Problem 3

Tuesday, June 11, 2024   4:16 PM

— QDA

— $p=1$

— class specific mean

— normal distribution

— class specific variance

— k^{th} class from $N(\mu_k, \sigma_k^2)$

~ $(4.16) = f_k(x) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)$

Prove Bayes classifier is not linear

$(4.15) = Pr(Y=k \mid X=x) = \frac{\pi_k f_k(x)}{\sum_{t=1}^{k} \pi_t f_t(x)}$

thus (4.16) into (4.15)

$\begin{array}{l}
4.17 \\
modified
\end{array} = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(\frac{-1}{-2\sigma_k^2}(x-\mu_k)^2\right)}{\sum_{t=1}^{k} \pi_t \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}$

based on Problem 2

$\begin{array}{l}
4.18 \\
modified
\end{array} = \frac{x\mu_k}{\sigma_k^2} - \frac{x^2}{2\sigma_k^2} - \frac{\mu_k^2}{2\sigma_k^2} + \log(\pi_k)$

∴ Thus it is quadratic from the $x + x^2$ terms, not linear.