UNIVERSITY OF TENNESSEE, DEPARTMENT OF MATHEMATICS

MATH 447 - Homework 4

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1 THE SET OF REAL NUMBERS IS UNCOUNTABLE

PROOF. Assume the set $I = [0,1]$ is countable. This implies its elements are enumerable. That is, $I = \{x_1, x_2, \ldots, x_n, \ldots\}$.

Now construct the set $I_1 = \{x \in I | x \neq x_1\}$ so that $I_1 \subset I$ and $I_1 = I \setminus \{x_1\}$. Clearly this construction can progress in a recursive fashion so that $I_j = I_{j-1} \setminus \{x_j\}$, and $I_j \supseteq I_{j+1} \supseteq \cdots$.

Thus, we have establshed a collection of nested intervals, so we know $\exists \xi \in I_n \forall n \in \mathbb{N}$. Suppose $\xi = x_k$ for some $k \in \mathbb{N}$, then $\xi \notin I_k$, which contradicts the Nested Intervals Property. Clearly this is absurd, so we refute our hypothesis that $I = [0,1]$ is countable.

Since $[0,1] \subseteq \mathbb{R}$ is uncountable, we know that $\mathbb{R}$ itself is uncountable.■

$$\begin{aligned}(x+y)^3 &= (x+y)^2(x+y) \\ &= (x^2+2xy+y^2)(x+y) \\ &= (x^3+2x^2y+xy^2)+(x^2y+2xy^2+y^3) \\ &= x^3+3x^2y+3xy^2+y^3\end{aligned} \tag{1.1}$$

1.1 HEADING ON LEVEL 2 (SUBSECTION)

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$$A = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix} \tag{1.2}$$

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1.1.1 Heading on level 3 (subsubsection)

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Heading on level 4 (paragraph) Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

2 Lists

2.1 Example of list (3*itemize)

- First item in a list
  - First item in a list
    - First item in a list
    - Second item in a list
  - Second item in a list
- Second item in a list

2.2 Example of list (enumerate)

1. First item in a list
2. Second item in a list
3. Third item in a list