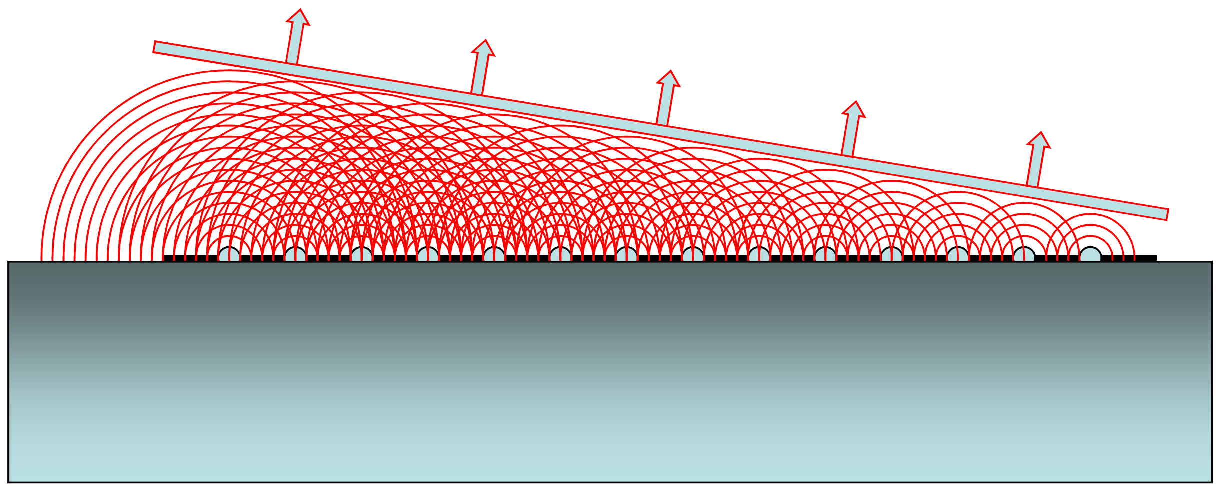
The Discrete Line Array

Steering the Beam Using a Soundspeed Gradient (refraction)

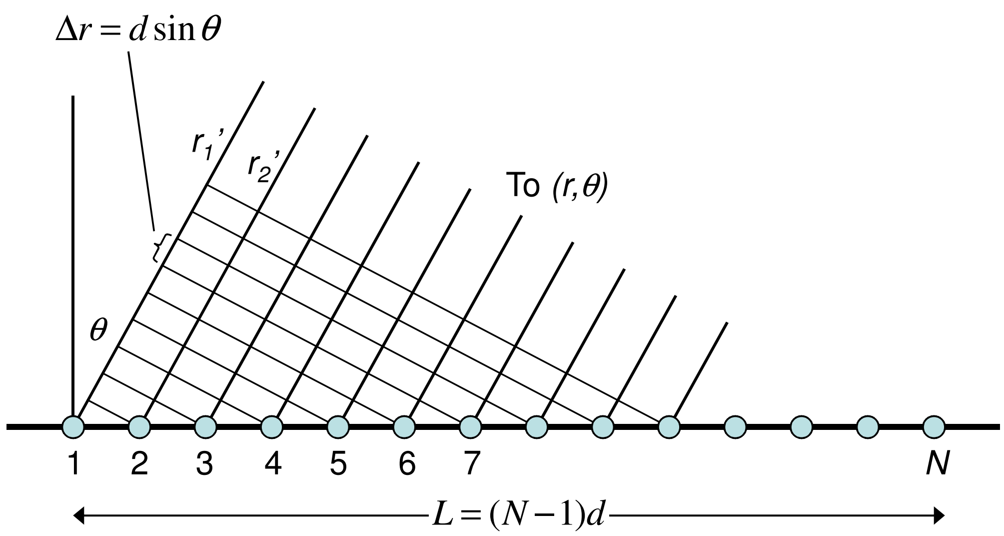


- Consider N simple sources spaced d apart, all possessing the same source strength and radiating in phase. The pressure in the far field…

$$\mathbf{p}_i = \frac{A}{r_i'} e^{-i(\omega t - k r_i')}$$

$$\mathbf{p}(r,\theta,t) = \sum_{i=1}^{N} \frac{A}{r_i'} e^{-i(\omega t - k r_i')} \qquad [3.11]$$

$$\mathbf{p}(r,\theta,t) = \frac{A}{r} e^{-i(\omega t - k r)} \left[\frac{\sin(\frac{N}{2} k \Delta r)}{\sin(\frac{1}{2} k \Delta r)} \right]$$



- The amplitude of the far field pressure can be written as the product of an axial pressure amplitude and a non-dimensional Directional Factor

$$P(r,\theta) = |\mathbf{p}(r,\theta,t)| = P_{ax}(r)H(\theta)$$

$$P_{ax}(r) = NA/r \qquad [3.12]$$

$$H(\theta) = \left| \frac{1}{N} \frac{\sin(\frac{N}{2} kd \sin\theta)}{\sin(\frac{1}{2} kd \sin\theta)} \right|$$

- Note that there are multiple main lobes, since there are multiple values for which $H(\theta) = 1$:

$$\tfrac{1}{2} kd \sin\theta = m\pi \quad \text{or} \quad |\sin\theta| = \frac{2m\pi}{kd} = m\frac{\lambda}{d} \quad ; \quad m = 0,1,2,\ldots\left\lfloor \frac{d}{\lambda} \right\rfloor$$

- The regions of zero pressure, or nodal surfaces, occur for:

$$\tfrac{N}{2} kd \sin\theta = n\pi \quad \text{or} \quad |\sin\theta| = \frac{2n\pi}{Nkd} = \frac{n}{m}\frac{\lambda}{d} \quad ; \quad \frac{n}{m} \neq m$$

- If we now insert a time delay $n\tau$ into the transmit signal for the n[th] element

$$\mathbf{p}_n = \frac{A}{r_n'} e^{-i(\omega(t+n\tau)-kr_i')}$$

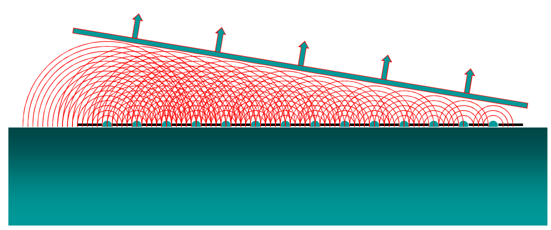
The Directional Factor becomes:

$$H(\theta) = \left| \frac{1}{N} \frac{\sin\left[\frac{N}{2} kd\left(\sin\theta - \frac{c\tau}{d}\right)\right]}{\sin\left[\frac{1}{2} kd\left(\sin\theta - \frac{c\tau}{d}\right)\right]} \right| \qquad [3.13]$$

- The 1st major lobe (m=0) now points in the direction of θ_o:

$$\sin\theta_o = \frac{c\tau}{d} \qquad [3.14]$$

which means that the beam is now steered in the direction of θ_o



<u>**BME2 – Biomedical Ultrasonics**</u>

Lecture 4: Sources of Sound – the Continuous Line Source and the Baffled Piston Source

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Department of Engineering Science

Contents

4.1 The continuous line source

Long, thin radially pulsating cylinder

Length: *L*

Surface velocity: $dQ = U_o 2\pi a\, dx$

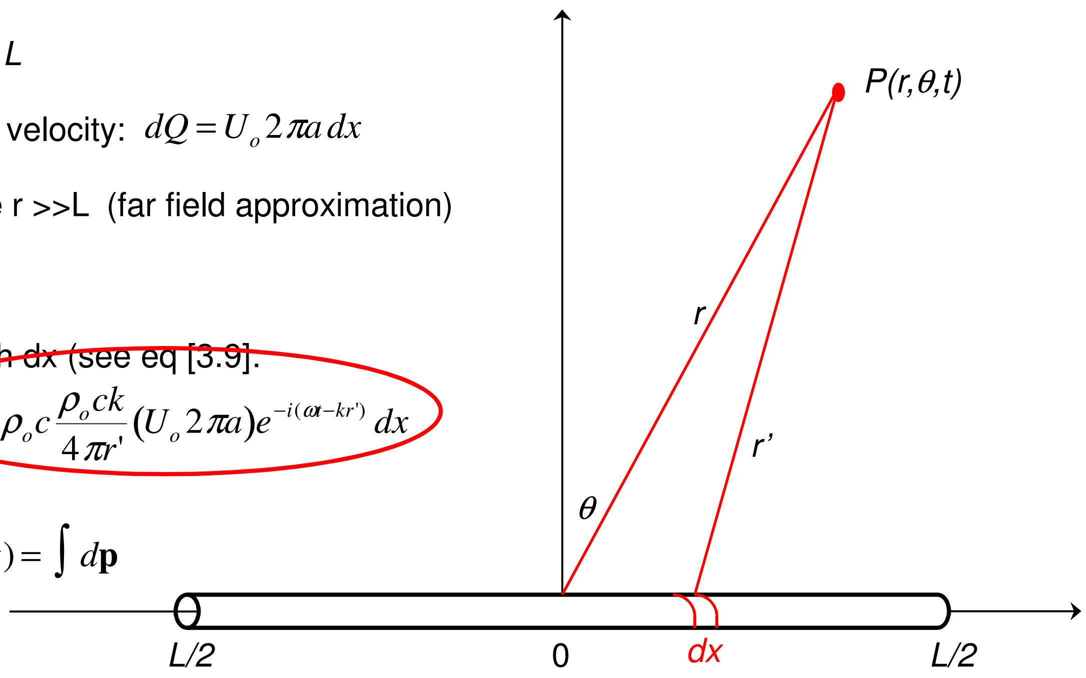
Assume r >>L (far field approximation)

For each dx (see eq [3.9]:

$$d\mathbf{p} = -i\rho_o c \frac{\rho_o ck}{4\pi r'}(U_o 2\pi a)e^{-i(\omega t - kr')}\, dx$$

and

$$\mathbf{p}(r,\theta,t) = \int d\mathbf{p}$$



- The radiated pressure field is axisymmetric.

- The amplitude of the far field pressure can be written as the product of an axial pressure amplitude and a non-dimensional Directional Factor

$$P(r,\theta) = |\mathbf{p}(r,\theta,t)| = P_{ax}(r)H(\theta)$$

$$P_{ax}(r) = \frac{1}{2}\rho_o c U_o \frac{a}{r} kL \qquad [4.1]$$

$$H(\theta) = \left|\frac{\sin\eta}{\eta}\right| \quad where \quad \eta = \tfrac{1}{2}kL\sin\theta$$

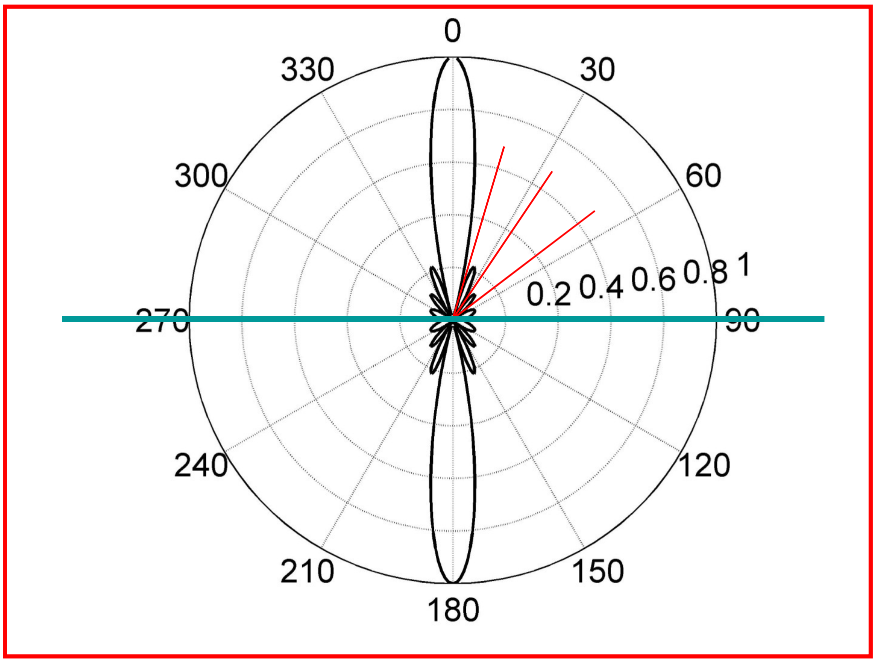
- Energy is projected into regions of pressure maxima called diffraction lobes (or main lobes).
- These lobes are bounded by conical nodal surfaces for which

$$\tfrac{1}{2}kL\sin\theta = \pm n\pi \quad \text{or} \quad |\sin\theta| = \frac{2n\pi}{kL} = \frac{n\lambda}{L} \quad ; \quad n = 1,2,..\left\lfloor \frac{L}{\lambda} \right\rfloor$$

- The larger the value of *kL,* the more narrow the lobes.

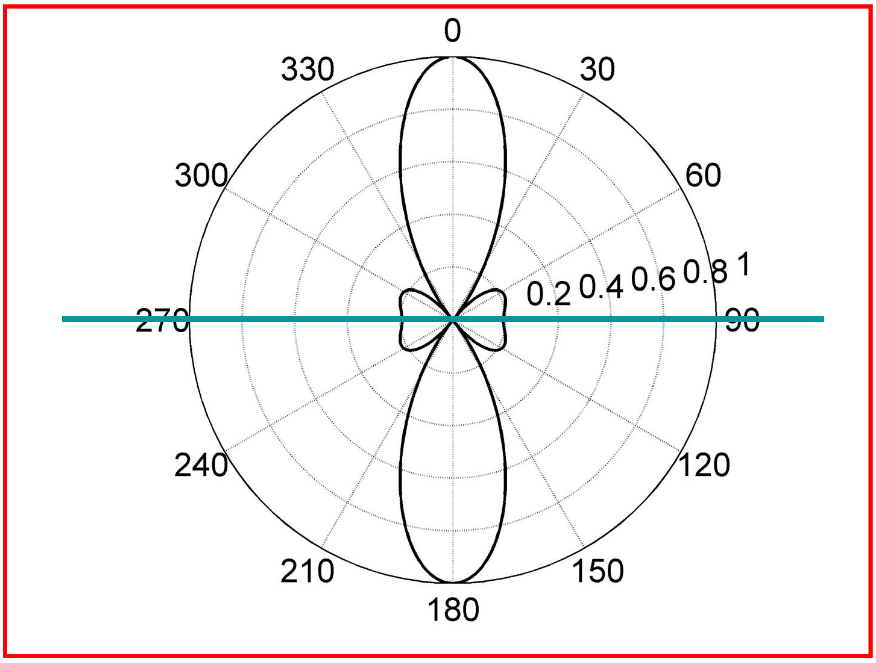
Beam Pattern For A Continuous Line Source

Directional Factor for $kL = 24$



Beam Pattern For A Continuous Line Source

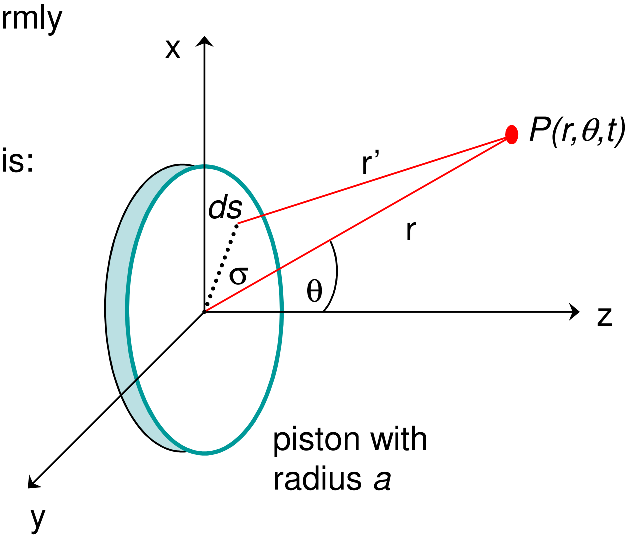
Directional Factor for $kL = 10$



4.2 The baffled piston source

- We follow the same procedure as for the line source, except that we now integrate over the surface of a disk and assume baffled simple sources (see Eq [3.10]).

- Radiating surface moves uniformly with speed $U_0 e^{-i\omega t}$



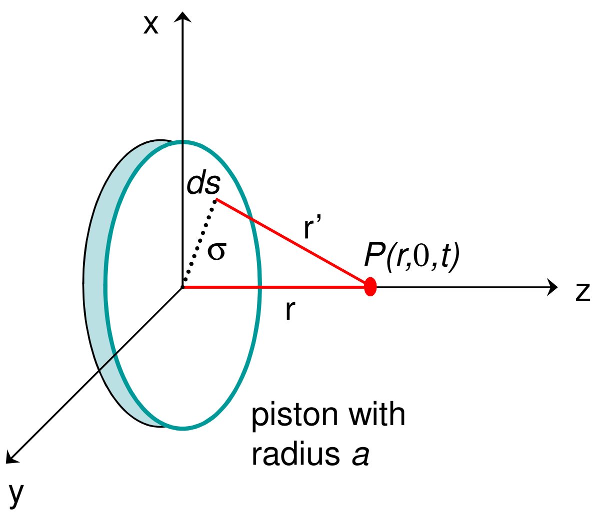
- The source strength therefore is:

$$dQ = U_0 ds$$

- Summing over all the cources:

$$\mathbf{p}(r,\theta,t) = -i\frac{\rho_o c U_o k}{2\pi}\int_S \frac{e^{-i(\omega t - kr')}}{r'} ds$$

- The field along the acoustic axis is given by:



$$\mathbf{p}(r,0,t) = \rho_o c U_o e^{-i\omega t}\left[e^{ikr} - e^{\left(ik\sqrt{r^2+a^2}\right)}\right]$$

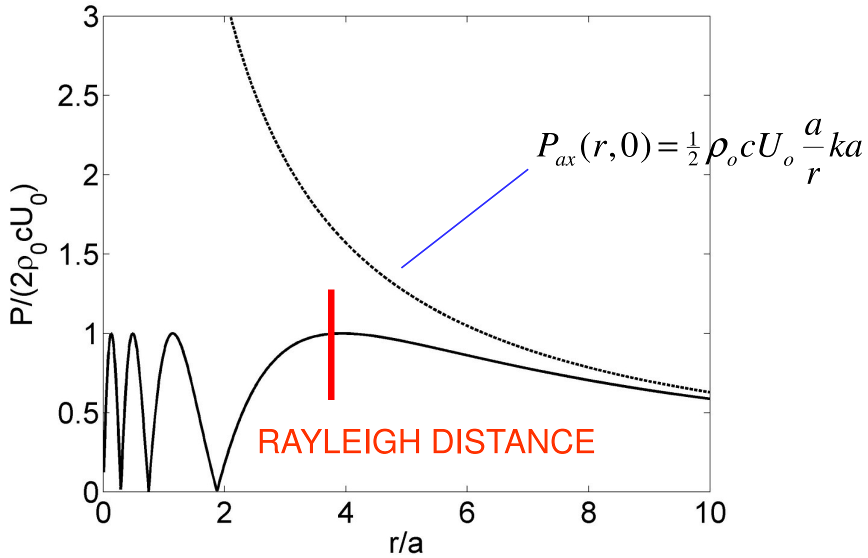
The pressure amplitude follows...

$$P_{ax}(r) = 2\rho_o c U_o \left|\sin\left(\tfrac{1}{2}kr\left[\sqrt{1+\frac{a^2}{r^2}}-1\right]\right)\right|$$

For $r/a >> 1$ and $r/a >> ka$

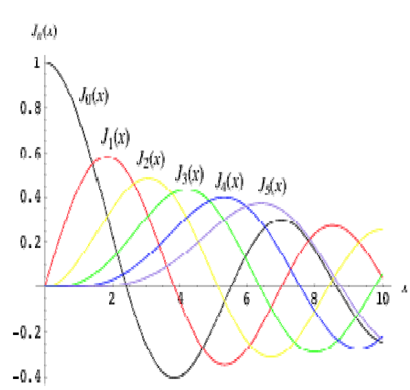
$$P_{ax}(r) = \tfrac{1}{2}\rho_o c U_o \frac{a}{r} ka \quad \text{(a simple source!)}$$

Axial Response of a Baffled Piston Source



$$\tfrac{1}{2}kr\left[\sqrt{1+\frac{a^2}{r^2}}-1\right]=m\frac{\pi}{2} \quad\Rightarrow\quad \frac{r_m}{a}=\frac{a}{\lambda}-\frac{m}{4}\frac{\lambda}{a} \quad\Rightarrow\quad \boxed{r_1=\frac{ka^2}{2\pi}-\frac{1}{4}\lambda\approx\frac{1}{2\pi}ka^2}$$

- The pressure in the far field can be written as the product of the axial pressure response and the Directional Factor



$$P(r,\theta)=\left|\mathbf{p}(r,\theta,t)\right|=P_{ax}(r)H(\theta)$$

$$\boxed{P_{ax}(r)=\tfrac{1}{2}\rho_o c U_o \frac{a}{r} ka}$$

$$\boxed{H(\theta)=\left|\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right|} \qquad [4.2]$$

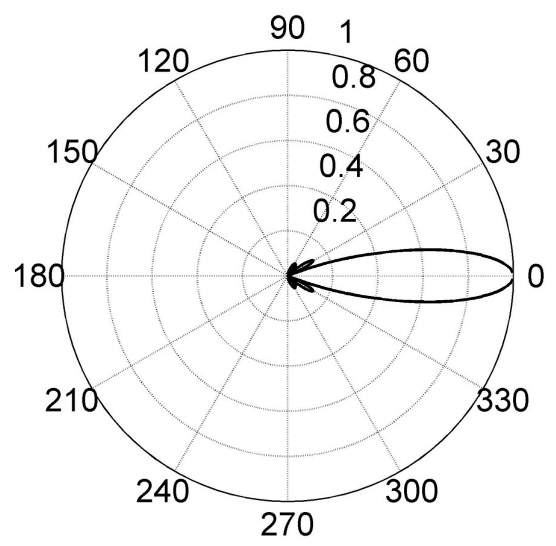
- There is a maximum in the beam pattern for $\theta = 0$ and nodal surfaces for $\theta = \theta_m$

$$ka\sin\theta_m = j_{1m}$$

where j_{1m} is the argument corresponding to the m^{th} zero of J_1. Note that the larger the value of ka, the more "spread out" the beam pattern is.

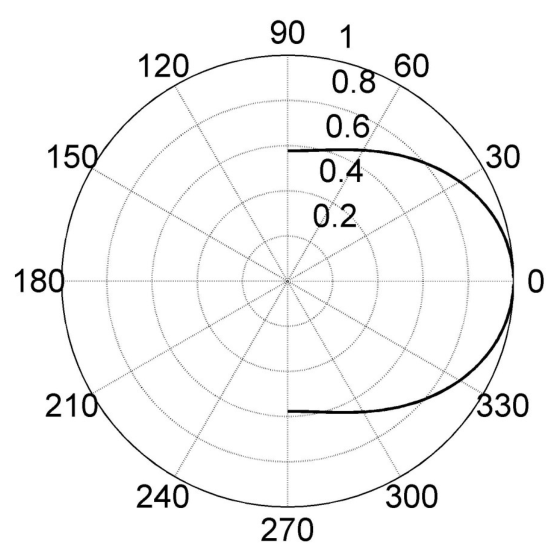
Beam Pattern of a Baffled Piston Source

Directional Factor for *ka* = 10



Beam Pattern of a Baffled Piston Source

Directional Factor for *ka* = 2

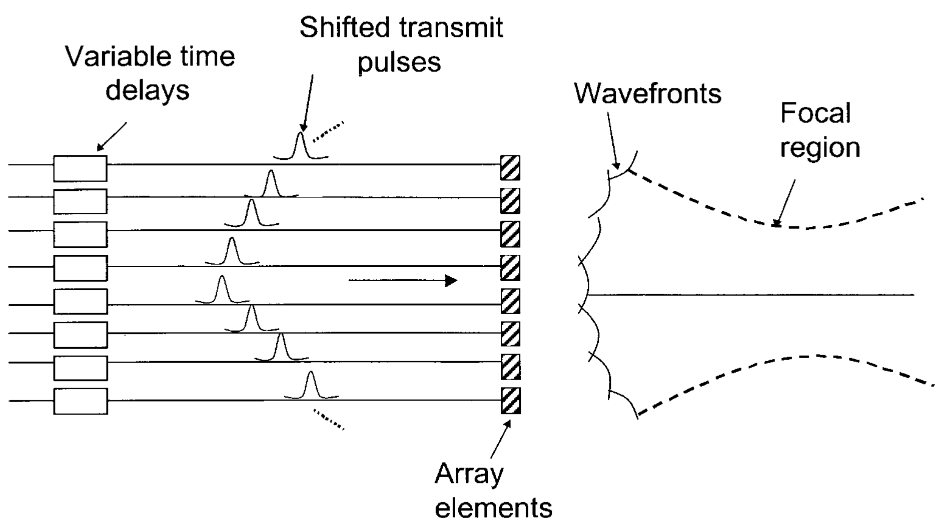


4.3 The focussed piston source

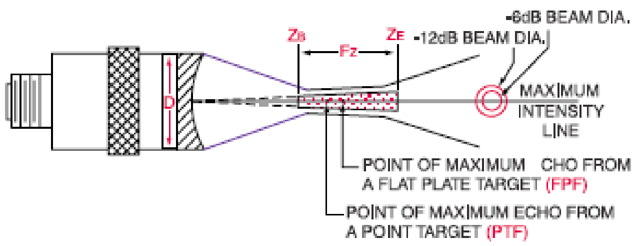
➢ Arrays can be focused by adding time delays that simulate the curved wavefront (The same thinking applies to a single radiating surface!)

r **= distance from origin to focal point**
x_n **= distance from origin to center of nth element (*np*)**
t_o **= constant delay added to avoid negative delays**

$$\tau_n = \frac{\left[r - \sqrt{(x_r - x_n)^2 + z_r^2}\right]}{c} + t_o$$



Diagnostic Focused Piston Transducers
Panametrics Corp.



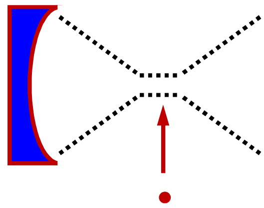
Z_B = Beginning of the Focal Zone
F_Z = Focal Zone
Z_E = End of the Focal Zone
D = Element Diameter
F_Z = Focal Zone
N = Near Field
S_F = Normalized Focal Length

$$BD_{-6dB} = 1.02\frac{Fc}{fD}$$

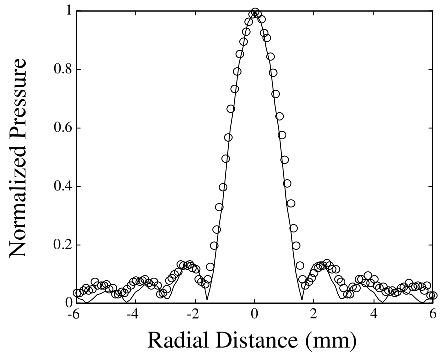
$$F_z = NS_F^2\left[\frac{2}{1+0.5S_F}\right] \quad ; \quad S_F = \frac{F}{N}$$

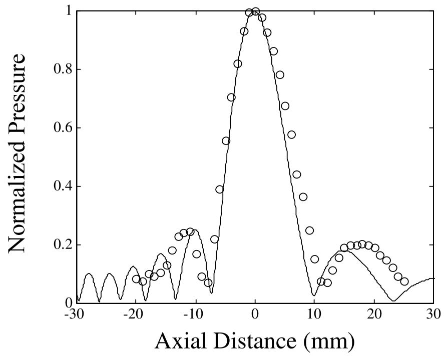
BD = Beam Diameter
F = Focal Length
c = Material Sound Velocity
f = Frequency
D = Element Diameter
S_F = Normalized Focal Length

Measured Beam Patterns for a Therapy Transducer



Sonic Concepts Model H101
Focal length: 62.64 mm
Aperture: 70 mm
Driven at 1 MHz in water





CASE STUDY:
Use of Speed of Sound, Acoustic Impedance and Attenuation to Measure Blood Haematocrit and Haemolysis in Artificial Circuits

Biomedical Ultrasonics and Biotherapy Laboratory
Institute of Biomedical Engineering
University of Oxford

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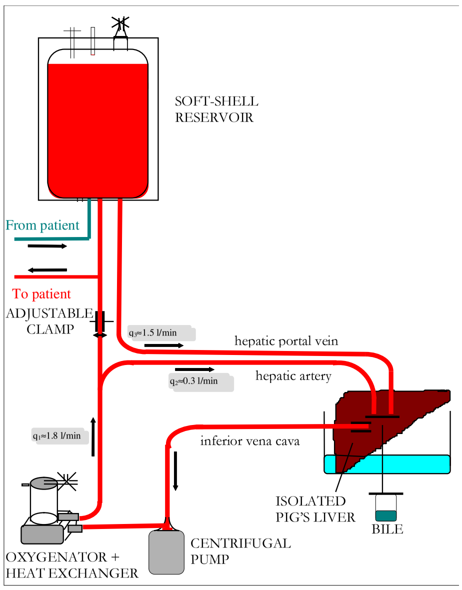
Motivation: monitoring of haemolysis



No effective method for measuring haemolysis **on-line** and **in real time.**

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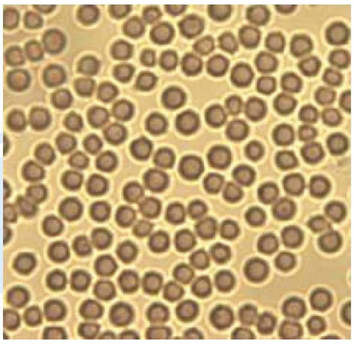
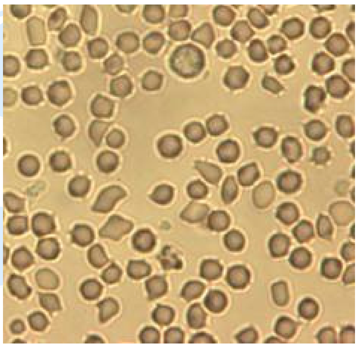
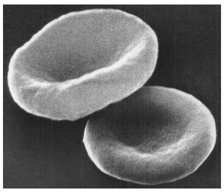
How does haemolysis ‘change’ Blood?

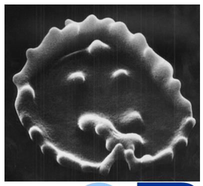




- Mechanical haemolysis causes:
  – A decresase in the concentration of healthy cells.
  – The appearance of irregularly shaped, damaged (ghost) cells.
  – The dissipation of free hemoglobin (fHb) in the plasma.
- In closed circuit of blood volume V, with initial number of cells N

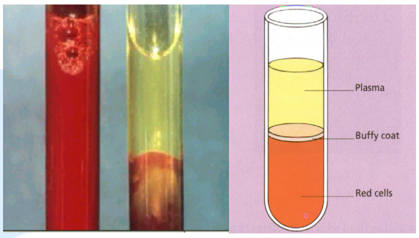
$$N=N_h + N_l$$





Current Methods for Measuring Hemolysis

- Measurement of Hematocrit.

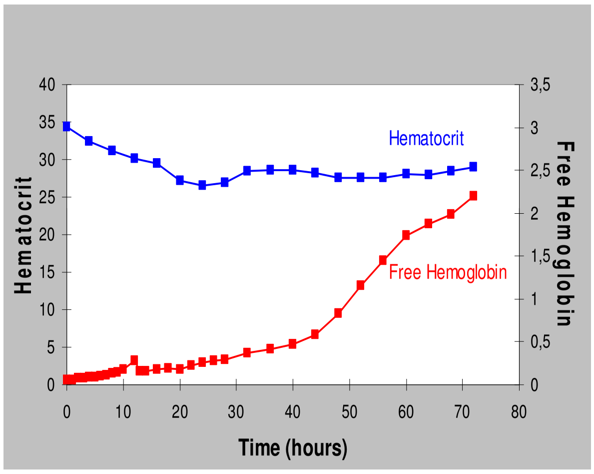


- Measurement of fHb in plasma by spectrophotometry.

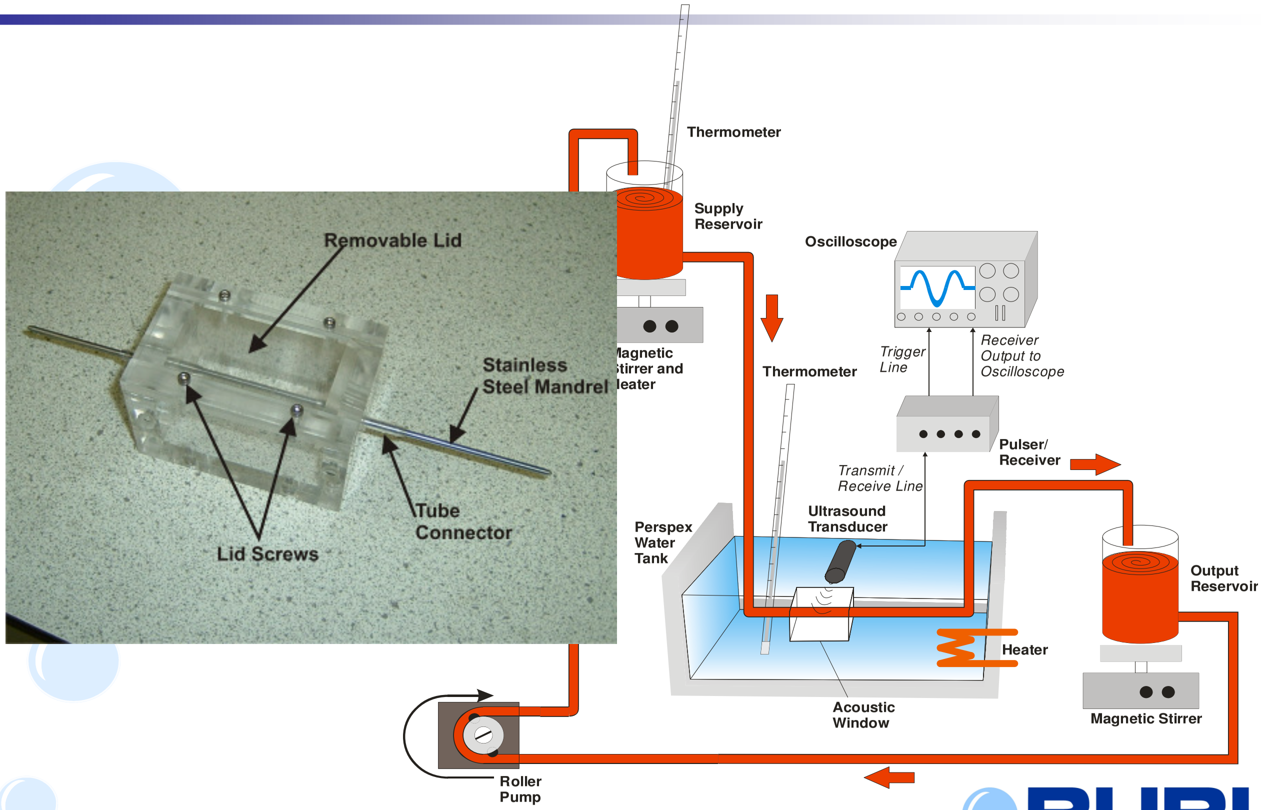
⇒ Both methods require sampling(unsuitable for on-line systems).

⇒ fHb measurements inaccurate.

- In Extracorporeal Liver Perfusion circuit:



Experimental Apparatus



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Mechanically haemolyzed samples

Sample	Haemolysis from Free Haemaglobin Data			Free Haemoglobin Data Predicted Haemolysis	Standard Deviation (%)	Average Haematocrit	Haematocrit Predicted Haemolysis
	Sample 1	Sample 2	Sample 3				
Prime (0)	0	0	0	0.00%	0	35%	0.00%
1	5.26	4.99	5.34	5.20%	0.18	33.67%	3.80%
2	10.70	10.60	11.03	10.78%	0.23	31.33%	10.49%
3	15.71	16.20	16.28	16.06%	0.31	30%	14.29%
4	31.18	30.72	31.42	31.10%	0.36	26.17%	25.23%
5	45.93	47.47	48.84	47.41%	1.45	23%	34.29%
6	56.99	50.88	52.50	53.46%	3.17	20%	42.86%
7	81.77	83.13	80.97	81.96%	1.09	17%	51.43%
8	83.91	86.43	83.36	84.57%	1.64	12.75%	63.57%
9	83.59	84.87	81.35	83.27%	1.78	8.50%	75.71%
10	95.96	97.59	95.79	96.45%	0.99	4%	88.57%
11	99.76	98.70	93.50	97.32%	3.35	1%	97.14%

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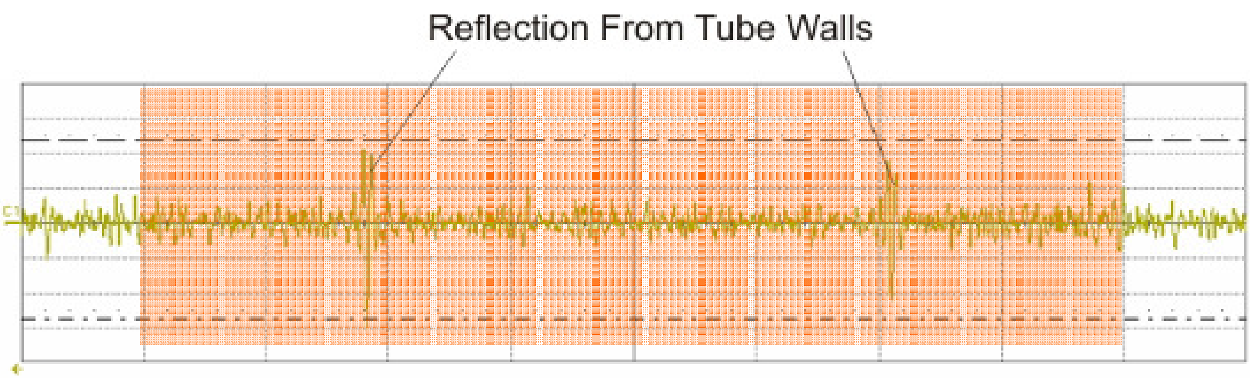
Measuring attenuation, speed of sound and acoustic impedance



$$Z_{bl} = \frac{1 - R_{ag-bl}}{1 + R_{ag-bl}} \qquad c = \frac{2d}{(\Delta t_{farwall} - \Delta t_{nearwall})}$$

$$\alpha = \frac{1}{2L} 20\log\left(\frac{1}{\beta}\frac{v_{ptp}(near)}{v_{ptp}(far)}\right) \qquad where \quad \beta = \frac{v_{ptp}(nearsaline)}{v_{ptp}(farsaline)}$$

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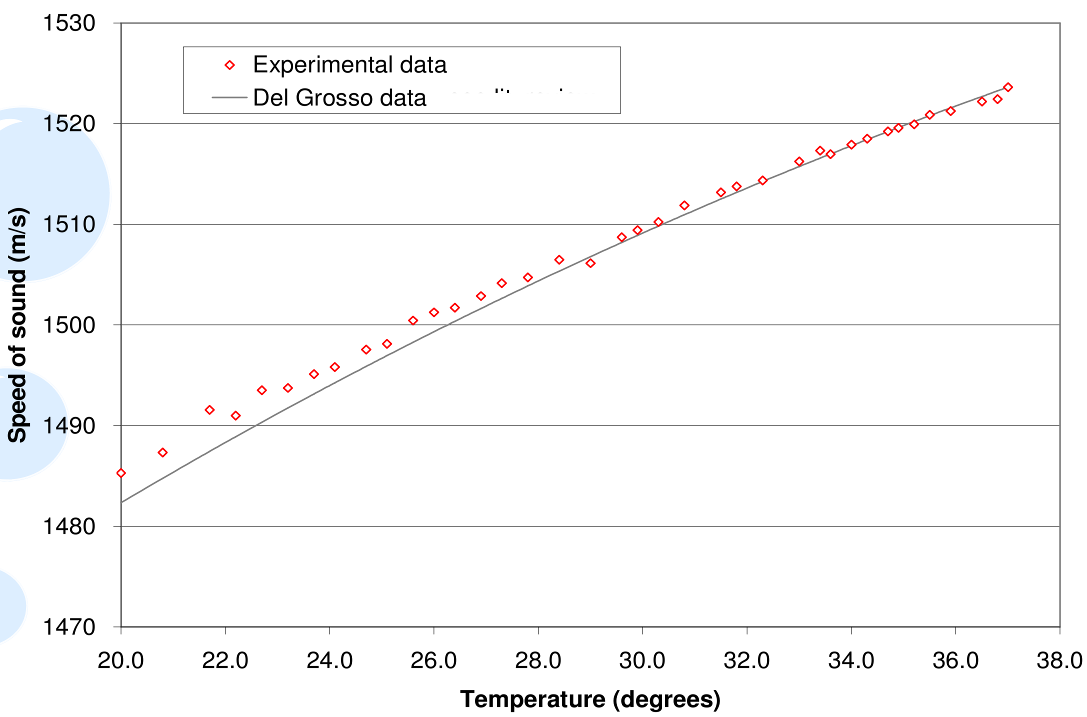
Validation: speed of sound in pure water



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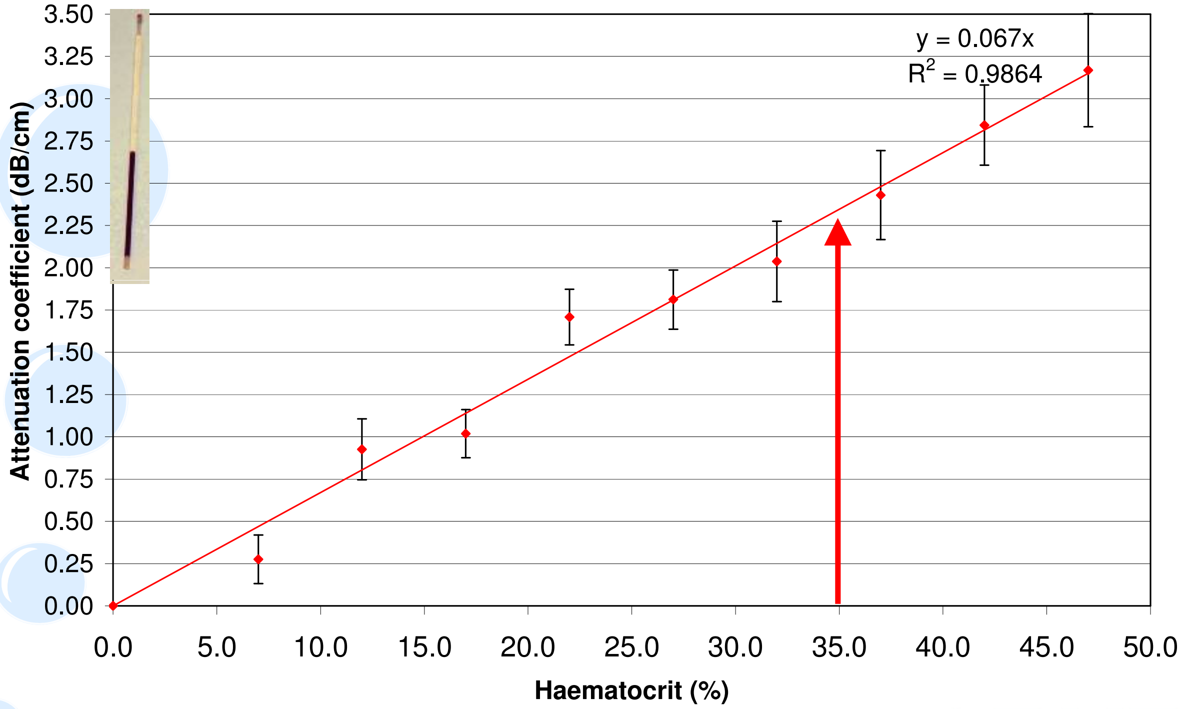
Attenuation through healthy blood at 15 MHz as a function of haematocrit



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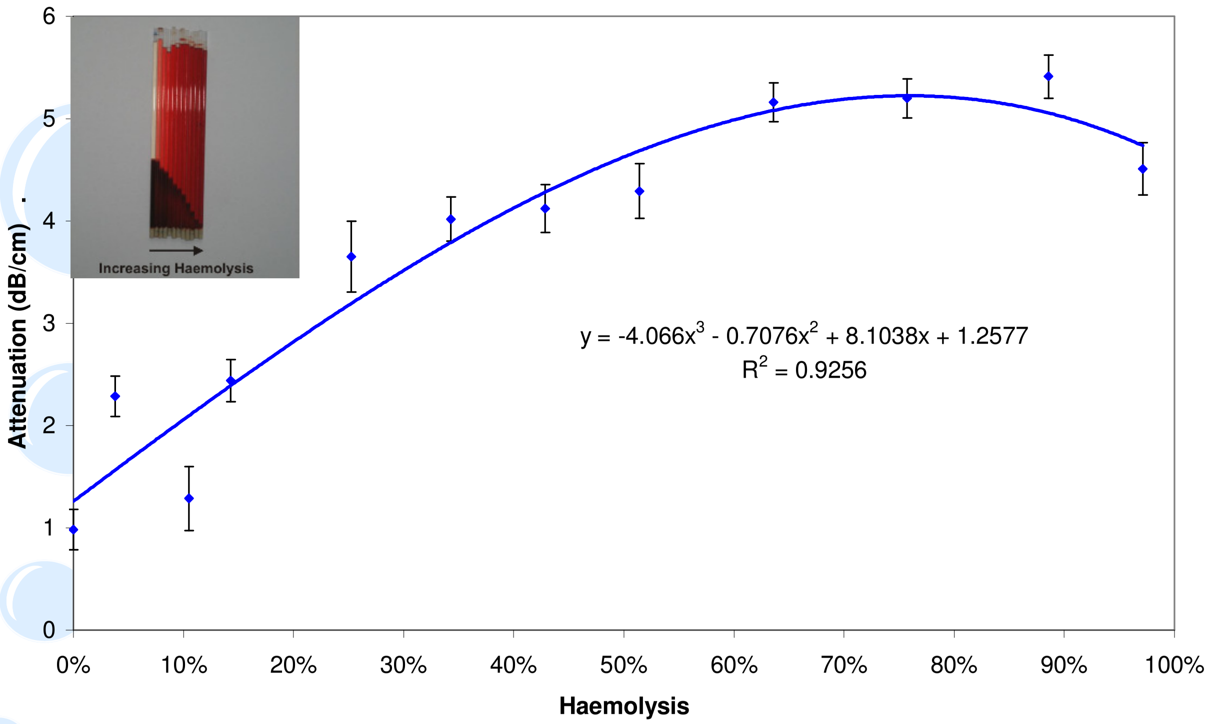
Attenuation through increasingly haemolyzed blood at 15 MHz (initial Hct: 35%)



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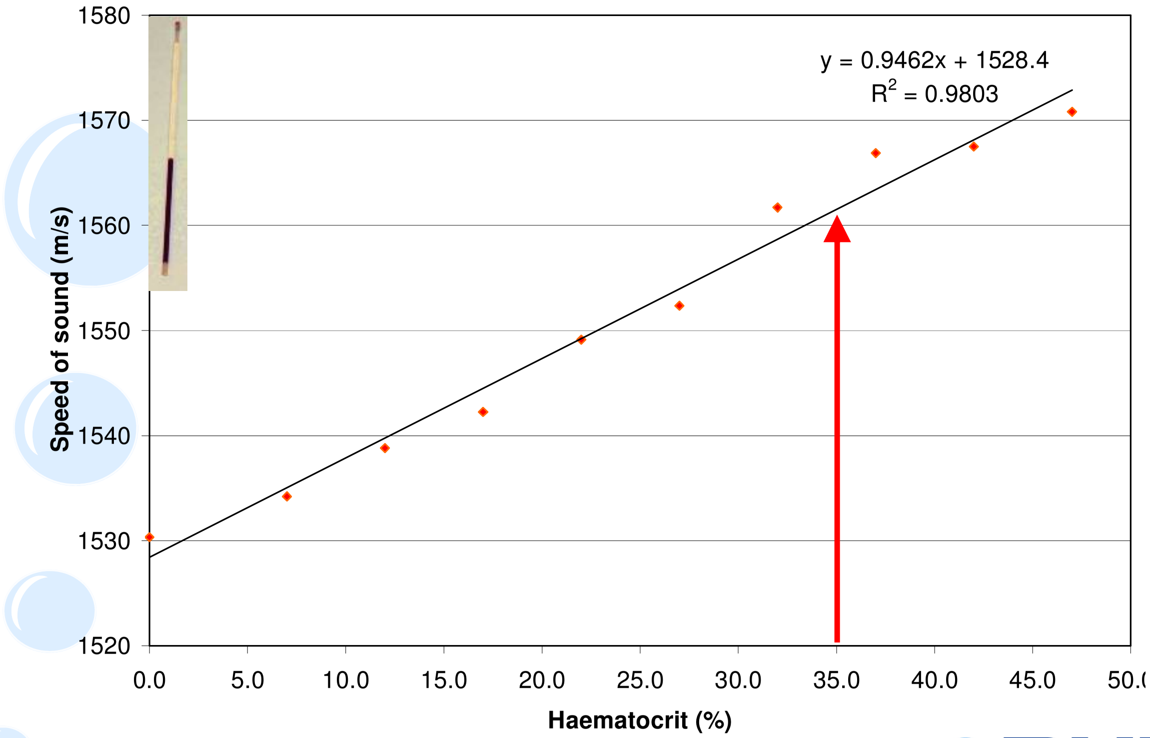
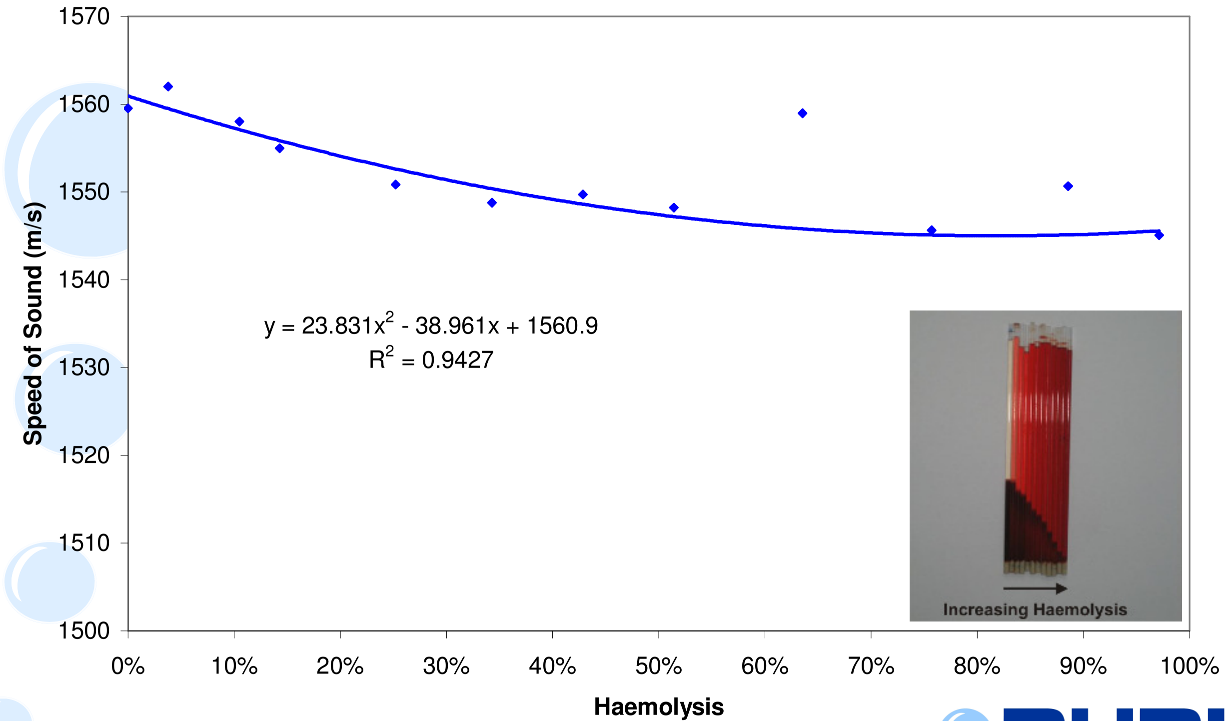
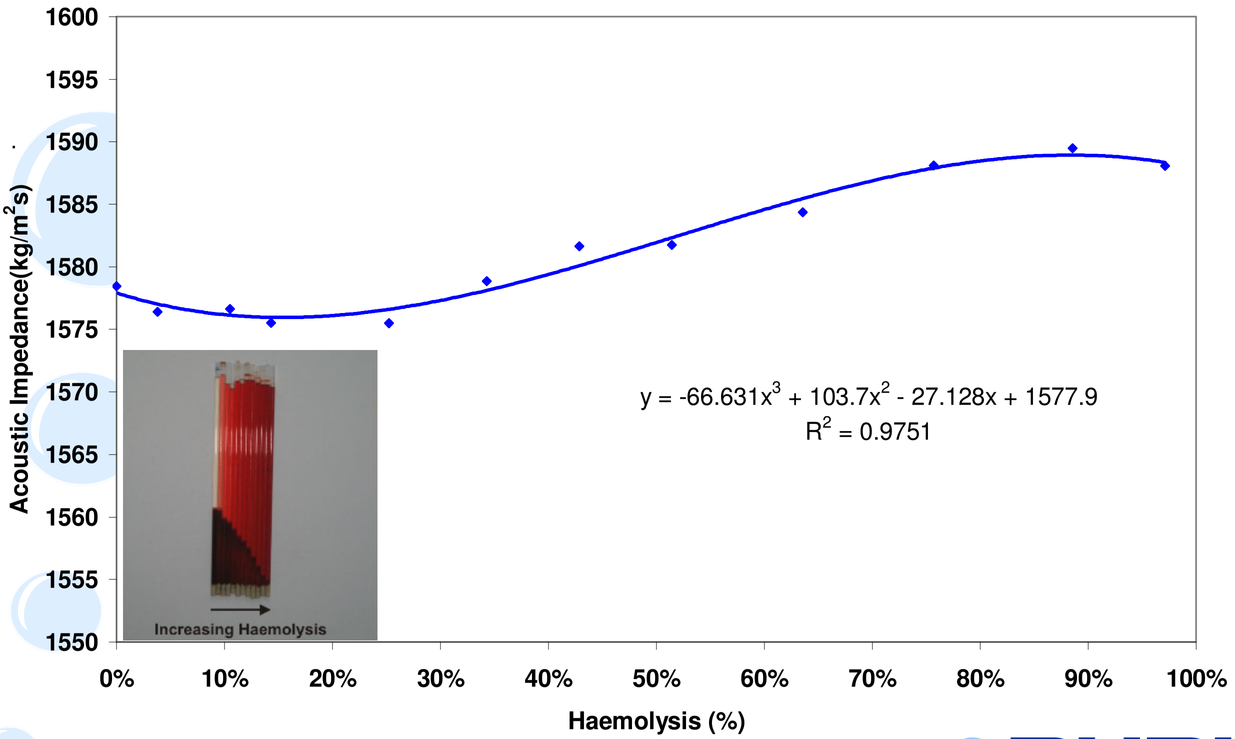
Speed of sound through healthy blood at 15 MHz as a function of haematocrit



Speed of sound through increasingly haemolyzed blood at 15 MHz (initial Hct: 35%)



Acoustic impedance of increasingly haemolyzed blood at 15 MHz



Conclusions & Future work

- Speed of sound, acoustic impedance and attenuation are equally sensitive to increasing red cell damage in the range 15% - 75% haemolysis, but acoustic impedance exhibits significant variation all the way up to 90% haemolysis and speed of sound varies most significantly at low haemolysis levels.
- Can combinations of parameters be used to solve inverse problem?

- *Acknowledgments: Adam White, Andrew Connelly, Andrew Talbot, Jim Fisk.*