Development of Equations for the Temperature Model

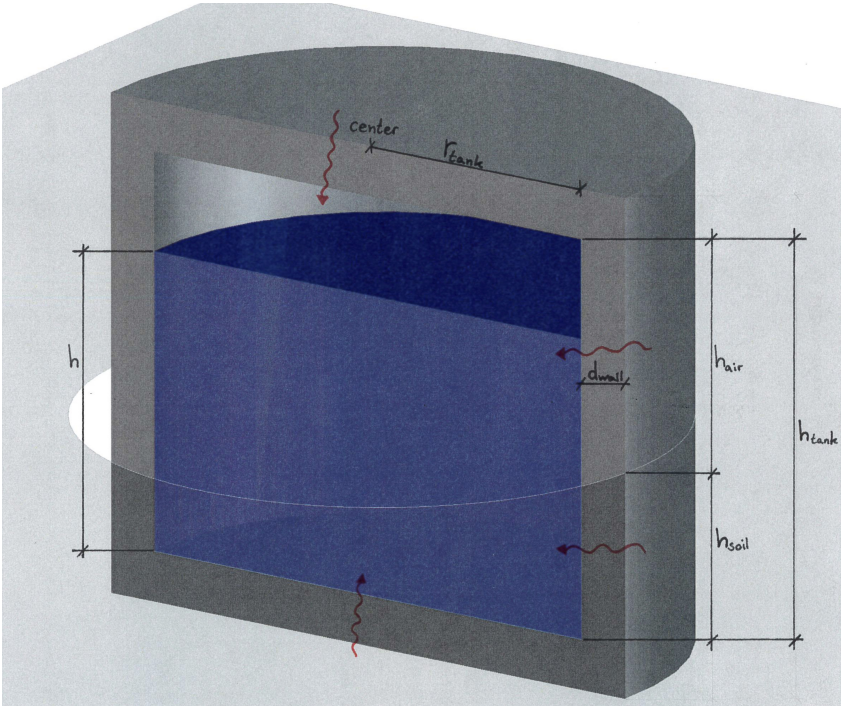

Figure 1: Draft of the citern, with relevant measures and temperature flow arrows

thermal conduction

$$\frac{\mathrm{d}T_{water}}{\mathrm{d}t} = \frac{T_{amb} - T_{water}}{R_{ia} * c_i}$$

$$R_{ia} = \frac{d}{\lambda * A} = \left[\frac{\mathrm{m}}{\frac{\mathrm{W}}{\mathrm{mK}}\mathrm{m}^2} = \frac{\mathrm{K}}{\mathrm{W}} = \frac{\mathrm{Ks}^3}{\mathrm{m}^2\mathrm{kg}}\right]$$

$$c_i = c_p * \rho V = \left[\frac{\mathrm{J}}{\mathrm{kg} * \mathrm{K}} * \frac{\mathrm{kg}}{\mathrm{m}^3} * \mathrm{m}^3 = \frac{\mathrm{J}}{\mathrm{K}} = \frac{\mathrm{m}^2\mathrm{kg}}{\mathrm{Ks}^2}\right]$$

$$c_i * R_{ia} = \left[\frac{\mathrm{Ks}^3}{\mathrm{m}^2\mathrm{kg}} * \frac{\mathrm{m}^2\mathrm{kg}}{\mathrm{Ks}^2} = \mathrm{s}\right]$$

$$c_i = c_p * \rho V = c_{p,water}\rho_{water}V_{water} + c_{p,wall}\rho_{wall}V_{wall}$$

$$\frac{\mathrm{d}T_{water}}{\mathrm{d}t} = \frac{T_{soil} - T_{water}}{R_{ia,bottom} * c_{i,bottom}} + \frac{T_{soil} - T_{water}}{R_{ia,side,soil} * c_{i,side,soil}}$$

$$+\frac{T_{air} - T_{water}}{R_{ia,side,air} * c_{i,side,air}} + \frac{T_{air} - T_{water}}{R_{ia,top} * c_{i,top}}$$

applying pollution equation for temperature (since inflow water has ambient temperature as well)

$$\text{Pollution Concentration in water tank: } c = \frac{Q_{in} * c_{in} + V * c - Q_{out} * c}{Q_{in} + V - Q_{out}}$$

$$\rightarrow \frac{\mathrm{d}c}{\mathrm{d}t} = \frac{Q_{in} * c_{in} + V * c - Q_{out} * c}{Q_{in} + V - Q_{out}} - c$$

$$\text{Pollution Concentration in water tank: } c = \frac{Q_{in}c_{in}\Delta t + Vc - Q_{out}c\Delta t}{Q_{in}\Delta t + V - Q_{out}\Delta t}$$

$$\rightarrow \Delta c = \frac{Q_{in} c_{in} \Delta t + V c - Q_{out} c \Delta t}{Q_{in} \Delta t + V - Q_{out} \Delta t} - c$$

$$\rightarrow \mathrm{d}c = \frac{Q_{in} c_{in}\,\mathrm{d}t + V c - Q_{out} c\,\mathrm{d}t}{Q_{in}\,\mathrm{d}t + V - Q_{out}\,\mathrm{d}t} - c$$

$$\rightarrow \mathrm{d}T_{water} = \frac{Q_{in} T_{in}\,\mathrm{d}t + V T_{water} - Q_{out} T_{water}\,\mathrm{d}t}{Q_{in}\,\mathrm{d}t + V - Q_{out}\,\mathrm{d}t} - T_{water}$$

combination

$$\frac{\mathrm{d}T_{water}}{\mathrm{d}t} = \frac{T_{soil} - T_{water}}{R_{ia,bottom} * c_{i,bottom}} + \frac{T_{soil} - T_{water}}{R_{ia,side,soil} * c_{i,side,soil}}$$

$$+ \frac{T_{air} - T_{water}}{R_{ia,side,air} * c_{i,side,air}} + \frac{T_{air} - T_{water}}{R_{ia,top} * c_{i,top}} + \frac{Q_{in} T_{in}\,\mathrm{d}t + V T_{water} - Q_{out} T_{water}\,\mathrm{d}t}{Q_{in}\,\mathrm{d}t + V - Q_{out}\,\mathrm{d}t} * \frac{\mathrm{d}}{\mathrm{d}t} - \frac{T_{water}}{\mathrm{d}t}$$