Solutions to Exercises 5 for *Introduction to Logic*

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Exercise 5

(The truth-values in bold are beneath the main connective and are therefore the truth-values for the whole formula.)

1. Tautology.

p	q	$(p \to q)$	$\lor$	$(q \to p)$
t	t	t	**t**	t
t	f	f	**t**	t
f	t	t	**t**	f
f	f	t	**t**	t

2. Contingent.

p	q	$\neg$	$(p \to q)$	$\lor$	$q \land$	$\neg p$
t	t	f	t	**f**	f	f
t	f	t	f	**t**	f	f
f	t	f	t	**t**	t	t
f	f	f	t	**f**	f	t

3. Contingent.

p	q	$\neg$	$(p \to q)$	$\land p$	$\to q$
t	t	f	t	t	**t**
t	f	t	f	f	**f**
f	t	t	t	f	**t**
f	f	t	t	f	**f**

4. Tautology.

p	q	r	$(p \to q)$	$\lor$	$(q \to r)$	$\lor$	$(r \to p)$
t	t	t	t	t	t	**t**	t
t	t	f	t	t	f	**t**	t
t	f	t	f	t	t	**t**	t
t	f	f	f	t	t	**t**	t
f	t	t	t	t	t	**t**	f
f	t	f	t	t	f	**t**	t
f	f	t	t	t	t	**t**	f
f	f	f	t	t	t	**t**	t

5. Tautology.

p	q	$(p \to q)$	$\lor$	$\neg p$	$\to q$
t	t	t	**t**	f	t
t	f	f	**t**	f	t
f	t	t	**t**	t	t
f	f	t	**t**	t	f

6. Contingent.

p	q	r	$(p \to q)$	$\to$	$((p \to r)$	$\to$	$(q \to r))$
t	t	t	t	**t**	t	t	t
t	t	f	t	**t**	f	t	f
t	f	t	f	**t**	t	t	t
t	f	f	f	**t**	f	t	t
f	t	t	t	**t**	t	t	t
f	t	f	t	**f**	t	f	f
f	f	t	t	**t**	t	t	t
f	f	f	t	**t**	t	t	t

7. Tautology.

p	q	r	$p \vee (\neg q \to r)$			$\to$	$q \vee (\neg p \to r)$			
t	t	t	t	f	t	**t**	t	f	t	
t	t	f	t	f	t	**t**	t	f	t	
t	f	t	t	t	t	**t**	t	f	t	
t	f	f	t	t	f	**t**	t	f	t	
f	t	t	t	f	t	**t**	t	t	t	
f	t	f	t	f	t	**t**	t	t	f	
f	f	t	t	t	t	**t**	t	t	t	
f	f	f	f	t	f	**t**	f	t	f	

8. Tautology.

p	q	r	$\neg(p \wedge (q \to \neg r))$				$\to$	$(p \to q) \wedge (p \to r)$		
t	t	t	t	f	f	f	**t**	t	t	t
t	t	f	f	t	t	t	**t**	t	f	f
t	f	t	f	t	t	f	**t**	f	f	t
t	f	f	f	t	t	t	**t**	f	f	f
f	t	t	t	f	f	f	**t**	t	t	t
f	t	f	t	f	t	t	**t**	t	t	t
f	f	t	t	f	t	f	**t**	t	t	t
f	f	f	t	f	t	t	**t**	t	t	t

9. Contingent.

p	q	r	$\neg(\neg p \to q \vee r)$				$\to$	$\neg(p \vee q) \wedge r$		
t	t	t	f	f	t	t	**t**	f	t	f
t	t	f	f	f	t	t	**t**	f	t	f
t	f	t	f	f	t	t	**t**	f	t	f
t	f	f	f	f	t	f	**t**	f	t	f
f	t	t	f	t	t	t	**t**	f	t	f
f	t	f	f	t	t	t	**t**	f	t	f
f	f	t	f	t	t	t	**t**	t	f	t
f	f	f	t	t	f	f	**f**	t	f	f

10. Tautology.

p	q	r	$p \wedge (q \leftrightarrow r)$		$\to$	$(p \wedge q \leftrightarrow p \wedge r)$		
t	t	t	t	t	**t**	t	t	t
t	t	f	f	f	**t**	t	f	f
t	f	t	f	f	**t**	f	f	t
t	f	f	t	t	**t**	f	t	f
f	t	t	f	t	**t**	f	t	f
f	t	f	f	f	**t**	f	t	f
f	f	t	f	f	**t**	f	t	f
f	f	f	f	t	**t**	f	t	f

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11. Tautology.

p	q	r	$p \vee (\neg q \rightarrow r) \leftrightarrow q \vee (\neg p \rightarrow r)$							
t	t	t	t	f	t	**t**	t	f	t	
t	t	f	t	f	t	**t**	t	f	t	
t	f	t	t	t	t	**t**	t	f	t	
t	f	f	t	t	f	**t**	t	f	t	
f	t	t	t	f	t	**t**	t	t	t	
f	t	f	t	f	t	**t**	t	t	f	
f	f	t	t	t	t	**t**	t	t	t	
f	f	f	f	t	f	**t**	f	t	f	

12. Tautology.

p	q	r	$\neg(p \wedge (q \rightarrow \neg r)) \leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$							
t	t	t	t	f	f	f	**t**	t	t	t
t	t	f	f	t	t	t	**t**	t	f	f
t	f	t	f	t	t	f	**t**	f	f	t
t	f	f	f	t	t	t	**t**	f	f	f
f	t	t	t	f	f	f	**t**	t	t	t
f	t	f	t	f	t	t	**t**	t	t	t
f	f	t	t	f	t	f	**t**	t	t	t
f	f	f	t	f	t	t	**t**	t	t	t

13. Contingent.

p	q	r	$(p \wedge q) \vee r$	
t	t	t	t	**t**
t	t	f	t	**t**
t	f	t	f	**t**
t	f	f	f	**f**
f	t	t	f	**t**
f	t	f	f	**f**
f	f	t	f	**t**
f	f	f	f	**f**

14. Contingent.

p	q	r	$\neg((p \vee q) \rightarrow r)$		
t	t	t	**f**	t	t
t	t	f	**t**	t	f
t	f	t	**f**	t	t
t	f	f	**t**	t	f
f	t	t	**f**	t	t
f	t	f	**t**	t	f
f	f	t	**f**	f	t
f	f	f	**f**	f	t

15. Contingent.

p	q	r	$\neg p \vee q \rightarrow r$		
t	t	t	f	t	**t**
t	t	f	f	t	**f**
t	f	t	f	f	**t**
t	f	f	f	f	**t**
f	t	t	t	t	**t**
f	t	f	t	t	**f**
f	f	t	t	t	**t**
f	f	f	t	t	**f**

16. Tautology.

p	$(p \to \neg p) \to \neg p$			
t		f f	**t**	f
f		t t	**t**	t

17. Contingent.

p	q	r	$\neg\neg\neg r \to p \vee q$				
t	t	t	f t f	**t**	t
t	t	f	t f t	**t**	t
t	f	t	f t f	**t**	t
t	f	f	t f t	**t**	t
f	t	t	f t f	**t**	t
f	t	f	t f t	**t**	t
f	f	t	f t f	**t**	f
f	f	f	t f t	**f**	f

18. Contradiction.

p	q	$(p \to q) \wedge \neg(\neg q \to \neg p)$					
t	t	t	**f**	f f	t	f
t	f	f	**f**	t t	f	f
f	t	t	**f**	f f	t	t
f	f	t	**f**	f t	t	t

19. Contingent.

p	q	r	$(p \to (q \to r)) \to p$		
t	t	t	t	t	**t**
t	t	f	f	f	**t**
t	f	t	t	t	**t**
t	f	f	t	t	**t**
f	t	t	t	t	**f**
f	t	f	t	f	**f**
f	f	t	t	t	**f**
f	f	f	t	t	**f**

20. Tautology.

p	q	r	$(p \to (q \to r)) \to ((p \to q) \to (p \to r))$					
t	t	t	t	t	**t**	t	t	t
t	t	f	f	f	**t**	t	f	f
t	f	t	t	t	**t**	f	t	t
t	f	f	t	t	**t**	f	t	f
f	t	t	t	t	**t**	t	t	t
f	t	f	t	f	**t**	t	t	t
f	f	t	t	t	**t**	t	t	t
f	f	f	t	t	**t**	t	t	t