ or $I=\dfrac{\mathscr{E}_{\text{battery, left}}+\mathscr{E}_{\text{battery, right}}}{R_{\text{circuit}}}$. We plug in values by considering the potential differences as we step clockwise around the circuit, thus obtaining $I=\dfrac{(+4.0\ \text{V})+(-9.0\ \text{V})}{(5\ \Omega)+(10\ \Omega)+(10\ \Omega)+(15\ \Omega)+(5\ \Omega)}=-1.1\times10^{-1}\ \text{A}$. Here, the negative sign indicates that the direction of the current is opposite the direction in which we took steps across circuit elements. Thus the current is $1.1\times10^{-1}\ \text{A}$ counterclockwise.

31.48. There is no logical inconsistency in the circuit in part (*a*). But this figure illustrates that if we had truly ideal wires, an arbitrarily small potential difference would produce infinite currents. In figure (*b*), it is clear that the wires do have non-zero resistance. Further, even as the resistance of the wires becomes extremely small, no divergence should be seen in the current, thanks to the internal resistance of the battery. Here, we expect $I = \dfrac{\mathscr{E}}{R_{\text{wire}} + R_{\text{batt}}}$.

31.49. The electromotive force is the same in either case. Thus we can use the result of Exercise 31.9 in *Principles* to express $\mathscr{E}_{\text{battery}}$ with the $2.0\ \Omega$ resistor (the initial resistance outside the battery) and also with the $1.0\ \Omega$ resistor (the final resistance outside the battery). Equating the two expressions yields $I_{\text{i}}(R_{\text{battery}} + R_{\text{i}}) = \mathscr{E}_{\text{battery}}$ $= I_{\text{f}}(R_{\text{battery}} + R_{\text{f}})$, or $R_{\text{battery}} = \dfrac{(I_{\text{f}}R_{\text{f}} - I_{\text{i}}R_{\text{i}})}{(I_{\text{i}} - I_{\text{f}})} = \dfrac{((3.0\text{ A})(1.0\ \Omega) - (2.0\text{ A})(2.0\ \Omega))}{((2.0\text{ A}) - (3.0\text{ A}))} = 1.0\ \Omega.$ Inserting this result back into the expression for the electromotive force yields $\mathscr{E}_{\text{battery}} = I_{\text{f}}(R_{\text{battery}} + R_{\text{f}}) = (3.0\text{ A})((1.0\ \Omega) + (1.0\ \Omega)) = 6.0\text{ V}.$ So $\mathscr{E}_{\text{battery}} = 6.0\text{ V}$ and $R_{\text{battery}} = 1.0\ \Omega.$

31.50. Call the resistors R_1 and R_2. We write the expressions for the equivalent resistance in each case: $R_{\text{series}} = R_1 + R_2$ and $R_{\text{parallel}} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)^{-1}$. Solving the series expression for R_1 and inserting this into the parallel expression, we find $\dfrac{1}{R_{\text{parallel}}} = \dfrac{1}{R_{\text{series}} - R_2} + \dfrac{1}{R_2}$ or $R_2^2 - R_2 R_{\text{series}} + R_{\text{series}} R_{\text{parallel}} = 0.$ This quadratic equation has solutions $R_2 = 2.0\ \Omega$ and $6.0\ \Omega$. If we insert one of these values for R_2, we find that R_1 is equal to the other value. Thus the two resistances are $6.0\ \Omega$ and $2.0\ \Omega$.

31.51. Starting at point a, the potential increases by 8.0 V across battery 1. In order for the potential difference between a and b to be 5.5 V as the problem states, the potential drop across the resistor R_1 must be 2.5 V. Thus we can write $V_{\text{resistor 1}} = IR_1 \Rightarrow I = \dfrac{V_{\text{resistor 1}}}{R_1} = \dfrac{(2.5\text{ V})}{(5.0\ \Omega)} = 0.50\text{ A}.$ Because the circuit consists of only a single loop, this is the current everywhere, including across resistor 2. Moving from point b across the bottom of the circuit toward a, we know the changes in electric potential obey $\mathscr{E}_{\text{battery,1}} + V_{\text{resistor 2}} = V_{\text{ba}}$, or $R_2 = \dfrac{(-5.5\text{ V}) - \mathscr{E}_{\text{battery}}}{-I}$ $= \dfrac{V_{\text{ba}} - \mathscr{E}_{\text{battery}}}{-I} = \dfrac{(-5.5\text{ V}) - (-4.0\text{ V})}{-(0.50\text{ A})} = 3.0\ \Omega.$

31.52. (*a*) A parallel arrangement would result in a lower current, since the batteries would be opposing each other in terms of the current they would produce. (*b*) With the batteries connected in series, as we go around the loop of wire in the direction of the current flow, we have $\mathscr{E}_{\text{batt,good}} + \mathscr{E}_{\text{batt,dead}} + V_{R\text{ good internal}} + V_{R\text{ dead internal}} = 0$, or $I = \dfrac{\mathscr{E}_{\text{batt,good}} + \mathscr{E}_{\text{batt,dead}}}{R_{\text{batt, good}} + R_{\text{batt, dead}}} = \dfrac{(12.0\text{ V}) + (11.0\text{ V})}{(0.0200\ \Omega) + (0.200\ \Omega)} = 105\text{ A}.$ (*c*) The key difference here is that the batteries are oriented such that their electromotive forces are not added, but one is subtracted from the other. Here we have $I = \dfrac{\mathscr{E}_{\text{batt,good}} + \mathscr{E}_{\text{batt,dead}}}{R_{\text{batt, good}} + R_{\text{batt, dead}}} = \dfrac{(12.0\text{ V}) + (-11.0\text{ V})}{(0.0200\ \Omega) + (0.200\ \Omega)} = 4.55\text{ A}.$

31.53. (*a*) We determine the current by using Equation 31.21 to write $\mathscr{E}_{\text{batt, 1}} + \mathscr{E}_{\text{batt, 2}} + V_{R_1} + V_{R_2} + V_{R_3} = 0.$ Since there is only one loop to this circuit, the current is the same everywhere, which allows us to write $\mathscr{E}_{\text{batt, 1}} + \mathscr{E}_{\text{batt, 2}} - I(R_1 + R_2 + R_3) = 0.$ We solve for the current and insert values assuming a clockwise current: $I = \dfrac{\mathscr{E}_{\text{batt, 1}} + \mathscr{E}_{\text{batt, 2}}}{(R_1 + R_2 + R_3)} = \dfrac{(10\text{ V}) + (-2.0\text{ V})}{((5.0\ \Omega) + (3.0\ \Omega) + (6.0\ \Omega))} = 0.6\text{ A}.$ (*b*) From point a to point b battery 1 increases the potential, and then there is a potential drop across resistor 1. We write this as $V_{\text{ab}} = \mathscr{E}_1 - IR_1 = (10\text{ V}) - (0.57\text{ A})(5.0\ \Omega) = 7\text{ V}.$

From point b to point c there is no battery, and no resistance. There can be no potential difference between points b and c: $V_{bc} = 0$. From point c to point d there is a potential drop across resistor 2 and there is a further potential drop across battery 2. Thus $V_{cd} = \mathcal{E}_1 - IR_2 = (-2.0\text{ V}) - (0.57\text{ A})(3.0\ \Omega) = -4\text{ V}$. Finally, from point d back to point a there is only a potential drop across resistor 3. So $V_{da} = -IR_3 = -3\text{ V}$.

31.54. It is clear that the voltage across the battery terminals changes as the battery ages. Let us assume that the electromotive force of the battery does not change (although this is certainly not true in many cases). With that assumption, we attribute the entire difference in the circuit's behavior to the change in the internal resistance of the battery.

In either case we can write the current in the circuit in terms of the potential difference across the load: $I = \dfrac{V_{load}}{R_{load}}$. Thus we write for the new battery: $\mathcal{E}_{batt} - I(R_{batt,i} + R_{load}) = 0$ or $\mathcal{E}_{batt} - \dfrac{V_{load,i}}{R_{load}}(R_{batt,i} + R_{load}) = 0 \Rightarrow R_{batt,i}$ $= \left(\dfrac{\mathcal{E}_{batt}}{V_{load,i}} - 1\right)R_{load} = \left(\dfrac{(12.0\text{ V})}{(11.9\text{ V})} - 1\right)(100\ \Omega) = 0.840\ \Omega.$

In the final case, once the circuit has operated for a while, we have

$$\mathcal{E}_{batt} - \frac{V_{load,f}}{R_{load}}(R_{batt,f} + R_{load}) = 0 \Rightarrow R_{batt,f} = \left(\frac{\mathcal{E}_{batt}}{V_{load,f}} - 1\right)R_{load} = \left(\frac{(12.0\text{ V})}{(11.5\text{ V})} - 1\right)(100\ \Omega) = 4.35\ \Omega$$

So, the internal resistance of the battery has increased by a factor of 5.18 (from $0.840\ \Omega$ to $4.35\ \Omega$).

31.55. (*a*) All three resistors are in series, so we simply add the resistances: $R_{eq} = R_1 + R_2 + R_3$ $= (200\ \Omega) + (900\ \Omega) + (100\ \Omega) = 1.200\times 10^3\ \Omega$. (*b*) We have $I = \dfrac{V_{circuit}}{R_{circuit}} = \dfrac{(12\text{ V})}{(1200\ \Omega)} = 1.0\times 10^{-2}\text{ A}$ clockwise. (*c*) The battery increases the potential from zero up to 12 V, so $V_a = 12\text{ V}$. Moving from point a to point b, there is a drop in potential across resistor 1 equal to $V_{ab} = -IR_1 = (0.010\text{ A})(200\ \Omega) = -2.0\text{ V}$. So the potential at b is 2.0 V lower than at a, and $V_b = 10\text{ V}$. Similarly, from b to c there is a potential drop across resistor 2 equal to $V_{bc} = -IR_2 = (0.010\text{ A})(900\ \Omega) = -9.0\text{ V}$. So the potential at c is 9.0 V lower than at b, meaning $V_c = 1\text{ V}$.

31.56. (*a*) The potential difference across resistor 1 is $V_1 = IR_1$, and the current can be written as $I = \dfrac{V_{circuit}}{R_{circuit}} = \dfrac{V_{batt}}{R_1 + R_2}$. Combining, we have $V_1 = \dfrac{V_{batt}R_1}{R_1 + R_2} = \dfrac{(4.5\text{ V})(40\ \Omega)}{(40\ \Omega) + (70\ \Omega)} = 1.6\text{ V}$. (*b*) We look at the expression we had for the current in part (*a*): $I = \dfrac{V_{circuit}}{R_{circuit}} = \dfrac{V_{batt}}{R_1 + R_2}$. Clearly, if the resistance of R_1 decreases, then the total resistance in the circuit decreases, meaning more current can flow. It is easy to see in the equation above that the current increases, since the denominator becomes smaller. It should be clear that decreasing the resistance of R_1 decreases the potential difference across R_1. If this is not obvious, consider rewriting the final expression in part (*a*) as $V_1 = \dfrac{V_{batt}}{1 + R_2/R_1}$. Now clearly, as R_1 becomes smaller the denominator becomes large and the potential difference decreases. Thus the current in the circuit increases, and the potential difference across resistor 1 decreases.

31.57. First, we determine the necessary current by looking at the potential difference across the bulb only: $I = \dfrac{V_{bulb}}{R_{bulb}} = \dfrac{(3.0\text{ V})}{(5.0\ \Omega)} = 0.60\text{ A}$. Now we can use Equation 31.21 to write $V_{batt} - I(R_{bulb} + R_{additional}) = 0$, or $R_{additional} = \dfrac{V_{batt}}{I} - R_{bulb} = \dfrac{(9.0\text{ V})}{(0.60\text{ A})} - (5.0\ \Omega) = 10\ \Omega.$

31.58. From the required current, it is easy to determine the necessary resistance R_{nichrome} of your nichrome resistor. We know $V_{\text{batt}} = \mathcal{E}_{\text{batt}} - IR_{\text{batt}}$, and once we add a load, this can be modified to $V_{\text{batt}} - IR_{\text{load}} = \mathcal{E}_{\text{batt}} - I(R_{\text{batt}} + R_{\text{load}}) = 0$ such that $R_{\text{load}} = \dfrac{\mathcal{E}_{\text{batt}} - IR_{\text{batt}}}{I}$. This resistance can be expressed in terms of conductivity as

$$R_{\text{load}} = \frac{\mathcal{E}_{\text{batt}} - IR_{\text{batt}}}{I} = \frac{\ell}{\sigma A} \qquad (1)$$

Since we must use the entire volume of nichrome that we are given, we can also write

$$\ell A = V \qquad (2)$$

Combining equations (1) and (2), we find

$$\ell = \sqrt{\left(\frac{\mathcal{E}_{\text{batt}}}{I} - R_{\text{batt}}\right)\sigma V} = \sqrt{\left(\frac{(10.0\text{ V})}{(0.300\text{ A})} - (18.0\ \Omega)\right)(6.7\times10^{5}\text{ A/(V}\cdot\text{m))}(20.0\times10^{-9}\text{ m}^3)} = 0.45\text{ m}$$

And the cross sectional area is then given by $A = \dfrac{V}{\ell} = \dfrac{(20.0\times10^{-9}\text{ m}^3)}{(0.453\text{ m})} = 4.4\times10^{-8}\text{ m}^2$. So the dimensions are $\ell = 0.45\text{ m}$ and $A = 4.4\times10^{-8}\text{ m}^2$.

31.59. Resistors R_2 and R_5 are in series, and can be combined into an equivalent resistance $R_{25} = R_2 + R_5$. This combination is in parallel with resistor R_4, allowing us to write the resistance of this parallel combination as $R_{245} = \left(\dfrac{1}{R_4} + \dfrac{1}{R_2 + R_5}\right)^{-1}$. This combination is in series with resistor R_6. We write $R_{2456} = R_6 + \left(\dfrac{1}{R_4} + \dfrac{1}{R_2 + R_5}\right)^{-1}$.

This combination is in parallel with R_3, such that $R_{23456} = \left(\dfrac{1}{R_3} + \left(R_6 + \left(\dfrac{1}{R_4} + \dfrac{1}{R_2 + R_5}\right)^{-1}\right)^{-1}\right)^{-1}$. Finally, we add resistor R_1 such that

$$\begin{aligned} R_{\text{eq}} &= R_1 + \left(\frac{1}{R_3} + \left(R_6 + \left(\frac{1}{R_4} + \frac{1}{R_2 + R_5}\right)^{-1}\right)^{-1}\right)^{-1} \\ &= (2.0\ \Omega) + \left(\frac{1}{(2.0\ \Omega)} + \left((1.5\ \Omega) + \left(\frac{1}{(1.5\ \Omega)} + \frac{1}{(1.5\ \Omega) + (2.0\ \Omega)}\right)^{-1}\right)^{-1}\right)^{-1} = 3.1\ \Omega \end{aligned}$$

31.60. We calculate the resistance of each arrangement.

$$R_a = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{R}{2} + \frac{R}{2} = R$$

$$R_b = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} + R + R = \frac{R}{2} + 2R = \frac{5}{2}R$$

$$R_c = R + R + R + R = 4R$$

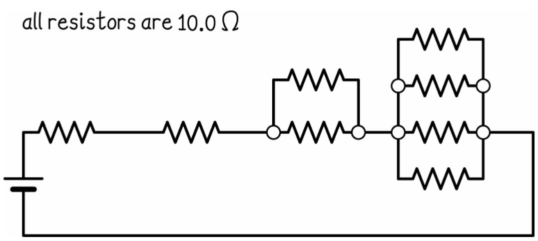
Circuit (*a*) has the smallest resistance, and circuit (*c*) has the largest resistance.

31.61. The current in the absence of the ammeter is $I_0 = \dfrac{V_{\text{batt}}}{R_{\text{load}}} = \dfrac{(3.00\text{ V})}{(40.0\ \Omega)} = 7.50\times10^{-2}\text{ A}$. With the ammeter in series with the load, we have $I_{\text{ammeter}} = \dfrac{V_{\text{batt}}}{R_{\text{load}} + R_{\text{ammeter}}} = \dfrac{(3.00\text{ V})}{(40.0\ \Omega) + (0.503\ \Omega)} = 7.407\times10^{-2}\text{ A}$. So the percent difference caused by the introduction of the ammeter is

$$\frac{I_{\text{ammeter}} - I_0}{I_0} = \frac{(7.407\times10^{-2}\ \text{A}) - (7.500\times10^{-2}\ \text{A})}{(7.500\times10^{-2}\ \text{A})} = -0.01 \text{ or } -1\ \%$$

31.62. (*a*) Bulbs A and B are in series; they must have the same current flowing through them. Thus A and B will have the same brightness. The potential difference between points a and b is zero, because they are connected by an ideal wire. This means the potential difference across bulb C is zero; it does not light at all. Thus $\text{A} = \text{B} > \text{C}$. (*b*) Once the wire is cut, A, B, and C are all in series and must all have the same current flowing through them. Thus $\text{A} = \text{B} = \text{C}$. (*c*) Now that current must flow through bulb C as well, the equivalent resistance of the circuit increases. Thus the total current flowing out of the battery decreases. Cutting the wire increases the brightness of bulb C, but decreases the brightness of bulbs A and B.

31.63. There are many correct answers, but one of the simplest is shown below.



31.64. (*a*) The resistance is given by $R_{\text{wire}} = \dfrac{\ell_{\text{wire}}}{\sigma A} = \dfrac{\ell_{\text{wire}}}{\sigma\pi(d/2)^2} = \dfrac{(10\ \text{m})}{(5.9\times10^7\ \text{A/V}\cdot\text{m})\ \pi((0.20\times10^{-3}\ \text{m})/2)^2} = 5.4\ \Omega.$

(*b*) The resistance of each segment of the cut wire is $R_{\text{segment}} = \dfrac{R_{\text{wire}}}{N}$, and the resistance of N such segments in parallel is $\left(\displaystyle\sum_{i=1}^{N}\frac{1}{R_{\text{seg}}}\right)^{-1} = \left(\dfrac{N}{R_{\text{seg}}}\right)^{-1} = \dfrac{R_{\text{seg}}}{N} = \dfrac{R_{\text{wire}}}{N^2}$. Thus we require $\dfrac{R_{\text{wire}}}{N^2} < R_{\text{resistor}} \Rightarrow N > \sqrt{\dfrac{R_{\text{wire}}}{R_{\text{resistor}}}} = \sqrt{\dfrac{(5.4\ \Omega)}{(1.0\ \Omega)}} = 2.3$. So the smallest possible value of N is 3.

31.65. (*a*) Near point a the $900\ \Omega$ resistor and the $300\ \Omega$ resistor are in series, making an equivalent $1200\ \Omega$ resistance. These are in parallel with the $1200\ \Omega$ resistor, making an equivalent resistance of $600\ \Omega$. This is in series with the remaining $900\ \Omega$ resistor, such that the equivalent resistance of the entire right side of the circuit is $R_{\text{right}} = (600\ \Omega) + (900\ \Omega) = 1500\ \Omega$. Finally, the left hand side and right hand side of the circuit are in series with each other, such that the total equivalent resistance for the circuit is $R_{\text{eq}} = \left(\dfrac{1}{R_{\text{right}}} + \dfrac{1}{R_{\text{left}}}\right)^{-1}$ $= \left(\dfrac{1}{(1500\ \Omega)} + \dfrac{1}{(1500\ \Omega)}\right)^{-1} = 750\ \Omega$. (*b*) The total current coming out of the battery is $I_{\text{batt}} = \dfrac{V_{\text{batt}}}{R_{\text{eq}}} = \dfrac{(30\ \text{V})}{(750\ \Omega)}$ $= 0.040\ \text{A}$. When the current leaves the battery it will split evenly between the paths to the left and to the right, because the resistance is the same to the left and to the right. Thus a current of 0.020 A will flow through the $900\ \Omega$ resistor across the top of the circuit. At that point the current divides again. Current can flow through the $1200\ \Omega$ resistor or through the combination of the $900\ \Omega$ resistor and the $300\ \Omega$ resistor. Note that this latter path also has an equivalent resistance of $1200\ \Omega$. So, purely by coincidence, these two paths have the same resistance, such that the current will again divide evenly and 0.010 A will flow through the $300\ \Omega$ resistor. It is important to remember that currently will not always split evenly at junctions; it is only the case here because the two possible paths have equivalent resistances. We now know the current through the $300\ \Omega$ resistor, so the potential difference across it is $V = IR = (0.010\ \text{A})(300\ \Omega) = 3.0\ \text{V}$. This is the difference between the potential at a and the lower wire. Since the lower wire is held at a potential of zero, the potential at the point a is $3.0\ \text{V}$. (*c*) In our discussion of currents in part (*b*), we determined the current flowing through each segment of wire. The current is 0.020 A down through the

$1500\ \Omega$ resistor, 0.020 A to the right across the $900\ \Omega$ resistor that is along the top wire, 0.010 A down through the $1200\ \Omega$ resistor, and 0.010 A downward through both the $900\ \Omega$ and the $300\ \Omega$ resistors along the far right.

31.66. One way of solving this problem is to introduce a few more currents in different segments of the circuit. The entire current I_2 flows through the $600\ \Omega$, but right after this resistor I_2 can split at the junction. Call the current that goes downward through the $300\ \Omega$ resistor I_3 and the current that goes to the right across the top wire I_4. Finally, call the current going downward through the right-most $600\ \Omega$ resistor I_5. We can use the loop rule and the junction rule to write five equations for our five unknown currents. There is not one unique set of equatioins, but they must given the same results. We apply the junction rule twice, and the loop rule to the three smallest loops to obtain

$$I_2 = I_3 + I_4 \tag{1}$$

$$I_5 = I_1 + I_4 \tag{2}$$

$$(24\ \text{V}) - I_2(600\ \Omega) - I_3(300\ \Omega) = 0 \tag{3}$$

$$I_3(300\ \Omega) - I_5(600\ \Omega) = 0 \tag{4}$$

$$I_5(600\ \Omega) - (6\ \text{V}) = 0 \tag{5}$$

Solving this system of equations for the five unknown currents is simple. Equation (5) immediately shows us that $I_5 = 0.01$ A. Inserting this into equation (4) yields $I_3 = 0.02$ A. This, in turn, can be inserted into equation (3) to yield $I_2 = 0.03$ A. We can now solve equation (1) to find $I_4 = I_2 - I_3 = (0.03\ \text{A}) - (0.02\ \text{A}) = 0.01$ A. Finally, we have from equation (2) $I_1 = I_5 - I_4 = (0.01\ \text{A}) - (0.01\ \text{A}) = 0.$

31.67. We apply the junction rule, Equation 31.27 to the junction at the top of the circuit (which is equivalent to the junction at the bottom). We then apply to loop rule, Equation 31.21 to the left and right loops of the circuit to obtain three equations:

$$I_1 = I_2 + I_3 \tag{1}$$

$$(12\ \text{V}) - I_1(300\ \Omega) + (9\ \text{V}) - I_2(1000\ \Omega) = 0 \tag{2}$$

$$I_2(1000\ \Omega) - (9\ \text{V}) - I_3(1200\ \Omega) + (1.5\ \text{V}) - I_3(600\ \Omega) + (1.5\ \text{V}) = 0 \tag{3}$$

We can use equation (2) to write I_1 in terms of I_2:

$$I_1 = -I_2\frac{(1000\ \Omega)}{(300\ \Omega)} + \frac{(21\ \text{V})}{(300\ \Omega)} = (0.070\ \text{A}) - (3.33)I_2 \tag{4}$$

Similarly, we can use equation (3) to write I_3 in terms of I_2:

$$I_3 = \frac{(1000\ \Omega)I_2 - (6\ \text{V})}{(1800\ \Omega)} = (0.556)I_2 - (0.0033\ \text{A}) \tag{5}$$

Now, inserting equations (4) and (5) into equation (1), we obtain an equation entirely in terms of I_2 which yields

$$I_2 = \frac{(0.0733\ \text{A})}{(4.89)} = 0.015\ \text{A}$$

Inserting this result back into equations (4) and (5) gives us the remaining two currents: $I_1 = 2.0\times10^{-2}$ A, $I_2 = 1.5\times10^{-2}$ A, $I_3 = 5.0\times10^{-3}$ A.

31.68. (*a*) After the circuit has been closed a long time, no current can flow onto the capacitors, as they are already fully charged. This means that the current through the $300\ \Omega$ resistor and the $100\ \Omega$ resistor on the left is zero. With no current flowing through these segments, we are left with current flowing through the $525\ \Omega$, $1500\ \Omega$, and the bottom $100\ \Omega$ resistors in series. Thus, applying Ohm's Law, we $I = \dfrac{V_{\text{batt}}}{R_{\text{eq}}} = \dfrac{(75\ \text{V})}{(525\ \Omega) + (1500\ \Omega) + (100\ \Omega)} = 0.035$ A.

So the current through the $525\ \Omega$, $1500\ \Omega$, and the bottom $100\ \Omega$ resistors is 35 mA. (*b*) With no current flowing to or from the $50\ \mu\text{F}$ capacitor, we know that the potential difference across this capacitor must be the same as the potential difference provided by the battery (because of their parallel arrangement). Thus $q = CV_{\text{batt}} = (50\ \mu\text{F})(75\ \text{V}) = 3.8$ mC on the $50\ \mu\text{F}$ capacitor. Since no current flows across the $300\ \Omega$ resistor, the potential difference across the $100\ \mu\text{F}$ capacitor must be the same as the potential difference across the $1500\ \Omega$ resistor (because of their parallel arrangement). Thus we have $q = CV_{1500\ \Omega} = CIR_{1500\ \Omega}$ $= (100\ \mu\text{F})(0.0353\ \text{A})(1500\ \Omega) = 5.3\ \text{mC}$ on the $100\ \mu\text{F}$ capacitor.

31.69. (*a*) We see that R_4 and R_5 are in series, and their combination is in parallel with resistors R_2 and R_3. So the equivalent resistance for these four resistors is $R_{2345} = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right)^{-1}$ $= \left(\frac{1}{(2.0\ \Omega)} + \frac{1}{(2.0\ \Omega)} + \frac{1}{(3.0\ \Omega) + (1.0\ \Omega)}\right)^{-1} = 0.80\ \Omega.$ This is in series with resistors R_1 and R_6 such that the equivalent resistance for the entire circuit is $R_{\text{eq}} = R_1 + R_{2345} + R_6 = (1.0\ \Omega) + (0.80\ \Omega) + (1.0\ \Omega) = 2.8\ \Omega.$ (*b*) Let I_n denote the current across resistor number *n*. The total current provided by the battery is $I = \frac{V_{\text{batt}}}{R_{\text{eq}}} = \frac{(14\ \text{V})}{(2.8\ \Omega)} = 5.0\ \text{A}.$ Clearly, all of this current must flow through resistors 1 and 6. So $I_1 = I_6 = 5.0\ \text{A}.$ Using the loop rules shows us that the potential difference across any of the three parallel segments must be $4\ \text{V}.$ For any segment, we simply write $I_{\text{segment}} = \frac{(4\ \text{V})}{R_{\text{segment}}}.$ Thus, we obtain the values for the currents across all resistors: $I_1 = 5.0\ \text{A},$ $I_2 = 2.0\ \text{A},$ $I_3 = 2.0\ \text{A},$ $I_4 = 1.0\ \text{A},$ $I_5 = 1.0\ \text{A},$ and $I_6 = 5.0\ \text{A}.$

31.70. We obtain three equations describing our three unknown currents using the loop and junction rules. We apply the junction rule to the junction at the top of the circuit (which is equivalent to the junction at the bottom). We apply the loop rule to the small right and left loops, as opposed to a loop going around the outer perimeter of the circuit. We find

$$I_1 = I_2 + I_3 \tag{1}$$
$$-\mathcal{E}_3 - \mathcal{E}_2 - I_3 R_2 = 0 \tag{2}$$
$$\mathcal{E}_1 + \mathcal{E}_2 - I_1 R_1 = 0 \tag{3}$$

Equations (2) and (3) can be immediately solved to yield $I_3 = \frac{-\mathcal{E}_2 - \mathcal{E}_3}{R_2} = \frac{-(6.0\ \text{V}) - (9.0\ \text{V})}{(8.0\ \Omega)} = -1.9\ \text{A}$ and $I_1 = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R_1} = \frac{(6.0\ \text{V}) + (6.0\ \text{V})}{(8.0\ \Omega)} = 1.5\ \text{A}.$ Inserting these results into equation (1), we obtain $I_2 = 3.4\ \text{A}.$ Thus $I_1 = 1.5\ \text{A}$ in the direction shown, $I_2 = 3.4\ \text{A}$ in the direction shown, and $I_3 = 1.9\ \text{A}$ in the direction opposite that shown.

31.71. On the far right, there is a single resistor in series with an arrangement of three resistors in parallel. The equivalent resistance of this right-side portion is $R_{\text{right}} = \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)^{-1} + R = \frac{4}{3}R.$ Any current passing through this right-side arrangement must pass through one of two resistors: the vertical resistor or the diagonal resistor that branches off from the lowest wire and slopes upward to the left. This diagonal resistor is in parallel with the vertical resistor, and their parallel arrangement is in series with $R_{\text{right}}.$ So the equivalent resistance at this intermediate step is $R_{\text{int}} = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} + R_{\text{right}} = \frac{11}{6}R.$ R_{int} is in parallel with the other diagonal resistor, and that equivalent parallel arrangement is in series with the final, horizontal resistor. Finally, we have $R_{\text{eq}} = R + \left(\frac{1}{R} + \frac{1}{R_{\text{int}}}\right)^{-1} = \frac{28}{17}R = \left(\frac{28}{17}\right)(200\ \Omega) = 329\ \Omega.$ The current drawn from the battery is given by $I = \frac{V_{\text{batt}}}{R_{\text{eq}}} = \frac{(28\ \text{V})}{(28/17)(200\ \Omega)} = 0.085\ \text{A}.$

31.72. (*a*) Call the current flowing from a to b I_1, the current flowing from c to f I_2, and the current flowing from e to d I_2. We obtain three equations describing our three unknown currents using the loop and junction rules. We apply the junction rule to the junction at point c (which is equivalent to the junction at point f). We apply the loop rule to the loop a-b-c-f-a and f-c-d-e-f. We find

$$I_1 + I_3 = I_2 \tag{1}$$

$$\mathscr{E}_1 - I_2R_1 - I_1R_3 = 0 \tag{2}$$

$$I_2R_1 - \mathscr{E}_2 + \mathscr{E}_3 + I_3R_2 = 0 \tag{3}$$

Equations (2) and (3) can be used to express currents I_1 and I_3 in terms of I_2:

$$I_1 = \frac{\mathscr{E}_1 - I_2R_1}{R_3} \tag{4}$$

$$I_3 = \frac{-I_2R_1 + \mathscr{E}_2 - \mathscr{E}_3}{R_2} \tag{5}$$

Inserting equations (4) and (5) into equation (1), we obtain an equation entirely in terms of I_2 and we find

$$I_2 = \frac{\left(\dfrac{\mathscr{E}_1}{R_3} + \dfrac{\mathscr{E}_2}{R_2} - \dfrac{\mathscr{E}_3}{R_2}\right)}{\left(1 + \dfrac{R_1}{R_2} + \dfrac{R_1}{R_3}\right)} = \frac{\left(\dfrac{(5.0\text{ V})}{(50\ \Omega)} + \dfrac{(5.0\text{ V})}{(50\ \Omega)} - \dfrac{(1.5\text{ V})}{(50\ \Omega)}\right)}{\left(1 + \dfrac{(50\ \Omega)}{(50\ \Omega)} + \dfrac{(50\ \Omega)}{(50\ \Omega)}\right)} = 0.057\text{ A}$$

Inserting this result into equations (4) and (5) gives us the other two currents:

$$I_1 = \frac{(5.0\text{ V}) - (0.0567\text{ A})(50\ \Omega)}{(50\ \Omega)} = 0.043\text{ A}$$

$$I_3 = \frac{-(0.0567\text{ A})(50\ \Omega) + (5.0\text{ V}) - (1.5\text{ V})}{(50\ \Omega)} = 0.013$$

These currents are sufficient for us to describe the current between any of two labeled points. Some are redundant, but here are a few specific examples. A current of 0.043 A flows from point a to b across the 5.0 V battery. A current of 0.057 A flows from c to f across the $50\ \Omega$ resistor. A current of 0.013 A flows from f across the $50\ \Omega$ resistor to point e, then across the two batteries to d. (*b*) Between points a and b, there is only a 5.0 V battery, so $V_{ab} = 5.0$ V. There is only ideal wire between points b and c, and also between points c and d. So $V_{bc} = 0$ and $V_{cd} = 0$. Between points d and e, there are two batteries oriented in opposite directions, yielding $V_{de} = -3.5$ V. Between points e and f a current of 0.013 A flows across the $50\ \Omega$ resistor, such that $V_{ef} = I_3R_2 = (0.0133)(50\ \Omega) = 0.67$ V. Finally, between points f and a there is a current of 0.043 A flowing across a $50\ \Omega$ resistor, such that $V_{fa} = -I_1R_3 = -(0.0433)(50\ \Omega) = -2.2$ V. Collecting these results, we have: $V_{ab} = 5.0$ V, $V_{bc} = 0$, $V_{cd} = 0$, $V_{de} = -3.5$ V, $V_{ef} = 0.67$ V, and $V_{fa} = -2.2$ V.

31.73. The resistance of each parallel segment consisting of one bulb and one resistor is $\left(\dfrac{1}{R_\text{b}} + \dfrac{1}{R_\text{p}}\right)^{-1}$. N such segments in series have a resistance of $N\left(\dfrac{1}{R_\text{b}} + \dfrac{1}{R_\text{p}}\right)^{-1}$. If one bulb goes out, the current that once split between two possible paths across R_p or R_b is now forced across R_p. This increases the resistance of the string, such that the remaining bulbs will become dimmer. If N is large, the effect may be small.

31.74. (*a*) The resistance (R_ext) of the $4.0\ \Omega$ resistor would cause a current of 0.75 A to come from the battery. Even when one adds in the internal resistance of the ammeter, the resistance only increases to $4.5\ \Omega$ which only lowers the current to 0.66 A. This is still six times the current that the ammeter is designed to handle. (*b*) There are many possible answers. We could insert an additional load resistance $R_\text{load} = 40.5\ \Omega$. This would make the total resistance $R_{\text{total}} = 45.0\ \Omega,$ a factor of 10 larger than the resistance of the initial resistor and the ammeter in series, meaning that the current will be exactly a factor of 10 lower with this additional load added.

31.75. The arrangement at the top of the diagram consists of three paths wired in parallel, with the resistances of the three paths being $R,$ $R,$ and $2R.$ Thus $R_{\text{top}} = \left(\dfrac{1}{R} + \dfrac{1}{R} + \dfrac{1}{2R}\right)^{-1} = \dfrac{2}{5}R.$ The arrangement along the bottom of the diagram consists of two parallel paths, each having resistance . Thus $R_{\text{bottom}} = \left(\dfrac{1}{2R} + \dfrac{1}{2R}\right)^{-1} = R.$ Now we have reduced the circuit to four resistors (in clockwise order) $R,$ $R_{\text{top}},$ $R,$ $R_{\text{bottom}},$ all of which are in series. Thus $R_{\text{eq}} = R + \dfrac{2}{5}R + R + R = \dfrac{17}{5}R = \dfrac{17}{5}(5.0\ \Omega) = 17\ \Omega.$

31.76. (*a*) After several minutes, the capacitors will be fully charged, and no current will flow toward them or away from them. Thus, any segments of the circuit with capacitors on them will act like broken wires. In this case the circuit is simplified such that R_1 and R_2 are in parallel, and their combination is in series with the remaining resistors through which current flows: $R_4,$ $R_5,$ and $R_6.$ Thus, the equivalent resistance of the circuit is

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} + R_4 + R_5 + R_6 = \left(\frac{1}{(2.0\ \Omega)} + \frac{1}{(1.5\ \Omega)}\right)^{-1} + (1.0\ \Omega) + (2.0\ \Omega) + (1.0\ \Omega) = 4.86\ \Omega$$

From this we can see that the current supplied by the battery is $I = \dfrac{V_{\text{batt}}}{R_{\text{eq}}} = \dfrac{(12.0\ \text{V})}{(4.86\ \Omega)} = 2.5\ \text{A}.$ We know the current will not split evenly between resistors 1 and 2. But since they are in parallel we know the potential difference across them must be the same. Thus we take the two equations:

$$I_1 + I_2 = I \qquad (1)$$

$$I_1R_1 = I_2R_2 \qquad (2)$$

and solve to obtain $I_2 = \dfrac{IR_1}{(R_1{+}R_2)} = \dfrac{(2.47\ \text{A})(2.0\ \Omega)}{((2.0\ \Omega){+}(1.5\ \Omega))} = 1.4\ \text{A},$ and $I_1 = 1.1\ \text{A}.$ Since the remaining resistors are in series with this parallel arrangement, the entire current from the battery must pass through them. Thus, we have: $I_1 = 1.4\ \text{A},$ $I_2 = 1.1\ \text{A},$ $I_3 = 0,$ $I_4 = I_5 = I_6 = 2.5\ \text{A}.$ (*b*) The potential difference across C_1 must be the same as the potential difference across R_1 because they are in parallel. Thus we can write $q_1 = C_1V_1 = C_1I_1R_1 = (20\times10^{-6}\ \text{F})(1.06\ \text{A})(2.0\ \Omega) = 4.2\times10^{-5}\ \text{C}.$ Similarly, C_2 is in parallel with the series arrangement of $R_4,$ $R_5,$ and $R_6.$ So the potential difference across C_2 must be the same as the potential difference across the entire series arrangement of $R_4,$ $R_5,$ and $R_6.$ Thus $q_2 = C_2V_2 = C_2I(R_4 + R_5 + R_6) = (40\times10^{-6}\ \text{F})(2.47\ \text{A})((1.0\ \Omega) + (2.0\ \Omega) + (1.0\ \Omega)) = 4.0\times10^{-4}\ \text{C}.$

31.77. We use the loop and junction rules to obtain five equations involving our five unknown currents. We apply the junction rule to the junction on the far right and to the junction on the bottom. We apply the loop rule to the three smallest possible loops. Thus we have

$$I_1 + I_2 + I_3 + I_4 = 0 \qquad (1)$$

$$I_3 + I_4 = I_5 \qquad (2)$$

$$2\mathcal{E} + I_1R = 0 \qquad (3)$$

$$\mathcal{E} - I_3R - I_5R = 0 \qquad (4)$$

$$-\mathcal{E} + I_3R - I_4R = 0 \qquad (5)$$

Solving this system of equations involves some algebra, but one finds: $I_1 = 0,$ $I_2 = 0.67\ \text{A}$ but opposite the direction shown, $I_3 = 1.3\ \text{A},$ $I_4 = 0.67\ \text{A}$ but opposite the direction shown, and $I_5 = 0.67\ \text{A}.$

31.78. If no current flows across the ammeter, then the potential at points a and c must be the same. This allows us to the relate the potential differences across the resistors on the top triangle: $I_L(70.0\ \Omega) = I_R(210\ \Omega)$ or $I_L = 3I_R$. We also know that the potential difference across the two resistors on the bottom triangle must be the same. Thus $I_L(185\ \Omega) = I_R R_6 = \frac{I_L}{3}R_6 \Rightarrow R_6 = 3(185\ \Omega) = 555\ \Omega$. Now that we have all resistances, we can determine the equivalent resistance and find the current supplied by the battery. The central bridge has a resistance $R_{\text{bridge}} = \left(\frac{1}{(70.0\ \Omega)+(185\ \Omega)} + \frac{1}{(210\ \Omega)+(555\ \Omega)}\right)^{-1} = 191\ \Omega$. This is in parallel with the $350\ \Omega$ resistor, and their parallel combination is in series with the $50.0\ \Omega$ and $150\ \Omega$ resistors. Thus the equivalent resistance of the circuit is $R_{\text{eq}} = \left(\frac{1}{(191\ \Omega)} + \frac{1}{(350\ \Omega)}\right)^{-1} + (150\ \Omega) + (50.0\ \Omega) = 323.7\ \Omega$. And it follows trivially that the current supplied by the battery is $I = \frac{V_{\text{batt}}}{R_{\text{eq}}} = \frac{(15.0\ \text{V})}{(323.7\ \Omega)} = 0.0463\ \text{A}$. All current leaving the battery must flow through the $(50.0\ \Omega)$ resistor and the $(150\ \Omega)$ resistor. The potential drop across these two resistors together will be $V = IR = (0.0463\ \text{A})((50.0\ \Omega)+(150\ \Omega)) = 9.27\ \text{V}$. Since the battery maintains a 15.0 V potential difference, there must be an additional drop of 5.73 V across the bridge. In particular, the potential difference across the left side of the bridge must be 5.73 V. So we require $I_L = \frac{V_L}{R_L} = \frac{(5.73\ \text{V})}{(70.0\ \Omega)+(185\ \Omega)} = 2.25\times10^{-2}\ \text{A}$ in the direction indicated.

31.79. (*a*) Using Equation 31.46 we can write $P = I^2R \Rightarrow I = \sqrt{\frac{P}{R}} = \sqrt{\frac{(9.0\ \text{W})}{(5.5\ \Omega)}} = 1.3\ \text{A}$. (*b*) Using Equation 31.44 we can write $P = \frac{V^2}{R} \Rightarrow V = \sqrt{PR} = \sqrt{(9.0\ \text{W})(5.5\ \Omega)} = 7.0\ \text{V}$.

31.80. $\Delta E = P\Delta t = I^2R\Delta t = (2.0\ \text{A})^2(10\ \Omega)(3600\ \text{s}) = 1.4\times10^5\ \text{J}$

31.81. (*a*) $P_1 = \frac{V^2}{R_1} = \frac{(8.00\ \text{V})^2}{(4.0\ \Omega)} = 16\ \text{W}$, $P_2 = \frac{V^2}{R_2} = \frac{(8.00\ \text{V})^2}{(6.0\ \Omega)} = 11\ \text{W}$. (*b*) The only two circuit elements that are dissipating energy are the two resistors described in part (*a*). Thus $P_{\text{Circuit}} = P_1 + P_2 = (16\ \text{W}) + (11\ \text{W}) = 27\ \text{W}$. (*c*) Now the equivalent resistance of the circuit is $R_{\text{eq}} = R_1 + R_2 = 10\ \Omega$, such that the current supplied by the battery is $I = 0.80\ \text{A}$. All of this current must flow through each of the light bulbs, such that we can write $P_1 = I^2R_1 = (0.80\ \text{A})^2(4.0\ \Omega) = 2.6\ \text{W}$, and $P_2 = I^2R_2 = (0.80\ \text{A})(6.0\ \Omega) = 3.8\ \text{W}$. The power dissipated in the circuit is just the sum of these: $P_{\text{Circuit}} = P_1 + P_2 = (2.6\ \text{W}) + (3.8\ \text{W}) = 6.4\ \text{W}$.

31.82. The battery can supply energy equivalent to $\Delta E = IV\Delta t = (40\ \text{A})(12\ \text{V})(1\ \text{h}) = 480\ \text{W}\cdot\text{h}$. The light is drawing current at a rate of 0.80 A across the 12 V potential difference. So the light will exhaust the energy in the battery after a time $\Delta t = \frac{\Delta E}{VI} = \frac{(480\ \text{W}\cdot\text{h})}{(12\ \text{V})(0.80\ \text{A})} = 50\ \text{h}$.

31.83. (*a*) Call the current up the left side of the circuit I_{left}, call the current up the center wire I_{center}, and call the current up the right side I_{right}. We use the junction rule and the loop rule to obtain three equations involving our three unknown currents. We find

$$I_{\text{left}} + I_{\text{center}} + I_{\text{right}} = 0 \quad (1)$$

$$\mathscr{E}_1 - I_{\text{left}}R_1 - \mathscr{E}_2 = 0 \quad (2)$$

$$\mathscr{E}_2 + I_{\text{right}}\left(R_2 + R_3 + R_4\right) = 0 \quad (3)$$

Equations (2) and (3) trivially yield $I_{\text{left}} = \dfrac{-\mathscr{E}_2 + \mathscr{E}_1}{R_1} = \dfrac{-(5.00\text{ V}) + (10.0\text{ V})}{(50.0\ \Omega)} = 0.10\text{ A}$ and $I_{\text{right}} = -\dfrac{\mathscr{E}_2}{(R_2 + R_3 + R_4)} = -\dfrac{(5.00\text{ V})}{((50.0\ \Omega) + (50.0\ \Omega) + (50.0\ \Omega))} = -0.0333\text{ A}$. Inserting these results into equation (1) yields $I_{\text{center}} = -0.067\text{ A}$. These three currents are sufficient to determine the current across any of the resistors. We have $I_1 = 0.10\text{ A}$ and $\Delta V_1 = I_{\text{left}} R_1 = (0.10\text{ A})(50.0\ \Omega) = 5.0\text{ V}$. $I_2 = I_3 = I_4 = 0.033\text{ A}$ clockwise around the right-most loop, and $\Delta V_2 = \Delta V_3 = \Delta V_4 = (0.033\text{ A})(50.0\ \Omega) = 1.7\text{ V}$. (*b*) $P_1 = I_1^2 R_1 = (0.10\text{ A})^2(50.0\ \Omega) = 0.50\text{ W}$, $P_2 = P_3 = P_4 = (0.033\text{ A})^2(50.0\ \Omega) = 0.056\text{ W}$. (*c*) ϵ_1 supplies a power of $P = VI = (10.0\text{ V})(0.10\text{ A}) = 1.0\text{ W}$ and ϵ_2 uses power at a rate of $P = IV = (0.067\text{ A})(5.00\text{ V}) = 0.33\text{ W}$.

31.84. We can determine the current through the light bulb using Equation 31.46 $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{(60\text{ W})}{(10.00\ \Omega)}} = 2.45\text{ A}$. We can also relate this current to the EMF of the battery according to the loop rule: $\mathscr{E} - IR_{\text{batt}} - IR_{\text{bulb}} = 0 \Rightarrow R_{\text{batt}} = \dfrac{\mathscr{E} - IR_{\text{bulb}}}{I} = \dfrac{(120.0\text{ V}) - (2.45\text{ A})(10.00\ \Omega)}{(2.45\text{ A})} = 39\ \Omega$.

31.85. After the circuit has been closed for several minutes, the capacitor will be fully charged and no more current can flow in the central wire (across the $75\ \Omega$ resistor). The circuit can then be treated as though it consists only of the four resistors around the outer perimeter of the circuit. The equivalent resistance is then $R_{\text{eq}} = (200\ \Omega) + (250\ \Omega) + (100\ \Omega) + (50\ \Omega) = 600\ \Omega$, which means the current supplied by the battery will be $I = \dfrac{V_{\text{batt}}}{R_{\text{eq}}} = \dfrac{(150\text{ V})}{(600\ \Omega)} = 0.250\text{ A}$. All of this current will pass through the resistors on the outer perimeter, and there is no current in the center $(75\ \Omega)$ resistor. The potential difference across the capacitor must be the same as the potential difference across the $50\ \Omega$ and $100\ \Omega$ resistors in series. Thus we can write $q = CV = (40\times10^{-6}\text{ F})(0.250\text{ A})((100\ \Omega) + (50\ \Omega)) = 1.5\times10^{-3}\text{ C}$. Collecting these results: the current through all the outer resistors $(250\ \Omega,\ 100\ \Omega,\ 50\ \Omega,\text{ and }200\ \Omega)$ is 0.25 A. The current through the center $(75\ \Omega)$ resistor is zero. $q = 1.5\text{ mC}$.

31.86. (*a*) We can determine the current in the resistor by considering the power: $P_1 = I^2 R_1 \Rightarrow I = \sqrt{\dfrac{P_1}{R_1}} = \sqrt{\dfrac{(0.75\text{ W})}{(12\ \Omega)}} = 0.25\text{ A}$. Let us assume that the current flows counterclockwise around the circuit. Using the loop rule, we can write $\mathscr{E}_2 - IR_1 - \mathscr{E}_1 - IR_2 = 0 \Rightarrow R_2 = \dfrac{\mathscr{E}_2 - IR_1 - \mathscr{E}_1}{I} = \dfrac{(8.0\text{ V}) - (0.25\text{ A})(12\ \Omega) - (4.5\text{ V})}{(0.25\text{ A})} = 2.0\ \Omega$. (*b*) $P_2 = I^2 R_2 = (0.25\text{ A})^2(2.0\ \Omega) = 0.13\text{ W}$. (*c*) Battery 2 maintains a larger potential difference across its terminals than battery 1. This will cause a current to flow counterclockwise. Thus ϵ_2 supplies power to the circuit. The amount of power supplied is given by $P_{\text{battery 2}} = I\mathscr{E}_2 = (0.25\text{ A})(8.0\text{ V}) = 2.0\text{ W}$. (*d*) The current in the circuit charges battery 1, such that battery 1 consumes power at the rate of $P_{\text{battery 1}} = I\mathscr{E}_1 = (0.25\text{ A})(4.5\text{ V}) = 1.1\text{ W}$.

31.87. The power dissipated is $P = I^2 R = \dfrac{I^2 \ell}{\sigma \pi r^2} = \dfrac{(20\text{ A})^2(1.0\times10^3\text{ m})}{(5.9\times10^7\text{ A/(V}\cdot\text{m}))\pi(1.2\times10^{-3}\text{ m})^2} = 1.5\text{ kW}$. Using a higher potential difference means you can deliver the same amount of power to a city at a lower current. Using the lower current means less power is dissipated in the transmission lines leading into the city.

31.88. (*a*) We are not given a value for the potential difference across battery terminals, so we must calculate all values symbolically. The equivalent resistance of the circuit is $R_{\text{eq}} = 77.9\ \Omega$, and we will use the symbol I to refer to the entire current supplied by the battery $I = \dfrac{V_{\text{batt}}}{R_{\text{eq}}}$. We write the power dissipated by each resistor in terms of this current. Since all current must flow through the $20\ \Omega$ resistor at the top of the circuit and the $10\ \Omega$ resistor at the bottom, we can determine the power dissipated in each: $P_{20\ \Omega,\text{top}} = I^2 R = (20\ \Omega)I^2$, and $P_{10\ \Omega,\text{bottom}} = I^2 R = (10\ \Omega)I^2$. Current will split evenly between the two $90\ \Omega$ resistors, so the power dissipated in each of them is $P_{90\ \Omega} = (I/2)^2 R = (22.5\ \Omega)I^2$. The current will split along three paths at the $10\ \Omega$, $5\ \Omega$, and $20\ \Omega$ resistors, but the potential differences across each of those resistors must be the same. This is only possible if $\dfrac{2}{7}I$ flows across the $10\ \Omega$ resistor in parallel, $\dfrac{4}{7}I$ flows across the $5\ \Omega$ resistor, and $\dfrac{1}{7}I$ flows across the $20\ \Omega$ resistor in the parallel arrangement. This allows us to write expressions for the power dissipated in each resistor: $P_{10\ \Omega,\ \text{parallel}} = (2I/7)^2 R = (0.82\ \Omega)I^2$, $P_{5\ \Omega} = (4I/7)^2 R = (1.6\ \Omega)I^2$, and $P_{20\ \Omega,\ \text{parallel}} = (I/7)^2 R = (0.41\ \Omega)I^2$. Comparing these results, we find that either of the two $90\ \Omega$ resistors dissipates power at the highest rate. (*b*) Comparing the results from part (*a*), we see that the power dissipated by the $20\ \Omega$ resistor in the parallel arrangement dissipates power at the lowest rate.

31.89. In Problem 31.67 we used the loop and junction rules to determine the currents flowing in each part of the circuit shown in Figure P31.67. We found $I_1 = 2.0\times10^{-2}$ A, $I_2 = 1.5\times10^{-2}$ A, and $I_3 = 5.0\times10^{-3}$ A. Using these values, we can easily see that power is being supplied by all batteries (none is using power to charge, for example). The rates at which the batteries supply power are $P_{12\ \text{V}} = I_1 V_{12\ \text{V}} = (2.0\times10^{-2}\ \text{A})(12\ \text{V}) = 0.24\ \text{W}$, $P_{9\ \text{V}} = I_2 V_{9\ \text{V}} = (1.5\times10^{-2}\ \text{A})(9\ \text{V}) = 0.1\ \text{W}$, and the power dissipated by either of the two 1.5 V batteries is $P_{1.5\ \text{V}} = I_3 V_{1.5\ \text{V}} = (5.0\times10^{-3}\ \text{A})(1.5\ \text{V}) = 7.5\ \text{mW}$. The same set of currents enables us to find the power dissipated by each resistor:

$$P_{300\ \Omega} = I_1^2 R_{300\ \Omega} = (2.0\times10^{-2}\ \text{A})^2(300\ \Omega) = 0.12\ \text{W}$$
$$P_{1000\ \Omega} = I_2^2 R_{1000\ \Omega} = (1.5\times10^{-2}\ \text{A})^2(1000\ \Omega) = 0.23\ \text{W}$$
$$P_{1200\ \Omega} = I_3^2 R_{1200\ \Omega} = (5.0\times10^{-3}\ \text{A})^2(1200\ \Omega) = 0.030\ \text{W}$$
$$P_{600\ \Omega} = I_3^2 R_{600\ \Omega} = (5.0\times10^{-3}\ \text{A})^2(600\ \Omega) = 0.015\ \text{W}$$

To the significant digits given, we find that the power supplied by the batteries, and the power dissipated by the resistors is equal to 0.4 W.

31.90. We know that the magnetic field inside a solenoid is

$$B_{\text{sol}} = \mu_0 n I = \frac{\mu_0 I}{d} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.0\ \text{A})}{(0.321\times10^{-3}\ \text{m})} = 12\ \text{mT}$$

The power dissipated is

$$P = I^2 R = \frac{I^2 \ell_{\text{wire}}}{\sigma\pi r_{\text{wire}}^2} = \frac{I^2(2\pi r_{\text{sol}})\ell_{\text{sol}}}{\sigma\pi r_{\text{wire}}^2 d_{\text{wire}}} = \frac{8I^2 r_{\text{sol}}\ell_{\text{sol}}}{\sigma d_{\text{wire}}^3} = \frac{8(3.0\ \text{A})^2(0.025\ \text{m})(0.20\ \text{m})}{(5.9\times10^7\ \text{A/(V}\cdot\text{m)})(0.321\times10^{-3}\ \text{m})^3} = 1.8\times10^2\ \text{W}$$

The simplest and most common way to increase the field is to fill the solenoid with an iron core. The small field from the solenoid causes the microscopic magnets in iron to align. The resulting field is much larger than the field of the solenoid alone. Another possibility would be to increase the number of windings by adding a second layer of wire windings over the first one, effectively doubling the number of turns. This can be repeated any number of times, and could be done in conjunction with the use of a thicker wire. The thicker wire will have less resistance and will dissipate less heat. One could use silver wire if one has the funds.

31.91. We use $P = \dfrac{V^2}{R}$ to write $R_{\text{hot}} = \dfrac{V^2}{P_{\text{hot}}} = \dfrac{(120\ \text{V})^2}{(100\ \text{W})} = 144\ \Omega$. This means the resistance has increased by a factor $\dfrac{R_{\text{hot}}}{R_{\text{cold}}} = \dfrac{(144\ \Omega)}{(9.5\ \Omega)} = 15.$

31.92. No, the bulb will not light up. Both terminals of the battery are connected to the same side of the filament.

31.93. There may be many correct answers. Examples are electrolytic solutions, solid state ionic conductors, etc.

31.94. The resistance is constant, so the ratio $\frac{V}{I}$ is constant. Thus we can write

$$\frac{V_{\text{i}}}{I_{\text{i}}} = \frac{V_{\text{f}}}{I_{\text{f}}} \Rightarrow V_{\text{f}} = \frac{I_{\text{f}}V_{\text{i}}}{I_{\text{i}}} = \frac{(3.5\text{ A})(12\text{ V})}{(1.0\text{ A})} = 42\text{ V}$$

31.95. Using the loop rule, we can write

$$\mathcal{E} - IR_{\text{batt}} - IR = 0 \Rightarrow R = \frac{\mathcal{E}}{I} - R_{\text{batt}} = \frac{(20.0\text{ V})}{(0.100\text{ A})} - (13.0\ \Omega) = 187\ \Omega$$

31.96. Looking first at the far right vertical wire with three resistors, we see that there is a potential difference between the top and bottom of this wire equal to $V_{\text{right}} = I_3R_{\text{right}} = 3I_3R.$ Clearly, the potential differences across the middle wire (with two resistors) and the left wire (with only one resistor) must be the same as that across the right wire, because all three paths are parallel to each other. Symbolically, we can write the potential differences across the left and center wire as $V_{\text{left}} = I_1R$ and $V_{\text{center}} = 2I_2R,$ respectively. Equating the potential differences yields $I_1 = 3I_3 = 3(2.0\text{ A}) = 6.0\text{ A},$ and $I_2 = \frac{3}{2}I_3 = \frac{3}{2}(2.0\text{ A}) = 3.0\text{ A}.$

31.97. (*a*) Here, the two batteries are in parallel. The potential difference across each one is 9.0 V, and the potential difference across their parallel combination is also 9.0 V. The potential must drop by 9.0 V across the light bulb in order to satisfy the loop rule. Thus, the potential difference across the bulb is 9.0 V. (*b*) Here the batteries are in series. The first battery causes an increase in potential of 9.0 V, and the second battery causes an additional potential difference of 9.0 V, resulting in a total potential difference across the batteries equal to 18.0 V. The loop rule tells us that the potential drop across the light bulb must be 18.0 V.

31.98. (*a*) The current is greater through the top (thicker) resistor. The two resistors have the same potential difference across them, because they are wired in parallel. The current is given by $I = \frac{V}{R}.$ Since the top (wider) resistor will have the smaller resistance, it will carry the larger current. (*b*) The potential differences across the two resistors are the same; they are wired in parallel. (*c*) The total resistance is smaller than either resistance of the top or bottom wire alone. The total resistance is given by $\frac{1}{R_{\text{total}}} = \frac{1}{R_{\text{top}}} + \frac{1}{R_{\text{bottom}}}.$

31.99. We would add a resistor in parallel to those already in the circuit. This always reduces the overall resistance of the circuit. We would add a resistor of large area like the top resistor.

31.100. (*a*) The only way for the potential difference across resistor 1 to be zero, is if no current flows across it. Of course, since there is only one loop in this circuit, that means no current will flow across resistor 2 either. In order to satisfy the loop rule, we write $\mathcal{E}_1 - \mathcal{E}_2 + \mathcal{E}_3 = 0 \Rightarrow \mathcal{E}_3 = \mathcal{E}_2 - \mathcal{E}_1 = (5.0\text{ V}) - (3.0\text{ V}) = 2.0\text{ V}.$ (*b*) As mentioned in part (*a*), the current in the loop is zero. So the current across resistor 2 is clearly zero.

31.101. The maximum current you can pull from the battery is 0.10 A. This happens when you fill the gap with a resistor that has such a small resistance that it is practically zero (like adding an ideal conductor). In that case, current would bypass the $100\ \Omega$ resistor in parallel with the conductor and the total resistance of the circuit would just be $100\ \Omega$ from the resistor that is not in series with anything. This leaves us with $I_{\text{max}} = \frac{V_{\text{batt}}}{R_{\text{top}}} = \frac{(10\text{ V})}{(100\ \Omega)} = 0.10\text{ A}.$ The smallest the current could ever be is 0.05 A. This happens when you close the gap with a resistor that has such a huge resistance that current is essentially prohibited from traversing it (or you could just leave the gap there, with nothing closing it). In that case the total resistance of the parallel arrangement would just be $R_{\text{par}} = \lim_{R_{\text{large}}\to\infty}\left(\frac{1}{(100\ \Omega)} + \frac{1}{R_{\text{large}}}\right)^{-1} = 100\ \Omega,$ and the total resistance of the circuit would be $200\ \Omega.$ This leaves us with $I_{\max} = \frac{V_{\text{batt}}}{R_{\text{eq}}} = \frac{(10\ \text{V})}{(200\ \Omega)} = 0.050\ \text{A}.$

31.102. Answers may vary slightly depending on how well one reads the plot. The lower potential corresponds to a resistance of approximately $90\ \Omega.$ The higher potential difference corresponds to a resistance of approximately $7\times10^4\ \Omega.$ The maximum power occurs when the product of the current and potential difference is maximized. This appears to happen near a potential difference of 4.5–5 V, and near a current of 0.9–1.0 mA, resulting in a maximum power output of 4–5 mW.

31.103. Let I_n be the current through resistor R_n. Since the circuit has been closed a long time, the capacitors will be fully charged and no additional current will flow to or from them. In particular, this means no current will flow across R_6 or the $30.0\ \mu\text{F}$ capacitor. Current can only flow through that parallel arrangement involving the capacitors by flowing through R_7 and R_8, which puts them in series with R_1 and R_2. This drastically simplifies the equivalent resistance of the circuit to

$$R_{\text{eq}} = R_1 + R_2 + \left(\frac{1}{R_3+R_4} + \frac{1}{R_5}\right)^{-1} + R_7 + R_8$$

$$= (5.00\ \Omega) + (5.00\ \Omega) + \left(\frac{1}{(4.00\ \Omega)+(6.00\ \Omega)} + \frac{1}{(10.0\ \Omega)}\right)^{-1} + (1.00\ \Omega) + (0.500\ \Omega) = 16.5\ \Omega$$

This allows us to determine the current supplied by the battery: $I_{\text{batt}} = \frac{V_{\text{batt}}}{R_{\text{eq}}} = \frac{(100\ \text{V})}{(16.5\ \Omega)} = 6.06\ \text{A}.$ Note that this must also be the current through resistors $R_1,$ $R_2,$ $R_7,$ and $R_8.$ In general, current will not split evenly between two parallel paths. But in this case, the upper and lower path across the parallel arrangement have equal resistances ($10.0\ \Omega$ each). So, in this case the current will split evenly between those two paths, such that the current is 3.03 A through $R_3,$ $R_4,$ and $R_5.$ The potential difference across the $30.0\ \mu\text{F}$ capacitor must be the same as the potential difference across $R_7,$ because they are in parallel. Thus, we can write $q_{30.0\ \mu\text{F}} = C_{30.0\ \mu\text{F}}V_{30.0\ \mu\text{F}} = C_{30.0\ \mu\text{F}}I_{\text{R}_7}R_7 = (30.0\ \mu\text{F})(6.06\ \text{A})(1.00\ \Omega) = 1.82\times10^{-4}\ \text{C}.$ Similarly, the potential difference across the $20.0\ \mu\text{F}$ capacitor must be the same as the potential difference across the series combination of R_7 and $R_8,$ because those paths are in parallel. Thus

$$q_{20.0\ \mu\text{F}} = C_{20.0\ \mu\text{F}}V_{20.0\ \mu\text{F}} = C_{20.0\ \mu\text{F}}I_{\text{R}_7}(R_7+R_8) = (20.0\ \mu\text{F})(6.06\ \text{A})((1.00\ \Omega)+(0.500\ \Omega)) = 1.82\times10^{-4}\ \text{C}$$

Collecting these results: $I_{\text{battery}} = I_1 = I_2 = I_7 = I_8 = 6.06\ \text{A},$ $I_3 = I_4 = I_5 = 3.03\ \text{A},$ and $I_6 = 0.$ The charge on the $30\ \mu\text{F}$ capacitor is $1.82\times10^{-4}\ \text{C}.$ The charge on the $20\ \mu\text{F}$ capacitor is also $1.82\times10^{-4}\ \text{C}.$

31.104. The 12 gauge wire is twice as thick as the 18 gauge wire. This means that a given length of wire will have a 4 times less resistance if it is made from 12 gauge wire as compared to 18 gauge. The actual potential difference across the length of the wire could be anything. But since people are using this cable for all their power needs, we might assume that they are coming close to maxing out the current that can be carried by the household wiring (close to tripping the breakers). This could correspond to as much as 20 A in a standard household circuit (though some may be higher for heavy electric appliances). If the third floor is drawing 20 A of current through the wire, then power is being dissipated in the extension cord according to $P = I^2R = \frac{I^2L}{\sigma A}.$ For the numbers given and assuming copper wiring, this corresponds to a power of 250 W using the 18 gauge wire, and 65 W using the 12 gauge wire. Obviously this is only a factor of four difference. But consider what is happening to this power “dissipated” in the cord. It is increasing the thermal energy of the cord. Since the 18 gauge wire is thinner, it will be made up of ¼ the material (presumably copper) used in the 12 gauge wire. So in the 18 gauge wire, one has four times the power being delivered to a volume of material that is four times smaller. This could lead to temperature increases in the 18 gauge wire that are 16 times larger than the increases in the 12 gauge wire. So, yes, the diameter of the wire could make a huge difference in terms of the risk of fire.

31.105. *Principles* Equation 31.9 refers to the movement of electrons in a solid. In that case it is an electron that is carrying charge and leading to conduction of a current. In seawater, the particle that carries charge is never a dissociated electron, but rather an entire ion. Even the lightest possible ion (H^+) is about 2000 times heavier than an electron, and the ions in water are typically from salts of much heavier ions. Hence, even if equation 31.9 were valid, it would not be the mass of the electron in the denominator, but the mass of the ion. This leads you to a much lower conductivity.

30

CHANGING ELECTRIC FIELDS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^8 m **2.** 10^0 m **3.** 10^7 Hz **4.** 10^{-8} V/m **5.** 10^{-1} s **6.** 10^{-4} A **7.** 10^0 V/m **8.** 10^{-7} T **9.** 10^2 $\text{N}\cdot\text{m}^2/\text{C}$
10. 10^8 W

Guided Problems

30.2 Electric from magnetic

1. Getting Started Many aspects of Worked Problem 30.1 can be useful here. For example, the solution to Worked Problem 30.1 shows us how to determine the direction of propagation of the wave, and how to relate magnitudes of electric and magnetic fields in an electromagnetic wave. The only difference is that in that problem we were given the electric field and asked for the magnetic field, and in the current problem we are given the magnetic field and asked for the electric field.

Looking at the functional form of the magnetic field: $\vec{B}(y,t) = B\cos(ky + \omega t)\hat{k}$. We can see that the argument of the cosine function changes in time and space. There is a peak at $y = 0$, and $t = 0$. As time increases the argument increases and spatial position $y = 0$ is no longer a peak in the magnetic field. The peak must now occur at some slightly negative position along the y axis, such that the total argument $ky + \omega t$ remains zero. Thus, we see that the wave is moving in the $-\hat{j}$ direction. The magnetic field is oscillating along the z axis.

2. Devise Plan The wavenumber k is related to the angular frequency by

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = c$$

If the wave is propagating in the vacuum, this reduces to

$$k = \frac{\omega}{c_0} \tag{1}$$

We can look to Figure 30.35 to remind ourselves of the relationship between the mutually orthogonal directions of $\vec{E}$, $\vec{B}$, and $\vec{S}$. We see that if the magnetic field is pointing along the $+z$ axis and the wave propagating along the $-y$ axis, the electric field must be pointing along the $+x$ axis.

Finally, the magnitudes of the electric and magnetic fields are related by $E = Bc_0$.

Combining the direction of propagation, the magnitude and the direction of the electric field at $t = 0$ and $y = 0$, we have

$$\vec{E}(y,t) = E\cos(ky + \omega t)\hat{i} = c_0 B\cos\left(\frac{\omega}{c_0}y + \omega t\right)\hat{i}$$

3. Execute Plan (*a*) Inserting numerical values for the wavenumber, we obtain

$$k = \frac{\omega}{c_0} = \frac{(3.0\times10^9\ \text{s}^{-1})}{(3.0\times10^8\ \text{m/s})} = 1.0\times10^1\ \text{m}^{-1}$$

(*b*) Inserting numerical values for the electric field magnitude, we find

$$E = Bc_0 = (2.0\times10^{-5}\ \text{T})(3.0\times10^8\ \text{m/s}) = 6.0\times10^3\ \text{T}\cdot\text{m/s}$$

The units of the electric field may appear strange. But recall that the rate of change of flux gives an electromotive force with units of Volts. That means $\text{T}\cdot\text{m}^2/\text{s} = \text{V} \Rightarrow \text{T}\cdot\text{m/s} = \text{V/m}$ which are the well-known units of an electric field.

Thus the electric field is

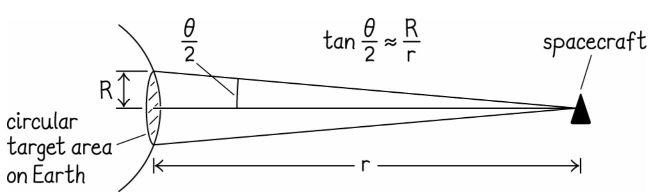
$$\vec{E}(y,t) = (6.0\times10^3\ \text{V/m})\cos((1.0\times10^1\ \text{m}^{-1})y + (3.0\times10^9\ \text{s}^{-1})t)\hat{i}$$

4. Evaluate Result The magnitude of the electric field in the EM wave is more than an order of magnitude larger than the common atmospheric electric field noted. The magnetic field in the EM wave is very close to the magnetic field at the equator, which is approximately 2.5×10^{-5} T.

30.4 Beam from outer space

1. Getting Started A directional antenna allows most of the energy in a signal to propagate in one particular direction, as opposed to the energy being emitted in all directions equally. Let us suppose that all 20 W of power being emitted by the antenna can be directed such that it is evenly distributed across the conical region shown in Figure WG30.1, and no power is emitted outside that conical region.

It is simple to relate the area over which the signal falls, on Earth, using simple geometry. Consider the figure below.



Clearly the radius of the circle is given by $R = r\tan(\theta/2)$, such that the area of the circular region on Earth on which the signal falls is $A = \pi R^2 = \pi r^2\tan^2(\theta/2)$.

2. Devise Plan Equation (30.40) lets us relate the intensity of an electromagnetic wave to the root-mean-squared values of the electric and magnetic fields:

$$S_{\text{av}} = \frac{1}{\mu_0}E_{\text{rms}}B_{\text{rms}} = \frac{E_{\text{rms}}^2}{c_0\mu_0} \qquad (1)$$

Since we know the minimal root-mean-quared value of the electric field, we can set equation (1) equal to the power delivered per unit area:

$$S_{\text{av}} = \frac{P}{A} = \frac{P}{\pi R^2} \qquad (2)$$

We first set (1) and (2) equal to each other to determine R. We then use that value and the relation $R = r\tan(\theta/2)$ to determine the angle over which the signal should be emitted.

3. Execute Plan Using equation (1), we find

$$S_{\text{av}} = \frac{E_{\text{rms}}^2}{c_0\mu_0} = \frac{(1.0\times10^{-3}\ \text{V/m})^2}{(3.0\times10^8\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})} = 2.65\times10^{-9}\ \text{W/m}^2 \tag{3}$$

Rearranging equation (2) and inserting the numerical result (3), we obtain

$$R = \sqrt{\frac{P}{\pi S_{\text{av}}}} = \sqrt{\frac{(20\ \text{W})}{\pi(2.65\times10^{-9}\ \text{W/m}^2)}} = 4.9\times10^4\ \text{m} \tag{4}$$

This yields

$$\theta = 2\tan^{-1}\left(\frac{R}{r}\right) = \tan^{-1}\left(\frac{(4.90\times10^4\ \text{m})}{(120)(1.5\times10^{11}\ \text{m})}\right) = 5.4\times10^{-9}\ \text{radians or } 1.6\times10^{-8}\ \text{degrees}$$

4. Evaluate Result The 34-m diameter dish would receive such a tiny fraction of the signal strength that it would likely not be able to **detect** the signal at all. The ratio of the signal strength detected to the total signal strength would equal the ratio of the dish area to the area over which the signal is spread:

$$\frac{P_{\text{det}}}{P} = \frac{A_{\text{det}}}{A} \Rightarrow P_{\text{det}} = P\frac{R_{\text{dish}}^2}{R^2} = (20\ \text{W})\frac{(17\ \text{m})^2}{(4.9\times10^4\ \text{m})^2} = 2.4\ \mu\text{W}$$

This is an extremely small amount of power.

30.6 Charge it!

1. Getting Started In order to determine the maximum power that flows into the empty volume between capacitor plates, we need to know the intensity into that volume, and the relevant surface area. The surface area bounding the volume is effectively given in the problem statement: it is just the area of the circular band bounding the outer edge of the plates. Thus $A = 2\pi Rd$. However the intensity is not given in the problem statement. The maximum magnetic field is given. But this is not an electromagnetic wave; the electric and magnetic fields do not have a trivial relationship.

2. Devise Plan Ampere's law (including the displacement current) can give us an expression for the magnetic field in terms of the changing electric field. Let us consider an Amerpian path that is circular and is concentric with the two circular metal disks. Let us further restrict our path to be between capacitor plates, such that no current is enclosed, but only the displacement current. Then

$$\oint_{\substack{\text{Amperian}\\ \text{path}}} \vec{B}\cdot d\vec{\ell}\mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

Clearly, the magnetic field will be largest when the rate of change of flux is largest. This corresponds to choosing an Amperian loop that encloses as much changing electric field as possible. In this case that corresponds to a circular Amperian loop of radius R, such that the loop is just along the circular band that describes the boundary of the capacitor. Because the magnetic field in this case will wrap in concentric circles and will always be parallel to our Amperian loop, and because the electric flux in this case is simply $\Phi_E = EA = E\pi R^2$, Ampere's law simplifies quite a bit to

$$B_{\max}(2\pi R) = \mu_0\varepsilon_0\pi R^2\frac{dE}{dt} \tag{1}$$

One could proceed by simply integrating this expression over time:

$$E = \frac{2B_{\max}c_0^2}{R}t + c$$

And noting that when $t = 0$ the current has not had time to deliver any charge to the capacitor plates, we see that $c = 0$, and we have

$$E = \frac{2B_{\max}c_0^2}{R}t \tag{2}$$

But we are asked to relate this to the current in the wire. So we note that the electric field between the capacitor plates can also be written as

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\pi R^2\varepsilon_0} \tag{3}$$

This was derived originally using Gauss's Law, but we simply quote the result here. Inserting this into equation (1), we have

$$B_{\max} = \frac{\mu_0}{(2\pi R)}\frac{dq}{dt} \tag{4}$$

And of course the rate of change of the charge on the capacitor plates is the current: $\frac{dq}{dt} = I$. Rearranging trivially we have $q_{\max} = It_{\max}$. Maximizing this current and inserting it into equation (3) yields the maximum electric field. So we have maximized the magnetic field by choosing the largest possible radius for our Amperian path (still inside the capacitor plates) and we have maximized the electric field by letting as much time pass as possible. Inserting there maximum values into equation (30.36) should yield the maximum intensity.

3. Execute Plan Modifying (4) as described above and rearranging, we have

$$I = \frac{B_{\max}(2\pi R)}{\mu_0}$$

This means that the maximum charge on the capacitor plates is

$$q_{\max} = \frac{B_{\max}(2\pi R)}{\mu_0}t_{\max}$$

And inserting this into (3) we obtain

$$E_{\max} = \frac{q_{\max}}{\pi R^2\varepsilon_0} = \frac{2B_{\max}}{\mu_0\varepsilon_0 R}t_{\max} = \frac{2B_{\max}c_0^2}{R}t_{\max}$$

This was exactly our result (2), which we obtained by simple integration.

Inserting this into equation (30.36), we find the magnitude of the maximum power flowing across the boundary is

$$S_{\max} = \frac{1}{\mu_0}E_{\max}B_{\max}$$

$$S_{\max} = \frac{1}{\mu_0}\frac{2B_{\max}^2c_0^2}{R}t_{\max}$$

$$S_{\max} = \frac{1}{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})}\frac{2(1.2\times10^{-8}\ \mathrm{T})^2(3.0\times10^{8}\ \mathrm{m/s})^2}{(0.200\ \mathrm{m})}(7.7\times10^{-6}\ \mathrm{s})$$

$$S_{\max} = 794\ \mathrm{W/m^2}$$

The area of the boundary (over which these fields exist) is $A = 2\pi Rd$, such that the total power crossing the boundary is $P_{\max} = S_{\max}(2\pi Rd) = (794\ \mathrm{W/m^2})(2\pi)(0.200\ \mathrm{m})(0.015\ \mathrm{m}) = 15\ \mathrm{W}$.

For concreteness, let us say that the current is flowing along the $+z$ axis. Then the electric field between the capacitor plates will also point in the $+z$ direction. The direction of the magnetic field will be around the z axis in the ϕ direction. The right hand rule for the Poynting vector shows us that the energy is flowing into the capacitor. Thus the maximum flow of power is 15 W into the capacitor.

4. Evaluate Result Inserting numerical values into our modified equation (4), we see that the current in the wire is

$$I = \frac{B_{\max}(2\pi R)}{\mu_0} = \frac{(1.2\times10^{-8}\ \mathrm{T})(2\pi)(0.200\ \mathrm{m})}{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})} = 0.012\ \mathrm{A}$$

This is a very reasonable current. In fact, it is small enough that it is reasonable to assume it could be kept constant for a few microseconds, particularly if it is flowing onto a very large capacitor.

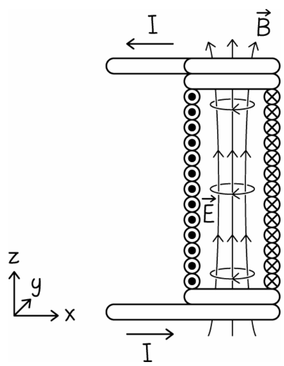
Both the magnitude of the power and the fact that the power is flowing into the capacitor make sense. We already know that capacitors can store a large amount of energy, and we know that energy must end up on the capacitor through the charging process.

30.8 Coiled again!

1. Getting Started In Worked Problem 30.7 the current was decreasing. That might lead one to conclude that energy was leaving the solenoid, and in fact the application of the right hand rule showed that was the case. In this problem the current is increasing, and we might expect that to mean that energy is flowing into the solenoid. We will check this with the Poynting vector.

If we assume that the solenoid is tightly wound, we can apply equation (28.6): $B(t) = \mu_0 n I(t)$. We can find the electric field by using Faraday's Law, which relates electric fields to changes in magnetic flux.

Consider the figure below.



With this choice of current direction, we see that the magnetic field will be upward in the $+z$ direction. Because the current is increasing in the solenoid, the magnitude of the magnetic field will be increasing. This means that the flux upward through any cross-section of the solenoid will also be increasing. Lenz's law tells us that this will cause an electric field clockwise as seen from above the solenoid. This direction can also be called $-\hat{\phi}$.

2. Devise Plan We can conveniently refer to the magnetic field anywhere in the interior of the solenoid, as it is uniform. However, the electric field will vary in space. Since we want to describe power entering or leaving the solenoid, we want the Poynting vector at the boundary of the solenoid. It therefore makes sense to find expressions for $\vec{E}(t)$ and $\vec{B}(t)$ at the outer edge of the solenoid. So

$$\vec{B}(t) = \mu_0 n I(t)\hat{k} \tag{1}$$

The electric field is given by Faraday's law applied to a circular path concentric with the axis of symmetry of the solenoid. Once we have the electric and magnetic fields, we obtain the magnitude and direction of the Poynting vector from equation (30.37). The Poynting vector describes the flow of power per unit area. If we want to know the power delivered to or from the solenoid, we simply multiply the magnitude of the Poynting vector by the surface area of the solenoid. If the magnetic and electric fields were not constant over that area, we could not use simple multiplication; we would have to integrate.

3. Execute Plan Applying Faraday's law to a circular loop inside the solenoid, concentric with the axis of the solenoid, we find

$$\oint \vec{E}\cdot d\vec{\ell} = \frac{d\Phi_B}{dt}$$

$$E(2\pi a) = (\pi a^2)\frac{dB(t)}{dt}$$

$$E = \frac{a}{2}\frac{d}{dt}(\mu_0 n I(t))$$

$$E = \frac{a\mu_0 n b}{2}$$

or

$$\vec{E} = -\frac{a\mu_0 nb}{2}\hat{\phi} \tag{2}$$

Inserting equations (1) and (2) into (30.37), we find

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$\vec{S} = -\frac{a\mu_0 n^2 b^2 t}{2}(\hat{\phi}\times\hat{k})$$

$$\vec{S} = -\frac{a\mu_0 n^2 b^2 t}{2}\hat{r}$$

We note that the energy is flowing in the $-\hat{r}$ direction, meaning radially inward toward the center of the solenoid, as we expected. The power is then given by

$$P = SA$$

$$P(t) = \mu_0 \pi a^2 n^2 b^2 ht$$

or equivalently

$$P(t) = \frac{\mu_0 \pi a^2 N^2 b^2}{h}t$$

Thus power is flowing into the solenoid at a rate of $P(t) = \dfrac{\mu_0 \pi a^2 N^2 b^2}{h}t.$

4. Evaluate Result We know the energy density stored in a magnetic field is $u_B = \dfrac{B^2}{2\mu_0},$ which is uniform inside the solenoid. So the total magnetic energy stored in the solenoid is $U^B = u_B V = \dfrac{B^2 \pi a^2 h}{2\mu_0}.$ Inserting the expression for the magnetic field, this becomes

$$U^B = \frac{\mu_0^2 n^2 I^2(t)\pi a^2 h}{2\mu_0}$$

$$U^B = \frac{\mu_0 \pi n^2 a^2 b^2 h}{2}t^2$$

Taking the first derivative of this energy stored in the magnetic field must yield the flow of energy into the solenoid:

$$\frac{dU^B}{dt} = \frac{\mu_0 \pi n^2 a^2 b^2 h}{2}\frac{d}{dt}t^2$$

$$\frac{dU^B}{dt} = \mu_0 \pi n^2 a^2 b^2 ht$$

This is exactly the expression we obtained for the Poynting vector.

Questions and Problems

30.1. The direction of the magnetic field that accompanies a changing electric field is the same as that of a magnetic field produced by a current in the same direction as $\Delta\vec{E}.$ The magnetic field at P would be produced by a current directed out of the page at the location of the electric field, so the electric field magnitude is increasing.

30.2. Because the direction of the electric field is into the page and its magnitude is decreasing, the direction of $\Delta\vec{E}$ is out of the page. A current directed out of the page would produce a magnetic field that encircles the current in a counterclockwise direction, and that is the direction of the magnetic field in both cases.

30.3. Because the electric field in the isolated capacitor is not changing, there is no magnetic field surrounding it.

30.4. (*a*) Because the electric field points out of the page and is decreasing, $\Delta\vec{E}$ points into the page. A current directed into the page would produce a magnetic field directed in clockwise circles, so that is the direction of the magnetic field accompanying the changing electric field. (*b*) This electric field could be the field inside a discharging capacitor. If we apply Ampère's law to a closed path inside or outside the volume, in either case we can construct a surface that intercepts the wire through which the capacitor is being discharged, so a magnetic field must exist both inside and outside the volume.

30.5. (*a*) As the capacitor is being charged, the right plate has a positive charge and the left plate has a negative charge, so the electric field is directed toward the left. (*b*) Because the electric field is increasing, the change in the electric field is also directed toward the left. (*c*) The magnetic field between the plates has the same direction as it does around the wires, clockwise as viewed in the direction of the current.

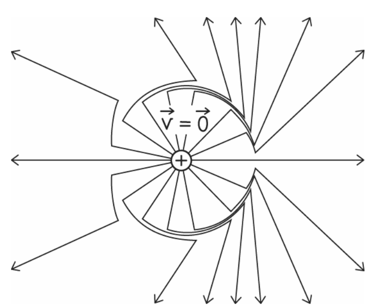
30.6. (*a*) The magnitude of the electric field is proportional to the emf induced by the changing magnetic field, and the induced emf is proportional to the rate of change of the magnetic field, so if the rate of change is increasing, the magnitude of the electric field must be increasing, too. (*b*) Because the electric field is changing, there must be an accompanying magnetic field. A current loop in the direction of the increasing electric field would produce a magnetic field directed toward the left in the region of the original magnetic field. This magnetic field counteracts the original magnetic field.

30.7. (*a*) Yes, the electric field lines around a stationary charged particle are straight lines. (*b*) Yes, the electric field lines around a charged particle moving at constant velocity are straight lines. (*c*) No, the electric field lines around an accelerating charged particle are not straight lines. See, for example, *Principles* Figure 30.9*f*, where the electric field lines produced when the particle was accelerating are curved.

30.8. (*a*) The pattern in (1) could represent the electric field lines of a charged particle, and the pattern in (2) could represent the electric field lines accompanying a changing magnetic field. The pattern in (3) could not represent electric field lines because the lines intersect each other. (*b*) The pattern in (2) could represent magnetic field lines because they form closed loops. The pattern in (1) could not represent magnetic field lines because they do not form closed loops, and the pattern in (3) could not represent magnetic field lines because the lines intersect each other.

30.9. A particle that moves in a circle is accelerating toward the center of the circle, which should create kinks in its electric field.

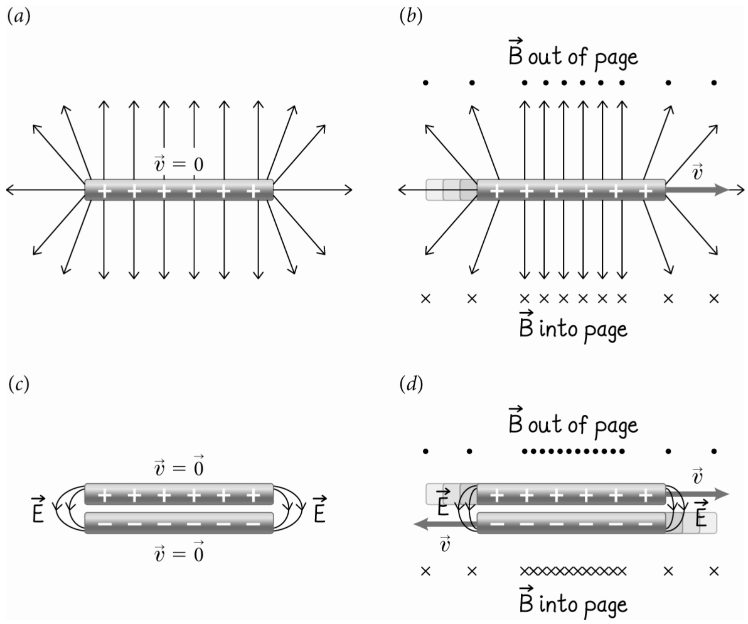
30.10.



30.11. (*a*) No. A wire with no surplus charge carrying a constant current produces a constant magnetic field, but no electric field. (*b*) Yes. Because the current in the wire varies with time, so does the magnetic field produced by the current, and an electric field accompanies a changing magnetic field. (*c*) Yes. As the bar magnet moves, the magnetic field it produces at a given location changes with time, and an electric field accompanies the changing magnetic field. (*d*) No. When the bar magnet rotates about an axis through its poles, the magnetic field does not change, and a constant magnetic field does not produce an electric field.

30.12. (*a*) Yes. The charge carriers in the sphere move in circles, so there are current loops in the rotating sphere, which produce magnetic fields. (*b*) Yes. The moving charged particle can be thought of as a tiny piece of current, which produces a magnetic field. Alternatively, the electric field of the particle at a given location changes as the particle moves, and changing electric fields are accompanied by magnetic fields. (*c*) Maybe, but not necessarily. If the speed of the particle is non-zero it produces a magnetic field. If the speed of the particle is zero (as it might be instantaneously when its velocity changes direction) it does not produce a magnetic field.

30.13.



30.14. To create a dipole field, the length of the antenna should be one-half the wavelength of the electromagnetic waves. The wavelength is related to the frequency by Eq. 16.10, $\lambda f = c,$ so the length should be

$$\ell = \tfrac{1}{2}\lambda = \frac{c}{2f} = \frac{3.0\times10^8\ \text{m/s}}{2(100\times10^6\ \text{Hz})} = 1.5\ \text{m}$$

30.15. The electric field points upward. At a point a little further to the east, the magnetic field will be increasing toward the south. This changing magnetic field is accompanied by an electric field that, according to Lenz's law, would create a current opposing the change in the magnetic field. That is, a current following the electric field would produce a northward-pointing magnetic field, so from the right-hand dipole rule, the electric field at our original point is directed upward.

30.16. (*a*) By definition, the polarization of the wave is the orientation of the electric field, which is parallel to the *x* axis. (*b*) The magnetic field lines are parallel to the *y* axis, perpendicular to both the electric field and the direction of propagation of the wave.

30.17. (*a*) The magnetic field at P is approximately zero. A short time later, as the wave moves to the left, the magnetic field at P will be upward. So, the direction of the change in magnetic field is upward. (*b*) The changing magnetic field is accompanied by an electric field that, according to Lenz's law, would create a current opposing the change in the magnetic field. A current that goes into the page to the left and out of the page to the right of P would create a downward pointing magnetic field at P. So, the direction of the electric field loop around P is into the page on the left and out of the page on the right. (*c*) Because the electric field oscillates into and out of the page, that is the orientation of the oscillating dipole.

30.18. (*a*) The wavelength is related to the frequency by Eq. 16.10, $\lambda f = c,$ so

$$\lambda = \frac{c}{f} = \frac{3.0\times10^8\ \text{m/s}}{2.4\times10^9\ \text{Hz}} = 0.13\ \text{m}$$

(*b*) A dipole antenna should be one-half a wavelength tall, or about 63 mm. Yes, this is comparable to the length of some router antennas.

30.19. A radio transmission tower is a large dipole antenna, and transmits most of its energy in the plane perpendicular to the oscillating dipole. In particular, it does not transmit any energy along the line of the oscillating dipole. So, a vertically oriented antenna transmits equally well in all horizontal directions, but a horizontally oriented antenna only transmits well in the direction perpendicular to the antenna. A horizontally oriented dipole antenna would also transmit a lot of its energy straight up, where there are no receivers.

30.20. The particle must initially be set in motion by an electric force, because it starts from rest and there is no magnetic force on a stationary particle. So, the electric field must have a component in the y direction, it must not have a component in the x direction, and it might or might not have a component in the z direction. Once the particle is moving, a magnetic field will exert forces on it, directed perpendicular to both the particle's velocity and the magnetic field. If the magnetic field had a component in the z direction, it would exert a force in the x direction on the particle as it moved in the y direction. As this does not happen, the magnetic field must have no z component. Because the electric and magnetic fields in an electromagnetic wave are perpendicular to each other, their scalar product is zero. The x component of the electric field is zero, as is the z component of the magnetic field, so $\vec{E} \cdot \vec{B} = E_y B_y = 0$. Because the y component of the electric field is non-zero, the y component of the magnetic field must be zero, that is, the magnetic field is in the x direction. The direction of propagation of the wave is perpendicular to both the electric and magnetic fields, so it must have a z component and it might or might not have a y component.

30.21. Because the kinks produce a downward current in the rod, the electric field in the kinks must point downward. That means the electric field points toward the particle in the other regions, which means that the particle is negatively charged.

30.22. (*a*) We would want an antenna in the shape of a loop, so we could measure the emf induced by the changing magnetic flux through the loop. (*b*) It would need to be oriented with the plane of the loop perpendicular to the magnetic field. That is, the loop should be oriented so that both the direction of the electric field and the direction of propagation of the wave are in the plane of the loop. (*c*) Faraday's law says that the induced emf is proportional to the rate of change of magnetic flux through the loop. For a given magnetic field, higher frequencies cause a greater rate of change of magnetic flux, so this antenna is a better detector of high-frequency waves.

30.23. We can use Ampère's law (Eq. 30.6) to calculate the magnetic field. If we choose a circular path of radius r parallel to and between the capacitor's plates, the line integral of the magnetic field is just $2\pi rB$ because the magnetic field must be constant around the loop by symmetry. If we choose a flat surface spanning the loop, it intercepts no current, but the electric flux through the surface changes as the capacitor charges. The electric field is uniform inside the capacitor, so the electric flux through the surface is just $\pi r^2 E$.

To relate the rate of change of electric flux through the surface to the charging current, we note that the magnitude of the electric field is proportional to the potential difference across the capacitor, which is proportional to the charge on the capacitor plates, and the rate of change of the charge on the plates is the charging current (see *Principles* Example 26.2). In that example, we also derived a formula for the capacitance of a parallel-plate capacitor. Combining these we have

$$\frac{d\Phi_E}{dt} = \pi r^2 \frac{dE}{dt} = \pi r^2 \left(\frac{1}{d} \frac{dV_{\text{cap}}}{dt} \right) = \frac{\pi r^2}{d} \left(\frac{1}{C} \frac{dq}{dt} \right)$$
$$= \frac{\pi r^2}{d} \frac{d}{\epsilon_0 A} I = \frac{\pi r^2 I}{\epsilon_0 A} = \frac{\pi r^2 I}{\epsilon_0 (\pi R^2)} = \frac{r^2 I}{\epsilon_0 R^2}$$

Equating the two sides of Eq. 30.6 gives

$$2\pi rB = \mu_0 \epsilon_0 \frac{r^2 I}{\epsilon_0 R^2}$$

$$B = \frac{\mu_0}{2\pi}\frac{rI}{R^2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.030\ \text{m})(5.0\ \text{A})}{2\pi(0.040\ \text{m})^2} = 1.9\times10^{-5}\ \text{T}$$

30.24. (*a*) The displacement current is, by definition, proportional to the rate of change of magnetic flux through a surface (Eq. 30.7), $I_{\text{disp}} = \epsilon_0 \frac{d\Phi_E}{dt}$. For the uniform field in the problem, the electric flux is the scalar product of the electric field and the area vector of the surface, and the displacement current is $I_{\text{disp}} = \epsilon_0 \frac{d}{dt}\vec{E}\cdot\vec{A} = \epsilon_0 \frac{d}{dt}EA\cos\theta$. So, the maximum displacement current as a function of time is

$$I_{\text{disp,max}} = \epsilon_0 A \frac{dE}{dt} = -\epsilon_0 A E_0 \omega \sin(\omega t)$$

which achieves its maximum instantaneous value $\epsilon_0 A E_0 \omega$ when $\sin(\omega t) = -1$,

$$I_{\text{disp,max}} = \epsilon_0 A E_0 \omega = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.50\ \text{m}^2)(10\ \text{V/m})(1.0\times10^7\ \text{s}^{-1}) = 4.4\times10^{-4}\ \text{A}$$

(*b*) To get this maximum displacement current, the area vector of the surface must be parallel to electric field.

30.25. Let's consider a circular Ampèrian path of radius *r* centered on and perpendicular to the axis of the wire. By symmetry, the magnetic field magnitude will be constant along the path and the line integral in Ampère's law (Eq. 30.6) is just $2\pi rB$. For the right-hand side, we can choose a cylindrical surface parallel to the wire's axis that passes some depth into the wire and is closed there by a circular disk. Because the current is uniformly distributed in the wire, the current intercepted by our surface is proportional to the area of the disk,

$$\frac{I_{\text{int}}}{\pi r^2} = \frac{I}{\pi R^2}$$

$$I_{\text{int}} = \frac{r^2}{R^2} I$$

and Ampère's law gives $2\pi rB = \mu_0 \frac{r^2}{R^2} I$, because the changing electric field in the gap is parallel to our surface, so the electric flux through it is zero. Solving for the magnetic field, we have

$$B = \frac{\mu_0 r I}{2\pi R^2} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.0\times10^{-3}\ \text{m})(2.0\ \text{A})}{2\pi(5.00\times10^{-3}\ \text{m})^2} = 3.2\times10^{-5}\ \text{T}$$

30.26. (*a*) If the area of the plates is much greater than their separation, we can assume that the electric field between them is uniform and described by the results we found in *Principles* Example 26.2,

$$E = \frac{V_{\text{cap}}}{d} = \frac{1}{d}\frac{q}{C} = \frac{1}{d}\frac{d}{\epsilon_0 A} q = \frac{q}{\epsilon_0 A}$$

The electric flux between the plates is equal to the product of the electric field and the area of the plates, $\Phi_E = EA = \frac{q}{\epsilon_0 A} A = \frac{q}{\epsilon_0}$, and its rate of change is

$$\frac{d\Phi_E}{dt} = \frac{1}{\epsilon_0}\frac{dq}{dt} = \frac{1}{\epsilon_0} I = \frac{5.0\ \text{A}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 5.6\times10^{11}\ \text{V}\cdot\text{m/s}$$

Note that other expressions for the units are acceptable, provided they are equivalent. This particular expression can be seen as

$$1\frac{\text{A}\cdot\text{N}\cdot\text{m}^2}{\text{C}^2} = 1\frac{\text{N}\cdot\text{m}^2}{\text{C}\cdot\text{s}} = 1\frac{\text{J}\cdot\text{m}}{\text{C}\cdot\text{s}} = 1\frac{\text{V}\cdot\text{m}}{\text{s}} = 1\frac{(\text{V/m})\cdot\text{m}^2}{\text{s}}$$

In SI base units, we have

$$1\frac{\text{A}\cdot\text{N}\cdot\text{m}^2}{\text{C}^2} = 1\frac{\text{A}\cdot(\text{kg}\cdot\text{m/s}^2)\cdot\text{m}^2}{(\text{A}\cdot\text{s})^2} = 1\frac{\text{kg}\cdot\text{m}^3}{\text{A}\cdot\text{s}^4}$$

(*b*) The displacement current between the plates is by definition (Eq. 30.7)

$$I_{\text{disp}} = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{1}{\epsilon_0} I = I = 5.0\ \text{A}$$

(*c*) If there is a dielectric between the plates, the electric field between the plates is reduced by a factor of κ, and so is the rate of change of the electric flux. So,

$$\frac{d\Phi_E}{dt} = \frac{1}{\kappa}\frac{1}{\epsilon_0}\frac{dq}{dt} = \frac{1}{\kappa}\frac{1}{\epsilon_0} I = \frac{5.0\ \text{A}}{(1.5)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.8\times10^{11}\ \text{V}\cdot\text{m/s}$$

In order to preserve Ampère's law in the form of Eq. 30.8, $\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_{\text{int}} + I_{\text{disp}})$, we must increase the displacement current by a factor of κ, so

$$I_{\text{disp}} = \kappa\epsilon_0 \frac{d\Phi_E}{dt} = \kappa\epsilon_0 \frac{1}{\kappa\epsilon_0} I = I = 5.0\ \text{A}$$

Because displacement current is defined without regard for dielectric properties of the material, we obtain the same answer with a dielectric.

30.27. [**NOTE:** In the problem statement, "center of the wire" is changed to "center of a long wire."] (*a*) Choosing a circular Ampèrian path of radius *r* centered on and perpendicular to the wire, and a flat surface spanning the path, Ampère's law gives

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0(I_{\text{int}} + I_{\text{disp}})$$

$$2\pi r B = \mu_0 I$$

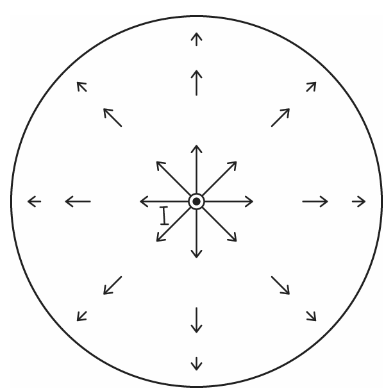
$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(7.0\times10^{-3}\ \text{m})(1.6\times10^{-8}\ \text{T})}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 5.6\times10^{-4}\ \text{A}$$

(*b*) The charge on the plate is uniformly distributed, so the amount of charge on the ring between radius *r* and the edge of the plate is proportional to the area of the ring,

$$\frac{q_r}{\pi(R^2 - r^2)} = \frac{q}{\pi R^2}$$

$$q_r = q\left(1 - \frac{r^2}{R^2}\right)$$

and the radial current flowing into the ring is $I_r = \frac{dq_r}{dt} = \frac{dq}{dt}\left(1 - \frac{r^2}{R^2}\right) = I\left(1 - \frac{r^2}{R^2}\right)$.



30.28. The displacement current between the capacitor's plates is equal to the current charging the capacitor. From the definition of capacitance (Eq. 26.1), we have

$$C = \frac{q}{V_{\text{cap}}}$$
$$q = CV_{\text{cap}}$$
$$\frac{dq}{dt} = \frac{d}{dt}CV_{\text{cap}}$$
$$I = C\frac{dV_{\text{cap}}}{dt}$$

so between the capacitor's plates $I_{\text{disp}} = I = C\dfrac{dV_{\text{cap}}}{dt}$.

30.29. (*a*) The electric field between the plates is uniform and at any instant is described by the results we found in *Principles* Example 26.2,

$$E = \frac{V_{\text{cap}}}{d} = \frac{1}{d}\frac{q}{C} = \frac{1}{d}\frac{d}{\epsilon_0 A}q = \frac{q}{\epsilon_0 A}$$

Because the capacitor is initially uncharged and the charging current is steady, the charge at time *t* is $q(t) = It$, so the electric field as a function of time is

$$E(t) = \frac{It}{\epsilon_0 A} = \frac{It}{\epsilon_0 \pi R^2} = \frac{(2.0\ \text{A})}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\pi(0.030\ \text{m})^2}t = (8.0\times10^{13}\ \text{V/m}\cdot\text{s})t$$

(*b*) The electric field between the plates is uniform, so the electric flux through the surface equals the product of the electric field and the area of the surface. The displacement current is then (Eq. 30.7)

$$I_{\text{disp}} = \epsilon_0\frac{d\Phi_E}{dt} = \epsilon_0(\pi r^2)\frac{dE}{dt} = \epsilon_0\pi r^2\frac{I}{\epsilon_0\pi R^2} = \frac{Ir^2}{R^2} = \frac{(2.0\ \text{A})(0.010\ \text{m})^2}{(0.030\ \text{m})^2} = 0.22\ \text{A}$$

(*c*) There is no electric field to the left of the plate, and the electric field between the plates is parallel to the surface, so the electric flux through the surface is zero and so is the displacement current. (*d*) By definition the displacement current depends on the surface chosen, so we have no reason to expect them to be the same. This must be the case if Ampère's law as modified by Maxwell (Eq. 30.8) is correct, because different surfaces will intercept different currents. The displacement current equals the current through the capacitor only when we choose a surface that passes completely through the capacitor and intercepts no current, which neither of these surfaces do.

30.30. Because of the symmetry of the problem, we expect the magnetic field to form circular loops around the wire in the *yz* plane. So, if we choose a circular path of radius *b* centered on the origin, we have for the left side of Ampère's law (Eq. 30.6) $2\pi bB$.

If we choose a flat surface bounded by the loop for the right side of Ampère's law, we have both an intercepted current and a displacement current. If we choose the positive *x* direction as our positive direction for current, the intercepted current is the current in the wire, which is equal to the rate of change of the charge on the sphere located at $x = a$, $I_{\text{int}} = I = \dfrac{dq}{dt}$. For the displacement current, we will follow the hint and divide the surface into concentric rings of width dr and area $2\pi r\,dr$. We calculated the electric field of a dipole in *Principles* Section 23.6; the electric field a distance *r* from the *x* axis is (Eq. 23.8, with variables changed to those of this problem)

$$E_x = -\frac{1}{4\pi\epsilon_0}\frac{2qa}{(r^2+a^2)^{3/2}}$$

so the electric flux through our surface is

$$\Phi_E = \int_0^b E_x 2\pi r\,dr = \int_0^b -\frac{q}{\epsilon_0}\frac{ar}{(r^2+a^2)^{3/2}}dr = \frac{q}{\epsilon_0}\left(\frac{a}{\sqrt{a^2+b^2}} - 1\right)$$

The displacement current is then (Eq. 30.7)

$$I_{\text{disp}} = \epsilon_0 \frac{d\Phi_E}{dt} = \left(\frac{a}{\sqrt{a^2+b^2}} - 1 \right) \frac{dq}{dt}$$

and Ampère's law gives

$$2\pi b B = \mu_0 \left[\frac{dq}{dt} + \left(\frac{a}{\sqrt{a^2+b^2}} - 1 \right) \frac{dq}{dt} \right] = \frac{\mu_0 I a}{\sqrt{a^2+b^2}}$$

$$B = \frac{\mu_0 I a}{2\pi b \sqrt{a^2+b^2}}$$

In *Principles* Example 28.6, we used the Biot-Savart law to calculate the magnetic field of an infinitely long, straight wire. We can use the same formula here, except that our limits of integration will be from $x = -a$ to $x = a$, and we want the magnetic field a distance b from the wire:

$$B = \frac{\mu_0 I b}{4\pi} \int_{-a}^{a} \frac{dx}{(x^2+b^2)^{3/2}} = \frac{\mu_0 I b}{4\pi} \frac{1}{b^2} \frac{2a}{\sqrt{a^2+b^2}} = \frac{\mu_0 I a}{2\pi b \sqrt{a^2+b^2}}$$

which is the same result we got using Ampère's law.

30.31. If we choose a circular loop of radius r concentric with the capacitor plates, the magnetic field will be tangent to the loop and have the same magnitude around the loop, so the left side of Ampère's law (Eq. 30.6) will be $2\pi r B$.

If we choose a flat surface spanning the loop, it intercepts no current, but there is electric flux through the surface. The electric field is uniform and equal in magnitude to the potential difference between the plates divided by their separation distance. So, the rate of change of electric flux through the surface is

$$\frac{d\Phi_E}{dt} = \frac{d}{dt} EA = \frac{d}{dt} \frac{V}{d} \pi r^2 = \frac{\pi r^2}{d} \frac{dV}{dt} = \frac{\pi r^2}{d} \frac{V_{\max}}{\tau_0} e^{-t/\tau_0}$$

and Ampère's law gives

$$2\pi r B = \mu_0 \epsilon_0 \frac{\pi r^2}{d} \frac{V_{\max}}{\tau_0} e^{-t/\tau_0}$$

$$B = \frac{\mu_0 \epsilon_0 r V_{\max}}{2 d \tau_0} e^{-t/\tau_0}$$

$$= (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) \times \frac{(0.20\ \text{m})(15\ \text{V})}{2(0.10 \times 10^{-3}\ \text{m})(3.0 \times 10^{-3}\ \text{s})} e^{-(1.30\ \text{ms})/(3.0\ \text{ms})}$$

$$= 3.6 \times 10^{-11}\ \text{T}$$

30.32. Yes, the equations still hold, but they don't provide us with much useful information.

Suppose we know the electric field inside a capacitor. If we integrate the electric field around the given path, we find that one leg of the path is parallel to the field, one leg is antiparallel, so their contributions cancel, and the other two legs are perpendicular to the field, so the scalar product is zero. That is, the line integral is zero, so the rate of change of magnetic flux through a surface spanning the path must be zero, too. But there are many ways in which the rate of change of magnetic flux through a surface could be zero, for example, the magnetic flux might be constant, or it might be increasing through one part of the surface and decreasing through another, or it might be parallel to the surface, or any number of other possibilities.

Supposing again that we know the electric field inside a capacitor, we find that the rate of change of electric flux through a surface spanning the path is zero, so the line integral of the magnetic field around the path must be zero, too. But, again, there are many ways for this to be the case.

30.33. To use Eq. 30.11, we must choose a Gaussian surface. Let's choose a thin box that straddles the surface in question, with two sides parallel to the surface and the other sides perpendicular to it. The components of the magnetic field parallel to the surface contribute nothing to the integral because, for the parallel components, every magnetic field line that enters our box also leaves it. For the normal component of the magnetic field, the flux through the top side must be equal and opposite to the flux through the bottom side in order for them to sum to zero, in accordance with Eq. 30.11. As we make our box thinner and thinner, the top and bottom sides approach the surface in question, and because the areas of the top and bottom sides are the same, the normal component of the magnetic field must be the same, too, in order for the fluxes to be equal and opposite.

30.34. The fluxes are only zero through closed surfaces. The fluxes in Eqs. 30.12 and 30.13 are not through closed surfaces.

30.35. [**NOTE:** In part (*a*) of the problem statement, "a conductor" is changed to "a perfect conductor."] (*a*) If the magnetic field in the conductor were not constant, then there would be some surface through which the magnetic flux was changing, and so the line integral of the electric field around the boundary of the surface would be nonzero. But in a perfect conductor, the electric field is zero everywhere, so the line integral of the electric field around any closed path must be zero, so the magnetic flux cannot be changing through any surface inside the conductor, which means that the magnetic field must be constant. What this means physically is that whenever an external source attempts to change the magnetic field inside the perfect conductor, currents are induced that exactly offset the attempted change. (*b*) Because the electric field is zero everywhere inside the perfectly conducting loop, the line integral of the electric field along a path coinciding with the loop is zero, too. So, by Faraday's law, the change in magnetic flux through the loop must also be zero. Any attempted change by an external source induces a current in the loop that exactly offsets it. (*c*) If the magnetic field is zero everywhere inside the superconductor, then the line integral of the magnetic field around any closed path within the superconductor must also be zero. The electric field is also zero inside the superconductor, so the electric flux through any surface within the superconductor and bounded by our closed path is also zero. So, by Maxwell's generalization of Ampère's law, because both the displacement current and the line integral of the magnetic field are zero, the surface cannot intercept any current. Because this is so for any closed path and surface bounded by it within the superconductor, there must be no currents anywhere within the bulk of the superconductor.

30.36. Yes, it is possible to adjust the potential difference so that there is no magnetic field. The potential difference must be adjusted so that, through a flat circular surface of radius r, centered on and perpendicular to the wire, the displacement current has exactly the same magnitude as the current in the wire but the opposite sign. In this case, Maxwell's generalization of Ampère's law gives zero magnetic field.

The electric field in the capacitor is uniform and equal in magnitude to the potential difference between the plates divided by their separation distance. So, the rate of change of electric flux through the surface is $\frac{d\Phi_E}{dt} = \frac{d}{dt}EA = \frac{d}{dt}\frac{V}{d}\pi r^2 = \frac{\pi r^2}{d}\frac{dV}{dt}$, and there will be no magnetic field when

$$\epsilon_0 \frac{d\Phi_E}{dt} = -I$$

$$\epsilon_0 \frac{\pi r^2}{d}\frac{dV}{dt} = -I$$

$$\frac{dV}{dt} = -\frac{Id}{\epsilon_0 \pi r^2} = -\frac{(0.0200\text{ A})(3.0\times10^{-3}\text{ m})}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)\pi(20\times10^{-3}\text{ m})^2} = -5.4\times10^9\text{ V/s}$$

30.37. For electromagnetic waves in vacuum, we have $f\lambda = c_0$. The speed of electromagnetic waves in air is very nearly the same as in vacuum, so the wavelength of AM radio waves ranges from $\lambda = c_0/f = (3.0\times10^8\text{ m/s})/(10^6\text{ Hz}) = 3\times10^2\text{ m}$ to $\lambda = c_0/f = (3.0\times10^8\text{ m/s})/(10^5\text{ Hz}) = 3\times10^3\text{ m}$, or order of magnitude 1 km, and the frequency of FM waves ranges from $f = c_0/\lambda = (3.0\times10^8\text{ m/s})/(10\text{ m}) = 3.0\times10^7\text{ Hz}$ to $f = c_0/\lambda = (3.0\times10^8\text{ m/s})/(1.0\text{ m}) = 3.0\times10^8\text{ Hz}$, or order of magnitude 100 MHz.

30.38. In *Principles* Example 30.8, we found the relationship between the wave speed and the dielectric constant, $c = \frac{c_0}{\sqrt{\kappa}}$, or

$$\kappa = \left(\frac{c_0}{c}\right)^2 = \left(\frac{3.00\times10^8 \text{ m/s}}{2.26\times10^8 \text{ m/s}}\right)^2 = 1.76$$

The dielectric constant varies with frequency, so this value based on wave propagation at high frequency differs substantially from the essentially zero-frequency value in *Principles* Table 26.1.

30.39. In *Principles* Example 30.8, we found the relationship between the wave speed and the dielectric constant, so the speed of light in the fiber is

$$c = \frac{c_0}{\sqrt{\kappa}} = \frac{3.0\times10^8 \text{ m/s}}{\sqrt{1.6}} = 2.4\times10^8 \text{ m/s}$$

The distance from California to New York is about 4000 km, so the time interval it takes for the signal to travel this distance is

$$\Delta t = \frac{\Delta x}{c} = \frac{4.0\times10^6 \text{ m}}{2.4\times10^8 \text{ m/s}} = 1.7\times10^{-2} \text{ s}$$

30.40. The phone call is transmitted to and from the satellite by radio waves, which travel at c_0 in the vacuum of space, so the time interval to make the round trip is

$$\Delta t = \frac{\Delta x}{c} = \frac{2(3.6\times10^7 \text{ m})}{3.0\times10^8 \text{ m/s}} = 0.24 \text{ s}$$

30.41. We learned in *Principles* Section 30.3 that a dipole antenna has a length of one-half the wavelength of the electromagnetic waves it transmits or receives. From the relationship between wave speed, wavelength, and frequency (Eq. 16.10), and assuming this is a dipole antenna, we have

$$f = \frac{c_0}{\lambda} = \frac{c_0}{2(\frac{1}{2}\lambda)} = \frac{3.0\times10^8 \text{ m/s}}{2(0.080 \text{ m})} = 1.9\times10^9 \text{ Hz}$$

30.42. (*a*) The magnitudes of the electric and magnetic fields in an electromagnetic wave are related by Eq. 30.24,

$$B = \frac{E}{c_0} = \frac{15 \text{ V/m}}{3.0\times10^8 \text{ m/s}} = 5.0\times10^{-8} \text{ T}$$

(*b*) If the potential difference of the antenna remains the same, the magnitude of the electric field will also remain the same, $E_{\text{damp,max}} = E_{\text{max}}$. (*c*) As we saw in *Principles* Section 26.7, in order for the potential difference to remain the same, a greater quantity of charge must be separated, which means that the current in the antenna will be greater by a factor of the dielectric constant of the damp air, increasing the magnetic field by the same factor, $B_{\text{damp,max}} = \kappa B_{\text{max}}$.

30.43. Assuming the usable area of the discs is the same, the number of pits that can be written on a disc is inversely proportional to the area of the pits, which is proportional to the square of the pits' diameters. If the wavelength of the laser light is proportional to the pits' diameters, we have

$$N_\text{B}A_\text{B} = N_\text{D}A_\text{D}$$
$$N_\text{B}d_\text{B}^2 = N_\text{D}d_\text{D}^2$$
$$N_\text{B}\lambda_\text{B}^2 = N_\text{D}\lambda_\text{D}^2$$
$$\lambda_\text{B} = \lambda_\text{D}\sqrt{\frac{N_\text{D}}{N_\text{B}}} = (650 \text{ nm})\sqrt{\frac{4.7 \text{ GB}}{25 \text{ GB}}} = 280 \text{ nm}$$

This wavelength corresponds to ultraviolet light. In fact, Blu-ray discs are read with 405 nm light, so our assumptions must not accurately describe all the differences between DVD and Blu-ray discs. But our result does correctly describe the trend, that greater storage densities can be achieved with shorter wavelengths.

30.44. The frequency of the light is the same in both media, but the speed is smaller in the dielectric, so the wavelength in the dielectric is also smaller. In *Principles* Example 30.8, we found a relationship between the speed of light in a dielectric medium and the dielectric constant of the medium, applying which we have

$$\lambda_\mathrm{d} = \frac{c_\mathrm{d}}{f} = \frac{c_0}{\sqrt{\kappa}}\frac{1}{f} = \frac{c_0}{\sqrt{\kappa}}\frac{\lambda}{c_0} = \frac{\lambda}{\sqrt{\kappa}} = \frac{430 \text{ nm}}{\sqrt{1.45}} = 357 \text{ nm}$$

30.45. If the energy of the radiation is kT, and the energy is proportional to the frequency, we have

$$\frac{kT}{f} = \frac{U_1}{f_1}$$

$$f = \frac{kTf_1}{U_1} = \frac{(1.38\times10^{-23} \text{ J/K})(3.0 \text{ K})(1.0\times10^{6} \text{ Hz})}{6.6\times10^{-28} \text{ J}} = 6.3\times10^{10} \text{ Hz}$$

This frequency is in the microwave part of the spectrum, so this radiation is also called the cosmic microwave background.

30.46. The amplitude of the wave decreases as $1/r$ it travels outward from the antenna, and we have a relationship between the magnitude of the electric and magnetic fields in Eq. 30.24. Combining these we get

$$B_{500} = \frac{E_{500}}{c_0} = \frac{E_{0.1}}{c_0}\frac{r_{0.1}}{r_{500}} = \frac{(6.0\times10^{5} \text{ V/m})(0.100 \text{ m})}{(3.0\times10^{8} \text{ m/s})(500 \text{ m})} = 4.0\times10^{-7} \text{ T}$$

30.47. In *Principles* Example 30.9, we found a relationship among the average energy density of an electromagnetic wave, the average Poynting vector, and the root-mean-square electric field magnitude, $u_\mathrm{av} = \dfrac{S_\mathrm{av}}{c_0} = \epsilon_0 E_\mathrm{rms}^2$. Assuming the wave is sinusoidal, we also have a relationship among the root-mean-square electric field magnitude and its maximum magnitude, $E_\mathrm{rms}^2 = (E^2)_\mathrm{av} = \frac{1}{2}E_\mathrm{max}^2$. Combining these we have

$$\frac{S_\mathrm{av}}{c_0} = \tfrac{1}{2}\epsilon_0 E_\mathrm{max}^2$$

$$E_\mathrm{max} = \sqrt{\frac{2S_\mathrm{av}}{\epsilon_0 c_0}} = \sqrt{\frac{2(8.00\times10^{-7} \text{ W/m}^2)}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^{8} \text{ m/s})}} = 2.45\times10^{-2} \text{ V/m}$$

30.48. The root-mean-square electric and magnetic fields are related in the same way as the instantaneous values (Eq. 30.24), so

$$E_\mathrm{rms} = c_0 B_\mathrm{rms} = (3.0\times10^{8} \text{ m/s})(1.5\times10^{-6} \text{ T}) = 4.5\times10^{2} \text{ V/m}$$

and the average intensity is (Eq. 30.40)

$$S_\mathrm{av} = \frac{1}{\mu_0}E_\mathrm{rms}B_\mathrm{rms} = \frac{(4.5\times10^{2} \text{ V/m})(1.5\times10^{-6} \text{ T})}{4\pi\times10^{-7} \text{ T}\cdot\text{m/A}} = 5.4\times10^{2} \text{ W/m}^2$$

30.49. The intensity of radiation from a source that spreads out in the usual way decreases with distance from the source by a factor of $1/r^2$. The intensity of the laser beam decreases much less with distance.

30.50. The intensity of a wave is the energy it delivers per unit time per unit area normal to the direction of propagation. So,

$$S = \frac{P}{A} = \frac{P}{\pi R^2} = \frac{1.0\times10^{-3} \text{ W}}{\pi(0.6\times10^{-3} \text{ m})^2} = 9\times10^{2} \text{ W/m}^2$$

30.51. The relationship between average intensity and average energy density is the same as between the instantaneous quantities (Eq. 30.55), so

$$u_\mathrm{av} = \frac{S_\mathrm{av}}{c_0} = \frac{2.0\times10^{20} \text{ W/mm}^2}{3.0\times10^{8} \text{ m/s}}\left(\frac{1000 \text{ mm}}{1 \text{ m}}\right)^2 = 6.7\times10^{17} \text{ J/m}^3$$

The energy delivered by one pulse to a target equals the product of the intensity, the pulse duration, and the target area,

$$U = S\Delta t A = (2.0\times10^{20}\ \text{W/mm}^2)\left(\frac{1000\ \text{mm}}{1\ \text{m}}\right)^2(30\times10^{-15}\ \text{s})\times(1.0\ \mu\text{m}^2)\left(\frac{1\ \text{m}}{10^6\ \mu\text{m}}\right)^2 = 6.0\ \text{J}$$

30.52. (*a*) In *Principles* Example 30.9, we found a relationship among the average energy density of an electromagnetic wave, the average Poynting vector, and the root-mean-square electric field magnitude, $u_{\text{av}} = \dfrac{S_{\text{av}}}{c_0} = \epsilon_0 E_{\text{rms}}^2$. Assuming the wave is sinusoidal, we also have a relationship among the root-mean-square electric field magnitude and its maximum magnitude, $E_{\text{rms}}^2 = (E^2)_{\text{av}} = \frac{1}{2}E_{\text{max}}^2$. Combining these we have

$$\frac{S_{\text{av}}}{c_0} = \tfrac{1}{2}\epsilon_0 E_{\text{max}}^2$$

$$E_{\text{max}} = \sqrt{\frac{2S_{\text{av}}}{\epsilon_0 c_0}} = \sqrt{\frac{2(10\times10^{-6}\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\ \text{m/s})}} = 8.7\times10^{-2}\ \text{V/m}$$

and (Eq. 30.24)

$$B_{\text{max}} = \frac{E_{\text{max}}}{c_0} = \frac{8.7\times10^{-2}\ \text{V/m}}{3.0\times10^8\ \text{m/s}} = 2.9\times10^{-10}\ \text{T}$$

(*b*) If the antenna emits radiation equally in all directions, the power it emits equals the product of the intensity at a given distance times the area of a spherical surface with that radius, $P_{\text{av}} = S_{\text{av}}A = S_{\text{av}}(4\pi R^2) = 4\pi(8.0\times10^3\ \text{m})^2(10\times10^{-6}\ \text{W/m}^2) = 8.0\times10^3\ \text{W}$.

30.53. In *Principles* Example 30.10 we used two methods to find an expression for the power delivered to a capacitor by a steady current, first by determining the magnitude of the Poynting vector at the boundary of the capacitor and integrating that over the surface, and then by determining the rate of change of the electric potential energy stored in the capacitor. In both cases we found $P = \dfrac{I^2 d\Delta t}{\epsilon_0 \pi R^2}$, which we can solve for Δt,

$$\Delta t = \frac{\epsilon_0 \pi R^2 P}{I^2 d} = \frac{\pi(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.075\ \text{m})^2(1.0\times10^6\ \text{W})}{(0.20\ \text{A})^2(0.200\times10^{-3}\ \text{m})} = 2.0\times10^{-2}\ \text{s}$$

30.54. (*a*) Assuming the light bulb radiates uniformly in all directions, the intensity at a given distance equals the power radiated by the bulb divided by the surface area of a sphere of that radius,

$$S_{\text{av}} = \frac{P_{\text{av}}}{A} = \frac{P_{\text{av}}}{4\pi R^2} = \frac{60\ \text{W}}{4\pi(2.0\ \text{m})^2} = 1.2\ \text{W/m}^2$$

(*b*) In *Principles* Example 30.9, we found a relationship among the average energy density of an electromagnetic wave, the average Poynting vector, and the root-mean-square electric field magnitude, $u_{\text{av}} = \dfrac{S_{\text{av}}}{c_0} = \epsilon_0 E_{\text{rms}}^2$. Assuming the wave is sinusoidal, we also have a relationship among the root-mean-square electric field magnitude and its maximum magnitude, $E_{\text{rms}}^2 = (E^2)_{\text{av}} = \frac{1}{2}E_{\text{max}}^2$. Combining these we have

$$\frac{S_{\text{av}}}{c_0} = \tfrac{1}{2}\epsilon_0 E_{\text{max}}^2$$

$$E_{\text{max}} = \sqrt{\frac{2S_{\text{av}}}{\epsilon_0 c_0}} = \sqrt{\frac{2(1.2\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\ \text{m/s})}} = 30\ \text{V/m}$$

(*c*) The magnetic field in an electromagnetic wave is proportional to the electric field (Eq. 30.24), so $B_{\text{max}} = \dfrac{E_{\text{max}}}{c_0} = \dfrac{30\ \text{V/m}}{3.0\times10^8\ \text{m/s}} = 1.0\times10^{-7}\ \text{T}$.

30.55. If the antenna radiates equally in all directions, then the intensity of the radiation at a given distance from the antenna equals the radiated power divided by the area of a spherical surface of that radius, $S_{\text{av}} = \dfrac{P_{\text{av}}}{A} = \dfrac{P_{\text{av}}}{4\pi R^2}$. In *Principles* Example 30.9, we found a relationship among the average energy density of an electromagnetic wave, the average Poynting vector, and the root-mean-square electric field magnitude, $u_{\text{av}} = \dfrac{S_{\text{av}}}{c_0} = \epsilon_0 E_{\text{rms}}^2$. Assuming the wave is sinusoidal, we also have a relationship between the root-mean-square electric field magnitude and its maximum magnitude, $E_{\text{rms}}^2 = (E^2)_{\text{av}} = \frac{1}{2}E_{\text{max}}^2$. Combining these we have

$$\frac{S_{\text{av}}}{c_0} = \tfrac{1}{2}\epsilon_0 E_{\text{max}}^2$$

$$\frac{P_{\text{av}}}{4\pi R^2 c_0} = \tfrac{1}{2}\epsilon_0 E_{\text{max}}^2$$

$$E_{\text{max}} = \sqrt{\frac{2P_{\text{av}}}{4\pi R^2 c_0 \epsilon_0}}$$

and (Eq. 30.24), $B_{\text{max}} = \dfrac{E_{\text{max}}}{c_0}$. (*a*) At a distance of 100 m from the antenna, we have

$$E_{\text{max}} = \sqrt{\frac{2P_{\text{av}}}{4\pi R^2 c_0 \epsilon_0}} = \sqrt{\frac{2(100\times10^3\ \text{W})}{4\pi(100\ \text{m})^2(3.00\times10^8\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}} = 24.5\ \text{V/m}$$

and

$$B_{\text{max}} = \frac{24.5\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 8.16\times10^{-8}\ \text{T}$$

(*b*) At a distance of 50 km from the antenna, we have

$$E_{\text{max}} = \sqrt{\frac{2P_{\text{av}}}{4\pi R^2 c_0 \epsilon_0}} = \sqrt{\frac{2(100\times10^3\ \text{W})}{4\pi(50\times10^3\ \text{m})^2(3.00\times10^8\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}} = 4.9\times10^{-2}\ \text{V/m}$$

and

$$B_{\text{max}} = \frac{4.9\times10^{-2}\ \text{V/m}}{3.0\times10^8\ \text{m/s}} = 1.6\times10^{-10}\ \text{T}$$

(*c*) If the receiving antenna is aligned with the electric field of the radio wave, then the maximum potential difference between its ends equals the product of the maximum electric field magnitude times the length of the antenna. So, for an antenna that is 1.0 m long, the maximum potential difference is 24 V at 100 m and 0.049 V at 50 km.

30.56. From the maximum value of the magnetic field, we can find the maximum value of the electric field (Eq. 30.24) and the intensity (Eq. 30.36). The power delivered is the product of the intensity and the beam area, so we have

$$P_{\text{max}} = S_{\text{max}}A = \left(\frac{1}{\mu_0}E_{\text{max}}B_{\text{max}}\right)(\pi R^2) = \frac{\pi R^2}{\mu_0}(c_0 B_{\text{max}})B_{\text{max}} = \frac{\pi R^2 c_0 B_{\text{max}}^2}{\mu_0}$$

$$= \frac{\pi(1.5\times10^{-3}\ \text{m})^2(3.0\times10^8\ \text{m/s})(5.0\times10^{-6}\ \text{T})^2}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = 4.2\times10^{-2}\ \text{W}$$

This is the maximum power. For a sinusoidal wave, the average power is one-half the maximum power, or 21 mW.

30.57. The intensity of a wave is the energy it delivers per unit time per unit area normal to the direction of propagation. Because the solar panel is inclined with respect to the sunlight, the intensity of the radiation incident on it is reduced by a factor of $\cos\theta$. The amount of power that can be harvested is the product of the incident intensity, the area of the panel, and the panel's efficiency,

$$P = S\cos\theta A\eta = (1.0\times10^3\ \text{W/m}^2)\cos(20°)(2.0\ \text{m})(1.6\ \text{m})(0.20) = 6.0\times10^2\ \text{W}$$

30.58. (*a*) In order to raise the water from room temperature (about $25\ °\text{C}$) to its boiling point ($100\ °\text{C}$) we must add an amount of thermal energy $Q = mc_V\Delta T$ (Eq. 20.49), where m is the mass of the water, c_V is its specific heat capacity (the value of which can be found in *Principles* Table 20.2), and ΔT is the change in its temperature. In order to accomplish this in a given time interval Δt, the average power required is

$$\begin{aligned} P_{\text{av}} &= \frac{Q}{\Delta t} = \frac{mc_V\Delta T}{\Delta t} = \frac{(\rho V)c_V\Delta T}{\Delta t} \\ &= \frac{(1.0\ \text{kg/L})(0.20\ \text{L})(4181\ \text{J/K}\cdot\text{kg})(75\ \text{K})}{(3.00\ \text{min})(60\ \text{s/1 min})} = 3.5\times10^{2}\ \text{W} \end{aligned}$$

(*b*) The average intensity of the microwave radiation equals the average power delivered divided by the area of the surface through which it is delivered,

$$S_{\text{av}} = \frac{P_{\text{av}}}{A} = \frac{P_{\text{av}}}{\pi R^2} = \frac{348\ \text{W}}{\pi(0.030\ \text{m})^2} = 1.2\times10^{5}\ \text{W/m}^2$$

(*c*) In *Principles* Example 30.9, we found a relationship among the average energy density of an electromagnetic wave, the average Poynting vector, and the root-mean-square electric field magnitude, $u_{\text{av}} = \dfrac{S_{\text{av}}}{c_0} = \epsilon_0 E_{\text{rms}}^2$, so

$$E_{\text{rms}} = \sqrt{\frac{S_{\text{av}}}{\epsilon_0 c_0}} = \sqrt{\frac{1.23\times10^{5}\ \text{W/m}^2}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^{8}\ \text{m/s})}} = 6.8\times10^{3}\ \text{V/m}$$

30.59. The simplest assumptions are that the light bulb radiates equally in all directions, so the intensity at a given distance equals the power radiated divided by the surface area of a sphere with that radius, and that the light is not affected by the intervening atmosphere. With these assumptions we have

$$S_{\text{min}} = \frac{P}{4\pi R^2}$$

$$R = \sqrt{\frac{P}{4\pi S_{\text{min}}}} = \sqrt{\frac{10\ \text{W}}{4\pi(10^{-12}\ \text{W/m}^2)}} = 9\times10^{5}\ \text{m}$$

This result, about 900 km, does not seem reasonable. Most likely, the air it passes through has some effect on the light, for example, absorbing it or scattering it (changing its direction).

30.60. If you double the distance from a light bulb, you reduce the intensity of the light reaching your eye by a factor of four, because the power radiated from the light bulb is now spread over four times the area. Because perceived brightness is a logarithmic relationship, the perceived brightness is reduced by a smaller factor. To see this, note that we are told that doubling the intensity of light does not double the perceived brightness, so doubling it again makes the perceived brightness less than four times what it was originally. Reversing the process, the perceived brightness is reduced by a smaller factor than the intensity is.

30.61. (*a*) The Poynting vector is parallel to the wave's propagation direction, so the wave is moving in the positive z direction. (*b*) The average of the sine squared function is one half, so the average intensity is $50.0\ \text{W/m}^2$, and the average power transmitted through $1.0\ \text{m}^2$ is 50 W. (*c*) The propagation direction of an electromagnetic wave is the same as the direction of the vector product $\vec{E}\times\vec{B}$, so if the electric field is in the positive x direction, the magnetic field must be in the positive y direction. (*d*) The constants in the equation multiplying z and t are the wave number and angular frequency, respectively, so the wavelength and frequency of the electromagnetic wave are

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1000\ \text{m}^{-1}} = 6.3\times10^{-3}\ \text{m}$$

and

$$f = \frac{\omega}{2\pi} = \frac{3.0\times10^{11}\ \text{s}^{-1}}{2\pi} = 4.8\times10^{10}\ \text{Hz}$$

(*e*) We don't have enough information to uniquely determine the electric and magnetic field vectors. We can calculate their amplitudes from Eqs. 30.35, 30.31, and 30.24,

$$\frac{S_{\max}}{c_0} = \epsilon_0 E_{\max}^2$$

$$E_{\max} = \sqrt{\frac{S_{\max}}{\epsilon_0 c_0}} = \sqrt{\frac{100\ \mathrm{W/m^2}}{(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(3.0\times10^{8}\ \mathrm{m/s})}} = 194\ \mathrm{V/m}$$

and

$$B_{\max} = \frac{E_{\max}}{c_0} = \frac{194\ \mathrm{V/m}}{3.0\times10^{8}\ \mathrm{m/s}} = 6.47\times10^{-7}\ \mathrm{T}$$

In part (*c*) we were told that at some instant the electric field is in the *x* direction. If we assume the radiation is linearly polarized, one possible set of equations is

$$\vec{E} = (194\ \mathrm{V/m})\sin\left[(1000\ \mathrm{m^{-1}})z - (3.0\times10^{11}\ \mathrm{s^{-1}})t\right]\hat{\imath}$$

$$\vec{B} = (6.47\times10^{-7}\ \mathrm{T})\sin\left[(1000\ \mathrm{m^{-1}})z - (3.0\times10^{11}\ \mathrm{s^{-1}})t\right]\hat{\jmath}$$

Another would be to add an initial phase of π to both equations.

30.62. (*a*) In a long solenoid, the magnetic field is uniform and directed parallel to the solenoid's axis, and it is zero outside the solenoid. Because the magnitude of the magnetic field is proportional to the current and the current is increasing, there is an accompanying electric field, the lines of which form circular loops centered on and perpendicular to the axis of the solenoid. By Lenz's law, the electric field is directed opposite to the increasing current. The Poynting vector is perpendicular to both the electric and magnetic fields, in the direction $\vec{E}\times\vec{B}$, so it is directed radially toward the solenoid's axis. (*b*) The magnitude of the Poynting vector is proportional to the magnitudes of the electric and magnetic fields (Eq. 30.36) $S = \frac{1}{\mu_0}EB$. The magnitude of the magnetic field in the solenoid is (Eq. 28.6) $B = \mu_0 nI = \frac{\mu_0 NI}{h}$, and the rate of change of magnetic flux through a flat, circular surface of radius *R* centered on and perpendicular to the axis of the solenoid is

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}AB = \frac{\pi R^2\mu_0 N}{h}\frac{dI}{dt} = \frac{\pi R^2\mu_0 N\alpha}{h}$$

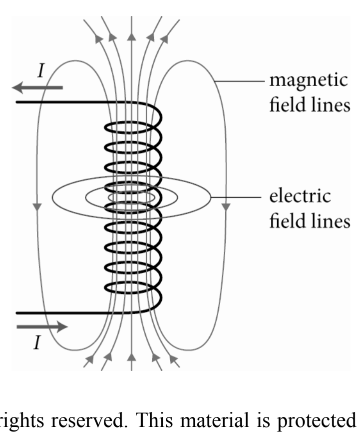
So, Faraday's law (Eq. 29.17) gives

$$2\pi RE = \left|\frac{d\Phi_B}{dt}\right| = \frac{\pi R^2\mu_0 N\alpha}{h}$$

$$E = \frac{\mu_0 RN\alpha}{2h}$$

and the magnitude of the Poynting vector is

$$S = \frac{1}{\mu_0}EB = \frac{1}{\mu_0}\frac{\mu_0 RN\alpha}{2h}\frac{\mu_0 NI}{h} = \frac{\mu_0 RN^2\alpha^2 t}{2h^2}$$



(c) The power input to the solenoid is the integral of the Poynting vector over the surface of the solenoid. Because the Poynting vector is uniform and directed radially, the integral equals the product of the Poynting vector and the surface area of the solenoid,

$$P = SA = \frac{\mu_0 R N^2 \alpha^2 t}{2h^2}(2\pi R h) = \frac{\pi \mu_0 R^2 N^2 \alpha^2 t}{h}$$

(d) In *Principles* Example 29.8, we calculated the inductance of a solenoid. Expressed in terms of the variables we have been using in this problem, the inductance is $L = \dfrac{\pi \mu_0 N^2 R^2}{h}$. The magnetic energy stored in an inductor is (Eq. 29.25) $U^B = \frac{1}{2} L I^2$, and its rate of change is

$$\frac{dU^B}{dt} = LI\frac{dI}{dt} = \frac{\pi \mu_0 N^2 R^2}{h}(\alpha t)\alpha = \frac{\pi \mu_0 N^2 R^2 \alpha^2 t}{h}$$

in agreement with our earlier result.

30.63. We are free to express the electric field in terms of any coordinate system we like. Let's choose one in which the x axis is parallel to polarizing axis of the filter, the z axis is perpendicular to the filter, and the y axis is oriented so that $\hat{i} \times \hat{j} = \hat{k}$. (In terms of Figure P30.63b, the x axis points in the direction of the vector labeled E_{after}, the z axis corresponds to the black arrow, and the y axis is directed upward and to the left.) After the light passes through the filter, its electric field has only an x component, $\vec{E}_{\text{after}} = E_{\text{after}\,x}\,\hat{i}$, which is equal to the x component of the electric field of the incoming wave, $E_{\text{after}\,x} = E_{\text{before}\,x}$.

Expressing the electric field of the incoming wave in terms of its components in this coordinate system we have $\vec{E}_{\text{before}} = E_{\text{before}\,x}\,\hat{i} + E_{\text{before}\,y}\,\hat{j} = E_{\text{before}}\cos(\theta)\hat{i} + E_{\text{before}}\sin(\theta)\,\hat{j}$, so $E_{\text{after}} = E_{\text{before}}\cos\theta$. The intensity of an electromagnetic wave is proportional to the square of the electric field magnitude (Eqs. 30.30 and 30.35), so

$$\frac{S_{\text{after}}}{E^2_{\text{after}}} = \frac{S_{\text{before}}}{E^2_{\text{before}}}$$

$$S_{\text{after}} = S_{\text{before}}\frac{E^2_{\text{after}}}{E^2_{\text{before}}} = S_{\text{before}}\frac{E^2_{\text{before}}\cos^2\theta}{E^2_{\text{before}}} = S_{\text{before}}\cos^2\theta = (200\ \text{W/m}^2)\cos^2(20.0°) = 177\ \text{W/m}^2$$

30.64. As we saw in *Principles* Example 30.8, the speed of an electromagnetic wave pulse in a dielectric medium is reduced from its speed in vacuum by a factor of the square root of the dielectric constant of the medium. So, in liquid argon, the speed of an electromagnetic wave pulse is

$$c = \frac{c_0}{\sqrt{\kappa}} = \frac{3.0\times10^8\ \text{m/s}}{\sqrt{1.5}} = 2.4\times10^8\ \text{m/s}$$

30.65. A half-wave antenna should be half as long as the wavelength of the electromagnetic radiation it emits. The wavelength is related to the frequency by Eq. 16.10, $\lambda f = c$, so we have

$$\ell = \tfrac{1}{2}\lambda = \tfrac{1}{2}\frac{c_0}{f} = \frac{3.0\times10^8\ \text{m/s}}{2(90.5\times10^6\ \text{Hz})} = 1.66\ \text{m}$$

30.66. Yes, it does. Wavelength is related to frequency by Eq. 16.10, $\lambda f = c$, so the frequency of radiation with wavelength 1.0 mm is

$$f = \frac{c_0}{\lambda} = \frac{3.0\times10^8\ \text{m/s}}{1.0\times10^{-3}\ \text{m}} = 3.0\times10^{11}\ \text{Hz} = 0.30\ \text{THz}$$

and the wavelength of radiation with frequency 1.0 THz is

$$\lambda = \frac{c_0}{f} = \frac{3.0\times10^8\ \text{m/s}}{1.0\times10^{12}\ \text{Hz}} = 3.0\times10^{-4}\ \text{m} = 0.30\ \text{mm}$$

30.67. A half-wave antenna should be half as long as the wavelength of the electromagnetic radiation it emits, and wavelength is related to frequency by Eq. 16.10, $\lambda f = c$, so we have for the home microwave

$$\lambda = \frac{c_0}{f} = \frac{3.0\times10^8 \text{ m/s}}{2.45\times10^9 \text{ Hz}} = 0.122 \text{ m}$$

and $\ell = \frac{1}{2}\lambda = \frac{1}{2}(0.1224 \text{ m}) = 0.0612 \text{ m}$. For the restaurant microwave, we have $\lambda = \dfrac{3.0\times10^8 \text{ m/s}}{915\times10^6 \text{ Hz}} = 0.328 \text{ m}$ and $\ell = \frac{1}{2}(0.328 \text{ m}) = 0.164 \text{ m}$.

30.68. The square of a sinusoidally oscillating function averages to one-half the square of the function's amplitude, so for a sinusoidal electromagnetic wave we have

$$E_{\text{rms}}^2 = \tfrac{1}{2}E_{\max}^2$$
$$E_{\text{rms}} = \frac{\sqrt{2}}{2}E_{\max} = \frac{\sqrt{2}}{2}(500 \text{ N/C}) = 354 \text{ N/C}$$

30.69. Combining Eqs. 30.30 and 30.35 gives us a relation between the intensity and electric field magnitude of an electromagnetic wave, $\dfrac{S}{c_0} = \epsilon_0 E^2$. So in order to ionize air, the intensity of the laser light must be at least

$$S = \epsilon_0 c_0 E_{\min}^2 = (8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8 \text{ m/s})(1.0\times10^6 \text{ N/C})^2 = 2.7\times10^9 \text{ W/m}^2$$

But this is the smallest value of the maximum instantaneous intensity sufficient to ionize air; the smallest value of the average intensity required is one-half this intensity, $1.3\times10^9 \text{ W/m}^2$.

30.70. (*a*) The same relationship holds between the root-mean-square electric and magnetic fields as between their instantaneous values, Eq. 30.24, so

$$E_{\text{rms}} = c_0 B_{\text{rms}} = (3.0\times10^8 \text{ m/s})(0.30\times10^{-6} \text{ T}) = 90 \text{ V/m}$$

(*b*) The relationship between the average energy density and the root-mean-square electric field has the same form as Eq. 30.30,

$$u_{\text{av}} = \epsilon_0 E_{\text{rms}}^2 = (8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(90 \text{ N/C})^2 = 7.2\times10^{-8} \text{ J/m}^3$$

(*c*) The relationship between the average intensity and the average energy density has the same form as Eq. 30.35, so $S_{\text{av}} = u_{\text{av}}c_0 = (7.17\times10^{-8} \text{ J/m}^3)(3.0\times10^8 \text{ m/s}) = 22 \text{ W/m}^2$.

30.71. The laser delivers 500 W to a spot of diameter 0.17 mm, so the average intensity, that is, the average magnitude of the Poynting vector, is

$$S_{\text{av}} = \frac{P_{\text{av}}}{A} = \frac{P_{\text{av}}}{\pi(d/2)^2} = \frac{500 \text{ W}}{\pi(\frac{1}{2}0.17\times10^{-3} \text{ m})^2} = 2.2\times10^{10} \text{ W/m}^2$$

Combining Eqs. 30.31 and 30.35, which apply to average as well as instantaneous values, we have

$$\frac{B_{\text{rms}}^2}{\mu_0} = \frac{S_{\text{av}}}{c_0}$$
$$B_{\text{rms}} = \sqrt{\frac{\mu_0 S_{\text{av}}}{c_0}} = \sqrt{\frac{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(2.2\times10^{10} \text{ W/m}^2)}{3.0\times10^8 \text{ m/s}}} = 9.6\times10^{-3} \text{ T}$$

and (Eq. 30.24) $E_{\text{rms}} = c_0 B_{\text{rms}} = (3.0\times10^8 \text{ m/s})(9.6\times10^{-3} \text{ T}) = 2.9\times10^6 \text{ V/m}$.

30.72. If the signal is emitted equally in all directions, then at any distance from the antenna the emitted power is distributed over the surface area of a sphere of that radius, so

$$S = \frac{P}{A} = \frac{P}{4\pi R^2} = \frac{20\times10^3 \text{ W}}{4\pi(10\times10^3 \text{ m})^2} = 1.6\times10^{-5} \text{ W/m}^2$$

30.73. Assuming the Sun radiates equally in all directions, at any distance from the Sun its power is evenly distributed over the surface of a sphere of that radius. The travel time of light is proportional to the distance it travels, and the surface area of a sphere is proportional to the square of its radius, so we have

$$S_{\text{E}}(\Delta t_{\text{E}})^2 = S_{\text{J}}(\Delta t_{\text{J}})^2$$

$$S_{\text{J}} = S_{\text{E}}\left(\frac{\Delta t_{\text{E}}}{\Delta t_{\text{J}}}\right)^2 = (1400\ \text{W/m}^2)\left(\frac{8\ \text{min}}{44\ \text{min}}\right)^2 = 5\times10^1\ \text{W/m}^2$$

30.74. (*a*) Combining Eqs. 30.30 and 30.35 gives us a relation between the intensity and electric field magnitude of an electromagnetic wave, $\dfrac{S}{c_0} = \epsilon_0 E^2$. This equation holds for either instantaneous or average values. Assuming the waves are sinusoidal, we have $E_{\text{rms}}^2 = \frac{1}{2}E_{\text{max}}^2$, and so

$$\frac{S_{\text{av}}}{c_0} = \epsilon_0\left(\tfrac{1}{2}E_{\text{max}}^2\right)$$

$$E_{\text{max}} = \sqrt{\frac{2S_{\text{av}}}{\epsilon_0 c_0}} = \sqrt{\frac{2(1.35\times10^3\ \text{W/m}^2)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.0\times10^8\ \text{m/s})}} = 1.01\times10^3\ \text{V/m}$$

and (Eq. 30.24)

$$B_{\text{max}} = \frac{E_{\text{max}}}{c_0} = \frac{1.01\times10^3\ \text{V/m}}{3.0\times10^8\ \text{m/s}} = 3.36\times10^{-6}\ \text{T}$$

(*b*) Assuming the Sun radiates equally in all directions, at any distance from the Sun its power is evenly distributed over the surface of a sphere of that radius, so

$$P = SA = S(4\pi R^2) = 4\pi(1.35\times10^3\ \text{W/m}^2)(1.5\times10^{11}\ \text{m})^2 = 3.8\times10^{26}\ \text{W}$$

(*c*) The power that would reach Earth is the product of the sunlight's intensity and the cross-sectional area of Earth,

$$P = SA = S(\pi R^2) = \pi(1.35\times10^3\ \text{W/m}^2)(6.37\times10^6\ \text{m})^2 = 1.72\times10^{17}\ \text{W}$$

30.75. Combining Eqs. 30.31 and 30.35 gives us a relation between the intensity and the magnetic field, $\dfrac{S}{c_0} = \dfrac{B^2}{\mu_0}$.

For the cell phone we have

$$B_{\text{rms}} = \sqrt{\frac{\mu_0 S_{\text{av}}}{c_0}} = \sqrt{\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(35\ \text{W/m}^2)}{3.0\times10^8\ \text{m/s}}} = 3.8\times10^{-7}\ \text{T}$$

and for the microwave oven

$$B_{\text{rms}} = \sqrt{\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(10\ \text{W/m}^2)}{3.0\times10^8\ \text{m/s}}} = 2.0\times10^{-7}\ \text{T}$$

For the Bluetooth headset, the magnetic field is

$$B_{\text{rms}} = \sqrt{\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.080\ \text{W/m}^2)}{3.0\times10^8\ \text{m/s}}} = 1.8\times10^{-8}\ \text{T}$$

When the cell phone is near the waist, it is about 0.75 m from the head, so the intensity of the radiation is reduced by the square of the ratio of the distances,

$$S_{\text{waist}} = S_{\text{head}}\left(\frac{r_{\text{head}}}{r_{\text{waist}}}\right)^2 = (35\ \text{W/m}^2)\left(\frac{0.050\ \text{m}}{0.75\ \text{m}}\right)^2 = 0.16\ \text{W/m}^2$$

and the magnetic field is

$$B_{\text{rms}} = \sqrt{\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(0.16\ \text{W/m}^2)}{3.0\times10^8\ \text{m/s}}} = 2.6\times10^{-8}\ \text{T}$$

These values are greater than those of the Bluetooth headset, but the sum of the intensities or magnetic fields of the Bluetooth headset and the cell phone at the waist is still much less than the values of the cell phone at the head.

The wavelength of the cell-phone radiation is $\lambda = \dfrac{c_0}{f} = \dfrac{2.998\times10^8 \text{ m/s}}{824.6\times10^6 \text{ Hz}} = 0.3636 \text{ m},$ which is significantly larger than a person's head, although half a wavelength is not a lot larger than a person's head. Considering that the wavelength is reduced in a dielectric medium, it could be possible to form standing waves (see *Principles* Sections 16.6) in a person's head if the radio waves were reflected from the air-head interface.

30.76. If the mesh provides 50 dB attenuation, then the incident and transmitted intensities are related by

$$50 = 10\log\left(\frac{S_\text{i}}{S_\text{t}}\right)$$

$$5 = \log\left(\frac{S_\text{i}}{S_\text{t}}\right)$$

$$10^5 = \frac{S_\text{i}}{S_\text{t}}$$

$$S_\text{t} = \frac{S_\text{i}}{10^5}$$

Because the intensity is proportional to the square of the field strength, the field strengths are reduced by a factor of the square root of 10^5, or about 300.

The wavelengths of the frequencies mentioned in the advertisement are

$$\lambda = \frac{c_0}{f} = \frac{3.0\times10^8 \text{ m/s}}{10\times10^6 \text{ Hz}} = 30 \text{ m}$$

and

$$\lambda = \frac{3.0\times10^8 \text{ m/s}}{3\times10^9 \text{ Hz}} = 0.1 \text{ m}$$

so the holes in the mesh must not be larger than about 0.1 m if it is to be effective at the higher frequency.

An ordinary window screen has holes on the order of a millimeter squared, that is, about 100 times smaller, so if a window screen were made of conducting material, it would block radiation of frequency up to about 300 GHz, although the attenuation might not be as great as with this product. We might compare this to the screen on the window of a microwave oven. The holes are on the order of a millimeter in diameter, but the screen appears to be made of much heavier material than an ordinary window screen, which might be necessary to achieve the required attenuation.

32

ELECTRONICS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

Developing a Feel

1. 10^{-6} m **2.** 10^{3} Ω **3.** 10^{-3} Ω **4.** 10^{7} Ω **5.** 10^{-5} F **6.** 10^{-2} H **7.** 10^{0} H **8.** 10^{1} W **9.** 10^{2} W **10.** 10^{0} H

Guided Problems

32.2 *RLC* circuit

1. Getting Started When switch 1 is closed, current can flow from the battery to the capacitor around the left-hand loop of the circuit. No current can flow in the right-most loop of the circuit as long as switch 2 is open. As current flows to the capacitor the charge on the capacitor plates increases and the potential difference across the plates also increases. As charge builds up on the capacitor, the current decreases. After a long time, the capacitor will become fully charged, at which point the potential difference across the capacitor plates is equal to the electromotive force of the battery.

When switch 1 is opened, no more current can flow from the battery. This means that the total energy in the circuit will be fixed from this point onward (because current will only flow now in regions of the loop where there is no resistance). When switch 2 is opened, charge will flow off of the capacitor. Note that the battery is no longer holding the charge there. The charge will flow around the right-most loop in the circuit, though the inductor. As charge leaves the capacitor, the current decreases.

2. Devise Plan The electric potential energy stored in a capacitor is given by

$$U^{C} = \frac{1}{2}CV_{\text{cap}}^{2} \tag{1}$$

It is very important to realize that V_{cap} is the potential difference across the capacitor plates, which is not necessarily equal to the potential difference across the battery.

The magnetic potential energy stored in an inductor is given by

$$U^{L} = \frac{1}{2}LI^{2} \tag{2}$$

At the instant switch 1 is closed and switch 2 is open, the only energy in the system is that stored on the capacitor. At that instant the potential difference across the capacitor happens to be equal to the electromotive force of the battery, such that $U_{\text{i}} = \frac{1}{2}C\mathcal{E}^{2}$. We are asked to find the current when half of the energy is in the form of magnetic potential energy. Thus we need only set half the above expression for the total potential energy equal to the energy stored in the inductor, and solve for the current.

3. Execute Plan Equating half the total energy to the magnetic potential energy, we find

$$U^L = \frac{1}{2}U_{\text{i}}$$

$$\frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{1}{2}C\varepsilon^2\right)$$

$$I = \varepsilon\sqrt{\frac{C}{2L}}$$

$$I = (4.0\text{ V})\sqrt{\frac{(0.50\times10^{-9}\text{ F})}{2(8.0/\pi^2)\times10^{-3}\text{ H}}}$$

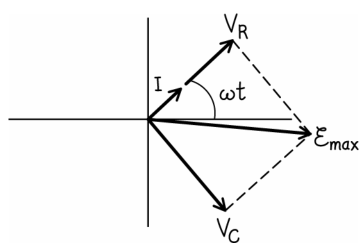
$$I = 2.2\text{ mA}$$

4. Evaluate Result The current we obtained is fairly easy to measure with a standard multi-meter.

If all the potential energy were magnetic potential energy, we would have set $U^L = U_{\text{i}}$ and we would have found $I_{\text{max}} = \varepsilon\sqrt{\frac{C}{L}}$, which is 3.1 mA. This is larger by a factor of $\sqrt{2}$, which is reasonable.

32.4 Four resistors

1. Getting Started We begin by drawing a phasor diagram showing the current, the potential differences across the resistor and capacitor and the maximum electromotive force.



2. Devise Plan Using equation (32.32) we can write the current amplitude as

$$I = \frac{\varepsilon_{\text{max}}}{\sqrt{R^2 + \chi_C^2}} = \frac{\varepsilon_{\text{max}}}{\sqrt{R^2 + (1/\omega C)^2}}$$

The phase difference is given by

$$\phi = \tan^{-1}\left(\frac{-1}{\omega RC}\right) \tag{1}$$

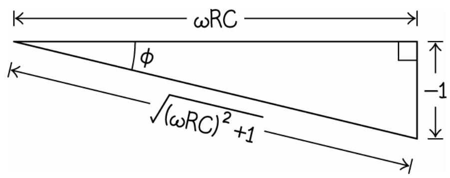
The time average of the power is given by equation (32.60)

$$P_{\text{av}} = \frac{1}{2}\varepsilon_{\text{max}} I\cos(\phi)$$

or

$$P_{\text{av}} = \frac{1}{2}\varepsilon_{\text{max}} I\cos\left(\tan^{-1}\left(\frac{-1}{\omega RC}\right)\right)$$

These nested trigonometric functions are difficult to deal with algebraically. But it is easy to simplify them if one pictures a triangle corresponding to equation (1) above:



It now becomes obvious that $\cos(\phi) = \dfrac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}$. Inserting this into equation (32.60), we have

$$P_{\text{av}} = \frac{1}{2}\mathcal{E}_{\max} I \left(\frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}} \right)$$

Also inserting equation (32.32) for the current amplitude, we have

$$P_{\text{av}} = \frac{1}{2}\mathcal{E}_{\max} \left(\frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (1/\omega C)^2}} \right) \left(\frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}} \right)$$

$$P_{\text{av}} = \frac{1}{2}\mathcal{E}_{\max}^2 \left(\frac{R}{R^2 + (1/\omega C)^2} \right) \tag{2}$$

We are told that the initial power at frequency ω_1 is twice that when the circuit is operated at frequency ω_1. Thus

$$\frac{P_{\text{av}}(\omega_1)}{P_{\text{av}}(\omega_2)} = 2 = \frac{\frac{1}{2}\mathcal{E}_{\max}^2 \left(\frac{R}{R^2 + (1/\omega_1 C)^2} \right)}{\frac{1}{2}\mathcal{E}_{\max}^2 \left(\frac{R}{R^2 + (1/\omega_2 C)^2} \right)}$$

$$2 = \frac{R^2 + (1/\omega_2 C)^2}{R^2 + (1/\omega_1 C)^2}$$

$$R^2 + 2/(\omega_1 C)^2 = (1/\omega_2 C)^2$$

$$\omega_2 = \frac{1}{\sqrt{(RC)^2 + 2/\omega_1^2}} \tag{3}$$

3. Execute Plan Inserting the numerical values given into equation (3), we have

$$\omega_2 = \frac{1}{\sqrt{((3.0\ \Omega)(5.0\times 10^{-2}\ \text{F}))^2 + \dfrac{2}{(100\ \text{s}^{-1})^2}}} = 6.6\ \text{s}^{-1}$$

4. Evaluate Result Since the power decreases upon changing ω_1 to ω_2, we might expect the current to decrease. Indeed that is the case. Inserting the initial and final frequencies into equation (32.32) we see

$$I_1 = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (1/\omega_1 C)^2}} = \frac{(10\ \text{V})}{\sqrt{(3.0\ \Omega)^2 + (1/(100\ \text{s}^{-1})(5.0\times 10^{-2}\ \text{F}))^2}} = 3.3\ \text{A}$$

$$I_2 = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (1/\omega_2 C)^2}} = \frac{(10\ \text{V})}{\sqrt{(3.0\ \Omega)^2 + (1/(6.63\ \text{s}^{-1})(5.0\times 10^{-2}\ \text{F}))^2}} = 2.4\ \text{A}$$

Thus we see the current amplitude decreased by a factor of $\dfrac{1}{\sqrt{2}}$ when the power decreased by a factor of $\dfrac{1}{2}$, which is perfectly reasonable. Equation (2) is a very reasonable expression for the power. Consider the limits when the frequency becomes very high. In that limit the term $(1/\omega C)^2$ is negligible compared to R^2, in which case the power equation would reduce to $P(\omega \to \infty) = \dfrac{\mathcal{E}_{\text{max}}^2}{2R}$ as it should. In the opposite limit, as the frequency becomes small, practically direct current, the capacitor will prevent current from flowing and the power will approach zero. This is also seen in equation (2). These limits would lead us to expect that reducing the power would correspond to a significant reduction in frequency. Thus our answer for ω_2 is consistent with our expectations.

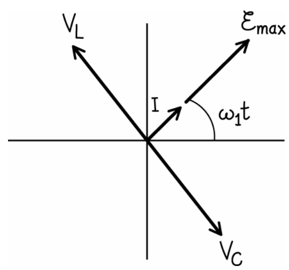
32.6 Unknown circuit element II

1. Getting Started If the square represented a capacitor, the current would lead the source emf. If the square represented an inductor, the current would lag behind the emf. The only possibility is that the square represents both an inductor and a capacitor.
Equation (32.48) relates the phase between the current and emf to the reactance:

$$\tan(\phi) = \frac{\omega L - \dfrac{1}{\omega C}}{R}$$

And we are told that the magnitude of the phase difference at ω_2 is $\pi/4$.

To help with our analysis we draw a phasor diagram for the circuit when operating at ω_1.



2. Devise Plan In general, the inductance and capacitance will be related to the phase difference according to equation (32.48). In this case we have two conditions. The first is

$$\tan(0) = \frac{\omega_1 L - \dfrac{1}{\omega_1 C}}{R} = 0 \Rightarrow \omega_1 = \frac{1}{\sqrt{LC}} \tag{1}$$

The second condition involves the phase difference at ω_2. Since $\omega_2 > \omega_1$, it is obvious from equation (32.48) that the phase difference at ω_2 will be positive. Thus

$$\tan(\pi/4) = \frac{\omega_2 L - \dfrac{1}{\omega_2 C}}{R} = 1 \Rightarrow L = \frac{R}{\omega_2} + \frac{1}{\omega_2^2 C} \tag{2}$$

Thus we have equations (1) and (2) relating our two unknowns. We can solve for L and C. Once those quantities are determined, the current amplitude can be found using equation (32.42):

$$I = \frac{\mathcal{E}_{\text{max}}}{\sqrt{R^2 + \left(\omega_1 L - \dfrac{1}{\omega_1 C}\right)^2}}$$

The power dissipated in the resistor is still given by $\frac{1}{2} I^2 R$, from (32.63). No power is dissipated in an inductor or in a capacitor.

3. **Execute Plan** Combining equations (1) and (2) we find

$$\frac{1}{C\omega_1^2} = \frac{R}{\omega_2} + \frac{1}{\omega_2^2 C}$$

$$\frac{1}{C}\left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right) = \frac{R}{\omega_2}$$

$$C = \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right)\frac{\omega_2}{R} = \left(\frac{1}{(1.0\ \text{s}^{-1})^2} - \frac{1}{(2.0\ \text{s}^{-1})^2}\right)\frac{(2.0\ \text{s}^{-1})}{(3.0\ \Omega)}$$

$$C = 0.50\ \text{F}$$

Inserting this back into equation (1), we find

$$L = \frac{1}{C\omega_1^2} = \frac{1}{(0.50\ \text{F})(1.0\ \text{s}^{-1})} = 2.0\ \text{H}$$

Using the values of the capacitance and the inductance in equation (32.42), we determine the current amplitude for each frequency:

$$I_1 = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(\omega_1 L - \dfrac{1}{\omega_1 C}\right)^2}} = \frac{(6.0\ \text{V})}{\sqrt{(3.0\ \Omega)^2 + \left((1.0\ \text{s}^{-1})(2.0\ \text{H}) - \dfrac{1}{(1.0\ \text{s}^{-1})(0.50\ \text{F})}\right)^2}} = 2.0\ \text{A}$$

$$I_2 = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(\omega_2 L - \dfrac{1}{\omega_2 C}\right)^2}} = \frac{(6.0\ \text{V})}{\sqrt{(3.0\ \Omega)^2 + \left((2.0\ \text{s}^{-1})(2.0\ \text{H}) - \dfrac{1}{(2.0\ \text{s}^{-1})(0.50\ \text{F})}\right)^2}} = 1.4\ \text{A}$$

Thus the power dissipated in each case is

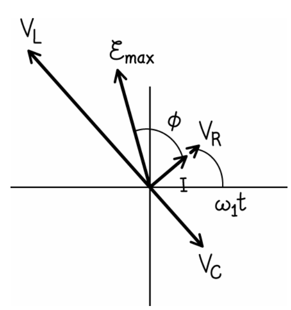
$$P_{\text{av},1} = \frac{1}{2}I_1^2 R = \frac{1}{2}(2.0\ \text{A})^2(3.0\ \Omega) = 6.0\ \text{W}$$

$$P_{\text{av},2} = \frac{1}{2}I_2^2 R = \frac{1}{2}(1.41\ \text{A})^2(3.0\ \Omega) = 3.0\ \text{W}$$

4. **Evaluate Result** The average power disspated at ω_1 was $6.0\ \text{W}$. Since the potential differences across the inductor and capacitor had to be equal in magnitude and opposite in sign, it was clear from the beginning that ω_1 corresponds to resonance. At resonance the impedance is just the resistance. Thus, one expects $P_{\text{av},1} = \frac{1}{2}\frac{\mathcal{E}_{\max}^2}{R} = \frac{1}{2}\frac{(6.0\ \text{V})^2}{(3.0\ \Omega)} = 6.0\ \text{W}$. So, yes, this is what we expected.

32.8 *RLC* circuit

1. **Getting Started** We begin by making a phasor diagram for the circuit.



2. Devise Plan In order to calculate the current amplitude using equation (32.42), we need only the frequency and numerical values associated with circuit elements such as capacitance, inductance, etc.

$$I = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

Equation (32.44) similarly gives the phase constant in terms of those same values.

$$\tan(\phi) = \frac{\omega L - \frac{1}{\omega C}}{R}$$

The amplitudes of the potential differences V_C and V_L are given by equations (32.15) and (32.27) respectively:

$$V_C = I\chi_C$$
$$V_L = I\chi_L$$

The reactances are simply $\chi_C = \frac{1}{\omega C}$ and $\chi_L = \omega L.$

3. Execute Plan The reactances are

$$\chi_C = \frac{1}{\omega C} = \frac{1}{(4000 \text{ s}^{-1})(2.0\times10^{-6} \text{ F})} = 125 \ \Omega$$
$$\chi_L = \omega L = (4000 \text{ s}^{-1})(0.400 \text{ H}) = 1.6 \text{ k}\Omega$$

The impedance is thus

$$Z = \sqrt{R^2 + (\chi_L - \chi_C)^2} = \sqrt{(10.0 \ \Omega)^2 + ((1600 \ \Omega) - (125 \ \Omega))^2} = 1.48 \text{ k}\Omega$$

Inserting this into equation (332.42), we find the current amplitude is

$$I = \frac{\mathcal{E}_{\max}}{Z} = \frac{(100 \text{ V})}{(1.48 \text{ k}\Omega)} = 68 \text{ mA}$$

We can find the phase constant using equation (32.44):

$$\phi = \tan^{-1}\left(\frac{\chi_L - \chi_C}{R}\right) = \tan^{-1}\left(\frac{(1600 \ \Omega) - (125 \ \Omega)}{(10.0 \ \Omega)}\right)$$
$$\phi = 1.6 \text{ rad}$$

The above value for ϕ is correct to the number of significant figures we have. However, physically a phase angle greater than $\pi/2$ does not really make sense. So, for the purposes of physical clarity, we also quote the value to high precision than we really know it, to emphasize that our answer is physically reasonable: $\phi = 1.56$ rad. This is just slightly less than $\pi/2 \approx 1.57$ rad.

Since the same current amplitude exists for all circuit elements, the ratio of the potential differences cross the inductor and capacitor will be the same as the ratio of their reactances:

$$\frac{V_L}{V_C} = \frac{I\chi_L}{I\chi_C} = \omega^2 LC = (4000 \text{ s}^{-1})^2(0.400 \text{ H})(2.0\times10^{-6} \text{ F})$$
$$\frac{V_L}{V_C} = 13$$

4. Evaluate Result The resonance angular frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.400 \text{ H})(2.0\times10^{-6} \text{ F})}} = 1.1\times10^3 \text{ s}^{-1}$$

Thus the circuit has been driven at an angular frequency greater than the resonance angular frequency. This means that the inductance will dominate in the impedance. The potential difference amplitude across the inductor will be much greater than the potential difference amplitude across the capacitor. It also tells us that the phase angle will be positive and will be very close to $\pi/2$. We saw this was the case.

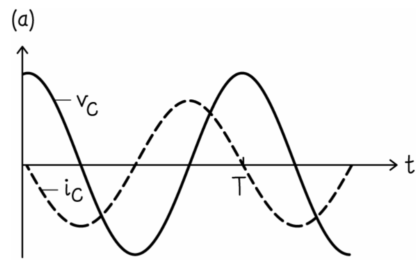
Questions and Problems

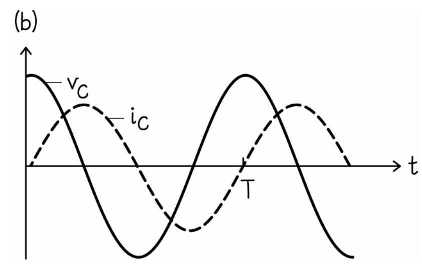
32.1. The root mean squared value is $B_{\text{rms}} = \dfrac{\mu_0 n I_{\max}}{\sqrt{2}}$.

32.2. (*a*) There are four such intervals in one full oscillation, meaning the period is $T = 4(0.020\text{ s}) = 0.080\text{ s}$, and the frequency is thus $f = \dfrac{1}{T} = \dfrac{1}{(0.080\text{ s})} = 13\text{ Hz}$. (*b*) The potential difference from one plate to another is just as often positive as it is negative. The average value of the potential difference is zero.

32.3. The energy stored in an inductor is maximal when the magnitude of the current flowing through the inductor is maximal. This happens at 0.25 s and 0.75 s and of course at any time that is an integer number of periods later than these times: 1.25 s, 1.75 s, etc.

32.4.





32.5. X could certainly contain a resistor, perhaps in series with an inductor or another capacitor. As a specific example, consider a circuit consisting of a charged capacitor, a resistor, and an inductor in series. The capacitor and inductor would lead to oscillatory behavior in the potential difference and the resistor would dissipate energy as heat, accounting for the decreasing amplitude of the potential difference. Y could also contain a resistor if there is a source of alternating current maintaining a fixed maximum potential difference.

32.6. The charge on a capacitor is related to the potential difference across the plates according to $q_C = Cv_C$, and the electric potential energy stored in a capacitor is given by $U^E = \dfrac{1}{2}Cv_C^2$, so $|v_C|$, $|q_C|$, and $|U^E|$ all reach maxima simultaneously. The current, magnetic field, and potential energy stored in a solenoid are similarly related by $B = \mu_0 ni$, and $U^B = \dfrac{1}{2}Li^2$, such that $|i|$, $|B|$, and $|U^B|$ all reach maxima simultaneously. The two sets of values (capacitor-related values and inductor related values) do not reach their maxima at the same time. This can be seen by considering that charge must flow away from the capacitor in order for the current through the inductor to become large.

32.7. The total energy in the circuit must be conserved, and this energy consists of two types: potential energy stored in the electric field in the capacitor and in the magnetic field in the inductor. Thus, $E_\text{i} = E_\text{f} \Rightarrow U^E = U_\text{f}^B + U_\text{f}^E$. Recall that the energy stored in a capacitor can be written $U^E = \dfrac{1}{2}\dfrac{q^2}{C}$. So when the charge is reduced to $q/2$, the final electrostatic potential energy in the capacitor becomes $U_\text{f}^E = \dfrac{1}{2}\dfrac{(q/2)^2}{C} = \dfrac{1}{4}U^E$. Thus we have $U_\text{f}^B = U^E - U_\text{f}^E = U^E - \dfrac{1}{4}U^E = \dfrac{3}{4}U^E$.

32.8. With a given potential difference, the power dissipated in a resistor is $P = V^2/R$. But here, the potential difference is constantly changing, sinusoidally. We can write $P(t) = V(t)^2/R = \dfrac{\left[V_{\max}\sin(\omega t)\right]^2}{R}$. Because the circuit goes through 45 oscillations in the time specified, we can use the average power: $P_{\text{av}} = \dfrac{V_{\max}^2\left[\sin^2(\omega t)\right]_{\text{av}}}{R} = \dfrac{V_{\max}^2}{2R}$.

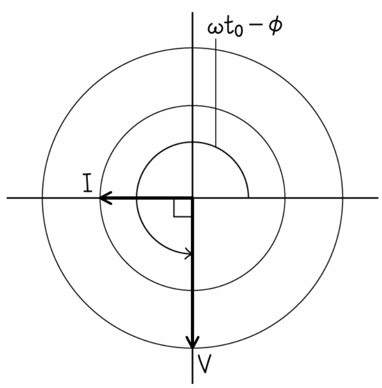
Now that we have the average power, we can write $\Delta E = P_{\text{av}} t = \dfrac{V_{\max}^2}{2R} t = \dfrac{(170\ \text{V})^2}{2(9.0\ \Omega)}(0.75\ \text{s}) = 1.2\ \text{kJ}$.

32.9. The capacitance of a parallel plate capacitor is $C = \dfrac{\epsilon \kappa A}{d}$, and $C = \dfrac{q}{v_C}$ for any capacitor. We can rearrange this to determine the initial (maximum) charge on the capacitor: $q = \dfrac{v_C \epsilon \kappa A}{d}$. The energy stored in a capacitor is $U^E = \dfrac{1}{2}\dfrac{q^2}{C}$, such that $q = \sqrt{2CU^E}$. So if the energy in the inductor is increased to 85% of its maximum, the energy in the capacitor is reduced to 15% of its maximum, and we can write $q_{\text{f}} = \sqrt{2C(0.15)U_{\max}^E} = \sqrt{(0.15)}q_{\max}$. Thus

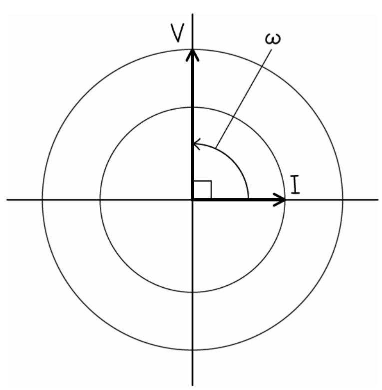
$$q_{\text{f}} = \sqrt{(0.15)}\frac{v_{C,\max}\epsilon \kappa A}{d} = \sqrt{(0.15)}\frac{(10\ \text{V})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1200)(1.00\times10^{-4}\ \text{m}^2)}{(0.100\times10^{-3}\ \text{m})} = 41.1\ \text{nC}$$

32.10. (*a*) The resistance of the wires is responsible for electrical energy being converted to thermal energy. (*b*) The graph will be roughly sinusoidal, but with an amplitude that decreases in time. (*c*) The total energy in the system will decrease exponentially in time.

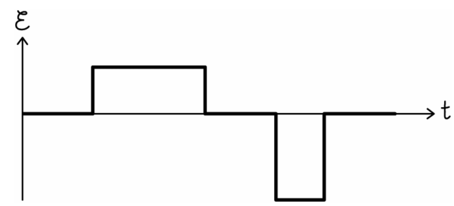
32.11.



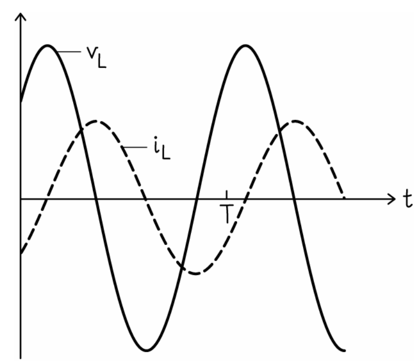
32.12.



32.13.



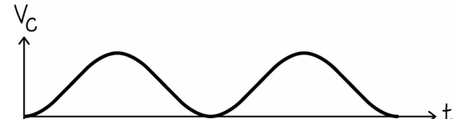
32.14. (*a*) The potential difference is leading the current, so this is an inductor.
(*b*)



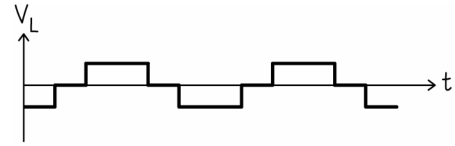
32.15. Because the current and potential difference are always in phase, we know the circuit contains only a resistor. From the magnitudes of the current and potential difference, we see that $R = \frac{V}{I} = \frac{(9.0\text{ V})}{(1.0\text{ A})} = 9.0\ \Omega.$ After 1.0 s has elapsed, we see the phasors at the position of 1/8 of a complete oscillation. This could mean that the frequency at which the circuit oscillates is $f = \frac{1}{8}$ Hz. But the circuit could also have gone through a full revolution plus an additional 1/8, or two full revolutions plus an additional 1/8, etc. So all we can definitely say about the frequency is that it obeys $f = \left(\frac{1}{8} + n\right)$ Hz, where n is any integer.

32.16. Here the current leads the potential difference. The circuit element is a capacitor.

32.17. (*a*) We draw the figure assuming that the capacitor is initially uncharged. The potential difference across the capacitor is given by $v_C = \frac{q}{C} = \frac{1}{C}\int_0^t i(t')dt'.$ As long as the current is increasing linearly in time, the charge will increase quadratically in time. When the current is constant, the charge will increase linearly in time, and so on.



(*b*) The potential difference across the inductor is proportional to the rate of change of the current.



32.18. Because the current leads the potential difference, we know the element must be a capacitor. Determining the capacitance is not so easy. We know the current is of the form $i(t) = I_{\max}\cos(\omega t)$, and we can read from the graph that the period of the oscillation in the circuit is 2.0 s. The maximum potential difference will be reached after the current has carried charge to the capacitor plates for one fourth of the period, as we see on the graph. Thus $q_{\max} = \int_0^{T/4} i(t)dt = I_{\max}\int_0^{T/4}\cos\left(\frac{2\pi t}{T}\right)dt = \frac{I_{\max}T}{2\pi}$. Of course, we also know $v_{C,\max} = \frac{q_{\max}}{C}$, such that $C = \frac{q_{\max}}{v_{C,\max}} = \frac{I_{\max}T}{2\pi v_{C,\max}} = \frac{(1.00\times10^{-3}\text{ A})(2.0\text{ s})}{2\pi(1.00\text{ V})} = 0.32\text{ mF}$. Thus, your friend is using the 0.32 mF capacitor.

32.19. (*a*) Each Si atom has four valence electrons. So the number of valence electrons in the sample is $N_e = 4N_{\text{Si}} = 4(3.0\times10^{22}) = 1.2\times10^{23}$. (*b*) None. All of these electrons are involved in bonding with neighbors.

32.20. Indium, gallium, boron, and aluminum all have three valence electrons. They will be able to bond with three of the neighboring silicon atoms, but not with the fourth. There will be an absence of a fourth electron, which we call a hole. Since the hole acts effectively as a positive charge-carrier (the absence of negative charge), this is called a p-type semiconductor. Similarly, phosphorous, arsenic, and antimony have five valence electrons. They can bond with all four neighboring silicon atoms, and still have a fifth electron to conduct charge. In this case, the charge carrier is negative, so we call this an n-type semiconductor.

32.21. Before doping, there are no free charge carriers in pure silicon (assuming no crystal impurities and very low temperatures). Hence the addition of even a small number of free (conducting) electrons is quite noticeable.

32.22. It recombines with an electron, and an equivalent hole enters from the opposite lead.

32.23. (*a*) Either of the doped semiconductors will conduct a current much better than the undoped semiconductor. (*b*) The *n*-type semiconductor is called "*n*-type" because there are negative charge carriers in the form of excess electrons. There are as many electrons as in the undoped semiconductor used in bonding, plus additional electrons that are able to conduct current. The *p*-type semiconductor has even fewer electrons than the intrinsic semiconductor. Thus the *n*-type semiconductor has the most electrons. (*c*) None. Holes do not leave the semiconductor. They recombine with an electron at one lead and are simultaneously created at the opposite lead, within the semiconducting material.

32.24. **[NOTE:** In the problem statement, 463.05 mg is changed to 465.9480 mg.] Let us allow for the possibility of contamination and call the fraction of the sample's volume that is silicon f_{Si}, and the fraction of the volume that is boron f_{B}. One might choose to work with fractions by mass, but we will calculate that at the end also. From the mass and volume measurements of the sample, we can say:

$$V_{\text{Si}} + V_{\text{B}} = V \text{ or } f_{\text{Si}} + f_{\text{B}} = 1 \quad (1)$$

$$m = m_{\text{Si}} + m_{\text{B}} = \rho_{\text{Si}}V_{\text{Si}} + \rho_{\text{B}}V_{\text{B}} = (\rho_{\text{Si}}f_{\text{Si}} + \rho_{\text{B}}f_{\text{B}})V \quad (2)$$

Inserting equation (1) into equation (2), we have

$$\begin{aligned} m = (\rho_{\text{Si}}(1-f_{\text{B}}) + \rho_{\text{B}}f_{\text{B}})\ell wh \Rightarrow f_{\text{B}} &= \frac{\left(\frac{m}{\ell wh} - \rho_{\text{Si}}\right)}{\rho_{\text{B}} - \rho_{\text{Si}}} \\ &= \frac{\left(\frac{(465.9408\times10^{-6}\text{ kg})}{(10.00\times10^{-3}\text{ m})(10.00\times10^{-3}\text{ m})(2.000\times10^{-3}\text{ m})} - (2329.6\text{ kg/m}^3)\right)}{(2340.0\text{ kg/m}^3) - (2329.6\text{ kg/m}^3)} \\ &= 0.0100 \end{aligned}$$

Here we have shown additional significant digits, only to emphasize that this fraction is slightly different than the fraction by mass (calculated below). But to the accuracy we are given, we can only say that the sample is about 1 % boron by volume. This is the fraction by volume; we may also be interested to know the fraction by mass, given by

$$f_{\text{B,mass}} = \frac{m_{\text{B}}}{m_{\text{total}}} = \frac{f_{\text{B}}V\rho_{\text{B}}}{m_{\text{total}}} = \frac{(0.010)(10.00\times10^{-3}\text{ m})(10.00\times10^{-3}\text{ m})(2.000\times10^{-3}\text{ m})(2340.0\text{ kg/m}^3)}{(465.9408\times10^{-6}\text{ kg})} = 0.0101$$

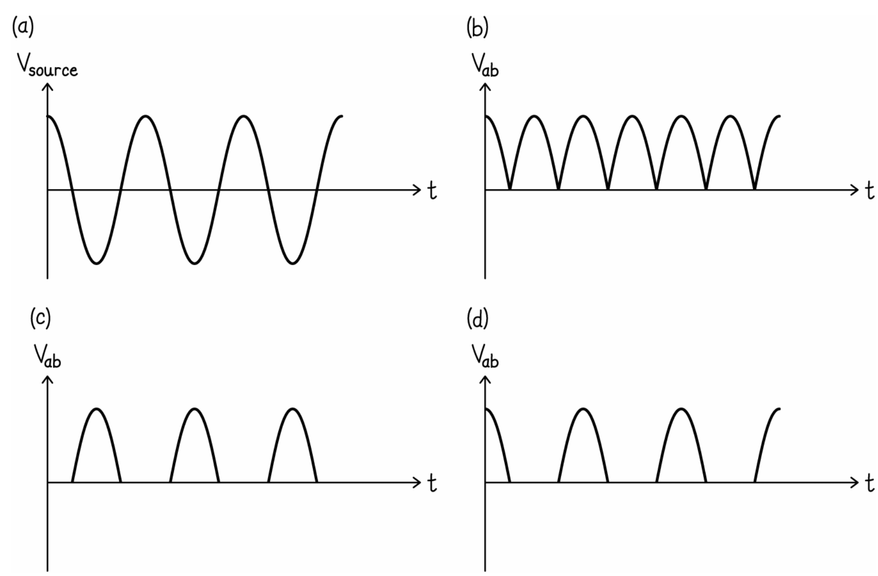
which is also about 1 % to the prescribed accuracy. Thus, the sample is approximately 1% boron by volume and 1% by mass. This is extremely high doping. The sample will have conductivity that differs drastically from that of pure silicon. You need a new sheet.

32.25. Circuit B produces the greatest current in the emitter. In A and C, the batteries can shift the depletion zone in the *p*-type semiconductor (the base), but the insulating depletion zone is still present and blocks current. In circuit B, a bias is applied to the base, restoring mobile holes to the *p*-type semiconductor, and allowing current to flow.

32.26. This is an AND gate. Current can only flow from the input to the output if current can cross both *npn* junctions, meaning both A and B must have a bias applied.

32.27. No. The symbol at the top of the circuit depicts a diode which allows current to flow only in the direction in which the triangle points along the wire. The battery is oriented so as to cause current in the other direction, and the diode blocks the current.

32.28. (*a*) See figure below. (*b*) The function of the rectifier is to ensure that the current across the resistor is always in the same direction, even though the source is AC. Thus, the potential difference across the resistor will always have the same sign (chosen here to be positive). (*c*) The function of the circuit will be the same, except that current will no longer be able to flow downward across the path that diode 1 formerly allowed. So, during half the time intervals the potential difference across the resistor will be the same as in part (*b*), and for the other half of the time intervals the potential difference will be zero. (*d*) This is identical to part (*c*), except that now the time intervals where the potential difference is zero and non-zero are switched.



32.29. No. The function of a rectifier is to convert AC currents to DC. In a normal rectifier diagram, the current is made to flow across the resistor in the same direction at all times. If the resistor were replaced with a capacitor, current would flow until the capacitor was fully charged, and the current would then stop.

32.30. Two transistors in a row must be biased (either A and B, or C and D) in order for the output to be Y. Thus, from top to bottom: Y, Y, Y, Y, N, N.

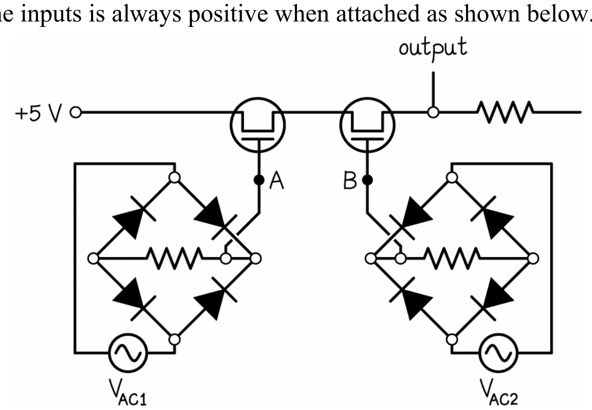
32.31. (*a*) B to A. The current flows when the positive terminal of the battery is connected to B. (*b*) A to B. One can either answer this by noting that "holes" are simply the absence of an electron, so electrons and holes must move in opposite directions, or by noting that current flows when the negative terminal of the battery is connected to A. (*c*) Charge will flow when the potential difference applied by the battery is in a direction opposite the potential difference due to the electron-hole recombination in the depletion zone. Since current flows when the positive battery terminal is connected to B, material B must have had a negative potential and therefore a net negative charge. This is the case for a *p*-type semiconductor (because negative electrons diffuse into the *p*-type semiconductor and fill the holes, leaving a net negative charge. The opposite arguments hold for material A. Thus A is *n*-type, and B is *p*-type. (*d*) A may contain phosphorous, and B may contain boron. This can seen either from examining the examples in *Principles*, or simply noting that boron has only three valence electron resulting in a hole when bonding when silicon, and phosphorous has five valence electrons resulting in an extra electron when bonding with silicon.

32.32. (*a*) The charge on the *p*-type region is negative, because electrons from the *n*-type regions will wander into the *p*-type region where they will combine with holes. Because the electrons linger in the *p*-type region, this gives the *p*-type region a net negative charge. (*b*) The *n*-type regions are positive. Electrons from the *n*-type regions wander into adjacent *p*-type regions. Since the *n*-type materials started out neutral, this loss of electrons makes the *n*-type regions positive.

32.33. In order for the bulb to light, at least one of the top two transistors (A or B or both) must be positively biased, and at least one of the bottom two transistors (C or D or both) must be positively biased. Thus, the possible combinations that cause the bulb to light are ABCD, ABC, ABD, ACD, BCD, AC, AD, BC, or BD.

32.34. In this case, we would need both transistors connected to either of the AND gates to be positively biased. That is, either both A and B must be biased, or both C and D must be biased. If one of those conditions is satisfied, the other transistors may be biased or not and the combination will still work. Thus, the possible combinations that make the bulb light are ABCD, ABC, ABD, AB, ACD, BCD, or CD.

32.35. The AC source would cause the bias to sometimes allow current to flow, and would not allow the current to flow at other times. If one knows to expect this, then seeing the transistor rapidly switch current on and off could tell you that the transistor is working, it is simply not being used the way a transistor is typically used. But this circuit is an AND gate. So, the only way for both transistors to allow a current to pass is for both to be positively biased at the same time. If one is using two different AC sources with possibly different phases or even different frequencies, it could be very difficult to glean any useful information from this. There may be many possible alterations to the circuit that could yield useful information. One possibility is to short each transistor (one at a time) so that they can be tested one at a time. In that case one would look for the rapid switching of a bulb or other test object from on to off. Alternatively, one could run an ideal conducting wire from the A bias to the B bias, and use only one AC source. This way the bias voltages are guaranteed to be in phase, such that one can test both transistors simultaneously. This preserves the AND function of the circuit. Finally, one might attach bridge rectifiers to the bias inputs at A and B. This would allow one to use two separate AC sources, and still preserve the AND nature of the circuit. Rectifiers can ensure that the potential on the inputs is always positive when attached as shown below.



32.36. The inductive reactance is given by Equation 32.26: $X_L = \omega L = 2\pi fL = 2\pi(120\text{ Hz})(50.0\times10^{-3}\text{ H}) =$ 37.7 Ω.

32.37. The capacitve reactive is given by Equation 32.14: $X_C = \dfrac{1}{\omega C} \Rightarrow \omega = \dfrac{1}{X_C C} = \dfrac{1}{(1.0\times10^4\text{ Hz})(30.0\times10^{-6}\text{ F})} =$ 3.3 s^{-1}.

32.38. Using Equation 32.14 we require that the capacitive reactance of the capacitor when the circuit operates at a certain frequency be the same as the resistance of the resistor. $R = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{RC} \Rightarrow f = \frac{1}{2\pi RC} = \frac{1}{2\pi(540\ \Omega)(2.40\times10^{-6}\ \text{F})} = 123\ \text{Hz}.$

32.39. (*a*) We know that $q(t) = Cv_C(t) = CV_{\max}\sin(\omega t)$. Taking the derivative of both sides yields $i(t) = CV_{\max}\omega\cos(\omega t)$, which obviously has a maximum value of $i_{\max}(t) = CV_{\max}\omega = (10\times10^{-3}\ \text{F})(5.0\ \text{V})(20\ \text{s}^{-1}) = 1.0\ \text{A}$. (*b*) The current amplitude increases linearly with the source frequency. This can either be answered intuitively, since a capacitor only resists the flow of charge as charge accumulates on the plates, or one can see it from the equation obtained in part (*a*): $i_{\max}(t) = CV_{\max}\omega$.

32.40. (*a*) $v_L = iX_L \Rightarrow i_{\max} = \frac{v_{\max,L}}{X_L} = \frac{v_{\max,L}}{\omega L} = \frac{(5.0\ \text{V})}{(200\ \text{s}^{-1})(10\times10^{-3}\ \text{H})} = 2.5\ \text{A}.$ (*b*) As the angular frequency decreases, the current amplitude increases like $1/\omega$. As the inductance decreases, the current amplitude increases like $1/L$.

32.41. (*a*) We know $v_R = iR$, and the equivalent resistance of the circuit is $R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$, so the maximum potential difference across the circuit is $v_{R,\max} = i_{\max}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = (2.03\ \text{A})\left(\frac{1}{(30.0\ \Omega)} + \frac{1}{(10.0\ \Omega)}\right)^{-1} = 15.2\ \text{V}.$ (*b*) $v_R = v_{R,\max}\sin(\omega t) = v_{R,\max}\sin(2\pi ft) = (15.23\ \text{V})\sin(2\pi(60.0\ \text{Hz})(20.2\times10^{-3}\ \text{s})) = 3.60\ \text{V}.$

32.42. (*a*) Capacitor. Since the current changes direction very rapidly, there will not be an opportunity for charge to accumulate on the plates of the capacitor, meaning the capacitor will not resist the flow of charge in an appreciable way. (*b*) Inductor. Since the current changes very slowly in time, the rate of change of the current will be small. It follows that the potential difference across the inductor will be small. (*c*) Inductor. Since the current is DC there will be no change of the current with time, so the inductor will not have a potential difference across it. (*d*) The best option would be a parallel arrangement of an inductor and a capacitor so that one will always allow current to pass with very low reactance. But if one must use only one element, then a resistor (with low resistance) is the best choice.

32.43. From the frequency, we see that the period of a full oscillation is $T = 0.0167\ \text{s}$, such that the 5.0 ms that have elapsed correspond to 30% of a full oscillation. At this time the source emf has increased from zero to $\mathcal{E}_{\max}\sin\left(\frac{2\pi}{T}t\right) = \mathcal{E}_{\max}\sin\left(\frac{2\pi}{0.0167\ \text{s}}(0.0050\ \text{s})\right) = (0.951)\mathcal{E}_{\max}$. So the source emf has already reached 95% of its maximum value, whereas the current has only reached 30.9% of its maximum value. Thus, the device lags behind the source emf (or leads the emf by more than $\pi/2$ radians, but this is less likely).

32.44. With the resistor, the current obeys $v_R = iR \Rightarrow i = \frac{1}{R}v_R = \frac{\mathcal{E}_{\max}}{R}\sin(\omega t + \phi_1)$, so the current is zero when $\sin(\omega t + \phi_1) = 0 \Rightarrow \omega t + \phi_1 = n\pi$ or $t = \frac{n\pi - \phi_1}{\omega}$. Inserting the numerical values given, we find that the current is zero at any time given by $t = \frac{n - 1/4}{60}\ \text{s}$, where n is an integer.

With the capacitor, the current obeys $i = \frac{d}{dt}q(t) = C\frac{d}{dt}v_C = -C\frac{d}{dt}\mathscr{E} = -C\frac{d}{dt}\mathscr{E}_{\max}\sin(\omega t + \phi_i) = -C\mathscr{E}_{\max}\cos(\omega t + \phi_i)$, so the current is zero when $\cos(\omega t + \phi_i) = 0 \Rightarrow \omega t + \phi_i = \left(n - \frac{1}{2}\right)\pi$ or $t = \frac{\left(n - \frac{1}{2}\right)\pi - \phi_i}{\omega}$. Inserting the numerical values given, we find that the current is zero at any time given by $t = \frac{n - 3/4}{60}$ s where n is an integer.

32.45. Because the capacitor and inductor are in parallel, they must always have the same potential difference across them. In particular, they must have the same amplitude of potential difference, so we can equate their inductive and capacitive reactances: $X_C = X_R \Rightarrow \frac{1}{\omega C} = \omega L$ or $\omega = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(20.0\times10^{-3}\text{ H})(10.0\times10^{-3}\text{ F})}} =$ 70.7 s^{-1}. We can now equate either of these reactances to the resistace; we choose the capacitive reactace. $R = \frac{1}{\omega C} = \frac{1}{(70.71\text{ s}^{-1})(10.0\times10^{-3}\text{ F})} = 1.41\ \Omega$. Thus $R = 1.41\ \Omega$ and $\omega = 70.7\text{ s}^{-1}$.

32.46. In this circuit there is no capacitance. One can follow the arguments used to go from Equation 32.40 to Equation 32.43 to obtain an expression for the impedance in an RL circuit. The impedance is $Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{(1.00\times10^3\ \Omega)^2 + (2\pi(1.00\times10^3\text{ Hz})(5.00\times10^{-1}\text{ H}))^2} = 3.30\text{ k}\Omega$.

32.47. From Equation 32.40 we know that $\mathscr{E}_{\max}^2 = V_R^2 + (V_L - V_C)^2 = (IR)^2 + (V_L - V_C)^2 = ((5.00\times10^{-3}\text{ A})(900\ \Omega))^2$ $+((29.0\text{ V}) - (13.0\text{ V}))^2 = 16.6$ V.

32.48. We use Equation 32.42 to write $I = \frac{\mathscr{E}_{\max}}{Z} = \frac{\mathscr{E}_{\max}}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{(5.00\text{ V})}{\sqrt{(20.0\ \Omega)^2 + ((10.0\text{ s}^{-1})(6.00\times10^{-3}\text{ F}))^{-2}}} =$ 0.192 A.

32.49. (*a*) The potential differences are given by Equations 32.35 and 32.36:

$$V_R = IR = \frac{\mathscr{E}_{\max}}{Z}R = \frac{\mathscr{E}_{\max}R}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{(35.0\text{ V})(1000\ \Omega)}{\sqrt{(1000\ \Omega)^2 + (2\pi(60\text{ Hz})(1.00\times10^{-6}\text{ F}))^{-2}}} = 12.3\text{ V}$$

and

$$V_C = IX_C = \frac{\mathscr{E}_{\max}/\omega C}{Z} = \frac{\mathscr{E}_{\max}/\omega C}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{(35.0\text{ V})/(2\pi(60\text{ Hz})(1.00\times10^{-6}\text{ F}))}{\sqrt{(1000\ \Omega)^2 + (2\pi(60\text{ Hz})(1.00\times10^{-6}\text{ F}))^{-2}}} = 32.8\text{ V}$$

We obtain the phase constant ϕ using Equation 32.37:

$$\phi = \tan^{-1}\left(-\frac{1}{\omega RC}\right) = \tan^{-1}\left(-\frac{1}{2\pi(60\text{ Hz})(1000\ \Omega)(1.00\times10^{-6}\text{ F})}\right) = -1.21\text{ radians or } -69.3°$$

(*b*) The potential differences are again given by Equations 32.35 and 32.36:

$$V_R = IR = \frac{\mathscr{E}_{\max}}{Z}R = \frac{\mathscr{E}_{\max}R}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{(35.0\text{ V})(1000\ \Omega)}{\sqrt{(1000\ \Omega)^2 + (2\pi(2100\text{ Hz})(1.00\times10^{-6}\text{ F}))^{-2}}} = 34.9\text{ V}$$

and

$$V_C = IX_C = \frac{\mathscr{E}_{\max}/\omega C}{Z} = \frac{\mathscr{E}_{\max}/\omega C}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{(35.0\text{ V})/(2\pi(2100\text{ Hz})(1.00\times10^{-6}\text{ F}))}{\sqrt{(1000\ \Omega)^2 + (2\pi(2100\text{ Hz})(1.00\times10^{-6}\text{ F}))^{-2}}} = 2.64\text{ V}$$

We obtain the phase constant ϕ using Equation 32.37:

$$\phi = \tan^{-1}\left(-\frac{1}{\omega RC}\right) = \tan^{-1}\left(-\frac{1}{2\pi(2100\text{ Hz})(1000\ \Omega)(1.00\times10^{-6}\text{ F})}\right) = -0.0756\text{ radians or } -4.33°$$

32.50. From Equation 32.35 we can relate the maximum potential difference across the resistor to the capacitance:

The potential differences are given by Equations 32.35 and 32.36:

$$V_R = \frac{\mathcal{E}_{\max} R}{\sqrt{R^2 + (1/\omega C)^2}} \Rightarrow C = \frac{1}{\omega}\left(\left(\frac{\mathcal{E}_{\max} R}{V_R}\right)^2 - R^2\right)^{-1/2}$$
$$= \frac{1}{(2\pi(1500\ \text{Hz}))}\left(\left(\frac{(18\ \text{V})(1200\ \Omega)}{(9.0\ \text{V})}\right)^2 - (1200\ \Omega)^2\right)^{-1/2}$$
$$= 5.1\times10^{-8}\ \text{F}$$

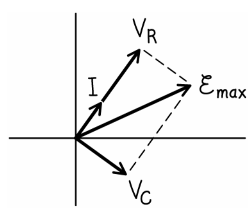
32.51. With the source and maximum current the same in each circuit, we know the impedance must be the same in each circuit. Since the resistance is the same in each circuit, we can equate the capacitive reactance in circuit 1 to the inductive reactance in circuit 2. Thus

$$\frac{1}{\omega C} = \omega L \Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi(60.0\ \text{Hz}))^2(4.50\times10^{-3}\ \text{F})} = 1.56\ \text{mH}$$

32.52. As in Example 32.8, we know $\omega_c = \frac{1}{RC} \Rightarrow C = \frac{1}{R\omega_c} = \frac{1}{(50\ \Omega)(2\pi(200\ \text{Hz}))} = 1.6\times10^{-5}\ \text{F}.$ To determine the fraction of the source emf that passes through the circuit at 60 Hz, we simply calculate $\frac{V_R}{\mathcal{E}_{\max}}$. We find

$$\frac{V_R}{\mathcal{E}_{\max}} = \frac{R}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{(50\ \Omega)}{\sqrt{(50\ \Omega)^2 + (2\pi(60\ \text{Hz})(1.59\times10^{-5}\ \text{F}))^{-2}}} = 0.29\ \text{ or } 29\%$$

32.53. (*a*)



(*b*) We use Equation 32.38 to determine the phase constant:

$$\phi = \tan^{-1}\left(-\frac{1}{\omega RC}\right)$$
$$= \tan^{-1}\left(-\frac{1}{\omega(R_1+R_2)(C_1+C_2)}\right)$$
$$= \tan^{-1}\left(-\frac{1}{(4000\ \text{s}^{-1})((200\ \Omega)+(400\ \Omega))((1.00\times10^{-6}\ \text{F})+(3.00\times10^{-6}\ \text{F}))}\right)$$
$$= -0.104\ \text{radians or } -5.95^\circ$$

32.54. The circuit originally has an impedance that is equal to the resistance, $Z_0 = R$. But the final impedance will be $Z = \sqrt{R^2 + (\omega L)^2}$. Thus the ratio of the maximum current after the inductor is added to that before the inductor is added is

$$\frac{I_\text{f}}{I_\text{i}} = \frac{\mathcal{E}_{\max}/Z_\text{f}}{\mathcal{E}_{\max}/Z_\text{i}} = \frac{R}{\sqrt{R^2+(\omega L)^2}} = \frac{(8.00\ \Omega)}{\sqrt{(8.00\ \Omega)^2 + (2\pi(1000\ \text{Hz})(10.0\times10^{-3}\ \text{H}))^2}} = 12.6\%$$

32.55. Because the two branches are in parallel, we know the potential difference across them is the same at any time. Since the maximum potential difference across the branches and the maximum current through them are the same, the impedances must be the same. We note immediately that (since the resistances of the branches are the same) the impedances can only be equal if the inductive reactance of the top branch equals the capacitive reactance of the bottom branch. Thus $L = \dfrac{1}{\omega^2 C} = \dfrac{1}{(80.0\ \text{s}^{-1})^2(50.0\times 10^{-6}\ \text{F})} = 3.13\ \text{H}.$

32.56. In an RC circuit, the phase constant is angle by which the current leads the source emf. The amount of time required for the system to oscillation through that phase constant is

$$\Delta t = \frac{|\phi|}{\omega} = \frac{\left|\tan^{-1}\left(-\dfrac{1}{\omega RC}\right)\right|}{\omega} = \frac{\left|\tan^{-1}\left(-\dfrac{1}{(150\ \text{s}^{-1})(20.0\ \Omega)(300\times 10^{-6}\ \text{F})}\right)\right|}{(150\ \text{s}^{-1})} = 5.59\ \text{ms}$$

32.57. From the maximum current and emf, we know

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \mathcal{E}_{\max} I \tag{1}$$

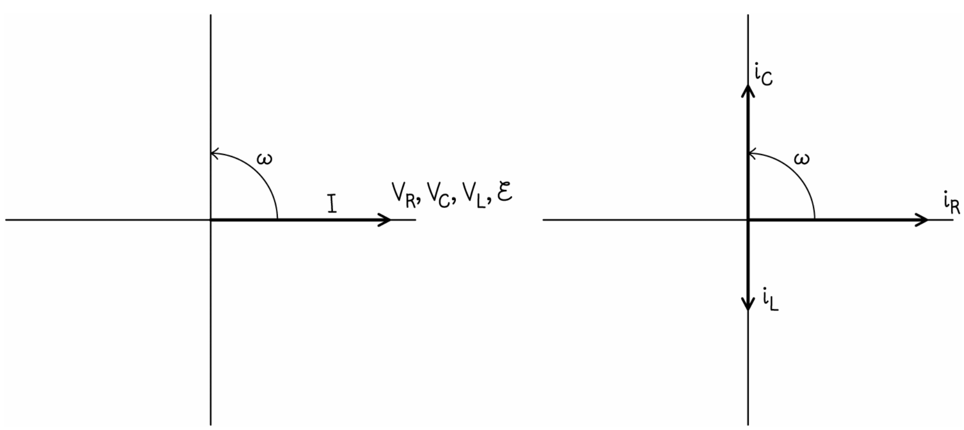
and from the phase constant information, we can say

$$\tan(\phi) = \frac{\omega L - 1/\omega C}{R} \tag{2}$$

Solving equation (1) for the quantity $\omega L - 1/\omega C,$ and inserting this result into equation (2), we obtain

$$\tan(\phi) = \frac{\sqrt{(\mathcal{E}_{\max} I)^2 - R^2}}{R} \Rightarrow R = \frac{(\mathcal{E}_{\max} I)}{\sqrt{(1+\tan^2(\phi))}} = \frac{((60\ \text{V})(1.0\ \text{A}))}{\sqrt{(1+\tan^2(-44°))}} = 43\ \Omega$$

32.58. (*a*) The potential differences must all be the same, and they must be equal to the emf provided by the source.
(*b*)



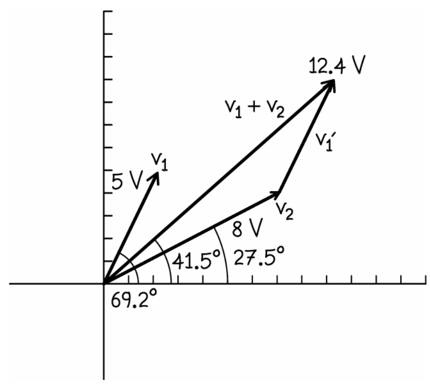
Because the resistor, inductor, and capacitor are wired in parallel, the potential difference across each element must be the same at all times. Thus, the phasor arrows for all three (and the source) overlap. For the purposes of discussion, let us call the counterclockwise motion of positive charge positive. Let us further consider an instant when the current through the resistor is positive (meaning the top of the resistor will be at a higher potential than the bottom), and as large as it ever becomes. At this instant, the source is providing the maximum possible potential difference across the inductor and capacitor as well. In order for the capacitor to have its maximum positive potential difference, it must have the maximum possible positive charge on the top plate. This means that the current to the capacitor at that instant must be zero, and it means that the current over to the capacitor must have recently been positive, and just reduced to zero. Thus, in the diagram below, we draw the current to the capacitor leading the current in the resistor. Similarly, when the potential difference is maximal across the inductor, the rate of change of current must be maximal. Since the current will be a sinusoidal function, this maximum change occurs at an instant when the current is zero. At this instant we are considering, the top of the inductor is at a more positive potential than the bottom, indicating that the inductor is trying to resist a current that in increasing upward from the bottom of the inductor to the top. This means that the current through the inductor is instantaneously zero, but was recently negative (bottom of inductor to top), and will shortly become positive. Thus we draw the current through the inductor lagging behind the current through the resistor. (*c*) Using the phasor diagram above, we can write

$$i_S^2(t) = i_R^2 + (i_C - i_L)^2 \text{ or } i_S(t) = \left(\left(\frac{\mathcal{E}_{\max}}{R}\sin(\omega t)\right)^2 + \left(\mathcal{E}_{\max}\,\omega C\sin(\omega t + \pi/2) - \frac{\mathcal{E}_{\max}}{\omega L}\sin(\omega t - \pi/2)\right)^2\right)^{1/2}$$

This can be written in other ways. (*d*) For a given source emf, and a given resistance, the operating frequency will not affect the current through the resistor. From the phasor diagram, one can see that the current across the resistor is in phase with the potential difference supplied by the emf, and that the other two currents (through the capacitor and the inductor) are perpendicular to this in the phase diagram. This is equivalent to saying those currents are out of phase with the current across the resistor by one quarter of an oscillation. Since we cannot affect the current in the resistor by changing the frequency, the minimum current in the circuit is exactly that current through the resistor. In that case, the currents through the capacitor and inductor must be equal (since they are exactly out of phase). Thus we require the current magnitude through the inductor and capacitor to be equal:

$$I_C = I_L \Rightarrow \mathcal{E}_{\max}\,\omega C = \frac{\mathcal{E}_{\max}}{\omega L} \Rightarrow \omega = \sqrt{\frac{1}{LC}}$$

32.59. (*a*) We draw a phase diagram and estimate graphically the quantities desired.



The magnitude of the resulting potential difference appears close to 12.4 V, and the phase appears close to $\phi = 41.5°$. We see $v_{\text{sum}}(t = 0.0500\text{ s}) = 8.3\text{ V}$. Answers could differ slightly depending on the quality of the diagram, but answers should be near these values. (*b*) For discussion, let us momentarily refer to the horizontal axis in the phasor diagram as the "hor" axis, and the vertical axis as the "vert" axis. We do not use x and y to avoid confusion, as the axis in phasor diagrams are not spatial axes. We find the components of each potential difference at $t = 0$. From the data given, we know

$$\phi_1 = \sin^{-1}\left(\frac{v_1}{V_1}\right) - \omega t = \sin^{-1}\left(\frac{(4.50\text{ V})}{(5.00\text{ V})}\right) - (10.0\text{ s}^{-1})(0) = 1.12\text{ radians}$$

and

$$\phi_2 = \sin^{-1}\left(\frac{v_2}{V_2}\right) - \omega t = \sin^{-1}\left(\frac{(3.70\text{ V})}{(8.00\text{ V})}\right) - (10.0\text{ s}^{-1})(0) = 0.481\text{ radians}$$

We are already given the vertical components at this time: $V_{\text{vert}} = v_1(t=0) + v_2(t=0) = (4.50\text{ V}) + (3.70\text{ V}) = (8.20\text{ V})$. The horizontal components combine to give us $V_{\text{vert}} = V_1\cos(\phi_1) + V_2\cos(\phi_2) = (5.00\text{ V})\cos(1.12)$ $+(8.00\text{ V})\cos(0.481) = (9.27\text{ V})$. Thus the total amplitude of the sum is $V = \sqrt{V_{\text{hor}}^2 + V_{\text{vert}}^2} = \sqrt{(9.27\text{ V})^2 + (8.20\text{ V})^2} = 12.4\text{ V}$. The phase constant is given by $\phi = \tan^{-1}\left(\frac{V_{\text{vert}}}{V_{\text{hor}}}\right) = \tan^{-1}\left(\frac{(8.20\text{ V})}{(9.27\text{ V})}\right) =$ $0.724\text{ radians or }41.5°$. Finally, the potential difference due to the sum at $t = 0.0500\text{ s}$ is

$v_{\text{sum}}(t = 0.0500\text{ s}) = V\sin(\omega t + \phi) = (12.38\text{ V})\sin((10.0\text{ s}^{-1})(0.0500\text{ s}) + 0.724) = 11.6\text{ V}$. These values are in very good agreement with the results from using phasors.

32.60. Since the three branches of the circuit are wired in parallel, the potential difference across all branches will be the same at all times. The only way for the current amplitude in all three branches to be the same is if the impedance of each branch is the same. Thus we equate the impedances and solve for the inductance and capacitance:

$$Z_{\text{top}} = Z_{\text{bottom}} \Rightarrow \sqrt{R_L^2 + (\omega L)^2} = R \Rightarrow L = \frac{1}{\omega}\sqrt{R^2 - R_L^2} = \frac{1}{(100\text{ s}^{-1})}\sqrt{(20.0\ \Omega)^2 - (15.0\ \Omega)^2} = 0.132\text{ H}$$

and

$$Z_{\text{mid}} = Z_{\text{bottom}} \Rightarrow \sqrt{R_C^2 + (\omega C)^{-2}} = R \Rightarrow C = \frac{1}{\omega\sqrt{R^2 - R_C^2}} = \frac{1}{(100\text{ s}^{-1})\sqrt{(20.0\ \Omega)^2 - (18.0\ \Omega)^2}} = 1.15\text{ mF}$$

32.61. Equation 32.47 given us the expression for the resonant angular frequency: $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(50\times10^{-3}\text{ H})(20\times10^{-6}\text{ F})}} = 1.0\times10^{-3}\text{ s}^{-1}$.

32.62. (*a*) Equation 32.47 given us the expression for the resonant angular frequency: $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(6.00\times10^{-3}\text{ H})(1.0\times10^{-6}\text{ F})}} = 1.3\times10^{4}\text{ s}^{-1}$. (*b*) At resonance $I = \frac{\mathcal{E}_{\text{max}}}{R} = \frac{(15\text{ V})}{(3.0\ \Omega)} = 5.0\text{ A}$.

32.63. Equation 32.47 given us the expression for the resonant angular frequency: $\omega_0 = \frac{1}{\sqrt{LC}}$ $\Rightarrow C = \frac{1}{\omega^2 L} = \frac{1}{(380\text{ s}^{-1})^2(0.80\text{ H})} = 8.7\times10^{-6}\text{ F}$.

32.64. At resonance $I = \frac{\mathcal{E}_{\text{max}}}{R} = \frac{(12\text{ V})}{(10\ \Omega)} = 1.2\text{ A}$.

32.65. Let us write the general expression for the resonant angular frequency in terms of the capacitive and inductive reactances:

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{\omega^2}{\omega L(\omega C)}} = \omega\sqrt{\frac{X_C}{X_L}}$$

Thus, if both X_C and X_L double, the expression for the resonant angular frequency is unchanged. Thus $\omega_{0\text{f}} = \omega_{0\text{i}}$.

32.66. Initially we have $I_{\text{i}} = \frac{\mathcal{E}_{\text{max}}}{Z_{\text{i}}} = \frac{\mathcal{E}_{\text{max}}}{R} = 1.0\text{ A}$. After the angular frequency is doubled, we know the capacitive reactance is halved, and the inductive reactance is doubled. Thus we have $I_{\text{f}} = \frac{\mathcal{E}_{\text{max}}}{Z_{\text{f}}} = \frac{\mathcal{E}_{\text{max}}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathcal{E}_{\text{max}}}{\sqrt{R^2 + (2R - R/2)^2}} = \frac{\mathcal{E}_{\text{max}}}{R\sqrt{3.25}} = \frac{1}{\sqrt{3.25}}I_{\text{i}} = \frac{(1.0\text{ A})}{\sqrt{3.25}} = 0.55\text{ A}$.

32.67. The inductance is determined using equation (32.47):

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega_0^2 C} = \frac{1}{(4.08\times10^{4}\text{ s}^{-1})^2(2.00\times10^{-6}\text{ F})} = 3.00\times10^{-4}\text{ H}$$

The current is stated as a single value, even though it is an AC current. Typically this would represent the rms value of current, so we assume that is the intent here. From the maximum source emf and the rms current, we know that at resonance ($Z = R$):

$$I_{\text{rms}} = \frac{\mathcal{E}_{\text{rms}}}{Z_{\text{resonance}}} = \frac{\mathcal{E}_{\max}}{\sqrt{2}R} \Rightarrow R = \frac{\mathcal{E}_{\max}}{\sqrt{2}I_{\text{rms}}} = \frac{(5.00 \text{ V})}{\sqrt{2}(0.400 \text{ A})} = 8.84 \ \Omega$$

32.68. Because the resonant frequency decreases with increasing capacitance, and because parallel capacitors add directly and series capacitors add inversely, it should be clear that circuit (*a*) has the lower resonant frequency. This can be checked explicitly by calculating the resonant frequency for circuit (*b*) as well. We calculate the resonant angular frequency for circuit (*a*) using Equation 32.47:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L(C_1 + C_2)}} = \frac{1}{\sqrt{(47.0\times10^{-3} \text{ H})((1.00\times10^{-6} \text{ F}) + (2.00\times10^{-6} \text{ F}))}} = 2.66\times10^3 \text{ s}^{-1}$$

32.69. (*a*) Equation 32.47 given us the expression for the resonant angular frequency: $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(4.0\times10^{-3} \text{ H})(8.0\times10^{-6} \text{ F})}} = 5.59\times10^3 \text{ s}^{-1}$. (*b*) At resonance, we have $I = \frac{\mathcal{E}_{\max}}{Z_{\text{resonance}}} = \frac{\mathcal{E}_{\max}}{R} = \frac{(30.0 \text{ V})}{(5.00 \ \Omega)} = 6.00 \text{ A}$. (*c*) Now, that we are off-resonance, we must use the full expression for the impedance:

$$\begin{aligned} I &= \frac{\mathcal{E}_{\max}}{Z} = \frac{\mathcal{E}_{\max}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \\ &= \frac{(30.0 \text{ V})}{\sqrt{(5.00 \ \Omega)^2 + (2\pi(1.00\times10^3 \text{ Hz})(4.00\times10^{-3} \text{ H}) - (2\pi(1.00\times10^3 \text{ Hz})(8.00\times10^{-6} \text{ Hz}))^{-1})^2}} \\ &= 4.14 \text{ A} \end{aligned}$$

32.70. (*a*) Equation 32.47 given us the expression for the resonant angular frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi(50.0 \text{ Hz}))^2(0.350 \text{ H})} = 2.89\times10^{-5} \text{ F}$$

(*b*) At resonance the potential differences across the inductor and capacitor must be the same. Using the definition of reactance, we can write

$$V_L = IX_L = \frac{\mathcal{E}_{\max}}{Z_{\text{resonance}}} X_L = \frac{\sqrt{2}\mathcal{E}_{\text{rms}}\omega L}{R} = \frac{\sqrt{2}(120 \text{ V})2\pi(50.0 \text{ Hz})(0.350 \text{ H})}{(25.0 \ \Omega)} = 746 \text{ V}$$

Thus $V_L = V_C = 7.46\times10^2$ V.

32.71. We want the capacitance to have a value such that the resonant frequency matches the broadcast frequency of the radio station. Thus $C = \frac{1}{\omega_0^2 L} = \frac{1}{(2\pi(870\times10^3 \text{ Hz}))^2(15.0\times10^{-6} \text{ H})} = 2.23\times10^{-9}$ F. The radio could be made more selective by lowering the resistance. Figure 32.63 illustrates how the selectivity of the circuit changes with resistance.

32.72. In order for the power to be the same, with the potential difference supplied by the sources the same, the impedance must be the same. Equating the expressions for the impedances, we find

$$\sqrt{R^2 + (\omega_1 L - 1/\omega_1 C)^2} = \sqrt{R^2 + (\omega_2 L - 1/\omega_2 C)^2}$$

$$\Rightarrow (\omega_1 L - 1/\omega_1 C)^2 = (\omega_2 L - 1/\omega_2 C)^2$$

or

$$(\omega_1 L - 1/\omega_1 C) = \pm(\omega_2 L - 1/\omega_2 C)$$

Choosing $(\omega_1 L - 1/\omega_1 C) = +(\omega_2 L - 1/\omega_2 C)$ simply yields $\omega_2 = \omega_1$, and we are looking for a different solution. So we choose

$$(\omega_1 L - 1/\omega_1 C) = -(\omega_2 L - 1/\omega_2 C)$$
$$L\left(\omega_1 - \frac{\omega_0^2}{\omega_1}\right) = -L\left(\omega_2 - \frac{\omega_0^2}{\omega_2}\right)$$
$$\omega_2^2 - \frac{3}{2}\omega_2\omega_0 - \omega_0^2 = 0$$

This quadratic equation has solutions $\omega_2 = 2\omega_0$, and $\omega_2 = -\dfrac{\omega_0}{2}$. The second solution is nonsense, because we cannot have a negative frequency. Thus, we obtain the same power output if we set the source angular frequency to $\omega_2 = 2\omega_0$.

32.73. The average power is given by

$$\begin{aligned} P_{\text{av}} &= I_{\text{rms}}^2 R = \frac{\mathcal{E}_{\text{rms}}^2 R}{Z^2} \\ &= \frac{\mathcal{E}_{\max}^2 R}{2(R^2 + (\omega L - 1/\omega C)^2)} \\ &= \frac{(170\text{ V})^2(200\ \Omega)}{2((200\ \Omega)^2 + ((2\pi(400\text{ Hz}))(3.00\text{ H}) - (2\pi(400\text{ Hz})(300\times 10^{-6}\text{ F}))^{-1})^2)} \\ &= 50.8\text{ mW} \end{aligned}$$

32.74. The power is given by $P_{\text{av}} = I_{\text{rms}}^2 R = I_{\text{rms}}\mathcal{E}_{\text{rms}}$, where the last step is applicable only because the circuit contains no other circuit elements but a resistor. Rearranging, we have $I_{\text{rms}} = \dfrac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}} = \dfrac{P_{\text{av}}}{\mathcal{E}_{\max}/\sqrt{2}} = \dfrac{\sqrt{2}(1.00\times 10^3\text{ W})}{(170\text{ V})} =$ 8.32 A.

32.75. $P_{\text{av}} = I_{\text{rms}}^2 R = \dfrac{I_{\max}^2}{2} R = \dfrac{(4.00\text{ A})^2}{2}(0.500\ \Omega) = 4.00\text{ W}.$

32.76. (*a*) Ignoring any capacitive and inductive reactances, the impedance is the same as the resistance, and we can write $P_{\text{av}} = I_{\text{rms}}^2 R = I_{\text{rms}}\mathcal{E}_{\text{rms}}$, such that $I_{\text{rms}} = \dfrac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}} = \dfrac{(1875\text{ W})}{(120\text{ V})} = 15.6\text{ A}$. From that if follows that $I_{\max} = \sqrt{2} I_{\text{rms}} = \sqrt{2}(15.63\text{ A}) = 22.1\text{ A}$. (*b*) Using the same arguments as in part (*a*), we have $I_{\text{rms}} = \dfrac{P_{\text{av}}}{\mathcal{E}_{\text{rms}}} = \dfrac{(1875\text{ W})}{(220\text{ V})} = 8.52\text{ A}$ and $I_{\max} = \sqrt{2} I_{\text{rms}} = \sqrt{2}(8.523\text{ A}) = 12.1\text{ A}$.

32.77. The source could be DC in which case the inductive reactance would be zero. If an AC source is used, the minimum inductive reactance is given by $(X_L)_{\min} = \omega_{\min} L = \left(\dfrac{2\pi}{T_{\max}}\right)L$. Note that the first 0.1000 s and the second 0.1000 s are equivalent, in terms of energy dissipation. This means that a plot of power as a function of time must be symmetric about a line through $t = 0.1000$ s. This can only be the case if $\Delta t = \dfrac{nT}{4}$ or $T = 4\Delta t/n$, where $\Delta t = 0.1000$ s. Clearly the maximum period is $T_{\max} = 4\Delta t/(1)$, such that $(X_L)_{\min} = \left(\dfrac{2\pi}{4\Delta t}\right)L = \left(\dfrac{2\pi}{4(0.1000\text{ s})}\right)(70.0\times 10^{-3}\text{ H}) = 1.10\ \Omega.$

32.78. At resonance, the average power is $P_{av} = I_{rms}^2 R = \left(\frac{\mathcal{E}_{rms}}{Z}\right)^2 R = \frac{\mathcal{E}_{max}^2}{2R} = \frac{(30.0 \text{ V})^2}{2(5.00\ \Omega)} = 90.0 \text{ W},$ and $\cos(\phi) = \frac{R}{Z} = \frac{R}{R} = 1.00.$ At 1.00 kHz.

$$\begin{aligned}P_{av} &= I_{rms}^2 R = \left(\frac{\mathcal{E}_{rms}}{Z}\right)^2 R = \frac{\mathcal{E}_{max}^2 R}{2(R^2 + (\omega L - 1/\omega C)^2)}\\ &= \frac{(30.0 \text{ V})^2 (5.00\ \Omega)}{2((5.00\ \Omega)^2 + (2\pi(1.00\times10^3 \text{ Hz})(4.00\times10^{-3} \text{ H}) - (2\pi(1.00\times10^3 \text{ Hz})(8.00\times10^{-6} \text{ F}))^{-1})^2)}\\ &= 42.9 \text{ W}\end{aligned}$$ and

$$\begin{aligned}\cos(\phi) &= \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\\ &= \frac{(5.00\ \Omega)}{\sqrt{(5.00\ \Omega)^2 + (2\pi(1.00\times10^3 \text{ Hz})(4.00\times10^{-3} \text{ H}) - (2\pi(1.00\times10^3 \text{ Hz})(8.00\times10^{-6} \text{ F}))^{-1})^2}}\\ &= 0.690\end{aligned}$$

32.79. The simplest calculation is probably $\mathcal{E}_{rms} = \frac{\mathcal{E}_{max}}{\sqrt{2}} = \frac{(30.0 \text{ V})}{\sqrt{2}} = 21.2 \text{ V}.$ The impedance is given by

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{\left((100\ \Omega)^2 + ((5000 \text{ s}^{-1})(6.00\times10^{-3} \text{ H}) - ((5000 \text{ s}^{-1})(4.00\times10^{-6} \text{ F}))^{-1}))^{-2}\right)} = 102\ \Omega$$

It is now easy to calculate the power factor $\cos(\phi) = \frac{R}{Z} = \frac{(100\ \Omega)}{(101.98\ \Omega)} = 0.981.$ And from this it is trivial to calculate the phase constant $\phi = \cos^{-1}\left(\frac{R}{Z}\right) = -0.197$ radians or $-11.3°$. We can now determine $I_{rms} = \frac{\mathcal{E}_{rms}}{Z} = \frac{(21.21 \text{ V})}{(101.98\ \Omega)} =$ 0.208 A. Finally, the average power dissipated is $P_{av} = I_{rms}^2 R = (0.208 \text{ A})^2 (100\ \Omega) = 4.33 \text{ W}.$

32.80. (*a*) The phase constant is given by $\phi = \tan^{-1}\left(-\frac{1}{\omega RC}\right) = \tan^{-1}\left(-\frac{X_C}{R}\right) = \tan^{-1}\left(-\frac{(35.0\ \Omega)}{(20.0\ \Omega)}\right) =$ -1.05 radians or $-60.3°$. (*b*) $P_{av} = I_{rms}^2 R = \left(\frac{\mathcal{E}_{rms}}{Z}\right)^2 R = \frac{\mathcal{E}_{max}^2 R}{2(R^2 + (X_C)^2)} = \frac{(120 \text{ V})^2 (20.0\ \Omega)}{2((20.0\ \Omega)^2 + (35.0\ \Omega)^2)} = 88.6 \text{ W}.$ (*c*) We know $\cos(\phi) = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_C^2}} \Rightarrow X_C = R\left(\frac{1}{\cos^2(\phi)} - 1\right)^{1/2} = (25.0\ \Omega)\left(\frac{1}{(0.25)^2} - 1\right)^{1/2} = 96.8\ \Omega.$

32.81. (*a*)

$$\begin{aligned}P_{av} &= I_{rms}^2 R = \left(\frac{\mathcal{E}_{rms}}{Z}\right)^2 R = \frac{\mathcal{E}_{max}^2 R}{2(R^2 + (\omega L - 1/\omega C)^2)}\\ &= \frac{(8.0 \text{ V})^2 (5.0\ \Omega)}{2((5.0\ \Omega)^2 + (2\pi(8.0 \text{ Hz})(0.015 \text{ H}) - (2\pi(8.0 \text{ Hz})(0.010 \text{ F}))^{-1})^2)}\\ &= 6.03 \text{ W}\end{aligned}$$

(*b*) At resonance, the capacitive and inductive reactances cancel one another, and we have

$$P_{av} = I_{rms}^2 R = \left(\frac{\mathcal{E}_{rms}}{Z}\right)^2 R = \frac{\mathcal{E}_{max}^2 R}{2(R^2 + (\omega L - 1/\omega C)^2)} = \frac{\mathcal{E}_{max}^2}{2R} = \frac{(8.0 \text{ V})^2}{2(5.0\ \Omega)} = 6.40 \text{ W}$$

32.82. You want to use high ϵ_{max}. There are two relevant powers in this problem. The is the power being transferred from the farm to the storage facility P_{farm}. That is a fixed quantity. The farm will continue to have the same power regardless of how you choose to transport the power. The power transported can also be written as $P_{\text{farm}} = I_{\text{rms, wires}}\epsilon_{\text{rms}}$. The second relevant power is the power that will be lost to heating of the wire used to transfer the power, P_{loss}. Clearly, we can write $P_{\text{loss}} = I^2_{\text{rms, wires}}R_{\text{wires}}$, and we can write the current in the wires using the expression above for the power delivered by the farm $\left(I_{\text{rms, wires}} = \dfrac{P_{\text{farm}}}{\epsilon_{\text{rms}}}\right)$ to obtain $P_{\text{loss}} = \left(\dfrac{P_{\text{farm}}}{\epsilon_{\text{rms}}}\right)^2 R_{\text{wires}}$. Clearly, the power lost to the heating of the wire will be small when the ϵ_{rms} (or equivalently, the ϵ_{max}) is as large as possible.

32.83. (*a*) We start with Equation 32.58, but we modify it to use cosine functions instead of sine functions because of the initial conditions given at $t = 0$. We write the instantaneous power as $p = \mathscr{E}_{\text{max}} I\cos(\omega t)\cos(\omega t - \phi) = \dfrac{\mathscr{E}^2_{\text{max}}}{Z}\cos(\omega t)\cos(\omega t - \phi)$. We know that

$$\begin{aligned}\phi &= \tan^{-1}\left(\frac{\omega L - 1/\omega C}{R}\right)\\ &= \tan^{-1}\left(\frac{2\pi(20.0\text{ Hz})(1.00\text{ H}) - (2\pi(20.0\text{ Hz})(3.00\times10^{-3}\text{ F}))^{-1}}{(450\ \Omega)}\right)\\ &= 0.2668\text{ radians or } 15.29^\circ\end{aligned}$$

and the impedance is

$$Z = \sqrt{(450\ \Omega)^2 + (2\pi(20.0\text{ Hz})(1.00\text{ H}) - (2\pi(20.0\text{ Hz})(3.00\times10^{-3}\text{ F}))^{-1})^2} = 466.5\ \Omega$$

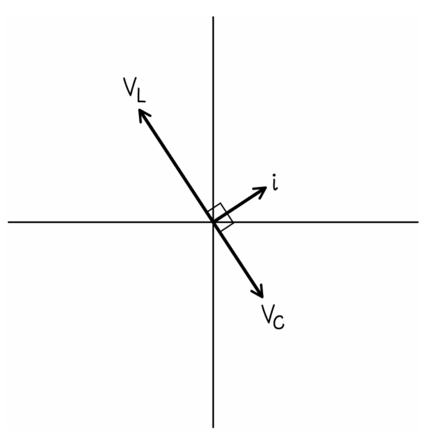
Inserting this into the above expression for the power, we have

$$\begin{aligned}p &= \frac{2\mathscr{E}^2_{\text{rms}}}{Z}\cos(\omega t)\cos(\omega t - \phi)\\ &= \frac{2(60.0\text{ V})}{(466.5\ \Omega)}\cos(2\pi(20.0\text{ Hz})(0.0200\text{ s}))\cos(2\pi(20.0\text{ Hz})(0.0200\text{ s}) - (0.2668))\\ &= 7.81\text{ W}\end{aligned}$$

(*b*) One can insert all values into the equation obtained in part (*a*). Alternatively, one can note that 0.0375 s is exactly ¾ of a period. Since the potential difference across the source started at its maximum, after ¾ of an oscillation the potential difference across the source will be zero. Hence the instantaneous power will be zero. This is verified by using the equation from part (*a*). (*c*) From part (*a*) we have

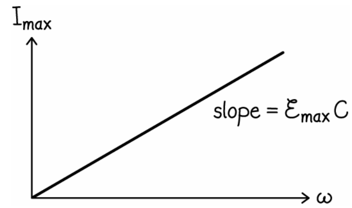
$$p = \frac{2\mathscr{E}^2_{\text{rms}}}{Z}\cos(\omega t)\cos(\omega t - \phi) = \frac{2(60.0\text{ V})}{(466.5\ \Omega)}\cos(2\pi(20.0\text{ Hz})(0.500\text{ s}))\cos(2\pi(20.0\text{ Hz})(0.500\text{ s}) - (0.2668)) = 14.9\text{ W}$$

32.84.



If each of the phasors shown are described by sine functions, then the initial phase of v_C is $\phi_C = \pi/2$, since it is at its maximum value. At this time, the inductor would have half an oscillation away, meaning it would have an initial phase of $\phi_L = 3\pi/2$ and $\phi_i = \pi$.

32.85.



32.86. There may be many possible answers. Diodes can be responsible for a great many phenomena BY controlling current direction. Very few applications can be said to have nothing to do with the current-direction control. But there are effects that are somewhat removed from the current in a steady circuit. For example, they can be used to make a rudimentary solar panel. They spontaneously develop a potential difference across their interface, effectively turning the kinetic energy of the wandering electrons into electric potential energy.

32.87. (*a*) (A,B) could be (Y,Y), (Y,N), (N,Y), or (N,N). The output is N for all combinations except (Y,Y), for which the output is Y. (*b*) The possible inputs for (A,B) are the same as in part (*a*): (Y,Y), (Y,N), (N,Y), or (N,N). Now the output is Y for all combinations except (N,N) for which the output is N.

32.88. Following Example 32.9 in *Principles*, we know

$$\tan(\phi) = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{\omega L}{R} \Rightarrow \phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{2\pi(60\text{ Hz})(6.0\times10^{-3}\text{ H})}{(3.0\ \Omega)}\right) = 37^\circ \text{ or } 0.65 \text{ radians}$$

The current is given by

$$I = \frac{\mathscr{E}_{\max}}{Z} = \frac{\mathscr{E}_{\max}}{\sqrt{R^2 + (\omega L)^2}} = \frac{(15\text{ V})}{\sqrt{(3.0\ \Omega)^2 + (2\pi(60\text{ Hz})(6.0\times10^{-3}\text{ H}))^2}} = 4.0\text{ A}$$

32.89. (*a*) We know

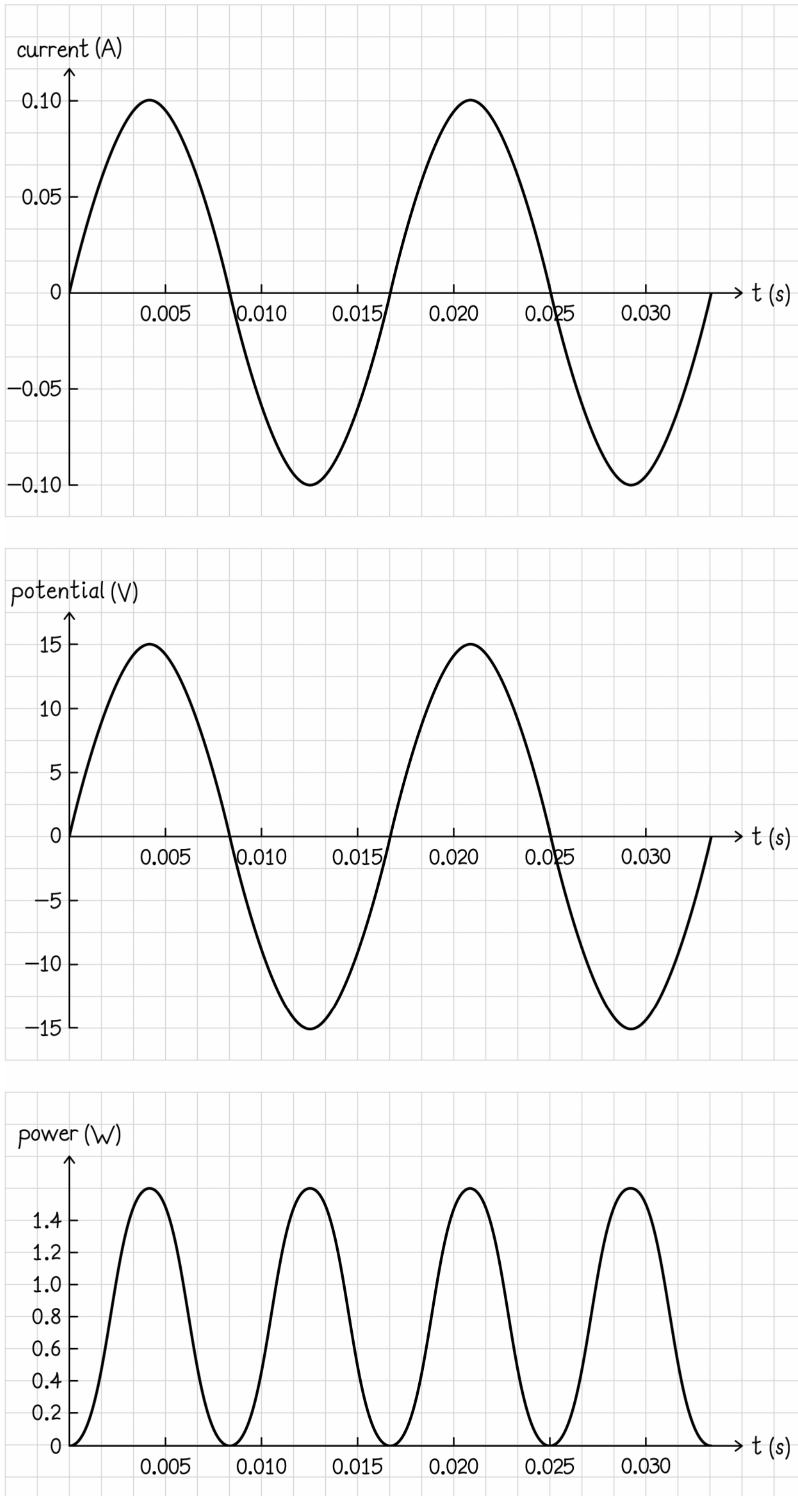
$$Z = \sqrt{R^2 + (\omega C)^{-2}} \text{ such that } \omega = \frac{1}{C\sqrt{Z^2 - R^2}} \Rightarrow$$

$$f = \frac{1}{2\pi C\sqrt{Z^2 - R^2}} = \frac{1}{2\pi(1.00\times10^{-6}\text{ F})\sqrt{((1330\ \Omega)^2 - (100\ \Omega)^2)}} = 120\text{ Hz}$$

(*b*) The phase constant is given by $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\pm\sqrt{Z^2 - R^2}}{R}\right)$. Here there is a subtlety. The root inside the inverse tangent function could be positive or negative. We choose the negative root, because of our understanding of phasors (we know the current will lead the potential difference). Thus $\phi = \tan^{-1}\left(\frac{-\sqrt{(1330\ \Omega)^2 - (100\ \Omega)^2}}{(100\ \Omega)}\right) = -85.7^\circ \text{ or } -1.50 \text{ radians.}$

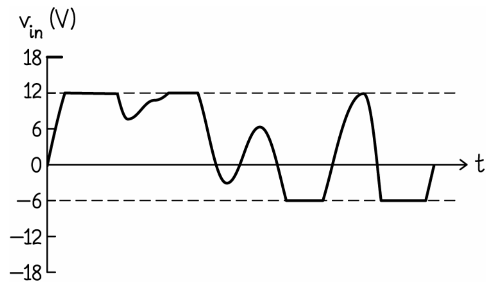
32.90. If the frequency is simply very high, all we can say with certainty is that the current will be significantly less than when the circuit is operated at resonance. At resonance, the current is $I = \frac{\mathscr{E}_{\max}}{R} = \frac{(12\text{ V})}{(20.0\ \Omega)} = 0.60\text{ A}$, so we know the current at high frequencies will be much less than this. In the limit as the frequency becomes infinite, the inductive reactance will also become infinite and the current will be zero.

32.91.



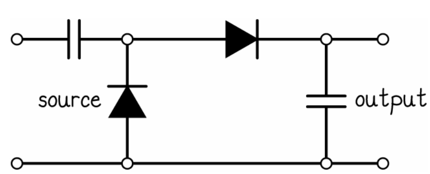
32.92. As long as the source can maintain the potential difference that it is supposed to, the brightness of the bulb should not change. However, as the frequency of the source becomes extremely fast the capacitor will effectively short the bulb and the current through the capacitor may become very high. It is likely that in this case the source may fail to operate normally (may fail to maintain a set potential difference amplitude across the bulb) and the bulb could go out.

32.93. If the potential difference is small and positive (top segment at higher potential than bottom), the right diode would allow no current to flow. The left diode could allow current to flow downward, but that would not happen because of the 12 V battery oriented so as to cause current to flow the wrong way through the diode. Only when the positive potential exceeds 12 V can current flow through the left diode, ensuring that the potential difference between the upper and lower segments remains 12 V. The diode on the right plays no role until the potential becomes negative. If the potential is small and negative, the 6 V battery ensures that no current flows across the right hand diode. But if the potential becomes more negative than –6 V, the net potential is such that current can flow across the diode, maintaining a potential difference of –6 V. Thus, potentials higher than positive 12 V and lower than –6 V are "cut off" as shown in the figure.



32.94 Because the current lags behind the potential difference, we know the element must be an inductor. We can read from the graph that the period is 0.50 s and the current amplitude is 0.30 A. We can use this to determine the current as a function of time: $i(t) = I\sin(2\pi t/T)$. We know that the potential difference across an inductor is given by $v_L = -LI\left(\frac{2\pi}{T}\right)\cos\left(\frac{2\pi t}{T}\right)$. In particular, let us equate the maxima of the left hand side and right hand side to obtain $V_L = LI\left(\frac{2\pi}{T}\right) \Rightarrow L = \frac{V_L T}{2\pi I} = \frac{(0.30\text{ V})(0.50\text{ s})}{2\pi(0.30\text{ A})} = 80$ mH. Thus the mystery element is an 80 mH inductor.

32.95. You can arrange the diodes and capacitors as shown below.



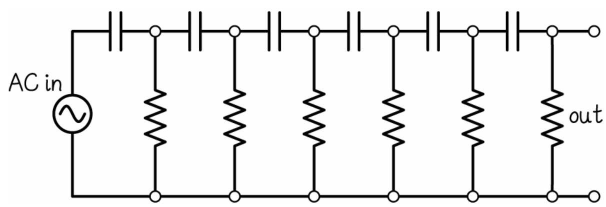
This setup is called a "voltage doubler", or more generally a "voltage multiplier".

32.96. In the circuit shown, the capacitor is in parallel with the series combination of the inductor and the radio. This means that the potential difference across the combination of the inductor and the radio must be that shown in (c). Clearly, the inductor will not resist the flow of a DC current, so only the "wiggle" will be affected by the inductor. If we wish to reduce the amplitude of the wiggle across the radio to half, then we want the other half of the potential difference "wiggle" across the inductor. Thus we require

$$V_{L,\text{wiggle}} = \frac{1}{2}V_{\text{wiggle}}$$
$$\frac{\mathcal{E}_{\text{max,wiggle}}}{Z}(\omega L) = \frac{1}{2}\mathcal{E}_{\text{max,wiggle}}$$
$$4(\omega L)^2 = (R^2 + (\omega L)^2)$$
$$L = \frac{R}{\omega\sqrt{3}} = \frac{(20.0\ \Omega)}{2\pi(120\ \text{Hz})\sqrt{3}} = 15.3\ \text{mH}$$

32.97. There may be many possible answers. Using only a single high-pass filter is not sufficient. The required attenuation of the "noise" (reduced by a factor of 10) would also reduce the desired signal by a factor of around 5. But there are many ways around that. For example, one could use six successive high-pass filters operating at a cutoff frequency of $1.7\ \text{kHz}$ to attenuate the noise by a factor of 10 times more than the desired signals are attenuated. One can then use voltage multipliers to regain the original signal strength.

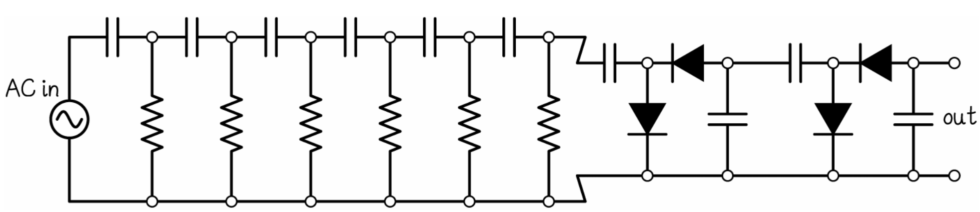
To see how this would work, consider the circuit below, consisting of six successive high-pass filters. The capacitance of each capacitor is $93.6\ \mu\text{F}$, and the resistance of each resistor is $1.00\ \Omega$.



After each successive filter, the potential is reduced by a factor $\frac{V_{\text{step}}}{V_{\text{in}}} = \frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$, such that the effect of the total circuit is $\frac{V_{\text{out}}}{V_{\text{in}}} = \left(1+\left(\frac{\omega_c}{\omega}\right)^2\right)^{-n/2}$. Inserting the angular frequency of the noise and the lowest of the desired frequencies, we find

$$\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)_{\text{noise}} = \left(1+\left(\frac{\omega_c}{\omega_{\text{noise}}}\right)^2\right)^{-n/2} = \left(1+\left(\frac{2\pi(1.7\ \text{kHz})}{2\pi(1.1\ \text{kHz})}\right)^2\right)^{-6/2} = 0.026$$
$$\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)_{\text{desired}} = \left(1+\left(\frac{\omega_c}{\omega_{\text{desired}}}\right)^2\right)^{-n/2} = \left(1+\left(\frac{2\pi(1.7\ \text{kHz})}{2\pi(2.0\ \text{kHz})}\right)^2\right)^{-6/2} = 0.196$$

Clearly, the noise is nearly gone and clearly the desired signal is also seriously attenuated. What is important here is that the noise has been attenuated by a factor of nearly 10 more than the desired signal. Adding a voltage multiplier to the circuit allows us to increase the overall strength until we have out desired signal strength. For a description of voltage multipliers, see Problem 32.95. The entire circuit is shown below.



An alternative would be to construct an RLC circuit that resonates in the range of your desired frequencies and has a fairly low resistance. In order for the resonance peak to be narrow enough to sufficiently attenuate the noise, it is not likely that the entire range of desired frequencies could work with just one such circuit. It is more likely that you would need to build in a variable inductor. As you sweep through different frequencies, one would need to adjust the inductor accordingly to keep the resonance peak at the operating frequency.

33

RAY OPTICS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

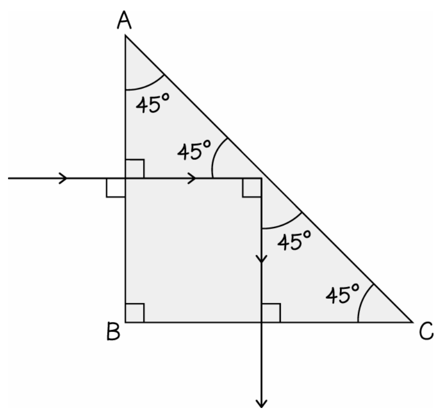
Developing a Feel

1. 10^3 m **2.** 10^6 m **3.** 10^{-3} m **4.** 10^0 rad or 10^2 degrees **5.** 10^{-2} rad or 10^0 degrees **6.** 10^2 s **7.** 10^{-3} m **8.** 10^0 m **9.** 10^{-1} m **10.** 10^4 m

Guided Problems

33.2 Prism in air or liquid

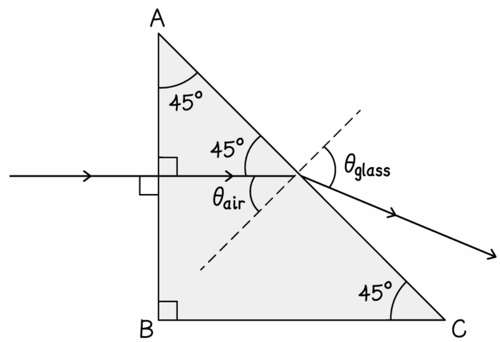
1. Getting Started We begin by sketching the path the light will follow in and around the prism when the prism is surrounded by air.



We are told that the beam undergoes total internal reflection when the prism is surrounded by air. Since the (initially horizontal) incident beam makes a $45°$ angle with the diagonal face of the prism, the reflected beam will also make a $45°$ angle with that face. Simple geometry tells us that the angle between the incident and reflected rays will thus be $90°$.

It is worth noting that this case was significantly simplified by the fact that the incoming ray of light was incident on the outside of the prism normal to the vertical face. That allowed us to ignore refraction at that first air-glass interface, and simply state that the incident ray continues in a straight light without changing direction. If the angle from normal of the incident ray had been anything but $0°$, we would have needed Snel's Law: $n_{\text{air}}\sin(\theta_{\text{air}}) = n_{\text{glass}}\sin(\theta_{\text{glass}})$.

2. Devise Plan The properties of the light ray inside the material of the prism cannot depend on what material surrounds the prism. The surrounding material can only play a role at the interface between the prism and the surrounding material. So the geometry inside the prism will be independent of the surrounding material except where the light strikes the diagonal face AC of the prism. At that point, the light could either be totally internally reflected, or it could exit the prism, refracting as it does so. In that case the refracted beam would look something like this:



In the figure above, we have shown $\theta_{\text{glass}} > \theta_{\text{air}}$, as is the case when the fluid has a lower index of refraction than the glass. Although there may be some exotic fluids with an index of refraction larger than that of glass, it is most common that $n_{\text{fluid}} < n_{\text{glass}}$.

The angle at which the ray strikes the face AC is determined entirely by the geometry of the prism, since the light enters the left face (AB) normal to the surface. It is easy to see that the angle between the incident ray and the dashed line normal to the face AC is $45°$. When this $45°$ angle is greater than the critical angle, there will be total internal reflection. When this $45°$ angle is less than the critical angle, light will escape the prism. To find the limiting case (the minimum fluid index of refraction) we set the incident angle of $45°$ equal to the critical angle. That is

$$\theta_{\text{incident}} = \theta_c = \sin^{-1}\left(\frac{n_{\text{fluid}}}{n_{\text{glass}}}\right) \tag{1}$$

We can solve equation (1) for the index of refraction of the fluid, trivially, if we look up the index of refraction of flint-glass. From Table 33.1, we see $n_{\text{glass}} = 1.65$.

3. Execute Plan Rearranging equation (1), we have

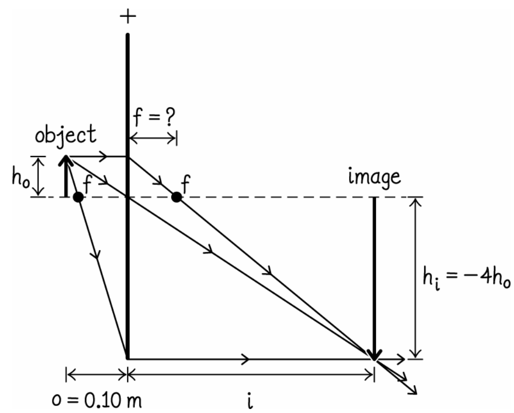
$$n_{\text{fluid}} = n_{\text{glass}} \sin(\theta_{\text{incident}})$$
$$n_{\text{fluid}} = (1.65)\sin(45°)$$
$$n_{\text{fluid}} = 1.17$$

4. Evaluate Result This is a physically plausible value of the index of refraction for a fluid. Indices of refraction for liquids tend to be somewhat higher; for example for water the value is 1.33. The index of refraction of air is lower (1.00029). But a dense gas, or a mixture of steam and air could certainly produce an index of refraction between those of air and water, such as the value of 1.17 that we obtained.

33.4 Image on paper

1. Getting Started To identify the lens, we wish to determine the magnitude and sign of its focal length, f. We know that the lens must be a converging lens, also called a convex lens. A convex lens collects light together and can form a real image. A diverging lens (also called a concave lens) causes light rays to diverge and can only form virtual images. The fact that the lens must be convex means that we expect its focal length to be positive.

We begin by constructing a simplified ray diagram.



2. Devise Plan The size of the image is related to the size of the object through the magnification. This quantity also relates the image and object distances:

$$m = \frac{h_i}{h_o} = -\frac{i}{o} \tag{1}$$

The object and image distances are related to the focal length through

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{2}$$

We can rearrange equation (1) and insert it into equation (2) to determine the focal length of the lens.

3. Execute Plan We rearrange equation (1) to yield $i = -o\dfrac{h_i}{h_o}$, and insert this into equation (2) to find

$$f = o\left(1 - \frac{h_o}{h_i}\right)^{-1} = (0.10\ \text{m})\left(1 - \frac{h_o}{-4h_o}\right)^{-1} = 0.080\ \text{m}$$

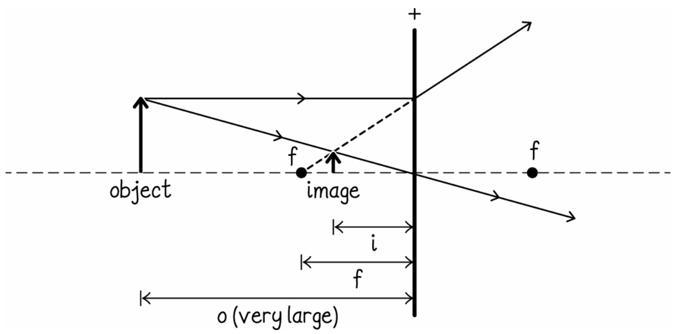
So the focal length is positive and equal to 80 mm.

4. Evaluate Result In drawing our simplified ray diagram, we found that the object had to be very close to the focal length to yield such a large magnification. Of course, in order to produce a real image, the object distance had to be larger than the focal length, but just barely. Numerically, we have an object distance of 100 mm, and we found a focal length of 80 mm. This is very consistent with our simplified ray diagrm.

33.6 Correcting near-sightedness

1. Getting Started A –3.00 diopter lens is diverging. We know this because of the sign of the focal length; a negative focal length always corresponds to a diverging lens. According to the problem statement, this lens "corrects" a person's vision by forming an image of distant objects at the person's far point.

We begin by constructing a simplified ray diagram for the diverging lens and a distant object. We are not told an exact object distance, we only know that the object distance will be large compared to other lengths in the problem. Eventually, we will take the limit as the object distance approaches infinity.



We see that the image appears to be very small, but this is reasonable for a very distant object. If the object distance truly became infinite, of course the image would disappear, since we would not expect a person to be able to see things infinitely far away.

We wish to determine the location of the image when the object is very distant.

2. Devise Plan The focal length can be determined directly from the lens strength:

$$f = \frac{1}{D} \tag{1}$$

The focal length is related to the object and image distances according to

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{2}$$

We wish to determine the focal length from equation (1) and insert that value into equation (2). We then take the limit as $o \to \infty$ to solve for the image distance i.

3. Execute Plan Inserting numbers, we find $f = \dfrac{1}{(-3.00 \text{ diopters})} = -0.333 \text{ m}$. Taking the limit as the object distance approaches infinity, we find

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \to \frac{1}{i}$$
$$i = f = -0.333 \text{ m}$$

So the image distance is –0.333 m, meaning the far point is 0.333 m in front of the lens (which is approximately the same as 0.333 m in front of the eyes). Thus we report the far point of the person's vision as 0.333 m.

4. Evaluate Result We know that the focal length is the distance from the lens to the point at which light will be focused when incident on the lens parallel to the lens axis. Or, in the case of the diverging lens, light parallel to the lens axis will emerge from the lens as though it emanated from the focal point. Light from a very distant object will reach the lens approximately parallel to the lens axis, and should be scattered such that it appears to come from the focal point. Thus it is perfectly reasonable to find that the image distance is the same (in magnitude) as the focal length. Of course, if the object were truly infinitely far away, the size of the image would also approach zero. In reality we expect a person to view objects that are very distant, but not infinitely far away, and we expect images to be formed very close to 0.33 m in front of their eyes, but not at exactly that point.

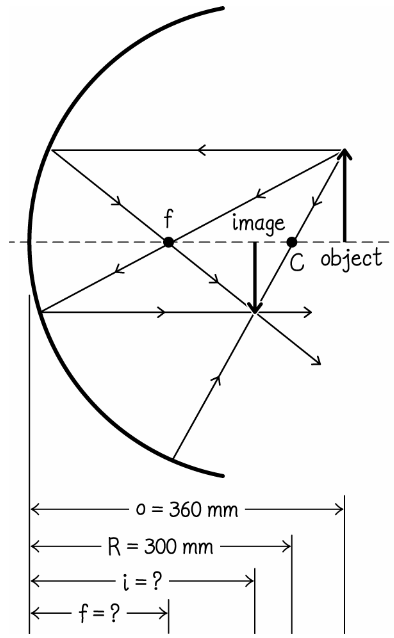
33.8 Reflecting flames

1. Getting Started In a real image, light from an object is collected together and actually reaches an object such as a viewing screen or sheet of paper. In a virtual image, light is scattered and diverges in such a way that it appears to have originated at a point that is the location of the virtual image. In the formation of a virtual image, light is not actually collected together; it merely appears to have originated from an image. Thus, a virtual image cannot be cast onto a sheet of paper. Only a real image can be.

Converging mirrors can focus light down to a point, and can be used to form a real image. Diverging mirrors scatter light and can only form virtual images.

Clearly, if we are using a mirror to cast an image onto a sheet, it must be a converging mirror.

2. Devise Plan We begin by making a simplified ray diagram.



Note that the image is inverted, which means it is real. We know that the focal length for a converging mirror is positive. We also know that the focal length is related to the radius of curvature of the mirror according to

$$f = R/2 \tag{1}$$

That focal length can then be used to relate the object and image distances through

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{2}$$

This is sufficient information to solve the problem; we need only insert given values.

3. Execute Plan Using the given radius of curvature in equation (1), we find $f = (300 \text{ mm})/2 = 150 \text{ mm}$. Inserting this focal length and the object distance into equation (2), we find

$$i = \left(\frac{1}{f} - \frac{1}{o}\right)^{-1} = \left(\frac{1}{(150 \text{ mm})} - \frac{1}{(360 \text{ mm})}\right)^{-1}$$

$$i = 257 \text{ mm}$$

This image corresponds to a magnification given by

$$\frac{h_\text{i}}{h_\text{o}} = m = -\frac{i}{o} = -\frac{(257 \text{ mm})}{(360 \text{ mm})} = -0.714$$

This means the image is inverted and reduced in size by about 71%.

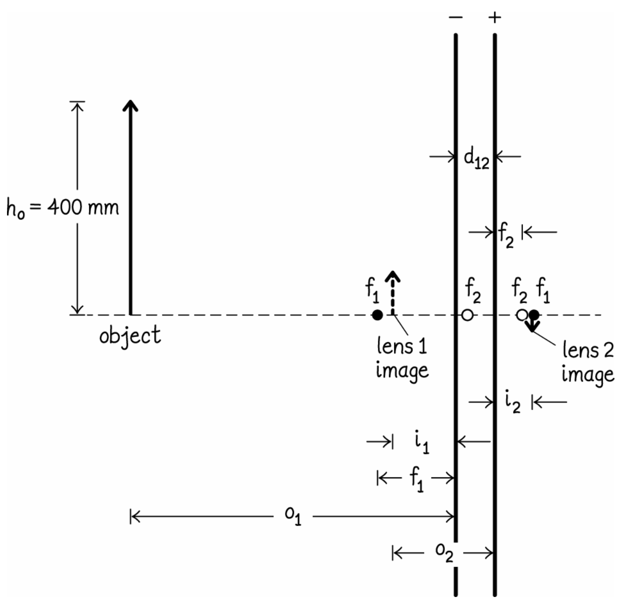
4. Evaluate Result In getting started, we pointed out that only a converging mirror can produce a real image. However, a converging lens need not always produce a real image. When an object is brought too close to the mirror (inside the focal length) the rays of light from the object are spreading out at such a rate that the mirror is not able to focus them into a real image. In that case, a virtual image is formed. Our calculations were based on a real image, so the image distance we calculated should be larger than the focal length. Indeed we see that is the case since the image distance of 257 mm is well outside the focal length of 150 mm.

The size of the image should be about 71% of the original object, and the image should be inverted. This is consistent with the ray diagram we constructed.

33.10 Double lens

1. Getting Started When light from the plant passes through the diverging lens, it will form a virtual image. Since light appears to have originated from that virtual image, that virtual image will play the role of the object for the second (converging) lens. That converging lens must then focus the light into a real image on the camera sensor.

We begin by drawing a diagram of the setup.



As the diagram suggests, the image cast onto the sensor of a camera will be much smaller than the object being photographed. That is a physical necessity, and lends credibility to our diagram. However, it also means that the ray diagram will become very compact, and difficult to read. It may be necessary to separate the problem into two ray diagrams, giving each portion its own relevant length scale.

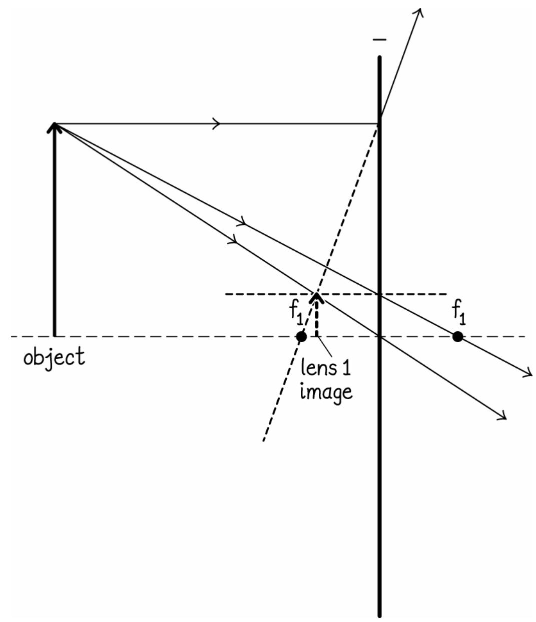
We want to find the location at which a clear image from the converging lens is formed (lens 2 image in the figure above). That is where the sensor must be in order for the camera to take a clear photograph.

2. Devise Plan We should draw a ray diagram showing the rays of light leaving the plant, reaching the diverging lens and forming a virtual image. We should then draw a separate ray diagram (with a different length scale) showing that image from the diverging lens acting as the object for the second, converging lens. In that diagram we should show the principle rays that form the final, real image to be cast on the camera sensor.

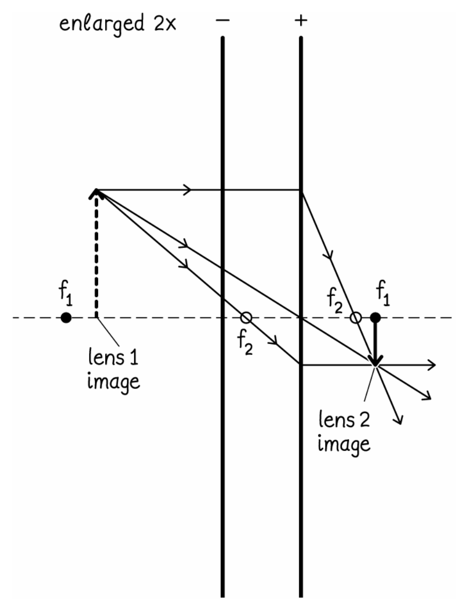
For each lens, we will find the image distance given the object distance and focal length using

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \tag{1}$$

3. **Execute Plan** We draw the ray diagram for the diverging lens first:



We now use the image from lens 1 as the object for lens 2. Note that the diagrams have different length scales.



The image distance for the diverging lens is given by

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o_1}\right)^{-1} = \left(\frac{1}{(-120\text{ mm})} - \frac{1}{(5.00\times 10^2\text{ mm})}\right)^{-1}$$

$$i_1 = -96.8\text{ mm}$$

Here, the negative sign means that the virtual image is formed on the same side of the lens as the object (which we knew from our ray diagram). When this image acts as the object for the second, converging lens, the object distance will be the distance from the image to the diverging lens, plus the spacing between lenses:

$o_2 = |i_1| + d_{12} = (96.8\text{ mm}) + (60\text{ mm}) = 156.9\text{ mm}.$ Here we have kept one additional significant digit, because this is an intermediate calculation. The image distance from the converging lens is then found according to

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1} = \left(\frac{1}{(42\text{ mm})} - \frac{1}{(156.8\text{ mm})}\right)^{-1}$$
$$i_2 = 57\text{ mm}$$

4. Evaluate Result The distance between the converging lens and the sensor is 57 mm, very similar to the spacing between lenses. This is consistent with our ray diagrams. The total distance from the sensor to the farthest lens would be 117 mm, which is about 4 ½ inches (for those accustomed to imperial units). This is a reasonable length for the lens structure of a camera in its most extended state. Many specialized cameras and lenses are much larger (such as telephoto lenses).

Questions and Problems

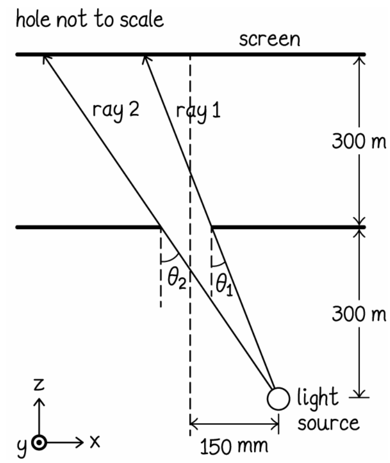
33.1. (*a*) Light from all sources (a-h) strikes A. Light will emanate radially outward from all sources, and there exists a straight line path (ray) from each light to detector A. (*b*) Now there is a possibility that light from some sources might not reach detector B, because it might lie in the shadow of detector A. Note that a line from the right-most edge of detector A to the right-most edge of detector B makes an angle $\theta_{\text{max}} = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{(0.29\text{ m})}{(0.20\text{ m})}\right)$ $= 55.4°$ with the vertical. If a ray from a source passes the right-most edge of detector A at an angle smaller than this, it will strike detector A. If light from a source passes the right-most edge of detector A at an angle greater than this, it will miss the detector. One can determine the maximum horizontal distance from the right most edge of detector A where a source could be placed, or one could simply check a few sources to see if they work. Simple geometry shows

$$\theta_{\text{A}} = \tan^{-1}\left(\frac{\Delta x_{\text{A}}}{\Delta y_{\text{A}}}\right) = \tan^{-1}\left(\frac{(0.41\text{ m})}{(0.20\text{ m})}\right) = 64.0° > \theta_{\text{max}}$$
$$\theta_{\text{B}} = \tan^{-1}\left(\frac{\Delta x_{\text{B}}}{\Delta y_{\text{B}}}\right) = \tan^{-1}\left(\frac{(0.31\text{ m})}{(0.20\text{ m})}\right) = 57.2° > \theta_{\text{max}}$$
$$\theta_{\text{C}} = \tan^{-1}\left(\frac{\Delta x_{\text{C}}}{\Delta y_{\text{C}}}\right) = \tan^{-1}\left(\frac{(0.21\text{ m})}{(0.20\text{ m})}\right) = 46.4° < \theta_{\text{max}}$$

Since the angles for all subsequent sources will be smaller than this, we can say with confidence that light from six sources (c-h) strikes B. Light from sources a and b is obscured by detector A and cannot reach detector B.

33.2. Since rays of light reach the same points on the screen and originate from the same source location, the outermost rays of light make the same angle with the z axis as when the first sheet was used. Call this widest angle at which rays of light pass through the hole in the cardboard θ_{max}. Call the side length of square hole in the first piece of cardboard ℓ_1, and that in the second piece ℓ_2. Call the side length of the bright image on the screen ℓ_{screen}. Since $\theta_{\text{max},1} = \theta_{\text{max},2}$, we can say $\tan(\theta_{\text{max},1}) = \tan(\theta_{\text{max},2}) \Rightarrow \frac{\ell_1}{\Delta z_1} = \frac{\ell_{\text{screen}}}{\Delta z_{\text{screen}}} = \frac{\ell_2}{\Delta z_2}$. Thus $\ell_2 = \frac{\Delta z_2 \ell_1}{\Delta z_1}$ $= \frac{(0.25\text{ m})(0.040\text{ m})}{(0.50\text{ m})} = 0.020\text{ m}.$ Because the hole is a square, the side length could refer to either the height or the width, so the dimensions are $(0.020\text{ m}) \times (0.020\text{ m})$.

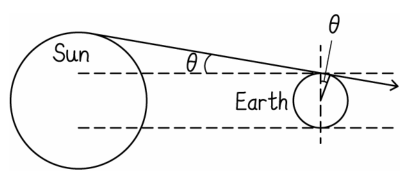
33.3. (*a*) Consider the figure below. Note that the dimensions of the hole in the screen have been somewhat exaggerated for clarity of the picture.



Because ray 1 travels the same distance in z before reaching the hole and between the hole and the screen, it will travel the same distance along the x axis before reaching the hole and between the hole and the screen. Clearly this distance along the x axis is 135 mm (from $x = 150$ mm to $x = 15$ mm). 135 mm to the left of the right-most edge of the hole corresponds to $x = -120$ mm on the screen. Similarly, ray 2 travels 165 mm parallel to the x axis before reaching the hole, and will reach a point on the screen $x = -180$ mm. Thus the width of the image in the x direction is 60 mm.

Similar arguments demonstrate that the highest (in the y direction) ray that passes through the hole will travel 15 mm parallel to the y axis on the way to the hole, and another 15 mm parallel to the y axis between the hole and the screen, meaning it will strike the screen at a position $y = 30$ mm. Similarly, the lowest ray of light that passes through the hole will strike the screen at the position $y = -30$ mm. Thus, the dimensions of the bright image on the screen are $(60 \text{ mm}) \times (60 \text{ mm})$. (*b*) The points that mark the corners of the perimeter are (–120 mm, 30 mm), (–120 mm, –30 mm), (–180 mm, 30 mm), (–180 mm, –30 mm). Alternatively, one could express the perimeter as the lines $x = -120$ mm, $x = -180$ mm, $y = -30$ mm, and $y = 30$ mm.

33.4. (*a*) Consider the figure below, in which the distances and sizes of objects have been drastically altered for clarity of geometry.

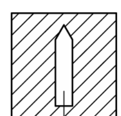


The umbra is nearly a half of a spherical shell, with the half of Earth farthest from the sun being in shadow. However, the fact that the sun is so much larger than Earth means that light emitted from the very outer edge of the sun may strike Earth slightly on the far side of Earth. Let the z axis point directly from the center of the sun through the center of the Earth, exiting the Earth on its dark side (the center of the umbra). The umbra extends across all points on the surface of the Earth from an angle of $0°$ from this z axis, to some angle very close to $90°$. We calculate the small angle away from $90°$ at which light may still strike the distant side of Earth (θ in the figure above). Simple geometry shows that this small angle is the same as the angle from the z axis to the outer edge of the sun: $\theta = \tan^{-1}\left(\dfrac{R_{\text{Sun}} - R_{\text{Earth}}}{D_{\text{Earth-Sun}}}\right) = \tan^{-1}\left(\dfrac{(7\times10^{8} \text{ m}) - (6.738\times10^{6} \text{ m})}{(1.496\times10^{11} \text{ m})}\right) = 0.27°$. Thus, the umbra extends across all points on the surface of Earth from an angle of $0°$ from the z axis to $89.73°$ from the z axis. Equivalently, the umbra is $0.27°$ shy of being a complete half-sphere.

The penumbra is a band of Earth's surface having an angular width of $0.53°$. This is simply twice the angle obtained in part (*a*), because we must consider light from the opposite side of the sun. This angular spread is equivalent to a band with a width of approximately 60 km. (*b*) A person can see part of the sun from the penumbra. Geometrically speaking, one cannot see the sun from the umbra, although light is bent significantly by Earth's atmosphere. There may be a very narrow region of the umbra in which the geometry of the solar system is not sufficient to explain why one can see some sunlight. One would have to consider the index of refraction of the atmosphere at different altitudes.

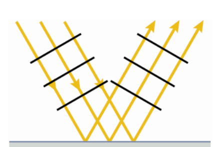
33.5. (*a*) In model A, light leaves the surface of the bulb radially outward from the center of the bulb. In model B, light leaves the bulb radially outward from a few specific points on the bulb.(*b*) Experiments could be done to see if there were a few key points emitting most of the light (such as the four points shown in the figure). An example of such an experiment would be to place a 2-sided solar panel pointed directly at the center of the light. If you move the panel around the surface (taking care to keep its length normal to the surface of the bulb) a disparity in the voltages produced by the panels on either side would indicate a region of higher intensity on one side than the other. If, however, the figure is suggesting that a continuum of points along the surface of the bulb can be thought of as emitting light (rather than the light emanating from the center of the bulb) then there is no experiment that can distinguish models A and B. This is Huygens' Principle.

33.6. Assume a vertex of the triangle points vertically upward. The bright region would be shaped like a tall rectangle with a pointed top, as shown below.

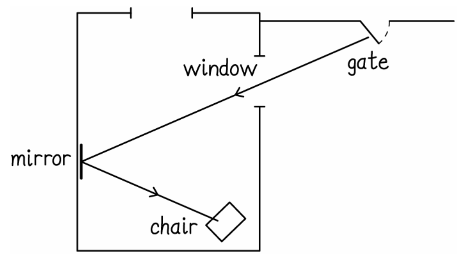


33.7. We are not told anything about the size of the hole. Let us initially assume that it is large on the scale of light rays (several millimeters or larger). In this case, light from throughout the flame would pass through the large hole and produce a bright spot in roughly the same of a circle. The position of the circle may fluctuate as variations in the candle's flame cause the brightest spot to move slightly. The boundary of the circle would not be sharp, as light comes from a 1-3 cm region of flame, rather than a single point source. If, instead, the hole is extremely small then (relatively) few light rays from a given region of the flame could pass through the hole. Consider two parts of the flame: region A and region B, with region B being higher than region A. Any rays of light from region A that passes through the hole will be nearly parallel, and will strike the screen in a small well-defined region. Also, simple geometry tells us that light rays from region B will strike the screen below the light rays from region A. This argument could be extended to describe light coming from many such regions of the flames. Bright regions would produce bright images on the screen, reddish regions would produce reddish regions on the screen, and so on, meaning that an image of the entire flame as a whole will appear on the screen. But as we noted with points A and B, the image must be inverted. If the candlestick or other nearby objects were very strongly illuminated, their images might also appear on the screen. This setup is known as a "pinhole camera".

33.8.

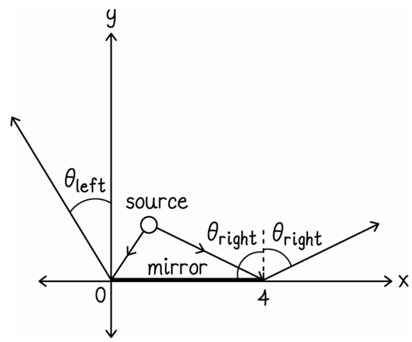


33.9.



33.10. The shirt is actually reflecting blue light. The shirt absorbs colors in the visible spectrum that are not blue. The "blueness" that the students sees is not in the shirt, nor did it originate in the shirt.

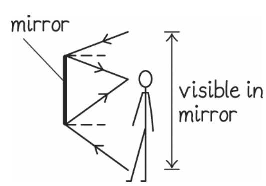
33.11. Consider the figure below.



The rays drawn are the right-most and left-most rays that can be reflected by the mirror. We can describe the left-most line by $y = -2x$, and the right-most line by $y = \frac{1}{3}(x-4)$, but only after the rays have been reflected. Thus the positions in the y direction at which the image can be seen for a given

$$x: \begin{cases} x > 4 & y > (x-4)/3 \\ 0 < x < 4 & \text{all } y > 0 \\ x < 0 & y > -x \end{cases}$$

33.12.



33.13. The reason we obtain a different result for viewing objects in a mirror as compared to viewing your own image, is that when viewing yourself the object distance and image distance is always the same. When viewing distant objects, the image distance and object distance can be drastically different. Assume you are looking into a mirror at the distant object behind you. Call the distance from you to the distant object d_o, call the distance from you to the mirror d_m, call the height of the distant object h_o, and call the height of the mirror h_m. Consider the angles of incidence and reflection for an arbitrary ray of light striking the mirror. Since they are identical, we can write

$$\frac{\Delta y_{\text{object-mirror}}}{\Delta x_{\text{object-mirror}}} = \tan(\theta_i) = \tan(\theta_r) = \frac{\Delta y_{\text{eye-mirror}}}{\Delta x_{\text{eye-mirror}}} \quad (1)$$

We can apply equation (1) to the highest ray and the lowest ray that make it from the object (reflecting off the mirror) and to the eye. This yields

$$\Delta y_{\text{eye-mirror,highest}} = \frac{\Delta x_{\text{eye-mirror}}}{\Delta x_{\text{object-mirror}}} \Delta y_{\text{object-mirror,highest}} = \frac{d_{\text{i}}}{d_{\text{o}}} \Delta y_{\text{object-mirror,highest}} \quad (2)$$

$$\Delta y_{\text{eye-mirror,lowest}} = \frac{\Delta x_{\text{eye-mirror}}}{\Delta x_{\text{object-mirror}}} \Delta y_{\text{object-mirror,lowest}} = \frac{d_{\text{i}}}{d_{\text{o}}} \Delta y_{\text{object-mirror,lowest}} \quad (3)$$

The highest ray and the lowest ray making it to the eye will cover a combined distance $\Delta y_{\text{eye-mirror,highest}} + \Delta y_{\text{eye-mirror,lowest}} = h_{\text{m}}$, and we are requiring that these rays can come from the entire height of the distant object, so $\Delta y_{\text{object-mirror,highest}} + \Delta y_{\text{object-mirror,lowest}} + h_{\text{m}} = h_{\text{o}}$. Using these facts and adding equations (2) and (3), we obtain

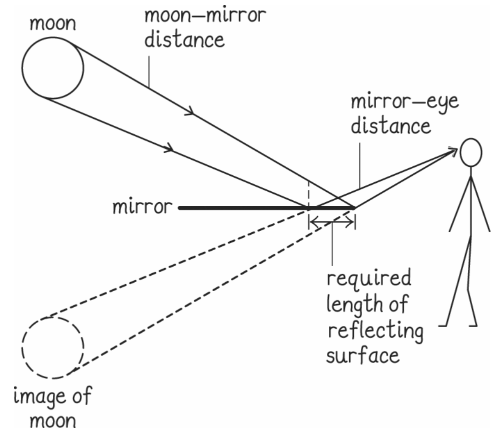
$$h_{\text{m,min}} = \Delta y_{\text{eye-mirror,lowest}} + \Delta y_{\text{eye-mirror,highest}} = \frac{d_{\text{i}}}{d_{\text{o}}}(\Delta y_{\text{object-mirror,lowest}} + \Delta y_{\text{object-mirror,highest}}) = \frac{d_{\text{i}}}{d_{\text{o}}}(h_{\text{o}} - h_{\text{m,min}}) \Rightarrow h_{\text{m,min}} = \frac{d_{\text{i}} h_{\text{o}}}{d_{\text{i}} + d_{\text{o}}}$$

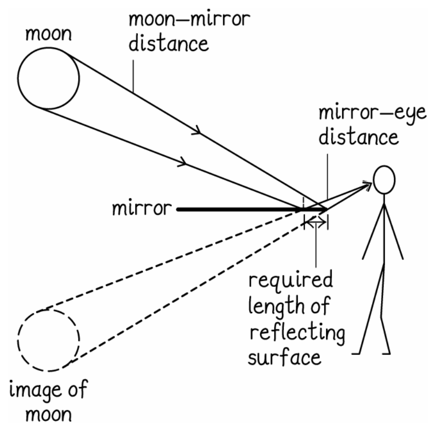
33.14. If the image appears to be 2.0 m behind the mirror, then the object is 2.0 m in front of the mirror. This means the flashlight is 1.0 m behind you. Heights and distances parallel to the plane of the mirror should be preserved, so the flashlight is 1.0 m above your head, and 3.0 m to your right.

33.15. There may be many correct answers. Let us suppose at first that we are interested in planar mirrors only. We might first try a very qualitative example. In that case, we might stand 1.0 m from a mirror and reach one hand out toward the mirror. If we slowly approach the mirror with outstretched fingers, hopefully our friend will agree that the reflection appears to get closer and closer to the actual fingers (approaching the plane of the mirror). If the reflection is approaching the plane of the mirror from behind, then clearly the image is behind the mirror by at least some small distance. Alternatively, we could get quantitative by taking two laser pointers, with a small fixed angle between them. Shining both laser pointers into the mirror would result in two reflected beams that continue to spread out. By measuring where the beams strike a nearby wall, and measuring where the bright spots on the glass of the mirror occur, you can determine the distance to the image. You will find that the image is behind the mirror. If we are not restricted to planar mirrors, we could demonstrate the difference between a real image and a virtual image by casting a real image onto a screen. Then place a screen on the mirror's surface. Note that no visible image appears on the screen. Hence, the image is not being formed at the mirror's surface.

33.16. (*a*) Both the boy and his reflection approach the mirror at a speed of 1.0 m/s. (*b*) The woman approaches the boy at 1.0 m/s, whereas his reflection approaches him at 2.0 m/s. This is because he and the mirror move closer together at a rate of 1.0 m/s, and the same is true of his reflection.

33.17. (*a*) The image of the moon contains all the same features, regardless of what reflective surface is used to view it. The image of the moon should not change size, significantly. But the surface area of reflective material required to view the entire image will change. (*b*) For a planar mirror, the image size is determined by the distance light travels from the object to your eye (from your eye to the reflecting surface, and by the distance from the reflecting surface to the object being viewed) and by the size of the original object. Because the Earth-moon distance is so much larger than any plausible distance from your eye to a reflecting surface, the size of the image will not change substantially. But the surface area of the reflective material required for you to see the object does depend on the distance from your eye to the surface. The figures below show you viewing the moon with your eyes about a meter from a reflecting surface, and just a few centimeters from the surface. Note that the image is unchanged, but the length of the reflecting surface required is very different.





33.18. (*a*) Light from the top of your hat strikes the very top of the mirror and reflects to your eyes. Since the angles of incidence and reflection are the same, such a ray of light must cross half the vertical distance to eyes on the way to the mirror, and half the vertical distance on the way from the mirror to the eyes. Thus, the vertical distance from the top of the mirror to the eyes is half the distance from the eyes to the top of the hat. The top of the mirror is 55.0 mm above the eyes. (*b*) By the same arguments as in (*a*), light from the belt buckle will travel half the vertical distance to the eyes before striking the mirror, and half the distance after striking the mirror. Thus, it must strike the bottom edge of the mirror 400 mm below the eyes. Since the bottom edge is 400 mm below the eyes and the top edge is 55 mm above the eyes, the entire height of the mirror is 455 mm.

33.19. The shadow is smaller when the tub is filled with water than when the tub is empty. Consider a ray of light that travels from the bulb to the very edge of the boat, and then down to the bottom of the tub. This ray of light will mark the edge of the shadow on the tub. When the tub is empty, the path this ray follows is a straight line all the way from the bulb, past the boat, and to the bottom of the tub. When the tub is filled with water, the light bends at the air-water interface. We know it bends toward a normal line, meaning its angle with the vertical decreases as it enters the water. Thus, the ray will not spread as far (horizontally) as it did in the absence of water. This argument applies for all the rays marking all edges of the shadow. Thus, the shadow will be smaller with water in the tub.

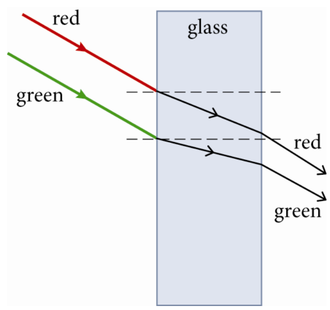
33.20. (*a*) We know from the chapter and from Figure 33.22b that short wavelength light (toward the blue end of the spectrum) bends more at an interface between two materials than does red light. Light at the red end of the spectrum (long wavelength) will bend less, and is more likely to go straight through an interface will little deflection. Thus, red light is your best bet. Since orange light also has a longer wavelength than yellow light, you might also try orange light. (*b*) No, the dependence on frequency of the index of refraction is relatively small.

33.21. Light will travel from the coin to each of the observer's eyes. As light leaves the surface of the water, it will bend away from the normal line. The eyes perceive depth by tracing back the light rays to their point of origin in a linear path. Since the rays bent at the interface, the eyes will trace them back to a point closer to the surface than the actual coin. Thus the water appears shallower to the observer than it actually is.

33.22. The plank appears to be bent at the air-water interface. The submerged part appears to be closer to the surface than it actually is. This happens because rays of light that exit the water bend away from the line normal to the surface of the water. The eyes perceive depth by tracing the rays back to an origin in a linear path. Due to the bending of rays at the surface, the eyes trace the light back to a point that is shallower than the actual plank.

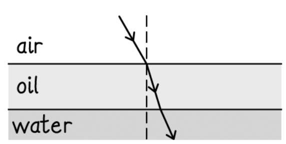
33.23. Case A: (*a*) Light travels faster in material 2. We know this because the light is closer to the normal line in material 1. (*b*) Light originates in material 2. The two rays in material 2 are the incident and reflected rays. If light had originated in material 1, there would be no way to explain why there are two rays in material 2. Case B: (*a*) Light travels faster in material 1. Total internal reflection can only keep light rays in a medium where light moves more slowly. (*b*) Light originates in material 2. In fact, it never leaves material 2. Case C: (*a*) One cannot tell in which material light moves more quickly from examining a ray that is incident normal to the interface. (*b*) One cannot tell in which material the light originates from examining a ray that is incident normal to the interface.

33.24.



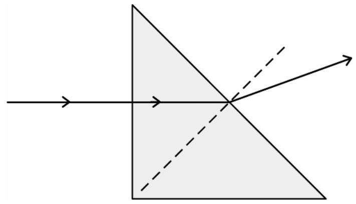
The red light bends less than the green light as it enters the glass. This results in the red light ray moving closer to the green light, and it causes the two rays to no longer be parallel inside the glass. But as the rays exit, again the red light is bent by less than the green light, and the rays end up being parallel again. They have, however, moved closer together.

33.25.



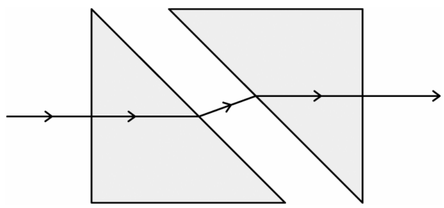
33.26. (*a*) Consider a ray of light that leaves the bulb and just barely misses the edge of the boat, making it up to the ceiling. This ray will mark the edge of the shadow. With water in the tub, the light will bend away from a line normal to the water-air interface before reaching the ceiling. This means the shadow on the ceiling is already larger than the shadow would be if the tub were empty. Now we replace the water with a material in which light travels faster than in water. This means the light will not bend away from normal as much as it previously did, when entering the air. Thus, the shadow on the ceiling shrinks. (*b*) We use the same approach and logic as in part (*a*). But now we replace the water with a material in which light moves more slowly than in water. This means the light will bend away from normal to an even greater extent than before when exiting the material and entering air. Thus, the shadow on the ceiling becomes larger.

33.27. The light will pass straight through the vertical edge into the prism. Light cannot undergo total internal reflection when it strikes the diagonal edge. Rather, light will exit the diagonal edge and bend toward the line normal to that diagonal edge. This is shown below.



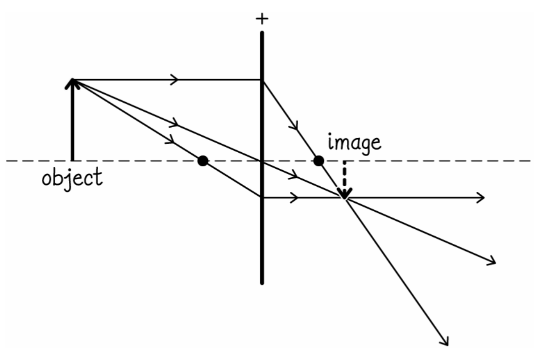
33.28. The air near black asphalt is typically hotter than the air around it. Light propagates slightly faster through hot air than cold air. Hence, light that is incident on this layer of hot air could undergo total internal reflection in the region of air (colder air) that has the higher index of refraction. This can result in specular reflection of oncoming vehicles or even the sky off of the layer of air above the road. This is sometimes called a road mirage.

33.29. Place a second identical prism next to the first. Invert the second prism, so that its vertex points downward as shown below.



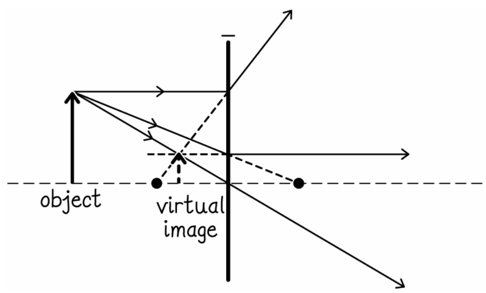
33.30. Light travels directly through the oil and through the glass cylinder without bending. If light bent at the interface, we would see differences in the light reach our eyes from near the interface. Since we see no such interface, light must not be bent there. Thus, light travels at the same speed through the glass and through the oil.

33.31. (*a*)



(*b*) The image is real. We know this because light is actually focused to a real point. (*c*) The image is inverted. This is clear from the figure. (*d*) The image shrinks. This is most easily seen by considering what happens to the principle ray that passes through the lens center, as the object is moved farther from the lens.

33.32. (*a*)



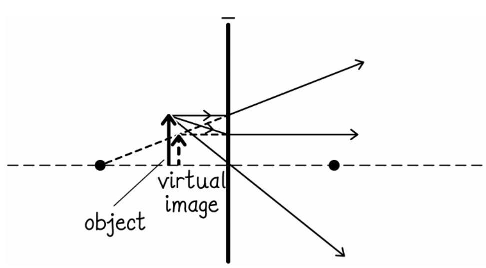
(*b*) The image is virtual. We know this because light is not actually focused to a real point. Light is spread by the lens such that our eyes would trace the rays back to a point. So the light appears to originate from that point, but light is not actually there. (*c*) The image is upright. This is clear from the figure. (*d*) The image shrinks. Consider the principle ray that passes through the focal point on the opposite side of the lens. If the object is moved away, this ray will clearly intersect the lens at a lower position. This will result in the dashed line over to the virtual image being lower (closer to the lens axis). Thus the image will be shorter.

33.33. The object is held closer to the lens than the focal point, such that the lens is not able to focus the light down to a single point.

33.34. The focal length is still 100 mm. The original focal length given already included the refraction at both sides of the lens. This is not changed by turning the lens around.

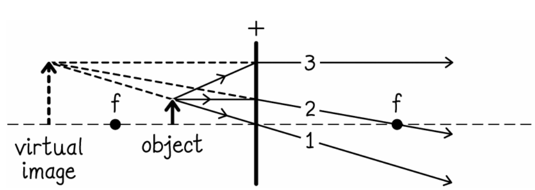
33.35. The image is on the same side as the object (opposite side from your eyes). If the image is magnified, the lens is converging. If the image is reduced, the lens is diverging.

33.36. (*a*)



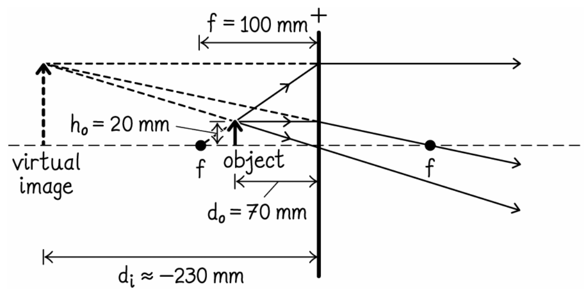
(*b*) The image is virtual. We know this because the light is not actually focused down to a point. The rays spread as they pass through the lens, such that our eyes trace the rays back to a single point. But light is not actually focused at that point. (*c*) The image is upright. This is clear from the figure. (*d*) The image shrinks. Consider the principle ray that passes through the focal point on the opposite side of the lens. If the object is moved away, this ray will clearly intersect the lens at a lower position. This will result in the dashed line over to the virtual image being lower (closer to the lens axis). Thus the image will be shorter.

33.37. (*a*)



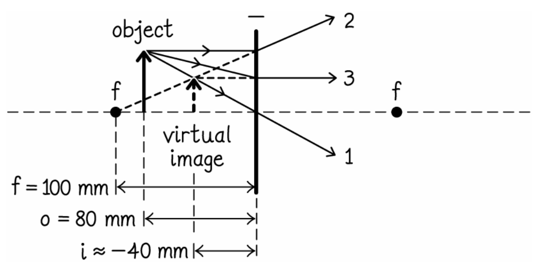
(*b*) The image is virtual. We know this because light is not actually focused down to a point. The rays move closer together as they pass through the lens, but not enough to collect them to a point. They still spread in such a way that our eyes can trace them back to a point behind the lens. Thus our eyes perceive an image at that point. But no light is actually focused at that point. (*c*) The image is upright. This is clear from the figure. (*d*) The image becomes larger. Consider the ray that travels upward to the lens and is parallel to the lens axis after being refracted. It is parallel to the lens axis upon refraction because it approached the lens from the direction of the focal point. As the object is moved closer to the focus, such a ray must still come from the direction of the focal point, which would cause it to strike the lens at a higher position (farther from the lens axis). This means the horizontal dashed line (that the eye traces back to form the image) would be higher, and the image would be larger.

33.38. (*a*)



(*b*) The image is virtual. We know it should be because the object is closer to the lens than the focal length. We can confirm that it is virtual, because no light is actually focused down to the image. Rather, the image is formed by our eyes tracing the refracted rays back to an apparent point of origin. (*c*) The image is upright. This is clear from the figure. (*d*) The image is larger than the object. This is clear from the figure.

33.39. (*a*)



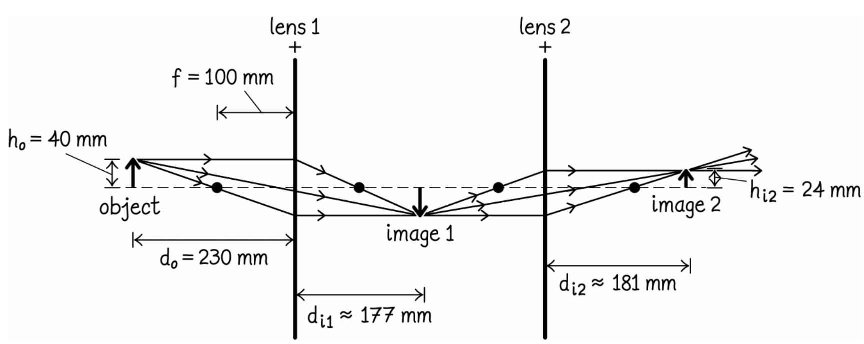
(*b*) The image is virtual. We know it should be because diverging lenses cannot focus light down to a point. We see that it is virtual from the figure, because light is not focused down to a point. Rather, the image is formed by our eyes tracing the refracted rays back to an apparent point of origin. (*c*) The image is upright. This is clear from the figure. (*d*) The image is smaller than the object. This is clear from the figure.

33.40. Each entry in the table can be determined from sketching a simplified ray diagram. Here are the correct table values for a converging lens:

Object Location	**Image Location**	**Image**		
		Real or virtual?	**Upright or inverted?**	**Larger or smaller than object?**
Between lens and focus	Between positive infinity and lens (same side)	Virtual	Upright	Larger
At focus	Infinity	NA	NA	NA
Between focus and twice the focal length	Between positive infinity and twice the focal point (opposite side from object)	Real	Inverted	Larger
At twice the focal length	$x = +2f$	Real	Inverted	Same size
Beyond twice the focal length	Between the focal point and twice the focal point (opposite side from the object)	Real	Inverted	Smaller
At infinity	f	Real (though focused down to a point, usually not called an image)	Inverted (usually not called an image)	Infinitesimal, essentially zero height

33.41. When parallel rays of light are refracted by the diverging lens, the resulting spreading rays can be traced back to a point 100 mm behind the diverging lens. By definition, the focal length of the diverging lens must be 100 mm. When these spreading rays pass through the converging lens, they emerge parallel to each other. The rays can be traced back to a point 200 mm from the converging lens. By definition, the focal length of the converging lens must be 200 mm. Thus $f_{\text{diverging}} = 100$ mm and $f_{\text{converging}} = 200$ mm.

33.42. (*a*) and (*b*)



(*c*) The image formed by the right lens is real. We know this because light is actually focused down to a point. (*d*) The image is upright. This is clear from the figure. (*e*) The image is smaller than the object. The exact factor by which the image is shrunken will depend on the quality of your diagram. But the image should be close to 60% the size of the object.

33.43. The index of refraction of such a material is given by $n = \dfrac{c_0}{c} = \dfrac{(3.00\times10^8 \text{ m/s})}{(1.24\times10^8 \text{ m/s})} = 2.42.$ While there may be many materials with this index of refraction, the only material in Table 33.1 with this index of refraction is diamond. The material is likely to be diamond.

33.44. The index of refraction is given by Equation 33.1: $n = \dfrac{c_0}{c}$, which in this case can be written $n = \dfrac{c_0}{(1-0.075)c_0} = 1.08.$

33.45. The frequency of the light is unchanged when it passes from one medium to another. Thus $f = f_0 = \dfrac{c_0}{\lambda_0} = \dfrac{(3.00\times10^8 \text{ m/s})}{(530\times10^{-9} \text{ m})} = 5.66\times10^{14}$ Hz. The wavelength does change as the light moves from one medium to another. The new wavelength is given by Equation 33.3:

$$\lambda_{\text{glass}} = \frac{\lambda_0}{n_{\text{glass}}} = \frac{(530 \text{ nm})}{1.65} = 321 \text{ nm}$$

33.46. (*a*) This minimum incident angle at which total internal reflection occurs is exactly the critical angle, given by Equation 33.9:

$$\theta_{\text{c}} = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.24}{1.45}\right) = 58.8°$$

(*b*) Total internal reflection is not possible in this case. Light cannot become trapped in the material in which it moves faster. This should be qualitatively understood from the discussion in the text, but it can also be seen by looking at what happens if you switch the indices of refraction in Equation 33.9. When the ratio in the inverse sine is greater than one there is no solution.

33.47. The minimum angle from normal at which total internal reflection will occur is the critical angle, given by Equation 33.9:

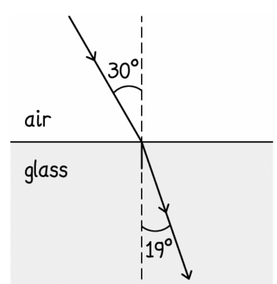
$$\theta_{\text{c}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.00}{1.37}\right) = 46.9°$$

Any angle greater than or equal to $46.9°$ would result in total internal reflection.

33.48. (*a*) Using Snel's Law, which is Equation 33.7, we can write $n_{\text{air}}\sin(\theta_{\text{air}}) = n_{\text{glass}}\sin(\theta_{\text{glass}})$, or

$$\theta_{\text{glass}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{glass}}}\sin(\theta_{\text{air}})\right) = \sin^{-1}\left(\frac{(1.0)}{(1.5)}\sin(30°)\right) = 19°$$

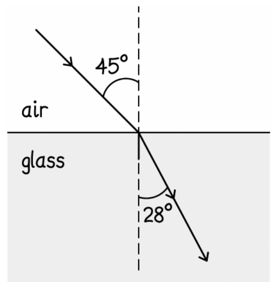
(*b*)



(*c*) The answer to (*a*) would be

$$\theta_{\text{glass}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{glass}}}\sin(\theta_{\text{air}})\right) = \sin^{-1}\left(\frac{(1.0)}{(1.5)}\sin(45°)\right) = 28°$$

meaning the figure in (*b*) would show a larger angle inside the glass. This is shown below.

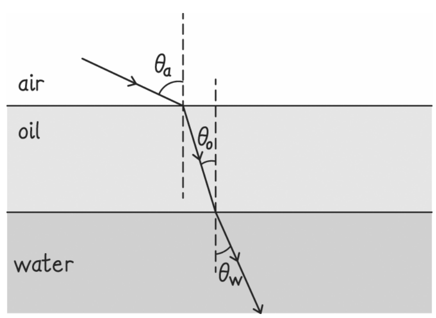


33.49. (*a*) The wavelength is reduced by 1.333 to $\lambda = \frac{\lambda_0}{n} = \frac{538 \text{ nm}}{1.333} = 404 \text{ nm}.$ This follows immediately from Equation 33.3. (*b*) The frequency does not change. It remains $f = \frac{c_0}{\lambda_0} = \frac{(3.00\times10^8 \text{ m/s})}{(538\times10^{-9} \text{ m})} = 5.58\times10^{14} \text{ Hz}.$ (*c*) The speed is reduced according to the definition of the index of refraction (Equation 33.1) to $c = \frac{c_0}{n} = \frac{(3.00\times10^8 \text{ m/s})}{1.333} = 2.25\times10^8 \text{ m/s}.$ (*d*) The angle from normal is reduced according to Equation 33.7 to

$$\theta_{\text{water}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\sin(\theta_{\text{air}})\right) = \sin^{-1}\left(\frac{(1.000)}{(1.333)}\sin(60°)\right) = 41°$$

(*e*) The wavelength, frequency, and speed would not be altered by changing the angle of entry. The refracted angle would change to $0°$; the light will continue on parallel to the normal line.

33.50. (*a*)



(*b*) We apply Snel's law twice: once at each interface. For the air-oil interface, we have

$$n_{\text{a}}\sin(\theta_{\text{a}}) = n_{\text{o}}\sin(\theta_{\text{o}}) \quad (1)$$

For the oil-water interface, we have

$$n_{\text{o}}\sin(\theta_{\text{o}}) = n_{\text{w}}\sin(\theta_{\text{w}}) \quad (2)$$

The right hand side of equation (1) and the left hand side of equation (2) are exactly the same. This allows us to equation the left hand side of equation (1) and the right hand side of equation (2): $n_{\text{a}}\sin(\theta_{\text{a}}) = n_{\text{w}}\sin(\theta_{\text{w}})$. (*c*) No, we cannot ignore the middle layer if we care about the displacement of the rays of light. The displacement in the middle layer (oil, in this case) is determined by the thickness of that layer and the angle made by the rays of light. The angle is of course determined using Snell's law which involves the index of refraction of this middle layer. Hence the optical properties of the oil are important for determining the position of the exiting rays of light.

33.51. (*a*) Applying Equation 33.7 we can write $n_{\text{air,blue}}\sin(\theta_{\text{air,blue}}) = n_{\text{glass,blue}}\sin(\theta_{\text{glass,blue}})$. Let us assume that the index of refraction for air is approximately 1.00 for all colors of light, and we focus only on the frequency dependence of the glass's index of refraction. Then we can write

$$\theta_{\text{air,blue}} = \sin^{-1}\left(\frac{n_{\text{glass,blue}}}{n_{\text{air}}}\sin(\theta_{\text{glass,blue}})\right) = \sin^{-1}\left(\frac{(1.66)}{(1.00)}\sin(30°)\right) = 56.1°$$

(*b*) We apply Equation 33.7 as in part (*a*) to obtain $\theta_{\text{air,red}} = \sin^{-1}\left(\frac{n_{\text{glass,red}}}{n_{\text{air}}}\sin(\theta_{\text{glass,red}})\right) = \sin^{-1}\left(\frac{(1.61)}{(1.00)}\sin(30°)\right)$ $= 53.6°$. (*c*) The angle between the ray of red light and blue light is $\Delta\theta = \theta_{\text{blue}} - \theta_{\text{red}} = (56.1°) - (53.6°) = (2.5°)$. Simple trigonometry tells us that $\tan(\Delta\theta) = \frac{\Delta y_{\text{screen}}}{\Delta x}$, where Δx is the orthogonal distance from the glass to the screen and Δy_{screen} is the distance between the two bright spots on the screen. Then $\Delta y_{\text{screen}} =$ $\Delta x \tan(\Delta\theta) = (0.50\text{ m})\tan(2.5°) = 22\text{ mm}$.

33.52. (*a*) Equation 33.9 tells us $\theta_{\text{c}} = \sin^{-1}\left(\frac{n_1}{n_2}\right) = \sin^{-1}\left(\frac{1.0}{1.4}\right) = 46°$. (*b*) As in part (*a*), we write $\theta_{\text{c}} = \sin^{-1}\left(\frac{n_1}{n_2}\right) = \sin^{-1}\left(\frac{1.0}{1.8}\right) = 34°$. (*c*) A small radius of curvature corresponds to a very sharp bend in the cable, this means a ray of light travelling (more or less) parallel to the cable will strike the curved edge of the cable at an angle closer to normal if the radius of curvature is small. Thus, the material with the smaller critical angle will allow the light to remain in the cable through total internal reflection with a smaller radius of curvature in the cable. Thus the cable with the index of refraction of 1.8 could have a smaller radius of curvature while maintaining total internal reflection.

33.53. We apply Snell's law twice: once at the top interface and once at the bottom interface. For the top interface, we have

$$n\sin(\theta) = n_1\sin(\theta_1) \tag{1}$$

Let us call the final angle at which the light exits the slab θ_{f}. Then, for the bottom interface, we have

$$n_1\sin(\theta_1) = n\sin(\theta_{\text{f}}) \tag{2}$$

Clearly, the right hand side of equation (1) and the left hand side of equation (2) are identical. Hence we can equate $n\sin(\theta) = n\sin(\theta_{\text{f}})$. Cancelling the index of refraction (which is the same above and below the slab), we have $\sin(\theta) = \sin(\theta_{\text{f}})$. Because both angles are less than $90°$, the only way the sines can be equal is if $\theta = \theta_{\text{f}}$. Hence the angles at which the light enters and leaves the slab are identical.

33.54. (*a*) The wavelength changes according to Equation 33.3: $\lambda_1 = \frac{\lambda_0}{n_1} = \frac{(700\text{ nm})}{(2.42)} = 289\text{ nm}$. The frequency does not change at the interface, such that

$$f = \frac{c_1}{\lambda_1} = \frac{c_0}{\lambda_0} = \frac{(3.00\times10^8\text{ m/s})}{(700\times10^{-9}\text{ m})} = 4.29\times10^{14}\text{ Hz}$$

(*b*) $\theta_{\text{diamond}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{diamond}}}\sin(\theta_{\text{air}})\right) = \sin^{-1}\left(\frac{(1.00)}{(2.42)}\sin(40.0°)\right) = 15.4°$

33.55. We require that the reflected ray and refracted ray be orthogonal. We can express both angles in terms of the incident angle using our knowledge that incident and reflected angles are the same, and using Equation 33.7 to describe the refracted angle. Thus we require $(90° - \theta_{\text{refracted}}) + (90° - \theta_{\text{reflected}}) = 90°$ or $\theta_{\text{reflected}} + \theta_{\text{refracted}} = 90°$, which we write as

$$\theta_{\text{incident}} + \sin^{-1}\left(\frac{n_1}{n_2}\sin(\theta_{\text{incident}})\right) = 90° \text{ or } \sin^{-1}\left(\frac{n_1}{n_2}\sin(\theta_{\text{incident}})\right) = 90° - \theta_{\text{incident}}$$

Taking the sine of both sides and noting that $\sin(90° - \theta) = \cos(\theta)$, we find

$$\tan(\theta_{\text{incident}}) = \frac{n_2}{n_1} \Rightarrow \theta_{\text{incident}} = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.5}{1.1}\right) = 54°$$

33.56. Basic geometry tells us that $\theta = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{(3.0\,\text{m})}{(4.0\,\text{m})}\right) = 36.9°$ (where we have kept an additional significant digit because this is an intermediate step). Equation 33.7 tells us that the refracted angle is $\theta_{\text{refracted}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\sin(\theta)\right) = \sin^{-1}\left(\frac{(1.00)}{(1.33)}\sin(36.9°)\right) = 26.8°$. Because we can see the buoy on the surface, we know the light must come from a horizontal distance of 3.0 m to the right of the point on the surface where the researcher is looking. Thus we can write $\tan(\theta_{\text{refracted}}) = \frac{\Delta x_{\text{water}}}{\Delta y_{\text{water}}} \Rightarrow \Delta y_{\text{water}} = \frac{\Delta x_{\text{water}}}{\tan(\theta_{\text{refracted}})} = \frac{(3.0\,\text{m})}{\tan(26.8°)} = 5.9\,\text{m}.$

33.57. (*a*) Let us call the angle of refraction just inside the flat end of the fiber on the far left $\theta_{\text{refracted}}$, and call the angle from normal at which the ray strikes the top flat surface of the cable $\theta_{\text{reflected}}$. The smallest that the reflected angle can be is the critical angle. Using Equation 33.9, we can write $\theta_{\text{reflected,min}} = \theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n}\right)$. Simple geometry shows us that $\theta_{\text{refracted}} = 90° - \theta_{\text{reflected}}$ or $\theta_{\text{refracted,max}} = 90° - \theta_{\text{reflected,min}} = 90° - \sin^{-1}\left(\frac{n_{\text{air}}}{n}\right)$. Now using Equation 33.7, we can write the incident angle in terms of this refracted ray

$$\theta = \sin^{-1}\left(\frac{n}{n_{\text{air}}}\sin(\theta_{\text{refracted}})\right) \Rightarrow \theta_{\text{max}} = \sin^{-1}\left(\frac{n}{n_{\text{air}}}\sin(\theta_{\text{refracted,max}})\right) = \sin^{-1}\left(\frac{n}{n_{\text{air}}}\sin\left(90° - \sin^{-1}\left(\frac{n_{\text{air}}}{n}\right)\right)\right)$$

Rewriting the sine in terms of cosine, we find

$$\theta_{\text{max}} = \sin^{-1}\left(\frac{n}{n_{\text{air}}}\sqrt{1-\cos^2\left(90° - \sin^{-1}\left(\frac{n_{\text{air}}}{n}\right)\right)}\right) = \sin^{-1}\left(\frac{n}{n_{\text{air}}}\sqrt{1-\sin^2\left(\sin^{-1}\left(\frac{n_{\text{air}}}{n}\right)\right)}\right) = \sin^{-1}\left(\frac{n}{n_{\text{air}}}\sqrt{1-\left(\frac{n_{\text{air}}}{n}\right)^2}\right)$$

Finally, inserting $n_{\text{air}} = 1.00$, we have $\theta_{\text{max}} = \sin^{-1}(\sqrt{n^2-1})$. (*b*) Using the result of part (*a*), we can see that the maximum angle reaches $90°$ when the argument of the inverse sine approaches one. Thus we require $\sqrt{n_0^2 - 1} = 1 \Rightarrow n_0 = \sqrt{2}$.

33.58. Let us choose a $+y$ axis that points vertically upward, perpendicular to the interface, and choose a $+x$ axis that points to the right. Let us assume that the ray of light strikes the interface such that some component of its momentum is along the $+x$ axis. If the ray were not bent at the interface, then it would continue at the same $40°$ angle from normal, and would exit the bottom of medium 2 a distance $x_{\text{unbent}} = d\tan(\theta_{\text{incident}}) = (12\text{ mm})\tan(40°) = 10.07\text{ mm}$ in the $+x$ direction away from the point of entry. Of course, we know that the light does bend at the interface, and it does so according to Snel's Law. Equation 33.7 tell us $\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin(\theta_1)\right) = \sin^{-1}\left(\frac{1.3}{1.6}\sin(40°)\right) = 31.48°$. So the point at which the ray of light will actually exit medium 2 is a distance along the x axis from the point of entry equal to $x_{\text{real}} = \Delta y\tan(\theta_2) = (12\text{ mm})\tan(31.48°) = 7.35\text{ mm}$. So the distance between where a straight (unbent) ray of light would exit and where light actually exits is $\Delta x = x_{\text{unbent}} - x_{\text{real}} = (10.07\text{ mm}) - (7.35\text{ mm}) = 2.72\text{ mm}$. This is the distance the ray is displaced along the plane of medium 2. The perpendicular distance from the ray to the path the unbent ray would follow is then given by $\cos(\theta_1) = \ell/\Delta x \Rightarrow \ell = \Delta x\cos(\theta_1) = (2.72\text{ mm})\cos(40°) = 2.1\text{ mm}$.

33.59. Let us call the height of the child's eyes above the water Δx_{air}, and the distance along the surface of the water from the edge of the pool to the point where light rays exit the water on their way to the child's eyes $\Delta y_{\text{surface}}$. Call the distance along the surface from that point at $\Delta y_{\text{surface}}$ to a point directly above the toy Δy_{water}, such that $\Delta y_{\text{toy}} = \Delta y_{\text{surface}} + \Delta y_{\text{water}}$. Let us note first that the child's eyes trace the ray of light back in a linear (unbent) fashion. This means that the child's brain processes the image assuming $\theta_{\text{air}} = \theta_{\text{water}}$ or $\dfrac{\Delta y_{\text{surface}}}{(h-d)} = \dfrac{(\Delta y_{\text{toy,apparent}} - \Delta y_{\text{surface}})}{d}$. Rearranging this to solve for $\Delta y_{\text{surface}}$, we obtain $\Delta y_{\text{surface}} = \dfrac{(h-d)\Delta y_{\text{toy,apparent}}}{h} = \dfrac{((3.5\text{ m})-(1.8\text{ m}))(4.2\text{ m})}{(3.5\text{ m})}$ $= 2.04$ m. We can determine the angle that light rays make with the normal line in air using simple geometry:

$$\theta_{\text{air}} = \tan^{-1}\left(\frac{\Delta y_{\text{surface}}}{(h-d)}\right) = \tan^{-1}\left(\frac{(2.04\text{ m})}{((3.5\text{ m})-(1.8\text{ m}))}\right) = 50.2°$$

The angle that the ray makes with the normal line in the water is given by Equation 33.7:

$$\theta_{\text{water}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\sin(\theta_{\text{air}})\right) = \sin^{-1}\left(\frac{(1.00)}{(1.33)}\sin(50.2°)\right) = 35.3°$$

Now the horizontal distance that the rays travel in the water is given by $\Delta y_{\text{water}} = d\tan(\theta_{\text{water}}) = (1.8\text{ m})\tan(35.3°)$ $= 1.27$ m. The light rays travel a horizontal distance of 2.04 m in the air, and 1.27 m in the water. Thus the toy is a horizontal distance of 3.3 m from the side of the pool.

33.60. We know that the focal length for a diverging lens is negative. Using Equation 33.22 we have $d = \dfrac{1\text{ m}}{f} = \dfrac{1\text{ m}}{(-0.400\text{ m})} = -2.5$ diopters.

33.61. For a diverging lens, the focal length is negative. Using the lens equation (*Principles* Equation 33.16) we can write $\dfrac{1}{i} = \dfrac{1}{f} - \dfrac{1}{o}$. The object distance is always positive. The only way for a real image to be formed is it the image distance is also positive. But clearly both terms on the right hand side of the above equation are negative. Hence $\dfrac{1}{i} < 0$, from which it immediately follows that $i < 0$. Since the image distance is always negative for a diverging lens, a diverging lens cannot form a real image.

33.62. Using Equation 33.22 we have $d = \dfrac{1\text{ m}}{f}$ or $f = \dfrac{1\text{ m}}{d} = \dfrac{1\text{ m}}{(+1.5\text{ diopters})} = 0.67$ m.

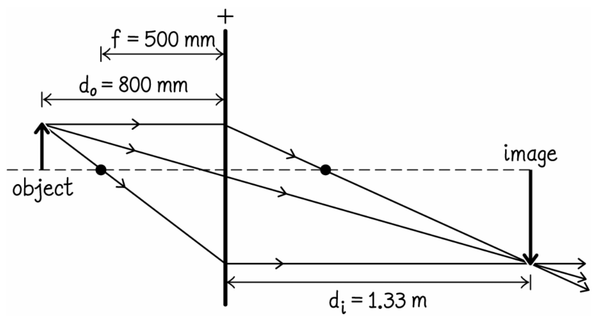
33.63. Using Equation 33.16 we have $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} = \left(\dfrac{1}{(0.100\text{ m})} - \dfrac{1}{(2.00\text{ m})}\right)^{-1} = 0.105$ m or 105 mm.

33.64. We know that the magnification of an image can be written either in terms of object and image distances or in terms of object and image heights. Equating the two expressions as in Equation 33.17, we obtain $o = -i\dfrac{h_{\text{o}}}{h_{\text{i}}} = -(5.8\text{ mm})\dfrac{(324\text{ m})}{(-10\text{ mm})} = 1.9\times10^2$ m.

33.65. Using the result of Example 33.9, we have $M_\theta = \left|\dfrac{f_1}{f_2}\right| \Rightarrow |f_2| = |f_{\text{eye}}| = \dfrac{|f_1|}{M_\theta} = \dfrac{(2.0\text{ m})}{(40)} = 50$ mm.

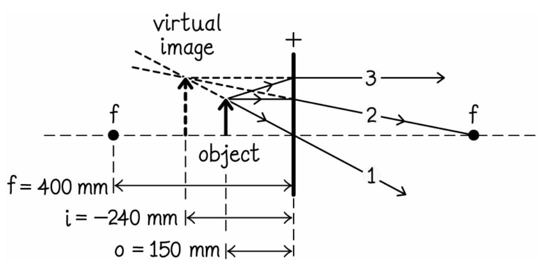
33.66. (*a*) Using Equation 33.16 we have $i=\left(\dfrac{1}{f}-\dfrac{1}{o}\right)^{-1}=\left(\dfrac{1}{(0.500\text{ m})}-\dfrac{1}{(0.800\text{ m})}\right)^{-1}=1.33\text{ m}.$ (*b*) Using Equation 33.17, we can write the magnification as $M=-\dfrac{i}{o}=-\dfrac{(1.33\text{ m})}{(0.800\text{ m})}=-1.67.$

(*c*)



33.67. (*a*) Using Equation 33.16 we have $i=\left(\dfrac{1}{f}-\dfrac{1}{o}\right)^{-1}=\left(\dfrac{1}{(400\text{ mm})}-\dfrac{1}{(150\text{ mm})}\right)^{-1}=-240\text{ mm}.$ (*b*) Using Equation 33.17, we can write the magnification as $M=-\dfrac{i}{o}=-\dfrac{(-240\text{ mm})}{(150\text{ mm})}=1.60.$

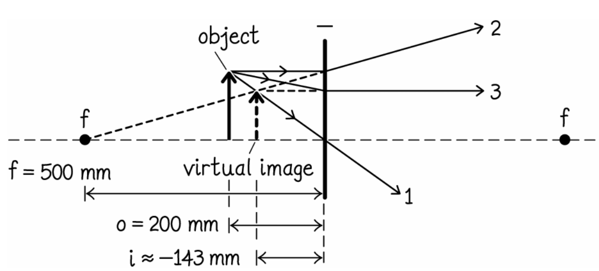
(*c*)



33.68. (*a*) Using Equation 33.16 we have $i=\left(\dfrac{1}{f}-\dfrac{1}{o}\right)^{-1}=\left(\dfrac{1}{(-300\text{ mm})}-\dfrac{1}{(600\text{ mm})}\right)^{-1}=-200\text{ mm}.$ (*b*) Using Equation 33.17, we can write the magnification as $M=-\dfrac{i}{o}=-\dfrac{(-200\text{ mm})}{(600\text{ mm})}=0.333.$

33.69. (*a*) Using Equation 33.16 we have $i=\left(\dfrac{1}{f}-\dfrac{1}{o}\right)^{-1}=\left(\dfrac{1}{(-500\text{ mm})}-\dfrac{1}{(200\text{ mm})}\right)^{-1}=-143\text{ mm}.$ (*b*) Using Equation 33.17, we can write the magnification as $M=-\dfrac{i}{o}=-\dfrac{(-143\text{ mm})}{(200\text{ mm})}=0.714.$

(*c*)



33.70. The image appears 250 mm from your eyes, which is 150 mm from the lens (but on the same side of the lens as the bug). This means the image must be virtual, and the image distance is $i = -150$ mm. The object distance is $o = 30$ mm. Using Equation 33.16, we have

$$f = \left(\frac{1}{i} + \frac{1}{o}\right)^{-1} = \left(\frac{1}{(-150\text{ mm})} + \frac{1}{(30\text{ mm})}\right)^{-1} = 38\text{ mm}$$

33.71. The way the eyeglasses work is by creating an image at the woman's near point of 400 mm, when the object is at a distance of the normal human near point of 250 mm. Since the image is still in front of the woman's eyes, the image distance will be negative. Thus, Equation 33.16 tells us

$$\frac{1}{f} = \left(\frac{1}{i} + \frac{1}{o}\right)^{-1} = \left(\frac{1}{(-0.400\text{ m})} + \frac{1}{(0.250\text{ m})}\right)^{-1} = 1.5\text{ diopters}$$

33.72. (*a*) We solve this problem by finding the position of the image from lens 1, and using that as the object for lens 2. Equation 33.16 tells us

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o_1}\right)^{-1} = \left(\frac{1}{(150\text{ mm})} - \frac{1}{(400\text{ mm})}\right)^{-1} = 240\text{ mm}$$

Since the lenses are 600 mm apart, this image is 360 mm from lens 2, so we set $o_2 = 360$ mm. A second application of Equation 33.16, this time to lens 2 yields

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1} = \left(\frac{1}{(200\text{ mm})} - \frac{1}{(360\text{ mm})}\right)^{-1} = 450\text{ mm}$$

and this distance is measured relative to lens 2. Thus the final image is 450 mm to the right of lens 2. (*b*) To find the overall magnification, we find the magnification due to each lens and multiply them:

$$M = M_1 M_2 = \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) = \left(-\frac{(240\text{ mm})}{(400\text{ mm})}\right)\left(-\frac{(450\text{ mm})}{(360\text{ mm})}\right) = 0.750$$

(*c*) From the sign of the magnification, we can see the image is upright. Physically, the image is upright because it is inverted once by lens 1, and then inverted again (flipped upright again) by lens 2.

33.73. (*a*) Since the two foci coincide, we know $f_1 + f_2 = d$. We obtain a second equation relating these quantities from Example 33.9: $M_\theta = \left|\frac{f_1}{f_2}\right|$. Solving the first equation for f_2 (the eyepiece) we find $M_\theta = \left|\frac{f_1}{d - f_1}\right| \Rightarrow f_1 = \frac{dM_\theta}{(1 + M_\theta)}$. (*b*) Inserting the answer to part (*a*) to either of the initial equations ($f_1 + f_2 = d$ or Example 33.9) we find $f_2 = d - f_1 = d - \frac{dM_\theta}{(1 + M_\theta)} = \frac{d}{M_\theta + 1}$.

33.74. (*a*) We first find the position of the image from lens 1 (the objective), and then use this image as the object for lens 2 (the eyepiece). We use Equation 33.16 to write

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o_1}\right)^{-1} = \left(\frac{1}{(25\text{ mm})} - \frac{1}{(30\text{ mm})}\right)^{-1} = 150\text{ mm}$$

Since the two lenses are 200 mm apart, this makes the object distance for lens 2 50 mm. A second application of Equation 33.16, this time to lens 2, yields

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1} = \left(\frac{1}{(63\text{ mm})} - \frac{1}{(50\text{ mm})}\right)^{-1} = -2.4\times10^2\text{ mm}$$

Thus the image formed by the eyepiece is 240 mm to the left of the eyepiece lens. (*b*) The overall magnification is the product of the magnification due to each lens:

$$M = M_1 M_2 = \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) = \left(-\frac{(150\text{ mm})}{(30\text{ mm})}\right)\left(-\frac{(-242\text{ mm})}{(50\text{ mm})}\right) = -24$$

33.75. (*a*) We want to know the image distance for lens 2 so that we can get a crisp clear image on our screen. We first find the position of the image from lens 1, and then use this image as the object for lens 2. We use Equation 33.16 to write

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o_1}\right)^{-1} = \left(\frac{1}{(100\text{ mm})} - \frac{1}{(50\text{ mm})}\right)^{-1} = -100\text{ mm}$$

which is 100 mm to the left of lens 1. Since the two lenses are 150 mm apart, this makes the object distance for lens 2 $o_2 = 250$ mm. A second application of Equation 33.16, this time to lens 2, yields

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1} = \left(\frac{1}{(180\text{ mm})} - \frac{1}{(250\text{ mm})}\right)^{-1} = 0.64\text{ m}$$

Thus the image formed by the eyepiece is 0.64 m to the right of lens 2. This is where we should place our screen. (*b*) The overall magnification is the product of the magnification due to each lens:

$$M = M_1 M_2 = \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) = \left(-\frac{(-100\text{ mm})}{(50\text{ mm})}\right)\left(-\frac{(642\text{ mm})}{(250\text{ mm})}\right) = -5.1$$

(*c*) We see from the sign of the magnification that the image is inverted. The first lens formed an upright virtual image, but the second lens inverted this image and formed a real image.

33.76. We solve this problem by finding the position of the image from lens 1, and using that as the object for lens 2. Equation 33.16 tells us

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o_1}\right)^{-1} = \left(\frac{1}{(100\text{ mm})} - \frac{1}{(150\text{ mm})}\right)^{-1} = 300\text{ mm}$$

Since the lenses are 550 mm apart, this image is 250 mm from lens 2, so we set $o_2 = 250$ mm. A second application of Equation 33.16, this time to lens 2, yields

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1} = \left(\frac{1}{(200\text{ mm})} - \frac{1}{(250\text{ mm})}\right)^{-1} = 1.00\text{ m}$$

and this distance is measured relative to lens 2. Thus the final image is 1.00 m to the right of lens 2.

33.77. It appears as though the position of the object relative to lens 1 is roughly twice the focal length. This means the image distance (created by lens 1) will also be twice the focal length. Thus the image due to the first lens will be the same size as the object, but inverted. This also means that the image created by lens 1 will be just inside the focal length of lens 2 (closer to lens 2 than its focal length). This means converging lens 2 will not be able to focus the rays into a real image; the image due to this lens (and therefore the overall image) is virtual. Since the image created by lens 1 was inverted, and lens 2 does not invert the image, the overall image is inverted. Lens 1 left the size of the image equal to the size of the object. So the overall magnification will be determined by the magnification due to lens 2. It appears that the image due to lens 1 (which is the object for lens 2) is very close to the focal point of lens 2. Looking at Equation 33.16, we see that because $\frac{1}{f_2} - \frac{1}{o_2}$ is small and negative, its inverse: $i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1}$ will be large and negative. Thus we expect the magnification due to lens 2 $M_2 = -\frac{i_2}{o_2}$ to be large and positive. Collecting all this together, we expect the image to be virtual, inverted, and enlarged.

33.78. (*a*) We solve this problem by finding the position of the image from lens 1, and using that as the object for lens 2. Equation 33.16 tells us

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o_1}\right)^{-1} = \left(\frac{1}{(100 \text{ mm})} - \frac{1}{(180 \text{ mm})}\right)^{-1} = 225 \text{ mm}$$

Since the lenses are 160 mm apart, this image is 65 mm to the right of lens 2. This image will act as the object for lens 2, but it is to the right of lens 2. Since light does not actually emanate from that point and pass through lens 2, this can be thought of as a "virtual object", and the object distance is negative. So we set $o_2 = -65$ mm. A second application of Equation 33.16, this time to lens 2, yields

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1} = \left(\frac{1}{(-80.0 \text{ mm})} - \frac{1}{(-65 \text{ mm})}\right)^{-1} = 3.5\times10^2 \text{ mm}$$

and this distance is measured relative to lens 2. Thus the final image is 0.35 m to the right of lens 2. (*b*) The overall magnification is the product of the magnification due to each lens:

$$M = M_1M_2 = \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) = \left(-\frac{(225 \text{ mm})}{(180 \text{ mm})}\right)\left(-\frac{(347 \text{ mm})}{(-65 \text{ mm})}\right) = -6.7$$

33.79. (*a*) We solve this problem by finding the position of the image from lens 1 (the left lens), and using that as the object for lens 2 (the right lens). Equation 33.16 tells us

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{o_1}\right)^{-1} = \left(\frac{1}{(100 \text{ mm})} - \frac{1}{(230 \text{ mm})}\right)^{-1} = 177 \text{ mm}$$

Since the lenses are 400 mm apart, this image is 223 mm to the left of lens 2. This image will act as the object for lens 2, so we set $o_2 = 223$ mm. A second application of Equation 33.16, this time to lens 2, yields

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{o_2}\right)^{-1} = \left(\frac{1}{(100 \text{ mm})} - \frac{1}{(223 \text{ mm})}\right)^{-1} = 181 \text{ mm}$$

and this distance is measured relative to lens 2. Thus the final image is 181 mm to the right of lens 2 (the right-most lens). (*b*) Because all image distances were positive, it is easy to see that the magnification due to the first lens will be negative, as will the magnification due to the second lens, making the product of the two magnifications (the total magnification) positive. Physically, this means that the first lens inverts the image, and the second lens inverts it again leaving it upright. (*c*) The final image distance is positive, which corresponds to a real image. (*d*) The overall magnification is the product of the magnification due to lens 1 and that due to lens 2:

$$M = M_1M_2 = \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) = \left(-\frac{(177 \text{ mm})}{(230 \text{ mm})}\right)\left(-\frac{(181 \text{ mm})}{(223 \text{ mm})}\right) = 0.625$$

33.80. Since we know the image distance from the second lens and the focal length of the final lens (which we call lens 2), we can find the object distance for the second lens using Equation 33.16:

$$o_2 = \left(\frac{1}{f_2} - \frac{1}{i_2}\right)^{-1} = \left(\frac{1}{(100 \text{ mm})} - \frac{1}{(300 \text{ mm})}\right)^{-1} = 150 \text{ mm}$$

This information can be used in a number of ways. Because this lens inverts the image and the total image must be upright, we know the image must also be inverted by the first lens (lens 1). This means a real image will be formed by the first lens, which allows us to relate the object distance for lens 2 to the image distance for lens 1, using the distance between lenses, d: $i_1 = d - o_2$. We apply Equation 33.16 a second time, this time to lens 1, to obtain

$$o_1 = \left(\frac{1}{f_1} - \frac{1}{i_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{d - o_2}\right)^{-1} \tag{1}$$

We also know the total magnification is $M = M_1M_2 = \left(-\frac{i_1}{o_1}\right)\left(-\frac{i_2}{o_2}\right) = 3.00 \Rightarrow o_1 = \frac{i_1 i_2}{(3.00)o_2}$ or

$$o_1 = \frac{(d - o_2)i_2}{(3.00)o_2} \tag{2}$$

Equating expressions (1) and (2) and solving for d yields $d = \frac{(3.00)o_2 f_1}{i_2} + o_2 + f_1 = \frac{(3.00)(150 \text{ mm})(100 \text{ mm})}{(300 \text{ mm})}$ $+(150 \text{ mm}) + (100 \text{ mm}) = 400 \text{ mm}.$ Inserting this result back into equation (2) yields $o_1 = \frac{(d - o_2)i_2}{(3.00)o_2}$ $= \frac{((400 \text{ mm}) - (150 \text{ mm}))(300 \text{ mm})}{(3.00)(150 \text{ mm})} = 167 \text{ mm}.$ Thus the lenses should be placed 400 mm apart, with the lens nearest the object being 167 mm from it.

33.81. From the information on the near point of the average human, we know that the minimum angle subtended by the image must be $\Delta\theta_{\text{min}} = 2\tan^{-1}\left(\frac{\Delta x/2}{r_{\text{near}}}\right) = 2\tan^{-1}\left(\frac{(0.1000 \text{ mm})/2}{(250 \text{ mm})}\right) = 0.02292°.$ We require that our telescope have a magnification that allows us to see a geosynchronous satellite, so we must determine the distance to the satellite (and the angle of our vision that it subtends) in order to determine the minimum required angular magnification. Recall from rotational motion and gravitation, that a geosynchronous satellite must be held in place by gravity and have the same period of rotation as Earth. From this one obtains

$$R = \left(\frac{Gm_{\text{Earth}}T^2}{4\pi^2}\right)^{1/3} = \left(\frac{(6.67\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24} \text{ kg})(24\times 3600 \text{ s})^2}{4\pi^2}\right)^{1/3} = 4.2227\times 10^4 \text{ km}$$

This distance is measured from the center of Earth, such that the distance from an observer on the surface of Earth to the satellite is 3.585×10^4 km. Thus the angle subtended by the satellite is $\Delta\theta_{\text{min}} = 2\tan^{-1}\left(\frac{\Delta x/2}{R_{\text{observer-satellite}}}\right)$ $= 2\tan^{-1}\left(\frac{(5.5 \text{ m})/2}{(3.585\times 10^7 \text{ m})}\right) = (8.790\times 10^{-6})°.$ Finally we can write two equations relating the focal lengths of the two lenses, using Example 33.9:

$$M_\theta = \frac{\Delta\theta_{\text{min}}}{\Delta\theta} = \left|\frac{f_1}{f_2}\right| \tag{1}$$

$$f_1 + f_2 = \ell \tag{2}$$

where ℓ is the length of the telescope. Solving equation (1) for lens 1 and inserting this into equation (2) yields

$$f_2 = \frac{\ell}{\left(1 + \frac{\Delta\theta_{\text{min}}}{\Delta\theta}\right)} = \frac{(1.000 \text{ m})}{\left(1 + \frac{(0.02292°)}{(8.790\times 10^{-6})°}\right)} = 0.3834 \text{ mm}$$

From this it follows trivially from equation (2) that $f_1 = 0.9996$ m. So this setup requires an objective lens with a focal length of 999.6 mm and an eyepiece lens with a focal length of 0.383 mm. The eyepiece focal length would be very difficult to achieve, and is not likely to be found in a home telescope. It would be much easier to use a longer telescope, and allow for a more complex setup involving multiple lenses and/or mirrors.

33.82. The sun's rays are approximately parallel, so they will be focused down to a point at the focal point of the mirror. Equation 33.23 tells us $R = 2f = 2(160 \text{ mm}) = 320 \text{ mm}.$

33.83. Because there is a positive radius of curvature, we know the focal length is also positive (which corresponds to a concave mirror). We use Equation 33.23 to write the focal length in terms of the radius of curvature, and we insert this into Equation 33.24 to obtain

$$i = \left(\frac{1}{f} - \frac{1}{o}\right)^{-1} = \left(\frac{2}{R} - \frac{1}{o}\right)^{-1} = \left(\frac{2}{(250 \text{ mm})} - \frac{1}{(200 \text{ mm})}\right)^{-1} = 333 \text{ mm}$$

So the image is 333 mm in front of the mirror.

33.84. (*a*) We use Equation 33.23 to write the focal length in terms of the radius of curvature, and we insert this into Equation 33.24 to obtain

$$i=\left(\frac{1}{f}-\frac{1}{o}\right)^{-1}=\left(\frac{2}{R}-\frac{1}{d_{\text{close}}}\right)^{-1}=\left(\frac{2}{(400\text{ mm})}-\frac{1}{(100\text{ mm})}\right)^{-1}=-200\text{ mm}$$

The sign of the image distance tells us that the image is virtual, and the magnification is $m=-\frac{i}{o}=-\frac{(-200\text{ mm})}{(100\text{ mm})}=2.00.$ So the image is virtual, upright, magnified by a factor of 2.00, and located 200 mm behind the mirror. (*b*) We proceed as in part (*a*), and write

$$i=\left(\frac{1}{f}-\frac{1}{o}\right)^{-1}=\left(\frac{2}{R}-\frac{1}{d_{\text{far}}}\right)^{-1}=\left(\frac{2}{(400\text{ mm})}-\frac{1}{(1200\text{ mm})}\right)^{-1}=240\text{ mm}$$

The sign of the image distance tells us that the image is real, and it is located 240 mm in front of the mirror. The magnification is $m=-\frac{i}{o}=-\frac{(240\text{ mm})}{(1200\text{ mm})}=-0.200,$ such that the image is inverted and 0.200 times as large as the object.

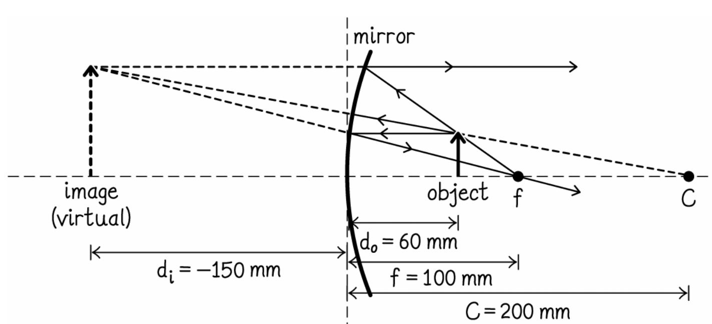
33.85. We use Equation 33.23 to write the focal length in terms of the radius of curvature, and we insert this into Equation 33.24 to obtain

$$i=\left(\frac{1}{f}-\frac{1}{o}\right)^{-1}=\left(\frac{2}{R}-\frac{1}{o}\right)^{-1}=\left(\frac{2}{(-0.800\text{ m})}-\frac{1}{(20.0\text{ m})}\right)^{-1}=-0.392\text{ mm}$$

The image appears 392 mm behind mirror. But because the image is reduced, it occupies a much smaller fraction of your field of view than the car itself would. The size of the image is given by $h_{\text{i}}=Mh_{\text{o}}=-\left(\frac{i}{o}\right)h_{\text{o}}=-\left(\frac{-0.392\text{ m}}{20.0\text{ m}}\right)h_{\text{o}}=0.0196h_{\text{o}}.$ The angle subtended by an object decreases linearly with distance (in the small angle approximation), so your brain perceives this image height as coming from $h_{\text{i}}=\frac{h_{\text{o}}}{r}\Rightarrow r=\frac{h_{\text{o}}}{h_{\text{i}}}=\frac{1}{0.0196}=51\text{ m}.$ Thus, the image occupies roughly the same angular spread in your field of view as a car that is 51 m away.

33.86. (*a*) We use Equation 33.23 to write the focal length in terms of the radius of curvature, and we insert this into Equation 33.24 to obtain $i=\left(\frac{1}{f}-\frac{1}{o}\right)^{-1}=\left(\frac{2}{R}-\frac{1}{o}\right)^{-1}=\left(\frac{2}{(200\text{ mm})}-\frac{1}{(60\text{ mm})}\right)^{-1}=-150\text{ mm}.$ So, the image is 150 mm behind the mirror. (*b*) The magnification is given by $m=-\frac{i}{o}=-\frac{(-150\text{ mm})}{(60\text{ mm})}=2.5.$ (*c*) The image is virtual; the image distance is negative, meaning it is behind the mirror. (*d*) The image is upright. This is clear from the positive magnification.

(*e*)

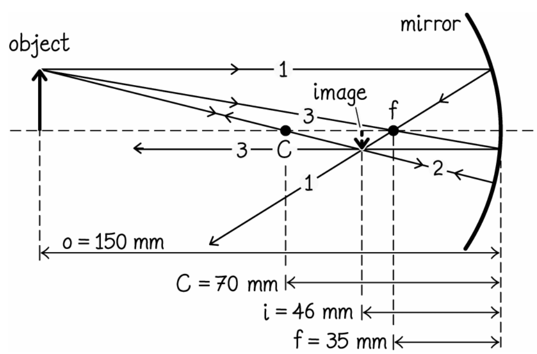


33.87. (*a*) We use Equation 33.23 to write the focal length in terms of the radius of curvature, and we insert this into Equation 33.24 to obtain the image distance:

$$i = \left(\frac{1}{f} - \frac{1}{o}\right)^{-1} = \left(\frac{2}{R} - \frac{1}{o}\right)^{-1} = \left(\frac{2}{(70.0\text{ mm})} - \frac{1}{(150\text{ mm})}\right)^{-1} = 45.65\text{ mm}$$

Now we can use the magnification to determine the image height: $\frac{h_i}{h_o} = m = -\frac{i}{o} \Rightarrow h_i = -\frac{i}{o}h_o$ $= -\frac{(-45.65\text{ mm})}{(150\text{ mm})}(20.0\text{ mm}) = -6.09\text{ mm},$ where the negative sign indicates that the image is inverted; the magnitude of the image height is 6.09 mm. (*b*) From the positive image distance, we see the image is real. (*c*) From the negative magnification, we see the image is inverted.

(*d*)



33.88. (*a*) Equation 33.23 can be solved for the image distance to obtain $i = \left(\frac{1}{f} - \frac{1}{o}\right)^{-1}$. Without inserting any values, we know the focal length is negative, meaning that both terms on the right hand side are negative and therefore the image distance is negative. Thus the image is virtual. (*b*) Since the magnification is given by $m = -\frac{i}{o}$, and we know the object distance is positive and the image distance is negative, clearly the magnification will have a positive sign. Thus the image is upright. (*c*) We use Equation 33.23 to write the focal length in terms of the radius of curvature, and we insert this into Equation 33.24 to obtain the image distance:

$$i = \left(\frac{1}{f} - \frac{1}{o}\right)^{-1} = \left(\frac{2}{R} - \frac{1}{o}\right)^{-1} = \left(\frac{2}{(-3.5\text{ m})} - \frac{1}{(0.50\text{ m})}\right)^{-1} = -0.39\text{ m}$$

Thus the image is located 0.44 m behind the mirror. (*d*) Your image is smaller than you by a factor of $m = -\frac{i}{o} = -\frac{(-0.389\text{ m})}{(0.50\text{ m})} = 0.78.$

33.89. The magnification tells us that $i = -mo,$ but we are not told whether the image is upright or inverted. Since the image distance is smaller than the object distance (in magnitude) a negative image distance would result in a negative focal length. But this cannot be the case, because we are told the mirror is a converging mirror. Thus, we set $i = -mo = (1/4)(1.0\text{ m}) = 0.25\text{ m}.$ Then we can insert this into Equation 33.23, which can be rearranged to yield

$$R = 2\left(\frac{1}{i} + \frac{1}{o}\right)^{-1} = 2\left(\frac{1}{(0.25\text{ m})} + \frac{1}{(1.0\text{ m})}\right)^{-1} = 0.40\text{ m}$$

33.90. Using the expression for the magnification gives us $m = N = -\dfrac{i}{o}$, and Equation 33.23 tells us $i = \left(\dfrac{2}{R} - \dfrac{1}{o}\right)^{-1}$. Inserting the latter equation into the former and solving for o yields $o = \left(1 - \dfrac{1}{N}\right)\dfrac{R}{2}$.

33.91. Since the image is real, we know the image distance is positive, which allows us to use the magnification to determine $i = -mo = -(-2)(0.750\text{ m}) = 1.50\text{ m}$. Then we can use Equation 33.23 to write $R = 2\left(\dfrac{1}{i} + \dfrac{1}{o}\right)^{-1} = 2\left(\dfrac{1}{(1.50\text{ m})} + \dfrac{1}{(0.750\text{ m})}\right)^{-1} = 1.00\text{ m}$.

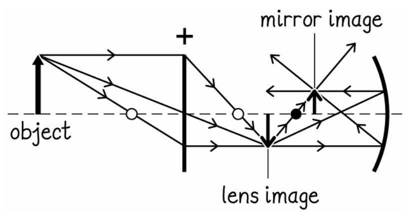
33.92. In all cases we use Equation 33.23 to determine the image distance: $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1}$, and we insert this into the magnification expression: $m = -\dfrac{i}{o}$. (*a*) $i = \left(\dfrac{1}{f} - \dfrac{1}{f/2}\right)^{-1} = -f = -300\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{-f}{f/2} = 2.00$. (*b*) The image position would go to infinity as the object approaches the focal point. The magnification is not defined in such a case, as no clear image is visible. (*c*) $i = \left(\dfrac{1}{f} - \dfrac{1}{3f/2}\right)^{-1} = 3f = 900\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{3f}{3f/2} = -2.00$. (*d*) $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} = \left(\dfrac{1}{f} - \dfrac{1}{2f}\right)^{-1} = 2f = 600\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{2f}{2f} = -1.00$. (*e*) $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} = \left(\dfrac{1}{f} - \dfrac{1}{3f}\right)^{-1}$ $= 3f/2 = 450\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{3f/2}{3f} = -0.500$. (*f*) In this case the image is formed at $i = 300\text{ mm}$, meaning all the light is focused down to a single point. One might call this a magnification of zero.

33.93. In all cases we use Equation 33.23 to determine the image distance: $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1}$, and we insert this into the magnification expression: $m = -\dfrac{i}{o}$. (*a*) $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} = \left(-\dfrac{1}{|f|} - \dfrac{1}{|f|/2}\right)^{-1} = -\dfrac{|f|}{3} = -100\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{-|f|/3}{|f|/2} = 0.667$. (*b*) $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} = \left(-\dfrac{1}{|f|} - \dfrac{1}{|f|}\right)^{-1} = -\dfrac{|f|}{2} = -150\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{-|f|/2}{|f|}$ $= 0.500$. (*c*) $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} = \left(-\dfrac{1}{|f|} - \dfrac{1}{3|f|/2}\right)^{-1} = -\dfrac{3|f|}{5} = -180\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{-3|f|/5}{3|f|/2} = 0.400$. (*d*) $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} = \left(-\dfrac{1}{|f|} - \dfrac{1}{2|f|}\right)^{-1} = -\dfrac{2|f|}{3} = -200\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{-2|f|/3}{2|f|} = 0.333$. (*e*) $i = \left(\dfrac{1}{f} - \dfrac{1}{o}\right)^{-1} =$ $\left(-\dfrac{1}{|f|} - \dfrac{1}{3|f|}\right)^{-1} = -\dfrac{3|f|}{4} = -225\text{ mm}$ and $m = -\dfrac{i}{o} = -\dfrac{-3|f|/4}{3|f|} = 0.250$. (*f*) In this case the image appears to be at $i = -300\text{ mm}$, meaning all the light is focused down to a single point. One might call this a magnification of zero.

For the converging mirror, some positions gave magnified images and some positions gave images reduced in size. But for diverging mirrors, all images were reduced in size. Converging mirrors can produce images that are either upright or inverted, whereas diverging mirrors can only produce images that are upright. Placing an object at the focal point of a converging mirror causes the image to be ill-defined (projected at infinity). Because we cannot place the object at the focal point of diverging mirror (which is behind the mirror) an analogous condition does not exist for a diverging mirror.

33.94. If the image appears 10.0 m behind the mirror, we know $i = -10.0$ m. From the magnification we find $m = -\frac{i}{o} \Rightarrow o = -\frac{i}{m} = -\frac{(-10.0\text{ m})}{(0.100)} = 100$ m. Inserting these distances into Equation 33.23 we find $R = 2f = 2\left(\frac{1}{i} + \frac{1}{o}\right)^{-1} = 2\left(\frac{1}{(-10.0\text{ m})} + \frac{1}{(100\text{ m})}\right)^{-1} = -22.2$ m.

33.95.



The final image is upright, smaller than the object, and is located a distance in front of the mirror equal to approximately $0.83R$.

33.96. Images made by diverging mirrors are always virtual, upright, behind the mirror, and shrunken. Images made by flat mirrors are always virtual, upright, behind the mirror, and the same size as the original object. Images made by converging mirrors may make real or virtual images depending on the location of the object relative to the focal length of the mirror. If the object is closer to the mirror than the focal length, then the image will be virtual, upright, behind the mirror, and magnified. If the object is farther from the mirror than the focal length, then the image will be real, inverted, and in front of the mirror. Whether the image is magnified or reduced depends again on where the object is. If the object is between the focal point and twice the focal length, then the object will be magnified. If the object is farther from the mirror than twice the focal length, the image will be reduced.

33.97. The first mirror must be concave, because a convex mirror cannot form a real image. The second mirror must be concave because a convex mirror cannot increase the size of an image compared to the object. Considering the magnification, for the first mirror we can write $i_1 = 2o_1$ which implies $R_1 = 2\left(\frac{1}{i_1} + \frac{1}{o_1}\right)^{-1} = \frac{4o_1}{3}$. In the case of the second mirror, we have $i_2 = -2o_2$, such that $R_2 = 2\left(\frac{1}{i_2} + \frac{1}{o_2}\right)^{-1} = 4o_1$. Thus $R_2/R_1 = 3$.

33.98. For each lens, we refer to Equation 33.36 and check to see if the sum $\frac{1}{R_1} + \frac{1}{R_2}$ is positive, negative, or zero. This will tell us if the focal length is positive, negative, or infinite (planar). (*a*) Both radii are positive, so the focal length is positive. The lens is converging. (*b*) One radius is positive and the other is infinite, making the focal length positive. The lens is converging. (*c*) One radius is positive and the other is negative. They appear to have roughly the same magnitude, making $\frac{1}{R_1} + \frac{1}{R_2} \approx 0$. This makes the focal length approximately infinite, meaning the lens is neighter converging nor diverging. (*d*) Both radii are negative, so the focal length is negative. The lens is diverging. (*e*) One radius is negative and the other is infinite, making the focal length negative. The lens is diverging. (*f*) One radius is negative and the other is positive, but the negative radius of curvature is larger than the positive radius. This tells us that $\frac{1}{R_1} + \frac{1}{R_2} > 0$ such that the lens is converging.

33.99. Using Equation 33.36 we have $f=\left[(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{-1}=\left[(1.40-1.00)\left(\frac{1}{(300\text{ mm})}+\frac{1}{(500\text{ mm})}\right)\right]^{-1}$ $=469\text{ mm}.$

33.100. Rearranging Equation 33.36, we have $R_2=\left(\frac{1}{f(n-1)}-\frac{1}{R_1}\right)^{-1}=\left(\frac{-3.0\text{ diopters}}{(1.5-1.0)}-\frac{1}{\infty}\right)^{-1}=-0.17\text{ m}.$

33.101. Rearranging Equation 33.36, we have $R_2=\left(\frac{1}{f(n-1)}-\frac{1}{R_1}\right)^{-1}=\left(\frac{1}{(0.170\text{ m})(1.6-1.0)}-\frac{1}{\infty}\right)^{-1}=0.10\text{ m}.$

33.102. Rearranging Equation 33.36, we have $R_2=\left(\frac{1}{f(n-1)}-\frac{1}{R_1}\right)^{-1}=\left(\frac{1}{(0.300\text{ m})(1.45-1.00)}-\frac{1}{(1.50\text{ m})}\right)^{-1}$ $=0.15\text{ m}.$

33.103. Rearranging Equation 33.36, we have $n=\frac{1}{f}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^{-1}+1=\frac{1}{(4.5\text{ mm})}\left(\frac{1}{(1.8\text{ mm})}+\frac{1}{(1.8\text{ mm})}\right)^{-1}$ $+1=1.2.$

33.104. (*a*) Using Equation 33.36 we have $f=\left[(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\right]^{-1}=\left[(1.50-1.00)\left(\frac{1}{(0.15\text{ m})}+\frac{1}{(-0.25\text{ m})}\right)\right]^{-1}$ $=0.75\text{ m}.$ (*b*) The focal length remains $0.75\text{ m}.$ The focal length does not change when the lens is turned around.

33.105. We can determine the image distance using the magnification: $m=-\frac{i}{o}\Rightarrow i=-mo=-\left(\frac{h_i}{h_o}\right)o$ $=-\left(\frac{(21.5\text{ mm})}{(10\text{ mm})}\right)(500\text{ mm})=-1.08\text{ m},$ where we have kept one additional significant digit because this is an intermediate step. Inserting this into Equation 33.16 gives us the focal length $f=\left(\frac{1}{i}+\frac{1}{o}\right)^{-1}=\left(\frac{1}{(-1.075\text{ m})}+\frac{1}{(0.500\text{ m})}\right)^{-1}=0.935\text{ m}.$ Finally, we rearrange Equation 33.36 to yield $R_2=\left(\frac{1}{f(n-1)}-\frac{1}{R_1}\right)^{-1}=\left(\frac{1}{(0.935\text{ m})(1.50-1.00)}-\frac{1}{(-0.350\text{ m})}\right)^{-1}=0.20\text{ m or }2.0\times10^2\text{ mm}.$

33.106. Equation 33.36 is derived for the specific case of a lens surrounded by air. One must work back through the derivation inserting an index of refraction for the medium in which the lens is submerged/imbedded. Call this n_{mat}, and call the index of refraction of the lens itself n_{lens}. This changes Equation 33.25 to now read: $n_{\text{lens}}\sin(\theta_1)=n_{\text{mat}}\sin(\theta_r),$ which changes the small angle approximation to read $n_{\text{lens}}\theta_1\approx n_{\text{mat}}\theta_r.$ This changes Equation 33.27 to $\theta_r-\theta_1=\left(\frac{n_{\text{lens}}}{n_{\text{mat}}}-1\right)\theta_1,$ and Equation 33.30 becomes $\frac{1}{f_1}=\frac{\left(\frac{n_{\text{lens}}}{n_{\text{mat}}}-1\right)}{R_1}.$ Clearly Equation 33.31 changes in an identical way, such that $\frac{1}{f_2}=\frac{\left(\frac{n_{\text{lens}}}{n_{\text{mat}}}-1\right)}{R_2}.$ Combining them as before, we obtain a more general lens maker's formula:

$$f = \left[\left(\frac{n_{\text{lens}}}{n_{\text{mat}}} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{-1} \tag{1}$$

We use equation (1) to write the ratio of the focal lengths in air and in water:

$$\frac{f_{\text{in air}}}{f_{\text{in water}}} = \frac{\left[\left(\frac{n_{\text{glass}}}{n_{\text{air}}} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{-1}}{\left[\left(\frac{n_{\text{glass}}}{n_{\text{water}}} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{-1}} = \frac{\left(\frac{n_{\text{glass}}}{n_{\text{water}}} - 1 \right)}{\left(\frac{n_{\text{glass}}}{n_{\text{air}}} - 1 \right)}$$

Or equivalently,

$$f_{\text{in water}} = \frac{\left(\frac{n_{\text{glass}}}{n_{\text{air}}} - 1 \right)}{\left(\frac{n_{\text{glass}}}{n_{\text{water}}} - 1 \right)} f_{\text{in air}} = \frac{\left(\frac{(1.55)}{(1.00)} - 1 \right)}{\left(\frac{(1.55)}{(1.33)} - 1 \right)} (0.500 \text{ m}) = 1.66 \text{ m}$$

33.107. (*a*) Using Equation 33.36 we have $f = \left[(n-1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{-1} = \left[(1.50 - 1.00) \left(\frac{1}{(40 \text{ mm})} + \frac{1}{\infty} \right) \right]^{-1} = 80 \text{ mm}.$

(*b*) Rearranging Equation 33.36, we find $n = \frac{1}{f} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} + 1 = \frac{1}{(100 \text{ mm})} \left(\frac{1}{(40 \text{ mm})} + \frac{1}{\infty} \right)^{-1} + 1 = 1.4.$

33.108. We use Equation 33.36 to write the ratio of the two focal lengths:

$$\frac{f_{\text{B}}}{f_{\text{A}}} = \frac{\left[(n_{\text{B}} - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{-1}}{\left[(n_{\text{A}} - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{-1}} = \frac{(n_{\text{A}} - 1)}{(n_{\text{B}} - 1)} = \frac{(n_{\text{A}} - 1)}{(2n_{\text{A}} - 1)}$$

So $f_{\text{B}} = \frac{(n_{\text{A}} - 1)}{(2n_{\text{A}} - 1)} f_{\text{A}} = \frac{((1.1) - 1.0)}{(2(1.1) - 1.0)} f_{\text{A}} = 0.083 f_{\text{A}}.$

33.109. (*a*) The lens is converging. One radius of curvature is positive and one is negative, but the positive radius is smaller in magnitude. This means the quantity $\left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ is positive, so the focal length is also positive. (*b*) Neither converging nor diverging. The quantity $\left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ would be zero, which would make the focal length infinite. (*c*) The lens would be diverging. The quantity $\left(\frac{1}{R_1} + \frac{1}{R_2} \right)$ would be negative, which would make the focal length negative.

33.110. We can rearrange Equation 33.36 to read $n = \frac{1}{f} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} + 1,$ which in this case can be simplified to $n = \frac{1}{f} \frac{R}{2} + 1.$ Here we have used the fact that the radius of curvature must be positive in order for the lens to form a real image. The fact that the magnification is −2, also lets us write the image distance as $i = -mo = 2R.$ Now the thin lens equation lets us determine the focal length of the lens: $f=\left(\frac{1}{i}+\frac{1}{o}\right)^{-1}=\left(\frac{1}{2R}+\frac{1}{R}\right)^{-1}=\frac{2R}{3}$. Inserting this into the simplification of Equation 33.36 above, we find $n=\frac{1}{f}\frac{R}{2}+1=\frac{3}{2R}\frac{R}{2}+1=1.75$.

33.111. You can only see light when it is reflected off of some object. In the air, there is no mechanism by which the laser light can be reflected to your eye. However, if you sprinkle powder into the path of the beam, the particles can scatter/reflect light to your eyes and the beam becomes partly visible.

33.112. When light leaves the fish and exits the water, it bends away from the normal line at the interface. The light rays thus bend such that they are closer to parallel to the water's surface. Since the fisherman's eyes trace those rays back in a linear fashion (ignoring refraction), the fisherman perceives the fish to be closer to the surface of the water (shallower than the fish actually is). Thus, the spear fisherman should aim below the image.

33.113. $\Delta t=\frac{d}{c}=\frac{dn}{c_0}=\frac{(4.0\times10^6\text{ m})(1.6)}{(3.00\times10^8\text{ m/s})}=0.021\text{ s}$

33.114. We would want the parallel rays of light from the sun to be focused down to a point right on the sails of the ship, 100 m away. So we would want a focal length of 100 m. This corresponds to a radius of curvature $R=2f=2(100\text{ m})=200\text{ m}$.

33.115. (*a*) This could be used to form a real image that is the same size as the original object. This follows from $i=\left(\frac{1}{f}-\frac{1}{o}\right)^{-1}=\left(\frac{1}{f}-\frac{1}{2f}\right)^{-1}=2f$, and $m=-\frac{i}{o}=-\frac{2f}{2f}=-1$. (*b*) This could be used to make light emanating from a point source travel in a straight line without spreading. This can be seen by writing $i=\left(\frac{1}{f}-\frac{1}{o}\right)^{-1}=\left(\frac{1}{f}-\frac{1}{f}\right)^{-1}\to\frac{1}{0}\Rightarrow i\to\infty$. (*c*) This could be used to focus light down to a single point (from the definition of a focal point), such as for heating a small region of space.

33.116. (*a*) We first use the magnification to determine the image distance. We know

$$\frac{h_\text{i}}{h_\text{o}}=m=-\frac{i}{o}\Rightarrow i=-o\left(\frac{h_\text{i}}{h_\text{o}}\right)=-(60\text{ mm})\left(\frac{(-80\text{ mm})}{(40\text{ mm})}\right)=1.2\times10^2\text{ mm}$$

(*b*) Now we can use the thin lens equation to determine the focal length: $f=\left(\frac{1}{i}+\frac{1}{o}\right)^{-1}=\left(\frac{1}{(120\text{ mm})}+\frac{1}{(60\text{ mm})}\right)^{-1}=40\text{ mm}$.

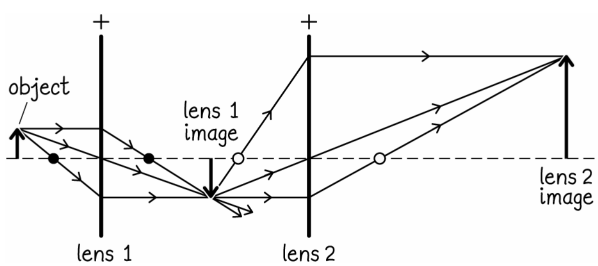
33.117. A converging lens can form either a real or a virtual image. Let us first assume the image is virtual (and upright). In that case the magnification tells us that $i=-mo=-3o$. Inserting this into the thin lens equation, we find $\frac{1}{f}=\frac{1}{i}+\frac{1}{o}=\frac{1}{-3o}+\frac{1}{o}\Rightarrow o=\frac{2f}{3}=\frac{2}{3}(50\text{ mm})=33\text{ mm}$. So, if the image is upright the object is 33 mm from the lens. Now let us consider the possibility that the image is real (and inverted). Then the magnification tells us $i=-mo=3o$, and the thin lens equation yields $\frac{1}{f}=\frac{1}{i}+\frac{1}{o}=\frac{1}{3o}+\frac{1}{o}\Rightarrow o=\frac{4f}{3}=\frac{4}{3}(50\text{ mm})=67\text{ mm}$. If the image is inverted, the object distance is 67 mm.

33.118. (*a*) The wavelength in this medium (call it medium 1) is $\lambda_1 = \frac{\lambda_{\text{vacuum}}}{n_1} = \frac{\lambda_{\text{vacuum}} c_1}{c_0} = \frac{(550\text{ nm})(2.4\times10^8\text{ m/s})}{(3.0\times10^8\text{ m/s})}$ $= 440\text{ nm}$. (*b*) The frequency is given by $f = \frac{c_0}{\lambda_0} = \frac{(3.0\times10^8\text{ m/s})}{(550\times10^{-9}\text{ m})} = 5.5\times10^{14}\text{ Hz}$. (*c*) The frequency does not change when the light goes from one medium to another. Thus the frequency is still $5.5\times10^{14}\text{ Hz}$.

33.119. (*a*) Dispersion. (*b*) It can be minimized, but not removed completely. (*c*) No.

33.120. (*a*) The image is virtual. A diverging mirror cannot form a real image. (*b*) The image is upright. The image distance is negative and the object distance is positive, making the overall magnification positive. This corresponds to an upright image. (*c*) We obtain the radius of curvature using Equation 33.24, inserting Equation 33.23 along the way: $R = 2f = 2\left(\frac{1}{i}+\frac{1}{o}\right)^{-1} = 2\left(\frac{1}{(-0.75\text{ m})}+\frac{1}{(1.2\text{m})}\right)^{-1} = -4.0\text{ m}$.

33.121.



33.122. The image distance can be obtained by the thin lens equation: $i = \left(\frac{1}{f}-\frac{1}{o}\right)^{-1} = \left(\frac{1}{(0.050\text{ m})}-\frac{1}{(29\text{ m})}\right)^{-1} = 0.05009\text{ mm}$ (where we have kept some extra digits, for this intermediate step; rounding to 0.050 gives us nonsense because that is the focal length). We now relate this to the image height through the magnification: $\frac{h_{\text{i}}}{h_{\text{o}}} = m = -\frac{i}{o} \Rightarrow h_{\text{i}} = -\frac{i}{o}h_{\text{o}} = -\frac{(0.05009\text{ m})}{(29\text{ m})}(1.7\text{ m}) = -2.9\text{ mm}$. Here the negative sign indicates that the image is inverted.

33.123. (*a*) We find the critical angle for the interface between diamond and flint glass: $\theta_{\text{c}} = \sin^{-1}\left(\frac{n_{\text{flint glass}}}{n_{\text{diamond}}}\right) = \sin^{-1}\left(\frac{(1.65)}{(2.42)}\right) = 43.0°$. (*b*) Now we find the critical angle for the air-flint glass interface, and then use Snel's law to find the incident angle in the diamond. For the air-flint glass interface, the critical angle is $\theta_{\text{c}} = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{flint glass}}}\right) = \sin^{-1}\left(\frac{(1.00)}{(1.65)}\right) = 37.31°$. This is the largest angle at which light in the flint glass can strike the air-glass interface and just barely make it through to the air. Now we assume light is indeed traveling at that angle from normal in the flint glass, and apply Snel's law to the diamond-glass interface: $\theta_{\text{diamond}} = \sin^{-1}\left(\frac{n_{\text{flint glass}}}{n_{\text{diamond}}}\sin(\theta_{\text{flint glass}})\right) = \sin^{-1}\left(\frac{(1.65)}{(2.42)}\sin(37.31°)\right) = 24.2°$.

33.124. You tell your friend to dive to the lake bottom while pulling a fishing line from your hand. This allows you to measure your friend's vertical depth. You tell your friend to shine the laser point just behind the boat so that light that exits the water hits the rear of the boat, just a few millimeters above the water line. While your friend remains stationary, you begin to paddle away from him, letting out fishing line attached to his floating marker as you go. As you drift away from your friend you watch the side of the boat as the bright spot sinks lower, closer to the water line. At the moment the bright spot disappears completely you stop letting out line and note the distance from you to your friend. Knowing your friend's depth and horizontal distance from you, you can calculate the incident angle of the beam on the surface $\theta_{\text{incident}} = \tan^{-1}(\Delta x/\Delta h)$. At the moment the beam vanished, it started undergoing total internal reflection, so the incident angle is also the critical angle. Hence we can write $\sin^{-1}(n_{\text{air}}/n_{\text{water}}) = \theta_{\text{c}} = \tan^{-1}(\Delta x/\Delta h)$. Equivalently, $n_{\text{water}} = n_{\text{air}}/\sin(\tan^{-1}(\Delta x/\Delta h))$. All quantities on the right hand side are known, so your goal is accomplished.

33.125. From the information given, we can determine the distance from the lens to the screen using the magnification: $\dfrac{h_{\text{i}}}{h_{\text{o}}} = m = -\dfrac{i}{o} \Rightarrow i = -\left(\dfrac{h_{\text{i}}}{h_{\text{o}}}\right)o$. At this point we note that a real image produced by a single lens is always inverted (the film is fed into the project with the scenes upside-down to correct for this), so the ratio of heights will be a negative number. Thus $i = -\left(\dfrac{h_{\text{i}}}{h_{\text{o}}}\right)o = -\left(\dfrac{-(1.450\text{ m})}{(0.02000\text{ m})}\right)(0.1000\text{ m}) = 7.250\text{ m}$. This means the distance from the film to the screen is $d = 7.350\text{ m}$, and this distance is fixed. So, we require a magnification that is three times greater than the initial magnification, or $m_{\text{final}} = 3m_{\text{i}} = 3\left(\dfrac{h_{\text{i}}}{h_{\text{o}}}\right) = 3\left(\dfrac{-(1.450\text{ m})}{(0.02000\text{ m})}\right) = -217.5$. We write the image distance in terms of the object distance and magnification: $i = -m_{\text{final}}o = -m_{\text{final}}(d-i) \Rightarrow i = \dfrac{-m_{\text{final}}d}{(1-m_{\text{final}})} = \dfrac{-(-217.5)(7.350\text{ m})}{(1-(-217.5))} = 7.316\text{ m}$, which corresponds to an object distance of $o = d - i = (7.350\text{ m}) - (7.3164\text{ m}) = 0.0336\text{ m}$. We can now determine the best lens to use by applying the thin lens equation: $f = \left(\dfrac{1}{i} + \dfrac{1}{o}\right)^{-1} = \left(\dfrac{1}{(7.3164\text{ m})} + \dfrac{1}{(0.0336\text{ m})}\right)^{-1} = 0.0335\text{ m}$. So, we must use a converging lens with a focal length of 33.5 mm. We must place the lens a distance 33.6 mm in front of the film holder. This will cause an image to be cast on the screen 7.316 m in front of the lens, and the image will be three times larger than your initial attempt, meaning the image will have a final height of 4.350 m.

33.126. Equation 33.36 is derived for the specific case of a lens surrounded by air. One must work back through the derivation inserting an index of refraction for the medium in which the lens is submerged/imbedded. Call this n_{mat}, and call the index of refraction of the lens itself n_{lens}. This changes Equation 33.25 to now read: $n_{\text{lens}}\sin(\theta_{\text{l}}) = n_{\text{mat}}\sin(\theta_{\text{r}})$, which changes the small angle approximation to read $n_{\text{lens}}\theta_{\text{l}} \approx n_{\text{mat}}\theta_{\text{r}}$. This changes Equation 33.27 to

$$\theta_{\text{r}} - \theta_{\text{l}} = \left(\frac{n_{\text{lens}}}{n_{\text{mat}}} - 1\right)\theta_{\text{l}}$$

and Equation 33.30 becomes

$$\frac{1}{f_1} = \frac{\left(\dfrac{n_{\text{lens}}}{n_{\text{mat}}} - 1\right)}{R_1}$$

Clearly Equation 33.31 changes in an identical way, such that

$$\frac{1}{f_2} = \frac{\left(\dfrac{n_{\text{lens}}}{n_{\text{mat}}} - 1\right)}{R_2}$$

Combining them as before, we obtain a more general lens maker's formula:

$$f = \left[\left(\frac{n_{\text{lens}}}{n_{\text{mat}}} - 1 \right) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \right]^{-1} \tag{1}$$

The only thing about equation (1) that changes if $n_{\text{medium}} > n_{\text{lens}}$ is the overall sign of the focal length. This means that a converging lens would suddenly behave like a diverging lens, and vice versa. Note also that for the special case that $n_{\text{medium}} = n_{\text{lens}}$, the focal length diverges as one would expect.

34

WAVE AND PARTICLE OPTICS

Solutions to Developing a Feel Questions, Guided Problems, and Questions and Problems

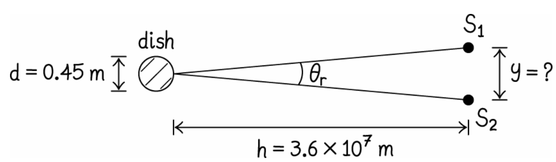
Developing a Feel

1. 10^3 m **2.** 10^0 rad or 10^1 degrees **3.** 10^{-1} rad or 10^1 degrees **4.** 10^{-5} m **5.** 10^{16} **6.** 10^{-10} N **7.** 10^{12}
8. 10^{-9} m **9.** 10^{-7} m **10.** 10^5 m/s

Guided Problems

34.2 Geosynchronous satellites

1. Getting Started We begin by sketching the dish and two satellites which will be the sources of the signals.



The condition for the two sources to be resolvable is Rayleigh's criterion for a circular aperture (because the satellite dish is circular). Rayleigh's criterion is given by equation (34.40) in PRAC:

$$\theta_r \approx \sin(\theta_r) = 1.22\frac{\lambda}{d}$$

Here, θ_r is the minimum resolvable angular separation of the two sources. d is the diameter of the dish, and λ is the wavelength of light being used by the satellites to transmit information.

2. Devise Plan In order to know how many satellites we can fit in the circular orbit described, we need to know the circumference of the circular orbit and the necessary minimum distance between adjacent satellites. The number of satellites is given by

$$N = \frac{C}{y} = \frac{2\pi R_{\text{orbit}}}{y}$$

$$N = \frac{2\pi(R_{\text{E}} + h)}{y} \quad (1)$$

In equation (1) R_E is the radius of Earth, and h is the height of the satellites above the surface of Earth. We can look up the radius of Earth and find $R_E = 6.38\times10^6$ km. Equation (34.40) can be used to give us the minimum resolving angle between satellites. We can relate that angular measure to the distance between satellites using basic geometry.

$$\tan(\theta_r/2) = y/2h$$

$$y = 2h\tan(\theta_r/2) \qquad (2)$$

In order to evaluate equation (34.40) and determine the resolving angle, we must know the wavelength of the light being used. We know that $c_0 = \lambda f$, so we simply insert $\lambda = c_0/f$. Using the results of equation (34.40) in equation (2) above, we have all necessary information to evaluate equation (1) and find the number of possible satellites.

3. Execute Plan Inserting numbers into equation (34.40) we find

$$\theta_r \approx \sin(\theta_r) = 1.22\frac{c_0}{df} = 1.22\frac{(3.00\times10^8 \text{ m/s})}{(0.45 \text{ m})(1.2\times10^{10} \text{ Hz})}$$

$$\theta_r \approx 0.0678$$

Inserting this into equation (2), we find

$$y = 2(3.6\times10^7 \text{ m})\tan(0.0678/2)$$

$$y = 2.44\times10^3 \text{ m}$$

Finally inserting this result and the radius of Earth into equation (1), we have

$$N = \frac{2\pi((6.38\times10^6 \text{ m}) + (3.6\times10^7 \text{ m}))}{(2.44\times10^3 \text{ m})}$$

$$N = 1.1\times10^2$$

Thus, to the number of significant digits given, 110 satellites can fit in the geosynchronous orbit.

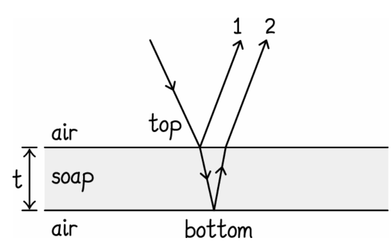
4. Evaluate Result This number is plausible. Note that not all satellites will operate at exactly 12 GHz, so this number is only an order of magnitude estimate. The actual number of geostationary satellites is somewhat larger.

34.4 Colorful soap film

1. Getting Started When light is incident on a soap film, some light will reflect off the first air-soap interface. Some light will pass through and then reflect off the second (back) soap-air interface. In order for a color of light to be brightly reflected from the surface of a film, the light waves reflected off the two interfaces must light up constructively once they leave the film. Thus we must consider how many wavelengths of light fit into the soap film. This means our answer should depend on the thickness of the film, and the index of refraction of the film. We must also take into account whether there are any phase shifts in the reflected rays due to reflection.

When the soap film becomes thinner, the wavelength of light that just barely fits in the film would also become smaller. So we expect the last color we see to be at the short-wavelength end of the visible spectrum.

Let us assume that we are looking at the soap film normal to its surface, or very close to normal. In the figure below, we draw the rays of light incident at an angle away from normal. We do this only for clarity in our picture. We will take the limit as this angle from normal approaches zero.



2. Devise Plan In order for the color to be strongly reflected from the film, the two reflected rays in our diagram (rays 1 and 2) must be in phase. We do not know the exact value of the index of refraction of the soap film, but we know it is very likely to be higher than that of air. So the ray that reflects off the top air-soap interface (ray 1) will have a phase shift due to reflection. Ray 2 will not have a phase shift due to reflection. This means that in order for the two rays to be in phase after leaving the film, ray 2 must travel an additional half wavelength. Thus

$$2t = \frac{\lambda_{\text{soap}}}{2} \tag{1}$$

And we know that the wavelength in the soap will be different than the wavelength in air according to $\lambda_{\text{soap}} = \lambda_{\text{air}}/n_{\text{soap}}$. Inserting this into equation (1) and rearranging, we have

$$t = \frac{\lambda_{\text{air}}}{4n_{\text{soap}}} \tag{2}$$

If we have an estimate of the wavelength and the index of refraction, we can obtain the thickness of the soap film.

3. Execute Plan We do not know the exact index of refraction of the soap bubble. But the index of refraction of water is $n_{\text{water}} = 1.33$. We will use the index of refraction of water for our calculation. However, many soapy films contain glycerin, which has an index of refraction of $n_{\text{glycerin}} = 1.46$. So estimates slightly larger than 1.33 are also reasonable. We approximate $n_{\text{soap}} = 1.33$.

(*a*) The smallest wavelength visible to the human eye (although this varies from person to person somewhat) is about 400 nm. Because the thinner film will strongly reflect shorter wavelengths of light, the last color of light we see will be violet light with a wavelength around 400 nm. (*b*) Inserting these values into equation (2), we obtain

$$t = \frac{(4.0\times10^{-7}\ \text{m})}{4(1.33)}$$
$$t = 75\ \text{nm}$$

4. Evaluate Result The thickness of a piece of paper is about 0.1 mm. We would expect the minimum thickness at which visible light is reflected strongly from the film to be orders of magnitude less than that. So our answer fits our expectation.

When a child fills the circular hole in a bubble-blowing wand (diameter approximately 0.01 m), the film reflects a swirl of color. Thus we expect the film is several times the wavelength of red light in thickness. We round our answer to $5\ \mu\text{m}$. That gives us a volume of liquid equal to $V = \pi\left(\frac{d}{2}\right)^2 t_{\text{in wand}} \approx 4\times10^{-10}\ \text{m}^3$. This volume of liquid might be used to blow a bubble with a radius around 3 cm. Using that assumption, we set the volume of liquid equal to the thickness of the bubble times the surface area of the bubble:

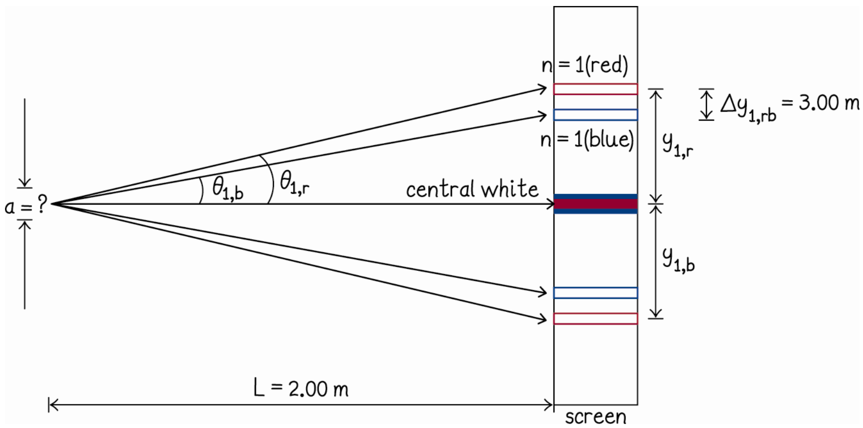
$$V = 4\pi r_{\text{bubble}}^2 t_{\text{bubble}}$$
$$t_{\text{bubble}} = \frac{V}{4\pi r_{\text{bubble}}^2}$$
$$t_{\text{bubble}} = \frac{(4\times10^{-10}\ \text{m}^3)}{4\pi(0.03\ \text{m})}$$
$$t_{\text{bubble}} = 35\ \text{nm}$$

This is a factor of two smaller than our answer. But the orders of magnitude are in perfect agreement.

34.6 Two colors

1. Getting Started When light passes through a narrow slit, the light will form bright lines (maxima) off to either side of the central bright spot, and dark regions (minima) between them. The distance from the central bright spot to those minima is large for large wavelengths, and small for small wavelengths. So, we expect any non-zero order minima (including the $n = 1$ minima discussed in this problem) to be closer to the center for blue light than for red light. So we expect that when the replacement is made and the blue laser is used, the minima should move 3.00 mm toward the center. We will check this more rigorously below.

We begin by drawing a diagram of the patterns of light created by the red and blue lasers. Marking bright and dark regions in two different colors on the same figure would become messy. So we mark only the minima, and we do so with thin line-like markers that are hollow inside (to distinguish them from bright lines or maxima).



In the figure above, the distances from the central white spot to the first-order red and blue minima are $y_{1,\mathrm{r}}$ and $y_{1,\mathrm{b}}$, respectively. The difference between the positions of the first order red and blue minima is $\Delta y_{1,\mathrm{rb}} = 3.00$ mm. We also label the angles from the center at which these minima occur. We are free to label any of these quantities either above or below the central bright spot, because of the symmetry about that point.

2. Devise Plan For a single slit, the locations of the minima are given by equation (34.26):

$$\sin(\theta_n) = \frac{n\lambda}{a}$$

We expect the angles at which these minima occur to be small enough that we are justified in using the small angle approximation, which is expressed in equation (34.27): $y_n = L\tan(\theta_n) \approx L\sin(\theta_n)$. Combining equations (34.26) and (34.27) we have

$$y_n = \frac{nL\lambda}{a} \tag{1}$$

We can apply equation (1) to the red light and the blue light, and take the difference between the two positions we obtain. That must be equal to the 3.00 mm spacing described in the problem.

3. Execute Plan (*a*) We can see from equation (34.26) that a smaller wavelength will correspond to a smaller angle for a given order of minima. This means our expectation was correct. When the blue laser replaces the red laser, the position of the first-order minima will move 3.00 mm toward the central white spot. (*b*) Writing out equation (1) for the red and blue light, and taking the difference, we obtain

$$\Delta y_{1,\mathrm{rb}} = y_{1,\mathrm{r}} - y_{1,\mathrm{b}} = \frac{(1)L(\lambda_\mathrm{r} - \lambda_\mathrm{b})}{\Delta y_{1,\mathrm{rb}}}$$

$$a = \frac{(1)(2.00\text{ m})((6.33\times10^{-7}\text{ m}) - (4.88\times10^{-7}\text{ m}))}{(3.00\times10^{-3}\text{ m})}$$

$$a = 9.67\times10^{-5}\text{ m}$$

4. Evaluate Result The width of the slit is two orders of magnitude larger than the wavelengths of light used. It is also about the width of a single sheet of paper (often taken to be 0.1 mm). This means our answer is plausible for two reasons. First, it is plausible that it could cause observable diffraction of light. Second, it is plausible that such a slit could be created with relative ease.

As an additional check, one might go back and insert the slit width into equation (1) or (34.26) to determine whether or not the small angle approximation was justified. We find

$$\theta_{1,\mathrm{r}} = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{(6.33\times10^{-7}\ \mathrm{m})}{(9.67\times10^{-5}\ \mathrm{m})}\right)$$

$$\theta_{1,\mathrm{r}} = 6.55\times10^{-3}\ \mathrm{rad\ or}\ 0.375^\circ$$

This is certainly small enough to apply the small angle approximation. So our method was sound.

34.8 Photoelectric effect with two light sources

1. Getting Started We can apply the conservation of energy to this problem. The energy initially in the laser light can be transformed into the kinetic energy of the ejected electrons, and into the change in potential energy of the electrons associated with removing the electrons from the metal. This latter energy is called the work function. Thus, for a given wavelength of incident light, a larger work function will correspond to lower kinetic energies left over.

2. Devise Plan With no potential difference between target and collector, some energy from the light will be used in ejecting the light from the metal and the remaining energy will be converted to kinetic energy of the electron:

$$\frac{hc}{\lambda} = E_0 + K \tag{1}$$

We need to write equation (1) for laser 1 and for laser 2. That will allow us to relate the two cases. We will rearrange variables to determine the work function E_0.

3. Execute Plan From the problem statement, we know

$$\frac{hc}{\lambda_1} = E_0 + K_1 \tag{2}$$

$$\frac{hc}{\lambda_2} = E_0 + K_2 \tag{3}$$

We also know that $\lambda_2 = 1.50\lambda_1$. Inserting this into equation (3), we find

$$\frac{hc}{1.50\lambda_1} = E_0 + K_2$$

We see that the above equation can be very easily combined with equation (2) if we multiply both sides by 1.5:

$$\frac{hc}{\lambda_1} = 1.50(E_0 + K_2) \tag{4}$$

Subtracting equation (2) from equation (4) we have

$$(0.50)E_0 + (1.50)K_2 - K_1 = 0$$
$$E_0 = 2.0(K_1 - (1.50)K_2)$$
$$E_0 = 2.0((2.8\ \mathrm{eV}) - (1.50)(1.1\ \mathrm{eV}))$$
$$E_0 = 2.3\ \mathrm{eV}$$

So the metal has a work function of 2.3 eV.

4. Evaluate Result The photoelectric experiment described is reasonable. So the best way to determine whether our answer is reasonable might be to compare it to known values of work functions of metals. We find there are quite a few metals with work functions close to this value, including Sr $(E_0 = 2.59\ \mathrm{eV})$, Cs $(E_0 = 2.14\ \mathrm{eV})$, and the closest match: Na $(E_0 = 2.36\ \mathrm{eV})$. So our answer is indeed reasonable.

Questions and Problems

34.1. There are many possible answers. Any material consisting of tiny pinholes or gaps is acceptable, such as the fabric of an umbrella. One can also observe diffraction at very sharp edges, such as that of a razor blade.

34.2. All light would diffract through the grating; the question is whether or not one would be readily able to observe the diffraction. An exact answer would depend on the screen and what experiment you do to detect the diffraction. We therefore give only order of magnitude estimates. The apertures in a window screen have a width on the order of 1×10^{-3} m. Diffraction may be observable for any wavelengths smaller than this. It should be noticeable for wavelengths on the order of 1 mm. Light of this wavelength has a frequency of 3×10^{11} Hz.

34.3. The wave properties of light predict that the center of the pattern should be bright. Since all rays diffracting around the edge of the circular obstacle are equidistant from the center, all the waves should strike the center in phase, and therefore interfere constructively.

34.4. (*a*) The diffracted waves would be bent less, such that a pattern cast on a screen would contract. The precise factor by which the pattern would contract is not constant for the entire pattern because of the sinusoidal dependence on the diffraction angle. But in the small angle approximation, the pattern would be half as wide when the aperture width is doubled. (*b*) The diffracted waves would be bent more, such that a pattern on a screen would spread out. As in (*a*) there is not one factor by which the entire pattern would spread. But in the small angle approximation (near the center of the pattern) the width of the pattern would approximately double.

34.5. Sound is also a wave and diffracts around barriers in a fashion similar to light. But the relevant wavelengths of sound waves and light waves are different by many orders of magnitude. Diffraction of a light wave around an object as thick as a tree is negligible, whereas the diffraction of sound around a tree is considerable.

34.6. (*a*) Let us call the ray of light that travels the shorter distance ray A, and let us call the ray that travels the longer distance ray B. The difference between the distances travelled by rays A and B is $\Delta\ell = d\sin(\theta)$. Since each full oscillation of the wave corresponds to a phase of 2π radians, we can write the total phase change as

$$\Delta\phi = 2\pi\left(\frac{\Delta\ell}{\lambda}\right) = \frac{2\pi d\sin(\theta)}{\lambda} = \frac{2\pi(100\times10^{-6}\text{ m})\sin(15.4°)}{(530\times10^{-9}\text{ m})} = 315\text{ rad}$$

which is equivalent to 50 full cycles plus an additional 0.659 rad. (*b*) Following the same procedure as part (*a*), we find $\Delta\phi = 2\pi\left(\frac{\Delta\ell}{\lambda}\right) = \frac{2\pi d\sin(\theta)}{\lambda} = \frac{2\pi(100\times10^{-6}\text{ m})\sin(20.7°)}{(530\times10^{-9}\text{ m})} = 419\text{ rad},$ which is equivalent to 66 full cycles plus an additional 4.36 rad.

34.7. The second order bright fringe occurs when light from one slit must travel exactly two wavelengths farther than the adjacent slit. Thus we write

$$\Delta\ell = d\sin(\theta) = 2\lambda \Rightarrow \theta = \sin^{-1}\left(\frac{2\lambda}{d}\right) = \sin^{-1}\left(\frac{2(589\times10^{-9}\text{ m})}{(4.00\times10^{-6}\text{ m})}\right) = 17.1°$$

34.8. Let us look at the position of an arbitrary m^{th} order bright fringe. The m^{th} order bright fringe occurs at a position such that light from one slit must travel m full wavelengths farther than light from an adjacent slit. Thus we write $\Delta\ell = d\sin(\theta) = m\lambda \Rightarrow \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$. Because the inverse sine function is monotonic in the region of interest $(0°\text{ to }90°)$, a larger argument in the inverse sine function corresponds to a larger angle. The above expression is valid for either of the wavelengths. Since red light has a longer wavelength, it will be diffracted to a larger angle than green light, for a given order of bright fringe. Since this holds for any value of m, the spacing for the bright fringes of red light is larger than that of green light, throughout the pattern.

34.9. The m^{th} order bright fringe occurs at a position such that light from one slit must travel m full wavelengths farther than light from an adjacent slit. Thus we write $\Delta\ell = d\sin(\theta) = m\lambda.$ Writing the center-to-center distance from one slit to the next as $d = \frac{w}{N},$ we find $\sin(\theta) = \frac{mN\lambda}{w}.$ Because the sine function is bounded above by one, we impose $m \le \frac{w}{N\lambda}$. Thus the criteria to determine the highest order bright fringe visible is $m \le \frac{w}{\lambda N}$, and the total number would be $2m_{\text{max}} + 1$.

34.10. Let us write the path length difference for rays from adjacent slits as $\Delta\ell = d\sin(\theta) = p\lambda$, where p is not necessarily an integer. We solve for p to determine whether the point is a bright fringe (integer), dark fringe (half integer), or something in between. From the distances given, we know $\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{394\text{ mm}}{1000\text{ mm}}\right) = 21.5°$. Inserting this into the equation above, we find $p = \frac{d\sin(\theta)}{\lambda} = \frac{(6.0\times10^{-6}\text{ m})\sin(21.5°)}{(400\times10^{-9}\text{ m})} = 5.5$. Note that the m^{th} order dark fringe occurs when the path length difference is $\left(m - \frac{1}{2}\right)\lambda$, such that this value of $p = 5.5$ corresponds to the sixth order dark fringe.

34.11. We can first use the information about the fifth order dark fringe to symbolically determine the spacing between slits in terms of the wavelength of light. We know from basic geometry that $\theta_{\text{fifth dark}} = \tan^{-1}\left(\frac{\Delta y_{\text{fifth dark}}}{\Delta x_{\text{screen}}}\right) = \tan^{-1}\left(\frac{45.0\text{ mm}}{500\text{ mm}}\right) = 5.143°$. Setting the path length difference to 4.5λ and rearranging yields $d = \frac{4.5\lambda}{\sin(\theta_{\text{fifth dark}})}$. This spacing between slits will be the same no matter what the angle of diffraction, so we equate $\frac{3.0\lambda}{\sin(\theta_{\text{third bright}})} = d = \frac{4.5\lambda}{\sin(\theta_{\text{fifth dark}})} \Rightarrow \theta_{\text{third bright}} = \sin^{-1}\left(\frac{3.0\lambda}{4.5\lambda}\sin(\theta_{\text{fifth dark}})\right) = \sin^{-1}\left(\frac{3.0}{4.5}\sin(5.143°)\right) = 3.426°$. From here, basic geometry tells us $\Delta y_{\text{third dark}} = \Delta x_{\text{screen}}\tan(\theta_{\text{third dark}}) = (500\text{ mm})\tan(3.426°) = 29.9\text{ mm}$.

34.12. (*a*) Initially, we equate the path length difference to the a full number (x) of wavelengths to obtain $d\sin(\theta_{\text{max}}) = x\lambda$, or equivalently $x = \frac{d\sin(\theta_{\text{max}})}{\lambda}$. From here it is trivial to see that doubling the wavelength halves the order of x. If x is an even number, then this simply means that the highest order bright fringe visible on the screen is now $x/2$, such that the total number of bright fringes on the screen is $2(x/2)+1 = x+1$. If however x is an even number, then the new fringe at the edge of the screen will be a dark fringe. The highest order bright fringe would then be written $(x-1)/2$ such that the number of total bright fringes would be $2(x-1)/2+1 = x$. (*b*) Starting from the expression obtained in part (*a*) $x = \frac{d\sin(\theta_{\text{max}})}{\lambda}$, we see that if the spacing is doubled so is the highest order of bright fringe on the screen. So the total number of bright fringes on the screen is then $2(2x)+1 = 4x+1$.

34.13. (*a*)

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(1)(546\times10^{-9}\text{ m})}{(1.0\times10^{-6}\text{ m})}\right) = 33°$$

(*b*) We use exactly the expression in part (*a*), but with the distance between slits oscillating in time:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{d_{\text{orig}} + A\sin(\omega t)}\right) = \sin^{-1}\left(\frac{546\times10^{-9}\text{ m}}{(1.0\times10^{-6}\text{ m}) + (0.25\times10^{-6}\text{ m})\sin((100\text{ s}^{-1})t)}\right)$$

(*c*) Because the sine function is monotonic between 0 and 90°, a smaller distance corresponds to a larger angle. Thus we want the distance to reach its minimum value of $d_{\text{orig}} - A$ which occurs after ¾ of an oscillation. Thus, we require $t = \frac{3}{4}T = \frac{3}{4}\left(\frac{2\pi}{\omega}\right) = \frac{3\pi}{200}$ s which is approximately 4.7×10^{-2} s. (*d*) Because the sine function is monotonic between 0 and 90°, a larger distance corresponds to a smaller angle. Thus we want the distance to reach its maximum value of $d_{\text{orig}} + A$ which occurs after ¼ of an oscillation. Thus, we require $t = \frac{1}{4}T = \frac{1}{4}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{200}$ s which is approximately 1.6×10^{-2} s. (*e*) Only A, λ, and d_{orig} affect the values of the maximum and minimum angles, although ω affects the time at which these maximum values are achieved.

34.14. There are several reasons. Visible light rays still undergo diffraction, but the wavelength of visible light is orders of magnitude larger than the interatomic spacing in crystalline solids. This means that in the expression $d\sin(\theta) = m\lambda$, the first order diffraction maximum would never be achieved at any angle. Basically, one might see a very slight attenuation of the central bright spot off near $\theta \to 90°$. But it would be very difficult to detect. In order to get bright peaks in intensity that are easy to identify and measure, one needs a few order of bright and dark spots. This means the wavelength has to be at least of the same order of magnitude as the interatomic spacing, if not smaller. Hence waves with a wavelength on the order of 10^{-10} m are used. These are X-rays. Another reason is that crystalline solids tend to absorb and reflect visible light. A greater intensity of light will make it through to a detector if X-rays are used.

34.15. Increasing the energy of an x-ray means increasing the quantity hf, or equivalently decreasing the wavelength. From the criterion for a bright fringe of arbitrary order $2d\cos(\theta) = m\lambda$, we see that decreasing the wavelength increases the angle of that diffraction order. Thus the bright spots will be farther apart.

34.16. (*a*) We first calculate the incident angle using $\theta = \cos^{-1}\left(\frac{m\lambda}{2d}\right) = \cos^{-1}\left(\frac{(1)(1.000\times10^{-10}\text{ m})}{2(2.815\times10^{-10}\text{ m})}\right) = 79.77°$. Now we relate the Bragg angle to this incident angle using $2\alpha + 2\theta = 180.0°$ or $\alpha = 90.00° - \theta = 10.23°$. (*b*) Since the Bragg angle is related to the atomic spacing and order by $2d\sin(\alpha) = m\lambda$, the order is bounded by $\frac{m\lambda}{2d} \le 1 \Rightarrow m \le \frac{2d}{\lambda} = \frac{2(2.815\times10^{-10}\text{ m})}{(1.000\times10^{-10}\text{ m})} = 5.63$, so the highest order is 5. The corresponding Bragg angle is $\alpha = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{5(1.000\times10^{-10}\text{ m})}{2(2.815\times10^{-10}\text{ m})}\right) = 62.64°$.

34.17. The largest Bragg angle corresponds to the largest angle of reflection (or incidence). We can relate the two angles through Figure 34.20 using $2\alpha + 2\theta = 180.0°$ or $\theta = 90.00° - \alpha = 90.00° - 35.00° = 55.00°$. Since this corresponds to the lowest order reflected ray, we can write $d = \frac{m\lambda}{2\cos(\theta)} = \frac{1(1.000\times10^{-9}\text{ m})}{2\cos(55.00°)} = 8.717\times10^{-10}$ m.

34.18. The Bragg angle is related to other quantities through $2d\sin(\alpha) = m\lambda$, which can be rearranged to yield $\sin(\alpha) = \frac{m\lambda}{2d}$, such that the order of the Bragg angle is clearly bounded by $m \le \frac{2d}{\lambda} = \frac{2(6.70\times10^{-10}\text{ m})}{(5.00\times10^{-10}\text{ m})} = 2.68$, so the two highest orders of Bragg angle are $m = 2$ and $m = 1$. From here it is trivial to plug numbers into the equation $\alpha = \sin^{-1}\left(\frac{m\lambda}{2d}\right)$ and obtain $\alpha_{\text{highest}} = \sin^{-1}\left(\frac{2(0.500\times10^{-10}\text{ m})}{2(6.70\times10^{-10}\text{ m})}\right) = 48.27°$ and $\alpha_{\text{2nd highest}} = \sin^{-1}\left(\frac{1(0.500\times10^{-10}\text{ m})}{2(6.70\times10^{-10}\text{ m})}\right)$ $= 21.91°$.

34.19. The DeBroglie wavelength is given by $\lambda = h/p$ or $\lambda = h/mv$ for nonrelativistic massive objects like people walking down the street. The order of magnitude of a person's mass is 1×10^{2} kg, and the order of magnitude of a person's speed while walking (as opposed to running) is right between 1×10^{0} m/s and 1×10^{1} m/s (around 3 m/s). Either of these orders is acceptable depending on whether it is a leisurely stroll or a brisk walk. The constant h is of order 1×10^{-33} J·s. So the order of magnitude of the wavelength of a person walking down the street could be 10^{-35} m or 10^{-36} m.

34.20. $\lambda = \dfrac{h}{mv} \Rightarrow v = \dfrac{h}{m\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(1.5\times10^{-10}\ \text{m})} = 2.6\times10^{3}\ \text{m/s}$

34.21. Yes, the electron has a non-zero chance of reaching the detector in all cases. But the chance may be very small if the obstacle is wide.

34.22. $\lambda = \dfrac{h}{mv} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(0.17\ \text{kg})(45\ \text{m/s})} = 8.7\times10^{-35}\ \text{m}$

34.23. We can relate the kinetic energy to the velocity according to

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(7.48\times10^{-13}\ \text{J})}{(6.645\times10^{-27}\ \text{kg})}} = 1.50\times10^{7}\ \text{m/s}$$

It would not be necessary to calculate the speed as an intermediate step; we carried out the calculation here to ensure that the speed is low enough that relativity can be ignored. At only 5% the speed of light, we can safely ignore relativity here. How the DeBroglie wavelength is given by $\lambda = \dfrac{h}{mv} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(6.645\times10^{-27}\ \text{kg})(1.50\times10^{7}\ \text{m/s})}$ $= 6.65\times10^{-15}$ m.

34.24. We simply calculate the wavelength of each particle or radiation type. The particles or radiation with the larger wavelengths with have greater distance between bright fringes. For light, we use the approximate values $\lambda_{\text{yellow}} = 550$ nm and $\lambda_{\text{blue}} = 440$ nm. For the particles, we use $\lambda = \dfrac{h}{mv}$. So $\lambda_{n^0} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(1.04\times10^{3}\ \text{m/s})} = 3.82\times10^{-10}$ m, and to the significant digits given this expression is exactly the same for a proton. However, we know that the proton is actually slightly less massive than a neutron. So even though $\lambda_{p^+} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(1.04\times10^{3}\ \text{m/s})} = 3.82\times10^{-10}$ m it's wavelength will be slightly larger than that of the neutron. Finally, for the electron we have $\lambda_{e^-} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(1.04\times10^{3}\ \text{m/s})} = 6.99\times10^{-7}$ m. So the order of increasing wavelengths, and therefore of increasing spacing between bright fringes is: n^0, p^+, blue, yellow, e^-.

34.25. For the entire diffraction pattern to have exactly 101 bright fringes, the $m = 50$ bright fringe must be cast at $\theta = 90°$. We know $d\sin(\theta) = m\lambda \Rightarrow \lambda = \dfrac{d\sin(\theta)}{m}$, and we know the wavelength in this equation is the DeBroglie wavelength of the electrons. Thus $\dfrac{h}{m_{e^-}v} = \dfrac{d\sin(\theta)}{m}$, where we have put an e^- subscript on the mass to distinguish it from the m^{th} order. Solving for the speed, we find $v = \dfrac{mh}{d\sin(\theta)m_{e^-}} = \dfrac{(50)(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(7.00\times10^{-6}\ \text{m})\sin(90°)(9.11\times10^{-31}\ \text{kg})}$ $= 5.20\times10^{3}$ m/s.

34.26. Blue light has higher energy per photon. This can be seen by noting that blue light has a higher frequency than red light (or equivalently a smaller wavelength than yellow light). The energy of a photon can be written $\dfrac{hc}{\lambda}$ or hf. So the higher frequency or lower wavelength light has the greater energy.

34.27. We know the energy for a single photon is given by $\dfrac{hc}{\lambda}$. So $N\dfrac{hc}{\lambda} = E_{\text{total}} \Rightarrow N = \dfrac{E_{\text{total}}\lambda}{hc} = \dfrac{(200\text{ J})(580\times10^{-9}\text{ m})}{(6.626\times10^{-34}\text{ m})(3.00\times10^{8}\text{ m/s})} = 5.84\times10^{20}.$

34.28. (*a*) The smallest amount of momentum that can be delivered by red light is the momentum in a single photon of red light: $p = \dfrac{h}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})}{(700\times10^{-9}\text{ m})} = 9.47\times10^{-28}\text{ kg}\cdot\text{m/s}.$ (*b*) The smallest amount of energy that can be delivered by red light is the energy of a single photon of red light: $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(700\times10^{-9}\text{ m})} = 2.84\times10^{-19}\text{ J}.$

34.29. Let us assume that all the power refers to the output power as opposed to internal power use from the batteries, and assume further the photons are not scattered or absorbed by any medium on their way to the wall. Then we can equate the energy delivered in the form of photon energies each second to the power rating of the laser: $P = E\Delta t = N\left(\dfrac{hc}{\lambda}\right)\Delta t \Rightarrow N = \dfrac{P\lambda}{hc\Delta t} = \dfrac{(5.00\times10^{-3}\text{ W})(532\times10^{-9}\text{ m})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})(1.00\text{ s})} = 1.34\times10^{16}$ photons strike the wall each second.

34.30. (*a*) We must assume some average value of the wavelength of light emitted by the sun. One might guess that it is the same as the average wavelength of light that reaches Earth's surface (550-580 nm). But the average wavelength emitted is actually somewhat lower, closer to 500 nm. But anything of this order is appropriate. Using this value, one obtains

$$P = E\Delta t = N\left(\frac{hc}{\lambda}\right)\Delta t \Rightarrow N = \frac{P\lambda}{hc\Delta t} = \frac{(3.83\times10^{26}\text{ W})(500\times10^{-9}\text{ m})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})(1.00\text{ s})} = 9.6\times10^{44}\text{ photons per second}$$

Slightly different answers are fine, but the answer should definitely be of the order 1×10^{45} photons emitted by the sun each second. (*b*) Again, we must choose a value for the average wavelength of the incident light. One might continue using the approximate value of part (*a*). Or one might happen to know that the average wavelength reaching the surface of Earth is around 580 nm. We will use that latter value for the purposes of calculation, but other estimates may be valid or even better under certain weather conditions. If we assume the sun emits photons equally in all directions, we can calculate the intensity at Earth's position in the solar system and find a value of $I = \dfrac{P}{A} = \dfrac{P}{4\pi R_{\text{ES}}^2} = \dfrac{(3.83\times10^{26}\text{ W})}{4\pi(1.496\times10^{11}\text{ m})^2} = 1.36\times10^{3}\text{ W/m}^2.$ However, one must make an assumption about how much of that intensity actually makes it through the atmosphere to the solar panel. This obviously depends on weather. But the World Meteorological Organization uses the value $1.2\times10^{2}\text{ W/m}^2$ as a typical value in sunshine. Whichever intensity we use, we find the number of photons using $P = E\Delta t = N\left(\dfrac{hc}{\lambda}\right)\Delta t \Rightarrow N = \dfrac{IA\lambda}{hc\Delta t}.$ Using value in the upper atmosphere yields

$$N = \frac{IA\lambda}{hc\Delta t} = \frac{(1.36\times10^{3}\text{ W/m}^2)(1.00\text{ m}^2)(580\times10^{-9}\text{ m})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})(1.00\text{ s})} = 3.97\times10^{21}\text{ photons per second}$$

and using the surface value yields

$$N = \frac{IA\lambda}{hc\Delta t} = \frac{(1.2\times10^{2}\text{ W/m}^2)(1.00\text{ m}^2)(580\times10^{-9}\text{ m})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})(1.00\text{ s})} = 3.5\times10^{20}\text{ photons per second}$$

Anything of the order of magnitude of these answers may be acceptable.

34.31. (*a*) $E=\dfrac{hc}{\lambda}=\dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(633\times10^{-9}\text{ m})}=3.14\times10^{-19}\text{ J}$ (*b*) The two lasers emit different numbers of photons each second. In order for laser A to have four times the power of laser B, laser A must emit four times as many photons each second as laser B does.

34.32. We know that $d\sin(\theta)=m\lambda$ for a bright fringe. Using the small angle approximation, we can write this as $d\dfrac{\Delta y}{\Delta x}=m\lambda.$ Applying this to two adjacent orders (m and $m+1$) and comparing, we find $\Delta y_{m+1}-\Delta y_m=\dfrac{(m+1)\Delta x}{d}\lambda-\dfrac{(m)\Delta x}{d}\lambda=\dfrac{\Delta x}{d}\lambda\Rightarrow\lambda=\dfrac{(\Delta y_{m+1}-\Delta y_m)d}{\Delta x}=\dfrac{(20\times10^{-3}\text{ m})(4.5\times10^{-6}\text{ m})}{(0.200\text{ m})}=4.5\times10^{-7}\text{ m}.$

34.33. Let us first determine the maximum angle to which light can be diffracted and still hit the screen:

$$\theta_{\max}=\tan^{-1}\left(\frac{\Delta y_{\max}}{\Delta x}\right)=\tan^{-1}\left(\frac{(200\text{ mm})}{(1000\text{ mm})}\right)=11.31^\circ$$

Now we determine what order of diffraction this angle corresponds to, keeping in mind that our order m' may not be an integer:

$$d\sin(\theta_{\max})=(m')\lambda\Rightarrow m'=\frac{d\sin(\theta_{\max})}{\lambda}=\frac{(4.000\times10^{-6}\text{ m})\sin(11.31^\circ)}{(650\times10^{-9}\text{ m})}=1.21$$

This means that we have the $m=0$ bright fringe, and two fringes on either side of the central bright fringe corresponding to $m=1$. Thus, there are three bright fringes on the screen. (*b*) The angle from the center to the first order bright fringe is $\theta=\sin^{-1}\left(\dfrac{\lambda}{d}\right)$, such that $\Delta y=\Delta x\tan(\theta)=\Delta x\tan\left(\sin^{-1}\left(\dfrac{\lambda}{d}\right)\right)$ $=(1.00\text{ m})\tan\left(\sin^{-1}\left(\dfrac{(650\times10^{-9}\text{ m})}{(4000\times10^{-9}\text{ m})}\right)\right)=0.165\text{ m}.$ Clearly, if there is 0.165 m between each bright fringe, only two such spacings (between three fringes) will fit on a screen 0.400 m wide.

34.34. $\lambda=\dfrac{d\sin(\theta)}{m}=d\sin\left(\tan^{-1}\left(\dfrac{\Delta y}{\Delta x}\right)\right)=(0.150\times10^{-3}\text{ m})\sin\left(\tan^{-1}\left(\dfrac{23.0\times10^{-3}\text{ m}}{5.0\text{ m}}\right)\right)=6.9\times10^{-7}\text{ m}.$

34.35. Let us consider the central and first order bright fringes, such that we can write

$$\lambda=\frac{d\sin(\theta)}{m}=d\sin\left(\tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)\right)=(5.00\times10^{-6}\text{ m})\sin\left(\tan^{-1}\left(\frac{170\times10^{-3}\text{ m}}{2.00\text{ m}}\right)\right)=4.23\times10^{-7}\text{ m}\ \text{ or 423 nm}$$

34.36. Using Equation 34.11 we can write

$$\frac{S_{\text{av}}}{S_{\text{center,av}}}=\frac{4S_{0,\text{av}}\cos^2\left(\dfrac{\pi d\sin(\theta)}{\lambda}\right)}{4S_{0,\text{av}}}\Rightarrow\theta=\sin^{-1}\left(\frac{\lambda}{\pi d}\cos^{-1}\left(\sqrt{\frac{S_{\text{av}}}{S_{\text{center,av}}}}\right)\right)=\sin^{-1}\left(\frac{(510\times10^{-9}\text{ m})}{\pi(2.30\times10^{-6}\text{ m})}\cos^{-1}(\sqrt{0.12})\right)=4.9^\circ$$

The distance on the screen is then given by $\Delta y=\Delta x\tan(\theta)=(450\text{ mm})\tan(4.92^\circ)=39\text{ mm}.$

34.37. In all cases, we find the distance on the screen from the center using $\lambda=\dfrac{d\sin(\theta)}{m}=\dfrac{d}{m}\sin\left(\tan^{-1}\left(\dfrac{\Delta y}{\Delta x}\right)\right)$ or equivalently $\Delta y=\Delta x\tan\left(\sin^{-1}\left(\dfrac{m\lambda}{d}\right)\right)$. Inserting numbers, we find

$$\Delta y_{1g} = (0.500\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(546.1\times10^{-9}\text{ m})}{(1.67\times10^{-6}\text{ m})}\right)\right) = 0.173\text{ m}$$

$$\Delta y_{2g} = (0.500\text{ m})\tan\left(\sin^{-1}\left(\frac{(2)(546.1\times10^{-9}\text{ m})}{(1.67\times10^{-6}\text{ m})}\right)\right) = 0.434\text{ m}$$

$$\Delta y_{1b} = (0.500\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(435.8\times10^{-9}\text{ m})}{(1.67\times10^{-6}\text{ m})}\right)\right) = 0.135\text{ m}$$

$$\Delta y_{2b} = (0.500\text{ m})\tan\left(\sin^{-1}\left(\frac{(2)(435.8\times10^{-9}\text{ m})}{(1.67\times10^{-6}\text{ m})}\right)\right) = 0.307\text{ m}$$

34.38. Yes, this could be due to interference. There would be a maximum in the interference directly in front of the center of the façade, and a first order minimum at $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(1)(0.300\text{ m})}{(0.500\text{ m})}\right) = 36.9°.$ This is equivalent to a distance from the center of the façade of $\Delta x = \ell\tan(\theta) = (10\text{ m})\tan(36.9°) = 7.5\text{ m}.$ If there were simply a double slit, there would be a single minimum between these maxima. But with a grating, the intensity of the signal is significantly reduced everywhere in between the maxima.

34.39. (*a*) We find the angle for each wavelength and take the difference. For the shorter wavelength, we have

$$\theta_{\text{shorter}} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(3)(589.0\times10^{-9}\text{ m})}{(2.00\times10^{-6}\text{ m})}\right) = 62.07°$$

and for the longer we have

$$\theta_{\text{longer}} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(3)(589.6\times10^{-9}\text{ m})}{(2.00\times10^{-6}\text{ m})}\right) = 62.18°$$

Thus the difference in angular positions is $\theta_{\text{longer}} - \theta_{\text{shorter}} = (62.18°) - (62.07°) = 0.11°.$ (*b*) If we assume it does exist and try to find the angle at which it appears, we see that

$$\theta_{m=4} = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(4)(589.6\times10^{-9}\text{ m})}{(2.00\times10^{-6}\text{ m})}\right) = \sin^{-1}(1.179)$$

which is not possible. There is no fourth order in the diffraction pattern.

34.40. (*a*) Using Equation 34.16 we have

$$\theta_{\text{maxima}} = \sin^{-1}\left(\frac{m\lambda}{d}\right),\ \text{such that}\ \theta_1 = \sin^{-1}\left(\frac{(1)(550\times10^{-9}\text{ m})}{(0.125\times10^{-3}\text{ m})}\right) = 0.252°$$

$$\theta_2 = \sin^{-1}\left(\frac{(2)(550\times10^{-9}\text{ m})}{(0.125\times10^{-3}\text{ m})}\right) = 0.504°$$

(*b*) Using Equation 34.17 we have $\theta_{\text{minima}} = \sin^{-1}\left(\frac{k\lambda}{Nd}\right),$ where k is not an integer multiple of the number of slits N. Thus, we expect $N-1$ dark spots in between the two bright spots and they occur at these angles:

$$\theta_1 = \sin^{-1}\left(\frac{(1)(550\times10^{-9}\text{ m})}{(6)(0.125\times10^{-3}\text{ m})}\right) = 0.0420°$$

$$\theta_2 = \sin^{-1}\left(\frac{(2)(550\times10^{-9}\text{ m})}{(6)(0.125\times10^{-3}\text{ m})}\right) = 0.0840°$$

$$\theta_3 = \sin^{-1}\left(\frac{(3)(550\times10^{-9}\text{ m})}{(6)(0.125\times10^{-3}\text{ m})}\right) = 0.126°$$

$$\theta_4 = \sin^{-1}\left(\frac{(4)(550\times10^{-9}\text{ m})}{(6)(0.125\times10^{-3}\text{ m})}\right) = 0.168°$$

$$\theta_5 = \sin^{-1}\left(\frac{(5)(550\times10^{-9}\text{ m})}{(6)(0.125\times10^{-3}\text{ m})}\right) = 0.210°$$

(*c*) There are five, as seen in part (*b*). (*d*) There are four secondary maxima, between each of the dark fringes from part (*b*).

34.41. Let us find the distance from the center of the pattern to the first order maximum of each color, and then take the difference. In general, the angular position of a bright fringe is $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$ such that the distance on the screen is $\Delta y = \Delta x\tan(\theta) = \Delta x\tan\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right)$. For red light, we have

$$\Delta y_{1,\text{red}} = (3.00\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(650\times10^{-9}\text{ m})}{(2.50\times10^{-6}\text{ m})}\right)\right) = 0.8078\text{ m}$$

and for green light we have

$$\Delta y_{1,\text{green}} = (3.00\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(532\times10^{-9}\text{ m})}{(2.50\times10^{-6}\text{ m})}\right)\right) = 0.6534\text{ m}$$

So the distance between these two bright fringes is $\Delta y_{1,\text{red}} - \Delta y_{1,\text{green}} = (0.8078\text{ m}) - (0.6534) = 0.154\text{ m}$.

34.42. No. This would require the sine function to increase linearly with its argument, which is true only at small angles. The setup described could be approximately satisfied at small angles, but not at large angles.

34.43. We first calculate the highest order of bright fringe that is produced by requiring that $\sin(\theta) = \frac{m\lambda}{d} \le 1 \Rightarrow m \le \frac{d}{\lambda} = \frac{(0.115\times10^{-3}\text{ m})}{(570\times10^{-9}\text{ m})} = 201.8$, so there are 201 orders of bright lines. Thus, the screen must extend to an angle $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(201)(570\times10^{-9}\text{ m})}{(0.115\times10^{-3}\text{ m})}\right) = 85.04°$. The vertical height of the screen above the center is then given by $\Delta y = \Delta x\tan(\theta) = (0.0450\text{ m})\tan(85.04°) = 0.519\text{ m}$. But this is only the distance from the center to the upper edge. The entire width of the screen is 1.04 m.

34.44. From $d\sin(\theta) = m\lambda$ it is easy to see that the longer wavelength light will be diffracted to a larger angle. Thus, if a bright red fringe exists in the pattern at a given order, that same order of blue fringe must also exist. Thus, we need only find the highest order at which red light has a maximum intensity. We require $\sin(\theta_{\text{red}}) = \frac{m\lambda}{d} \le 1 \Rightarrow m \le \frac{d}{\lambda} = \frac{(2.50\times10^{-6}\text{ m})}{(700\times10^{-9}\text{ m})} = 3.57$. So the highest order bright fringe of red light is three. Thus there are three complete spectra on one side of the slits and three complete spectra on the other side. The entire spectrum exists at the center, but not dispersed into different colors. All overlap at the center.

34.45. We can determine the spacing between adjacent slits using $d = \frac{m\lambda}{\sin(\theta)} = \frac{(2)(566.0\times10^{-9}\text{ m})}{\sin(22.00°)} = 3.022\times10^{-6}\text{ m}$. If this is the spacing between each pair of slits, then the number of slits is $N = \frac{w_{\text{grating}}}{d} = \frac{(24.00\times10^{-3}\text{ m})}{(3.022\times10^{-6}\text{ m})} = 7.942\times10^{3}$ slits.

34.46. From $d\sin(\theta) = m\lambda$ it is easy to see that the longer wavelength light will be diffracted to a larger angle. Overlap will occur if $m\lambda_{\text{red}} \geq (m+1)\lambda_{\text{blue}}$. To find the lowest order at which that happens, we change ≥ to = and solve for the order

$$m = \frac{1}{\frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} - 1} = \frac{1}{\frac{750\text{ nm}}{390\text{ nm}} - 1} = 1.08$$

This means that the first order bright fringes will be distinct, but higher orders will overlap to some extent.

34.47. (*a*) Let us find the distance from the center of the pattern to the first order maximum of each color, and then take the difference. In general, the angular position of a bright fringe is $\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$ such that the distance on the screen from the center to the bright fringe is $\Delta y = \Delta x \tan(\theta) = \Delta x \tan\left(\sin^{-1}\left(\frac{m\lambda}{d}\right)\right)$. For red light, we have

$$\Delta y_{1,\text{red}} = (2.10\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(656.3\times10^{-9}\text{ m})}{(1.00\times10^{-4}\text{ m})}\right)\right) = 0.01378\text{ m}$$

and for blue-green light we have

$$\Delta y_{1,\text{blue-green}} = (2.10\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(486.1\times10^{-9}\text{ m})}{(1.00\times10^{-4}\text{ m})}\right)\right) = 0.01021\text{ m}$$

So the distance between these two bright fringes is $\Delta y_{1,\text{red}} - \Delta y_{1,\text{blue-green}} = (0.01378\text{ m}) - (0.01021\text{ m}) = 3.6\text{ mm}$. We would consider these two to be resolvable if the distance from the red peak to the blue peak is greater than the distance from the red peak to the first order minimum in red light. The position of the first order minimum of red light is given by

$$\Delta y = \Delta x \tan(\theta) = \Delta x \tan\left(\sin^{-1}\left(\frac{\left(m + \frac{1}{2}\right)\lambda}{d}\right)\right)$$

or

$$\Delta y_{1,\text{red min}} = (2.10\text{ m})\tan\left(\sin^{-1}\left(\frac{(1/2)(656.3\times10^{-9}\text{ m})}{(1.00\times10^{-4}\text{ m})}\right)\right) = 0.00689\text{ m}$$

The distance from the first order maximum of red light to the first order minimum of red light is $\Delta y_{1,\text{red max}} - \Delta y_{1,\text{red min}} = (0.01378\text{ m}) - (0.00689) = 6.9\text{ mm}$. This means that the packet of bright red light is wider than the separation between the red and blue peaks, such that the two packets of bright light are not resolvable. We could just as easily have considered the width of the bright blue region, rather than the red. (*b*) We repeat the process in part (*a*), now using $d = \frac{(1.00\times10^{-3}\text{ m})}{600} = 1.67\times10^{-6}\text{ m}$. For red light, we have

$$\Delta y_{1,\text{red}} = (2.10\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(656.3\times10^{-9}\text{ m})}{(1.667\times10^{-6}\text{ m})}\right)\right) = 0.8996\text{ m}$$

and for blue-green light we have

$$\Delta y_{1,\text{blue-green}} = (2.10\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(486.1\times10^{-9}\text{ m})}{(1.667\times10^{-6}\text{ m})}\right)\right) = 0.6403\text{ m}$$

So the distance between these two bright fringes is $\Delta y_{1,\text{red}} - \Delta y_{1,\text{blue-green}} = (0.8996\text{ m}) - (0.6403\text{ m}) = 0.259\text{ m}$.

34.48. We know the longer wavelength will be diffracted (at a given order) to a larger angle. Call the angle between the long-wavelength first order bright light and the adjacent dark line $\Delta\theta_{\text{long bright dark}} = \theta_{\text{long, }m=1\text{, bright}} - \theta_{\text{long, }m=1\text{, dark}}$, and call the angle between the peaks of the long and short wavelength first order maxima

$\Delta\theta_{\text{long short}} = \theta_{\text{long, }m=1\text{, bright}} - \theta_{\text{short, }m=1\text{, bright}}$. We require that $\Delta\theta_{\text{long bright dark}} \le \Delta\theta_{\text{long short}}$ in order for the two maxima to be resolvable. Taking the limiting case where the two maxima are just barely resolvable, the condition can be written

$$\theta_{\text{long, }m=1\text{, bright}} - \theta_{\text{long, }m=1\text{, dark}} = \theta_{\text{long, }m=1\text{, bright}} - \theta_{\text{short, }m=1\text{, bright}}$$

$$\sin^{-1}\left(\frac{m\lambda_{\text{long}}}{d}\right) - \sin^{-1}\left(\frac{\left(\frac{k}{N}\right)\lambda_{\text{long}}}{d}\right) = \sin^{-1}\left(\frac{m\lambda_{\text{long}}}{d}\right) - \sin^{-1}\left(\frac{m\lambda_{\text{short}}}{d}\right)$$

$$\frac{\left(\frac{k}{N}\right)\lambda_{\text{long}}}{d} = \frac{m\lambda_{\text{short}}}{d}$$

In order to ensure that the peaks are just barely resolvable, we want to choose the value of k that corresponds to the dark fringe just next to the bright peak of the long wavelength light. Thus $k = N - 1$, and we have

$$N = \left(1 - \frac{m\lambda_{\text{short}}}{\lambda_{\text{long}}}\right)^{-1} = \left(1 - \frac{(1)610\text{ nm}}{615\text{ nm}}\right)^{-1} = 123\text{ lines}$$

34.49. As light reflects from the top of the glass a phase shift due to reflection occurs, but for light reflecting from the bottom of the glass there will be no such phase shift due to reflection. Thus, the path length difference required to produce constructive interference is described by

$$\Delta\ell = 2t = \frac{\left(m - \frac{1}{2}\right)\lambda_0}{n} \Rightarrow \lambda_0 = \frac{2tn}{\left(m - \frac{1}{2}\right)} = \frac{2(0.500\times10^{-6}\text{ m})(1.52)}{\left(m - \frac{1}{2}\right)}$$

This holds for any order m. Inserting $m = 1$ or 2 yields wavelengths in the infrared. But inserting $m = 3$ and 4 yield two wavelengths in the visible spectrum: 608 nm and 434 nm, respectively.

34.50. As light is reflected off the reflective coating material, there will be a phase shift due to reflection because we can assume the material will have an index of refraction greater than 1.00 (that of air). However, we do not know a priori whether there will be a phase shift in light that is reflected from the interface between the reflective coating and the glass underneath. Let us initially guess that the index of refraction of the coating is greater than that of glass such that this second ray will not experience a phase shift upon reflection. In this case there is a net phase shift of one-half wavelength between the two reflected rays even without accounting for the phase shift due to thickness. Thus even a very small thickness (approaching zero) should work. Why not try 10 nm? After all, the material is expensive and we want to use as little as possible! But the problem clearly states that we have determined that 104 nm is the absolute minimum we could use. Therefore the coating material must not have an index of refraction larger than that of glass.

Alternatively, we could have guessed that the index of refraction for the coating material could be less than that of the glass, such that the light reflected off the coating-glass interface also experiences a phase shift upon reflection. In that case there is no net phase shift due to reflection, and we must obtain a half-wavelength of phase shift from the thickness. The light will cancel if

$$\Delta\ell = 2t = \frac{\left(m - \frac{1}{2}\right)\lambda_0}{n} \Rightarrow n = \frac{\left(m - \frac{1}{2}\right)\lambda_0}{2t} = \frac{(1/2)(550\times10^{-9}\text{ m})}{2(104\times10^{-9}\text{ m})} = 1.32.$$

This is consistent with our guess about the relative indices of refraction. Thus there is only one valid answer: $n = 1.32$.

34.51. Light rays reflecting off the air-coating interface or off the coating-glass interface will both experience phase shifts due to reflection. Thus the condition for constructive interference can be written $\lambda_0 = \frac{2tn}{m}$. We are asked for the longest wavelength that satisfies this, which clearly corresponds to $m = 1$ such that $\lambda_0 = 2tn = 2(90.6\times10^{-9}\text{ m})(1.38) = 250\text{ nm}$.

34.52. Light reflected off the outer air-bubble interface will experience a phase shift upon reflection, which light reflecting off the inner bubble-air interface will not. Thus, the condition for especially bright constructive interference of the red light is $t=\left(m-\frac{1}{2}\right)\frac{\lambda_0}{2n}$.

34.53. We do not know a priori if the index of refraction of the fluid is greater than or less than that of the fluorite. Let us initially assume that the index of refraction of the fluid is less than that of the fluorite, and we return to check if our answer agrees with this assumption. In that case, light reflecting off the air-fluid interface, and light reflecting off the fluid-fluorite interface will experience a phase shift upon reflection. Thus, in order for green light to be strongly reflected, we know $n=m\frac{\lambda_{0,\text{green}}}{2t}$, where m could be any order. Since the next wavelength that has virtually zero reflection is the red light, we can write $n=\left(m-\frac{1}{2}\right)\frac{\lambda_{0,\text{red}}}{2t}$, where m refers to the same order. Taking the ratio of the these two conditions, we find

$$\left(\frac{m}{m-\frac{1}{2}}\right)\left(\frac{\lambda_{0,\text{green}}}{\lambda_{0,\text{red}}}\right)=1\Rightarrow m=-\frac{\lambda_{0,\text{red}}}{2\lambda_{0,\text{green}}}\left(1-\frac{\lambda_{0,\text{red}}}{\lambda_{0,\text{green}}}\right)^{-1}=-\frac{(750\text{ nm})}{2(510\text{ nm})}\left(1-\frac{(750\text{ nm})}{(510\text{ nm})}\right)^{-1}=1.56$$

This does not correspond to an integer value. Thus there is no solution corresponding to our assumption that the index of refraction of the fluid is less than that of the fluorite.

Let us now assume that the index of refraction of the fluid is greater than that of the fluorite. In that case, there will be a phase shift upon reflection for light reflecting off the air-fluid interface, but not for light reflecting off the fluid-fluorite interface. This means the rays are already out of phase, and the condition for green light to be strongly reflected is $n=\left(m-\frac{1}{2}\right)\frac{\lambda_{0,\text{green}}}{2t}$. Similarly, the condition for red light not to be reflected at all is $n=(m-1)\frac{\lambda_{0,\text{red}}}{2t}$. Note here that we had to use $(m-1)$, because $n=(m)\frac{\lambda_{0,\text{red}}}{2t}$ could not possibly hold if the condition for green light is satisfied since red light has a longer wavelength than green light. Taking the ratio of these conditions yields

$$\left(\frac{m-\frac{1}{2}}{m-1}\right)\left(\frac{\lambda_{0,\text{green}}}{\lambda_{0,\text{red}}}\right)=1\Rightarrow m=\left(-\frac{\lambda_{0,\text{red}}}{\lambda_{0,\text{green}}}+\frac{1}{2}\right)\left(1-\frac{\lambda_{0,\text{red}}}{\lambda_{0,\text{green}}}\right)^{-1}=\left(-\frac{(750\text{ nm})}{(510\text{ nm})}+\frac{1}{2}\right)\left(1-\frac{(750\text{ nm})}{(510\text{ nm})}\right)^{-1}=2.06$$

while this is not precisely an integer, it is close enough that an experimental observer could easily report this as being strongly reflected green light and red light that is hardly reflected at all. If we let $m=2$, then the index of refraction is $n=(m-1)\frac{\lambda_{0,\text{red}}}{2t}=\frac{(750\text{ nm})}{2(158\text{ nm})}=2.37$, which is consistent with our assumption that the index of refraction of the fluid is greater than that of the fluorite. Thus, the only possible value of the index of refraction is 2.37.

34.54. Because the index of refraction of the film is smaller than that of the glass, we see that light reflected off of either interface will experience a phase shift upon reflection. Thus, for light to be strongly reflected, a full number of wavelengths must fit in the additional path length travelled by one ray compared to the other $2t=m\frac{\lambda_0}{n}$. Since this is not observed for a thinner film, we know $m=1$, such that $n_{\text{film}}=\frac{\lambda_0}{2t}$. We find the angle of refraction using Snel's law:

$$n_{\text{film}}\sin(\theta_{\text{film}})=n_{\text{air}}\sin(\theta_{\text{air}})\Rightarrow\theta_{\text{film}}=\sin^{-1}\left(\frac{2tn_{\text{air}}}{\lambda_0}\sin(\theta_{\text{air}})\right)=\sin^{-1}\left(\frac{2(175\text{ nm})(1.00)}{(473\text{ nm})}\sin(51.0°)\right)=35.1°$$

34.55. From Example 34.8, we can use equation (1) to write $a = \dfrac{2\lambda L}{w} = \dfrac{2(633\times10^{-9}\text{ m})(0.90\text{ m})}{(0.012\text{ m})} = 9.50\times10^{-5}\text{ m}.$

34.56. From Equation 34.26 we have $a\sin(\theta) = m\lambda \Rightarrow a = \dfrac{m\lambda}{\sin(\theta)} = \dfrac{(1)(650\times10^{-9}\text{ m})}{\sin(30.0°)} = 1.30\times10^{-6}\text{ m}.$

34.57. If the distance between first order dark fringes is $\Delta y,$ then the angular position of the first order dark fringe is related to the distance to the screen by $\tan(\theta) = \dfrac{\Delta y}{2\ell}$ or $\ell = \dfrac{\Delta y}{2\tan(\theta)}.$ Further, we know from Equation 34.26 that $a\sin(\theta) = \lambda \Rightarrow \theta = \sin^{-1}\left(\dfrac{\lambda}{a}\right).$ Combining these two conditions yields

$$\ell = \frac{\Delta y}{2\tan\left(\sin^{-1}\left(\dfrac{\lambda}{a}\right)\right)} = \frac{(10.0\times10^{-3}\text{ m})}{2\tan\left(\sin^{-1}\left(\dfrac{(400\times10^{-9}\text{ m})}{(5.00\times10^{-4}\text{ m})}\right)\right)} = 6.25\text{ m}$$

34.58. From Equation 34.26 we know $a\sin(\theta) = \lambda \Rightarrow \theta = \sin^{-1}\left(\dfrac{\lambda}{a}\right).$ From simple geometry, we know

$$\tan(\theta) = \frac{\Delta y}{\ell} \text{ or } \Delta y = \ell\tan(\theta) = \ell\tan\left(\sin^{-1}\left(\frac{\lambda}{a}\right)\right) = (2.000\text{ m})\tan\left(\sin^{-1}\left(\frac{(656.5\times10^{-9}\text{ m})}{(1.00\times10^{-4}\text{ m})}\right)\right) = 13.1\text{ mm}$$

34.59. We first find the angle at which the fourth order dark spot occurs using Equation 34.26: $a\sin(\theta) = m\lambda \Rightarrow \theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right).$ Simple geometry allows us to relate this to the distance along the screen, such that

$$\tan(\theta) = \frac{\Delta y}{\ell} \text{ or } \Delta y = \ell\tan(\theta) = \ell\tan\left(\sin^{-1}\left(\frac{m\lambda}{a}\right)\right) = (0.710\text{ m})\tan\left(\sin^{-1}\left(\frac{4(545\times10^{-9}\text{ m})}{(15\times10^{-6}\text{ m})}\right)\right) = 0.10\text{ m}.$$

34.60. When the slit width equal the wavelength, the diffraction pattern is spread across all angles from $-\pi/2$ $(-90°)$ to $+\pi/2$ $(+90°)$. The first order dark fringes appear at exactly $(-90°)$ and $(+90°)$, mathematically speaking. In practice, the dark fringes would not be visible because no flat screen can extend out to those angles. The distance between two minima on the screen is infinite regardless of screen size or source-screen distance.

34.61. Many numerical answers are possible depending on exact wavelength of light, width of the door, and distance from the door to the far wall of the cabin. But the crux of the answer is that the pattern will be extremely small, too small to observe with the naked eye, and too small to observe even with instruments if light from other sources is present. One would not even call this a diffraction pattern. As an example, let us assume that the light has a wavelength of 700 nm, that the door is 1.0 m wide, and that the distance from the door to the far wall of your cabin in 10 m. We can use Equation 34.26 to determine the angle at which the first order dark fringe would be found: $a\sin(\theta) = \lambda \Rightarrow \theta = \sin^{-1}\left(\dfrac{\lambda}{a}\right).$ And using simple geometry, we find the distance along the wall to this dark fringe is

$$\Delta y = \ell\tan(\theta) = \ell\tan\left(\sin^{-1}\left(\frac{\lambda}{a}\right)\right) = (10\text{ m})\tan\left(\sin^{-1}\left(\frac{(700\times10^{-9}\text{ m})}{(1.0\text{ m})}\right)\right) = 7.0\times10^{-6}\text{ m}$$

There is no way for you to realistically observe a brightness variation over a distance of 7.0 μm from the central bright spot.

34.62. In order for the central bright spot on the wall to have the same width as the slit, the angle at which the first order minimum occurs must be given by $\theta = \tan^{-1}\left(\dfrac{a}{2\ell}\right)$. We also know from Equation 34.26 $\sin(\theta) = \dfrac{m\lambda}{a}$. Because the aperture of a laser is likely much smaller than the distance from the laser to the screen, we use the small angle approximation to write

$$\frac{a}{2\ell} \approx \theta \approx \frac{\lambda}{a} \Rightarrow a \approx \sqrt{2\lambda\ell} = \sqrt{2(550\times10^{-9}\ \text{m})(0.50\ \text{m})} = 7.4\times10^{-4}\ \text{m}$$

34.63. We find the angular position of the first order bright fringe using Equation 34.26: $a\sin(\theta) = \lambda \Rightarrow \theta = \sin^{-1}\left(\dfrac{\lambda}{a}\right)$. We use simple geometry to relate this to a distance along the screen:

$$\Delta y = \ell\tan(\theta) = \ell\tan\left(\sin^{-1}\left(\frac{\lambda}{a}\right)\right) = (0.450\ \text{m})\tan\left(\sin^{-1}\left(\frac{(440\times10^{-9}\ \text{m})}{(75\times10^{-6}\ \text{m})}\right)\right) = 2.64\times10^{-3}\ \text{m}$$

where we have kept one additional significant digit for this intermediate step. This is only the distance from the center to one first order dark spots. The distance between the two is thus 5.3 mm.

34.64. We can use Equation 34.26 to determine the angle at which the first order dark fringe would be found, and then use simple geometry to relate this to a distance along the screen. We do this for both wavelengths and then compare the positions. For either wavelength, we can write $a\sin(\theta) = m\lambda \Rightarrow \theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right)$. And using simple geometry, we find the distance along the wall to this dark fringe is $\Delta y = \ell\tan(\theta) = \ell\tan\left(\sin^{-1}\left(\dfrac{m\lambda}{a}\right)\right)$. Applying this equation to each wavelength yields

$$\Delta y_{\text{shorter}} = (0.220\ \text{m})\tan\left(\sin^{-1}\left(\frac{2(482.0\times10^{-9}\ \text{m})}{(2.470\times10^{-6}\ \text{m})}\right)\right) = 9.33\times10^{-2}\ \text{m}$$

$$\Delta y_{\text{longer}} = (0.220\ \text{m})\tan\left(\sin^{-1}\left(\frac{2(517.3\times10^{-9}\ \text{m})}{(2.470\times10^{-6}\ \text{m})}\right)\right) = 1.01\times10^{-1}\ \text{m}$$

The difference between these positions is 8 mm.

34.65. **[NOTE:** The second line of the problem statement "The *n* = 1 and *n* = 1 dark fringes…." is changed to "The *n* = 1 and *n* = –1 dark fringes…."] (*a*) Let us call the distance between the two first order dark fringes w. Then the angle at which either dark fringe is cast is $\theta = \tan^{-1}\left(\dfrac{w}{2\ell}\right)$, and this angle is related to the aperture width through Equation 34.26: $a\sin(\theta) = m\lambda$. Combining, we find

$$a = \frac{\lambda}{\sin\left(\tan^{-1}\left(\dfrac{w}{2\ell}\right)\right)} = \frac{(485\times10^{-9}\ \text{m})}{\sin\left(\tan^{-1}\left(\dfrac{(22.4\times10^{-3}\ \text{m})}{2(0.320\ \text{m})}\right)\right)} = 1.39\times10^{-5}\ \text{m}$$

(*b*) Rearranging Equation 34.26 we find $\sin(\theta) = \dfrac{m\lambda}{a} \le 1$, such that $m \le \dfrac{a}{\lambda} = \dfrac{(1.387\times10^{-5}\ \text{m})}{(485\times10^{-9}\ \text{m})} = 28.6$. Thus the highest order dark fringe corresponds to $m = 28$. For this order, the angle is

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{(28)(485\times10^{-9}\ \text{m})}{(1.3866\times10^{-5}\ \text{m})}\right) = 78.4°.$$

34.66. We use the information about the two slits to determine the distance d between them. Then, once the cardboard between slits is removed, the aperture width a is the same as the former value of d. Using the double slits, we have $d\sin(\theta) = m\lambda \Rightarrow d = \dfrac{m\lambda}{\sin(\theta)} = \dfrac{3(620\times10^{-9}\text{ m})}{\sin(4.27°)} = 2.50\times10^{-5}\text{ m}.$ Now we find the positions of the dark fringes by simply inserting difference orders into the expression $\theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right).$ Thus

$$\theta_1 = \sin^{-1}\left(\frac{1(620\times10^{-9}\text{ m})}{(2.50\times10^{-5}\text{ m})}\right) = 1.42°$$

$$\theta_2 = \sin^{-1}\left(\frac{2(620\times10^{-9}\text{ m})}{(2.50\times10^{-5}\text{ m})}\right) = 2.85°$$

$$\theta_3 = \sin^{-1}\left(\frac{3(620\times10^{-9}\text{ m})}{(2.50\times10^{-5}\text{ m})}\right) = 4.27°$$

34.67. We can use Equation 34.26 to determine the angle at which a given order dark fringe would be found. We do this for both orders and then compare the positions. For either order, we can write $a\sin(\theta) = m\lambda \Rightarrow \theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right).$ Applying this equation to each order yields

$$\theta_3 = \sin^{-1}\left(\frac{3(800\times10^{-9}\text{ m})}{(45\times10^{-6}\text{ m})}\right) = 3.06°$$

$$\theta_5 = \sin^{-1}\left(\frac{5(800\times10^{-9}\text{ m})}{(45\times10^{-6}\text{ m})}\right) = 5.10°$$

Thus the angular separation is $2.0°.$

34.68. The answer depends on the wavelength and aperture size. Consider for example two aperture sizes for the same set of other parameters. Case 1: $a = 2.50\times10^{-6}\text{ m},$ $\lambda = 500\text{ nm},$ and the distance to the screen $D = 1.00\text{ m}.$ The width of the central (zeroth-order) maximum in this case is

$$w = 2\Delta y = 2\ell\tan(\theta) = 2\ell\tan\left(\sin^{-1}\left(\frac{m\lambda}{a}\right)\right) = 2(1.00\text{ m})\tan\left(\sin^{-1}\left(\frac{(1)(500\times10^{-9}\text{ m})}{(2.50\times10^{-6}\text{ m})}\right)\right) = 0.408\text{ m}$$

whereas the width of the third-order maximum is

$$\begin{aligned} y_4 - y_3 &= \ell\tan\left(\sin^{-1}\left(\frac{4\lambda}{a}\right)\right) - \ell\tan\left(\sin^{-1}\left(\frac{3\lambda}{a}\right)\right) \\ &= (1.00\text{ m})\left(\tan\left(\sin^{-1}\left(\frac{4(500\times10^{-9}\text{ m})}{(2.50\times10^{-6}\text{ m})}\right)\right) - \tan\left(\sin^{-1}\left(\frac{3(500\times10^{-9}\text{ m})}{(2.50\times10^{-6}\text{ m})}\right)\right)\right) = 0.583\text{ m} \end{aligned}$$

such that the third-order bright fringe is wider. If we now change only the aperture size (Case 2) to $a = 3.50\times10^{-6}\text{ m},$ we find the width of the central maximum to be 0.289 m and we find $y_4 - y_3 = 0.222\text{ m}.$ In this second case, the central bright envelope is wider.

34.69. From the interference pattern we can deduce the wavelength of the light being used. According to Equation 34.26, $\lambda = \dfrac{a\sin(\theta)}{m} = \dfrac{(1500\times10^{-9}\text{ m})\sin(25.0°)}{(1)} = 633.9\text{ nm}.$ Now we turn our attention to the reflection. Because the soap film has a higher index of refraction than water, light reflecting off the air-film interface will experience a phase shift due to reflection but light reflecting off the film-water interface will not. Thus, the two rays are already out of phase simply due to reflection. Stating that that the film have zero thickness is not a valid answer, because then no film would be present. We thus require that the film have the smallest non-zero thickness that keeps the rays out of phase, which corresponds to fitting one full wavelength in the additional path length travelled by one ray. Thus $t = \dfrac{\lambda}{2n} = \dfrac{(633.9\text{ nm})}{2(1.40)} = 226\text{ nm}.$

34.70. Equation 34.29 tells us that the first order minimum in intensity occurs at the angle given by $\sin(\theta_1) = 1.22\dfrac{\lambda}{d}$, where d is the diameter of the aperture. Simple geometry tells us that the distance from the center of the pattern to this first order minimum is

$$\Delta y = \ell \tan(\theta_1) = \ell \tan\left(\sin^{-1}\left(1.22\frac{\lambda}{d}\right)\right) = (1.5\text{ m})\tan\left(\sin^{-1}\left(1.22\frac{(550\times10^{-9}\text{ m})}{(2.0\times10^{-4}\text{ m})}\right)\right) = 5.03\text{ mm}$$

Of course, this is only half the diameter of the Airy disk. The full width is 1.0×10^{-2} m.

34.71. Equation 34.29 tells us that the first order minimum in intensity occurs at the angle given by $\sin(\theta_1) = 1.22\dfrac{\lambda}{d}$, where d is the diameter of the aperture. Simple geometry tells us that the distance from the center of the pattern to this first order minimum is

$$\Delta y = \ell \tan(\theta_1) = \ell \tan\left(\sin^{-1}\left(1.22\frac{\lambda}{d}\right)\right) = (1.200\text{ m})\tan\left(\sin^{-1}\left(1.22\frac{(550\times10^{-9}\text{ m})}{(0.200\text{ m})}\right)\right) = 4.03\ \mu\text{m}$$

34.72. Using Equation 34.31 we know the radius of the Airy disk is given by $y_\text{r} = 1.22\dfrac{\lambda f}{d}$. Since all lenses are being used to focus the same beam of light, we simply choose the lens with the smallest ratio of f/d, which is lens B. Thus lens B will focus the light down to the smallest disk.

34.73. We use Equation 34.30 to write the minimum angular separation between two objects as $\theta_\text{r} = 1.22\dfrac{\lambda}{d}$. Inserting values for each extremum of wavelength, we find

$$\theta_{\text{r,shortest }\lambda} = 1.22\frac{(3.00\times10^{-6}\text{ m})}{(0.85\text{ m})} = 4.3\times10^{-6}\text{ rad or }(2.5\times10^{-4})°,\text{ and}$$

$$\theta_{\text{r,longest }\lambda} = 1.22\frac{(180\times10^{-6}\text{ m})}{(0.85\text{ m})} = 2.6\times10^{-4}\text{ rad or }(1.5\times10^{-2})°$$

34.74. (*a*) Using Equation 34.30 we can write the minimum angular separation between two objects as $\theta_\text{r} = 1.22\dfrac{\lambda}{d} = 1.22\dfrac{(400\times10^{-9}\text{ m})}{(3.0\times10^{-3}\text{ m})} = 1.6\times10^{-4}\text{ rad or }(9.3\times10^{-3})°$. (*b*) As in part (*a*) $\theta_\text{r} = 1.22\dfrac{\lambda}{d}$ $= 1.22\dfrac{(650\times10^{-9}\text{ m})}{(3.0\times10^{-3}\text{ m})} = 2.6\times10^{-4}\text{ rad or }(1.5\times10^{-2})°$. (*c*) Simple geometry tells us $\Delta y = \ell\tan(\theta)$ $= (100\text{ m})\tan((9.32\times10^{-3})°) = 16\text{ mm}$. (*d*) As in (*c*), $\Delta y = \ell\tan(\theta) = (100\text{ m})\tan((1.51\times10^{-2})°) = 26\text{ mm}$.

34.75. Equation 34.29 tells us that the first order minimum in intensity occurs at the angle given by $\sin(\theta_1) = 1.22\dfrac{\lambda}{d}$, where d is the diameter of the aperture. Simple geometry tells us that the distance from the center of the pattern to this first order minimum is

$$\Delta y = \ell \tan(\theta_1) = \ell \tan\left(\sin^{-1}\left(1.22\frac{\lambda}{d}\right)\right) = (0.800\text{ m})\tan\left(\sin^{-1}\left(1.22\frac{(530\times10^{-9}\text{ m})}{(4.00\times10^{-4}\text{ m})}\right)\right) = 1.29\text{ mm}$$

But this is only the radius of the disk; the diameter is 2.59 mm. One could also have used the small angle approximation here, without affecting the results.

34.76. The radius of an Airy disk produced by a lens of diameter Equation 34.29 tells us that the first order minimum in intensity occurs at the angle given by $\sin(\theta_1) = 1.22\dfrac{\lambda}{d}$, where d is the diameter of the aperture. Simple geometry tells us that the distance from the center of the pattern to this first order minimum is d is given by Equation 34.31 as Equation 34.29 tells us that the first order minimum in intensity occurs at the angle given by $\sin(\theta_1) = 1.22\dfrac{\lambda}{d}$, where d is the diameter of the aperture. Simple geometry tells us that the distance from the center of the pattern to this first order minimum is $y_\mathrm{r} = 1.22\dfrac{\lambda f}{d}$. So the largest Airy disk corresponds to a large wavelength and small pupil diameter; in that case: $y_{\mathrm{r,max}} = 1.22\dfrac{(750\times 10^{-9}\text{ m})(0.025\text{ m})}{(2.00\times 10^{-3}\text{ m})} = 11.4\ \mu\text{m}$. The smallest Airy disk corresponds to a short wavelength and a large pupil diameter; in that case: $y_{\mathrm{r,max}} = 1.22\dfrac{(390\times 10^{-9}\text{ m})(0.025\text{ m})}{(8.00\times 10^{-3}\text{ m})} = 1.49\ \mu\text{m}$.

34.77. Let us assume we can work in the small angle approximation. In that case $\theta_1 \approx \sin(\theta_1) = 1.22\dfrac{\lambda}{d}$ and $\theta_1 \approx \tan(\theta_1) = \dfrac{y_\mathrm{r}}{\ell}$ can be equated to yield $y_\mathrm{r} = 1.22\dfrac{\lambda\ell}{d}$. We know that when the setup is submerged, the distance to the screen and the pinhole aperture will be unchanged, whereas the wavelength will decrease according to $\lambda_{\text{water}} = \dfrac{\lambda_{\text{vac}}}{n_{\text{water}}}$. Thus $y_{\mathrm{r,water}} = \dfrac{y_{\mathrm{r,vac}}}{n_{\text{water}}} = \dfrac{y_{\mathrm{r,vac}}}{1.33}$.

34.78. We determine the radius of the Airy disk through $y_\mathrm{r} = \ell\tan(\theta_1)$, where we insert $\theta_1 = \sin^{-1}\left(1.22\dfrac{\lambda}{d}\right)$ from Equation 34.29. We obtain

$$y_\mathrm{r} = \ell\tan\left(\sin^{-1}\left(1.22\frac{\lambda}{d}\right)\right) = (0.350\text{ m})\tan\left(\sin^{-1}\left(1.22\frac{(500\times 10^{-9}\text{ m})}{(30.0\times 10^{-6}\text{ m})}\right)\right) = 7.12\times 10^{-3}\text{ m}$$

Then the area of the disk is given by $A = \pi r^2 = \pi(7.12\times 10^{-3}\text{ m})^2 = 1.59\times 10^{-4}\text{ m}^2$.

34.79. The angular separation of the two objects is $\theta = 2\tan^{-1}\left(\dfrac{\Delta x}{2L}\right)$, and we require that this be just barely equal to the minimum resolvable angle given by $\theta_1 = \sin^{-1}\left(1.22\dfrac{\lambda}{d}\right)$. Since these angles will be small, we can write $L = \dfrac{d\Delta x}{1.22\lambda} = \dfrac{(6.00\times 10^{-3}\text{ m})(30.0\times 10^{-3}\text{ m})}{1.22(550\times 10^{-9}\text{ m})} = 268\text{ m}$.

34.80. We know from Equation 34.30 that the minimum angle at which two objects can be resolved is $\theta_1 = 1.22\dfrac{\lambda}{d}$. We are given the diameter of the telescope, but all we are told about the light is that it is in the visible spectrum. The minimum possible angular separation will correspond to the minimum wavelength that could be used. The shortest wavelength that people can see is approximately 4.0×10^{-7} m, although it varies slightly from person to person. We will use a cutoff of exactly 400 nm for the visible spectrum for the purposes of this calculation; answers that use a slightly smaller value (around 390 nm) could also be correct. $\theta_{1,\min} = 1.22\dfrac{\lambda_{\min}}{d} = 1.22\dfrac{(400\times 10^{-9}\text{ m})}{(60.0\times 10^{-3}\text{ m})} = 8.13\times 10^{-6}$ radians or $(4.66\times 10^{-4})^\circ$.

34.81. Equation 34.30 gives us the minimum angular separation between sides of the object for the two sides to be resolvable: $\theta_1 = 1.22\dfrac{\lambda}{d} = 1.22\dfrac{(525\times10^{-9}\text{ m})}{(2.75\text{ m})} = 2.33\times10^{-9}$ rad. Simple geometry tells us this is related to the diameter D of the object through

$$\tan(\theta_1/2) = \frac{D}{2L} \Rightarrow D = 2L\tan(\theta_1/2) = 2(2.50\times10^{7}\text{ m})\tan\left(\frac{(2.33\times10^{-9}\text{ rad})}{2}\right) = 5.82\text{ m}$$

34.82. We know from Equation 34.29 that the angle from the bright center of an Airy disk to the first order minimum is $\theta_1 = \sin^{-1}\left(1.22\dfrac{\lambda}{d}\right)$. Simple geometry allows us to relate this to the radius of the disk on the screen through

$$\Delta y_{\text{r}} = \ell\tan(\theta_1) = \ell\tan\left(\sin^{-1}\left(1.22\frac{\lambda}{d}\right)\right) \Rightarrow d = \frac{1.22\lambda}{\sin\left(\tan^{-1}\left(\dfrac{\Delta y_{\text{r}}}{\ell}\right)\right)} = \frac{1.22(650\times10^{-9}\text{ m})}{\sin\left(\tan^{-1}\left(\dfrac{(0.139\text{ m})/2}{(0.350\text{ m})}\right)\right)} = 4.07\times10^{-6}\text{ m}$$

34.83. Let us assume that the wavelength of the light emitted by the brake lights is approximately 650 nm. We can calculate the size of the Airy disk produced by such light passing through this lens using Equation 34.31: $y_{\text{r}} = 1.22\dfrac{\lambda f}{d} = 1.22\dfrac{(650\times10^{-9}\text{ m})(0.050\text{ m})}{(4.00\times10^{-3}\text{ m})} = 9.91\times10^{-6}$ m. In order for the two lights to be resolvable, their Airy disks must overlap, so the distance between the Airy disk centers on the camera sensors must be greater than or equal to 1.98×10^{-5} m. Since the object distance is very large, the image distance will be very close to the focal length, and we can approximate the minimum angle at the camera lens between the two images as

$$\theta_{\text{min}} = 2\tan^{-1}\left(\frac{\Delta x/2}{f}\right) = 2\tan^{-1}\left(\frac{(1.98\times10^{-5}\text{ m})/2}{(0.050\text{ m})}\right) = 3.96\times10^{-4}\text{ rad}$$

From the placement of the lights, we can determine the angle at the camera lens between the two light sources: $\theta = 2\tan^{-1}\left(\dfrac{\Delta x/2}{L}\right) = 2\tan^{-1}\left(\dfrac{(2.00\text{ m})/2}{(300\text{ m})}\right) = 6.67\times10^{-3}$ rad. This is about 17 times larger than the minimum angular separation calculated. So it should be possible to resolve the two Airy disks. It is also worth asking whether the sensors in the camera are small enough to resolve the two objects. One can look up the size of pixels in a digital camera sensor, but we are told in Example 34.10 that the width is around $2.0\ \mu$m. So the radius of the Airy disk produced by either light is five times the pixel size, and the camera should have no trouble resolving the two lights. So, yes, the two lights are likely to be resolved in the photograph.

34.84. (*a*) Considering the shadow only, the smallest bright spot that could be cast on the film is 0.30 mm. Considering the wave properties of light, the diameter of the Airy disk would be $D = 2\Delta y_{\text{r}} = 2(1.22)\dfrac{\lambda f}{d} = 2(1.22)\dfrac{(390\times10^{-9}\text{ m})(0.100\text{ m})}{(0.300\times10^{-3}\text{ m})} = 0.317$ mm. (*b*) You would use the Raleigh Criterion. The optimal pinhole size is the largest size that still gives you the required resolving power. The largest possible size is desirable so as to allow the maximum amount of light through. But suppose you also need to distinguish certain features in your picture such that you require a minimum resolvable angle $\theta_{r,\text{min}}$. Presumably you want see the features in all visible colors, so make sure it works with the longest possible wavelength (approximately 750 nm) Then using the Raleigh Criterion tells you $d = 1.22\lambda_{\text{max}}/\sin(\theta_{r,\text{min}})$.

34.85. (*a*) The kinetic energy is reduced by a factor of 1/4, because $K_{\text{i}} = \dfrac{1}{2}mv_{\text{i}}^2$ and $K_{\text{i}} = \dfrac{1}{2}mv_{\text{f}}^2 = \dfrac{1}{2}m\left(\dfrac{v_{\text{i}}}{2}\right)^2 = \dfrac{1}{4}K_{\text{i}}$. (*b*) There is no change. The energy of a photon depends only on its frequency and the frequency is fixed when a photon crosses from one medium to another. It is only the wavelength that changes.

34.86. (*a*) Using Equation 34.35 we know $E = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(400\times10^{-9}\ \text{m})} = 4.97\times10^{-19}\ \text{J}$ or $3.10\ \text{eV}$. (*b*) As in part (*a*), we write $E = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(700\times10^{-9}\ \text{m})} = 2.84\times10^{-19}\ \text{J}$ or $1.77\ \text{eV}$.

34.87. We know $E = hf = \dfrac{hc}{\lambda} \Rightarrow \lambda = \dfrac{hc}{E} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(8.0\times10^{-14}\ \text{J})} = 2.5\times10^{-12}\ \text{m or } 2.5\ \text{pm}.$

34.88. From Equation 34.37 we know $p = \dfrac{E}{c_0} = \dfrac{(8.0\times10^{-14}\ \text{J})}{(3.00\times10^{8}\ \text{m/s})} = 2.7\times10^{-22}\ \text{kg}\cdot\text{m/s}.$

34.89. Rewriting Equation 34.36 in terms of the wavelength, we have $\dfrac{hc}{\lambda} = E_0 + eV_{\text{stop}}$. Clearly if the wavelength is very small, a very high stopping potential will be required. But we are asked about the maximum wavelength that can eject an electron. That means it is just barely able to eject an electron and no additional potential difference is required to stop the ejected electron. Thus our equation reduces to $\lambda_{\text{max}} = \dfrac{hc}{E_0} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(6.54\times10^{-19}\ \text{J})} =$ $304\ \text{nm}$.

34.90. If red light corresponds to the longest wavelength that can eject an electron from the metal, then we can find the work function of the metal by inserting $V_{\text{stop}} = 0$ in Equation 34.36. We are left with

$$E_0 = \frac{hc}{\lambda_{\text{max}}} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(700\times10^{-9}\ \text{m})} = 2.84\times10^{-19}\ \text{J}$$

Now we use violet light and determine the energy left over after the electron is ejected from the metal:

$$K_{\text{max}} = \frac{hc}{\lambda} - E_0 = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(400\times10^{-9}\ \text{m})} - (2.84\times10^{-19}\ \text{J}) = 2.13\times10^{-19}\ \text{J}$$

34.91. Writing Equation 34.35 in terms of the wavelength of light, we have $E_0 = \dfrac{hc}{\lambda} - K_{\text{max}}$ $= \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(410\times10^{-9}\ \text{m})} - (3.5\times10^{-20}\ \text{J}) = 4.5\times10^{-19}\ \text{J or } 2.8\ \text{eV}.$

34.92. (*a*) A large stopping potential difference means the ejected electrons had a large kinetic energy after being ejected. That means that a much lower frequency of light could have been used and an electron still would have been ejected. Since material 1 had the largest stopping potential, the energy in the light *hf* exceeded the minimum energy required to eject an electron from material 1 by a greater amount than it exceeded the minimum energies for other materials. This means material 1 has the lowest minimum energy required to eject an electron. Thus material 1 has the lowest frequency below which no electrons are ejected.
(*b*) By the same argument as in part (*a*), material 3 has the highest frequency below which no electrons are ejected.

34.93. Applying Equation 34.36 we find

$$E_0 = \frac{hc}{\lambda} - eV_{\text{stop}} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(140\times10^{-9}\ \text{m})} - (1.60\times10^{-19}\ \text{C})(3.4\ \text{V}) = 8.8\times10^{-19}\ \text{J}$$

34.94. (*a*) We proceed assuming that relativity can be ignored, and we check our final electron speed to make sure our assumption was correct. Using Equation 34.35 we find

$$K_{\text{max}} = \frac{hc}{\lambda} - E_0 = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(390\times10^{-9}\ \text{m})} - (4.6\times10^{-19}\ \text{J}) = 5.0\times10^{-20}\ \text{J}$$

Now the electron's speed is given by

$$v = \sqrt{\frac{2K_{\text{max}}}{m}} = \sqrt{\frac{2(4.97\times10^{-20}\text{ J})}{(9.11\times10^{-31}\text{ kg})}} = 3.3\times10^{5}\text{ m/s}$$

Since this speed is much less than the speed of light in a vacuum, we were justified in ignoring relativity.

(*b*) One might try proceeding exactly as in part (*a*), and one would find $K_{\text{max}} = \dfrac{hc}{\lambda} - E_0$ $= \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(750\times10^{-9}\text{ m})} - (4.6\times10^{-19}\text{ J}) = -1.9\times10^{-19}\text{ J}.$ Clearly, a negative kinetic energy is nonsense. The resolution of this is that $\dfrac{hc}{\lambda} < E_0,$ such that the light does not have enough energy to even eject an electron from the metal, let alone leave it with kinetic energy. One might call this a kinetic energy of zero, but the most accurate answer is: Light with a wavelength of 750 nm does not have sufficient energy to break an electron from this metal. There is no kinetic energy or speed of an ejected electron.

34.95. We set the minimum energy required to eject electrons equal to the work function to find $E_0 = hf_{\text{min}} - = (6.626\times10^{-34}\text{ J}\cdot\text{s})(7.20\times10^{14}\text{ Hz}) = 4.77\times10^{-19}\text{ J}.$ Now, when a higher frequency is used, we require that the electron be left with a kinetic energy: $K_{\text{max}} = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(9.11\times10^{-31}\text{ kg})(8.50\times10^{5}\text{ m/s})^2 = 3.29\times10^{-19}\text{ J}.$ Finally, we use Equation 34.35 to determine the new required frequency:

$$f = \frac{K_{\text{max}} + E_0}{h} = \frac{(3.29\times10^{-19}\text{ J}) + (4.77\times10^{-19}\text{ J})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})} = 1.22\times10^{15}\text{ Hz}$$

34.96. (*a*) The energy of a photon is just $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(633\times10^{-9}\text{ m})} = 3.14\times10^{-19}\text{ J}.$ (*b*) We know the power output of the laser, so we equate $NE_{\text{photon}} = P\Delta t \Rightarrow N = \dfrac{P\Delta t}{E_{\text{photon}}} = \dfrac{(2.50\times10^{-4}\text{ W})(1.0\text{ s})}{(3.14\times10^{-19}\text{ J})} =$ 7.96×10^{14} photons. (*c*) From Equation 34.37 we know $p_{\text{photon}} = \dfrac{E_{\text{photon}}}{c_0} = \dfrac{(3.14\times10^{-19}\text{ J})}{(3.00\times10^{8}\text{ m/s})} = 1.05\times10^{-27}\text{ kg}\cdot\text{m/s}.$ (*d*) We know from mechanics $P = \dfrac{F}{A} = \dfrac{|\Delta p|}{\Delta t A} = \dfrac{Np_{\text{photon}}}{\Delta t \pi r^2} = \dfrac{(7.96\times10^{14}\text{ photons})(1.05\times10^{-27}\text{ kg}\cdot\text{m/s})}{(1.0\text{ s})\pi(1.00\times10^{-3}\text{ m})^2} =$ 2.65×10^{-7} Pa.

34.97. (*a*) In the case of a photon we relate the wavelength to the energy via $\lambda_{\text{photon}} = \dfrac{hc}{E_{\text{photon}}}.$ For an electron (which we assume is not moving at relativistic speeds), the wavelength is given by $\lambda_{\text{e}} = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2mE_{\text{e}}}}.$ Thus the ratio of wavelengths is

$$\frac{\lambda_{\text{e}}}{\lambda_{\text{photon}}} = \frac{\dfrac{h}{\sqrt{2mE_{\text{e}}}}}{\dfrac{hc}{E_{\text{photon}}}} = \frac{E_{\text{photon}}}{c\sqrt{2mE_{\text{e}}}} = \frac{1}{c}\sqrt{\frac{E_{\text{photon}}}{2m}} = \frac{1}{(3.00\times10^{8}\text{ m/s})}\sqrt{\frac{(2.00\times10^{-18}\text{ J})}{2(9.11\times10^{-31}\text{ kg})}} = 3.49\times10^{-3}$$

meaning the wavelength of the photon is much longer. (*b*) Using the relationships between wavelength and energy from part (*a*), we can write the ratio of energies:

$$\frac{E_{\text{photon}}}{E_{\text{e}}}=\frac{\dfrac{hc}{\lambda_{\text{photon}}}}{\dfrac{h^2}{2m\lambda_{\text{e}}^2}}=\frac{2mc\lambda_{\text{e}}}{h}=\frac{2(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})(250\times10^{-9}\text{ m})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})}=2.06\times10^{5}$$

meaning the photon has a much larger energy.

34.98. The energy of a single photon of this light is $E=\dfrac{hc}{\lambda}=\dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(216\times10^{-9}\text{ m})}=9.20\times10^{-19}\text{ J}.$ We note first that this is sufficient energy to eject electrons from the target. Given the power output of the laser, we can determine the number of photons emitted by the laser in a one-second interval: $NE_{\text{photon}}=P\Delta t\Rightarrow N=\dfrac{P\Delta t}{E_{\text{photon}}}=\dfrac{(3.50\text{ W})(1.0\text{ s})}{(9.02\times10^{-19}\text{ J})}=3.80\times10^{18}$ photons. Since each one of these photons is capable of ejecting an electron, the maximum number of electrons ejected per second is 3.80×10^{18}.

34.99. We know $F=\dfrac{dp}{dt}\rightarrow\dfrac{\Delta p}{\Delta t}$ since the rate of delivery of photons is constant. Since the photons are absorbed as opposed to reflected, we can write $F=ma=\dfrac{Np_{\text{photon}}}{\Delta t}\Rightarrow a=\dfrac{NE_{\text{photon}}}{mc\Delta t}=\dfrac{P}{mc}=\dfrac{IA}{mc}=\dfrac{(60\text{ W/m}^2)(4.5\times10^{-6}\text{ m}^2)}{(2.3\times10^{-6}\text{ kg})(3.00\times10^{8}\text{ m/s})}$ $=3.9\times10^{-7}\text{ m/s}^2$. Thus the acceleration is $3.9\times10^{-7}\text{ m/s}^2$ in the initial direction of the photons.

34.100. The energy of a single photon is $E=\dfrac{hc}{\lambda}=\dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(400\times10^{-9}\text{ m})}=4.97\times10^{-19}\text{ J}.$ We can find the number of photons striking the target each second:

$$NE_{\text{photon}}=P\Delta t=IA\Delta t\Rightarrow N=\frac{IA\Delta t}{E_{\text{photon}}}=\frac{(5.50\text{ W/m}^2)(600\times10^{-6}\text{ m}^2)(1.0\text{ s})}{(4.97\times10^{-19}\text{ J})}=6.64\times10^{15}\text{ photons}$$

Each of these photons could eject an electron, meaning a charge of Nq_{e} could flow across the vacuum tube each second. Thus the maximum current is

$$I_{\max}=\frac{Nq_{\text{e}}}{\Delta t}=\frac{(6.64\times10^{15}\text{ photons})(1.60\times10^{-19}\text{ C})}{(1.0\text{ s})}=1.06\text{ mA}$$

34.101. We use the information about the light to determine the aperture width: $a=\dfrac{m\lambda}{\sin(\theta)}=\dfrac{(1)(550\times10^{-9}\text{ m})}{\sin(32.5°)}$ $=1.024\ \mu\text{m}$. Now the electrons must have the same total energy as the photons, which allows us to write the wavelength of the electrons:

$$\lambda_{\text{e}}=\frac{h}{mv}=\frac{h}{\sqrt{2mK_{\text{e}}}}=\frac{h}{\sqrt{2mE_{\text{photon}}}}=\sqrt{\frac{h\lambda_{\text{photon}}}{2mc}}=\sqrt{\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(550\times10^{-9}\text{ m})}{2(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})}}=8.165\times10^{-10}\text{ m}$$

Now we make use of the aperture width to write

$$\theta=\sin^{-1}\left(\frac{m\lambda}{a}\right)=\sin^{-1}\left(\frac{(1)(8.165\times10^{-10}\text{ m})}{(1.024\times10^{-6}\text{ m})}\right)=0.0457°$$

34.102. Because the index of refraction of the film is between that of air and of glass, light reflecting off of either interface (air-film or film-glass) will experience a phase shift due to reflection. This leaves the two in phase. In order to be out of phase, one ray must travel a greater distance than the other by half of one wavelength. Thus

$2t = \left(\frac{1}{2}\right)\frac{\lambda_0}{n_{\text{film}}} \Rightarrow \lambda_0 = 4tn_{\text{film}} = 4(123\text{ nm})(1.30) = 639.6\text{ nm}.$ The energy of each photon with that wavelength is $E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(639.6\times10^{-9}\text{ m})} = 3.11\times10^{-19}\text{ J}.$

34.103. The longer the wavelength, the wider the angle at which the first minimum occurs. This is clear from $\sin(\theta_1) = \frac{(1)\lambda}{a}.$ Thus, the 600 nm light is diffracted the most.

34.104. Since $L \gg d$ we can use the small angle approximation to write $\theta_m = \frac{m\lambda}{d}$ and $\theta_m = \frac{y_m}{L}.$ This tells us that the distance along the screen from the center to the m^{th} bright fringe is $y_m = \frac{m\lambda L}{d}.$ This holds for either wavelength of light. Thus the distance between two adjacent bright fringes can be written $y_{m+1} - y_m = \frac{(m+1)\lambda L}{d} - \frac{m\lambda L}{d} = \frac{\lambda L}{d}.$ This expression for the distance is also valid for either wavelength. So we can write relate the distances between bright fringes of difference colors as follows:

$$\frac{y_{\text{r}}}{y_{\text{g}}} = \frac{\frac{\lambda_{\text{r}}L}{d}}{\frac{\lambda_{\text{g}}L}{d}} = \frac{\lambda_{\text{r}}}{\lambda_{\text{g}}} \Rightarrow y_{\text{r}} = \frac{\lambda_{\text{r}}}{\lambda_{\text{g}}}y_{\text{g}} = \frac{(650\text{ nm})}{(532\text{ nm})}y_{\text{g}} = 1.22y_{\text{g}}$$

34.105. Since we can use the small angle approximation, we write $\theta_m = \frac{m\lambda}{a}$ and $\theta_m = \frac{y_m}{L}.$ This tells us that the distance along the screen from the center to the m^{th} dark fringe is $y_m = \frac{m\lambda L}{a}.$ Thus the distance between two adjacent dark fringes can be written $y_{m+1} - y_m = \frac{(m+1)\lambda L}{d} - \frac{m\lambda L}{d} = \frac{\lambda L}{d}.$ Rearranging, we find $\lambda = \frac{(y_{m+1} - y_m)d}{L}$ $= \frac{(31\times10^{-3}\text{ m})(5.0\times10^{-5}\text{ m})}{(2.5\text{ m})} = 6.2\times10^{-7}\text{ m}.$

34.106. From Figure 34.18, and the accompanying text, we know that strong peaks in x-ray diffraction occur when $2d\cos(\theta) = m\lambda.$ Thus $d = \frac{\lambda}{2\cos(\theta)} = \frac{(0.155\times10^{-9}\text{ m})}{2\cos(51.7°)} = 1.25\times10^{-10}\text{ m}.$

34.107. Babinet's principle means that the diffraction pattern should have the same geometry as if the light had passed through a slit of the same thickness as the hair. Thus we can relate the angle at which the first dark fringe appears to an aperture width $a,$ which in this case will correspond to the width of the hair. We know $a = \frac{m\lambda}{\sin(\theta)}$ and simple geometry gives us the angle $\theta = \tan^{-1}\left(\frac{\Delta y}{L}\right).$ So

$$a = \frac{\lambda}{\sin\left(\tan^{-1}\left(\frac{\Delta y}{L}\right)\right)} = \frac{(690\times10^{-9}\text{ m})}{\sin\left(\tan^{-1}\left(\frac{(16\times10^{-3}\text{ m})}{(1.2\text{ m})}\right)\right)} = 5.2\times10^{-5}\text{ m}$$

34.108. From Equation 34.30 we know the minimum angle at which the planet can be resolved is $\theta_{\text{r}} = 1.22\dfrac{\lambda}{d}$, where d is the diameter of the mirror. The angular width of the planet is given by $\theta = \dfrac{D_{\text{planet}}}{L}$, where D_{planet} is the diameter of the planet, and L is the distance to the planet from Earth. Thus

$$\frac{D_{\text{planet}}}{L} = 1.22\frac{\lambda}{d} \Rightarrow d = \frac{1.22\lambda L}{D_{\text{planet}}} = \frac{1.22(525\times10^{-9}\text{ m})(10\text{ y})(3.00\times10^{8}\text{ m/s})(365.25\text{ days/y})(24\text{ h/day})(3600\text{ s/h})}{(1.27\times10^{7}\text{ m})}$$
$$= 4.8\times10^{3}\text{ m}$$

which is much larger than any existing telescopes.

34.109. (*a*) Increasing the frequency is equivalent to decreasing the wavelength. The angle at which the m^{th} order bright fringe occurs is given by $\theta_m = \sin^{-1}\left(\dfrac{m\lambda}{d}\right)$. Since the sine function is monotonic in the range $0 < \theta < 90°$, we see that decreasing the wavelength decreases the angle at which an arbitrary bright fringe appears. This means that the overall pattern is being compressed. (*b*) The pattern would appear brighter, but the geometry would not change. (*c*) A brighter light must have a greater intensity, meaning more energy leaves the surface of the source per second. Since the frequency is unchanged, the energy of each photon is unchanged. The only way for more energy to leave the surface is for more photons to leave the surface.

34.110. The oil has a greater index of refraction than the water. This means that light incident on the air-oil interface will experience a phase shift upon reflection, but light incident on the oil-water interface will not. This leaves the two rays out of phase when one considers reflection only. We require that green light be strongly reflected, such that the additional path length travelled by one ray of light causes the rays to be in phase again. This means $\Delta L = \dfrac{2t}{\cos(\theta_{\text{in oil}})} = \left(m - \dfrac{1}{2}\right)\dfrac{\lambda_0}{n_{\text{oil}}}$. We know from Snel's law that making the incident angle as small as possible corresponds to making the refracted angle $\theta_{\text{in oil}}$ as small as possible. So we wish to minimize

$$\theta_{\text{in oil}} = \cos^{-1}\left(\frac{2t}{\left(m - \frac{1}{2}\right)\frac{\lambda_0}{n_{\text{oil}}}}\right)$$

which means minimizing m. But note that we cannot choose $m = 1$, because the oil film is so much thicker than a wavelength of light. In fact, the thickness of the oil is $\dfrac{tn_{\text{oil}}}{\lambda} = \dfrac{(1.00\times10^{-5}\text{ m})(1.48)}{(510\times10^{-9}\text{ m})} = 29.0$ times the wavelength of the light in oil. So the path length difference can never be less than 58.0 wavelengths, and the smallest value that m can have while still satisfying the constructive interference condition is 59. Thus,

$$\theta_{\text{in oil}} = \cos^{-1}\left(\frac{2(1.00\times10^{-5}\text{ m})}{\left(59 - \frac{1}{2}\right)\frac{(510\times10^{-9}\text{ m})}{(1.48)}}\right) = 7.20°$$

This is the angle in the oil. We use Snel's law to relate this to the incident angle: $\theta_{\text{incident}} = \sin^{-1}\left(\dfrac{n_{\text{oil}}}{n_{\text{air}}}\sin(\theta_{\text{in oil}})\right)$ $= \sin^{-1}\left(\dfrac{(1.48)}{(1.00)}\sin(7.20°)\right) = 10.7°.$

34.111. We can write the condition on the ninth bright fringe as $\sin(\theta_9) = \sin(\theta_8 + 10°) = \dfrac{9\lambda}{a}$. Invoking a trigonometric identity yields

$$\sin(\theta_8)\cos(10°) + \sin(10°)\cos(\theta_8) = \frac{9\lambda}{a}$$
$$\left(\frac{8\lambda}{a}\right)\cos(10°) + \sin(10°)\sqrt{1-\left(\frac{8\lambda}{a}\right)^2} = \frac{9\lambda}{a}$$

which is a quadratic equation in the ratio $\left(\dfrac{\lambda}{a}\right)$, with solution $\left(\dfrac{\lambda}{a}\right) = 0.097.$

34.112. Let us call the slits slit 1 and slit 2. We know that the energy of a wave is proportional to the square of the amplitude, so we can write $I = CA^2$, where C is a constant with units of $\dfrac{W}{m^4}$. Then the amplitude of light through each slit can be written $A_1 = \sqrt{\dfrac{I_1}{C}}$ and $A_2 = \sqrt{\dfrac{I_2}{C}}$. When the waves are in phase, these amplitudes will add, giving us a total amplitude of $A_1 + A_2$. This means we can write the intensity of a bright spot as $I_{\text{bright}} =$ $C(A_1 + A_2)^2 = (\sqrt{C}A_1 + \sqrt{C}A_2)^2 = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{(0.010 \text{ mW/mm}^2)} + \sqrt{(0.030 \text{ mW/mm}^2)})^2 = 0.075 \text{ mW/mm}^2.$
In the case of destructive interference, we have an amplitude of $|A_1 - A_2|$, such that the intensity is

$$I_{\text{dark}} = C(A_1 - A_2)^2 = (\sqrt{C}A_1 - \sqrt{C}A_2)^2 = (\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{(0.010 \text{ mW/mm}^2)} - \sqrt{(0.030 \text{ mW/mm}^2)})^2$$
$$= 5.4 \times 10^{-3} \text{ mW/mm}^2$$

34.113. (*a*) We equate the increase in kinetic energy to the magnitude of the change in potential energy:

$$K_{\text{f}} = e\Delta V \Rightarrow v_{\text{f}} = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2(1.60\times10^{-19} \text{ C})(2.0\times10^3 \text{ V})}{(9.11\times10^{-19} \text{ kg})}} = 2.7\times10^7 \text{ m/s}$$

Note that this is about 9% the speed of light, and around the upper limit before one has to consider relativity. Relativity would give a small correction here, but when working with only two significant digits it is acceptable to ignore it. (*b*) The de Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})}{(9.11\times10^{-31} \text{ kg})(2.65\times10^7 \text{ m/s})} = 2.7\times10^{-11} \text{ m}$$

(*c*) Let us calculate the angle at which the first order maximum would occur:

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\left(\frac{(2.74\times10^{-11} \text{ m})}{(1.23\times10^{-10} \text{ m})}\right) = 13°$$

This should be quite easily detected. Yes, the diffraction pattern will be visible.

34.114. (*a*) You require $\Delta\theta = \theta_{\text{red}} - \theta_{\text{blue}} = 12°$. Since $\sin(\theta_{1,\text{red}}) = \sin(\theta_{1,\text{blue}} + \Delta\theta) = \dfrac{\lambda_{\text{red}}}{d}$ and $\sin(\theta_{1,\text{blue}}) = \dfrac{\lambda_{\text{blue}}}{d}$, we have

$$\frac{\sin(\theta_{1,\text{blue}} + \Delta\theta)}{\sin(\theta_{1,\text{blue}})} = \frac{\lambda_{\text{red}}}{\lambda_{\text{green}}} \text{ or } \frac{\sin(\theta_{1,\text{blue}})\cos(\Delta\theta) + \sin(\Delta\theta)\cos(\theta_{1,\text{blue}})}{\sin(\theta_{1,\text{blue}})} = \frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}}$$

This can be solved to yield

$$\theta_{1,\text{blue}} = \tan^{-1}\left[\left(\frac{1}{\sin(\Delta\theta)}\left(\frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} - \cos(\Delta\theta)\right)\right)^{-1}\right] = \tan^{-1}\left[\left(\frac{1}{\sin(12°)}\left(\frac{(700 \text{ nm})}{(400 \text{ nm})} - \cos(12°)\right)\right)^{-1}\right] = 15.1°$$

Finally, we can reinsert this angle into $\sin(\theta_{1,\text{blue}}) = \dfrac{\lambda_{\text{blue}}}{d}$ to obtain $d = \dfrac{\lambda_{\text{blue}}}{\sin(\theta_{1,\text{blue}})} = \dfrac{(400\times10^{-9}\text{ m})}{\sin(15.1°)} = 1.54\times10^{-6}\text{ m}.$ With lines of that width, the grating would have 650 lines/mm. (*b*) The red light will always be diffracted to a larger angle. So if any color within a particular order runs off the edge of the screen, it would be red light. So let us calculate the maximum order of diffraction of red light. $\sin(\theta_{\text{m,red}}) = \dfrac{m\lambda_{\text{red}}}{d} \le 1 \Rightarrow m \le \dfrac{d}{\lambda_{\text{red}}} = \dfrac{(1.54\times10^{-6}\text{ m})}{(700\times10^{-9}\text{ m})} = 2.20.$ So there will be two full orders of diffraction on either side of the central maximum. This means there are four complete spectra.

34.115. From Equation 34.30 and simple trigonometry we know

$$y_{\text{r}} = L\tan\left(\sin^{-1}\left(\frac{1.22\lambda}{d}\right)\right) \Rightarrow \lambda = \frac{d}{1.22}\sin\left(\tan^{-1}\left(\frac{y_{\text{r}}}{L}\right)\right) = \frac{2(1.36\times10^{-6}\text{ m})}{1.22}\sin\left(\tan^{-1}\left(\frac{(33.3\times10^{-3}\text{ m})}{(0.120\text{ m})}\right)\right) = 596\text{ nm}$$

Given this wavelength, the energy transferred per photon is $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(5.96\times10^{-7}\text{ m})}$ $= 3.33\times10^{-19}\text{ J},$ and the momentum transferred per photon is $p_{\text{photon}} = \dfrac{E_{\text{photon}}}{c} = \dfrac{(3.33\times10^{-19}\text{ J})}{(3.00\times10^{8}\text{ m/s})}$ $= 1.11\times10^{-27}\text{ kg}\cdot\text{m/s}.$

34.116. Let the letter subtend an angle $\theta_{\text{letter}} = \tan^{-1}\left(\dfrac{h_{\text{o}}}{d_0}\right) \approx \dfrac{h_{\text{o}}}{d_0}$ in her view, where h_{o} is the height of the letter and d_{o} is the distance from her eye to the letter. Let rays A and B come from the top and bottom of the letter respectively. Rays A and B form the angle θ_{letter} as they enter the eye. We assume the rays pass through the center of the lens, such that the angle between the refracted rays is still θ_{letter}. In this case, we can use the magnification to write $\left|\dfrac{h_{\text{i}}}{d_{\text{i}}}\right| = \dfrac{h_{\text{o}}}{d_{\text{o}}} \Rightarrow \theta_{\text{letter}} \approx \left|\dfrac{h_{\text{i}}}{d_{\text{i}}}\right|.$ Light from the bottom of the letter and from the top of the letter will form Airy disks on the back of the eye. The minimum angle between the centers of the disks in order for the two ends of the letter to be resolvable is given by Equation 34.30: $\theta_{\text{r}} = \dfrac{1.22\lambda}{d}.$ Setting this minimum resolvable angle equal to θ_{letter}, we can determine the minimum height of the letter: $\dfrac{h_{\text{o,min}}}{d_{\text{o}}} = \dfrac{1.22\lambda}{d} \Rightarrow h_{\text{o,min}} = \dfrac{1.22\lambda d_{\text{o}}}{d}$ $= \dfrac{1.22(500\times10^{-9}\text{ m})(0.25\text{ m})}{(3.0\times10^{-3}\text{ m})} = 5.1\times10^{-5}\text{ m}.$

34.117. Because of the cylindrical shape of the material, a ray of light passing through that slit and striking the screen (call this ray A) will always leave the material normal to the surface. Thus, there will be no effects due to refraction at the interface. However, the index of refraction in the material is slightly larger than that of air. This means that for a particular beam of light, wavelengths will be reduced upon entering the material. This means we can fit more wavelengths in that region of space than if the space were occupied by mere air. In order to shift bright spots to dark and dark spots to bright, we wish to increase the number of wavelengths making up the path of ray A by 1/2. Call the radius of the material r. In air, the number of wavelengths that would fit in this space is $N_{\text{air}} = \dfrac{r}{\lambda_0}.$ With the material, this number becomes $N_{\text{material}} = \dfrac{rn_{\text{material}}}{\lambda_0}.$ We require that the difference be

$$N_{\text{material}} - N_{\text{air}} = \frac{r}{\lambda_0}(n_{\text{material}} - 1) = \frac{1}{2} \Rightarrow r = \frac{\lambda_0}{2(n_{\text{material}} - 1)} = \frac{(750\times10^{-9}\text{ m})}{2(1.001-1)} = 4\times10^{-4}\text{ m}$$

34.118. The Bragg angle is related to the lattice constant through $2d\cos(\theta) = m\lambda = 2d\cos(90-\alpha) = 2d\sin(\alpha) \Rightarrow d = \dfrac{m\lambda}{2\sin(\alpha)} = \dfrac{(1)(0.154\times10^{-9}\text{ m})}{2\sin(22.4°)} = 2.02\times10^{-10}\text{ m}$. This is the spacing for LiF, so you have the sample of LiF.

34.119. We know that gravity must be holding the satellite in orbit, so we can equate the acceleration due to gravity to the necessary centripetal acceleration and obtain

$$\frac{Gm_{\text{mars}}}{r^2} = \frac{v^2}{r} = \frac{(2\pi r)^2}{rT^2} \Rightarrow r = \left(\frac{Gm_{\text{mars}}T^2}{4\pi}\right)^{1/3} = \left(\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\text{ kg})((17\text{ h})(3600\text{ s/h}))^2}{4\pi}\right)^{1/3}$$
$$= 2.34\times10^7\text{ m}$$

So the distance from the center of mars to the satellite is 2.34×10^7 m. But we are interested in viewing the surface of mars, so the really relevant distance is that from the satellite to the surface: $d_{\text{surface}} = r - R_{\text{mars}} = (2.34\times10^7\text{ m}) - (3.40\times10^6\text{ m}) = 2.00\times10^7\text{ m}$. We wish to resolve objects 2.00 m across a distance 2.00×10^7 m away, meaning the resolvable angle must be $\theta_{\text{r}} \approx 1.00\times10^{-7}$ rad. From here, we rearrange Equation 34.31 to relate the required lens diameter and focal lengths to the resolvable angle: $\theta_{\text{r}} \approx \dfrac{y_{\text{r}}}{f} \approx \dfrac{1.22\lambda}{d}$. Because of our limited space, we clearly need to use the smallest wavelength of visible light possible. This minimum varies slightly from person to person, but we will assume a value of 400 nm. Solving for the required diameter of the apparatus, we find $d = \dfrac{1.22\lambda}{\theta_{\text{r}}} = \dfrac{1.22(400\times10^{-9}\text{ m})}{(1.00\times10^{-7}\text{ rad})} = 4.9\text{ m}$. One might use a different wavelength of light, or one might try to construct arguments using a grating instead of a lens, but the apparatus must be considerably larger than 1.0 m.