

# Contents

<b>1 Basic</b>	<b>1</b>	<b>6 String</b>	<b>21</b>
1.1 .vimrc	1	6.1 PalTree 7280a6	21
1.2 hash.sh	1	6.2 SAIS 4379c7	22
1.3 Custom Hash a296c3	1	6.3 SuffixAutomata 815370	22
1.4 Fast Division/Modular 9b4495	1	6.4 Z Value 391d23	22
1.5 python-related	1	6.5 BWT f8c84c	22
<b>2 flow</b>	<b>2</b>	6.6 ZValue Palindrome 44a8c2	23
2.1 ISAP ab2177	2	6.7 Smallest Rotation 28fe26	23
2.2 MinCostFlow 8b28ab	2	6.8 Cyclic LCS bcba38	23
2.3 Dinic aa63d9	2	<b>7 Data Structure</b>	<b>23</b>
2.4 Kuhn Munkres e807c8	2	7.1 Link-Cut Tree f3eedd	23
2.5 SW min-cut 9beb62	3	<b>8 Others</b>	<b>24</b>
2.6 Max Cost Circulation 932772	3	8.1 Find max tangent(x,y is increasing) 8fea15	24
2.7 Gomory-Hu Tree 796cf8	3	8.2 Exact Cover Set 91d0af	24
2.8 Max flow with lower/upper bound cee539	3	8.3 Binary Next Permutation b7a40a	24
2.9 HLPPA 719919	4	8.4 Hilbert Curve 15d26a	24
2.10Flow Method	4	<b>1 Basic</b>	<b>4</b>
<b>3 Math</b>	<b>4</b>	<b>1.1 .vimrc</b>	<b>4</b>
3.1 FFT c7d0f0	4	syn on	5
3.2 NTT 65f1a7	5	se ai nu rnu ru cul mouse=a	5
3.3 Fast Walsh Transform c77328	5	se cin et ts=2 sw=2 sts=2	6
3.4 FFT Mod 96080e	6	so \$VIMRUNTIME/mswin.vim	6
3.5 Poly operator d70dd4	6	colo desert	6
3.6 Poly Interpolation 6e3604	6	filet plugin indent on	7
3.7 Linear Recurrence 29d614	7	no <F5> :!./a.out<CR>	7
3.8 BerlekampMassey 868031	7	no <F9> :!g++ -O2 -std=c++17 % -g -fsanitize=undefined -	7
3.9 Miller Rabin 82c8fa	7	Wall -Wextra -Wshadow -Wno-unused-result<CR>	7
3.10Simplex c2c23c	7	se undofile undodir=~/.vim/undodir " mkdir manually	8
3.11Faulhaber 862da1	7	<b>1.2 hash.sh</b>	<b>8</b>
3.12Chinese Remainder 94874a	8	#!/bin/bash	8
3.13Pollard Rho 6b641a	8	cpp -dD -P -fpreprocessed \$1   tr -d '[:space:]'   md5sum	8
3.14ax+by=gcd 5a651f	8	lcut -c-6	8
3.15Discrete sqrt f12a09	8	<b>1.3 Custom Hash a296c3</b>	<b>9</b>
3.16Discrete logarithm 3aac04	8	struct custom_hash {	9
3.17Romberg 6dc94c	8	static uint64_t splitmix64(uint64_t x) {	10
3.18Simpson 3fec8d	8	x += 0x9e3779b97f4a7c15;	10
3.19Prefix Inverse 9e8ee9	8	x = (x ^ (x >> 30)) * 0xbfb58476d1ce4e5b9;	10
3.20Roots of Polynomial 20a9c9	8	x = (x ^ (x >> 27)) * 0x94d049bb133111eb;	10
3.21Sum of Division/Modular 1adbfe	8	return x ^ (x >> 31);	10
3.22Fraction Binary Search 38ec70	8	}	10
3.23Closest Fraction ef6b46	8	size_t operator()(uint64_t x) const {	10
3.24Primes and $\mu$ function d79d2b	9	static const uint64_t FIXED_RANDOM = chrono::	11
3.25Subset Convolution 84a3e0	9	steady_clock::now().time_since_epoch().count();	11
3.26Result fd0b69	9	return splitmix64(x + FIXED_RANDOM);	11
<b>4 Geometry</b>	<b>10</b>	}	11
4.1 Intersection of 2 lines 3db65e	10	};	12
4.2 halfPlaneIntersection 1d54e0	10	<b>1.4 Fast Division/Modular 9b4495</b>	<b>12</b>
4.3 Intersection of 2 segments b7e393	10	struct FastDiv{	13
4.4 Banana de5c4e	10	ull b,m;	13
4.5 Intersection of circle and line 73c7f5	10	FastDiv(ull _b):b(_b),m(-1ULL/_b){}	14
4.6 Intersection of polygon and circle 868772	10	pair<ull,ull> div(ull a){ // (a/b,a%b)	14
4.7 Intersection of 2 circles 57edf8	10	ull q=((__uint128_t(m)*a)>>64),r=a-q*b;	14
4.8 Circle cover 4b4772	11	return r>=b?make_pair(q+1,r-b):make_pair(q,r);	14
4.9 Li Chao Segment Tree 01810b	11	}	15
4.10Convex Hull trick 66a3a1	11	};	15
4.11Rotating Sweep Line 3971d4	12	<b>1.5 python-related</b>	<b>16</b>
4.12Tangent line of two circles a45324	12	from fractions import Fraction	16
4.13Tangent line of point and circle 35a7bf	12	from decimal import Decimal, getcontext	16
4.14Min distance of two convex 55abcb	12	getcontext().prec = 250 # set precision	17
4.15Poly Union 7a6b24	12	itwo,two,N = Decimal(0.5),Decimal(2),200	17
4.16Lower Concave Hull dd665b	13	def angle(cost):	18
4.17Delaunay Triangulation 8ee9b3	13	"""given cos(theta) in decimal return theta"""	19
4.18Min Enclosing Circle 0de93f	14	for i in range(N):	19
4.19Min Enclosing Ball 64186c	14	cost = ((cost + 1) / two) ** itwo	19
4.20Minkowski sum 3ce748	14	sint = (1 - cost * cost) ** itwo	20
4.21Min dist on Cuboid 95b46d	14	return sint * (2 ** N)	20
4.22Heart of Triangle 4da867	15	pi = angle(Decimal(-1))	21
<b>5 Graph</b>	<b>15</b>		
5.1 DominatorTree 0e5706	15		
5.2 Directed MST(ElogE) 4b46a2	15		
5.3 MaximalClique 7e84df	15		
5.4 MaxCliqueDyn e0119d	16		
5.5 Strongly Connected Component 10c233	16		
5.6 Dynamic MST 21e59f	16		
5.7 Maximum General graph Matching a15e63	17		
5.8 Minimum General Weighted Matching 91b293	17		
5.9 Maximum General Weighted Matching 0baee5	17		
5.10Minimum Steiner Tree 837386	18		
5.11BCC based on vertex 4ea1ac	19		
5.12Min Mean Cycle f5f8be	19		
5.13Directed Graph Min Cost Cycle c448cd	19		
5.14K-th Shortest Path 355040	20		
5.15Chordal Graph a96ac6	20		
5.16Matroid Intersection d1fe1e	21		
5.17Graph Hash	21		
5.18Graph Method	21		

## 2 flow

### 2.1 ISAP ab2177

```
#define SZ(c) ((int)(c).size())
struct Maxflow{
    static const int MAXV=50010;
    static const int INF =1000000;
    struct Edge{
        int v,c,r;
        Edge(int _v,int _c,int _r):v(_v),c(_c),r(_r){}
    };
    int s,t; vector<Edge> G[MAXV];
    int iter[MAXV],d[MAXV],gap[MAXV],tot;
    void init(int n,int _s,int _t){
        tot=n,s=_s,t=_t;
        for(int i=0;i<=tot;i++){
            G[i].clear(); iter[i]=d[i]=gap[i]=0;
        }
    }
    void add_edge(int u,int v,int c){
        G[u].push_back(Edge(v,c,SZ(G[v])));
        G[v].push_back(Edge(u,0,SZ(G[u])-1));
    }
    int DFS(int p,int flow){
        if(p==t) return flow;
        for(int &i=iter[p];i<SZ(G[p]);i++){
            Edge &e=G[p][i];
            if(e.c>0&&d[p]==d[e.v]+1){
                int f=DFS(e.v,min(flow,e.c));
                if(f){ e.c-=f; G[e.v][e.r].c+=f; return f; }
            }
        }
        if(--gap[d[p]]==0) d[p]=tot;
        else{ d[p]++; iter[p]=0; ++gap[d[p]]; }
        return 0;
    }
    int flow(){
        int res=0;
        for(res=0,gap[0]=tot;d[s]<tot;res+=DFS(s,INF));
        return res;
    }
} flow;
```

### 2.2 MinCostFlow 8b28ab

```
struct zkwflow{
    static const int maxN=10000;
    struct Edge{ int v,f,re; ll w; };
    int n,s,t,ptr[maxN]; bool vis[maxN]; ll dis[maxN];
    vector<Edge> E[maxN];
    void init(int _n,int _s,int _t){
        n=_n,s=_s,t=_t;
        for(int i=0;i<n;i++) E[i].clear();
    }
    void add_edge(int u,int v,int f,ll w){
        E[u].push_back({v,f,(int)E[v].size(),w});
        E[v].push_back({u,0,(int)E[u].size()-1,-w});
    }
    bool SPFA(){
        fill_n(dis,n,LLONG_MAX); fill_n(vis,n,false);
        queue<int> q; q.push(s); dis[s]=0;
        while (!q.empty()){
            int u=q.front(); q.pop(); vis[u]=false;
            for(auto &it:E[u]){
                if(it.f>0&&dis[it.v]>dis[u]+it.w){
                    dis[it.v]=dis[u]+it.w;
                    if(!vis[it.v]){
                        vis[it.v]=true; q.push(it.v);
                    }
                }
            }
        }
        return dis[t]!=LLONG_MAX;
    }
    int DFS(int u,int nf){
        if(u==t) return nf;
        int res=0; vis[u]=true;
        for(int &i=ptr[u];i<(int)E[u].size();i++){
            auto &it=E[u][i];
            if(it.f>0&&dis[it.v]==dis[u]+it.w&&!vis[it.v]){
                int tf=DFS(it.v,min(nf,it.f));
                res+=tf,nf-=tf,it.f-=tf;
                E[it.v][it.re].f+=tf;
                if(nf==0){ vis[u]=false; break; }
            }
        }
    }
```

```
    }
    }
    return res;
}
pair<int,ll> flow(){
    int flow=0; ll cost=0;
    while (SPFA()){
        fill_n(ptr,n,0);
        int f=DFS(s,INT_MAX);
        flow+=f; cost+=dis[t]*f;
    }
    return{ flow,cost };
}
} flow;
```

### 2.3 Dinic aa63d9

```
struct Dinic{
    static const int MXN=10000;
    struct Edge{ int v,f,re; };
    int n,s,t,level[MXN]; vector<Edge> E[MXN];
    void init(int _n,int _s,int _t){
        n=_n; s=_s; t=_t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u,int v,int f){
        E[u].push_back({v,f,(int)E[v].size()});
        E[v].push_back({u,0,(int)E[u].size()-1});
    }
    bool BFS(){
        for (int i=0;i<n;i++) level[i]=-1;
        queue<int> que; que.push(s); level[s]=0;
        while(!que.empty()){
            int u=que.front(); que.pop();
            for (auto &it:E[u]){
                if(it.f>0&&level[it.v]==-1){
                    level[it.v]=level[u]+1; que.push(it.v);
                }
            }
        }
        return level[t]!=-1;
    }
    int DFS(int u,int nf){
        if(u==t) return nf;
        int res=0;
        for (auto &it:E[u]){
            if(it.f>0&&level[it.v]==level[u]+1){
                int tf=DFS(it.v,min(nf,it.f));
                res+=tf; nf-=tf; it.f-=tf;
                E[it.v][it.re].f+=tf;
                if(nf==0) return res;
            }
        }
        if(!res) level[u]=-1;
        return res;
    }
    int flow(int res=0){
        while(BFS()) res+=DFS(s,2147483647);
        return res;
    }
} flow;
```

### 2.4 Kuhn Munkres e807c8

```
struct KM{ // max weight, for min negate the weights
    static const int MXN=2001; // 1-based
    static const ll INF=0x3f3f3f3f;
    int n,mx[MXN],my[MXN],pa[MXN],la[MXN]; bool vx[MXN],vy[MXN];
    ll g[MXN][MXN],lx[MXN],ly[MXN],sy[MXN];
    void init(int _n){
        n=_n; for(int i=1;i<=n;i++) fill(g[i],g[i]+n+1,0);
    }
    void addEdge(int x,int y,ll w){ g[x][y]=w; }
    void augment(int y){
        for(int x,z;y;z=x) x=pa[y],z=mx[x],my[y]=x,mx[x]=y;
    }
    void bfs(int st){
        for(int i=1;i<=n;i++) sy[i]=INF,vx[i]=vy[i]=0;
        queue<int> q;q.push(st);
        for(;;){
            while(q.size()){
                int x=q.front(); q.pop(); vx[x]=1;
                for(int y=1;y<=n;y++) if(!vy[y]){
                    ll t=lx[x]+ly[y]-g[x][y];

```

```

    if(t==0){
        pa[y]=x;
        if(!my[y]){ augment(y); return; }
        vy[y]=1,q.push(my[y]);
    }else if(sy[y]>t) pa[y]=x,sy[y]=t;
    }
    ll cut=INF;
    for(int y=1;y<=n;++y)
        if(!vy[y]&&cut>sy[y]) cut=sy[y];
    for(int j=1;j<=n;++j){
        if(vx[j]) lx[j]-=cut;
        if(vy[j]) ly[j]+=cut;
        else sy[j]-=cut;
    }
    for(int y=1;y<=n;++y) if(!vy[y]&&sy[y]==0){
        if(!my[y]){ augment(y); return; }
        vy[y]=1,q.push(my[y]);
    } } }
    ll solve(){
        fill(mx,mx+n+1,0);fill(my,my+n+1,0);
        fill(ly,ly+n+1,0);fill(lx,lx+n+1,-INF);
        for(int x=1;x<=n;++x) for(int y=1;y<=n;++y)
            lx[x]=max(lx[x],g[x][y]);
        for(int x=1;x<=n;++x) bfs(x);
        ll ans=0;
        for(int y=1;y<=n;++y) ans+=g[my[y]][y];
        return ans;
    }
}graph;

```

## 2.5 SW min-cut 9beb62

```

const int INF=0x3f3f3f3f;
template<typename T>
struct stoer_wagner{// 0-base
    static const int MAXN=501;
    T g[MAXN][MAXN],dis[MAXN]; int nd[MAXN],n,s,t;
    void init(int _n){
        n=_n;
        for(int i=0;i<n;++i) for(int j=0;j<n;++j) g[i][j]=0;
    }
    void add_edge(int u,int v,T w){ g[u][v]=g[v][u]+=w; }
    T min_cut(){
        T ans=INF;
        for(int i=0;i<n;++i) nd[i]=i;
        for(int ind,tn=n;tn>1;--tn){
            for(int i=1;i<tn;++i)dis[ind[i]]=0;
            for(int i=1;i<tn;++i){
                ind=i;
                for(int j=i;j<tn;++j){
                    dis[ind[j]]+=g[ind[i-1]][nd[j]];
                    if(dis[ind[j]]<dis[ind[j]])ind=j;
                }
                swap(nd[ind],nd[i]);
            }
            if(ans>dis[nd[ind]])
                ans=dis[nd[ind]],s=nd[ind-1];
            for(int i=0;i<tn;++i)
                g[nd[ind-1]][nd[i]]=g[nd[i]][nd[ind-1]]
                    +=g[nd[i]][nd[ind]];
        }
        return ans;
    }
};

```

## 2.6 Max Cost Circulation 932772

```

struct MaxCostCirc {
    static const int MAXN=33;
    struct Edge { int v,w,c,r; };
    vector<Edge> g[MAXN];
    int dis[MAXN],prv[MAXN],prve[MAXN];
    int n,m,ans; bool vis[MAXN];
    void init(int _n,int _m) : n(_n),m(_m) {}
    void adde(int u,int v,int w,int c) {
        g[u].push_back({v,w,c,(int)g[v].size()});
        g[v].push_back({u,-w,0,(int)g[u].size()-1});
    }
    bool poscyc() {
        fill(dis,dis+n+1,0); fill(prv,prv+n+1,0);
        fill(vis,vis+n+1,0); int tmp=-1;
    }
};

```

```

for(int t=0;t<=n;t++) {
    for(int i=1;i<=n;i++) {
        for(int j=0;j<(int)g[i].size();j++) {
            Edge& e=g[i][j];
            if(e.c&&dis[e.v]<dis[i]+e.w) {
                dis[e.v]=dis[i]+e.w;
                prv[e.v]=i; prve[e.v]=j;
                if(t==n) { tmp=i; break; }
            } } }
    if(tmp==n) return 0;
    int cur=tmp;
    while(!vis[cur]) { vis[cur]=1; cur=prv[cur]; }
    int now=cur,cost=0,df=100000;
    do{
        Edge &e=g[prv[now]][prve[now]];
        df=min(df,e.c); cost+=e.w; now=prv[now];
    }while(now!=cur);
    ans+=df*cost; now=cur;
    do{
        Edge &e=g[prv[now]][prve[now]];
        Edge &re=g[now][e.r];
        e.c-=df; re.c+=df; now=prv[now];
    }while(now!=cur);
    return 1;
} circ;

```

## 2.7 Gomory-Hu Tree 796cf8

```

//n,Dinic::flow must be filled
//result:e[u][v]=u-v mincut;p[u]:u's parent on cut tree
int n,e[MAXN][MAXN],p[MAXN];
void gomory_hu(){
    fill(p,p+n,0); fill(e[0],e[n],INF);
    for(int s=1;s<n;s++){
        int t=p[s]; Dinic F; F.init(n,s,t);
        copy(flow.E,flow.E+MXN,F.E); int tmp=F.flow();
        for(int i=0;i<s;i++){
            e[s][i]=e[i][s]=min(tmp,e[t][i]);
            for(int is=s+1;i<n;i++){
                if(p[i]==t&&F.level[i]!=-1) p[i]=s;
            }
        }
    }
}

```

## 2.8 Max flow with lower/upper bound cee539

```

// Max flow with lower/upper bound on edges
// use with ISAP
int in[N],out[N],l[M],r[M],a[M],b[M];
int solve(int n, int m, int s, int t){
    flow.init(n);
    for(int i=0;i<m;i++){
        in[r[i]]+=a[i]; out[l[i]]+=a[i];
        flow.addEdge(l[i],r[i],b[i]-a[i]);
        // flow from l[i] to r[i] must in [a[i], b[i]]
    }
    int nd=0;
    for(int i=0;i<=n;i++){
        if(in[i]<out[i]){
            flow.addEdge(i,flow.t,out[i]-in[i]);
            nd+=out[i]-in[i];
        }
        if(out[i]<in[i])
            flow.addEdge(flow.s,i,in[i]-out[i]);
    }
    // original sink to source
    flow.addEdge(t,s,INF);
    if( flow.solve() != nd ) return -1; // no solution
    int ans=flow.G[s].back().c; // source to sink
    flow.G[s].back().c=flow.G[t].back().c=0;
    // take out super source and super sink
    for(size_t i=0;i<flow.G[flow.s].size();i++){
        flow.G[flow.s][i].c=0;
        Maxflow::Edge &e=flow.G[flow.s][i];
        flow.G[e.v][e.r].c=0;
    }
    for(size_t i=0;i<flow.G[flow.t].size();i++){
        flow.G[flow.t][i].c=0;
        Maxflow::Edge &e=flow.G[flow.t][i];
        flow.G[e.v][e.r].c=0;
    }
}

```

```

flow.addEdge(flow.s,s,INF);
flow.addEdge(t,flow.t,INF);
flow.reset(); // set iter,d,gap to 0
return ans + flow.solve();
}

```

## 2.9 HLPPA 719919

```

template <int MAXN, class T = int>
struct HLPP {
    const T INF = numeric_limits<T>::max();
    struct Edge { int to, rev; T f; };
    int n, s, t; T ef[MAXN]; vector<Edge> adj[MAXN];
    deque<int> lst[MAXN]; vector<int> gap[MAXN];
    int ptr[MAXN], h[MAXN], cnt[MAXN], work, hst=0; // highest
    void init(int _n, int _s, int _t) {
        n=_n+1; s=_s; t=_t;
        for(int i=0; i<n; i++) adj[i].clear();
    }
    void add_edge(int u, int v, T f, bool isDir = true) {
        adj[u].push_back({v, adj[v].size(), f});
        adj[v].push_back({u, adj[u].size()-1, isDir?0:f});
    }
    void updHeight(int v, int nh) {
        work++;
        if(h[v] != n) cnt[h[v]]--;
        h[v] = nh;
        if(nh == n) return;
        cnt[nh]++; hst = nh; gap[nh].push_back(v);
        if(ef[v]>0) lst[nh].push_back(v), ptr[nh]++;
    }
    void globalRelabel() {
        work = 0; fill(h, h+n, n); fill(cnt, cnt+n, 0);
        for(int i=0; i<=hst; i++)
            lst[i].clear(), gap[i].clear(), ptr[i] = 0;
        queue<int> q({t}); h[t] = 0;
        while(!q.empty()) {
            int v = q.front(); q.pop();
            for(auto &e : adj[v])
                if(h[e.to] == n && adj[e.to][e.rev].f > 0)
                    q.push(e.to), updHeight(e.to, h[v] + 1);
            hst = h[v];
        }
    }
    void push(int v, Edge &e) {
        if(ef[e.to] == 0)
            lst[h[e.to]].push_back(e.to), ptr[h[e.to]]++;
        T df = min(ef[v], e.f);
        e.f -= df, adj[e.to][e.rev].f += df;
        ef[v] -= df, ef[e.to] += df;
    }
    void discharge(int v) {
        int nh = n;
        for(auto &e : adj[v]) {
            if(e.f > 0) {
                if(h[v] == h[e.to] + 1) {
                    push(v, e);
                    if(ef[v] <= 0) return;
                }
                else nh = min(nh, h[e.to] + 1);
            }
        }
        if(cnt[h[v]] > 1) updHeight(v, nh);
        else {
            for(int i = h[v]; i < n; i++) {
                for(auto j : gap[i]) updHeight(j, n);
                gap[i].clear(), ptr[i] = 0;
            }
        }
    }
    T flow() {
        fill(ef, ef+n, 0); ef[s] = INF, ef[t] = -INF;
        globalRelabel();
        for(auto &e : adj[s]) push(s, e);
        for(; hst >= 0; hst--) {
            while(!lst[hst].empty()) {
                int v=lst[hst].back(); lst[hst].pop_back();
                discharge(v);
                if(work > 4 * n) globalRelabel();
            }
        }
        return ef[t] + INF;
    }
};

```

## 2.10 Flow Method

Maximize  $c^T x$  subject to  $Ax \leq b$ ,  $x \geq 0$ ;  
with the corresponding symmetric dual problem,  
Minimize  $b^T y$  subject to  $A^T y \geq c$ ,  $y \geq 0$ .

Maximize  $c^T x$  subject to  $Ax \leq b$ ;  
with the corresponding asymmetric dual problem,  
Minimize  $b^T y$  subject to  $A^T y = c$ ,  $y \geq 0$ .

Maximize  $\sum x$  subject to  $x_i + x_j \leq A_{ij}$ ,  $x \geq 0$ ;  
=> Maximize  $\sum x$  subject to  $x_i + x_j \leq A_{ij}$ ;  
=> Minimize  $A^T y = \sum A_{ij} y_{ij}$  subject to for all  $v$ ,  
 $\sum_{i=v \text{ or } j=v} y_{ij} = 1$ ,  $y_{ij} \geq 0$   
=> possible optimal solution:  $y_{ij} = \{0, 0.5, 1\}$   
=>  $y' = 2y$ :  $\sum_{i=v \text{ or } j=v} y'_{ij} = 2$ ,  $y'_{ij} = \{0, 1, 2\}$   
=> Minimum Bipartite perfect matching/2 ( $V_1=X, V_2=X, E=A$ )

General Graph:

$|Max \text{ Ind. Set}| + |Min \text{ Vertex Cover}| = |V|$   
 $|Max \text{ Ind. Edge Set}| + |Min \text{ Edge Cover}| = |E|$

Bipartite Graph:

$|Max \text{ Ind. Set}| = |Min \text{ Edge Cover}|$   
 $|Max \text{ Ind. Edge Set}| = |Min \text{ Vertex Cover}|$

To reconstruct the minimum vertex cover, dfs from each unmatched vertex on the left side and with unused edges only. Equivalently, dfs from source with unused edges only and without visiting sink. Then, a vertex is chosen iff. it is on the left side and without visited or on the right side and visited through dfs.

Minimum Weighted Bipartite Edge Cover:

Construct new bipartite graph with  $n+m$  vertices on each side:

for each vertex  $u$ , duplicate a vertex  $u'$  on the other side  
for each edge  $(u,v,w)$ , add edges  $(u,v,w)$  and  $(v',u',w)$   
for each vertex  $u$ , add edge  $(u,u',2w)$  where  $w$  is min edge connects to  $u$   
then the answer is the minimum perfect matching of the new graph (KM)

Maximum density subgraph ( $\sum W_e + \sum W_v$ ) /  $|V|$   
Binary search on answer:

For a fixed  $D$ , construct a Max flow model as follow:  
Let  $S$  be Sum of all weight (or inf)  
1. from source to each node with cap =  $S$   
2. For each  $(u,v,w)$  in  $E$ ,  $(u \rightarrow v, \text{cap}=w)$ ,  $(v \rightarrow u, \text{cap}=w)$   
3. For each node  $v$ , from  $v$  to sink with cap =  $S + 2 * D - \text{deg}[v] - 2 * (W \text{ of } v)$   
where  $\text{deg}[v] = \sum \text{weight of edge associated with } v$   
If  $\text{maxflow} < S * |V|$ ,  $D$  is an answer.

Requiring subgraph: all vertex can be reached from source with edge whose cap > 0.

Maximum closed subgraph

1. connect source with positive weighted vertex (capacity = weight)
2. connect sink with negative weighted vertex (capacity = -weight)
3. make capacity of the original edges = inf
4. ans = sum(positive weighted vertex weight) - (max flow)

Minimum Path Cover of DAG

1. For each vertex  $v$ , split it to  $v_{in}$  and  $v_{out}$ .
2. For each edge  $(u \rightarrow v)$ , add an edge between  $u_{out}$  and  $v_{in}$
3.  $|Minimum \text{ Path Cover}| = |V| - |Maximum \text{ Matching}|$  of the new bipartite graph

## 3 Math

### 3.1 FFT c7d0f0

```

const int MXN=1048576; // (must be 2^k)
// before any usage, run pre_fft() first
typedef long double ld;
typedef complex<ld> cplx;

```

```

const ld PI=acosl(-1); const cplx I(0,1);
cplx omega[MXN+1];
void pre_fft(){
    for(int i=0;i<=MXN;i++){
        omega[i]=exp(i*2*PI/MXN*I);
    }
}
// n must be 2^k; fft(cplx(a+b,a-b)); Re(iff(c))/4+0.5;
void fft(int n,cplx a[],bool inv=false){
    int basic=MXN/n,theta=basic;
    for(int m=n;m>=2;m>=1) {
        int mh=m>>1;
        for(int i=0;i<mh;i++) {
            cplx w=omega[inv?MXN-(i*theta%MXN):i*theta%MXN];
            for(int j=i;j<n;j+=m) {
                int k=j+mh; cplx x=a[j]-a[k];
                a[j]+=a[k]; a[k]=w*x;
            }
        }
        theta=(theta*2)%MXN;
    }
    int i=0;
    for(int j=1;j<n-1;j++) {
        for(int k=n>>1;k>=(i^k);k>=1);
        if(j<i) swap(a[i],a[j]);
    }
    if(inv) for(i=0;i<n;i++) a[i]/=n;
}

```

### 3.2 NTT 65f1a7

```

/* p=a*2^k+1
p      a      k      root
998244353      119      23      3
1107296257      33      25      10
2013265921      15      27      31
2061584302081      15      37      7
2748779069441      5      39      3
1945555039024054273      27      56      5 */
template<ll P,ll root,int MAXK,int MAXN>
struct NTT{
    static ll powi(ll a,ll b){
        ll ret=1;
        for(;b>=1;a=mul(a, a, P)){
            if(b&1) ret=mul(ret, a, P);
        }
        return ret;
    }
    static ll inv(ll a,ll b){
        if(a==1) return 1;
        return (((a-inv(b%a,a))*b+1)/a)%b; // overflow
    }
    ll omega[MAXK+1],inv_omega[MAXK+1];
    NTT(){
        omega[MAXK]=powi(root,(P-1)>>MAXK);
        for(int i=MAXK-1;i>=0;i--){
            omega[i]=mul(omega[i+1], omega[i+1], P);
        }
        for(int i=0;i<=MAXK;i++){
            inv_omega[i]=inv(omega[i],P);
        }
    }
    void tran(int n,ll a[],bool inv_ntt=false){//n=2^i
        for(int i=1,j=0;i<n;i++){
            for(int k=n>>1;k>=(i^k)&k>=1);
            if(i<j) swap(a[i],a[j]);
        }
        ll *G=(inv_ntt?inv_omega:omega);
        for(int k=2,t=1;k<=n;k<=1){
            int k2=k>>1;ll dw=G[t++];
            for(int j=0;j<n;j+=k){
                ll w=1;
                for(int i=j;i<j+k2;i++){
                    ll x=a[i],y=mul(a[i+k2], w, P);
                    a[i]=x+y; if(a[i]>=P) a[i]-=P;
                    a[i+k2]=x-y; if(a[i+k2]<0) a[i+k2]+=P;
                    w=mul(w, dw, P);
                }
            }
        }
        if(inv_ntt){
            ll inv_n=inv(n,P);
            for(int i=0;i<n;i++) a[i]=mul(a[i], inv_n, P);
        }
    }
};
const int MAXN=4194304,MAXK=22; //MAXN=2^k

```

```

const ll P=2013265921,root=31;
NTT<P,root,MAXK,MAXN> ntt;

```

### 3.3 Fast Walsh Transform c77328

```

/* xor convolution:
*x=(x0,x1),y=(y0,y1)
*z=(x0y0+x1y1,x0y1+x1y0)
*=>
*x'=(x0+x1,x0-x1),y'=(y0+y1,y0-y1)
*z'=((x0+x1)(y0+y1),(x0-x1)(y0-y1))
*z=(1/2)*z''
*or convolution:
*x=(x0,x0+x1),inv=(x0,x1-x0) w/o final div
*and convolution:
*x=(x0+x1,x1),inv=(x0-x1,x1) w/o final div
*ternary xor convolution:
*x=(x0+x1+x2,x0+x1w+x2w^2,x0+x1w^2+x2w)
*inv=(1/3)*(x0+x1+x2,x0+x1w^2+x2w,x0+x1w+x2w^2)
*where w^3=1 and w^2=-w-1 */
typedef long long ll;
const int MAXN=(1<<20)+10; const ll MOD=1e9+7;
inline ll pw(ll x,ll k) {
    ll res=1;
    for(ll bs=x;k>=1,bs=(bs*bs)%MOD){
        if(k&1) res=(res*bs) % MOD;
        return res;
    }
}
inline ll invf(ll x) { return pw(x,MOD-2); }
inline void fwt(ll x[MAXN],int N,bool inv=0) {
    for(int d=1;d<N;d<=1) {
        int d2=d<<1;
        for(int s=0;s<N;s+=d2){
            for(int i=s,j=s+d;i<s+d;i++,j++){
                ll ta=x[i],tb=x[j]; x[i]=ta+tb; x[j]=ta-tb;
                if(x[i]>=MOD) x[i]-=MOD;
                if(x[j]<0) x[j]+=MOD;
            }
        }
        ll invN=invf(N);
        if(inv)
            for(int i=0;i<N;i++) { x[i] *= invN; x[i] %= MOD; }
    }
}

```

### 3.4 FFT Mod 96080e

```

void fftmod(ll a[],int n,ll b[],int m,ll c[],ll mod){
    int B=32-__builtin_clz(n+m-1),N=1<<B,cut=sqrt(mod);
    vector<cplx> L(N),R(N),outs(N),outl(N);
    for(int i=0;i<n;i++) L[i]=cplx(a[i]/cut,a[i]%cut);
    for(int i=0;i<m;i++) R[i]=cplx(b[i]/cut,b[i]%cut);
    fft(N,L.data()); fft(N,R.data());
    for(int i=0;i<N;i++){
        int j=-i&(N-1);
        outl[j]=(L[i]+conj(L[j]))*R[i]/(2.0L*N);
        outs[j]=(L[i]-conj(L[j]))*R[i]/(2.0L*N)/1i;
    }
    fft(N,outl.data()); fft(N,outs.data());
    for(int i=0;i<n+m-1;i++){
        ll av=real(outl[i])+.5,cv=imag(outs[i])+.5;
        ll bv=(ll)(imag(outl[i])+.5)+(ll)(real(outs[i])+.5);
        c[i]=((av%mod*cut+bv)%mod*cut+cv)%mod;
    }
} // NlogN*mod < 8.6e14 (maybe >=1e16 in practice)

```

### 3.5 Poly operator d70dd4

```

struct PolyOp {
#define FOR(i,c) for (int i=0; i<(c); ++i)
    NTT<P,root,MAXK,MAXN> ntt;
    static int nxt2k(int x) {
        int i=1; for (; i<x; i<= 1); return i;
    }
    void Mul(int n,ll a[],int m,ll b[],ll c[]) {
        static ll aa[MAXN],bb[MAXN]; int N=nxt2k(n+m);
        copy(a,a+n,aa); fill(aa+n,aa+N,0);
        copy(b,b+m,bb); fill(bb+m,bb+N,0);
        ntt.tran(N,aa); ntt.tran(N,bb);
        FOR(i,N) c[i]=aa[i]*bb[i]%P;
        ntt.tran(N,c,1);
    }
    void Inv(int n,ll a[],ll b[]) {

```



```

// ab=aa^-1=1 mod x^(n/2)
// (b-a^-1)^2=0 mod x^n
// bb+a^-2-2 ba^-1=0
// bba+a^-1-2b=0
// a^-1=2b-bba
static ll tmp[MAXN];
if(n == 1) { b[0]=ntt.inv(a[0],P); return; }
Inv((n+1)/2,a,b); int N=nxt2k(n*2);
copy(a,a+n,tmp); fill(tmp+n,tmp+N,0);
fill(b+n,b+N,0); ntt.tran(N,tmp); ntt.tran(N,b);
FOR(i,N) {
    ll t1=(2-b[i]*tmp[i])%P;
    if(t1<0) t1+=P;
    b[i]=b[i]*t1%P;
}
ntt.tran(N,b,1); fill(b+n,b+N,0);
}
void Div(int n,ll a[],int m,ll b[],ll d[],ll r[]){
    // Ra=Rb*Rd mod x^(n-m+1)
    // Rd=Ra*Rb^-1 mod
    static ll aa[MAXN],bb[MAXN],ta[MAXN],tb[MAXN];
    if(n<m) { copy(a,a+n,r); fill(r+n,r+m,0); return; }
    // d: n-1-(m-1)=n-m (n-m+1 terms)
    copy(a,a+n,aa); copy(b,b+m,bb);
    reverse(aa,aa+n); reverse(bb,bb+m);
    Inv(n-m+1,bb,tb); Mul(n-m+1,ta,n-m+1,tb,d);
    fill(d+n-m+1,d+n,0); reverse(d,d+n-m+1);
    // r: m-1-1=m-2 (m-1 terms)
    Mul(m,b,n-m+1,d,ta);
    FOR(i,n) { r[i]=a[i]-ta[i]; if(r[i]<0) r[i]+=P; }
}
void dx(int n,ll a[],ll b[]){
    for(int i=1;i<=n-1;i++) b[i-1]=i*a[i]%P;
}
void Sx(int n,ll a[],ll b[]){
    b[0]=0; FOR(i,n) b[i+1]=a[i]*ntt.inv(i+1,P)%P;
}
void Ln(int n,ll a[],ll b[]){
    // Integral a' a^-1 dx
    static ll a1[MAXN],a2[MAXN],b1[MAXN];
    int N=nxt2k(n*2); dx(n,a,a1); Inv(n,a,a2);
    Mul(n-1,a1,n,a2,b1); Sx(n+n-1-1,b1,b);
    fill(b+n,b+N,0);
}
void Exp(int n,ll a[],ll b[]){
    // Newton method to solve g(a(x))=ln(b(x))-a(x)=0
    // b'=b-g(b(x)) / g'(b(x))
    // b'=b (1-lnb+a)
    static ll lnb[MAXN],c[MAXN],tmp[MAXN];
    assert(a[0] == 0); // dont know exp(a[0]) mod P
    if(n == 1) { b[0]=1; return; }
    Exp((n+1)/2,a,b); fill(b+(n+1)/2,b+n,0);
    Ln(n,b,lnb); fill(c,c+n,0); c[0]=1;
    FOR(i,n) {
        c[i]=a[i]-lnb[i]; if(c[i]<0) c[i]+=P;
        if(c[i]>=P) c[i]-=P;
    }
    Mul(n,b,n,c,tmp); copy(tmp,tmp+n,b);
}
bool Sqrt(int n,ll a[],ll b[]){
    // Square root of a : b*b=a (mod x^n)
    // bb=a mod x^(n/2)
    // (bb-a)^2=0 mod x^n
    // (bb+a)^2=4 bba
    // ((bb+a) / 2b)^2=a
    // sqrt(a)=b / 2a / 2b
    static ll c[MAXN]; int ind=0,x,y,p=1;
    while(a[ind]==0) ind++;
    for(int i=0;i<n;i++) a[i]=a[i+ind];
    if((ind&1) || !dsqrt(a[0],mod,x,y)) // discrete sqrt
        return 0;
    b[0]=min(x,y);
    while(p<n) p<<=1;
    for(int t=2;t<=p;t<=1){
        Inv(t,b,c); Mul(t,a,t,c,c);
        for(int i=0;i<t;i++)
            b[i]=(b[i]+c[i])*inv(2)%mod;
    }
    if(ind){
        for(int i=p-1;i>=ind/2;i--) b[i]=b[i-ind/2];
        for(int i=0;i<ind/2;i++) b[i]=0;
        for(int i=p-1;i>=ind;i--) a[i]=a[i-ind];
    }
}

```

```

    for(int i=0;i<ind;i++) a[i]=0;
}
}
} polyop;

```

### 3.6 Poly Interpolation 6e3604

```

typedef vector<double> poly;
poly interpolate(poly x,poly y,int n){
    poly res(n),temp(n);
    for(int k=0;k<n-1;k++) for(int i=k+1;i<n;i++)
        y[i]=(y[i]-y[k])/(x[i]-x[k]);
    double last=0; temp[0]=1;
    for(int k=0;k<n;k++) for(int i=0;i<n;i++){
        res[i]+=y[k]*temp[i];
        swap(last,temp[i]); temp[i]-=last*x[k];
    }
    return res;
}

```

### 3.7 Linear Recurrence 29d614

```

// Usage: linearRec({0, 1}, {1, 1}, k) //k'th fib
typedef vector<ll> Poly;
ll linearRec(Poly&& S, Poly&& tr, ll k) {
    int n=tr.size();
    auto combine=[&](Poly& a, Poly& b) {
        Poly res(n*2+1);
        for(int i=0;i<=n;i++) for(int j=0;j<=n;j++)
            res[i+j]=(res[i+j]+a[i]*b[j])%mod;
        for(int i=2*n;i>n;--i) for(int j=0;j<n;j++)
            res[i-1-j]=(res[i-1-j]+res[i]*tr[j])%mod;
        res.resize(n+1);
        return res;
    }; // combine: a * b mod (x^n-tr)
    Poly pol(n+1), e(pol);
    pol[0]=e[1]=1;
    for(++k;k/2){
        if(k%2) pol=combine(pol,e);
        e=combine(e,e);
    }
    ll res=0;
    for(int i=0;i<n;i++) res=(res+pol[i+1]*S[i])%mod;
    return res;
}

```

### 3.8 BerlekampMassey 868031

```

// find shortest linear recurrence relation O(n^2)
// example: BM({1,1,2,3,5,8,13,21})
// 2*len terms for uniqueness
inline vector<ll> BM(const vector<ll> &x) {
    vector<ll> ls, cur; int lf; ll ld;
    for(int i=0;i<x.size();i++) {
        ll t=0;
        for(int j=0;j<cur.size();j++)
            t=(t+x[i-j-1]*cur[j])%mod;
        if((t-x[i])%mod==0) continue;
        if(!cur.size()) {
            cur.resize(i+1); lf=i; ld=(t-x[i])%mod; continue;
        }
        ll k=-(x[i]-t)*inv(ld, mod)%mod;
        vector<ll> c(i-lf-1); c.push_back(k);
        for(auto j:ls) c.push_back(-j*k%mod);
        if(c.size()<cur.size()) c.resize(cur.size());
        for(int j=0;j<cur.size();j++)c[j]=(c[j]+cur[j])%mod;
        if(i-lf+(int)ls.size()>=(int)cur.size())
            ls=cur,lf=i,ld=(t-x[i])%mod;
        cur=move(c);
    }
    for(auto& xx:cur) xx=(xx%mod+mod)%mod;
    return cur;
}

```

### 3.9 Miller Rabin 82c8fa

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pirmes <= 13
// n < 2^64              7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
bool witness(ll a,ll n,ll u,int t){

```

```

if(!(a%n)) return 0;
ll x=mypow(a,u,n);
for(int i=0;i<t;i++) {
    ll nx=mul(x,x,n);
    if(nx==1&&x!=1&&x!=n-1) return 1;
    x=nx;
}
return x!=1;
}
bool miller_rabin(ll n,int s=100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if(n<2) return 0;
    if(!(n&1)) return n == 2;
    ll u=n-1; int t=0;
    while(!(u&1)) u>>=1, t++;
    while(s--){
        ll a=randll()%(n-1)+1;
        if(witness(a,n,u,t)) return 0;
    }
    return 1;
}

```

### 3.10 Simplex c2c23c

```

/*target:
max \sum_{j=1}^n A_{0,j} * x_j
condition:
\sum_{j=1}^n A_{i,j} * x_j <= A_{i,0}  li=1~m
x_j >= 0 lj=1~n
VDB=vector<double>*/
template<class VDB>
VDB simplex(int m,int n,vector<VDB> a){
    vector<int> left(m+1),up(n+1);
    iota(left.begin(),left.end(),n);
    iota(up.begin(),up.end(),0);
    auto pivot=[&](int x,int y){
        swap(left[x],up[y]);
        auto k=a[x][y];a[x][y]=1; vector<int> pos;
        for(int j=0;j<=n;++j){
            a[x][j]/=k;
            if(a[x][j]!=0) pos.push_back(j);
        }
        for(int i=0;i<=m;++i){
            if(a[i][y]==0||i==x) continue;
            k=a[i][y],a[i][y]=0;
            for(int j:pos) a[i][j] -= k*a[x][j];
        }
    };
    for(int x,y;;){
        for(int i=x+1;i<=m;++i) if(a[i][0]<a[x][0]) x=i;
        if(a[x][0]>=0) break;
        for(int j=y+1;j<=n;++j) if(a[x][j]<a[x][y]) y=j;
        if(a[x][y]>=0) return VDB(); // infeasible
        pivot(x,y);
    }
    for(int x,y;;){
        for(int j=y+1;j<=n;++j) if(a[0][j]>a[0][y]) y=j;
        if(a[0][y]<=0) break;
        x=-1;
        for(int i=1;i<=m;++i) if(a[i][y]>0)
            if(x==-1||a[i][0]/a[i][y]<a[x][0]/a[x][y]) x=i;
        if(x==-1) return VDB(); // unbounded
        pivot(x,y);
    }
    VDB ans(n + 1);
    for(int i=1;i<=m;++i)
        if(left[i]<=n) ans[left[i]]=a[i][0];
    ans[0]=-a[0][0];
    return ans;
}

```

### 3.11 Faulhaber 862da1

```

/* faulhaber' s formula -
* cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK],inv[MAXK+1]; // bernoulli number,inverse
int cm[MAXK+1][MAXK+1]; // combinatorics
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i
inline int getinv(int x) {

```

```

int a=x,b=mod,a0=1,a1=0,b0=0,b1=1;
while(b) {
    int q,t; q=a/b; t=b; b=a-b*q; a=t;
    t=b0; b0=a0-b0*q; a0=t; t=b1; b1=a1-b1*q; a1=t;
}
return a0<0?a0+mod:a0;
}
inline void pre() {
    for(int i=0;i<=MAXK;i++) {
        cm[i][0]=cm[i][i]=1;
        for(int j=1;j<i;j++)
            cm[i][j]=add(cm[i-1][j-1],cm[i-1][j]);
    }
    for(int i=1;i<=MAXK;i++) inv[i]=getinv(i);
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2;i<=MAXK;i++) {
        if(i&1) { b[i]=0; continue; }
        b[i]=1;
        for(int j=0;j<i;j++)
            b[i]=sub(b[i],mul(cm[i][j],mul(b[j],inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1,j)*Bj*n^(p-j+1)}
    for(int i=1;i<=MAXK;i++) {
        co[i][0]=0;
        for(int j=0;j<=i;j++)
            co[i][i-j+1]=mul(inv[i+1],mul(cm[i+1][j],b[j]));
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n,int p) {
    int sol=0,m=n;
    for(int i=1;i<=p+1;i++) {
        sol=add(sol,mul(co[p][i],m)); m=mul(m, n);
    }
    return sol;
}

```

### 3.12 Chinese Remainder 94874a

```

ll crt(ll x1, ll m1, ll x2, ll m2) {
    ll g = __gcd(m1, m2); // or std::gcd
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<ll,ll> p = gcd(m1, m2);
    ll lcm = m1 * m2 * g;
    ll res=p.first*(x2-x1)%lcm*m1%lcm+x1; // overflow m^3
    return (res % lcm + lcm) % lcm;
}

```

### 3.13 Pollard Rho 6b641a

```

// does not work when n is prime
ll f(ll x,ll mod){ return add(mul(x,x,mod),1,mod); }
ll pollard_rho(ll n) {
    if(!(n&1)) return 2;
    while(true){
        ll y=2,x=rand()%(n-1)+1,res=1,tmp=1;
        for(int sz=2;res==1;sz*=2){
            for(int i=0,t=0;i<sz&&res<=1;i++,t++){
                x=f(x,n); tmp=mul(tmp,abs(x-y),n);
                if(!(t&31)||i+1==sz) res=__gcd(tmp,n),tmp=1;
            }
            y=x;
        }
        if(res!=0&&res!=n) return res;
    }
}

```

### 3.14 ax+by=gcd 5a651f

```

pair<ll,ll> gcd(ll a, ll b){
    if(b == 0) return {1, 0};
    pair<ll,ll> q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

```

### 3.15 Discrete sqrt f12a09

```

void calch(ll &t,ll &h,const ll p){
    ll tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}

```

```

}
// solve equation x^2 mod p=a where p is a prime
bool dsqrt(ll a,ll p,ll &x,ll &y){
    a%=p; if(p==2){ x=y=a; return true; }
    ll p2=p/2,tmp=mypow(a,p2,p);
    if(tmp==p-1) return false;
    if((p+1)%4==0){
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else{
        ll t,h,b,pb=0; calcH(t,h,p);
        if(t>=2){
            do{ b=rand()%(p-2)+2; }while(mypow(b,p/2,p)!=p-1);
            pb=mypow(b,h,p);
        }
        ll s=mypow(a,h/2,p);
        for(int step=2; step<=t; step++){
            ll ss=mul(mul(s,s,p),a,p);
            for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
            if(ss+1==p) s=mul(s,pb,p);
            pb=mul(pb,pb,p);
        }
        x=mul(s,a,p); y=p-x;
    }
    return true;
}

```

### 3.16 Discrete logarithm 3aac04

```

ll dlog(ll x,ll y,ll m){
    if(y==1||m==1) return 0;
    ll s=max((int)sqrt(m),1)+2,g=1;
    unordered_map<ll,ll> mp;
    for(ll i=0;i<s;i++,g=g*x%m){
        if(g==y) return i;
        mp[g*y%m]=i;
    }
    if(gcd(g,m)!=gcd(y,m)) return -1; // std::gcd
    for(ll i=1,h=g;i<s;i++,h=h*g%m){
        if(mp.count(h)) return i*s-mp[h];
    }
    return -1;
}

```

### 3.17 Romberg 6dc94c

```

// Estimates the definite integral of \int_a^b f(x) dx
template<class T>
double romberg(T& f,double a,double b,double eps=1e-8){
    vector<double>t; double h=b-a,last,curr; int k=1,i=1;
    t.push_back(h*(f(a)+f(b))/2);
    do{ last=t.back(); curr=0; double x=a+h/2;
        for(int j=0;j<k;j++) curr+=f(x), x+=h;
        curr=(t[0]+h*curr)/2; double k1=4.0/3.0,k2=1.0/3.0;
        for(int j=0;j<i;j++){ double temp=k1*curr-k2*t[j];
            t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1;
        }
        t.push_back(curr); k*=2; h/=2; i++;
    }while( fabs(last-curr)>eps);
    return t.back();
}

```

### 3.18 Simpson 3fec8d

```

template<class F>
ld quad(ld a,ld b,F f,const int n=1000) {
    ld h=(b-a)/2/n,v=f(a)+f(b);
    for(int i=1;i<n*2;++i) v+=f(a+i*h)*(i&1?4:2);
    return v*h/3;
}

```

### 3.19 Prefix Inverse 9e8ee9

```

void solve(int m){
    inv[1]=1;
    for(int i=2;i<m;i++) inv[i]=((ll)(m-m/i)*inv[m%i])%m;
}

```

### 3.20 Roots of Polynomial 20a9c9

```

const double eps=1e-12,inf=1e+12;
double a[10],x[10]; // a[0..n](coef) must be filled
int n; // degree of polynomial must be filled

```

```

int sign(double x){ return (x<-eps)?(-1):(x>eps); }
double f(double a[],int n,double x){
    double tmp=1,sum=0;
    for(int i=0;i<=n;i++) { sum=sum+a[i]*tmp; tmp=tmp*x; }
    return sum;
}
double binary(double l,double r,double a[],int n){
    int sl=sign(f(a,n,l)),sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2; int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n,double a[],double x[],int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10],dx[10]; int ndx;
    for(int i=n;i>=1;i--) da[i-1]=a[i]*i;
    solve(n-1,da,dx,ndx); nx=0;
    if(ndx==0){
        double tmp=binary(-inf,inf,a,n);
        if(tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp=binary(-inf,dx[1],a,n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1;i<=ndx-1;i++){
        tmp=binary(dx[i],dx[i+1],a,n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx],inf,a,n);
    if(tmp<inf) x[++nx]=tmp;
} // roots are stored in x[1..nx]

```

### 3.21 Sum of Division/Modular 1adbfe

```

ull sumsq(ull n){ return n/2*((n-1)|1); }
// sum i=0~n-1 floor((ki+c)/m)
ull divsum(ull n,ull k,ull c,ull m){
    ull res=k/m*sumsq(n)+c/m*n; k%=m; c%=m;
    if(!k) return res;
    ull n2=(n*k+c)/m;
    return res+(n-1)*n2-divsum(n2,m,m-1-c,k);
}
// sum i=0~n-1 (ki+c)%m
ll modsum(ull n, ll k, ll c, ll m){
    c=(c%m+m)%m; k=(k%m+m)%m;
    return n*c+k*sumsq(n)-m*divsum(n,k,c,m);
}

```

### 3.22 Fraction Binary Search 38ec70

```

//find smallest p/q in [0,1] s.t. f(p/q)=1&&p,q<=N
struct Frac{ll p,q;};
Frac fracBS(function<bool>(Frac)> f,ll N) {
    bool dir=1,A=1,B=1;
    Frac lo{0,1},hi{1,1}; // set hi to 1/0 to search (0,N]
    if(f(lo)) return lo;
    assert(f(hi));
    while(A or B){
        ll adv=0,step=1; // move hi if dir, else lo
        for(int si=0;step;(step*=2)>=si){
            adv+=step; Frac m{lo.p*adv+hi.p,lo.q*adv+hi.q};
            if(abs(m.p)>N or m.q>N or dir==!f(m))
                adv-=step,si=2;
        }
        hi.p+=lo.p*adv; hi.q+=lo.q*adv;
        dir=!dir; swap(lo,hi); A=B; B=!adv;
    }
    return dir?hi:lo;
}

```

### 3.23 Closest Fraction ef6b46

```

// x>=0, find closest p/q with p,q <= N. |p/q-x| <= 1/qN
pair<ll,ll> approximate(ld x,ll N) {
    ll LP=0,LQ=1,P=1,Q=0,inf=LLONG_MAX; ld y=x;
    for(;;){
        ll lim=min(P?(N-LP)/P:inf, Q?(N-LQ)/Q:inf),

```



```

    a=(ll)floor(y),b=min(a,lim),NP=b*P+LP,NQ=b*Q+LQ;
    if(a>b)
        return (abs(x-(ld)NP/(ld)NQ)<abs(x-(ld)P/(ld)Q))?)
        make_pair(NP,NQ):make_pair(P,Q);
    if(abs(y=1/(y-(ld)a))>3*N) return {NP,NQ};
    LP=P; P=NP; LQ=Q; Q=NQ;
}
}

```

### 3.24 Primes and $\mu$ function d79d2b

```

/* 12721, 13331, 14341, 75577, 123457, 222557, 556679
* 999983, 1097774749, 1076767633, 100102021, 999997771
* 1001010013, 1000512343, 987654361, 999991231
* 999888733, 98789101, 987777733, 999991921, 1010101333
* 1010102101, 1000000000039, 100000000000037
* 2305843009213693951, 4611686018427387847
* 9223372036854775783, 18446744073709551557 */
int mu[N],p_tbl[N]; // mobius, min prime factor
vector<int> primes;
void sieve() { // calculate multiplicative function f
    mu[1]=p_tbl[1]=1;
    for(int i=2;i<N;i++){
        if(!p_tbl[i]){
            p_tbl[i]=i; primes.push_back(i);
            mu[i]=-1; // f(i)=... where i is prime
        }
        for(int p:primes){
            int x=i*p;
            if(x>=N) break;
            p_tbl[x]=p; mu[x]=-mu[i];
            if(i%p==0){ // f(x)=f(i)/f(p^(k-1))*f(p^k)
                mu[x]=0; break;
            } // else f(x)=f(i)*f(p) where gcd(i,p)=1
        }
    }
}
vector<int> factor(int x){
    vector<int> fac{ 1 };
    while(x > 1){
        int fn=fac.size(),p=p_tbl[x],pos=0;
        while(x%p==0){
            x/=p;
            for(int i=0;i<fn;i++) fac.push_back(fac[pos++]*p);
        }
    }
    return fac;
}

```

### 3.25 Subset Convolution 84a3e0

```

// h(s)=\sum_{s'} \subseq s f(s')g(s\backslash s')
vector<int> SubsetConv(int n,const vector<int> &f,const
    vector<int> &g){
    const int m=1<<n;
    vector<vector<int>> a(n+1,vector<int>(m)),b=a;
    for(int i=0;i<m;++i){
        a[__builtin_popcount(i)][i]=f[i];
        b[__builtin_popcount(i)][i]=g[i];
    }
    for(int i=0;i<=n;++i){
        for(int j=0;j<n;++j){
            for(int s=0;s<m;++s){
                if(s>>j&1){
                    a[i][s]=a[i][s^(1<<j)];
                    b[i][s]=b[i][s^(1<<j)];
                }
            }
        }
    }
    vector<vector<int>> c(n+1,vector<int>(m));
    for(int s=0;s<m;++s){
        for(int i=0;i<=n;++i){
            for(int j=0;j<=i;++j) c[i][s]=a[j][s]*b[i-j][s];
        }
    }
    for(int i=0;i<=n;++i){
        for(int j=0;j<n;++j){
            for(int s=0;s<m;++s){
                if(s>>j&1) c[i][s]=c[i][s^(1<<j)];
            }
        }
    }
    vector<int> res(m);
    for(int i=0;i<m;++i)
        res[i]=c[__builtin_popcount(i)][i];
    return res;
}

```

### 3.26 Result fd0b69

- Lucas' Theorem :  
For  $n, m \in \mathbb{Z}^*$  and prime  $P$ ,  $C(m, n) \bmod P = \prod C(m_i, n_i)$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- 1st Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \prod_{i=0}^{n-1} (x+i)$   
 $S(n+1, k) = nS(n, k) + S(n, k-1)$
- 2nd Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
 $S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$   
 $S(n+1, k) = kS(n, k) + S(n, k-1)$
- Calculate  $f(x+n)$  where  $f(x) = \sum_{i=0}^{n-1} a_i x^i$ :  
$$f(x+n) = \sum_{i=0}^{n-1} a_i (x+n)^i = \sum_{i=0}^{n-1} x^i \cdot \frac{1}{i!} \sum_{j=i}^{n-1} \frac{a_j}{j!} \cdot \frac{n^{j-i}}{(j-i)!}$$
- Calculate  $c[i-j] + = a[i] \times b[j]$  for  $a[n], b[m]$   
1.  $a = \text{reverse}(a)$ ;  $c = \text{mul}(a, b)$ ;  $c = \text{reverse}(c[1:n])$ ;  
2.  $b = \text{reverse}(b)$ ;  $c = \text{mul}(a, b)$ ;  $c = \text{rshift}(c, m-1)$ ;
- Eulerian number(permutation  $1 \sim n$  with  $m$   $a[i] > a[i-1]$ ):  
 $A(n, m) = \sum_{i=0}^m (-1)^i \binom{n+1}{i} (m+1-i)^n$   
 $A(n, m) = (n-m)A(n-1, m-1) + (m+1)A(n-1, m)$
- Derangement:  
 $D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n$
- Pick's Theorem :  $A = i + b/2 - 1$
- Euler Characteristic:  
planar graph:  $V - E + F - C = 1$   
convex polyhedron:  $V - E + F = 2$   
 $V, E, F, C$ : number of vertices, edges, faces(regions), and components
- Kirchhoff's theorem :  
- number of spanning tree of undirected graph:  
degree matrix  $D_{ii} = \deg(i)$ ,  $D_{ij} = 0$   
adjacency matrix  $G_{ij} = \# \text{ of } (i, j) \in E$ ,  $G_{ii} = 0$ ,  
let  $A = D - G$ , delete any one row, one column, and cal  $\det(A')$   
- number of spanning tree of directed graph:  
in-degree matrix  $D_{ii}^{\text{in}} = \text{indeg}(i)$ ,  $D_{ij}^{\text{in}} = 0$   
out-degree matrix  $D_{ii}^{\text{out}} = \text{outdeg}(i)$ ,  $D_{ij}^{\text{out}} = 0$   
let  $L^{\text{in}} = D^{\text{in}} - G$ ,  $L^{\text{out}} = D^{\text{out}} - G$ , delete the  $i$ -th row and column  
 $\det(L_i^{\text{in}})$  and  $\det(L_i^{\text{out}})$  is the number of spanning tree from/to root  $i$
- Tutte Matrix:  
A graph  $G = (V, E)$  has a perfect matching iff  $\det(A) \neq 0$  where  
 $A_{ij} = ((i, j) \in E ? (i < j ? x_{ij} : -x_{ji}) : 0)$  and  $x_{ij}$  are random numbers.
- Erdős-Gallai theorem:  
There exists a simple graph with degree sequence  $d_1 \geq \dots \geq d_n$  iff  
 $\sum_{i=1}^n d_i$  is even and  $\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k)$ ,  $\forall 1 \leq k \leq n$
- Burnside Lemma:  $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- Polya theorem:  $|Y^x/G| = \frac{1}{|G|} \sum_{g \in G} m^{c(g)}$   
 $m = |Y|$ : num of colors,  $c(g)$ : num of cycle
- Prüfer Sequence:  
There is a bijection between the set of labeled trees with  $n$  vertices and the set of sequences of length  $n-2$  containing  $1 \sim n$ .  
Property: Each vertex  $i$  exists  $d_i - 1$  times in the sequence.  
Tree to sequence: iterate  $n-2$  times to remove a leaf with smallest id and append its adjacent vertex's id to the end of the sequence.  
Sequence to tree: iterate through  $i = 1 \sim n-2$  and connect  $a_i$  with the smallest id that doesn't exist in  $a_{i+1}, \dots, a_{n-2}$  and haven't been used yet. Also connect the remaining two unused vertices at last.
- Anti SG (the person who has no strategy wins) :  
first player wins iff either  
1. SG value of ALL subgame  $\leq 1$  and SG value of the game  $= 0$   
2. SG value of some subgame  $> 1$  and SG value of the game  $\neq 0$
- Möbius inversion formula :  
 $g(n) = \sum_{d|n} f(d)$  for every integer  $n \geq 1$ , then  
 $f(n) = \sum_{d|n} \mu(d)g(\frac{n}{d}) = \sum_{d|n} \mu(\frac{n}{d})g(d)$  for every integer  $n \geq 1$   
Dirichlet convolution :  $f * g = g * f = \sum_{d|n} f(d)g(\frac{n}{d}) = \sum_{d|n} f(\frac{n}{d})g(d)$   
 $g = f * 1 \Leftrightarrow f = g * \mu$ ,  $\epsilon = \mu * 1$ ,  $Id = \phi * 1$ ,  $d = 1 * 1$ ,  $\sigma = Id * 1 = \phi * d$ ,  
 $\sigma_k = Id_k * 1$  where  $\epsilon(n) = [n=1]$ ,  $1(n) = 1$ ,  $Id(n) = n$ ,  $Id_k(n) = n^k$ ,  
 $d(n) = \#(\text{divisor})$ ,  $\sigma(n) = \sum \text{divisor}$ ,  $\sigma_k(n) = \sum \text{divisor}^k$
- Find a Primitive Root of  $n$ :  
 $n$  has primitive roots iff  $n = 2, 4, p^k, 2p^k$  where  $p$  is an odd prime.  
1. Find  $\phi(n)$  and all prime factors of  $\phi(n)$ , says  $P = \{p_1, \dots, p_m\}$   
2.  $\forall g \in [2, n]$ , if  $g^{\frac{\phi(n)}{p_i}} \neq 1, \forall p_i \in P$ , then  $g$  is a primitive root.  
3. Since the smallest one isn't too big, the algorithm runs fast.  
4.  $n$  has exactly  $\phi(\phi(n))$  primitive roots.

- Sum of Two Squares Thm (Legendre):  
For a given positive integer  $N$ , let  
 $D1 = (\# \text{ of } d \in N \text{ dividing } N \text{ that } d \equiv 1 \pmod{4})$   
 $D3 = (\# \text{ of } d \in N \text{ dividing } N \text{ that } d \equiv 3 \pmod{4})$   
then  $N$  can be written as a sum of two squares in exactly  $R(N) = 4(D1 - D3)$  ways.
- Difference of  $D1 - D3$  Thm:  
let  $N = 2^t \times [p_1^{e_1} \times \dots \times p_r^{e_r}] \times [q_1^{f_1} \times \dots \times q_s^{f_s}]$   
where  $p_i \in \text{mod } 4 = 1 \text{ prime}$ ,  $q_i \in \text{mod } 4 = 3 \text{ prime}$   
then  $D1 - D3 = \begin{cases} (e_1 + 1)(e_2 + 1) \dots (e_r + 1) & \text{if } f_i \text{ all even} \\ 0 & \text{if any } f_i \text{ is odd} \end{cases}$
- Sherman-Morrison formula:  
suppose  $A \in \mathbb{R}^{n \times n}$  is invertible and  $u, v \in \mathbb{R}^n$   
 $A + uv^T$  is invertible if and only if  $1 + v^T A^{-1} u \neq 0$   
 $(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}$
- Pohlig-Hellman algorithm (discrete log):  
Given an order  $n$  group, generator  $g$ , element  $h$ , find  $x$  s.t.  $g^x = h$ .  
If  $n = p^e$ :  
\* let  $x_0 = 0$ ,  $\gamma = g^{p^{e-1}}$  where  $\gamma$  has order  $p$ .  
\* for  $k = 0 \sim e - 1$ :  
1. let  $h_k = (g^{-x_k} h)^{p^{e-1-k}}$  whose order divide  $p \implies h_k \in \langle \gamma \rangle$ .  
2. find  $d_k$  s.t.  $\gamma^{d_k} = h_k$  with baby-step giant-step in  $O(\sqrt{p})$ .  
3. set  $x_{k+1} = x_k + p^k d_k$   
\* return  $x_e$  in total time complexity  $O(e\sqrt{p})$   
If  $n = \prod_{i=1}^r p_i^{e_i}$ :  
\* for each  $i = 1 \sim r$ :  
1. let  $g_i = g^{n/p_i^{e_i}}$  having order  $p_i^{e_i}$ ,  $h_i = h^{n/p_i^{e_i}}$  where  $h_i \in \langle g_i \rangle$ .  
2. find  $x_i$  s.t.  $g_i^{x_i} = h_i$  using above algorithm.  
\* return  $x = CRT(\{x_i \bmod p_i^{e_i}\})$

## 4 Geometry

### 4.1 Intersection of 2 lines 3db65e

```
Pt LLIntersect(Line a, Line b) {
    Pt p1 = a.s, p2 = a.e, q1 = b.s, q2 = b.e;
    ld f1 = (p2-p1)^(q1-p1), f2 = (p2-p1)^(p1-q2), f;
    if(dcmp(f=f1+f2) == 0)
        return dcmp(f1)?Pt(NAN,NAN):Pt(INFINITY,INFINITY);
    return q1*(f2/f) + q2*(f1/f);
}
```

### 4.2 halfPlaneIntersection 1d54e0

```
// for point or line solution, change > to >=
bool onleft(Line L, Pt p) {
    return dcmp(L.v^(p-L.s)) > 0;
}
// assume that lines intersect
vector<Pt> HPI(vector<Line>& L) {
    sort(L.begin(), L.end()); // sort by angle
    int n=L.size(), fir, las; Pt *p=new Pt[n];
    vector<Line> q(n); q[fir=las=0]=L[0];
    for(int i=1; i<n; i++) {
        while(fir<las&&!onleft(L[i], p[las-1])) las--;
        while(fir<las&&!onleft(L[i], p[fir])) fir++;
        q[++las]=L[i];
        if(dcmp(q[las].v^q[las-1].v) == 0) {
            las--;
            if(onleft(q[las], L[i].s)) q[las]=L[i];
        }
        if(fir<las) p[las-1]=LLIntersect(q[las-1], q[las]);
    }
    while(fir<las&&!onleft(q[fir], p[las-1])) las--;
    if(las-fir<=1) return {};
    p[las]=LLIntersect(q[las], q[fir]);
    int m=0; vector<Pt> ans(las-fir+1);
    for(int i=fir; i<=las; i++) ans[m++] = p[i];
    return ans;
}
```

### 4.3 Intersection of 2 segments b7e393

```
bool onseg(Pt p, Line L) {
    Pt x = L.s-p, y = L.e-p;
    return dcmp(x^y) == 0 && dcmp(x*y) <= 0;
} // inseg: dcmp(x^y)==0&&dcmp(x*y)<0
// assume a.s != a.e != b.s != b.e
Pt SSIntersect(Line a, Line b) {
    Pt p = LLIntersect(a, b);
```

```
if(isinf(p.x)&&(onseg(a.s,b)||onseg(a.e,b)||onseg(b.s,
    a)||onseg(b.e,a))) return p; // overlap
if(isfinite(p.x)&&onseg(p,a)&&onseg(p,b)) return p;
return {NAN,NAN}; // non-intersect
}
```

### 4.4 Banana de5c4e

```
int ori(const Pt& o, const Pt& a, const Pt& b){
    ll ret=(a-o)^(b-o);
    return (ret>0)-(ret<0);
}
// p1==p2||q1==q2 need to be handled
bool banana(const Pt& p1, const Pt& p2,
    const Pt& q1, const Pt& q2){
    if(((p2-p1)^(q2-q1))==0){ // parallel
        if(ori(p1,p2,q1)) return false;
        return ((p1-q1)*(p2-q1))<=0||((p1-q2)*(p2-q2))<=0||
            ((q1-p1)*(q2-p1))<=0||((q1-p2)*(q2-p2))<=0;
    }
    return (ori(p1,p2,q1)*ori(p1,p2,q2)<=0)&&
        (ori(q1,q2,p1)*ori(q1,q2,p2)<=0);
}
```

### 4.5 Intersection of circle and line 73c7f5

```
vector<Pt> CLInter(const Line &a, const Circle &c){
    Pt p=a.s+(c.o-a.s)*a.v/norm2(a.v)*a.v;
    ld d=c.r*c.r-norm2(c.o-p);
    if(d<-eps) return {};
    if(d<eps) return {p};
    Pt v=a.v/norm(a.v)*sqrt(d);
    return {p+v,p-v};
}
```

### 4.6 Intersection of polygon and circle 868772

```
ld PCIntersect(vector<Pt> v, Circle cir) {
    for(int i=0; i<(int)v.size(); ++i) v[i]=v[i]-cir.o;
    ld ans=0, r=cir.r; int n=v.size();
    for(int i=0; i<n; ++i) {
        Pt pa=v[i], pb=v[(i+1)%n];
        if(norm(pa)<norm(pb)) swap(pa,pb);
        if(dcmp(norm(pb))==0) continue;
        ld s,h,theta,a=norm(pb),b=norm(pa),c=norm(pb-pa);
        ld cosB=(pb*(pb-pa))/a/c, B=acos(cosB);
        if(cosB>1) B=0; else if(cosB<-1) B=PI;
        ld cosC=(pa*pb)/a/b, C=acos(cosC);
        if(cosC>1) C=0; else if(cosC<-1) C=PI;
        if(a>r){
            s=(C/2)*r*r; h=a*b*sin(C)/c;
            if(h<r&&B<PI/2) s-=acos(h/r)*r*r-h*sqrt(r*r-h*h);
        }
        else if(b>r) {
            theta=PI-B-asin(sin(B)/r*a);
            s=0.5*a*r*sin(theta)+(C-theta)/2*r*r;
        }
        else s=0.5*sin(C)*a*b;
        ans+=abs(s)*dcmp(v[i]^v[(i+1)%n]);
    }
    return abs(ans);
}
```

### 4.7 Intersection of 2 circles 57edf8

```
vector<Pt> CCinter(Circle& a, Circle& b){
    Pt o1=a.o, o2=b.o; ld r1=a.r, r2=b.r;
    if(norm(o1-o2)>r1+r2) return {};
    if(norm(o1-o2)<max(r1,r2)-min(r1,r2)) return {};
    ld d2=(o1-o2)*(o1-o2), d=sqrt(d2);
    if(d>r1+r2) return {};
    Pt u=(o1+o2)*0.5+(o1-o2)*((r2*r2-r1*r1)/(2*d2));
    ld A=sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
    Pt v=Pt(o1.y-o2.y, -o1.x+o2.x)*A/(2*d2);
    return {u+v,u-v};
}
```

## 4.8 Circle cover 4b4772

```
#define N 1021
struct CircleCover{
    // overlap
    int C; Circle c[N]; bool g[N][N], over[N][N];
    // Area[i] : area covered by at least i circles
    ld Area[N];
    void init(int _C){ C=_C; }
    struct Teve {
        Pt p; ld ang; int add;
        Teve() {}
        Teve(Pt _a, ld _b, int _c): p(_a), ang(_b), add(_c) {}
        bool operator<(const Teve &a) const {
            return ang<a.ang; }
    } eve[N*2];
    // strict: x=0, otherwise x=-1
    bool disjunct(Circle& a, Circle& b, int x) {
        return sign(norm(a.o-b.o)-a.r-b.r)>x; }
    bool contain(Circle& a, Circle& b, int x) {
        return sign(a.r-b.r-norm(a.o-b.o)>x; }
    bool contain(int i, int j){
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].r-c[j].r)>0 || (sign(c[i].r-c[j].r)
            ==0 && i<j)) && contain(c[i], c[j], -1);
    }
    void solve(){
        for(int i=0; i<=C+1; i++) Area[i]=0;
        for(int i=0; i<C; i++) for(int j=0; j<C; j++)
            over[i][j]=contain(i, j);
        for(int i=0; i<C; i++) for(int j=0; j<C; j++)
            g[i][j]=!(over[i][j] || over[j][i] || disjunct(c[i],
                c[j], -1));
        for(int i=0; i<C; i++){
            int E=0, cnt=1;
            for(int j=0; j<C; j++) if(j!=i && over[j][i]) cnt++;
            for(int j=0; j<C; j++)
                if(i!=j && g[i][j]){
                    vector<Pt> v=CCinter(c[i], c[j]);
                    ld A=atan2(v[0].y-c[i].o.y, v[0].x-c[i].o.x);
                    ld B=atan2(v[1].y-c[i].o.y, v[1].x-c[i].o.x);
                    eve[E++]=Teve(v[1], B, 1);
                    eve[E++]=Teve(v[0], A, -1);
                    if(B>A) cnt++;
                }
            if(E==0) Area[cnt]+=pi*c[i].r*c[i].r;
            else{
                sort(eve, eve+E); eve[E]=eve[0];
                for(int j=0; j<E; j++){
                    cnt+=eve[j].add;
                    Area[cnt]+=(eve[j].p^eve[j+1].p)*.5;
                    ld theta=eve[j+1].ang-eve[j].ang;
                    if(theta<0) theta+=2.*pi;
                    Area[cnt]+=(theta-sin(theta))*c[i].r*c[i].r/2;
                }
            }
        }
    }
} } } }
```

## 4.9 Li Chao Segment Tree 01810b

```
struct LiChao_min{
    struct line{
        ll m, c;
        line(ll _m=0, ll _c=0){ m=_m; c=_c; }
        ll eval(ll x){ return m*x+c; } // overflow
    };
    struct node{
        node *l, *r; line f;
        node(line v){ f=v; l=r=NULL; }
    };
    typedef node* pnode;
    pnode root; ll sz, ql, qr;
    #define mid ((l+r)>>1)
    void insert(line v, ll l, ll r, pnode &nd){
        /* if(ql<=l && r<=qr){
            if(!nd) nd=new node(line(0, INF));
            if(ql<=mid) insert(v, l, mid, nd->l);
            if(qr>mid) insert(v, mid+1, r, nd->r);
            return;
        } */
        // used for adding segment */
        if(!nd){ nd=new node(v); return; }
        ll trl=nd->f.eval(l), trr=nd->f.eval(r);
        ll vl=v.eval(l), vr=v.eval(r);
        if(trl<=vl && trr<=vr) return;
        if(trl>vl && trr>vr) { nd->f=v; return; }
```

```
        if(trl>vl) swap(nd->f, v);
        if(nd->f.eval(mid)<v.eval(mid))
            insert(v, mid+1, r, nd->r);
        else swap(nd->f, v), insert(v, l, mid, nd->l);
    }
    ll query(ll x, ll l, ll r, pnode &nd){
        if(!nd) return INF;
        if(l==r) return nd->f.eval(x);
        if(mid>=x)
            return min(nd->f.eval(x), query(x, l, mid, nd->l));
        return min(nd->f.eval(x), query(x, mid+1, r, nd->r));
    }
    /* -sz<=ll query_x<=sz */
    void init(ll _sz){ sz=_sz+1; root=NULL; }
    void add_line(ll m, ll c, ll l=-INF, ll r=INF){
        line v(m, c); ql=l; qr=r; insert(v, -sz, sz, root);
    }
    ll query(ll x) { return query(x, -sz, sz, root); }
};
```

## 4.10 Convex Hull trick 66a3a1

```
/* Given a convex hull, answer queries in O(lg N)
CH should not contain identical points, the area should
be>0, min pair(x, y) should be listed first */
double det(const Pt& p1, const Pt& p2)
{ return p1.x*p2.y-p1.y*p2.x; }
struct Conv{
    int n; vector<Pt> a, upper, lower;
    Conv(vector<Pt> _a): a(_a){
        n=a.size(); int ptr=0;
        for(int i=1; i<n; ++i) if(a[ptr]<a[i]) ptr=i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    } // sign: modify when changing to double
    int sign(ll x){ return x<0?-1:x>0; }
    pair<ll, int> get_tang(vector<Pt> &conv, Pt vec){
        int l=0, r=(int)conv.size()-2;
        while(l+1<r){
            int mid=(l+r)/2;
            if(sign(det(conv[mid+1]-conv[mid], vec))>0) r=mid;
            else l=mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0]-p, a[id]-p)>0) i0=id;
        if(det(a[i1]-p, a[id]-p)<0) i1=id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l==r) return;
        upd_tang(p, l, i0, i1);
        int sl=sign(det(a[l%n]-p, a[(l+1)%n]-p));
        while(l+1<r){
            int mid=(l+r)/2;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if(smid==sl) l=mid; else r=mid;
        }
        upd_tang(p, r, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r){
        int sl=sign(det(v-u, a[l%n]-u));
        while(l+1<r){
            int mid=(l+r)/2, smid=sign(det(v-u, a[mid%n]-u));
            if(smid==sl) l=mid; else r=mid;
        }
        return l%n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p){
        if(p.x<lower[0].x || p.x>lower.back().x) return 0;
        int id=lower_bound(lower.begin(), lower.end(), Pt(p.x,
            -INF))-lower.begin();
        if(lower[id].x==p.x){
            if(lower[id].y>p.y) return 0;
        } else if(det(lower[id-1]-p, lower[id]-p)<0) return 0;
        id=lower_bound(upper.begin(), upper.end(), Pt(p.x, INF),
            greater<Pt>())-upper.begin();
        if(upper[id].x==p.x){
            if(upper[id].y<p.y) return 0;
```

```

    }else if(det(upper[id-1]-p,upper[id]-p)<0) return 0;
    return 1;
}
// 2. Find 2 tang pts on CH of a given outside point
// return true with i0,i1 as index of tangent points
// return false if inside CH
bool get_tang(Pt p,int &i0,int &i1){
    if(contain(p)) return false;
    i0=i1=0;
    int id=lower_bound(lower.begin(),lower.end(),p)-
        lower.begin();
    bi_search(0,id,p,i0,i1);
    bi_search(id,(int)lower.size(),p,i0,i1);
    id=lower_bound(upper.begin(),upper.end(),p,greater<
        Pt>())-upper.begin();
    bi_search((int)lower.size()-1,(int)lower.size()-1+id
        ,p,i0,i1);
    bi_search((int)lower.size()-1+id,(int)lower.size()
        -1+(int)upper.size(),p,i0,i1);
    return true;
}
// 3. Find tangent points of a given vector
// ret the idx of vertex has max cross value with vec
int get_tang(Pt vec){
    pair<ll,int> ret=get_tang(upper,vec);
    ret.second=(ret.second+(int)lower.size()-1)%n;
    ret=max(ret,get_tang(lower,vec));
    return ret.second;
}
// 4. Find intersection point of a given line
// return 1 and intersection is on edge (i,next(i))
// return 0 if no strictly intersection
bool get_intersection(Pt u,Pt v,int &i0,int &i1){
    int p0=get_tang(u-v),p1=get_tang(v-u);
    if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u)<0){
        if(p0>p1) swap(p0,p1);
        i0=bi_search(u,v,p0,p1); i1=bi_search(u,v,p1,p0+n);
        return 1;
    }
    return 0;
}
};

```

#### 4.11 Rotating Sweep Line 3971d4

```

void rotatingSweepLine(vector<Pt> &ps){
    int n=ps.size(); vector<int> id(n),pos(n);
    vector<pair<int,int>> line(n*(n-1)/2); int m=0;
    for(int i=0;i<n;++i)
        for(int j=i+1;j<n;++j) line[m++]=make_pair(i,j);
    sort(line.begin(),line.end(),[&](const pair<int,int> &a,
        const pair<int,int> &b)->bool{
        if(ps[a.first].x==ps[a.second].y) return 0;
        if(ps[b.first].x==ps[b.second].y) return 1;
        return (double)(ps[a.first].y-ps[a.second].y)/(ps[a.
            first].x-ps[a.second].x)<(double)(ps[b.first].y-
            ps[b.second].y)/(ps[b.first].x-ps[b.second].x);
    }); // change to use multiply for better precision
    for(int i=0;i<n;++i) id[i]=i;
    sort(id.begin(),id.end(),[&](const int &a,const int &b
        ){ return ps[a]<ps[b]; }); // tie(x,y)
    for(int i=0;i<n;++i) pos[id[i]]=i;
    for(int i=0;i<m;++i){ pair<int,int> l=line[i];
        // do something: Line(ps[l.first],ps[l.second]);
        // id: sorted id of ps w.r.t dis(l,p) (dis<0 for
        // points in -y direction); pos[id[i]]=i;
        tie(pos[l.first],pos[l.second],id[pos[l.first]],id[
            pos[l.second]])=make_tuple(pos[l.second],pos[l.
            first],l.second,l.first);
    }
}

```

#### 4.12 Tangent line of two circles a45324

```

vector<Line> go(const Circle& c1,const Circle& c2,int
    sign1){
    // sign1=1 for outer tang,-1 for inter tang
    vector<Line> ret;
    double d_sq=norm2(c1.o-c2.o);
    if(d_sq<eps) return ret;
    double d=sqrt(d_sq); Pt v=(c2.o-c1.o)/d;
    double c=(c1.r-sign1*c2.r)/d;

```

```

    if(c*c>1) return ret;
    double h=sqrt(max(0.0,1.0-c*c));
    for(int sign2=1;sign2>=-1;sign2-=2){
        Pt n={v.x*c-sign2*h*v.y, v.y*c+sign2*h*v.x};
        Pt p1=c1.o+n*c1.r,p2=c2.o+n*(c2.r*sign1);
        if(fabs(p1.x-p2.x)<eps and fabs(p1.y-p2.y)<eps)
            p2=p1+perp(c2.o-c1.o);
        ret.push_back({p1,p2});
    }
    return ret;
}

```

#### 4.13 Tangent line of point and circle 35a7bf

```

vector<Line> PCTangent(const Circle& C,const Pt& P){
    vector<Line> ans; Pt u=C.o-P; double dist=norm(u);
    if(dist<C.r) return ans;
    else if(abs(dist)<eps){
        ans.push_back({P,P+rotate(u,M_PI/2)});
        return ans;
    }
    else{
        double ang=asin(C.r/dist);
        ans.push_back({P,P+rotate(u,-ang)});
        ans.push_back({P,P+rotate(u,+ang)});
        return ans;
    }
}

```

#### 4.14 Min distance of two convex 55abcb

```

double TwoConvexHullMinDis(Pt P[],Pt Q[],int n,int m){
    int mn=0,mx=0; double tmp,ans=1e9;
    for(int i=0;i<n;++i) if(P[i].y<P[mn].y) mn=i;
    for(int i=0;i<m;++i) if(Q[i].y>Q[mx].y) mx=i;
    P[n]=P[0]; Q[m]=Q[0];
    for (int i=0;i<n;++i) {
        while(tmp=((Q[mx+1]-P[mn+1])^(P[mn]-P[mn+1]))>((Q[mx]
            -P[mn+1])^(P[mn]-P[mn+1]))) mx=(mx+1)%m;
        if(tmp<0) // pt to segment distance
            ans=min(ans,dis(Line(P[mn],P[mn+1]),Q[mx]));
        else // segment to segment distance
            ans=min(ans,dis(Line(P[mn],P[mn+1]),Line(Q[mx],Q[
                mx+1])));
        mn=(mn+1)%n;
    }
    return ans;
}

```

#### 4.15 Poly Union 7a6b24

```

struct PY{
    int n; Pt pt[5]; double area;
    Pt& operator[](const int x){ return pt[x]; }
    void init(){ //n,pt[0~n-1] must be filled
        area=pt[n-1]^pt[0];
        for(int i=0;i<n-1;i++) area+=pt[i]^pt[i+1];
        if((area/=2)<0)reverse(pt,pt+n),area=-area;
    }
};
PY py[500]; pair<double,int> c[5000];
inline double segP(Pt &p,Pt &p1,Pt &p2){
    if(dcmp(p1.x-p2.x)==0) return (p.y-p1.y)/(p2.y-p1.y);
    return (p.x-p1.x)/(p2.x-p1.x);
}
double polyUnion(int n){ //py[0~n-1] must be filled
    int i,j,ii,jj,ta,tb,r,d; double z,w,s,sum=0,tc,td;
    for(i=0;i<n;i++) py[i][py[i].n]=py[i][0];
    for(i=0;i<n;i++){
        for(ii=0;ii<py[i].n;ii++){
            r=0;
            c[r++]=make_pair(0.0,0); c[r++]=make_pair(1.0,0);
            for(j=0;j<n;j++){
                if(i==j) continue;
                for(jj=0;jj<py[j].n;jj++){
                    ta=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj]));
                    tb=dcmp(tri(py[i][ii],py[i][ii+1],py[j][jj+1]));
                );
                if(ta==0 && tb==0){
                    if((py[j][jj+1]-py[j][jj])*(py[i][ii+1]-py[i][
                        ii]))>0&&j<i){

```



```

        c[r++]=make_pair(segP(py[j][jj],py[i][ii],
        py[i][ii+1]),1);
        c[r++]=make_pair(segP(py[j][jj+1],py[i][ii
        ],py[i][ii+1]),-1);
    }
    }else if(ta>=0 && tb<0){
        tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
        td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
        c[r++]=make_pair(tc/(tc+td),1);
    }else if(ta<0 && tb>=0){
        tc=tri(py[j][jj],py[j][jj+1],py[i][ii]);
        td=tri(py[j][jj],py[j][jj+1],py[i][ii+1]);
        c[r++]=make_pair(tc/(tc+td),-1);
    } } }
    sort(c,c+r);
    z=min(max(c[0].first,0.0),1.0); d=c[0].second; s
    =0;
    for(j=1;j<r;j++){
        w=min(max(c[j].first,0.0),1.0);
        if(!d) s+=w-z;
        d+=c[j].second; z=w;
    }
    sum+=(py[i][ii]^py[i][ii+1])*s;
}
}
return sum/2;
}

```

#### 4.16 Lower Concave Hull dd665b

```

const ll is_query=-(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b!=is_query) return m<rhs.m;
        const Line* s=succ();
        return s?b-s->b<(s->m-m)*rhs.m:0;
    }
}; // maintain upper hull for maximum
struct HullDynamic:public multiset<Line> {
    bool bad(iterator y) {
        auto z=next(y);
        if (y==begin()) {
            if (z==end()) return 0;
            return y->m==z->m&&y->b<=z->b;
        }
        auto x=prev(y);
        if (z==end()) return y->m==x->m&&y->b<=x->b;
        return
            (x->b-y->b)*(z->m-y->m)>=(y->b-z->b)*(y->m-x->m);
    }
    void insert_line(ll m, ll b) {
        auto y=insert({m, b});
        y->succ=[=]{ return next(y)==end()?0:&*next(y); };
        if (bad(y)) { erase(y); return; }
        while(next(y)!=end()&&bad(next(y))) erase(next(y));
        while(y!=begin()&&bad(prev(y))) erase(prev(y));
    }
    ll eval(ll x) {
        auto l=*lower_bound((Line) { x, is_query });
        return l.m*x + l.b;
    }
};

```

#### 4.17 Delaunay Triangulation 8ee9b3

**/\* Delaunay Triangulation:**  
 Given a sets of points on 2D plane, find a triangulation such that no points will strictly inside circumcircle of any triangle.

find: return a triangle contain given point  
 add\_point: add a point into triangulation

A Triangle is in triangulation iff. its has\_chd is 0.  
 Region of triangle u: iterate each u.edge[i].tri,  
 each points are u.p[(i+1)%3], u.p[(i+2)%3]

Voronoi diagram: for each triangle in triangulation,  
 the bisector of all its edges will split the region.  
 nearest point will belong to the triangle containing it

```

/*
typedef double T; // T is integer: eps=0
const int N=100000+5; const T inf=1e9, eps=1e-8;
T sqr(T x) { return x*x; }
// return p4 is in circumcircle of tri(p1,p2,p3)
bool in_cc(const Pt& p1, const Pt& p2, const Pt& p3,
const Pt& p4){
    T u11=p1.x-p4.x; T u21=p2.x-p4.x; T u31=p3.x-p4.x;
    T u12=p1.y-p4.y; T u22=p2.y-p4.y; T u32=p3.y-p4.y;
    T u13=sqr(p1.x)-sqr(p4.x)+sqr(p1.y)-sqr(p4.y);
    T u23=sqr(p2.x)-sqr(p4.x)+sqr(p2.y)-sqr(p4.y);
    T u33=sqr(p3.x)-sqr(p4.x)+sqr(p3.y)-sqr(p4.y);
    T det=-u13*u22*u31+u12*u23*u31+u13*u21*u32
        -u11*u23*u32-u12*u21*u33+u11*u22*u33;
    return det > eps;
}
T side(const Pt& a, const Pt& b, const Pt& p)
{ return (b-a)^(p-a); }
typedef int SdRef; struct Tri; typedef Tri* TriRef;
struct Edge {
    TriRef tri; SdRef side;
    Edge():tri(0), side(0){}
    Edge(TriRef _tri, SdRef _side):tri(_tri), side(_side)
    {}
};
struct Tri {
    Pt p[3]; Edge edge[3]; TriRef chd[3];
    Tri() {}
    Tri(const Pt& p0, const Pt& p1, const Pt& p2) {
        p[0]=p0; p[1]=p1; p[2]=p2; chd[0]=chd[1]=chd[2]=0;
    }
    bool has_chd() const { return chd[0]!=0; }
    int num_chd() const {
        return chd[0]==0?(chd[1]==0?1:chd[2]==0?2:3);
    }
    bool contains(Pt const& q) const {
        for(int i=0;i<3;i++){
            if(side(p[i],p[(i+1)%3],q)<-eps) return false;
            return true;
        }
    }
} pool[N*10], *tris;
void edge(Edge a, Edge b){
    if(a.tri) a.tri->edge[a.side]=b;
    if(b.tri) b.tri->edge[b.side]=a;
}
struct Trig { // Triangulation
    void init() { // Tri should at least contain all points
        the_root=new(tris++)Tri(Pt(-inf,-inf),Pt(+inf+inf,-
        inf),Pt(-inf,+inf+inf));
    }
    TriRef find(Pt p)const{ return find(the_root,p); }
    void add_point(const Pt& p)
    { add_point(find(the_root,p),p); }
    TriRef the_root;
    static TriRef find(TriRef root, const Pt& p) {
        while(true){
            if(!root->has_chd()) return root;
            for(int i=0;i<3&&root->chd[i];++i)
                if (root->chd[i]->contains(p)) {
                    root=root->chd[i]; break;
                }
        }
        assert(false); // "point not found"
    }
    void add_point(TriRef root, Pt const& p) {
        TriRef tab,tbc,tca; // split it into three triangles
        tab=new(tris++) Tri(root->p[0],root->p[1],p);
        tbc=new(tris++) Tri(root->p[1],root->p[2],p);
        tca=new(tris++) Tri(root->p[2],root->p[0],p);
        edge(Edge(tab,0), Edge(tbc,1));
        edge(Edge(tbc,0), Edge(tca,1));
        edge(Edge(tca,0), Edge(tab,1));
        edge(Edge(tab,2), root->edge[2]);
        edge(Edge(tbc,2), root->edge[0]);
        edge(Edge(tca,2), root->edge[1]);
        root->chd[0]=tab;root->chd[1]=tbc;root->chd[2]=tca;
        flip(tab,2); flip(tbc,2); flip(tca,2);
    }
    void flip(TriRef tri, SdRef pi) {
        TriRef trj=tri->edge[pi].tri; if (!trj) return;
        int pj=tri->edge[pi].side;
    }
}

```



```

    if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[pj])
        ) return;
    /* flip edge between tri, trj */
    TriRef trk=new(tris++) Tri(tri->p[(pi+1)%3], trj->p[
        pj], tri->p[pi]);
    TriRef trl=new(tris++) Tri(trj->p[(pj+1)%3], tri->p[
        pi], trj->p[pj]);
    edge(Edge(trk,0), Edge(trl,0));
    edge(Edge(trk,1), tri->edge[(pi+2)%3]);
    edge(Edge(trk,2), trj->edge[(pj+1)%3]);
    edge(Edge(trl,1), trj->edge[(pj+2)%3]);
    edge(Edge(trl,2), tri->edge[(pi+1)%3]);
    tri->chd[0]=trk; tri->chd[1]=trl; tri->chd[2]=0;
    trj->chd[0]=trk; trj->chd[1]=trl; trj->chd[2]=0;
    flip(trk,1); flip(trk,2); flip(trl,1); flip(trl,2);
}
}tri; // the triangulation structure
vector<TriRef> triang; // vector of all triangle
set<TriRef> vst;
void go(TriRef now){ // store all tri into triang
    if(vst.find(now)!=vst.end()) return;
    vst.insert(now);
    if(!now->has_chd()){
        triang.push_back(now); return;
    }
    for(int i=0; i<now->num_chd(); i++) go(now->chd[i]);
}
void build(int n, Pt* ps){ // build triangulation
    tris=pool; triang.clear(); vst.clear();
    random_shuffle(ps, ps+n); tri.init();
    for(int i=0; i<n; ++i) tri.add_point(ps[i]);
    go(tri.the_root());
}

```

#### 4.18 Min Enclosing Circle 0de93f

```

struct Mec{ // return pair of center and r
    static const int N=101010;
    int n; Pt p[N], cen; double r2;
    void init(int _n, Pt _p[]){
        n=_n; memcpy(p, _p, sizeof(Pt)*n);
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2){
        Pt a=p1-p0, b=p2-p0;
        double c1=norm2(a)*0.5, c2=norm2(b)*0.5, d=a^b;
        double x=p0.x+(c1*b.y-c2*a.y)/d;
        double y=p0.y+(a.x*c2-b.x*c1)/d;
        return Pt(x,y);
    }
    pair<Pt, double> solve(){ // expected O(n)
        random_shuffle(p, p+n); r2=0;
        for (int i=0; i<n; i++){
            if (norm2(cen-p[i])<=r2) continue;
            cen=p[i]; r2=0;
            for (int j=0; j<i; j++){
                if (norm2(cen-p[j])<=r2) continue;
                cen=Pt((p[i].x+p[j].x)/2, (p[i].y+p[j].y)/2);
                r2=norm2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (norm2(cen-p[k])<=r2) continue;
                    cen=center(p[i], p[j], p[k]); r2=norm2(cen-p[k]);
                }
            }
            return {cen, sqrt(r2)};
        }
    }
}mec;

```

#### 4.19 Min Enclosing Ball 64186c

```

// Pt:{x,y,z}
#define N 202020
int n, nouter; Pt pt[N], outer[4], res; double radius, tmp;
double det(double m[3][3]){
    return m[0][0]*m[1][1]*m[2][2]+m[0][1]*m[1][2]*m[2][0]
        +m[0][2]*m[1][0]*m[2][1]-m[0][2]*m[1][1]*m[2][0]
        -m[0][1]*m[1][0]*m[2][2]-m[0][0]*m[1][2]*m[2][1];
}
void ball(){
    Pt q[3]; double m[3][3], sol[3], L[3], d;
    int i, j; res.x=res.y=res.z=radius=0;
    switch(nouter){
        case 1: res=outer[0]; break;

```

```

        case 2:
            res=(outer[0]+outer[1])/2;
            radius=norm2(res, outer[0]); break;
        case 3:
            for(i=0; i<2; ++i) q[i]=outer[i+1]-outer[0];
            for(i=0; i<2; ++i)
                for(j=0; j<2; ++j) m[i][j]=(q[i]*q[j])*2;
            for(i=0; i<2; ++i) sol[i]=(q[i]*q[i]);
            if(fabs(d=m[0][0]*m[1][1]-m[0][1]*m[1][0])<eps)
                return;
            L[0]=(sol[0]*m[1][1]-sol[1]*m[0][1])/d;
            L[1]=(sol[1]*m[0][0]-sol[0]*m[1][0])/d;
            res=outer[0]+q[0]*L[0]+q[1]*L[1];
            radius=norm2(res, outer[0]); break;
        case 4:
            for(i=0; i<3; ++i)
                q[i]=outer[i+1]-outer[0], sol[i]=(q[i]*q[i]);
            for(i=0; i<3; ++i)
                for(j=0; j<3; ++j) m[i][j]=(q[i]*q[j])*2;
            d=det(m); if(fabs(d)<eps) return;
            for(j=0; j<3; ++j){
                for(i=0; i<3; ++i) m[i][j]=sol[i];
                L[j]=det(m)/d;
                for(i=0; i<3; ++i) m[i][j]=(q[i]*q[j])*2;
            }
            res=outer[0]; for(i=0; i<3; ++i) res=res+q[i]*L[i];
            radius=norm2(res, outer[0]);
    }
}
void minball(int n){
    ball();
    if(nouter<4) for(int i=0; i<n; i++){
        if(norm2(res, pt[i])-radius>eps){
            outer[nouter++]=pt[i]; minball(i); --nouter;
            if(i>0){ Pt Tt=pt[i];
                memmove(&pt[1], &pt[0], sizeof(Pt)*i); pt[0]=Tt;
            }
        }
    }
    double solve(){ // n points in pt
        random_shuffle(pt, pt+n); radius=-1;
        for(int i=0; i<n; i++) if(norm2(res, pt[i])-radius>eps)
            nouter=1, outer[0]=pt[i], minball(i);
        return sqrt(radius);
    }
}

```

#### 4.20 Minkowski sum 3ce748

```

vector<Pt> minkowski(vector<Pt> p, vector<Pt> q){
    int n=p.size(), m=q.size(); Pt c=Pt(0,0);
    for(int i=0; i<m; i++) c=c+q[i];
    c=c/m; int cur=-1;
    for(int i=0; i<m; i++) q[i]=q[i]-c;
    for(int i=0; i<m; i++) if((q[i]^p[0]-p[n-1]))>-eps)
        if((cur== -1 || (q[i]^p[0]-p[n-1]))>
            (q[cur]^p[0]-p[n-1]))) cur=i;
    vector<Pt> h; p.push_back(p[0]);
    for(int i=0; i<n; i++){
        while(true){
            h.push_back(p[i]+q[cur]);
            int nxt=(cur+1==m ? 0:cur+1);
            if((q[cur]^p[i+1]-p[i]))<-eps) cur=nxt;
            else if((q[nxt]^p[i+1]-p[i]))>
                (q[cur]^p[i+1]-p[i])) cur=nxt;
            else break;
        }
        for(auto &i:h) i=i+c;
        return convex_hull(h);
    }
}

```

#### 4.21 Min dist on Cuboid 95b46d

```

typedef ll T; T r;
void turn(T i, T j, T x, T y, T z, T x0, T y0, T L, T W, T H){
    if (z==0){ T R=x*y+y*y; if (R<r) r=R; return; }
    if(i>=0&&i<2)
        turn(i+1, j, x0+L+z, y, x0+L-x, x0+L, y0, H, W, L);
    if(j>=0&&j<2)
        turn(i, j+1, x, y0+W+z, y0+W-y, x0, y0+W, L, H, W);
    if(i<=0&&i>=-2) turn(i-1, j, x0-z, y, x-x0, x0-H, y0, H, W, L);
    if(j<=0&&j>=-2) turn(i, j-1, x, y0-z, y-y0, x0, y0-H, L, H, W);
}
T solve(T L, T W, T H, T x1, T y1, T z1, T x2, T y2, T z2){
    if(z1!=0&&z1!=H){

```

```

    if(y1==0||y1==W) swap(y1,z1),swap(y2,z2),swap(W,H);
    else swap(x1,z1),swap(x2,z2),swap(L,H);
}
if (z1==H) z1=0,z2=H-z2;
r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
return r;
}

```

## 4.22 Heart of Triangle 4da867

```

Pt inCenter(Pt &A,Pt &B,Pt &C) { // 内心
    double a=norm(B-C),b=norm(C-A),c=norm(A-B);
    return (A*a+B*b+C*c)/(a+b+c);
}
Pt circumCenter(Pt &a,Pt &b,Pt &c) { // 外心
    Pt bb=b-a,cc=c-a;
    double db=norm2(bb),dc=norm2(cc),d=2*(bb^cc);
    return a-Pt(bb.Y*dc-cc.Y*db,cc.X*db-bb.X*dc)/d;
}
Pt othroCenter(Pt &a,Pt &b,Pt &c) { // 垂心
    Pt ba=b-a,ca=c-a,bc=b-c;
    double Y=ba.Y*ca.Y*bc.Y,A=ca.X*ba.Y-ba.X*ca.Y,
    x0=(Y+ca.X*ba.Y*bc.X-ba.X*ca.Y*c.X)/A,
    y0=-ba.X*(x0-c.X)/ba.Y+ca.Y;
    return Pt(x0, y0);
}

```

## 5 Graph

### 5.1 DominatorTree 0e5706

```

const int MAXN=100010;
struct DominatorTree { // 1-based
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n,s; vector<int> g[MAXN],pred[MAXN],cov[MAXN];
    int dfn[MAXN],nfd[MAXN],ts,par[MAXN];
    int sdom[MAXN],idom[MAXN],mom[MAXN],mn[MAXN];
    inline bool cmp(int u,int v){ return dfn[u] < dfn[v]; }
    int eval(int u){
        if(mom[u]==u) return u;
        int res=eval(mom[u]);
        if(cmp(sdom[mn[mom[u]]],sdom[mn[u]]))
            mn[u]=mn[mom[u]];
        return mom[u]=res;
    }
    void init(int _n,int _s){
        ts=0; n=_n; s=_s;
        REP(i,1,n) g[i].clear(),pred[i].clear();
    }
    void addEdge(int u,int v){
        g[u].push_back(v); pred[v].push_back(u);
    }
    void dfs(int u){
        ts++; dfn[u]=ts; nfd[ts]=u;
        for(int v:g[u]) if(dfn[v]==0){ par[v]=u; dfs(v); }
        // x dominates y <=> path s to y must go through x
    }
    void build(){ // <=> x is an ancestor of y in the tree
        REP(i,1,n){ // result tree edges: idom[i] -> i
            dfn[i]=nfd[i]=0; cov[i].clear();
            mom[i]=mn[i]=sdom[i]=i;
        }
        dfs(s);
        REPD(i,n,2){
            int u=nfd[i];
            if(u==0) continue;
            for(int v:pred[u]) if(dfn[v]){
                eval(v);
                if(cmp(sdom[mn[v]],sdom[u]))sdom[u]=sdom[mn[v]];
            }
            cov[sdom[u]].push_back(u); mom[u]=par[u];
            for(int w:cov[par[u]]){
                eval(w);
                if(cmp(sdom[mn[w]],par[u])) idom[w]=mn[w];
                else idom[w]=par[u];
            }
            cov[par[u]].clear();
        }
        REP(i,2,n){
            int u=nfd[i];
            if(u==0) continue;
            if(idom[u]!=sdom[u]) idom[u]=idom[idom[u]];
        }
    }
}

```

```

}
}
}domT;

```

### 5.2 Directed MST(ElogE) 4b46a2

```

struct RollbackUF {
    vi e; vector<pii> st;
    RollbackUF(int n):e(n,-1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x]<0?x:find(e[x]); }
    int time() { return st.size(); }
    void rollback(int t) {
        for(int i=time();i-->t;)e[st[i].first]=st[i].second;
        st.resize(t);
    }
    bool join(int a,int b) {
        a=find(a),b=find(b);
        if(a==b) return false;
        if(e[a]>e[b]) swap(a,b);
        st.push_back({a,e[a]}); st.push_back({b,e[b]});
        e[a]+=e[b]; e[b]=a;
        return true;
    }
};
struct Edge {int a,b; ll w;};
struct Node { // lazy skew heap node
    Edge key; Node *l,*r; ll d;
    void prop() {
        key.w+=d; if(l) l->d+=d; if(r) r->d+=d; d=0;
    }
    Node(Edge e):key(e),l(0),r(0),d(0){}
    Edge top() { prop(); return key; }
};
Node *merge(Node *a,Node *b) {
    if(!a||!b) return a?a:b;
    a->prop(); b->prop();
    if(a->key.w>b->key.w) swap(a,b);
    swap(a->l,(a->r=merge(b,a->r)));
    return a;
}
void pop(Node*& a){ a->prop(); a=merge(a->l,a->r); }
pair<ll,vi> dmst(int n,int r,vector<Edge>& g){
    RollbackUF uf(n); vector<Node*> pq(n);
    for(Edge e:g) pq[e.b]=merge(pq[e.b],new Node(e));
    ll res=0; vi seen(n,-1),path(n),par(n); seen[r]=r;
    vector<Edge> Q(n),in(n,{-1,-1,0});
    deque<tuple<int,int,vector<Edge>>> cys;
    for(int s=0;s<n;s++){
        int u=s,qi=0,w;
        while(seen[u]<0){
            if(!pq[u]) return {-1,{};};
            Edge e=pq[u]->top(); pq[u]->d=-e.w,pop(pq[u]);
            Q[qi]=e,path[qi++]=u,seen[u]=s;
            res+=e.w,u=uf.find(e.a);
            if(seen[u]==s) { // found cycle,contract
                Node* cyc=0; int end=qi,t=uf.time();
                do cyc=merge(cyc,pq[w=path[--qi]]);
                while(uf.join(u,w));
                u=uf.find(u),pq[u]=cyc,seen[u]=-1;
                cys.push_front({u,t,{&Q[qi],&Q[end]}});
            }
        }
        for(int i=0;i<qi;i++) in[uf.find(Q[i].b)]=Q[i];
    }
    for(auto& [u,t,comp]:cys) { // restore sol
        uf.rollback(t); Edge inEdge=in[u];
        for(auto& e:comp) in[uf.find(e.b)]=e;
        in[uf.find(inEdge.b)]=inEdge;
    }
    for(int i=0;i<n;i++) par[i]=in[i].a;
    return {res,par};
}

```

### 5.3 MaximalClique 7e84df

```

#define N 80
struct MaxClique { // 0-base
    typedef bitset<N> Int;
    Int lnk[N],v[N]; int n;
    void init(int _n){
        n=_n;
    }
}

```

```

    for(int i=0;i<n;i++){
        lnk[i].reset(); v[i].reset();
    }
}
void addEdge(int a,int b) { v[a][b]=v[b][a]=1; }
int ans,stk[N],id[N],di[N],deg[N]; Int cans;
void dfs(int elem_num,Int candi,Int ex){
    if(candi.none()&&ex.none()){
        cans.reset();
        for(int i=0;i<elem_num;i++) cans[id[stk[i]]]=1;
        ans=max(ans,elem_num); // cans is a maximal clique
        return;
    }
    int pivot=(candi.ex).Find_first();
    Int smaller_candi=candi&(~lnk[pivot]);
    while(smaller_candi.count()){
        int nxt=smaller_candi.Find_first();
        candi[nxt]=smaller_candi[nxt]=0;
        ex[nxt]=1; stk[elem_num]=nxt;
        dfs(elem_num+1,candi&lnk[nxt],ex&lnk[nxt]);
    }
}
int solve(){
    for(int i=0;i<n;i++){id[i]=i; deg[i]=v[i].count();}
    sort(id,id+n,[&](int id1,int id2)
        { return deg[id1]>deg[id2]; });
    for(int i=0;i<n;i++) di[id[i]]=i;
    for(int i=0;i<n;i++) for(int j=0;j<n;j++){
        if(v[i][j]) lnk[di[i]][di[j]]=1;
    }
    ans=0; cans.reset(); cans[0]=1;
    dfs(0,Int(string(n,'1')),0);
    return ans;
}
}graph;

```

## 5.4 MaxCliqueDyn e0119d

```

#define N 150
struct MaxClique{ // Maximum Clique
    bitset<N> a[N],cs[N]; int ans,sol[N],q,cur[N],d[N],n;
    void init(int _n){
        n=_n; for(int i=0;i<n;i++) a[i].reset();
    }
    void addEdge(int u,int v){ a[u][v]=a[v][u]=1; }
    void csort(vector<int> &r,vector<int> &c){
        int mx=1,km=max(ans-q+1,1),t=0,m=r.size();
        cs[1].reset(); cs[2].reset();
        for(int i=0;i<m;i++){
            int p=r[i],k=1;
            while((cs[k]&a[p]).count()) k++;
            if(k>mx){ mx++; cs[mx+1].reset(); }
            cs[k][p]=1; if(k<km) r[t++]=p;
        }
        c.resize(m); if(t) c[t-1]=0;
        for(int k=km;k<=mx;k++){
            for(int p=cs[k].Find_first();p<N;p=cs[k].Find_next(p)){
                r[t]=p; c[t]=k; t++;
            }
        }
    }
    void dfs(vector<int> &r,vector<int> &c,int l,bitset<N> mask){
        while(!r.empty()){
            int p=r.back(); r.pop_back(); mask[p]=0;
            if(q+c.back()<=ans) return;
            cur[q++]=p;
            vector<int> nr,nc; bitset<N> nmask=mask&a[p];
            for(int i:r) if(a[p][i]) nr.push_back(i);
            if(!nr.empty()){
                if(l<4){
                    for(int i:nr) d[i]=(a[i]&nmask).count();
                    sort(nr.begin(),nr.end(),
                        [&](int x,int y){return d[x]>d[y];});
                }
                csort(nr,nc); dfs(nr,nc,l+1,nmask);
            }
            else if(q>ans){ ans=q; copy(cur,cur+q,sol); }
            c.pop_back(); q--;
        }
    }
    // vertex mask
    int solve(bitset<N> mask=bitset<N>(string(N,'1'))){
        vector<int> r,c; ans=q=0;
        for(int i=0;i<n;i++) if(mask[i]) r.push_back(i);
    }
}

```

```

    for(int i=0;i<n;i++) d[i]=(a[i]&mask).count();
    sort(r.begin(),r.end(),
        [&](int i,int j){return d[i]>d[j];});
    csort(r,c); dfs(r,c,1,mask);
    return ans; // vertices set: sol[0 ~ ans-1]
}
}graph;

```

## 5.5 Strongly Connected Component 10c233

```

void dfs(int i){
    V[i]=low[i]=++ts,stk[top++]=i,instk[i]=1;
    for(auto x:E[i]){
        if(!V[x])dfs(x),low[i]=min(low[i],low[x]);
        else if(instk[x])low[i]=min(low[i],V[x]);
    }
    if(V[i]==low[i]){
        int j;
        do{j=stk[--top],instk[j]=0,scc[j]=i;
        }while(j!=i);
    }
}
}

```

## 5.6 Dynamic MST 21e59f

```

/* Dynamic MST O(Q lg^2 Q)
n nodes, m edges, Q query
(u[i], v[i], w[i])->edge
(qid[i], qw[i])->chg weight of edge No.qid[i] to qw[i]
delete an edge: (i, \infty)
add an edge: change from \infty to specific value */
const int M=1e5,MXQ=1e5,SZ=M+3*MXQ; int a[N],*tz;
int find(int x){ return x==a[x]?x:a[x]=find(a[x]); }
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt,vd[N],id[M],app[M],cur;
long long answer[MXQ]; // answer after ith query
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,int
    *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[qx[0]]=qy[0]; tz=z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        answer[cur++]=ans; return;
    }
    int ri,rj,tm=0,n2=0; kt=0;
    //contract
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]);
        if(ri!=rj) a[ri]=rj;
    }
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[qx[i]]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){
        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans+=z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[find(kx[i])]=find(ky[i]);
    for(int i=1;i<=n;i++) if(a[i]==0) vd[i]=++n2;
    for(int i=1;i<=n;i++) if(a[i]) vd[i]=vd[find(i)];
    int m2=0,*Nx=x+m1,*Ny=y+m1,*Nz=z+m1;
    for(int i=0;i<m1;i++) app[i]=-1;
    for(int i=0;i<Q;i++) if(app[qx[i]]!=-1){
        Nx[m2]=vd[x[qx[i]]]; Ny[m2]=vd[y[qx[i]]];
        Nz[m2]=z[qx[i]]; app[qx[i]]=m2; m2++;
    }
    for(int i=0;i<Q;i++){z[qx[i]]=qy[i];qx[i]=app[qx[i]];}
    for(int i=1;i<=n2;i++) a[i]=0;
    for(int i=0;i<tm;i++){
        ri=find(vd[x[id[i]]]); rj=find(vd[y[id[i]]]);
    }
}

```

```

    if(ri!=rj){
        a[ri]=rj; Nx[m2]=vd[x[id[i]]];
        Ny[m2]=vd[y[id[i]]]; Nz[m2]=z[id[i]]; m2++;
    }
}
int mid=Q/2;
solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
} // fill these variables and call work()
int u[SZ],v[SZ],w[SZ],qid[MXQ],qw[MXQ],n,m,Q;
void work(){if(Q) cur=0,solve(qid,qw,Q,n,u,v,w,m,0);}

```

## 5.7 Maximum General graph Matching a15e63

```

// should shuffle vertices and edges
const int N=100005,E=(2e5)*2+40;
struct Graph{ // 1-based; match: i <-> lnk[i]
    int to[E],bro[E],head[N],e,lnk[N],vis[N],stp,n;
    void init(int _n){
        stp=0; e=1; n=_n;
        for(int i=1;i<=n;i++) head[i]=lnk[i]=vis[i]=0;
    }
    void add_edge(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(!lnk[v]){ lnk[x]=v,lnk[v]=x; return true; }
        }
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(vis[lnk[v]]<stp){
                int w=lnk[v]; lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w)) return true;
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans=0;
        for(int i=1;i<=n;i++) if(!lnk[i]) stp++,ans+=dfs(i);
        return ans;
    }
}graph;

```

## 5.8 Minimum General Weighted Matching 91b293

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN=105;
    int n,edge[MXN][MXN],match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n=_n;
        for(int i=0;i<n;i++)
            for(int j=0;j<n;j++) edge[i][j]=0;
    }
    void add_edge(int u,int v,int w)
    { edge[u][v]=edge[v][u]=w; }
    bool SPFA(int u){
        if(onstk[u]) return true;
        stk.push_back(u); onstk[u]=1;
        for(int v=0;v<n;v++){
            if(u!=v&&match[u]!=v&&!onstk[v]){
                int m=match[v];
                if(dis[m]>dis[u]-edge[v][m]+edge[u][v]){
                    dis[m]=dis[u]-edge[v][m]+edge[u][v];
                    onstk[v]=1; stk.push_back(v);
                    if(SPFA(m)) return true;
                    stk.pop_back(); onstk[v]=0;
                }
            }
        }
        onstk[u]=0; stk.pop_back();
        return false;
    }
    int solve() { // find a match
        for(int i=0;i<n;i+=2){ match[i]=i+1;match[i+1]=i; }
        while(true){

```

```

            int found=0;
            for(int i=0;i<n;i++) onstk[i]=dis[i]=0;
            for(int i=0;i<n;i++){
                stk.clear();
                if(!onstk[i]&&SPFA(i)){
                    found=1;
                    while((int)stk.size()>=2){
                        int u=stk.back();stk.pop_back();
                        int v=stk.back();stk.pop_back();
                        match[u]=v;match[v]=u;
                    }
                }
                if(!found) break;
            }
            int ret=0;
            for(int i=0;i<n;i++) ret+=edge[i][match[i]];
            return ret/2;
        }
    }graph;

```

## 5.9 Maximum General Weighted Matching 0baee5

```

struct WeightGraph {
    static const int INF=INT_MAX,N=514;
    struct edge{
        int u,v,w; edge(){}
        edge(int ui,int vi,int wi):u(ui),v(vi),w(wi){}
    };
    int n,n_x,lab[N*2],match[N*2],slack[N*2],st[N*2];
    int pa[N*2],flo_from[N*2][N+1],S[N*2],vis[N*2];
    edge g[N*2][N*2]; vector<int> flo[N*2]; queue<int> q;
    int e_delta(const edge &e){
        return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
    }
    void update_slack(int u,int x){
        if(!slack[x]||e_delta(g[u][x])<
            e_delta(g[slack[x]][x])) slack[x]=u;
    }
    void set_slack(int x){
        slack[x]=0;
        for(int u=1;u<=n;u++)
            if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
                update_slack(u,x);
    }
    void q_push(int x){
        if(x<=n) q.push(x);
        else for(size_t i=0;i<flo[x].size();i++)
            q.push(flo[x][i]);
    }
    void set_st(int x,int b){
        st[x]=b;
        if(x>n) for(size_t i=0;i<flo[x].size();i++)
            set_st(flo[x][i],b);
    }
    int get_pr(int b,int xr){
        int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].begin();
        if(pr%2==1){
            reverse(flo[b].begin()+1,flo[b].end());
            return (int)flo[b].size()-pr;
        }else return pr;
    }
    void set_match(int u,int v){
        match[u]=g[u][v].v; if(u<=n) return; edge e=g[u][v];
        int xr=flo_from[u][e.u],pr=get_pr(u,xr);
        for(int i=0;i<pr;i++){
            set_match(flo[u][i],flo[u][i^1]);
            set_match(xr,v); rotate(flo[u].begin(),flo[u].begin()
                +pr,flo[u].end());
        }
    }
    void augment(int u,int v){
        for(;;){
            int xnv=st[match[u]]; set_match(u,v);
            if(!xnv) return;
            set_match(xnv,st[pa[xnv]]); u=st[pa[xnv]],v=xnv;
        }
    }
    int get_lca(int u,int v){
        static int t=0;
        for(++t;u!=v;swap(u,v)){
            if(u==0) continue; if(vis[u]==t) return u;
            vis[u]=t; u=st[match[u]]; if(u) u=st[pa[u]];

```



```

    }
    return 0;
}
void add_blossom(int u, int lca, int v){
    int b=n+1; while(b<=n_x&&st[b])++b; if(b>n_x)++n_x;
    lab[b]=0, S[b]=0; match[b]=match[lca];
    flo[b].clear(); flo[b].push_back(lca);
    for(int x=u, y; x!=lca; x=st[pa[y]])
        flo[b].push_back(x),
        flo[b].push_back(y=st[match[x]]), q_push(y);
    reverse(flo[b].begin()+1, flo[b].end());
    for(int x=v, y; x!=lca; x=st[pa[y]])
        flo[b].push_back(x),
        flo[b].push_back(y=st[match[x]]), q_push(y);
    set_st(b, b);
    for(int x=1; x<=n_x; ++x) g[b][x].w=g[x][b].w=0;
    for(int x=1; x<=n; ++x) flo_from[b][x]=0;
    for(size_t i=0; i<flo[b].size(); ++i){
        int xs=flo[b][i];
        for(int x=1; x<=n_x; ++x)
            if(g[b][x].w==0 || e_delta(g[xs][x])<e_delta(g[b][x])) g[b][x]=g[xs][x], g[x][b]=g[x][xs];
        for(int x=1; x<=n; ++x)
            if(flo_from[xs][x]) flo_from[b][x]=xs;
    }
    set_slack(b);
}
void expand_blossom(int b){
    for(size_t i=0; i<flo[b].size(); ++i)
        set_st(flo[b][i], flo[b][i]);
    int xr=flo_from[b][g[b][pa[b]].u], pr=get_pr(b, xr);
    for(int i=0; i<pr; i+=2){
        int xs=flo[b][i], xns=flo[b][i+1];
        pa[xs]=g[xns][xs].u; S[xs]=1, S[xns]=0;
        slack[xs]=0, set_slack(xns); q_push(xns);
    }
    S[xr]=1, pa[xr]=pa[b];
    for(size_t i=pr+1; i<flo[b].size(); ++i){
        int xs=flo[b][i]; S[xs]=-1, set_slack(xs);
    }
    st[b]=0;
}
bool on_found_edge(const edge &e){
    int u=st[e.u], v=st[e.v];
    if(S[v]==-1){
        pa[v]=e.u, S[v]=1; int nu=st[match[v]];
        slack[v]=slack[nu]=0; S[nu]=0, q_push(nu);
    } else if(S[v]==0){
        int lca=get_lca(u, v);
        if(!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}
bool matching(){
    memset(S+1, -1, sizeof(int)*n_x);
    memset(slack+1, 0, sizeof(int)*n_x); q=queue<int>();
    for(int x=1; x<=n_x; ++x)
        if(st[x]==x&&!match[x]) pa[x]=0, S[x]=0, q_push(x);
    if(q.empty()) return false;
    for(;;){ while(q.size()){
        int u=q.front(); q.pop(); if(S[st[u]]==1) continue;
        for(int v=1; v<=n; ++v)
            if(g[u][v].w>0&&st[u]!=st[v]){
                if(e_delta(g[u][v])==0){
                    if(on_found_edge(g[u][v])) return true;
                } else update_slack(u, st[v]);
            }
        }
        int d=INF;
        for(int b=n+1; b<=n_x; ++b)
            if(st[b]==b&&S[b]==1) d=min(d, lab[b]/2);
        for(int x=1; x<=n_x; ++x) if(st[x]==x&&slack[x]){
            if(S[x]==-1) d=min(d, e_delta(g[slack[x]][x]));
            else if(S[x]==0)
                d=min(d, e_delta(g[slack[x]][x])/2);
        }
        for(int u=1; u<=n; ++u){ if(S[st[u]]==0){
            if(lab[u]<=d) return 0; lab[u]=-d;
            } else if(S[st[u]]==1) lab[u]+=d;
        }
        for(int b=n+1; b<=n_x; ++b) if(st[b]==b){

```

```

            if(S[st[b]]==0) lab[b]+=d*2;
            else if(S[st[b]]==1) lab[b]-=d*2;
        }
        q=queue<int>();
        for(int x=1; x<=n_x; ++x)
            if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[slack[x]][x])>0)
                if(on_found_edge(g[slack[x]][x])) return true;
        for(int b=n+1; b<=n_x; ++b) if(st[b]==b&&S[b]==1&&lab[b]==0) expand_blossom(b);
    }
    return false;
}
pair<long long, int> solve(){
    memset(match+1, 0, sizeof(int)*n); n_x=n;
    int n_matches=0, w_max=0; long long tot_weight=0;
    for(int u=0; u<=n; ++u) st[u]=u, flo[u].clear();
    for(int u=1; u<=n; ++u) for(int v=1; v<=n; ++v){
        flo_from[u][v]=(u==v?0:0);
        w_max=max(w_max, g[u][v].w);
    }
    for(int u=1; u<=n; ++u) lab[u]=w_max;
    while(matching()) ++n_matches;
    for(int u=1; u<=n; ++u) if(match[u]&&match[u]<u)
        tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight, n_matches);
}
void add_edge(int ui, int vi, int wi)
{ g[ui][vi].w=g[vi][ui].w=wi; }
void init(int _n){
    n=_n;
    for(int u=1; u<=n; ++u) for(int v=1; v<=n; ++v)
        g[u][v]=edge(u, v, 0);
}
}graph;

```

## 5.10 Minimum Steiner Tree 837386

```

// Minimum Steiner Tree  $O(V^3 \Delta T + V \Delta^2 \Delta T)$ 
// shortest_path() should be called before solve()
// w: vertex weight, default 0
const int V=66, T=10; const ll INF=1023456789;
struct SteinerTree{
    int n, dst[V][V], dp[1<<T][V], tdst[V], w[V];
    void init(int _n){
        n=_n; fill(w, w+n, 0);
        for(int i=0; i<n; i++){
            for(int j=0; j<n; j++) dst[i][j]=INF;
            dst[i][i]=0;
        }
    }
    void add_edge(int ui, int vi, int wi){
        dst[ui][vi]=min(dst[ui][vi], wi);
        dst[vi][ui]=min(dst[vi][ui], wi);
    }
    void shortest_path(){
        for(int i=0; i<n; i++){
            for(int j=0; j<n; j++)
                if(i!=j&&dst[i][j]!=INF) dst[i][j]+=w[i];
            for(int k=0; k<n; k++){
                for(int i=0; i<n; i++){
                    for(int j=0; j<n; j++){
                        dst[i][j]=min(dst[i][j], dst[i][k]+dst[k][j]);
                    }
                }
            }
        }
        for(int i=0; i<n; i++){
            for(int j=0; j<n; j++){
                if(dst[i][j]!=INF) dst[i][j]+=w[j];
            }
        }
    }
    int solve(const vector<int>& ter){
        int t=(int)ter.size();
        for(int i=0; i<(1<<t); i++){
            for(int j=0; j<n; j++) dp[i][j]=INF;
            for(int i=0; i<n; i++) dp[0][i]=0;
            for(int msk=1; msk<(1<<t); msk++){
                if(msk==(msk&(-msk))){
                    int who=__lg(msk);
                    for(int i=0; i<n; i++) dp[msk][i]=dst[ter[who]][i];
                    continue;
                }
                for(int i=0; i<n; i++){
                    for(int submsk=(msk-1)&msk; submsk; submsk=(submsk-1)&msk)
                        dp[msk][i]=min(dp[msk][i], dp[submsk][i]+dp[msk^submsk][i]-w[i]);
                }
                for(int i=0; i<n; i++){
                    tdst[i]=INF;

```



```

    for(int j=0;j<n;j++){
        min(tdst[i],dp[msk][j]+dst[j][i]-w[j]);
    }
    for(int i=0;i<n;i++) dp[msk][i]=tdst[i];
}
int ans=INF;
for(int i=0;i<n;i++) ans=min(ans,dp[(1<t)-1][i]);
return ans;
}
} solver;

```

### 5.11 BCC based on vertex 4ea1ac

```

struct BccVertex{
    int n,nBcc,step,dfn[MXN],low[MXN],top,stk[MXN];
    vector<int> E[MXN],bccv[MXN];
    // vector<pair<int,int>> bcce[MXN];
    // pair<int,int> estk[MXN]; // max edge number
    // int etop,id[MXN],pos[MXN];
    void init(int _n){
        n=_n;nBcc=step=0; for(int i=0;i<n;i++) E[i].clear();
    }
    void addEdge(int u,int v)
    { E[u].push_back(v);E[v].push_back(u);}
    void DFS(int u,int f){
        dfn[u]=low[u]=step++; stk[top++]=u;
        for(auto v:E[u]){
            if(v==f) continue;
            if(dfn[v]==-1){
                // estk[etop++]={u,v};
                DFS(v,u); low[u]=min(low[u],low[v]);
                if(low[v]>=dfn[u]){
                    int z;bccv[nBcc].clear();//bcce[nBcc].clear();
                    // pair<int,int> ez;
                    // do{
                    //     ez=estk[--etop];bcce[nBcc].push_back(ez);
                    // }while(ez.first!=u);
                    do{
                        z=stk[--top]; bccv[nBcc].push_back(z);
                        // id[z]=nBcc;pos[z]=bccv[nBcc].size();
                    }while(z!=v);
                    bccv[nBcc++].push_back(u);
                }
            }else{
                low[u]=min(low[u],dfn[v]);
                // if(dfn[v]<dfn[u]) estk[etop++]={u,v};
            }
        }
    }
    vector<vector<int>> solve(){
        vector<vector<int>> res;
        for(int i=0;i<n;i++) dfn[i]=low[i]=-1;
        for(int i=0;i<n;i++) if(dfn[i]==-1){
            top=0; DFS(i,i); // etop=0;
        }
        for(int i=0;i<nBcc;i++) res.push_back(bccv[i]);
        return res;
    }
    /* bccv[first][.second.first,.second.second]=={u,v}
    pair<int,pair<int,int>> getpos(int u,int v){
        if(dfn[u]>dfn[v]) swap(u,v);
        int cid=id[v];
        if(id[u]==cid) return{cid,{pos[v],pos[u]}};
        else return{cid,{pos[v],bccv[cid].size()-1}};
    } */
}graph;

```

### 5.12 Min Mean Cycle f5f8be

```

/* minimum mean cycle O(VE) */
const int E=101010,V=1021;
const double inf=1e9,eps=1e-8;
struct MMC{
    struct Edge{ int v,u; double c; };
    int n,m,prv[V][V],prve[V][V],vst[V]; Edge e[E];
    vector<int> edgeID,cycle,rho; double d[V][V];
    void init(int _n){ n=_n; m=0; }
    // WARNING: TYPE MATTERS
    void addEdge(int vi,int ui,double ci)
    { e[m++]={vi,ui,ci}; }
    void bellman_ford(){
        for(int i=0;i<n;i++) d[0][i]=0;
        for(int i=0;i<n;i++){
            fill(d[i+1],d[i+1]+n,inf);

```

```

        for(int j=0;j<m;j++){
            int v=e[j].v,u=e[j].u;
            if(d[i][v]<inf&&d[i+1][u]>d[i][v]+e[j].c){
                d[i+1][u]=d[i][v]+e[j].c;
                prv[i+1][u]=v; prve[i+1][u]=j;
            }
        }
    }
}
double solve(){
    // returns inf if no cycle,mmc otherwise
    double mmc=inf; int st=-1; bellman_ford();
    for(int i=0;i<n;i++){
        double avg=-inf;
        for(int k=0;k<n;k++){
            if(d[n][i]<inf-eps)
                avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if(avg<mmc) tie(mmc,st)=tie(avg,i);
    }
    if(st==-1) return inf;
    FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
    for(int i=n;!vst[st];st=prv[i--][st]){
        vst[st]++; edgeID.push_back(prve[i][st]);
        rho.push_back(st);
    }
    while(vst[st]!=2){
        int v=rho.back(); rho.pop_back();
        cycle.push_back(v); vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize((int)cycle.size());
    return mmc;
}
}mmc;

```

### 5.13 Directed Graph Min Cost Cycle c448cd

```

const int N=5010,M=200010; const ll INF=(1ll<<55);
struct edge{
    int to; ll w;
    edge(int a=0,ll b=0):to(a),w(b){}
};
struct node{
    ll d; int u,next;
    node(ll a=0,int b=0,int c=0): d(a),u(b),next(c){}
}b[M];
struct DirectedGraphMinCycle{ // works in O(NM)
    vector<edge> g[N],grev[N]; ll dp[N][N],p[N],d[N],mu;
    bool inq[N]; int n,bn,bsz,hd[N];
    void b_insert(ll d,int u){
        int i=d/mu; if(i>=bn) return;
        b[++bsz]=node(d,u,hd[i]); hd[i]=bsz;
    }
    void init(int _n){
        n=_n; for(int i=1;i<=n;i++) g[i].clear();
    }
    void addEdge(int ai,int bi,ll ci)
    { g[ai].push_back(edge(bi,ci)); }
    ll solve(){
        fill(dp[0],dp[0]+n+1,0);
        for(int i=1;i<=n;i++){
            fill(dp[i+1],dp[i+1]+n+1,INF);
            for(int j=1;j<=n;j++) if(dp[i-1][j]<INF){
                for(int k=0;k<(int)g[j].size();k++){
                    dp[i][g[j][k].to]=min(dp[i][g[j][k].to],dp[i-1][j]+g[j][k].w);
                }
            }
        }
        mu=INF; ll bunbo=1;
        for(int i=1;i<=n;i++) if(dp[n][i]<INF){
            ll a=-INF,b=1;
            for(int j=0;j<=n-1;j++) if(dp[j][i]<INF){
                if(a*(n-j)<b*(dp[n][i]-dp[j][i])){
                    a=dp[n][i]-dp[j][i]; b=n-j;
                }
            }
            if(mu*b>bunbo*a) mu=a,bunbo=b;
        }
        if(mu<0) return -1; // negative cycle
        if(mu==INF) return INF; // no cycle
        if(mu==0) return 0;
        for(int i=1;i<=n;i++)
            for(int j=0;j<(int)g[i].size();j++)

```

```

    g[i][j].w*=bunbo;
    memset(p,0,sizeof(p)); queue<int> q;
    for(int i=1;i<=n;i++){ q.push(i); inq[i]=true; }
    while(!q.empty()){
        int i=q.front(); q.pop(); inq[i]=false;
        for(int j=0;j<(int)g[i].size();j++){
            if(p[g[i][j].to]>p[i]+g[i][j].w-mu){
                p[g[i][j].to]=p[i]+g[i][j].w-mu;
                if(!inq[g[i][j].to]){
                    q.push(g[i][j].to); inq[g[i][j].to]=true;
                }
            }
        }
    }
    for(int i=1;i<=n;i++) grev[i].clear();
    for(int i=1;i<=n;i++){
        for(int j=0;j<(int)g[i].size();j++){
            g[i][j].w+=p[i]-p[g[i][j].to];
            grev[g[i][j].to].push_back(edge(i,g[i][j].w));
        }
    }
    ll mlcd=n*mu;
    for(int i=1;i<=n;i++){
        bn=mlcd/mu,bsz=0; memset(hd,0,sizeof(hd));
        fill(d+i+1,d+n+1,INF); b_insert(d[i]=0,i);
        for(int j=0;j<=bn-1;j++){
            for(int k=hd[j];k;k=b[k].next){
                int u=b[k].u; ll du=b[k].d;
                if(du>d[u]) continue;
                for(int l=0;l<(int)g[u].size();l++){
                    if(g[u][l].to>i){
                        if(d[g[u][l].to]>du+g[u][l].w){
                            d[g[u][l].to]=du+g[u][l].w;
                            b_insert(d[g[u][l].to],g[u][l].to);
                        }
                    }
                }
            }
        }
        for(int j=0;j<(int)grev[i].size();j++){
            if(grev[i][j].to>i)
                mlcd=min(mlcd,d[grev[i][j].to]+grev[i][j].w);
        }
    }
    return mlcd/bunbo;
}
}
graph;

```

## 5.14 K-th Shortest Path 355040

```

// time: O(|E| \lg |E|+|V| \lg |V|+K)
// memory: O(|E| \lg |E|+|V|)
struct KSP{ // 1-base
    struct nd{
        int u,v; ll d;
        nd(int ui=0,int vi=0,ll di=INF){ u=ui; v=vi; d=di; }
    };
    struct heap{ nd* edge; int dep; heap* chd[4]; };
    static int cmp(heap* a,heap* b)
    { return a->edge->d > b->edge->d; }
    struct node{
        int v; ll d; heap* H; nd* E;
        node(){ }
        node(ll _d,int _v,nd* _E){ d=_d; v=_v; E=_E; }
        node(heap* _H,ll _d){ H=_H; d=_d; }
        friend bool operator<(node a,node b)
        { return a.d>b.d; }
    };
    int n,k,s,t,dst[N]; nd *nxt[N];
    vector<nd*> g[N],rg[N]; heap *nullNd,*head[N];
    void init(int _n,int _k,int _s,int _t){
        n=_n; k=_k; s=_s; t=_t;
        for(int i=1;i<=n;i++){
            g[i].clear(); rg[i].clear();
            nxt[i]=NULL; head[i]=NULL; dst[i]=-1;
        }
    }
    void addEdge(int ui,int vi,ll di){
        nd* e=new nd(ui,vi,di);
        g[ui].push_back(e); rg[vi].push_back(e);
    }
    queue<int> dfsQ;
    void dijkstra(){
        while(dfsQ.size()) dfsQ.pop();
        priority_queue<node> Q; Q.push(node(0,t,NULL));
        while (!Q.empty()){
            node p=Q.top(); Q.pop(); if(dst[p.v]!=-1)continue;
            dst[p.v]=p.d; nxt[p.v]=p.E; dfsQ.push(p.v);
            for(auto e:rg[p.v]) Q.push(node(p.d+e->d,e->u,e));
        }
    }
}

```

```

heap* merge(heap* curNd,heap* newNd){
    if(curNd==nullNd) return newNd;
    heap* root=new heap;memcpy(root,curNd,sizeof(heap));
    if(newNd->edge->d<curNd->edge->d){
        root->edge=newNd->edge;
        root->chd[2]=newNd->chd[2];
        root->chd[3]=newNd->chd[3];
        newNd->edge=curNd->edge;
        newNd->chd[2]=curNd->chd[2];
        newNd->chd[3]=curNd->chd[3];
    }
    if(root->chd[0]->dep<root->chd[1]->dep)
        root->chd[0]=merge(root->chd[0],newNd);
    else root->chd[1]=merge(root->chd[1],newNd);
    root->dep=max(root->chd[0]->dep,
        root->chd[1]->dep)+1;
    return root;
}
vector<heap*> V;
void build(){
    nullNd=new heap; nullNd->dep=0; nullNd->edge=new nd;
    fill(nullNd->chd,nullNd->chd+4,nullNd);
    while(not dfsQ.empty()){
        int u=dfsQ.front(); dfsQ.pop();
        if(!nxt[u]) head[u]=nullNd;
        else head[u]=head[nxt[u]->v];
        V.clear();
        for(auto&& e:g[u]){
            int v=e->v;
            if(dst[v]==-1) continue;
            e->d+=dst[v]-dst[u];
            if(nxt[u]!=e){
                heap* p=new heap;fill(p->chd,p->chd+4,nullNd);
                p->dep=1; p->edge=e; V.push_back(p);
            }
        }
        if(V.empty()) continue;
        make_heap(V.begin(),V.end(),cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
        for(size_t i=0;i<V.size();i++){
            if(L(i)<V.size()) V[i]->chd[2]=V[L(i)];
            else V[i]->chd[2]=nullNd;
            if(R(i)<V.size()) V[i]->chd[3]=V[R(i)];
            else V[i]->chd[3]=nullNd;
        }
        head[u]=merge(head[u],V.front());
    }
}
vector<ll> ans;
void first_KC(){
    ans.clear(); priority_queue<node> Q;
    if(dst[s]==-1) return;
    ans.push_back(dst[s]);
    if(head[s]!=nullNd)
        Q.push(node(head[s],dst[s]+head[s]->edge->d));
    for(int _=1;_<k and not Q.empty();_++){
        node p=Q.top();q; Q.pop(); ans.push_back(p.d);
        if(head[p.H->edge->v]!=nullNd){
            q.H=head[p.H->edge->v]; q.d=p.d+q.H->edge->d;
            Q.push(q);
        }
    }
    for(int i=0;i<4;i++){
        if(p.H->chd[i]!=nullNd){
            q.H=p.H->chd[i];
            q.d=p.d-p.H->edge->d+p.H->chd[i]->edge->d;
            Q.push(q);
        }
    }
}
void solve() { // ans[i] stores the i-th shortest path
    dijkstra(); build();
    first_KC(); // ans.size() might less than k
}
}
solver;

```

## 5.15 Chordal Graph a96ac6

```

struct Chordal{
    static const int MXN=100010;
    vector<int> E[MXN],V[MXN];
    int n,f[MXN],rk[MXN],order[MXN],stk[MXN],nsz[MXN];
    bool vis[MXN],isMaximalClique[MXN];
    void init(int _n){

```

```

n=_n;
for(int i=0;i<=n;++i){
    E[i].clear(),V[i].clear();
    f[i]=rk[i]=order[i]=vis[i]=0;
}
}
void addEdge(int x,int y){
    E[x].push_back(y),E[y].push_back(x);
}
void mcs(){
    for(int i=1;i<=n;++i) V[0].push_back(i);
    for(int i=n,M=0;i>=1;--i){
        for(;;){
            while(V[M].size()&&vis[V[M].back()])
                V[M].pop_back();
            if(V[M].size()) break; else M--;
        }
        auto x=V[M].back();order[i]=x;rk[x]=i;vis[x]=1;
        for(auto y:E[x]) if(!vis[y])
            f[y]++,V[f[y]].push_back(y),M=max(M,f[y]);
    }
}
bool isChordal(){
    for(int i=0;i<=n;++i) vis[i]=stk[i]=0;
    for(int i=n;i>=1;--i){
        int top=0,cnt=0,m=n+1;
        for(auto x:E[order[i]]) if(rk[x] > i)
            stk[top++] = x,vis[x]=1,m=min(m,rk[x]);
        if(m==n+1) continue;
        for(auto x:E[order[m]]) if(vis[x]) ++cnt;
        for(int j=0;j<top;++j) vis[stk[j]]=0;
        if(cnt+1!=top) return 0;
    }
    return 1;
}
void getMaximalClique(){
    for(int i=n;i>=1;--i){
        int M=n+1,w=order[i],v=0;
        nsz[w]=0;isMaximalClique[w]=1;
        for(auto x:E[w]) if(rk[x]>i){
            nsz[w]++; if(rk[x]<M) M=rk[x],v=x;
        }
        if(v) isMaximalClique[v]&=nsz[v]+1>nsz[w];
    }
}
int getMaximumClique(){
    int res=0;
    for(int i=1;i<=n;++i) res=max(res,f[i]+1);
    return res;
}
int getMaximumIndependentSet(){
    for(int i=0;i<=n;++i) vis[i]=0;
    int res=0;
    for(int i=1;i<=n;++i) if(!vis[order[i]]){
        res++,vis[order[i]]=1;
        for(auto x:E[order[i]]) vis[x]=1;
    }
    return res;
}
};

```

## 5.16 Matroid Intersection d1fe1e

```

/* Matroid Definition:
* 1. Empty set is ind. 2. Subset of ind. set is ind.
* 3. If set A, B are ind. and |A| < |B|,
*    there exists x in B\A s.t. A ∪ {x} is ind.
* Max Weighted Matroid Intersection: (memorize testInd)
* Let vertex weight l(x) = (x is chosen ? w(x) : -w(x))
* Find shortest aug. path with SPFA, based on minimize
* tie(sum of l(x), number of edges) on the path. */
struct MatroidIntersection {
    int n; // Elem: bool chosen, int p, info...
    vector<Elem> GS; // Ground Set.
    vector<int> indSet; // Current chosen ind. set
    bool testInd1(int add){} // indSet ∪ {a}
    bool testInd1(int add,int removed){} // ind\{r} ∪ {a}
    bool testInd2(int add){}
    bool testInd2(int add,int removed){}
    bool augment(){ // prepareInd1(), prepareInd2();
        for(auto &x:GS) x.p=-2; // init l,dis,len,inque
        int ep=-3;queue<int> q;

```

```

for(int i=0;i<=n;++i) if(!GS[i].chosen&&testInd1(i))
    GS[i].p=-1,q.push(i);
while(!q.empty()){ // bfs -> SPFA
    int cur=q.front(); q.pop();
    if(GS[cur].chosen){ // SPFA dont check .p != -2
        for(int nxt=0;nxt<=n;++nxt){
            if(GS[nxt].chosen or GS[nxt].p!=-2) continue;
            if(!testInd1(nxt,cur)) continue;
            GS[nxt].p=cur; q.push(nxt);
        }
    }else{ // SPFA record nearest ep, dont break
        if(testInd2(cur)){ ep=cur; break; }
        for(auto nxt:indSet){
            if(GS[nxt].p!=-2 or !testInd2(cur,nxt))
                continue;
            GS[nxt].p=cur;q.push(nxt);
        }
    }
    if(ep==3) return false;
    do{ GS[ep].chosen^=1; ep=GS[ep].p; } while(ep!=-1);
    indSet.clear();
    for(int i=0;i<=n;i++) if(GS[i].chosen)
        indSet.push_back(i);
    return true;
}
void solve(){ n=GS.size(); while(augment()); }
}MI;

```

## 5.17 Graph Hash

$$F_t(i) = (F_{t-1}(i) \times A + \sum_{j \rightarrow i} F_{t-1}(j) \times B + \sum_{j \leftarrow i} F_{t-1}(j) \times C + D \times (i = a)) \bmod P$$

for each node i, iterate t times. t, A, B, C, D, P are hash parameter

## 5.18 Graph Method

Manhattan MST

For each point, consider the points that surround it(8 octants). Then, connect it with the closest point. For example, consider 45~90. For each point p, the closest point is  $\min\{x+y \mid x-y \geq p.x-p.y, x \geq p.x\}$ . Finally, the answer is [this new graphs\(E=4N\)](#) MST.

## 6 String

### 6.1 PalTree 7280a6

```

const int MXN = 1000010;
struct PalT{
    int nxt[MXN][26],fail[MXN],len[MXN];
    int tot,lst,n,state[MXN],cnt[MXN],num[MXN];
    int diff[MXN],sfail[MXN],fac[MXN],dp[MXN];
    char s[MXN]={-1};
    int newNode(int l,int f){
        len[tot]=l,fail[tot]=f,cnt[tot]=num[tot]=0;
        memset(nxt[tot],0,sizeof(nxt[tot]));
        diff[tot]=(l>0?l-len[f]:0);
        sfail[tot]=(l>0&&diff[tot]==diff[f]?sfail[f]:f);
        return tot++;
    }
    int getfail(int x){
        while(s[n-len[x]-1]!=s[n]) x=fail[x];
        return x;
    }
    int getmin(int v){
        dp[v]=fac[n-len[sfail[v]]-diff[v]];
        if(diff[v]==diff[fail[v]])
            dp[v]=min(dp[v],dp[fail[v]]);
        return dp[v]+1;
    }
    int push(){
        int c=s[n]-'a',np=getfail(lst);
        if(!lst==nxt[np][c]){
            lst=newNode(len[np]+2,nxt[getfail(fail[np])][c]);
            nxt[np][c]=lst; num[lst]=num[fail[lst]]+1;
        }
        fac[n]=n;
        for(int v=lst;len[v]>0;v=sfail[v])
            fac[n]=min(fac[n],getmin(v));
        return ++cnt[lst],lst;
    }
    void init(const char *_s){

```

```

    tot=lst=n=0; newNode(0,1),newNode(-1,1);
    for(;;_s[n];) s[n+1]=_s[n],++n,state[n-1]=push();
    for(int i=tot-1;i>1;i--) cnt[fail[i]]+=cnt[i];
}
}palt;

```

## 6.2 SAIS 4379c7

```

const int N=300010;
struct SA{
#define REP(i,n) for(int i=0;i<int(n);i++)
#define REP1(i,a,b) for(int i=(a);i<=int(b);i++)
    bool _t[N*2]; int _s[N*2],_sa[N*2];
    int _c[N*2],x[N],_p[N],_q[N*2],hei[N],r[N];
    int operator [](int i){ return _sa[i]; }
    void build(int *s,int n,int m){
        memcpy(_s,s,sizeof(int)*n);
        sais(_s,_sa,_p,_q,_t,_c,n,m); mkhei(n);
    }
    void mkhei(int n){
        REP(i,n) r[_sa[i]]=i;
        hei[0]=0;
        REP(i,n) if(r[i]){
            int ans=i>0?max(hei[r[i-1]]-1,0):0;
            while(_s[i+ans]==_s[_sa[r[i]-1]+ans]) ans++;
            hei[r[i]]=ans;
        }
    }
    void sais(int *s,int *sa,int *p,int *q,bool *t,int *c,
        int n,int z){
        bool uniq=t[n-1]=true,neq;
        int nn=0,nmxz=-1,*nsa=sa+n,*ns=s+n,lst=-1;
#define MS0(x,n) memset((x),0,n*sizeof(*(x)))
#define MAGIC(XD) MS0(sa,n);\
        memcpy(x,c,sizeof(int)*z); XD;\
        memcpy(x+1,c,sizeof(int)*(z-1));\
        REP(i,n) if(sa[i]&&!t[sa[i]-1]) sa[x[s[sa[i]-1]]++]=sa[i]-1;\
        memcpy(x,c,sizeof(int)*z);\
        for(int i=n-1;i>=0;i--) if(sa[i]&&t[sa[i]-1]) sa[--x[s[sa[i]-1]]]=sa[i]-1;
        MS0(c,z); REP(i,n) uniq&=++c[s[i]]<2;
        REP(i,z-1) c[i+1]+=c[i];
        if(uniq) { REP(i,n) sa[--c[s[i]]]=i; return; }
        for(int i=n-2;i>=0;i--){
            t[i]=(s[i]==s[i+1]?t[i+1]:s[i]<s[i+1]);
            MAGIC(REP1(i,1,n-1) if(t[i]&&!t[i-1]) sa[--x[s[i]]]=p[q[i]=nn++]=i);
            REP(i,n) if(sa[i]&&t[sa[i]]&&!t[sa[i]-1]){
                neq=lst<0||memcmp(s+sa[i],s+lst,(p[q[sa[i]]+1]-sa[i])*sizeof(int));
                ns[q[lst=sa[i]]]=nmxz+neq;
            }
        }
        sais(ns,nsa,p+nn,q+n,t+n,c+z,nn,nmxz+1);
        MAGIC(for(int i=nn-1;i>=0;i--) sa[--x[s[p[nsa[i]]]]]=p[nsa[i]]);
    }
}sa;
int H[N],SA[N],RA[N];
void suffix_array(int* ip,int len){
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len]=0
    ip[len++]=0; sa.build(ip,len,128);
    memcpy(H,sa.hei+1,len<<2); memcpy(SA,sa._sa+1,len<<2);
    for(int i=0;i<len;i++) RA[i]=sa.r[i]-1;
    // resulting height, sa array \in [0,len)
}

```

## 6.3 SuffixAutomata 815370

```

// any path start from root forms a substring of S
// occurrence of P: iff SAM can run on input word P
// number of different substring: ds[1]-1
// total length of all different substring: dsl[1]
// max/min length of state i: mx[i]/mx[mom[i]]+1
// assume a run on input word P end at state i:
// number of occurrences of P: cnt[i]
// first occurrence position of P: fp[i]-|P|+1
// all position: !clone nodes in dfs from i through rmom
const int MXM=1000010;
struct SAM{

```

```

    int tot,root,lst,mom[MXM],mx[MXM]; // ind[MXM]
    int nxt[MXM][33]; // cnt[MXM],ds[MXM],dsl[MXM],fp[MXM]
    // bool v[MXM],clone[MXM]
    int newNode(){
        int res=++tot; fill(nxt[res],nxt[res]+33,0);
        mom[res]=mx[res]=0; // cnt=ds=dsl=fp=v=clone=0
        return res;
    }
    void init(){ tot=0;root=newNode();lst=root; }
    void push(int c){
        int p=lst,np=newNode(); // cnt[np]=1,clone[np]=0
        mx[np]=mx[p]+1; // fp[np]=mx[np]-1
        for(;;p&&nxt[p][c]==0;p=mom[p]) nxt[p][c]=np;
        if(p==0) mom[np]=root;
        else{
            int q=nxt[p][c];
            if(mx[p]+1==mx[q]) mom[np]=q;
            else{
                int nq=newNode(); // fp[nq]=fp[q],clone[nq]=1
                mx[nq]=mx[p]+1;
                for(int i=0;i<33;i++) nxt[nq][i]=nxt[q][i];
                mom[nq]=mom[q]; mom[q]=nq; mom[np]=nq;
                for(;;p&&nxt[p][c]==q;p=mom[p]) nxt[p][c]=nq;
            }
        }
        lst=np;
    }
    void calc(){
        calc(root); iota(ind,ind+tot,1);
        sort(ind,ind+tot,[&](int i,int j){return mx[i]<mx[j];});
        for(int i=tot-1;i>=0;i--){
            cnt[mom[ind[i]]]+=cnt[ind[i]];
        }
    }
    void calc(int x){
        v[x]=ds[x]=1;dsl[x]=0; // rmom[mom[x]].push_back(x);
        for(int i=0;i<26;i++){
            if(nxt[x][i]){
                if(!v[nxt[x][i]]) calc(nxt[x][i]);
                ds[x]+=ds[nxt[x][i]];
                dsl[x]+=dsl[nxt[x][i]]+dsl[nxt[x][i]];
            }
        }
    }
    void push(char *str){
        for(int i=0;str[i];i++) push(str[i]-'a');
    }
} sam;

```

## 6.4 Z Value 391d23

```

void z_value(const char *s,int len,int *z){
    z[0]=len;
    for(int i=1,l=0,r=0;i<len;i++){
        z[i]=i<r?(i-l+z[i-l]<z[l]?z[i-l]:r-i):0;
        while(i+z[i]<len&&s[i+z[i]]==s[z[i]]) ++z[i];
        if(i+z[i]>r) l=i,r=i+z[i];
    }
}

```

## 6.5 BWT f8c84c

```

const int SIGMA=26; const char BASE='a';
struct BurrowsWheeler{
    vector<int> v[SIGMA];
    void BWT(char* ori,char* res){
        // make ori -> ori+ori and then build suffix array
    }
    void iBWT(char* ori,char* res){
        for(int i=0;i<SIGMA;i++) v[i].clear();
        int len=strlen(ori); vector<int> a;
        for(int i=0;i<len;i++) v[ori[i]-BASE].push_back(i);
        for(int i=0,ptr=0;i<SIGMA;i++){
            for(auto j:v[i]){
                a.push_back(j); ori[ptr++]=BASE+i;
            }
        }
        for(int i=0,ptr=0;i<len;i++){
            res[i]=ori[a[ptr]]; ptr=a[ptr];
        }
        res[len]=0;
    }
} bwt;

```



## 6.6 ZValue Palindrome 44a8c2

```
void z_value_pal(char *s,int len,int *z){
    len=(len<<1)+1; z[0]=1;
    for(int i=len-1;i>=0;i--) s[i]=i&1?s[i>>1]:'0';
    for(int i=1,l=0,r=0;i<len;i++){
        z[i]=i<r?min(z[l+1-i],r-i):1;
        while(i-z[i]>=0&&i+z[i]<len&&s[i-z[i]]==s[i+z[i]])
            ++z[i];
        if(i+z[i]>r) l=i,r=i+z[i];
    }
}
```

## 6.7 Smallest Rotation 28fe26

```
//rotate(begin(s),begin(s)+minRotation(s),end(s))
int minRotation(string s) {
    int a = 0, N = s.size(); s += s;
    rep(b,0,N) rep(k,0,N) {
        if(a+k == b || s[a+k] < s[b+k])
            {b += max(0, k-1); break;}
        if(s[a+k] > s[b+k]) {a = b; break;}
    } return a;
}
```

## 6.8 Cyclic LCS bcba38

```
const int L=0,LU=1,U=2,mov[3][2]={0,-1,-1,-1,-1,0};
int al,bl,dp[MAXL*2][MAXL];
char a[MAXL*2],b[MAXL*2],pred[MAXL*2][MAXL]; // 0-based
inline int lcs_length(int r) {
    int i=r+al,j=bl,l=0;
    while(i>r){
        char dir=pred[i][j]; if(dir==LU) l++;
        i+=mov[dir][0]; j+=mov[dir][1];
    }
    return l;
}
inline void reroot(int r){ // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl){
        if(pred[i+1][j]==U) i++; pred[i][j]=L;
        else if(j<bl&&pred[i+1][j+1]==LU){
            i++; j++; pred[i][j]=L;
        } else j++;
    }
}
int cyclic_lcs(){
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl){
        swap(al,bl);strcpy(tmp,a);strcpy(a,b);strcpy(b,tmp);
    }
    strcpy(tmp,a); strcat(a,tmp);
    // basic lcs
    for(int i=0;i<=2*al;i++){ dp[i][0]=0; pred[i][0]=U; }
    for(int j=0;j<=bl;j++){ dp[0][j]=0; pred[0][j]=L; }
    for(int i=1;i<=2*al;i++){ for(int j=1;j<=bl;j++){
        if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
        else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
        if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
        else if(a[i-1]==b[j-1]) pred[i][j]=LU;
        else pred[i][j]=U;
    }
    }
    int clcs=0; // do cyclic lcs
    for(int i=0;i<al;i++){
        clcs=max(clcs,lcs_length(i)); reroot(i+1);
    }
    a[al]='\0'; // recover a
    return clcs;
}
```

## 7 Data Structure

### 7.1 Link-Cut Tree f3eedd

```
const int MEM=100005;
struct Splay {
    static Splay nil,mem[MEM],*pmem; Splay *ch[2],*f;
    int val,rev,size; // int sum,vir,tot;
    Splay(int _val=-1:val(_val),rev(0),size(1)
    { f=ch[0]=ch[1]=nil; }
    bool isr(){ return f->ch[0]!=this&&f->ch[1]!=this; }
    int dir(){ return f->ch[0]!=this; }
    void setCh(Splay *c,int d){
        ch[d]=c; if(c!=&nil) c->f=this; pull();
    }
    void push(){
        if(!rev) return; swap(ch[0],ch[1]);
        if(ch[0]!=&nil) ch[0]->rev^=1;
        if(ch[1]!=&nil) ch[1]->rev^=1;
        rev=0;
    }
    void pull(){
        size=ch[0]->size+ch[1]->size+1;
        // sum={ch[0]->sum,ch[1]->sum,val}; tot={sum,vir};
        if(ch[0]!=&nil) ch[0]->f=this;
        if(ch[1]!=&nil) ch[1]->f=this;
    }
}; Splay::nil,Splay::mem[MEM],*Splay::pmem=Splay::mem;
Splay *nil=&Splay::nil; vector<Splay*> splayVec;
void rotate(Splay *x){
    Splay *p=x->f; int d=x->dir();
    if(!p->isr()) p->f->setCh(x,p->dir()); else x->f=p->f;
    p->setCh(x->ch[!d],d); x->setCh(p,!d);
}
void splay(Splay *x){
    splayVec.clear();
    for(Splay *q=x; q=q->f){
        splayVec.push_back(q);
        if(q->isr()) break;
    }
    reverse(begin(splayVec),end(splayVec));
    for(auto it:splayVec) it->push();
    while(!x->isr()){
        if(x->f->isr()) rotate(x);
        else if(x->dir()==x->f->dir())
            rotate(x->f),rotate(x);
        else rotate(x),rotate(x);
    }
}
int id(Splay *x){ return x-Splay::mem+1; }
Splay* access(Splay *x){
    Splay *q=nil;
    for(;x!=nil;x=x->f){
        splay(x); // x->vir+=x->ch[0]->tot-{-q->tot};
        x->setCh(q,1); q=x;
    }
    return q;
}
void chroot(Splay *x){ access(x); splay(x); x->rev^=1; }
void link(Splay *x,Splay *y){
    chroot(y); access(x); splay(x); y->f=x;
    // x->vir+={y->tot};
}
void cut_p(Splay *y){
    access(y);splay(y); y->ch[0]=y->ch[0]->f=nil;
}
void cut(Splay *x,Splay *y){ chroot(x); cut_p(y); }
Splay* get_root(Splay *x) {
    x=access(x);
    for(;x->ch[0]!=nil;x=x->ch[0]) x->push();
    splay(x); return x;
}
bool conn(Splay *x,Splay *y){
    return get_root(x)==get_root(y);
}
Splay* lca(Splay *x,Splay *y){
    access(x); return access(y);
}
/* query(Splay *x,Splay *y){ // path
    setroot(y),x=access(x); return x->size; // x->sum;
} */
/* query(Splay *x,Splay *y){ // path
    Splay *p=lca(x,y);
    return 1+p->ch[1]->size+(x!=p?x->size:0);
    // {p->val,p->ch[1]->sum,x!=p?x->sum:0};
} */
```



```
/* query(Splay *x){ // subtree
    access(x); return {x->val,x->vir};
} */
```

## 8 Others

### 8.1 Find max tangent(x,y is increasing) 8fea15

```
const int MAXN=100010;
Pt sum[MAXN],pnt[MAXN],ans,calc;
inline bool cross(Pt a,Pt b,Pt c){
    return (c.y-a.y)*(c.x-b.x)>(c.x-a.x)*(c.y-b.y);
} // pt[0]=(0,0);pt[i]=(i,pt[i-1].y+dy[i-1]),i=1~n;dx>=1
double find_max_tan(int n,int l,LL dy[]){
    int np,st,ed,now; sum[0].x=sum[0].y=np=st=ed=0;
    for(int i=1,v;i<=n;i++){
        sum[i].x=i,sum[i].y=sum[i-1].y+dy[i-1];
        ans.x=now=1,ans.y=-1;
        for(int i=0;i<=n-l;i++){
            while(np>1&&cross(pnt[np-2],pnt[np-1],sum[i])) np--;
            if(np<now&&np!=0) now=np;
            pnt[np++]=sum[i];
            while(now<np&&!cross(pnt[now-1],pnt[now],sum[i+l]))
                now++;
            calc=sum[i+l]-pnt[now-1];
            if(ans.y*calc.x<ans.x*calc.y)
                ans=calc,st=pnt[now-1].x,ed=i+l;
        }
        return (double)(sum[ed].y-sum[st].y)/(sum[ed].x-sum[st].x);
    }
}
```

### 8.2 Exact Cover Set 91d0af

```
// given n*m 0-1 matrix, find a set of rows s.t.
// for each column, there's exactly one 1
const int N=1024,M=1024,NM=((N+2)*(M+2)) // row,col
bool A[N][M]; // n*m 0-1 matrix
bool used[N]; // answer: the row used
int id[N][M];
int L[NM],R[NM],D[NM],U[NM],C[NM],S[NM],ROW[NM];
void remove(int c){
    L[R[c]]=L[c]; R[L[c]]=R[c];
    for(int i=D[c];i!=c;i=D[i]){
        for(int j=R[i];j!=i;j=R[j]){
            U[D[j]]=U[j]; D[U[j]]=D[j]; S[C[j]]--;
        }
    }
}
void resume(int c){
    for(int i=D[c];i!=c;i=D[i]){
        for(int j=L[i];j!=i;j=L[j]){
            U[D[j]]=D[U[j]]=j; S[C[j]]++;
        }
    }
    L[R[c]]=R[L[c]]=c;
}
bool dfs(){
    if(R[0]==0) return 1;
    int md=100000000,c;
    for(int i=R[0];i!=0;i=R[i]) if(S[i]<md){md=S[i]; c=i;}
    if(md==0) return 0;
    remove(c);
    for(int i=D[c];i!=c;i=D[i]){
        used[ROW[i]]=1;
        for(int j=R[i];j!=i;j=R[j]) remove(C[j]);
        if(dfs()) return 1;
        for(int j=L[i];j!=i;j=L[j]) resume(C[j]);
        used[ROW[i]]=0;
    }
    resume(c); return 0;
}
bool exact_cover(int n,int m){
    for(int i=0;i<=m;i++){
        R[i]=i+1; L[i]=i-1; U[i]=D[i]=i; S[i]=0; C[i]=i;
    }
    R[m]=0; L[0]=m; int t=m+1;
    for(int i=0;i<n;i++){
        int k=-1;
        for(int j=0;j<m;j++){
            if(!A[i][j]) continue;
            if(k==-1) L[t]=R[t]=t;
        }
    }
}
```

```
else{ L[t]=k; R[t]=R[k]; }
k=t; D[t]=j+1; U[t]=U[j+1];
L[R[t]]=R[L[t]]=U[D[t]]=D[U[t]]=t;
C[t]=j+1; S[C[t]]++; ROW[t]=i; id[i][j]=t++;
}
}
for(int i=0;i<n;i++) used[i]=0;
return dfs();
}
```

### 8.3 Binary Next Permutation b7a40a

```
ull next_perm(ull v){
    ull t=vl(v-1);
    return (t+1)|(((~t&~t)-1)>>(__builtin_ctzll(v)+1));
}
```

### 8.4 Hilbert Curve 15d26a

```
long long hilbert(int n,int x,int y){
    long long res=0;
    for(int s=n/2;s>=1;s--){
        int rx=(x&s)>0,ry=(y&s)>0; res+=s*1ll*s*((3*rx)^ry);
        if(ry==0){ if(rx==1) x=s-1-x,y=s-1-y; swap(x,y); }
    }
    return res;
}
```