Solutions to Exercises 4 for *Introduction to Logic*

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Exercise 4.1

1. No, not a wff. (Needs brackets.)
2. No. (Unbalanced brackets.)
3. Yes, it's a wff.
4. Yes.
5. Yes.
6. No. ($\Rightarrow$ is a gatecrasher to our sentential party.)
7. Yes.
8. No. (Because *our* sentential variables must be lower-case.)
9. Yes.
10. Yes.
11. No. (Where are the brackets?)
12. Yes.
13. No. (Note the double brackets around the whole.)
14. Yes.
15. Yes.
16. Yes. (We *do* allow subscripts.)
17. No. (Because '(s)' is forbidden.)

Exercise 4.2

1. No, not an argument. (The conclusion must be a single formula.)
2. Yes, it's an argument. (We allow arguments with no premises.)
3. No. (All the premises and conclusion aren't formulas of sentential logic.)
4. Yes.
5. Yes. (This is fine: repetition is allowed.)
6. No. (Arguments need a conclusion.)
7. No. (No comma between the premises.)

Exercise 4.3

3. $(p \wedge q) \vee r$
4. $\neg(p \vee q) \rightarrow r$
5. $\neg(p \vee q \rightarrow r)$
7. $\neg p \vee q \rightarrow r$
9. $(p \rightarrow \neg p) \rightarrow \neg p$
10. $\neg\neg\neg\neg\neg r \rightarrow p \vee q$
12. $(p \rightarrow q) \wedge \neg(\neg q \rightarrow \neg p)$

14. $p \to (q \to (r \to (s \to t)))$

15. $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

16. $\neg(\neg\neg(\neg((p_{12} \vee \neg p_9) \wedge p_8) \vee p_7 \to p_6) \leftrightarrow p_5 \vee \neg\neg p_{13})$

Exercise 4.4

1. $(p \vee q)$

2. $((p \wedge q) \to r)$

3. $(p \to (q \vee r))$

4. $((p \vee q) \to ((p \wedge r) \vee \neg s))$

5. $((\neg p \vee (q \wedge \neg r)) \to (\neg(p \vee \neg s) \vee \neg(q \to s)))$

Exercise 4.5

1. No: not correctly applied. (No brackets from either $p \to (q \to r)$ or $(p \to q) \to r$ can be removed.)

2. Yes. (If the original brackets were as in $((p \vee q) \to (r \wedge s))$ then they have been correctly removed to reach the given formula. Yet if the original formula had been, e.g., $(p \vee ((q \to r) \wedge s))$, then they were *incorrectly* removed.)

3. No. (Neither (CP1) nor (CP2) allow us to remove brackets within a long conjunction.)

4. No. (The consequent here requires some brackets which must not be removed.)

5. No. (This is ambiguous between $p \to (((q \to r) \vee s) \to q)$ and $(p \to ((q \to r) \vee s)) \to q$.)