Computational Physics Project B1
A Log Likelihood fit for extracting the D^0 lifetime

Robert Stein

December 15, 2016

1 Abstract

Finite Differential Methods (FDMs) are methods to solve

2 Aims and Implementation

The basis for this analysis is a dataset containing many measurements with varying associated errors. A simple histogram shows the heavily-smeared raw distribution in Figure 1. It arises from a convolution of the true exponential decay distribution and Gaussian smearing arising from measurement uncertainty.

From the data, we wish to obtain the most probable value for the decay constant τ. We choose a τ which maximises the likelihood of obtaining the results, and thus minimises the Negative Log Likelihood (NLL) of the dataset. A convenient consequence of considering NLL rather than Likelihood directly is that the NLL of a dataset is simply a sum of the component from each measurement, rather than a product of many values.

The experimental value of τ can be obtained through a simple 1D minimisation of the function $NLL(\tau)$, using a parabolic minimiser. A parabolic minimiser begins with three starting values x_0, x_1 and x_2. With $y_i = f(x_i)$ for a function f, we can calculate the values x_3 and y_3.

$$x_3 = \frac{1}{2}\frac{(x_2^2 - x_1^2)y_0 + (x_0^2 - x_2^2)y_1 + (x_1^2 - x_0^2)y_2}{(x_2 - x_1)y_0 + (x_0 - x_2)y_1 + (x_1 - x_0)y_2}$$

The parameter x_i with the largest corresponding y_i value is eliminated, and the remaining three values are relabelled. The process is repeated until convergence on a minimum occurs.

Alternatively, the small number of background events N_{bkg} can also be considered. In general, we define $a = \frac{N_{sig}}{N_{bkg}+N_{sig}}$. We can consider the total likelihood of each event as the weighted sum of probabilities:

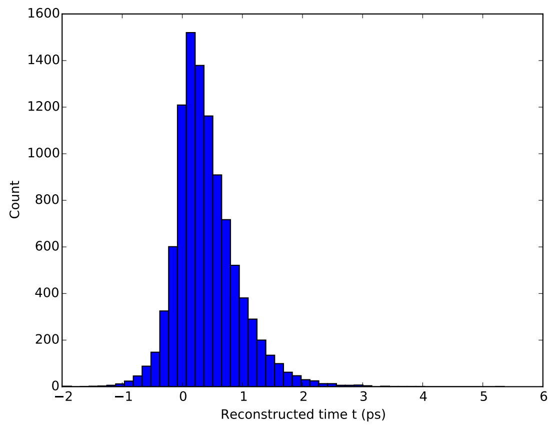

Figure 1: The reconstructed decay times of D^0 mesons.

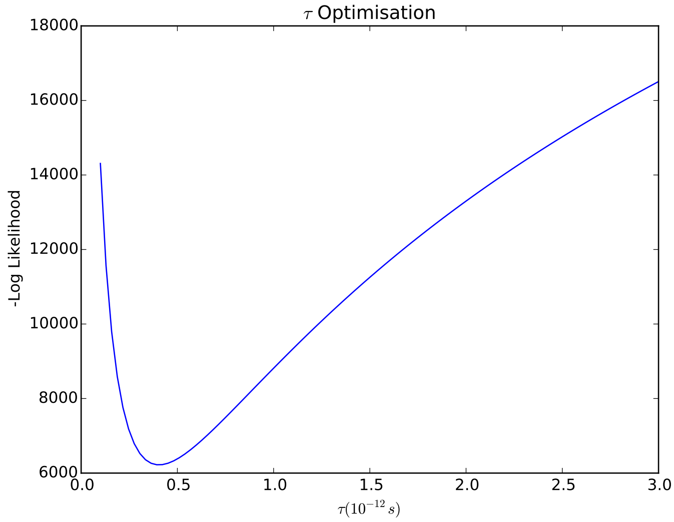

Figure 2: The function $NLL(\tau)$ against τ. The minimum clearly lies in the region of $0.4 < \tau < 0.5$.

$$Likelihood = aP_{sig}(t \mid \tau, \sigma) + (1-a)P_{bkg}(t \mid \tau, \sigma)$$

The NLL is again found for the purposes of minimisation. A Quasi-Newton minimiser was implemented for 2D minimisation, following an iterative process, with G_0 being the 2D identity matrix.

$$\mathbf{x_{n+1}} = \mathbf{x_n} - \alpha G_n \cdot \nabla f(\mathbf{x_n})$$

$$G_{n+1} = G_n + \frac{(\delta_\mathbf{n} \otimes \delta_\mathbf{n})}{\gamma_\mathbf{n} \cdot \delta_\mathbf{n}} - \frac{G_n \cdot (\delta_\mathbf{n} \otimes \delta_\mathbf{n}) \cdot G_n}{\gamma_\mathbf{n} \cdot G_n \cdot \gamma_\mathbf{n}}$$

$$\delta_\mathbf{n} \equiv \mathbf{x_{n+1}} - \mathbf{x_n}$$

$$\gamma_\mathbf{n} \equiv \nabla f(\mathbf{x_{n+1}}) - \nabla f(\mathbf{x_n})$$

In both cases, a definition of convergence is required to terminate the iterative process. in terms of change in NLL rather than parameter space distance.

We find a minimum $\mathbf{x_{min}} = \sum_i x_i$ for dimensions i. From the definition of

3 Code Validation

Other function. cosh(x)

An effective way to validate the minimisation process is to produce a graph of the function NLL. This is done in Figures 2 and 3 for the 1D and 2D minimsations respectively. In both cases, a broad minimisation region is clear, providing a useful sanity check on minimiser results. However, the production of such graphs is extremely resource-intensive. The iterative process required to obtain precise results in this manner is thus inefficient, limiting the effectiveness of such validation to perhaps one order of magnitude.

4 Results

etc.

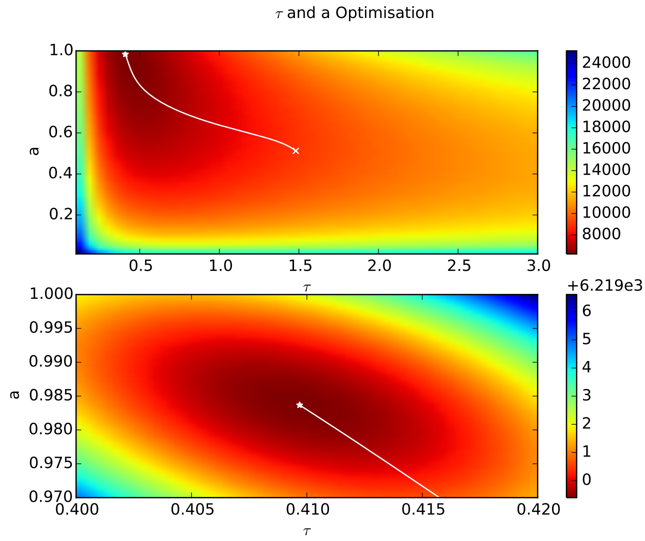

Figure 3: Graphs of a against τ, with the colour scale representing function $NLL(\tau, a)$. The above graph covers the entire minimisation region, showing the minimum lying in the region $0.2 < \tau < 1.0$ and $0.5 < a < 1.0$. The lower graph focuses exclusively on this low-NLL region to provide higher contrast. With the new colour scale, it is clear that $0.3 < \tau < 0.6$ and $0.7 < a < 1.0$. We expect the minimisation solution to lie in this region. The path of the minimisation is shown in white, beginning at the cross and ending at the star.

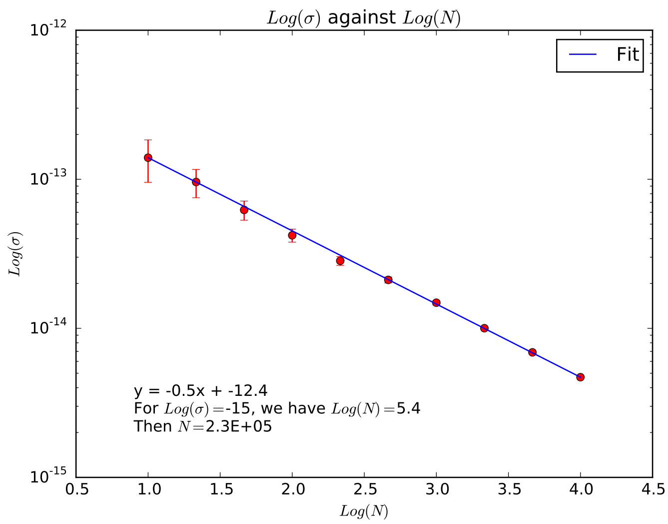

Figure 4: A graph of $\log(\sigma)$ against $\log(n)$

5 discussion

moar

6 Conclusion

end