Modular Forms

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Defining Modular Forms

In this presentation, I will be working with the modular group $SL_2(\mathbb{Z}) = \Gamma_1$

Definition: Modular form

f is a modular form of weight k if

1. f is holomorphic on $\mathbb{H}$
2. f continues to be holomorphic as $z \longrightarrow i\infty$
3. $f(\frac{az+b}{cz+d}) = (cz+d)^k f(z) \quad \forall \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \Gamma_1$

Dimension of $M_k(\Gamma_1)$

Modular forms of a fixed weight form a vector space over the Complex numbers. The dimension of $M_k(\Gamma_1)$ is 0 for all negative and odd values of k. Otherwise the dimension is given by the following formula:

$$dim(M_k(\Gamma_1)) = \begin{cases} \lfloor \frac{k}{12} \rfloor + 1 & \text{if } k \not\equiv 12 \ mod(12) \\ \lfloor \frac{k}{12} \rfloor & \text{if } k \equiv 12 \ mod(12) \end{cases}$$