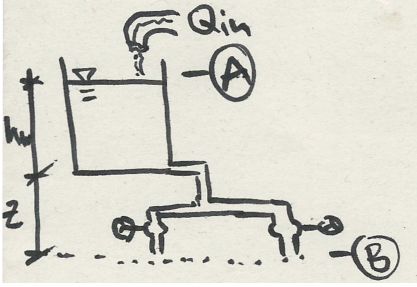


Development of Equations for the Inflow/Outflow Micro-Model

numeric model/difference equations

building blocks model

static model



A = Surface of the water in tank

B = Height of outflow at water taps

Bernoulli $A \rightarrow B$:

$$h + z + \frac{\alpha * v_A^2}{2g} = \frac{\alpha * v_B^2}{2g} \rightarrow v_B = \sqrt{\frac{2g}{\alpha} \left(h + z + \frac{\alpha * v_A^2}{2g} \right)}$$

$$Q_{out} = n_{taps} * v_B * \pi r_{taps}^2 = n_{taps} * \pi r_{taps}^2 \sqrt{\frac{2g}{\alpha} \left(h + z + \frac{\alpha * v_A^2}{2g} \right)}$$

dynamic model

$$Q_{out,t} = n_{taps} * \pi r_{taps}^2 \sqrt{\frac{2g}{\alpha} \left(h_t + z + \frac{\alpha * v_A^2}{2g} \right)}$$

$$\Delta h_t = \frac{Q_{in} - Q_{out,t}}{\pi r_{tank}^2} \Delta t$$

$$\rightarrow h_t = h_{t-1} + \frac{Q_{in} - Q_{out,t-1}}{\pi r_{tank}^2} \Delta t$$

$$n_{bottles} = \frac{1}{V_{bottles}} \int_0^t Q_{out} dt \rightarrow \frac{dn_{bottles}}{dt} = \frac{Q_{out}}{V_{bottles}}$$

asymptotic dynamic model

$$\alpha \approx 1, v_A \approx 0$$

$$\rightarrow Q_{out,t} = n_{taps} * \pi r_{taps}^2 \sqrt{2g(h_t + z)}$$

$$h_t = F(Q_{out,t-1}, h_{t-1}) = h_{t-1} + \frac{Q_{in} - Q_{out,t-1}}{\pi r_{tank}^2} \Delta t$$

$$Q_{out,t} = G(Q_{out,t-1}, h_{t-1}) = n_{taps} * \pi r_{taps}^2 \sqrt{2g \left(h_{t-1} + z + \frac{Q_{in} - Q_{out,t-1}}{\pi r_{tank}^2} \Delta t \right)}$$

differential equation model

$$\frac{dh}{dt} = \frac{Q_{in} - Q_{out}}{\pi r_{tank}^2}$$

$$\frac{dQ_{out}}{dt} = g * n_{taps} * \pi r_{taps}^2 (2g(h + z))^{-1/2} \frac{dh}{dt}$$

$$\frac{dn_{bottles}}{dt} = \frac{Q_{out}}{V_{bottles}}$$