# Contents



# 1 Basic

## 1.1 Default Code [d41d8c]

```cpp
#include <bits/stdc++.h>

using namespace std;
using ll = long long;

void solve() {

}

int main() {
    ios_base::sync_with_stdio(false);
    cin.tie(nullptr);
    int t = 1;
    cin >> t;
    while (t--) {
        solve();
    }
    return 0;
}
```

## 1.2 Compare Fuction [d41d8c]

```cpp
// 1. sort, 二分搜刻在函式內 lambda 就好
// 2. priority queue 小到大是 >, set 是 <
// 3. set 不能 = , multiset 必須 =
// 4. 確保每個成員都要比到
// 5. pbds_multiset 不要用 lower_bound
// 6. 如果要用 find, 插入 inf 後使用 upper_bound
// 7. multiset 可以跟 set 一樣使用, 但請注意第 3、4 點

auto cmp = [](int i, int j) { return i > j; };
priority_queue<int, vector<int>, decltype(cmp)> pq(cmp);

vector<int> a {1, 2, 5, 4, 3}; // 小心不要改到 a
auto cmp = [&a](int i, int j) { return a[i] > a[j]; };
priority_queue<int, vector<int>, decltype(cmp)> pq(cmp);
```

## 1.3 Pbds [d41d8c]

```cpp
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<class T>
using pbds_set = tree<T, null_type,
      less<T>, rb_tree_tag, tree_order_statistics_node_update>;
template<class T>
using pbds_multiset = tree<T, null_type, less_equal
     <T>, rb_tree_tag, tree_order_statistics_node_update>;
```

## 1.4 Double [7db939]

```cpp
struct D {
    double x;
    D() : x{0} {}
    D(double x) : x{x} {}
    constexpr static double eps = 1E-12;
    explicit operator double() const { return x; }
    D operator-() const {
        return D(-x);
    }
    D &operator*=(D rhs) & {
        x *= rhs.x; return *this;
    }
    D &operator+=(D rhs) & {
        x += rhs.x; return *this;
    }
    D &operator-=(D rhs) & {
        x -= rhs.x; return *this;
    }
    D &operator/=(D rhs) & {
        assert(fabs(rhs.x) > eps);
        x /= rhs.x; return *this;
    }
    friend D operator*(D lhs, D rhs) {
        return lhs *= rhs;
    }
    friend D operator+(D lhs, D rhs) {
        return lhs += rhs;
    }
    friend D operator-(D lhs, D rhs) {
        return lhs -= rhs;
    }
    friend D operator/(D lhs, D rhs) {
        return lhs /= rhs;
    }
    friend istream &operator>>(istream &is, D &a) {
        double v; is >> v; a = D(v); return is;
    }
    friend ostream &operator<<(ostream &os, const D &a) {
        return os << fixed << setprecision(10)
                  << a.x + (a.x > 0 ? eps : a.x < 0 ? -eps : 0);
    } // eps should < precision
    friend bool operator<(D lhs, D rhs) {
        return lhs.x - rhs.x < -eps;
    }
    friend bool operator>(D lhs, D rhs) {
        return lhs.x - rhs.x > eps;
    }
    friend bool operator==(D lhs, D rhs) {
        return fabs(lhs.x - rhs.x) < eps;
    }
    friend bool operator!=(D lhs, D rhs) {
        return fabs(lhs.x - rhs.x) > eps;
    }
    friend bool operator<=(D lhs, D rhs) {
        return lhs < rhs || lhs == rhs;
    }
    friend bool operator>=(D lhs, D rhs) {
        return lhs > rhs || lhs == rhs;
    }
};
```

## 1.5 Int128 [85923a]

```cpp
using i128 = __int128_t; // 1.7E38
istream &operator>>(istream &is, i128 &a) {
    i128 sgn = 1; a = 0;
    string s; is >> s;
    for (auto c : s) {
        if (c == '-') {
            sgn = -1;
        } else {
            a = a * 10 + c - '0';
        }
    }
    a *= sgn;
    return is;
}
ostream &operator<<(ostream &os, i128 a) {
    string res;
    if (a < 0) os << '-', a = -a;
    while (a) {
        res.push_back(a % 10 + '0');
        a /= 10;
    }
    reverse(res.begin(), res.end());
    os << res;
    return os;
}
```

### 1.6 Rng [401544]

```
mt19937_64 rng
    (chrono::steady_clock::now().time_since_epoch().count());
ll x = rng();
shuffle(a.begin(), a.end(), rng);
```

# 2 Graph

## 2.1 DFS And BFS [e2d856]

```
int main() {
    int n;
    vector<vector<int>> adj(n);
    // dfs_graph
    vector<bool> vis(n);
    auto dfs = [&](auto self, int u) -> void {
        if (vis[u]) return;
        vis[u] = true;
        for (auto v: adj[u]) {
            self(self, v);
        }
    };
    dfs(dfs, 0);
    // bfs
    vector<int> depth(n, 1e9);
    queue<int> q;
    auto bfs = [&](auto self, int s) -> void {
        vis[s] = true, depth[s] = 0;
        q.push(s);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (auto v : adj[u]) {
                if (vis[v]) continue;
                vis[v] = true;
                depth[v] = depth[u] + 1;
                q.push(v);
            }
        }
    };
    bfs(bfs, 0);
}
```

## 2.2 Prim [7e2d87]

```
auto prim =
     [&](int n, vector<vector<pair<int, int>>> &adj) -> bool {
    int sz = 0;
    priority_queue<pair<int, int>,
          vector<pair<int, int>>, greater<pair<int, int>>> pq;
    pq.emplace(0, 0); // w, vertex
    vector<bool> vis(n);
    while (!pq.empty()) {
        auto [u, w] = pq.top();
        pq.pop();
        if (vis[u]) continue;
        vis[u] = true;
        sz++;
        for (auto v : adj[u])
            if (!vis[v.first])
                pq.emplace(v.second, v.first);
    }
    if (sz == n) return true;
    return false;
};
```

## 2.3 Bellman-Ford [430ded]

```
// 用 Bellman Ford 找負環
int main() {
    int n, m; cin >> n >> m;
    vector<array<int, 3>> e;
    for (int i = 0; i < m; i++) {
        int u, v, w; cin >> u >> v >> w;
        u--, v--; e.push_back({u, v, w});
    }
    vector<ll> dis(n, inf), par(n);
    int t = -1; dis[0] = 0;
    for (int i = 1; i <= n; i++) {
        for (auto [u, v, w] : e) {
            if (dis[v] > dis[u] + w) {
                dis[v] = dis[u] + w;
                par[v] = u;
                if (i == n) t = v;
            }
        }
    }
    if (t == -1) { cout << "NO\n"; return; }
    for (int i = 1; i < n; i++) t = par[t];
    vector<int> ans {t};
    int i = t;
    do {
        i = par[i];
        ans.push_back(i);
    } while (i != t);
    reverse(ans.begin(), ans.end());
    cout << "YES\n";
    for (auto x : ans) cout << x + 1 << " ";
}
```

## 2.4 Floyd-Warshall [da23ad]

```
constexpr ll inf = 1E18;
void FloydWarshall(int n, int m) {
    int n, m; cin >> n >> m;
    vector<vector<int>> dis(n, vector<int>(n, inf));
    for (int i = 0; i < m; i++) {
        int u, v, w; cin >> u >> v >> w;
        dis[u][v] = min(dis[u][v], w);
        dis[v][u] = min(dis[v][u], w);
    }
    for (int i = 0; i < n; i++) dis[i][i] = 0;
    for (int k = 0; k < n; k++)
        for (int i = 0; i < n; i++)
            for (int j = 0; j < n; j++)
                dis[i][j
                    ] = min(dis[i][j], dis[i][k] + dis[k][j]);
}

const int N = 500; // Floyd 封包
void Floyd(int n, vector<bitset<N>> &dp) {
    for (int k = 0; k < n; k++)
        for (int i = 0; i < n; i++)
            if (dp[i][k])
                dp[i] |= dp[k];
}
```

## 2.5 Euler [4177dc]

```
// 1. 無向圖是歐拉圖：
// 非零度頂點是連通的
// 頂點的度數都是偶數

// 2. 無向圖是半歐拉圖(有路沒有環)：
// 非零度頂點是連通的
// 恰有 2 個奇度頂點

// 3. 有向圖是歐拉圖：
// 非零度頂點是強連通的
// 每個頂點的入度和出度相等

// 4. 有向圖是半歐拉圖(有路沒有環)：
// 非零度頂點是弱連通的
// 至多一個頂點的出度與入度之差為 1
// 至多一個頂點的入度與出度之差為 1
// 其他頂點的入度和出度相等
vector<int> ans;
auto dfs = [&](auto &&self, int u) -> void {
    while (g[u].size()) {
        int v = *g[u].begin();
        g[u].erase(v);
        self(self, v);
    }
    ans.push_back(u);
};
dfs(dfs, 0);
reverse(ans.begin(), ans.end());
```

## 2.6 DSU [b7ac4a]

```
struct DSU {
    int n;
    vector<int> boss, siz;
    DSU(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_; boss.resize(n);
        iota(boss.begin(), boss.end(), 0);
        siz.assign(n, 1);
    }
    int find(int x) {
        if (boss[x] == x) return x;
        return boss[x] = find(boss[x]);
    }
    bool same(int x, int y) {
        return find(x) == find(y);
    }
    bool merge(int x, int y) {
        x = find(x); y = find(y);
        if (x == y) return false;
        if (siz[x] < siz[y]) swap(x, y);
        siz[x] += siz[y];
        boss[y] = x;
        n--;
        return true;
    }
    int size(int x) {
        return siz[find(x)];
    }
};

struct DSU {
    int n;
    vector<int> boss, siz, stk;
    DSU(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_;
        boss.resize(n);
        iota(boss.begin(), boss.end(), 0);
        siz.assign(n, 1);
```

```
        stk.clear();
    }
    int find(int x) {
        return x == boss[x] ? x : find(boss[x]);
    }
    bool same(int x, int y) {
        return find(x) == find(y);
    }
    bool merge(int x, int y) {
        x = find(x); y = find(y);
        if (x == y) return false;
        if (siz[x] < siz[y]) swap(x, y);
        siz[x] += siz[y];
        boss[y] = x;
        n--;
        stk.push_back(y);
        return true;
    }
    void undo(int x) {
        while (stk.size() > x) {
            int y = stk.back();
            stk.pop_back();
            n++;
            siz[boss[y]] -= siz[y];
            boss[y] = y;
        }
    }
    int size(int x) {
        return siz[find(x)];
    }
};
```

## 2.7 SCC [26d711]

```
struct SCC {
    int n, cur, cnt;
    vector<vector<int>> adj;
    vector<int> stk, dfn, low, bel;
    SCC(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_;
        adj.assign(n, {});
        dfn.assign(n, -1), low.resize(n);
        bel.assign(n, -1), stk.clear();
        cur = cnt = 0;
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
    }
    void dfs(int x) {
        dfn[x] = low[x] = cur++;
        stk.push_back(x);
        for (auto y : adj[x]) {
            if (dfn[y] == -1) {
                dfs(y);
                low[x] = min(low[x], low[y]);
            } else if (bel[y] == -1) {
                low[x] = min(low[x], dfn[y]);
            }
        }
        if (dfn[x] == low[x]) {
            int y;
            do {
                y = stk.back();
                bel[y] = cnt;
                stk.pop_back();
            } while (y != x);
            cnt++;
        }
    }
    vector<int> work() {
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) dfs(i);
        return bel;
    }
    struct Graph {
        int n;
        vector<pair<int, int>> edges;
        vector<int> siz, cnte;
    };
    Graph compress() {
        Graph g;
        g.n = cnt;
        g.siz.resize(cnt);
        g.cnte.resize(cnt);
        for (int i = 0; i < n; i++) {
            g.siz[bel[i]]++;
            for (auto j : adj[i]) {
                if (bel[i] != bel[j]) {
                    g.edges.emplace_back(bel[i], bel[j]);
                } else {
                    g.cnte[bel[i]]++;
                }
            }
        }
        return g;
    }
};
```

## 2.8 VBCC [2d1f9d]

```
struct VBCC {
    int n, cur, cnt;
    vector<vector<int>> adj, bcc;
    vector<int> stk, dfn, low;
    vector<bool> ap;
    VBCC(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_;
        adj.assign(n, {});
        dfn.assign(n, -1), low.resize(n);
        bcc.assign(n, {}), ap.assign(n, false);
        stk.clear();
        cur = cnt = 0;
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void dfs(int x, int p) {
        dfn[x] = low[x] = cur++;
        stk.push_back(x);
        int child = 0;
        for (auto y : adj[x]) {
            if (y == p) continue;
            if (dfn[y] == -1) {
                dfs(y, x), child++;
                low[x] = min(low[x], low[y]);
                if (low[y] >= dfn[x]) {
                    int v;
                    do {
                        v = stk.back();
                        bcc[v].push_back(cnt);
                        stk.pop_back();
                    } while (v != y);
                    bcc[x].push_back(cnt);
                    cnt++;
                }
                if (low[y] >= dfn[x] && p != -1)
                    ap[x] = true;
            } else {
                low[x] = min(low[x], dfn[y]);
            }
        }
        if (p == -1 && child > 1)
            ap[x] = true;
    }
    vector<bool> work() {
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) dfs(i, -1);
        return ap;
    }
    struct Graph {
        int n;
        vector<pair<int, int>> edges;
        vector<int> bel, siz, cnte;
    };
    Graph compress() {
        Graph g; // 壓完是一棵樹，但不一定每個 bel 都有節點
        g.bel.resize(n);
        g.siz.resize(cnt);
        g.cnte.resize(cnt);
        for (int u = 0; u < n; u++) {
            if (ap[u]) {
                g.bel[u] = cnt++;
                g.siz.emplace_back();
                g.cnte.emplace_back();
                for (auto v : bcc[u]) {
                    g.edges.emplace_back(g.bel[u], v);
                }
            } else if (bcc[u].size() == 1) {
                g.bel[u] = bcc[u][0];
            }
            g.siz[g.bel[u]]++;
        }
        g.n = cnt;
        for (int i = 0; i < n; i++)
            for (auto j : adj[i])
                if (g.bel[i] == g.bel[j] && i < j)
                    g.cnte[g.bel[i]]++;
        return g;
    }
};
```

## 2.9 EBCC [9d70fc]

```
struct EBCC { // CF/contest/1986/pF
    int n, cur, cnt;
    vector<vector<int>> adj;
    vector<int> stk, dfn, low, bel;
    vector<pair<int, int>> bridges; // 關鍵邊
    EBCC(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_;
        adj.assign(n, {});
        dfn.assign(n, -1), low.resize(n);
        bel.assign(n, -1), stk.clear();
        bridges.clear();
        cur = cnt = 0;
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
```

```
    void dfs(int x, int p) {
        dfn[x] = low[x] = cur++;
        stk.push_back(x);
        for (auto y : adj[x]) {
            if (y == p) continue;
            if (dfn[y] == -1) {
                dfs(y, x);
                low[x] = min(low[x], low[y]);
                if (low[y] > dfn[x]) {
                    bridges.emplace_back(x, y);
                }
            } else if (bel[y] == -1) {
                low[x] = min(low[x], dfn[y]);
            }
        }
        if (dfn[x] == low[x]) {
            int y;
            do {
                y = stk.back();
                bel[y] = cnt;
                stk.pop_back();
            } while (y != x);
            cnt++;
        }
    }
    vector<int> work() { // not connected
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) dfs(i, -1);
        return bel;
    }
    struct Graph {
        int n;
        vector<pair<int, int>> edges;
        vector<int> siz, cnte;
    };
    Graph compress() {
        Graph g;
        g.n = cnt;
        g.siz.resize(cnt);
        g.cnte.resize(cnt);
        for (int i = 0; i < n; i++) {
            g.siz[bel[i]]++;
            for (auto j : adj[i]) {
                if (bel[i] < bel[j]) {
                    g.edges.emplace_back(bel[i], bel[j]);
                } else if (i < j) {
                    g.cnte[bel[i]]++;
                }
            }
        }
        return g;
    }
};
```

## 2.10 2-SAT [28688f]

```
struct TwoSat {
    int n; vector<vector<int>> e;
    vector<bool> ans;
    TwoSat(int n) : n(n), e(2 * n), ans(n) {}
    void addClause(int u, bool f, int v, bool g) {
        e[2 * u + !f].push_back(2 * v + g);
        e[2 * v + !g].push_back(2 * u + f);
    }
    void ifThen(int u, bool f, int v, bool g) {
        // 必取 A: not A -> A
        e[2 * u + !f].push_back(2 * v + g);
    }
    bool satisfiable() {
        vector<int
             > id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1);
        vector<int> stk;
        int now = 0, cnt = 0;
        function<void(int)> tarjan = [&](int u) {
            stk.push_back(u);
            dfn[u] = low[u] = now++;
            for (auto v : e[u]) {
                if (dfn[v] == -1) {
                    tarjan(v);
                    low[u] = min(low[u], low[v]);
                } else if (id[v] == -1) { // in stk
                    low[u] = min(low[u], dfn[v]);
                }
            }
            if (dfn[u] == low[u]) {
                int v;
                do {
                    v = stk.back();
                    stk.pop_back();
                    id[v] = cnt;
                } while (v != u);
                ++cnt;
            }
        };
        for (int i
              = 0; i < 2 * n; ++i) if (dfn[i] == -1) tarjan(i);
        for (int i = 0; i < n; ++i) {
            if (id[2 * i] == id[2 * i + 1]) return false;
            ans[i] = id[2 * i] > id[2 * i + 1];
        }
        return true;
    }
    vector<bool> answer() { return ans; }
};
```

## 2.11 Funtional Graph [e8fd64]

```
constexpr int N = 2E5 + 5;
int cht[N][31]; // 倍增表, 放外面不然 TLE
struct FuntionalGraph {
    int n, cnt;
    vector<int> g, bel, id, len, in, top;
    FuntionalGraph() : n(0) {}
    FuntionalGraph(vector<int> g_) { init(g_); }
    void init(vector<int> g_) {
        n = g_.size(); cnt = 0;
        g = g_; bel.assign(n, -1);
        id.resize(n); len.clear();
        in.assign(n, 0); top.assign(n, -1);
        build();
    }
    void build() {
        for (int i = 0; i < n; i++) {
            cht[i][0] = g[i];
            in[g[i]]++;
        }
        for (int i = 1; i <= 30; i++)
            for (int u = 0; u < n; u++)
                cht[u][i] = cht[cht[u][i - 1]][i - 1];
        for (int i = 0; i < n; i++)
            if (in[i] == 0) label(i);
        for (int i = 0; i < n; i++)
            if (top[i] == -1) label(i);
    }
    void label(int u) {
        vector<int> p; int cur = u;
        while (top[cur] == -1) {
            top[cur] = u;
            p.push_back(cur);
            cur = g[cur];
        }
        auto s = find(p.begin(), p.end(), cur);
        vector<int> cyc(s, p.end());
        p.erase(s, p.end()); p.push_back(cur);
        for (int i = 0; i < (int)cyc.size(); i++) {
            bel[cyc[i]] = cnt;
            id[cyc[i]] = i;
        }
        if (!cyc.empty())
            ++cnt, len.push_back(cyc.size());
        for (int i = p.size() - 1; i > 0; i--)
            id[p[i - 1]] = id[p[i]] - 1;
    }
    int jump(int u, int k) {
        for (int b = 0; k > 0; b++) {
            if (k & 1) u = cht[u][b];
            k >>= 1;
        }
        return u;
    }
};
```

# 3 Data Structure

## 3.1 Fenwick [d41d8c]

```
template<class T>
struct Fenwick { // 全部以 0 based 使用
    int n; vector<T> a;
    Fenwick(int n_ = 0) {
        init(n_);
    }
    void init(int n_) {
        n = n_;
        a.assign(n, T{});
    }
    void add(int x, const T &v) {
        for (int i = x + 1; i <= n; i += i & -i)
            a[i - 1] = a[i - 1] + v;
    }
    T sum(int x) { // 左閉右開查詢
        T ans{};
        for (int i = x; i > 0; i -= i & -i)
            ans = ans + a[i - 1];
        return ans;
    }
    T rangeSum(int l, int r) { // 左閉右開查詢
        return sum(r) - sum(l);
    }
    int select(const T &k, int start = 0) {
        // 找到最小的 x, 使得 sum(x + 1) - sum(start) > k
        int x = 0; T cur = -sum(start);
        for (int i = 1 << __lg(n); i; i /= 2) {
            if (x + i <= n && cur + a[x + i - 1] <= k) {
                x += i;
                cur = cur + a[x - 1];
            }
        }
        return x;
    }
};
template<class T>
```

```
struct TwoDFenwick {  // 全部以 0 based 使用
    int nx, ny;  // row, col 個數
    vector<vector<T>> a;
    TwoDFenwick(int nx_ = 0, int ny_ = 0) {
        init(nx_, ny_);
    }
    void init(int nx_, int ny_) {
        nx = nx_; ny = ny_;
        a.assign(nx, vector<T>(ny, T{}));
    }
    void add(int x, int y, const T &v) {
        for (int i = x + 1; i <= nx; i += i & -i)
            for (int j = y + 1; j <= ny; j += j & -j)
                a[i - 1][j - 1] = a[i - 1][j - 1] + v;
    }
    T sum(int x, int y) { // 左閉右開查詢
        T ans{};
        for (int i = x; i > 0; i -= i & -i)
            for (int j = y; j > 0; j -= j & -j)
                ans = ans + a[i - 1][j - 1];
        return ans;
    }
    T rangeSum
        (int lx, int ly, int rx, int ry) { // 左閉右開查詢
        return sum(
            rx, ry) - sum(lx, ry) - sum(rx, ly) + sum(lx, ly);
    }
};
```

## 3.2 RangeFenwick [d41d8c]

```
template<class T>
struct rangeFenwick { // 全部以 0 based 使用
    int n;
    vector<T> d, di;
    rangeFenwick(int n_ = 0) {
        init(n_);
    }
    void init(int n_) {
        n = n_;
        d.assign(n, T{});
        di.assign(n, T{});
    }
    void add(int x, const T &v) {
        T vi = v * (x + 1);
        for (int i = x + 1; i <= n; i += i & -i) {
            d[i - 1] = d[i - 1] + v;
            di[i - 1] = di[i - 1] + vi;
        }
    }
    void rangeAdd(int l, int r, const T &v) {
        add(l, v); add(r, -v);
    }
    T sum(int x) { // 左閉右開查詢
        T ans{};
        for (int i = x; i > 0; i -= i & -i) {
            ans = ans + T(x + 1) * d[i - 1];
            ans = ans - di[i - 1];
        }
        return ans;
    }
    T rangeSum(int l, int r) { // 左閉右開查詢
        return sum(r) - sum(l);
    }
    int select(const T &k, int start = 0) {
        // 找到最小的 x, 使得 sum(x + 1) - sum(start) > k
        int x = 0; T cur = -sum(start);
        for (int i = 1 << __lg(n); i; i /= 2) {
            if (x + i <= n) {
                T val = T(
                    x + i + 1) * d[x + i - 1] - di[x + i - 1];
                if (cur + val <= k) {
                    x += i;
                    cur = cur + val;
                }
            }
        }
        return x;
    }
};
template<class T>
struct rangeTwoDFenwick {  // 全部以 0 based 使用
    int nx, ny;  // row, col 個數
    vector<vector<T>> d, di, dj, dij;
    rangeTwoDFenwick(int nx_ = 0, int ny_ = 0) {
        init(nx_, ny_);
    }
    void init(int nx_, int ny_) {
        nx = nx_; ny = ny_;
        d.assign(nx, vector<T>(ny, T{}));
        di.assign(nx, vector<T>(ny, T{}));
        dj.assign(nx, vector<T>(ny, T{}));
        dij.assign(nx, vector<T>(ny, T{}));
    }
    void add(int x, int y, const T &v) {
        T vi = v * (x + 1);
        T vj = v * (y + 1);
        T vij = v * (x + 1) * (y + 1);
        for (int i = x + 1; i <= nx; i += i & -i) {
            for (int j = y + 1; j <= ny; j += j & -j) {
                d[i - 1][j - 1] = d[i - 1][j - 1] + v;
                di[i - 1][j - 1] = di[i - 1][j - 1] + vi;
                dj[i - 1][j - 1] = dj[i - 1][j - 1] + vj;
                dij[i - 1][j - 1] = dij[i - 1][j - 1] + vij;
            }
        }
    }
    void rangeAdd(int lx, int ly, int rx, int ry, const T &v) {
        add(rx, ry, v);
        add(lx, ry, -v);
        add(rx, ly, -v);
        add(lx, ly, v);
    }
    T sum(int x, int y) { // 左閉右開查詢
        T ans{};
        for (int i = x; i > 0; i -= i & -i) {
            for (int j = y; j > 0; j -= j & -j) {
                ans = ans
                      + T(x * y + x + y + 1) * d[i - 1][j - 1];
                ans = ans - T(y + 1) * di[i - 1][j - 1];
                ans = ans - T(x + 1) * dj[i - 1][j - 1];
                ans = ans + dij[i - 1][j - 1];
            }
        }
        return ans;
    }
    T rangeSum
        (int lx, int ly, int rx, int ry) { // 左閉右開查詢
        return sum(
            rx, ry) - sum(lx, ry) - sum(rx, ly) + sum(lx, ly);
    }
};
```

## 3.3 Segment Tree [d41d8c]

```
template<class Info>
struct Seg {    // 左閉右開寫法
    int n;
    vector<Info> info;
    Seg() : n(0) {}
    Seg(int n_, Info v_ = Info()) {
        init(n_, v_);
    }
    template<class T>
    Seg(vector<T> init_) { init(init_); }
    void init(int n_, Info v_ = Info()) {
        init(vector(n_, v_));
    }
    template<class T>
    void init(vector<T> init_) {
        n = init_.size();
        info.assign(4 << __lg(n), Info());
        function <void(
            int, int, int)> build = [&](int p, int l, int r) {
            if (r - l == 1) {
                info[p] = init_[l];
                return;
            }
            int m = (l + r) / 2;
            build(p * 2, l, m);
            build(p * 2 + 1, m, r);
            pull(p);
        };
        build(1, 0, n);
    }
    void pull(int p) {
        info[p] = info[p * 2] + info[p * 2 + 1];
    }
    void modify(int p, int l, int r, int x, const Info &v) {
        if (r - l == 1) {
            info[p] = v;
            return;
        }
        int m = (l + r) / 2;
        if (x < m) {
            modify(2 * p, l, m, x, v);
        } else {
            modify(2 * p + 1, m, r, x, v);
        }
        pull(p);
    }
    void modify(int p, const Info &i) {
        modify(1, 0, n, p, i);
    }
    Info query(int p, int l, int r, int ql, int qr) {
        if (qr <= l || ql >= r) return Info();
        if (ql <= l && r <= qr) return info[p];
        int m = (l + r) / 2;
        return query(p *
            2, l, m, ql, qr) + query(p * 2 + 1, m, r, ql, qr);
    }
    Info query(int ql, int qr) {
        return query(1, 0, n, ql, qr);
    }
    template<class F>   // 尋找區間內，第一個符合條件的
    int findFirst
        (int p, int l, int r, int x, int y, F &&pred) {
        if (l >= y || r <= x) return -1;
        if (l >= x && r <= y && !pred(info[p])) return -1;
        if (r - l == 1) return l;
        int m = (l + r) / 2;
```

```
        int res = findFirst(2 * p, l, m, x, y, pred);
        if (res == -1) {
            res = findFirst(2 * p + 1, m, r, x, y, pred);
        }
        return res;
    }
    template<class F>   // 若要找 last，先右子樹遞迴即可
    int findFirst(int l, int r, F &&pred) {
        return findFirst(1, 0, n, l, r, pred);
    }
};

struct Info {
    int n = 1;
    int sum = 0;
};
Info operator+(const Info &a, const Info &b) {
    return { a.n + b.n, a.sum + b.sum };
}
```

## 3.4 Lazy Segment Tree [d41d8c]

```
template<class Info, class Tag>
struct LazySeg {    // 左閉右開寫法
    int n;
    vector<Info> info;
    vector<Tag> tag;
    LazySeg() : n(0) {}
    LazySeg(int n_, Info v_ = Info()) {
        init(n_, v_);
    }
    template<class T>
    LazySeg(vector<T> init_) {
        init(init_);
    }
    void init(int n_, Info v_ = Info()) {
        init(vector(n_, v_));
    }
    template<class T>
    void init (vector<T> init_) {
        n = init_.size();
        info.assign(4 << __lg(n), Info());
        tag.assign(4 << __lg(n), Tag());
        function <void(
             int, int, int)> build = [&](int p, int l, int r) {
            if (r - l == 1) {
                info[p] = init_[l];
                return;
            }
            int m = (l + r) / 2;
            build(p * 2, l, m);
            build(p * 2 + 1, m, r);
            pull(p);
        };
        build(1, 0, n);
    }
    void pull(int p) {
        info[p] = info[p * 2] + info[p * 2 + 1];
    }
    void apply(int p, int l, int r, const Tag &v) {
        info[p].apply(l, r, v);
        tag[p].apply(v);
    }
    void push(int p, int l, int r) {
        int m = (l + r) / 2;
        if (r - l >= 1) {
            apply(p * 2, l, m, tag[p]);
            apply(p * 2 + 1, m, r, tag[p]);
        }
        tag[p] = Tag();
    }
    void modify(int p, int l, int r, int x, const Info &v) {
        if (r - l == 1) {
            info[p] = v;
            return;
        }
        int m = (l + r) / 2;
        push(p, l, r);
        if (x < m) {
            modify(2 * p, l, m, x, v);
        } else {
            modify(2 * p + 1, m, r, x, v);
        }
        pull(p);
    }
    void modify(int p, const Info &i) {
        modify(1, 0, n, p, i);
    }
    Info query(int p, int l, int r, int ql, int qr) {
        if (qr <= l || ql >= r) return Info();
        if (ql <= l && r <= qr) return info[p];
        int m = (l + r) / 2;
        push(p, l, r);
        return query(p *
             2, l, m, ql, qr) + query(p * 2 + 1, m, r, ql, qr);
    }
    Info query(int ql, int qr) {
        return query(1, 0, n, ql, qr);
    }
    void range_apply
         (int p, int l, int r, int ql, int qr, const Tag &v) {
        if (qr <= l || ql >= r) return;
        if (ql <= l && r <= qr) {
            apply(p, l, r, v);
            return;
        }
        int m = (l + r) / 2;
        push(p, l, r);
        range_apply(p * 2, l, m, ql, qr, v);
        range_apply(p * 2 + 1, m, r, ql, qr, v);
        pull(p);
    }
    void range_apply(int l, int r, const Tag &v) {
        range_apply(1, 0, n, l, r, v);
    }
    template<class F>   // 尋找區間內，第一個符合條件的
    int findFirst
         (int p, int l, int r, int x, int y, F &&pred) {
        if (l >= y || r <= x) return -1;
        if (l >= x && r <= y && !pred(info[p])) return -1;
        if (r - l == 1) return l;
        int m = (l + r) / 2;
        push(p);
        int res = findFirst(2 * p, l, m, x, y, pred);
        if (res == -1) {
            res = findFirst(2 * p + 1, m, r, x, y, pred);
        }
        return res;
    }
    template<class F>   // 若要找 last，先右子樹遞迴即可
    int findFirst(int l, int r, F &&pred) {
        return findFirst(1, 0, n, l, r, pred);
    }
};
struct Tag { // 有些 Tag 不用 push 例如 sweepLine
    int set_val; int add;
    void apply(const Tag& v) {
        if (v.set_val) {
            set_val = v.set_val;
            add = v.add;
        }
        else {
            add += v.add;
        }
    }
};
struct Info {
    int sum;
    void apply(int l, int r, const Tag &v) {
        if (v.set_val) {
            sum = (r - l) * v.set_val;
        }
        sum += (r - l) * v.add;
    }
    // Info &operator=(const Info &rhs) {
    //     // 部分 assignment 使用
    //     return *this;
    // }
};
Info operator+(const Info &a, const Info &b) {
    return { a.sum + b.sum };
}
```

## 3.5 Persistent Segment Tree [d41d8c]

```
template<class Info>
struct PST {
    struct Node {
        Info info = Info();
        int lc = 0, rc = 0;
    };
    vector<Node> nd;
    int n = 0;
    vector<int> rt;
    PST() : n(0) {}
    PST(int n_, Info v_ = Info()) { init(n_, v_); }
    template<class T>
    PST(vector<T> init_) { init(init_); }
    void init(int n_, Info v_ = Info()) {
        init(vector<Info>(n_, v_));
    }
    template<class T>
    void init(vector<T> init_) {
        n = init_.size();
        nd.clear(); rt.clear();
        nd.emplace_back(); // 讓 root 指向 1-based
        rt.push_back(build(0, n, init_));
    }
    int build(int l, int r, vector<Info> &init_) {
        int id = nd.size();
        nd.emplace_back();
        if (r - l == 1) {
            nd[id].info = init_[l];
            return id;
        }
        int m = (l + r) >> 1;
        nd[id].lc = build(l, m, init_);
        nd[id].rc = build(m, r, init_);
        pull(nd[id]);
        return id;
    }
    void pull(Node &t) {
        t.info = nd[t.lc].info + nd[t.rc].info;
```

```
    }
    int copy(int t) { // copy 一個 node
        nd.push_back(nd[t]);
        return nd.size() - 1;
    }
    int generate() { // 創立新的 node
        nd.emplace_back();
        return nd.size() - 1;
    }
    int modify(int t, int l, int r, int x, const Info &v) {
        t = t ? copy(t) : generate();
        if (r - l == 1) {
            nd[t].info = v;
            return t;
        }
        int m = (l + r) >> 1;
        if (x < m) {
            nd[t].lc = modify(nd[t].lc, l, m, x, v);
        } else {
            nd[t].rc = modify(nd[t].rc, m, r, x, v);
        }
        pull(nd[t]);
        return t;
    }
    void modify(int ver, int pos, const Info &val) {
        if (int(rt.size()) <= ver) rt.resize(ver + 1);
        rt[ver] = modify(rt[ver], 0, n, pos, val);
    }
    Info query(int t, int l, int r, int ql, int qr) {
        if (l >= qr || r <= ql) return Info();
        if (ql <= l && r <= qr) return nd[t].info;
        int m = (l + r) >> 1;
        return query(nd[t].
             lc, l, m, ql, qr) + query(nd[t].rc, m, r, ql, qr);
    }
    Info query(int ver, int ql, int qr) {
        return query(rt[ver], 0, n, ql, qr);
    }
    void createVersion(int ori_ver) {
        rt.push_back(copy(rt[ori_ver]));
    }
    void reserve(int n, int q) {
        nd.reserve(n + q * (2 * __lg(n) + 1));
        rt.reserve(q + 1);
    }
    void resize(int n) {
        rt.resize(n);
    }
};
struct Info {
    int sum = 0;
};
Info operator+(const Info &a, const Info &b) {
    return { a.sum + b.sum };
}
```

## 3.6 Treap [d41d8c]

```
struct Treap {
    Treap *lc, *rc;
    int pri, siz; bool rev_valid;
    int val; int min;
    Treap(int val_) {
        min = val = val_;
        pri = rand();
        lc = rc = nullptr;
        siz = 1; rev_valid = 0;
    }
    void pull() { // update siz or other information
        siz = 1;
        min = val;
        for (auto c : {lc, rc}) {
            if (!c) continue;
            siz += c->siz;
            min = std::min(min, c->min);
        }
    }
    void push() {
        if (rev_valid) {
            swap(lc, rc);
            if (lc) lc->rev_valid ^= 1;
            if (rc) rc->rev_valid ^= 1;
        }
        rev_valid = false;
    }
    int find(int k) { // 找到 min 是 k 的位置 (1-based)
        push();
        int ls = (lc ? lc->siz : 0) + 1;
        if (val == k) return ls;
        if (lc && lc->min == k) return lc->find(k);
        else return rc->find(k) + ls;
    }
};
int size(Treap *t) {
    return t ? t->siz : 0;
}
Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) return a ? a : b;
    a->push(); b->push();
    if (a->pri > b->pri) {
        a->rc = merge(a->rc, b);
        a->pull();
        return a;
    }
    else {
        b->lc = merge(a, b->lc);
        b->pull();
        return b;
    }
}
pair<Treap*, Treap*> split(Treap *t, int k) {
    // 分割前 k 個在 first，剩下的在 second
    if (t == nullptr) return {nullptr, nullptr};
    t->push();
    if (size(t->lc) < k) {
        auto [a, b] = split(t->rc, k - size(t->lc) - 1);
        t->rc = a;
        t->pull();
        return {t, b};
    }
    else {
        auto [a, b] = split(t->lc, k);
        t->lc = b;
        t->pull();
        return {a, t};
    }
}
void Print(Treap *t) {
    if (!t) return;
    t->push();
    Print(t->lc);
    cout << t->val;
    Print(t->rc);
}
```

## 3.7 RMQ [d41d8c]

```
template<class T, class Cmp = less<T>>
struct RMQ {
    const Cmp cmp = Cmp();
    static constexpr unsigned B = 64;
    using u64 = unsigned long long;
    int n;
    vector<vector<T>> a;
    vector<T> pre, suf, ini;
    vector<u64> stk;
    RMQ() {}
    RMQ(const vector<T> &v) { init(v); }
    void init(const vector<T> &v) {
        n = v.size();
        pre = suf = ini = v;
        stk.resize(n);
        if (!n) return;
        const int M = (n - 1) / B + 1;
        const int lg = __lg(M);
        a.assign(lg + 1, vector<T>(M));
        for (int i = 0; i < M; i++) {
            a[0][i] = v[i * B];
            for (int j = 1; j < B && i * B + j < n; j++) {
                a[0][i] = min(a[0][i], v[i * B + j], cmp);
            }
        }
        for (int i = 1; i < n; i++) {
            if (i % B) {
                pre[i] = min(pre[i], pre[i - 1], cmp);
            }
        }
        for (int i = n - 2; i >= 0; i--) {
            if (i % B != B - 1) {
                suf[i] = min(suf[i], suf[i + 1], cmp);
            }
        }
        for (int j = 0; j < lg; j++) {
            for (int i = 0; i + (2 << j) <= M; i++) {
                a[j + 1][i
                     ] = min(a[j][i], a[j][i + (1 << j)], cmp);
            }
        }
        for (int i = 0; i < M; i++) {
            const int l = i * B;
            const int r = min(1U * n, l + B);
            u64 s = 0;
            for (int j = l; j < r; j++) {
                while (s && cmp(v[j], v[__lg(s) + l])) {
                    s ^= 1ULL << __lg(s);
                }
                s |= 1ULL << (j - l);
                stk[j] = s;
            }
        }
    }
    T operator()(int l, int r) {
        if (l / B != (r - 1) / B) {
            T ans = min(suf[l], pre[r - 1], cmp);
            l = l / B + 1;
            r = r / B;
            if (l < r) {
                int k = __lg(r - l);
                ans = min
                     ({ans, a[k][l], a[k][r - (1 << k)]}, cmp);
            }
            return ans;
        } else {
            int x = B * (l / B);
```

```
            return ini
                [__builtin_ctzll(stk[r - 1] >> (l - x)) + l];
        }
    }
};
```

### 3.8 Mo [d41d8c]

```
struct query {
    int l, r, id;
};
void Mo(vector<query> &q) {
    int blk = sqrt(q.size());
    sort(q.begin
         (), q.end(), [&](const query &a, const query &b) {
        int x = a.l / blk, y = b.l / blk;
        return x == y ? a.r < b.r : x < y;
    });
}
```

## 4 Flow Matching

### 4.1 Dinic [d41d8c]

```
template<class T>
struct Dinic {
    struct _Edge {
        int to;
        T f, cap; // 流量跟容量
    };
    int n, m, s, t;
    const T INF_FLOW = 1LL << 60;
    vector<vector<int>> g;
    vector<_Edge> e;
    vector<int> h, cur;
    Dinic(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_; m = 0;
        h.resize(n); cur.resize(n);
        g.assign(n, {});
        e.clear();
    }
    void add_edge(int u, int v, T cap) {
        e.push_back({v, 0, cap});
        e.push_back({u, 0, 0});
        g[u].push_back(m++);
        g[v].push_back(m++);
    }
    bool bfs() {
        fill(h.begin(), h.end(), -1);
        h[s] = 0; queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (int id : g[u]) {
                auto [v, f, cap] = e[id];
                if (f == cap) continue;
                if (h[v] == -1) {
                    h[v] = h[u] + 1;
                    if (v == t) return true;
                    q.push(v);
                }
            }
        }
        return false;
    }
    T dfs(int u, T flow) {
        if (flow == 0) return 0;
        if (u == t) return flow;
        for (int &i = cur[u]; i < g[u].size(); i++) {
            int j = g[u][i];
            auto [v, f, cap] = e[j];
            if (h[u] + 1 != h[v]) continue;
            if (f == cap) continue;
            T mn = dfs(v, min(flow, cap - f));
            if (mn > 0) {
                e[j].f += mn;
                e[j ^ 1].f -= mn;
                return mn;
            }
        }
        return 0;
    }
    T work(int s_, int t_) {
        s = s_; t = t_; T f = 0;
        while (bfs()) {
            fill(cur.begin(), cur.end(), 0);
            while (true) {
                T res = dfs(s, INF_FLOW);
                if (res == 0) break;
                f += res;
            }
        }
        return f;
    }
    void reset() {
        for (int i = 0; i < m; i++) e[i].f = 0;
    }
    void reuse(int n_) { // 走殘留網路, res += f
        while (n < n_) {
            g.emplace_back();
            h.emplace_back();
            cur.emplace_back();
            n += 1;
        }
    }
};
```

### 4.2 Min Cut [d41d8c]

```
// CSES Police Chase
int main() {
    int n, m; cin >> n >> m;
    Dinic<int> g(n);
    for (int i = 0; i < m; i++) {
        int u, v, cap = 1;
        cin >> u >> v;
        u--; v--;
        g.add_edge(u, v, cap);
        g.add_edge(v, u, cap);
    }
    int res = g.work(0, n - 1);
    cout << res << "\n";
    if (res == 0) return;

    vector<int> vis(n);
    auto find = [&](auto self, int u) -> void {
        if (!vis[u]) {
            vis[u] = 1;
            for (int id : g.adj[u]) {
                auto e = g.edges[id];
                if (e.cap - e.flow > 0) {
                    self(self, e.to);
                }
            }
        }
    };
    find(find, 0);
    for (int i = 0; i < n; i++) {
        if (!vis[i]) continue;
        for (int id : g.adj[i]) {
            if (id & 1) continue;
            auto e = g.edges[id];
            if (!vis[e.to]) {
                cout << i + 1 << " " << e.to + 1 << "\n";
            }
        }
    }
}
```

### 4.3 MCMF [d41d8c]

```
template<class Tf, class Tc>
struct MCMF {
    struct _Edge {
        int to;
        Tf f, cap; // 流量跟容量
        Tc cost;
    };
    int n, m, s, t;
    const Tf INF_FLOW = 1 << 30;
    const Tc INF_COST = 1 << 30;
    vector<_Edge> e;
    vector<vector<int>> g;
    vector<Tc> dis;
    vector<int> rt, inq;
    MCMF(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_; m = 0;
        e.clear();
        g.assign(n, {});
    }
    void add_edge(int u, int v, Tf cap, Tc cost) {
        e.push_back({v, 0, cap, cost});
        e.push_back({u, 0, 0, -cost});
        g[u].push_back(m++);
        g[v].push_back(m++);
    }
    bool spfa() {
        dis.assign(n, INF_COST);
        rt.assign(n, -1), inq.assign(n, 0);
        queue<int> q; q.push(s);
        dis[s] = 0;
        while (!q.empty()) {
            int u = q.front(); q.pop();
            inq[u] = 0;
            for (int id : g[u]) {
                auto [v, f, cap, cost] = e[id];
                Tc ndis = dis[u] + cost;
                if (f < cap && dis[v] > ndis) {
                    dis[v] = ndis, rt[v] = id;
                    if (!inq[v])
                        q.push(v), inq[v] = 1;
                }
            }
        }
        return dis[t] != INF_COST;
    }
    // 限定 flow, 最小化 cost
    pair<Tf, Tc> work_flow(int s_, int t_, Tf need) {
        s = s_, t = t_;
        Tf flow{}; Tc cost{};
        while (spfa()) {
```

```
            Tf f = need;
            for (int i = t; i != s; i = e[rt[i] ^ 1].to)
                f = min(f, e[rt[i]].cap - e[rt[i]].f);
            for (int i = t; i != s; i = e[rt[i] ^ 1].to)
                e[rt[i]].f += f, e[rt[i] ^ 1].f -= f;
            flow += f, need -= f;
            cost += f * dis[t];
            if (need == 0) break;
        }
        return {flow, cost};
    }
    // 限定 cost, 最大化 flow
    pair<Tf, Tc> work_budget(int s_, int t_, Tc budget) {
        s = s_, t = t_;
        Tf flow{}; Tc cost{};
        while (spfa()) {
            Tf f = budget / dis[t];
            for (int i = t; i != s; i = e[rt[i] ^ 1].to)
                f = min(f, e[rt[i]].cap - e[rt[i]].f);
            for (int i = t; i != s; i = e[rt[i] ^ 1].to)
                e[rt[i]].f += f, e[rt[i] ^ 1].f -= f;
            flow += f, budget -= f * dis[t];
            cost += f * dis[t];
            if (budget == 0 || f == 0) break;
        }
        return {flow, cost};
    }
    void reset() {
        for (int i = 0; i < m; i++) e[i].f = 0;
    }
};
```

## 4.4 Hungarian [d41d8c]

```
struct Hungarian { // 0-based, O(VE)
    int n, m;
    vector<vector<int>> adj;
    vector<int> used, vis;
    vector<pair<int, int>> match;
    Hungarian(int n_ = 0, int m_ = 0) {
        init(n_, m_);
    }
    void init(int n_, int m_) {
        n = n_; m = m_;
        adj.assign(n + m, {});
        used.assign(n + m, -1);
        vis.assign(n + m, 0);
    }
    void addEdge(int u, int v) {
        adj[u].push_back(n + v);
        adj[n + v].push_back(u);
    }
    bool dfs(int u) {
        int sz = adj[u].size();
        for (int i = 0; i < sz; i++) {
            int v = adj[u][i];
            if (vis[v] == 0) {
                vis[v] = 1;
                if (used[v] == -1 || dfs(used[v])) {
                    used[v] = u;
                    return true;
                }
            }
        }
        return false;
    }
    vector<pair<int, int>> work() {
        match.clear();
        used.assign(n + m, -1);
        vis.assign(n + m, 0);
        for (int i = 0; i < n; i++) {
            fill(vis.begin(), vis.end(), 0);
            dfs(i);
        }
        for (int i = n; i < n + m; i++)
            if (used[i] != -1)
                match.emplace_back(used[i], i - n);
        return match;
    }
};
```

## 4.5 Theorem [d41d8c]

```
// 有向無環圖:

// 最小不相交路徑覆蓋:
// 最小路徑數 = 頂點數 - 最大匹配數

// 最小相交路徑覆蓋:
// 先用
     Floyd 求傳遞封包，有連邊就建邊，然後再套最小不相交路徑覆蓋

// 二分圖:

// 最小點
     覆蓋: 選出一些點，讓所有邊至少有一個端點在點集中的最少數量
// 最小點覆蓋 = 最大匹配數
// 還原解，flow 的作法是從源點開始 dfs，只走 cap - flow > 0
// 的邊，最後挑選左邊還沒被跑過的點和右邊被跑過的點當作覆蓋的點

// 最少邊覆蓋: 選出一些邊，讓所有點都覆蓋到的最少數量
// 最少邊覆蓋 = 點數 - 最大匹配數

// 最大獨立集: 選出一些點，使這些點兩兩沒有邊連接的最大數量
// 最大獨立集 = 點數 - 最大匹配數
```

# 5 String

## 5.1 Hash [852711]

```
constexpr int B = 59;
vector<Z> Hash(string &s) {
    vector<Z> ans {0};
    for (auto c : s) {
        ans.push_back(ans.back() * B + (c - 'a' + 1));
    }
    return ans;
}
void solve() {
    string s, sub;
    cin >> s >> sub;
    auto a = Hash(s);
    auto q = Hash(sub);
    auto find = q.back();
    int ans = 0;
    int l = 1, r = sub.size(), len = sub.size();
    while (r <= s.size()) {
        if (a[r] - a[l - 1] * power(Z(B), len) == find) {
            ans++;
        }
        l++, r++;
    }
    cout << ans << "\n";
}
```

## 5.2 KMP [731acf]

```
struct KMP {
    string sub;
    vector<int> fail;
    // fail 存匹配失敗時，移去哪
    // 也就是 sub(0, i) 的最長共同前後綴長度
    // ex : a   b  c a b c
    //      -1 -1 -1 0 1 2
    KMP() {}
    KMP(const string &sub_) {
        build(sub_);
    }
    vector<int> build(const string &sub_) {
        sub = sub_, fail.resize(sub.size(), -1);
        for (int i = 1; i < sub.size(); i++) {
            int now = fail[i - 1];
            while (now != -1 && sub[now + 1] != sub[i])
                now = fail[now];
            if (sub[now + 1] == sub[i])
                fail[i] = now + 1;
        }
        return fail;
    }
    vector<int> match(const string &s) {
        vector<int> match;
        for (int i = 0, now = -1; i < s.size(); i++) {
            while (s[i] != sub[now + 1] && now != -1)
                now = fail[now];
            if (s[i] == sub[now + 1]) now++;
            if (now + 1 == sub.size()) {
                match.push_back(i - now);
                now = fail[now];
            }
        }
        return match;
    }
};
```

## 5.3 Z Function [5b63dc]

```
// z[i] 表示 s 和 s[i, n - 1] (以 s[i] 開頭的後綴)
// 的最長公共前綴 (LCP) 的長度
vector<int> Z(const string &s) {
    int n = s.size();
    vector<int> z(n);
    z[0] = n; // lcp(s, s), -1 or n
    for (int i = 1, j = 1; i < n; i++) {
        z[i] = max(0, min(j + z[j] - i, z[i - j]));
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            z[i]++;
        if (i + z[i] > j + z[j]) j = i;
    }
    return z;
}
```

## 5.4 Manacher [958661]

```
// 找到對於每個位置的迴文半徑
vector<int> manacher(const string &s) {
    string t = "#";
    for (auto c : s) {
        t += c;
        t += '#';
```

```
    }
    int n = t.size();
    vector<int> r(n);
    for (int i = 0,
         j = 0; i < n; i++) { // i 是中心, j 是最長回文字串中心
        if (2 * j - i >= 0 && j + r[j] > i)
            r[i] = min(r[2 * j - i], j + r[j] - i);
        while (i - r[i] >=
               0 && i + r[i] < n && t[i - r[i]] == t[i + r[i]])
            r[i] += 1;
        if (i + r[i] > j + r[j])
            j = i;
    }
    return r;
}
// # a # b # a #
// 1 2 1 4 1 2 1
// # a # b # b # a #
// 1 2 1 2 5 2 1 2 1
// 值 -1 代表原回文字串長度
// (id - val + 1) / 2 可得原字串回文開頭
```

## 5.5 Trie [72392f]

```
constexpr int N = 1E7;
int tot = 0;
int trie[N][26], cnt[N];
void reset() {
    tot = 0, fill_n(trie[0], 26, 0);
}
int newNode() {
    int x = ++tot;
    cnt[x] = 0, fill_n(trie[x], 26, 0);
    return x;
}
void add(const string &s) {
    int p = 0;
    for (auto c : s) {
        int &q = trie[p][c - 'a'];
        if (!q) q = newNode();
        p = q;
    }
    cnt[p] += 1;
}
int find(const string &s) {
    int p = 0;
    for (auto c : s) {
        int q = trie[p][c - 'a'];
        if (!q) return 0;
        p = q;
    }
    return cnt[p];
}
```

## 5.6 SA [f9b5d1]

```
struct SuffixArray {
    int n; string s;
    vector<int> sa, rk, lc;
    // n: 字串長度
    // sa: 後綴數組，sa[i] 表示第 i 小的後綴的起始位置
    // rk: 排名數組，rk[i] 表示從位置 i 開始的後綴的排名
    // lc: LCP
         數組，lc[i] 表示 sa[i] 和 sa[i + 1] 的最長公共前綴長度
    SuffixArray(const string &s_) {
        s = s_; n = s.length();
        sa.resize(n);
        lc.resize(n - 1);
        rk.resize(n);
        iota(sa.begin(), sa.end(), 0);
        sort(sa.begin(), sa.
             end(), [&](int a, int b) { return s[a] < s[b]; });
        rk[sa[0]] = 0;
        for (int i = 1; i < n; i++)
            rk[sa[i]]
                = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
        int k = 1;
        vector<int> tmp, cnt(n);
        tmp.reserve(n);
        while (rk[sa[n - 1]] < n - 1) {
            tmp.clear();
            for (int i = 0; i < k; i++)
                tmp.push_back(n - k + i);
            for (auto i : sa)
                if (i >= k)
                    tmp.push_back(i - k);
            fill(cnt.begin(), cnt.end(), 0);
            for (int i = 0; i < n; i++)
                ++cnt[rk[i]];
            for (int i = 1; i < n; i++)
                cnt[i] += cnt[i - 1];
            for (int i = n - 1; i >= 0; i--)
                sa[--cnt[rk[tmp[i]]]] = tmp[i];
            swap(rk, tmp);
            rk[sa[0]] = 0;
            for (int i = 1; i < n; i++)
                rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa
                    [i - 1]] < tmp[sa[i]] || sa[i - 1] + k ==
                    n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
            k *= 2;
        }
        for (int i = 0, j = 0; i < n; i++) {
            if (rk[i] == 0) {
                j = 0;
            } else {
                for (j -=
                     j > 0; i + j < n && sa[rk[i] - 1] + j < n
                     && s[i + j] == s[sa[rk[i] - 1] + j]; j++);
                lc[rk[i] - 1] = j;
            }
        }
    }
};
RMQ<int> rmq(sa.lc);
auto lcp = [&](int i, int j) { // [i, j]
    i = sa.rk[i];
    j = sa.rk[j];
    if (i > j) swap(i, j);
    assert(i != j);
    return rmq(i, j);
};
```

## 5.7 SAM [3bdfeb]

```
struct SAM {
    // 1 -> initial state
    static constexpr int ALPHABET_SIZE = 26;
    struct Node {
        int len;
        int link;
        array<int, ALPHABET_SIZE> next;
        Node() : len{}, link{}, next{} {}
    };
    vector<Node> t;
    SAM() {
        init();
    }
    void init() {
        t.assign(2, Node());
        t[0].next.fill(1);
        t[0].len = -1;
    }
    int newNode() {
        t.emplace_back();
        return t.size() - 1;
    }
    int extend(int p, int c) {
        if (t[p].next[c]) {
            int q = t[p].next[c];
            if (t[q].len == t[p].len + 1) {
                return q;
            }
            int r = newNode();
            t[r].len = t[p].len + 1;
            t[r].link = t[q].link;
            t[r].next = t[q].next;
            t[q].link = r;
            while (t[p].next[c] == q) {
                t[p].next[c] = r;
                p = t[p].link;
            }
            return r;
        }
        int cur = newNode();
        t[cur].len = t[p].len + 1;
        while (!t[p].next[c]) {
            t[p].next[c] = cur;
            p = t[p].link;
        }
        t[cur].link = extend(p, c);
        return cur;
    }
};
void solve() {
    string s; cin >> s;
    int n = s.length();
    vector<int> last(n + 1); // s[i - 1] 的後綴終點位置
    last[0] = 1;
    SAM sam;
    for (int i = 0; i < n; i++)
        last[i + 1] = sam.extend(last[i], s[i] - 'a');
    int sz = sam.t.size();
    vector<int> cnt(sz);
    for (int i = 1; i <= n; i++)
        cnt[last[i]]++; // 去重 = 1
    vector<vector<int>> order(sz);
    for (int i = 1; i < sz; i++)
        order[sam.t[i].len].push_back(i);
    for (int i = sz - 1; i > 0; i--)
        for (int u : order[i])
            if (sam.t[u].link != -1)
                cnt[sam.t[u].link] += cnt[u];
    vector<ll> dp(sz, -1);
    auto dfs = [&](auto self, int u) -> void {
        dp[u] = cnt[u];
        for (int c = 0; c < SAM::ALPHABET_SIZE; c++) {
            int v = sam.t[u].next[c];
            if (v) {
                if (dp[v] == -1) self(self, v);
                dp[u] += dp[v];
            }
        }
    };
```

```
    dfs(dfs, 1);
}
```

## 5.8 Palindrome Tree [77b763]

```
struct PAM {
    // 0 -> even root, 1 -> odd root
    static constexpr int ALPHABET_SIZE = 26;
    struct Node {
        int len;
        int fail;
        array<int, ALPHABET_SIZE> next;
        Node() : len{}, fail{}, next{} {}
    };
    vector<int> s;
    vector<Node> t;
    PAM() {
        init();
    }
    void init() {
        t.assign(2, Node());
        s.clear();
        t[0].len = 0;
        t[1].len = -1;
        t[0].fail = 1;
    }
    int newNode() {
        t.emplace_back();
        return t.size() - 1;
    }
    int extend(int p, int c) {
        int n = s.size();
        s.push_back(c);
        while (s[n - t[p].len - 1] != c)
            p = t[p].fail;
        if (!t[p].next[c]) {
            int r = newNode();
            t[r].len = t[p].len + 2;
            int cur = t[p].fail;
            while (s[n - t[cur].len - 1] != c)
                cur = t[cur].fail;
            t[r].fail = t[cur].next[c];
            t[p].next[c] = r;
        }
        p = t[p].next[c];
        return p;
    }
};
void solve() {
    string s; cin >> s;
    int n = s.length();
    vector<int> last(n + 1);
    last[0] = 1;
    PAM pam;
    for (int i = 0; i < n; i++)
        last[i + 1] = pam.extend(last[i], s[i] - 'a');
    int sz = pam.t.size();
    vector<int> cnt(sz);
    for (int i = 1; i <= n; i++)
        cnt[last[i]]++; // 去重 = 1
    for (int i = sz - 1; i > 1; i--)
        cnt[pam.t[i].fail] += cnt[i];
}
```

## 5.9 Duval [f9dcca]

```
// duval_algorithm
// 將字串分解成若干個非嚴格遞減的非嚴格遞增字串
vector<string> duval(string s) {
    int i = 0, n = s.size();
    vector<string> res;
    while (i < n) {
        int k = i, j = i + 1;
        while (s[k] <= s[j] && j < n) {
            if (s[k] < s[j]) k = i;
            else k++;
            j++;
        }
        while (i <= k) {
            res.push_back(s.substr(i, j - k));
            i += j - k;
        }
    }
    return res;
}
// 最小旋轉字串
string min_round(string s) {
    s += s;
    int i = 0, n = s.size();
    int start = i;
    while (i < n / 2) {
        start = i;
        int k = i, j = i + 1;
        while (s[k] <= s[j] && j < n) {
            if (s[k] < s[j]) k = i;
            else k++;
            j++;
        }
        while (i <= k) {
            i += j - k;
        }
    }
    return s.substr(start, n / 2);
}
```

# 6 Math

## 6.1 Modulo [e2fbf0]

```
template<class T>
T power(T a, ll b) {
    T res {1};
    for (; b; b /= 2, a *= a)
        if (b & 1) res *= a;
    return res;
}
ll mul(ll a, ll b, ll p) { // 大模數再抄
    ll res = a * b - ll(1.L * a * b / p) * p;
    res %= p;
    if (res < 0) res += p;
    return res;
}
template<ll P>
struct MInt {
    ll x;
    MInt() : x {0} {}
    MInt(ll x) : x {norm(x % getMod())} {}
    static ll Mod;
    static ll getMod() {
        return P > 0 ? P : Mod;
    }
    static void setMod(ll Mod_) {
        Mod = Mod_;
    }
    ll norm(ll x) const {
        if (x < 0) x += getMod();
        if (x >= getMod()) x -= getMod();
        return x;
    }
    MInt operator-() const {
        return MInt(norm(getMod() - x));
    }
    MInt inv() const {
        return power(*this, getMod() - 2);
    }
    MInt &operator*=(MInt rhs) & {
        if (getMod() < (1ULL << 31)) {
            x = x * rhs.x % int(getMod());
        } else {
            x = mul(x, rhs.x, getMod());
        }
        return *this;
    }
    MInt &operator+=(MInt rhs) & {
        x = norm(x + rhs.x);
        return *this;
    }
    MInt &operator-=(MInt rhs) & {
        x = norm(x - rhs.x);
        return *this;
    }
    MInt &operator/=(MInt rhs) & {
        return *this *= rhs.inv();
    }
    friend MInt operator*(MInt lhs, MInt rhs) {
        return lhs *= rhs;
    }
    friend MInt operator+(MInt lhs, MInt rhs) {
        return lhs += rhs;
    }
    friend MInt operator-(MInt lhs, MInt rhs) {
        return lhs -= rhs;
    }
    friend MInt operator/(MInt lhs, MInt rhs) {
        return lhs /= rhs;
    }
    friend istream &operator>>(istream &is, MInt &a) {
        ll v; is >> v; a = MInt(v); return is;
    }
    friend ostream &operator<<(ostream &os, const MInt &a) {
        return os << a.x;
    }
    friend bool operator==(MInt lhs, MInt rhs) {
        return lhs.x == rhs.x;
    }
    friend bool operator!=(MInt lhs, MInt rhs) {
        return lhs.x != rhs.x;
    }
    friend bool operator<(MInt lhs, MInt rhs) {
        return lhs.x < rhs.x;
    }
};
template<>
ll MInt<0>::Mod = 998244353;
constexpr ll P = 1E9 + 7;
using Z = MInt<P>;
```

## 6.2 Combination [6aa734]

```
struct Comb {
    ll n; vector<Z> _fac, _invfac, _inv;
    Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}
```

```
    Comb(ll n) : Comb() { init(n); }
    void init(ll m) {
        m = min(m, Z::getMod() - 1);
        if (m <= n) return;
        _fac.resize(m + 1);
        _invfac.resize(m + 1);
        _inv.resize(m + 1);
        for (int i = n + 1; i <= m; i++) {
            _fac[i] = _fac[i - 1] * i;
        }
        _invfac[m] = _fac[m].inv();
        for (int i = m; i > n; i--) {
            _invfac[i - 1] = _invfac[i] * i;
            _inv[i] = _invfac[i] * _fac[i - 1];
        }
        n = m;
    }
    Z fac(ll m) {
        if (m > n) init(2 * m);
        return _fac[m];
    }
    Z invfac(ll m) {
        if (m > n) init(2 * m);
        return _invfac[m];
    }
    Z inv(ll m) {
        if (m > n) init(2 * m);
        return _inv[m];
    }
    Z binom(ll n, ll m) {
        if (n < m || m < 0) return 0;
        return fac(n) * invfac(m) * invfac(n - m);
    }
    Z lucas(ll n, ll m) { // Mod 要在 1E5 左右
        if (m == 0) return 1;
        return binom(n % Z::getMod(), m % Z::getMod
            ()) * lucas(n / Z::getMod(), m / Z::getMod());
    }
} comb; // 注意宣告，若要換模數需重新宣告
```

## 6.3 Sieve [37ae54]

```
vector<int> primes, minp;
void sieve(int n) {
    minp.assign(n + 1, 0);
    primes.clear();
    // minp[i] == i, 質數
    for (int i = 2; i <= n; i++) {
        if (minp[i] == 0) {
            minp[i] = i;
            primes.push_back(i);
        }
        for (auto p : primes) {
            if (i * p > n) break;
            minp[i * p] = p;
            if (p == minp[i]) break;
        }
    }
}
// a ^ (m-1) = 1 (Mod m)
// a ^ (m-2) = 1/a (Mod m)
// Exp2: cout << power(x, power(y, p, Mod - 1), Mod)
// Num = (x+1) * (y+1) * (z+1)...
// Sum = (a^0 + a^1+...+ a^x) * (b^0 +...+ b^y)
// Mul = N * (x+1) * (y+1) * (z+1) / 2
```

## 6.4 MillerRabinPollardRho [40f4c1]

```
template<class T>
constexpr T power(T a, ll b, ll p) {
    T res {1};
    for (; b; b /= 2, a = mul(a, a, p)) {
        if (b & 1) {
            res = mul(res, a, p);
        }
    }
    return res;
}
constexpr ll mul(ll a, ll b, ll p) {
    ll res = a * b - ll(1.L * a * b / p) * p;
    res %= p;
    if (res < 0) res += p;
    return res;
}
vector<ll
     > chk {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
bool check(ll a, ll d, int s, ll n) {
    a = power(a, d, n);
    if (a <= 1) return 1;
    for (int i = 0; i < s; i++, a = mul(a, a, n)) {
        if (a == 1) return 0;
        if (a == n - 1) return 1;
    }
    return 0;
}
bool IsPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) {
        d /= 2, s++;
    }
    for (ll i : chk) {
        if (!check(i, d, s, n)) return 0;
    }
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll FindFactor(ll n) {
    if (IsPrime(n)) return 1;
    for (ll p : small) {
        if (n % p == 0) return p;
    }
    ll x, y = 2, d, t = 1;
    auto f = [&](ll a) {
        return (mul(a, a, n) + t) % n;
    };
    for (int l = 2; ; l *= 2) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; ++j) {
                y = f(y), d = mul(d, abs(x - y), n);
            }
            ll g = gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> res;
void PollardRho(ll n) {
    if (n == 1) return;
    if (IsPrime(n)) {
        res[n]++;
        return;
    }
    ll d = FindFactor(n);
    PollardRho(n / d), PollardRho(d);
}
```

## 6.5 CRT [d41d8c]

```
ll exgcd(ll a, ll b, ll &x, ll &y) {
    if (!b) {
        x = 1, y = 0;
        return a;
    }
    ll g = exgcd(b, a % b, y, x);
    y -= a / b * x;
    return g;
}
ll inv(ll x, ll m) {
    ll a, b;
    exgcd(x, m, a, b);
    a %= m;
    if (a < 0) a += m;
    return a;
}
// remain, mod
ll CRT(vector<pair<ll, ll>> &a) {
    ll prod = 1;
    for (auto x : a) {
        prod *= x.second;
    }
    ll res = 0;
    for (auto x : a) {
        auto t = prod / x.second;
        res += x.first * t % prod * inv(t, x.second) % prod;
        if (res >= prod) res -= prod;
    }
    return res;
}
```

## 6.6 Matrix [2856cb]

```
template<class T>
vector<vector<T>> operator*(
     const vector<vector<T>> &a, const vector<vector<T>> &b) {
    int n = a.size(), k = a[0].size(), m = b[0].size();
    assert(k == b.size());
    vector<vector<T>> res(n, vector<T>(m));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            for (int l = 0; l < k; l++)
                res[i][j] += a[i][l] * b[l][j];
    return res;
}
template<class T>
vector<vector<T>> unit(int n) {
    vector<vector<T>> res(n, vector<T>(n));
    for (int i = 0; i < n; i++)
        res[i][i] = 1;
    return res;
}
template<class T>
vector<vector<T>> power(vector<vector<T>> a, ll b) {
    int n = a.size();
    assert(n == a[0].size());
```

```
    auto res = unit<T>(n);
    for (; b; b /= 2, a = a * a)
        if (b % 2) res = res * a;
    return res;
}
using Matrix = vector<vector<Z>>;
```

## 6.7 Mex [14628f]

```
template<class T>
int mex(vector<T> &v) {
    unordered_set<T> s;
    for (auto e : v) s.insert(e);
    for (T i = 0; ; i++)
        if (s.find(i) == s.end()) return i;
}
```

## 6.8 Game Theorem

- $sg$ 值為 0 代表先手必敗
- 當前 $sg$ 值 = 可能的後繼狀態的 $mex$ (例如拿一個或拿兩個, 就等於兩者的 $sg$ 值 $mex$), 若有互相依賴就兩個後繼狀態 $xor$ 當作一組 $sg$ 值 (例如切開成兩半, 只算一次)
- 單組基礎 $nim$ 的 $sg$ 值為本身的原因: $f(0)=0, f(1)=mex(f(0))=1, f(2)=mex(f(0),f(1))=2...$, 都是自己
- 多組賽局可以把 $sg$ 值 $xor$ 起來, 當成最後的 $sg$ 值, nim 也是一樣, 且由於 $xor$ 性質, 如果可以快速知道 $sg(1)g(2)...g(n)$, 就可以用 $xor$ 性質處理不連續組合

## 6.9 Integer Partition [005dc3]

```
// CSES_Sum_of_Divisors
const int mod = 1E9 + 7;
const int inv_2 = 500000004;
// n / 1 * 1 + n / 2 * 2 + n / 3 * 3 + ... + n / n * n
int main() {
    ll ans = 0;
    ll n; cin >> n;
    for (ll l = 1, r; l <= n; l = r + 1) {
        r = n / (n / l);
        ll val = n / l;     // n / l 到 n / r 一樣的值
        ll sum = (((l + r) % mod) *
             ((r - l + 1) % mod)) % mod * inv_2;     // l 加到 r
        val %= mod; sum %= mod;
        ans += val * sum;
        ans %= mod;
    }
    cout << ans << "\n";
}
```

## 6.10 Mobius Theorem

- 數論分塊可以快速計算一些含有除法向下取整的和式，就是像 $\sum_{i=1}^{n} f(i)g(\lfloor \frac{n}{i} \rfloor)$ 的和式。當可以在 $O(1)$ 內計算 $f(r)-f(l)$ 或已經預處理出 f 的前綴和時，數論分塊就可以在 $O(\sqrt{n})$ 的時間內計算上述和式的值。
- 迪利克雷捲積 $h(x)=\sum_{d|x} f(d)g(\frac{x}{d})$
- 積性函數
  - 莫比烏斯函數
    1. 定義
    $$\sum_{d|n}\mu(d)=\begin{cases}1 & \text{for } n=1\\ 0 & \text{for } n\neq 0\end{cases}$$
    2. $\mu$ 是常數函數 1 的反元素
    $\Rightarrow \mu*1=\epsilon$，$\epsilon(n)$只在$n=1$時為 1，其餘情況皆為 0。
  - $\phi$ 歐拉函數: $x$ 以下與 $x$ 互質的數量

$$\begin{aligned}\phi*1&=\sum_{d|n}\phi(\frac{n}{d}) \text{ 質因數分解}\\ &=\sum_{i=0}^{c}\phi(p^i)\\ &=1+p^0(p-1)+p^1(p-1)+...+p^{c-1}(p-1)\\ &=p^c\\ &=id\end{aligned}$$

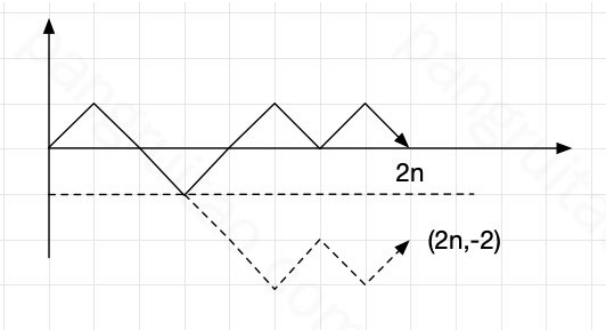
- 莫比烏斯反演公式
  - $f(n)=\sum_{d|n} g(d) \Leftrightarrow g(n)=\sum_{d|n}\mu(d)f(\frac{n}{d})$
  - $f(n)=\sum_{n|d} g(d) \Leftrightarrow g(n)=\sum_{n|d}\mu(\frac{d}{n})f(d)$
- 例子

$$\begin{aligned}&\sum_{i=a}^{b}\sum_{j=c}^{d}[gcd(i,j)=k]\\ &\Rightarrow\sum_{i=1}^{x}\sum_{j=1}^{y}[gcd(i,j)=k]\\ &=\sum_{i=1}^{\lfloor\frac{x}{k}\rfloor}\sum_{j=1}^{\lfloor\frac{y}{k}\rfloor}\epsilon(gcd(i,j))\\ &=\sum_{i=1}^{\lfloor\frac{x}{k}\rfloor}\sum_{j=1}^{\lfloor\frac{y}{k}\rfloor}\sum_{d|gcd(i,j)}\mu(d)\\ &=\sum_{d=1}^{\infty}\mu(d)\sum_{i=1}^{\lfloor\frac{x}{k}\rfloor}[d|i]\sum_{j=1}^{\lfloor\frac{y}{k}\rfloor}[d|j] \text{ d 可整除 i 時為 1}\\ &=\sum_{d=1}^{min(\lfloor\frac{x}{k}\rfloor,\lfloor\frac{y}{k}\rfloor)}\mu(d)\lfloor\frac{x}{kd}\rfloor\lfloor\frac{y}{kd}\rfloor\end{aligned}$$

## 6.11 Mobius Inverse [d41d8c]

```
const int maxn = 2E5;
ll mobius_pref[maxn];
void init() {
    mobius_pref[1] = 1;
    vector<ll> wei
        (maxn);  // wei = 0 代表是質數，-1 代表可被平方數整除
    for (ll i = 2; i < maxn; i++) {
        if (wei[i] == -1) {
            mobius_pref[i] = mobius_pref[i - 1];
            continue; // 包含平方
        }
        if (wei[i] == 0) {
            wei[i] = 1;
            for (ll j = 2; i * j < maxn; j++) {
                if (j % i == 0) wei[i * j] = -1;
                else if (wei[i * j] != -1) wei[i * j]++;
            }
        }
        mobius_pref[i]
            = mobius_pref[i - 1] + (wei[i] % 2 == 0 ? 1 : -1);
    }
}
void solve() {
    ll a, b, c, d, k; cin >> a >> b >> c >> d >> k;
    auto cal = [&](ll x, ll y) -> int {
        int res = 0;
        for (int l = 1, r; l <= min(x, y); l = r + 1) {
            r = min(x / (x / l), y / (y / l));
            res += (mobius_pref[r] - mobius_pref[l
                - 1]) * (x / l) * (y / l);  // 代推出來的式子
        }
        return res;
    };
    cout << cal
        (b / k, d / k) - cal((a - 1) / k, d / k) - cal(b / k,
         (c - 1) / k) + cal((a - 1) / k, (c - 1) / k) << "\n";
}
```

## 6.12 Catalan Theorem




1. $n$ 個往上 $n$ 個往下，先枚舉所有情況 $\frac{(2n)!}{n!n!}=C_n^{2n}$
2. 扣掉非法的，有多少種可能讓最後的點落在 $(2n,-2)$

假設往上有 $x$ 個，往下有 $y$ 個，會有:

$$\begin{cases}x+y=2n\\ y-x=2\end{cases}\Rightarrow\begin{cases}x=n-1\\ y=n+1\end{cases}$$

所以只要扣掉 $C_{n-1}^{2n}$ 即可

### 6.13 Burnside's Lemma

$|X/G| = \frac{1}{|G|}\sum_{g\in G}|X^g|$

- $G$：各種翻轉操作所構成的置換群
- $X/G$：本質不同的方案的集合
- $X^g$：對於某一種操作 $g$，所有方案中，經過 $g$ 這種翻轉後保持不變的方案的集合
- 集合取絕對值代表集合數

# 7 Search and Gready

## 7.1 Binary Search [d41d8c]

```
int main() {
    // 二分找上界
    while (lo < hi) {
        int x = (lo + hi + 1) / 2;
        if (check(x)) lo = x;
        else hi = x - 1;
    }
    cout << lo; // 保證有解

    while (lo <= hi) {
        int x = (lo + hi) / 2;
        if (check(x)) lo = x + 1;
        else hi = x - 1;
    }
    cout << hi; // 範圍外代表無解

    // 二分找下界
    while (lo < hi) {
        int x = (lo + hi) / 2;
        if (check(m)) hi = x;
        else lo = x + 1;
    }
    cout << lo; // 保證有解

    while (lo <= hi) {
        int x = (lo + hi) / 2;
        if (check(m)) hi = x - 1;
        else lo = x + 1;
    }
    cout << lo; // 範圍外代表無解
}
```

## 7.2 Ternary Search [d41d8c]

```
int main() {
    int lo = 0, hi = 10;
    while (lo <= hi) {
        int xl = lo + (hi - lo) / 3;
        int xr = hi - (hi - lo) / 3;
        int ansl = check(xl), ansr = check(xr);
        if (ansl < ansr) {
            lo = xl + 1;
        } else {
            hi = xr - 1;
        }
        // record ans and index
    }
}
```

# 8 Tree

## 8.1 Binary Lifting LCA [4273df]

```
const int Q = 20; // log(q) or log(n)
vector<vector<int>> par;
vector<int> dep, dfn;
void build(int n, vector<vector<int>> &tree, int u = 0) {
    par.assign(n, vector<int>(Q + 1, -1));
    dep.assign(n, 0), dfn.assign(n, 0);
    int cur = 0;
    auto dfs = [&](auto self, int x, int p) -> void {
        dfn[x] = cur++;
        for (auto y : tree[x]) {
            if (y == p) continue;
            par[y][0] = x;
            dep[y] = dep[x] + 1;
            self(self, y, x);
        }
    };
    par[u][0] = u;
    dfs(dfs, 0, -1);
    for (int i = 1; i <= Q; i++)
        for (int j = 0; j < n; j++)
            par[j][i] = par[par[j][i - 1]][i - 1];
}
int lca(int a, int b) {
    if (dep[a] < dep[b]) swap(a, b);
    int pull = dep[a] - dep[b];
    for (int i = 0; i <= Q; i++)
        if (pull & (1 << i))
            a = par[a][i];
    if (a == b) return a;
    for (int i = Q; i >= 0; i--)
        if (par[a][i] != par[b][i])
            a = par[a][i], b = par[b][i];
    return par[a][0];
}
int jump(int x, int k) {
    for (int i = Q; i >= 0; i--)
        if (k >> i & 1)
            x = par[x][i];
    return x;
}
```

## 8.2 Centroid Decomposition [c40feb]

```
#include <bits/stdc++.h>
using namespace std;
struct CenDecom {
    int n;
    vector<vector<int>> adj;
    vector<bool> vis;
    vector<int> siz;
    CenDecom(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_;
        adj.assign(n, {});
        vis.assign(n, false);
        siz.assign(n, 1);
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void get_siz(int x, int p = -1) {
        siz[x] = 1;
        for (int y : adj[x]) {
            if (y == p || vis[y]) continue;
            get_siz(y, x);
            siz[x] += siz[y];
        }
    }
    int get_cen(int x, int sz, int p = -1) {
        for (int y : adj[x]) {
            if (y == p || vis[y]) continue;
            if (siz[y] * 2 > sz)
                return get_cen(y, sz, x);
        }
        return x;
    }
    void get_ans(int x, int p) {
        // do something
        for (int y : adj[x]) {
            if (y == p || vis[y]) continue;
            get_ans(y, x);
        }
    }
    void work(int x = 0) {
        get_siz(0, x);
        int cen = get_cen(x, siz[x]);
        vis[cen] = true;
        for (int y : adj[cen]) {
            if (vis[y]) continue;
            get_ans(y, cen);
        }
        for (int y : adj[cen]) {
            if (vis[y]) continue;
            work(y);
        }
    }
};
```

## 8.3 Heavy Light Decomposition [41d99e]

```
struct HLD {
    int n, cur;
    vector<int> siz, top, dep, parent, in, out, seq;
    vector<vector<int>> adj;
    HLD(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_; cur = 0;
        siz.resize(n); top.resize(n); dep.resize(n);
        parent.resize(n); in.resize(n); out.resize(n);
        seq.resize(n); adj.assign(n, {});
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void work(int rt = 0) {
        top[rt] = rt;
        dep[rt] = 0;
        parent[rt] = -1;
        dfs1(rt); dfs2(rt);
    }
    void dfs1(int u) {
        if (parent[u] != -1)
            adj[u].erase(find
                (adj[u].begin(), adj[u].end(), parent[u]));
        siz[u] = 1;
        for (auto &v : adj[u]) {
            parent[v] = u, dep[v] = dep[u] + 1;
            dfs1(v);
            siz[u] += siz[v];
            if (siz[v] > siz[adj[u][0]]) {
                swap(v, adj[u][0]);
            } // 讓 adj[u][0] 是重子節點
        }
    }
```

```
    void dfs2(int u) {
        in[u] = cur++;
        seq[in[u]] = u; // dfn 對應的編號
        for (auto v : adj[u]) {
            top[v] = v == adj[u][0] ? top[u] : v;
            dfs2(v);
        }
        out[u] = cur;
    }
    int lca(int u, int v) {
        while (top[u] != top[v]) {
            if (dep[top[u]] > dep[top[v]]) {
                u = parent[top[u]];
            } else {
                v = parent[top[v]];
            }
        }
        return dep[u] < dep[v] ? u : v;
    }
    int dist(int u, int v) {
        return dep[u] + dep[v] - 2 * dep[lca(u, v)];
    }
    int jump(int u, int k) {
        if (dep[u] < k) return -1;
        int d = dep[u] - k;
        while (dep[top[u]] > d)
            u = parent[top[u]];
        return seq[in[u] - dep[u] + d];
    }
    bool isAncester(int u, int v) {
        return in[u] <= in[v] && in[v] < out[u];
    }
    int rootedParent(int rt, int v) {
        swap(rt, v);
        if (rt == v) return rt;
        if (!isAncester(rt, v)) return parent[rt];
        auto it = upper_bound(adj[
             rt].begin(), adj[rt].end(), v, [&](int x, int y) {
            return in[x] < in[y];
        }) - 1;
        return *it;
    }
    int rootedSize(int rt, int v) {
        if (rt == v) return n;
        if (!isAncester(v, rt)) return siz[v];
        return n - siz[rootedParent(rt, v)];
    }
    int rootedLca(int rt, int a, int b) {
        return lca(rt, a) ^ lca(a, b) ^ lca(b, rt);
    }
};
```

## 8.4 Link Cut Tree [96c213]

```
template<class Info, class Tag>
struct LinkCutTree { // 1-based
    struct Node {
        Info info = Info();
        Tag tag = Tag();
        bool rev = false;
        int size = 0;
        int ch[2], p = 0;
    };
    vector<Node> nd;
    LinkCutTree(int n = 0) { init(n); }
    void init(int n) {
        nd.clear();
        nd.emplace_back();
        resize(n);
    }
    void resize(int n) {
        nd.resize(n + 1);
    }
    bool isrt(int t) {
        return !nd[t].p || (
             nd[nd[t].p].ch[0] != t && nd[nd[t].p].ch[1] != t);
    }
    void make_rev(int t) {
        swap(nd[t].ch[0], nd[t].ch[1]);
        nd[t].rev ^= true;
    }
    void apply(int t, const Tag &v) {
        nd[t].info.apply(nd[t].size, v);
        nd[t].tag.apply(v);
    }
    void push(int t) {
        if (nd[t].rev) {
            if (nd[t].ch[0]) make_rev(nd[t].ch[0]);
            if (nd[t].ch[1]) make_rev(nd[t].ch[1]);
            nd[t].rev = false;
        }
        if (nd[t].ch[0]) apply(nd[t].ch[0], nd[t].tag);
        if (nd[t].ch[1]) apply(nd[t].ch[1], nd[t].tag);
        nd[t].tag = Tag();
    }
    void pull(int t) {
        nd[t].size
             = 1 + nd[nd[t].ch[0]].size + nd[nd[t].ch[1]].size;
        nd[t].info
             .pull(nd[nd[t].ch[0]].info, nd[nd[t].ch[1]].info);
    }
    int pos(int t) {
        return nd[nd[t].p].ch[1] == t;
    }
    void pushAll(int t) {
        if (!isrt(t)) {
            pushAll(nd[t].p);
        }
        push(t);
    }
    void rotate(int t) {
        int q = nd[t].p;
        int x = !pos(t);
        nd[q].ch[!x] = nd[t].ch[x];
        if (nd[t].ch[x]) nd[nd[t].ch[x]].p = q;
        nd[t].p = nd[q].p;
        if (!isrt(q)) nd[nd[q].p].ch[pos(q)] = t;
        nd[t].ch[x] = q;
        nd[q].p = t;
        pull(q);
    }
    void splay(int t) {
        pushAll(t);
        while (!isrt(t)) {
            if (!isrt(nd[t].p)) {
                if (pos(t) == pos(nd[t].p)) {
                    rotate(nd[t].p);
                } else {
                    rotate(t);
                }
            }
            rotate(t);
        }
        pull(t);
    }
    void access(int t) { // access 後自動 splay
        for (int i = t, q = 0; i; q = i, i = nd[i].p) {
            splay(i);
            nd[i].ch[1] = q;
            pull(i);
        }
        splay(t);
    }
    void makeRoot(int t) {
        access(t);
        make_rev(t);
    }
    int findRoot(int t) {
        access(t);
        int x = t;
        while (nd[x].ch[0]) {
            push(x);
            x = nd[x].ch[0];
        }
        access(x);
        return x;
    }
    bool connected(int x, int y) {
        return findRoot(x) == findRoot(y);
    }
    bool neighber(int x, int y) {
        makeRoot(x);
        access(y);
        if (nd[y].ch[0] != x || nd[x].ch[1]) return false;
        return true;
    }
    void split(int rt, int y) {
        makeRoot(y);
        access(rt);
    }
    void link(int x, int y) {
        makeRoot(x);
        if (findRoot(y) != x)
            nd[x].p = y;
    }
    void cut(int x, int y) {
        makeRoot(x);
        access(y);
        nd[y].ch[0] = nd[nd[y].ch[0]].p = 0;
        pull(x);
        pull(y);
    }
    void modify(int x, const Info &v) {
        access(x);
        nd[x].info = v;
    }
    void path_apply(int x, int y, const Tag &v) {
        assert(connected(x, y));
        split(x, y);
        apply(x, v);
    }
    Info path_query(int x, int y) {
        assert(connected(x, y));
        split(x, y);
        return nd[x].info;
    }
};
constexpr int Mod = 51061;
struct Tag {
    ll add = 0; ll mul = 1;
    void apply(const Tag &v) {
        mul = mul * v.mul % Mod;
        add = (add * v.mul % Mod + v.add) % Mod;
    }
```

```
};
struct Info {
    ll val = 0; ll sum = 0;
    void apply(int size, const Tag &v) {
        val = (val * v.mul % Mod + v.add) % Mod;
        sum = (sum * v.mul % Mod + v.add * size % Mod) % Mod;
    }
    void pull(const Info &l, const Info &r) {
        sum = (l.sum + r.sum + val) % Mod;
    }
};
```

## 8.5 Virtual Tree [41e291]

```
// 多次詢問給某些關鍵點，虛樹可達成快速樹 DP (前處理每個點)
// 例如這題是有權樹，給一些關鍵點，求跟 vertex 1 隔開的最小成本
// 前處理 root 到所有點的最小邊權
vector<int> stk;
void insert(int key, vector<vector<int>> &vt) {
    if (stk.empty()) {
        stk.push_back(key);
        return;
    }
    int l = lca(stk.back(), key);
    if (l == stk.back()) {
        stk.push_back(key);
        return;
    }
    while (
        stk.size() > 1 && dfn[stk[stk.size() - 2]] > dfn[l]) {
        vt[stk[stk.size() - 2]].push_back(stk.back());
        stk.pop_back();
    }
    if (stk.size() < 2 || stk[stk.size() - 2] != l) {
        vt[l].push_back(stk.back());
        stk.back() = l;
    } else {
        vt[l].push_back(stk.back());
        stk.pop_back();
    }
    stk.push_back(key);
}
int work(vector<vector<int>> &vt) {
    while (stk.size() > 1) {
        vt[stk[stk.size() - 2]].push_back(stk.back());
        stk.pop_back();
    }
    int rt = stk[0];
    stk.clear();
    return rt;
}
void solve() {
    int n; cin >> n;
    vector<vector<int>> g(n);
    vector<vector<pair<int, int>>> wg(n);
    vector<vector<int>> vt(n);
    for (int i = 1; i < n; i++) {
        int u, v, w;
        cin >> u >> v >> w;
        u--, v--;
        g[u].push_back(v), g[v].push_back(u);
        wg[u].emplace_back(v, w), wg[v].emplace_back(u, w);
    }
    build(n, g); // build LCA
    vector<int> dis(n, 1E9); // root 到各點的最小邊權
    auto dfs_dis = [&](auto &&self, int x, int p) -> void {
        for (auto [y, w] : wg[x]) {
            if (y == p) continue;
            dis[y] = min(w, dis[x]);
            self(self, y, x);
        }
    };
    dfs_dis(dfs_dis, 0, -1);

    vector<bool> iskey(n);
    vector<ll> dp(n);
    int q; cin >> q;
    while (q--) {
        int m; cin >> m;
        vector<int> key(m);
        for (int i = 0; i < m; i++) {
            cin >> key[i];
            key[i] -= 1;
            iskey[key[i]] = true;
        }
        key.push_back(0); // 固定 0 為 root, 看題目需求
        sort(key.begin(), key.end(), [&](int a, int b) {
            return dfn[a] < dfn[b];
        }); // 要 sort 再 insert
        for (auto x : key) insert(x, vt);
        work(vt);
        auto dfs = [&](auto &&self, int x) -> void {
            for (auto y : vt[x]) {
                self(self, y);
                if (iskey[y]) { // 直接砍了
                    dp[x] += dis[y];
                } else { // 不砍 or 砍
                    dp[x] += min<ll>(dp[y], dis[y]);
                } // 記得 reset
                iskey[y] = dp[y] = 0;
            }
            vt[x].clear(); // 記得 reset
        };
        dfs(dfs, 0);
        cout << dp[0] << "\n";
        dp[0] = 0; // 最後 reset root
    }
}
```

## 8.6 Dominator Tree [0b03d9]

```
// dom
    存起點到達此點的必經的上個節點(起點 = 自己), 無法到達 = -1
struct Dominator_tree {
    int n, id;
    vector<vector<int>> adj, radj, bucket;
    vector<int> sdom, dom, vis, rev, pa, rt, mn, res;
    Dominator_tree(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_, id = 0;
        adj.assign(n, {});
        radj.assign(n, {});
        bucket.assign(n, {});
        sdom.resize(n), dom.assign(n, -1);
        vis.assign(n, -1), rev.resize(n);
        pa.resize(n), rt.resize(n);
        mn.resize(n), res.resize(n);
    }
    void add_edge(int u, int v) {
        adj[u].push_back(v);
    }
    int query(int v, int x) {
        if (rt[v] == v) return x ? -1 : v;
        int p = query(rt[v], 1);
        if (p == -1) return x ? rt[v] : mn[v];
        if (sdom[mn[v]] > sdom[mn[rt[v]]])
            mn[v] = mn[rt[v]];
        rt[v] = p;
        return x ? p : mn[v];
    }
    void dfs(int v) {
        vis[v] = id, rev[id] = v;
        rt[id] = mn[id] = sdom[id] = id, id++;
        for (int u : adj[v]) {
            if (vis[u] == -1)
                dfs(u), pa[vis[u]] = vis[v];
            radj[vis[u]].push_back(vis[v]);
        }
    }
    vector<int> build(int s) {
        dfs(s);
        for (int i = id - 1; i >= 0; i--) {
            for (int u : radj[i])
                sdom[i] = min(sdom[i], sdom[query(u, 0)]);
            if (i) bucket[sdom[i]].push_back(i);
            for (int u : bucket[i]) {
                int p = query(u, 0);
                dom[u] = sdom[p] == i ? i : p;
            }
            if (i) rt[i] = pa[i];
        }
        res.assign(n, -1);
        for (int i = 1; i < id; i++)
            if (dom[i] != sdom[i])
                dom[i] = dom[dom[i]];
        for (int i = 1; i < id; i++)
            res[rev[i]] = rev[dom[i]];
        res[s] = s;
        for (int i = 0; i < n; i++)
            dom[i] = res[i];
        return dom;
    }
};
```

# 9 DP

## 9.1 LCS [087c0d]

```
int main() {
    int m, n; cin >> m >> n;
    string s1, s2; cin >> s1 >> s2;
    int L = 0;
    vector<vector<int>> dp(m + 1, vector<int>(n + 1, 0));
    for (int i = 1; i <= m; i++) {
        for (int j = 1; j <= n; j++) {
            if (s1[i - 1] == s2[j - 1]) {
                dp[i][j] = dp[i - 1][j - 1] + 1;
            } else {
                dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
            }
        }
    }
    int length = dp[m][n];
    cout << length << "\n";
    string s(length, 'c'); // backtracking
    while (m >= 1 && n >= 1) {
        if (s1[m - 1] == s2[n - 1]) {
            s[length - 1] = s1[m - 1];
            m--, n--, length--;
        }
        else {
```

```
            if (dp[m - 1][n] > dp[m][n - 1]) m--;
            else n--;
        }
    }
    cout << s << "\n";
}
```

## 9.2 LIS [91741b]

```
int main() {
    int n; cin >> n;
    vector<int> v(n);
    for (int i = 0; i < n; i++) cin >> v[i];
    int dp[n], L = 1;
    dp[0] = 1;
    vector<int> stk {v[0]};
    for (int i = 1; i < n; i++) {
        if (v[i] > stk.back()) {
            stk.push_back(v[i]);
            dp[i] = ++L;
        } else {
            auto it
                = lower_bound(stk.begin(), stk.end(), v[i]);
            *it = v[i]; dp[i] = it - stk.begin() + 1;
        }
    }
    vector<int> ans; cout << L << "\n";
    for (int i = n - 1; i >= 0; i--)
        if (dp[i] == L)
            ans.push_back(v[i]), L--;
    reverse(ans.begin(), ans.end());
    for (auto i : ans) cout << i << " ";
}
```

## 9.3 Edit Distance [308023]

```
int main() {
    string s1, s2; cin >> s1 >> s2;
    int n1 = s1.size(), n2 = s2.size();
    // dp[i][j] 為 s1 的前 i 個字元，跟 s2 的前 j 個字元
    vector<int> dp(n2 + 1);
    iota(dp.begin(), dp.end(), 0);
    for (int i = 1; i <= n1; i++) {
        vector<int> cur(n2 + 1); cur[0] = i;
        for (int j = 1; j <= n2; j++) {
            if (s1[i - 1] == s2[j - 1]) {
                cur[j] = dp[j - 1];
            } else {
                // s1 新增等價於 s2 砍掉
                // dp[i][j] = min(s2 新增, 修改, s1 新增);
                cur[j]
                    = min({cur[j - 1], dp[j - 1], dp[j]}) + 1;
            }
        }
        swap(dp, cur);
    }
    cout << dp[n2] << "\n";
}
```

## 9.4 Bitmask [da8000]

```
void hamiltonianPath() {
    int n, m; cin >> n >> m;
    vector<vector<int>> adj(n);
    for (int i = 0; i < m; i++) {
        int u, v; cin >> u >> v;
        adj[--v].push_back(--u);
    }
    // 以...為終點，走過...
    vector dp(n, vector<int>(1 << n));
    dp[0][1] = 1;
    for (int mask = 1; mask < 1 << n; mask++) {
        if ((mask & 1) == 0) continue;
        for (int i = 0; i < n; i++) {
            if ((mask >> i & 1) == 0) continue;
            if (i == n - 1 && mask != (1 << n) - 1) continue;
            int pre = mask ^ (1 << i);
            for (int j : adj[i]) {
                if ((pre >> j & 1) == 0) continue;
                dp[i][mask] = (dp[i][mask] + dp[j][pre]) % Mod;
            }
        }
    }
    cout << dp[n - 1][(1 << n) - 1] << "\n";
}
void elevatorRides() {
    int n, x; cin >> n >> x;
    vector<int> a(n);
    for (int i = 0; i < n; i++) {
        cin >> a[i];
    }
    vector<int> dp(1 << n), f(1 << n);
    dp[0] = 1; // 次數、已使用人數
    for (int mask = 1; mask < 1 << n; mask++) {
        dp[mask] = 2E9;
        for (int i = 0; i < n; i++) {
            if ((mask >> i & 1) == 0) continue;
            int pre = mask ^ (1 << i);
            if (f[pre] + a[i] <= x) {
                if (dp[pre] < dp[mask] || dp[pre]
                        == dp[mask] && f[pre] + a[i] < f[mask]) {
                    dp[mask] = dp[pre];
                    f[mask] = f[pre] + a[i];
                }
            } else if (dp[pre] + 1 < dp[mask] ||
                  dp[pre] + 1 == dp[mask] && a[i] < f[mask]) {
                dp[mask] = dp[pre] + 1;
                f[mask] = a[i];
            }
        }
    }
    cout << dp[(1 << n) - 1] << "\n";
}
void minClique() { // 移掉一些邊，讓整張圖由最少團組成
    int n, m;
    cin >> n >> m;
    vector<bitset<N>> g(n);
    for (int i = 0; i < m; i++) {
        int u, v;
        cin >> u >> v;
        u--; v--;
        g[u][v] = g[v][u] = 1;
    }
    vector<int> dp(1 << n, inf);
    dp[0] = 1;
    for (int mask = 0; mask < 1 << n; mask++) { // 先正常 dp
        for (int i = 0; i < n; i++) {
            if (mask & (1 << i)) {
                int pre = mask ^ (1 << i);
                if (dp[pre]
                      == 1 && (g[i] & bitset<N>(pre)) == pre) {
                    dp[mask] = 1; // i 有連到所有 pre
                }
            }
        }
    }
    for (int
          mask = 0; mask < 1 << n; mask++) { // 然後枚舉子集 dp
        for (int sub = mask; sub; --sub &= mask) {
            dp[mask] = min(dp[mask], dp[sub] + dp[mask ^ sub]);
        }
    }
    cout << dp[(1 << n) - 1] << "\n";
}
```

## 9.5 Projects [ca09b1]

```
int main() { // 排程有權重問題，輸出價值最多且時間最少
    struct E {
        int from, to, w, id;
    };
    int n; cin >> n; vector<E> a(n + 1);
    for (int i = 1; i <= n; i++) {
        int u, v, w;
        cin >> u >> v >> w;
        a[i] = {u, v, w, i};
    }
    vector<array<ll, 2>> dp(n + 1); // w, time
    vector<array<int, 2>> rec(n + 1); // 有沒選，上個是誰
    sort(a.begin(), a.end());
    for (int i = 1; i <= n; i++) {
        int id = --
             lower_bound(all(a), {0, a[i].from}, [](E x, E y) {
            return x.to < y.to;
        }) - a.begin();
        dp[i] = dp[i - 1];
        ll nw = dp[id][0] + a[i].w;
        ll nt = dp[id][1] + a[i].to - a[i].from;
        if (dp[i][0] < nw || dp[i][0] == nw && dp[i][1] > nt) {
            dp[i] = {nw, nt};
            rec[i] = {1, id};
        }
    }
    vector<int> ans;
    for (int i = n; i != 0;) {
        if (rec[i][0]) {
            ans.push_back(a[i].id);
            i = rec[i][1];
        } else {
            i--;
        }
    }
}
```

## 9.6 Removal Game [588f62]

```
// 兩個人比賽，每個人輪流取一個數字且只能是頭尾
// 問兩人都選得好，第一出手的人可取得的最大分數
int main() {
    int n; cin >> n;
    vector<ll> a(n);
    for (int i = 0; i < n; i++) cin >> a[i];
    vector<vector<ll>> dp(n, vector<ll>(n));
    // i 到 j 區間的最大 diff
    for (int i = n - 1; i >= 0; i--) {
        dp[i][i] = a[i];
        for (int j = i + 1; j < n; j++)
            dp[i][j] =
                 max(a[i] - dp[i + 1][j], a[j] - dp[i][j - 1]);
    }
    // x + y = sum; // x - y = dp[0][n - 1]
```

```
    cout << (accumulate
         (a.begin(), a.end(), 0LL) + dp[0][n - 1]) / 2 << "\n";
}
```

## 9.7 Monotonic Queue [f4976d]

```
// 應用: dp(i) = h(i) + max(A(j)), for l(i)≤j≤r(i)
// A(j) 可能包含 dp(j), h(i) 可 O(1)
void Bounded_Knapsack() {
    int n, k; // O(nk)
    vector<int> w(n), v(n), num(n);
    deque<int> q;
    // 於是我們將同餘的數分在同一組
    // 每次取出連續 num[i] 格中最大值
    // g_x = max(_{k=0}^num[i] (g'_{x-k} + v_i*k))
    // G_x = g'_{x} - v_i*x
    // x 代 x-k => v_i*(x-k)
    // g_x = max(_{k=0}^num[i] (G_{x-k} + v_i*x))
    vector<vector<ll>> dp(2, vector<ll>(k + 1));
    for (int i = 0; i < n; i++) {
        for (int r = 0; r < w[i]; r++) { // 餘數
            q.clear(); // q 記錄在 x = i 時的 dp 有單調性
            for (int x = 0; x * w[i] + r <= k; x++) {
                while (!q.empty() && q.front() < x - num[i])
                    q.pop_front(); // 維護遞減
                ll nxt = dp[0][x * w[i] + r] - x * v[i];
                while (!q.empty() && dp[0][q.back
                     () * w[i] + r] - q.back() * v[i] < nxt)
                    q.pop_back();
                q.push_back(x);
                dp[1][x * w[i] + r] = dp[0][q.front()
                     * w[i] + r] - q.front() * v[i] + x * v[i];
            }
        }
        swap(dp[0], dp[1]);
    }
    cout << dp[0][k] << "\n";
}
```

## 9.8 SOS [7a4936]

```
// 使用情況: 跟 bit 與(被)包含有關, 且 x 在 1E6 左右
// 題目: 一數組, 問有多少所有數 & 起來為 0 的集合數
// dp[
    x] 代表包含 x 的 y 個數(比 x 大且 bit 1 全包含 x 的有幾個)
// 答案應該包含在 dp[0] 內，但是有重複元素，所以考慮容斥
// => ans = \sum _{i=0}^{n} (-1)^{pop_count(i)} 2^{dp[i]-1}
// => 全
    部為 0 的個數 - 至少一個為 1 的個數 + 至少兩個為 1 的個數
void solve() {
    int n; cin >> n; Z ans = 0;
    vector<int> a(n);
    for (int i = 0; i < n; i++) cin >> a[i];
    int m = __lg(*max_element(a.begin(), a.end())) + 1;
    // 定義 dp[mask] 為 mask 被包含於 a[i] 的 i 個數
    vector<Z> dp(1 << m);
    for (int i = 0; i < n; i++)
        dp[a[i]] += 1;
    for (int i = 0; i < m; i++) {
        for (int mask = 0; mask < 1 << m; mask++) {
            if (mask >> i & 1) {
                int pre = mask ^ (1 << i);
                dp[pre] += dp[mask];
            }
        }
    }
    for (int mask = 0; mask < 1 << m; mask++) {
        int sgn = __builtin_popcount(mask) & 1 ? -1 : 1;
        ans += sgn * (power(Z(2), dp[mask].val()) - 1);
    }
    cout << ans << "\n";
}

// x | y = x, 代表包含於 x 的 y 個數, 定義為 dp[x][0]
// x & y = x, 代表包含 x 的 y 個數, 定義為 dp[x][1]
// x & y != 0, 代表至
    少有一個位元都為 1 的 y 個數, = n - 與自己相同 - ~dp[x][0]
void solve() {
    int n; cin >> n;
    vector<int> a(n);
    map<int, int> mp;
    for (int i = 0; i < n; i++) {
        cin >> a[i];
        mp[a[i]]++;
    }
    int m = __lg(*max_element(a.begin(), a.end())) + 1;
    vector<array<ll, 2>> dp(1 << m);
    for (int i = 0; i < n; i++) {
        dp[a[i]][0] += 1;
        dp[a[i]][1] += 1;
    }
    for (int i = 0; i < m; i++) {
        for (int mask = 0; mask < 1 << m; mask++) {
            if (mask >> i & 1) {
                int pre = mask ^ (1 << i);
                dp[mask][0] += dp[pre][0];
                dp[pre][1] += dp[mask][1];
            }
        }
    }
    for (int i = 0; i < n; i++) {
        cout << dp[a[i]][0] << " " << dp[a[i]][1] <<
             " " << n - (dp[((1 << m) - 1) ^ a[i]][0]) << "\n";
    }
}
```

## 9.9 CHT [5f5c25]

```
// 應用: dp(i) = h(i) + min/max(A(j)X(i) + B(j)), for j≤r(i)
// A(j), B(j) 可能包含 dp(j)，分別就是 m 跟 b
struct Line {
    ll m, b;
    Line(ll m = 0, ll b = 0) : m(m), b(b) {}
    ll eval(ll x) {
        return m * x + b;
    }
};
struct CHT { // 用在查詢單調斜率也單調
    int n, lptr, rptr;
    vector<Line> hull;
    CHT(int n_ = 0, Line init_ = Line()) {
        init(n_, init_);
    }
    void init(int n_ = 0, Line init_ = Line()) {
        n = n_; hull.resize(n); reset(init_);
    }
    void reset(Line init_ = Line()) {
        lptr = rptr = 0; hull[0] = init_;
    }
    bool pop_front(Line &l1, Line &l2, ll x) {
        // 斜率遞減、查詢遞增，因此只要左直線的 Y >= 右直線的 Y
        // 代表查詢的當下，右線段的高度已經低於左線段了
        return l1.eval(x) >= l2.eval(x);
    }
    bool pop_back(Line &l1, Line &l2, Line &l3) {
        // 本題斜率遞減、上凸包
        // 因此只要 l2 跟
            l3 的 X 交點 <= l1 跟 l3 的 X 交點，l2 就用不到了
        return (l3.b - l2.b)
            * (l1.m - l3.m) <= (l3.b - l1.b) * (l2.m - l3.m);
    }
    void insert(Line L) {
        while (rptr - lptr
            > 0 && pop_back(hull[rptr - 1], hull[rptr], L))
            rptr--;
        hull[++rptr] = L;
    }
    ll query(ll x) {
        while (rptr - lptr
            > 0 && pop_front(hull[lptr], hull[lptr + 1], x))
            lptr++;
        return hull[lptr].eval(x);
    }
};
```

## 9.10 DNC [d2ed4d]

```
// 應用: 切 k 段問題, 且滿足四邊形不等式
// w(a,c) + w(b,d) ≤(≥) w(a,d) + w(b,c)
// dp[k][j] = min(dp[k - 1][i] + cost[i][j])
// cost: (i, j]
constexpr int N = 3E3 + 5;
constexpr ll inf = 4E18;
ll dp[N][N]; // 1-based
ll get_cost(int l, int r) {}
void DNC(int k, int l, int r, int optl, int optr) {
    if (l > r) return;
    int m = (l + r) >> 1, opt = -1;
    dp[k][m] = inf;
    for (int i = max(k, optl); i <= min(m, optr); i++) {
        // 注意 i 的範圍、 get_cost 與 dp 的邊界
        ll cur = dp[k - 1][i] + get_cost(i, m);
        if (cur < dp[k][m])
            dp[k][m] = cur, opt = i;
    }
    DNC(k, l, m - 1, optl, opt);
    DNC(k, m + 1, r, opt, optr);
}
int main() {
    // first build cost...
    for (int i = 1; i <= n; i++) {
        // init dp[1][i]
    }
    for (int i = 2; i <= k; i++)
        DNC(i, 1, n, 1, n);
    cout << dp[k][n] << "\n";
}
```

## 9.11 LiChao Segment Tree [588aa3]

```
// 應用: dp(i) = h(i) + min/max(A(j)X(i) + B(j)), for j ≤ r(i)
//        y    =  c   +          m   x   +   b
constexpr ll inf = 4E18;
struct Line {
    ll m, b;
    Line(ll m = 0, ll b = inf) : m(m), b(b) {}
    ll eval(ll x) const {
        return m * x + b;
```

```
    }
};
struct LiChaoSeg { // 取 max 再變換就好
    int n;
    vector<Line> info;
    LiChaoSeg(int n_ = 0) { init(n_); }
    void init(int n_) {
        n = n_;
        info.assign(4 << __lg(n), Line());
    }
    void update(Line line, int node, int l, int r) {
        int m = (l + r) / 2;
        bool left = line.eval(l) < info[node].eval(l);
        bool mid = line.eval(m) < info[node].eval(m);
        if (mid) swap(info[node], line); // 如果新線段比較好
        if (r - l == 1) return;
        else if (left != mid) update(line, 2 * node, l, m);
        // 代表左半有交點
        else update(line, 2 * node + 1, m, r);
        // 代表如果有交點一定在右半
    }
    void add_line(Line line) { update(line, 1, 0, n); }
    ll query(int x, int node, int l, int r) {
        if (r - l == 1) return info[node].eval(x);
        int m = (l + r) / 2;
        if (x < m) {
            return min(
                info[node].eval(x), query(x, 2 * node, l, m));
        } else {
            return min(info
                [node].eval(x), query(x, 2 * node + 1, m, r));
        }
    }
    ll query(int x) {
        return query(x, 1, 0, n);
    }
};
```

## 9.12 Codeforces Example [a0184a]

```
// CF 1932 pF
// 給你很多區間，你可以選一些點，重疊到的線段得到 1 分
// 請問在線段不重複的情況下，最多獲得幾分
int main() {
    int n, m;
    cin >> n >> m;
    // 記錄每點有幾個線段
    // 再一個紀錄，包含這個點的左界
    vector<int> l_side(n + 1, inf), cnt(n + 5, 0);
    for (int i = 0; i < m; i++) {
        int l, r; cin >> l >> r;
        l_side[r] = min(l_side[r], l);
        cnt[l]++;
        cnt[r + 1]--;
    }
    for (int i = 2; i <= n; i++)
        cnt[i] += cnt[i - 1];
    for (int i = n; i >= 2; i--)
        l_side[i - 1] = min(l_side[i - 1], l_side[i]);
    vector<int> dp(n + 1);
    dp[0] = 0;
    for (int i = 1; i <= n; i++) {
        dp[i] = cnt[i];
        if (l_side[i] != inf)
            dp[i] += dp[l_side[i] - 1];
        dp[i] = max(dp[i], dp[i - 1]);
    }
    cout << dp[n] << "\n";
}

// CF 1935 pC
// 給你每個事件的 a, b，挑事件會把 a 全部加起來
// 再加上 max(bi) - min(bi)
int main() {
    int n, k, ans = 0; cin >> n >> k;
    vector<pii> v(n + 1);
    for (int i = 1; i <= n; i++) {
        int a, b; cin >> a >> b;
        v[i] = {a, b};
        if (a <= k) ans = 1;
    }
    sort(v.begin() + 1, v.end(), [](pii &a, pii &b) {
        return a.second < b.second;
    }); // 用 bi 來排，考慮第 i 個時可以先扣
    vector<vector<int>> dp(n + 1, vector<int>(n + 1, inf));
    // 考慮 v[i] 時，選 j 個的 sum(ai) - min(bi)
    for (int i = 1; i <= n; i++) { // 滾動 dp
        for (int j = n; j >= 2; j--) {
            dp[i][j] = min
                (dp[i - 1][j], dp[i - 1][j - 1] + v[i].first);
            // min(不選, 選)
            if (dp[i
                - 1][j - 1] + v[i].first + v[i].second <= k) {
                // 假如可以選，更新 ans 時再加回去 bi
                ans = max(ans, j);
            }
        }
        dp[i][1] = min(dp[i - 1][1], v[i].first - v[i].second);
    }
    cout << ans << "\n";
}
```

# 10 Geometry

## 10.1 Basic [d41d8c]

```
template<class T>
struct Point {
    T x, y;
    Point(const T &x_ = 0, const T &y_ = 0) : x(x_), y(y_) {}
    template<class U>
    operator Point<U>() {
        return Point<U>(U(x), U(y));
    }
    Point &operator+=(const Point &p) & {
        x += p.x; y += p.y; return *this;
    }
    Point &operator-=(const Point &p) & {
        x -= p.x; y -= p.y; return *this;
    }
    Point &operator*=(const T &v) & {
        x *= v; y *= v; return *this;
    }
    Point &operator/=(const T &v) & {
        x /= v; y /= v; return *this;
    }
    Point operator-() const {
        return Point(-x, -y);
    }
    friend Point operator+(Point a, const Point &b) {
        return a += b;
    }
    friend Point operator-(Point a, const Point &b) {
        return a -= b;
    }
    friend Point operator*(Point a, const T &b) {
        return a *= b;
    }
    friend Point operator/(Point a, const T &b) {
        return a /= b;
    }
    friend Point operator*(const T &a, Point b) {
        return b *= a;
    }
    friend bool operator==(const Point &a, const Point &b) {
        return a.x == b.x && a.y == b.y;
    }
    friend istream &operator>>(istream &is, Point &p) {
        return is >> p.x >> p.y;
    }
    friend ostream &operator<<(ostream &os, const Point &p) {
        return os << "(" << p.x << ", " << p.y << ")";
    }
};
template<class T>
T dot(const Point<T> &a, const Point<T> &b) {
    return a.x * b.x + a.y * b.y;
}
template<class T>
T cross(const Point<T> &a, const Point<T> &b) {
    return a.x * b.y - a.y * b.x;
}
template<class T>
T square(const Point<T> &p) {
    return dot(p, p);
}
template<class T>
double length(const Point<T> &p) {
    return sqrt(double(square(p)));
}
template<class T>
Point<T> normalize(const Point<T> &p) {
    return p / length(p);
}
template<class T>
Point<T> rotate(const Point<T> &a) {
    return Point(-a.y, a.x);
}
template<class T>
int sgn(const Point<T> &a) {
    return a.y > 0 || (a.y == 0 && a.x > 0) ? 1 : -1;
}

template<class T>
struct Line {
    Point<T> a;
    Point<T> b;
    Line(const Point<T> &a_ = Point<T>()
         , const Point<T> &b_ = Point<T>()) : a(a_), b(b_) {}
};
template<class T>
double length(const Line<T> &l) {
    return length(l.a - l.b);
}
template<class T>
bool parallel(const Line<T> &l1, const Line<T> &l2) {
    return cross(l1.b - l1.a, l2.b - l2.a) == 0;
}
template<class T>
double distance(const Point<T> &a, const Point<T> &b) {
    return length(a - b);
```

```
}
template<class T>
double distancePL(const Point<T> &p, const Line<T> &l) {
    return abs(cross(l.a - l.b, l.a - p)) / length(l);
}
template<class T>
double distancePS(const Point<T> &p, const Line<T> &l) {
    if (dot(p - l.a, l.b - l.a) < 0)
        return distance(p, l.a);
    if (dot(p - l.b, l.a - l.b) < 0)
        return distance(p, l.b);
    return distancePL(p, l);
}
template<class T>
bool pointOnLineLeft(const Point<T> &p, const Line<T> &l) {
    return cross(l.b - l.a, p - l.a) > 0;
}
template<class T>
Point<T
    > lineIntersection(const Line<T> &l1, const Line<T> &l2) {
    return l1.a + (l1.b - l1.a) * (cross(l2.b -
        l2.a, l1.a - l2.a) / cross(l2.b - l2.a, l1.a - l1.b));
}
template<class T>
bool pointOnSegment(const Point<T> &p, const Line<T> &l) {
    return cross(p - l.a, l.b - l.a) == 0 &&
        min(l.a.x, l.b.x) <= p.x && p.x <= max(l.a.x, l.b.x)
        && min
            (l.a.y, l.b.y) <= p.y && p.y <= max(l.a.y, l.b.y);
}
template<class T>
bool pointInPolygon
    (const Point<T> &a, const vector<Point<T>> &p) {
    int n = p.size(), t = 0;
    for (int i = 0; i < n; i++)
        if (pointOnSegment(a, Line(p[i], p[(i + 1) % n])))
            return true;
    for (int i = 0; i < n; i++) {
        auto u = p[i];
        auto v = p[(i + 1) % n];
        if (u.x < a.
            x && v.x >= a.x && pointOnLineLeft(a, Line(v, u)))
            t ^= 1;
        if (u.x >= a
            .x && v.x < a.x && pointOnLineLeft(a, Line(u, v)))
            t ^= 1;
    }
    return t == 1;
}
// 0 : strictly outside
// 1 : on boundary
// 2 : strictly inside
template<class T>
int pointInConvexPolygon
    (const Point<T> &a, const vector<Point<T>> &p) {
    int n = p.size();
    if (n == 0) {
        return 0;
    } else if (n <= 2) {
        return pointOnSegment(a, Line(p[0], p.back()));
    }
    if (pointOnSegment(a, Line(p[0],
        p[1])) || pointOnSegment(a, Line(p[0], p[n - 1]))) {
        return 1;
    } else if (pointOnLineLeft(a, Line(p[1],
        p[0])) || pointOnLineLeft(a, Line(p[0], p[n - 1]))) {
        return 0;
    }
    int lo = 1, hi = n - 2;
    while (lo < hi) {
        int x = (lo + hi + 1) / 2;
        if (pointOnLineLeft(a, Line(p[0], p[x]))) {
            lo = x;
        } else {
            hi = x - 1;
        }
    }
    if (pointOnLineLeft(a, Line(p[lo], p[lo + 1]))) {
        return 2;
    } else {
        return pointOnSegment(a, Line(p[lo], p[lo + 1]));
    }
}
template<class T>
bool lineIntersectsPolygon
    (const Line<T> &l, const vector<Point<T>> &p) {
    int n = p.size();
    Point<T> a = l.a, b = l.b;
    for (int i = 0; i < n; i++) {
        Line<T> seg(p[i], p[(i + 1) % n]);
        if (cross(b - a
            , seg.a - a) == 0 || cross(b - a, seg.b - a) == 0)
            return true;
        if (cross(b
            - a, seg.a - a) > 0 ^ cross(b - a, seg.b - a) > 0)
            return true;
    }
    return false;
}
// 0 : not intersect
// 1 : strictly intersect
// 2 : overlap
// 3 : intersect at endpoint
template<class T>
tuple<int, Point<T>, Point<T>> segmentIntersection
    (const Line<T> &l1, const Line<T> &l2) {
    if (max(l1.a.x, l1.b.x) < min(l2.a.x, l2.b.x))
        return {0, Point<T>(), Point<T>()};
    if (min(l1.a.x, l1.b.x) > max(l2.a.x, l2.b.x))
        return {0, Point<T>(), Point<T>()};
    if (max(l1.a.y, l1.b.y) < min(l2.a.y, l2.b.y))
        return {0, Point<T>(), Point<T>()};
    if (min(l1.a.y, l1.b.y) > max(l2.a.y, l2.b.y))
        return {0, Point<T>(), Point<T>()};
    if (cross(l1.b - l1.a, l2.b - l2.a) == 0) {
        if (cross(l1.b - l1.a, l2.a - l1.a) != 0) {
            return {0, Point<T>(), Point<T>()};
        } else {
            auto maxx1 = max(l1.a.x, l1.b.x);
            auto minx1 = min(l1.a.x, l1.b.x);
            auto maxy1 = max(l1.a.y, l1.b.y);
            auto miny1 = min(l1.a.y, l1.b.y);
            auto maxx2 = max(l2.a.x, l2.b.x);
            auto minx2 = min(l2.a.x, l2.b.x);
            auto maxy2 = max(l2.a.y, l2.b.y);
            auto miny2 = min(l2.a.y, l2.b.y);
            Point<T> p1(max(minx1, minx2), max(miny1, miny2));
            Point<T> p2(min(maxx1, maxx2), min(maxy1, maxy2));
            if (!pointOnSegment(p1, l1))
                swap(p1.y, p2.y);
            if (p1 == p2) {
                return {3, p1, p2};
            } else {
                return {2, p1, p2};
            }
        }
    }
    auto cp1 = cross(l2.a - l1.a, l2.b - l1.a);
    auto cp2 = cross(l2.a - l1.b, l2.b - l1.b);
    auto cp3 = cross(l1.a - l2.a, l1.b - l2.a);
    auto cp4 = cross(l1.a - l2.b, l1.b - l2.b);
    if ((cp1 > 0 && cp2 > 0) || (cp1 < 0 && cp2
        < 0) || (cp3 > 0 && cp4 > 0) || (cp3 < 0 && cp4 < 0))
        return {0, Point<T>(), Point<T>()};
    Point p = lineIntersection(l1, l2);
    if (cp1 != 0 && cp2 != 0 && cp3 != 0 && cp4 != 0) {
        return {1, p, p};
    } else {
        return {3, p, p};
    }
}
template<class T>
double distanceSS(const Line<T> &l1, const Line<T> &l2) {
    if (get<0>(segmentIntersection(l1, l2)) != 0)
        return 0.0;
    return min({distancePS(l1.a, l2), distancePS(l1
        .b, l2), distancePS(l2.a, l1), distancePS(l2.b, l1)});
}
template<class T>
bool segmentInPolygon
    (const Line<T> &l, const vector<Point<T>> &p) {
    int n = p.size();
    if (!pointInPolygon(l.a, p)) return false;
    if (!pointInPolygon(l.b, p)) return false;
    for (int i = 0; i < n; i++) {
        auto u = p[i];
        auto v = p[(i + 1) % n];
        auto w = p[(i + 2) % n];
        auto [t, p1, p2] = segmentIntersection(l, Line(u, v));
        if (t == 1) return false;
        if (t == 0) continue;
        if (t == 2) {
            if (pointOnSegment(v, l) && v != l.a && v != l.b)
                if (cross(v - u, w - v) > 0)
                    return false;
            } else {
                if (p1 != u && p1 != v) {
                    if (pointOnLineLeft(l.a, Line(v, u))
                        || pointOnLineLeft(l.b, Line(v, u)))
                        return false;
                } else if (p1 == v) {
                    if (l.a == v) {
                        if (pointOnLineLeft(u, l)) {
                            if (pointOnLineLeft(w, l)
                                && pointOnLineLeft(w, Line(u, v)))
                                return false;
                        } else {
                            if (pointOnLineLeft(w, l)
                                || pointOnLineLeft(w, Line(u, v)))
                                return false;
                        }
                    } else if (l.b == v) {
                        if (pointOnLineLeft(u, Line(l.b, l.a))) {
                            if (pointOnLineLeft(w, Line(l.b, l.a))
                                && pointOnLineLeft(w, Line(u, v)))
                                return false;
                        } else {
                            if (pointOnLineLeft(w, Line(l.b, l.a))
                                || pointOnLineLeft(w, Line(u, v)))
                                return false;
                        }
                    } else {
                        if (pointOnLineLeft(u, l)) {
                            if (pointOnLineLeft(w, Line(l.b, l.a))
```

```
                            || pointOnLineLeft(w, Line(u, v)))
                            return false;
                    } else {
                        if (pointOnLineLeft(w, l)
                            || pointOnLineLeft(w, Line(u, v)))
                            return false;
                    }
                }
            }
        }
    }
    return true;
}
template<class T>
vector<Point<T>> convexHull(vector<Point<T>> a) {
    sort(a.begin()
         , a.end(), [](const Point<T> &l, const Point<T> &r) {
        return l.x == r.x ? l.y < r.y : l.x < r.x;
    });
    a.resize(unique(a.begin(), a.end()) - a.begin());
    if (a.size() <= 1) return a;
    vector<Point<T>> h(a.size() + 1);
    int s = 0, t = 0;
    for (int i = 0; i < 2; i++, s = --t) {
        for (Point<T> p : a) {
            while (t >= s + 2 && cross
                 (h[t - 1] - h[t - 2], p - h[t - 2]) <= 0) t--;
            h[t++] = p;
        }
        reverse(a.begin(), a.end());
    }
    return {h.begin(), h.begin() + t};
}
template<class T>
vector<Point<T>> hp(vector<Line<T>> lines) {
    sort(lines.begin(), lines.end(), [&](auto l1, auto l2) {
        auto d1 = l1.b - l1.a;
        auto d2 = l2.b - l2.a;
        if (sgn(d1) != sgn(d2))
            return sgn(d1) == 1;
        return cross(d1, d2) > 0;
    });
    deque<Line<T>> ls;
    deque<Point<T>> ps;
    for (auto l : lines) {
        if (ls.empty()) {
            ls.push_back(l);
            continue;
        }
        while (!ps.empty() && !pointOnLineLeft(ps.back(), l))
            ps.pop_back(), ls.pop_back();
        while (!ps.empty() && !pointOnLineLeft(ps[0], l))
            ps.pop_front(), ls.pop_front();
        if (cross(l.b - l.a, ls.back().b - ls.back().a) == 0) {
            if (dot
                 (l.b - l.a, ls.back().b - ls.back().a) > 0) {
                if (!pointOnLineLeft(ls.back().a, l)) {
                    assert(ls.size() == 1);
                    ls[0] = l;
                }
                continue;
            }
            return {};
        }
        ps.push_back(lineIntersection(ls.back(), l));
        ls.push_back(l);
    }
    while (!ps.empty() && !pointOnLineLeft(ps.back(), ls[0]))
        ps.pop_back(), ls.pop_back();
    if (ls.size() <= 2) return {};
    ps.push_back(lineIntersection(ls[0], ls.back()));
    return vector(ps.begin(), ps.end());
}
using P = Point<ll>;
```

## 10.2 Min Euclidean Distance [82650f]

```
void solve() {
    int n; cin >> n;
    constexpr ll inf = 8E18;
    vector<Point<ll>> a(n);
    for (int i = 0; i < n; i++) {
        ll x, y;
        cin >> x >> y;
        a[i] = Point<ll>(x, y);
    }
    struct sortY {
        bool operator
             ()(const Point<ll> &a, const Point<ll> &b) const {
            return a.y < b.y;
        }
    };
    struct sortXY {
        bool operator
             ()(const Point<ll> &a, const Point<ll> &b) const {
            return a.x == b.x ? a.y < b.y : a.x < b.x;
        }
    };
    sort(a.begin(), a.end(), sortXY());
    vector<Point<ll>> t(n);
    auto devide = [&](auto &&self, int l, int r) -> ll {
        if (l == r) return inf;
        int m = (l + r) / 2;
        ll ans = min(self(self, l, m), self(self, m + 1, r));
        ll midval = a[m].x;
        ll p = 0;
        for (int i = l; i <= r; i++)
            if ((midval - a[i].x) * (midval - a[i].x) <= ans)
                t[p++] = a[i];
        sort(t.begin(), t.begin() + p, sortY());
        for (int i = 0; i < p; i++) {
            for (int j = i + 1; j < p; j++) {
                ans = min(ans, square(t[i] - t[j]));
                if ((t[i].y -
                     t[j].y) * (t[i].y - t[j].y) > ans) break;
            }
        }
        return ans;
    };
    cout << devide(devide, 0, n - 1) << "\n";
}
```

## 10.3 Max Euclidean Distance [5abbe1]

```
template<class T>
tuple<T, int, int> mxdisPair(vector<Point<T>> a) {
    auto get = [&](const Point<T> &p, const Line<T> &l) -> T {
        return abs(cross(l.a - l.b, l.a - p));
    };
    T res = 0; int n = a.size(), x, y, id = 2;
    a.push_back(a.front());
    if (n <= 2) return {square(a[0] - a[1]), 0, 1};
    for (int i = 0; i < n; i++) {
        while (get(a[id], Line(a[i], a[i + 1])
             ) <= get(a[(id + 1) % n], Line(a[i], a[i + 1])))
            id = (id + 1) % n;
        if (res < square(a[i] - a[id])) {
            res = square(a[i] - a[id]);
            x = i, y = id;
        }
        if (res < square(a[i + 1] - a[id])) {
            res = square(a[i + 1] - a[id]);
            x = i + 1, y = id;
        }
    }
    return {res, x, y};
}
```

## 10.4 Lattice Points [b14b2b]

```
int main() {
    // Area 求法與 Polygun 內整數點數
    int n; cin >> n;
    vector<Point<ll>> polygon(n);
    for (int i = 0; i < n; i++) cin >> polygon[i];
    ll area = 0;
    for (int i = 0; i < n; i++)
        area += cross(polygon[i], polygon[(i + 1) % n]);
    area = abs(area);
    auto countBoundaryPoints
         = [](const vector<Point<ll>>& polygon) -> ll {
        ll res = 0;
        int n = polygon.size();
        for (int i = 0; i < n; i++) {
            ll dx = polygon[(i + 1) % n].x - polygon[i].x;
            ll dy = polygon[(i + 1) % n].y - polygon[i].y;
            res += std::gcd(abs(dx), abs(dy));
        }
        return res;
    };
    ll res = countBoundaryPoints(polygon);
    ll ans = (area - res + 2) / 2;
    cout << ans << " " << res << "\n";
}
```

## 10.5 Min Circle Cover [02619b]

```
template<class T>
pair<T, Point<T>> minCircle(vector<Point<T>> &a) {
    random_shuffle(a.begin(), a.end());
    int n = a.size();
    Point<T> c = a[0]; T r = 0;
    for (int i = 1; i < n; i++) {
        if (T(length(c - a[i]) - r) > 0.0) {
            c = a[i], r = 0;
            for (int j = 0; j < i; j++) {
                if (T(length(c - a[j]) - r) > 0.0) {
                    c = (a[i] + a[j]) / 2.0;
                    r = length(c - a[i]);
                    for (int k = 0; k < j; k++) {
                        if (T(length(c - a[k]) - r) > 0.0) {
                            Point<T> p = (a[j] + a[i]) / 2;
                            Point<T> q = (a[j] + a[k]) / 2;
                            if (cross(a[j] - a[i],
                                  a[k] - a[j]) == 0) continue;
                            c = lineIntersection(Line(p,
                                 p + rotate(a[j] - a[i])), Line
                                 (q, q + rotate(a[k] - a[j])));
                            r = length(c - a[i]);
                        }
                    }
                }
            }
        }
    }
```

```
    }
    return {r, c};
}
```

## 10.6 Min Rectangle Cover [fb3bca]

```
template<class T>
pair<T, vector<Point<T>>> minRectangle(vector<Point<T>> a) {
    if (a.size() <= 2) return {0, {}};
    auto get = [&](const Point<T> &p, const Line<T> &l) -> T {
        return abs(cross(l.a - l.b, l.a - p).x);
    };
    int n = a.size(), j = 2, l = 1, r = 1;
    a.push_back(a.front());
    D th, tw, area = numeric_limits<double>::infinity();
    vector<Point<T>> ans;
    for (int i = 0; i < n; i++) {
        while (get(a[j], Line(a[i], a[i +
             1])) <= get(a[(j + 1) % n], Line(a[i], a[i + 1])))
            j = (j + 1) % n;
        while (dot(a[i + 1] - a[i], a[r] - a[i])
            <= dot(a[i + 1] - a[i], a[(r + 1) % n] - a[i]))
            r = (r + 1) % n;
        if (i == 0) l = j;
        while (dot(a[i + 1] - a[i], a[l] - a[i])
            >= dot(a[i + 1] - a[i], a[(l + 1) % n] - a[i]))
            l = (l + 1) % n;
        D th = get(a[j], Line(a[i], a[i + 1]));
        D tw = dot(a[i] - a[i + 1],
              a[l] - a[i]) + dot(a[i + 1] - a[i], a[r] - a[i]);
        if (th * tw / square(a[i + 1] - a[i]) < area) {
            ans.clear
                 (), area = th * tw / square(a[i + 1] - a[i]);
            Line l1(a[i], a[i + 1]);
            for (auto p : {a[r], a[j], a[l], a[i]}) {
                Line l2 = Line(p, p + rotate(l1.a - l1.b));
                if (cross(l1.a - l1.b, p - l1.a) == 0) {
                    ans.push_back(p);
                    l1 = Line(p, p + rotate(l1.a - l1.b));
                } else {
                    Point<T> res = lineIntersection(l1, l2);
                    ans.push_back(res);
                    l1.a = res, l1.b = p;
                }
            }
        }
    }
    return {area, ans};
}
```

# 11 Polynomial

## 11.1 FFT [9172ce]

```
const double PI = acos(-1.0);
using cd = complex<double>;
vector<int> rev;
void fft(vector<cd> &a, bool inv) {
    int n = a.size();
    if (int(rev.size()) != n) {
        int k = __builtin_ctz(n) - 1;
        rev.resize(n);
        for (int i = 0; i < n; i++)
            rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
    }
    for (int i = 0; i < n; i++)
        if (rev[i] < i)
            swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2) {
        double ang = (inv ? -1 : 1) * PI / k;
        cd wn(cos(ang), sin(ang));
        for (int i = 0; i < n; i += 2 * k) {
            cd w(1);
            for (int j = 0; j < k; j++, w = w * wn) {
                cd u = a[i + j];
                cd v = a[i + j + k] * w;
                a[i + j] = u + v;
                a[i + j + k] = u - v;
            }
        }
    }
    if (inv) for (auto &x : a) x /= n;
}
template<class T>
vector<double> mulT(const vector<T> &a, const vector<T> &b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 2 << __lg(a.size() + b.size());
    fa.resize(n), fb.resize(n);
    fft(fa, false), fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] = fa[i] * fb[i];
    fft(fa, true);
    vector<double> res(n);
    for (int i = 0; i < n; i++)
        res[i] = fa[i].real();
    return res; // use llround if need
}
```

## 11.2 NTT [7decc9]

```
template<int V, ll P>
MInt<P> CInv = MInt<P>(V).inv();

vector<ll> rev;
template<ll P>
vector<MInt<P>> roots{0, 1};

template<int P>
MInt<P> findPrimitiveRoot() {
    MInt<P> i = 2;
    int k = __builtin_ctz(P - 1);
    while (true) {
        if (power(i, (P - 1) / 2) != 1) break;
        i += 1;
    }
    return power(i, (P - 1) >> k);
}

template<ll P>
MInt<P> primitiveRoot = findPrimitiveRoot<P>();
template<>
MInt<998244353> primitiveRoot<998244353> {31};

template<ll P>
void dft(vector<MInt<P>> &a) {
    int n = a.size();
    if (int(rev.size()) != n) {
        int k = __builtin_ctz(n) - 1;
        rev.resize(n);
        for (int i = 0; i < n; i++)
            rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
    }
    for (int i = 0; i < n; i++)
        if (rev[i] < i) swap(a[i], a[rev[i]]);
    if (roots<P>.size() < n) {
        int k = __builtin_ctz(roots<P>.size());
        roots<P>.resize(n);
        while ((1 << k) < n) {
            auto e = power(primitiveRoot
                 <P>, 1 << (__builtin_ctz(P - 1) - k - 1));
            for (int i = 1 << (k - 1); i < (1 << k); i++) {
                roots<P>[2 * i] = roots<P>[i];
                roots<P>[2 * i + 1] = roots<P>[i] * e;
            }
            k++;
        }
    }
    for (int k = 1; k < n; k *= 2) {
        for (int i = 0; i < n; i += 2 * k) {
            for (int j = 0; j < k; j++) {
                MInt<P> u = a[i + j];
                MInt<P> v = a[i + j + k] * roots<P>[k + j];
                a[i + j] = u + v;
                a[i + j + k] = u - v;
            }
        }
    }
}

template<ll P>
void idft(vector<MInt<P>> &a) {
    int n = a.size();
    reverse(a.begin() + 1, a.end());
    dft(a);
    MInt<P> inv = (1 - P) / n;
    for (int i = 0; i < n; i++) a[i] *= inv;
}

template<ll P = 998244353>
struct Poly : public vector<MInt<P>> {
    using Value = MInt<P>;
    Poly() : vector<Value>() {}
    explicit Poly(int n) : vector<Value>(n) {}
    explicit Poly(const vector<Value> &a) : vector<Value>(a) {}
    Poly(const
          initializer_list<Value> &a) : vector<Value>(a) {}
    template<class InputIt, class = _RequireInputIter<InputIt>>
    explicit Poly(InputIt
          first, InputIt last) : vector<Value>(first, last) {}
    template<class F>
    explicit Poly(int n, F f) : vector<Value>(n) {
        for (int i = 0; i < n; i++)
            (*this)[i] = f(i);
    }
    Poly shift(int k) const {
        if (k >= 0) {
            auto b = *this;
            b.insert(b.begin(), k, 0);
            return b;
        } else if (this->size() <= -k) {
            return Poly();
        } else {
            return Poly(this->begin() + (-k), this->end());
        }
    }
    Poly trunc(int k) const {
        Poly f = *this;
        f.resize(k);
        return f;
    }
    friend Poly operator+(const Poly &a, const Poly &b) {
        Poly res(max(a.size(), b.size()));
        for (int i = 0; i < a.size(); i++)
```

```
            res[i] += a[i];
        for (int i = 0; i < b.size(); i++)
            res[i] += b[i];
        return res;
    }
    friend Poly operator-(const Poly &a, const Poly &b) {
        Poly res(max(a.size(), b.size()));
        for (int i = 0; i < a.size(); i++)
            res[i] += a[i];
        for (int i = 0; i < b.size(); i++)
            res[i] -= b[i];
        return res;
    }
    friend Poly operator-(const Poly &a) {
        vector<Value> res(a.size());
        for (int i = 0; i < int(res.size()); i++)
            res[i] = -a[i];
        return Poly(res);
    }
    friend Poly operator*(Poly a, Poly b) {
        if (a.size() == 0 || b.size() == 0)
            return Poly();
        if (a.size() < b.size()) swap(a, b);
        int n = 1, tot = a.size() + b.size() - 1;
        while (n < tot) n *= 2;
        if (((P - 1) & (n - 1)) != 0 || b.size() < 128) {
            Poly c(a.size() + b.size() - 1);
            for (int i = 0; i < a.size(); i++)
                for (int j = 0; j < b.size(); j++)
                    c[i + j] += a[i] * b[j];
            return c;
        }
        a.resize(n), b.resize(n);
        dft(a), dft(b);
        for (int i = 0; i < n; ++i)
            a[i] *= b[i];
        idft(a);
        a.resize(tot);
        return a;
    }
    friend Poly operator*(Value a, Poly b) {
        for (int i = 0; i < int(b.size()); i++)
            b[i] *= a;
        return b;
    }
    friend Poly operator*(Poly a, Value b) {
        for (int i = 0; i < int(a.size()); i++)
            a[i] *= b;
        return a;
    }
    friend Poly operator/(Poly a, Value b) {
        for (int i = 0; i < int(a.size()); i++)
            a[i] /= b;
        return a;
    }
    Poly &operator+=(Poly b) {
        return (*this) = (*this) + b;
    }
    Poly &operator-=(Poly b) {
        return (*this) = (*this) - b;
    }
    Poly &operator*=(Poly b) {
        return (*this) = (*this) * b;
    }
    Poly &operator*=(Value b) {
        return (*this) = (*this) * b;
    }
    Poly &operator/=(Value b) {
        return (*this) = (*this) / b;
    }
    Poly deriv() const {
        if (this->empty()) return Poly();
        Poly res(this->size() - 1);
        for (int i = 0; i < this->size() - 1; ++i)
            res[i] = (i + 1) * (*this)[i + 1];
        return res;
    }
    Poly integr() const {
        Poly res(this->size() + 1);
        for (int i = 0; i < this->size(); ++i)
            res[i + 1] = (*this)[i] / (i + 1);
        return res;
    }
    Poly inv(int m) const {
        Poly x{(*this)[0].inv()};
        int k = 1;
        while (k < m) {
            k *= 2;
            x = (x * (Poly{2} - trunc(k) * x)).trunc(k);
        }
        return x.trunc(m);
    }
    Poly log(int m) const {
        return (deriv() * inv(m)).integr().trunc(m);
    }
    Poly exp(int m) const {
        Poly x{1};
        int k = 1;
        while (k < m) {
            k *= 2;
            x = (x * (Poly{1} - x.log(k) + trunc(k))).trunc(k);
        }
        return x.trunc(m);
    }
    Poly pow(int k, int m) const {
        int i = 0;
        while (i < this->size() && (*this)[i] == 0) i++;
        if (i == this->size() || 1LL * i * k >= m)
            return Poly(m);
        Value v = (*this)[i];
        auto f = shift(-i) * v.inv();
        return (f.log(m - i *
             k) * k).exp(m - i * k).shift(i * k) * power(v, k);
    }
    Poly sqrt(int m) const {
        Poly x{1};
        int k = 1;
        while (k < m) {
            k *= 2;
            x = (x +
                  (trunc(k) * x.inv(k)).trunc(k)) * CInv<2, P>;
        }
        return x.trunc(m);
    }
    Poly mulT(Poly b) const {
        if (b.size() == 0) return Poly();
        int n = b.size();
        reverse(b.begin(), b.end());
        return ((*this) * b).shift(-(n - 1));
    }
    vector<Value> eval(vector<Value> x) const {
        if (this->size() == 0)
            return vector<Value>(x.size(), 0);
        const int n = max(x.size(), this->size());
        vector<Poly> q(4 * n);
        vector<Value> ans(x.size());
        x.resize(n);
        function<void(
             int, int, int)> build = [&](int p, int l, int r) {
            if (r - l == 1) {
                q[p] = Poly{1, -x[l]};
            } else {
                int m = (l + r) / 2;
                build(2 * p, l, m);
                build(2 * p + 1, m, r);
                q[p] = q[2 * p] * q[2 * p + 1];
            }
        };
        build(1, 0, n);
        function<void(int, int, int, const Poly &)>
             work = [&](int p, int l, int r, const Poly &num) {
            if (r - l == 1) {
                if (l < int(ans.size()))
                    ans[l] = num[0];
            } else {
                int m = (l + r) / 2;
                work(2 * p, l,
                      m, num.mulT(q[2 * p + 1]).resize(m - l));
                work(2 * p + 1,
                      m, r, num.mulT(q[2 * p]).resize(r - m));
            }
        };
        work(1, 0, n, mulT(q[1].inv(n)));
        return ans;
    }
};

template<ll P = 998244353>
Poly<P> berlekampMassey(const Poly<P> &s) {
    Poly<P> c, oldC;
    int f = -1;
    for (int i = 0; i < s.size(); i++) {
        auto delta = s[i];
        for (int j = 1; j <= c.size(); j++)
            delta -= c[j - 1] * s[i - j];
        if (delta == 0) continue;
        if (f == -1) {
            c.resize(i + 1);
            f = i;
        } else {
            auto d = oldC;
            d *= -1;
            d.insert(d.begin(), 1);
            MInt<P> df1 = 0;
            for (int j = 1; j <= d.size(); j++)
                df1 += d[j - 1] * s[f + 1 - j];
            assert(df1 != 0);
            auto coef = delta / df1;
            d *= coef;
            Poly<P> zeros(i - f - 1);
            zeros.insert(zeros.end(), d.begin(), d.end());
            d = zeros;
            auto temp = c;
            c += d;
            if (i - temp.size() > f - oldC.size()) {
                oldC = temp;
                f = i;
            }
        }
    }
    c *= -1;
    c.insert(c.begin(), 1);
    return c;
}
```

```cpp
template<ll P = 998244353>
MInt<P> linearRecurrence(Poly<P> p, Poly<P> q, ll n) {
    int m = q.size() - 1;
    while (n > 0) {
        auto newq = q;
        for (int i = 1; i <= m; i += 2)
            newq[i] *= -1;
        auto newp = p * newq;
        newq = q * newq;
        for (int i = 0; i < m; i++)
            p[i] = newp[i * 2 + n % 2];
        for (int i = 0; i <= m; i++)
            q[i] = newq[i * 2];
        n /= 2;
    }
    return p[0] / q[0];
}
```

# 12 Else

## 12.1 Python [6f660a]

```python
from decimal import * # 無誤差浮點數
from fractions import * # 分數
from random import *
from math import *
# set decimal prec if it could overflow in precision
setcontext(Context(prec=10, rounding=ROUND_FLOOR))
# read and print
x = int(input())
a, b, c = list(map(Fraction, input().split()))
arr = list(map(Decimal, input().split()))
print(x)
print(a, b, c)
print(*arr)
# set
S = set(); S.add((a, b)); S.remove((a, b))
if not (a, b) in S:
# dict
D = dict(); D[(a, b)] = 1; del D[(a, b)]
for (a, b) in D.items():
# random
arr = [randint(l, r) for i in range(size)]
choice([8, 6, 4, 1]) # random pick one
shuffle(arr)
```

## 12.2 BigNumber [a73fbc]

```cpp
string Add(const string &a, const string &b) {
    int n = a.length() - 1, m = b.length() - 1, car = 0;
    string res;
    while (n >= 0 || m >= 0 || car) {
        int x = (n >= 0 ?
             a[n] - '0' : 0) + (m >= 0 ? b[m] - '0' : 0) + car;
        res += (x % 10) + '0', car = x / 10;
        n--, m--;
    }
    while (
         res.length() > 1 && res.back() == '0') res.pop_back();
    reverse(res.begin(), res.end());
    return res;
}
string Minus(const string &a, const string &b) {
    // Assume a >= b
    int n = a.length() - 1, m = b.length() - 1, bor = 0;
    string res;
    while (n >= 0) {
        int x = a[n] - '0' - bor, y = m >= 0 ? b[m] - '0' : 0;
        bor = 0;
        if (x < y) x += 10, bor = 1;
        res += x - y + '0';
        n--, m--;
    }
    while (
         res.length() > 1 && res.back() == '0') res.pop_back();
    reverse(res.begin(), res.end());
    return res;
}
string Multiple(const string &a, const string &b) {
    string res = "0";
    int n = a.length() - 1, m = b.length() - 1;
    for (int i = m; i >= 0; i--) {
        string add; int car = 0;
        for (int j = n; j >= 0 || car; j--) {
            int x = (
                 j >= 0 ? a[j] - '0' : 0) * (b[i] - '0') + car;
            add += (x % 10) + '0', car = x / 10;
        }
        while (add.
             length() > 1 && add.back() == '0') add.pop_back();
        reverse(add.begin(), add.end());
        res = Add(res, add + string(m - i, '0'));
    }
    return res;
}
```

## 12.3 Fraction [3f8970]

```cpp
template<class T>
struct Fraction {
    T n, d;
    void reduce() {
        T g = gcd(abs(n), abs(d));
        n /= g, d /= g;
        if (d < 0) n = -n, d = -d;
    }
    Fraction(T n_ = 0, T d_ = 1) : n(n_), d(d_) {
        assert(d != 0);
        reduce();
    }
    Fraction(const string &str) {
        istringstream ss(str);
        char slash;
        if (str.find('/') != -1) {
            ss >> n >> slash >> d;
        } else {
            ss >> n;
            d = 1;
        }
        Fraction(n, d);
    }
    Fraction operator+=(Fraction rhs) & {
        n = n * rhs.d + rhs.n * d;
        d *= rhs.d;
        reduce();
        return *this;
    }
    Fraction operator-=(Fraction rhs) & {
        n = n * rhs.d - rhs.n * d;
        d *= rhs.d;
        reduce();
        return *this;
    }
    Fraction operator*=(Fraction rhs) & {
        n *= rhs.n;
        d *= rhs.d;
        reduce();
        return *this;
    }
    Fraction operator/=(Fraction rhs) & {
        assert(rhs.n != 0);
        n *= rhs.d;
        d *= rhs.n;
        reduce();
        return *this;
    }
    friend Fraction operator+(Fraction lhs, Fraction rhs) {
        return lhs += rhs;
    }
    friend Fraction operator-(Fraction lhs, Fraction rhs) {
        return lhs -= rhs;
    }
    friend Fraction operator*(Fraction lhs, Fraction rhs) {
        return lhs *= rhs;
    }
    friend Fraction operator/(Fraction lhs, Fraction rhs) {
        return lhs /= rhs;
    }
    friend istream &operator>>(istream &is, Fraction &f) {
        string s;
        is >> s;
        f = Fraction(s);
        return is;
    }
    friend
         ostream &operator<<(ostream &os, const Fraction &f) {
        if (f.d == 1) {
            os << f.n;
        } else {
            os << f.n << "/" << f.d;
        }
        return os;
    }
    friend bool operator==(Fraction lhs, Fraction rhs) {
        return lhs.n * rhs.d == rhs.n * lhs.d;
    }
    friend bool operator!=(Fraction lhs, Fraction rhs) {
        return lhs.n * rhs.d != rhs.n * lhs.d;
    }
    friend bool operator<(Fraction lhs, Fraction rhs) {
        return lhs.n * rhs.d < rhs.n * lhs.d;
    }
};
```

## 12.4 Gaussian Elimination [76d62d]

```cpp
// 找反矩陣
    就開 2n，右邊放單位矩陣，做完檢查左半是不是單位，回傳右半
// 0  : no solution
// -1 : infinity solution
// 1  : one solution
template<class T>
tuple<T,
     int, vector<T>> gaussianElimination(vector<vector<T>> a) {
    int n = a.size(), m = a[0].size(), rk = 0, sgn = 1;
    T det = 1;
    bool zero_det = false;
    for (int c = 0; c < n; c++) {
        int p = -1;
        for (int r = rk; r < n; r++) {
            if (a[r][c] != 0) {
                p = r;
```

```
                break;
            }
        }
        if (p == -1) {
            zero_det = true;
            continue;
        }
        if (p != rk) swap(a[rk], a[p]), sgn *= -1;
        det *= a[rk][c];
        T inv = 1 / a[rk][c];
        for (int j = c; j < m; j++) a[rk][j] *= inv;
        for (int r = 0; r < n; r++) {
            if (r == rk || a[r][c] == 0) continue;
            T fac = a[r][c];
            for (int j = c; j < m; j++)
                a[r][j] -= fac * a[rk][j];
        }
        rk++;
    }
    det = (zero_det ? 0 : det * sgn);
    for (int r = rk; r < n; r++)
        if (a[r][m - 1] != 0) return {det, 0, {}};
    if (rk < n) return {det, -1, {}};
    vector<T> ans(n);
    for (int i = 0; i < n; i++) ans[i] = a[i][m - 1];
    return {det, 1, ans};
}
template<class T>
tuple<int, vector
     <T>, vector<vector<T>>> findBasis(vector<vector<T>> a) {
    int n = a.size(), m = a[0].size(), rk = 0;
    vector<int> pos(m - 1, -1);
    for (int c = 0; c < m - 1; c++) {
        int p = -1;
        for (int r = rk; r < n; r++) {
            if (a[r][c] != 0) {
                p = r;
                break;
            }
        }
        if (p == -1) continue;
        if (p != rk) swap(a[rk], a[p]);
        pos[c] = rk;
        T inv = 1 / a[rk][c];
        for (int j = c; j < m; j++) a[rk][j] *= inv;
        for (int r = 0; r < n; r++) {
            if (r == rk || a[r][c] == 0) continue;
            T fac = a[r][c];
            for (int j = c; j < m; j++)
                a[r][j] -= fac * a[rk][j];
        }
        rk++;
    }
    vector<T> sol(m - 1);
    vector<vector<T>> basis;
    for (int r = rk; r < n; r++)
        if (a[r][m - 1] != 0)
            return {-1, sol, basis};
    for (int c = 0; c < m - 1; c++)
        if (pos[c] != -1)
            sol[c] = a[pos[c]][m - 1];
    for (int c = 0; c < m - 1; c++)
        if (pos[c] == -1)
            vector<T> v(m - 1);
            v[c] = 1;
            for (int j = 0; j < m - 1; j++)
                if (pos[j] != -1)
                    v[j] = -a[pos[j]][c];
            basis.push_back(v);
    return {rk, sol, basis};
}
template<class T>
using Matrix = vector<vector<T>>;
```