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## 前言

### 1.1 Attention

```
// Construct VScode
// 1. instaint
     vscode & msys2, check desktop path of vscode
// 2. open mingw64, not ucrt64, "pacman
     -S -- needed base - devel mingw - w64 - x86_64 - toolchain"
// 3. add C:\\msys64\\mingw64\\bin to environment path
// notebook-generator ./ --author "Salmon" --initials
    Salmon --columns 3 --output "CodeBook.pdf" --size 8
// Init
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#define all(x) (x).begin(), (x).end()
#define pii pair<int, int>
#define endl "\n"
#define int long long
using namespace std;
using namespace __gnu_pbds;
template<typename T>
using pbds_set = tree<T, null_type, less_equal<T</pre>
    >, rb_tree_tag, tree_order_statistics_node_update>;
template < typename T>
using pbds_multiset = tree<T, null_type, less_equal<T</pre>
>, rb_tree_tag, tree_order_statistics_node_update>;
struct cmp { // 在有 template 的資結使用
    bool operator()(const int &a, const int &b) const {
        return a < b;</pre>
    // sort, bound 不用 struct
    // priority queue 小到大是 > , set 是 <
// set 不能 = , multiset 要 =
    // 每個元素都要比到,不然會不見
struct edge {
    int from; int to;
    int weight;
struct node {
    int sum;
const int llinf = 4e18;
const int inf = 2e9;
```

```
const int mod = 1e9 + 7;
const int maxn = 2e5 + 5;
void solve(){
signed main(){
    ios_base::sync_with_stdio(0);
    cin.tie(nullptr);
    int t = 1;
    cin >> t;
    while(t--){
        solve();
}
```

#### 動態規劃 2

### 2.1 背包問題

```
#include <bits/stdc++.h>
using namespace std;
int dp[1005][100005];
       // 考慮前 i 個,預算有 j 塊錢的最多 page
vector<int> Page(1005, 0);
vector<int> Price(1005, 0);
int main(){
    int n, bud;
    cin >> n >> bud;
    for(int i = 1; i <= n; i++){</pre>
        int tmp; cin >> tmp;
        Price[i] = tmp;
    for(int i = 1; i <= n; i++){</pre>
        int tmp; cin >> tmp;
        Page[i] = tmp;
    for(int i = 1; i <= n; i++){</pre>
        for(int j = 1; j <= bud; j++){</pre>
            if(j >= Price[i]){ // 買得起
                // 不買或買
                dp[i][j] = max(dp[i - 1][j],
                    dp[i - 1][j - Price[i]] + Page[i]);
            else {
                dp[i][j] = dp[i - 1][j];
        }
    cout << dp[n][bud] << "\n";
```

#### 2.2 Bitmask DP

```
#include <bits/stdc++.h>
using namespace std;
const int mod = 1e9 + 7;
void travel_exactly_once(){
   // [走過的路][終點]
   vector
       <vector<int>> dp(1 << 20, vector<int> (20, 0));
    vector<int> rev_adj[20];
    int n, m; cin >> n >> m;
    for(int i = 0; i < m; i++){</pre>
       int u, v; cin >> u >> v;
       rev_adj[--v].push_back(--u);
    dp[1][0] = 1;
   for(int road = 0; road < (1 << n); road++){</pre>
        // 沒經過起點,不用走
       if(road & 1 == 0) continue;
       // 有終點但沒全部走過
       if(road & (1 << (n
            - 1)) && road != ((1 << n) - 1)) continue;
        // DP,隨便選定一個當前路徑的終點
       for (int end = 0; end < n; end++) {</pre>
           // 路徑沒包含假定的 end
     if ((road & (1 << end)) == 0) continue;</pre>
           // 去除終點,得到 pre_road
     int pre_road = road - (1 << end);</pre>
           // 從 rev_adj 找 pre_road 的終點
      for (int pre_road_end : rev_adj[end]) {
       if ((road & (1 << pre_road_end))) {</pre>
         dp[road][end] += dp[pre_road][pre_road_end];
          dp[road][end] %= mod;
```

```
}
    }
    cout << dp[(1 << n) - 1][n - 1];</pre>
void elevator_rides(){
    int n, k; cin >> n >> k;
    vector<int> passenger(n);
    for(int i = 0; i < n; i++) cin >> passenger[i];
    vector<int> used
        (1 << n, 0);
                          // 最後載完人的電梯用了多少空間
    vector < int > dp(1 << n, 1); // bitset
for(int i = 1; i < 1 << n; i++){</pre>
        used[i] = dp[i] = 2e9;
        for(int j = 0; j < n; j++){</pre>
             if(i & (1 << j)){ // 有 j
int pre = i ^ (1 << j);
                 // 最後的電梯還能載 j
                 if(used[pre] + passenger[j] <= k){</pre>
                      // 電梯數先比,再來比用掉的空間
                      if(dp[pre] < dp[i] ||
                           (dp[pre] == dp[i] && used[pre
                          ] + passenger[j] < used[i])){</pre>
                          used[i]
                              = used[pre] + passenger[j];
                          dp[i] = dp[pre];
                 // 搭新的電梯
                 else {
                     if(dp[pre] + 1
                           < dp[i] || (dp[pre] + 1 == dp[
                          i] && passenger[j] < used[i])){
                          used[i] = passenger[j];
                          dp[i] = dp[pre] + 1;
                     }
                 }
            }
        }
    cout << dp[(1 << n) - 1];</pre>
int main(){
    travel_exactly_once();
    elevator_rides();
```

#### 2.3 硬幣

```
#include <bits/stdc++.h>
using namespace std;
const int mod = 1e9 + 7;
void coin_combination_II(){
   // 有 n 種錢幣,求組合為 x 的組數,順序不可顛倒
   // 可顛倒的話只要一維,先 x 迴圈,再 coin[i] 去加
   int n, x; cin >> n >> x;
   vector<int> coin(n + 1);
   // dp[i][j] 為考慮前 i 個硬幣,組合為 i 的組數
   vector<vector<int>> dp(2, vector<int>(x + 1, 0));
   dp[0][0] = 1;
   for(int i = 1; i <= n; i++) cin >> coin[i];
   for(int i = 1; i <= n; i++){</pre>
       for(int j = 0; j <= x; j++){</pre>
           // 壓到 2 * n
           dp[i \& 1][j] = dp[!(i \& 1)][j];
           if(j >= coin[i]){
               (dp[i & 1][j]
                    += dp[i & 1][j - coin[i]]) %= mod;
       }
   cout << dp[n & 1][x];</pre>
void minimize_coins_nums(){
   // 有 n 種錢幣,求組合為 x 的最小硬幣數
   int n, x; cin >> n >> x;
   vector<int> coin(n);
   for(int i = 0; i < n; i++) cin >> coin[i];
   // dp[i] 是組合為 i 的最小硬幣數
    vector < int > dp(x + 1, 0);
   for(int i = 1; i <= x; i++){</pre>
       dp[i] = 2e9;
       for(auto &j : coin){
```

### 2.4 編輯距離

```
#include <bits/stdc++.h>
using namespace std;
void solve(){
     string s1, s2; cin >> s1 >> s2;
     int size1 = s1.size(), size2 = s2.size();
         [i][j] 為 s1 的前 i 個字元,跟 s2 的前 j 個字元
     vector<vector<
    int>> dp(size1 + 1, vector<int>(size2 + 1, 0));
s1 = "0" + s1, s2 = "0" + s2;
for(int i = 1; i <= size1; i++) dp[i][0] = i;</pre>
     for(int i = 1; i <= size2; i++) dp[0][i] = i;</pre>
     for(int i = 1; i <= size1; i++){</pre>
         for(int j = 1; j <= size2; j++){</pre>
              if(s1[i] == s2[j]){
                   dp[i][j] = dp[i - 1][j - 1];
              else {
                   // s1 新增等價於 s2 砍掉
                   // dp[i
                   ][j] = min(修改, s1 新增, s2 新增);
dp[i][j] = min({dp[i - 1][j - 1],
                          dp[i - 1][j], dp[i][j - 1]}) + 1;
              }
         }
     cout << dp[size1][size2];</pre>
```

#### 2.5 LCS

```
#include <bits/stdc++.h>
using namespace std;
int main(){
    int m, n; cin >> m >> n;
    string s1, s2;
    cin >> s1 >> s2;
    s1.insert(s1.begin(), '1');
    s2.insert(s2.begin(), '1');
    int L = 0;
    vector
         <vector<int>> dp(m + 1, vector<int>(n + 1, 0));
    for(int i = 1; i <= m; i++){</pre>
         for(int j = 1; j <= n; j++){
   if(s1[i] == s2[j]){</pre>
                  dp[i][j] = dp[i-1][j-1] + 1;
             else {
                  dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
        }
    int length = dp[m][n];
    cout << length << "\n";</pre>
    vector < char > s(length);
    // along to dp to trace back
    while(m >= 1 && n >= 1){
         if(s1[m] == s2[n]){
             s[length - 1] = s1[m];
             m - -;
             n - -;
             length--;
         else {
             if(dp[m-1][n] > dp[m][n-1]){
             else n--;
         }
    }
```

```
for(auto c : s){
    cout << c;
}
</pre>
```

#### 2.6 LIS

```
#include <bits/stdc++.h>
using namespace std;
// Rec Sequence LIS
void solve(){
    int n; cin >> n;
    vector < int > v(n);
for(int i = 0; i < n; i++){</pre>
         cin >> v[i];
     int dp[n]; vector<int> mono;
    mono.push_back(v[0]);
    dp[0] = 1; int L = 1;
for(int i = 1; i < n; i++){</pre>
         if(v[i] > mono.back()){
              mono.push_back(v[i]);
              dp[i] = ++L;
          else {
              auto it = lower_bound
                  (mono.begin(), mono.end(), v[i]);
              *it = v[i];
              dp[i] = it - mono.begin() + 1;
         }
     vector<int> ans;
    cout << L << endl;
for(int i = n - 1; i >= 0; i--){
         if(dp[i] == L){
              ans.push_back(v[i]);
              L - -:
         }
     reverse(ans.begin(), ans.end());
    for(auto i : ans){
    cout << i << " ";</pre>
```

## 2.7 Projects

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
#define pii pair<int, int>
const int maxn = 2e5+5;
typedef struct {
    int u, v, w;
} project:
void compress(vector<int> &sorted, vectorproject
    > &projects, vector<vector<pii>> &EndProjects){
    sort(sorted.begin(), sorted.end());
    sorted.erase(unique
    (sorted.begin(), sorted.end());
for(int i = 0; i < projects.size(); i++){</pre>
        EndProjects[lower_bound(sorted.begin(), sorted.
             end(), projects[i].v) - sorted.begin() + 1]
         .push_back({lower_bound(sorted.begin(), sorted.
        end(), projects[i].u) - sorted.begin() + 1,
projects[i].w});
    }
signed main(){
    int n; cin >> n;
    vector<project> projects(n);
    vector<vector<pii>> EndProjects(2 * n + 1);
    vector<<mark>int</mark>> nums;
    for(int i = 0; i < n; i++){</pre>
        cin >> projects
            [i].u >> projects[i].v >> projects[i].w;
        nums.push_back(projects[i].u);
        nums.push_back(projects[i].v);
    compress(nums, projects, EndProjects);
    vector<int> dp(nums.size() + 1, 0);
    for(int end = 1; end <= nums.size(); end++){</pre>
        dp[end] = dp[end - 1];
        for(auto [from, gain] : EndProjects[end]){
            dp[end
                 ] = max(dp[end], dp[from - 1] + gain);
```

```
}
cout << dp[nums.size()];
}
// Monotonic DP in campus contest, use monotonic stack
// first is lowest mountain, second is pref in stack</pre>
```

#### 2.8 Removal Game

```
#include <bits/stdc++.h>
using namespace std;
int dp[5005][5005];
void solve(){
    int n; cin >> n;
    int pref = 0;
    vector<int> v(n+1);
    for(int i = 1; i <= n; i++){</pre>
         cin >> v[i];
         pref += v[i];
     // dp[i][j] = max_diff(i to j);
     for(int i = n; i > 0; i--){
         for(int j = 1; j <= n; j++){</pre>
              if(i > j) continue;
else if(i == j){
                  dp[i][j] = v[i];
              else {
                   dp[i][j] = max(v[
                       i] - dp[i+1][j], v[j] - dp[i][j-1])
                          // i+1, j-1, care dp's order
         }
    // x + y = sum, dp[1][n] = x - y;
cout << (pref + dp[1][n]) / 2;
```

## 3 最大流

### 3.1 Dinic

```
#include <bits/stdc++.h>
using namespace std;
bool vis[505];
int lev[505], n, m, ans;
typedef struct {
   int to, w, rev_ind;
} edge;
vector<edge> adj[505];
bool label_level(){ //
    Tag the depth, if can't reach end => return false memset(lev, -1, sizeof(lev));
    lev[1] = 0;
    queue<int> q;
                   q.push(1);
    while(!q.empty()){
        int u = q.front(); q.pop();
        for(auto i : adj[u]){
            if(i.w > 0 && lev[i.to] == -1){
                q.push(i.to);
                lev[i.to] = lev[u] + 1;
            }
        }
    return (lev[n] == -1 ? false : true);
int dfs(int u, int flow){
    if(u == n) return flow;
    for(auto &i : adj[u]){
        vis[i.to] = true;
            int ret = dfs(i.to, min(flow, i.w));
            if(ret > 0) {
                i.w -= ret:
                adj[i.to][i.rev_ind].w += ret;
                return ret;
       }
    }
                // if can't reach end => return 0
    return 0;
void dinic(){
    while(label level()){
       while(1){
            init(vis, 0);
```

```
int tmp = dfs(1, inf);
            if(tmp == 0) break;
            ans += tmp;
        }
    }
void build(){
    for(int i = 1; i <= m; i++) {</pre>
        int u, v, w; cin >> u >> v >> w;
        adj[u].push_back({v, w,
            (int)adj[v].sz});
                                // inverse flow's index
        adj[v].push_back({u, 0, (int)adj
            [u].sz-1}); // have pushed one, need to -1
// Police Chase, need to open
     adj to Augment && ori to determine what pb give
// Dinic `dfs2, then use reach as u, if the edge pb has
     given && w == 0 && v is not in reach, is the ans
void dfs2(int now, unordered_set<int> &reach){
    if(!vis[now]){
        vis[now] = 1;
        reach.insert(now);
        for(auto i : adj[now]){
            if(i.w > 0){
                dfs2(i.to, reach);
        }
    }
// two two pair // School Dance
// Dinic
    , then w == 0's edge, which pb has given is the ans
// Distinct Route
// edge set valid var, if we need to argument
     pos road, the reverse edge set true valid;
// if we need argument the argumented
     edge, both set false. Last, from v dfs ans times
bool get_road
    (int now, vector<int> &ans, vector<bool> &vis){
    if(now == 1) return true;
    for(auto &v : adj[now]){
        if(v.arg_valid && !vis[v.to]){
            ans.push_back(v.to);
            vis[v.to] = true;
            bool flag = get_road(v.to, ans, vis);
            if(flag){
                v.arg_valid = false;
                return true:
            ans.pop_back();
        }
    return false;
}
```

#### 3.2 MCMF

```
// Ceiled MinCostMaxFlow, if not, use dinic
typedef struct {
    int from, to, w, cost;
} edge;
int n, m, parcel;
vector<edge> adj; // set num to each edge vector<int> p[505]; // p[u] has edge's num
int now_edge = 0;
void add_edge(int u, int v, int w, int cost){
    adj.push_back({u, v, w, cost});
    p[u].push_back(now_edge);
    now_edge++;
    adj.push_back({
         v, u, ⊙, -cost});
                               // argumenting path use -
    p[v].push_back(now_edge);
    now_edge++;
ll Bellman_Ford(){
    vector<ll> dis(n+1, inf); dis[1] = 0;
    vector<int> par(m);
    vector < int > flow_rec(n + 1, 0); flow_rec[1] = 1e9;
    for(int i = 1; i < n; i++){</pre>
        bool flag = 1;
         int size = adj.sz;
        for(int i = 0; i < size; i++){</pre>
             auto &[from, to, w, cost] = adj[i];
             if(w > 0 && dis[to] > dis[from] + cost){
```

```
flag = 0;
                 dis[to] = dis[from] + cost;
                 par[to] = i;
                               // record num
                  flow_rec[to] = min(flow_rec[from], w);
         if(flag) break;
     if(dis[n] == 1e9) return 0;
     int mn_flow = flow_rec[n];
     int v = n;
     while(v != 1){
         int u = adj[par[v]].from;
         adj[par[v]].w -= mn_flow;
         adj[par[v] ^ 1].w += mn_flow;
         v = u;
     mn_flow = min(mn_flow, parcel);
     parcel -= mn_flow;
     return mn_flow * dis[n];
void solve(){
    cin >> n >> m >> parcel;
     ll ans = 0;
     for(int i = 1; i <= m; i++){</pre>
         int u, v, w, cost; cin >> u >> v >> w >> cost;
         add_edge(u, v, w, cost);
     while(parcel > 0){
         int tmp = Bellman_Ford();
         if(tmp == 0) break;
ans += tmp;
     cout << (parcel > 0 ? -1 : ans);
}
```

### 4 向量

### 4.1 Cross Product

```
const double EPS = 1e-9;
struct point{
    double x, v;
    point operator * (ll a){return {a * x, a * y};}
    point operator
         + (point b){return {x + b.x, y + b.y};}
    point operator
          - (point b){return {x - b.x, y - b.y};}
        operator * (point b){return x * b.x + y * b.y;}
    double
        operator ^ (point b){return x * b.y - y * b.x;}
    bool operator <
        (point b){return x == b.x ? y < b.y : x < b.x;}
};
// len
double
     abs(point a){return sqrt(a.x * a.x + a.y * a.y);}
int sign(double a){
    if(abs(a) < EPS)</pre>
       return 0:
    else
        return (a > 0 ? 1 : -1);
//cross product
int ori(point a,point b,point c){
    return sign((b - a) ^ (c - a));
bool colinear(point a,point b,point c){
    return sign((b - a) \wedge (c - a)) == 0;
bool between
    (point a, point b, point c){ // c between a and b
    if(!colinear(a,b,c))
        return false;
    return sign((a - c) * (b - c)) <= θ;
bool intersect(point
     a,point b,point c,point d){ // line(a,b) line(c,d)
    int abc = ori(a,b,c);
    int abd = ori(a,b,d);
    int cda = ori(c,d,a);
    int cdb = ori(c,d,b);
    if(abc == 0 || abd == 0)
        return between(a,b,c) || between(
            a,b,d) || between(c,d,a) || between(c,d,b);
```

```
return abc * abd <= 0 && cda * cdb <= 0;
}
int main(){
   int n;
   cin >> n;
   point p[1010];
   cin >> p[0].x >> p[0].y;
   ll ans = 0;
   for(int i = 1;i < n;i++){
      cin >> p[i].x >> p[i].y;
      ans += (p[i] ^ p[i - 1]);
}
ans += (p[0] ^ p[n - 1]);
cout << abs(ans) << '\n';
return 0;
}</pre>
```

### 4.2 Convex Hull

```
vector<pii> P, L, U;
ll cross(pii o, pii a, pii b){ // OA OB >0 counterclock
    return (a.first - o.first) * (b.second - o.second
        ) - (a.second-o.second) * (b.first-o.first);
ll Andrew_monotone_chain(ll n){
    sort(P.begin(), P.end());
    ll l = 0, u = 0; // upper and lower hull
    for (ll i=0; i<n; ++i){</pre>
        while (l
              >= 2 && cross(L[l-2], L[l-1], P[i]) <= 0){
             1--;
             L.pop_back();
        while (u
              >= 2 && cross(U[u-2], U[u-1], P[i]) >= 0){
             u - -;
             U.pop_back();
        l++;
        u++;
        L.push_back(P[i]);
        U.push_back(P[i]);
    cout << l << ' ' << u << '\n';
    return l + u;
int main(){
    ll n,x,y;
    cin >> n;
for(ll i = 0;i < n;i++){</pre>
        cin >> x >> y;
        P.push_back({x,y});
    ll ans = Andrew_monotone_chain(n) - 2;
    cout << ans << "\n";
    return 0;
}
```

# 5 圖論

#### 5.1 2-SAT

```
#include <bits/stdc++.h>
using namespace std;
// +(-) u or +(-) v
const int maxn = 1e5 + 5;
vector<int> adj[2 * maxn], rev_adj[2 * maxn];
vector<int> order;
int cat[2 * maxn];
int k = 1;
bool vis[2 * maxn];
void dfs(int now){
    if (!vis[now]){
         vis[now] = 1;
for (auto v : adj[now]){
             dfs(v);
         order.push_back(now);
    }
void rev_dfs(int now){
    if (!vis[now]){
         cat[now] = k;
         vis[now] = 1;
         for (auto v : rev_adj[now]){
```

```
rev_dfs(v);
         }
    }
int main(){
    int n, m;
    cin >> m >> n;
     for(int i = 1; i <= m; i++){</pre>
         int u, v;
         char a, b;
         cin >> a >> u >> b >> v;
if (a == '-'){
              u = 2 * n - u + 1; // reverse
         if (b == '-'){
              v = 2 * n - v + 1; // reverse
         adj[2 * n - u + 1].push_back
         (v); // from -u to v; // if -u, then v adj[2 * n - v + 1].push_back
         (u); // from -v to u; // if -v, then u rev_adj[v].push_back(2 * n - u + 1);
         rev_adj[u].push_back(2 * n - v + 1);
     for(int i = 1; i <= 2 * n; i++){</pre>
         if (!vis[i]){
             dfs(i);
         }
    memset(vis, 0, sizeof(vis));
     reverse(order.begin(), order.end());
     for (auto i : order){
         if (!vis[i]){
              rev_dfs(i);
              k++;
         }
     char ans[2 * n + 1];
     for(int i = 1; i <= n; i++){</pre>
         if (cat[i] == cat[2 * n - i + 1]){
              cout << "IMPOSSIBLE";
              return:
         if (cat[i] > cat[2 * n - i + 1]){
              ans[i] = '+';
         else ans[i] = '-';
    for(int i = 1; i <= n; i++){</pre>
         cout << ans[i] << " ";
}
```

### 5.2 DFS跟BFS

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 1e6;
vector<int> adj[maxn];
bool vis[maxn];
void DFS(int s){
    if(vis[s]) return;
    vis[s] = true;
    for(auto u: adj[s]){
        DFS(u);
queue < int > q;
int dis[maxn];
void BFS(int x){
    vis[x] = true;
    dis[x] = 0;
    q.push(x);
    while(!q.empty()){
         int now = q.front();q.pop();
         for(auto nxt : adj[now]){
             if(vis[nxt]) continue;
             vis[nxt] = true;
dis[nxt] = dis[now] + 1;
             q.push(nxt);
        }
    }
}
```

### 5.3 **DSU**

```
#include <bits/stdc++.h>
using namespace std;
// After each day, print the number of components
// and the size of the largest component
struct DSU {
    vector<int> boss, siz;
    DSU (int n){
         boss.resize(n + 1);
         iota(boss.begin(), boss.end(), 0);
         siz.assign(n + 1, 1);
     int find_boss(int x){
         if(boss[x] == x) return x;
         return boss[x] = find_boss(boss[x]);
    bool same(int x, int y) {
    return find_boss(x) == find_boss(y);
     bool merge(int x, int y){
         x = find_boss(x);
         y = find_boss(y);
         if (x == y) {
              return false;
         if(siz[x] < siz[y]) swap(x, y);</pre>
         siz[x] += siz[y];
         boss[y] = x;
         return true;
     int size(int x){
         return siz[find_boss(x)];
};
```

### 5.4 EulerRoad

```
#include <bits/stdc++.h>
using namespace std;
// Undirected: check adj[i].sz
     == odd => IMPOSSIBLE ' road.sz != m+1 => IMPOSSIBLE
// Directed: minimize
     to 1 -> 2, so check in_degree == out_degree
int n, m;
const int maxn = 1e5 + 5;
set < int > adj[maxn]; // rev_adj[maxn];
int in[maxn];
void dfs(int now, vector<int> &road){
    while(!adj[now].empty()){
         int nxt = *adj[now].begin();
         adj[now].erase(nxt);
        dfs(nxt, road);
    road.push back(now);
void solve(){
    cin >> n >> m;
    memset(in, sizeof(in), 0);
for(int i = 1; i <= m; i++){</pre>
         int u, v; cin >> u >> v;
         adj[u].insert(v);
        in[v]++;
    in[1]++;
    in[n]--;
    for(int i = 1; i <= n; i++){</pre>
        if(adj[i].size() != in[i]){
             cout << "IMPOSSIBLE";</pre>
             return;
        }
    vector<int> road;
    dfs(1, road);
    if(road.size() != m+1){
        cout << "IMPOSSIBLE";</pre>
        return;
    reverse(road.begin(), road.end());
    for(auto i : road) cout << i <<</pre>
```

## 5.5 FloydWarshall

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 505;
int graph[maxn][maxn];
```

```
int dis[maxn][maxn];
int n, m, q; int a, b, c;
const int INF = 1e18;
int main(){
    cin >> n >> m >> q;
    for(int i = 0; i <= n; i++) {
    for(int j = 0; j <= n; j++) {</pre>
              graph[i][j] = INF;
    for(int i = 0; i < m; i++){</pre>
         cin >> a >> b >> c;
         graph[a][b] = min(graph[a][b], c);
         graph[b][a] = min(graph[b][a], c);
    for(int i = 0; i <= n; i++) {</pre>
         for(int j = 0; j <= n; j++) {</pre>
              dis[i][j] = graph[i][j];
    for(int i = 0; i <= n; i++) // self to self is 0</pre>
         dis[i][i] = 0;
    for(int k = 1; k <= n; k++){</pre>
         for(int i = 1; i <= n; i++){
    for(int j = 1; j <= n; j++){</pre>
                   dis[i][j] = min
                        (dis[i][j], dis[i][k] + dis[k][j]);
         }
    for(int i = 0; i < q; i++){</pre>
         cin >> a >> b;
         cout << (
              dis[a][b] >= INF ? -1 : dis[a][b]) << "\n";
    }
```

### 5.6 用 Bellman 找負環

```
#include <bits/stdc++.h>
using namespace std;
typedef struct{
    int from; int to;
    int weight;
} edge;
// NegCyc_Finding_Road
vector<edge> graph;
int main(){
    int src = 0;
                 cin >> n >> m;
    int n, m;
    vector < int > par(n + 1), dis(n + 1);
    for(int i = 0; i < m; i++){</pre>
         int a, b, w; cin >> a >> b >> w;
        graph.push_back({a, b, w});
    for(int i = 1; i <= n; i++){</pre>
        dis[i] = 1e9 + 5;
    dis[1] = 0;
    for(int i = 0; i <= n; i++){</pre>
        src = 0;
         for(auto [a, b, w] : graph){
             if(dis[b] > dis[a] + w){
                 dis[\bar{b}] = dis[a] + w;
                 par[b] = a;
                  src = b;
        }
    if(src){
         vector<int> ans;
         cout << "YES" << endl;</pre>
         for(int i = 0; i <= n; i++) src = par[src];</pre>
        ans.push_back(src);
        for(int i = par[src]; i != src; i = par[i]){
             ans.push_back(i);
        ans.push_back(src);
        reverse(ans.begin(), ans.end());
        for (auto i : ans){
    cout << i << " ";</pre>
    }
    else {
         cout << "NO" << endl;
```

## **5.7** 最大距離

```
#include <bits/stdc++.h>
using namespace std;
// Max_Dis, Use Topo, Use queue
/// If 1 can
't reach n, still may be relaxed 'Should dis[n] < 0
// Only Directed Graph
void print_ans(int n, vector<int> &par){
    deque<int> ans;
    int now = n;
    while(now != 1){
        ans.push_front(now);
        now = par[now];
    ans.push_front(1);
    cout << ans.size() << endl;</pre>
    cout << i <<
void solve(){
    int n, m;
    cin >> n >> m;
    vector<int> dis(n + 1, -1e9); dis[1] = 0;
    vector<int> graph[n+1];
    vector < bool > vis(n+1, 0);
    vector<int> par(n+1);
    vector<int> in(n+1, 0);
    queue<int> q;
    for(int i = 1; i <= m; i++){</pre>
        int u, v; cin >> u >> v;
        graph[u].push_back(v);
        in[v]++;
    for(int i = 1; i <= n; i++){</pre>
        if(in[i] == 0) q.push(i);
    while(!q.empty()){
        int u = q.front(); q.pop();
        for(auto nxt : graph[u]){
    if(dis[nxt] < dis[u] + 1){</pre>
                 dis[nxt] = dis[u] + 1;
                 par[nxt] = u;
             in[nxt]--; if(in[nxt] == 0) q.push(nxt);
        vis[u] = 1;
    if(dis[n] < 0){
        cout << "IMPOSSIBLE";</pre>
    else print_ans(n, par);
```

## 5.8 負權最大距離

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2505;
typedef struct {
    int u, v, w;
} edge;
int m, n;
vector<edge> graph;
vector<pair<int, int>> adj[maxn];
vector<int> rev_adj[maxn];
int dis[maxn];
bool vis[maxn] = {0};
bool nvis[maxn] = {0};
void dfs(int par, int now){
    if (vis[now] == 1) return;
    vis[now] = 1;
    for (auto [i, w] : adj[now]){
        if (i != par){
            dfs(now, i);
        }
   }
void rev_dfs(int par, int now){
   if (nvis[now] == 1) return;
    nvis[now] = 1;
    for (auto i : rev_adj[now]){
```

```
if (i != par){
              rev_dfs(now, i);
    }
void solve(){
    cin >> n >> m;
    for(int i = 1; i <= m; i++){</pre>
         int u, v, w;
         cin >> u >> v >> w;
         graph.push_back({u, v, w});
         adj[u].push_back({v, w});
         rev_adj[v].push_back(u);
    for(int i = 1; i <= n; i++) dis[i] = -1e9;</pre>
    dis[1] = 0;
    for(int i = 1; i <= n; i++){</pre>
         for (auto [u, v, w] : graph){
    if (dis[u] + w > dis[v]){
                   dis[v] = dis[u] + w;
         }
    dfs(0, 1);
    rev_dfs(0, n);
    for (auto [u, v, w] : graph){
    if (dis[u] + w > dis[v]
              && nvis[u] && nvis[v] && vis[u] && vis[v]){
              cout << -1;
              return;
         }
    }
    cout << dis[n];
```

### 5.9 Planet Queries II

```
#include <bits/stdc++.h>
using namespace std;
// now
     on a and want to reach b, the min steps, directed
int n, q;
const int maxn = 2e5 + 5;
int dp[30][maxn];
vector<vector<int>> cycles;
int no[maxn]; // Order & Can be in cycle, or out
int cycle_idx[maxn];
bool vis[maxn];
void set_out_of_cycle_no
    (int now, unordered_set < int > &done) {
if (done.find(now) != done.end())
         return;
    set_out_of_cycle_no(dp[0][now], done);
    done.insert(now);
    no[now] = no[dp[\theta][now]] - 1;
int wiint_go_to
    (int u, int k){ // return the node when walk k
for(int i = 0; i <= 18; i++){
   if (k & (1 << i)){</pre>
             u = dp[i][u];
         }
    return u;
void find_cycle(int now){
    unordered_set < int > appear;
    vector<int> vec;
    bool flag = true;
    while (appear.find(now) == appear.end()){
         appear.insert(now);
         vec.push_back(now);
         if (vis[now]){ // Didn't Find Cycle
             flag = false;
             break;
        now = dp[0][now];
    for (auto i : vec) vis[i] = true;
    if (!flag) return;
    int z = find(vec.begin(), vec.end(), now
         ) - vec.begin(); // start pushing from last now
    int m = vec.size();
    vector<int> cycle;
    for (int i = z; i < m; i++){</pre>
         cycle.push_back(vec[i]);
```

```
cycles.push_back(cycle);
void solve(){
    cin >> n >> q;
for(int u = 1; u <= n; u++){
   cin >> dp[0][u];
     for(int i = 1; i <= 18; i++){ // Make Chart</pre>
         for(int u = 1; u <= n; u++){</pre>
             dp[i][u] = dp[i - 1][dp[i - 1][u]];
     for(int i = 1; i <= n; i++){</pre>
         if (!vis[i]) find_cycle(i);
     int idx = 0;
     memset(no, -1, sizeof(no));
    memset(cycle_idx, -1, sizeof(cycle_idx));
unordered_set<int> done;
     for (auto &i : cycles){
         int c = 0;
         for (auto &j : i){
              no[j] = c++;
              cycle_idx[j] = idx;
              done.insert(j);
         idx++;
     for(int i
    = 1; i <= n; i++) set_out_of_cycle_no(i, done);
for(int i = 1; i <= q; i++){
         int u, v; cin >> u >> v;
         // Same Cycle
         if (cycle_idx[u] == cycle_idx[v] &&
               cycle_idx[u] != -1 && cycle_idx[v] != -1){
              int cyc_size = cycles[cycle_idx[u]].size();
              cout << (no[v]
                  - no[u] + cyc_size) % cyc_size << "\n";</pre>
         else if (cycle_idx[u] == -1 && cycle_idx
[v] == -1){ // Both are not in a Cycle
              if (no[u] > no[v]){
                   cout << -1 <<
                  continue;
              int jump = no[v] - no[u];
              if (wiint_go_to(u, jump) == v){
                  cout << jump << "\n";
              else cout << -1 << "\n";
               (cycle_idx[u] == -1 && cycle_idx[v] != -1)
              { // v is in cycle, Smainter Binary Search
              int l = -1, r = n;
              while (l <= r){
                   int m = (l + r) / 2;
                   if (cycle_idx[wiint_go_to
                        (u, m)] == cycle_idx[v]){
                       r = m - 1;
                   else
                       l = m + 1;
              if (l != -1 && l <= n){
                   int in_cycle_of_u = wiint_go_to(u, l);
                   int cycle_size
                  = cycles[cycle_idx[v]].size();
cout << l + (no[v] - no[in_cycle_of_u]
+ cycle_size) % cycle_size << "\n";
              else cout << -1 << "\n";
         else { // u is death in the cycle, can't reach
              cout << -1 << "\n";
         }
    }
}
```

## 5.10 Planets Cycles

```
#include <bits/stdc++.h>
using namespace std;
vector <int> dis, v;
vector <bool> vis;
```

```
int step;
queue < int > path;
void dfs(int x){
    path.push(x);
    if (vis[x]){
        step += dis[x];
        return:
    vis[x] = true;
    step++;
    dfs(v[x]);
// count path_dis to rep
int main(){
    int n; cin >> n;
    v.assign(n + 1, 0);
dis.assign(n + 1, 0);
    vis.assign(n + 1, false);
    for (int i = 1; i <= n; i++){</pre>
        cin >> v[i];
    for (int i = 1; i <= n; i++){</pre>
        step = 0;
        int is_outof_cycle = 1;
        dfs(i);
        while (!path.empty()){
            if (path.front() == path.back()){
                is_outof_cycle = 0;
            dis[path.front()] = step;
            step -= is_outof_cycle;
            path.pop();
        }
    cout << dis[i] <<
    cout << '\n';
```

#### 5.11 找環

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 1e5+5;
vector<int> graph[maxn];
int color[maxn], parent[maxn];
bool vis[maxn];
int n, m;
void print_ans(int ori){
    int now = parent[ori];
    deque<int> ans;
    ans.push_front(ori);
    while(now != ori){
        ans.push front(now);
        now = parent[now];
    ans.push_front(ori);
    cout << ans.size() << endl;</pre>
    cout << i <<
    }
    exit(0);
void dfs(int now){
    color[now] = 1;
    vis[now] = 1;
    for(auto nxt : graph[now]){
        parent[nxt] = now;
        if(color[nxt] == 1){
            print_ans(nxt);
        else if(color[nxt] == 0){
            dfs(nxt);
    color[now] = 2;
void solve(){
    cin >> n >> m;
    for(int i = 1; i <= m; i++){</pre>
        int u, v; cin >> u >> v;
        graph[u].push_back(v);
    for(int i = 1; i <= n; i++){</pre>
        if(!vis[i])
```

```
dfs(i);
}
cout << "IMPOSSIBLE";
}</pre>
```

#### 5.12 Prim

```
#include <bits/stdc++.h>
using namespace std;
#define pii pair<int, int>
int n, m;
int ans = 0;
const int maxn = 2e5 + 5;
vector<pair<int, int>> adj[maxn];
bool Prim(){
    int node_sz = 0;
    priority_queue<pii, vector<pii>, greater<pii>> pq;
    pq.push({0, 1});
    bool vis[maxn] = {false};
    while(!pq.empty()){
        auto [cost, u] = pq.top(); pq.pop();
        if(vis[u]) continue;
        vis[u] = true;
        ans += cost;
        node_sz++;
        for(auto [v, cost] : adj[u]){
             if(!vis[v])
                pq.push({cost, v});
    if(node_sz == n) return true;
    return false;
void solve(){
    cin >> n >> m;
    for(int i = 1; i <= m; i++){</pre>
        int u, v, cost; cin >> u >> v >> cost;
         adj[u].push_back({v, cost});
        adj[v].push_back({u, cost});
    if(Prim()) cout << ans;</pre>
    else cout << "IMPOSSIBLE";</pre>
}
```

### 5.13 SCC 跟拓樸 DP

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
// 找到所有 SCC 然後結合原圖重建一個 DAG,然後拓樸 DP
void dfs(int u, vector<int> &vis
     vector<int> &kosaraju, vector<vector<int>> &adj){
   if(!vis[u]){
       vis[u] = 1;
       for(auto v : adj[u]){
           dfs(v, vis, kosaraju, adj);
       kosaraju.push_back(u);
   }
void rev_dfs(int u, vector<int> &vis, vector<int> &
   order, vector<vector<int>> &rev_adj, int &scc_num){
   if(!vis[u]){
       vis[u] = 1;
       order[u] = scc_num;
       for(auto v : rev_adj[u]){
           rev_dfs(v, vis, order, rev_adj, scc_num);
   }
signed main(){
   int n, m, scc_num = 0;
   cin >> n >> m;
   vector
   vector<int>()), rev_adj(n + 1, vector<int>());
    vector<<mark>int</mark>> kosaraju;
   for(int i = 1; i <= n; i++){</pre>
       cin >> coin[i];
   for(int i = 1; i <= m; i++){</pre>
       int u, v; cin >> u >> v;
       adj[u].push back(v);
       rev_adj[v].push_back(u);
   }
```

```
for(int i = 1; i <= n; i++){</pre>
    if(!vis[i]){
        dfs(i, vis, kosaraju, adj);
reverse(kosaraju.begin(), kosaraju.end());
vis.assign(n + 1, 0);
for(auto &u : kosaraju){
    if(!vis[u]){
        scc_num++;
        rev_dfs(u, vis, order, rev_adj, scc_num);
    }
}
// 重新建 DAG
vector
    <vector<int>> DAG(scc_num + 1, vector<int>());
vector<int> in_degree(scc_num + 1, 0);
vector<int> sum_coin
    (scc_num + 1, 0), dp_coin(scc_num + 1, 0);
set<pair<int, int>> st;
int ans = -1e9;
for(int i = 1; i <= n; i++){</pre>
    sum_coin[order[i]] += coin[i];
    for(auto j : adj[i]){
        if(order[i] != order[j] && st.find
             ({order[i], order[j]}) == st.end()){
             DAG[order[i]].push_back(order[j]);
             in_degree[order[j]]++;
             st.insert({order[i], order[j]});
        }
    }
}
// 對 DAG 拓蹼 DP
queue < int > q;
for(int i = 1; i <= scc_num; i++){</pre>
    if(in_degree[i] == 0){
        q.push(i);
while(!q.empty()){
    int now = q.front(); q.pop();
    dp_coin[now] += sum_coin[now];
    ans = max(ans, dp_coin[now]);
    for(auto v : DAG[now]){
        in_degree[v]--;
        dp_coin[v] = max(dp_coin[v], dp_coin[now]);
        if(in_degree[v] == 0) q.push(v);
    }
cout << ans;
```

### 5.14 狀態 Dijkstra

```
#include <bits/stdc++.h>
using namespace std;
#define pii pair<int, int>
// Flight Discount
int n, m;
const int maxn = 2e5 + 5;
vector<pii> graph[maxn];
int dis[maxn][2];
                      // 0 for not used
void dijkstra(){
    priority_queue < vector
         <int>, vector<vector<int>>>, greater<vector<</pre>
    int>>> pq; // 0 for w, 1 for u, 2 for discount
for(int i = 1; i <= n; i++){</pre>
         dis[i][0] = dis[i][1] = 1e9;
    dis[1][0] = dis[1][1] = 0;
    pq.push({0, 1, 0});
    while(!pq.empty()){
         auto nxt = pq.top(); pq.pop();
         int dist
               = nxt[0], u = nxt[1]; bool us = nxt[2];
         if(dis[u][us
             ] < dist) continue; // is out of time, pass
         if(us){
             for(auto [v, w] : graph[u]){
                  if(dis[u][1] + w < dis[v][1]){
    dis[v][1] = dis[u][1] + w;</pre>
                       pq.push({dis[v][1], v, 1});
             }
         }
         else {
```

```
for(auto [v, w] : graph[u]){
                     if(dis[u][0] + w < dis[v][0]){
    dis[v][0] = dis[u][0] + w;
                          pq.push({dis[v][0], v, 0});
                     if(dis[u][0] + w / 2 < dis[v][1]){
    dis[v][1] = dis[u][0] + w / 2;</pre>
                          pq.push({dis[v][1], v, 1});
               }
          }
     cout << min(dis[n][0], dis[n][1]);</pre>
void solve(){
     cin >> n >> m;
     for(int i = 1; i <= m; i++){</pre>
          int u, v, w;
          cin >> u >> v >> w;
          graph[u].push_back({v, w});
     dijkstra();
}
```

### 5.15 Vis Dijkstra

```
#include <bits/stdc++.h>
using namespace std;
void solve(){
    int n, m, noon, night;
    cin >> n >> m >> noon >> night;
    int dis[n + 1];
    vector<int> graph[n + 1];
    bool vis[n + 1];
    for(int i = 1; i <= m; i++){</pre>
        int u, v, w; cin >> u >> v >> w;
        graph[u].push_back({v, w});
        graph[v].push_back({u, w});
    priority_queue < vector < int > ,
         vector<vector<int>>> pq;
    // noon is -
    for(int i = 1; i <= n; i++){</pre>
        dis[i] = 1e9; vis[i] = 0;
    pq.push({0, -noon, 1});
    dis[1] = 0;
    while(!pq.empty()){
        vector < int > now = pq.top(); pq.pop();
int now_noon = -now[1], u = now[2];
        if(vis[u]) continue;
        for(auto [nxt, w] : graph[u]){
            if(noon < w) continue; // never pass</pre>
            int tmp = dis[u] + (now_noon
                 >= w ? w : now_noon + night + w);
            if(tmp < dis[nxt]){</pre>
                dis[nxt] = tmp;
                pq.push({dis[nxt], -(now_noon >= w
                      ? now_noon - w : noon - w), nxt});
            }
        vis[u] = true;
    if(dis[n] == 1e9) cout << -1 << endl;
    else cout << dis[n] << endl;</pre>
// Investigation
void Investigation(){
    vector<vector<
    for(auto [v, w] : graph[u]){
        if(dis[u] + w < dis[v]){
            dis[v] = dis[u] + w;
            pq.push({dis[v], v});
            min_price_nums[v] = min_price_nums[u];
            max_dis_min_price
                 [v] = max_dis_min_price[u] + 1;
            min_dis_min_price
                 [v] = min_dis_min_price[u] + 1;
        else if(dis[u] + w == dis[v]){
            min_price_nums[v] = (min_price_nums
                 [u] + min_price_nums[v]) % mod;
            max dis min price
                 [v] = max(max_dis_min_price
                 [u] + 1, max_dis_min_price[v]);
```

## 6 數學

### 6.1 質因數分解

```
#include <bits/stdc++.h>
using namespace std;
// a^(m-1) □ 1 (mod m)
// a^(m-2) [] 1/a (mod m)
// EXP2
     : cout << fast_exp(x, fast_exp(y, p, MOD - 1), MOD)</pre>
// Filter +
    DP; DP save min factor, recur, factor decomposition
// FacNums = (x+1)(y+1)(z+1)...
// FacSum = (a^0+a^1...+a^x)(b^0+...+b^y)
// FacMul = N(x+1)(y+1)(z+1)/2
int main(){
    vector<int> is_prime(2e6 + 1, 1);
    // 1 代表是質數,非 1 不是
    for(int i = 2; i <= 1000; i++){</pre>
        if(is_prime[i] == 1){
            for(int j = i + i; j <= 1000000; j += i){</pre>
                 is_prime[j] = i;
        }
    int ans = 1;
    int q; cin >> q;
    map<int, int> mp;
    while(is_prime[q] != 1){
        mp[is_prime[q]]++;
        q /= is_prime[q];
    if(q != 1) mp[q]++;
    for(auto [a, b] : mp){
        ans *= b + 1;
    cout << ans << "\n";
}
```

### 6.2 盧卡斯定理

```
struct nCr {
     int mod;
     nCr(int mod) : mod(mod){};
     int inverse(int num){
          if(num == 1) return 1;
          return (mod - ((mod
               / num) * inverse(mod % num)) % mod) % mod;
     int fast_exp(int x, int p){
          int ans = 1;
          while(p > 0){
              if(p & 1) ans = (ans * x) % mod;
x = x * x % mod;
              p >>= 1;
          }
         return ans:
     }
     vector<int> fac;
     void BuildLucas(int n){
          fac.resize(n + 1);
          fac[0] = 1;
         for(int i = 1; i <= n; i++){</pre>
              fac[i] = fac[i - 1] * i % mod;
     int C(int m, int n){
          return m < n ? 0 : fac[m] * inverse</pre>
              (fac[n]) % mod * inverse(fac[m - n]) % mod;
     int Lucas(int m, int n){
         return n == 0 ? 1 % mod : Lucas(m / mod, n / mod) * C(m % mod, n % mod) % mod;
     }
};
```

### 7 Oueries

### 7.1 BIT

```
#include <bits/stdc++.h>
using namespace std;
struct BIT {
    int n;
    vector<int> bit;
    BIT(int n) {
        this -> n = n:
        bit.resize(n + 1, 0);
    void modify(int i, int val) {
    for (; i <= n; i += i & -i) {</pre>
            bit[i] += val;
    int query(int r) {
      int ans = 0;
      for (; r; r -= r & -r) ans += bit[r];
      return ans;
struct TwoDimensionBIT {
    int nx, ny;
    vector<vector<int>> bit;
    TwoDimensionBIT(int x, int y) {
        nx = x; ny = y;
        bit.resize(x + 1, vector \langle int \rangle(y + 1, 0));
    void modify(int x, int y, int mod){
        for(; x \le nx; x += x & -x){
             for(int
                  tmp = y; tmp \ll ny; tmp += tmp \& -tmp){
                 bit[x][tmp] += mod;
        }
    int query(int r1, int r2){
         int ans = 0;
         for(; r1; r1 -= r1 & -r1){
             for(int tmp = r2; tmp; tmp -= tmp & -tmp){
                 ans += bit[r1][tmp];
             }
         return ans;
    }
};
```

## 7.2 Increasing Array Queries

#include <bits/stdc++.h>

```
using namespace std;
const int maxn = 2e5+5;
int n, q;
int nums[maxn],
    prefix[maxn], ans[maxn], BIT[maxn], contrib[maxn];
vector<pair<int, int>> queries[maxn];
void update(int pos, int val) +
 for (; pos <= n; pos += pos & -pos) BIT[pos] += val;</pre>
int query(int a, int b) {
 int ans = 0;
  for (; b; b -= b&-b) ans += BIT[b];
 for (a--; a; a -= a&-a) ans -= BIT[a];
  return ans;
void solve(){
   cin >> n >> q;
for(int i = 1; i <= n; i++){</pre>
        cin >> nums[i];
        prefix[i] = prefix[i-1] + nums[i];
    nums[n + 1] = 1e9;
    prefix[n + 1] = 2e18;
    for(int i = 1; i <= q; i++){</pre>
        int a, b; cin >> a >> b;
        queries[a].push_back({b, i});
    deque<int> mono; mono.push_front(n+1);
    for(int i = n; i > 0;
        i--){ // question from start at n to start at 1
        while (nums[i] >= nums[mono.front()]) {
        update(mono.front(), -contrib[mono.front
            ()]); // mono.front's contrib become 0
```

```
11
         mono.pop front();
       contrib[i] = (mono.front() - 1 - i) * nums
       [i] - (prefix[mono.front() - 1] - prefix[i]);
update(i, contrib[i]);
       mono.push_front(i);
       for (auto j : queries[
    i]) { // pos is the index in mono <= end's
int pos = upper_bound(mono.begin</pre>
            (), mono.end(), j.first) - mono.begin() - 1;
       ans[j.second] = (pos ? query(i, mono
            [pos - 1]) : \theta) // smainter than y's mono
                                // mono
                                       to y caculate directly
                       + (j.first
                              - mono[pos]) * nums[mono[pos]]
                              - (prefix[j.first
                                   ] - prefix[mono[pos]]);
    for(int i = 1; i <= q; i++){</pre>
         cout << ans[i] << endl;</pre>
7.3 線段樹
#include <bits/stdc++.h>
using namespace std;
template <class Node, class Lazy>
struct Seg {
    int n;
    vector < Node > tree;
```

```
template <typename T>
    Seg (vector<T> init_){
        n = init_.size() - 1;
        tree.resize(4 * n);
        function <void(int, int,</pre>
             int)> build = [&](int now, int l, int r) {
             if (l == r) {
                tree[now] = init_[l];
                 return;
            int m = (l + r) / 2;
            build(now << 1, 1, m);</pre>
            build((now << 1) + 1, m + 1, r);
            pull(now);
        build(1, 1, n);
    Node query(int l, int r, int ql, int qr, int now){
        int m = (l + r) >> 1;
        if(qr < l || ql > r){
//
                       ----out of
     range, return what-----
            return {0};
        if(ql <= l && r <= qr){
            return tree[now];
      return query(l, m, ql, qr, now <<
1) + query(m + 1, r, ql, qr, (now << 1) + 1);
    Node query
        (int l, int r) { return query(1, n, l, r, 1); }
    void pull(int now){
        tree[now
            ] = tree[now << 1] + tree[(now << 1) + 1];
    void modify
        (int l, int r, int idx, int now, int add){
        if(l == r){
    how to modify ?-----
            tree[now].sum = add;
            return;
        }
        int m = (l + r) >> 1;
```

```
if(idx <= m){</pre>
           modify(l, m, idx, now << 1, add);
           modify(m + 1, r, idx, (now << 1) + 1, add);</pre>
       pull(now);
    void modify(int
        idx, int add) { modify(1, n, idx, 1, add); }
  -----define
    structure and info plus-----
struct Node {
   int sum;
Node operator+(const Node &a, const Node &b) {
   return {{a.sum + b.sum}};
   // use lc `rc to undate now
   // tree[now].sum = tree[lc].sum + tree[rc].sum;
   // tree[now].prefix = max(
       tree[lc].sum+tree[rc].prefix, tree[lc].prefix);
   // tree[now].suffix = max(
        tree[lc].suffix+tree[rc].sum, tree[rc].suffix);
   // tree[now].middle_max
        = max(max(tree[lc].middle_max, tree[rc].
       middle_max), tree[lc].suffix+tree[rc].prefix);
   // tree
       [now].middle_max = max(max(tree[now].middle_max
       , tree[now].prefix), tree[now].suffix);
}
//
// pizza_queries
// 左邊的店(s < t): dis_l = (pizza[s] - s) + t;
// 右邊的店(t < s): dis_r = (pizza[s] + s) - t;
// 實作: 建左查詢線段樹跟右查詢線段樹,用最小值pull
// 答案是
    min(left_query(1, s) + t, right_query(s, end) + t);
// List Removals
// 維護區間內有幾個數字被選過
// 用二分搜找右
    區間最小位,使得 ans - query == 1~ans 被選過的數量
```

### **7.4** 懶標線段樹

#include <bits/stdc++.h>

```
using namespace std;
template <class Node, class Lazy>
struct LazySeg {
    int n;
    vector < Node > tree;
    vector<Lazy> lazy;
    template <typename T>
    LazySeg (vector <T> init_){
    n = init_.size() - 1;
        tree.resize(4 * n);
         lazy.resize(4 * n);
         function <void(int, int,</pre>
              int)> build = [&](int now, int l, int r) {
             if (l == r) {
                 tree[now] = init_[l];
                 return;
             int m = (l + r) / 2;
build(now << 1, l, m);</pre>
             build((now << 1) + 1, m + 1, r);
             pull(now);
        }:
        build(1, 1, n);
    Node query(int l, int r, int ql, int qr, int now){
        int m = (l + r) >> 1;
        if(qr < l || ql > r){
   ----out of
     range, return what-----
             return Node();
//
```

```
push(now, l, r);
if(ql <= l && r <= qr){</pre>
         return tree[now];
  return query(l, m, ql, qr, now <<
    1) + query(m + 1, r, ql, qr, (now << 1) + 1);</pre>
Node query
    (int l, int r) { return query(1, n, l, r, 1); }
void pull(int now){
    tree[now
         ] = tree[now << 1] + tree[(now << 1) + 1];
void modify_add(int
    l, int r, int ql, int qr, int now, int add){
int m = (l + r) >> 1;
    if(qr < l || ql > r){
         return;
    if(ql <= l && r <= qr){
how to modify ?-----
         lazy[now].add += add;
    push(now, l, r);
modify_add(l, m, ql, qr, now << 1, add);</pre>
    modify_add
    (m + 1, r, ql, qr, (now << 1) + 1, add);

push(now << 1, l, m);

push((now << 1) + 1, m + 1, r);
    pull(now);
void modify_add(int l, int
    r, int add) { modify_add(1, n, l, r, 1, add); }
void modify_set(int
     l, int r, int ql, int qr, int now, int val){
    int m = (l + r) >> 1;
    if(qr < l || ql > r){
         return;
    if(ql <= l && r <= qr){
how to modify ?-----
         lazy[now].set_val = val;
         lazy[now].add = 0;
         return:
    }
    push(now, l, r);
    modify_set(l, m, ql, qr, now << 1, val);</pre>
    modify_set
    (m + 1, r, ql, qr, (now << 1) + 1, val); push(now << 1, l, m);
    push((now << 1) + 1, m + 1, r);
    pull(now);
void modify_set(int l, int
    r, int val) { modify_set(1, n, l, r, 1, val); }
void push(int now, int l, int r){
   apply(now, l, r);
 push down ?-----
    if(l != r){
         if(lazy[now].set_val){
             lazy[now
                   << 1].set_val = lazy[now].set_val;
             lazy[(now << 1)]
                   + 1].set_val = lazy[now].set_val;
             lazy[now << 1].add = lazy[now].add;</pre>
             lazy[(
                  now << 1) + 1].add = lazy[now].add;</pre>
         else {
             lazy[now << 1].add += lazy[now].add;</pre>
             lazy[(now
```

<< 1) + 1].add += lazy[now].add;

} |};

```
int size(Treap *treap) {
                                                               if (treap == NULL) return 0;
//
                                                               return treap->subsize;
                                                           // lazy
                                                           void push(Treap *t){
        lazy[now] = Lazy();
                                                               if(!t) return;
                                                               if(t->rev_valid){
    void apply(int now, int l, int r){
        if(lazy[now].set_val){
                                                                   swap(t->l, t->r);
                                                                   if(t->l) t->l->rev_valid ^= 1;
            tree[now].
                sum = (r - l + 1) * lazy[now].set_val;
                                                                   if(t->r) t->r->rev_valid ^= 1;
                                                               t->rev_valid = false;
        tree[now].sum += (r - l + 1) * lazy[now].add;
    }
                                                           Treap *merge(Treap *a, Treap *b){
// -----define
                                                               if(!a || !b) return a ? a : b;
    structure and info plus-----
                                                                                       // lazy
                                                                // push(a); push(b);
struct Node {
                                                               if(a->pri > b->pri){}
                                                                   a->r = merge(a->
    int sum;
                                                                       r, b); // a -> r = new, inorder, make sense
                                                                   a->pull();
struct Lazy {
    int set_val; int add;
                                                                   return a;
Node operator+(const Node &a, const Node &b) {
    return {{a.sum + b.sum}};
                                                                   b->l = merge(a,
                                                                        b->l); // new->l=a, inorder, make sense
//
                                                                   b->pull();
                                                                   return b;
// polynomial queries
                                                           pair < Treap*,
// 設置梯形的底跟加了幾次, apply_tag時底為l的合,
                                                                Treap*> split(Treap *root, int k) { // find 1~k
                                                             if (root == nullptr) return {nullptr, nullptr};
    d為加給次,所以sum += (底*2 + 次*區間) * 區間 / 2;
                                                               // push(root); // lazy
                                                             if (size(root->l) < k) {</pre>
7.5 莫隊
                                                               auto
                                                                   [a, b] = split(root->r, k - size(root->l) - 1);
#include <bits/stdc++.h>
                                                               root->r = a;
using namespace std;
                                                               root->pull();
struct query {
                                                               return {root, b};
    int l, r, id;
} typedef query;
void MO(int n, vector<query> &queries){
                                                               auto [a, b] = split(root->l, k);
    int block = sqrt(n);
                                                               root - > l = b;
    function <bool
                                                               root->pull();
        (query, query)> cmp = [&](query a, query b) {
                                                               return {a, root};
        int block_a = a.l / block;
int block_b = b.l / block;
        if(block a
                                                           void Print(Treap *t){
             != block_b) return block_a < block_b;
                                                               if(t){
        return a.r < b.r;</pre>
                                                                   // push(t);
                                                                                  // lazy
                                                                   Print(t->l);
    sort(queries.begin(), queries.end(), cmp);
                                                                   cout << t->val;
                                                                   Print(t->r);
void compress(vector<int> &nums){
                                                               }
    vector<int> sorted = nums;
    sort(sorted.begin(), sorted.end());
                                                           void substring_rev(){
    sorted.erase(unique
                                                               int n, m; cin >> n >> m;
        (sorted.begin(), sorted.end());
                                                               Treap *root = nullptr;
    for(int i = 0; i < nums.size(); i++){
   nums[i] = lower_bound(sorted.begin(), sorted</pre>
                                                               string str; cin >> str;
                                                               for(auto c : str){
            .end(), nums[i]) - sorted.begin() + 1;
                                                                   root = merge(root, new Treap(c));
}
                                                               for(int i = 1; i <= m; i++){</pre>
                                                                   int x, y; cin >> x >> y;
7.6 Treap
                                                                   auto [a,
                                                                        b] = split(root, x-1); // a: 1~x-1, b: x~n
#include <bits/stdc++.h>
                                                                   auto [
using namespace std;
                                                                       c, d] = split(b, y-x+1); // Use b to split
struct Treap {
                                                                    // c->rev_valid ^= true;
    Treap *l, *r;
                                                                   real ge(a, d); // Notice the order root = merge(b, c);
    int pri, subsize; char val; bool rev_valid;
    Treap(int val){
        this->val = val;
        pri = rand();
                                                               Print(root);
        l = r = nullptr;
                                                           }
        subsize = 1; rev_valid = 0;
                                                                搜尋與貪心
    void pull
              // update subsize or other information
        (){
                                                           8.1 二分搜
        subsize = 1;
        for(auto i : {l, r}){
            if(i) subsize += i->subsize;
                                                           int main(){
                                                               int l = 1, r = 10;
```

// 1 to tar, find tar

while(l <= r){</pre>

```
int m = (l + r) / 2;
if(check(m)) l = m + 1;
else r = m - 1;
}
cout << r;
// tar to end
while(l <= r){
   int m = (l + r) / 2;
   if(check(m)) r = m - 1;
   else l = m + 1;
}
cout << l;
}</pre>
```

#### 8.2 Concert Ticket

```
// Better than Binary Search
int main(){
    int n, m; cin >> n >> m;
    multiset<int> tik;
    for(int i = 0; i < n; i++){</pre>
        int tmp; cin >> tmp;
        tik.insert(tmp);
    while(m--){
        int x; cin >> x;
        auto it = tik.upper_bound(x);
        if(it == tik.begin()){
            cout << -1 <<
            continue:
        it--;
        cout << *it << " ";
        tik.erase(it);
    }
}
```

#### 8.3 Restaurant Customers

```
int main(){
    vector<pair<int, int>> times;
    int n; cin >> n;
    for(int i = 0; i < n; i++){
        int u, v; cin >> u >> v;
        times.push_back({u, 1});
        times.push_back({v, -1});
    }
    sort(times.begin(), times.end());
    int now_people = 0, ans = 0;
    for(auto [t, x]: times){
        ans = max(ans, (now_people += x));
    }
    cout << ans;
}</pre>
```

## 9 字串演算法

### 9.1 KMP

```
#include <bits/stdc++.h>
using namespace std;
struct KMP {
    string sub;
    vector<int> failure;
    KMP(string &sub) {
        this -> sub = sub:
        failure.resize(sub.size(), -1);
        buildFailFunction();
    void buildFailFunction() {
        for(int i = 1; i < sub.size(); i++) {
    int now = failure[i - 1];</pre>
             while(now != -1 && sub[
                 now + 1] != sub[i]) now = failure[now];
             if (sub[now
                  + 1] == sub[i]) failure[i] = now + 1;
        }
    vector<int> KMPmatching(string &s) {
        vector<int> match;
        for(int i = 0, now = -1; i < s.size(); i++) {</pre>
             // now is the compare sucessed length -1
             while (s[i] != sub[now
                  + 1] && now != -1) now = failure[now];
             // f stores
                  if comparison fail, move to where
```

```
if (s[i] == sub[now + 1]) now++;
    if (now + 1 == sub.size()) {
        match.push_back(i - now);
        now = failure[now];
    }
    return match;
}

return match;
}

int main(){
    string s = "xxtxxtxtx";
    string sub = "tx";
    KMP kmp(sub);
    vector <int> ans = kmp.KMPmatching(s);
    for(auto &i : ans) cout << i << " ";
}</pre>
```

```
9.2 Trie
#include <bits/stdc++.h>
using namespace std;
#define all(x) (x).begin(), (x).end()
#define endl "\n"
#define int long long
typedef pair<int, int> pii;
const int llinf = 4e18;
const int inf = 2e9;
const int mod = 1e9 + 7;
const int maxn = 2e5 + 5;
struct Trie {
    struct trie_node {
        bool is_word;
        vector<trie_node *> children;
        trie_node() {
            is_word = false;
            children.resize(26, NULL);
        }
    trie_node *root = new trie_node();
    void insert(string &s) {
        trie_node *cur = root;
        for (int i = 0; i < s.size(); i++) {</pre>
            int idx = s[i] - 'a'
            if (cur->children[idx] == NULL) {
                 cur->children[idx] = new trie_node();
            cur = cur->children[idx];
        cur->is_word = true;
    bool is_in_trie(string &s) {
        trie_node *cur = root;
        for(int i = 0; i < s.size(); i++) {</pre>
            if(cur->children
                [s[i] - 'a'] == nullptr) return false;
            cur = cur->children[s[i] - 'a'];
        }
        return true;
    int search_i_start
        (string &s, int i, vector<int> &dp) {
        trie_node *cur = root;
        int sz = s.size(), ans = 0;
        for(int j = i; j < sz; j++) {</pre>
            if(cur->children
                [s[j] - 'a'] == nullptr) return ans;
            cur = cur->children[s[j] - 'a'];
            if(cur->is_word)
                 (ans += dp[j + 1]) \% = mod;
        return ans;
    }
void solve(){
    // 找到 sub 集合裡,可以重複用,組成 s 的組數
    Trie trie;
    string s; cin >> s;
    int sz = s.size();
    // dp 代表 i 開頭到最後的配對總數
    // 找到有結尾為 stop 的 dp[i] += dp[j + 1]
    int n; cin >> n;
    vector<int> dp(sz + 1, 0);
    for(int i = 0; i < n; i++){</pre>
```

string sub; cin >> sub;

trie.insert(sub);

```
}
dp[sz] = 1;
for(int i = sz - 1; i >= 0; i--){
    dp[i] = trie.search_i_start(s, i, dp);
}
cout << dp[0] << endl;
}
signed main(){
    ios_base::sync_with_stdio(0);
    cin.tie(nullptr);
    int t = 1;
    // cin >> t;
    while(t--){
        solve();
    }
}
```

## 10 樹論

### 10.1 LCA

```
#include <bits/stdc++.h> // LCA from 1
using namespace std;
const int maxn = 2e5+5;
int boss[maxn];
int height[maxn];
int arr[18][maxn];
vector<int> tree[maxn];
void Calculate_H(int now_node){
     for(auto nxt_node : tree[now_node]){
         height[nxt_node] = height[now_node] + 1;
         Calculate_H(nxt_node);
    }
int Find_Ancestor(int k, int h){
   for(int i = 0; i <= 17; i++){
      if(h & (1 << i)) k = arr[i][k];</pre>
     return k;
int main(){
    memset(arr, 0, sizeof(arr));
     int n, q; cin >> n >> q;
     boss[1] = 1;
     for(int i = 2; i <= n; i++){</pre>
         int tmp; cin >> tmp; // tmp to i
         boss[i] = tmp;
         tree[tmp].push_back(i);
     Calculate_H(1);
     for(int i = 2; i <= n; i++){</pre>
         arr[0][i] = boss[i];
         int i = 1; i <= 17; i++){    // make chart
for(int j = 1; j <= n; j++){
    arr[i][j] = arr[i - 1][arr[i - 1][j]];</pre>
     for(int i = 1; i <= 17; i++){</pre>
         }
     while(q--){
         int a, b; cin >> a >> b;
         if(height[a] < height[b]) swap(a, b);</pre>
         a = Find_Ancestor(a, height
              [a] - height[b]); // same depth from 1
         if(a == b){ // same point
              cout << a << "\n";
              continue;
         for(int i = 17; i >= 0; i--){
              if(arr[i][a] != arr[i][b]){
                   // if a, b up 2 ^ i not the same point
                   a = arr[i][a];
                   b = arr[i][b];
              }
         cout << arr[0][a] << "\n"; // more one</pre>
    }
}
```

### 10.2 子樹 DP

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector<int> tree[maxn];
int sub[maxn];
void dfs(int now){
```

```
for(auto nxt : tree[now]){
    dfs(nxt);
    sub[now] += sub[nxt] + 1;
}
int main(){
    memset(sub, 0, sizeof(sub));
    int n; cin >> n;
    for(int i = 2; i <= n; i++){
        int b; cin >> b;
        tree[b].push_back(i);
    }
    dfs(1);
    for(int i = 1; i <= n; i++){
        cout << sub[i] << " ";
}
}</pre>
```

### 10.3 樹重心

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector<int> tree[maxn];
int cen = 0, n;
int dfs(int par, int now){
    bool flag = 1;
    int size = 0;
    for(auto nxt : tree[now]){
         if(par != nxt){
             int subsize = dfs(now, nxt);
             if(subsize > n / 2) flag = false;
             size += subsize;
        }
    if(n - 1 - size > n / 2) flag = false;
    if(flag) cen = now;
    return size + 1;
int main(){
    cin >> n;
    for(int i = 1; i < n; i++){</pre>
         int u, v; cin >> u >> v;
         tree[u].push_back(v);
         tree[v].push_back(u);
    for(int i = 1; i <= n; i++){
    for(auto nxt : tree[i])</pre>
             dfs(i, nxt);
         if(cen) break;
    }
```

### 10.4 節點距離總和

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector<int> tree[maxn];
vector<int> subtree(maxn, 1);
long long ans[maxn];
int n;
void dfs(int par, int now, int depth){
    ans[1] += depth;
    for(auto nxt : tree[now]){
   if(par != nxt) {
             dfs(now, nxt, depth + 1);
             subtree[now] += subtree[nxt];
        }
    }
void find_ans(int par, int now){
// each sub's dis make - 1, non subnode + 1
    for(auto nxt : tree[now]){
        if(par != nxt){
             ans[nxt] = ans[now
                 ] + (n - subtree[nxt]) - subtree[nxt];
             find_ans(now, nxt);
        }
    }
int main(){
    cin >> n;
    for(int i = 1; i < n; i++){</pre>
        int u, v; cin >> u >> v;
```

tree[u].push\_back(v);

```
tree[v].push_back(u);
     dfs(0, 1, 0);
     find_ans(0, 1);
     for(int i = 1; i <= n; i++){
    cout << ans[i] << " ";</pre>
}
         無權樹直徑
10.5
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
int dep1[maxn], dep2[maxn];
vector<int> tree[maxn];
int start, maxdep = 1, End;
void dfs1(int par, int now){
   for(auto nxt : tree[now]){
      if(par != nxt){
              dep1[nxt] = dep1[now] + 1;
              if(dep1[nxt] > maxdep){
                   maxdep = dep1[nxt];
                   start = nxt;
              dfs1(now, nxt);
         }
    }
void find_depth1(int par, int now){
     for(auto nxt : tree[now]){
         if(par != nxt){
              dep1[nxt] = dep1[now] + 1;
              if(dep1[nxt] > maxdep){
                   maxdep = dep1[nxt];
                   End = nxt;
               find_depth1(now, nxt);
         }
     }
void find_depth2(int par, int now){
    for(auto nxt : tree[now]){
   if(par != nxt){
              dep2[nxt] = dep2[now] + 1;
               find_depth2(now, nxt);
         }
    }
int main(){
     int n; cin >> n;
     for(int i = 1; i < n; i++){
   int u, v; cin >> u >> v;
          tree[u].push_back(v);
         tree[v].push_back(u);
     dep1[1] = 1;
     dfs1(0, 1);
     dep1[start] = 1;
     maxdep = 1;
     find_depth1(start, start);
     dep2[End] = 1;
     find_depth2(End, End);
     for(int i = 1; i <= n; i++){</pre>
          cout << max(dep1[i], dep2[i]) - 1 << " ";</pre>
}
10.6 有權樹直徑
#include <bits/stdc++.h> // weighted tree centroid
using namespace std;
const int maxn = 1e5+5;
using ll = long long;
vector<pair<int, int>> tree[maxn];
ll dp[maxn];
ll ans = 0;
void DP(int now, int par){
    ll mx1 = 0; ll mx2 = 0;
     for(auto [nxt, w] : tree[now]){
          if(nxt == par) continue;
         DP(nxt, now);
```

if(mx1

}

< w + dp[nxt]){ // mx2 = mx1 , mx1 = new mx

mx2 = mx1; mx1 = w + dp[nxt];

```
else if(mx2 < w + dp[nxt]){ // mx2 = new
            mx2 = w + dp[nxt];
    dp[now] = mx1;
    ans = max(ans, mx1 + mx2);
int main(){
    int n; cin >> n;
    memset(dp, 0, sizeof(dp));
    for(int i = 1; i < n; i++){</pre>
        int u, v, w; cin >> u >> v >> w;
        tree[u].push_back({v, w});
        tree[v].push_back({u, w});
    DP(1, 0);
    cout << (ans < 0 ? 0 : ans);
```