# Problem 10

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4.32 $\log\left(\frac{Pr(Y=k \mid X=x)}{Pr(Y=K \mid X=x)}\right)$ $p>1$

$p>1$ < $\mu_k$ = mean k is p dim vector
$\Sigma$ = shared covariance $p \times p$ matrix

$p=1$ $\mu_1 \dots \mu_K$ > scalars
$\sigma^2$

(4.32) want $a_k, b_{kj}$ in terms of $\pi_k$ $\pi_K$, $\mu_k$ $\mu_K$, $\sigma^2$

$$Pr(Y=k \mid X=x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{K} \pi_l f_l(x)} \quad ①$$

$$f_k(x) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right) \quad ②$$

$$\log\left(\frac{Pr(Y=k \mid X=x)}{Pr(Y=K \mid X=x)}\right) = \log\left(\frac{\frac{\pi_k f_k(x)}{\cancel{\sum_{t=1}^{K} \pi_t f_t(x)}}}{\frac{\pi_K f_K(x)}{\cancel{\sum_{t=1}^{K} \pi_t f_t(x)}}}\right) \quad \text{by ①}$$

$$= \log\left(\frac{\pi_k f_k(x)}{\pi_K f_K(x)}\right) \quad \text{by cancelation of terms}$$

$$= \log\left(\frac{\pi_k \cancel{\frac{1}{\sqrt{2\pi}\sigma_k}} \exp\left(-\frac{1}{2\sigma_k^2}(x-\mu_k)^2\right)}{\pi_K \cancel{\frac{1}{\sqrt{2\pi}\sigma_K}} \exp\left(-\frac{1}{2\sigma_K^2}(x-\mu_K)^2\right)}\right) \quad \text{by ②}$$

$$= \log\left(\frac{\pi_k}{\pi_K}\right) - \frac{1}{2\sigma^2}(x-\mu_k)^2 + \frac{1}{2\sigma^2}(x-\mu_K)^2$$

$\sigma_k^2 = \sigma_K^2$ based on given

$$= \log\left(\frac{\pi_k}{\pi_K}\right) - \frac{1}{2\sigma^2}\left((x-\mu_k)^2 - (x-\mu_K)^2\right)$$

$$= \log\left(\frac{\pi_k}{\pi_K}\right) - \frac{1}{2\sigma^2}\left(x^2 - 2\mu_k x + \mu_k^2 - x^2 + 2\mu_K x - \mu_K^2\right)$$

$$= \log\left(\frac{\pi_k}{\pi_K}\right) - \frac{1}{2\sigma^2}\left(-2\mu_k x + \mu_k^2 + 2\mu_K x - \mu_K^2\right)$$

$$= \log\left(\frac{\pi_k}{\pi_K}\right) + \frac{\mu_k - \mu_K}{\sigma^2}(x) + \frac{-\mu_k^2 + \mu_K^2}{2\sigma^2}$$

$a = \log\left(\frac{\pi_k}{\pi_K}\right) + \frac{-\mu_k^2 + \mu_K^2}{2\sigma^2}$ ← based not on (x)

$b = \frac{\mu_k - \mu_K}{\sigma^2}$ ← based on (x)