Which Control Chart Should You Use?

EACH TYPE HAS BENEFITS AND DISADVANTAGES, DEPENDING ON THE SITUATION.

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One of the primary statistical process control tools used to evaluate the state of a process is the control chart. A major decision for Six Sigma practitioners is what type of control chart to use and the corresponding type of data to measure and track.

A control chart provides a graphic comparison of performance data, such as sample statistics, to compute control limits drawn on the chart. It is used to detect assignable causes of variation in the process.

A process operating without assignable causes of variation is said to be either stable, predictable or in a state of statistical control. Control limits are calculated in such a way that variation outside these limits is presumed to be due not to random causes but to assignable ones.

While control charts can be an effective tool for evaluating performance and improvement in a Six Sigma project, it is critical that Six Sigma project managers carefully evaluate different charts available to ensure the selection provides the most useful information for the project. This decision should take into account a variety of factors including sample size, data characteristics and whether the data is attribute or variable.

Attribute Data

Control charts fall broadly into two categories: attribute and variable. An attribute is a characteristic appraised in terms of whether it does or does not exist (for example, go or no-go) with respect to a given requirement.

The two classes of attribute data are based on:

- The Poisson distribution.
- The binomial distribution.

In the Poisson distribution, the term "defect" or "nonconformity" describes the occurrence of an attribute such as a blemish, scratch, burn, error or omission that appears on an object.[1] A defect does not necessarily make the object unusable or unacceptable.

Stated in more general terms, for this class of attribute, we are describing the occurrences of any operationally defined event—desirable or undesirable—within a given sample space. For example, you might wish to monitor the number of trees in an acre of forest or the number of students who receive passing grades in a particular class.

The control charts for monitoring defects include *c*, *u* and individual and moving range (*XmR*) charts.

A *c* chart monitors the number of occurrences of an operationally defined event per sample. When using a *c* chart, the sample size must remain constant. A *u* chart monitors the number of occurrences of an operationally defined event per unit. When using a *u* chart, the sample size may vary.

In certain cases, variable control charts may be used to evaluate processes in which attribute data is collected. Donald Wheeler describes how it is generally easier to use variable charts to monitor attribute data than to attempt to verify the statistical conditions necessary for use of an attribute chart (see Table 1).[2] For example, an *X* or individual chart is a variable chart that also may be used for attribute data in place of a *c* or *u* chart.

Sample size must remain constant when using *XmR* charts to monitor counts—to substitute for a *c* chart. Sample size may vary when using the *XmR* charts to monitor rates—to substitute for a *u* chart.

In the second class of attribute data, the term "defective" or "nonconforming item" describes the occurrence of a product or service flawed beyond use or acceptability.[3]

Stated in more general terms, an item meets—or fails to meet—some previously established operationally defined criteria for acceptance. For example, you could monitor a basketball player's percentage of successful or unsuccessful free throws.

The appropriate charts for defectives include *np*, *p* and *np* charts. An *np* chart monitors the number of defectives in a sample. A *p* chart monitors the percentage of defectives in a sample. *XmR* charts also may be used in place of *np* or *p* charts.

An *np* chart requires a constant sample size, but sample size may vary when using a *p* chart. When using *XmR* charts to monitor counts (as a substitute for an *np* chart), sample size must remain constant. But sample size may vary when using an *YmR* chart to monitor rates (as a substitute for a *p* chart).

The following example illustrates the distinction between the terms "defect" and "defective." An appliance dealer might have a washing machine with a scratch, which, although an undesirable quality characteristic, clearly would not affect the performance of the product. The product could still be sold. This is an example of a defect in the product.

However, if the machine had so many scratches and dents it would not be sold, it could be described as defective. It would not meet the dealer's operationally defined criteria for salability (see Table 1).

Variable Data

Variable data are quantitative data in the form of a measurement such as length, height, temperature, density, weight or time.[4] Variable control charts

Table 1. **Required Conditions For Poisson and Binomial Charts**

Poisson
• The counts are of discrete events.
• The discrete events occur within some well-defined, finite region of space, time or product. This finite region is the area of opportunity for the count.
• The events occur independently of each other, and the likelihood of an event is proportional to the size of area of opportunity.
• The events are rare.
Binomial
• The area of opportunity for the count *Y* must consist of *n* distinct items.
• Each n distinct item must be classified as possessing or not posessing some attribute. This attribute is usually some type of nonconformance to specifications.
• The value of the probability that an item has the attribute being counted must be the same for all *n* items in any one sample.
• The likelihood of an item's possessing the attribute will not be affected by whether the preceding item possessed the attribute.

Source: Donald J. Wheeler, *Advanced Topics in Statistical Process Control*, SPC Press, 1995.

usually are used in pairs: One chart monitors the central tendency of the process, and the other monitors the variability of the process.

The sample size usually determines which pair of charts is used. The most common combinations are *XmR*, sample average and range, and sample average and sample standard deviation charts. Although in practice sample size usually remains constant, there are techniques that enable sample size to vary with variable control charts.

Chart Selection

All processes exhibit variation. A control chart user must select the chart that provides the most effective picture of the variation in the system's performance. The following hypothetical scenario illustrates this:

A Six Sigma analyst is interested in evaluating the room cleaning process of a large hotel. He is trying to decide which control chart to use to determine whether the process is stable and to monitor improvements in performance. He lists options.

- He considers each room as an attribute in which 10 tasks must be accomplished correctly for the room to meet his quality standard: Therefore the room is considered an entity that is either acceptable or not. He selects a sample of 50 rooms to be evaluated each week. In this case, he uses a constant sample size and is monitoring defectives, so he selects an *np* chart.
- He considers each room as an attribute in which 10 tasks must be accomplished correctly for the room to meet his quality standard. Therefore the room is considered as an entity that is either acceptable or not. Recognizing there is considerable variation in hotel occupancy, he varies sample size by selecting a sample of 10% of all rooms occupied during the week. In this case, he uses a varying sample size and is monitoring defectives, so he selects a *p* chart.
- He considers each room as an attribute in which 10 tasks must be accomplished correctly for the room to meet his quality standard. Therefore the room is considered as an entity that is either acceptable or not. He selects a sample of 50 rooms to be evaluated each week and treats each improperly cleaned room as a defective item. However, rather than using an attribute control chart, he selects *XmR* charts to plot the number of improperly cleaned rooms per sample.
- He decides to evaluate how many of the 10 tasks are improperly performed in each room and treats each improperly performed task as a defect. He selects a sample of 50 rooms each week to evaluate. The number of defects is the total number of improperly performed tasks in the sample of 50 rooms. In this case, he uses a constant sample size and is monitoring defects, so he selects a *c* chart.
- He decides to evaluate how many of the 10 tasks are improperly performed in each room. He treats each improperly performed task as a defect. Recognizing there is considerable variation in hotel occupancy, he varies sample size by selecting a sample representing 10% of all rooms occupied during the week. In this case, he uses a varying sample size and is monitoring defects, so he selects a *u* chart.
- He decides to evaluate how many of the 10 tasks are improperly performed in each room, treating each improperly performed task as a defect. Recognizing there is considerable variation in hotel occupancy, he varies sample size by selecting a sample representing 10% of all rooms occupied during the week. However, rather than use an attribute control chart, he selects *XmR* charts to plot the number of improperly performed tasks per room.
- He decides there is a scalable difference in the performance level for each of the 10 tasks he has selected to study. For each room, he grades the performance of each task using a 10-point scale. To ensure consistency, he also uses a descriptive assessment scale to operationally define the relationship between performance and the evaluation scale. He selects a sample of 50 rooms each week to evaluate. He uses a separate $\bar{x}$ and S chart for each task being monitored. Each value plotted on the $\bar{x}$ chart will be an average measurement for a particular task. Each value plotted on an S chart will be the standard deviation for the 50 measurements for the task.
- He decides there is a scalable difference in the performance level of each of the 10 tasks selected for study. For each room, he grades the performance of each task using a 10-point scale. To ensure consistency, he also uses a descriptive assessment scale to operationally define the rela-

tionship between performance and the evaluation scale. He selects a sample representing 10% of all rooms occupied during the week to evaluate and uses a separate $\bar{x}$ and S chart for each task being monitored. Each value plotted on the $\bar{x}$ chart will be an average measurement for a particular task. Each value plotted on an S chart will be the standard deviation for the 50 measurements for the task. However, with varying sample size, it will be necessary to continuously recalculate the control limits for each sample.

Which Chart Is Best?

Each of the options described provides the Six Sigma project manager means to evaluate the performance of the housekeeping process. However, the potential to capture useful information varies considerably from chart to chart.

The analyst could use either an *np* or a *p* chart to evaluate the number of rooms in a 50-room sample that meet the criteria of having all 10 tasks performed correctly. However, if he uses either chart, he will not capture the possible variation in the number of tasks improperly performed in the rooms that fail the criteria. Therefore, the system performance could change without any indication on the chart.

The *np* chart also does not enable the analyst to vary sample size to reflect the usual variations in occupancy. Because the *p* chart monitors the percentage of rooms that meet the criteria of having all 10 tasks performed correctly, it allows the analyst to vary sample size. Before he uses either the *p* or *np* chart, he must first verify the relevant binomial conditions are met.

The *c* chart enables the analyst to evaluate how many of the tasks are performed improperly in a sample of 50 rooms. Each data point represents the total number of improperly performed tasks (defects) in the sample of 50 rooms.

If the analyst wants to take into consideration the usual variation in occupancy, he could substitute a *u* chart for the *c* chart. Each data point now represents the average number of improperly performed tasks per room. Before the analyst uses either the *c* or *u* chart, he must first verify that the relevant Poisson conditions are met.

If the analyst were to use any of the variable control charts (*XmR* or $\bar{x}$ and S charts), he would not be required to ensure the binomial or Poisson conditions necessary for the *p, np, c* or *u* chart have been met for the data collected.

Also, the *p, np, c* and *u* charts assume dispersion is a function of location and therefore use theoretical control limits. That is, they assume a theoretical relationship between the average and the dispersion because the dispersion is estimated directly from the estimate of the average. The individual chart, however, uses empirical limits. If the theory is wrong, the theoretical limits will be wrong while the empirical limits will be correct.[5, 6]

Using the $\bar{x}$ and S charts will undoubtedly require the most effort to collect and evaluate data. The analyst also will have to develop operational definitions for the descriptive assessment scales and train data collectors to use them.

If the analyst wishes to vary sample size, he also will have to rely on computer software to continuously adjust control limits. On the other hand, this approach could provide the most comprehensive picture of the variation in the performance of each activity in the room cleaning process.

Table 2. **Variable and Attribute Control Charts**

Variable Control Charts (All in Pairs)	
X–MR:	Monitor individual data from a sample of one and the ranges between consecutive sample values.
$\bar{x}$*–R:*	Monitor sample average and sample range. Sample size usually falls between 2 and 10.
$\bar{x}$*–S:*	Monitor sample average and sample standard deviations. Sample size usually is greater than 10.
Attribute Control Charts (All Single Charts)	
np:	Monitors the number of items within a constant sample size that meet or fail to meet some operationally defined criteria.
p:	Monitors the percentage of items within a sample that meet or fail to meet some operationally defined criteria.
c:	Monitors the number of occurrences of an operationally defined event within a fixed sample space.
u:	Monitors the number of occurrences per unit of an operationally defined event.

Table 3. **Advantages and Disadvantages of Each Type of Chart**

Chart type	*Np* chart	*P* chart	*C* chart	*U* chart	*XmR* charts	$\bar{x}$–*S* charts
Use	Monitors items meeting or failing to meet operationally defined criteria—defectives.	Monitors items meeting or failing to meet operationally defined criteria—defectives.	Monitors the occurrence of operationally defined events—defects.	Monitors the occurrence of operationally defined events—defects.	Monitors individual variable measurements (X chart) and ranges between an n number of consecutive variable measurements. Also monitors the number or percentage of items meeting or failing to meet operationally defined criteria—defectives. Also monitors the occurrence of operationally defined events—defects.	Monitors sample averages and sample standard deviations for variable data. Also monitors the number or percentage of items meeting or failing to meet operationally defined criteria—defectives. Also monitors the occurrence of operationally defined events—defects.
Data requirements	Data must meet binomial conditions.	Data must meet binomial conditions.	Data must meet Poisson conditions.	Data must meet Poisson conditions.	Data does not have to meet binomial or Poisson conditions.	Data does not have to meet binomial or Poisson conditions.
Sample size	Must remain constant—all samples must have the same sized area of opportunity.	May vary to reflect changes in the area of opportunity, but user must recalculate control limits if area of opportunities varies.	Must remain constant—all samples must have the same sized area of opportunity.	May vary to reflect changes in the area of opportunity—requires user to recalculate control limits if area of opportunity varies.	Must remain constant when using to monitor counts or to substitute for an *np* or *c* chart. May vary when using to monitor rates or to substitute for a *p* or *u* chart.	Sample size requires user to recalculate control limits if sample sizes vary. Requires calculation of each sample's standard deviations.
Other		Chart's use requires counts to be divided by their areas of opportunity to determine proportion.		Chart's use requires each count to be divided by its area of opportunity to determine rate.		

No One Right Answer

So which chart is best? There is no universal answer to this question. Wheeler writes that to understand data, we must first know the context for the data.[7] The example shows several possibilities for the analyst to consider before selecting a chart to monitor and evaluate the process.

In selecting an appropriate chart for a particular situation, we advise readers to enumerate the chart selection possibilities and consider the benefits and disadvantages for each possible selection.

As our example shows, the issue is not simply whether you are evaluating defects or defectives or even whether you are using attribute or variable data. The primary criteria should be the knowledge that would be gained by using any particular chart. Resource availability to monitor the charts also may be a factor.

Control charts are used to understand the degree of control, or stability, of the process under initial study. If a chart shows the process is unpredictable, the Six Sigma project should focus on determining the sources of assignable cause variation, which prevent the process from being in control.

If the chart shows the process is predictable, or in cases in which the Six Sigma project has been successful in eliminating the sources of special cause variation occurring in the process, then the practitioner must determine next whether that process is capable of meeting the customer's specifications through capability analysis.

Control charts also may be used when examining a process from a higher level. Here the focus is to compile and analyze data to provide a long-term view of the capability and performance of the process relative to meeting the needs of customers. This approach may support the long-term control of a stable process to ensure it remains predictable.[8]

Table 2 (p. 29) summarizes the most common control charts and their primary characteristics. The list is not exhaustive. Donald Wheeler's *Advanced Topics in Statistical Process Control* provides a detailed survey of control charts and their characteristics.[9]

Table 3 summarizes the technical requirements, advantages and disadvantages of each of the charts listed in Table 2. This table may be used as a reference to help the reader select an appropriate chart for a Six Sigma project.

What Data Are Available?

Practitioners should consider the availability of the necessary data and the resources required to collect and evaluate that data. They also must understand the distinct roles control charts may play in a Six Sigma project.

In an initial study of a process, use the understanding of that process to select the measurement system that will provide the most useful information to evaluate whether it is stable. Such understanding only comes after detailed study using other quality tools such as flowcharts and cause and effect analysis.

For a refined, capable process, practtioners must consider how it can most efficiently and effectively be monitored over time to prevent reaction to common cause issues as though they were special cause.

There often are no cookbook solutions to the dilemma of chart selection. A variety of options may be available, so don't fall into the habit of selecting the most obvious chart without careful consideration of which will work best.

REFERENCES

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