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## 1 Basic

#### 1.1 Basic [4ef69d]

```
// 如何安裝 vscode
// 1. 下載 vscode & msys2
// 2. 在跳出的 terminal 中 / 或打開 ucrt64,打上
      "pacman -S --needed base-devel mingw-w64-x86_64-toolchain"
// 3. 環境變數加上 C:\\msys64\\ucrt64\\bin
// 4. 重開 vscode, 載 C/C++, 運行, 編譯器選擇 g++
// 5. 打開 settings -> compiler -> add compilerPath
-> 在 "" 裡打上 C:\\msys64\\ucrt64\\bin\\g++.exe
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#Include <ext/pb_ds/tree_policy.npp>
#define all(x) (x).begin(), (x).end()
#define pii pair<int, int>
#define endl "\n"
#define int long long
using namespace std;
    uct cmp {  // 在有 template 的資結使用
bool operator()(const int &a, const int &b) const {
struct cmp {
         return a < b;
     // sort, bound 不用 struct
    // priority queue 小到大是 > , set 是 <
     // set 不能 = , multiset 要 =
     // 每個元素都要比到,不然會不見
    // pbds_multiset 的 upper_bound 跟 lower_bound
           功能相反,如果要 find,插入 inf 後使用 upper_bound
     // 內建 multiset
           可以跟 set 一樣正常使用, 自定義比較結構就比照以上
}:
struct cmp {
   vector<int> &v;
     cmp(vector<int>& vec) : v(vec) {}
     bool operator() (int a, int b) const {
              部向量來比較元素的優先級,記得不要改到比較 vector
         return v[a] > v[b];
    }
};
// main:
// cmp cmp1(vector);
// priority_queue<int, vector<int>, cmp> pq(cmp1);
const int llinf = 4e18;
const int inf = 2e9;
const int mod = 1e9 + 7;
const int maxn = 2e5 + 5;
void solve() {
}
```

```
signed main() {
    ios_base::sync_with_stdio(0);
    cin.tie(nullptr);
    int t = 1;
    cin >> t;
    while (t--) {
        solve();
    }
}
```

## 2 Graph 2.1 DFS 跟 BFS [2a332e]

```
#include <bits/stdc++.h>
using namespace std;
int main() {
      vector<vector<int>> adj(n + 1, vector<int>());
      // dfs_graph
vector<bool> vis(n + 1, 0);
      auto dfs = [&](auto self, int u) -> void {
   if (vis[u]) return;
   vis[u] = true;
   for (auto v: adj[u]) {
      self(self, v);
   }
}
           }
      dfs(dfs, 1);
      // bfs
       vector<<mark>int</mark>> deep(n + 1, 1e9);
      queue<int> q;
auto bfs = [&](auto self, int u) -> void {
   vis[u] = true;
            deep[u] = 0;
            q.push(u):
             while (!q.empty()) {
                  int now = q.front(); q.pop();
for (auto nxt : adj[now]) {
   if (vis[nxt]) continue;
                         vis[nxt] = true;
                         deep[nxt] = deep[now] + 1;
                         q.push(nxt);
                  }
           }
      bfs(bfs, 1);
```

#### 2.2 DSU [99b9f3]

```
#include <bits/stdc++.h>
using namespace std;
struct DSU {
      vector<int> boss, siz;
      DSU(int n) { // 1 based
boss.resize(n + 1);
iota(boss.begin(), boss.end(), θ);
siz.assign(n + 1, 1);
      int find_boss(int x) {
   if(boss[x] == x) return x;
   return boss[x] = find_boss(boss[x]);
      bool same(int x, int y) {
            return find_boss(x) == find_boss(y);
      bool merge(int x, int y) {
            x = find_boss(x);
y = find_boss(y);
            if (x == y) {
    return false;
            if(siz[x] < siz[y]) swap(x, y);
siz[x] += siz[y];</pre>
             boss[y] = x;
             return true:
      int size(int x) {
    return siz[find_boss(x)];
      }
}:
```

## 2.3 最短距離算法 - Dijkstra [b8dfdb]

```
#include <bits/stdc++.h>
using namespace std;
#define pii pair<int, int>
// Flight Discount
void dijkstra() {
   int n, m; cin >> n >> m;
   vector<vector<pii>>> adj(n + 1, vector<pii>(n + 1));
   vector<vector<int>>
        dis(n + 1, vector<int>(2, 2e9)); // 0 for not used
   for (int i = 1; i <= m; i++) {
      int u, v, w;
        cin >> u >> v >> w;
        adj[u].push_back({v, w});
}
```

```
while (!pq.empty()) {
         }
         else {
              for (auto [v, w] : adj[u]) {
   if (dis[u][0] + w < dis[v][0]) {
      dis[v][0] = dis[u][0] + w;
}</pre>
                       pq.push({dis[v][0], v, 0});
                  if (dis[u][0] + w / 2 < dis[v][1]) {
   dis[v][1] = dis[u][0] + w / 2;
   pq.push({dis[v][1], v, 1});</pre>
             }
        }
    cout << min(dis[n][0], dis[n][1]);
}
```

## 2.4 最小生成樹 - Prim [5318c2]

```
#include <bits/stdc++.h>
using namespace std;
#define pii pair<int, int>
int n, m;
int ans = 0;
const int maxn = 2e5 + 5;
vector<pair<int, int>> adj[maxn];
bool Prim() {
      int node_sz = 0;
      priority_queue<pii, vector<pii>>, greater<pii>>> pq;
pq.push({0, 1});
bool vis[maxn] = {false};
      while (!pq.empty()) {
    auto [cost, u] = pq.top(); pq.pop();
    if (vis[u]) continue;
             vis[u] = true;
            if (node_sz == n) return true;
return false;
void solve() {
     cin >> n >> m;
for(int i = 1; i <= m; i++) {
   int u, v, cost; cin >> u >> v >> cost;
   adj[u].push_back({v, cost});
   adj[v].push_back({u, cost});
     if (Prim()) cout << ans;
else cout << "IMPOSSIBLE";</pre>
```

## 2.5 正權找環 [2a579d]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 1e5+5;
vector<int> graph[maxn];
 int color[maxn], parent[maxn];
bool vis[maxn];
int n. m:
 void print_ans(int ori) {
      int now = parent[ori];
deque < int > ans;
ans.push_front(ori);
      while (now != ori) {
    ans.push_front(now);
    now = parent[now];
      ans.push_front(ori);
      cout << ans.size() << endl;
for (auto i : ans) {
   cout << i << " ";</pre>
      exit(0);
print_ans(nxt);
```

```
else if (color[nxt] == 0) {
               dfs(nxt);
         }
     color[now] = 2;
void solve() {
     cin >> n >> m;
for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
          graph[u].push_back(v);
     for (int i = 1: i <= n: i++) {
          if (!vis[i])
               dfs(i);
     cout << "IMPOSSIBLE";
}
```

## **2.6** 負權找負環 [a27f3b]

```
// 用 Bellman Ford 找負環
#include <bits/stdc++.h>
using namespace std;
vector<array<int, 3>> graph; // u, v, w
int main() {
  int src = 0;
  int n, m;  cin >> n >> m;
  vector<int> par(n + 1), dis(n + 1, 1e9);
  for (int i = 0; i < m; i++) {
    int a, b, w; cin >> a >> b >> w;
}
            graph.push_back({a, b, w});
      dis[1] = 0;
for (int i = 0; i <= n; i++) {
            src = 0;
            for (auto [u, v, w] : graph) {
   if (dis[v] > dis[u] + w) {
      dis[v] = dis[u] + w;
      par[v] = u;
}
                         src = v;
                  }
            }
      if (src) { // 到第 n + 1 次還在鬆弛
vector<int> ans;
cout << "YES" << endl;
            for (int
            . i = 0; i <= n; i++) src = par[src]; // 找那個負環 ans.push_back(src);
            for (int
                   i = par[src]; i != src; i = par[i]) { // 輸出負環
                   ans.push_back(i);
            ans.push back(src):
            reverse(ans.begin(), ans.end());
            for (auto i : ans) {
    cout << i << " ";
            cout << "NO" << "\n";
      }
}
```

## 2.7 正權最大距離 [a3879a]

```
#include <bits/stdc++.h>
using namespace std;
// 只能用在 DAG, 用拓樸按順序鬆弛
// 如果 1 不能到達 n, n 也有
        可能被鬆弛,所以要看的是 dis[n] < 0,不能只看有沒有 = -1e9
void print_ans(int n, vector<int> &par) {
      deque <int> ans;
int now = n;
while(now != 1) {
    ans.push_front(now);
}
             now = par[now];
       ans.push_front(1);
      cout << ans.size() << endl;
for(auto i : ans) {
    cout << i << " ";</pre>
void solve() {
      int n, m;
cin >> n >> m;
       vector < int > dis(n + 1, -1e9); dis[1] = 0;
      vector<int> dts(n + 1, -1e9), dts[1] = 0,
vector<vector<int>> graph(n + 1, vector<int>());
vector<body>
vector<int> par(n + 1);
vector<int> in(n + 1, 0);
      queue int> q;
for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
   int u, v; cin >> u >> v;
             graph[u].push_back(v);
              in[v]++:
       for (int i = 1; i <= n; i++) {
```

```
if(in[i] == 0) q.push(i);
}
while (!q.empty()) {
    int u = q.front(); q.pop();
    for (auto nxt : graph[u]) {
        if (dis[nxt] < dis[u] + 1) { // 縣地
            dis[nxt] = dis[u] + 1;
            par[nxt] = u;
        }
        in[nxt] --;
        if (in[nxt] == 0) q.push(nxt);
        }
        vis[u] = 1;
    }
    if (dis[n] < 0) {
        cout << "IMPOSSIBLE";
    }
    else print_ans(n, par);
}</pre>
```

## 2.8 負權最大距離 [8372e8]

```
#include <bits/stdc++.h>
#define int long long
using namespace std;
// CSES High Score
const int maxn = 2505;
void dfs(int u, vector<int> &vis, vector<vector<int>> &adj) {
   if (vis[u]) return;
     vis[u] = 1;
for (int v : adj[u]) {
    dfs(v, vis, adj);
void bellman_ford
      (int n, int s, vector<int> &vis, vector<int> &dis
, vector<array<int, 3>> edge, vector<vector<int>> &adj) {
fill(dis.begin(), dis.end(), -1e18);
      dis[s] = 0;
     if (i == n) {
    dfs(v, vis, adj);
                 }
           }
     }
signed main() {
   int n, m; cin >> n >> m;
     vector < array < int, 3 >> edge;
vector < vector < int >> adj(n + 1);
      vector < int > dis(n + 1), vis(n + 1);
      while (m--) {
           int u, v, w;
cin >> u >> v >> w;
edge.push_back({u, v, w});
            adj[u].push_back(v);
     bellman_ford(n, 1, vis, dis, edge, adj);
if (vis[n]) cout << -1;
else cout << dis[n];</pre>
}
```

## 2.9 FloydWarshall [410f48]

## 2.10 歐拉環與歐拉路 [1dc3a5]

}

```
#include <bits/stdc++.h>
using namespace std;
// 無向圖、尤拉環: 檢查每個點的出度為偶數
// 有向圖、
      尤拉路:可以看成 1 走到 n,所以檢查所有點的出度等於入度
vector<set<int>> adj;
vector<int> in;
void dfs(int now, vector<int> &road) {
     while (!adj[now].empty()) {
   int nxt = *adj[now].begin();
           adj[now].erase(nxt);
           dfs(nxt, road);
      road.push_back(now);
void solve() {
      cin >> n >> m;
      in.assign(n + 1, 0);
adj.assign(n + 1, set<int>());
for (int i = 1; i <= m; i++) {</pre>
           int u, v; cin >> u >> v;
adj[u].insert(v);
           in[v]++;
      in[1]++;
     in[n]--;
for (int i = 1; i <= n; i++) {
    if(adj[i].size() != in[i]) {
        cout << "IMPOSSIBLE";
}</pre>
          }
      vector<int> road;
      dfs(1, road);
      if (road.size() != m + 1) {
    cout << "IMPOSSIBLE";</pre>
           return:
      reverse(road.begin(), road.end());
for(auto i : road) cout << i << " ";</pre>
```

#### 2.11 SCC 結合拓樸 DP [382a7f]

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
    找到所有 SCC 然後結合原圖重建一個 DAG, 然後拓樸 DP
void dfs(int u, vector<int</pre>
     > &vis, vector<int> &kosaraju, vector<vector<int>> &adj) {
if (!vis[u]) {
           vis[u] = 1;
           for (auto v : adj[u]) {
                dfs(v, vis, kosaraju, adj);
           kosaraju.push_back(u); // finish time 小到大排列
     }
void rev_dfs(int u, vector<int> &vis, vector<
    int> &order, vector<vector<int>> &rev_adj, int &scc_num) {
    if (!vis[u]) {
          vis[u] = 1;
order[u] = scc_num;
for (auto v : rev_adj[u]) {
    rev_dfs(v, vis, order, rev_adj, scc_num);
}
     }
signed main() {
     int n, m, scc_num = 0;
cin >> n >> m;
vector<int> coin(n + 1), order(n + 1), vis(n + 1, 0);
vector<vector<int>> adj(n + 1), rev_adj(n + 1);
     vector <int> kosaraju;
for (int i = 1; i <= n; i++) {
    cin >> coin[i];
     for (int i = 1; i <= m; i++) {
  int u, v; cin >> u >> v;
  adj[u].push_back(v);
           rev_adj[v].push_back(u);
     for (int i = 1; i <= n; i++) {
           if (!vis[i]) {
                 dfs(i, vis, kosaraju, adj);
     reverse(kosaraju.begin(), kosaraju
     (-1,0) .end()); // 轉過來,從 finish time 大的開始做 dfs vis.assign(n + 1, 0);
     for (auto &u : kosaraju) {
           if (!vis[u]) {
                scc_num++;
                rev_dfs(u, vis, order, rev_adj, scc_num);
```

```
·
// 重新建 DAG,根據原圖,如果不再同個 SCC,對 order 加邊
       vector<vector<int>> DAG(scc_num + 1, vector<int>());
vector<int> in_degree(scc_num + 1, 0);
       vector<int
       DAG[order[i]].push_back(order[j]);
                            in_degree[order[j]]++;
                           st.insert({order[i], order[j]});
                    }
             }
       }
      q.push(i);
       for confined; i= sar_cotfinew;;
for (auto v : DAG[now]) {
   in_degree[v]--;
   dp_coin[v] = max(dp_coin[v], dp_coin[now]);
   if (in_degree[v] == 0) q.push(v);
}
              }
       cout << ans;
2.12 2-SAT [7b512c]
#include <bits/stdc++.h>
#Include <pts/stoc++.n>
using namespace std;
// +(-) u or +(-) v
const int maxn = 1e5 + 5;
vector <int> adj[2 * maxn], rev_adj[2 * maxn];
vector <int> order;
int cat[2 * maxn];
int k = 1;
back vis[2 * maxe];
bool vis[2 * maxn];
void dfs(int now) {
    if (!vis[now]) {
              vis[now] = 1;
for (auto v : adj[now]) {
    dfs(v);
              order.push_back(now);
      }
void rev_dfs(int now) {
      if (!vis[now]) {
   cat[now] = k;
   vis[now] = 1;
   for (auto v : rev_adj[now]) {
       rev_dfs(v);
}
      }
int main() {
       int n, m;
       cin >> m >> n;
for (int i = 1; i <= m; i++) {</pre>
              int u, v;
             char a, b;
cin >> a >> u >> b >> v;
if (a == '-') {
    u = 2 * n - u + 1; // reverse
              if (b == '-') {
    v = 2 * n - v + 1; // reverse
             }
adj[2 * n - u + 1].
    push_back(v); // from -u to v; // if -u, then v
adj[2 * n - v + 1].
    push_back(u); // from -v to u; // if -v, then u
rev_adj[v].push_back(2 * n - u + 1);
rev_adj[u].push_back(2 * n - v + 1);
       for (int i = 1; i <= 2 * n; i++) {
    if (!vis[i]) {</pre>
                    dfs(i):
             }
       ,
memset(vis, 0, sizeof(vis));
reverse(order.begin(), order.end());
for (auto i : order) {
    if (!vis[i]) {
                     rev_dfs(i);
              }
```

```
char ans[2 * n + 1];
for (int i = 1; i <= n; i++) {
    if (cat[i] == cat[2 * n - i + 1]) {
        cout << "IMPOSSIBLE";
        return;
    }
    if (cat[i] > cat[2 * n - i + 1]) {
        ans[i] = '+';
    }
    else ans[i] = '-';
}
for (int i = 1; i <= n; i++) {
    cout << ans[i] << " ";
}</pre>
```

### 2.13 Planets Cycles [391e2a]

```
#include <bits/stdc++.h>
using namespace std;
vector<int> dis, v;
vector<bool> vis;
int step;
queue < int > path;
void dfs(int x) {
      path.push(x);
      if (vis[x]) {
    step += dis[x];
            return;
      vis[x] = true:
      step++;
      dfs(v[x]);
// count path_dis to rep
int main() {
   int n; cin >> n;
      v.assign(n + 1, 0);
      dis.assign(n + 1, 0);
vis.assign(n + 1, false);
for (int i = 1; i <= n; i++) {
            cin >> v[i];
      for (int i = 1; i <= n; i++) {
            step = 0;
            int is_outof_cycle = 1;
            int tage
dfs(i);
while (!path.empty()) {
    if (path.front() == path.back()) {
        if outof cycle = 0;
}
                  dis[path.front()] = step;
                  step -= is_outof_cycle;
path.pop();
           }
      for (int i = 1; i <= n; i++) {
    cout << dis[i] << ' ';</pre>
      cout << '\n';
}
```

### 2.14 Planet Queries II [8c2a64]

```
#include <bits/stdc++.h>
using namespace std;
// now on a and want to reach b, the min steps, directed
int n, q;
const int maxn = 2e5 + 5;
int dp[30][maxn];
vector<vector<int>> cycles;
int no[maxn]; // Order & Can be in cycle, or out
int cycle_idx[maxn];
bool vis[maxn];
void set_out_of_cycle_no(int now, unordered_set<int> &done) {
    if (done.find(now) != done.end())
      set_out_of_cycle_no(dp[0][now], done);
done.insert(now);
      no[now] = no[dp[0][now]] - 1;
int wiint_go_to(int u, int k) { // return the node when walk k
    for (int i = 0; i <= 18; i++) {
        if (k & (1 << i)) {</pre>
                u = dp[i][u];
           }
      return u;
void find_cycle(int now) {
     unordered_set <int> appear;
vector <int> vec;
bool flag = true;
      while (appear.find(now) == appear.end()) {
           appear.insert(now);
           vec.push_back(now);
           if (vis[now]) { // Didn't Find Cycle
                 flag = false;
                 break;
           now = dp[0][now];
```

```
for (auto i : vec) vis[i] = true;
     int m = vec.size();
     vector < int > cycle;
for (int i = z; i < m; i++) {</pre>
           cycle.push_back(vec[i]);
     cycles.push_back(cycle);
void solve() {
     cin >> n >> q;
for (int u = 1; u <= n; u++) {</pre>
           cin >> dp[0][u];
     for (int i = 1; i <= 18; i++) { // Make Chart
    for (int u = 1; u <= n; u++) {
        dp[i][u] = dp[i - 1][dp[i - 1][u]];</pre>
     for (int i = 1; i <= n; i++) {
    if (!vis[i]) find_cycle(i);</pre>
      int idx = 0;
     memset(no, -1, sizeof(no));
memset(cycle_idx, -1, sizeof(cycle_idx));
unordered_set<int> done;
     for (auto &i : cycles) {
   int c = 0;
           for (auto &j : i) {
               no[j] = c++;
cycle_idx[j] = idx;
done.insert(j);
           idx++:
     for (int i = 1; i <= n; i++) set_out_of_cycle_no(i, done);
for (int i = 1; i <= q; i++) {
   int u, v; cin >> u >> v;
           int u, v;
           // Same Cycle
           if (cycle_idx[u] == cycle_idx
    [v] && cycle_idx[u] != -1 && cycle_idx[v] != -1) {
    int cyc_size = cycles[cycle_idx[u]].size();
                cout <<
     (no[v] - no[u] + cyc_size) % cyc_size << "\n";</pre>
           continue;
                int jump = no[v] - no[u];
                if (wiint_go_to(u, jump) == v) {
    cout << jump << "\n";</pre>
                else cout << -1 << "\n";
           else if (cycle_idx[u] == -1 && cycle_idx[v]
                != -1) { // v is in cycle, Smainter Binary Search
int l = -1, r = n;
while (l <= r) {
    int m = (l + r) / 2;
    if (real-int)
                      if (cycle_idx
                            [wiint_go_to(u, m)] == cycle_idx[v]) {
                     else
l = m + 1;
                else cout << -1 << "\n";
           else { // u is death in the cycle, can't reach
                cout << -1 << "\n";
     }
}
```

#### 3 Data Structure

#### 3.1 BIT [d41d8c]

```
bit.resize(n + 1, 0);
for (int i = 1; i <= n; i++) {
                   modify(i, init[i]);
       void modify(int i, int val) {
    for (; i <= n; i += i & -i) {
        bit[i] += val;
}</pre>
      int query(int r) {
            int ans = 0;
for (; r; r -= r & -r) ans += bit[r];
             return ans:
      int query(int l, int r) {
    return query(r) - query(l - 1);
      }
struct TwoDimensionBIT {
      int nx, ny;
vector<vector<int>>> bit;
      TwoDimensionBIT(int x, int y) {
            nx = x; ny = y;
bit.resize(x + 1, vector<int>(y + 1, 0));
      for (; x <= nx; x += x & -x) {
    for (int tmp = y; tmp <= ny; tmp += tmp & -tmp) {
        bit[x][tmp] += mod;
}</pre>
            }
      int query(int r1, int r2) {
             int ans = 0;
            for (; r1; r1 -= r1 & -r1) {
    for (int tmp = r2; tmp; tmp -= tmp & -tmp) {
        ans += bit[r1][tmp];
}
             return ans:
      }
};
```

### 3.2 Increasing Array Queries [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
int n, q;
int nums
[maxn], prefix[maxn], ans[maxn], BIT[maxn], contrib[maxn];
vector<pair<int, int>> queries[maxn];
void update(int pos, int val) {
   for (; pos <= n; pos += pos & -pos) BIT[pos] += val;</pre>
int query(int a, int b) {
                int ans = 0;
for (; b; b -= b&-b) ans += BIT[b];
                 for (a--; a; a -= a&-a) ans -= BIT[a];
void solve() {
                cin >> n >> q;
for (int i = 1; i <= n; i++) {
                                cin >> nums[i];
prefix[i] = prefix[i-1] + nums[i];
                nums[n + 1] = 1e9;
                nums[n + 1] = 1ey;
prefix[n + 1] = 2e18;
for (int i = 1; i <= q; i++) {
   int a, b; cin >> a >> b;
   queries[a].push_back({b, i});
                deque<int> mono; mono.push_front(n+1);
               deque<int> mono; mo
                                 contrib[i] = (mono.front() - 1 - i) *
                                                      nums[i] - (prefix[mono.front() - 1] - prefix[i]);
                            - mono[pos]) * nums[mono[pos]]
- (prefix
                                                                                                                             [j.first] - prefix[mono[pos]]);
                               }
                for (int i = 1; i <= q; i++) {
                                 cout << ans[i] << endl;
```

# 3.3 線段樹 [d41d8c]

| }

```
#include <bits/stdc++.h>
 using namespace std;
template <class Node>
 struct Seg {
   int n;
      vector < Node > tree;
     Seg (vector<Node> init_) {
          n = init_.size() - 1;
tree.resize(4 * n);
function < void(int</pre>
               , int, int)> build = [&](int now, int l, int r) {
if (l == r) {
    tree[now] = init_[l];
                    return:
              int m = (l + r) / 2;
build(now << 1, l, m);
build((now << 1) + 1, m + 1, r);
pull(now);</pre>
          build(1, 1, n);
      Node query(int l, int r, int ql, int qr, int now) {
          int m = (l + r) >> 1;
if (qr < l || ql > r) {
               return Node();
          if (ql <= l && r <= qr) {
               return tree[now];
          Node query(int l, int r) { return query(1, n, l, r, 1); }
void pull(int now) {
    tree[now] = tree[now << 1] + tree[(now << 1) + 1];
      void modify(int l, int r, int idx, int now, int add) {
    if (l == r) {
      how to modify ?-----
               tree[now].sum = add;
               return:
           int m = (l + r) >> 1;
          if (idx <= m) {
               modify(l, m, idx, now << 1, add);</pre>
               modify(m + 1, r, idx, (now << 1) + 1, add);</pre>
          pull(now);
     void modify
           (int idx, int add) { modify(1, n, idx, 1, add); }
      define structure and info plus-----
 struct Node {
     int sum;
Node () {
          sum = 0:
 Node operator + (const Node &a, const Node &b) {
     Node c;
     c.sum = a.sum + b.sum;
     return c;
// use lc rc to undate now
// tree[now].sum = tree[lc].sum + tree[rc].sum;
// tree[now].prefix
            = max(tree[lc].sum+tree[rc].prefix, tree[lc].prefix);
     [rc].middle_max), tree[lc].suffix+tree[rc].prefix);
     // tree[now].middle_max = max(max(tree[
    now].middle_max, tree[now].prefix), tree[now].suffix);
 // pizza_queries
 // 左邊的店(s < t): dis_l = (pizza[s] - s) + t;
 // 右邊的店(t < s): dis_r = (pizza[s] + s) - t;
// 實作: 建左查詢線段樹跟右查詢線段樹,用最小值pull
 // 答案是 min(left_query(1, s) + t, right_query(s, end) + t);
 // List Removals
// 維護區間內有幾個數字被選過
```

```
|// 用二
| 分搜找右區間最小位,使得 ans - query == 1~ans 被選過的數量
```

## 3.4 懶標線段樹 [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
template <class Node, class Lazy>
struct LazySeg {
      int n;
vector<Node> tree;
      vector<Lazy> lazy;
      template <typename T>
      LazySeg (vector<T> init_) { // 必須是 1-based
n = init_.size() - 1;
tree.resize(4 * n);
lazy.resize(4 * n);
            function <void(int
    , int, int) > build = [&](int now, int l, int r) {
    if (l == r) {
        tree[now] = init_[l];
}
                        return:
                  int m = (l + r) / 2;
build(now << 1, l, m);
build((now << 1) + 1, m + 1, r);</pre>
                  pull(now);
            build(1, 1, n);
      Node query(int l, int r, int ql, int qr, int now) {
   int m = (l + r) >> 1;
   if (qr < l || ql > r) {
       out of range, return what-----
return Node();
//
            push(now, l, r);
if (ql <= l && r <= qr) {</pre>
                  return tree[now];
            return query(l, m, ql, qr, now
<< 1) + query(m + 1, r, ql, qr, (now << 1) + 1);
      Node query(int l, int r) { return query(1, n, l, r, 1); } void pull(int now) {
            tree[now] = tree[now << 1] + tree[(now << 1) + 1];
      void modify_add
            (int |, int r, int ql, int qr, int now, int add) {
int m = (l + r) >> 1;
if (qr < l || ql > r) {
            if (ql <= l && r <= qr) {
       how to modify ?-----
                  lazy[now].add += add;
       .....
                  return:
            push(now, l, r);
modify_add(l, m, ql, qr, now << 1, add);
modify_add(m + 1, r, ql, qr, (now << 1) + 1, add);
push(now << 1, l, m);
push((now << 1) + 1, m + 1, r);</pre>
            pull(now);
      void modify add(int
      l, int r, int add) { modify_add(1, n, l, r, 1, add); }
void modify_set
            (int l, int r, int ql, int qr, int now, int val) {
int m = (l + r) >> 1;
if (qr < l || ql > r) {
                  return:
            if (ql <= l && r <= qr) {
       how to modify ?-----
                  lazy[now].set_val = val;
                  lazy[now].add = 0;
                  return;
            push(now, l, r);
modify_set(l, m, ql, qr, now << 1, val);
modify_set(m + 1, r, ql, qr, (now << 1) + 1, val);
push(now << 1, l, m);
push((now << 1) + 1, m + 1, r);</pre>
            pull(now);
      }
```

}

int size(Treap \*treap) {

if (treap == NULL) return 0;

```
void modify_set(int
    l, int r, int val) { modify_set(1, n, l, r, 1, val); }
void push(int now, int l, int r) {
    apply(now, l, r);
                                                                                               return treap->subsize:
                                                                                          // lazy
                                                                                          void push(Treap *t) {
   if (!t) return;
                                                                                               if (t->rev_valid) {
      how to push down ?-----
                                                                                                     swap(t->l, t->r);
if (t->l) t->l->rev_valid ^= 1;
if (t->r) t->r->rev_valid ^= 1;
           if (l != r) {
                if (lazy[now].set_val) {
                     lazy[now << 1].set_val = lazy[now].set_val;
lazy[(now</pre>
                          << 1) + 1].set_val = lazy[now].set_val;
lazy[now << 1].add = lazy[now].add;
lazy[(now << 1) + 1].add = lazy[now].add;</pre>
                                                                                               t->rev valid = false:
                                                                                          Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) return a ? a : b;
    // push(a); push(b); // lazy
                     lazy[now << 1].add += lazy[now].add;
lazy[(now << 1) + 1].add += lazy[now].add;
                                                                                                if (a->pri > b->pri) {
                                                                                                     a->r = merge
                                                                                                           (a->r, b); // a->r = new, inorder, make sense
          }
                                                                                                     a->pull();
//
                                                                                                     return a;
                                                                                                     b->l = merge
                                                                                                     (a, b->l); // new->l = a, inorder, make sense
b->pull();
           lazv[now] = Lazv():
     void apply(int now, int l, int r) {
                                                                                                     return b;
          if (lazy[now].set_val) {
    tree[now].sum = (r - l + 1) * lazy[now].set_val;
                                                                                               }
                                                                                          tree[now].sum += (r - l + 1) * lazy[now].add;
     }
                                                                                                     auto [a, b] = split(root->r, k - size(root->l) - 1);
root->r = a;
     define structure and info plus-----
struct Node {
    int sum;
                                                                                                     root->pull();
                                                                                                     return {root, b};
struct Lazy {
     int set_val; int add;
                                                                                                else {
                                                                                                     auto [a, b] = split(root->l, k);
root->l = b;
Node operator+(const Node &a. const Node &b) {
     return {{a.sum + b.sum}};
                                                                                                     root->pull();
                                                                                                     return {a, root};
                                                                                               }
                                                                                          void Print(Treap *t) {
                                                                                               if (t) {
    // push(t);
                                                                                                                        // lazy
                                                                                                     Print(t->l);
// polynomial queries
                                                                                                     cout << t->val;
Print(t->r);
// 設置梯形的底跟加了幾次, apply_tag時底為
     l的合, d為加給次, 所以sum += (底*2 + 次*區間) * 區間 / 2;
                                                                                               }
3.5 莫隊 [d41d8c]
                                                                                          #include <bits/stdc++.h>
wsing namespace std;
struct query {
   int l, r, id;
} typedef query;
void MO(int n, vector<query> &queries) {
   int block = sqrt(n);
   function sheel(query | query)> cmn = |
                                                                                                     root = merge(root, new Treap(c));
                                                                                                for(int i = 1; i <= m; i++) {
                                                                                                    int t = 1; t <= m; i++) {
int x, y; cin >> x >> y;
auto [a, b] = split(root, x-1); // a: 1~x-1, b: x~n
auto [c, d] = split(b, y-x+1); // Use b to split
// c->rev_valid ^= true;
// push(c);
b = merge(a d): // Nation the first
     function <bool(query, query)> cmp = [&](query a, query b) {
   int block_a = a.l / block;
   int block_b = b.l / block;
           if (block_a != block_b) return block_a < block_b;</pre>
           return a.r < b.r;
                                                                                                     b = merge(a, d); // Notice the order
root = merge(b, c);
     sort(queries.begin(), queries.end(), cmp);
                                                                                               Print(root);
void compress(vector<int> &nums) {
                                                                                          }
     vector < int > sorted = nums;
sort(sorted.begin(), sorted.end());
                                                                                                 Flow
                                                                                          4
     sorted.erase
            (unique(sorted.begin(), sorted.end()), sorted.end());
                                                                                          4.1 Dinic [4d1a72]
     for (int i = 0; i < nums.size(); i++) {</pre>
           nums[i] = lower_bound(sorted.begin
                                                                                          #include <bits/stdc++.h>
                 (), sorted.end(), nums[i]) - sorted.begin() + 1;
                                                                                          using namespace std;
bool vis[505];
                                                                                          int lev[505], n, m, ans;
struct edge {
3.6 Treap [d41d8c]
                                                                                              int to, w, rev_ind;
#include <bits/stdc++.h>
using namespace std;
struct Treap {
    Treap *l, *r;
                                                                                          vector<edge> adj[505];
                                                                                          bool label_level
                                                                                               () { // Tag the depth, if can't reach end => return false
memset(lev, -1, sizeof(lev));
lev[1] = 0;
queue<int> q; q.push(1);
     Treap *l, *r;
int pri, subsize; char val; bool rev_valid;
     Treap(int val) {
          this - > val = val;
                                                                                               while (!q.empty()) {
    int u = q.front(); q.pop();
           pri = rand();
           l = r = nullptr;
                                                                                                     for (auto i : adj[u]) {
    if (i.w > 0 && lev[i.to] == -1) {
           subsize = 1; rev_valid = 0;
     void pull() {
                                                                                                                q.push(i.to);
                           // update subsize or other information
          subsize = 1;
for(auto i : {l, r}) {
    if (i) subsize += i->subsize;
                                                                                                                lev[i.to] = lev[u] + 1;
                                                                                                          }
                                                                                                    }
```

return (lev[n] == -1 ? false : true);

int dfs(int u, int flow) {

if(u == n) return flow;

```
for (auto &i : adj[u]) {
   if (lev[i.to] == lev[u] + 1 && !vis[i.to] && i.w > θ) {
      vis[i.to] = true;
}
                   int ret = dfs(i.to, min(flow, i.w));
                   if (ret > 0) {
                         adj[i.to][i.rev_ind].w += ret;
                         return ret;
                  }
            }
      return 0; // if can't reach end => return 0
void dinic(){
      while (label_level()) {
            while (1) {
                  init(vis, 0);
int tmp = dfs(1, inf);
if(tmp == 0) break;
ans += tmp;
            }
      }
void build() {
    for(int i = 1; i <= m; i++) {</pre>
            int u, v, w; cin >> u >> v >> w;
            adj[u].push_back({
    v, w, (int)adj[v].sz}); // inverse flow's index
            adj[v].push_back({u, 0, (int )adj[u].sz - 1}); // have pushed one, need to -1
// Police Chase, need
to open adj to Augment && ori to determine what pb give
// Dinic \ dfs2, then use reach as u, if the edge
pb has given && w == 0 && v is not in reach, is the ans
void dfs2(int now, unordered_set<int> &reach) {
   if(!vis[now]){
            vis[now] = 1;
            reach.insert(now);
for(auto i : adj[now]){
    if(i.w > 0){
                       dfs2(i.to, reach);
                  }
            }
      }
// two two pair // School Dance
// Dinic, then w == 0's edge, which pb has given is the ans
   ' Distinct Route
// bistinct Route
// edge set valid var, if we need
to argument pos road, the reverse edge set true valid;
// if we need argument the argumented
edge both set false. Last, from v dfs ans times
bool get_road(int now, vector<int> &ans, vector<bool> &vis) {
   if(now == 1) return true;
   for(auto &v : adj[now]){
            if(v.arg_valid && !vis[v.to]) {
                  ans.push_back(v.to);
                  vis[v.to] = true;
                   bool flag = get_road(v.to, ans, vis);
                  if(flag){
                         v.arg_valid = false;
                  ans.pop_back();
            }
      return false:
}
```

## 4.2 MCMF [40d5b7]

```
dis[to] = dis[from] + cost;
par[to] = i;    // record num
flow_rec[to] = min(flow_rec[from], w);
                 }
            if(flag) break;
       if(dis[n] == 1e9) return 0;
       int mn_flow = flow_rec[n];
       int v = n;
      while(v != 1){
            int u = adj[par[v]].from;
            adj[par[v]].w -= mn_flow;
adj[par[v] ^ 1].w += mn_flow;
      mn_flow = min(mn_flow, parcel);
      parcel -= mn_flow;
       return mn_flow * dis[n];
void solve(){
   cin >> n >> m >> parcel;
      ll ans = 0;
      for(int i = 1; i <= m; i++){
   int u, v, w, cost; cin >> u >> v >> w >> cost;
            add_edge(u, v, w, cost);
       while(parcel > 0){
            int tmp = Bellman_Ford();
if(tmp == 0) break;
ans += tmp;
      cout << (parcel > 0 ? -1 : ans);
1
```

## 5 String

#### 5.1 KMP [11be97]

```
#include <bits/stdc++.h>
 using namespace std;
struct KMP {
       string sub;
vector<<mark>int</mark>> failure;
        KMP(string &sub) {
              this->sub = sub;
              failure.resize(sub.size(), -1);
              buildFailFunction();
        void buildFailFunction() {
              for(int i = 1; i < sub.size(); i++) {
   int now = failure[i - 1];</pre>
                     while(now != -1
                     && sub[now + 1] != sub[i]) now = failure[now];
if (sub[now + 1] == sub[i]) failure[i] = now + 1;
        vector<int> KMPmatching(string &s) {
              for(int i = 0, now = -1; i < s.size(); i++) {
    // now is the compare successed length -1
    while (s[i] !=</pre>
                     sub[now + 1] && now != -1) now = failure[now];
// f stores if comparison fail, move to where
if (s[i] == sub[now + 1]) now++;
if (now + 1 == sub.size()) {
                           match.push_back(i - now);
now = failure[now];
                    }
              return match;
       }
 int main() {
        string s = "xxtxxtxtx";
        string sub = "tx";
        KMP kmp(sub);
        vector < int > ans = kmp.KMPmatching(s);
for(auto &i : ans) cout << i << " ";</pre>
}
```

#### 5.2 Manacher [3ad367]

```
r[i] += 1;
}
if (i + r[i] > j + r[j]) {
    j = i;
}

return r;
// # a # b # a #
// 1 2 1 4 1 2 1
// index 為奇數代表中心點在字元上(即回文字串長度是奇數)
}
```

#### 5.3 Trie [b84198]

```
#include <bits/stdc++.h>
using namespace std;
#define all(x) (x).begin(), (x).end()
#define endl "\n"
#define endl "|n"
#define int long long
typedef pair<int, int> pii;
const int llinf = 4e18;
const int mod = 1e9 + 7;
const int maxn = 2e5 + 5;
struct Trie {
      struct trie_node {
            bool is_word;
vector<trie_node *> children;
trie_node() {
   is_word = false;
   children.resize(26, NULL);
      trie_node *root = new trie_node();
              insert(string &s) {
            trie_node *cur = root;
for (int i = 0; i < s.size(); i++) {
   int idx = s[i] - 'a';</pre>
                   if (cur->children[idx] == NULL) {
                         cur->children[idx] = new trie_node();
                   cur = cur->children[idx];
            cur->is word = true:
      bool is_in_trie(string &s) {
             trie_node *cur = root;
for (int i = 0; i < s.size(); i++) {</pre>
                   children[s[i] - 'a'] == nullptr) return false;
cur = cur->children[s[i] - 'a'];
             return true:
      int search_i_start(string &s, int i, vector<int> &dp) {
            trie_node *cur = root;
int sz = s.size(), ans = 0;
for (int j = i; j < sz; j++) {</pre>
                   if (cur
                   ->children[s[j] - 'a'] == nullptr) return ans;
cur = cur->children[s[j] - 'a'];
                   if (cur->is_word)
                         (ans += dp[j + 1]) %= mod;
             return ans;
     }
};
void solve() {
      // 找到 sub 集合裡,可以重複用,組成 s 的組數
      Trie trie;
      string s; cin >> s;
int sz = s.size();
      // dp 代表 i 開頭到最後的配對總數
      // 找到有結尾為 stop 的 dp[i] += dp[j + 1]
      int n; cin >> n;
vector <int> dp(sz + 1, 0);
for (int i = 0; i < n; i++) {
    string sub; cin >> sub;
    trie.insert(sub);
      dp[sz] = 1;
for (int i = sz - 1; i >= 0; i--) {
    dp[i] = trie.search_i_start(s, i, dp);
      cout << dp[0] << endl;</pre>
signed main() {
   ios_base::sync_with_stdio(θ);
   cin.tie(nullptr);
      int t = 1;
      // cin >> t;
while (t--) {
            solve();
```

#### 6 Math

## **6.1** 質因數分解 [b535c8]

```
9
#include <bits/stdc++.h>
#include <bits/stdc++.h>
using namespace std;
// a^(m-1) {\triple_equal} 1 (mod m)
// a^(m-2) {\triple_equal} 1/a (mod m)
// EXP2: cout << fast_exp(x, fast_exp(y, p, MOD - 1), MOD)
// Filter + DP; DP save min factor recur rector decomposition
// FacNums = (x+1)(y+1)(z+1)...
// FacSum = (a^0+a^1...+a^x)(b^0+...+b^y)
// FacMul = N(x+1)(y+1)(z+1)/2
int main() {</pre>
int main() {
    vector < int > is_prime(2e6 + 1, 1);
    // 1 代表是質數,非 1 不是
    for (int i = 2; i <= 1000; i++) {
             if (is_prime[i] == 1) {
    for (int j = i + i; j <= 1000000; j += i) {</pre>
                           is_prime[j] = i;
             }
       int ans = 1;
      int q; cin >> q;
map<int, int> mp;
while (is_prime[q] != 1) {
    mp[is_prime[q]]++;
             q /= is_prime[q];
       if (q != 1) mp[q]++;
       for (auto [a, b]: mp) {
    ans *= b + 1;
       cout << ans << "\n";
6.2 矩陣快速幕 [d41d8c]
#include <bits/stdc++.h>
using namespace std;
#define int long long
const int mod = 1e9 + 7;
struct Mat {
       vector<vector<int>> matrix:
       Mat(int n) {
             this->n = n;
             matrix.resize(n);
for (int i = 0; i < n; i++) {
    matrix[i].resize(n);</pre>
       Mat(vector<vector<int>> matrix) {
             this->matrix = matrix:
       res.matrix[i][i] = 1;
       void mul(Mat b) {
            i][k] * b.matrix[k][j] % mod)) %= mod;
                   }
             matrix = ans.matrix:
       void pow(int p) {
    Mat x = *this;
    *this = unit(n);
              while (p > 0) {
   if (p & 1)
                          mul(x);
                    x.mul(x);
                    p >>= 1;
```

}

}

signed main() {

mat.mul(x);

cout << ans << "\n";

int n, ans; cin >> n; if (n <= 4) { vector < int >> v = {0, 1, 1, 2, 4};

ans = mat.matrix[0][0];

Mat mat({{4, 2, 1}, {2, 1, 1}, {1, 1, 0}});

Mat x(3);
x.matrix = {{1, 1, 0}, {1, 0, 1}, {1, 0, 0}};
x.pow(n - 4);

```
// 初始矩陣
// f4 f3 f2
// f3 f2 f1
// f2 f1 f0
// 轉移式
// 1 0 0
// 1 0 0
// f5 f4 f3
// f4 f3 f2
// f3 f2 f1
```

## **6.3** 盧卡斯定理 [cf624d]

```
#include <bits/stdc++.h>
using namespace std;
struct nCr {
    int mod;
    nCr(int mod) : mod(mod) {};
int inverse(int num) {
         if (num == 1) return 1;
return (mod
               - ((mod / num) * inverse(mod % num)) % mod) % mod;
    int fast_exp(int x, int p) {
         int ans = 1;
while (p > 0) {
              if (p & 1) ans = (ans * x) % mod;
x = x * x % mod;
         return ans;
     vector<int> fac;
    void BuildLucas(int n) {
         fac.resize(n + 1);
         fac[0] = 1;

for(int i = 1; i <= n; i++) {

fac[i] = fac[i - 1] * i % mod;
    int C(int m, int n) {
    return m < n ? 0 : fac[m] *</pre>
               inverse(fac[n]) % mod * inverse(fac[m - n]) % mod;
    };
```

## 6.4 樹論分塊 [fe6b55]

#### 6.5 Theorem

- 數論分塊可以快速計算一些含有除法向下取整的和式,就是像  $\sum_{i=1}^n f(i)g(\left\lfloor \frac{n}{i} \right\rfloor)$  的和式。當可以在O(1)內計算f(r)-f(l)或已經預處理出f的前綴和時,數論分塊就可以在 $O(\sqrt{n})$ 的時間內計算上述和式的值。
- 迪利克雷捲積  $h(x) = \sum_{d|x} f(d)g(\frac{x}{d})$
- 積性函數
  - 莫比烏斯函數
    - 1. 定義

$$\sum_{d\mid n}\!\!\mu(d)\!=\!\begin{cases} 1 & \text{for } n\!=\!1\\ 0 & \text{for } n\!\neq\!0 \end{cases}$$

2.  $\mu$ 是常數函數 1 的反元素  $\Rightarrow \mu*1=\epsilon$ ,  $\epsilon(n)$ 只在n=1時為 1, 其餘情況皆為 0。

 $-\phi$ 歐拉函數: x以下與x互質的數量

$$\phi*1 = \sum_{d|n} \phi(\frac{n}{d})$$
 質因數分解
$$= \sum_{i=0}^{c} \phi(p^{i})$$
$$= 1 + p^{0}(p-1) + p^{1}(p-1) + \dots + p^{c-1}(p-1)$$
$$= p^{c}$$
$$= id$$

• 莫比烏斯反演公式

- 
$$f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$$

- 
$$f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$$

例子

$$\begin{split} \sum_{i=a}^{b} \sum_{j=c}^{d} [gcd(i,j) = k] \\ \Rightarrow \sum_{i=1}^{x} \sum_{j=1}^{y} [gcd(i,j) = k] \\ &= \sum_{i=1}^{x} \sum_{j=1}^{y} \epsilon(gcd(i,j)) \\ &= \sum_{i=1}^{x} \sum_{j=1}^{y} \sum_{d|gcd(i,j)} \mu(d) \\ &= \sum_{d=1}^{\infty} \sum_{j=1}^{y} \sum_{d|gcd(i,j)} \mu(d) \\ &= \sum_{d=1}^{\infty} \mu(d) \sum_{i=1}^{\left \lfloor \frac{x}{k} \right \rfloor} [d|i] \sum_{j=1}^{y} \left \lfloor \frac{y}{k} \right \rfloor [d|j] \mathbf{d} \, \text{可整除 i 時為 1} \\ &= \sum_{d=1}^{min(\left \lfloor \frac{x}{k} \right \rfloor, \left \lfloor \frac{y}{k} \right \rfloor)} \mu(d) \left \lfloor \frac{x}{kd} \right \rfloor \left \lfloor \frac{y}{kd} \right \rfloor \end{split}$$

## 6.6 莫比烏斯反演 [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 5e4 + 5;
int mobius_pref[maxn];
void init() {
      mobius_pref[1] = 1;
vector<<mark>int</mark>> wei
      (maxn); // wei = 0 代表是質數, -1 代表可被平方數整除
for (int i = 2; i < maxn; i++) {
    if (wei[i] == -1) {
        mobius_pref[i] = mobius_pref[i - 1];
                  continue; // 包含平方
            if (wei[i] == 0) {
                  wei[i] -- 0, {
wei[i] = 1;
for (int j = 2; i * j < maxn; j++) {
    if (j % i == 0) wei[i * j] = -1;
    else if (wei[i * j] != -1) wei[i * j]++;</pre>
            mobius_pref[i]
                   = mobius_pref[i - 1] + (wei[i] % 2 == 0 ? 1 : -1);
     }
void solve() {
      int a, b, c, d, k; cin >> a >> b >> c >> d >> k;
auto cal = [&](int x, int y) -> int {
           - 1]) * (x / l) * (y / l); // 代推出來的式子
            return res;
      }:
      cout << cal
             (b / k, d / k) - cal((a - 1) / k, d / k) - cal(b / k,
(c - 1) / k) + cal((a - 1) / k, (c - 1) / k) << "\n";
```

# 7 Search and Gready

#### **7.1** 二分搜 [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
int main() {
   int l = 1, r = 10;
```

```
// 1 to tar, find tar
while (l <= r) {
   int m = (l + r) / 2;
   if (check(m)) l = m + 1;
           else r = m -
cout << r;
cout << r;
// tar to end
while (l <= r) {
   int m = (l + r) / 2;
   if (check(m)) r = m - 1;
   else l = m + 1;</pre>
cout << l;
```

## 7.2 三分搜 [d41d8c]

```
// 找極值問題,遞增遞減
#include <bits/stdc++.h>
using namespace std;
     solve() {
    int l = 0, r = 10, ans = 0; // ans 紀錄答案
    while (l <= r) {
        int d = (r - l) / 3; // 差
        int ml = l + d, mr = r - d; // mr 要用減的
auto cal = [&](int m) -> int {
            int x = 0;
            return x; // 計算答案
        int ansl = cal(ml), ansr = cal(mr);
if (ansl < ansr) {</pre>
             l = ml + 1;
         else r = mr - 1;
}
```

## 7.3 Concert Ticket [d41d8c]

```
// Better than Binary Search
#include <bits/stdc++.h>
using namespace std;
int x; cin >> x;
auto it = tik.upper_bound(x);
if (it == tik.begin()) {
              cout << -1 << '
              continue:
          cout << *it << " ";
          tik.erase(it);
}
```

#### 7.4 Restaurant Customers [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
int main() {
       vector<pair<int, int>> times;
      for (int i = 0; i < n; i++) {
   int u, v; cin >> u >> v;
   times.push_back({u, 1});
   times.push_back({v, -1});
       sort(times.begin(), times.end());
      int now_people = 0, ans = 0;
for (auto [t, x] : times) {
    ans = max(ans, (now_people += x));
       cout << ans:
```

### **8 Тгее**

## 8.1 LCA [ca194c]

```
#include <bits/stdc++.h> // LCA from 1
using namespace std;
int main() {
   int n, q; cin >> n >> q;
   vector<vector<int>> tree(n + 1);
        for (int i = 1; i < n; i++) {
   int u, v; cin >> u >> v;
   tree[u].push_back(v);
   tree[v].push_back(u);
        vector<vector<int>> par(n + 1, vector<int>(18));
vector<int> depth(n + 1);
auto dfs = [&](auto self, int u, int pre) -> void {
```

```
for (auto v : tree[u]) {
   if (v == pre) continue;
   par[v][0] = u; // 2 ^ 0
   depth[v] = depth[u] + 1;
   self(self, v, u);
}
        auto lca = [&](int a, int b) -> int {
   if (depth[a] < depth[b]) swap(a, b);</pre>
                 int pull = depth[a] - depth[b];
for (int i = 0; i < 18; i++) {
   if (pull & (1 << i)) {</pre>
                                  a = par[a][i];
                         }
                }
if (a == b) return a;
for (int i = 17; i >= 0; i--) {
    if (par[a][i] != par[b][i]) {
        a = par[a][i], b = par[b][i];
}
                 return par[a][0];
}
```

### 8.2 樹 DFS [b6cb9a]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector<int> depth;
void dfs(vector<vector<int>> &tree, int u, int pre) {
      for(auto v : tree[u]){
  if(v == pre)
  depth[v] = depth[u] + 1;
  dfs(tree, v, u);
}
```

## 8.3 樹重心 [2771f3]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector < int > tree[maxn];
int cen = 0, n;
int dfs(int par, int now) {
      size += subsize;
               }
       if (n - 1 - size > n
if (flag) cen = now;
return size + 1;
                  - 1 - size > n / 2) flag = false;
int main() {
       for (int i = 1; i < n; i++) {
   int u, v; cin >> u >> v;
   tree[u].push_back(v);
   tree[v].push_back(u);
}
        for (int i = 1; i <= n; i++) {
    for (auto nxt : tree[i])
        dfs(i, nxt);</pre>
               if (cen) break;
       }
}
```

#### 8.4 節點距離總和 [3bfb86]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector < int > tree[maxn];
vector < int > subtree(maxn, 1);
 long long ans[maxn];
int n;
void dfs(int par, int now, int depth) {
    ans[1] += depth;
    for (auto nxt : tree[now]) {
        if (par != nxt) {
            dfs(now, nxt, depth + 1);
            subtree[now] += subtree[nxt];
}
int n;
                   }
        }
// youd find_ans(int par, int now) {
// each sub's dis make - 1, non subnode + 1
    for (auto nxt : tree[now]) {
                   if (par != nxt) {
```

### 8.5 有權樹直徑 [98f093]

## 8.6 樹壓平 [03946b]

```
|// 父節
      點加值 = 所有子節點區間加值,求單點,使用 bit,做前綴差分
 // CSES 1138_Path Queries
 #include <bits/stdc++.h>
 #define int long long
using namespace std;
 struct BIT { // BIT 都是 1-based 的查詢
      int n;
      vector<int> bit;
      BIT(int n) { // 有幾個數
this->n = n;
           bit.resize(n + 1, 0);
           vector<int> &init) { // 必須是 1-based this->n = init.size() - 1;
      BIT(vector<int> &init) {
           bit.resize(n + 1, 0);
for (int i = 1; i <= n; i++) {
   modify(i, init[i]);</pre>
      int query(int r) {
           int ans = 0;
for (; r; r -= r & -r) ans += bit[r];
return ans;
      int query(int l, int r) {
    return query(r) - query(l - 1);
      }
 void solve(){
      int n, q; cin >> n >> q;
vector <int> node_value(n + 1), euler_ordered_value(n);
for (int i = 1; i <= n; i++) {</pre>
           cin >> node_value[i];
      vector<vector<int>> tree(n +
      for (int i = 1; i < n; i++) {
    int u, v; cin >> u >> v;
```

```
tree[u].push_back(v);
tree[v].push_back(u);
       vector<pair<int, int>> tree_mapping(n + 1);
       int cnt = 0;
       auto dfs = [%](auto self, int u, int par) -> void {
    euler_ordered_value[++cnt] = node_value[u];
    tree_mapping[u].first = cnt;
    for (auto v : tree[u]) {
        if (v == par) continue;
        self(self, v, u);
}
             tree_mapping[u].second = cnt;
       dfs(dfs, 1, 0);
      for (int i = 1; i <= n; i++) {
   bit.modify(tree_mapping[i].first, node_value[i]);
}</pre>
             if (tree_mapping[i].first < n) {</pre>
                    bit.modify
                           (tree mapping[i].second + 1. -node value[i]):
       for (int i = 0; i < q; i++) {
    int op; cin >> op;
             if (op == 1) {
   int s, x; cin >> s >> x;
   int add = x
                              - euler_ordered_value[tree_mapping[s].first];
                    euler_ordered_value[tree_mapping[s].first] = x;
bit.modify(tree_mapping[s].first, add);
if (tree_mapping[s].first < n) {</pre>
                          bit.modify(tree_mapping[s].second + 1, -add);
             else {
    int node; cin >> node;
                   cout <<
                            bit.query(tree_mapping[node].first) << "\n";</pre>
      }
}
```

### 9 DP

## 9.1 背包問題 [9457ef]

## 9.2 Bitmask DP [c130ec]

```
#include <bits/stdc++.h>
using namespace std;
const int mod = 1e9 + 7;
void travel_exactly_once(){
    // [走過的路][終點]
    vector<vector<int>> dp(1 << 20, vector<int>> (20, 0));
    vector<iint> rev_adj[20];
    int n, m; cin >> n >> m;
    for(int i = 0; i < m; i++){
        int u, v; cin >> u >> v;
        rev_adj[--v].push_back(--u);
    }
    dp[1][0] = 1;
    for (int road = 0; road < (1 << n); road++) {
        // 沒經過起點, 不用走
        if (road & 1 == 0) continue;
        // 有終點但沒全部走過
        if (road & (1 << n) - 1)) continue;
        // DP,隨便選定一個當前路徑的終點
```

```
for (int end = 0; end < n; end++) {
    // 路徑沒包含假定的 end
                 if ((road & (1 << end)) == 0) continue;
                 // 去除終點,得到 pre_road
int pre_road = road - (1 << end);
                 // 從 rev_adj 找 pre_road 的終點
for (int pre_road_end : rev_adj[end]) {
                      if ((road & (1 << pre_road_end))) {
                            dp[road
                                  ][end] += dp[pre_road][pre_road_end];
                            dp[road][end] %= mod;
                }
          }
     cout << dp[(1 << n) - 1][n - 1];
void elevator_rides(){
     int n, k; cin >> n >> k;
vector <int> passenger(n);
for (int i = 0; i < n; i++) cin >> passenger[i];
      vector<int
     > used(1 << n, 0);  // 最後載完人的電梯用了多少空間
vector <int> dp(1 << n, 1);  // bitset
for (int i = 1; i < 1 << n; i++) {
    used[i] = dp[i] = 2e9;
           for (int j = 0; j < n; j++) {
                if (i & (1 << j)) { // 有 j int pre = i ^ (1 << j); // 最後的電梯還能載 j if (used[pre] + passenger[j] <= k) {
                            // 電梯數先比,再來比用掉的空間
                            if (dp
                                  r
[pre] < dp[i] || (dp[pre] == dp[i] &&
used[pre] + passenger[j] < used[i])) {
                                 used[i] = used[pre] + passenger[j];
dp[i] = dp[pre];
                           }
                      }
                      // 搭新的電梯
                      else
                            if (dp[pre] + 1 < dp[i] || (dp[pre] + 1
                                  == dp[i] && passenger[j] < used[i])) {
used[i] = passenger[j];
                                  dp[i] = dp[pre] + 1;
                            }
                      }
                }
           }
     cout << dp[(1 << n) - 1];
int main(){
              _exactly_once();
      travel
      elevator_rides();
9.3 硬幣 [d41d8c]
```

```
#include <bits/stdc++.h>
using namespace std;
const int mod = 1e9 + 7;
void coin_combination_II(){
    // 有 n 種錢幣,求組合為 x 的組數,順序不可顛倒
     // 可顛倒的話只要一維,先 x 迴圈,再 coin[i] 去加
    int n, x; cin >> n >> x;
vector<int> coin(n + 1);
     // dp[i][j] 為考慮前 i 個硬幣,組合為 i 的組數
vector<vector<int>> dp(2, vector<int>(x + 1, 0));
    for (int i = 1; i <= n; i++) {
    for (int j = 0; j <= x; j++) {</pre>
               // 壓到 2 * n
                dp[i & 1][j] = dp[!(i & 1)][j];
                if (j
                       >= coin[i]) {
                     (dp[i
                           & 1][j] += dp[i & 1][j - coin[i]]) %= mod;
               }
          }
     cout << dp[n & 1][x];
void minimize_coins_nums(){
     // 有 n 種錢幣, 求組合為 x 的最小硬幣數
     int n, x; cin >> n >> x;
    vector <int> coin(n);

for (int i = 0; i < n; i++) cin >> coin[i];

// dp[i] 是組合為 i 的最小硬幣數

vector <int> dp(x + 1, 0);

for (int i = 1; i <= x; i++) {
          dp[i] = 2e9;
for(auto &j : coin){
    if(j <= i){</pre>
                     dp[i] = min(dp[i], dp[i - j] + 1);
          }
     cout << (dp[x] == 2e9 ? -1 : dp[x]);
```

```
}
int main(){
    coin_combination_II();
    minimize_coins_nums();
}
```

## 9.4 編輯距離 [80c4dc]

## 9.5 LCS [937a28]

```
#include <bits/stdc++.h>
using namespace std;
int main(){
       int m, n; cin >> m >> n;
       string s1, s2;
cin >> s1 >> s2;
int L = 0;
       vector < vector < int >> dp(m + 1, vector < int >(n + 1, 0));
       for (int i = 1; i <= m; i++) {
   for (int j = 1; j <= n; j++) {
     if (s1[i - 1] == s2[j - 1]) {
          dp[i][j] = dp[i - 1][j - 1] + 1;
     }</pre>
                             dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
                     }
              }
       int length = dp[m][n];
cout << length << "\n"</pre>
       string s('a', length);
// along to dp to trace back
while (m >= 1 && n >= 1) {
    if (s1[m - 1] == s2[n - 1]) {
        s[length - 1] = s1[m - 1]
}
                      length--;
                      if (dp[m - 1][n] > dp[m][n - 1]){
                             m - -:
              }
       cout << s << "\n";
}
```

#### 9.6 LIS [f23284]

```
#include <bits/stdc++.h>
using namespace std;
// Rec Sequence LIS
void solve(){
     int n; cin >> n;
vector < int >> v(n);
for (int i = 0; i < n; i++) {</pre>
          cin >> v[i];
     int dp[n]; vector<int> mono;
     mono.push_back(v[0]);
     dp[0] = 1; int L = 1;
for (int i = 1; i < n; i++)
    if (v[i] > mono.back())
               mono.push_back(v[i]);
               dp[i] = ++L;
          else {
               auto it
                    = lower_bound(mono.begin(), mono.end(), v[i]);
               *it = v[i];
               dp[i] = it - mono.begin() + 1;
```

```
}
}
vector<int> ans;
cout << L << endl;
for (int i = n - 1; i >= 0; i--) {
    if (dp[i] == L) {
        ans.push_back(v[i]);
        L--;
    }
}
reverse(ans.begin(), ans.end());
for (auto i : ans) {
    cout << i << " ";
}
}</pre>
```

## 9.7 Projects [c03e88]

```
#include <bits/stdc++.h>
using namespace std;
#define all(x) (x).begin(), (x).end()
#define endl "\n"
#define int long long
const int maxn = 2e5 + 5;
struct project {
    int from, end, gain, id;
void solve(){
     int n; cin >> n;
vectorvectorcproject
projects(n + 1);
for (int i = 1; i <= n; i++) {
          cin >> projects
               [i].from >> projects[i].end >> projects[i].gain;
          projects[i].id = i;
     if (a.end == b.end) return a.gain < b.gain;
return a.end < b.end;</pre>
    vector<array
          }) - projects.begin(); // 二分搜最接近 from 的 end // cerr << idx << "\n"; dp[i] = dp[i - 1]; par[i] = i - 1;
          par[i] = i - 1;
if (dp[i][1] < dp[idx][1] + projects[i].gain ||</pre>
          (dp[i][1]
                     dp[idx][1] + projects[i].gain && dp[i][2] >
               [idx][2] + projects[i].end - projects[i].from)) {
dp[i] = [dp[idx
                              1, dp[idx][1] + projects[i].gain,
                     idx][2] + projects[i].end - projects[i].from};
               par[i] = idx;
add[i] = projects[i].id;
          }
    cout << dp
           [n][0] << " " << dp[n][1] << " " << dp[n][2] << endl;
     for (int now = n; now > 0; now = par[now]) {
   if (add[now] != -1)
               ans.push_back(add[now]);
    sort(all(ans));
for(auto &i : ans) cout << i << " ";</pre>
```

#### 9.8 Removal Game [45a446]

# 10 Geometry 10.1 Cross Product [c37c89]

```
#include <bits/stdc++.h>
using namespace std;
const double eps = 1e-8;
struct point {
       double x, y;
point operator * (int a){ return {a * x, a * y}; }
point operator + (point b){ return {x + b.x, y + b.y}; }
point operator - (point b){ return {x - b.x, y - b.y}; }
double operator * (point b){ return x * b.x + y * b.y; }
double operator * (point b){ return x * b.y - y * b.x; }
       double operator ^ (point b){ return x * b.y - y * b.x; }
       bool operator
                 < (point b) { return x == b.x ? y < b.y : x < b.x; }
double abs(point a) { return sqrt(a * a); }
int sign
        (double a) { return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
int ori(point
              point b, point c) { return sign((b - a) ^ (c - a)); }
bool colinear(point a,
point b, point c) { return sign((b - a) ^ (c - a)) == 0; }
bool between(point a, point b, point c){ // c between a and b
    if (!colinear(a, b, c)) return false;
    return sign((a - c) * (b - c)) <= 0;</pre>
bool intersect(point
       a, point b, point c, point d){ // line(a, b) line(c, d)
int abc = ori(a, b, c);
int abd = ori(a, b, d);
       int cda = ori(c, d, a);
       int cdb = ori(c, d, b);
       if(abc == 0 || abd == 0)
    return between(a, b, c) || between
       (a, b, d) || between(c, d, a) || between(c, d, b);
return abc * abd <= 0 && cda * cdb <= 0;
}
```

### 10.2 Convex Hull [e8ad24]

```
vector<pii> P, L, U;
Il Andrew_monotone_chain(ll n){
   L.pop_back();
       while (u >= 2 && cross(U[u-2], U[u-1], P[i]) >= 0){
          U.pop_back();
      1++:
      u++;
       L.push_back(P[i]);
      U.push_back(P[i]);
   cout << l << ' ' << u << '\n';
   return l + u;
int main(){
   ll n,x,y;
   cin >> n;
for(ll i = 0;i < n;i++){</pre>
      cin >> x >> y;
P.push_back({x,y});
   ll ans = Andrew_monotone_chain(n) - 2;
cout << ans << "\n";</pre>
   return 0;
```