Semi-Endogenous Shocks

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1 Introduction

Semi-endogenous shocks involves allowing the 'exogenous' shocks to depend on the endogenous state. So while a standard (infinite-horizon) value function iteration problem would look like,

$$V(a,z) = \max_{d,a'}\{F(d,a',a,z) + \beta E[V(a',z')|a,z]\}$$

subject to

$$z' = \pi(z)$$

where $z \equiv$ vector of exogenous state variables, $a \equiv$ vector of endogenous state variables, $d \equiv$ vector of decision variables. For a semi-endogenous shock we have the only difference being that now we replace $z' = \pi(z)$ with $z' = \pi(a,z)$. z is no longer an exogenous state variable, it is a *semi-endogenous* state variable.

In terms of implementation notice that the difference in terms of the value funcion problem is just in terms of computing the expection term.[1]

VFI Toolkit implements this using $vfoptions$. It can also use *simoptions* to then solve for the stationary distribution and to simulate panel data. This can only be done for infinite-horizon. If you have a need/use for semi-endogenous shocks in another situation, please contact me and I can implement them.

The rest of this document describes the specific example of the stochastic neoclassical growth model with semi-endogenous shocks. Code implementing this can be found at:
https://github.com/robertdkirkby/semiendogenousshocks

In writing this document I am assuming you are familiar with basic value function iteration.

2 Stochastic Neo-classical Growth model with semi-endogenous shocks

The only difference between this model and the standard stochastic neo-classical growth model is that the productivity shock, z, is semi-endogenous.

The value function problem to be solved is,

$$V(k,z) = \sup_{k'}\{\frac{c^{1-\gamma}}{1-\gamma} + \beta * E[V(k',z')|z]\}$$

[1] As long as the same grid is used for z, independent of the value of a; so transition probabilities for z depend on a, but the discretized grid does not.

subject to

$$c + i = exp(z)k^{\alpha} \tag{1}$$

$$k' = (1 - \delta)k + i \tag{2}$$

$$z' = \rho z + \epsilon' \quad , \epsilon \overset{iid}{\sim} N(0, \sigma_\epsilon(k)^2) \tag{3}$$

where k is physical capital, i is investment, c is consumption, z is a productivity shock that follows an AR(1) process; $exp(z)k^{\alpha}$ is the production function, δ is the depreciation rate. Notice in the semi-endogenous shocks z that the innovations ϵ have a variance that depends on k, namely $\sigma_\epsilon(k)^2$.

The only change from the standard stochastic neo-classical growth model is precisely the $N(0, \sigma_\epsilon(k)^2)$ which makes z semi-endogenous.

To implement this in the codes we have to use a set grid on z (*z_grid* in the codes) which will be the same for all capital levels. And then set the transition probabilities (*pi_z_semiendog* in codes) in a way that allows them to depend on k. Specifically we will set $\sigma_\epsilon(k) = \bar{\sigma}_\epsilon(1 - log(k)/maxlogk)$, so that the standard deviation of the shocks is decreasing in k. Note that $\bar{\sigma}_\epsilon$ denotes a constant, and that $maxlogk$ is set as the maximum $log(k)$ for the grid on k (this is arbitrary, it likely makes more sense to set it based on, e.g., the steady-state level of capital in a model without shocks, rather than on the max grid point). All of this is implemented in the code inside *SemiEndogShockFn.m*.

3 Stochastic Neo-classical Growth model with exogenous shocks

This is just the standard stochastic neo-classical growth model.[2] For a full explanation check Díaz-Gímenez (2001).[3] Code solving it can be found at
https://github.com/vfitoolkit/VFItoolkit-matlab-examples

The value function problem to be solved is,

$$V(k, z) = \sup_{k'} \{ \frac{c^{1-\gamma}}{1-\gamma} + \beta * E[V(k', z')|z] \}$$

subject to

$$c + i = exp(z)k^{\alpha} \tag{4}$$

$$k' = (1 - \delta)k + i \tag{5}$$

$$z' = \rho z + \epsilon' \quad , \epsilon \overset{iid}{\sim} N(0, \sigma_\epsilon^2) \tag{6}$$

[2] Start with the Solow-Swan neoclassical growth model, add 'savings decisions' to get Ramsey growth model, then add stochastic shock to get the stochastic neoclassical growth model. If you further added an endogenous labor decision you would get the basic Real Business Cycle model.

[3] Same model is solved in Aldrich, Fernandez-Villaverde, Gallant, and Rubio-Ramirez (2011).

where k is physical capital, i is investment, c is consumption, z is a productivity shock that follows an AR(1) process; $exp(z)k^{\alpha}$ is the production function, δ is the depreciation rate.

We will use the Tauchen Method to discretize the AR(1) shock.

References

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