

Development of Equations for the Bacterial Growth Model

Exponential bacterial growth - due to bacteria fission:

$$\frac{dc(t)}{dt} = kc \rightarrow \Delta c = kc\Delta t$$

Bacterial growth regarding temperature dependence - inactivation rate k at specific temperature T in reference to 20degree outside and a chosen water source (see [1]):

$$\frac{k}{k_{20}} = Q_{10}^{(T-T_{20})/10} \rightarrow k = k_{20} * Q_{10}^{(T-T_{20})/10}$$

Bacterial concentration in water tank (compare [4] - Pollution concentration in liquid tank):

$$\begin{aligned} c &= \frac{Q_{in}c_{in}\Delta t + Vc - Q_{out}c\Delta t}{Q_{in}\Delta t + V - Q_{out}\Delta t} \rightarrow \Delta c = \frac{Q_{in}c_{in}\Delta t + Vc - Q_{out}c\Delta t}{Q_{in}\Delta t + V - Q_{out}\Delta t} - c \\ &\rightarrow dc = \frac{Q_{in}c_{in} dt + Vc - Q_{out}c dt}{Q_{in} dt + V - Q_{out} dt} - c \end{aligned}$$

Bacterial concentration in water tank and bacterial growth:

$$\begin{aligned} \Delta c &= \frac{Q_{in}c_{in}\Delta t + Vc - Q_{out}c\Delta t}{Q_{in}\Delta t + V - Q_{out}\Delta t} + kc\Delta t - c \\ &\rightarrow dc = \frac{Q_{in}c_{in} dt + Vc - Q_{out}c dt}{Q_{in} dt + V - Q_{out} dt} + kc dt - c \end{aligned}$$

Combination - bacteria concentration in water tank and bacterial growth including inactivation rate k depending on temperature:

$$\begin{aligned} \Delta c &= \frac{Q_{in}c_{in}\Delta t + Vc - Q_{out}c\Delta t}{Q_{in}\Delta t + V - Q_{out}\Delta t} + kc\Delta t - c \\ &\rightarrow dc = \left[\left(\frac{Q_{in}c_{in} dt + Vc - Q_{out}c dt}{Q_{in} dt + V - Q_{out} dt} \right) + \left(k_{20}Q_{10}^{(T-T_{20})/10}c \right) dt \right] - c \end{aligned}$$