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## 1 Basic

#### 1.1 Basic [6a5860]

```
// 如何安裝 vscode
// 1. 下載 vscode & msys2
// 2. 在跳出的 terminal 中 / 或打開 ucrt64,打上
"pacman -5 --needed base-devel mingw-w64-x86_64-toolchain"
// 3. 環境變數加上 C:\\msys64\\ucrt64\\bin
// 4. 重開 vscode, 載 C/C++, 運行, 編譯器選擇 g++
// 5. 打開 settings -> compiler -> add compilerPath
          -> 在 "" 裡打上 C:\\msys64\\ucrt64\\bin\\g++.exe
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
#Include <ext/pb_ds/tree_policy.npp>
#define all(x) (x).begin(), (x).end()
#define pit pair<int, int>
#define endl "\n"
#define int long long
using namespace std;
using namespace __gnu_pbds;
template<typename T>
using pbds_set = tree<T, null_type,
    less<T>, rb_tree_tag, tree_order_statistics_node_update>;
template < typename T>
using pbds_multiset = tree<T, null_type, less_equal
    <T>, rb_tree_tag, tree_order_statistics_node_update>;
      uct cmp {  // 在有 template 的資結使用
bool operator()(const int &a, const int &b) const {
struct cmp {
            return a < b;
      // sort, bound 不用 struct
      // priority queue 小到大是 > , set 是 <
      // set 不能 = , multiset 要 = 
// 每個元素都要比到,不然會不見
      // pbds_multiset 的 upper_bound 跟 lower_bound
               功能相反,如果要 find,插入 inf 後使用 upper_bound
       // 內建 multiset
               可以跟 set 一樣正常使用, 自定義比較結構就比照以上
};
const int llinf = 4e18;
const int inf = 2e9;
const int mod = 1e9 + 7;
const int maxn = 2e5 + 5;
void solve() {
}
signed main() {
   ios_base::sync_with_stdio(0);
      cin.tie(nullptr);
      int t = 1;
cin >> t;
      while (t--) {
```

```
2 Graph
2.1 DFS跟BFS [2a332e]
```

solve():

```
#include <bits/stdc++.h>
using namespace std;
int main() {
       int n;
       vector<vector<int>> adj(n + 1, vector<int>());
       vector <vector <tint>> adj(n + 1, vector <tint>)
// dfs_graph
vector <book > vis(n + 1, 0);
auto dfs = [&](auto self, int u) -> void {
    if (vis[u]) return;
    vis[u] = true;
    for (auto v: adj[u]) {
        self(self, v);
    }
}
              }
       dfs(dfs, 1);
        // bfs
        vector<int> deep(n + 1, 1e9);
       queue<int> q;
auto bfs = [&](auto self, int u) -> void {
    vis[u] = true;
               deep[u] = 0;
               q.push(u);
               while (!q.empty()) {
                      int now = q.front(); q.pop();
for (auto nxt : adj[now]) {
   if (vis[nxt]) continue;
                              vis[nxt] = true;
deep[nxt] = deep[now] + 1;
                              q.push(nxt);
                      }
              }
       bfs(bfs, 1);
```

#### **2.2 DSU** [99b9f3]

## 2.3 最短距離算法 - Dijkstra [b8dfdb]

```
#include <bits/stdc++.h>
using namespace std;
#define pii pair<int, int>
// Flight Discount
void dijkstra() {
    int n, m; cin >> n >> m;
    vector<vector<pi>>> adj(n + 1, vector<pi>>(n + 1));
    vector vector<int>>
        dis(n + 1, vector<int>(2, 2e9)); // 0 for not used
    for (int i = 1; i <= m; i++) {
        int u, v, w;
        cin >> u >> v >> w;
        adj[u].push_back({v, w});
    }
    priority_queue
        <array<int, 3>> pq; // 0 for w, 1 for u, 2 for discount
    dis[1][0] = dis[1][1] = 0;
    pq.push({0, 1, 0});
    while (!pq.empty()) {
        auto [dist, u, us] = pq.top(); pq.pop();
        if (dis[u][us] < dist) continue;</pre>
```

```
if (us) {
                  us) {
for (auto [v, w] : adj[u]) {
   if (dis[u][1] + w < dis[v][1]) {
      dis[v][1] = dis[u][1] + w;
      pq.push({dis[v][1], v, 1});
}</pre>
                           }
                 }
                  for (auto [v, w] : adj[u]) {
   if (dis[u][0] + w < dis[v][0]) {
      dis[v][0] = dis[u][0] + w;
      pq.push({dis[v][0], v, 0});
}</pre>
                            if (dis[u][0] + w / 2 < dis[v][1]) {
                                    dis[v][1] = dis[u][0] + w / 2;
pq.push({dis[v][1], v, 1});
                           }
                }
        }
cout << min(dis[n][0], dis[n][1]);</pre>
```

## 2.4 最小生成樹 - Prim [5318c2]

```
#include <bits/stdc++.h>
using namespace std;
#define pii pair<int, int>
int n, m;
int ans = 0;
const int maxn = 2e5 + 5;
vector<pair<int, int>> adj[maxn];
bool Prim() {
       l Prim() {
  int node_sz = 0;
  priority_queue<pii, vector<pii>, greater<pii>> pq;
  pq.push({0, 1});
  bool vis[maxn] = {false};
  while (!pq.empty()) {
    auto [cost, u] = pq.top(); pq.pop();
    if (vis[u]) continue;
    vis[u] = true;
    ans += cost;
    node sz++:
                 node_sz++;
for(auto [v, cost] : adj[u]) {
    if (!vis[v])
                                  pq.push({cost, v});
                 }
         if (node_sz == n) return true;
         return false;
void solve() {
        for(int i = 1; i <= m; i++) {
   int u, v, cost; cin >> u >> v >> cost;
   adj[u].push_back({v, cost});
   adj[v].push_back({u, cost});
}
        if (Prim()) cout << ans;
else cout << "IMPOSSIBLE";</pre>
}
```

## 2.5 正權找環 [2a579d]

```
#include <bits/stdc++.h>
  using namespace std;
const int maxn = 1e5+5;
  vector < int > graph[maxn];
int color[maxn], parent[maxn];
    bool vis[maxn];
bool vis[maxn];
int n, m;
void print_ans(int ori) {
   int now = parent[ori];
   deque<int> ans;
   ans.push_front(ori);
   while (now != ori) {
      ans.push_front(now);
      ans.push_front(now);

                                                               now = parent[now];
                                   ans.push_front(ori);
                                cout << ans.size() << endl;
for (auto i : ans) {
    cout << i << " ";</pre>
                                exit(0);
    void dfs(int now) {
                                color[now] = 1;
vis[now] = 1;
                                 vis[now] = 1;
for (auto nxt : graph[now]) {
    parent[nxt] = now;
    if (color[nxt] == 1) {
        print_ans(nxt);
    }
}
                                                                else if (color[nxt] == 0) {
                                                                                               dfs(nxt);
                                color[now] = 2;
 void solve() {
```

```
for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
   graph[u].push_back(v);
for (int i = 1; i <= n; i++) {
      if (!vis[i])
            dfs(i);
cout << "IMPOSSIBLE";</pre>
```

## 2.6 負權找負環 [a27f3b]

```
用 Bellman Ford 找負環
#include <bits/stdc++.h>
using namespace std;
vector<array<int, 3>> graph; // u, v, w
int main() {
       main() {
int src = 0;
int n, m; cin >> n >> m;
vector <int > par(n + 1), dis(n + 1, 1e9);
for (int i = 0; i < m; i++) {
   int a, b, w; cin >> a >> b >> w;
   graph.push_back({a, b, w});
}
       dis[1] = 0;
for (int i = 0; i <= n; i++) {
    src = 0;
    for (auto [u, v, w] : graph) {
        if (dis[v] > dis[u] + w) {
            dis[v] = dis[u] + w;
            conful = u;
}
       dis[1] = 0;
                             par[v] = u;
                             src = v;
                     }
              }
       ,
if (src) { // 到第 n + 1 次還在鬆弛
    vector < int > ans;
    cout << "YES" << endl;
    for (int</pre>
                        i = 0; i <= n; i++) src = par[src]; // 找那個負環
               ans.push_back(src);
              for (int
                      i = par[src]; i != src; i = par[i]) { // 輸出負環
                     ans.push_back(i);
              ans.push_back(src);
               reverse(ans.begin(), ans.end());
              for (auto i : ans) {
    cout << i << " ";</pre>
       else {
               cout << "NO" << "\n";
}
```

#### 正權最大距離 [a3879a] 2.7

```
#include <bits/stdc++.h>
using namespace std;
// 只能用在 DAG, 用拓樸按順序鬆弛 // 如果 1 不能到達 n, n 也有
        可能被鬆弛,所以要看的是 dis[n] < 0,不能只看有沒有 = -1e9
void print_ans(int n, vector<int> &par) {
    deque<int> ans;
       int now = n;
while(now != 1) {
             ans.push_front(now);
now = par[now];
       ans.push front(1);
       cout << ans.size() << endl;
for(auto i : ans) {
    cout << i << " ";</pre>
void solve() {
       int n, m;
cin >> n >> m;
vector<int> dis(n + 1, -1e9); dis[1] = 0;
       vector<int> dis(n + 1, -1e9); dis[1] = 0;
vector<vector<int>> graph(n + 1, vector<int>());
vector<body>
vector<int> par(n + 1);
vector<int> in(n + 1, 0);
       queue<int> q;
       for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
   graph[u].push_back(v);
       for (int i = 1; i <= n; i++) {
   if(in[i] == 0) q.push(i);</pre>
       while (!q.empty()) {
              int u = q.front(); q.pop();
              for (auto nxt : graph[u]) {
    if (dis[nxt] < dis[u] + 1) {       // 鬆弛
        dis[nxt] = dis[u] + 1;
```

```
| par[nxt] = u;
| in[nxt]--;
| if (in[nxt] == 0) q.push(nxt);
| vis[u] = 1;
| if (dis[n] < 0) {
| cout << "IMPOSSIBLE";
| else print_ans(n, par);
| 2.8 負權最大距離[8372e8]
```

```
#define int long long
using namespace std;
// CSES High Score
const int maxn = 2505;
void dfs(int u, vector<int> &vis, vector<vector<int>> &adj) {
   if (vis[u]) return;
     vis[u] = 1;
for (int v : adj[u]) {
    dfs(v, vis, adj);
     }
void bellman_ford
      (int n, int s, vector<int> &vis, vector<int> &dis
, vector<array<int, 3>> edge, vector<vector<int>> &adj) {
fill(dis.begin(), dis.end(), -1e18);
      dis[s] = 0;
     dfs(v, vis, adj);
                      }
                }
           }
     }
signed main() {
   int n, m; cin >> n >> m;
     vector<array<int, 3>> edge;
vector<vector<int>> adj(n + 1);
      vector < int > dis(n + 1), vis(n + 1);
      while (m--) {
   int u, v, w;
   cin >> u >> v >> w;
           edge.push_back({u, v, w});
           adj[u].push_back(v);
```

#### 2.9 FloydWarshall [410f48]

bellman\_ford(n, 1, vis, dis, edge, adj);
if (vis[n]) cout << -1;
else cout << dis[n];</pre>

## **2.10** 歐拉環與歐拉路 [1dc3a5]

```
#include <bits/stdc++.h>
using namespace std;
```

```
1// 無向圖、尤拉環: 檢查每個點的出度為偶數
// 有向圖、
       尤拉路:可以看成 1 走到 n, 所以檢查所有點的出度等於入度
int n, m;
const int maxn = 1e5 + 5;
 vector<set<int>> adj;
 vector<int> in;
 void dfs(int now, vector<int> &road) {
       while (!adj[now].empty()) {
            int nxt = *adj[now].begin();
adj[now].erase(nxt);
dfs(nxt, road);
       road.push_back(now);
 void solve() {
      cin >> n >> m;
in.assign(n + 1, 0);
adj.assign(n + 1, set<int>());
for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
   adj[u].insert(v);
   int u = n; interpretation
            in[v]++;
       in[1]++;
       in[n]--;
       for (int i = 1; i <= n; i++) {
    if(adj[i].size() != in[i]) {</pre>
                 cout << "IMPOSSIBLE
                  return;
            }
       vector<int> road;
       dfs(1, road);
       if (road.size() != m + 1) {
            cout << "IMPOSSIBLE";</pre>
            return:
       reverse(road.begin(), road.end());
road.cout < i << " ";</pre>
       for(auto i : road) cout << i <<</pre>
```

# 2.11 SCC 結合拓樸 DP [382a7f]

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
// 找到所有 SCC 然後結合原圖重建一個 DAG, 然後拓樸 DP
void dfs(int u, vector<int</pre>
    > &vis, vector<int> &kosaraju, vector<vector<int>> &adj) {
if (!vis[u]) {
         vis[u]
         for (auto v : adj[u]) {
             dfs(v, vis, kosaraju, adj);
         kosaraju.push_back(u); // finish time 小到大排列
vis[u] = 1:
         order[u] = scc_num;
         for (auto v : rev_adj[u]) {
             rev_dfs(v, vis, order, rev_adj, scc_num);
    }
signed main() {
   int n, m, scc_num = 0;
   cin >> n >> m;
   vector<int> coin(n + 1), order(n + 1), vis(n + 1, 0);
    vector < vector < int >> adj(n + 1), rev_adj(n + 1);
    vector < int > kosaraju;
for (int i = 1; i <= n; i++) {</pre>
        cin >> coin[i];
    for (int i = 1; i <= m; i++) {
  int u, v; cin >> u >> v;
  adj[u].push_back(v);
         rev_adj[v].push_back(u);
    for (int i = 1; i <= n; i++) {
         if (!vis[i]) {
             dfs(i, vis, kosaraju, adj);
    reverse(kosaraju.begin(), kosaraju
    .end()); // 轉過來,從 finish time 大的開始做 dfs vis.assign(n + 1, 0); for (auto &u : kosaraju) {
        if (!vis[u]) {
             scc_num++;
             rev_dfs(u, vis, order, rev_adj, scc_num);
    // 重新建 DAG,根據原圖,如果不再同個 SCC,對 order 加邊
    vector<vector<int>> DAG(scc_num + 1, vector<int>());
    vector<int> in_degree(scc_num + 1, 0);
         > sum_coin(scc_num + 1, 0), dp_coin(scc_num + 1, 0);
```

```
set<pair<int. int>> st:
         int ans = -1e9;
for (int i = 1; i <= n; i++) {
                  (Int i = 1; i <= n; i++) {
sum_coin[order[i]] += coin[i];
for (auto j : adj[i]) {
    // 如果不是在同一個 scc 且 order 邊還沒加過
    if (order[i] != order[j] &&
        st.find({order[i], order[j]}) == st.end()) {
        DAG[order[i]].push_back(order[j]);
        in_degree[order[j]]++;
        in_degree[order[j]]++;</pre>
                                    st.insert({order[i], order[j]});
                          }
                 }
         // 對 DAG 拓蹼 DP
         queue <int> q;
for (int i = 1; i <= scc_num; i++) {
    if (in_degree[i] == 0) {</pre>
                        q.push(i);
         while (!q.empty()) {
                  int now = q.front(); q.pop();
dp_coin[now] += sum_coin[now];
ans = max(ans, dp_coin[now]);
for (auto v : DAG[now]) {
                           in_degree[v]-;
dp_coin[v] = max(dp_coin[v], dp_coin[now]);
if (in_degree[v] == 0) q.push(v);
         cout << ans;
}
2.12 2-SAT [7b512c]
#include <bits/stdc++.h>
```

```
using namespace std;
// +(-) u or +(-) v
// '(-/ u u/ +(-/ v
const int maxn = 1e5 + 5;
vector <int> adj[2 * maxn], rev_adj[2 * maxn];
vector <int> order;
int cat[2 * maxn];
int k = 1;
heal vis[2 * ----];
order.push_back(now);
        }
 void rev_dfs(int now) {
        if (!vis[now]) {
   cat[now] = k;
   vis[now] = 1;
   for (auto v : rev_adj[now]) {
                         rev_dfs(v);
        }
 int main() {
         int n, m;
cin >> m >> n;
for (int i = 1; i <= m; i++) {</pre>
                  int u, v;
                 char a, b;
                 cin >> a >> u >> b >> v;
if (a == '-') {
    u = 2 * n - u + 1; // reverse
                  if (b == '-') {
    v = 2 * n - v + 1; // reverse
                  adj[2 * n - u +
                                                  1].
                 adj[2 * n - u + 1].
    push_back(v); // from -u to v; // if -u, then v
adj[2 * n - v + 1].
    push_back(u); // from -v to u; // if -v, then u
rev_adj[v].push_back(2 * n - u + 1);
rev_adj[u].push_back(2 * n - v + 1);
         for (int i = 1; i <= 2 * n; i++) {
                  if (!vis[i]) {
                         dfs(i);
         fmemset(vis, 0, sizeof(vis));
reverse(order.begin(), order.end());
for (auto i : order) {
   if (!vis[i]) {
      rev_dfs(i);
      k++;
}
                 }
         char ans[2 * n + 1];
for (int i = 1; i <= n; i++) {
    if (cat[i] == cat[2 * n - i + 1]) {
        cout << "IMPOSSIBLE";</pre>
                          return;
                 }
```

```
if (cat[i] > cat[2 * n - i + 1]) {
    ans[i] = '+';
      else ans[i] = '-';
for (int i = 1; i <= n; i++) {
    cout << ans[i] << " ";</pre>
```

## 2.13 Planets Cycles [391e2a]

```
#include <bits/stdc++.h>
using namespace std;
vector<int> dis, v;
vector<bool> vis;
int step;
queue < int > path;
void dfs(int x) {
     path.push(x);
     if (vis[x]) {
          step += dis[x];
          return;
     vis[x] = true;
     step++;
     dfs(v[x]);
// count path_dis to rep
int main() {
   int n; cin >> n;
     tht n; ctn >> n;
v.assign(n + 1, 0);
dis.assign(n + 1, 0);
vis.assign(n + 1, false);
for (int i = 1; i <= n; i++) {
    cin >> v[i];
      for (int i = 1; i <= n; i++) {
          step = 0;
           int is_outof_cycle = 1;
          step -= is_outof_cycle;
path.pop();
          }
     for (int i = 1; i <= n; i++) {
    cout << dis[i] << ' ';</pre>
     cout << '\n';
```

## 2.14 Planet Queries II [8c2a64]

```
#include <bits/stdc++.h>
using namespace std;
// now on a and want to reach b, the min steps, directed
int n, q;
const int maxn = 2e5 + 5;
int dp[30][maxn];
vector<vector<int>> cycles;
int no[maxn]; // Order & Can be in cycle, or out
int cycle_idx[maxn];
bool vis[maxn];
void set_out_of_cycle_no(int now, unordered_set<int> &done) {
   if (done.find(now) != done.end())
      set_out_of_cycle_no(dp[0][now], done);
done.insert(now);
no[now] = no[dp[0][now]] - 1;
int wiint_go_to(int u, int k) { // return the node when walk k
    for (int i = 0; i <= 18; i++) {
        if (k & (1 << i)) {
            u = dp[i][u];
        }
}</pre>
            }
      return u;
void find_cycle(int now) {
      unordered_set<int> appear;
      vector<int> vec;
bool flag = true;
while (appear.find(now) == appear.end()) {
            appear.insert(now);
            appear.insert(now);
vec.push_back(now);
if (vis[now]) { // Didn't Find Cycle
    flag = false;
    break;
            now = dp[0][now];
      for (auto i : vec) vis[i] = true;
      if (!flag) return;
      int z = find(vec.begin(), vec.end
      (), now) - vec.begin(); // start pushing from last now
int m = vec.size();
      vector<int> cycle;
```

```
for (int i = z; i < m; i++) {
    cycle.push_back(vec[i]);</pre>
               cycles.push_back(cycle);
void solve() {
              cin >> n >> q;
for (int u = 1; u <= n; u++) {
    cin >> dp[0][u];
               for (int i = 1; i <= 18; i++) { // Make Chart
    for (int u = 1; u <= n; u++) {
        dp[i][u] = dp[i - 1][dp[i - 1][u]];
    }</pre>
               for (int i = 1; i <= n; i++) {
    if (!vis[i]) find_cycle(i);</pre>
               int idx = 0;
               memset(no, -1, sizeof(no));
memset(cycle_idx, -1, sizeof(cycle_idx));
unordered_set<int> done;
               for (auto &i : cycles) {
   int c = 0;
   for (auto &j : i) {
                                            no[j] = c++;
cycle_idx[j] = idx;
done.insert(j);
                              idx++:
               }
for (int i = 1; i <= n; i++) set_out_of_cycle_no(i, done);</pre>
               for (int i = 1; i <= q; i++) {
   int u, v; cin >> u >> v;
                                       Same Cycle
                              if (cycle_idx[u] == cycle_idx
    [v] && cycle_idx[u] != -1 && cycle_idx[v] != -1) {
    int cyc_size = cycles[cycle_idx[u]].size();
                                                               (no[v] - no[u] + cyc_size) % cyc_size << "\n";</pre>
                              else if (cycle_idx[u] == -1 &&
                                             cycle_idx[v] == -1) { // Both are not in a Cycle
if (no[u] > no[v]) {
   cout << -1 << "\n";</pre>
                                                             continue;
                                             int jump = no[v] - no[u];
if (wiint_go_to(u, jump) == v) {
    cout << jump << "\n";</pre>
                                              else cout << -1 << "\n";
                              lse if (cycle_idx[u] == -1 && cycle_idx[v]
    != -1) { // v is in cycle, Smainter Binary Search
    int l = -1, r = n;
    while (l <= r) {
        int m = (l + r) / 2;
        int m =
                                                             if (cycle_idx
                                                                              [wiint_go_to(u, m)] == cycle_idx[v]) {
                                                                           r = m - 1:
                                                            else
l = m + 1;
                                             if (l != -1 && l <= n) {
   int in_cycle_of_u = wiint_go_to(u, l);
   int cycle_size = cycles[cycle_idx[v]].size();</pre>
                                                            cout << l + (no[v] - no[in_cycle_of_u
    ] + cycle_size) % cycle_size << "\n";</pre>
                                             else cout << -1 << "\n";
                              else { // u is death in the cycle, can't reach
                                             cout << -1 << "\n";
             }
```

## 3 Data Structure

#### 3.1 BIT [d41d8c]

```
bit[i] += val:
       int query(int r) {
            int ans = 0;
            for (; r; r -= r & -r) ans += bit[r];
            return ans:
       int query(int l, int r) {
    return query(r) - query(l - 1);
 struct TwoDimensionBIT {
      int nx, ny;
vector<vector<int>> bit;
       TwoDimensionBIT(int x, int y) {
            nx = x; ny = y;
bit.resize(x + 1, vector<int>(y + 1, 0));
       for (; x <= nx; x += x & -x) {
    for (int tnp = y; tmp <= ny; tmp += tmp & -tmp) {</pre>
                       bit[x][tmp] += mod;
       int query(int r1, int r2) {
   int ans = 0;
            for (; r1; r1 -= r1 & -r1) {
    for (int tmp = r2; tmp; tmp -= tmp & -tmp) {
        ans += bit[r1][tmp];
    }
}
            return ans:
};
```

## 3.2 Increasing Array Queries [d41d8c]

```
using namespace std;
const int maxn = 2e5+5;
int nums
[maxn], prefix[maxn], ans[maxn], BIT[maxn], contrib[maxn];
vector<pair<int, int>> queries[maxn];
void update(int pos, int val) {
   for (; pos <= n; pos += pos & -pos) BIT[pos] += val;</pre>
int query(int a, int b) {
     int ans = 0;
for (; b; b -= b&-b) ans += BIT[b];
     for (a--; a; a -= a&-a) ans -= BIT[a];
     return ans:
void solve() {
     cin >> n >> q;
for (int i = 1; i <= n; i++) {
          cin >> nums[i];
          prefix[i] = prefix[i-1] + nums[i];
     nums[n + 1] = 1e9;
     prefix[n + 1] = 2e18;
for (int i = 1; i <= q; i++) {
   int a, b; cin >> a >> b;
          queries[a].push_back({b, i});
    deque < int > mono; mono.push_front(n+1);
          contrib[i] = (mono.front() - 1 - i) *
          nums[i] - (prefix[mono.front() - 1] - prefix[i]);
update(i, contrib[i]);
mono.push_front(i);
         - mono[pos]) * nums[mono[pos]]
                                 - (prefix
                                       [j.first] - prefix[mono[pos]]);
     for (int i = 1; i <= q; i++) {
    cout << ans[i] << endl;</pre>
```

# 3.3 線段樹 [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
template <class Node>
```

```
struct Seg {
                                                                                      struct LazySeg {
     int n;
                                                                                           int n;
     vector<Node> tree;
Seg (vector<Node> init_) {
    n = init_.size() - 1;
                                                                                           vector < Node > tree;
                                                                                           vector < Lazy > lazy;
template < typename T >
          LazySeg (vector<T> init_) { // 必須是 1-based
                                                                                                n = init_.size() - 1;
                                                                                                tree.resize(4 * n);
lazy.resize(4 * n);
                     tree[now] = init_[l];
                                                                                                function <void(int
                    return:
                                                                                                     , int, int)> build = [&](int now, int l, int r) {
if (l == r) {
               int m = (l + r) / 2;
build(now << 1, l, m);
build((now << 1) + 1, m + 1, r);</pre>
                                                                                                          tree[now] = init_[l];
                                                                                                     fint m = (l + r) / 2;
build(now << 1, l, m);
build((now << 1) + 1, m + 1, r);</pre>
               pull(now);
          build(1, 1, n);
                                                                                                     pull(now):
     Node query(int l, int r, int ql, int qr, int now) {
          int m = (l + r) >> 1;
if (qr < l || ql > r) {
                                                                                                build(1, 1, n);
               return Node();
                                                                                           Node query(int l, int r, int ql, int qr, int now) {
                                                                                                int m = (l + r) >> 1;
if (qr < l || ql > r) {
          if (ql <= l && r <= qr) {
               return tree[now];
                                                                                           out of range, return what
    return Node();
          Node query(int l, int r) { return query(1, n, l, r, 1); } void pull(int now) {
          tree[now] = tree[now << 1] + tree[(now << 1) + 1];</pre>
                                                                                                push(now, l, r);
if (ql <= l && r <= qr) {
    return tree[now];</pre>
     void modify(int l, int r, int idx, int now, int add) {
          if (l == r) {
// ---
      how to modify ?-----
                                                                                                return query(l, m, ql, qr, now
<< 1) + query(m + 1, r, ql, qr, (now << 1) + 1);
               tree[now].sum = add;
//
                                                                                           Node query(int l, int r) { return query(1, n, l, r, 1); }
void pull(int now) {
    tree[now] = tree[now << 1] + tree[(now << 1) + 1];
               return:
                                                                                           void modify_add
                                                                                                (int l, int r, int ql, int qr, int now, int add) {
int m = (l + r) >> 1;
if (qr < l || ql > r) {
          int m = (l + r) >> 1;
          if (idx <= m) {
    modify(l, m, idx, now << 1, add);</pre>
                                                                                                     return;
               modify(m + 1, r, idx, (now << 1) + 1, add);
                                                                                                if (ql <= l && r <= qr) {
          pull(now);
                                                                                            how to modify ?-----
                                                                                                     lazy[now].add += add;
     void modify
          (int idx, int add) { modify(1, n, idx, 1, add); }
      define structure and info plus-----
                                                                                                     return:
struct Node {
     int sum;
                                                                                                push(now, l, r);
                                                                                                pusn(now, t, i),
modify_add(l, m, ql, qr, now << 1, add);
modify_add(m + 1, r, ql, qr, (now << 1) + 1, add);
push(now << 1, l, m);
push((now << 1) + 1, m + 1, r);</pre>
     Node () {
         sum = 0;
     }
Node operator + (const Node &a, const Node &b) {
                                                                                                pull(now);
    Node c;
c.sum = a.sum + b.sum;
                                                                                           l, int r, int add) { modify_add(1, n, l, r, 1, add); }
void modify_set
     return c;
// use lc > rc to undate now
// tree[now].sum = tree[lc].sum + tree[rc].sum;
                                                                                                (int l, int r, int ql, int qr, int now, int val) {
int m = (l + r) >> 1;
if (qr < l || ql > r) {
     // tree[now].prefix
     = max(tree[lc].sum+tree[rc].prefix, tree[lc].prefix);
// tree[now].suffix
                                                                                                     return;
               max(tree[lc].suffix+tree[rc].sum, tree[rc].suffix);
     // tree[now].middle_max = max(max(tree[lc].middle_max, tree
[rc].middle_max), tree[lc].suffix+tree[rc].prefix);
                                                                                                if (ql <= l && r <= qr) {
     // tree[now].middle_max = max(max(tree[
                                                                                            how to modify ?----
lazy[now].set_val = val;
           now].middle_max, tree[now].prefix), tree[now].suffix);
                                                                                                     lazy[now].add = 0;
                                                                                                     return;
// pizza_queries
                                                                                                push(now, l, r);
modify_set(l, m, ql, qr, now << 1, val);
modify_set(m + 1, r, ql, qr, (now << 1) + 1, val);
push(now << 1, l, m);
push((now << 1) + 1, m + 1, r);
pull(now);</pre>
// 左邊的店(s < t): dis_l = (pizza[s] - s) + t;
// 右邊的店(t < s): dis_r = (pizza[s] + s) - t;
// 實作: 建左查詢線段樹跟右查詢線段樹,用最小值pull
// 答案是 min(left_query(1, s) + t, right_query(s, end) + t);
// 維護區間內有幾個數字被選過
                                                                                           l, int r, int val) { modify_set(1, n, l, r, 1, val); }
void push(int now, int l, int r) {
   apply(now, l, r);
                                                                                           void modify_set(int
// 用
      分搜找右區間最小位,使得 ans - query == 1~ans 被選過的數量
3.4 懶標線段樹 [d41d8c]
                                                                                            how to push down ?----
                                                                                                if (l != r) {
    if (lazy[now].set_val) {
#include <bits/stdc++.h>
using namespace std;
template <class Node, class Lazy>
                                                                                                           lazy[now << 1].set_val = lazy[now].set_val;</pre>
```

swap(t->l, t->r);
if (t->l) t->l->rev\_valid ^= 1;
if (t->r) t->r->rev\_valid ^= 1;

```
lazy[(now
                       << 1) + 1].set_val = lazy[now].set_val;
lazy[now << 1].add = lazy[now].add;
lazy[(now << 1) + 1].add = lazy[now].add;</pre>
                                                                                                      t->rev_valid = false;
                                                                                               Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) return a ? a : b;
    // push(a); push(b); // lazy
    if (a->pri > b->pri) {
        a->r = merge
                       lazy[now << 1].add += lazy[now].add;
lazy[(now << 1) + 1].add += lazy[now].add;</pre>
                                                                                                                 (a->r, b); // a->r = new, inorder, make sense
                                                                                                           a->pull();
           }
11
                                                                                                           return a:
       -----
                                                                                                      else {
                                                                                                           b->l = merge
                                                                                                                  (a, b->l); // new->l = a, inorder, make sense
           lazy[now] = Lazy();
                                                                                                           b->pull();
      void apply(int now, int l, int r) {
   if (lazy[now].set_val) {
      tree[now].sum = (r - l + 1) * lazy[now].set_val;
                                                                                                           return b;
                                                                                                     }
                                                                                               tree[now].sum += (r - l + 1) * lazy[now].add;
                                                                                                           auto [a, b] = split(root->r, k - size(root->l) - 1);
      define structure and info plus-----
struct Node {
                                                                                                           root->r = a;
                                                                                                           root->pull();
     int sum;
                                                                                                           return {root, b};
struct Lazy {
     int set_val; int add;
                                                                                                      else {
                                                                                                           auto [a, b] = split(root->l, k);
root->l = b;
root->pull();
Node operator+(const Node &a, const Node &b) {
     return {{a.sum + b.sum}};
                                                                                                           return {a, root};
                                                                                                void Print(Treap *t) {
                                                                                                     if (t) {
    // push(t);
    Print(t->l);
    cout << t->val;
// polynomial queries
// 設置梯形的底跟加了幾次, apply_tag時底為
                                                                                                           Print(t->r);
     l的合, d為加給次,所以sum += (底*2 + 次*區間) * 區間 / 2;
                                                                                                     }
3.5 莫隊 [d41d8c]
                                                                                                void substring_rev() {
                                                                                                     int n, m; cin >> n >> m;
Treap *root = nullptr;
string str; cin >> str;
for(auto c : str) {
    root = merge(root, new Treap(c));
}
#include <bits/stdc++.h>
using namespace std;
struct query {
  int l, r, id;
} typedef query;
void MO(int n, vector<query> &queries) {
  int block = sqrt(n);
  function
                                                                                                      for(int i = 1; i <= m; i++) {
                                                                                                           int x, y; cin >> x >> y;
auto [a, b] = split(root, x-1); // a: 1~x-1, b: x~n
auto [c, d] = split(b, y-x+1); // Use b to split
      function <bool(query, query)> cmp = [&](query a, query b) {
  int block_a = a.l / block;
  int block_b = b.l / block;
                                                                                                           // c->rev_valid ^= true;
// push(c);
b = merge(a, d); // N
            if (block_a != block_b) return block_a < block_b;</pre>
                                                                                                           reciye(a, d); // Notice the order root = merge(b, c);
           return a.r < b.r;</pre>
      sort(queries.begin(), queries.end(), cmp);
                                                                                                      Print(root);
void compress(vector<int> &nums) {
     vector < int > sorted = nums;
sort(sorted.begin(), sorted.end());
                                                                                                4
                                                                                                       Flow
      sorted.erase
             (unique(sorted.begin(), sorted.end()), sorted.end());
     4.1 Dinic [4d1a72]
                                                                                                #include <bits/stdc++.h>
                                                                                                using namespace std;
bool vis[505];
}
                                                                                               int lev[505], n, m, ans;
struct edge {
3.6 Treap [d41d8c]
                                                                                                    int to, w, rev_ind;
#include <bits/stdc++.h>
using namespace std;
struct Treap {
    Treap *l, *r;
                                                                                                vector<edge> adj[505];
                                                                                               bool label_level
   () { // Tag the depth, if can't reach end => return false
     Treap {
Treap *l, *r;
int pri, subsize; char val; bool rev_valid;
Treap(int val) {
    this->val = val;
                                                                                                      memset(lev, -1, sizeof(lev));
                                                                                                      lev[1] = 0;
queue < int > q;
                                                                                                                             q.push(1);
                                                                                                      queue<int> q;  q.pusn(1);
while (!q.empty()) {
   int u = q.front(); q.pop();
   for (auto i : adj[u]) {
      if (i.w > 0 && lev[i.to] == -1) {
           pri = rand();
            l = r = nullptr;
            subsize = 1; rev_valid = 0;
      void pull() { // update subsize or
    subsize = 1;
    for(auto i : {l, r}) {
        if (i) subsize += i->subsize;
                                                                                                                       q.push(i.to);
                             // update subsize or other information
                                                                                                                       lev[i.to] = lev[u] + 1;
                                                                                                           }
           }
                                                                                                      return (lev[n] == -1 ? false : true);
     }
int size(Treap *treap) {
    if (treap == NULL) return 0;
                                                                                               int dfs(int u, int flow) {
   if(u == n) return flow;
   for (auto &i : adj[u]) {
      return treap->subsize;
                                                                                                           if (lev[i.to] == lev[u] + 1 && !vis[i.to] && i.w > 0) {
   vis[i.to] = true;
   int ret = dfs(i.to, min(flow, i.w));
   if (ret > 0) {
// lazy
void push(Treap *t) {
   if (!t) return;
   if (t->rev_valid) {
```

i.w -= ret:

return ret;

}

adj[i.to][i.rev\_ind].w += ret;

```
return 0; // if can't reach end => return 0
void dinic(){
      while (label_level()) {
   while (1) {
      init(vis, 0);
      int tmp = dfs(1, inf);
      if(tmp == 0) break;
      ans += tmp;
}
            }
      }
void build() {
      for(int i = 1; i <= m; i++) {
   int u, v, w; cin >> u >> v >> w;
   adj[u].push_back({
       v, w, (int)adj[v].sz}); //
                                                            // inverse flow's index
             adj[v].push_back({u, 0, (int )adj[u].sz - 1}); // have pushed one, need to -1
// Police Chase, need
to open adj to Augment && ori to determine what pb give
// Dinic \ dfs2, then use reach as u, if the edge
    pb has given && w == 0 && v is not in reach, is the ans
void dfs2(int now, unordered_set<int> &reach) {
      if(!vis[now]){
    vis[now] = 1;
             reach.insert(now);
for(auto i : adj[now]){
    if(i.w > 0){
                         dfs2(i.to, reach);
            }
      }
// two two pair // School Dance
// Dinic, then w == 0's edge, which pb has given is the ans
// edge set valid var, if we need
to argument pos road, the reverse edge set true valid;
// if we need argument the argumented
if(v.arg_valid && '!vis[v.to]) {
                  ans.push_back(v.to);
                   vis[v.to] = true;
bool flag = get_road(v.to, ans, vis);
if(flag){
                          v.arg_valid = false;
                          return true;
                   ans.pop_back();
            }
      return false;
}
```

# 4.2 MCMF [40d5b7]

```
// Ceiled MinCostMaxFlow <sup>,</sup> if not, use dinic
typedef struct {
   int from, to, w, cost;
} edge;
int n. m. parcel:
vector <edge > adj; // set num to each edge vector <int > p[505]; // p[u] has edge's num
int now_edge = 0;
void add_edge(int u, int v, int w, int cost){
      adj.push_back({u, v, w, cost});
      p[u].push_back(now_edge);
now_edge++;
       adj.push_back
      ({v, u, 0, -cost});
p[v].push_back(now_edge);
                                                     // argumenting path use -
       now_edge++;
Il Bellman_Ford(){
    vector<ll> dis(n+1, inf); dis[1] = 0;
      vector<int> par(m);
vector<int> par(m);
vector<int> flow_rec(n + 1, 0); flow_rec[1] = 1e9;
for(int i = 1; i < n; i++){
   bool flag = 1;
}</pre>
             bool rlag = 1;
int size = adj.sz;
for(int i = 0; i < size; i++){
    auto &[from, to, w, cost] = adj[i];
    if(w > 0 && dis[to] > dis[from] + cost){
        flag = 0;
        dis[from] + cost;
                           dis[to] = dis[from] + cost;
                           par[to] = i; // record num
flow_rec[to] = min(flow_rec[from], w);
                    }
             if(flag) break;
       if(dis[n] == 1e9) return 0;
       int mn_flow = flow_rec[n];
```

```
int v = n;
while(v != 1){
   int u = adj[par[v]].from;
   adj[par[v]].w -= mn_flow;
   adj[par[v] ^ 1].w += mn_flow;
       mn_flow = min(mn_flow, parcel);
       parcel -= mn_flow;
return mn_flow * dis[n];
void solve(){
       cin >> n >> m >> parcel;
ll ans = 0;
       for(int i = 1; i <= m; i++){</pre>
             int u, v, w, cost; cin >> u >> v >> w >> cost;
add_edge(u, v, w, cost);
       while(parcel > 0){
   int tmp = Bellman_Ford();
   if(tmp == 0) break;
   ans += tmp;
       cout << (parcel > 0 ? -1 : ans);
```

#### 5 String

## 5.1 KMP [11be97]

```
#include <bits/stdc++.h>
using namespace std;
struct KMP {
       string sub;
vector<<mark>int</mark>> failure;
       KMP(string &sub) {
              this -> sub = sub:
              failure.resize(sub.size(), -1);
              buildFailFunction();
        void buildFailFunction() {
              for(int i = 1; i < sub.size(); i++) {
   int now = failure[i - 1];</pre>
                     while (now != -1
                     && sub[now + 1] != sub[i]) now = failure[now];
if (sub[now + 1] == sub[i]) failure[i] = now + 1;
             }
        ·
vector<<mark>int</mark>> KMPmatching(string &s) {
              for(int int) match;
for(int i = 0, now = -1; i < s.size(); i++) {
    // now is the compare successed length -1
    while (s[i] !=</pre>
                     sub[now + 1] && now != -1) now = failure[now];
// f stores if comparison fail, move to where
if (s[i] == sub[now + 1]) now++;
if (now + 1 == sub.size()) {
   match.push_back(i - now);
                             now = failure[now];
                     }
               return match;
      }
int main() {
       string s = "xxtxxtxtx";
string sub = "tx";
        KMP kmp(sub);
       vector < int > ans = kmp.KMPmatching(s);
for(auto &i : ans) cout << i << " ";</pre>
```

## 5.2 Trie [b84198]

```
#include <bits/stdc++.h>
##Intude voits/staterring
using namespace std;
#define all(x) (x).begin(), (x).end()
#define end! "|n"
#define int long long
typedef pair<int, int> pii;
const int linf = 4e18;
const int inf = 2e9;
const int mod = 1e9 + 7;
const int maxn = 2e5 + 5;
struct Trie {
    struct trie_node {
                     bool is_word;
                      vector<trie_node *> children;
                     trie_node() {
   is_word = false;
                                 children.resize(26, NULL);
                     }
          };
trie_node *root = new trie_node();
void insert(string &s) {
    trie_node *cur = root;
    for (int i = 0; i < s.size(); i++) {
        int idx = s[i] - 'a';
        if (cur->children[idx] == NULL) {
            cur->children[idx] = new trie_node();
        }
}
```

```
cur = cur->children[idx]:
           cur->is_word = true;
     bool is_in_trie(string &s) {
           trie_node *cur = root;

for (int i = 0; i < s.size(); i++) {

    if (cur->
                children[s[i] - 'a'] == nullptr) return false;
cur = cur->children[s[i] - 'a'];
     int search_i_start(string &s, int i, vector<int> &dp) {
    trie_node *cur = root;
           int sz = s.size(), ans = 0;
for (int j = i; j < sz; j++) {
    if (cur</pre>
                ->children[s[j] - 'a'] == nullptr) return ans;
cur = cur->children[s[j] - 'a'];
                if (cur->is_word)
     (ans += dp[j + 1]) %= mod;
           return ans:
    }
void solve() {
     // 找到 sub 集合裡,可以重複用,組成 s 的組數 Trie trie; string s; cin >> s;
     int sz = s.size();
     // dp 代表 i 開頭到最後的配對總數
     ..
// 找到有結尾為 stop 的 dp[i] += dp[j + 1]
     int n; cin >> n;
     vector < int > dp(sz + 1, 0);
for (int i = 0; i < n; i++) {
    string sub; cin >> sub;
           trie.insert(sub);
     dp[sz] = 1;
     for (int i = sz - 1; i >= 0; i--) {
    dp[i] = trie.search_i_start(s, i, dp);
     cout << dp[0] << endl;
signed main() {
     ios_base::sync_with_stdio(0);
     cin.tie(nullptr);
     int t = 1;
// cin >> t;
     while (t--) {
          solve();
```

## Math

# 質因數分解 [b535c8]

```
#include <bits/stdc++.h>
#include <bits/stdc++.h>
using namespace std;
/ a^(m-1) {\triple_equal} 1 (mod m)
// a^(m-2) {\triple_equal} 1/a (mod m)
// EXP2: cout << fast_exp(x, fast_exp(y, p, MOD - 1), MOD)
// Filter + DP; DP save min factor recur rector decomposition
// FacNums = (x+1)(y+1)(z+1)...
// FacSum = (a^0+a^1...+a^x)(b^0+...+b^y)
// FacMul = N(x+1)(y+1)(z+1)/2
int main() {</pre>
int main() {
    vector<int> is_prime(2e6 + 1, 1);

           for (int i = 2; i <= 1000; i++) {
    if (is_prime[i] == 1) {
        for (int j = i + i; j <= 1000000; j += i) {
            is_prime[j] = i;
            }
                       }
            int ans = 1;
           int ais = 1;
int q; cin >> q;
map<int, int> mp;
while (is_prime[q] != 1) {
    mp[is_prime[q]]++;
    q /= is_prime[q];
}
            if (q != 1) mp[q]++;
           for (auto [a, b] : mp) {
    ans *= b + 1;
            cout << ans << "\n";
}
```

## 6.2 矩陣快速冪 [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
const int mod = 1e9 + 7:
struct Mat {
```

```
int n:
       vector < vector < int >> matrix;
      Mat(int n) {
            this -> n = n:
            matrix.resize(n);
            for (int i = 0; i < n; i++) {
    matrix[i].resize(n);</pre>
      Mat(vector<vector<int>> matrix) {
            this->n = matrix.size():
            this -> matrix = matrix;
      Mat unit(int n) { // 單位矩陣
            Mat res(n);

for (int i = 0; i < n; i++) {
                  res.matrix[i][i] = 1;
            return res:
       void mul(Mat b) {
           j mul(Mat v, \( \)
Mat ans(n);
for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
        for (int k = 0; k < n; k++) {
            (ans.matrix[i][j] += (matrix[i][k]] * b.matrix[k][j] %</pre>
                                     i][k] * b.matrix[k][j] % mod)) %= mod;
                  }
            matrix = ans.matrix;
      void pow(int p) {
    Mat x = *this;
    *this = unit(n);
    while (p > 0) {
                  if (p & 1) {
                        mul(x);
                  x.mul(x);
            }
      }
int n, ans; cin >> n;
if (n <= 4) {</pre>
            vector < int > v = \{0, 1, 1, 2, 4\};
            ans = v[n];
       else {
            Mat mat({{4, 2, 1}, {2, 1, 1}, {1, 1, 0}});
            Mat x(3);
            x.matrix = {{1, 1, 0}, {1, 0, 1}, {1, 0, 0}};
x.pow(n - 4);
            mat.mul(x);
            ans = mat.matrix[0][0];
      cout << ans << "\n";
}
// 初始矩陣
// f4 f3 f2
// f3 f2 f1
// f2 f1 f0
 // 轉移式
// 1 1 0
// 1 0 1
// 1 0 0
// =>
// f5 f4 f3
// f4 f3 f2
// f3 f2 f1
        盧卡斯定理 [cf624d]
6.3
#include <bits/stdc++.h>
```

```
using namespace std;
struct nCr
     int mod;
     nCr(int mod) : mod(mod) {};
int inverse(int num) {
   if (num == 1) return 1;
           return (mod
                  - ((mod / num) * inverse(mod % num)) % mod) % mod;
     int fast_exp(int x, int p) {
           int ans = 1;
while (p > 0) {
   if (p & 1) ans = (ans * x) % mod;
   x = x * x % mod;
                 p >>= 1;
           return ans;
     vector<int> fac;
     void BuildLucas(int n) {
    fac.resize(n + 1);
           fac[0] = 1;
for(int i = 1; i <= n; i++) {
                 fac[i] = fac[i - 1] * i % mod;
```

## 6.4 樹論分塊 [fe6b55]

```
// CSES_Sum_of_Divisors
#include <bits/stdc++.h>
using namespace std;
#define int long long
const int mod = 1e9 + 7;
const int inv_2 = 5000000004;
// n / 1 * 1 + n / 2 * 2 + n / 3 * 3 + ... + n / n * n
signed main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);

    int ans = 0;
    int n; cin >> n;
    for (int l = 1, r; l <= n; l = r + 1) {
        r = n / (n / l);
        int val = n / l; // n / l 到 n / r 一樣的值
        int sum = (((l + r) % mod)) * mod * inv_2; // l 加到 r
        val %= mod; sum %= mod;
        ans += val * sum;
        ans %= mod;
    }
    cout << ans << "\n";
}</pre>
```

#### 6.5 Theorem

- 數論分塊可以快速計算一些含有除法向下取整的和式,就是像  $\sum_{i=1}^n f(i)g(\left\lfloor \frac{n}{i} \right\rfloor)$  的和式。當可以在O(1)內計算f(r)-f(l)或已經預處理出f的前綴和時,數論分塊就可以在 $O(\sqrt{n})$ 的時間內計算上述和式的值。
- 迪利克雷捲積 $h(x) = \sum_{d|x} f(d)g(\frac{x}{d})$
- 積性函數
  - 莫比烏斯函數

1.

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{for } n=1\\ 0 & \text{for } n \neq 0 \end{cases}$$

- 2.  $\mu$ 是常數函數 1 的反元素  $\Rightarrow \mu*1=\epsilon$ , $\epsilon(n)$ 只在n=1時為 1,其餘情況皆為 0。
- $-\phi$ 歐拉函數: x以下與x互質的數量

$$\phi*1 = \sum_{d|n} \phi(\frac{n}{d})$$
 質因數分解
$$= \sum_{i=0}^{c} \phi(p^{i})$$

$$= 1 + p^{0}(p-1) + p^{1}(p-1) + \dots + p^{c-1}(p-1)$$

$$= p^{c}$$

$$= id$$

• 莫比烏斯反演公式

- 
$$f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})$$
  
-  $f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)$ 

• 例子  $\sum_{i=aj=c}^{b} \sum_{j=1}^{d} [gcd(i,j) = k]$   $\Rightarrow \sum_{i=1}^{x} \sum_{j=1}^{y} [gcd(i,j) = k]$   $= \sum_{i=1}^{\left\lfloor \frac{x}{k} \right\rfloor} \left\lfloor \frac{y}{k} \right\rfloor$   $= \sum_{i=1}^{x} \sum_{j=1}^{y} \epsilon(gcd(i,j))$   $= \sum_{i=1}^{\left\lfloor \frac{x}{k} \right\rfloor} \left\lfloor \frac{y}{k} \right\rfloor$   $= \sum_{d=1}^{\infty} \mu(d) \sum_{i=1}^{\left\lfloor \frac{x}{k} \right\rfloor} [d|i] \sum_{j=1}^{y} \lfloor \frac{y}{k} \rfloor [d|j] d \text{ 可整除 } i \text{ 時為 } 1$   $= \sum_{d=1}^{\min(\left\lfloor \frac{x}{k} \right\rfloor, \left\lfloor \frac{y}{k} \right\rfloor)}$   $= \sum_{d=1}^{\min(\left\lfloor \frac{x}{k} \right\rfloor, \left\lfloor \frac{y}{k} \right\rfloor)} \mu(d) \left\lfloor \frac{x}{kd} \right\rfloor \left\lfloor \frac{y}{kd} \right\rfloor$ 

# 7 Search and Gready

## 7.1 二分搜 [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
int main() {
    int l = 1, r = 10;
    // 1 to tar, find tar
    while (l <= r) {
        int m = (l + r) / 2;
        if (check(m)) l = m + 1;
        else r = m - 1;
    }
    cout << r;
    // tar to end
    while (l <= r) {
        int m = (l + r) / 2;
        if (check(m)) r = m - 1;
        else l = m + 1;
    }
    cout << l;
}</pre>
```

## 7.2 三分搜 [d41d8c]

## 7.3 Concert Ticket [d41d8c]

```
// Better than Binary Search
#include <bits/stdc++.h>
using namespace std;
int main() {
   int n, m; cin >> n >> m;
   multiset <int> tik;
   for (int i = 0; i < n; i++) {
      int tmp; cin >> tmp;
      tik.insert(tmp);
   }
   while (m--) {
      int x; cin >> x;
      auto it = tik.upper_bound(x);
   if (it == tik.begin()) {
      cout << -1 << " ";
      continue;
   }
   it--;
   cout << *it << " ";
   tik.erase(it);
   }
}</pre>
```

#### 7.4 Restaurant Customers [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
int main() {
    vector <pair <int, int >> times;
    int n; cin >> n;
    for (int i = 0; i < n; i++) {
        int u, v; cin >> u >> v;
        times.push_back({u, 1});
        times.push_back({v, -1});
    }
    sort(times.begin(), times.end());
    int now_people = 0, ans = 0;
    for (auto [t, x] : times) {
        ans = max(ans, (now_people += x));
    }
    cout << ans;
}</pre>
```

## 8 Tree

## 8.1 LCA [ca194c]

```
#include <bits/stdc++.h> // LCA from 1
using namespace std;
int main() {
       int n, q; cin >> n >> q;
       vector <vector <int>> tree(n + 1);
for (int i = 1; i < n; i++) {
   int u, v; cin >> u >> v;
               tree[u].push_back(v);
               tree[v].push_back(u);
       vector < vector < int >> par(n + 1, vector < int > (18));
       vector <int> depth(n + 1);
auto dfs = [&](auto self, int u, int pre) -> void {
              for (auto v : tree[u]) {
    if (v == pre) continue;
    par[v][0] = u; // 2 ^ 0
    depth[v] = depth[u] + 1;
    self(self, v, u);
}
              }
       };
dfs(dfs, 1, 0);
for (int i = 1; i < 18; i++) {
    for (int j = 1; j <= n; j++) {
        par[j][i] = par[par[j][i - 1]][i - 1];
    }
}</pre>
       auto lca = [&](int a, int b) -> int {
    if (depth[a] < depth[b]) swap(a,</pre>
               int pull = depth[a] - depth[b];
for (int i = 0; i < 18; i++) {
   if (pull & (1 << i)) {</pre>
                              a = par[a][i];
               if (a == b) return a;
for (int i = 17; i >= 0; i--) {
    if (par[a][i] != par[b][i] }
                              a = par[a][i], b = par[b][i];
               return par[a][0];
```

## 8.2 樹 DFS [b6cb9a]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector <int> depth;
void dfs(vector <vector <int>> &tree, int u, int pre) {
    for(auto v : tree[u]){
        if(v == pre)
        depth[v] = depth[u] + 1;
        dfs(tree, v, u);
    }
}
```

## 8.3 樹重心 [2771f3]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+5;
vector<int> tree[maxn];
int cen = 0, n;
int dfs(int par, int now) {
   bool flag = 1;
   int size = 0;
   for (auto nxt : tree[now]) {
      if (par != nxt) {
        int subsize = dfs(now, nxt);
        if (subsize > n / 2) flag = false;
        size += subsize;
    }
   if (n - 1 - size > n / 2) flag = false;
   if (flag) cen = now;
```

```
return size + 1;
}
int main() {
    cin >> n;
    for (int i = 1; i < n; i++) {
        int u, v; cin >> u >> v;
        tree[u].push_back(v);
        tree[v].push_back(u);
}
    for (int i = 1; i <= n; i++) {
        for (auto nxt : tree[i])
            dfs(i, nxt);
        if (cen) break;
}</pre>
```

## 8.4 節點距離總和 [3bfb86]

```
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2e5+
vector < int > tree[maxn];
vector < int > subtree(maxn, 1);
long long ans[maxn];
int n;
void dfs(int par, int now, int depth) {
   ans[1] += depth;
   for (auto nxt : tree[now]) {
      if (par != nxt) {
         dfs(now, nxt, depth + 1);
         subtree[now] += subtree[nxt];
}
        }
// void find_ans(int par, int now) {
// each sub's dis make - 1, non subnode + 1
    for (auto nxt : tree[now]) {
        if (par != nxt) {
                          ans[nxt] =
                                      ans[now] + (n - subtree[nxt]) - subtree[nxt];
                           find ans(now, nxt);
        }
int main() {
        maln() {
  cin >> n;
  for (int i = 1; i < n; i++) {
    int u, v; cin >> u >> v;
    tree[u].push_back(v);
    tree[v].push_back(u);
}
         dfs(0, 1,
         find_ans(0, 1);
for (int i = 1; i <= n; i++) {
                  cout << ans[i] <<
}
```

## 8.5 有權樹直徑 [98f093]

## 8.6 樹壓平 [03946b]

```
|// 父節

點加值 = 所有子節點區間加值,求單點,使用 bit,做前綴差分

// CSES 1138_Path Queries

#include <bits/stdc++.h>

#define int long long
```

```
using namespace std;
struct BIT { // BIT 都是 1-based 的查詢
      int n;
       vector<int> bit;
      BIT(int n) { // 有幾個數
this->n = n;
             bit.resize(n + 1, 0);
      BIT(vector < int > & init) { // 必須是 1-based this - >n = init.size() - 1;
             bit.resize(n + 1, 0);
for (int i = 1; i <= n; i++) {
    modify(i, init[i]);</pre>
       void modify(int i, int val) {
    for (; i <= n; i += i & -i) {
        bit[i] += val;
    }
}</pre>
       int query(int r) {
             int ans = 0;
for (; r; r -= r & -r) ans += bit[r];
return ans;
       int query(int l, int r) {
    return query(r) - query(l - 1);
      }
};
void solve(){
      int n, q; cin >> n >> q;
vector <int> node_value(n + 1), euler_ordered_value(n);
for (int i = 1; i <= n; i++) {</pre>
             cin >> node_value[i];
       vector<vector<int>> tree(n + 1):
       for (int i = 1; i < n; i++) {
   int u, v; cin >> u >> v;
             tree[u].push_back(v);
tree[v].push_back(u);
       vector<pair<int, int>> tree_mapping(n + 1);
       int cnt = 0;
       auto dfs = [&](auto self, int u, int par) -> void {
    euler_ordered_value[++cnt] = node_value[u];
    tree_mapping[u].first = cnt;
             for (auto v : tree[u]) {
   if (v == par) continue;
   self(self, v, u);
             tree_mapping[u].second = cnt;
      dfs(dfs, 1, 0);
      for (int i = 1; i <= n; i++) {
   bit.modify(tree_mapping[i].first, node_value[i]);
}</pre>
             if (tree_mapping[i].first < n) {</pre>
                   bit.modify
                          (tree_mapping[i].second + 1, -node_value[i]);
      for (int i = 0; i < q; i++) {
   int op; cin >> op;
             if (op == 1) {
   int s, x; cin >> s >> x;
   int add = x
                             - euler_ordered_value[tree_mapping[s].first];
                   euler_ordered_value[tree_mapping[s].first] = x;
bit.modify(tree_mapping[s].first, add);
if (tree_mapping[s].first < n) {</pre>
                          bit.modify(tree_mapping[s].second + 1, -add);
             else {
   int node; cin >> node;
                   cout <<
                            bit.query(tree_mapping[node].first) << "\n";</pre>
      }
}
```

#### 9 DP

## 9.1 背包問題 [9457ef]

```
#include <bits/stdc++.h>
using namespace std;
// 考慮前 i 個,預算有 j 塊錢的最多 page
int main(){
   int n, bud;
   cin >> n >> bud;

    vector <vector <int>> dp(n + 1, vector <int>(bud + 1));
vector <int> Page(n + 1, 0);
     vector<int> Price(n + 1, 0);
    for(int i = 1; i <= n; i++){</pre>
         cin >> Price[i];
     for(int i = 1: i <= n: i++){
         cin >> Page[i];
```

```
for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= bud; j++) {</pre>
      if (j >= Price[i]) { // 買得起
           // 不買或買
          else {
          dp[i][j] = dp[i - 1][j];
   }
cout << dp[n][bud] << "\n";
```

## 9.2 Bitmask DP [c130ec]

```
#include <bits/stdc++.h>
using namespace std;
const int mod = 1e9 + 7;
void travel_exactly_once(){
     // [走過的路][終點]
      vector < vector < int >> dp(1 << 20, vector < int > (20, 0));
     vector <int> up(1 << 20),
vector <int> rev_adj[20];
int n, m; cin >> n >> m;
for(int i = 0; i < m; i++){
   int u, v; cin >> u >> v;
   rev_adj[--v].push_back(--u);
}
     for (int road = 0; road < (1 << n); road++) {
    // 沒經過起點,不用走
    if (road & 1 == 0) continue;</pre>
           // DP,隨便選定一個當前路徑的終點
           for (int end = 0; end < n; end++) {
    // 路徑沒包含假定的 end
                 if ((road & (1 << end)) == 0) continue;</pre>
                // 去除終點,得到 pre_road
int pre_road = road - (1 << end);
                 // 從 rev_adj 找 pre_road 的終點
                 for (int pre_road_end : rev_adj[end]) {
   if ((road & (1 << pre_road_end))) {</pre>
                           dp[road
                           [][end] += dp[pre_road][pre_road_end];
dp[road][end] %= mod;
                      }
                }
          }
     cout << dp[(1 << n) - 1][n - 1];
void elevator_rides(){
   int n, k; cin >> n >> k;
   vector <int > passenger(n);
   for (int i = 0; i < n; i++) cin >> passenger[i];
     vector<int
     if (used[pre] + passenger[j] <= k) {</pre>
                            ---[r·-] · pubberilg []] <= K)
// 電梯數先比,再來比用掉的空間
if (dp
                                 (pre] < dp[i] || (dp[pre] == dp[i] &&
    used[pre] + passenger[j] < used[i])) {
    used[i] = used[pre] + passenger[j];
    dp[i] = dp[pre];</pre>
                           }
                      }
                      // 搭新的電梯
                      }
                }
          }
     cout << dp[(1 << n) - 1];
int main(){
     travel exactly once();
     elevator_rides();
```

#### 9.3 硬幣 [d41d8c]

```
#include <bits/stdc++.h>
using namespace std;
const int mod = 1e9 + 7;
```

// along to dp to trace back while (m >= 1 && n >= 1) { if (s1[m - 1] == s2[n - 1]) { s[length - 1] = s1[m - 1]

```
void coin_combination_II(){
      // 有 n 種錢幣,求組合為 x 的組數,順序不可顛倒
      length--;
                                                                                                                       if (dp[m - 1][n] > dp[m][n - 1]){
      vector < vector < int >> dp(2, vector < int > (x + 1, 0));
      dp[0][0] = 1;
                                                                                                                       else n--;
      for (int i = 1; i <= n; i++) cin >> coin[i];
for (int i = 1; i <= n; i++){
    for (int j = 0; j <= x; j++) {</pre>
                                                                                                                 }
                  // 壓到 2 * n
                                                                                                    }
                  dp[i & 1][j] = dp[!(i & 1)][j];
                  if (j >= coin[i]) {
                                                                                                     9.6 LIS [f23284]
                        (dp[i
                               & 1][j] += dp[i & 1][j - coin[i]]) %= mod;
                                                                                                    #include <bits/stdc++.h>
                  }
                                                                                                     using namespace std;
// Rec Sequence LIS
            }
                                                                                                     void solve(){
      cout << dp[n & 1][x];
                                                                                                           int n; cin >> n;
vector<int> v(n);
void minimize_coins_nums(){
                                                                                                           for (int i = 0;
                                                                                                                                  í < n; i++) {
         有 n 種錢幣,求組合為 x 的最小硬幣數
      // 有 n 種錢幣, 氷組 a 向 int n, x; cin >> n >> x;
                                                                                                                 cin >> v[i];
      vector < int > coin(n);
                                                                                                           int dp[n]; vector<int> mono;
      for (int i = 0; i < n; i++) cin >> coin[i];
// dp[i] 是組合為 i 的最小硬幣數
                                                                                                           mono.push_back(v[0]);
dp[0] = 1; int L = 1;
for (int i = 1; i < n; i++)
      vector <int> dp(x + 1, 0);
      for (int i = 1; i <= x; i++) {
                                                                                                                 if (v[i] > mono.back())
            dp[i] = 2e9;
for(auto &j : coin){
    if(j <= i){</pre>
                                                                                                                       mono.push_back(v[i]);
                                                                                                                       dp[i] = ++L;
                        dp[i] = min(dp[i], dp[i - j] + 1);
                                                                                                                 else {
                                                                                                                       auto it
                                                                                                                           = lower_bound(mono.begin(), mono.end(), v[i]);
            }
                                                                                                                       *it = v[i];
dp[i] = it - mono.begin() + 1;
      cout << (dp[x] == 2e9 ? -1 : dp[x]);
int main(){
      coin_combination_II();
                                                                                                           vector<int> ans:
                                                                                                           cout << L << endl;</pre>
      minimize_coins_nums();
                                                                                                           for (int i = n - 1; i >= 0; i--) {
   if (dp[i] == L) {
      ans.push_back(v[i]);
}
9.4 編輯距離 [80c4dc]
                                                                                                                }
#include <bits/stdc++.h>
using namespace std;
void solve(){
                                                                                                           reverse(ans.begin(), ans.end());
                                                                                                           for (auto i : ans) {
   cout << i << " "</pre>
      string s1, s2; cin >> s1 >> s2;
      int size1 = s1.size(), size2 = s2.size();
      // dp[i][j] 為 s1 的前 i 個字元,跟 s2 的前 j 個字元
      vector <
             vector<int>> dp(size1 + 1, vector<int>(size2 + 1, 0));
                                                                                                    9.7 Projects [c03e88]
      vector<int>> dp(size1 + 1, vector<int>(size1)
s1 = "0" + s1, s2 = "0" + s2;
for (int i = 1; i <= size1; i++) dp[i][0] = i;
for (int i = 1; i <= size2; i++) dp[0][i] = i;
for (int i = 1; i <= size1; i++){
    for (int j = 1; j <= size2; j++) {
        if (s1[i] == s2[j]) {
            dp[i][j] = dp[i - 1][j - 1];
        }
}</pre>
                                                                                                     #include <bits/stdc++.h>
                                                                                                    struct project {
                                                                                                           int from, end, gain, id;
                        // s1 新增等價於 s2 砍掉
                        // dp[i][j] = min(修改, s1 新增, s2 新增);
dp[i][j] = min({dp[i - 1][
j - 1], dp[i - 1][j], dp[i][j - 1]}) + 1;
                                                                                                     void solve(){
                                                                                                           int n; cin >> n;
vectorprojects(n + 1);
for (int i = 1; i <= n; i++) {</pre>
                                                                                                                 cin >> projects
    [i].from >> projects[i].end >> projects[i].gain;
projects[i].id = i;
            }
      cout << dp[size1][size2];</pre>
                                                                                                           sort(all(projects), [](project a, project b) {
   if (a.end == b.end) return a.gain < b.gain;
   return a.end < b.end;</pre>
9.5 LCS [937a28]
#include <bits/stdc++.h>
                                                                                                           vector<array
 using namespace std;
                                                                                                                  <int, 3>> dp(n + 1, {0, 0, 0}); // nums, gain, time
int main(){
                                                                                                           vector <int> par(n + 1, {0, 0, 0}); // nums, gatn, time
vector <int> par(n + 1, 0), ans, add(n + 1, -1);
for (int i = 1; i <= n; i++) {
   int idx = --upper_bound(projects.begin(), projects.
      begin() + i, project({0, projects[i].from, 0, 0}),
   [](const project &a, const project &b) -> bool {
      return a.end < b.end;
}</pre>
      int m, n; cin >> m >> n;
string s1, s2;
      cin >> s1 >> s2;
int L = 0;
      vector<vector<int>> dp(m + 1, vector<int>(n + 1, 0));
     for (int i = 1; i <= m; i++) {
   for (int j = 1; j <= n; j++) {
      if (s1[i - 1] == s2[j - 1]) {
            dp[i][j] = dp[i - 1][j - 1] + 1;
      }
}</pre>
                                                                                                                 }) - projects.begin();
// cerr << idx << "\n";</pre>
                                                                                                                                                      // cerr << idx <<
dp[i] = dp[i - 1];
                                                                                                                 par[i] = i - 1;
if (dp[i][1] < dp[idx][1] + projects[i].gain ||</pre>
                                                                                                                 (dp[i][1]
                                                                                                                       == dp[idx][1] + projects[i].gain && dp[i][2] > dp
[idx][2] + projects[i].end - projects[i].from)) {
dp[i] = {dp[idx
                        dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
                  }
            }
                                                                                                                              ][0] + 1, dp[idx][1] + projects[i].gain, dp[idx][2] + projects[i].end - projects[i].from};
      int length = dp[m][n];
cout << length << "\n";
string s('a', length);</pre>
                                                                                                                       par[i] = idx;
                                                                                                                       add[i] = projects[i].id;
```

}

[n][0] << " " << dp[n][1] << " " << dp[n][2] << endl;

```
for (int now = n; now > 0; now = par[now]) {
   if (add[now] != -1)
                                                                                                            cout << l << ' ' << u << '\n';
                  ans.push_back(add[now]);
                                                                                                            return l + u;
      sort(all(ans));
                                                                                                      int main(){
      for(auto &i : ans) cout << i << " ";</pre>
                                                                                                            ll n,x,y;
                                                                                                            cin >> n;
for(ll i = 0;i < n;i++){</pre>
}
9.8 Removal Game [45a446]
                                                                                                                  cin >> x >>
                                                                                                                  P.push_back({x,y});
#include <bits/stdc++.h>
                                                                                                            il ans = Andrew_monotone_chain(n) - 2;
cout << ans << "\n";</pre>
using namespace std;
void solve(){
                                                                                                            return 0:
      int n; cin >> n;
      vector<vector<int>> dp(n + 1, vector<int>(n + 1));
      int pref = 0:
      vector < int > v(n + 1);
      for (int i = 1; i <= n; i++) {</pre>
            cin >> v[i];
pref += v[i];
      // dp[i][j] = max_diff(i to j);
for (int i = n; i > 0; i--) {
    for (int j = 1; j <= n; j++) {
        if (i > j) continue;
        else if (i == j) {
            dp[i][j] = v[i];
        }
}
                        dp[i][j] = max(v[i] - dp[i + 1][j], v[j] - dp[
i][j - 1]); // i+1, j-1, care dp's order
                  }
           }
      // x + y = sum, dp[1][n] = x - y;
cout << (pref + dp[1][n]) / 2;
```

# 10 Geometry

## 10.1 Cross Product [c37c89]

# 10.2 Convex Hull [e8ad24]