Sentential Natural Deduction: Additional Exercises B

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Exercise B.1

With premises $\neg p$ and $\neg\neg q \to \neg\neg p$, give as many kinds of derivation possible of 6 lines or fewer. (6 lines, including the premises. So apply rules in as many different ways as you can, in the space of 4 extra lines.)

Exercise B.2

Show the following.

1. $p \to q \vdash \neg p \vee q$. (Use any of the derived rules on pp. 18–19 of the notes, as well as any non-derived rules.)

2. $\neg(p \to q) \vdash p$. (Just use the regular, non-derived rules.)

3. Show $\neg\neg p \vdash p$ **without** using (DN2), (ECQ), or (IP).

Exercise B.3

Show that the following derivable rules, from pp. 18–19 of the notes, obtain. (Once you've derived a rule, you can use it for the proofs in later parts of this exercise.)

1. (IDEMP2-$\vee$) $A \vee A \vdash A$.

2. (DM1) $\neg(A \wedge B) \vdash \neg A \vee \neg B$.

3. (DM2) $\neg(A \vee B) \vdash \neg A \wedge \neg B$.

4. (DIST1) $A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$.

5. (DIST2) $A \vee (B \wedge C) \vdash (A \vee B) \wedge (A \vee C)$.

Exercise B.4

In the notes (p. 19), it's shown that (DIS), (R), and (MT) are unnecessary: if we remove them all together, we have an extensionally equivalent system. Show the following other rules are also unnecessary (i.e., derive the following without using any of (DIS), (R), (MT), (ECQ), (DS2)). **NOTE**: don't use any derived rules for this part.

1. (ECQ) $A, \neg A \vdash B$.

2. (DS2) $A \vee B, \neg B \vdash A$.