Chen (2010) - A Life Cycle Analysis of Social Security with Housing

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Brief description of the model of Chen (2010). I make very few notational changes. Because of how VFI Toolkit works the write-up of the model looks quite different, but is the exact same model. For full details on the model framework see Chen (2010).

One difference between the model solved here and Chen2010. Chen2010 has deterministic productivity growth $g = 0.015$. This involves renormalizing the model (around a balanced growth path) before solving it. Here we just take the lazy approach and solve the model with $g = 0$ to skip having to bother with the renormalization. Is just some analytical derivations prior to solving the model.[1]

The model is a general equilibrium OLG model. The finite-horizon value function problem has one exogenous state (stochastic component of earnings), two endogenous state (assets and housing), and 66 periods. The household value function problem is given by

$$V(a,h,z,j) = \max_{c,d,a',h'} \frac{[(c^\theta d^{1-\theta}]^{1-\sigma}}{1-\sigma} + \beta s_j E_j[V(a',h',z',j+1)|z]$$

subject to

$$\tau_{h,h'} = \begin{cases} 0 & \text{if } h' = h \\ \phi h & \text{if } h' \neq h \end{cases}$$

homeowner ($h' > 0$):

$$d = h' \quad \textit{(housing services come from home-owned)}$$
$$c + h' = (1+r)a + (1-\tau_p)w\kappa_j z + (1-\delta_o)h - \tau_{h,h'} + Tr$$
$$a' \geq -(1-\gamma)h' \quad \textit{(borrowing (mortgage) as fraction of house)}$$

renter ($h' = 0$):

$$c + pd = (1+r)a + (1-\tau_p)w\kappa_j z + (1-\delta_o)h - \tau_{h,h'} + Tr + \mathbb{I}_{j \geq Jr} b$$
$$a' \geq 0$$

There are $J = 66$ periods (representing ages 20 to 85) and $V(a,h,i,J+1) = 0$ for all a, h & z. So household faces idiosyncratic labor productivity shocks (z) and solve a consumption-savings problem of choosing consumption c and next period assets a', as well as a decision on how much housing to buy h'. A household that chooses $h' > 0$ is called a 'home owner', and a household that chooses $h' = 0$ is called a renter. Households get utility from consumption and housing-services. Home-owners get housing services from the home they own ($d = h'$), while renters can buy housing

[1] E.g., Intro to Life-Cycle Models explains what to do in model 22; and Intro to OLG Models in model 8.

services, d at price p. Tr are lump-sum transfers, and represent the receipt of accidental bequests. $\tau_{h,h'}$ is a (proportional) transaction cost that is paid when the amount of housing owned changes.

The earnings shock process log of z follows an AR(1) process. Following the Fortran codes of Chen2010 this is discretized using 7 states, and with 'tauchen q' equal to $sqrt(7/2)$; the Tauchen method is used to discretize $log(z)$, and then take the exponential of the grid and normalize so that the process is mean 1. The standard deviation of the innovations to z are $\sigma_{z,\epsilon} = \sqrt{0.045}$; the paper of Chen2010 has a typo and says 0.045 is the standard deviation when it is in fact the variance of the innovations.[2] I have switched to use the KFTT quadrature method in the codes, as it is better than Tauchen method.

The initial distribution of agents at birth is for them to have zero assets, zero housing, and for z to be log-normally distributed with a standard deviation of $sqrt(0.38)$.[3]

The model has six general equilibrium constraints: (i) capital market (interest rate equals the marginal product of capital, net of depreciation;), (ii) labor market (wage equals marginal product of labor), (iii) asset market (physical capital equals assets minus housing services), (iv) housing services (price of housing services relates to interest rate), (v) pensions (pension spending equals pension revenue), (vi) accidental bequests (accidental bequests received equals those left). Four of these can be hard-coded: (ii) and (iv) can be done in the return function (as $w = (1-\alpha) * ((r + \delta_k)/\alpha)^(\alpha/(\alpha-1))$ and $p = (r + \delta_r)/(1 + r)$ respectively), (v) can be precomputed as the labor supply is exogenous, (iii) can be substituted into (i) (substitute $K = A - Hr$). This leaves two general equilibrium conditions, (i) and (vi), for capital markets and the accidental bequests that have to be solved for as such.

We will use the same setup for the grid on housing as Chen2010 codes, namely 35 evenly spaced points from 0 to 20 (just for convenience, this is not important). The minimum assets is going to be $-(1-\gamma)hmax$, due to the collateral constraint, and we have chosen 20 as $hmax$, we set the max assets to 100 (which is what Chen2010 codes do).

Notational differences from Chen (2010), originals in parentheses: Deterministic earnings as a function of age are κ_j (ϵ_j), the AR(1) process on earnings is z (η).

There were three tricks to solving the model, first is relevant to VFI Toolkit, other two are general.
i) If you look closely at model as it is written in Chen (2010), the distinction between "renter" and "owner" is just that a renter is someone choosing hprime=0 (zero next period housing) and an owner is someone choosing hprime¿0. This can be easily handled by VFI Toolkit with an *if-else* statement inside the ReturnFn. Hence the state-space of the model is just (a,h,z,j)
ii) Renters choose how much to spend on consumption, c, and how much on housing services, d. It is possible to define cspend=c+pd. Then budget constraint can be used to calculate cspend, and the split of cspend into c and d can be solved analytically. See pdf in the github repo, as that does the derivation, the formula is then used in the ReturnFn.
iii) Paper describes the utility function as a combining c and hprime: $u(c,x) = \frac{[(\theta c^{\upsilon} + (1-\theta)x^{\upsilon})^{1/\upsilon}]^{1-\sigma}}{1-\sigma}$, where x is the housing services ($x = h'$ for owners and $x = d$ for renters). But then calibrates $\upsilon = 0$. $\upsilon = 0$ means the formula in the paper (the inner part) will just be incorrect, and it should be simplified to a Cobb-Douglas. So the actual utility function becomes $u(c,x) = \frac{[(c^{\theta}x^{1-\theta}]^{1-\sigma}}{1-\sigma}$

[2] Special thanks to Alessandro Di Nola who found all this in the Fortran codes.

[3] Chen2010 paper does not clearly explain this. Thanks again to Alessandro Di Nola who found this in the Fortran codes.

There are some small errors/typos in , the following explain what they are and how we found the correct implementations in the codes:

Tr, the lump-sum transfers (of accidental bequests) are received by both home-owners and renters. Chen2010 paper has them going only to renters, but his codes give them to both (which makes more sense).

The standard deviation of the innovations to (log) z is $sqrt(0.045)$. Chen2010 paper describes 0.045 as the standard deviation, but it is in fact the variance (this was clear from his codes).

The initial distribution of agents is log-normal on z with a variance of 0.38 (this was omitted from paper but clear from his codes).

I think there is a typo in the Chen (2010) appendix. Reading section 2.7 about the financial intermediary, it reads like the financial intermediary takes assets A and turns it into a combination of rental housing services Hr and physical capital K. This agrees with Appendix 1 7c equation for "Asset Markets Clear". Where I think there is an error is equation A.3 in Appendix 1, for "Housing Markets Clear", which reads $H = Hr + H0$, all three of which are defined on the same page. But definition for H is $H = \int h'(s)d\mu$ and definition of $H0$ is $H0 = \int_{h'>0} h'(s)d\mu$, since $h' \geq 0$, it follows that $H = H0$ (as H is just H0 plus some zeros). This would leave $Hr = 0$, which is clearly stupid and in conflict with the rest of the model. Thus I think the "Housing Markets Clear" should just be erased entirely (it would be $H = H0$ but this is just degenerate, so just delete the equation as it is trivially satisfied); this makes sense as based on Section 2.7 the rental housing is not coming out of housing, it is coming out of assets. [The housing markets clear eqn was not used in my codes anyway, so no difference there.]

General comment: this model has a perfectly elastic housing supply —you can see this as it is just 'c+hprime' in the budget constraint so the relative price of housing is always equal to one (unit of consumption). You could add a house price and limit the supply of housing, this would essentially just be an extra general equilibrium equation (note that currently there is no GE condition for the housing market in the codes).

References

Kaiji Chen. A life-cycle analysis of social security with housing. Review of Economic Dynamics, 13, 2010. doi: https://doi.org/10.1016/j.red.2009.10.001.