Problem 2

Tuesday, June 11, 2024   4:00 PM

Assume observation is drawn from kth class from a $N(\mu_k, \sigma^2)$ distribution.

(4.17)
$$p_k(x) = \frac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}}{\sum_{l=1}^{K} \pi_l \frac{1}{\sqrt{2\pi}\sigma} e^{\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}}$$

(4.18)
$$\delta_k(x) = x\frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

4.17
$$\frac{\pi_k \; e^{\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}}{\sum_l^K \pi_l \; e^{\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}}$$

$$\underbrace{\log\left(\pi_k e^{\left(-\frac{1}{2\sigma^2}(x-\mu_k)^2\right)}\right)}_{\text{varies based on k}} - \underbrace{\log\left(\sum_{l=1}^{K} \pi_l e^{\left(-\frac{1}{2\sigma^2}(x-\mu_l)^2\right)}\right)}_{\text{based on L}}$$

→ $p_k(x)$ is max for terms based on k

$$= \log(\pi_k) + -\frac{1}{2\sigma^2}(x-\mu_k)^2$$
$$= \log(\pi_k) + -\frac{1}{2\sigma^2}(x^2 - 2\mu_k x + \mu_k^2)$$
$$= \log(\pi_k) + -\frac{x^2}{2\sigma^2} + \frac{\cancel{2}\mu_k x}{\cancel{2}\sigma^2} - \frac{\mu_k^2}{2\sigma^2}$$
$$= x\frac{\mu_k}{\sigma^2} - \frac{x^2}{2\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

$\frac{x^2}{2\sigma^2}$ is a constant since no k value thus can be removed

$$\boxed{\therefore \; 4.17 = 4.18}$$

✓

$$\delta(x) = \frac{x\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$