De Nardi, French & Jones (2010) - Why do the Elderly Save? The Role of Medical Expenses

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Brief description of replication of De Nardi, French, and Jones (2010). I make few notational changes, mainly that age is j here instead of t in the original. This is not intended as a full description of the model, so much as making it clear how the model is expressed in terms of the VFI Toolkit; for a full description of the model you should consult the original paper.

The model is a 31 period life-cycle model, representing ages 70 to 100. The model is essentially a standard consumption-savings exogenous life-cycle model, with some more complicated shock processes and parameters. The model has three exogenous states, a markov and an i.i.d on medical expenses, and a further markov on health. The model has ten permanent types, representing two genders (female and male) crossed with five income quintiles.

The household problem is,

$$V(a,h,\zeta,\xi) = \max_{c,a'} (1+\delta h)\frac{c^{1-\upsilon}-1}{1-\upsilon} + s_{i,j,h}\beta E[V(a',h',\zeta',\xi')|h,\zeta] + (1-s_{i,j,h})\beta\theta\frac{(estate+k)^{1-\upsilon}}{1-\upsilon}$$

subject to

$$\begin{aligned}
&c + a' = a + aftertaxincome + govtransfers - m \\
&govtransfers = max(0, cfloor + m - aftertaxincome) \\
&taxableincome = r * a + earnings_{i,j} \\
&m = exp(m_coeff_{i,j,h} + sigma_coeff_{i,j,h} * \psi), \quad \psi = \zeta + \xi \\
&\zeta' = \rho\zeta + \epsilon,\ \epsilon \sim N(0, \sigma^2_{\zeta,\epsilon}) \\
&\xi \sim N(0, \sigma^2_{\xi}) \\
&h' \sim \pi(h'|h;i,j) \text{is a markov process} \\
&a' \geq 0
\end{aligned}$$

where c is consumption, a is assets, $m_{i,j,h}$ is medical expenses. $earnings_{i,j}$ is exogenous earnings, and depends on age, gender and income quintile. h is health status and follows a markov process (that differs by age, gender and income quintile).

The calculation of $aftertaxincome$ from $taxableincome$ is omitted here but is just a straightforward calculation based on seven tax brackets and associated marginal tax rates.

Note, the government lump-sum transfer $govtransfers$ is typically zero, and will only be positive is it is needed for the household to reach the consumption floor $cfloor$ (after accounting for m and

$aftertaxincome$). If $govtransfers > 0$, then it is enforced that $c = cfloor$ and $a' = 0$ (not shown in eqns above).

$estate$ is a' after deducting estate taxes, $estate = max(0, (1 - \tau_e) * (a' - estateexemption))$; estate taxes have a tax rate τ_e and an exemption $estateexemption$ (DFJ2010 called these $tautilde$ and $xtilde$, respectively).

Because VFI Toolkit cannot handle conditional survival probabilities that depend on the state, we have to implement $s_{i,j,h}$ in an alternative fashion. We use just β as the discount factor. We extend h to have two further values, $h = 2$ is death and $h = 3$ is dead (we also need to give the warm-glow of bequests with probability $s_{i,j,h}$, hence we need to distinguish the period of death, when the warm-glow of bequests is received, from 'dead' which just gives return of zero). We then set up the transitions between $h = 0$ (good health) and $h = 1$ (bad health) following the health transition probabilities of DFJ2010, $\pi(h'|h; i, j)$, and set transition probabilities from $h = 0, 1$ to $h = 2$ following (one minus the) conditional survival probabilities $s_{i,j,h}$. Transitions from $h = 2$ to $h = 3$ occur with probability one, and $h = 3$ is an absorbing state (so probability 1 of staying in it once you are there). This is slightly wasteful computationally (we will end up carrying around all the dead households), but model is easy enough to solve that it won't be an issue.

The main complication for setting up this model is that many of these parameters depend on age, permanent type (both gender and income quintile), and some even depend on health status (good or bad). This means that the number of parameters in this model is over two-thousand. This is not difficult per se (you largely just set up the parameters as age-depedent vectors, and set different ones for each permanent type, the code will handle them trivially)[1] but it does take quite a lot of lines of code to write out.

The initial distribution of agents is based on the data distribution for 1996 (as this is the first year of data for the GMM estimation).

0.1 Normalization of psi

DFJ2010 discuss the calibration of $m_{i,j,h}$ and describe how $m\_coeff_{i,j,h}$ and $sigma\_coeff_{i,j,h}$ are estimated from medical expenses data. They also describe how ζ and ξ (and hence ψ) are estimated from medical expenses data. But this would then mean that when we take $m = exp(m\_coeff_{i,j,h} + sigma\_coeff_{i,j,h} * \psi)$ we would be squaring the standard deviation of the empirical medical expenses when we do the $sigma\_coeff_{i,j,h} * \psi$ since the way they are estimated each of $sigma\_coeff_{i,j,h}$ and ψ on their own captures the standard deviation of medical expenses.

Presumably the paper of DFJ2010 is missing a mention of a renormalization, and so in the codes implementing the model ψ is renormalized to have standard deviation of 1, so that $m = exp(m\_coeff_{i,j,h} + sigma\_coeff_{i,j,h} * \psi)$ has a standard deviation in line with the medical expenses data. In the codes this appears as $\psi/normalizepsi$ where ever ψ should be, and with $normalizepsi$ being a parameter that has the same standard deviation as ψ, so that $\psi/normalizepsi$ has standard deviation of one.

[1]Depdendence on health status is trickier, but since it is just good or bad we just create two parameters, and then in the ReturnFn etc. we juse have an if-statement on healt status that selects which of the two is relevant.

0.2 Warm-glow of bequests (PROBLEM)

DFJ2010 used FOCs/Euler eqns to estimate the model. Hence they do not use the warm-glow of bequests function itself, only it's derivative. Getting from the derivative to the original function, there will be an added constant. DFJ2010 write the eqns in the paper missing this constant in the warm-glow of bequests function, which did not matter for their code (as they only need the derivative) but means that in the codes where the warm-glow of bequests function is used directly, the warm-glow is immense and people save way too many assets.

0.3 GMM Estimation

Estimation is done in two-steps.

In the first step, many exogenous processes and parameters are estimated from the data — specifically, $earnings_{i,j}$, $s_{i,j,h}$, $\pi(h'|h;i,j)$, and everything for the process $m_{i,j,h}$— which essentially involves running various logit and log-linear regressions. I omit this here, and just take the parameter estimates from the replication materials of DFJ2010.[2] DFJ2010 refer to these parameters estimated in the first stage as χ.[3]

In the second step, the parameters υ, β, δ, θ, k, and $cfloor$ are found by GMM estimation of the Life-Cycle Model.[4] DFJ2010 refer to these parameters estimated in the first stage as Δ. The target moments for this second step to match median assets by cohort, age, and income quintile (note: not by gender).

"Our decision to use conditional medians rather than means reflects sample size considerations; in some pqv cells, changes in one or two individuals can lead to sizable changes in mean wealth. Sample size considerations also lead us to combine men and women in a single moment condition.". Comment: Cagetti (2003) also targets the conditional median of wealth (while estimating essentially the model of Carroll (1997), which is the same one as Gourinchas and Parker (2002) estimated targeting mean consumption. Cagetti (2003) motivates it on the ground that because wealth is so skewed, targeting the mean wealth in a model with something like a 'mean earnings process' would be doomed to fail.

Note: DFJ2010 set up the medians as the moment conditions:
$E[\mathbb{I}\{a_{iv} \leq a_{pqv}(\Delta,\chi)\} - 1/2 \,|\, p, q, v, \text{ individual } i \text{ alive at time } v] = 0$
where $a_{pqv}(\Delta,\chi)$ is the model-predicted median asset level in calendar year v for an individual of cohort p who was in the qth income quintile.
Whereas VFI Toolkit will set it up as the separable moment condition that the data median, m^d is equal to the model median, m^m. That is, $\{m^d : E[\mathbb{I}(a_{iv}) \leq m^d| \text{ individual } i \text{ alive at time } v] = 1/2\} - \{m^m : E[\mathbb{I}(a_{pqv}(\Delta,\chi)\} \leq m^m)|\, p, q, v] = 1/2\}$
both of these should lead to the same solution, I just mention it as it likely means that some code internals will be slightly different.

[2] These constitute some 2000+ parameter values (due to the dependence on age, gender, income quintile, and often health status.

[3] Off topic, the estimations of the income quintile fixed effects are based on dividing the cross-section for each years data, not on dividing the lifetime incomes. Hence they ignore any income mobility between the quintiles and thus overstate the permanent differences in income between the quintiles.

[4] υ=CES utility fn param, β=discount factor, δ=disutility of bad health, θ=bequest intensity, k=bequest curvature (luxury good), $cfloor$=floor on consumption (from gov transfers). Note, these six parameters are all scalars.

References

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