Portfolio-Choice with Housing and Unmodifiable-Mortgages

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This model extends Life-Cycle Model 35, which has Epsteinz-Zin preferences in a model with housing and portfolio-choice (in VFI Toolkit terms, one standard endogenous state and one riskyasset endogenous state). In that model, mortgages could be modified at any time, but sometimes people prefer to model mortgages as an unmodifiable commitment to a stream of payments. This model extends Life-Cycle Model 35 by adding three things: mortgages cannot be modified (requires that we keep track of years since house was purchased), house prices fluctuate (both before and after purchase of house), and downpayment can vary (model 35 also allowed this, but adding unmodifiable mortages means it would not be possible and less we then re-add it back in). All of these three get modelled as a semi-exogenous state, $semiz$, which is like a markov state except that the transition probabilities can be influenced by a decision variable d. In this model the decision to buy/hold/sell a house will influence all four of the $semiz$ (house prices fluctuations requires two semi-exo states, one for price changes before purchase and one for price changes after purchase). Note, if you wanted a version in which house prices could be modified, you could take Life-Cycle Model 35, and only add the house price fluctuations using the semi-exogenous state. There is one other important difference from Life-Cycle Model 35, namely that the model period here is 5 years, rather than annual. This was necessary because the model is huge, and especially because we want to keep track of the number of years since house was purchased (for mortgages) up to 30 years, which would be 31 points in an annual model but just 7 in a 5-yr model (note, we need 30yrs, plus an absorbing 'fully paid off' point).

Households get utility from consumption, c and housing services s. Homeowners get housing services as a fraction of their house value/size, renters get housing services equal to half the housing services of owning the smallest house size and have to pay rent. When households buy/sell a house there are housing transaction costs, htc.

When a household buys a house they will be getting a mortgage that is unmodifiable, so they much choose 'how big' a mortgage to take out, which is done implicitly by choosing the size of their downpayment, which is 20, 40 or 60% of the value of the house (you could allow more by adding more grid points).

Houses come in two different sizes (three grid points as also have 0 house). The price of all houses can go up and down, and we keep track of this by $pbefore$ (the price prior to buying a house, which we need to keep after the house is purchased because it will influence the size of the mortgage) and $pafter$ (the price changes since the house was purchased). These can go up and down with some probabilities. $pbefore$ only varies prior to a house being purchased, and then is fixed after. $pafter$ is irrelvant (and fixed in codes) before a house is purchased, and then is 1 at purchase and varies after purchase. Thus the current house price (after purchasing) is $pafter$ times $pbefore$; if you sell a house and buy a new one then we update $pbefore$ to include all house price changes that occured while you owned the first house.

buyhouse takes five values, and these are interpreted as: $buyhouse = 0$ means have no house next period (either don't have and don't but, or have buy selling and not buying), $buyhouse = 1, 2, 3$ means buying a house (either don't have, or had a different size) and correspond to the choice of downpayment being 20, 40 oand 60%, $buyhouse = 4$ means holding onto current house (which is not zero).

Our household problem is,

$$
\begin{aligned}
& V(h, a, z, pbefore, pafter, yearsowned, downpayment, j) \\
= & \max_{c, savings, riskyshare, hprime, buyhouse} \frac{(c^{1-\sigma_h} s^{\sigma_h})^{1-\sigma}}{1-\sigma} \\
& \quad -\beta E[-s_j V(hprime, aprime, pbeforeprime, pafterprime, yearsownedprime, ... \\
& \qquad downpaymentprime, zprime, j+1)^{1+\phi} | z]^{\frac{1}{1+\phi}}
\end{aligned}
$$

$$
\begin{aligned}
& \text{if } j < Jr: \; c + savings = a + w\kappa_j z + (h - hprime) - rentalcosts - htc \\
& \text{if } j >= Jr: \; c + savings = a + pension + (h - hprime) - rentalcosts - htc \\
& \text{if } h > 0: \; s = housingservices * h, \; rentalcosts = 0 \\
& \text{if } h = 0: \; s = 0.5 * housingservices * minhouse \, rentalcosts = rentalprice \\
& \text{if } hprime = h: \; htc = 0 \\
& \text{if } hprime \neq h: htc = f\_htc(h + hprime) \\
& \quad aprime = (1+r)(1 - riskyshare) savings + (1+u) \, riskyshare \, savings \\
& 0 \leq riskyshare \leq 1 \\
& savings \geq 0 \\
& log(zprime) = \rho_z log(z) + \epsilon, \; \epsilon \sim N(0, \sigma^2_{\epsilon,z}) \\
& u \sim N(rp, \sigma^2_u) \\
& \text{if } hprime = h \text{ and } hprime = 0: \; buyhouse = 0 \\
& \text{if } hprime = h \text{ and } hprime > 0: \; buyhouse = 4 \\
& \text{if } hprime \neq h \text{ and } hprime > 0: \; buyhouse = 1, 2, 3 \\
& \text{if } buyhouse = 1: downpaymentprime = 1 \\
& \text{if } buyhouse = 2: downpaymentprime = 2 \\
& \text{if } buyhouse = 3: downpaymentprime = 3 \\
& \text{if } buyhouse = 0, 4: downpaymentprime = downpayment \\
& \text{if } buyhouse = 4: yearsownedprime = yearsowned + 1 \text{ (note, there is a max yearsowend, so roughly)} \\
& \text{if } buyhouse = 1, 2, 3: yearsownedprime = 0 \\
& \text{if } buyhouse = 0: pbeforeprime = \zeta(pbefore) \\
& \text{if } buyhouse = 1, 2, 3, 4: pbeforeprime = pbefore * pafter \\
& \text{if } buyhouse = 0: pafterprime = pafter \\
& \text{if } buyhouse = 1, 2, 3: pafterprime = 1 \\
& \text{if } buyhouse = 4: pafterprime = \zeta(pafter)
\end{aligned}
$$

Notice that utility is now based on $c^{1-\sigma_h} s^{\sigma_h}$, which is an aggregate of consumption and housing services s. *minhouse* is the smallest (non-zero) house size/value.

The function $\zeta()$ which controls how house prices increase and decrease is coded by setting up grids on $pbefore$ and $pafter$ and then having a *probhousepricerise* that you move up one grid points, a $probhousepricefall$ that you move down one grid point, and $1 - probhousepricerise - probhousepricefall$ that you stay at current grid point; extending to allow probabilties of moving up/down two grid points could easily be done.

Because of how the toolkit works, the riskyasset must be the 'last' (here second) endogenous state. You have to use $vfoptions.refine\_d$ for the decision variables, which can be four types: $d1$ is in ReturnFn but not aprimeFn, $d2$ is in aprimeFn but not ReturnFn, $d3$ is in ReturnFn and aprimeFn, $d4$ is in ReturnFn and influences semi-exo states (so is in SemiExoStateFn). This model has one $d2$ (riskyshare), one $d3$ (savings) and one $d4$ (buyhouse).

PS. If you don't care about the unmodifiable mortgages (and are happy with modifiable mortgages as in Life-Cycle Model 35) you could eliminate both *yearsowned* and *downpayment* from the model, add a collateral constraint back into the model (there is one in model 35), and then also modify *buyhouse* appropriately (three values instead of the current five, as we no longer have *downpayment*. This will make the model much easier and faster to run, and let you put lot's more points on things like h, $pbefore$ and $pafter$.

References