

## APPENDIX C - BASIC RELIABILITY EQUATIONS

$$F(t) = \int_0^t f(\tau) d\tau ; 1.0 - R(t) ; 1.0 - e^{-\int_0^t h(\tau) d\tau} ; 1.0 - e^{-H(t)} ; 1.0 - e^{-\int_0^t \frac{1+L'(\tau)}{L(\tau)} d\tau}$$

An equation in which the independent variable is "time" and in which the dependent variable is the fraction failed is known as the **Cumulative distribution function or C.D.F.** Denoted by: **F (t)** = Fraction failed within total time "t".

Probabilistically: **P(T ≤ t)**

Properties: (1) F (t) is continuous for all "t".

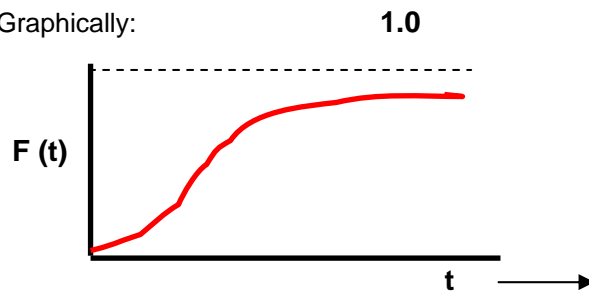
(2) F (t) < F (t') for all t < t'.

(3) F (t) is a monotonically increasing function on the interval [0, ∞] so that

$$F(\infty) = 1.0$$

**F(0) = 0** ==> Nothing has failed at time "0". ==> Everything has failed at time "∞"

Graphically:



$$f(t) = F'(t) ; -R'(t) ; -\frac{d}{dt}[e^{-H(t)}] ; -\frac{d}{dt}[e^{-\int_0^t h(\tau) d\tau}] ;$$

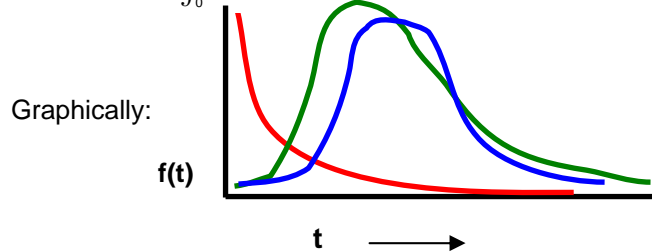
$$h(t) * R(t) ; \frac{1 + L'(t)}{L(t)} e^{-\int_0^t \frac{1+L'(\tau)}{L(\tau)} d\tau}$$

If we differentiate a C.D.F. with respect to "t", we obtain a **Probability Density function or p.d.f.** Denoted by:  $f(t)$  = Rate of change of the C.D.F.; It measures the "slope" of the cumulative curve.

Probabilistically:  **$P(T = t)$**

Properties: (1)  $f(t)$  is continuous for all "t".  
(2)  $f(t) > 0$  for all t.

(3)  $\int_0^t f(t)dt = 1.0$  area under the curve such that the area under the curve from "0" to "a" =F (a).



$$\text{slope} = f(t) = \frac{dF(t)}{dt}$$

$$R(t) = 1.0 - F(t) ; 1.0 - \int_0^t f(t)dt ; e^{-H(t)} ; e^{-\int_0^t h(t)dt} ; f(t)/h(t) ;$$

$$F'(t)/h(t) ; e^{-\int_0^t \frac{1+L'(\tau)}{L(\tau)}d\tau}$$

If we take the complement of the C.D.F. "F (t)" at some point in time "t", we obtain the **"Reliability function" at "t"**, or the fraction unfailed at time "t". Denoted by:  $R(t)$  = Fraction unfailed at time "t."

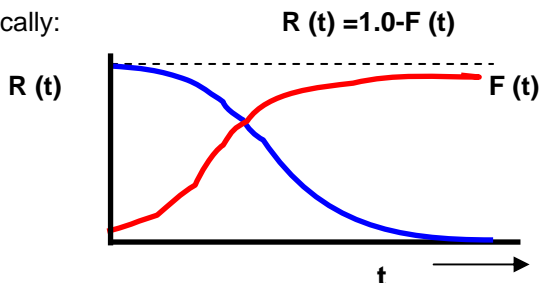
Probabilistically:  **$P(T > t)$**

Properties: (1)  $R(t)$  is continuous for all "t".  
(2)  $R(t) < R(t')$  for all  $t < t'$ .  
(3)  $R(t)$  is a monotonically decreasing function on the interval  $[0, \infty]$

Such that  $R(0) = 1.0 \implies$  Nothing has failed at time "0".

$R(\infty) = 0.0 \implies$  Everything has failed at time " $\infty$ ."

Graphically:

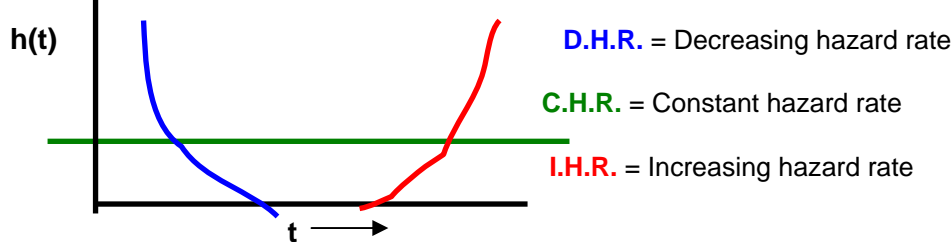


The **instantaneous hazard rate** at any point in time is  $f(x)/F(x)$ . Like a density function, a failure rate is a complete descriptor of a life distribution. Denoted by:  $h(t)$  = Rate of change of failure at time "t."

Probabilistically:  **$P(T = t)/P(T > t)$**

Properties: (1)  $h(t)$  is piece-wise continuous for all "t" greater than or equal to "0."  
(2)  $h(t)$  is an increasing / decreasing /constant function on the interval  $[0, \infty]$

Graphically:

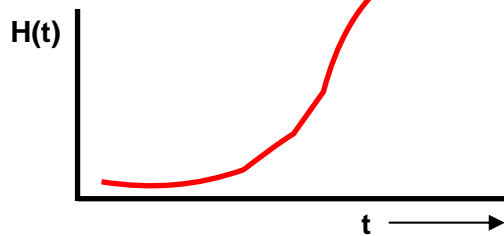


$$H(t) = \int_0^t h(\tau) d\tau ; \int_0^t \frac{f(\tau)}{1.0 - F(\tau)} d\tau ; \ln \frac{1.0}{R(t)} ; -\ln[R(t)] ; \ln F(t) ; \int_0^t \frac{1 + L'(\tau)}{L(\tau)} d\tau$$

The **cumulative hazard rate** at any point in time is  $\int_0^t h(\tau) d\tau$  denoted by:  $H(t)$  = Cumulative failure rate at time "t."

Properties: (1)  $H(t)$  is piece-wise continuous for all "t" greater than or equal to "0."  
(2)  $H(t)$  is an increasing function on the interval  $[0, \infty]$

The average hazard rate on an interval  $(t_1, t_2]$  is  $AHR(t_1, t_2) = \frac{H(t_2) - H(t_1)}{t_2 - t_1}$



$$L(t) = \frac{\int_t^\infty f(\tau) d\tau}{\int_t^\infty f(\tau) d\tau} - t ; 1.0 - \frac{1}{R(t)} \int_t^\infty R(\tau) d\tau ; \frac{1}{R(t)} \int_t^\infty R(\tau) d\tau ; \frac{\int_t^\infty e^{-\int_0^\tau h(y) dy} d\tau}{e^{-\int_0^t h(\tau) d\tau}} ; e^{H(t)} * \int_t^\infty e^{H(\tau)} d\tau$$

At time "t", the probability that the component or system has **remaining (residual) life** "t\*" is denoted by:  
 $L(t)$  = Remaining / residual life at "t\*".

Probabilistically:  $P(T > t + t^*) / P(T > t) = R(t + x) / R(x)$

**Special Conditional Random Variables**

Probability of a component or system of age "t" surviving some future time period "t\*":

$$P(T > t^* | T > t) = R(t^*) / R(t)$$

Probability that surviving life after burn in is greater than the warranty period

where  $t_b$  = tentative burn in time and

$t_w$  = warranty time/period

$$P(t - t_b > t_w | t > t_b) = \frac{R(t_w + t_b)}{R(t_b)}$$

## BASIC MEASURES OF CENTRAL TENDENCY

$$E[T] = \mu = \int_0^{\infty} t * f(t) dt ; \int_0^{\infty} R(t) dt \quad \text{If it exists. Think "Center of Mass."}$$

If "T" is a continuous random variable signifying Time to Failure, then the **expected or Mean time to failure** random variable is used frequently as an indicator for system reliability. In general, **the first moment** about the

origin is defined as the mean. Denoted by:  $E[T] = \text{Mean} = \text{M.T.T.F.}$  or

Discrete case:  $E(T) = \sum_{i=1}^n x_i * p(x_i)$

$$E(T) = \mu = \int_{-\infty}^{+\infty} t * f(t) dt$$

### Properties:

$$E[c] = c$$

$$E[c + X] = c + E[X]$$

$$E[cX] = c * E[X]$$

$$E[X + Y] = E[X] + E[Y]$$

$$E[X - Y] = E[X] - E[Y]$$

$$E(XY) = E(X) * E(Y)$$

$$E[c * g(T)] = c * E[g(T)]$$

$$E[g_1(T) + g_2(T)] = E[g_1(T)] + E[g_2(T)]$$

for "c" a constant .

Used in statistical tolerancing.

If X and Y are statistically independent.

### Sample Mean or Arithmetic Mean

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

### Population Mean

$$\mu = \frac{\sum_{i=1}^N x_i}{N}$$

### Geometric Mean

$$\bar{x}_g = \sqrt[N]{\prod_{i=1}^N x_i}$$

$$= \text{antilog}\left(\frac{1}{N} \sum_{i=1}^N \log(x_i)\right)$$

### Harmonic Mean

$$\bar{x}_h = \frac{N}{\sum_{i=1}^N \frac{1}{x_i}}$$

**Mode** (Measure of central tendency on a nominal scale).

On a p.d.f. plot, the location on the x-axis corresponding to the highest failure frequency is the mode. Also referenced as that value measurement of a sample that appears most frequently.

**Median:** On the C.D.F. plot, the location on the x-axis where  $F(x) = .50$  is known as the median. For "ordered" sample data, it corresponds to the following position within the ordered data values -

For "n" even

$$Md = \frac{\tilde{x}_{(N/2)} + \tilde{x}_{(N/2+1)}}{2}$$

For "n" odd

$$Md = \tilde{x}_{((N+1)/2)}$$

## BASIC MEASURES OF DISPERSION

$$V[T] = \sigma^2 = \int_{-\infty}^{\infty} (t - \mu)^2 * f(t) dt$$

Think "Distance from Center of Mass."

The second moment of a random variable about the origin is defined as the variance.

Denoted by:  $V[T] = E[(T - E(T))^2] =$   
 $E[T^2] - (E[T])^2 = \sigma^2$

The standard deviation is denoted by:

$$\sqrt{E[(T - E(T))^2]} = \sigma [T]$$

**Properties:**  
except

- (1)  $V[c] = 0$
- (2)  $V[(c * T) + b] = c^2 * V[T]$
- (3)  $V[c + T] = V[T]$
- (4)  $V[T_1 \pm T_2] = V[T_1] + V[T_2]$  whenever  $T_1$  and  $T_2$  are independent.  
 $= V[T_1] + V[T_2] + 2 * Cov(T_1, T_2)$  otherwise.

**Properties:** Same as Variance case

take the square root. Property (3) of the Variance case used in statistical tolerancing.

### Sample Variance Deviation

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1} = \frac{SS}{n-1}$$

### Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}} = \sqrt{\frac{SS}{n-1}}$$

### Sample Average

$$AD = \frac{\sum_{i=1}^n |X_i - Md|}{n}$$

### Population Variance

$$\sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N} = \frac{SS}{N}$$

### Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} = \sqrt{\frac{SS}{N}}$$

### Sample Mean Deviation

$$MD = \frac{\sum_{i=1}^n |X_i - \bar{X}|}{n}$$

where SS = Sum of Squares.

### Population Average Deviation

$$AD = .7979\sigma$$

### Population Mean Deviation

$$MD = E |x - \mu|$$

### Range (on an interval)

$$X_{\max} - X_{\min}$$

## BASIC MEASURES OF DISPERSION (cont.)

### Coefficient of Variation Standard

$$CV = \eta = \frac{\sigma}{\mu} \approx \frac{\sigma_{\bar{x}}}{\bar{x}} = \frac{s}{\bar{x}}$$

NOTE:  $1/CV = S/N$  Ratio

### Sample Standard error of the Mean

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

### Population

### error of the mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

## Skewness

The third moment of a random variable about the origin divided by the cube of the standard deviation is defined as the skewness.

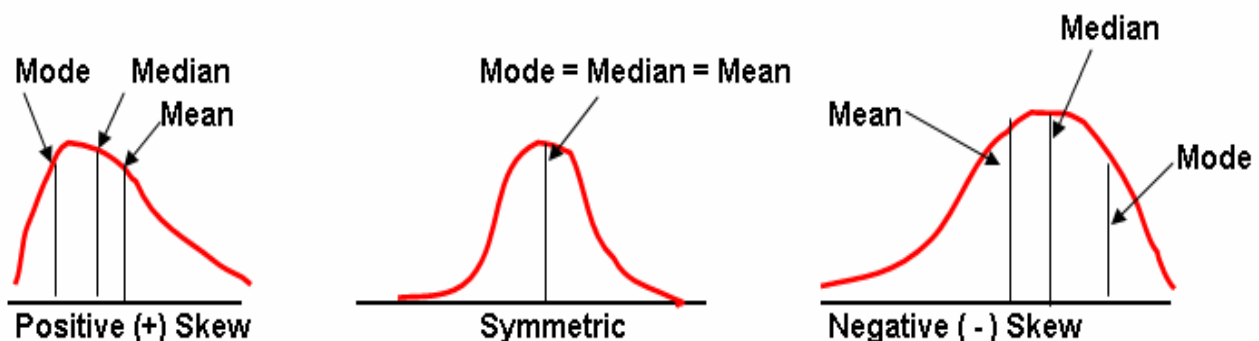
Population coefficient of skewness denoted by:

$$\alpha_3 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\int_{-\infty}^{\infty} (t - \mu)^3 * f(t) dt}{\sigma^3}$$

Sample coefficient of skewness denoted by:

$$\hat{\alpha}_3 = \frac{3 * (\bar{X} - Md)}{s} = \frac{\sum x_i^3 - [3 \sum x_i * \sum x_i^2 / N] + [2(\sum x_i)^3 / N^3]}{N}$$

It is an indicator of where a majority of the data lies within a distribution (p.d.f.). It conveys Location.



## BASIC MEASURES OF DISPERSION (cont.)

### Kurtosis

The fourth moment of a random variable about the origin divided by the variance squared is defined as the kurtosis.

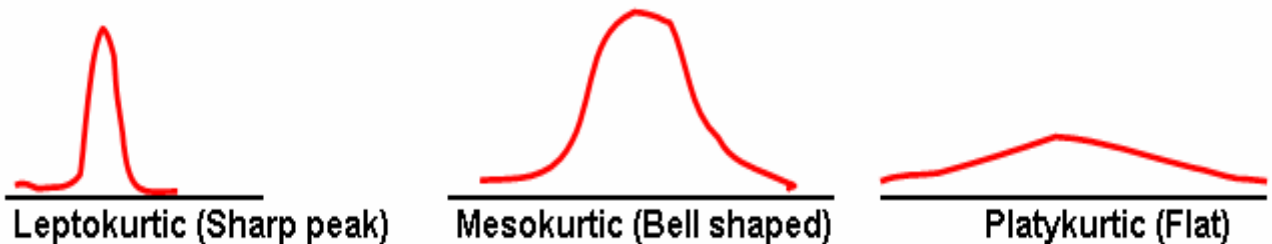
Population coefficient of kurtosis denoted by:

$$\alpha_4 = \frac{\mu_4}{\mu_2^2} = \frac{\int_{-\infty}^{\infty} (t - \mu)^4 * f(t) dt}{\sigma^4}$$

Sample coefficient of kurtosis denoted by:

$$\hat{\alpha}_4 = \frac{\sum x_i^4 - [4 \sum x_i * \sum x_i^3 / N] + 6((\sum x_i)^2 * \sum x_i^2 / N^2) - [3(\sum x_i)^4 / N^4]}{N}$$

It is an indicator of where a majority of the data lies within a distribution (p.d.f.). It conveys Location.



## Estimators (Biased / Unbiased)

An estimator is said to be Biased if and only if

An estimator is said to be Unbiased if and only if

$$\mathbf{b}(\theta) = \mathbf{E}(\hat{\theta}) - \theta \neq 0 \quad \text{or} \quad \mathbf{E}(\hat{\theta}) \neq \theta \quad \mathbf{E}(\hat{\theta}) - \theta \equiv 0 \quad \text{or} \quad \mathbf{E}(\hat{\theta}) \equiv \theta \quad \text{or} \quad \mathbf{b}(\theta) = 0.$$

Probability is the theory of modeling uncertainty. The problem in probability is to say something about an outcome when given a probabilistic model. The set of all possible outcomes is known to follow a certain probability law. On the other hand, the problem in Statistics is to say something about a population given a sample of outcomes; The goal of statistics is the following: "We take a sample from a population and estimate its statistics (parameters) so that we may attempt to make an inference about the population's behavior (parameters)." The "behavior" of the population may be defined with a "probabilistic model" (distribution). Examples of estimators for a population parameter vs. the "true" population parameters:

From  $N(\mu, \sigma)$  we have  $\hat{\theta} = (\bar{X}, s, s^2)$  vs.  $\theta = (\mu, \sigma, \sigma^2)$

## Maximum Likelihood Estimators

For a given "p.d.f.", the most probable estimator value (if it exists) for a given parameter. A "supremum".  
Properties (if it exists for a given distribution):

- It is invariant.
- It is the Uniformly Minimum Variance Unbiased Estimator (UMVUE).
- It is unique.
- It is asymptotically normal.
- It is asymptotically efficient.

