Lec 4

C3A - Display Technology

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Lecture 4

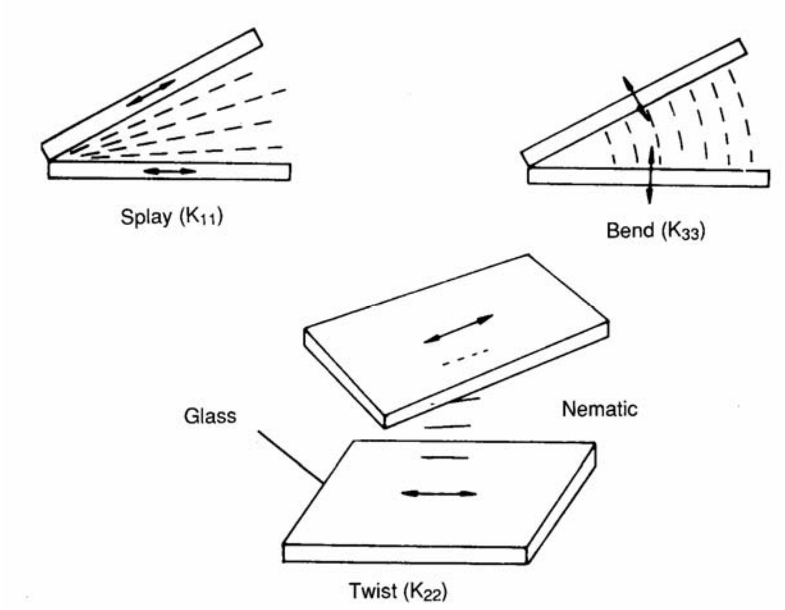
- Minimising the total energy
  - 1-D solutions
  - Critical voltage for Freedericksz transition
  - Useful analytical solutions for TN devices
- Introduction to Supertwisted Nematic LCDs (STN)

Continuum Theory – Energy Due to Distortions

- Distortion energy density of director n (Frank 1958)

$$U_k = 1/2\left\{k_{11}(div\,\underline{\mathrm{n}})^2 + k_{22}(\underline{\mathrm{n}}.curl\,\underline{\mathrm{n}})^2 + k_{33}(\underline{\mathrm{n}}\,x\,curl\,\underline{\mathrm{n}})^2\right\}$$

- Elastic constants
  - k_{11} splay, k_{22} twist and k_{33} bend
  - Magnitude $\approx 10^{-11}$ N
  - Units



Continuum Theory – Adding an Electric Field

- Dielectric energy density: $U_E = -1/2\,\underline{D}.\underline{E}$
- Total energy density: $U = U_k + U_E$

$$\Rightarrow \qquad U = 1/2 \left\{ \begin{array}{l} k_{11}(div\,\underline{\mathrm{n}})^2 + k_{22}(\underline{\mathrm{n}}.curl\,\underline{\mathrm{n}})^2 \\ +k_{33}(\underline{\mathrm{n}}\,x\,curl\,\underline{\mathrm{n}})^2 - \underline{D}.\underline{E} \end{array} \right\}$$

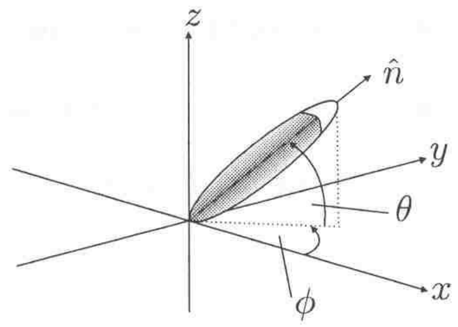
- Minus sign due to constant voltage on electrodes
- Want to find solutions for $\underline{n}(x, y, z)$
  - which minimise total energy $\int U\,dx\,dy\,dz$
  - with relevant boundary conditions imposed

Coordinates

- Cartesian: $\underline{n} = n_x, n_y, n_z$ with $n_x^2 + n_y^2 + n_z^2 = 1$

- Change to spherical polar:

$$\underline{n} = \cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta$$

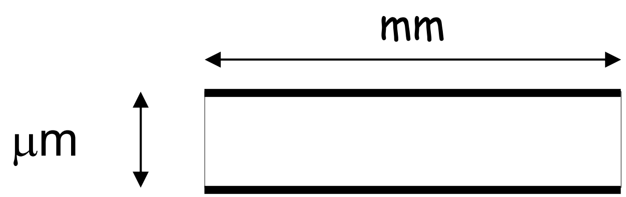


- Angles easily understood
  - θ is tilt angle of director to plane of layer
  - ϕ is twist angle of director within layer

Some Simplifications

- A 'large' pixel, so assume

$$\frac{\partial}{\partial x} = 0 \;\; and \;\; \frac{\partial}{\partial y} = 0$$



and consider only $\dfrac{\partial}{\partial z}$

- Consider a layer with no twist

$$\phi = 0 \quad \Rightarrow \quad \underline{n} = \cos\theta,\, 0,\, \sin\theta$$

Vector Differentials

- $\underline{n} = \cos\theta, 0, \sin\theta$ and $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$
- $div\,\underline{n} = \cos\theta\left(\frac{\partial\theta}{\partial z}\right)$

  $\Rightarrow \boxed{(div\,\underline{n})^2 = \cos^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2}$
- $curl\,\underline{n} = 0, \sin\theta\left(\frac{\partial\theta}{\partial z}\right), 0$ $\Rightarrow$ $\boxed{\underline{n}.curl\,\underline{n} = 0}$

  and $\underline{n}\,x\,curl\,\underline{n} = \sin^2\theta\left(\frac{\partial\theta}{\partial z}\right), 0, \sin\theta\cos\theta\left(\frac{\partial\theta}{\partial z}\right)$

  $\Rightarrow \boxed{(\underline{n}\,x\,curl\,\underline{n})^2 = \sin^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2}$

Energy Density

- $$U = 1/2\left\{k_{11}(div\,\underline{\mathrm{n}})^2 + k_{22}(\underline{\mathrm{n}}.curl\,\underline{\mathrm{n}})^2 + k_{33}(\underline{\mathrm{n}}\,x\,curl\,\underline{\mathrm{n}}\,)^2 - \underline{D}.\underline{E}\right\}$$

- Substituting

$$\Rightarrow \quad U = 1/2\left\{k_{11}\cos^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 + k_{33}\sin^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 - \underline{D}.\underline{E}\right\}$$

Dielectric Term

- Assume no free charge, Maxwell's equations:

$$div\,\underline{D} = 0 \;\; and \;\; curl\,\underline{E} = 0$$

- 'Large' pixel:

$$\frac{\partial}{\partial x} = 0\,,\;\; \frac{\partial}{\partial y} = 0 \;\; and \;\; E_x = E_y = 0$$

- Dielectric energy:

$$U_E = -\frac{1}{2}\underline{D}.\underline{E} = -\frac{1}{2}D_z\,E_z = -\frac{1}{2}\,\varepsilon_0\,\varepsilon_{zz}\,E_z^2$$

$$\varepsilon_{zz} = \varepsilon_\perp + \Delta\varepsilon\sin^2\theta$$

$$\Rightarrow \quad \boxed{U_E = -\frac{1}{2}\,E_z^2\,\varepsilon_0\,\Delta\varepsilon\sin^2\theta}$$

ignoring constant $\varepsilon_\perp$ term

Total Energy Density

- $$U = 1/2\left\{ k_{11}\cos^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 + k_{33}\sin^2\theta\left(\frac{\partial\theta}{\partial z}\right)^2 - E_z^2\,\varepsilon_0\,\Delta\varepsilon\sin^2\theta \right\}$$

- How to find (θ, ϕ) corresponding to minimum energy?

Euler-Lagrange Equation

- Calculus of variations
- Stephenson pp 454-455
- For LC energy:

$$I(\theta) = \int f\left(z, \theta, \theta^{'}\right) dz$$

where: $\theta^{'} = \left(\dfrac{\partial \theta}{\partial Z}\right)$

- Euler-Lagrange gives:

$$\boxed{\frac{\partial f}{\partial \theta} - \frac{\partial}{\partial z}\left(\frac{\partial f}{\partial \theta^{'}}\right) = 0}$$

Calculus of Variations [25.2]

function y such that the integral

$$I[y] = \int_{x_1}^{x_2} f(x, y, y')\, dx \qquad (2)$$

is stationary, where $y' = dy/dx$. It is assumed here that the function f is a given differentiable function of the three variables x, y and y', and that $y = y_1$ at $x = x_1$, $y = y_2$ at $x = x_2$, where x_1, x_2 are given (constant) limits of integration. Suppose $y(x)$ is any curve passing through the two points $(x_1\ y_1)$, (x_2, y_2) (see Fig. 25.1). Consider

Since $\eta(x)$ is an arbitrary function, (10) can only be satisfied if

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0. \qquad (11)$$

This is Euler's equation for the function y which must be satisfied if (2) is to have a stationary value. There are several cases in which Euler's equation may be simplified, as we shall now see.

Applying Euler-Lagrange

- $$\left\{k_{11}\cos^2\theta + k_{33}\sin^2\theta\right\}\left(\frac{\partial^2\theta}{\partial z^2}\right) + \left\{(k_{33} - k_{11})\left(\frac{\partial\theta}{\partial z}\right)^2 + E_z^2\,\varepsilon_0\,\Delta\varepsilon\right\}\sin\theta\cos\theta = 0$$

- Just above threshold, both θ *and* $\left(\frac{\partial\theta}{\partial z}\right)$ are small

  $$\Rightarrow \quad k_{11}\left(\frac{\partial^2\theta}{\partial z^2}\right) + E_z^2\,\varepsilon_0\,\Delta\varepsilon\,\sin\theta\cos\theta = 0$$

- Guess a solution of form: $\theta = \theta_m \sin\left(\frac{\pi z}{d}\right)$ with θ_m small

  $$\Rightarrow \quad \theta_m\left\{\varepsilon_0\,\Delta\varepsilon\,E_z^2 - k_{11}\left(\frac{\pi}{d}\right)^2\right\}\sin\left(\frac{\pi z}{d}\right) = 0$$ which has two solutions:

  - $\theta_m = 0$ ie below threshold and no distortion
  - $\theta_m \neq 0$ *and* $\varepsilon_0\,\Delta\varepsilon\,E_z^2 = k_{11}\left(\frac{\pi}{d}\right)^2 \quad \Rightarrow \quad \boxed{V_c^2 = \frac{k_{11}\pi^2}{\varepsilon_0\,\Delta\varepsilon}}$

Threshold Condition

- Threshold voltage
- Typical value:

$$k_{11} \approx 10\, pN,\ \Delta\varepsilon \approx 10 \quad \Rightarrow \quad V_c \approx 1\, Volt$$

- Different geometries have similar results
  - Different k_{ii} or combinations
  - Fields within layer produce threshold field

Extending the Analysis

- Expand to 1st order terms

$$\left\{k_{11}\cos^2\theta + k_{33}\sin^2\theta\right\}\left(\frac{\partial^2\theta}{\partial z^2}\right) + \left\{\left(k_{33} - k_{11}\right)\left(\frac{\partial\theta}{\partial z}\right)^2 + E_z^2\,\varepsilon_0\,\Delta\varepsilon\right\}\sin\theta\cos\theta = 0$$

  - $\Rightarrow$ Initial slope

- Twisted layers (eg TN)
  - Euler-Lagrange equations for both θ *and* ϕ
  - Initial slope of twisted cells

- Finite pitch (P) in a chiral nematic

$$k_{22}\left(\underline{n}.curl\,\underline{n}\right)^2 \Rightarrow k_{22}\left(\underline{n}.curl\,\underline{n} - \frac{2\pi}{P}\right)^2$$

- Numerical solutions

Calculating the Electric Field

- Electric field problem
  - Not simply given by $E = -\dfrac{V}{d}$
- Maxwell's equation and no free charge

$$div\,\underline{D} = 0 \quad \Rightarrow \quad D_z = constant$$

$$\Rightarrow \quad -V = \frac{D_z}{\varepsilon_0}\int_0^d \frac{dz}{\varepsilon_{zz}} \quad \Rightarrow \quad \boxed{E_z = \frac{-V}{\varepsilon_{zz}\displaystyle\int_0^d \frac{dz}{\varepsilon_{zz}}}}$$

Results for Untwisted Planar Layers

- Threshold

$$V_c^2 = \frac{k_{11}\pi^2}{\varepsilon_0\,\Delta\varepsilon}$$

- Initial slope

$$\theta_m^2 = \frac{4(V - V_c)/V_c}{k_{33}/k_{11} + \Delta\varepsilon/\varepsilon_\perp}$$

$\Rightarrow$ material design for multiplexing

$\Rightarrow$ small $\left(\frac{k_{33}}{k_{11}} + \frac{\Delta\varepsilon}{\varepsilon_\perp}\right)$

Results for Twisted Nematic Devices

- Threshold

$$V_c^2 = \frac{k_{11}\pi^2 + \left(\pi^2/4\right)\left\{k_{33} - 2k_{22}\left(1 - 4\pi d/P\right)\right\}}{\varepsilon_0 \Delta\varepsilon}$$

- Initial slope, more complicated, but still approximates to:

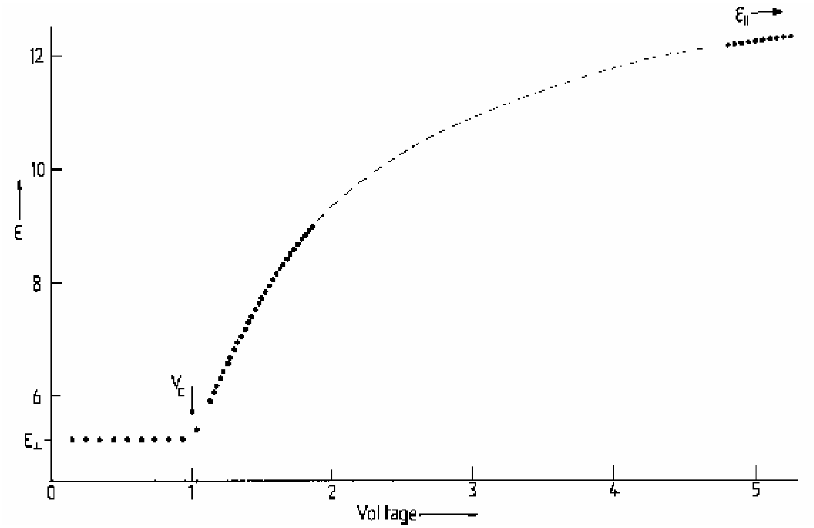
$$\theta_m^2 \approx \frac{4\left(V - V_c\right)/V_c}{k_{33}/k_{11} + \Delta\varepsilon/\varepsilon_\perp}$$

$\Rightarrow$ still want small $\left(\frac{k_{33}}{k_{11}} + \frac{\Delta\varepsilon}{\varepsilon_\perp}\right)$ for multiplexing

$\Rightarrow$ used in design of materials for TN displays

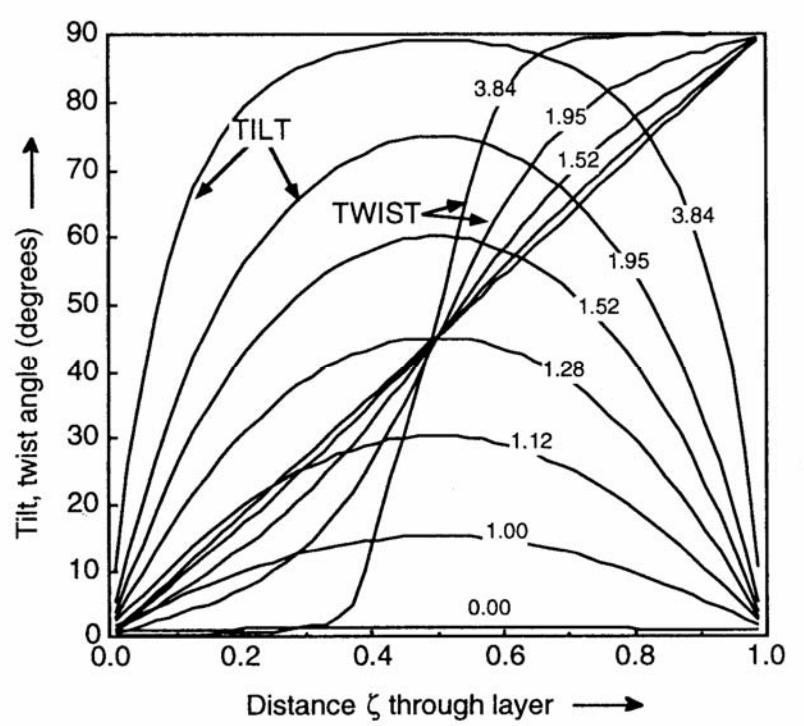
Some Numerical Results

- Untwisted planar layer
- Voltage dependence of permittivity
- Agreement between
  - Modelling data (curve)
  - Experimental data (points)
- Used in reverse to measure k_{11} and k_{33}



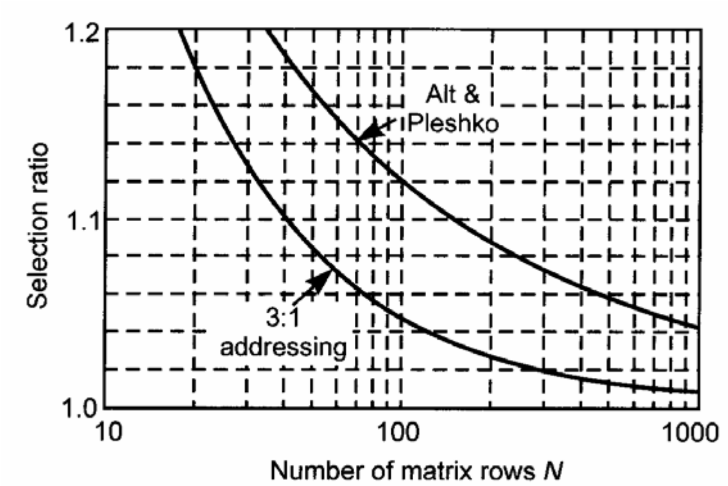
Some Numerical Results for TN

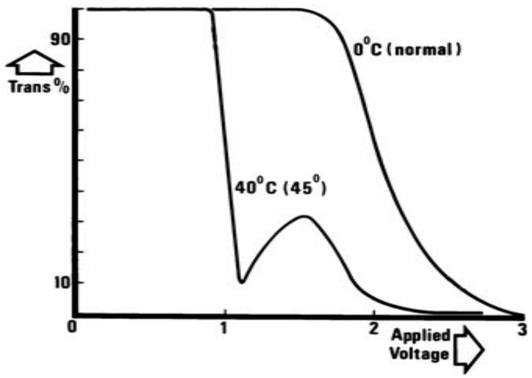
- Tilt (θ) and twist (ϕ)
  - Through layer
  - Voltages normalised to Vc
- Nothing for V < Vc
- For large V
  - Tilt approaches $\pi/2$ in centre
  - Twist concentrates in centre



Reminder of TN Multiplexing

$$\frac{V_{ON}}{V_{OFF}} = \sqrt{\frac{\sqrt{N}+1}{\sqrt{N}-1}}$$





Typical TN performance: ~ x 2

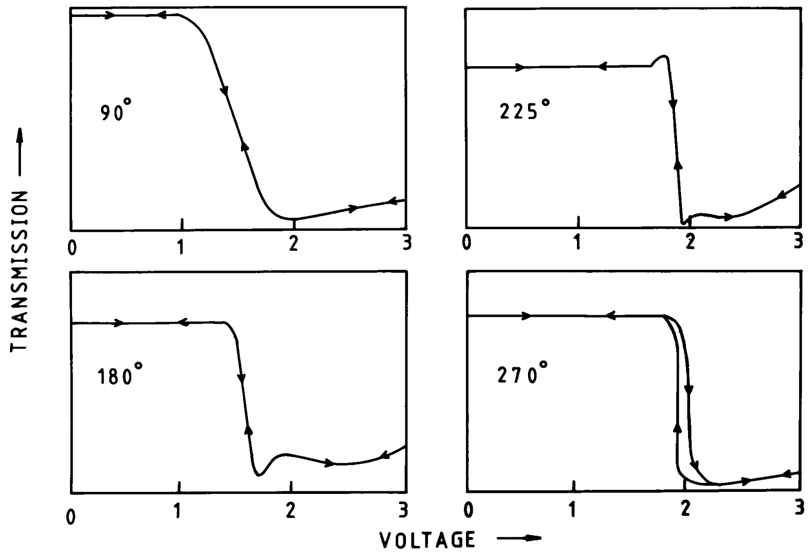
$\Rightarrow$ N ~ 5

Supertwisted Nematic (STN)

- Waters and Raynes 1982
- Increased twist angle *(ϕ)*
  - $\phi \sim 180^\circ$ - 270°
  - Surface alignment + pitch

$$\frac{\phi}{2\pi} - 0.25 \leq \left(\frac{d}{P}\right) \leq \frac{\phi}{2\pi} + 0.25$$

- Near infinite slope possible
  - Depends on
    - *ϕ, d/P*
    - LC material
- Ideal for RMS multiplexing



Next Lecture

- Supertwisted nematic LCDs
  - Basic device performance
  - Director modeling
  - Material optimisation
- TFT displays
  - Device construction
  - Addressing