

Solutions to Exercises 8 for *Introduction to Logic*

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Exercise 8.1

1. $p \vee (q \wedge \neg q) \vdash p$.

1. $p \vee (q \wedge \neg q)$	(P1)
2. $\parallel \neg p$	(Ass,IP)
3. $\parallel q \wedge \neg q$	1,2,(DS1)
4. p	2-3,(IP)

2. $p \vee q, p \vee \neg q \vdash p$.

1. $p \vee q$	(P1)
2. $p \vee \neg q$	(P2)
3. $\parallel \neg p$	(Ass,IP)
4. $\parallel q$	1,3,(DS1)
5. $\parallel \neg q$	2,3,(DS1)
6. $\parallel q \wedge \neg q$	4,5,(CON)
7. p	3-6,(IP)

3. $p \rightarrow q, p \rightarrow \neg q \vdash \neg p$.

1. $p \rightarrow q$	(P1)
2. $p \rightarrow \neg q$	(P2)
3. $\parallel \neg \neg p$	(Ass,IP)
4. $\parallel p$	3,(DN2)
5. $\parallel q$	4,1,(MP)
6. $\parallel \neg q$	4,2,(MP)
7. $\parallel q \wedge \neg q$	5,6,(CON)
8. $\neg p$	3-7,(IP)

4. $p \rightarrow q \vdash \neg(p \wedge \neg q)$.

1. $p \rightarrow q$	(P1)
2. $\parallel \neg \neg(p \wedge \neg q)$	(Ass,IP)
3. $\parallel p \wedge \neg q$	2,(DN2)
4. $\parallel p$	3,(SIMP1)
5. $\parallel q$	4,1,(MP)
6. $\parallel \neg q$	3,(SIMP2)
7. $\parallel q \wedge \neg q$	5,6,(CON)
8. $\neg(p \wedge \neg q)$	2-7,(IP)

5. $\neg p \vee \neg q \vdash \neg(p \wedge q)$.

1. $\neg p \vee \neg q$	(P1)
2. $\parallel \neg \neg(p \wedge q)$	(Ass,IP)
3. $\parallel p \wedge q$	2,(DN2)
4. $\parallel p$	3,(SIMP1)
5. $\parallel \neg \neg p$	4,(DN1)
6. $\parallel \neg q$	1,5,(DS1)
7. $\parallel q$	3,(SIMP2)
8. $\parallel q \wedge \neg q$	7,6,(CON)
9. $\neg(p \wedge q)$	2-8,(IP)

6. $p \rightarrow (q \rightarrow r) \vdash p \wedge q \rightarrow r$.

1.	$p \rightarrow (q \rightarrow r)$	(P1)
2.	$\parallel p \wedge q$	(Ass,CP)
3.	$\parallel p$	2,(SIMP1)
4.	$\parallel q \rightarrow r$	3,1,(MP)
5.	$\parallel q$	2,(SIMP2)
6.	$\parallel r$	5,4,(MP)
7.	$p \wedge q \rightarrow r$	2-6,(CP)

7. $p \rightarrow q \vdash p \wedge r \rightarrow q \wedge r$.

1.	$p \rightarrow q$	(P1)
2.	$\parallel p \wedge r$	(Ass,CP)
3.	$\parallel p$	2,(SIMP1)
4.	$\parallel q$	3,1,(MP)
5.	$\parallel r$	2,(SIMP2)
6.	$\parallel q \wedge r$	4,5,(CON)
7.	$p \wedge r \rightarrow q \wedge r$	2-6,(CP)

8. $\neg(q \vee r) \vdash q \vee p \rightarrow p$.

1.	$\neg(q \vee r)$	(P1)
2.	$\parallel q \vee p$	(Ass,CP)
3.	$\parallel \parallel \neg p$	(Ass,IP)
4.	$\parallel \parallel q$	2,3,(DS2)
5.	$\parallel \parallel q \vee r$	4,(ADD1)
6.	$\parallel \parallel (q \vee r) \wedge (q \vee \neg r)$	5,1,(CON)
7.	$\parallel p$	3-6,(IP)
8.	$q \vee p \rightarrow p$	2-7,(CP)

Alternative:

1.	$\neg(q \vee r)$	(P1)
2.	$\parallel \neg \neg q$	(Ass,IP)
3.	$\parallel q$	2,(DN2)
4.	$\parallel q \vee r$	3,(ADD1)
5.	$\parallel (q \vee r) \wedge \neg(q \vee r)$	4,1,(CON)
6.	$\neg q$	2-5,(IP)
7.	$\parallel q \vee p$	(Ass,CP)
8.	$\parallel p$	7,6,(DS1)
9.	$q \vee p \rightarrow p$	7-8,(CP)

Alternative alternative:

1.	$\neg(q \vee r)$	(P1)
2.	$\parallel q \vee p$	(Ass,CP)
3.	$\parallel \parallel \neg \neg q$	(Ass,IP)
4.	$\parallel \parallel q$	3,(DN2)
5.	$\parallel \parallel q \vee r$	4,(ADD1)
6.	$\parallel \parallel (q \vee r) \wedge \neg(q \vee r)$	5,1,(CON)
7.	$\parallel \neg q$	3-6,(IP)
8.	$\parallel p$	1,7,(DS1)
9.	$q \vee p \rightarrow p$	7-8,(CP)