Attanasio, Low & Sanchez-Marcos (2008) - Explaining Changes in Female Labor Supply in a Life-Cycle Model

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The explanation of the model in ALSM2008 is a bit of a mess in the sense the equations and parameters are all over the place and I had to look at the original codes to figure some things out, so this is just a pdf with a clear explanation of what the model is.

Two small notational changes from ALSM2008: I call z (z_m and z_f) what they call υ (the shocks to earnings). I call h_m what they call $exp(h_m)$; I do this so that h_f and h_m are analagous.

Households live for 50 periods, representing ages 23 to 52. Each household is a couple, and makes joint decisions on consumption, savings, and female labor supply. The husband is essentially just an exogenous shock, z_m, representing a stochastic income. The female can choose to work or not work (participation P equals 1 or 0). If the female works she earns income that depends on an exogenous shocks z_f, and on her 'history of labor force participation', h_f; h_f reflects the females history of labor force participation decisions. If the female does not work her income is zero (household income will still be non-zero as husband always works).

There are three permament types household: 'no child', 'young mothers', 'older mothers'. For 'no child' there is nothing much to add beyond what we already explained. The 'young mothers' and 'older mothers' households differ from 'no child' households in the values of two parameters: e, which is consumption-equivalance units that weight consumption, and *childcarecost* which are a fixed costs associated with the female choosing to work (choosing $P = 1$). The 'no child' household has a constant $e = 1.67$ at all ages, representing two adults, and has $childcarecost = 0$ at all ages, representing no children. The 'young mother' household has a profile on e that starts and ends at 1.67, but with a hump at young ages, and has *childcarecost* that start and end at zero, with a hump at young ages, with these humps representing the present of children in the household. The 'older mother' households are analagous to the 'young mother' households, except that the humps occur in a later period (when the mother is 'older').

So a household has state (a, h_f, z_m, z_f, j): assets, history of female labor force participation, stochastic component of male earnings, stochastic component of female earnings, and age. Everything about children is simply a parameter that depends on age, and which differs across permanent types.

The model is,

$$
\begin{aligned}
V_i(a, h_f, z_m, z_f, j) = & \max_{a', p} \frac{(c/e_i)^{1-\gamma}}{1-\gamma} exp(\phi_1 P) - \phi_2 P + \beta E_j[V_i(a', h_f', z_m', z_f', j+1)] \\
& \text{subject to } c + a'/R = a + (y_f - childcarecost_i)P + y_m \\
& y_f = y_{f0} h_f exp(z_f) \\
& y_m = y_{m0} h_m exp(z_m) \\
& log(h_f') = log(h_f) + h_{f,accum} P - \delta(1-P)
\end{aligned}
$$

Notice that male earnings y_m are received every period and are a function of a constant $y_m 0$ (normalized to 1), an age-dependent parameter h_m, and a markov shock z_m. Female earnings y_f are only earned if $P = 1$ (and then are net of $childcarecost$), and depend on a constant $y_f 0$ (used to generate a gender-wage gap), the history of labor force participation h_f, and a markov shock z_f. Note that h_m and h_f are very different.

Labor force participation history accumulates (in logs) by age-dependent parameter $h_{f,accum}$ when the female chooses participation P, and depreciates at constant δ when the female is out of the labor force $P = 0$.

Paper explains roughly how to calculate e_i and $childcarecost_i$ which depend on the permanent type, and you can find all the details in the code implementing the model (roughly lines 110-250, includes a graph of them).[1]

The shock process for z_f and z_m follows a joint random walk (with correlated innovations). In principle, the way to solve a model with unit-root shocks is to renormalize so that they disappear from state-space and just become stochastic discount factors (which is what original code would have done, see related example here). Here we do the lazy but much more computationally difficult thing of just treating them as regular markov shocks. The advantage of this is that changing them to any other process is trivial (and unit-roots for earnings are empirically unrealistic). Hence we write the joint-process as a VAR(1),

$$
\begin{aligned}
z_f(j) &= mu_{zeta,f} + rho_{z_f} z_f(j-1) + 0 * z_m(j-1) + zeta_f \\
z_m(j) &= mu_{zeta,m} + 0 * z_f(j-1) + rho_{z_m} z_m(j-1) + zeta_m
\end{aligned}
$$

So $z(j) = Mew + Rhoz(j-1) + zeta$ with $zeta \sim N(0, Sigma^2_{zeta})$, and the parameters used by ALSM2008 are,
$Mew = [-\sigma^2_{zeta,f}/2, -\sigma^2_{zeta,m}/2]$
$Rho = [rho_{z_f}, 0; 0, rho_{z_m}]$
$Sigma^2_{zeta} = [\sigma^2_{zeta,f}, Cov(\zeta_f, \zeta_m); Cov(\zeta_f, \zeta_m), \sigma^2_{zeta,m}]$
While this VAR(1) is non-stationary, we can still use/discretize it just fine because the model is finite horizon. Note that ALSM2008 write Mew as the mean of the shocks, rather than the constant term in a VAR(1).

Notice that this has $zeta$ with a negative mean ($mu_{zeta,f}$ and $mu_{zeta,f}$ are negative; ALSM2008 put these as the mean of $zeta$, I here rewrite them to be the constant term in the VAR(1) as that is how we need to think of them in the VAR(1)). This means that z_f and z_m drift negative as agents age, and hence $exp(z_m)$ and $exp(z_f)$ decrease with age. The formula ALSM2008 use for

[1] Paper did not explain details, but they could be recovered from the original codes. The example codes are clear on all the details.

the mean of *zeta* is what you would use for the mean of a random variable U if you have a iid normally distributed U and you want $exp(U)$ to be zero mean; which is not what ALSM2008 are doing here, they have log(z) rather than the innovations to z but I think they are just using it as an approximation/fudge.

The results of the codes here look somewhat different to ALSM2008 (same general results, but here mothers almost never work while paper has participation of mothers of children under 3 years old as 0.47). This is likely due to differences in parametrization (the paper was written such that it was difficult to pin down certain details from their explanation). Also, if you look at results in code for assets they look rather odd. ALSM2008 paper does not report any results about assets, so not sure if they had same if theirs look quite different.

References

Orazio Attanasio, Hamish Low, and Viriginia Sanchez-Marcos. Explaining changes in female labor supply in a life-cycle model. American Economic Review, 98(4):1517–1552, 2008.