Additional Exercises on Sentential Natural Deduction

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Exercise A.1

Show that the following obtain by giving derivations.

1. $\neg p \vdash p \to q$.

1. $\neg p$	(P1)
2. $\parallel p$	(Ass,CP)
3. $\parallel q$	2,1,(ECQ)
4. $p \to q$	2–3,(CP)

2. $p \vdash \neg\neg p$, **without** using (DN1).

1. p	(P1)
2. $\parallel \neg\neg\neg p$	(Ass,IP)
3. $\parallel \neg p$	2,(DN2)
4. $\parallel p \wedge \neg p$	1,3,(CON)
5. $\neg\neg p$	2–4,(IP)

3. $\vdash p \vee p \to p$. (Notice that there are no premises.)

1. $\parallel p$	(Ass,CP)
2. $\parallel p$	1,(R)
3. $p \to p$	1–2,(CP)
4. $p \vee p \to p$	3,3,(DIS)

4. $\vdash (p \to q) \to (\neg q \to \neg p)$. (Notice that there are no premises.)

1. $\parallel p \to q$	(Ass,CP)
2. $\parallel \parallel \neg q$	(Ass,CP)
3. $\parallel \parallel \neg p$	1,2,(MT)
4. $\parallel \neg q \to \neg p$	2–3,(CP)
5. $(p \to q) \to (\neg q \to \neg p)$	1–4,(CP)

5. $\neg(p \to q) \vdash p$.

1. $\neg(p \to q)$	(P1)
2. $\parallel \neg p$	(Ass,IP)
3. $\parallel \parallel p$	(Ass,CP)
4. $\parallel \parallel q$	3,2,(ECQ)
5. $\parallel p \to q$	3–4,(CP)
6. $\parallel (p \to q) \wedge \neg(p \to q)$	5,1,(CON)
7. p	2–6,(IP)

6. $\neg r \rightarrow \neg(p \land q) \vdash p \rightarrow \neg\neg(q \rightarrow r)$.

1.	$\neg r \rightarrow \neg(p \land q)$	(P1)
2.	$\parallel p$	(Ass,CP)
3.	$\parallel \parallel q$	(Ass,CP)
4.	$\parallel \parallel p \land q$	2,3,(CON)
5.	$\parallel \parallel \neg\neg(p \land q)$	4,(DN1)
6.	$\parallel \parallel \neg\neg r$	5,1,(MT)
7.	$\parallel \parallel r$	6,(DN2)
8.	$\parallel q \rightarrow r$	3–7,(CP)
9.	$\parallel \neg\neg(q \rightarrow r)$	8,(DN1)
10.	$p \rightarrow \neg\neg(q \rightarrow r)$	2–9,(CP)

7. $p \lor q \vdash (p \rightarrow q) \rightarrow q$.

1.	$p \lor q$	(P1)
2.	$\parallel p \rightarrow q$	(Ass,CP)
3.	$\parallel \parallel \neg q$	(Ass,IP)
4.	$\parallel \parallel p$	1,3,(DS2)
5.	$\parallel \parallel q$	4,2,(MP)
6.	$\parallel \parallel q \land \neg q$	5,3,(CON)
7.	$\parallel q$	3–6,(IP)
8.	$(p \rightarrow q) \rightarrow q$	2–7,(CP)

8. $\vdash p \lor \neg p$. (No premises.)

1.	$\parallel \neg(p \lor \neg p)$	(Ass,IP)
2.	$\parallel \parallel \neg\neg p$	(Ass,IP)
3.	$\parallel \parallel p$	2,(DN2)
4.	$\parallel \parallel p \lor \neg p$	3,(ADD1)
5.	$\parallel \parallel (p \lor \neg p) \land \neg(p \lor \neg p)$	4,1,(CON)
6.	$\parallel \neg p$	2–5,(IP)
7.	$\parallel p \lor \neg p$	6,(ADD2)
8.	$\parallel (p \lor \neg p) \land \neg(p \lor \neg p)$	7,1,(CON)
9.	$p \lor \neg p$	1–8,(IP)

9. $p \lor q, p \rightarrow r, q \rightarrow s \vdash r \lor s$.

1.	$p \lor q$	(P1)
2.	$p \rightarrow r$	(P2)
3.	$q \rightarrow s$	(P3)
4.	$\parallel \neg(r \lor s)$	(Ass,IP)
5.	$\parallel \parallel \neg\neg r$	(Ass,IP)
6.	$\parallel \parallel r$	5,(DN2)
7.	$\parallel \parallel r \lor s$	6,(ADD1)
8.	$\parallel \parallel (r \lor s) \land \neg(r \lor s)$	7,4,(CON)
9.	$\parallel \neg r$	5–8,(IP)
10.	$\parallel \neg p$	2,9,(MT)
11.	$\parallel q$	1,10,(DS1)
12.	$\parallel s$	11,3,(MP)
13.	$\parallel r \lor s$	12,(ADD2)
14.	$\parallel (r \lor s) \land \neg(r \lor s)$	13,4,(CON)
15.	$r \lor s$	4–14,(IP)

10. $p \lor q, \neg p \lor r \vdash q \lor r$, **without** using Case Distinction (CD).

1.	$p \lor q$	(P1)
2.	$\neg p \lor r$	(P2)
3.	$\parallel p$	(Ass,CP)
4.	$\parallel \neg\neg p$	3,(DN1)
5.	$\parallel r$	2,4(DS1)
6.	$\parallel q \lor r$	5,(ADD2)
7.	$p \rightarrow q \lor r$	2–6,(CP)
8.	$\parallel \neg p$	(Ass,CP)
9.	$\parallel q$	1,8,(DS1)
10.	$\parallel q \lor r$	9,(ADD1)
11.	$\neg p \rightarrow q \lor r$	8–10,(CP)
12.	$p \lor \neg p \rightarrow q \lor r$	7,11,(DIS)
13.	$\parallel \neg(p \lor \neg p)$	(Ass,IP)
14.	$\parallel \parallel \neg\neg p$	(Ass,IP)
15.	$\parallel \parallel p$	14,(DN2)
16.	$\parallel \parallel p \lor \neg p$	15,(ADD1)
17.	$\parallel \parallel (p \lor \neg p) \land \neg(p \lor \neg p)$	16,13,(CON)
18.	$\parallel \neg p$	14–17,(IP)
19.	$\parallel p \lor \neg p$	18,(ADD2)
20.	$\parallel (p \lor \neg p) \land \neg(p \lor \neg p)$	19,13,(CON)
21.	$p \lor \neg p$	13–20,(IP)
22.	$q \lor r$	12,21,(MP)

11. $\vdash ((p \rightarrow q) \rightarrow p) \rightarrow p$. (No premises.)

1.	$\parallel (p \rightarrow q) \rightarrow p$	(Ass,CP)
2.	$\parallel \parallel \neg p$	(Ass,IP)
3.	$\parallel \parallel \parallel p$	(Ass,CP)
4.	$\parallel \parallel \parallel q$	3,2,(ECQ)
5.	$\parallel \parallel p \rightarrow q$	3–4,(CP)
6.	$\parallel \parallel p$	5,1,(MP)
7.	$\parallel \parallel p \land \neg p$	6,2,(CON)
8.	$\parallel p$	2–7,(IP)
9.	$((p \rightarrow q) \rightarrow p) \rightarrow p$	1–8,(CP)

3