

Contents

1 Basic	1	6.10 BSGS	13
1.1 Default Code	1	6.11 Game Theorem	13
1.2 Debug	1	6.12 Gaussian Elimination	13
1.3 Compare Fuction	1	6.13 XOR Basis	14
1.4 Pbds	1	6.14 Pisano Period	14
1.5 Int128	2	6.15 Integer Partition	14
1.6 Rng	2	6.16 Mobius Theorem	14
		6.17 Mobius Inverse	14
		6.18 Catalan Theorem	15
		6.19 Burnside's Lemma	15
2 Graph	2	7 Search and Greedy	15
2.1 Prim	2	7.1 Binary Search	15
2.2 Bellman-Ford	2	7.2 Ternary Search	15
2.3 Euler	2		
2.4 DSU	2	8 Tree	15
2.5 SCC	2	8.1 Binary Lifting LCA	15
2.6 VBCC	3	8.2 Centroid Decomposition	15
2.7 EBCC	3	8.3 Heavy Light Decomposition	15
2.8 2-SAT	3	8.4 Link Cut Tree	16
2.9 Functional Graph	4	8.5 Virtual Tree	16
		8.6 Dominator Tree	17
3 Data Structure	4	9 DP	17
3.1 Segment Tree	4	9.1 LIS	17
3.2 Persistent Segment Tree	5	9.2 Projects	17
3.3 Static Kth-element	5	9.3 Bitmask	18
3.4 Dynamic Kth-element	5	9.4 Monotonic Queue	18
3.5 Fenwick	6	9.5 Digit	18
3.6 Range Fenwick	6	9.6 SOS	18
3.7 KDTree	6	9.7 CHT	19
3.8 Treap	7	9.8 DNC	19
3.9 RMQ	7	9.9 LiChao Segment Tree	19
3.10 Mo	7	10 Geometry	19
		10.1 Basic	19
4 Flow Matching	8	10.2 Min Euclidean Distance	21
4.1 Dinic	8	10.3 Max Euclidean Distance	21
4.2 Min Cut	8	10.4 Lattice Points	21
4.3 MCMF	8	10.5 Min Circle Cover	21
4.4 Hungarian	9	10.6 Min Rectangle Cover	22
4.5 Theorem	9	10.7 Polygon Union Area	22
5 String	9	11 Polynomial	22
5.1 Hash	9	11.1 FFT	22
5.2 KMP	9	11.2 NTT	22
5.3 Z Function	9	11.3 FWT	23
5.4 Manacher	9	11.4 Barlekamp Massey	24
5.5 Trie	9	11.5 Linear Recurrence	24
5.6 SA	9		
5.7 AC	10	12 Else	24
5.8 SAM	10	12.1 Python	24
5.9 Palindrome Tree	11	12.2 Fraction	24
5.10 Duval	11	12.3 Bigint	24
		12.4 Multiple	25
6 Math	11	12.5 Division	25
6.1 Mint	11	12.6 Division-Python	25
6.2 Combination	11		
6.3 Sieve	12		
6.4 Matrix	12		
6.5 Miller Rabin Pollard Rho	12		
6.6 Primitive Root	12		
6.7 CRT	12		
6.8 exLucas	13		
6.9 Quadratic Residue	13		

1 Basic

1.1 Default Code [d41d8c]

```
#include <bits/stdc++.h>

using namespace std;
using ll = long long;

const int Mod = 1E9 + 7;
int add(int a, int b) { a += b; if (a >= Mod) a -= Mod; return a; }
int sub(int a, int b) { a -= b; if (a < 0) a += Mod; return a; }
int mul(int a, int b) { return 1LL * a * b % Mod; }
int power(int a, ll b) {
    int ans = 1;
    for (; b > 0; b >>= 1, a = mul(a, a))
        if (b & 1) ans = mul(ans, a);
    return ans;
}

void solve() {
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(nullptr);
    auto s = chrono::high_resolution_clock::now();
    int t = 1;
    cin >> t;
    while (t--) {
        solve();
    }
}
```

```
auto e = chrono::high_resolution_clock::now();
cerr << chrono::duration_cast<chrono::milliseconds>(e - s).count() << " ms\n";
return 0;
}
```

1.2 Debug [33ccce]

```
# 對拍
CODE1="a"
CODE2="ac"
set -e
g++ $CODE1.cpp -o $CODE1
g++ $CODE2.cpp -o $CODE2
g++ gen.cpp -o gen
for ((i=1;;i++))
do
    echo "--- Testing: Case #$i ---"
    ./gen > input
    # python3 gen.py > input
    ./CODE1 < input > $CODE1.out
    ./CODE2 < input > $CODE2.out
    cmp $CODE1.out $CODE2.out || break
done
# 多重解, ifstream in(argv[1]);
CODE="a"
set -e
g++ $CODE.cpp -o $CODE
g++ gen.cpp -o gen
g++ checker.cpp -o checker
for ((i=1;;i++))
do
    ./gen > input
    ./CODE < input > $CODE.out
    ./checker $CODE.out < input || break
done
# 互動
CODE="a"
set -e
g++ $CODE.cpp -o $CODE
g++ checker.cpp -o checker
PIPE_IN="in"
PIPE_OUT="out"
trap 'rm -f $PIPE_IN $PIPE_OUT' EXIT
mkfifo $PIPE_IN $PIPE_OUT
for ((i=1;;i++))
do
    echo "--- Testing: Case #$i ---"
    ./CODE < $PIPE_IN > $PIPE_OUT &
    (exec 3>$PIPE_IN 4<$PIPE_OUT; ./checker <&4 >&3) || break
done
# 參考 checker
ll AC(int n) { return ans; }
void WA(string log = "") { cerr << log << endl; exit(1); }
void checkAC(string log = "") {
    string trash;
    if (cin >> trash) WA("redundant output\n" + log);
}

int main() {
    int n = uniform_int_distribution<int>(1, 10)(rng);
    ll sol = AC(n);
    stringstream log;
    cout << n << endl;
    log << n << endl;
    log << "judge: " << endl;
    log << "team: " << endl;
    WA(log.str());
    checkAC();
    return 0;
}
```

1.3 Compare Fuction [d41d8c]

```
// 1. sort, 二分搜刻在函式內 lambda 就好
// 2. priority queue 小到大是 >, set 是 <
// 3. set 不能 =, multiset 必須 =
// 4. 確保每個成員都要比到
// 5. pbds_multiset 不要用 lower_bound
// 6. 如果要用 find, 插入 inf 後使用 upper_bound
// 7. multiset 可以跟 set 一樣使用, 但請注意第 3、4 點
auto cmp = [](int i, int j) { return a[i] > a[j]; };
priority_queue<int, vector<int>, decltype(cmp)> pq(cmp);

vector<int> a {1, 2, 5, 4, 3}; // 小心不要改到 a
auto cmp = [&a](int i, int j) { return a[i] > a[j]; };
priority_queue<int, vector<int>, decltype(cmp)> pq(cmp);

vector<int> v {1, 2, 3, 4, 5};
upper_bound(v.begin(), v.end(), 2, [](int a, int b)
{ return a < b; }); // find first b that a < b, a is 2
lower_bound(v.begin(), v.end(), 2, [](int a, int b)
{ return a < b; }); // find first a that a < b fail, b is 2
```

1.4 Pbds [d41d8c]

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<class T>
using pbds_set = tree<T, null_type,
less<T>, rb_tree_tag, tree_order_statistics_node_update>;
```

```
template<class T>
using pbds_multiset = tree<T, null_type, less_equal
    <T>, rb_tree_tag, tree_order_statistics_node_update>;
```

1.5 Int128 [85923a]

```
using i128 = __int128_t; // 1.7E38
istream &operator>>(istream &is, i128 &a) {
    i128 sgn = 1; a = 0;
    string s; is >> s;
    for (auto c : s) {
        if (c == '-' || c == '+') {
            sgn = -1;
        } else {
            a = a * 10 + c - '0';
        }
    }
    a *= sgn;
    return is;
}
ostream &operator<<(ostream &os, i128 a) {
    string res;
    if (a < 0) os << "- ", a = -a;
    while (a) {
        res.push_back(a % 10 + '0');
        a /= 10;
    }
    reverse(res.begin(), res.end());
    os << res;
    return os;
}
```

1.6 Rng [401544]

```
mt19937_64 rng
    (chrono::steady_clock::now().time_since_epoch().count());
ll x = rng();
shuffle(a.begin(), a.end(), rng);
```

2 Graph

2.1 Prim [cefbbf]

```
auto prim =
    [&](int n, vector<vector<pair<int, int>>> &adj) -> bool {
        int sz = 0; ll ans = 0;
        priority_queue<pair<int, int>,
            vector<pair<int, int>>, greater<pair<int, int>>> pq;
        pq.emplace(0, 0); // w, vertex
        vector<bool> vis(n);
        while (!pq.empty()) {
            auto [w, u] = pq.top(); pq.pop();
            if (vis[u]) continue;
            vis[u] = true;
            ans += w, sz++;
            for (auto [v, w] : g[u])
                if (!vis[v]) pq.emplace(w, v);
        }
        if (sz == n) return true;
        return false;
    };
```

2.2 Bellman-Ford [430de2]

```
// 用 Bellman Ford 找負環
void bellmanFord() {
    int n, m; cin >> n >> m;
    vector<array<int, 3>> e;
    for (int i = 0; i < m; i++) {
        int u, v, w; cin >> u >> v >> w;
        u--, v--; e.push_back({u, v, w});
    }
    vector<ll> dis(n, inf), par(n);
    int t = -1; dis[0] = 0;
    for (int i = 1; i <= n; i++) {
        for (auto [u, v, w] : e) {
            if (dis[v] > dis[u] + w) {
                dis[v] = dis[u] + w;
                par[v] = u;
                if (i == n) t = v;
            }
        }
    }
    if (t == -1) { cout << "NO\n"; return; }
    for (int i = 1; i < n; i++) t = par[t];
    vector<int> ans {t};
    int i = t;
    do {
        i = par[i];
        ans.push_back(i);
    } while (i != t);
    reverse(ans.begin(), ans.end());
    cout << "YES\n";
    for (auto x : ans) cout << x + 1 << " ";
}
```

2.3 Euler [4177dc]

```
// 1. 無向圖是歐拉圖：
// 非零度頂點是連通的
// 頂點的度數都是偶數
```

```
// 2. 無向圖是半歐拉圖(有路沒有環)：
// 非零度頂點是連通的
// 恰有 2 個奇度頂點
```

```
// 3. 有向圖是歐拉圖：
// 非零度頂點是強連通的
// 每個頂點的入度和出度相等
```

```
// 4. 有向圖是半歐拉圖(有路沒有環)：
// 非零度頂點是弱連通的
// 至多一個頂點的出度與入度之差為 1
// 至多一個頂點的入度與出度之差為 1
// 其他頂點的入度和出度相等
```

```
vector<int> ans;
auto dfs = [&](auto &&self, int u) -> void {
    while (g[u].size()) {
        int v = *g[u].begin();
        g[u].erase(v);
        self(self, v);
    }
    ans.push_back(u);
};
dfs(dfs, 0);
reverse(ans.begin(), ans.end());
```

2.4 DSU [fde0cf]

```
struct DSU {
    int n;
    vector<int> f, siz;
    DSU(int n) : n(n), f(n), siz(n, 1) {
        iota(f.begin(), f.end(), 0);
    }
    int find(int x) {
        if (f[x] == x) return x;
        return f[x] = find(f[x]);
    }
    bool same(int x, int y) {
        return find(x) == find(y);
    }
    bool merge(int x, int y) {
        x = find(x); y = find(y);
        if (x == y) return false;
        if (siz[x] < siz[y]) swap(x, y);
        siz[x] += siz[y];
        f[y] = x;
        n--;
        return true;
    }
    int size(int x) { return siz[find(x)]; }
};

struct DSU {
    int n;
    vector<int> f, siz, stk;
    DSU(int n) : n(n), f(n), siz(n, 1) {
        iota(f.begin(), f.end(), 0);
        stk.clear();
    }
    int find(int x) {
        return x == f[x] ? x : find(f[x]);
    }
    bool same(int x, int y) {
        return find(x) == find(y);
    }
    bool merge(int x, int y) {
        x = find(x); y = find(y);
        if (x == y) return false;
        if (siz[x] < siz[y]) swap(x, y);
        siz[x] += siz[y];
        f[y] = x;
        n--;
        stk.push_back(y);
        return true;
    }
    void undo(int x) {
        while (stk.size() > x) {
            int y = stk.back();
            stk.pop_back();
            n++;
            siz[f[y]] -= siz[y];
            f[y] = y;
        }
    }
    int size(int x) { return siz[find(x)]; }
};
```

2.5 SCC [3ac1cb]

```
struct SCC {
    int n, cur, cnt;
    vector<vector<int>> adj;
    vector<int> stk, dfn, low, bel;
    SCC(int n) : n(n), cur
        (0), cnt(0), adj(n), dfn(n, -1), low(n), bel(n, -1) {}
    void addEdge(int u, int v) { adj[u].push_back(v); }
    void dfs(int x) {
        dfn[x] = low[x] = cur++;
        stk.push_back(x);
        for (auto y : adj[x]) {
            if (dfn[y] == -1) {
                dfs(y);
                low[x] = min(low[x], low[y]);
            }
        }
    }
};
```

```

    } else if (bel[y] == -1) {
        low[x] = min(low[x], dfn[y]);
    }
}
if (dfn[x] == low[x]) {
    int y;
    do {
        y = stk.back();
        bel[y] = cnt;
        stk.pop_back();
    } while (y != x);
    cnt++;
}
}
vector<int> work() {
    for (int i = 0; i < n; i++)
        if (dfn[i] == -1) dfs(i);
    return bel;
}
struct Graph {
    int n;
    vector<pair<int, int>> edges;
    vector<int> siz, cnte;
};
Graph compress() {
    Graph g; g.n = cnt;
    g.siz.resize(cnt); g.cnte.resize(cnt);
    for (int i = 0; i < n; i++) {
        g.siz[bel[i]]++;
        for (auto j : adj[i]) {
            if (bel[i] != bel[j]) {
                g.edges.emplace_back(bel[i], bel[j]);
            } else {
                g.cnte[bel[i]]++;
            }
        }
    }
    return g;
}
};

```

2.6 VBCC [95997d]

```

struct VBCC {
    int n, cur, cnt;
    vector<vector<int>> adj, bcc;
    vector<int> stk, dfn, low;
    vector<bool> ap;
    VBCC(int n) : n(n), cur(0)
        , cnt(0), adj(n), bcc(n), ap(n), low(n), dfn(n, -1) {}
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void dfs(int x, int p) {
        dfn[x] = low[x] = cur++;
        stk.push_back(x);
        int ch = 0;
        for (auto y : adj[x]) {
            if (y == p) continue;
            if (dfn[y] == -1) {
                dfs(y, x), ch++;
                low[x] = min(low[x], low[y]);
                if (low[y] >= dfn[x]) {
                    int v;
                    do {
                        v = stk.back();
                        bcc[v].push_back(cnt);
                        stk.pop_back();
                    } while (v != y);
                    bcc[x].push_back(cnt);
                    cnt++;
                }
                if (low[y] >= dfn[x] && p != -1)
                    ap[x] = true;
            } else {
                low[x] = min(low[x], dfn[y]);
            }
        }
        if (p == -1 && ch > 1) ap[x] = true;
    }
    vector<bool> work() {
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) dfs(i, -1);
        return ap;
    }
    struct Graph {
        int n;
        vector<pair<int, int>> edges;
        vector<int> bel, siz, cnte;
    };
    Graph compress() {
        Graph g; // 壓完是一棵樹，但不一定每個 bel 都有節點
        g.bel.resize(n);
        g.siz.resize(cnt); g.cnte.resize(cnt);
        for (int u = 0; u < n; u++) {
            if (ap[u]) {
                g.bel[u] = cnt++;
                g.siz.emplace_back(cnt);
                g.cnte.emplace_back(cnt);
                for (auto v : bcc[u]) {
                    g.edges.emplace_back(g.bel[u], v);
                }
            }
        }
    }
};

```

```

    }
    } else if (bcc[u].size() == 1) {
        g.bel[u] = bcc[u][0];
    }
    g.siz[g.bel[u]]++;
}
g.n = cnt;
for (int i = 0; i < n; i++)
    for (auto j : adj[i])
        if (g.bel[i] == g.bel[j] && i < j)
            g.cnte[g.bel[i]]++;
return g;
}
};

```

2.7 EBCC [12a170]

```

struct EBCC { // CF/contest/1986/pF
    int n, cur, cnt;
    vector<vector<int>> adj;
    vector<int> stk, dfn, low, bel;
    vector<pair<int, int>> bridges; // 關鍵邊
    EBCC(int n) : n(n), cur
        (0), cnt(0), adj(n), low(n), dfn(n, -1), bel(n, -1) {}
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void dfs(int x, int p) {
        dfn[x] = low[x] = cur++;
        stk.push_back(x);
        for (auto y : adj[x]) {
            if (y == p) continue;
            if (dfn[y] == -1) {
                dfs(y, x);
                low[x] = min(low[x], low[y]);
                if (low[y] > dfn[x]) {
                    bridges.emplace_back(x, y);
                }
            } else if (bel[y] == -1) {
                low[x] = min(low[x], dfn[y]);
            }
        }
        if (dfn[x] == low[x]) {
            int y;
            do {
                y = stk.back();
                bel[y] = cnt;
                stk.pop_back();
            } while (y != x);
            cnt++;
        }
    }
    vector<int> work() { // not connected
        for (int i = 0; i < n; i++)
            if (dfn[i] == -1) dfs(i, -1);
        return bel;
    }
    struct Graph {
        int n;
        vector<pair<int, int>> edges;
        vector<int> siz, cnte;
    };
    Graph compress() {
        Graph g; g.n = cnt;
        g.siz.resize(cnt); g.cnte.resize(cnt);
        for (int i = 0; i < n; i++) {
            g.siz[bel[i]]++;
            for (auto j : adj[i]) {
                if (bel[i] < bel[j]) {
                    g.edges.emplace_back(bel[i], bel[j]);
                } else if (i < j) {
                    g.cnte[bel[i]]++;
                }
            }
        }
        return g;
    }
};

```

2.8 2-SAT [f17517]

```

struct TwoSat {
    int n; vector<vector<int>> e;
    vector<bool> ans;
    TwoSat(int n) : n(n), e(2 * n), ans(n) {}
    void addClause(int u, bool f, int v, bool g) {
        e[2 * u + !f].push_back(2 * v + g);
        e[2 * v + !g].push_back(2 * u + f);
    }
    void ifThen(int u, bool f, int v, bool g) {
        // 必取 A: not A -> A
        e[2 * u + f].push_back(2 * v + g);
    }
    bool satisfiable() {
        vector<int>
            > id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1);
        vector<int> stk;
        int now = 0, cnt = 0;
        function<void(int)> tarjan = [&](int u) {
            stk.push_back(u);
            dfn[u] = low[u] = now++;
        };
    };
};

```

```

        for (auto v : e[u]) {
            if (dfn[v] == -1) {
                tarjan(v);
                low[u] = min(low[u], low[v]);
            } else if (id[v] == -1) { // in stk
                low[u] = min(low[u], dfn[v]);
            }
        }
        if (dfn[u] == low[u]) {
            int v;
            do {
                v = stk.back();
                stk.pop_back();
                id[v] = cnt;
            } while (v != u);
            ++cnt;
        }
    };
    for (int i = 0; i < 2 * n; ++i) if (dfn[i] == -1) tarjan(i);
    for (int i = 0; i < n; ++i) {
        if (id[2 * i] == id[2 * i + 1]) return false;
        ans[i] = id[2 * i] > id[2 * i + 1];
    }
    return true;
}
vector<bool> answer() { return ans; }
};

```

2.9 Functional Graph [c314w3]

```

const int N = 2E5;
const int Lg = __lg(N); // __lg(max(n, qi)), [0, Lg]
int cht[N][Lg];
struct FunctionalGraph {
    int n, cnt;
    vector<int> g, bel, id, cycsz, in, top, hei;
    FunctionalGraph(const vector<int> &g) : n(g.size()), cnt(0) {
        g(g), bel(n, -1), id(n), in(n), top(n, -1), hei(n) {
            for (int i = 0; i < n; i++)
                cht[i][0] = g[i], in[g[i]]++;
            for (int i = 1; i <= Lg; i++)
                for (int u = 0; u < n; u++) {
                    int nxt = cht[u][i - 1];
                    cht[u][i] = cht[nxt][i - 1];
                }
            for (int i = 0; i < n; i++)
                if (in[i] == 0) label(i);
            for (int i = 0; i < n; i++)
                if (top[i] == -1) label(i);
        }
        void label(int u) {
            vector<int> p; int cur = u;
            while (top[cur] == -1) {
                top[cur] = u;
                p.push_back(cur);
                cur = g[cur];
            }
            auto s = find(p.begin(), p.end(), cur);
            vector<int> cyc(s, p.end());
            p.erase(s, p.end()); p.push_back(cur);
            for (int i = 0; i < (int)cyc.size(); i++)
                bel[cyc[i]] = cnt, id[cyc[i]] = i, hei[cyc[i]] = cyc.size();
            if (!cyc.empty())
                ++cnt, cycsz.push_back(cyc.size());
            for (int i = p.size() - 1; i > 0; i--)
                id[p[i] - 1] = id[p[i]] - 1, hei[p[i] - 1] = hei[p[i]] + 1;
        }
        int jump(int u, int k) {
            for (int b = 0; k > 0; b++) {
                if (k & 1) u = cht[u][b];
                k >>= 1;
            }
            return u;
        }
    };
};

```

3 Data Structure

3.1 Segment Tree [d41d8c]

```

template<class Info, class Tag = bool()>
struct SegmentTree { // [l, r), uncomment /**/ to lazy
    int n;
    vector<Info> info;
    vector<Tag> tag;
    /**
    template<class T>
    SegmentTree(const vector<T> &init) {
        n = init.size();
        info.assign(4 << __lg(n), Info());
        /**
        tag.assign(4 << __lg(n), Tag());
        */
        function<void(
            int, int, int)> build = [&](int p, int l, int r) {
            if (r - l == 1) {
                info[p] = init[l];
                return;
            }

```

```

                }
                int m = (l + r) / 2;
                build(2 * p, l, m);
                build(2 * p + 1, m, r);
                pull(p);
            };
            build(1, 0, n);
        }
        void pull(int p) {
            info[p] = info[2 * p] + info[2 * p + 1];
        }
        /**
        void apply(int p, int l, int r, const Tag &v) {
            info[p].apply(l, r, v);
            tag[p].apply(v);
        }
        void push(int p, int l, int r) {
            int m = (l + r) / 2;
            if (r - l >= 1) {
                apply(2 * p, l, m, tag[p]);
                apply(2 * p + 1, m, r, tag[p]);
            }
            tag[p] = Tag();
        }
        /**
        void modify(int p, int l, int r, int x, const Info &v) {
            if (r - l == 1) {
                info[p] = v;
                return;
            }
            int m = (l + r) / 2;
            /**
            push(p, l, r);
            */
            if (x < m) {
                modify(2 * p, l, m, x, v);
            } else {
                modify(2 * p + 1, m, r, x, v);
            }
            pull(p);
        }
        void modify(int p, const Info &i) {
            modify(1, 0, n, p, i);
        }
        Info query(int p, int l, int r, int ql, int qr) {
            if (qr <= l || ql >= r) return Info();
            if (ql <= l && r <= qr) return info[p];
            int m = (l + r) / 2;
            /**
            push(p, l, r);
            */
            return query(2 * p, l, m, ql, qr) + query(2 * p + 1, m, r, ql, qr);
        }
        Info query(int ql, int qr) {
            return query(1, 0, n, ql, qr);
        }
        /**
        void rangeApply
        (int p, int l, int r, int ql, int qr, const Tag &v) {
            if (qr <= l || ql >= r) return;
            if (ql <= l && r <= qr) {
                apply(p, l, r, v);
                return;
            }
            int m = (l + r) / 2;
            push(p, l, r);
            rangeApply(2 * p, l, m, ql, qr, v);
            rangeApply(2 * p + 1, m, r, ql, qr, v);
            pull(p);
        }
        void rangeApply(int l, int r, const Tag &v) {
            rangeApply(1, 0, n, l, r, v);
        }
        /**
        template<class F> // 尋找區間內，第一個符合條件的
        int findFirst
        (int p, int l, int r, int x, int y, F &&pred) {
            if (l >= y || r <= x) return -1;
            if (l >= x && r <= y && !pred(info[p])) return -1;
            if (r - l == 1) return l;
            int m = (l + r) / 2;
            /**
            push(p, l, r);
            */
            int res = findFirst(2 * p, l, m, x, y, pred);
            if (res == -1)
                res = findFirst(2 * p + 1, m, r, x, y, pred);
            return res;
        }
        template<class F> // 若要找 last，先右子樹遞迴即可
        int findFirst(int l, int r, F &&pred) {
            return findFirst(1, 0, n, l, r, pred);
        }
    };
};
// 有些 Tag 不用 push 例如 sweepLine
/**
struct Tag {
    int setVal = 0;
    int add = 0;
    void apply(const Tag &t) & {

```

```

        if (t.setVal) {
            setVal = t.setVal;
            add = t.add;
        } else {
            add += t.add;
        }
    }
};
*/
struct Info {
    ll sum = 0;
    /*
    void apply(int l, int r, const Tag &t) & {
        if (t.setVal) {
            sum = (r - l) * t.setVal;
        }
        sum += (r - l) * t.add;
    }
    */
    // 部分 assignment 使用
    // Info &operator=(const Info &i) & {
    //     return *this;
    // }
    Info &operator=(const ll &x) & {
        sum = x;
        return *this;
    }
};
Info operator+(const Info &a, const Info &b) {
    Info c;
    c.sum = a.sum + b.sum;
    return c;
}

```

3.2 Persistent Segment Tree [d41d8c]

```

template<class Info>
struct PST {
    struct Node {
        Info info = Info();
        int lc = 0, rc = 0;
    };
    int n;
    vector<Node> nd;
    vector<int> rt;
    template<class T>
    PST(const vector<T> &init) {
        n = init.size();
        nd.assign(1, Node());
        rt.clear();
        function<int(int, int)> build = [&](int l, int r) {
            int id = nd.size();
            nd.emplace_back();
            if (r - l == 1) {
                nd[id].info = init[l];
                return id;
            }
            int m = (l + r) >> 1;
            nd[id].lc = build(l, m);
            nd[id].rc = build(m, r);
            pull(nd[id]);
            return id;
        };
        rt.push_back(build(0, n));
    }
    void pull(Node &t) {
        t.info = nd[t.lc].info + nd[t.rc].info;
    }
    int copy(int t) { // copy 一個 node
        nd.push_back(nd[t]);
        return nd.size() - 1;
    }
    int generate() { // 創立新節點
        nd.emplace_back();
        return nd.size() - 1;
    }
    int modify(int t, int l, int r, int x, const Info &v) {
        t = t ? copy(t) : generate();
        if (r - l == 1) {
            nd[t].info = v;
            return t;
        }
        int m = (l + r) / 2;
        if (x < m) {
            nd[t].lc = modify(nd[t].lc, l, m, x, v);
        } else {
            nd[t].rc = modify(nd[t].rc, m, r, x, v);
        }
        pull(nd[t]);
        return t;
    }
    void modify(int ver, int p, const Info &i) {
        if (int(rt.size()) <= ver) rt.resize(ver + 1);
        rt[ver] = modify(rt[ver], 0, n, p, i);
    }
    Info query(int t, int l, int r, int ql, int qr) {
        if (l >= qr || r <= ql) return Info();
        if (ql <= l && r <= qr) return nd[t].info;
        int m = (l + r) / 2;
        return query(nd[t].lc, l, m, ql, qr) + query(nd[t].rc, m, r, ql, qr);
    }
}

```

```

Info query(int ver, int ql, int qr) {
    return query(rt[ver], 0, n, ql, qr);
}
void createVersion(int ori_ver) {
    rt.push_back(copy(rt[ori_ver]));
}
void reserve(int n, int q) {
    nd.reserve(n + q * (2 * __lg(n) + 1));
    rt.reserve(q + 1);
}
void resize(int n) { rt.resize(n); }
};
struct Info {
    ll sum = 0;
};
Info operator+(const Info &a, const Info &b) {
    return { a.sum + b.sum };
}

```

3.3 Static Kth-element [d41d8c]

```

template<class T> struct StaticKth : PST<int> {
    int dct(T x) {
        return lower_bound(s.begin(), s.end(), x) - s.begin();
    }
    vector<T> v, s; // array, sorted
    map<T, int> cnt;
    StaticKth(const vector<T> &v_) {
        s = v = v_;
        sort(s.begin(), s.end());
        s.resize(unique(s.begin(), s.end()) - s.begin());
        init(s.size());
        for (int i = 0; i < v.size(); i++) {
            createVersion(i);
            int d = dct(v[i]);
            modify(i + 1, d, ++cnt[d]);
        }
    }
    int work(int a, int b, int l, int r, int k) {
        if (r - l == 1) return l;
        int x = nd[nd[b].lc].info - nd[nd[a].lc].info;
        int m = (l + r) / 2;
        if (x >= k) {
            return work(nd[a].lc, nd[b].lc, l, m, k);
        } else {
            return work(nd[a].rc, nd[b].rc, m, r, k - x);
        }
    }
    int work(int l, int r, int k) { // [l, r], k > 0
        return s[work(rt[l], rt[r], 0, n, k)];
    }
};

```

3.4 Dynamic Kth-element [d41d8c]

```

// Fenwick(rt-indexed) 包線段樹
template<class T>
struct DynamicKth : PST<int> {
    int dct(T x) {
        return lower_bound(s.begin(), s.end(), x) - s.begin();
    }
    vector<T> v, s; // array, sorted
    DynamicKth(const vector<T> &v_, const vector<T> &s_)
        : PST<int>(vector<int>(s_.size(), 0)) {
        assert(is_sorted(s_.begin(), s_.end()));
        v = v_, s = s_;
        rt.resize(v.size());
        for (int i = 0; i < v.size(); i++) add(i, dct(v[i]), 1);
    }
    int modify(int t, int l, int r, int x, int v) {
        t = t ? t : generate();
        if (r - l == 1) {
            nd[t].info += v;
            return t;
        }
        int m = (l + r) / 2;
        if (x < m) {
            nd[t].lc = modify(nd[t].lc, l, m, x, v);
        } else {
            nd[t].rc = modify(nd[t].rc, m, r, x, v);
        }
        pull(nd[t]);
        return t;
    }
    void add(int p, int x, int val) {
        for (int i = p + 1; i <= rt.size(); i += i & -i)
            rt[i - 1] = modify(rt[i - 1], 0, s.size(), x, val);
    }
    void modify(int p, int y) {
        add(p, dct(v[p]), -1);
        v[p] = y;
        add(p, dct(v[p]), 1);
    }
    int work(
        vector<int> &a, vector<int> &b, int l, int r, int k) {
        if (r - l == 1) return l;
        int m = (l + r) / 2;
        int res = 0;
        for (auto x : a) res -= nd[nd[x].lc].info;
        for (auto x : b) res += nd[nd[x].lc].info;
        if (res >= k) {

```

```

        for (auto &x : a) x = nd[x].lc;
        for (auto &x : b) x = nd[x].lc;
        return work(a, b, l, m, k);
    } else {
        for (auto &x : a) x = nd[x].rc;
        for (auto &x : b) x = nd[x].rc;
        return work(a, b, m, r, k - res);
    }
}
int work(int l, int r, int k) { // [l, r), k > 0
    vector<int> a, b;
    for (int i = l; i > 0; i -= i & -i)
        a.push_back(rt[i - 1]);
    for (int i = r; i > 0; i -= i & -i)
        b.push_back(rt[i - 1]);
    return s[work(a, b, 0, s.size(), k)];
}
};

```

3.5 Fenwick [d41d8c]

```

template<class T> struct Fenwick {
    int n; vector<T> a;
    Fenwick(int n) : n(n), a(n) {}
    void add(int x, const T &v) {
        for (int i = x + 1; i <= n; i += i & -i)
            a[i - 1] = a[i - 1] + v;
    }
    T sum(int x) {
        T ans{};
        for (int i = x; i > 0; i -= i & -i)
            ans = ans + a[i - 1];
        return ans;
    }
    T rangeSum(int l, int r) {
        return sum(r) - sum(l);
    }
    int select(const T &k, int start = 0) {
        // 找到最小的 x, 使得 sum(x + 1) - sum(start) > k
        // prefix sum 要有單調性
        int x = 0; T cur = -sum(start);
        for (int i = 1 << __lg(n); i; i /= 2) {
            if (x + i <= n && cur + a[x + i - 1] <= k) {
                x += i;
                cur = cur + a[x - 1];
            }
        }
        return x;
    }
};

template<class T> struct TwoDFenwick {
    int nx, ny; // row, col 個數
    vector<vector<T>> a;
    TwoDFenwick(int nx, int ny) : nx(nx), ny(ny) {
        a.assign(nx, vector<T>(ny, T{}));
    }
    void add(int x, int y, const T &v) {
        for (int i = x + 1; i <= nx; i += i & -i)
            for (int j = y + 1; j <= ny; j += j & -j)
                a[i - 1][j - 1] = a[i - 1][j - 1] + v;
    }
    T sum(int x, int y) {
        T ans{};
        for (int i = x; i > 0; i -= i & -i)
            for (int j = y; j > 0; j -= j & -j)
                ans = ans + a[i - 1][j - 1];
        return ans;
    }
    T rangeSum(int lx, int ly, int rx, int ry) {
        return sum(
            rx, ry) - sum(lx, ry) - sum(rx, ly) + sum(lx, ly);
    }
};

```

3.6 Range Fenwick [d41d8c]

```

template<class T> struct RangeFenwick {
    int n;
    vector<T> d, di;
    RangeFenwick(int n) : n(n), d(n), di(n) {}
    void add(int x, const T &v) {
        T vi = v * (x + 1);
        for (int i = x + 1; i <= n; i += i & -i) {
            d[i - 1] = d[i - 1] + v;
            di[i - 1] = di[i - 1] + vi;
        }
    }
    void rangeAdd(int l, int r, const T &v) {
        add(l, v); add(r, -v);
    }
    T sum(int x) { // 左閉右開查詢
        T ans{};
        for (int i = x; i > 0; i -= i & -i) {
            ans = ans + T(x + 1) * d[i - 1];
            ans = ans - di[i - 1];
        }
        return ans;
    }
    T rangeSum(int l, int r) { // 左閉右開查詢
        return sum(r) - sum(l);
    }
};

```

```

int select(const T &k, int start = 0) {
    // 找到最小的 x, 使得 sum(x + 1) - sum(start) > k
    int x = 0; T cur = -sum(start);
    for (int i = 1 << __lg(n); i; i /= 2) {
        if (x + i <= n) {
            T val = T(
                x + i + 1) * d[x + i - 1] - di[x + i - 1];
            if (cur + val <= k) {
                x += i;
                cur = cur + val;
            }
        }
    }
    return x;
}

template<class T> struct RangeTwoDFenwick {
    int nx, ny; // row, col 個數
    vector<vector<T>> d, di, dj, dij;
    RangeTwoDFenwick(int x, int y) : nx(x), ny(y) {
        d.assign(nx, vector<T>(ny, T{}));
        di.assign(nx, vector<T>(ny, T{}));
        dj.assign(nx, vector<T>(ny, T{}));
        dij.assign(nx, vector<T>(ny, T{}));
    }
    void add(int x, int y, const T &v) {
        T vi = v * (x + 1);
        T vj = v * (y + 1);
        T vij = v * (x + 1) * (y + 1);
        for (int i = x + 1; i <= nx; i += i & -i) {
            for (int j = y + 1; j <= ny; j += j & -j) {
                d[i - 1][j - 1] = d[i - 1][j - 1] + v;
                di[i - 1][j - 1] = di[i - 1][j - 1] + vi;
                dj[i - 1][j - 1] = dj[i - 1][j - 1] + vj;
                dij[i - 1][j - 1] = dij[i - 1][j - 1] + vij;
            }
        }
    }
    void rangeAdd(int lx, int ly, int rx, int ry, const T &v) {
        add(rx, ry, v);
        add(lx, ry, -v);
        add(rx, ly, -v);
        add(lx, ly, v);
    }
    T sum(int x, int y) { // 左閉右開查詢
        T ans{};
        for (int i = x; i > 0; i -= i & -i) {
            for (int j = y; j > 0; j -= j & -j) {
                ans = ans
                    + T(x * y + x + y + 1) * d[i - 1][j - 1];
                ans = ans - T(y + 1) * di[i - 1][j - 1];
                ans = ans - T(x + 1) * dj[i - 1][j - 1];
                ans = ans + dij[i - 1][j - 1];
            }
        }
        return ans;
    }
    T rangeSum(
        int lx, int ly, int rx, int ry) { // 左閉右開查詢
        return sum(
            rx, ry) - sum(lx, ry) - sum(rx, ly) + sum(lx, ly);
    }
};

```

3.7 KDTree [d41d8c]

```

struct Info {
    static constexpr int DIM = 2;
    array<int, DIM> x, l, r;
    int v = 0, sum = 0;
    void pull(const Info &l, const Info &r) {
        sum = v + l.sum + r.sum;
    }
};

struct KDTree {
    static constexpr int DIM = Info::DIM;
    vector<Info> info;
    vector<int> rt, l, r, p;
    KDTree(int n) : n(n), lg(
        __lg(n)), info(1), rt(lg + 1), l(n + 1), r(n + 1) {}
    void pull(int p) {
        info[p].l = info[p].R = info[p].x;
        info[p].pull(info[l[p]], info[r[p]]);
        for (int ch : {l[p], r[p]}) {
            if (!ch) continue;
            for (int k = 0; k < DIM; k++) {
                info[p]
                    .l[k] = min(info[p].l[k], info[ch].l[k]);
                info[p]
                    .r[k] = max(info[p].r[k], info[ch].r[k]);
            }
        }
    }
    int rebuild(int l, int r, int dep = 0) {
        if (r == l) return 0;
        int m = (l + r) / 2;
        nth_element(
            p.begin() + l, p.begin() + m, p.begin() + r,
            [&](int x, int y)
                { return info[x].x[dep] < info[y].x[dep]; });
        int x = p[m];
    }
};

```

```

    this->l[x] = rebuild(l, m, (dep + 1) % DIM);
    this->r[x] = rebuild(m + 1, r, (dep + 1) % DIM);
    pull(x);
    return x;
}
void append(int &x) {
    if (!x) return;
    p.push_back(x);
    append(l[x]);
    append(r[x]);
    x = 0;
}
void addNode(const Info &i) {
    p.assign(1, info.size());
    info.push_back(i);
    for (int j = 0; j < j++;) {
        if (!rt[j]) {
            rt[j] = rebuild(0, p.size());
            break;
        } else {
            append(rt[j]);
        }
    }
}
Info query(int p,
    const array<int, DIM> &l, const array<int, DIM> &r) {
    if (!p) return Info();
    bool inside = true;
    for (int k = 0; k < DIM; k++) {
        inside &= (
            l[k] <= info[p].L[k] && info[p].R[k] <= r[k]);
    }
    if (inside) return info[p];
    for (int k = 0; k < DIM; k++) {
        if (info[p].R[k] < l[k] || r[k] < info[p].L[k]) {
            return Info();
        }
    }
    Info ans;
    inside = true;
    for (int k = 0; k < DIM; k++) {
        inside &= (
            l[k] <= info[p].x[k] && info[p].x[k] <= r[k]);
    }
    if (inside) ans = info[p];
    ans.pull(
        query(this->l[p], l, r), query(this->r[p], l, r));
    return ans;
}
Info query
    (const array<int, DIM> &l, const array<int, DIM> &r) {
    Info res;
    for (int i = 0; i <= lg; i++) {
        res.pull(res, query(rt[i], l, r));
    }
    return res;
}
};

```

3.8 Treap [d41d8c]

```

template<class Info, class Tag = bool()>
struct Treap { // 0 -> initial root
    vector<Info> info;
    // vector<Tag> tag;
    vector<int> siz, par, rev, pri;
    vector<array<int, 2>> ch;
    Treap(int n) : info(n + 1), siz(n
        + 1), par(n + 1), rev(n + 1), pri(n + 1), ch(n + 1) {
        // tag.resize(n + 1);
        for (int i = 1; i <= n; i++)
            siz[i] = 1, pri[i] = gen();
    }
    // void apply(int t, const Tag &v) {
    //     info[t].apply(siz[t], v);
    //     tag[t].apply(v);
    // }
    void push(int t) {
        if (rev[t]) {
            swap(ch[t][0], ch[t][1]);
            if (ch[t][0]) rev[ch[t][0]] ^= 1;
            if (ch[t][1]) rev[ch[t][1]] ^= 1;
            rev[t] = 0;
        }
        // apply(ch[t][0], tag[t]);
        // apply(ch[t][1], tag[t]);
        // tag[t] = Tag();
    }
    void pull(int t) {
        siz[t] = 1 + siz[ch[t][0]] + siz[ch[t][1]];
        info[t].pull(info[ch[t][0]], info[ch[t][1]]);
    }
    int merge(int a, int b) {
        if (!a || !b) return a ? a : b;
        push(a), push(b);
        if (pri[a] > pri[b]) {
            ch[a][1] = merge(ch[a][1], b);
            pull(a); return a;
        } else {
            ch[b][0] = merge(a, ch[b][0]);
            pull(b); return b;
        }
    }
};

```

```

}
pair<int, int> split(int t, int k) {
    if (!t) return {0, 0};
    push(t);
    if (siz[ch[t][0]] >= k) {
        auto [a, b] = split(ch[t][0], k);
        ch[t][0] = b, pull(t);
        return {a, t};
    } else {
        auto [a
            , b] = split(ch[t][1], k - siz[ch[t][0]] - 1);
        ch[t][1] = a, pull(t);
        return {t, b};
    }
}
template<class F> // 尋找區間內，第一個符合條件的
int findFirst(int t, F &&pred) {
    if (!t) return 0;
    push(t);
    if (!pred(info[t])) return 0;
    int idx = findFirst(ch[t][0], pred);
    if (!idx) idx
        = 1 + siz[ch[t][0]] + findFirst(ch[t][1], pred);
    return idx;
}
int getPos(int rt, int t) { // get t's index in array
    int res = siz[t] + 1;
    while (t != rt) {
        int p = par[t];
        if (ch[p][1] == t) res += siz[ch[p][0]] + 1;
        t = p;
    }
    return res;
}
void getArray(int t, vector<Info> &a) {
    if (!t) return;
    push(t);
    getArray(ch[t][0], a);
    a.push_back(info[t]);
    getArray(ch[t][1], a);
}
};
struct Tag {
    int setVal; ll add;
    void apply(const Tag &t) {
        if (t.setVal) {
            setVal = t.setVal;
            add = t.add;
        } else {
            add += t.add;
        }
    }
};
struct Info {
    ll val, sum;
    void apply(int siz, const Tag &t) {
        if (t.setVal) {
            val = t.setVal;
            sum = 1LL * siz * t.setVal;
        }
        val += t.add;
        sum += 1LL * siz * t.add;
    }
    void pull(const Info &l, const Info &r) {
        sum = val + l.sum + r.sum;
    }
};

```

3.9 RMQ [d41d8c]

```

template<class T, class F = less<T>>
struct RMQ { // [l, r)
    int n;
    F cmp = F();
    vector<vector<T>> g;
    RMQ() {}
    RMQ(const vector<T> &a, F cmp = F()) : cmp(cmp) {
        init(a);
    }
    void init(const vector<T> &a) {
        n = a.size();
        int lg = __lg(n);
        g.resize(lg + 1);
        g[0] = a;
        for (int j = 1; j <= lg; j++) {
            g[j].resize(n - (1 << j) + 1);
            for (int i = 0; i <= n - (1 << j); i++)
                g[j][i] = min(g[j - 1][i], g[j - 1][i + (1 << (j - 1))], cmp);
        }
    }
    T operator()(int l, int r) {
        assert(0 <= l && l < r && r <= n);
        int lg = __lg(r - l);
        return min(g[lg][l], g[lg][r - (1 << lg)], cmp);
    }
};

```

3.10 Mo [d41d8c]

```

struct Query { int id, l, r; };
void mo(vector<Query> &q) {

```

```

int blk = sqrt(q.size());
sort(q.begin
    (), q.end(), [&](const Query &a, const Query &b) {
        int x = a.l / blk, y = b.l / blk;
        return x == y ? a.r < b.r : x < y;
    });
}
int nl = 0, nr = -1;
for (auto [id, l, r] : qry) {
    while (nr < r) nr++, addR();
    while (l < nl) nl--, addL();
    while (r < nr) delR(), nr--;
    while (nl < l) delL(), nl++;
}
}

```

4 Flow Matching

4.1 Dinic [d41d8c]

```

template<class T>
struct Dinic {
    // argument time: O(VE), O(E) for unit capacity,
    // argument number: O(V), min(O(E^0.5), O(V^2/3)) for unit
    // capacity, O(V^0.5) for deg_in(u) or deg_out(u) <= 1
    // so bipartite matching: O(EV^0.5)
    struct Edge {
        int to;
        T f, cap; // 流量跟容量
    };
    int n, m, s, t;
    const T INF_FLOW = numeric_limits<T>::max() / 2;
    vector<vector<int>>> g;
    vector<Edge> e;
    vector<int> h, cur;
    Dinic(int n) : n(n), m(0), g(n), h(n), cur(n) {}
    void addEdge(int u, int v, T cap) {
        e.push_back({v, 0, cap});
        e.push_back({u, 0, 0});
        g[u].push_back(m++);
        g[v].push_back(m++);
    }
    bool bfs() {
        fill(h.begin(), h.end(), -1);
        h[s] = 0; queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (int id : g[u]) {
                auto [v, f, cap] = e[id];
                if (f == cap) continue;
                if (h[v] == -1) {
                    h[v] = h[u] + 1;
                    if (v == t) return true;
                    q.push(v);
                }
            }
        }
        return false;
    }
    T dfs(int u, T flow) {
        if (flow == 0) return 0;
        if (u == t) return flow;
        for (int &i = cur[u]; i < g[u].size(); i++) {
            int j = g[u][i];
            auto [v, f, cap] = e[j];
            if (h[u] + 1 != h[v]) continue;
            if (f == cap) continue;
            T mn = dfs(v, min(flow, cap - f));
            if (mn > 0) {
                e[j].f += mn;
                e[j ^ 1].f -= mn;
                return mn;
            }
        }
        return 0;
    }
    T work(int s_, int t_) {
        s = s_; t = t_; T f = 0;
        while (bfs()) {
            fill(cur.begin(), cur.end(), 0);
            while (true) {
                T res = dfs(s, INF_FLOW);
                if (res == 0) break;
                f += res;
            }
        }
        return f;
    }
    void reuse(int n_) { // 走殘留網路, res += f
        while (n < n_) {
            g.emplace_back();
            h.emplace_back();
            cur.emplace_back();
            n += 1;
        }
    }
};

```

4.2 Min Cut [d41d8c]

```

void minCut(int n, int m, Dinic<int> d) {

```

```

    int ans = d.work(0, n - 1);
    vector<int> vis(n);
    auto dfs = [&](auto self, int u) -> void {
        if (vis[u]) continue;
        vis[u] = 1;
        for (int id : d.g[u]) {
            auto [to, f, cap] = d.e[id];
            if (cap - f > 0) self(self, to);
        }
    };
    dfs(dfs, 0);
    for (int i = 0; i < n; i++) {
        if (!vis[i]) continue;
        for (int id : d.g[i]) {
            if (id & 1) continue;
            auto e = d.e[id];
            if (!vis[e.to])
                cout << i + 1 << " " << e.to + 1 << " | n";
        }
    }
}

```

4.3 MCMF [d41d8c]

```

template<class Tf, class Tc>
struct MCMF {
    struct Edge {
        int to;
        Tf f, cap; // 流量跟容量
        Tc cost;
    };
    int n, m, s, t;
    const Tf INF_FLOW = numeric_limits<Tf>::max() / 2;
    const Tc INF_COST = numeric_limits<Tc>::max() / 2;
    vector<Edge> e;
    vector<vector<int>>> g;
    vector<Tc> dis, pot;
    vector<int> rt, inq;
    MCMF(int n) : n(n), m(0), g(n) {}
    void addEdge(int u, int v, Tf cap, Tc cost) {
        e.push_back({v, 0, cap, cost});
        e.push_back({u, 0, 0, -cost});
        g[u].push_back(m++);
        g[v].push_back(m++);
    }
    bool spfa() { // O(FVE)
        dis.assign(n, INF_COST);
        rt.assign(n, -1), inq.assign(n, 0);
        queue<int> q; q.push(s);
        dis[s] = 0, inq[s] = 1;
        while (!q.empty()) {
            int u = q.front(); q.pop();
            inq[u] = 0;
            for (int id : g[u]) {
                auto [v, f, cap, cost] = e[id];
                Tc ndis = dis[u] + cost + pot[u] - pot[v];
                if (f < cap && dis[v] > ndis) {
                    dis[v] = ndis, rt[v] = id;
                    if (!inq[v])
                        q.push(v), inq[v] = 1;
                }
            }
        }
        return dis[t] != INF_COST;
    }
    bool dijkstra() { // O(FE log V)
        dis.assign(n, INF_COST), rt.assign(n, -1);
        priority_queue<pair<Tc, int>,
            vector<pair<Tc, int>>, greater<pair<Tc, int>>> pq;
        dis[s] = 0; pq.emplace(dis[s], s);
        while (!pq.empty()) {
            auto [d, u] = pq.top(); pq.pop();
            if (dis[u] < d) continue;
            for (int id : g[u]) {
                auto [v, f, cap, cost] = e[id];
                Tc ndis = dis[u] + cost + pot[u] - pot[v];
                if (f < cap && dis[v] > ndis) {
                    dis[v] = ndis, rt[v] = id;
                    pq.emplace(ndis, v);
                }
            }
        }
        return dis[t] != INF_COST;
    }
    pair<Tf, Tc> work(int s_, int t_, Tf need) {
        s = s_, t = t_; pot.assign(n, 0);
        Tf flow{}; Tc cost{}; int fr = 0;
        while (fr++ ? dijkstra() : spfa()) {
            for (int i = 0; i < n; i++)
                dis[i] += pot[i] - pot[s];
            Tf f = need;
            for (int i = t; i != s; i = e[rt[i] ^ 1].to)
                f = min(f, e[rt[i]].cap - e[rt[i]].f);
            for (int i = t; i != s; i = e[rt[i] ^ 1].to)
                e[rt[i]].f += f, e[rt[i] ^ 1].f -= f;
            flow += f, need -= f;
            cost += f * dis[t];
            swap(dis, pot);
            if (need == 0) break;
        }
        return {flow, cost};
    }
};

```

```

void reset() {
    for (int i = 0; i < m; i++) e[i].f = 0;
}
};

```

4.4 Hungarian [d41d8c]

```

struct Hungarian { // 0-based, 0(VE)
    int n, m;
    vector<vector<int>> adj;
    vector<int> used, vis;
    vector<pair<int, int>> match;
    Hungarian(int n, int m) : n(n), m(m) {
        adj.assign(n + m, {});
        used.assign(n + m, -1);
        vis.assign(n + m, 0);
    }
    void addEdge(int u, int v) {
        adj[u].push_back(n + v);
        adj[n + v].push_back(u);
    }
    bool dfs(int u) {
        int sz = adj[u].size();
        for (int i = 0; i < sz; i++) {
            int v = adj[u][i];
            if (vis[v] == 0) {
                vis[v] = 1;
                if (used[v] == -1 || dfs(used[v])) {
                    used[v] = u;
                    return true;
                }
            }
        }
        return false;
    }
    vector<pair<int, int>> work() {
        match.clear();
        used.assign(n + m, -1);
        vis.assign(n + m, 0);
        for (int i = 0; i < n; i++) {
            fill(vis.begin(), vis.end(), 0);
            dfs(i);
        }
        for (int i = n; i < n + m; i++)
            if (used[i] != -1)
                match.emplace_back(used[i], i - n);
        return match;
    }
};

```

4.5 Theorem [d41d8c]

```

// 有向無環圖：
// 最小不相交路徑覆蓋：
// 最小路徑數 = 頂點數 - 最大匹配數
// 最小相交路徑覆蓋：
// 先用
//   Floyd 求傳遞封包，有連邊就建邊，然後再套最小不相交路徑覆蓋
// 二分圖：
// 最小點
//   覆蓋：選出一些點，讓所有邊至少有一個端點在點集中的最少數量
// 最小點覆蓋 = 最大匹配數
// 還原解，flow 的作法是從源點開始 dfs，只走 cap - flow > 0
// 的邊，最後挑選左邊還沒被跑過的點和右邊被跑過的點當作覆蓋的點
// 最少邊覆蓋：選出一些邊，讓所有點都覆蓋到的最少數量
// 最少邊覆蓋 = 點數 - 最大匹配數
// 最大獨立集：選出一些點，使這些點兩兩沒有邊連接的最大數量
// 最大獨立集 = 點數 - 最大匹配數

```

5 String

5.1 Hash [234076]

```

const int D = 59;
vector<int> rollingHash(string &s) {
    vector<int> a {0};
    for (auto c : s)
        a.push_back(mul(a.back(), D) + (c - 'A' + 1));
    return a;
}
int qryHash(vector<int> &h, int l, int r) { // [l, r)
    return sub(h[r], mul(h[l], power(D, r - l)));
}

```

5.2 KMP [e3717b]

```

struct KMP {
    string sub;
    vector<int> fail;
    // fail 存匹配失敗時，移去哪
    // 也就是 sub(0, i) 的最長共同前後綴長度
    // ex : a b c a b c
    //      -1 -1 -1 0 1 2
    KMP(const string &sub_) { build(sub_); }
    vector<int> build(const string &sub_) {
        sub = sub_; fail.resize(sub.size(), -1);
        for (int i = 1; i < sub.size(); i++) {
            int now = fail[i - 1];

```

```

            while (now != -1 && sub[now + 1] != sub[i])
                now = fail[now];
            if (sub[now + 1] == sub[i])
                fail[i] = now + 1;
        }
        return fail;
    }
    vector<int> match(const string &s) {
        vector<int> match;
        for (int i = 0, now = -1; i < s.size(); i++) {
            while (s[i] != sub[now + 1] && now != -1)
                now = fail[now];
            if (s[i] == sub[now + 1]) now++;
            if (now + 1 == sub.size()) {
                match.push_back(i - now);
                now = fail[now];
            }
        }
        return match;
    }
};

```

5.3 Z Function [5b63dc]

```

// z[i] 表示 s 和 s[i, n - 1] (以 s[i] 開頭的后綴)
// 的最長公共前綴 (LCP) 的長度
vector<int> Z(const string &s) {
    int n = s.size();
    vector<int> z(n);
    z[0] = n; // lcp(s, s), -1 or n
    for (int i = 1, j = 1; i < n; i++) {
        z[i] = max(0, min(j + z[j] - i, z[i - j]));
        while (i + z[i] < n && s[z[i]] == s[i + z[i]]) z[i]++;
        if (i + z[i] > j + z[j]) j = i;
    }
    return z;
}

```

5.4 Manacher [1eb30d]

```

// 找到對於每個位置的迴文半徑
vector<int> manacher(const string &s) {
    string t = "#";
    for (auto c : s) t = t + c + '#';
    int n = t.size();
    vector<int> r(n);
    for (int i = 0,
        j = 0; i < n; i++) { // i 是中心, j 是最長回文串中心
        if (2 * j - i >= 0 && j + r[j] > i)
            r[i] = min(r[2 * j - i], j + r[j] - i);
        while (i - r[i] >=
            0 && i + r[i] < n && t[i - r[i]] == t[i + r[i]])
            r[i] += 1;
        if (i + r[i] > j + r[j]) j = i;
    }
    return r;
}
// # a # b # a #
// 1 2 1 4 1 2 1
// # a # b # b # a #
// 1 2 1 2 5 2 1 2 1
// 值 -1 代表原回文字串長度
// (id - val + 1) / 2 可得原字串回文開頭

```

5.5 Trie [6c7186]

```

const int N = 1E7; // 0 -> initial state
const int ALPHABET_SIZE = 26;
int tot = 0;
int trie[N][ALPHABET_SIZE], cnt[N];
void reset() {
    tot = 0, fill_n(trie[0], ALPHABET_SIZE, 0);
}
int newNode() {
    int x = ++tot;
    cnt[x] = 0, fill_n(trie[x], ALPHABET_SIZE, 0);
    return x;
}
void add(const string &s) {
    int p = 0;
    for (auto c : s) {
        int &q = trie[p][c - 'a'];
        if (!q) q = newNode();
        p = q;
    }
    cnt[p] += 1;
}
int find(const string &s) {
    int p = 0;
    for (auto c : s) {
        int q = trie[p][c - 'a'];
        if (!q) return 0;
        p = q;
    }
    return cnt[p];
}

```

5.6 SA [b04578]

```

struct SuffixArray {
    int n;

```

```

vector<int> sa, rk, lc;
// n: 字串長度
// sa: 後綴數組, sa[i] 表示第 i 小的後綴的起始位置
// rk: 排名數組, rk[i] 表示從位置 i 開始的後綴的排名
// lc: LCP
// 數組, lc[i] 表示 sa[i] 和 sa[i + 1] 的最長公共前綴長度
SuffixArray(const string &s) {
    n = s.length();
    sa.resize(n);
    lc.resize(n - 1);
    rk.resize(n);
    iota(sa.begin(), sa.end(), 0);
    sort(sa.begin(), sa.end(), [&](int a, int b) { return s[a] < s[b]; });
    rk[sa[0]] = 0;
    for (int i = 1; i < n; i++)
        rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
    int k = 1;
    vector<int> tmp, cnt(n);
    tmp.reserve(n);
    while (rk[sa[n - 1]] < n - 1) {
        tmp.clear();
        for (int i = 0; i < k; i++) tmp.push_back(n - k + i);
        for (auto i : sa) if (i >= k) tmp.push_back(i - k);
        fill(cnt.begin(), cnt.end(), 0);
        for (int i = 0; i < n; i++) cnt[rk[i]]++;
        for (int i = 1; i < n; i++) cnt[i] += cnt[i - 1];
        for (int i = n - 1; i >= 0; i--) sa[--cnt[rk[tmp[i]]]] = tmp[i];
        swap(rk, tmp); rk[sa[0]] = 0;
        for (int i = 1; i < n; i++) rk[sa[i]] = rk[sa[i - 1]] +
            (tmp[sa[i - 1]] < tmp[sa[i]] || sa[i - 1] + k
             == n || tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
        k *= 2;
    }
    for (int i = 0, j = 0; i < n; i++) {
        if (rk[i] == 0) {
            j = 0;
        } else {
            for (j -= 1; j > 0; j++)
                if (i + j < n && sa[rk[i] - 1] + j < n
                    && s[i + j] == s[sa[rk[i] - 1] + j]; j++);
            lc[rk[i] - 1] = j;
        }
    }
};
RMQ<int> rmq(sa, lc);
auto lcp = [&](int i, int j) { // [i, j]
    i = sa.rk[i], j = sa.rk[j];
    if (i > j) swap(i, j);
    assert(i != j);
    return rmq(i, j);
};

```

5.7 AC [5d4167]

```

struct AC {
    static constexpr int ALPHABET_SIZE = 26;
    struct Node {
        int fail; // 指向最長後綴
        int cnt; // 有多少模式字串是自己的後綴
        array<int, ALPHABET_SIZE> ch, next;
        // next 是補全後的轉移
    };
    vector<Node> t;
    AC() : t(1) {}
    int newNode() {
        t.emplace_back();
        return t.size() - 1;
    }
    int insert(const string &s) {
        int u = 0;
        for (char c : s) {
            if (!t[u].ch[c - 'a'])
                t[u].ch[c - 'a'] = newNode();
            u = t[u].ch[c - 'a'];
        }
        t[u].cnt++;
        return u;
    }
    void build() {
        queue<int> q;
        for (int c = 0; c < ALPHABET_SIZE; c++) {
            if (t[0].ch[c]) {
                q.push(t[0].ch[c]);
                t[0].next[c] = t[0].ch[c];
            }
        }
        while (!q.empty()) {
            int u = q.front(); q.pop();
            for (int c = 0; c < ALPHABET_SIZE; c++) {
                if (t[u].ch[c]) {
                    int v = t[u].ch[c], f = t[u].fail;
                    while (f && !t[f].ch[c]) f = t[f].fail;
                    if (t[f].ch[c]) f = t[f].ch[c];
                    t[v].fail = f;
                    t[v].cnt += t[f].cnt;
                }
            }
        }
    }
};

```

```

        t[u].next[c] = v;
        q.push(v);
    } else {
        t[u].next[c] = t[t[u].fail].next[c];
    }
}
};

```

5.8 SAM [50a2d0]

```

struct SAM {
    // 0 -> initial state
    static constexpr int ALPHABET_SIZE = 26;
    // node -> strings with the same endpos set
    // link -> longest suffix with different endpos set
    // len -> state's longest suffix
    // fpos -> first endpos
    // strlen range -> [len(link) + 1, len]
    struct Node {
        int len, link = -1, fpos;
        array<int, ALPHABET_SIZE> next;
    };
    vector<Node> t;
    SAM() : t(1) {}
    int newNode() {
        t.emplace_back();
        return t.size() - 1;
    }
    int extend(int p, int c) {
        int cur = newNode();
        t[cur].len = t[p].len + 1;
        t[cur].fpos = t[p].len - 1;
        while (p != -1 && t[p].next[c]) {
            t[p].next[c] = cur;
            p = t[p].link;
        }
        if (p == -1) {
            t[cur].link = 0;
        } else {
            int q = t[p].next[c];
            if (t[p].len + 1 == t[q].len) {
                t[cur].link = q;
            } else {
                int r = newNode();
                t[r].len = t[q].len;
                t[r].fpos = t[p].len + 1;
                while (p != -1 && t[p].next[c] == q) {
                    t[p].next[c] = r;
                    p = t[p].link;
                }
                t[q].link = t[cur].link = r;
            }
        }
        return cur;
    }
};
void solve(int n, string s, ll k) { // Substring Order II
    vector<int> last(n + 1);
    SAM sam;
    for (int i = 0; i < n; i++)
        last[i + 1] = sam.extend(last[i], s[i] - 'a');
    int sz = sam.t.size();

    vector<int> cnt(sz); // endpos size
    for (int i = 1; i <= n; i++) cnt[last[i]]++;
    vector<vector<int>> g(sz);
    for (int i = 1; i < sz; i++)
        g[sam.t[i].link].push_back(i);
    auto dfs = [&](auto self, int u) -> void {
        for (auto v : g[u])
            self(self, v), cnt[u] += cnt[v];
    }; dfs(dfs, 0);

    vector<ll> dp(sz, -1);
    // for any path from root
    // , how many substring's prefix is the the path string
    auto rec = [&](auto self, int u) -> ll {
        if (dp[u] != -1) return dp[u];
        dp[u] = cnt[u]; // distinct: = 1
        for (int c = 0; c < SAM::ALPHABET_SIZE; c++) {
            int v = sam.t[u].next[c];
            if (v) dp[u] += self(self, v);
        }
        return dp[u];
    };
    rec(rec, 0);

    int p = 0; string ans;
    while (k > 0) { // 1-based
        for (int c = 0; c < SAM::ALPHABET_SIZE; c++) {
            int v = sam.t[p].next[c];
            if (v) {
                if (k > dp[v]) {
                    k -= dp[v];
                } else {
                    ans.push_back('a' + c);
                    k -= cnt[v]; // distinct: --
                    p = v; break;
                }
            }
        }
    }
}

```

```

    }
    } cout << ans << "\n";
}

```

5.9 Palindrome Tree [e5a1ed]

```

struct PAM {
    // 0 -> even root, 1 -> odd root
    static constexpr int ALPHABET_SIZE = 26;
    // fail -> longest prefix(suffix) palindrome
    // number end at i = end at link[last[i]] + 1
    struct Node {
        int len, fail, cnt;
        array<int, ALPHABET_SIZE> next;
        Node() : len{0}, fail{0}, next{} {}
    };
    vector<int> s;
    vector<Node> t;
    PAM() {
        t.assign(2, Node());
        t[0].len = 0, t[0].fail = 1;
        t[1].len = -1;
    }
    int newNode() {
        t.emplace_back();
        return t.size() - 1;
    }
    int getFail(int p, int i) {
        while (i - t[p].len < 1 || s[i - t[p].len - 1] != s[i])
            p = t[p].fail;
        return p;
    }
    int extend(int p, int c) {
        int i = s.size();
        s.push_back(c);
        p = getFail(p, i);
        if (!t[p].next[c]) {
            int r = newNode();
            int v = getFail(t[p].fail, i);
            t[r].len = t[p].len + 2;
            t[r].fail = t[v].next[c];
            t[p].next[c] = r;
        }
        return p = t[p].next[c];
    }
};

void solve() {
    string s; cin >> s;
    int n = s.length();
    vector<int> last(n + 1);
    last[0] = 1;
    PAM pam;
    for (int i = 0; i < n; i++)
        last[i + 1] = pam.extend(last[i], s[i] - 'a');
    int sz = pam.t.size();
    vector<int> cnt(sz);
    for (int i = 1; i <= n; i++)
        cnt[last[i]]++; // 去重 = 1
    for (int i = sz - 1; i > 1; i--)
        cnt[pam.t[i].fail] += cnt[i];
}

```

5.10 Duval [aed467]

```

// duval_algorithm
// 將字串分解成若干個非嚴格遞減的非嚴格遞增字串
vector<string> duval(string s) {
    int i = 0, n = s.size();
    vector<string> res;
    while (i < n) {
        int k = i, j = i + 1;
        while (s[k] <= s[j] && j < n) {
            if (s[k] < s[j]) k = i;
            else k++;
            j++;
        }
        while (i <= k) {
            res.push_back(s.substr(i, j - k));
            i += j - k;
        }
    }
    return res;
}

// 最小旋轉字串
string minRound(string s) {
    s += s;
    int i = 0, n = s.size(), start = i;
    while (i < n / 2) {
        start = i;
        int k = i, j = i + 1;
        while (s[k] <= s[j] && j < n) {
            if (s[k] < s[j]) k = i;
            else k++;
            j++;
        }
        while (i <= k) i += j - k;
    }
    return s.substr(start, n / 2);
}

```

6 Math

6.1 Mint [d13dad]

```

ll mul(ll a, ll b, ll p) { // P 超過 int 再用，慢
    ll res = a * b - ll(1.L * a * b / p) * p;
    res %= p;
    if (res < 0) res += p;
    return res;
}

template<class T> constexpr T power(T a, ll b) {
    T res {1};
    for (; b > 0; b >= 1, a = a * a)
        if (b & 1) res = res * a;
    return res;
}

template<int P> struct Mint {
    static int Mod;
    static int getMod() { return P > 0 ? P : Mod; }
    static void setMod(int Mod_) { Mod = Mod_; }
    ll x;
    Mint(ll x = 0) : x {norm(x % getMod())} {}
    ll norm(ll x) const {
        if (x < 0) x += getMod();
        if (x >= getMod()) x -= getMod();
        return x;
    }
    explicit operator int() const { return x; }
    Mint operator-() const { return getMod() - x; }
    Mint inv() const { return power(*this, getMod() - 2); }
    Mint operator+(Mint a) const { return x + a.x; }
    Mint operator-(Mint a) const { return x - a.x; }
    Mint operator*(Mint a) const { return x * a.x; }
    Mint operator/(Mint a) const { return *this * a.inv(); }

    Mint &operator+=(Mint a) { return *this = *this + a; }
    Mint &operator-=(Mint a) { return *this = *this - a; }
    Mint &operator*=(Mint a) { return *this = *this * a; }
    Mint &operator/=(Mint a) { return *this = *this / a; }

    friend istream &operator>>(istream &is, Mint &a)
    { ll v; is >> v; a = Mint(v); return is; }
    friend ostream &operator<<(ostream &os, Mint a)
    { return os << a.x; }
    bool operator==(Mint y) const { return x == y.x; }
    bool operator!=(Mint y) const { return x != y.x; }
};

template<> int Mint<0>::Mod = 998244353;
constexpr int P = 1E9 + 7;
using Z = Mint<P>;

```

6.2 Combination [0981be]

```

// C(n, m) = C(n, m - 1) * (n - m + 1) / m
// C(n + 1, m) = C(n, m) + C(n, m - 1)
// C(n, k) = 1 (mod 2) <=> all bit of k <= all bit of n in binary
struct Comb {
    int n;
    vector<Z> _fac, _invfac, _inv;
    Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}
    Comb(int n) : Comb() { init(n); }
    void init(int m) {
        m = min(m, Z::getMod() - 1);
        if (m <= n) return;
        _fac.resize(m + 1);
        _invfac.resize(m + 1);
        _inv.resize(m + 1);
        for (int i = n + 1; i <= m; i++) {
            _fac[i] = _fac[i - 1] * i;
        }
        _invfac[m] = _fac[m].inv();
        for (int i = m; i > n; i--) {
            _invfac[i - 1] = _invfac[i] * i;
            _inv[i] = _invfac[i] * _fac[i - 1];
        }
        n = m;
    }
    Z fac(int m) {
        if (m > n) init(2 * m);
        return _fac[m];
    }
    Z invfac(int m) {
        if (m > n) init(2 * m);
        return _invfac[m];
    }
    Z inv(int m) {
        if (m > n) init(2 * m);
        return _inv[m];
    }
    Z binom(int n, int m) {
        if (n < m || m < 0) return 0;
        return fac(n) * invfac(m) * invfac(n - m);
    }
    Z fastfac(ll n) { // O(p + T log(n)), p is prime
        return n ? power(Z(-1), n / Z::getMod()) * fac
            (n % Z::getMod()) * fastfac(n / Z::getMod()) : 1;
    }
    Z lucas(ll n, ll m) { // O(p + T log(n)), p is prime
        return m ? binom(n % Z::getMod(), m % Z::getMod
            ()) * lucas(n / Z::getMod(), m / Z::getMod()) : 1;
    }
} comb; // 若要換模數需重新宣告

```

6.3 Sieve [7331f6]

```
vector<int> minp, primes;
vector<int> phi, mu, pnum; // 質因數種類數
vector<int> mpnum, dnum; // 最小質因數的幕次數, 約數數量
vector<int> powpref, dsum; // 約數和
// dmul[i] = i ^ (dnum[i] / 2) for dnum[i] even
// dmul[i] = k ^ dnum[i], k * k = i else
void sieve(int n) {
    minp.resize(n + 1);
    phi.resize(n + 1);
    mu.resize(n + 1);
    pnum.resize(n + 1);

    mpnum.resize(n + 1);
    dnum.resize(n + 1);

    powpref.resize(n + 1);
    dsum.resize(n + 1);

    phi[1] = mu[1] = 1;
    dsum[1] = 1;
    powpref[1] = dsum[1] = 1;
    for (int i = 2; i <= n; i++) {
        if (!minp[i]) {
            minp[i] = i;
            primes.push_back(i);

            phi[i] = i - 1;
            mu[i] = -1;
            pnum[i] = 1;

            mpnum[i] = 1;
            dnum[i] = 2;

            powpref[i] = i + 1;
            dsum[i] = i + 1;
        }
        for (int p : primes) {
            if (i * p > n) break;
            minp[i * p] = p;
            if (p == minp[i]) {
                phi[i * p] = phi[i] * p;
                mu[i * p] = 0;
                pnum[i * p] = pnum[i];

                mpnum[i * p] = mpnum[i] + 1;
                dnum[i * p] = dnum[i] / mpnum[i * p] * (mpnum[i * p] + 1);

                powpref[i * p] = powpref[i] * p + 1;
                dsum[i * p] = dsum[i] / powpref[i] * powpref[i * p];
                break;
                // i * p = (p * x) * p
                // i * q = (p * x) * q
                // 到達 x * q 再用 p 篩掉就好
            } else {
                phi[i * p] = phi[i] * (p - 1);
                mu[i * p] = -mu[i];
                pnum[i * p] = pnum[i] + 1;

                mpnum[i * p] = 1;
                dnum[i * p] = dnum[i] * 2;

                powpref[i * p] = p + 1;
                dsum[i * p] = dsum[i] * (p + 1);
            }
        }
    }
    // a ^ (m-1) = 1 (Mod m)
    // a ^ (m-2) = 1/a (Mod m)
    // exp2: cout << power(x, power(y, p, Mod - 1), Mod)
    // num = (x+1) * (y+1) * (z+1) ...
    // sum = (a^0 + a^1 + ... + a^x) * (b^0 + ... + b^y)
    // mul = N ^ ((x+1) * (y+1) * (z+1) / 2)
```

6.4 Matrix [6b2cbc]

```
using Matrix = vector<vector<Z>>;
Matrix operator*(const Matrix &a, const Matrix &b) {
    int n = a.size(), k = a[0].size(), m = b[0].size();
    assert(k == b.size());
    Matrix res(n, vector<Z>(m));
    for (int i = 0; i < n; i++)
        for (int j = 0; j < m; j++)
            for (int l = 0; l < k; l++)
                res[i][j] += a[i][l] * b[l][j];
    return res;
}
Matrix power(Matrix a, ll b) {
    int n = a.size();
    Matrix res(n, vector<Z>(n));
    for (int i = 0; i < n; i++) res[i][i] = 1;
    for (; b > 0; b >>= 1, a = a * a)
        if (b & 1) res = res * a;
    return res;
}
```

6.5 Miller Rabin Pollard Rho [394cfb]

```
ll mul(ll a, ll b, ll p) {
    ll res = a * b - ll(1.L * a * b / p) * p;
    res %= p;
    if (res < 0) res += p;
    return res;
}
ll power(ll a, ll b, ll p) {
    ll res {1};
    for (; b; b /= 2, a = mul(a, a, p))
        if (b & 1) res = mul(res, a, p);
    return res;
}
vector<ll>
> chk {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
bool check(ll a, ll d, int s, ll n) {
    a = power(a, d, n);
    if (a <= 1) return 1;
    for (int i = 0; i < s; i++, a = mul(a, a, n)) {
        if (a == 1) return 0;
        if (a == n - 1) return 1;
    }
    return 0;
}
bool isPrime(ll n) {
    if (n < 2) return 0;
    if (n % 2 == 0) return n == 2;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) d /= 2, s++;
    for (ll i : chk)
        if (!check(i, d, s, n)) return 0;
    return 1;
}
const vector<ll> small = {2, 3, 5, 7, 11, 13, 17, 19};
ll findFactor(ll n) {
    if (isPrime(n)) return 1;
    for (ll p : small)
        if (n % p == 0) return p;
    ll x, y = 2, d, t = 1;
    auto f = [&](ll a) {
        return (mul(a, a, n) + t) % n;
    };
    for (int l = 2; ; l *= 2) {
        x = y;
        int m = min(l, 32);
        for (int i = 0; i < l; i += m) {
            d = 1;
            for (int j = 0; j < m; j++)
                y = f(y), d = mul(d, abs(x - y), n);
            ll g = __gcd(d, n);
            if (g == n) {
                l = 1, y = 2, ++t;
                break;
            }
            if (g != 1) return g;
        }
    }
}
map<ll, int> res;
void pollardRho(ll n) {
    if (n == 1) return;
    if (isPrime(n)) {
        res[n]++;
        return;
    }
    ll d = findFactor(n);
    pollardRho(n / d), pollardRho(d);
}
```

6.6 Primitive Root [b0ba96]

```
int findPrimitiveRoot(ll m) {
    Mlong<0>::setMod(m); // Mlong if needed
    ll phi = m; // m - 1 if m prime
    res.clear();
    pollardRho(m);
    for (auto [p, _] : res) phi = phi / p * (p - 1);
    vector<ll> pr; // prime factors of phi
    res.clear();
    pollardRho(phi);
    for (auto &[p, _] : res) pr.push_back(p);
    for (int i = 1; i <= m; i++) {
        bool ok = true;
        for (int j = 0; j < int(pr.size()) && ok; j++)
            ok &= power(Mlong<0>(i), phi / pr[j]).x != 1;
        if (ok) return i;
    }
    return -1; // m != 1, 2, 4, p^k, 2(p^k)
}
```

6.7 CRT [bdb847]

```
// ax = b (mod m) 的前提是 gcd(a, m) | b
// a * p.first + b * p.second = gcd(a, b)
pair<ll, ll> exgcd(ll a, ll b) {
    if (b == 0) return {1, 0};
    auto [y, x] = exgcd(b, a % b);
    return {x, y - (a / b) * x};
}
// smallest non-negative solution
using i128 = __int128_t;
```

```

pair<ll, ll> CRT(ll r1, ll m1, ll r2, ll m2) {
    ll g = __gcd(m1, m2);
    if ((r2 - r1) % g) return {-1, g};
    m1 /= g, m2 /= g;
    auto [p1, p2] = exgcd(m1, m2);
    i128 lcm = i128(m1) * m2 * g;
    i128 res = i128(p1) * (r2 - r1) * m1 + r1;
    return {(res % lcm + lcm) % lcm, lcm};
}

ll EXCRT(vector<pair<int, int>> a) {
    ll R = 0, M = 1;
    for (auto [r, m] : a) {
        auto [res, lcm] = CRT(R, M, r, m);
        if (res == -1) return -1;
        R = res, M = lcm;
    }
    return R;
}

// gcd(mod) = 1, support 3 1E9 Mod
i128 CRT(vector<pair<int, int>> a) {
    i128 s = 1, res = 0;
    for (auto [r, m] : a) s *= m;
    for (auto [r, m] : a) {
        i128 t = s / m;
        res = (res + r * t % s * exgcd(t, m).first % s) % s;
    }
    return (res + s) % s;
}

```

6.8 exLucas [565958]

```

ll legendre(ll n, int p) { // n! 中質數 p 的幕次
    ll res = 0;
    while (n) { n /= p, res += n; }
    return res;
}

Z C_pe(ll n, ll m, int p, int e) {
    int pe = power(p, e);
    Z::setMod(pe); // inv 要用 exgcd
    vector<Z> fac(pe); fac[0] = 1;
    for (int j = 1; j < pe; j++) {
        if (j % p == 0) fac[j] = fac[j - 1];
        else fac[j] = fac[j - 1] * j;
    }
    Z wilson = p
        == 2 && e >= 3 ? 1 : -1; // (p^e)! = wilson (mod p^e)
    function<Z(ll)> fastFac = [&](ll n)
    { return n ? power(
        wilson, n / pe) * fac[n % pe] * fastFac(n / p) : 1; };
    ll vp =
        legendre(n, p) - legendre(m, p) - legendre(n - m, p);
    if (vp >= e) return 0;
    return power(Z
        (p), vp) * fastFac(n) / (fastFac(m) * fastFac(n - m));
}

ll exlucas(ll n, ll m, int mod) {
    vector<pair<int, int>> a;
    for (int i = 2; i * i <= mod; i++) {
        if (mod % i) continue;
        int e = 0, p = i;
        while (mod % i == 0) mod /= i, e++, p *= i;
        a.emplace_back(C_pe(n, m, i, e).x, p);
    }
    if (mod != 1) a.emplace_back(C_pe(n, m, mod, 1).x, mod);
    return CRT(a);
}

```

6.9 Quadratic Residue [da805f]

```

int jacobi(int x, int p) {
    int s = 1;
    for (; p > 1; ) {
        x %= p;
        if (x == 0) return 0;
        const int r = __builtin_ctz(x);
        if ((r & 1) && ((p + 2) & 4)) s = -s;
        x >>= r;
        if (x & p & 2) s = -s;
        swap(x, p);
    }
    return s;
}

template<class Z>
int quadraticResidue(Z x) {
    int p = Z::getMod();
    if (p == 2) return x.x & 1;
    const int jc = jacobi(x.x, p);
    if (jc == 0) return 0;
    if (jc == -1) return -1;
    Z b, d;
    while (true) {
        b = rand(), d = b * b - x;
        if (jacobi(d.x, p) == -1) break;
    }
    Z f0 = b, f1 = 1, g0 = 1, g1 = 0, tmp;
    for (int e = (p + 1) >> 1; e; e >>= 1) {
        if (e & 1) {
            tmp = g0 * f0 + d * (g1 * f1);
            g1 = g0 * f1 + g1 * f0, g0 = tmp;
        }
        tmp = f0 * f0 + d * (f1 * f1);
        f1 = f0 * f1 * 2, f0 = tmp;
    }
}

```

```

}
return min(g0.x, p - g0.x);
}

```

6.10 BSGS [ed42da]

```

// a^x = b (mod m)
// x = A * sq - B (0 <= A, B <= sq)
vector<int> BSGS(int a, int b, int m) { // gcd(a, m) = 1
    Z::setMod(m);
    unordered_map<int, int> mp;
    ll sq = 1; while (sq * sq < m) sq++;
    Z rhs = b; vector<int> res;
    for (int B = 0; B <= sq; B++) {
        mp[rhs.x] = B;
        rhs *= a;
    }
    Z a_pow_sq = power(Z(a), sq), lhs = a_pow_sq;
    for (int A = 1; A <= sq; A++, lhs *= a_pow_sq)
        if (mp.find(lhs.x) != mp.end()) {
            int B = mp[lhs.x];
            res.push_back(A * sq - B);
        }
    return res;
}

int exBSGS(int a, int b, int m) {
    if (b == 1 || m == 1) return 0;
    if (!a) return b ? -1 : 1;
    int k = 0;
    ll tmp = 1;
    while (__gcd(a, m) != 1) {
        int d = __gcd(a, m);
        if (b % d != 0) return -1;
        b /= d, m /= d, k++;
        tmp = tmp * (a / d) % m;
        if (tmp == b) return k;
    }
    Z::setMod(m);
    Z b_ = Z(b) * Z(tmp).inv();
    auto xs = BSGS(a, b_.x, m);
    return xs.empty() ? -1 : *min_element(xs.begin(), xs.end()) + k;
}

```

6.11 Game Theorem

- sg 值為 0 代表先手必敗
- 當前 sg 值 = 可能的後繼狀態的 mex (例如拿一個或拿兩個, 就等於兩者的 sg 值 mex), 若有互相依賴就兩個後繼狀態 xor 當作一組 sg 值 (例如切成兩半, 只算一次)
- 單組基礎 nim 的 sg 值為本身的原因: $f(0) = 0, f(1) = mex(f(0)) = 1, f(2) = mex(f(0), f(1)) = 2 \dots$ 都是自己
- 多組結局可以把 sg 值 xor 起來, 當成最後的 sg 值, nim 也是一樣, 且由於 xor 性質, 如果可以快速知道 $sg(1)g(2) \dots g(n)$, 就可以用 xor 性質處理不連續組合

6.12 Gaussian Elimination [5d1aa7]

```

// 找反矩陣
    就開 2n, 右邊放單位矩陣, 做完檢查左半是不是單位, 回傳右半
// 0 : no solution
// -1 : infinity solution
// 1 : one solution
template<class T>
tuple<T,
    int, vector<T>> gaussianElimination(vector<vector<T>> a) {
    T det = 1;
    bool zeroDet = false;
    int n = a.size(), m = a[0].size(), rk = 0, sgn = 1;
    for (int c = 0; c < n; c++) {
        int p = -1;
        for (int r = rk; r < n; r++) {
            if (a[r][c] != 0) {
                p = r;
                break;
            }
        }
        if (p == -1) {
            zeroDet = true;
            continue;
        }
        if (p != rk) swap(a[rk], a[p]), sgn *= -1;
        det *= a[rk][c];
        T inv = 1 / a[rk][c];
        for (int j = c; j < m; j++) a[rk][j] *= inv;
        for (int r = 0; r < n; r++) {
            if (r == rk || a[r][c] == 0) continue;
            T fac = a[r][c];
            for (int j = c; j < m; j++)
                a[r][j] -= fac * a[rk][j];
        }
        rk++;
    }
    det = (zeroDet ? 0 : det * sgn);
    for (int r = rk; r < n; r++)
        if (a[r][m - 1] != 0) return {det, 0, {}};
    if (rk < n) return {det, -1, {}};
    vector<T> ans(n);
    for (int i = 0; i < n; i++) ans[i] = a[i][m - 1];
    return {det, 1, ans};
}

```

```

template<class T>
tuple<int, vector<T>, vector<vector<T>>> findBasis(vector<vector<T>> a) {
    int n = a.size(), m = a[0].size(), rk = 0;
    vector<int> pos(m - 1, -1);
    for (int c = 0; c < m - 1; c++) {
        int p = -1;
        for (int r = rk; r < n; r++) {
            if (a[r][c] != 0) {
                p = r;
                break;
            }
        }
        if (p == -1) continue;
        if (p != rk) swap(a[rk], a[p]);
        pos[c] = rk;
        T inv = 1 / a[rk][c];
        for (int j = c; j < m; j++) a[rk][j] *= inv;
        for (int r = 0; r < n; r++) {
            if (r == rk || a[r][c] == 0) continue;
            T fac = a[r][c];
            for (int j = c; j < m; j++)
                a[r][j] -= fac * a[rk][j];
        }
        rk++;
    }
    vector<T> sol(m - 1);
    vector<vector<T>> basis;
    for (int r = rk; r < n; r++)
        if (a[r][m - 1] != 0)
            return {-1, sol, basis};
    for (int c = 0; c < m - 1; c++)
        if (pos[c] != -1)
            sol[c] = a[pos[c]][m - 1];
    for (int c = 0; c < m - 1; c++)
        if (pos[c] == -1) {
            vector<T> v(m - 1);
            v[c] = 1;
            for (int j = 0; j < m - 1; j++)
                if (pos[j] != -1)
                    v[j] = -a[pos[j]][c];
            basis.push_back(v);
        }
    return {rk, sol, basis};
}
template<class T>
using Matrix = vector<vector<T>>;

```

6.13 XOR Basis [02f0c0c]

```

auto add = [&](vector<int> &bas, int x) {
    for (auto i : bas) x = min(x, x ^ i);
    if (x) bas.push_back(x);
};

sort(bas.begin(), bas.end()); // 最簡化列梯
for (auto i = bas.begin(); i != bas.end(); i++) {
    for (auto j = next(i); j != bas.end(); j++) {
        *j = min(*j, *j ^ *i);
    }
}

// [l, r] 的區間 xor max (1-indexed)
vector<array<int, B>> bas(n + 1, lst(n + 1));
for (int i = 1; i <= n; i++) {
    int x = a[i - 1], p = i;
    bas[i] = bas[i - 1], lst[i] = lst[i - 1];
    for (int j = B - 1; j >= 0; j--)
        if (x >> j & 1) {
            if (!bas[i][j]) {
                bas[i][j] = x, lst[i][j] = p;
                break;
            }
            if (lst[i][j] < p)
                swap(lst[i][j], p), swap(x, bas[i][j]);
            x ^= bas[i][j];
        }
}

auto qry = [&](int l, int r) -> int {
    int mx = 0;
    for (int i = B - 1; i >= 0; i--)
        if (lst[r][i] >= l)
            mx = max(mx, mx ^ bas[r][i]);
    return mx;
};

```

6.14 Pisano Period

- $\pi(ab) = \text{lcm}(\pi(a), \pi(b))$ ($\gcd(a, b) = 1$)
- $\pi(p^e) | \pi(p) \cdot p^{e-1}$
- $\pi(p) | p^2 - 1$ ($p \neq 2, 5$)
- $\pi(2) = 3, \pi(5) = 20$
- so can deal with $p \approx 10^9$ in long long

6.15 Integer Partition [83bc9d]

```

// CSES_Sum_of_Divisors
const int Mod = 1E9 + 7;
const int inv_2 = 500000004;
// n / 1 * 1 + n / 2 * 2 + n / 3 * 3 + ... + n / n * n
void integerPartition() {
    ll ans = 0, n; cin >> n;
    for (ll l = 1, r; l <= n; l = r + 1) {
        r = n / (n / l);

```

```

        ll val = n / l; // n / l 到 n / r 一樣的值
        ll sum = (((l + r) % Mod)
            * ((r - l + 1) % Mod)) % Mod * inv_2; // l 加到 r
        val %= Mod; sum %= Mod;
        ans += val * sum;
        ans %= Mod;
    }
    cout << ans << "\n";
}

```

6.16 Mobius Theorem

- 數論分塊可以快速計算一些含有除法向下取整的和式，就是像 $\sum_{i=1}^n f(i)g(\lfloor \frac{n}{i} \rfloor)$ 的和式。當可以在 $O(1)$ 內計算 $f(r) - f(l)$ 或已經預處理出 f 的前綴和時，數論分塊就可以在 $O(\sqrt{n})$ 的時間內計算上述和式的值。
- 迪利克雷捲積 $h(x) = \sum_{d|x} f(d)g(\frac{x}{d})$
- 積性函數

- 莫比烏斯函數

1. 定義

$$\mu(d) = \begin{cases} 1 & \text{for } n=1 \\ 0 & \text{for } n \neq 1 \end{cases}$$

2. μ 是常數函數 1 的反元素

$\Rightarrow \mu * 1 = \epsilon$, $\epsilon(n)$ 只在 $n=1$ 時為 1，其餘情況皆為 0。

- ϕ 歐拉函數: x 以下與 x 互質的數量

$$\begin{aligned}
 \phi * 1 &= \sum_{d|n} \phi\left(\frac{n}{d}\right) \text{ 質因數分解} \\
 &= \sum_{i=0}^c \phi(p^i) \\
 &= 1 + p^0(p-1) + p^1(p-1) + \dots + p^{c-1}(p-1) \\
 &= p^c \\
 &= id
 \end{aligned}$$

- 莫比烏斯反演公式

$$\begin{aligned}
 - f(n) &= \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) \\
 - f(n) &= \sum_{n|d} g(d) \Leftrightarrow g(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) f(d)
 \end{aligned}$$

- 例子

$$\begin{aligned}
 &\sum_{i=a}^b \sum_{j=c}^d [gcd(i, j) = k] \\
 &\Rightarrow \sum_{i=1}^{\frac{x}{k}} \sum_{j=1}^{\frac{y}{k}} [gcd(i, j) = 1] \\
 &= \sum_{i=1}^{\frac{x}{k}} \sum_{j=1}^{\frac{y}{k}} \epsilon(gcd(i, j)) \\
 &= \sum_{i=1}^{\frac{x}{k}} \sum_{j=1}^{\frac{y}{k}} \sum_{d|gcd(i, j)} \mu(d) \\
 &= \sum_{d=1}^{\infty} \mu(d) \sum_{i=1}^{\frac{x}{kd}} \sum_{j=1}^{\frac{y}{kd}} [d|i] [d|j] \text{ } d \text{ 可整除 } i \text{ 時為 } 1 \\
 &= \sum_{d=1}^{\min(\frac{x}{k}, \frac{y}{k})} \mu(d) \left\lfloor \frac{x}{kd} \right\rfloor \left\lfloor \frac{y}{kd} \right\rfloor
 \end{aligned}$$

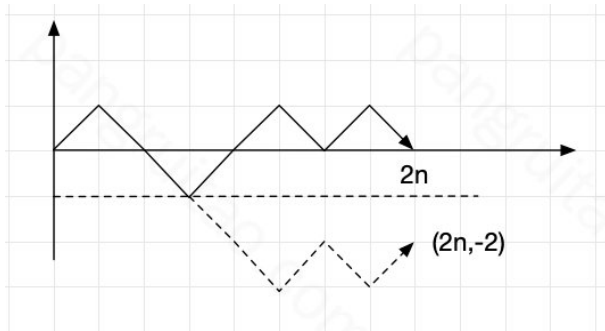
6.17 Mobius Inverse [d41d8c]

```

void solve() { // pref: pref of mu
    ll a, b, c, d, k; cin >> a >> b >> c >> d >> k;
    sieve(N);
    auto cal = [&](ll x, ll y) -> int {
        int res = 0;
        for (int l = 1, r; l <= min(x, y); l = r + 1) {
            r = min(x / (x / l), y / (y / l));
            res += (pref[r] - pref[l - 1]) * (x / l) * (y / l);
        }
        return res;
    };
    cout << cal
        (b / k, d / k) - cal((a - 1) / k, d / k) - cal(b / k,
            (c - 1) / k) + cal((a - 1) / k, (c - 1) / k) << "\n";
}

```

6.18 Catalan Theorem



1. n 個往上 n 個往下，先枚舉所有情況 $\frac{(2n)!}{n!n!} = C_n^{2n}$
2. 扣掉非法的，有多少種可能讓最後的點落在 $(2n, -2)$
假設往上有 x 個，往下有 y 個，會有：

$$\begin{cases} x+y=2n \\ y-x=2 \end{cases} \Rightarrow \begin{cases} x=n-1 \\ y=n+1 \end{cases}$$

所以只要扣掉 C_{n-1}^{2n-1} 即可

6.19 Burnside's Lemma

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

- G ：各種翻轉操作所構成的置換群
- X/G ：本質不同的方案的集合
- X^g ：對於某一種操作 g ，所有方案中，經過 g 這種翻轉後保持不變的方案集合
- 集合取絕對值代表集合數

7 Search and Greedy

7.1 Binary Search [d41d8c]

```
void binarySearch() {
    // 二分找上界
    // 如果無解會 = 原 lo, lo 要先 - 1
    while (lo < hi) {
        int x = (lo + hi + 1) / 2;
        if (check(x)) lo = x;
        else hi = x - 1;
    }
    cout << lo;
    // 二分找下界
    // 如果無解會 = 原 hi, hi 要先 + 1
    while (lo < hi) {
        int x = (lo + hi) / 2;
        if (check(n)) hi = x;
        else lo = x + 1;
    }
    cout << lo;
}
```

7.2 Ternary Search [d41d8c]

```
void ternarySearch() {
    int lo = 0, hi = 10;
    while (lo < hi) {
        int xl = lo + (hi - lo) / 3;
        int xr = hi - (hi - lo) / 3;
        int resl = calc(xl), resr = calc(xr);
        if (resl < resr) {
            lo = xl + 1;
        } else {
            hi = xr - 1;
        }
    }
}
```

8 Tree

8.1 Binary Lifting LCA [fd4743]

```
const int N = 2E5;
const int Lg = __lg(N); // __lg(max(N, Qi)), [0, Lg]
int up[N][Lg + 1];
vector<int> dep, dfn;
void build(int n, vector<vector<int>> &g, int rt = 0) {
    dep.assign(n, 0); dfn.assign(n, 0);
    int cur = 0;
    auto dfs = [&](auto self, int x, int p) -> void {
        dfn[x] = cur++;
        up[x][0] = p;
        for (int i = 1; i <= Lg; i++) {
            int nxt = up[x][i - 1];
            up[x][i] = up[nxt][i - 1];
        }
        for (auto y : g[x]) {
            if (y == p) continue;
            up[y][0] = x;
            dep[y] = dep[x] + 1;
            self(self, y, x);
        }
    };
    dfs(0, 0);
}
```

```
};
dfs(dfs, rt, rt);
}
int lca(int a, int b) {
    if (dep[a] < dep[b]) swap(a, b);
    int pull = dep[a] - dep[b];
    for (int i = 0; i <= Lg; i++)
        if (pull & (1 << i)) a = up[a][i];
    if (a == b) return a;
    for (int i = Lg; i >= 0; i--)
        if (up[a][i] != up[b][i])
            a = up[a][i], b = up[b][i];
    return up[a][0];
}
int jump(int x, int k) {
    for (int i = Lg; i >= 0; i--)
        if (k >= (1 << i)) x = up[x][i];
    return x;
}
int dist(int a, int b) {
    return dep[a] + dep[b] - 2 * dep[lca(a, b)];
}
```

8.2 Centroid Decomposition [2ec4c4]

```
vector<bool> vis(n);
vector<int> siz(n), par(n, -1);
auto findSize = [&](auto self, int u, int p) -> int {
    siz[u] = 1;
    for (int v : g[u]) {
        if (v == p || vis[v]) continue;
        siz[u] += self(self, v, u);
    }
    return siz[u];
};
auto findCen = [&](auto self, int u, int p, int sz) -> int {
    for (int v : g[u]) {
        if (v == p || vis[v]) continue;
        if (siz[v] * 2 > sz) return self(self, v, u, sz);
    }
    return u;
};
auto buildCen = [&](auto self, int u, int p) -> void {
    findSize(self, u, p);
    int c = findCen(self, u, p, siz[u]);
    vis[c] = true, par[c] = p;
    for (int v : g[c]) if (!vis[v]) self(self, v, c);
};
buildCen(buildCen, 0, -1);
```

8.3 Heavy Light Decomposition [9facc3]

```
struct HLD {
    int n, cur;
    vector<int> siz, top, dep, parent, in, out, seq;
    vector<vector<int>> adj;
    HLD(int n) : n(n), cur(0) {
        siz.resize(n); top.resize(n); dep.resize(n);
        parent.resize(n); in.resize(n); out.resize(n);
        seq.resize(n); adj.assign(n, {});
    }
    void addEdge(int u, int v) {
        adj[u].push_back(v);
        adj[v].push_back(u);
    }
    void work(int rt = 0) {
        top[rt] = rt;
        dep[rt] = 0;
        parent[rt] = -1;
        dfs1(rt); dfs2(rt);
    }
    void dfs1(int u) {
        if (parent[u] != -1)
            adj[u].erase(find(
                adj[u].begin(), adj[u].end(), parent[u]));
        siz[u] = 1;
        for (auto &v : adj[u]) {
            parent[v] = u, dep[v] = dep[u] + 1;
            dfs1(v);
            siz[u] += siz[v];
            if (siz[v] > siz[adj[u][0]]) {
                swap(v, adj[u][0]);
            } // 讓 adj[u][0] 是重子節點
        }
    }
    void dfs2(int u) {
        in[u] = cur++;
        seq[in[u]] = u; // dfn 對應的編號
        for (auto v : adj[u]) {
            top[v] = v == adj[u][0] ? top[u] : v;
            dfs2(v);
        }
        out[u] = cur;
    }
    int lca(int u, int v) {
        while (top[u] != top[v]) {
            if (dep[top[u]] > dep[top[v]])
                u = parent[top[u]];
            else
                v = parent[top[v]];
        }
    }
}
```

```

    return dep[u] < dep[v] ? u : v;
}
int dist(int u, int v) {
    return dep[u] + dep[v] - 2 * dep[lca(u, v)];
}
int jump(int u, int k) {
    if (dep[u] < k) return -1;
    int d = dep[u] - k;
    while (dep[top[u]] > d) u = parent[top[u]];
    return seq[in[u] - dep[u] + d];
}
bool isAncestor(int u, int v) {
    return in[u] <= in[v] && in[v] < out[u];
}
int rootedParent(int rt, int v) {
    if (rt == v) return rt;
    if (!isAncestor(v, rt)) return parent[v];
    auto it = upper_bound(adj[v].begin(), adj[v].end(), rt,
        [&](int x, int y) {
            return in[x] < in[y];
        }) - 1;
    return *it;
}
int rootedSize(int rt, int v) {
    if (rt == v) return n;
    if (!isAncestor(v, rt)) return siz[v];
    return n - siz[rootedParent(rt, v)];
}
int rootedLca(int rt, int a, int b) {
    return lca(rt, a) ^ lca(a, b) ^ lca(b, rt);
}
};

```

8.4 Link Cut Tree [544e55]

```

// 有用到 pathApply 才需要 apply 有關的
// 需要 pathQuery 才需要 pathInfo 有關的
// 需要 subtreeQuery 才需要 info, subtreeInfo
const int Mod = 51061;
struct Tag {
    ll add = 0, mul = 1;
    void apply(const Tag &v) {
        mul = mul * v.mul % Mod;
        add = (add * v.mul % Mod + v.add) % Mod;
    }
};
struct Info {
    int siz = 0;
    ll val = 0, sum = 0;
    void apply(const Tag &v) {
        val = (val * v.mul % Mod + v.add) % Mod;
        sum = (sum * v.mul % Mod + v.add * siz % Mod) % Mod;
    }
    void pull(const Info &l, const Info &r) {
        siz = 1 + l.siz + r.siz;
        sum = (l.sum + r.sum + val) % Mod;
    }
    Info &operator+=(const Info &i) {
        siz += i.siz;
        sum = (sum + i.sum) % Mod;
        return *this;
    }
    Info &operator-=(const Info &i) {
        siz -= i.siz;
        sum = (sum - (i.sum % Mod) + Mod) % Mod;
        return *this;
    }
};
struct LinkCutTree { // 1-based
    vector<Info> info, pathInfo, subtreeInfo;
    vector<Tag> tag;
    vector<array<int, 2>> ch;
    vector<int> p, rev;
    LinkCutTree(
        int n) : info(n + 1), pathInfo(n + 1), subtreeInfo(
            n + 1), tag(n + 1), ch(n + 1), p(n + 1), rev(n + 1) {}
    bool isrt(int x) {
        return ch[p[x]][0] != x && ch[p[x]][1] != x;
    }
    int pos(int x) { // x 是其 par 的左/右
        return ch[p[x]][1] == x;
    }
    void applyRev(int x) {
        swap(ch[x][0], ch[x][1]);
        rev[x] ^= 1;
    }
    void apply(int x, const Tag &v) {
        info[x].apply(v);
        pathInfo[x].apply(v);
        tag[x].apply(v);
    }
    void push(int x) {
        if (rev[x]) {
            if (ch[x][0]) applyRev(ch[x][0]);
            if (ch[x][1]) applyRev(ch[x][1]);
            rev[x] = 0;
        }
        if (ch[x][0]) apply(ch[x][0], tag[x]);
        if (ch[x][1]) apply(ch[x][1], tag[x]);
        tag[x] = Tag();
    }
    void pull(int x) {

```

```

        if (!x) return;
        pathInfo
            [x].pull(pathInfo[ch[x][0]], pathInfo[ch[x][1]]);
        info[x].pull(info[ch[x][0]], info[ch[x][1]]);
        info[x] += subtreeInfo[x];
    }
    void pushAll(int x) {
        if (!isrt(x)) pushAll(p[x]);
        push(x);
    }
    void rotate(int x) { // x 與其 par 交換位置
        int f = p[x], r = pos(x);
        ch[f][r] = ch[x][!r];
        if (ch[x][!r]) p[ch[x][!r]] = f;
        p[x] = p[f];
        if (!isrt(f)) ch[p[f]][pos(f)] = x;
        ch[x][!r] = f, p[f] = x;
        pull(f), pull(x);
    }
    void splay(int x) { // x 旋轉到當前的根
        pushAll(x);
        for (int f = p[x]; f = p[x], !isrt(x); rotate(x))
            if (!isrt(f)) rotate(pos(x) == pos(f) ? f : x);
    }
    // 第二次 access 可以回傳 LCA
    int access(int x) { // 根到 x 換成實鏈
        int c;
        for (c = 0; x; c = x, x = p[x]) {
            splay(x);
            subtreeInfo[x] += info[ch[x][1]];
            subtreeInfo[x] -= info[c];
            ch[x][1] = c;
            pull(x);
        }
        return c;
    }
    void makeRoot(int x) { // x 變成所在樹的根
        access(x), splay(x), applyRev(x);
    }
    int findRoot(int x) {
        access(x), splay(x);
        while (ch[x][0]) x = ch[x][0];
        splay(x); return x;
    }
    void split(int rt, int x) {
        makeRoot(x), access(rt), splay(rt);
    }
    void link(int rt, int x) {
        makeRoot(rt);
        access(x), splay(x);
        p[rt] = x;
        subtreeInfo[x] += info[rt];
        pull(x);
    }
    void cut(int rt, int x) {
        split(rt, x);
        ch[rt][0] = p[x] = 0;
        pull(rt);
    }
    bool connected(int x, int y) {
        return findRoot(x) == findRoot(y);
    }
    bool neighbor(int x, int y) {
        if (!connected(x, y)) return false;
        split(x, y);
        return pathInfo[x].siz == 2;
    }
    void modify(int x, const Info &v) {
        splay(x);
        info[x] = pathInfo[x] = v, pull(x);
    }
    void pathApply(int x, int y, const Tag &v) {
        assert(connected(x, y));
        split(x, y), apply(x, v);
    }
    Info pathQuery(int x, int y) {
        assert(connected(x, y));
        split(x, y); return pathInfo[x];
    }
    Info subtreeQuery(int rt, int x) {
        assert(connected(rt, x));
        split(rt, x);
        auto res = subtreeInfo[x];
        return res += pathQuery(x, x);
    }
};

```

8.5 Virtual Tree [c3a0b3]

```

// 多次詢問給某些關鍵點，虛樹可達成快速樹 DP (前處理每個點)
// 例如這題是有權樹，給一些關鍵點，求跟 vertex 1 隔開的最小成本
// 前處理 root 到所有點的最小邊權
vector<int> stk;
void insert(int key, vector<vector<int>> &vt) {
    if (stk.empty()) {
        stk.push_back(key);
        return;
    }
    int l = lca(stk.back(), key);
    if (l == stk.back()) {
        stk.push_back(key);
    }
}

```

```

    return;
}
while (
    stk.size() > 1 && dfn[stk[stk.size() - 2]] > dfn[l]) {
    vt[stk[stk.size() - 2]].push_back(stk.back());
    stk.pop_back();
}
if (stk.size() < 2 || stk[stk.size() - 2] != l) {
    vt[l].push_back(stk.back());
    stk.back() = l;
} else {
    vt[l].push_back(stk.back());
    stk.pop_back();
}
stk.push_back(key);
}
int work(vector<vector<int>> &vt) {
    while (stk.size() > 1) {
        vt[stk[stk.size() - 2]].push_back(stk.back());
        stk.pop_back();
    }
    int rt = stk[0];
    stk.clear();
    return rt;
}
void solve() {
    int n; cin >> n;
    vector<vector<int>> g(n);
    vector<vector<pair<int, int>>> wg(n);
    vector<vector<int>> vt(n);
    for (int i = 1; i < n; i++) {
        int u, v, w;
        cin >> u >> v >> w;
        u--, v--;
        g[u].push_back(v), g[v].push_back(u);
        wg[u].emplace_back(v, w), wg[v].emplace_back(u, w);
    }
    build(n, g); // build LCA
    vector<int> dis(n, 1E9); // root 到各點的最小邊權
    auto dfs_dis = [&](auto &&self, int x, int p) -> void {
        for (auto [y, w] : wg[x]) {
            if (y == p) continue;
            dis[y] = min(w, dis[x]);
            self(self, y, x);
        }
    };
    dfs_dis(dfs_dis, 0, -1);

    vector<bool> isKey(n);
    vector<ll> dp(n);
    int q; cin >> q;
    while (q--) {
        int m; cin >> m;
        vector<int> key(m);
        for (int i = 0; i < m; i++) {
            cin >> key[i];
            key[i] -= 1;
            isKey[key[i]] = true;
        }
        key.push_back(0); // 固定 0 為 root, 看題目需求
        sort(key.begin(), key.end()), [&](int a, int b) {
            return dfn[a] < dfn[b];
        }); // 要 sort 再 insert
        for (auto x : key) insert(x, vt);
        work(vt);
        auto dfs = [&](auto &&self, int x) -> void {
            for (auto y : vt[x]) {
                self(self, y);
                if (isKey[y]) { // 直接砍了
                    dp[x] += dis[y];
                } else { // 不砍 or 砍
                    dp[x] += min<ll>(dp[y], dis[y]);
                } // 記得 reset
                isKey[y] = dp[y] == 0;
            }
            vt[x].clear(); // 記得 reset
        };
        dfs(dfs, 0);
        cout << dp[0] << "\n";
        dp[0] = 0; // 最後 reset root
    }
}

```

8.6 Dominator Tree [d7608d]

```

// dom
存起點到達此點的必經的上個節點(起點 = 自己), 無法到達 = -1
struct DominatorTree {
    int n, id;
    vector<vector<int>> adj, radj, bucket;
    vector<int> sdom, dom, vis, rev, pa, rt, mn, res;
    DominatorTree(int n) : n(n), id(0) {
        sdom.resize(n), rev.resize(n);
        pa.resize(n), rt.resize(n);
        mn.resize(n), res.resize(n);
        bucket.assign(n, {});
        adj.assign(n, {}), radj.assign(n, {});
        dom.assign(n, -1), vis.assign(n, -1);
    }
    void addEdge(int u, int v) { adj[u].push_back(v); }
}

```

```

int query(int v, int x) {
    if (rt[v] == v) return x ? -1 : v;
    int p = query(rt[v], 1);
    if (p == -1) return x ? rt[v] : mn[v];
    if (sdom[mn[v]] > sdom[mn[rt[v]]])
        mn[v] = mn[rt[v]];
    rt[v] = p;
    return x ? p : mn[v];
}
void dfs(int v) {
    vis[v] = id, rev[id] = v;
    rt[id] = mn[id] = sdom[id] = id, id++;
    for (int u : adj[v]) {
        if (vis[u] == -1)
            dfs(u), pa[vis[u]] = vis[v];
        radj[vis[u]].push_back(vis[v]);
    }
}
vector<int> build(int s) {
    dfs(s);
    for (int i = id - 1; i >= 0; i--) {
        for (int u : radj[i])
            sdom[i] = min(sdom[i], sdom[query(u, 0)]);
        if (i) bucket[sdom[i]].push_back(i);
        for (int u : bucket[i]) {
            int p = query(u, 0);
            dom[u] = sdom[p] == i ? i : p;
        }
        if (i) rt[i] = pa[i];
    }
    res.assign(n, -1);
    for (int i = 1; i < id; i++)
        if (dom[i] != sdom[i])
            dom[i] = dom[dom[i]];
    for (int i = 1; i < id; i++)
        res[rev[i]] = rev[dom[i]];
    res[s] = s;
    for (int i = 0; i < n; i++)
        dom[i] = res[i];
    return dom;
}
}

```

9 DP

9.1 LIS [3018f4]

```

vector<int> LIS(const vector<int> &v) { // strictly
    int n = v.size(), L = 1;
    vector<int> dp(n); dp[0] = 1;
    vector<int> stk {v[0]};
    for (int i = 1; i < n; i++) {
        if (v[i] > stk.back()) { // >=
            stk.push_back(v[i]);
            dp[i] = ++L;
        } else { // upper
            auto it
                = lower_bound(stk.begin(), stk.end(), v[i]);
            *it = v[i];
            dp[i] = it - stk.begin() + 1;
        }
    }
    vector<int> ans;
    for (int i = n - 1; i >= 0; i--)
        if (dp[i] == L) ans.push_back(v[i]), L--;
    reverse(ans.begin(), ans.end());
    return dp;
}

```

9.2 Projects [8aa468]

```

void projects() { // 排程有權重問題, 輸出價值最多且時間最少
    struct E { int from, to, w, id; };
    int n; cin >> n; vector<E> a(n + 1);
    for (int i = 1; i <= n; i++) {
        int u, v, w; cin >> u >> v >> w;
        a[i] = {u, v, w, i};
    }
    vector<array<ll, 2>> dp(n + 1); // w, time
    vector<array<int, 2>> rec(n + 1); // 有沒選, 上個是誰
    sort(a.begin(), a.end());
    for (int i = 1; i <= n; i++) {
        int id = prev(
            lower_bound(all(a), {0, a[i].from}, [](E x, E y) {
                return x.to < y.to;
            }) - a.begin());
        dp[i] = dp[id - 1];
        ll nw = dp[id][0] + a[i].w;
        ll nt = dp[id][1] + a[i].to - a[i].from;
        if (dp[i][0] < nw || dp[i][0] == nw && dp[i][1] > nt) {
            dp[i] = {nw, nt};
            rec[i] = {1, id};
        }
    }
    vector<int> ans;
    for (int i = n; i != 0; i--) {
        if (rec[i][0]) {
            ans.push_back(a[i].id);
            i = rec[i][1];
        } else i--;
    }
}

```

9.3 Bitmask [60bdb9]

```
void hamiltonianPath() {
    int n, m; cin >> n >> m;
    vector<vector<int>> adj(n);
    for (int i = 0; i < m; i++) {
        int u, v; cin >> u >> v;
        adj[--v].push_back(--u);
    }
    // 以...為終點，走過...
    vector dp(n, vector<int>(1 << n));
    dp[0][1] = 1;
    for (int mask = 1; mask < 1 << n; mask++) {
        if ((mask & 1) == 0) continue;
        for (int i = 0; i < n; i++) {
            if ((mask >> i & 1) == 0) continue;
            if (i == n - 1 && mask != (1 << n) - 1) continue;
            int pre = mask ^ (1 << i);
            for (int j : adj[i]) {
                if ((pre >> j & 1) == 0) continue;
                dp[i][mask] = (dp[i][mask] + dp[j][pre]) % Mod;
            }
        }
    }
    cout << dp[n - 1][(1 << n) - 1] << "\n";
}

void elevatorRides() {
    int n, x; cin >> n >> x;
    vector<int> a(n);
    for (int i = 0; i < n; i++) cin >> a[i];
    vector<int> dp(1 << n), f(1 << n);
    dp[0] = 1; // 次數、已使用人數
    for (int mask = 1; mask < 1 << n; mask++) {
        dp[mask] = 2E9;
        for (int i = 0; i < n; i++) {
            if ((mask >> i & 1) == 0) continue;
            int pre = mask ^ (1 << i);
            if (f[pre] + a[i] <= x) {
                if (dp[pre] < dp[mask] || dp[pre]
                    == dp[mask] && f[pre] + a[i] < f[mask]) {
                    dp[mask] = dp[pre];
                    f[mask] = f[pre] + a[i];
                }
            } else if (dp[pre] + 1 < dp[mask] ||
                dp[pre] + 1 == dp[mask] && a[i] < f[mask]) {
                dp[mask] = dp[pre] + 1;
                f[mask] = a[i];
            }
        }
    }
    cout << dp[(1 << n) - 1] << "\n";
}

void minClique() { // 移掉一些邊，讓整張圖由最少團組成
    int n, m;
    cin >> n >> m;
    vector<bitset<N>> g(n);
    for (int i = 0; i < m; i++) {
        int u, v; cin >> u >> v;
        u--; v--; g[u][v] = g[v][u] = 1;
    }
    vector<int> dp(1 << n, inf);
    dp[0] = 1;
    for (int mask = 0; mask < 1 << n; mask++) { // 先正常 dp
        for (int i = 0; i < n; i++) {
            if (mask & (1 << i)) {
                int pre = mask ^ (1 << i);
                if (dp[pre]
                    == 1 && (g[i] & bitset<N>(pre)) == pre)
                    dp[mask] = 1; // i 有連到所有 pre
            }
        }
    }
    for (int
        mask = 0; mask < 1 << n; mask++) // 然後枚舉子集 dp
        for (int sub = mask; sub; --sub &= mask)
            dp[mask] = min(dp[mask], dp[sub] + dp[mask ^ sub]);
    cout << dp[(1 << n) - 1] << "\n";
}
```

9.4 Monotonic Queue [c9ba14]

```
// 應用:  $dp(i) = h(i) + \max(A(j))$ , for  $l(i) \leq j \leq r(i)$ 
//  $A(j)$  可能包含  $dp(j)$ ,  $h(i)$  可  $O(1)$ 
void boundedKnapsack() {
    int n, k; //  $O(nk)$ 
    vector<int> w(n), v(n), num(n);
    deque<int> q;
    // 將同餘的數分在同一組
    // 每次取出連續 num[i] 格中最大值
    //  $g_x = \max_{k=0}^{\text{num}[i]} (g'_{x-k} + v_i \cdot k)$ 
    //  $G_x = g'_{\{x\}} - v_i \cdot x$ 
    //  $x$  代  $x-k \Rightarrow v_i \cdot (x-k)$ 
    //  $g_x = \max_{k=0}^{\text{num}[i]} (G_{\{x-k\}} + v_i \cdot x)$ 
    vector<vector<ll>> dp(2, vector<ll>(k + 1));
    for (int i = 0; i < n; i++) {
        for (int r = 0; r < w[i]; r++) { // 餘數
            q.clear(); // q 記錄在  $x = i$  時的 dp 有單調性
            for (int x = 0; x * w[i] + r <= k; x++) {
                while (!q.empty() && q.front()
                    < x - num[i]) q.pop_front(); // 維護遞減
```

```
                ll nxt = dp[0][x * w[i] + r] - x * v[i];
                while (!q.empty() && dp[0][q.back()] * w[i] +
                    r - q.back() * v[i] < nxt) q.pop_back();
                q.push_back(x);
                dp[1][x * w[i] + r] = dp[0][q.front()]
                    * w[i] + r - q.front() * v[i] + x * v[i];
            }
            swap(dp[0], dp[1]);
        }
        cout << dp[0][k] << "\n";
    }
}
```

9.5 Digit [c42c49]

```
// 只含 0, 1, 8 的回文數
const int N = 44;
vector<vector<ll>> memo(N + 1, vector<ll>(N + 1, -1));
ll solve(string n) {
    vector<int> a {0};
    string tmp = n;
    while (!tmp.empty()) {
        a.push_back(tmp.back() - '0');
        tmp.pop_back();
    }
    n = tmp;
    int len = a.size() - 1;
    // 當前 digit 不會有貢獻，交給 dfs 處理答案就好
    vector<int> now(len + 1, -1); // 紀錄目前的數字
    auto dfs = [&](
        auto self, int p, int eff, bool f0, bool lim) -> ll {
        if (!p) { // 記得想好要回傳 0 還是 1
            return 1;
        }
        if (!lim && !f0 && memo[p][eff] != -1) {
            return memo[p][eff];
        }
        ll res = 0;
        int lst = lim ? a[p] : 9; // or 1 for binary
        for (int i = 0; i <= lst; i++) {
            if (i != 0 && i != 1 && i != 8) continue;
            bool nlim = lim && i == lst;
            now[p] = i;
            if (f0 && i == 0) { // 處理前導零
                res += self(self, p - 1, eff - 1, true, nlim);
            } else if (p > eff / 2) { // 前半段
                res += self(self, p - 1, eff, false, nlim);
            } else if (now[p] == now[eff - p + 1]) {
                res += self(self, p - 1, eff, false, nlim);
            }
        }
        if (!lim && !f0) memo[p][eff] = res;
        return res;
    };
    return dfs(dfs, len, len, true, true);
}
```

9.6 SOS [be203d]

```
// 使用情況: 跟 bit 與(被)包含有關，且  $x$  在  $1E6$  左右
// 題目: 一數組，問有多少所有數 & 起來為 0 的集合數
// dp[
    //  $x]$  代表包含  $x$  的  $y$  個數(比  $x$  大且 bit 1 全包含  $x$  的有幾個)
// 答案應該包含在  $dp[0]$  內，但是有重複元素，所以考慮容斥
//  $\Rightarrow \text{ans} = \sum_{i=0}^n (-1)^{\text{pop\_count}(i)} 2^{\text{dp}[i]-1}$ 
//  $\Rightarrow$  全
    // 部為 0 的個數 - 至少一個為 1 的個數 + 至少兩個為 1 的個數
void solve() {
    int n; cin >> n; Z ans = 0;
    vector<int> a(n);
    for (int i = 0; i < n; i++) cin >> a[i];
    int m = __lg(*max_element(a.begin(), a.end())) + 1;
    // 定義  $dp[\text{mask}]$  為  $\text{mask}$  被包含於  $a[i]$  的  $i$  個數
    vector<ll> dp(1 << m);
    for (int i = 0; i < n; i++) dp[a[i]]++;
    for (int i = 0; i < m; i++) {
        for (int mask = 0; mask < 1 << m; mask++) {
            if (mask >> i & 1) {
                int pre = mask ^ (1 << i);
                dp[pre] += dp[mask];
            }
        }
    }
    for (int mask = 0; mask < 1 << m; mask++) {
        int sgn = __builtin_popcount(mask) & 1 ? -1 : 1;
        ans += sgn * (power(Z(2), dp[mask]) - 1);
    }
}

//  $x / y = x$ , 代表包含於  $x$  的  $y$  個數，定義為  $dp[x][0]$ 
//  $x \& y = x$ , 代表包含  $x$  的  $y$  個數，定義為  $dp[x][1]$ 
//  $x \& y$ 
    //  $\neq 0$ , 代表至少有一個位元都為 1 的  $y$  個數,  $= n - dp[\sim x][0]$ 
void solve() {
    int n; cin >> n;
    vector<int> a(n);
    map<int, int> mp;
    for (int i = 0; i < n; i++) {
        cin >> a[i]; mp[a[i]]++;
    }
}
```

```

int m = __lg(*max_element(a.begin(), a.end())) + 1;
vector<array<ll, 2>> dp(1 << m);
for (int i = 0; i < n; i++) dp[a[i]][0]++, dp[a[i]][1]++;
for (int i = 0; i < m; i++) {
    for (int mask = 0; mask < 1 << m; mask++) {
        if (mask >> i & 1) {
            int pre = mask ^ (1 << i);
            dp[mask][0] += dp[pre][0];
            dp[pre][1] += dp[mask][1];
        }
    }
}
for (int i = 0; i < n; i++) {
    cout << dp[a[i]][0] << " " << dp[a[i]][1] <<
    " " << n - (dp[(1 << m) - 1] ^ a[i])[0] << "\n";
}
}

```

9.7 CHT [ce439f]

```

// 應用:  $dp(x) = C(x) + \min/\max(A(i) * x + B(i))$ , for  $i < x$ 
struct Line { // x 盡量從 1 開始
    ll m, b;
    Line(ll m = 0, ll b = 0) : m(m), b(b) {}
    ll eval(ll x) { return m * x + b; }
};

struct CHT { // 斜率單調
    int lptr = 0, rptr = 0;
    vector<Line> hull;
    CHT(Line init = Line()) { hull.push_back(init); }
    bool frontBad(Line &l1, Line &l2, ll x) {
        // 斜率遞減、查詢遞增, 因此只要左直線的  $y \geq$  右直線的  $y$ 
        // 代表查詢的當下, 右線段的高度已經低於左線段了
        return l1.eval(x) >= l2.eval(x);
    }
    bool backBad(Line &l1, Line &l2, Line &l3) {
        // 斜率遞減、上凸包、取 min
        // 因此只要 l2 跟
        // l3 的 x 交點 <= l1 跟 l3 的 x 交點, l2 就用不到了
        return (l3.b - l2.b) * (l1.m - l3.m) <= (l3.b - l1.b) * (l2.m - l3.m);
    }
    void addLine(Line l) {
        while (rptr - lptr > 0 && backBad(hull[rptr] - 1, hull[rptr], l)) hull.pop_back(), rptr--;
        hull.push_back(l), rptr++;
    }
    ll query(ll x) { // 查詢沒單調性需要二分搜
        while (rptr - lptr > 0 && frontBad(hull[lptr], hull[lptr + 1], x)) lptr++;
        return hull[lptr].eval(x);
    }
};

```

9.8 DNC [096d5a]

```

// 應用: 切 k 段問題, 且滿足四邊形不等式
//  $w(a,c) + w(b,d) \leq w(a,d) + w(b,c)$ 
//  $dp[k][j] = \min(dp[k-1][i] + cost[i][j])$ 
//  $O(K N \log N)$ 
ll dp[K+1][N+1];
ll getCost(int l, int r) { // (l, r], 1-based
void rec(int k, int l, int r, int optl, int opt) {
    if (l > r) return;
    int m = (l + r) / 2, opt = 0;
    dp[k][m] = inf;
    for (int i = max(k-1, optl); i <= min(m-1, opt); i++) {
        ll x = dp[k-1][i] + getCost(i, m);
        if (x < dp[k][m]) dp[k][m] = x, opt = i;
    }
    rec(k, l, m-1, optl, opt);
    rec(k, m+1, r, opt, opt);
}

void DNC() { // first build cost...
    for (int i = 1; i <= n; i++) dp[1][i] = getCost(0, i);
    for (int i = 2; i <= k; i++) rec(i, 1, n, 1, n);
    cout << dp[k][n] << "\n";
}

```

9.9 LiChao Segment Tree [2a9325]

```

// 應用:  $dp(i) = h(i) + \min/\max(A(j)X(i) + B(j))$ , for  $j \leq r(i)$ 
//  $y = c + mx + b$ 
template<class T, class F = less<ll>>
struct LiChaoSeg {
    F cmp = F();
    static const T inf = max(numeric_limits<T>::lowest() / 2, numeric_limits<T>::max() / 2, F());
    struct Line {
        T m, b;
        Line(T m = 0, T b = inf) : m(m), b(b) {}
        T eval(T x) const { return m * x + b; }
    };
    struct Node {
        Line line;
        ll l = -1, r = -1;
    };
    ll n;
    vector<Node> nd;

```

```

LiChaoSeg(ll n) : n(n) { newNode(); }
void addLine(Line line) { update(0, 0, n, line); }
void rangeAddLine(Line line, ll ql, ll qr) { rangeUpdate(0, 0, n, ql, qr, line); }
T query(ll x) { return query(x, 0, 0, n); }

private:
int newNode() {
    nd.emplace_back();
    return nd.size() - 1;
}

void update(int p, ll l, ll r, Line line) {
    ll m = (l + r) / 2;
    bool left = cmp(line.eval(l), nd[p].line.eval(l));
    bool mid = cmp(line.eval(m), nd[p].line.eval(m));
    if (mid) swap(nd[p].line, line);
    if (r - l == 1) return;
    if (left != mid) {
        if (nd[p].l == -1) nd[p].l = newNode();
        update(nd[p].l, l, m, line);
    } else {
        if (nd[p].r == -1) nd[p].r = newNode();
        update(nd[p].r, m, r, line);
    }
}

void rangeUpdate(int p, ll l, ll r, ll ql, ll qr, Line line) {
    if (r <= ql || l >= qr) return;
    if (ql <= l && r <= qr) return update(p, l, r, line);
    if (nd[p].l == -1) nd[p].l = newNode();
    if (nd[p].r == -1) nd[p].r = newNode();
    ll m = (l + r) / 2;
    rangeUpdate(nd[p].l, l, m, ql, qr, line);
    rangeUpdate(nd[p].r, m, r, ql, qr, line);
}

T query(ll x, int p, ll l, ll r) {
    if (p == -1) return inf;
    ll m = (l + r) / 2;
    if (x < m) return min(
        nd[p].line.eval(x), query(x, nd[p].l, l, m), cmp);
    else return min(
        nd[p].line.eval(x), query(x, nd[p].r, m, r), cmp);
}
};

```

10 Geometry

10.1 Basic [d41d8c]

```

const double eps = 1E-9;
template<class T>
struct Pt {
    T x, y;
    Pt(T x = 0, T y = 0) : x(x), y(y) {}
    Pt operator-() const { return Pt(-x, -y); }
    Pt operator+(Pt p) const { return Pt(x + p.x, y + p.y); }
    Pt operator-(Pt p) const { return Pt(x - p.x, y - p.y); }
    Pt operator*(T k) const { return Pt(x * k, y * k); }
    Pt operator/(T k) const { return Pt(x / k, y / k); }
    bool operator==(Pt p) const { return x == p.x && y == p.y; }
    bool operator!=(Pt p) const { return x != p.x || y != p.y; }
    friend istream &operator>>(istream &is, Pt &p) {
        return is >> p.x >> p.y;
    }
    friend ostream &operator<<(ostream &os, const Pt &p) {
        return os << "(" << p.x << ", " << p.y << ")";
    }
};

int sign(double x) {
    return fabs(x) <= eps ? 0 : (x > 0 ? 1 : -1);
}
using P = Pt<double>;

struct Line { P a, b; };
double dot(P a, P b) { return a.x * b.x + a.y * b.y; }
double cross(P a, P b) { return a.x * b.y - a.y * b.x; }
double square(P p) { return dot(p, p); }
double abs(P p) { return sqrt(square(p)); }
double dist(P a, P b) { return abs(a - b); }
double abs(Line l) { return abs(l.a - l.b); }
int dir(P p, Line l) // left -1, right 1, on 0
{ return -sign(cross(l.b - l.a, p - l.a)); }
bool btw(P p, Line l) // c on segment ab?
{ return dir(p, l) == 0 && sign(dot(p - l.a, p - l.b)) <= 0; }
P norm(P p) { return p / abs(p); }
P rot(P p) { return { -p.y, p.x }; } // 90 degree CCW
P rot(P p, double d) {
    double c = cos(d), s = sin(d);
    return { p.x * c - p.y * s, p.x * s + p.y * c };
}

bool parallel(Line l1, Line l2) {
    return cross(l1.b - l1.a, l2.b - l2.a) == 0;
}
P lineIntersection(Line l1, Line l2) {
    return l1.a + (l1.b - l1.a) * (cross(l2.b - l2.a, l1.a - l1.b)) / cross(l2.b - l2.a, l1.a - l1.b);
}
bool pointOnSegment(P p, Line l) {
    return dir(p, l) == 0 && sign(dot(p - l.a, p - l.b)) <= 0;
}
P projvec(P p, Line l) {
    P v = l.b - l.a;
    return l.a + v * (dot(p - l.a, v) / square(v));
}

// 0 : not intersect
// 1 : strictly intersect

```

```

// 2 : overlap
// 3 : intersect at endpoint
tuple<int, P, P> segmentIntersection(Line l1, Line l2) {
    if (max(l1.a.x, l1.b.x) < min(l2.a.x, l2.b.x) ||
        min(l1.a.x, l1.b.x) > max(l2.a.x, l2.b.x) ||
        max(l1.a.y, l1.b.y) < min(l2.a.y, l2.b.y) ||
        min(l1.a.y, l1.b.y) > max(l2.a.y, l2.b.y))
        return {0, {}, {}};
    if (cross(l1.b - l1.a, l2.b - l2.a) == 0) {
        if (cross(l1.b - l1.a, l2.a - l1.a) != 0) {
            return {0, {}, {}};
        } else {
            auto maxx1 = max(l1.a.x, l1.b.x);
            auto minx1 = min(l1.a.x, l1.b.x);
            auto maxy1 = max(l1.a.y, l1.b.y);
            auto miny1 = min(l1.a.y, l1.b.y);
            auto maxx2 = max(l2.a.x, l2.b.x);
            auto minx2 = min(l2.a.x, l2.b.x);
            auto maxy2 = max(l2.a.y, l2.b.y);
            auto miny2 = min(l2.a.y, l2.b.y);
            P p1(max(minx1, minx2), max(miny1, miny2));
            P p2(min(maxx1, maxx2), min(maxy1, maxy2));
            if (!pointOnSegment(p1, l1)) swap(p1.y, p2.y);
            if (p1 == p2) return {3, p1, p2};
            else return {2, p1, p2};
        }
    }
    auto cp1 = cross(l2.a - l1.a, l2.b - l1.a);
    auto cp2 = cross(l2.a - l1.b, l2.b - l1.b);
    auto cp3 = cross(l1.a - l2.a, l1.b - l2.a);
    auto cp4 = cross(l1.a - l2.b, l1.b - l2.b);
    if ((cp1 > 0 && cp2 > 0) || (cp1 < 0 && cp2 < 0) || (cp3 > 0 && cp4 > 0) || (cp3 < 0 && cp4 < 0)) return {0, P(), P()};
    P p = lineIntersection(l1, l2);
    if (cp1 != 0 && cp2 != 0 && cp3 != 0 && cp4 != 0) return {1, p, p};
    else return {3, p, p};
}

vector<P> convexHull(vector<P> a) {
    sort(a.begin(), a.end(), [](const P &l, const P &r) {
        return l.x == r.x ? l.y < r.y : l.x < r.x;
    });
    a.resize(unique(a.begin(), a.end()) - a.begin());
    if (a.size() <= 1) return a;
    vector<P> h(a.size() + 1);
    int s = 0, t = 0;
    for (int i = 0; i < 2; i++, s = --t) {
        for (P p : a) {
            while (t >= s + 2 && cross
                (h[t - 1] - h[t - 2], p - h[t - 2]) <= 0) t--;
            h[t++] = p;
        }
        reverse(a.begin(), a.end());
    }
    return {h.begin(), h.begin() + t};
}

double distPL(P &p, Line &l) {
    return abs(cross(l.a - l.b, l.a - p)) / abs(l);
}
double distancePS(P &p, Line &l) {
    if (dot(p - l.a, l.b - l.a) < 0) return dist(p, l.a);
    if (dot(p - l.b, l.a - l.b) < 0) return dist(p, l.b);
    return distPL(p, l);
}
double distanceSS(Line l1, Line l2) {
    if (get<0>(segmentIntersection(l1, l2)) != 0) return 0.0;
    return min({distancePS(l1.a, l2), distancePS(l1.b, l2), distancePS(l2.a, l1), distancePS(l2.b, l1)});
}

bool lineIntersectsPolygon(Line l, const vector<P> &p) {
    int n = p.size();
    P a = l.a, b = l.b;
    for (int i = 0; i < n; i++) {
        Line seg {p[i], p[(i + 1) % n]};
        if (cross(b - a, seg.a - a) == 0 || cross(b - a, seg.b - a) == 0) return true;
        if ((cross(b - a, seg.a - a) > 0) ^ (cross(b - a, seg.b - a) > 0)) return true;
    }
    return false;
}

bool pointInPolygon(P a, const vector<P> &p) {
    int n = p.size(), t = 0;
    for (int i = 0; i < n; i++)
        if (pointOnSegment(a, {p[i], p[(i + 1) % n]})) return true;
    for (int i = 0; i < n; i++) {
        P u = p[i], v = p[(i + 1) % n];
        if (u.x < a.x && v.x >= a.x && dir(a, {v, u}) < 0) t ^= 1;
        if (u.x >= a.x && v.x < a.x && dir(a, {u, v}) < 0) t ^= 1;
    }
    return t == 1;
}

// 0 : strictly outside
// 1 : on boundary
// 2 : strictly inside
int pointInConvexPolygon(P a, const vector<P> &p) {

```

```

    int n = p.size();
    if (n == 0) return 0;
    else if (n <= 2) return pointOnSegment(a, {p[0], p.back()});
    if (pointOnSegment(a, {p[0], p[1]}))
        || pointOnSegment(a, {p[0], p[n - 1]})) return 1;
    else if (dir(a, {p[0], p[1]}) < 0 || dir(a, {p[0], p[n - 1]}) < 0) return 0;
    int lo = 1, hi = n - 2;
    while (lo < hi) {
        int x = (lo + hi + 1) / 2;
        if (dir(a, {p[0], p[x]}) < 0) lo = x;
        else hi = x - 1;
    }
    if (dir(a, {p[lo], p[lo + 1]}) < 0) return 2;
    else return pointOnSegment(a, {p[lo], p[lo + 1]});
}

bool segmentInPolygon(Line l, const vector<P> &p) {
    int n = p.size();
    if (!pointInPolygon(l.a, p)) return false;
    if (!pointInPolygon(l.b, p)) return false;
    for (int i = 0; i < n; i++) {
        auto u = p[i];
        auto v = p[(i + 1) % n];
        auto w = p[(i + 2) % n];
        auto [t, p1, p2] = segmentIntersection(l, {u, v});
        if (t == 1) return false;
        if (t == 0) continue;
        if (t == 2) {
            if (pointOnSegment(v, l) && v != l.a && v != l.b && cross(u - v, w - v) < 0) return false;
        } else {
            if (p1 != u && p1 != v) {
                if (dir(l.a, {v, u}) < 0 || dir(l.b, {v, u}) < 0) return false;
            } else if (p1 == v) {
                if (l.a == v) {
                    if (dir(u, l) < 0) {
                        if (dir(w, l) < 0 && dir(w, {u, v}) < 0) return false;
                    } else if (dir(w, l) < 0 || dir(w, {u, v}) < 0) return false;
                } else if (l.b == v) {
                    if (dir(u, {l.b, l.a}) < 0) {
                        if (dir(w, {l.b, l.a}) < 0 && dir(w, {u, v}) < 0) return false;
                    } else if (dir(w, {l.b, l.a}) < 0 || dir(w, {u, v}) < 0) return false;
                } else {
                    if (dir(u, l) < 0) {
                        if (dir(w, {l.b, l.a}) < 0 || dir(w, {u, v}) < 0) return false;
                    } else if (dir(w, l) < 0 || dir(w, {u, v}) < 0) return false;
                }
            }
        }
    }
    return true;
}

vector<P> hp(vector<Line> lines) {
    auto sgn = [](P p) {
        return p.y > 0 || (p.y == 0 && p.x > 0) ? 1 : -1;
    };
    sort(lines.begin(), lines.end(), [&](auto l1, auto l2) {
        auto d1 = l1.b - l1.a;
        auto d2 = l2.b - l2.a;
        if (sgn(d1) != sgn(d2))
            return sgn(d1) == 1;
        return cross(d1, d2) > 0;
    });
    deque<Line> ls;
    deque<P> ps;
    for (auto l : lines) {
        if (ls.empty()) {
            ls.push_back(l);
            continue;
        }
        while (!ps.empty() && dir
            (ps.back(), l) >= 0) ps.pop_back(), ls.pop_back();
        while (!ps.empty() && dir
            (ps[0], l) >= 0) ps.pop_front(), ls.pop_front();
        if (cross(l.b - l.a, ls.back().b - ls.back().a) == 0) {
            if (dot
                (l.b - l.a, ls.back().b - ls.back().a) > 0) {
                if (dir(ls.back().a, l) >= 0) {
                    assert(ls.size() == 1);
                    ls[0] = l;
                }
                continue;
            }
            return {};
        }
        ps.push_back(lineIntersection(ls.back(), l));
        ls.push_back(l);
    }
    while (!ps.empty() && dir(ps.back(), ls[0]) >= 0)
        ps.pop_back(), ls.pop_back();
    if (ls.size() <= 2) return {};
    ps.push_back(lineIntersection(ls[0], ls.back()));
    return vector(ps.begin(), ps.end());
}

```

10.2 Min Euclidean Distance [cfb429]

```
// recursive solution
void minEuclideanDistance() {
    int n; cin >> n;
    const ll inf = 8E18;
    vector<P> a(n);
    for (int i = 0; i < n; i++) {
        ll x, y; cin >> x >> y;
        a[i] = P(x, y);
    }
    struct sortY { bool operator()(const P &a, const P &b) const { return a.y < b.y; } };
    struct sortXY {
        bool operator()(const P &a, const P &b) const {
            return a.x == b.x ? a.y < b.y : a.x < b.x;
        }
    };
    sort(a.begin(), a.end(), sortXY());
    vector<P> t(n);
    auto divide = [&](auto &&self, int l, int r) -> ll {
        if (l == r) return inf;
        int m = (l + r) / 2;
        ll ans = min(self(self, l, m), self(self, m + 1, r));
        ll midval = a[m].x;
        ll p = 0;
        for (int i = l; i <= r; i++)
            if ((midval - a[i].x) * (midval - a[i].x) <= ans)
                t[p++] = a[i];
        sort(t.begin(), t.end(), sortY());
        for (int i = 0; i < p; i++) {
            for (int j = i + 1; j < p; j++) {
                ans = min(ans, square(t[i].x - t[j].x));
                if ((t[i].y - t[j].y) * (t[i].y - t[j].y) > ans) break;
            }
        }
        return ans;
    };
    cout << divide(divide, 0, n - 1) << "\n";
}

// K-D tree solution
struct Info {
    static constexpr int DIM = 2;
    array<ll, DIM> x, L, R;
    ll distl, distr;
    ll f(const Info &i) {
        ll ret = 0;
        if (i.L[0] > x[0]) ret += (i.L[0] - x[0]) * (i.L[0] - x[0]);
        if (i.R[0] < x[0]) ret += (x[0] - i.R[0]) * (x[0] - i.R[0]);
        if (i.L[1] > x[1]) ret += (i.L[1] - x[1]) * (i.L[1] - x[1]);
        if (i.R[1] < x[1]) ret += (x[1] - i.R[1]) * (x[1] - i.R[1]);
        return ret;
    }
    void pull(const Info &l, const Info &r) {
        distl = f(l), distr = f(r);
    }
};

struct KDTree { // 1-indexed
    static constexpr int DIM = Info::DIM;
    int n, rt;
    vector<Info> info;
    vector<int> l, r;
    KDTree(const vector<Info> &info) : n(info.size()), info(info), l(n + 1), r(n + 1) {
        rt = build(1, n);
    }
    void pull(int p) {
        info[p].L = info[p].R = info[p].x;
        info[p].pull(info[l[p]], info[r[p]]);
        for (int ch : {l[p], r[p]}) {
            if (!ch) continue;
            for (int k = 0; k < DIM; k++) {
                info[p].L[k] = min(info[p].L[k], info[ch].L[k]);
                info[p].R[k] = max(info[p].R[k], info[ch].R[k]);
            }
        }
    }
    int build(int l, int r) {
        if (r == l) return 0;
        int m = (l + r) / 2;
        array<double, DIM> av = {}, va = {};
        for (int i = l; i < r; i++)
            for (int d = 0; d < DIM; d++)
                av[d] += info[i].x[d];
        for (int d = 0; d < DIM; d++)
            av[d] /= (double)(r - l);
        for (int i = l; i < r; i++)
            for (int d = 0; d < DIM; d++)
                va[d] += (info[i].x[d] - av[d]) * (info[i].x[d] - av[d]);
        int dep = max_element(va.begin(), va.end()) - va.begin();
        nth_element(info.begin() + l, info.begin() + m, info.begin() + r,
```

```
[&](const Info &x, const Info &y) { return x.x[dep] < y.x[dep]; });
this->l[m] = build(l, m);
this->r[m] = build(m + 1, r);
pull(m); return m;
}
ll ans = 9E18;
ll dist(int a, int b) {
    return (info[a].x[0] - info[b].x[0]) * (info[a].x[0] - info[b].x[0]) +
        (info[a].x[1] - info[b].x[1]) * (info[a].x[1] - info[b].x[1]);
}
void query(int p, int x) {
    if (!p) return;
    if (p != x) ans = min(ans, dist(x, p));
    ll distl = info[x].f(info[l[p]]);
    ll distr = info[x].f(info[r[p]]);
    if (distl < ans && distr < ans) {
        if (distl < distr) {
            query(l[p], x);
            if (distr < ans) query(r[p], x);
        } else {
            query(r[p], x);
            if (distl < ans) query(l[p], x);
        }
    } else {
        if (distl < ans) query(l[p], x);
        if (distr < ans) query(r[p], x);
    }
}
};
```

10.3 Max Euclidean Distance [4e338a]

```
tuple<ll, int, int> maxEuclideanDistance(vector<P> a) {
    auto get = [&](P p, Line l) -> ll {
        return abs(cross(l.a - l.b, l.a - p));
    };
    ll res = 0; int n = a.size(), x, y, id = 2;
    a.push_back(a.front());
    if (n <= 2) return {abs2(a[0] - a[1]), 0, 1};
    for (int i = 0; i < n; i++) {
        while (get(a[id], {a[i], a[i + 1]}) <= get(a[id + 1] % n, {a[i], a[i + 1]}))
            id = (id + 1) % n;
        if (res < abs2(a[i] - a[id])) {
            res = abs2(a[i] - a[id]);
            x = i, y = id;
        }
        if (res < abs2(a[i + 1] - a[id])) {
            res = abs2(a[i + 1] - a[id]);
            x = i + 1, y = id;
        }
    }
    return {res, x, y};
}
```

10.4 Lattice Points [52a028]

```
void latticePoints() {
    // 求 Area 與 Polygon 內整數點數
    int n; cin >> n;
    vector<P> polygon(n);
    for (int i = 0; i < n; i++) cin >> polygon[i];
    ll area = 0;
    for (int i = 0; i < n; i++)
        area += cross(polygon[i], polygon[(i + 1) % n]);
    area = abs(area);
    auto countBoundaryPoints =
        [&](const vector<P> &polygon) -> ll {
        ll res = 0;
        int n = polygon.size();
        for (int i = 0; i < n; i++) {
            ll dx = polygon[(i + 1) % n].x - polygon[i].x;
            ll dy = polygon[(i + 1) % n].y - polygon[i].y;
            res += __gcd(abs(dx), abs(dy));
        }
        return res;
    };
    ll res = countBoundaryPoints(polygon);
    ll ans = (area - res + 2) / 2;
}
```

10.5 Min Circle Cover [71b50f]

```
pair<double, P> minCircleCover(vector<P> a) {
    shuffle(a.begin(), a.end(), rng);
    int n = a.size();
    P c = a[0]; double r = 0;
    for (int i = 1; i < n; i++) {
        if (sign(abs(c - a[i]) - r) > 0) {
            c = a[i], r = 0;
            for (int j = 0; j < i; j++) {
                if (sign(abs(c - a[j]) - r) > 0) {
                    c = (a[i] + a[j]) / 2.0;
                    r = abs(c - a[i]);
                    for (int k = 0; k < j; k++) {
                        if (sign(abs(c - a[k]) - r) > 0) {
                            P p = (a[j] + a[k]) / 2;
                            P q = (a[j] + a[k]) / 2;
```

```

        if (cross(a[j] - a[i],
                  a[k] - a[j]) == 0) continue;
        c = lineIntersection
            ({p, p + rot(a[j] - a[i])},
            {q, q + rot(a[k] - a[j])});
        r = abs(c - a[i]);
    }
}
}
}
return {r, c};
}

```

10.6 Min Rectangle Cover [bde8ee6]

```

pair<double, vector<P>> minRectangleCover(vector<P> p) {
    if (p.size() <= 2) return {0, {}};
    auto get = [&](P p, Line l) -> double {
        return abs(cross(l.a - l.b, l.a - p));
    }; // line 到 p 圍成的四邊形面積
    int n = p.size(), j = 2, l = 1, r = 1;
    p.push_back(p.front());
    double ans = 8E18;
    vector<P> ps;
    for (int i = 0; i < n; i++) {
        while (get(p[j], {p[i], p[
            i + 1]}) <= get(p[(j + 1) % n], {p[i], p[i + 1]}))
            j = (j + 1) % n;
        while (dot(p[i + 1] - p[i], p[r] - p[i])
            <= dot(p[i + 1] - p[i], p[(r + 1) % n] - p[i]))
            r = (r + 1) % n;
        if (i == 0) l = j;
        while (dot(p[i + 1] - p[i], p[l] - p[i])
            >= dot(p[i + 1] - p[i], p[(l + 1) % n] - p[i]))
            l = (l + 1) % n;
        double area = get(p[j], {p[i], p[i + 1]});
        double w = dot(p[i] - p[i + 1],
            p[l] - p[i]) + dot(p[i + 1] - p[i], p[r] - p[i]);
        area *= w / square(p[i + 1] - p[i]);
        if (area < ans) {
            ps.clear(), ans = area;
            Line l1 {p[i], p[i + 1]};
            for (auto u : {p[r], p[j], p[l], p[i]}) {
                if (u == l1.b) {
                    ps.push_back(u);
                    l1 = {u, u + rot(l1.b - l1.a)};
                } else {
                    Line l2 = {u, u + rot(l1.b - l1.a)};
                    P res = lineIntersection(l1, l2);
                    ps.push_back(res);
                    l1 = {res, u};
                }
            }
        }
    }
    return {ans, ps};
}

```

10.7 Polygon Union Area [dc0989]

```

double polygonUnion(vector<vector<P>> ps) { // CCW needed
    int n = ps.size();
    for (auto &v : ps) v.push_back(v[0]);
    double res = 0;
    auto seg = [&](P o, P a, P b) -> double {
        if (b.x - a.x == 0) return (o.y - a.y) / (b.y - a.y);
        return (o.x - a.x) / (b.x - a.x);
    };
    for (int pi = 0; pi < n; pi++) {
        for (int i = 0; i + 1 < ps[pi].size(); i++) {
            vector<pair<double, int>> e;
            e.emplace_back(0, 0);
            e.emplace_back(1, 0);
            for (int pj = 0; pj < n; pj++) {
                if (pi == pj) continue;
                for (int j = 0; j + 1 < ps[pj].size(); j++) {
                    auto c1 = cross(ps[pi][i + 1]
                        - ps[pi][i], ps[pj][j] - ps[pi][i]);
                    auto c2 = cross(ps[pi][i + 1]
                        - ps[pi][i], ps[pj][j + 1] - ps[pi][i]);
                    if (c1 == 0 && c2 == 0) {
                        if (dot(ps[pi][i]
                            + 1] - ps[pi][i], ps[pj][j + 1]
                            - ps[pj][j]) > 0 && (pi - pj) > 0) {
                            e.emplace_back(seg(ps[pj][j],
                                ps[pi][i], ps[pi][i + 1]), 1);
                            e.emplace_back
                                (seg(ps[pj][j + 1], ps
                                    [pi][i], ps[pi][i + 1]), -1);
                        }
                    } else {
                        auto s1 = cross(ps[pj][j + 1] -
                            ps[pj][j], ps[pi][i] - ps[pj][j]);
                        auto s2 = cross(ps[pj][j + 1] - ps[pj]
                            [j], ps[pi][i + 1] - ps[pj][j]);
                        if (c1 >= 0 && c2 < 0)
                            e.emplace_back(s1 / (s1 - s2), 1);
                        else if (c1 < 0 && c2 >= 0)
                            e.emplace_back(s1 / (s1 - s2), -1);
                    }
                }
            }
        }
    }
}

```

```

    }
    sort(e.begin(), e.end());
    double pre = clamp(e[0].first, 0.0, 1.0), sum = 0;
    int cov = e[0].second;
    for (int j = 1; j < e.size(); j++) {
        double now = clamp(e[j].first, 0.0, 1.0);
        if (!cov) sum += now - pre;
        cov += e[j].second;
        pre = now;
    }
    res += cross(ps[pi][i], ps[pi][i + 1]) * sum;
}
return res / 2;
}

```

11 Polynomial

11.1 FFT [8d8ca2]

```

const double PI = acos(-1.0);
using cd = complex<double>;
vector<int> rev;
void fft(vector<cd> &a, bool inv = false) {
    int n = a.size();
    if (int(rev.size()) != n) {
        int k = __builtin_ctz(n) - 1;
        rev.resize(n);
        for (int i = 0; i < n; i++)
            rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
    }
    for (int i = 0; i < n; i++)
        if (rev[i] < i) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2) {
        double ang = (inv ? -1 : 1) * PI / k;
        cd wn(cos(ang), sin(ang));
        for (int i = 0; i < n; i += 2 * k) {
            cd w(1);
            for (int j = 0; j < k; j++, w = w * wn) {
                cd u = a[i + j];
                cd v = a[i + j + k] * w;
                a[i + j] = u + v;
                a[i + j + k] = u - v;
            }
        }
        if (inv) for (auto &x : a) x /= n;
    }
}
template<class T>
vector<T> Multiple(const vector<T> &a, const vector<T> &b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1, tot = a.size() + b.size() - 1;
    while (n < tot) n *= 2;
    fa.resize(n), fb.resize(n);
    fft(fa), fft(fb);
    for (int i = 0; i < n; i++)
        fa[i] = fa[i] * fb[i];
    fft(fa, true);
    vector<T> res(tot);
    for (int i = 0; i < tot; i++)
        res[i] = fa[i].real(); // use llround if need
    return res;
}

```

11.2 NTT [535f81]

```

template<int P = 998244353, int G = 3>
struct Poly : public vector<Mint<P>> {
    using Z = Mint<P>;
    static vector<int> rev;
    static vector<Z> w;
    static void ntt(vector<Z> &a, bool inv = false) {
        int n = a.size();
        if (rev.size() != n) {
            int k = __builtin_ctz(n) - 1;
            rev.resize(n);
            for (int i = 0; i < n; i++)
                rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
        }
        for (int i = 0; i < n; i++) if (rev[i] < i) swap(a[i], a[rev[i]]);
        if (w.size() < n) {
            int k = __builtin_ctz(w.size());
            w.resize(n);
            while ((1 << k) < n) {
                Z u = power(Z(G), (P - 1) >> (k + 1));
                for (int i = 1 << (k - 1); i < (1 << k); i++) {
                    w[i * 2] = w[i];
                    w[i * 2 + 1] = w[i] * u;
                }
                k++;
            }
        }
        for (int k = 1; k < n; k *= 2) {
            for (int i = 0; i < n; i += 2 * k) {
                for (int j = 0; j < k; j++) {
                    Z u =
                        a[i + j], v = a[i + j + k] * w[k + j];
                    a[i + j] = u + v; a[i + j + k] = u - v;
                }
            }
        }
    }
}

```

```

    }
    if (inv) {
        reverse(a.begin() + 1, a.end());
        Z inv_n = Z(n).inv();
        for (auto &x : a) x *= inv_n;
    }
}
explicit Poly(int n = 0) : vector<Z>(n) {}
Poly(const vector<Z> &a) : vector<Z>(a) {}
Poly(const initializer_list<Z> &a) : vector<Z>(a) {}
template<class InputIt, class = _RequireInputIter<InputIt>>
Poly(InputIt first, InputIt last) : vector<Z>(first, last) {}
Poly operator-() const {
    vector<Z> res(this->size());
    for (int i = 0; i < int(res.size()); i++) res[i] = -(*this)[i];
    return Poly(res);
}
friend Poly operator+(Poly a, Poly b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < b.size(); i++) a[i] += b[i];
    return a;
}
friend Poly operator-(Poly a, Poly b) {
    a.resize(max(a.size(), b.size()));
    for (int i = 0; i < b.size(); i++) a[i] -= b[i];
    return a;
}
friend Poly operator*(Poly a, Poly b) {
    if (a.empty() || b.empty()) return Poly();
    if (a.size() < b.size()) swap(a, b);
    int n = 1, tot = a.size() + b.size() - 1;
    while (n < tot) n *= 2;
    a.resize(n), b.resize(n);
    ntt(a), ntt(b);
    for (int i = 0; i < n; i++) a[i] *= b[i];
    ntt(a, true);
    a.resize(tot);
    return a;
}
friend Poly operator*(Poly a, Z x) {
    for (int i = 0; i < int(a.size()); i++) a[i] *= x;
    return a;
}
friend Poly operator/(Poly a, Z x) {
    for (int i = 0; i < int(a.size()); i++) a[i] /= x;
    return a;
}
Poly &operator+=(Poly a) {
    return (*this) = (*this) + a;
}
Poly &operator-=(Poly a) {
    return (*this) = (*this) - a;
}
Poly &operator*=(Poly a) {
    return (*this) = (*this) * a;
}
Poly &operator*=(Z a) {
    return (*this) = (*this) * a;
}
Poly &operator/=(Z a) {
    return (*this) = (*this) / a;
}
Poly shift(int k) const {
    if (k >= 0) {
        auto b = *this;
        b.insert(b.begin(), k, 0);
        return b;
    } else if (this->size() <= -k) {
        return Poly();
    } else {
        return Poly(this->begin() + (-k), this->end());
    }
}
Poly trunc(int k) const {
    Poly f = *this; f.resize(k); return f;
}
Poly deriv() const {
    if (this->empty()) return Poly();
    Poly res(this->size() - 1);
    for (int i = 0; i < this->size() - 1; i++)
        res[i] = (*this)[i + 1] * (i + 1);
    return res;
}
Poly integr() const {
    Poly res(this->size() + 1);
    for (int i = 0; i < this->size(); i++)
        res[i + 1] = (*this)[i] / (i + 1);
    return res;
}
Poly inv(int m) const {
    Poly x((*this)[0].inv());
    int k = 1;
    while (k < m) {
        k *= 2;
        x = (x * (Poly{2} - trunc(k) * x)).trunc(k);
    }
    return x.trunc(m);
}
Poly log(int m) const {
    return (deriv() * inv(m)).integr().trunc(m);
}
Poly pow(ll k, int m) const {
    if (k == 0) { Poly res(m); res[0] = 1; return res; }
    int i = 0;
    while (i < this->size() && (*this)[i] == 0) i++;

```

```

    if (i == this->size() || i > 0 && k > (m - 1) / i) return Poly(m);
    Z v = (*this)[i];
    auto f = shift(-i) * v.inv();
    return (f.log(m - i * k) * Z(k)).exp(m - i * k).shift(i * k) * power(v, k);
}
Poly sqrt(int m) const { // need quadraticResidue
    int k = 0;
    while (k < this->size() && (*this)[k] == 0) k++; // 找前導零
    if (k == this->size()) return Poly(m); // 全零多項式
    if (k % 2 != 0) return Poly(); // 無解: 最低次項為奇數
    int s = quadraticResidue((*this)[k]);
    if (s == -1) return Poly(); // 無解: 係數無平方根
    int oft = k / 2, r = m - oft;
    if (r <= 0) return Poly(m);
    Poly h = this->shift(-k) * (*this)[k].inv(), g{1};
    for (int i = 1; i < r; i <= 1) {
        int len = i < 1;
        g = (g + h.trunc(len) * g.inv(len)).trunc(len) / 2;
    }
    g.resize(r);
    g = (g * Z(s)).shift(oft).trunc(m);
    return g;
}
Poly exp(int m) const {
    Poly x{1};
    int k = 1;
    while (k < m) {
        k *= 2;
        x = (x * (Poly{1} - x.log(k) + trunc(k))).trunc(k);
    }
    return x.trunc(m);
}
Poly mult(Poly b) const {
    if (b.empty()) return Poly();
    int n = b.size();
    reverse(b.begin(), b.end());
    return ((*this) * b).shift(-(n - 1));
}
vector<Z> eval(vector<Z> x) const {
    if (this->size() == 0) return vector<Z>(x.size(), 0);
    const int n = max(x.size(), this->size());
    vector<Poly> q(4 * n);
    vector<Z> ans(x.size());
    x.resize(n);
    function<void(int, int, int)> build = [&](int p, int l, int r) {
        if (r - l == 1) {
            q[p] = Poly{1, -x[l]};
        } else {
            int m = (l + r) / 2;
            build(2 * p, l, m);
            build(2 * p + 1, m, r);
            q[p] = q[2 * p] * q[2 * p + 1];
        }
    };
    build(1, 0, n);
    function<void(int, int, int, const Poly &)> work = [&](int p, int l, int r, const Poly &num) {
        if (r - l == 1) {
            if (l < int(ans.size())) ans[l] = num[0];
        } else {
            int m = (l + r) / 2;
            work(2 * p, l, m, num.mult(q[2 * p + 1]).resize(m - l));
            work(2 * p + 1, m, r, num.mult(q[2 * p]).resize(r - m));
        }
    };
    work(1, 0, n, mult(q[1].inv(n)));
    return ans;
}
}
template<int P, int G> vector<int> Poly<P, G>::rev;
template<int P, int G> vector<Mint<P>> Poly<P, G>::w = {0, 1};

```

11.3 FWT [73ee9]

```

void fwt(vector<ll> &a, bool inv = false) {
    // Or: [[1, 0], [1, 1]] -> [[1, 0], [-1, 1]]
    // And: [[1, 1], [0, 1]] -> [[1, -1], [0, 1]]
    // Xor: [[1, 1], [1, -1]] -> [[0.5, 0.5], [0.5, -0.5]]
    int n = _lg(int(a.size()));
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < 1 << n; ++j) {
            if (j > i & 1) { // And
                ll x = a[j ^ (1 << i)];
                ll y = a[j];
                if (!inv) {
                    a[j ^ (1 << i)] = x + y;
                    a[j] = y;
                } else {
                    a[j ^ (1 << i)] = x - y;
                    a[j] = y;
                }
            }
        }
    }
}

```

11.4 Barlekamp Massey [853989]

```
template<int P = 998244353>
Poly<P> berlekampMassey(const Poly<P> &s) {
    Poly<P> c, oldC;
    int f = -1;
    for (int i = 0; i < s.size(); i++) {
        auto delta = s[i];
        for (int j = 1; j <= c.size(); j++)
            delta -= c[j - 1] * s[i - j];
        if (delta == 0) continue;
        if (f == -1) {
            c.resize(i + 1);
            f = i;
        } else {
            auto d = oldC;
            d *= -1;
            d.insert(d.begin(), 1);
            Mint<P> df1 = 0;
            for (int j = 1; j <= d.size(); j++)
                df1 += d[j - 1] * s[f + 1 - j];
            assert(df1 != 0);
            auto coef = delta / df1;
            d *= coef;
            Poly<P> zeros(i - f - 1);
            zeros.insert(zeros.end(), d.begin(), d.end());
            d = zeros;
            auto temp = c;
            c += d;
            if (i - temp.size() > f - oldC.size()) {
                oldC = temp;
                f = i;
            }
        }
    }
    c *= -1;
    c.insert(c.begin(), 1);
    return c;
}
```

11.5 Linear Recurrence [0272a8]

```
template<int P = 998244353>
Mint<P> linearRecurrence(Poly<P> p, Poly<P> q, ll n) {
    // assure: p(x) = \sum a_i x^i q(x) (mod x^d)
    //          q(x) = 1 - \sum c_i x^i
    int m = q.size() - 1;
    while (n > 0) {
        auto newq = q;
        for (int i = 1; i <= m; i += 2)
            newq[i] *= -1;
        auto newp = p * newq;
        newq = q * newq;
        for (int i = 0; i < m; i++)
            p[i] = newp[i * 2 + n % 2];
        for (int i = 0; i <= m; i++)
            q[i] = newq[i * 2];
        n /= 2;
    }
    return p[0] / q[0];
}
```

12 Else

12.1 Python [7c66a4]

```
from decimal import * # 高精度浮點數
from fractions import * # 分數
from random import *
from math import *
# set decimal prec bigger if it could overflow in precision
setcontext(
    Context(prec=10, Emax=MAX_EMAX, rounding=ROUND_FLOOR))
# read and print
x = int(input())
a, b, c = list(map(Fraction, input().split()))
arr = list(map(Decimal, input().split()))
print(*arr)
# set
st = set(); st.add((a, b)); st.remove((a, b))
if not (a, b) in st:
    # dict
    d = dict(); d[(a, b)] = 1; del d[(a, b)]
    for (a, b) in d.items():
        # random
        arr = [randint(1, r) for i in range(size)]
        choice([0, 6, 4, 1]) # random pick one
        shuffle(arr)
```

12.2 Fraction [62f33d]

```
template<class T>
struct Fraction {
    T n, d;
    void reduce() {
        T g = gcd(abs(n), abs(d));
        n /= g, d /= g;
        if (d < 0) n = -n, d = -d;
    }
    Fraction(T n = 0, T d = 1) : n(n), d(d)
    { assert(d != 0); reduce(); }
```

```
Fraction(const string &str) {
    char slash;
    if (str.find('/') != -1) {
        string x = str.substr(0, str.find('/'));
        string y = str.substr(str.find('/') + 1);
        n = stoBint(x), d = stoBint(y);
    } else {
        n = stoBint(str), d = 1;
    }
    Fraction(n, d);
}
Fraction operator+(Fraction rhs) const
{ return Fraction(n * rhs.d + rhs.n * d, d * rhs.d); }
Fraction operator-(Fraction rhs) const
{ return Fraction(n * rhs.d - rhs.n * d, d * rhs.d); }
Fraction operator*(Fraction rhs) const
{ return Fraction(n * rhs.n, d * rhs.d); }
Fraction operator/(Fraction rhs) const {
    assert(rhs.n != 0);
    return Fraction(n * rhs.d, d * rhs.n);
}
friend istream &operator>>(istream &is, Fraction &f) {
    string s; is >> s;
    f = Fraction(s);
    return is;
}
friend ostream &operator<<(ostream &os, const Fraction &f) {
    if (f.d == 1) os << f.n;
    else os << f.n << "/" << f.d;
    return os;
}
bool operator==(Fraction b) const
{ return n * b.d == b.n * d; }
bool operator!=(Fraction b) const
{ return n * b.d != b.n * d; }
bool operator<(Fraction b) const
{ return n * b.d < b.n * d; }
```

12.3 Bgint [c581e1]

```
struct Bgint { // not support hex division
private:
    static const int digit = 9; // hex: 7
    static const int base = 10; // hex: 16
    static const int B = power(ll(base), digit);
    Bgint(vector<int> x, int sgn) : x(x), sgn(sgn) {}
    template<class U> vector<int> norm(vector<U> a) {
        if (a.empty()) return {};
        for (int i = 0; i < a.size(); i++) {
            U c = a[i];
            a[i] = c % B;
            c /= B;
            if (c) {
                if (i == a.size() - 1) a.push_back(c);
                else a[i + 1] += c;
            }
        }
        while (a.size() > 1 && a.back() == 0) a.pop_back();
        return {a.begin(), a.end()};
    }
    void resign() { sgn = x.back() == 0 ? 1 : sgn; }
    int toInt(char c) const {
        if (isdigit(c)) return c - '0';
        else return c - 'A' + 10;
    }
    char toChar(int c) const {
        if (c < 10) return c + '0';
        else return c - 10 + 'A';
    }
public:
    int sgn = 1;
    vector<int> x; // 反著存
    Bgint() : x {0}, sgn(1) {}
    Bgint(ll a) { *this = Bgint(std::to_string(a)); }
    Bgint(string s) {
        if (s.empty()) { *this = Bgint(); return; }
        if (s[0] == '-') s.erase(s.begin()), sgn = -1;
        int add = 0, cnt = 0, b = 1;
        while (s.size()) {
            if (cnt == digit) {
                x.push_back(add), add = cnt = 0;
                b = 1;
            }
            add += toInt(s.back()) * b;
            cnt++, b *= base;
            s.pop_back();
        }
        if (add) x.push_back(add);
        x = norm(x);
    }
    int size() const { return x.size(); }
    Bgint abs() const { return Bgint(x, 1); }
    string to_string() const {
        string res;
        for (int i = 0; i < x.size(); i++) {
            string add; int v = x[i];
            for (int j = 0; j < digit; j++)
                add += toChar(v % base), v /= base;
            res += add;
        }
    }
```

```

    while (res
        .size() > 1 && res.back() == '0') res.pop_back();
    if (sgn == -1) res += '-';
    reverse(res.begin(), res.end());
    return res;
}
Bigint operator-(const Bigint(x, -sgn); }
friend Bigint operator+(Bigint a, Bigint b) {
    if (a.sgn != b.sgn) return a - (-b);
    int n = max(a.size(), b.size());
    a.x.resize(n), b.x.resize(n);
    for (int i = 0; i < n; i++) a.x[i] += b.x[i];
    a.x = a.norm(a.x), a.resign();
    return a;
}
friend Bigint operator-(Bigint a, Bigint b) {
    if (a.sgn != b.sgn) return a + (-b);
    if (a.abs() < b.abs()) return -(b - a);
    int n = max(a.size(), b.size());
    a.x.resize(n), b.x.resize(n);
    for (int i = 0; i < n; i++) {
        a.x[i] -= b.x[i];
        if (a.x[i] < 0) a.x[i] += B, a.x[i + 1]--;
    }
    a.x = a.norm(a.x), a.resign();
    return a;
}
friend istream &operator>>(istream &is, Bigint &a)
{ string v; is >> v; a = Bigint(v); return is; }
friend ostream &operator<<(ostream &os, Bigint a)
{ os << a.to_string(); return os; }
friend bool operator<(Bigint a, Bigint b) {
    if (a.sgn != b.sgn) return a.sgn < b.sgn;
    if (a.x.size() != b.x.size()) {
        return a.x.size() < b.x.size();
    } else {
        for (int i = a.x.size() - 1; i >= 0; i--)
            if (a.x[i] != b.x[i]) return a.x[i] < b.x[i];
    }
    return 0;
}
friend bool operator>(Bigint a, Bigint b) {
    if (a.sgn != b.sgn) return a.sgn > b.sgn;
    if (a.x.size() != b.x.size()) {
        return a.x.size() > b.x.size();
    } else {
        for (int i = a.x.size() - 1; i >= 0; i--)
            if (a.x[i] != b.x[i]) return a.x[i] > b.x[i];
    }
    return 0;
}
friend bool operator==(const Bigint &a, const Bigint &b)
{ return a.sgn == b.sgn && a.x == b.x; }
friend bool operator!=(const Bigint &a, const Bigint &b)
{ return a.sgn != b.sgn || a.x != b.x; }
friend bool operator>=(const Bigint &a, const Bigint &b)
{ return a == b || a > b; }
friend bool operator<=(const Bigint &a, const Bigint &b)
{ return a == b || a < b; }
};
Bigint abs(const Bigint &a) { return a.abs(); }
Bigint stoBigint(const string &s) { return Bigint(s); }

```

12.4 Multiple [a8c792]

```

// Require:
// Mint, NTT ~constructor and * operator
using i128 = __int128_t;
const int P1 = 1045430273; // 2^20 * 997 + 1
const int P2 = 1051721729; // 2^20 * 1003 + 1
const int P3 = 1053818881; // 2^20 * 1007 + 1
using Poly1 = Poly<1045430273, 3>;
using Poly2 = Poly<1051721729, 6>;
using Poly3 = Poly<1053818881, 7>;
const i128 T1 = i128(P2) * P3;
const i128 T2 = i128(P1) * P3;
const i128 T3 = i128(P1) * P2;
const int I1 = Mint<P1>(T1).inv().x;
const int I2 = Mint<P2>(T2).inv().x;
const int I3 = Mint<P3>(T3).inv().x;
const i128 M = i128(P1) * P2 * P3;
vector<i128> arbitraryMult
    (const vector<int> &a, const vector<int> &b) {
    int n = a.size(), m = b.size();
    Poly1 x =
        Poly1(a.begin(), a.end()) * Poly1(b.begin(), b.end());
    Poly2 y =
        Poly2(a.begin(), a.end()) * Poly2(b.begin(), b.end());
    Poly3 z =
        Poly3(a.begin(), a.end()) * Poly3(b.begin(), b.end());
    vector<i128> res(x.size());
    for (int i = 0; i < x.size(); i++)
        res[i] = (x[i].x * T1 % M * I1 +
            y[i].x * T2 % M * I2 +
            z[i].x * T3 % M * I3) % M;
    return res;
}
public:
friend Bigint operator*(Bigint a, Bigint b) {
    a.x = a.norm(arbitraryMult(a.x, b.x));
    a.sgn *= b.sgn, a.resign();
    return a;
}

```

12.5 Division [79e100]

```

private:
vector<int> smallDiv(vector<int> a, int v) {
    ll add = 0;
    for (int i = a.size() - 1; i >= 0; i--) {
        add = add * B + a[i];
        int q = add / v;
        a[i] = q, add %= v;
    }
    return norm(a);
}
friend Bigint operator<<(Bigint a, int k) {
    if (!a.x.empty()) {
        vector<int> add(k, 0);
        a.x.insert(a.x.begin(), add.begin(), add.end());
    }
    return a;
}
friend Bigint operator>>(Bigint a, int k) {
    a.x = vector<int>(
        a.x.begin() + min(k, int(a.x.size())), a.x.end());
    a.x = a.norm(a.x);
    return a;
}
public:
friend Bigint operator/(Bigint a, Bigint b) {
    a = a.abs(), b = b.abs();
    a.sgn *= b.sgn;
    if (a < b) return Bigint();
    if (b.size() == 1) {
        a.x = a.smallDiv(a.x, b.x[0]);
    } else {
        Bigint inv = 1LL * B * B / b.x.back();
        Bigint pre = 0, res = 0;
        int d = a.size() + 1 - b.size();
        int cur = 2, bcur = 1;
        while (inv != pre || bcur < b.size()) {
            bcur = min(bcur << 1, b.size());
            res.x = {b.x.end() - bcur, b.x.end()};
            pre = inv;
            inv = inv * ((Bigint
                (2) << (cur + bcur - 1)) - inv * res);
            cur = min(cur << 1, d);
            inv.x = {inv.x.end() - cur, inv.x.end()};
        }
        inv.x = {inv.x.end() - d, inv.x.end()};
        res = (a * inv) >> a.size();
        Bigint mul = res * b;
        while (mul + b <= a) res = res + 1, mul = mul + b;
        a.x = a.norm(res.x);
    }
    return a;
}
friend Bigint operator%(Bigint a, Bigint b)
{ return a - (a / b) * b; }
Bigint gcd(Bigint a, Bigint b) {
    while (b != 0) {
        Bigint r = a % b;
        a = b, b = r;
    }
    return a;
}

```

12.6 Division-Python [110bd8]

```

from decimal import * # 無誤差浮點數
setcontext(
    Context(prec=4000000, Emax=4000000, rounding=ROUND_FLOOR))
t = int(input())
for i in range(t):
    a, b = map(Decimal, input().split())
    d, m = divmod(a, b)
    print(d, m)

```