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Basic

1.1 install vscode [d41d8c]

```
// 如何安裝 vscode
// 1. 下載 vscode & msys2
// 2. 在跳出的 terminal 中 / 或打開 ucrt64,打上
"pacman -S --needed base-devel mingw-w64-x86_64-toolchain"
// 3. 環境變數加上 C:\\msys64\\ucrt64\\bin
// 4. 重開 vscode, 載 C/C++, 運行, 編譯器選擇 g++
// 5. 打開 settings -> compiler -> add compilerPath
      -> 在 "" 裡打上 C:\\msys64\\ucrt64\\bin\\g++.exe
```

1.2 default code [bee7dd]

```
#include <bits/stdc++.h>
#define all(x) (x).begin(), (x).end()
#define pii pair <int, int>
#define endl "\n"
#define int long long
using namespace std;
const int llinf = 4e18;
const int inf = 2e9;
const int mod = 1e9 + 7;
const int maxn = 2e5 + 5;
void solve() {
}
signed main() {
   ios_base::sync_with_stdio(0);
     cin.tie(nullptr);
     int t = 1;
     cin >> t;
while (t--) {
          solve();
}
```

1.3 compare fuction [4bc3e0]

```
// 在有 template 的資結使用
    bool operator()(const int &a, const int &b) const {
    return a < b;</pre>
 // sort, bound 不用 struct
// priority queue 小到大是 > , set 是 <
// set 不能 = , multiset 要 =
 // 每個元素都要比到,不然會不見
// pbds_multiset 不要用 lower_bound
// 如果要 find, 插入 inf 後使用 upper_bound
// 內建 multiset 可以跟 set 一樣正常使用
```

```
// 如果有自定義比較結構就比照以上
};
struct cmp {
   vector < int > &v;
                  cmp(vector<int>& vec) : v(vec) {}
                  bool operator() (int a, int b) const {
    return v[a] > v[b];
// main: cmp cmp1(vector);
// priority_queue<int, vector<int>, cmp> pq(cmp1);
};
 1.4 pbds [e28ae8]
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template < typename T >
using pbds_set = tree < T, null_type,</pre>
 less<T>, rb_tree_tag, tree_order_statistics_node_update>;
template<typename T>
 2
                       Graph
2.1 DFS 跟 BFS [cdd1d5]
 int main() {
                  int n;
                  vector < vector < int >> adj(n + 1, vector < int >());
                  vector (vector vector ve
                                                 self(self, v);
                                }
                  dfs(dfs, 1);
                  // bfs
                  vector < int > depth(n + 1, 1e9);
                  queue<int> q;
auto bfs = [&](auto self, int u) -> void {
   vis[u] = true;
                                   depth[u] = 0;
                                   q.push(u);
                                  q.pusn(u);
while (!q.empty()) {
   int u = q.front(); q.pop();
   for (auto v : adj[u]) {
      if (vis[v]) continue;
      vis[v] = true;
      depth[v] = depth[u] + 1;
      a push(v);
```

2.2 Dijkstra [4e0023]

}

}

}

bfs(bfs, 1);

q.push(v):

```
Flight Discount
int main() {
     int n, m; cin >> n >> m;
vector<vector<pair<</pre>
            int, int>>> adj(n + 1, vector<pair<int, int>>(n + 1));
     vector < vector < int >>
     dis(n + 1, vector<int>(2, 2e9));  // 0 for not used
for (int i = 1; i <= m; i++) {
  int u, v, w;</pre>
           cin >> u >> v >> w
           adj[u].push_back(\{v, w\});
     distrif[0] = distrif[1] = 0,
pq.push({0, 1, 0});
while (!pq.empty()) {
    auto [dist, u, us] = pq.top(); pq.pop();
    if (dis[u][us] < dist) continue;</pre>
          }
                 for (auto [v, w] : adj[u]) {
    if (dis[u][0] + w < dis[v][0]) {
        dis[v][0] = dis[u][0] + w;
}</pre>
                             pq.push({dis[v][0], v, 0});
                       fif (dis[u][0] + w / 2 < dis[v][1])
    dis[v][1] = dis[u][0] + w / 2;
    pq.push({dis[v][1], v, 1});</pre>
                                                        < dis[v][1]) {
```

```
}
     cout << min(dis[n][0], dis[n][1]);
2.3 Prim [e54eda]
int n, m, ans = 0;
const int maxn = 2e5 + 5;
vector<pair<int, int>> adj[maxn];
bool Prim() {
     int node_sz = 0;
     priority_queue<pit, vector<pit>, greater<pit>> pq;
pq.push({0, 1});
     while (!pq.empty()) {
   auto [cost, u] = pq.top(); pq.pop();
   if (vis[u]) continue;
   vis[u] = true;
   ans += cost;
   node syate
     bool vis[maxn] = {false};
           node_sz++;
for(auto [v, cost] : adj[u]) {
    if (!vis[v])
                     pq.push({cost, v});
          }
     if (node_sz == n) return true;
     return false;
void solve() {
```

2.4 正權找環 [0e0fdf]

if (Prim()) cout << ans;
else cout << "IMPOSSIBLE";</pre>

adj[v].push_back({u, cost});

```
const int maxn = 1e5+5;
vector<int> graph[maxn];
int color[maxn], parent[maxn];
bool vis[maxn];
int n, m;
void print_ans(int ori) {
  int now = parent[ori];
  deque<int> ans;
  ans.push_front(ori);
      while (now != ori) {
    ans.push_front(now);
    now = parent[now];
      ans.push_front(ori);
      cout << ans.size() << endl;
for (auto i : ans) {
    cout << i << " ";</pre>
      exit(0);
void dfs(int now) {
      color[now] = 1;
      vis[now] = 1;
      for (auto nxt : graph[now]) {
   parent[nxt] = now;
   if (color[nxt] == 1) {
                   print_ans(nxt);
             else if (color[nxt] == 0) {
                   dfs(nxt);
            }
      color[now] = 2;
void solve() {
      cin >> n >> m;
for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
            graph[u].push_back(v);
      for (int i = 1; i <= n; i++) {
    if (!vis[i])</pre>
                   dfs(i);
      cout << "IMPOSSIBLE";</pre>
```

2.5 BellmanFord [02f480]

```
// 用 Bellman Ford 找負環
vector<array<int, 3>> graph;
                       // u, v, w
int main() {
   int src = 0;
  graph.push_back({a, b, w});
```

```
dis[1] = 0;
for (int i = 0; i <= n; i++) {
          src = 0:
           for (auto [u, v, w] : graph) {
                if (dis[v] > dis[u] + w) {
    dis[v] = dis[u] + w;
    par[v] = u;
                }
          }
           src) { // 到第 n + 1 次還在鬆弛
vector<int> ans;
     if (src) {
          cout << "YES" << endl;
for (int</pre>
          i = 0; i <= n; i++) src = par[src]; // 找那個負環ans.push_back(src);
          for (int
                 i = par[src]; i != src; i = par[i]) { // 輸出負環
                ans.push_back(i);
          ans.push_back(src);
reverse(ans.begin(), ans.end());
          for (auto i : ans) {
   cout << i << " ";</pre>
     else {
           cout << "NO" << "\n";
}
```

2.6 正權最大距離 [454dba]

```
// CSES Longest Flight Route
 // 只能用在 DAG,用拓樸按順序鬆弛
void print_ans(int n, vector<int> &par) {
        deque < int > ans;
       int now = n;
while(now != 1) {
             ans.push_front(now);
             now = par[now];
        ans.push front(1):
        cout << ans.size() << "\n";
for(auto i : ans) {
    cout << i << " ";</pre>
 int main() {
       main() {
int n, m; cin >> n >> m;
vector<vector<int>> graph(n + 1);
vector<int> dis(n + 1, -1e9); dis[1] = 0;
vector<int> par(n + 1), in(n + 1);
        queue<int> q;
        for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
             graph[u].push_back(v);
        for (int i = 1; i <= n; i++) {
    if (in[i] == 0) q.push(i);</pre>
       while (!q.empty()) {
   int u = q.front(); q.pop();
              for (auto v : graph[u]) {
                    if (dis[v] < dis[u] + 1) { // 鬆弛
dis[v] = dis[u] + 1;
par[v] = u;
                    in[v]-
                    if (in[v] == 0) q.push(v);
        if (dis[n] == -1e9) {
    // 如果 1 不能到達 n,n 也有可能被鬆弛
              // 所以要看的是 dis[n] < 0
              cout << "IMPOSSIBLE";</pre>
        else print_ans(n, par);
```

2.7 負權最大距離 [2148ca]

```
// CSES High Score
void dfs(int u, vector<int> &vis, vector<vector<int>> &adj) {
   if (vis[u]) return;
       vis[u] = 1;
for (int v : adj[u]) {
    dfs(v, vis, adj);
signed main() {
  int n, m; cin >> n >> m;
  vector<array<int, 3>> edges;
       vector < vector < int >> adj(n + 1);
vector < int > dis(n + 1), vis(n + 1);
while (m--) {
              int u, v, w;
```

2.8 FloydWarshall [206b76]

```
| const int inf = 1e18;
| int main() {
| int n, m, q; cin >> n >> m >> q;
| vector <vector <int>>> graph(n + 1, vector <int>(n + 1, inf));
| vector <vector <int>>> dis(n + 1, vector <int>(n + 1));
| for (int i = 0; i < m; i++) {
| int u, v, w; cin >> u >> v >> w;
| cin >> u >> v >> w;
| graph[u][v] = min(graph[u][v], w);
| graph[v][u] = min(graph[v][u], w);
| }
| for (int i = 0; i <= n; i++) {
| for(int j = 0; j <= n; j++) {
| dis[i][j] = graph[i][j];
| }
| }
| for (int k = 1; k <= n; k++) {
| for (int i = 1; i <= n; i++) {
| dis[i][j] = n;
| for (int i = 1; i <= n; i++) {
| dis[i][j] | j <= n; j++) {
| cout <= (dis[u][v] >= inf ? -1 : dis[u][v]) << "\n";
| }
| }
```

2.9 **歐拉環與歐拉路** [0911ed]

```
| // 無向圖、尤拉環:檢查每個點的出度為偶數
 // 有向圖、
       尤拉路: 可以看成 1 走到 n, 所以檢查所有點的出度等於入度
 int n, m;
const int maxn = 1e5 + 5;
 vector<set<int>> adj;
 vector<int> in;
 void dfs(int now, vector<int> &road) {
       while (!adj[now].empty()) {
  int nxt = *adj[now].begin();
  adj[now].erase(nxt);
             dfs(nxt, road);
       road.push_back(now);
 void solve() {
       cin >> n >> m;
       in.assign(n + 1, 0);
adj.assign(n + 1, set<int>());
for (int i = 1; i <= m; i++) {</pre>
             int u, v; cin >> u >> v;
             adj[u].insert(v);
             in[v]++;
       in[1]++;
       in[n]--;
       for (int i = 1; i <= n; i++) {
    if(adj[i].size() != in[i]) {
        cout << "IMPOSSIBLE";</pre>
                  return;
            }
       vector <int>
dfs(1, road);
if (road.size() != m + 1) {
    cout << "IMPOSSIBLE";</pre>
       reverse(road.begin(), road.end());
for(auto i : road) cout << i << " ";</pre>
 }
```

2.10 Kosaraju 與拓模 DP [8036c2]

```
// 找到所有 SCC 然後結合原圖重建一個 DAG, 然後拓樸 DP
if (!vis[u]) {
                      vis[u] = 1;
                      for (auto v : adi[u]) {
                                  dfs(v, vis, kosaraju, adj);
                       kosaraju.push_back(u); // finish time 小到大排列
          }
void rev_dfs(int u, vector<int> &vis, vector<</pre>
             int> &order, vector<vector<int>> &rev_adj, int &scc_num) {
lf (!vis[u]) {
           if (!vis[u])
                      vis[u] = 1;
                      vts[u] = 'scc_num;
for (auto v : rev_adj[u]) {
    rev_dfs(v, vis, order, rev_adj, scc_num);
           }
signed main() {
           int n, m, scc_num = 0;
cin >> n >> m;
vector<int> coin(n + 1), order(n + 1), vis(n + 1, 0);
           vector<vector<int>> adj(n + 1), rev_adj(n + 1);
vector<int> kosaraju;
for (int i = 1; i <= n; i++) {</pre>
                      cin >> coin[i];
           for (int i = 1; i <= m; i++) {
   int u, v; cin >> u >> v;
   adj[u].push_back(v);
   rev_adj[v].push_back(u);
}
           for (int i = 1; i <= n; i++) {
    if (!vis[i]) {</pre>
                                  dfs(i, vis, kosaraju, adj);
           reverse(kosaraju.begin(), kosaraju
           .end()); // 轉過來,從 finish time 大的開始做 dfs
vis.assign(n + 1, 0);
for (auto &u: kosaraju) {
                      if (!vis[u]) {
                                  scc_num++;
rev_dfs(u, vis, order, rev_adj, scc_num);
                     }
           }
            // 重新建 DAG,根據原圖,如果不再同個 SCC,對 order 加邊
           アルマン (A TAN A TA
            vector<int
                       > sum_coin(scc_num + 1, 0), dp_coin(scc_num + 1, 0);
           set<pair<int, int>> st;
           int ans = -1e9;
for (int i = 1; i <= n; i++) {
    sum_coin[order[i]] += coin[i];</pre>
                       for (auto j : adj[i]) {
                                   // 如果不是在同一個 SCC 且 order 邊還沒加過
if (order[i] != order[j] &&
                                              st.find({order[i], order[j]}) == st.end()) {
DAG[order[i]].push_back(order[j]);
in_degree[order[j]]++;
                                              st.insert({order[i], order[j]});
                                 }
                     }
           // 對 DAG 拓蹼 DP
           queue<int> q;
for (int i = 1; i <= scc_num; i++) {
    if (in_degree[i] == 0) {</pre>
                                  q.push(i);
                      }
           while (!q.empty()) {
                      int now = q.front(); q.pop();
dp_coin[now] += sum_coin[now];
ans = max(ans, dp_coin[now]);
for (auto v : DAG[now]) {
                                  in_degree[v]--;
dp_coin[v] = max(dp_coin[v], dp_coin[now]);
if (in_degree[v] == 0) q.push(v);
                      }
           cout << ans:
```

2.11 Tarjan 與 2-SAT [eeddc1]

```
// CSES Giant Pizza
struct TwoSat {
    int n;
    vector<vector<int>> e;
    vector<bool> ans;
    TwoSat(int n) : n(n), e(2 * n), ans(n) {}
    void addClause(int u, bool f, int v, bool g) {
        e[2 * u + !f].push_back(2 * v + g);
        e[2 * v + !g].push_back(2 * u + f);
}
```

```
bool satisfiable() {
            \verb|vector<| \textbf{int}|
             > id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1);
vector<int> stk;
            int now = 0, cnt = 0;
function < void(int) > tarjan = [&](int u) {
                  stk.push_back(u);
dfn[u] = low[u] = now++;
for (auto v : e[u]) {
    if (dfn[v] == -1) {
                               tarjan(v);
                         low[u] = min(low[u], low[v]);

} else if (id[v] == -1) {
                               .se if (id[v] == -1) {
low[u] = min(low[u], dfn[v]);
                   if (dfn[u] == low[u]) {
                         do {
                               v = stk.back();
                        stk.pop_back();
id[v] = cnt;
} while (v != u);
                  }
            };
for (int i
            return true:
      vector < bool > answer() { return ans; }
};
int main() {
      int m, n; cin >> m >> n;
TwoSat ts(n);
for (int i = 0; i < m; ++i) {</pre>
            int u, v; char x, y;
cin >> x >> u >> y >> v;
ts.addClause(u - 1, x == '+', v - 1, y == '+');
      if (ts.satisfiable()) {
    for (int i = 0; i < n; ++i) {
        cout << (ts.answer()[i] ? '+' : '-') << " ";</pre>
      else cout << "IMPOSSIBLE\n";</pre>
```

2.12 Planets Cycles [71ac0e]

```
vector<int> dis, v;
vector<bool> vis;
int step;
queue < int > path;
void dfs(int x) {
      path.push(x);
      if (vis[x]) {
    step += dis[x];
            return;
      vis[x] = true;
      step++:
      dfs(v[x]);
// count path_dis to rep
int main() {
   int n; cin >> n;
     v.assign(n + 1, 0);
dis.assign(n + 1, 0);
vis.assign(n + 1, false);
for (int i = 1; i <= n; i++) {
    cin >> v[i];
      for (int i = 1; i <= n; i++) {
            step = 0;
            int is_outof_cycle = 1;
           dis[path.front()] = step;
step -= is_outof_cycle;
                  path.pop();
      for (int i = 1; i <= n; i++) {
    cout << dis[i] << ' ';</pre>
      cout << '\n';
}
```

2.13 Planet Queries II [872f72]

```
| // 在有向圖中,從 A 到 B 的最短距離
// 保證出度是 1 所以對 1 個點來說,從他出發只可能遇到一個環 int n, q;
```

```
int dp[200005][30]:
                              // 倍增表
vector<vector<int>> cycles;
vector<int
>> no, cycle_idx, vis; // Order & Can be in cycle, or out
void set_out_of_cycle_no(int now, unordered_set<int> &done) {
     // 把不在環內的也編號, v 是 u 的編號 -1 if (done.find(now)!= done.end()) return;
     set_out_of_cycle_no(dp[now][0], done);
done.insert(now); // post order
     no[now] = no[dp[now][0]] - 1;
return u;
void find_cycle(int now) {
    unordered_set<int> appear;
     vector<int> v;
     bool flag = true; // 代表有環
while (appear.find(now) == appear.end()) {
           appear.insert(now);
v.push_back(now);
           if (vis[now]) {
                 flag = false;
                 break:
           now = dp[now][0];
     for (auto i : v) vis[i] = true;
if (!flag) return;
      // now 是環的起點,我們先找到他在 v 的哪裡
     int z = find(v.begin(), v.end(), now) - v.begin();
vector <int> cycle(v.begin() + z, v.end());
     cycles.push_back(cycle);
int main() {
    cin >> n >> q;
    no.assign(n + 1,
     no.assign(n + 1, -1);
cycle_idx.assign(n + 1, -1);
     for (int u = 1; u <= n; u++) cin >> dp[u][0];
     for (int i = 1; i <= 18; i++) // 倍增表
for (int u = 1; u <= n; u++)
     dp[u][i] = dp[dp[u][i - 1]][i - 1];
for (int i = 1; i <= n; i++) {
   if (!vis[i]) find_cycle(i);</pre>
     int idx = 0;
     unordered_set < int > done;
for (auto &i : cycles) {
           int c = 0;
           for (auto &j : i) {
    no[j] = c++;
    cycle_idx[j] = idx;
                 done.insert(j);
     for (int i = 1; i <= n; i++) set_out_of_cycle_no(i, done);
for (int i = 1; i <= q; i++) {</pre>
           int u, v; cin >> u >> v;
           // 在同個環內
           if (cycle_idx[u] == cycle_idx
    [v] && cycle_idx[u] != -1 && cycle_idx[v] != -1) {
    int cyc_size = cycles[cycle_idx[u]].size();
                       (no[v] - no[u] + cyc_size) % cyc_size << "\n";
           // 都不再環內
           continue;
                if (wiint_go_to(u, no[v] - no[u]) == v) {
    cout << no[v] - no[u] << "\n";</pre>
                 else cout << -1 << "\n":
           else if (cycle_idx[u]
                == -1 && cycle_idx[v] != -1) { // v 在環內,二分搜 int l = -1, r = n; while (l <= r) { int m = (l + r) / 2;
                      if (l <= n) {
                                      // 如果 n 步內可以到
                      int in_cycle_of_u = wiint_go_to(u, l);
                      int cycle_size = cycles[cycle_idx[v]].size();
cout << l + (no[v] - no[in_cycle_of_u
] + cycle_size) % cycle_size << "|n";</pre>
                 else cout << -1 << "\n";
```

```
}
else { // u 在環內 b 不在,直接不可能
cout << -1 << "\n";
}
}
```

3 Data Structure

3.1 BIT [d41d8c]

```
struct BIT {
                      // BIT 都是 1-based 的查詢
      vector < int > bit;
     BIT(int n) { // 有幾個數
this->n = n;
            bit.resize(n + 1, 0);
            vector<int> &init) { // 必須是 1-based this->n = init.size() - 1;
      BIT(vector<int> &init) {
            bit.resize(n + 1, 0);
for (int i = 1; i <= n; i++) {</pre>
                 modify(i, init[i]);
     void modify(int i, int val) {
    for (; i <= n; i += i & -i) {
        bit[i] += val;
}</pre>
     int query(int r) {
   int ans = 0;
   for (; r; r -= r & -r) ans += bit[r];
            return ans;
      int query(int l, int r) {
            return query(r) - query(l - 1);
     }
struct TwoDimensionBIT {
     int nx, ny;
vector<vector<int>> bit;
      TwoDimensionBIT(int x, int y) {
           nx = x; ny = y;
bit.resize(x + 1, vector<int>(y + 1, 0));
      void modify(int x, int y, int mod) {
    for (; x <= nx; x += x & -x) {
        for (int tmp = y; tmp <= ny; tmp += tmp & -tmp) {</pre>
                       bit[x][tmp] += mod;
           }
      int query(int r1, int r2) {
            int ans = 0;
            for (; r1; r1 -= r1 & -r1) {
    for (int tmp = r2; tmp; tmp -= tmp & -tmp) {
        ans += bit[r1][tmp];
}
            return ans:
```

3.2 DSU [d41d8c]

3.3 Increasing Array Queries [d41d8c]

```
const int maxn = 2e5+5;
int n, q;
```

```
int nums
[maxn], prefix[maxn], ans[maxn], BIT[maxn], contrib[maxn];
vector<pair<int, int>> queries[maxn];
void update(int pos, int val) {
   for (; pos <= n; pos += pos & -pos) BIT[pos] += val;</pre>
int query(int a, int b) {
      int ans = 0;
for (; b; b -= b&-b) ans += BIT[b]
      for (a--; a; a -= a&-a) ans -= BIT[a];
      return ans:
void solve() {
     cin >> n >> q;
for (int i = 1; i <= n; i++) {</pre>
           cin >> nums[i];
           prefix[i] = prefix[i-1] + nums[i];
     fums[n + 1] = 1e9;
prefix[n + 1] = 2e18;
for (int i = 1; i <= q; i++) {
   int a, b; cin >> a >> b;
           queries[a].push_back({b, i});
      deque<int> mono; mono.push_front(n+1);
     contrib[i] = (mono.front() - 1 - i) *
    nums[i] - (prefix[mono.front() - 1] - prefix[i]);
update(i, contrib[i]);
           mono.push_front(i);
           for (auto j : queries
   [i]) { // pos is the index in mono <= end's
   int pos = upper_bound(mono.begin</pre>
                - mono[pos]) * nums[mono[pos]]
                                     - (prefix
                                           [j.first] - prefix[mono[pos]]);
          }
      for (int i = 1; i <= q; i++) {
    cout << ans[i] << endl;</pre>
```

3.4 線段樹 [d41d8c]

```
template <class Node>
struct Seg {
     int n;
     vector<Node> tree;
Seg (vector<Node> init_) {
         n = init_.size() -
tree.resize(4 * n);
          function < void(int
              , int, int)> build = [&](int now, int l, int r) {
if (l == r) {
    tree[now] = init_[l];
              int m = (l + r) / 2;
build(now << 1, l, m);</pre>
              build((now << 1) + 1, m + 1, r);
              pull(now);
         build(1, 1, n);
    Node query(int l, int r, int ql, int qr, int now) {
  int m = (l + r) >> 1;
  if (qr < l || ql > r) {
              return Node():
         if (ql <= l && r <= qr) {
              return tree[now];
         Node query(int l, int r) { return query(1, n, l, r, 1); }
     void pull(int now) {
         tree[now] = tree[now << 1] + tree[(now << 1) + 1];</pre>
     void modify(int l, int r, int idx, int now, int add) {
if (l == r) {
// ---how to modify ?---
             tree[now].sum = add;
             return;
         int m = (l + r) >> 1;
         if (idx <= m) {
              modify(l, m, idx, now << 1, add);
```

```
modify(m + 1, r, idx, (now << 1) + 1, add);
       pull(now);
    void modify
        (int idx, int add) { modify(1, n, idx, 1, add); }
  ---define structure and info plus---
struct Node {
   int sum;
   Node () {
sum = 0;
Node operator + (const Node &a, const Node &b) {
   Node c;
c.sum = a.sum + b.sum;
   // use lc > rc to undate now
}
// ---pizza_queries---
// 左邊的店(s < t): dis_l = (pizza[s] - s) + t;
// 右邊的店(t < s): dis_r = (pizza[s] + s) - t;
// 實作: 建左查詢線段樹跟右查詢線段樹, 用最小值pull
// 答案是 min(left_query(1, s) + t, right_query(s, end) + t);
// ---List Removals---
// 維護區間內有幾個數字被選過
// 用二分
    搜找右區間最小位,使得 ans - query == 1 ~ ans 被選過的數量
  ---CSES subarray queries:---
// tree[now].prefix
     = max(tree[lc].sum + tree[rc].prefix, tree[lc].prefix);
// tree[now].suffix
     = max(tree[lc].suffix+tree[rc].sum, tree[rc].suffix);
// tree[now].middle_max
     = max(lc 中, rc 中, lc 後 + rc 前, now 前, now 後)
3.5 懶標線段樹 [d41d8c]
```

```
template <class Node, class Lazy>
struct LazySeg {
       int n;
      vector < Node > tree;
vector < Lazy > lazy;
       template <typename T>
      LazySeg (vector<T> init_) { // 必須是 1-based
n = init_.size() - 1;
tree.resize(4 * n);
lazy.resize(4 * n);
             function <void(int</pre>
                    , int, int)> build = [&](int now, int l, int r) {
if (l == r) {
                          tree[now] = init_[l];
                   int m = (l + r) / 2;
build(now << 1, l, m);
build((now << 1) + 1, m + 1, r);</pre>
                    pull(now);
             build(1, 1, n);
Node query(int l, int r, int ql, int qr, int now) {
    int m = (l + r) >> 1;
    if (qr < l || ql > r) {
// ---out of range, return what---
    return Node();
// -----
             push(now, l, r);
if (ql <= l && r <= qr) {
                   return tree[now];
             Node query(int l, int r) { return query(1, n, l, r, 1); }
void pull(int now) {
    tree[now] = tree[now << 1] + tree[(now << 1) + 1];</pre>
       void modify_add
             (int l, int r, int ql, int qr, int now, int add) {
int m = (l + r) >> 1;
if (qr < l || ql > r) {
                    return:
             if (ql <= l && r <= qr) {
// ---how to modify ?---
lazy[now].add += add;
             push(now, l, r);
             push(now, t, r);
modify_add(l, m, ql, qr, now << 1, add);
modify_add(m + 1, r, ql, qr, (now << 1) + 1, add);
push(now << 1, l, m);
push((now << 1) + 1, m + 1, r);</pre>
             pull(now);
```

```
void modify_add(int
          l, int r, int add) { modify_add(1, n, l, r, 1, add); }
    return;
         if (ql <= l && r <= qr) {
// ---how to modify ?---
lazy[now].set_val = val;
             lazy[now].add = 0;
         push(now, l, r);
modify_set(l, m, ql, qr, now << 1, val);
modify_set(m + 1, r, ql, qr, (now << 1) + 1, val);
push(now << 1, l, m);
push((now << 1) + 1, m + 1, r);</pre>
lazy[(now
                  << 1) + 1].set_val = lazy[now].set_val;
lazy[now << 1].add = lazy[now].add;
lazy[(now << 1) + 1].add = lazy[now].add;</pre>
              else {
                  lazy[now << 1].add += lazy[now].add;</pre>
                  lazy[(now << 1) + 1].add += lazy[now].add;</pre>
              }
         }
// ----
         lazy[now] = Lazy();
     void apply(int now, int l, int r) {
         if (lazy[now].set_val) {
    tree[now].sum = (r - l + 1) * lazy[now].set_val;
         tree[now].sum += (r - l + 1) * lazy[now].add;
    }
};
// ---define structure and info plus---
struct Node {
   int sum;
struct Lazy {
    int set_val; int add;
Node operator+(const Node &a, const Node &b) {
    return {{a.sum + b.sum}};
// -----/// polynomial queries
// 設置梯形的底跟加了幾次, apply_tag時底為
      l的合, d為加給次, 所以 sum += (底*2 + 次*區間) * 區間 / 2;
3.6 莫隊 [d41d8c]
```

3.7 Treap [d41d8c]

```
struct Treap {
    Treap *l, *r;
    int pri, subsize; char val; bool rev_valid;
    Treap(int val) {
        this->val = val;
        pri = rand();
        l = r = nullptr;
```

```
subsize = 1; rev_valid = 0;
      void pull() {
           pull() {      // update subsize or other information
subsize = 1;
for(auto i : {l, r}) {
                if (i) subsize += i->subsize;
     }
int size(Treap *treap) {
   if (treap == NULL) return 0;
      return treap->subsize;
// lazy
void push(Treap *t) {
    if (!t) return;
     if (t->rev_valid) {
    swap(t->l, t->r);
    if (t->l) t->l->rev_valid ^= 1;
    if (t->r) t->r->rev_valid ^= 1;
      t->rev_valid = false;
Treap *merge(Treap *a, Treap *b) {
    if (!a || !b) return a ? a : b;
      // push(a); push(b);
                                     // lazy
     if (a->pri > b->pri) {
    a->r = merge
                (a->r, b); // a->r = new, inorder, make sense
           a->pull();
           return a:
     else {
           b->l = merge
                 (a, b->l); // new->l = a, inorder, make sense
           b->pull();
           return b;
     }
     if (root == nullptr) return {nullptr, nullptr};
// push(root): // lazu
pair<Treap*, Treap*> split(Treap *root, int k) {
     // push(root); // lazy
if (size(root->l) < k) {</pre>
           auto [a, b] = split(root->r, k - size(root->l) - 1);
           root->r = a;
           root->pull();
           return {root, b};
     else {
           auto [a, b] = split(root->l, k);
           root->pull();
           return {a, root};
void Print(Treap *t) {
     if (t) {
// push(t);
           Print(t->1);
cout << t->val;
           Print(t->r);
     }
}
void substring_rev() {
   int n, m; cin >> n >> m;
   Treap *root = nullptr;
   string str; cin >> str;
   for(auto c : str) {
      root = merge(root, new Treap(c));
   }
}
      for(int i = 1; i <= m; i++) {
           Print(root);
```

4 Flow

4.1 Dinic [db7233]

```
if (i.w > 0 && lev[i.to] == -1) {
                                                         q.push(i.to);
                                                         lev[i.to] = lev[u] + 1;
                           }
               return (lev[n] == -1 ? false : true);
 int dfs(int u, int flow) {
               fit(u = n) return flow;
for (auto &i : adj[u]) {
   if (lev[i.to] == lev[u] + 1 && !vis[i.to] && i.w > 0) {
      vis[i.to] = true;
      it | for the print flow in the left flow in the le
                                           int ret = dfs(i.to, min(flow, i.w));
                                           if (ret > 0) {
                                                         i.w -= ret;
                                                         adj[i.to][i.rev_ind].w += ret;
                                                         return ret;
                            }
                                                   // if can't reach end => return 0
 void dinic(){
               while (label_level()) {
                            while (1) {
                                         vis.assign(505, 0);
int tmp = dfs(1, inf);
if(tmp == 0) break;
ans += tmp;
                            }
              }
 void build() {
    for(int i = 1; i <= m; i++) {</pre>
                            int u, v, w; cin >> u >> v >> w;
adj[u].push_back({
    v, w, (int)adj[v].sz});  // inverse flow's index
adj[v].push_back({u, 0, (int
    )adj[u].sz - 1}); // have pushed one, need to -1
                                                                                                                                   // inverse flow's index
              }
// Police Chase, need
to open adj to Augment && ori to determine what pb give
// Dinic `dfs2, then use reach as u, if the edge
pb has given && w == 0 && v is not in reach, is the ans
 void dfs2(int now, unordered_set < int > & reach) {
   if(!vis[now]){
                            vis[now] = 1;
                            reach.insert(now);
for(auto i : adj[now]){
    if(i.w > 0){
                                                         dfs2(i.to, reach);
                            }
             }
// two two pair // School Dance
// Dinic, then w == 0's edge, which pb has given is the ans
 // Distinct Route
// DISTINCT ROUTE
// edge set valid var, if we need
    to argument pos road, the reverse edge set true valid;
// if we need argument the argumented
    edge ' both set false. Last, from v dfs ans times
bool get_road(int now, vector<int> &ans, vector<bool> &vis) {
   if(now == 1) return true;
   for(auto &v : adj[now]){
                            if(v.arg_valid && !vis[v.to]) {
                                          ans.push_back(v.to);
vis[v.to] = true;
                                                          flag = get_road(v.to, ans, vis);
                                           if(flag){
                                                         v.arg_valid = false;
                                                         return true;
                                           ans.pop back();
                            }
               return false;
```

4.2 MCMF [7f63db]

```
// Ceiled MinCostMaxFlow' if not, use dinic
typedef struct {
    int from, to, w, cost;
} edge;
int n, m, parcel;
vector<edge> adj; // set num to each edge
vector<int> p[505]; // p[u] has edge's num
int now_edge = 0;
void add_edge(int u, int v, int w, int cost){
    adj.push_back({u, v, w, cost});
    p[u].push_back(now_edge);
    now_edge++;
    adj.push_back(now_edge);
    row_edge++;
} p[v].push_back(now_edge);
now_edge++;
}
int Bellman_Ford(){
```

```
vector<int> dis(n+1, inf); dis[1] = 0;
        vector <int> dis(n+1, inr); dis[1] = 0;
vector <int> par(m);
vector <int> flow_rec(n + 1, 0); flow_rec[1] = 1e9;
for(int i = 1; i < n; i++){
    bool flag = 1;
}</pre>
                 bool flag = 1;
int size = adj.sz;
for(int i = 0; i < size; i++){
    auto &[from, to, w, cost] = adj[i];
    if(w > 0 && dis[to] > dis[from] + cost){
        flag = 0;
        dis[to] = dis[from] + cost;
        par[to] = i; // record num
        flow_rec[to] = min(flow_rec[from], w);
}
                         }
                 if(flag) break;
         if(dis[n] == 1e9) return 0;
         int mn_flow = flow_rec[n];
        int v = n;
while(v != 1){
                 int u = adj[par[v]].from;
adj[par[v]].w -= mn_flow;
adj[par[v] ^ 1].w += mn_flow;
        mn_flow = min(mn_flow, parcel);
parcel -= mn_flow;
return mn_flow * dis[n];
void solve(){
    cin >> n >> m >> parcel;
        int ans = 0;
for(int i = 1; i <= m; i++){
   int u, v, w, cost; cin >> u >> v >> w >> cost;
                 add_edge(u, v, w, cost);
        while(parcel > 0){
                 int tmp = Bellman_Ford();
if(tmp == 0) break;
ans += tmp;
        cout << (parcel > 0 ? -1 : ans);
```

5 String

5.1 KMP [132b98]

```
struct KMP {
       string sub;
vector<int> failure;
       KMP(string &sub) {
               this -> sub = sub;
               failure.resize(sub.size(), -1);
buildFailFunction();
       void buildFailFunction() {
    for(int i = 1; i < sub.size(); i++) {
        int now = failure[i - 1];
}</pre>
                       while(now != -1
                      && sub[now + 1] != sub[i]) now = failure[now];
if (sub[now + 1] == sub[i]) failure[i] = now + 1;
       vector<int> KMPmatching(string &s) {
               for (int i = 0, now = -1; i < s.size(); i++) {
    // now is the compare sucessed length -1
    while (s[i] !=</pre>
                      sub[now + 1] && now != -1) now = failure[now];
// f stores if comparison fail, move to where
if (s[i] == sub[now + 1]) now++;
if (now + 1 == sub.size()) {
                              match.push_back(i - now);
now = failure[now];
               return match;
      }
int main() {
    string s = "xxtxxtxtx";
       string sub = "tx";
       KMP kmp(sub);
vector <int> ans = kmp.KMPmatching(s);
for(auto &i : ans) cout << i << " ";</pre>
```

5.2 Manacher [9c9ca6]

```
// 找到對於每個位置的廻文半徑
vector < int > manacher(string s) {
    string t = "#";
    for (auto c : s) {
        t += c;
        t += '#';
    }
    int n = t.size();
    vector < int > r(n);
    for (int i = 0, j =
        0; i < n; i++) {        // i 是中心, j 是最長回文字串中心</pre>
```

```
if (2 * j - i >= 0 && j + r[j] > i) {
     r[i] = min(r[2 * j - i], j + r[j] - i);
}
while (i - r[i] >=
     0 && i + r[i] < n && t[i - r[i]] == t[i + r[i]]) {
     r[i] += 1;
}
if (i + r[i] > j + r[j]) {
     j = i;
}
return r;
// # a # b # a #
// 1 2 1 4 1 2 1
// index 為奇數代表中心點在字元上(即回文字串長度是奇數)
}
```

5.3 Trie [3b3aa0]

```
struct trie_node {
   bool is_word;
   vector<trie_node *> children;
   trie_node() {
      is_word = false;
      children.resize(26, NULL);
}
     trie_node *root = new trie_node();
      void insert(string &s) {
           trie_node *cur = root;
for (int i = 0; i < s.size(); i++) {
   int idx = s[i] - 'a';
                 if (cur->children[idx] == NULL) {
                       cur->children[idx] = new trie_node();
                 cur = cur->children[idx];
           cur->is_word = true;
     bool is_in_trie(string &s) {
    trie_node *cur = root;
           for (int i = 0; i < s.size(); i++) {</pre>
                 if (cur->
                 children[s[i] - 'a'] == nullptr) return false;
cur = cur->children[s[i] - 'a'];
           return true:
      int search_i_start(string &s, int i, vector<int> &dp) {
           trie_node *cur = root;
int sz = s.size(), ans = 0;
for (int j = i; j < sz; j++) {</pre>
                 if (cur
                       ->children[s[j]
                                                   'a'] == nullptr) return ans;
] - 'a'];
                 cur = cur->children[s[j] -
                 if (cur->is_word)
                       (ans += dp[j + 1]) \% = mod;
     }
int main() {
     // 找到 sub 集合裡,可以重複用,組成 s 的組數
Trie trie;
     string s; cin >> s;
int sz = s.size();
     // dp 代表 i 開頭到最後的配對總數
      // 找到有結尾為 stop 的 dp[i] += dp[j + 1]
     int n; cin >> n;
vector<int> dp(sz + 1, 0);
for (int i = 0; i < n; i++) {
    string sub; cin >> sub;
           trie.insert(sub);
     dp[sz] = 1;
for (int i = sz - 1; i >= 0; i--) {
    dp[i] = trie.search_i_start(s, i, dp);
     cout << dp[0] << endl;
```

6 Math

6.1 質因數分解 [91ef59]

```
| // a^(m-1) = 1 (mod m)
| // a^(m-2) = 1/a (mod m)
| // EXP2: cout << fast_exp(x, fast_exp(y, p, MOD - 1), MOD)
| // Filter + DP; DP save min factor 'recur' factor decomposition
| // FacNums = (x+1)(y+1)(z+1)...
| // FacSum = (a^0+a^1...+a^x)(b^0+...+b^y)
| // FacMul = N(x+1)(y+1)(z+1)/2
| int main() {
| vector < int > is_prime(2e6 + 1, 1);
| // 1 代表是質數,非 1 不是
| for (int i = 2; i <= 1000; i++) {
| if (is_prime[i] == 1) {
| for (int j = i + i; j <= 1000000; j += i) {
| is_prime[j] = i;
```

```
}
}
int ans = 1;
int q; cin >> q;
map<int, int> mp;
while (is_prime[q]]++;
    q /= is_prime[q];
}
if (q != 1) mp[q]++;
for (auto [a, b] : mp) {
    ans *= b + 1;
}
cout << ans << "\n";
}</pre>
```

6.2 矩陣快速幂 [d41d8c]

```
struct Mat {
     int n;
vector<vector<int>> matrix;
     Mat(int n) {
    this->n = n;
           matrix.resize(n);
           for (int i = 0; i < n; i++) {</pre>
                matrix[i].resize(n);
     Mat(vector<vector<int>> matrix) {
          this->n = matrix.size();
this->matrix = matrix;
     Mat unit(int n) { // 單位矩陣
          Mat res(n);
for (int i = 0; i < n; i++) {
    res.matrix[i][i] = 1;</pre>
          return res;
     Mat operator * (Mat b) {
          Mat ans(n);
for (int i = 0; i < n; i++) {</pre>
               }
          }
           return ans;
             *= (Mat b) { *this = *this * b; return *this; }
     Mat operator ^ (int p) {
   Mat x = *this;
   Mat ans = unit(n);
           while (p > 0) {
   if (p & 1) {
                     ans *= x;
                x *= x;
               p >>= 1;
           return ans;
             ^= (int p) { *this = *this ^ p; return *this; }
signed main() {
     int n, ans; cin >> n;
if (n <= 4) {
    vector < int >> v = {0, 1, 1, 2, 4};
     else {
           Mat init({{4, 2, 1}, {2, 1, 1}, {1, 1, 0}});
          T.matrix = {{1, 1, 0}, {1, 0, 1}, {1, 0, 0}};

T ^= n - 4;

init *= T;
          ans = init.matrix[0][0];
     cout << ans << "\n";
// 初始矩陣
// f4 f3 f2
// f3 f2 f1
// f2 f1 f0
                  轉移式
                 1 1 0  f5 f4 f3
1 0 1 => f4 f3 f2
1 0 0  f3 f2 f1
```

6.3 盧卡斯定理 [c92c05]

```
while (p > 0) {
    if (p & 1) ans = (ans * x) % mod;
    x = x * x % mod;
    p >>= 1;
}

return ans;
}
vector<int> fac;
void buildFac(int n) {
    fac.resize(n + 1);
    fac[0] = 1;
    for(int i = 1; i <= n; i++) {
        fac[i] = fac[i - 1] * i % mod;
    }
}
int C(int m, int n) {
    return m < n ? 0 : fac[m] *
        inverse(fac[n]) % mod * inverse(fac[m - n]) % mod;
}
int Lucas(int m, int n) {
    return n == 0 ? 1 % mod : Lucas
        (m / mod, n / mod) * C(m % mod, n % mod) % mod;
}
};</pre>
```

6.4 樹論分塊 [a2f49c]

6.5 Mobius Theorem

- 數論 分塊 可以快速 計算 一些 含有除法向下取整的和式,就是像 $\sum_{i=1}^n f(i)g(\left\lfloor \frac{n}{i} \right\rfloor)$ 的和式。當可以在 O(1) 內計算 f(r)-f(l) 或已經預處理 出 f 的前綴和時,數論分塊就可以在 $O(\sqrt{n})$ 的時間內計算上述和式的值。
- 迪利克雷捲積 $h(x) = \sum_{d|x} f(d)g(\frac{x}{d})$
- 積性函數
 - 莫比烏斯函數
 - 1. 定義

$$\sum_{d\mid n} \mu(d) = \begin{cases} 1 & \text{for } n=1\\ 0 & \text{for } n\neq 0 \end{cases}$$

- 2. μ 是常數函數 1 的反元素 $\Rightarrow \mu*1=\epsilon$, $\epsilon(n)$ 只在n=1時為 1,其餘情況皆為 0。
- φ歐拉函數: x以下與 x 互質的數量

$$\begin{split} \phi \! * \! 1 \! &= \! \sum_{d \mid n} \! \phi(\frac{n}{d}) \text{ 質因數分解} \\ &= \! \sum_{i=0}^{c} \! \phi(p^i) \\ &= \! 1 \! + \! p^0(p \! - \! 1) \! + \! p^1(p \! - \! 1) \! + \! \dots \! + \! p^{c-1}(p \! - \! 1) \\ &= \! p^c \\ &= \! id \end{split}$$

• 莫比烏斯反演公式

```
- f(n) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f(\frac{n}{d})

- f(n) = \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu(\frac{d}{n}) f(d)
```

例子

$$\begin{split} &\sum_{i=a}^{b} \sum_{j=c}^{d} [gcd(i,j) = k] \\ &\Rightarrow \sum_{i=1}^{x} \sum_{j=1}^{y} [gcd(i,j) = k] \\ &= \sum_{i=1}^{\left\lfloor \frac{x}{k} \right\rfloor} \left\lfloor \frac{y}{k} \right\rfloor \\ &= \sum_{i=1}^{x} \sum_{j=1}^{y} \epsilon(gcd(i,j)) \\ &= \sum_{i=1}^{x} \sum_{j=1}^{y} \sum_{d|gcd(i,j)} \mu(d) \\ &= \sum_{d=1}^{\infty} \mu(d) \sum_{i=1}^{\left\lfloor \frac{x}{k} \right\rfloor} \left\lfloor \frac{y}{k} \right\rfloor \\ &= \sum_{d=1}^{min(\left\lfloor \frac{x}{k} \right\rfloor, \left\lfloor \frac{y}{k} \right\rfloor)} \\ &= \sum_{d=1}^{min(\left\lfloor \frac{x}{k} \right\rfloor, \left\lfloor \frac{y}{k} \right\rfloor)} \mu(d) \left\lfloor \frac{x}{kd} \right\rfloor \left\lfloor \frac{y}{kd} \right\rfloor \end{split}$$

6.6 莫比烏斯反演 [d41d8c]

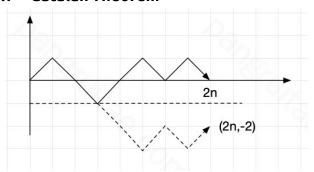
```
const int maxn = 2e5;
int mobius_pref[maxn];
void init() {
      mobius_pref[1] = 1;
      vector<int> wei
      (maxn); // wei = 0 代表是質數, -1 代表可被平方數整除
for (int i = 2; i < maxn; i++) {
    if (wei[i] == -1) {
        mobius_pref[i] = mobius_pref[i - 1];
    }
                   continue; // 包含平方
             if (wei[i] == 0) {
    wei[i] = 1;
    for (int j = 2; i * j < maxn; j++) {</pre>
                          if (j % i == 0) wei[i * j] = -1;
else if (wei[i * j] != -1) wei[i * j]++;
                   }
             mobius_pref[i]
                    = mobius_pref[i - 1] + (wei[i] % 2 == 0 ? 1 : -1);
void solve() {
      int a, b, c, d, k; cin >> a >> b >> c >> d >> k;
auto cal = [&](int x, int y) -> int {
             int res = 0;

for (int l = 1, r; l <= min(x, y); l = r + 1) {

    r = min(x / (x / l), y / (y / l));

    res += (mobius_pref[r] - mobius_pref[l
                            - 1]) * (x / l) * (y / l); // 代推出來的式子
             return res;
      cout << cal
              (b / k, d / k) - cal((a - 1) / k, d / k) - cal(b / k, (c - 1) / k) + cal((a - 1) / k, (c - 1) / k) << "\n";
}
```

6.7 Catalan Theorem



- 1. n 個往上n 個往下,先枚舉所有情況 $\frac{(2n)!}{n!n!} = C_n^{2n}$
- 2. 扣掉非法的,有多少種可能讓最後的點落在 (2n,-2)

假設往上有 x 個,往下有 y 個,會有:

$$\begin{cases} x+y=2n \\ y-x=2 \end{cases} \Rightarrow \begin{cases} x=n-1 \\ y=n+1 \end{cases}$$

6.8 Burnside's Lemma

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

- G:各種翻轉操作所構成的置換群
- · X/G:本質不同的方案的集合
- X^g : 對於某一種操作 g,所有方案中,經過 g 這種翻轉後保持不變的方案 的集合
- 集合取絕對值代表集合數

7 Search and Gready

7.1 二分搜 [d41d8c]

```
int main() {
    int l = 1, r = 10;
    // 1 to tar, find tar
    while (l <= r) {
        int m = (l + r) / 2;
        if (check(m)) l = m + 1;
        else r = m - 1;
    }
    cout << r;
    // tar to end
    while (l <= r) {
        int m = (l + r) / 2;
        if (check(m)) r = m - 1;
        else l = m + 1;
    }
    cout << l;
}</pre>
```

7.2 三分搜 [d41d8c]

8 Tree

8.1 LCA [2be9ca]

8.2 樹 DFS [7b2c0c]

```
const int maxn = 2e5+5;
vector <int> depth;
void dfs(vector <vector <int>>> &tree, int u, int pre) {
    for(auto v : tree[u]){
        if(v == pre)
        depth[v] = depth[u] + 1;
        dfs(tree, v, u);
    }
}
```

8.3 樹重心 [833d90]

```
const int maxn = 2e5+5;
vector<int> tree[maxn];
int cen = 0, n;
int dfs(int par, int now) {
    bool flag = 1;
    int size = 0;
    for (auto nxt : tree[now]) {
        if (par != nxt) {
            int subsize = dfs(now, nxt);
            if (subsize > n / 2) flag = false;
            size += subsize;
        }
    }
    if (n - 1 - size > n / 2) flag = false;
    if (flag) cen = now;
    return size + 1;
}
int main() {
    cin >> n;
    for (int i = 1; i < n; i++) {
        int u, v; cin >> u >> v;
        tree[u].push_back(v);
        tree[v].push_back(u);
    }
    for (int i = 1; i <= n; i++) {
        for (auto nxt : tree[i])
            dfs(i, nxt);
        if (cen) break;
    }
}</pre>
```

8.4 節點距離總和 [52870c]

```
const int maxn = 2e5+5;
vector < int > tree[maxn];
vector < int > subtree(maxn, 1);
long long ans[maxn];
int n:
void dfs(int par, int now, int depth) {
     ans[1] += depth;
for (auto nxt : tree[now]) {
    if (par != nxt) {
        dfs(now, nxt, depth + 1);
        subtree[now] += subtree[nxt];
}
             }
      }
// void find_ans(int par, int now) {
// each sub's dis make - 1, non subnode + 1
    for (auto nxt : tree[now]) {
      if (par != nxt) {
                    ans[nxt] =
                             ans[now] + (n - subtree[nxt]) - subtree[nxt];
                    find_ans(now, nxt);
             }
      }
int main() {
       cin >> n;
      for (int i = 1; i < n; i++) {
   int u, v; cin >> u >> v;
   tree[u].push_back(v);
             tree[v].push_back(u);
      cout << ans[i] <<
```

8.5 有權樹直徑 [ca20c3]

```
mx2 = w + dp[nxt];
}
dp[now] = mx1;
ans = max(ans, mx1 + mx2);
}
int main(){
  int n; cin >> n;
  memset(dp, 0, sizeof(dp));
  for(int i = 1; i < n; i++){
     int u, v, w; cin >> u >> v >> w;
     tree[u].push_back({v, w});
     tree[v].push_back({u, w});
}
DP(1, 0);
cout << (ans < 0 ? 0 : ans);
}</pre>
```

```
8.6 樹壓平 [83ba92]
|// 父節
        點加值 = 所有子節點區間加值,求單點,使用 bit,做前綴差分
 // CSES 1138_Path Queries
                     // BIT 都是 1-based 的查詢
 struct BIT {
       int n;
       vector<int> bit;
       BIT(int n) { // 有幾個數
this->n = n;
             bit.resize(n + 1, 0);
       BIT(vector < int > & init) { // 必須是 1-based this - >n = init.size() - 1;
             bit.resize(n + 1, 0);
for (int i = 1; i <= n;
    modify(i, init[i]);</pre>
       void modify(int i, int val) {
    for (; i <= n; i += i & ·i) {
        bit[i] += val;
    }</pre>
        int query(int r) {
             int ans = 0;
for (; r; r -= r & -r) ans += bit[r];
             return ans;
       int query(int l, int r) {
   return query(r) - query(l - 1);
 };
void solve(){
    int n, q; cin >> n >> q;
    vector vint > node_value(n + 1), euler_ordered_value(n);
    for (int i = 1; i <= n; i++) {
        cin >> node_value[i];
    }
}
       vector<vector<int>> tree(n + 1);
       for (int i = 1; i < n; i++) {
   int u, v; cin >> u >> v;
             tree[u].push_back(v);
tree[v].push_back(u);
        ·
vector<pair<int, int>> tree_mapping(n + 1);
       int cnt = 0;
auto dfs = [&](auto self, int u, int par) -> void {
    euler_ordered_value[++cnt] = node_value[u];
             tree_mapping[u].first = cnt;
             for (auto v : tree[u]) {
   if (v == par) continue;
   self(self, v, u);
             tree_mapping[u].second = cnt;
       dfs(dfs, 1, 0);
       if (tree_mapping[i].first < n) {
   bit.modify</pre>
                          (tree_mapping[i].second + 1, -node_value[i]);
             }
        for (int i = 0; i < q; i++) {
             int op; cin >> op;
if (op == 1) {
                   int s, x; cin >> s >> x;
int add = x
                   - euler_ordered_value[tree_mapping[s].first];
euler_ordered_value[tree_mapping[s].first] = x;
                   it modify(tree_mapping[s].first, add);
if (tree_mapping[s].first < n) {
   bit.modify(tree_mapping[s].second + 1, -add);
}</pre>
             else {
    int node; cin >> node;
                   cout <<
                           bit.query(tree_mapping[node].first) << "\n";</pre>
```

9 DP

| }

9.1 背包問題 [6d6b63]

```
考慮前 i 個,預算有 j 塊錢的最多 page
int main(){
    int n, bud;
    cin >> n >> bud:
    vector<vector<int>> dp(n + 1, vector<int>(bud + 1));
    vector <int> Page(n + 1, 0);
vector <int> Price(n + 1, 0);
    for(int i = 1; i <= n; i++){
    cin >> Price[i];
    for(int i = 1; i <= n; i++){
        cin >> Page[i];
    for (int i = 1; i <= n; i++) {
    for (int j = 1; j <= bud; j++) {</pre>
             if (j >= Price[i]) { // 買得起
                 .
// 不買或買
                 dp[i][j] = dp[i - 1][j];
        }
    cout << dp[n][bud] << "\n";</pre>
}
```

9.2 Bitmask DP [b18541]

```
void travel exactly once(){
    // [走過的路][終點]
    vector<vector<int>> dp(1 << 20, vector<int> (20, 0));
    vector < int > rev_adj[20];
    int n, m; cin >> n >> m;
for(int i = 0; i < m; i++){</pre>
         int u, v; cin >> u >> v;
rev_adj[--v].push_back(--u);
    dp[1][0] = 1;
for (int road = 0; road < (1 << n); road++) {
    // 沒經過起點,不用走
    if (road & 1 == 0) continue;
         // 有終點但沒全部走過
         // DP, 隨便選定一個當前路徑的終點
         for (int end = 0; end < n; end++) {</pre>
              // 路徑沒包含假定的 end
              if ((road & (1 << end)) == 0) continue;</pre>
             // 去除終點,得到 pre_road
int pre_road = road - (1 << end);
              // 從 rev_adj 找 pre_road 的終點
              for (int pre_road_end : rev_adj[end]) {
                  if ((road & (1 << pre_road_end))) {</pre>
                      dp[road
                            ][end] += dp[pre_road][pre_road_end];
                      dp[road][end] %= mod;
                 }
             }
        }
    cout << dp[(1 << n) - 1][n - 1];
void elevator_rides(){
    int n, k; cin >> n >> k;
vector <int> passenger(n);
for (int i = 0; i < n; i++) cin >> passenger[i];
    vector<int
    for (int j = 0; j < n; j++) {</pre>
             if (i & (1 << j)) { // 有 j
int pre = i ^ (1 << j);
                  // 最後的電梯遷能載 j
if (used[pre] + passenger[j] <= k) {
                       // 電梯數先比,再來比用掉的空間
                       if (dp
                           up
[pre] < dp[i] || (dp[pre] == dp[i] &&
  used[pre] + passenger[j] < used[i])) {
used[i] = used[pre] + passenger[j];
dp[i] = dp[pre];</pre>
                      }
                  }
                  // 搭新的電梯
```

```
dp[i] = dp[pre] + 1;
                   }
              }
         }
     cout << dp[(1 << n) - 1];
      travel_exactly_once();
     elevator_rides();
 9.3 硬幣 [d41d8c]
void coin_combination_II(){
     // 有 n 種錢幣, 求組合為 x 的組數, 順序不可顛倒
      ...
// 可顛倒的話只要一維, 先 x 廻圈, 再 coin[i] 去加
     int n, x; cin >> n >> x;
vector<int> coin(n + 1);
      // dp[i][j] 為考慮前 i 個硬幣,組合為 i 的組數
      vector < vector < int >> dp(2, vector < int >(x + 1, 0));
     for (int i = 1; i <= n; i++) cin >> coin[i];
for (int i = 1; i <= n; i++){
    for (int j = 0; j <= x; j++) {</pre>
               // 壓到 2 * n
               dp[i & 1][j] = dp[!(i & 1)][j];
               if (j >= coin[i]) {
    (dp[i
                        & 1][j] += dp[i & 1][j - coin[i]]) %= mod;
               }
          }
     cout << dp[n & 1][x];
 void minimize_coins_nums(){
     // 有 n 種錢幣, 求組合為 x 的最小硬幣數
      int n, x; cin >> n >> x;
     vector <int> coin(n);
for (int i = 0; i < n; i++) cin >> coin[i];
      // dp[i] 是組合為 i 的最小硬幣數
     dp[i] = min(dp[i], dp[i - j] + 1);
          }
     cout << (dp[x] == 2e9 ? -1 : dp[x]);
 int main(){
     coin_combination_II();
     minimize_coins_nums();
 9.4 編輯距離 [4d4a6d]
 int main() {
    string s1, s2; cin >> s1 >> s2;
    int size1 = s1.size(), size2 = s2.size();
      // dp[i][j] 為 s1 的前 i 個字元,跟 s2 的前 j 個字元
     if (s1[i] == s2[j]) {
    dp[i][j] = dp[i - 1][j - 1];
                   // s1 新增等價於 s2 砍掉
                   // dp[i][j] = min(修改, s1 新增, s2 新增);
dp[i][j] = min({dp[i - 1][
                           - 1], dp[i - 1][j], dp[i][j - 1]}) + 1;
          }
     cout << dp[size1][size2];</pre>
 9.5 LCS [087c0d]
 int main() {
     int m, n; cin >> m >> n;
     string s1, s2;
cin >> s1 >> s2;
     vector<vector<int>> dp(m + 1, vector<int>(n + 1, 0));
     for (int i = 1; i <= m; i++) {
    for (int j = 1; j <= n; j++) {
        if (s1[i - 1] == s2[j - 1]) {
            dp[i][j] = dp[i - 1][j - 1] + 1;
        }
}</pre>
```

else {

add[i] = projects[i].id;

if (add[now] != -1)

[n][0] << " " << dp[n][1] << " " << dp[n][2] << "\n";

for (int now = n; now > 0; now = par[now]) {

} cout << dp

```
dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
                                                                                                  ans.push_back(add[now]);
          }
                                                                                         sort(all(ans));
                                                                                        for (auto &i : ans) cout << i << " ";
     int length = dp[m][n];
     int length = dp[m][n];
cout << length << "|n";
string s(length, 'c');
// along to dp to trace back
while (m == 1 && n >= 1) {
    if (s1[m - 1] == s2[n - 1]) {
        s[length - 1] = s1[m - 1];
        m--, n--, length--;
}
                                                                                   9.8 Removal Game [211de0]
                                                                                  1// 兩個人比賽,每個人輪流取一個數字且只能是頭尾
                                                                                   // 問兩人都選得好,第一個人可取得的最大分數 int main() {
                                                                                        int n; cin >> n;
                                                                                        vector<vector<int>> dp(n + 1, vector<int>(n + 1));
          else {
    if (dp[m - 1][n] > dp[m][n - 1]) m--;
                                                                                        int pref = 0;
                                                                                        vector<int> v(n + 1);
                                                                                        for (int i = 1; i <= n; i++) {</pre>
                                                                                             cin >> v[i];
pref += v[i];
     cout << s << "\n";
                                                                                        // dp[i][j] 是 i 到 j 區間選完,的最大分數差
                                                                                        for (int i = n; i > 0; i--) {
    for (int j = i; j <= n; j++) {
        if (i == j) {
9.6 LIS [668131]
int main() {
    int n; cin >> n;
                                                                                                       dp[i][j] = v[i];
     vector<int> v(n);
                                                                                                  else {
     for (int i = 0; i < n; i++) {
    cin >> v[i];
                                                                                                        // 選左差距大,還是選右差距大
                                                                                                       int dp[n]; vector<int> mono;
     mono.push_back(v[0]);
dp[0] = 1; int L = 1;
for (int i = 1; i < n; i++) {
    if (v[i] > mono.back()) {
        mono.push_back(v[i]);
    }
}
                                                                                             }
                                                                                        // x + y = sum, dp[1][n] = x - y;
cout << (pref + dp[1][n]) / 2;
               dp[i] = ++L;
                                                                                   9.9 Max overlap [07fccd]
          else {
                    = lower_bound(mono.begin(), mono.end(), v[i]); |// CF 1932 pF
               *it = v[i]; dp[i] = it - mono.begin() + 1;
                                                                                  // 給你很多區間,你可以選一些點,重疊到的線段得到 1 分
                                                                                   // 請問在線段不重複的情況下,最多獲得幾分
          }
                                                                                   int main() {
                                                                                        int n, m;
cin >> n >> m;
     vector<int> ans;
     cout << L << "\n";
for (int i = n - 1; i >= 0; i--) {
   if (dp[i] == L) {
                                                                                        // 記錄每點有幾個線段
                                                                                        // 再一個紀錄,包含這個點的左界
vector<int> l_side(n + 1, inf), cnt(n + 5, 0);
for (int i = 0; i < m; i++) {
   int l, r; cin >> l >> r;
   l_side[r] = min(l_side[r], l);
               ans.push_back(v[i]);
          }
                                                                                             cnt[l]++;
     reverse(ans.begin(), ans.end());
for (auto i : ans) {
    cout << i << " ";</pre>
                                                                                             cnt[r + 1]--;
                                                                                        for (int i = 2; i <= n; i++) {
                                                                                             cnt[i] += cnt[i - 1];
}
9.7 Projects [479ba0]
                                                                                        for (int i = n; i >= 2; i--) {
                                                                                             l_side[i - 1] = min(l_side[i - 1], l_side[i]);
struct project {
   int from, end, gain, id;
                                                                                        vector<int> dp(n + 1);
                                                                                        dp[0] = 0;
for (int i = 1; i <= n; i++) {</pre>
int main() {
    int n; cin >> n;
                                                                                             dp[i] = cnt[i];
if (l_side[i] != inf)
     vector < project > projects(n + 1);
for (int i = 1; i <= n; i++) {</pre>
                                                                                                  dp[i] += dp[l_side[i] - 1];
          cin >> projects
                                                                                             dp[i] = max(dp[i], dp[i - 1]);
                [i].from >> projects[i].end >> projects[i].gain;
          projects[i].id = i;
                                                                                        cout << dp[n] << "\n";
     if (a.end == b.end) return a.gain < b.gain;
return a.end < b.end;</pre>
                                                                                   10 Geometry
                                                                                   10.1 Cross Product [8113ac]
     const double eps = 1e-8;
                    return a.end < b.end;</pre>
               }) - projects
                     .begin();
                                   // 二分搜最接近 from 的 end
          dp[i] = dp[i - 1];
par[i] = i - 1;
                                                                                                . (point b){ return x == b.x ? y < b.y : x < b.x; }</pre>
           if (dp[i][1] < dp[idx][1] + projects[i].gain ||
                                                                                   double abs(point a) { return sqrt(a * a); }
          (dp[i][1]
                 == dp[idx][1] + projects[i].gain && dp[i][2] > dp
[idx][2] + projects[i].end - projects[i].from)) {
                                                                                   int sign
               (double a) { return fabs(a) < eps ? 0 : a > 0 ? 1 : -1; }
                                                                                   int ori(point
                                                                                          a, point b, point c) { return sign((b - a) ^ (c - a)); }
                                                                                   bool colinear(point a,
                                                                                   point b, point c) { return sign((b - a) ^ (c - a)) == 0; }
bool between(point a, point b, point c) { // c between a and b
   if (!colinear(a, b, c)) return false;
   return sign((a - c) * (b - c)) <= 0;</pre>
               par[i] = idx;
```

bool intersect(point

a, point b, point c, point d){ // line(a, b) line(c, d) int abc = ori(a, b, c);

10.2 Convex Hull [e84f76]

```
}
int Andrew_monotone_chain(int n){
     sort(P.begin(), P.end());
int l = 0, u = 0;  // upper and lower hull
for (int i=0; i<n; ++i){</pre>
          while (l >= 2 && cross(L[l-2], L[l-1], P[i]) <= 0){
    l--;</pre>
               L.pop_back();
          while (u >= 2 && cross(U[u-2], U[u-1], P[i]) >= 0){
               U.pop_back();
          }
l++;
          u++;
          L.push_back(P[i]);
U.push_back(P[i]);
     cout << l << ' ' << u << '\n';
return l + u;
}
int main(){
     int n, x, y;
cin >> n;
for(int i = 0;i < n;i++){</pre>
         cin >> x >> y;
P.push_back({x, y});
     int ans = Andrew_monotone_chain(n) - 2;
cout << ans << "\n";</pre>
     return 0;
```