C3A-Optoelectronic Systems

Frank Payne MT 2007

Organisation

4 lectures Michaelmas Term, 8 lectures Hilary Term

3 Tutorials sheets

3 Exam questions

Related courses

Other C3 papers

AIM

This course covers some of the principles behind the design of fibre optic systems, components and networks. We start with the basic theory of the optical fibre, and then study some of the components needed in a fibre optic network. We then go on to study several examples of fibre optic systems and their design. Finally, we look at some of the principles of integrated optics and optical waveguide components.

Reading List

- Optoelectronics, An Introduction. J.Wilson, J.F.B.Hawkes, Prentice Hall.
- Optical communication systems. John Gowar.Prentice Hall,
- Optical fibre communications systems. Senior, Prentice Hall, 2nd Ed. (The most comprehensive on fibre systems)
- Optics. Eugene Hecht,Addison Wesley (3rd Ed?).
- Fiber-Optic Communication Systems. Govind P. Agrawal. Wiley (2nd Ed)

Related reading

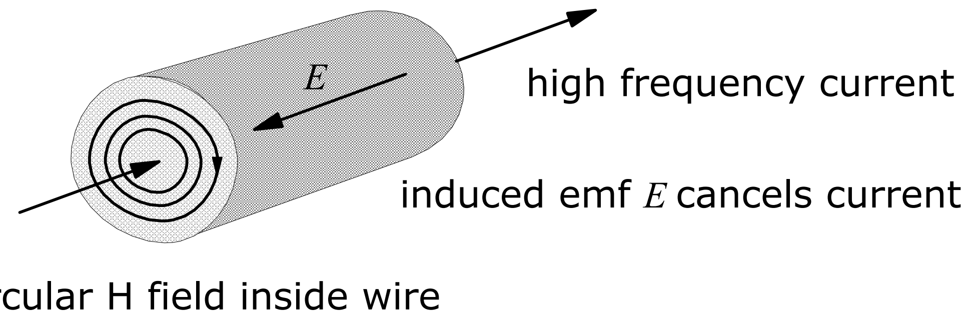
- Introduction to Fourier optics. J.W. Goodman. McGraw-Hill.
- Selected papers on optical computing. H. John Caulfield, Gregory Gheen, Proc.SPIE v. 1142 (RSL Eng. Per. 51 (v. 1142) (For enthusiasts)
- Applied Optics (RSL Phys. Per. 8g). (Contains Optoelectronic systems papers)(For enthusiasts)

Contents

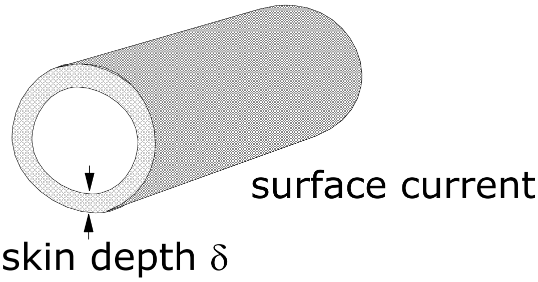
The Optical Fibre Communication Channel

The limitations of electrical cables

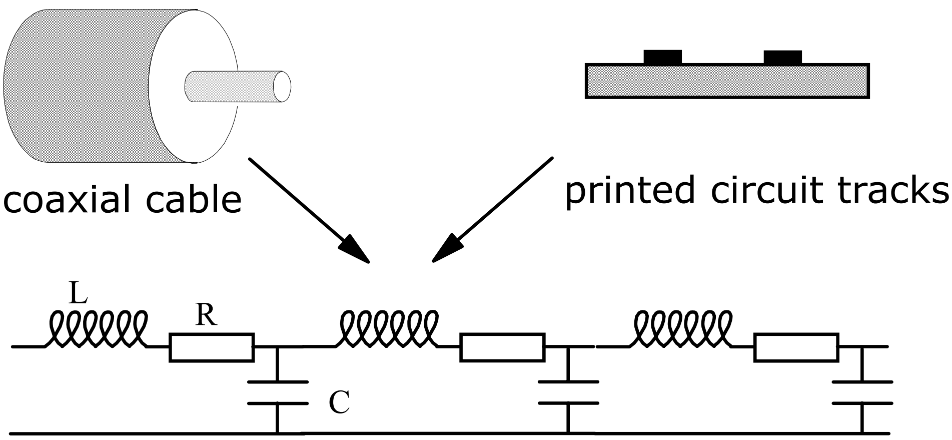
At high frequency f the skin effect increases the resistance of conductors with frequency



$$\boxed{\delta = \frac{1}{\sqrt{\pi\mu_0\sigma f}}}$$



We can model a cable as a distributed circuit



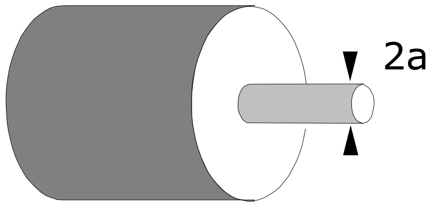
L, R, and C are per until length

$$\text{Bandwidth} \sim \frac{1}{RC(length)^2}$$

$$\text{Attenuation} \sim \frac{R}{2}\sqrt{\frac{C}{L}}$$

The bandwidth is approximated by $\frac{1}{RC(length)^2}$ where the resistance R is given by

$$R = \frac{1}{2\pi a\sigma\delta}$$

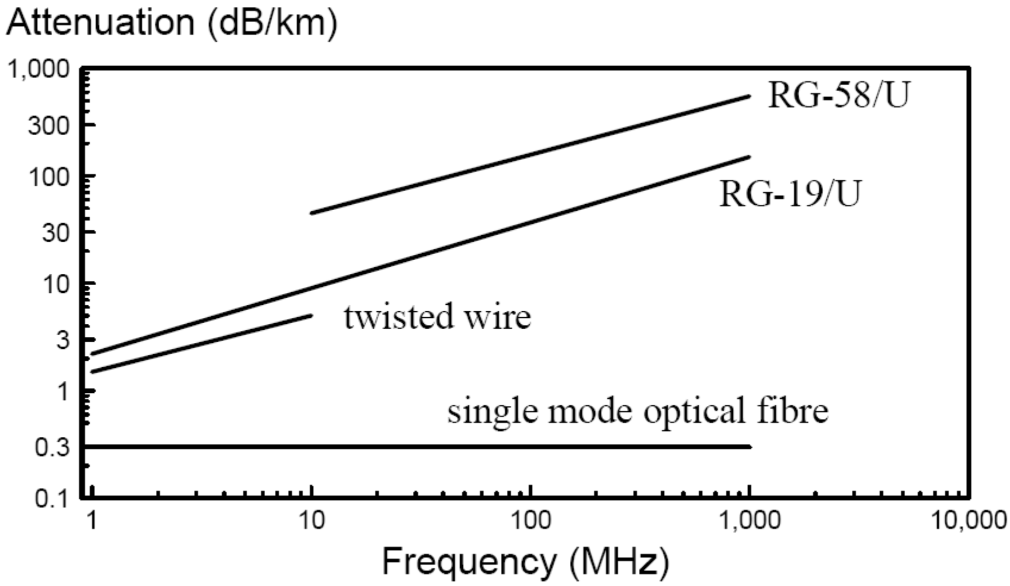


Assume bandwidth ~ f then we find

$$f^{3/2} \approx \frac{2a}{(length)^2 C}\sqrt{\frac{\pi\sigma}{\mu_0}}$$

For a cable with capacitance C = 100pF/m, radius a = 1/2mm and length = 10m, we find maximum transmission frequency f ~ 100MHz.

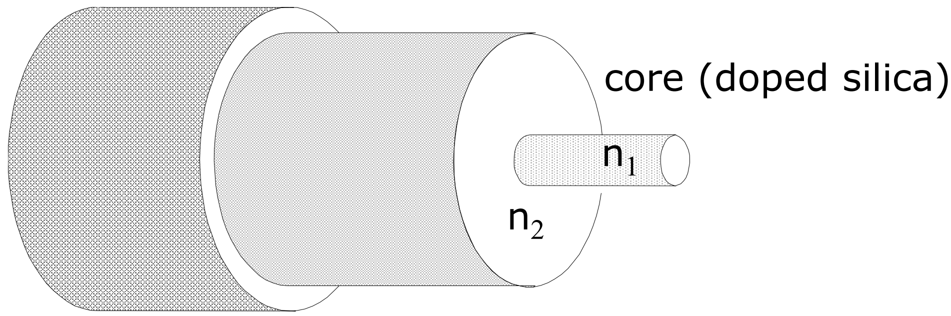
Cable attenuation also increases strongly with frequency:

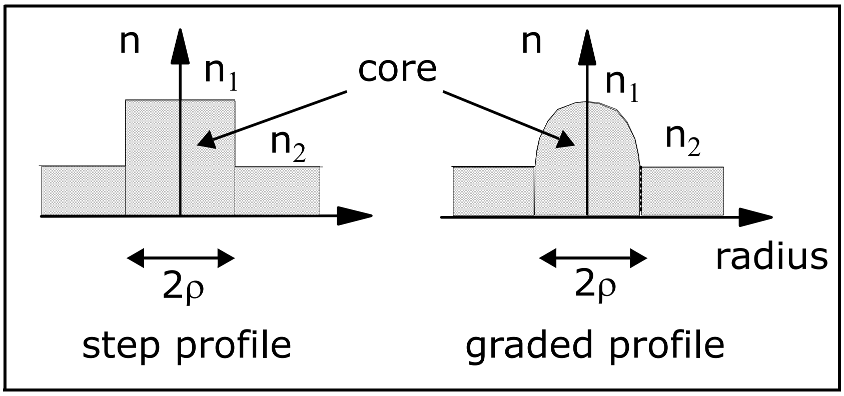


- cable attenuation imposes severe restrictions at high frequencies.
- attenuation of optical fibre is almost independent of data frequency.

Properties of the Optical Fibre Channel

Structure of Optical Fibres





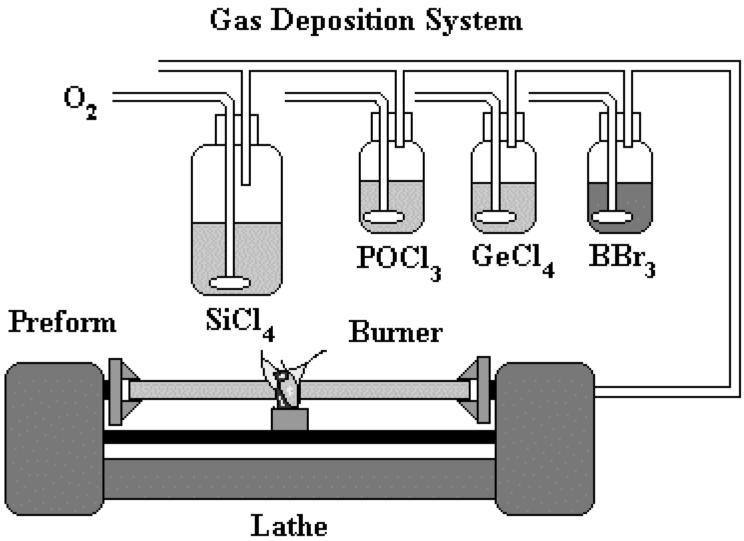
Optical fibres are classified as multimode or single mode, depending on the core radius. Multimode fibres have either a step index or graded index profile for the refractive index variation of the core. Single mode fibres have a step index profile. Multimode fibres are typically used for short distance low bandwidth data comms links, single mode fibres are used for high bandwidth telecommunications networks. Some typical parameters:

Multimode fibre	Single mode fibre
$12.5 < \rho < 100\ \mu m$	$2 < \rho < 5\mu m$
$0.8\mu m < \lambda < 1.6\mu m$	$0.8\mu m < \lambda < 1.6\mu m$
$0.01 < \Delta < 0.03$	$0.003 < \Delta < 0.01$

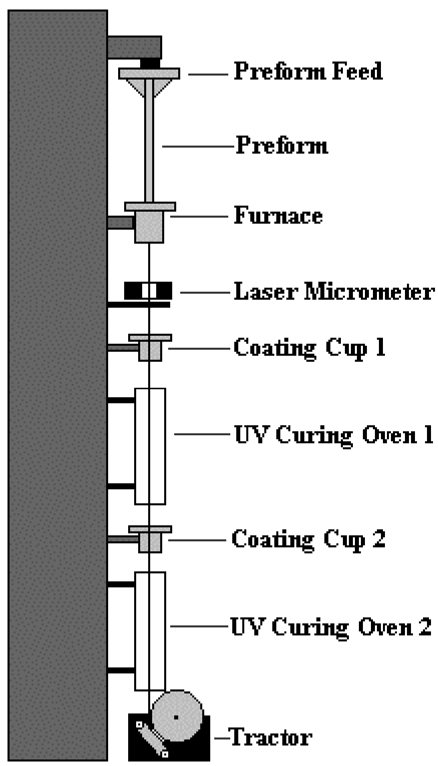
- λ is the free space wavelength
- $\Delta = (n_1 - n_2)/n_2$

Manufacture of Optical Fibre - The MCVD Method

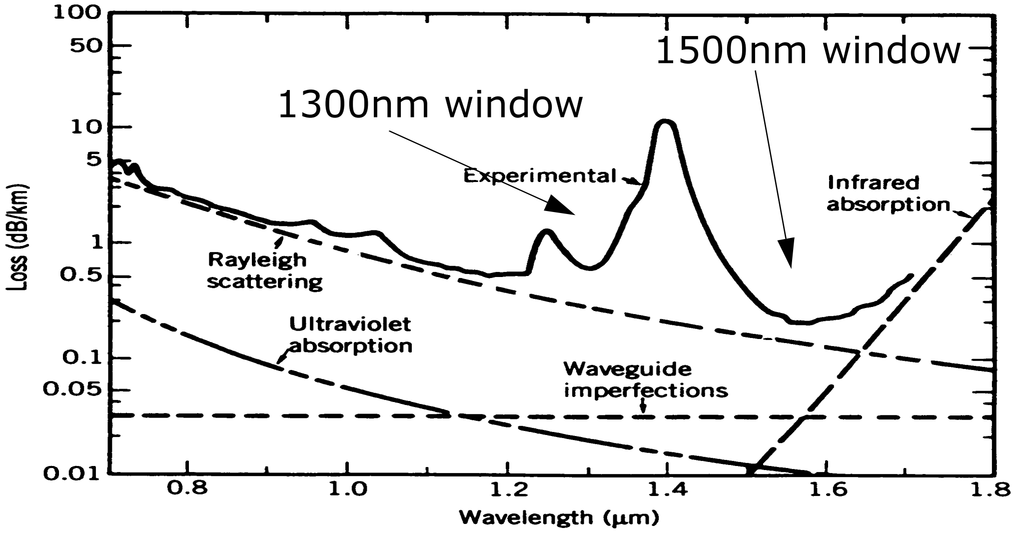
In Modified Chemical Vapour Deposition (MCVD), the vapours of various glass-producing halides are burned in oxygen and the resulting oxides sintered at high temperature. The optical structure (ie. refractive index profile) of the preform is built-up on the inside of a rotating substrate tube of synthetic silica, mounted in a lathe, by controlling the flows and proportions of the different halide vapours.



Typical halides include $SiCl_4$, phosphorus oxychloride $POCl_3$, and germanium tetrachloride $GeCl_4$ Each dopant introduces a different characteristic into the glass (eg. refractive index, expansion coefficient or melt-viscosity). At the end of the fabrication process the small hole remaining down the centre of the tube is closed-up, at an increased temperature, to create a solid preform. The preform is lowered into a graphite resistance element furnace with a hot-zone temperature of c.2200 °C. As the tip of the preform enters the hot-zone, it softens, allowing fibre to be drawn from it by the tractor located at the foot of the tower. The high surface tension of molten silica ensures that the optical structure of the preform is duplicated exactly in the fibre.

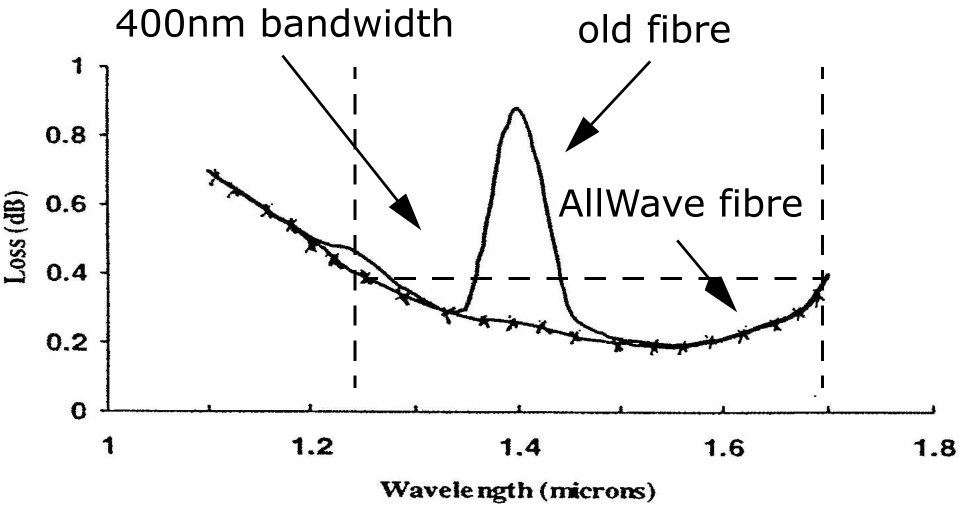


Fibre Attenuation



The solid line above shows attenuation of older (pre 1995) fibre, much of which is still in use. The loss peak at 1400nm is due to OH^- impurity ions in the glass and is known as the water peak. The ultimate limits on loss are set by Rayleigh scattering, which arises from small refractive index fluctuations in the glass. Rayleigh loss varies as $1/\lambda^4$.

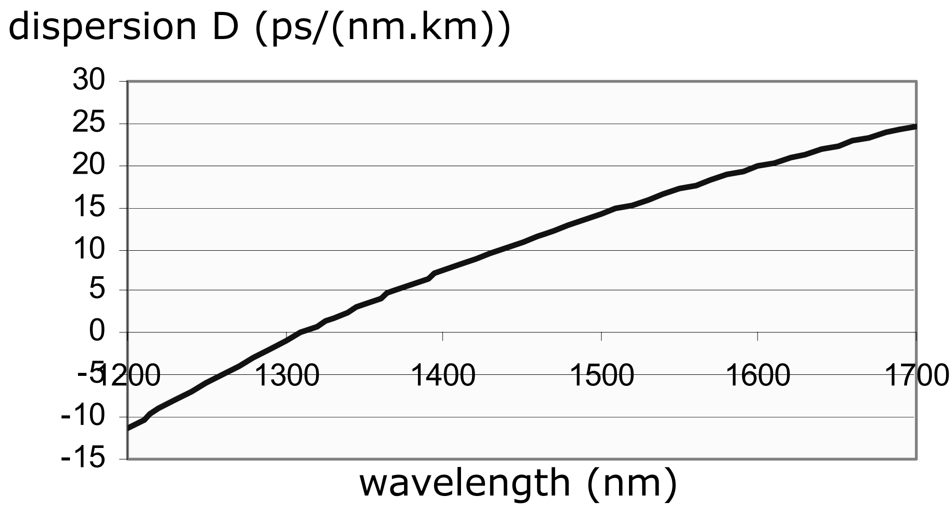
Modern fibre (post 1995) has improved substantially in terms of loss and a typical loss characteristic is shown below. This is the OFS (ex Lucent) Allwave fibre but Corning fibre (SMF-28) has similar loss characteristics. The bandwidth for loss < 0.4dB is around 400nm (about 50THz!)



improved fibre design results in much wider transmission window:	<0.4dB/km	400nm
	<0.3dB/km	300nm
	<0.2dB/km	200nm

Fibre dispersion

We will meet this in detail later. Fibre dispersion refers to the fact that the different wavelengths in an optical pulse travel at a slightly different speeds along the fibre causing the pulse width to increase. The graph below shows dispersion for a standard single mode fibre.



- Note that near 1300nm dispersion is zero
- Units of dispersion D are ps/(nm.km)

Increase in pulse width over distance L is given by:

$$\Delta t = D \cdot L \cdot \Delta\lambda$$

where $\Delta\lambda$ is source spectral width.

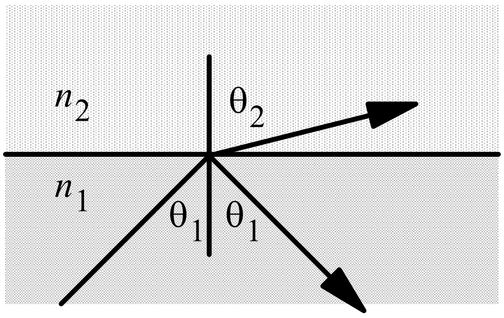
An accurate approximation for D is given by

$$D(\lambda) = \frac{S_0}{4} \cdot \left[\lambda - \frac{\lambda_0^4}{\lambda^3}\right] \text{ps/(nm.km)}$$

- S_0 is the zero dispersion slope, λ_0 is the zero dispersion wavelength, and λ is the wavelength (both in nm).
- For standard fibre λ_0 = 1310nm, S_0 = 0.09 ps/nm^2 km
- At 1550nm D = 17 ps/(nm.km)

Numerical aperture

Numerical aperture is a measure of the light gathering capacity of an optical fibre. It is derived using Snell's law of refraction:

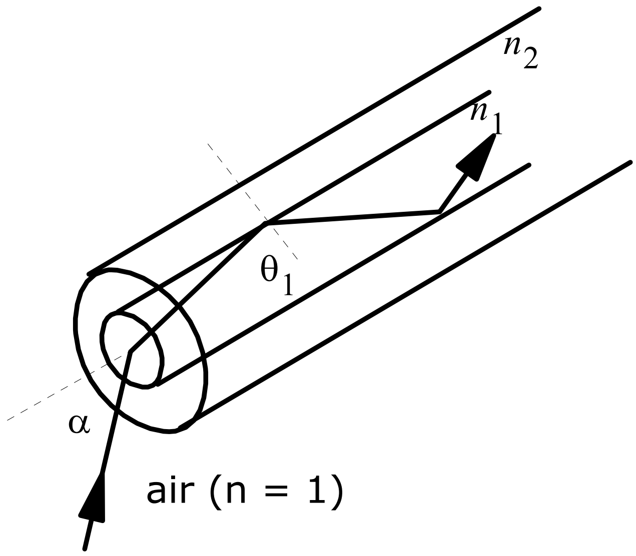


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

For total internal reflection $\theta_2 = \pi/2$:

$$\sin\theta_1 = \frac{n_2}{n_1}$$

Now consider light incident on the end face of a fibre at maximum angle of incidence α:



Applying Snell's Law twice:

$$1 \times \sin\alpha = n_1 \sin(\pi/2 - \theta_1) = n_1 \cos\theta_1$$

$$\sin\theta_1 = \frac{n_2}{n_1}$$

Eliminating θ_1 we find maximum possible angle of incidence for light to be captured by the fibre is:

$$\sin\alpha = \sqrt{n_1^2 - n_2^2}$$

The numerical aperture (NA) is defined as:

$$\boxed{\mathrm{NA} = \sqrt{n_1^2 - n_2^2}}$$

Worked Example

Calculate the numerical aperture for the following fibres, and the maximum angle of incidence at which light is captured into the fibre, (i) an unclad fibre with n_1=1.5, n_2=1.0; (ii) a fibre with n_1=1.45, n_2=1.445;

(i)

$$NA = \sqrt{1.5^2 - 1}$$
$$= 1.12$$

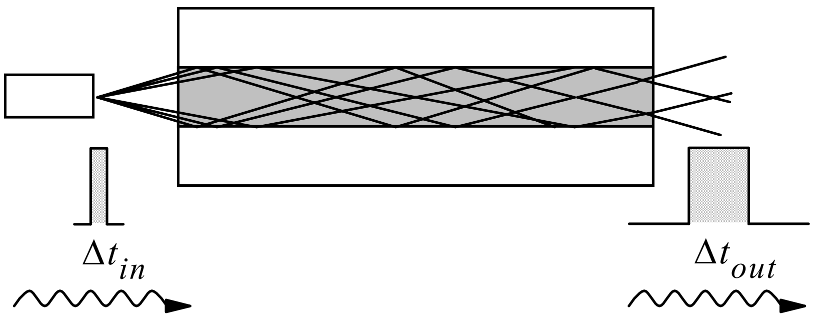
maximum angle of incidence = $90°$, since NA>1.

(ii)

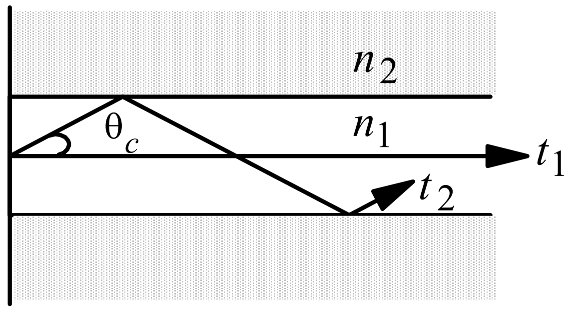
$$NA = \sqrt{1.45^2 - 1.445^2}$$
$$= 0.12$$

maximum angle of incidence = $\sin^{-1}(NA) = 6.9°$

Pulse broadening in a step index multimode fibre



Input pulse is distributed over many different propagation angles along fibre, causing pulse spreading. This is referred to as modal or multipath dispersion.



speed of light in medium n_1 is $v = c/n_1$

We consider two extreme rays propagating along the fibre. One along the axis of the fibre with travel time t_1 , and the other at the critical angle θ_c for total internal reflection and taking time t_2 .

Consider a length L of fibre

$$t_1 = \frac{L}{v} = \frac{Ln_1}{c}$$

$$t_2 = \frac{L}{v\cos\theta_c} = \frac{Ln_1}{c\cos\theta_c}$$

From Snell's Law:

$$\cos\theta_c = \frac{n_2}{n_1}$$

Spread of propagation times is

$$\Delta t = t_2 - t_1 = \frac{L}{c} \cdot \frac{n_1}{n_2} \cdot \frac{n_1^2 - n_2^2}{n_1 + n_2}$$

Define bandwidth B as

$$B = \frac{1}{2\Delta t}$$

Then we find that the bandwidth-length product is

$$\boxed{BL = c \cdot \frac{n_2}{2n_1} \cdot \frac{n_1 + n_2}{n_1^2 - n_2^2}}$$

For telecommunication fibre $n_1 \approx n_2 \approx n$, and recalling definition of numerical aperture $NA = \sqrt{n_1^2 - n_2^2}$, we can rewrite BL as

$$\boxed{BL \approx \frac{nc}{(NA)^2}}$$

Worked Example

Estimate the bandwidth-length products for the following fibres: (i) n_1 = 1.5, n_2 = 1.0; (ii) n_1=1.45, n_2=1.445.

(i)

$$BL = c \cdot \frac{n_2}{2n_1} \cdot \frac{n_1 + n_2}{n_1^2 - n_2^2} = 3 \times 10^8 \times \frac{1}{2 \times 1.5} \times \frac{1.5 + 1}{1.5^2 - 1}$$

$$= 200 Kbs^{-1} km$$

This is an extremely low data rate.

(ii)

$$BL \approx \frac{nc}{(NA)^2} = \frac{1.45 \times 3 \times 10^8}{(0.12)^2}$$

$$= 30 Mbs^{-1} km$$

The smaller index difference improves the bandwidth, but still only suitable for short distance data comms.

RMS pulse spreading

The above analysis gives the spreading of an impulse of light. More useful is the rms pulse spreading which can be shown to be given by:

$$\sigma = \frac{\Delta t}{2\sqrt{3}}$$

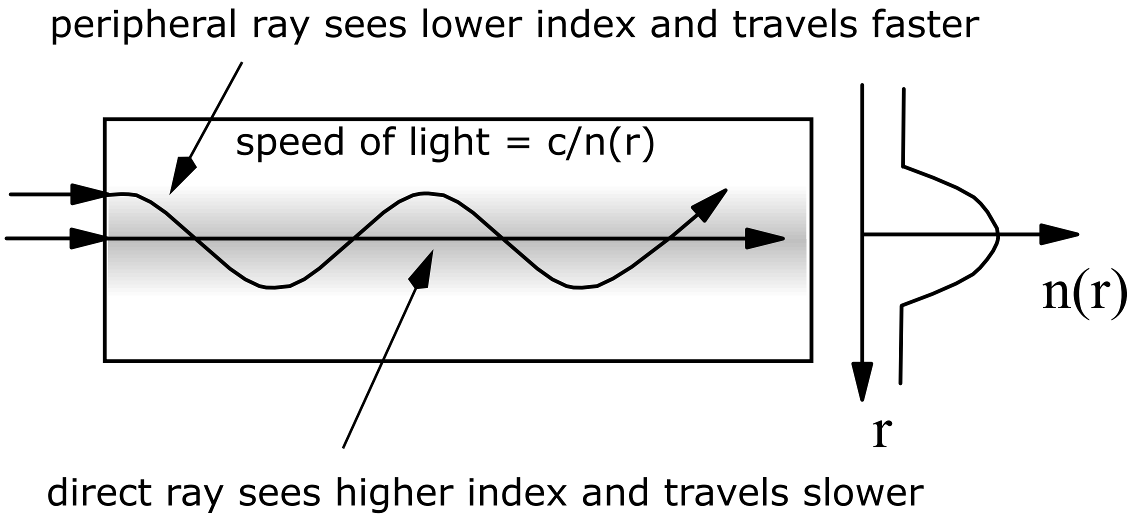
This can be written as

$$\sigma = \frac{L(NA)^2}{4\sqrt{3}cn}$$

Multimode fibres - increasing the bandwidth

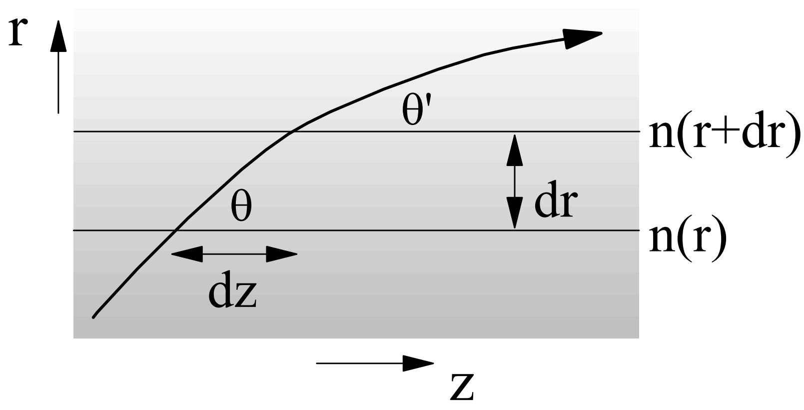
Path equalisation in graded index fibres

By grading the refractive index the peripheral rays, which travel longer distances, go faster than the direct rays along the axis of the fibre , allowing equalisation of the propagation times and reduction in pulse spreading.



Propagation through a graded index medium

We consider a ray of light travelling through a medium with gradually changing refractive index. We can obtain the equation for the ray by applying Snell's law. To do so, we break the refractive index up into a large number of thin layers, within each of which we consider the refractive index constant.



Apply Snell’s law to the boundary between n(r) and n(r+dr):

$$n(r)\cos\theta = n(r)\cos\theta' \equiv \bar{\beta}$$

Note $\bar{\beta}$ is an invariant for all rays.

From the geometry:

$$\left(\frac{dr}{dz}\right)^2 = \tan^2\theta$$

From the first equation we find

$$\tan^2\theta = \sec^2\theta - 1 = \frac{n^2(r) - \bar{\beta}^2}{\bar{\beta}^2}$$

Giving us the following equation

$$\left(\frac{dr}{dz}\right)^2 = \frac{n^2(r) - \bar{\beta}^2}{\bar{\beta}^2}$$

Differentiate both sides with respect to z,

$$2 \cdot \frac{dr}{dz} \cdot \frac{d^2 r}{dz^2} = \frac{1}{\bar{\beta}^2} \cdot \frac{d}{dr}n^2(r) \cdot \frac{dr}{dz}$$

Finally:

$$\boxed{\frac{d^2 r}{dz^2} = \frac{1}{2\bar{\beta}^2} \cdot \frac{d}{dr}n^2(r)}$$

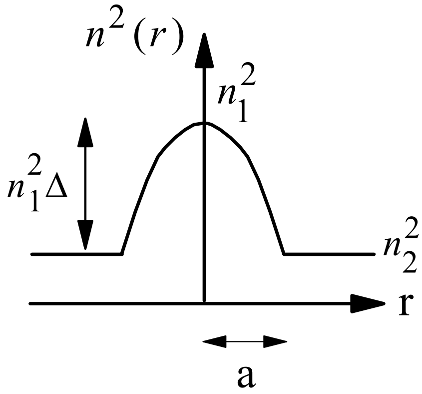
Propagation through a square law medium

We assume n(r) varies quadratically with r:

$$n^2(r) = n_1^2\left(1 - \Delta \cdot \frac{r^2}{a^2}\right) \quad \text{for } r \le a$$

$$n^2(r) = n_2^2 \quad \text{for } r > a$$

where $\Delta = \dfrac{n_1^2 - n_2^2}{n_1^2}$ (see diagram below)



Differentiating n^2 :

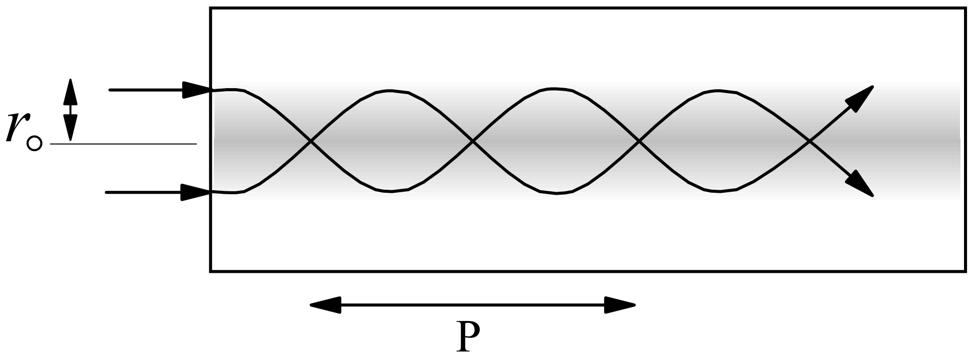
$$\frac{d}{dr}n^2(r) = \frac{-2n_1^2\Delta r}{a^2}$$

The ray equation is then

$$\frac{d^2 r}{dz^2} = -\frac{n_1^2\Delta}{a^2\bar{\beta}^2}\cdot r$$

The solution of this is:

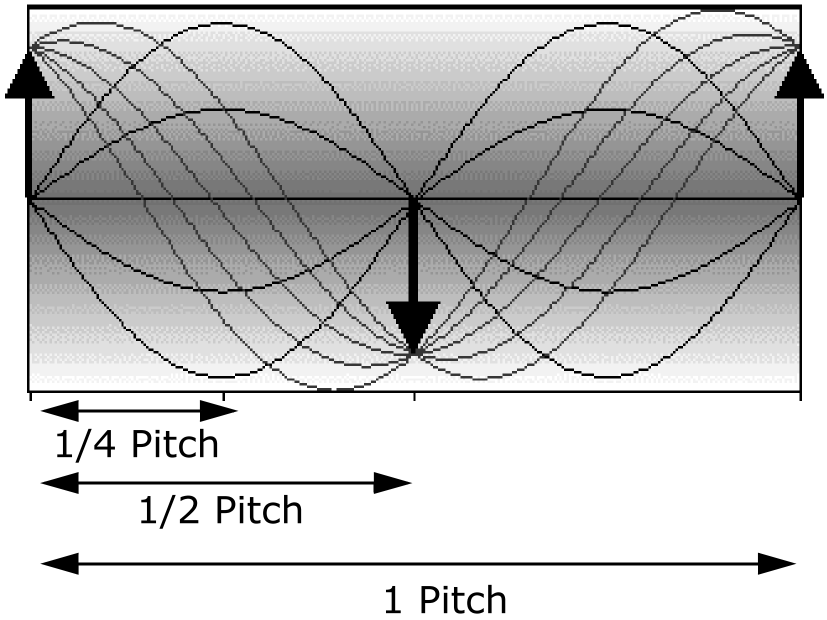
$$r = r_0\cos\left(\frac{n_1\sqrt{\Delta}}{a\bar{\beta}}\cdot z\right)$$



The graded index fibre periodically focusses the light with a pitch P:

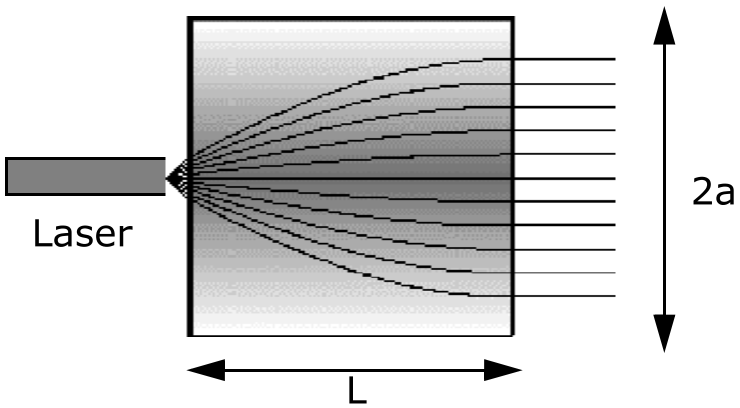
$$P = \frac{2a\bar{\beta}\pi}{n_1\sqrt{\Delta}} \quad \text{where} \quad \bar{\beta} = n(r_0)$$

As well as their use for communications, short lengths of graded index fibres are also used as lenses, known as GRIN (graded index) lenses, or SELFOC lenses. The imaging properties of such a lens are shown below.



Worked Example

The emitting region of a semiconductor laser is very small. One way to obtain a parallel beam is to use a GRIN lens. Calculate the length of lens required if a = 240μm , n_1 = 1.624 , and n_2 = 1.545 (this data corresponds to the lenses made by GRIN TECH).



The length L should be 1/4 the Pitch P above:

$$L = \frac{P}{4} = \frac{a\bar{\beta}\pi}{2n_1\sqrt{\Delta}}$$

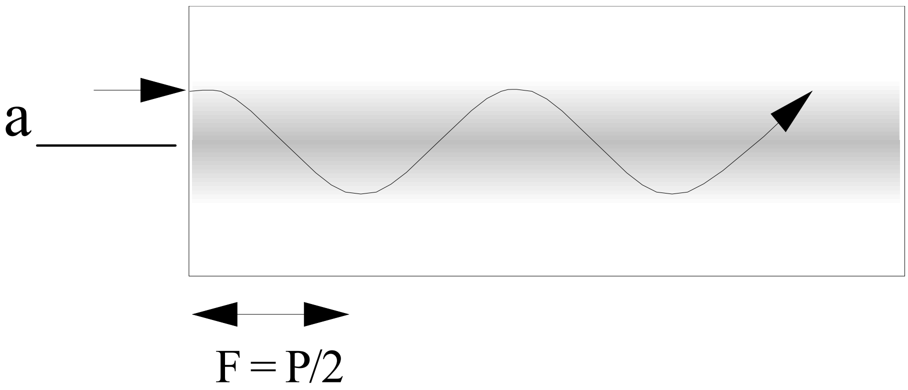
$$\bar{\beta} = n(a) = n_2 = 1.545$$

Inserting the parameters given, $\Delta = 0.0948$:

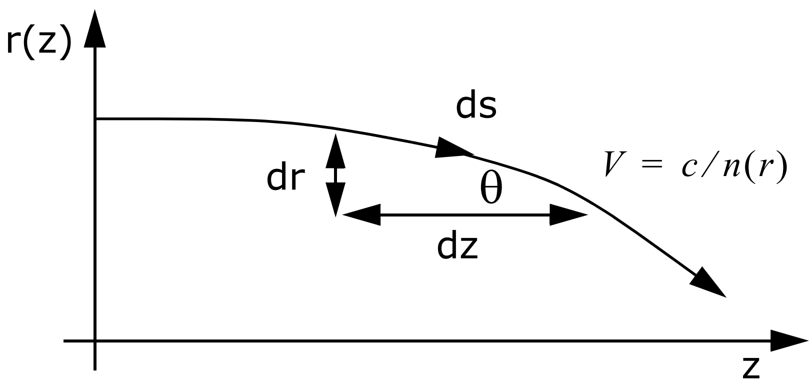
$$L = \frac{239 \times 1.545 \times \pi}{2 \times 1.624 \times \sqrt{0.0949}} = 1.16mm$$

Estimate of pulse spreading in graded index fibres

To estimate pulse spreading we consider the propagation times of an extreme ray launched at r = a, and a ray that travels along the axis of the fibre.



At an arbitrary point r(z) along the trajectory the speed of the ray is $V = c/n(r)$, (see diagram below)



Then the time t_F to travel one period is given by

$$t_F = \int_0^F \frac{1}{V} ds = \frac{1}{c} \cdot \int_0^F n(r) ds$$

$$= \frac{1}{c} \cdot \int_0^F n(r) \frac{ds}{dz} \cdot \mathrm{d}z = \frac{1}{c} \cdot \int_0^F \frac{n(r)}{\cos\theta} dz$$

We saw earlier that along a graded index

$$\bar{\beta} = n(r)\cos\theta = \text{invariant along ray}$$

When $\theta = 0$ we have $\bar{\beta} = n(a) = n_2$ giving:

$$n(r)\cos\theta = n_2$$

Substitute for $\cos\theta$ into integral, giving

$$t_F = \frac{1}{cn_2}\int_0^F n^2(r)dz$$

We know that along the ray trajectory r(z) is given by

$$r(z) = a\cos\left(\frac{\pi z}{F}\right)$$

and that n(r) is given by

$$n^2(r) = n_1^2\left(1 - \Delta \cdot \frac{r^2}{a^2}\right) = n_1^2\left(1 - \Delta \cdot \cos^2\left(\frac{\pi z}{F}\right)\right)$$

so that t_F is

$$t_F = \frac{n_1^2}{cn_2} \cdot \int_0^F \left(1 - \Delta \cdot \cos^2\left(\frac{\pi z}{F}\right)\right)dz$$

Doing the integral, and substituting for Δ gives

$$t_F = \frac{F}{2cn_2} \cdot (n_1^2 + n_2^2)$$

This is the time to travel distance F, so to travel distance L is

$$t_1 = \frac{L}{F} \cdot t_F$$

$$= \frac{L}{2cn_2} \cdot (n_1^2 + n_2^2)$$

Now consider a ray along the fibre axis. The time of travel is

$$t_2 = \frac{L}{c} \cdot n_1$$

Difference between t_1 and t_2 gives pulse spreading

$$\Delta t = t_1 - t_2$$

$$= \frac{L}{2n_1 c} \cdot \frac{(n_1^2 - n_2^2)^2}{(n_1 + n_2)^2}$$

For most fibres $n_1 \approx n_2 = n$. Then Δt is

$$\Delta t = \frac{L}{8n^3 c} \cdot (NA)^4$$

where NA is the numerical aperture. Defining bandwdth as

$$B = \frac{1}{2\Delta t}$$

we find the bandwidth length product is

$$\boxed{B \cdot L = \frac{4n^3 c}{(NA)^4}}$$

Worked Example

The data sheet for the Samsung SF-MM6 graded index multi-mode fibre specifies n_1 - n_2 = 0.02. Estimate the bandwidth-length product for this fibre.

Assume $n_1 \approx n_2 = 1.46$

$$NA = \sqrt{n_1^2 - n_2^2} = 0.24$$

$$B \cdot L = \frac{4n^3 c}{(NA)^4} = \frac{4 \times 1.46^3 \times 3 \times 10^8}{(0.24)^4}$$

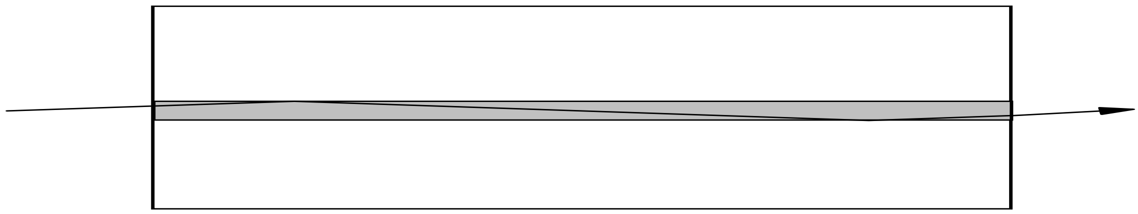
$$BL = 1125 \text{ MHz km}$$

The data sheet gives BL > 500 MHz km.

This is a **much** higher bandwidth than can achieved with a step index multimode fibre.

Single mode optical fibre

Multipath dispersion can be completely eliminated in an optical fibre by reducing the core diameter until only one mode can propagate. Under these conditions the light propagates along the fibre at a very small grazing angel of incidence.



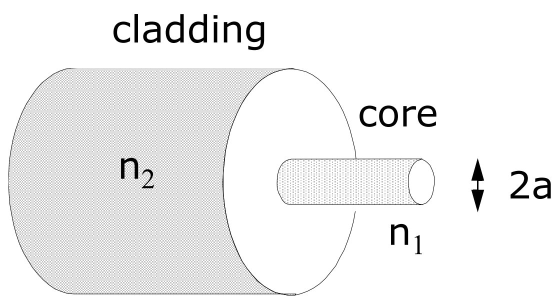
small diameter of core allows only one ray path along fibre

A key parameter in designing a single mode fibre is the normalised frequency:

Normalised frequency V

$$\boxed{V = \frac{2\pi a}{\lambda} \cdot \sqrt{n_1^2 - n_2^2}}$$

see diagram below



Condition for single mode design is

$$\boxed{V < 2.4}$$

Worked Example

The Corning single mode fibre SMF-28 is specified to have a numerical aperture NA = 0.14 . Estimate the maximum core diameter if the fibre is to be single moded at both λ = 1300nm and λ = 1550nm.

Single mode condition is:

$$V = \frac{2\pi a}{\lambda} \cdot \sqrt{n_1^2 - n_2^2} \le 2.4$$

$$2a \le 2.4 \times \frac{\lambda}{\pi} \times \frac{1}{\sqrt{n_1^2 - n_2^2}}$$

$$2a \le 2.4 \times \frac{1.55}{\pi} \times \frac{1}{0.14}$$

$$\text{diameter } \le 8.5\ \mu\text{m}$$

The Corning data sheet specifies a core diameter = 8.2 μm .

Single mode fibres - modal properties

The core-cladding refractive index difference is very small in an optical fibre. Under these conditions the modal electric and magnetic fields are linearly polarised. For a vertically polarised mode the electric field is described by:

$$E_y = \psi(r)\cos(n\theta)e^{j(\omega t - \beta z)}$$

where r and θ are cylindrical coordinates:

The modes are referred to as LPnm modes where n refers to the angular variation and m to the radial variation. Each LPnm mode has a cutoff value of V below which it does not propagate. Some cut off values are given below:

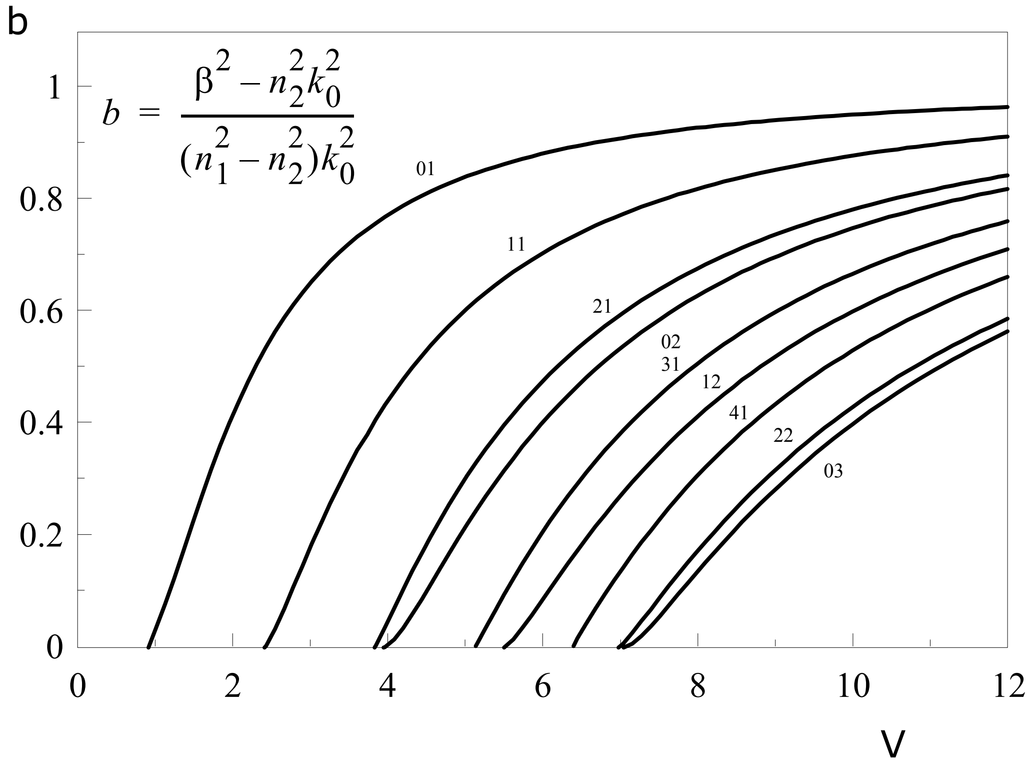
Mode cutoffs	V
LP_{11}	2.4
LP_{21} }	3.8
LP_{02} }	
LP_{31}	5.1
LP_{12}	5.5

$$V = \frac{2\pi a}{\lambda} \cdot \sqrt{n_1^2 - n_2^2}$$

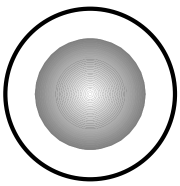
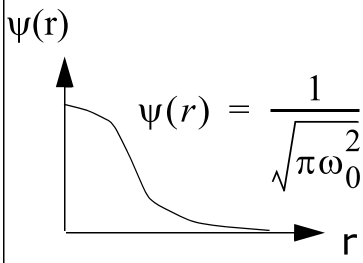
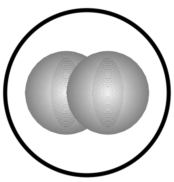
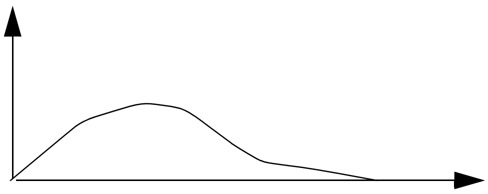
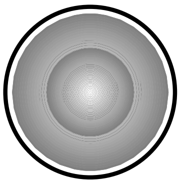
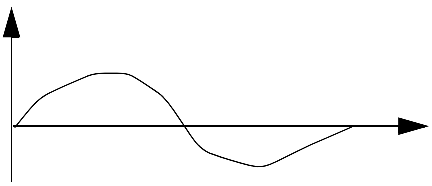
Each LPnm mode has a different propagation constant β which is a function of wavelength λ . The normalised propagation constant b is often used:

$$b = \frac{\beta^2 - n_2^2 k_0^2}{(n_1^2 - n_2^2)k_0^2} \quad \text{where } k_0 = \frac{2\pi}{\lambda}$$

A graph of b and V for the first few LPnm modes is shown below



The optical intensity of the first three LPnm modes is shown below

LP_{nm}	intensity	$\psi(r)$
LP_{01}		 $\psi(r) = \frac{1}{\sqrt{\pi\omega_0^2}} e^{-\frac{r^2}{2\omega_0^2}}$ $\omega_0 = \frac{a}{\sqrt{2\log_e V}}$
LP_{11}		
LP_{02}		

Note that the modal field for the LP01 mode above has been normalised so that

$$2\pi\int_0^\infty \psi^2(r)rdr = 1$$

Worked Example

The Corning SMF-28 single mode fibre has a mode diameter of 10.4μm at 1550nm, a core diameter of 8.2μm and an NA = 0.14 . Estimate the fraction of power within the core for the fundamental LP01 mode. If the optical power in the fibre is 100mW, estimate the peak electric field.

$$\omega_0 = \frac{1}{\sqrt{2}} \cdot \frac{10.4}{2} = 3.68 \ \ \mu m$$

Fraction of power in core η :

$$\eta = \frac{2\pi\int_0^a \psi^2(r)rdr}{2\pi\int_0^\infty \psi^2(r)rdr}$$

where 2a = 8.2μm. This gives

$$\eta = 1 - e^{-\frac{a^2}{\omega_0^2}}$$

$$\eta = 71\ \%$$

Write electric field as

$$E_y = E_0 \cdot e^{-\frac{r^2}{2\omega_0^2}}$$

Then magnetic field H_x is approximated by

$$\frac{E_y}{H_x} = Z_0 = \sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}} = \frac{1}{n} \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} = 258\Omega$$

where n =1.46 is refractive index of silica.

Poynting vector N is

$$N = \frac{1}{2} \cdot E_y H_x = \frac{E_0^2}{2Z_0} e^{-\frac{r^2}{\omega_0^2}}$$

Total power in fibre is

$$P = 2\pi \int_0^\infty N r dr = \frac{E_0^2}{2Z_0} \cdot \pi\omega_0^2$$

The peak electric field is

$$E_0 = \sqrt{\frac{2Z_0 P}{\pi\omega_0^2}} = \sqrt{\frac{2 \times 258 \times 100 \times 10^{-3}}{\pi \times (3.68 \times 10^{-6})^2}} = 1.1 \times 10^6 \text{ V/m}$$

This size of electric field is large enough to cause non-linear effects in a fibre.

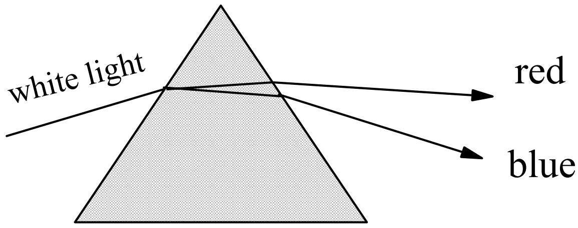
Dispersion in optical fibres

Single mode fibres do not suffer multipath (modal) dispersion. However, they do suffer from dispersion and this has two main causes arising from the spectral width of the optical pulses:

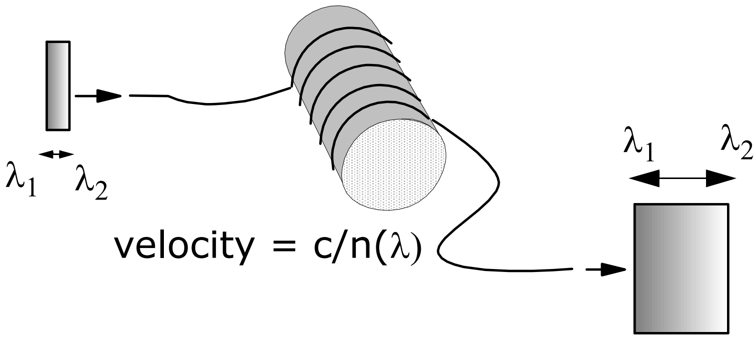
- material or chromatic dispersion.
- waveguide dispersion.

Material dispersion

The refractive index of glass varies with wavelength:



This causes different wavelength components in a pulse to travel at different speeds:

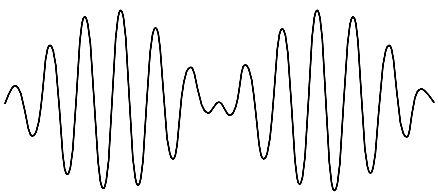


Consider an optical pulse with just two frequencies $\omega \pm \delta\omega$ (or wavelengths):

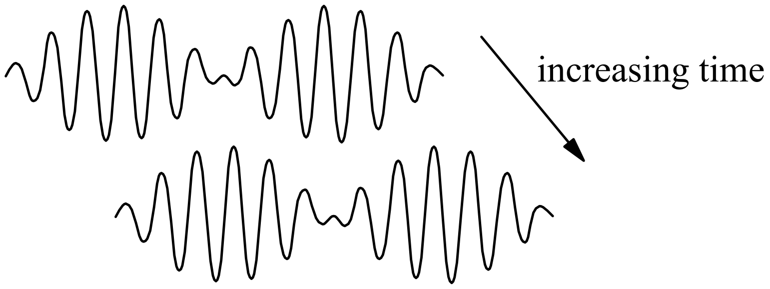
$$\cos((\omega + \delta\omega)t - (\beta + \delta\beta)z) + \cos((\omega - \delta\omega)t - (\beta - \delta\beta)z)$$

$$= 2\cos(\omega t - \beta z) \cdot \cos(\delta\omega t - \delta\beta z)$$

The following diagram shows the pulse at a fixed moment in time



As time increases the pulse moves



We see that the pulse envelope moves at the group velocity:

$$\boxed{v_g = \frac{\delta\omega}{\delta\beta} = \frac{\partial\omega}{\partial\beta}}$$

The following approximate relation now holds

$$\frac{\omega}{\beta} = \frac{c}{n(\lambda)}$$

hence

$$\beta = \frac{\omega}{c} \cdot n(\lambda) = \frac{2\pi}{\lambda} \cdot n(\lambda)$$

Group velocity is

$$v_g = \frac{\partial\omega}{\partial\beta} = \frac{\partial\omega}{\partial\lambda} \Big/ \frac{\partial\beta}{\partial\lambda}$$

$$v_g = -\frac{2\pi c}{\lambda^2} \cdot \frac{1}{\left(-\frac{2\pi n}{\lambda^2} + \frac{2\pi}{\lambda} \cdot n\right)}$$

$$\boxed{v_g = \frac{c}{n - \lambda \cdot \frac{\partial n}{\partial\lambda}}}$$

Consider the time for the pulse to travel distance L. This is called the group delay

$$t_{mat} = \frac{L}{v_g} = \frac{L}{c} \cdot \left(n - \lambda \cdot \frac{\partial n}{\partial\lambda}\right)$$

If the spectral width of the pulse is σ_λ, then the pulse width will increase by

$$\Delta\tau_{mat} = \frac{\partial t_{mat}}{\partial\lambda} \cdot \sigma_\lambda = -\frac{L}{c} \cdot \lambda \cdot \frac{\partial^2 n}{\partial\lambda^2} \cdot \sigma_\lambda$$

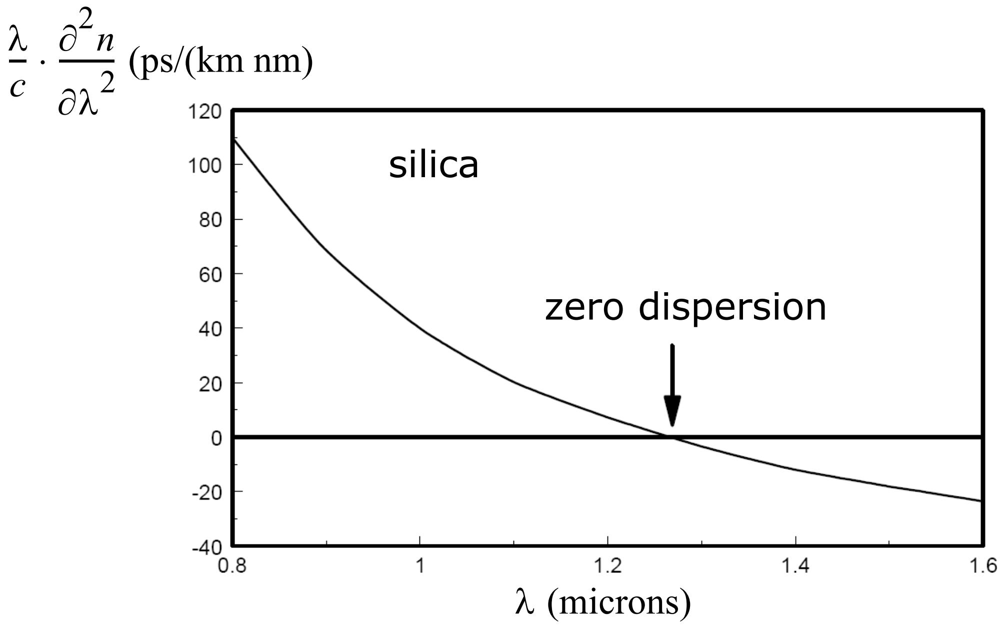
This is usually written as

$$\boxed{\Delta\tau_{mat} = L \cdot D_{mat} \cdot \sigma_\lambda}$$

where the material dispersion D_{mat} is defined as

$$\boxed{D_{mat} = -\frac{\lambda}{c} \cdot \frac{\partial^2 n}{\partial\lambda^2}}$$

A graph of $\frac{\lambda}{c} \cdot \frac{\partial^2 n}{\partial\lambda^2}$ for silica is plotted below:



- Units of D_{mat} are ps/(km nm)
- Material dispersion vanishes for λ at 1.27μm.
- D_{mat} is negative below 1.27μm and positive above.
- The graph is for pure silica only. Waveguide geometry and material shifts zero dispersion wavelength to around 1.27μm - 1.29μm.

Waveguide dispersion

The phase velocity ω/β along a single mode fibre varies with both wavelength and waveguide dimensions, as can be seen from the b-V curves in previous lecture. This gives rise to waveguide dispersion, and is present even if refractive index is wavelength independent. Recall definition of b

$$b = \frac{\beta^2 - n_2^2 k_0^2}{(n_1^2 - n_2^2)k_0^2}$$

This gives

$$\beta^2 = n_2^2 k_0^2 + b \cdot (n_1^2 - n_2^2)k_0^2$$

Since $(n_1 - n_2) \ll 1$ we can approximate β as

$$\beta = n_2 k_0 + \frac{b \cdot (n_1^2 - n_2^2)}{2n_2} \cdot k_0 \quad \text{where} \quad k_0 = \frac{2\pi}{\lambda}$$

We now calculate $\partial\beta/\partial\omega$ ignoring wavelength dependence of refractive indices

$$\frac{\partial\beta}{\partial\omega} = \frac{\partial k_0}{\partial\omega} \cdot \frac{\partial\beta}{\partial k_0} = \frac{n_2}{c} + \frac{(n_1^2 - n_2^2)}{2n_2 c} \cdot \frac{\partial}{\partial k_0}(bk_0)$$

Recall definition of V

$$V = k_0 a \cdot \sqrt{n_1^2 - n_2^2}$$

Then

$$\frac{\partial}{\partial k_0} = a \cdot \sqrt{n_1^2 - n_2^2} \cdot \frac{\partial}{\partial V}$$

Hence

$$\frac{\partial\beta}{\partial\omega} = \frac{1}{c} \cdot \left(n_2 + \frac{(n_1^2 - n_2^2)}{2n_2} \cdot \frac{\partial}{\partial V}(bV) \right)$$

The group delay arising from waveguide dispersion is then

$$t_{wg} = \frac{L}{c} \cdot \left(n_2 + \frac{(n_1^2 - n_2^2)}{2n_2} \cdot \frac{\partial}{\partial V}(bV) \right)$$

For a signal source with spectral width σ_λ the pulse spreading is

$$\Delta\tau_{wg} = \sigma_\lambda \cdot \frac{\partial t_{wg}}{\partial \lambda}$$

To evaluate this we make use of the following

$$\frac{\partial}{\partial \lambda} = \frac{\partial V}{\partial \lambda} \cdot \frac{\partial}{\partial V} = -\frac{V}{\lambda} \cdot \frac{\partial}{\partial V}$$

Giving

$$\Delta\tau_{wg} = -\frac{V}{\lambda} \cdot \sigma_\lambda \cdot \frac{\partial t_{wg}}{\partial V}$$

$$= -L \cdot \sigma_\lambda \cdot \frac{(n_1^2 - n_2^2)}{2n_2 \lambda c} \cdot V \cdot \frac{d^2}{dV^2}(bV)$$

This pulse spreading be written as

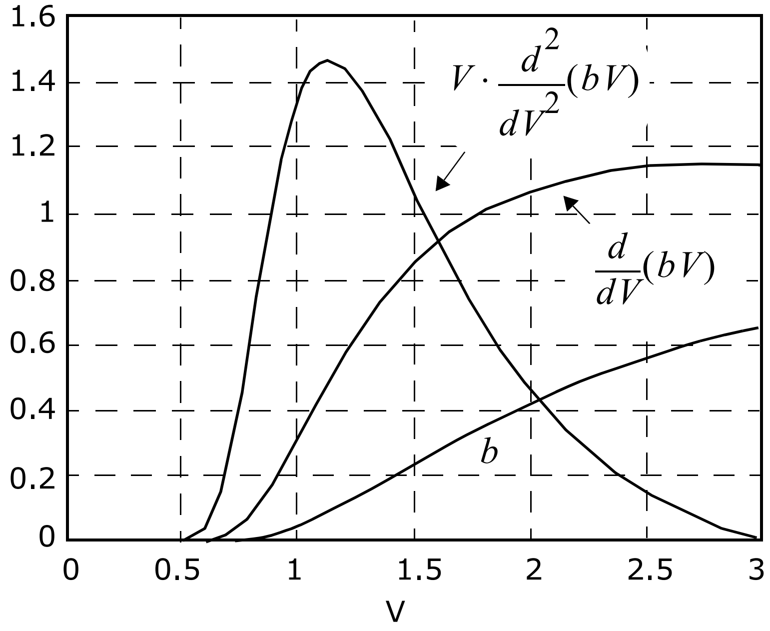
$$\Delta\tau_{wg} = L \cdot \sigma_\lambda \cdot D_{wg}$$

where the waveguide dispersion D_{wg} is given by

$$\boxed{D_{wg} = -\frac{(n_1^2 - n_2^2)}{2n_2 \lambda c} \cdot V \cdot \frac{d^2}{dV^2}(bV)}$$

A graph of b, $\frac{d}{dV}(bV)$ and $V \cdot \frac{d^2}{dV^2}(bV)$ plotted against V is shown below.

Note that this graph appears in many textbooks, but often only approximately calculated.



We note from this graph and the expression for D_{wg} that waveguide dispersion is always negative.

Total Dispersion

Single mode Fibres

Combining material and waveguide dispersion, the total pulse broadening is given by

$$\Delta\tau = \Delta\tau_m + \Delta\tau_{wg}$$

$$\boxed{\Delta\tau = \sigma_\lambda \cdot L \cdot D}$$

where the total dispersion D is

$$\boxed{D = D_m + D_{wg}}$$

Multimode Fibres

Multimode fibres have both material and modal dispersion. Total rms pulse broadening is given by

$$\sigma = \sqrt{\sigma_{mode}^2 + \sigma_{material}^2}$$

Worked Example

Estimate the total dispersion at 1550nm for the Corning SMF28 fibre, for which the numerical aperture NA = 0.14, core radius a = 4.1μm, and cladding index n_2 *= 1.444.*

From material dispersion graph we find

$$D_m = 20 \text{ ps/(nm.km)}$$

$$V = k_0 a \cdot \sqrt{n_1^2 - n_2^2} = k_0 a \cdot NA = 2.3$$

From above graph we estimate $V \cdot \frac{d^2}{dV^2}(bV) = 0.2$

Then waveguide dispersion is

$$D_{wg} = -\frac{NA^2}{2n_2\lambda c} \cdot V \cdot \frac{d^2}{dV^2}(bV) = -3 \text{ ps/(nm.km)}$$

Total dispersion is then

$$D = D_m + D_{wg} = (20 - 3) \text{ ps/(nm.km)}$$

$$\boxed{D = 17 \text{ ps/(nm.km)}}$$

The Corning data sheet gives D = 16 ps/(nm.km). Most fibre currently in use is of this type.

Worked Example

Estimate the total dispersion for a single mode fibre with the following parameters: NA = 0.17, a = 2.3 μm, n_2 *= 1.447,* λ *= 1550nm.*

$$D_m = 20 \text{ ps/(nm.km)}$$

$$V = k_0 a \cdot \sqrt{n_1^2 - n_2^2} = k_0 a \cdot NA = 1.6$$

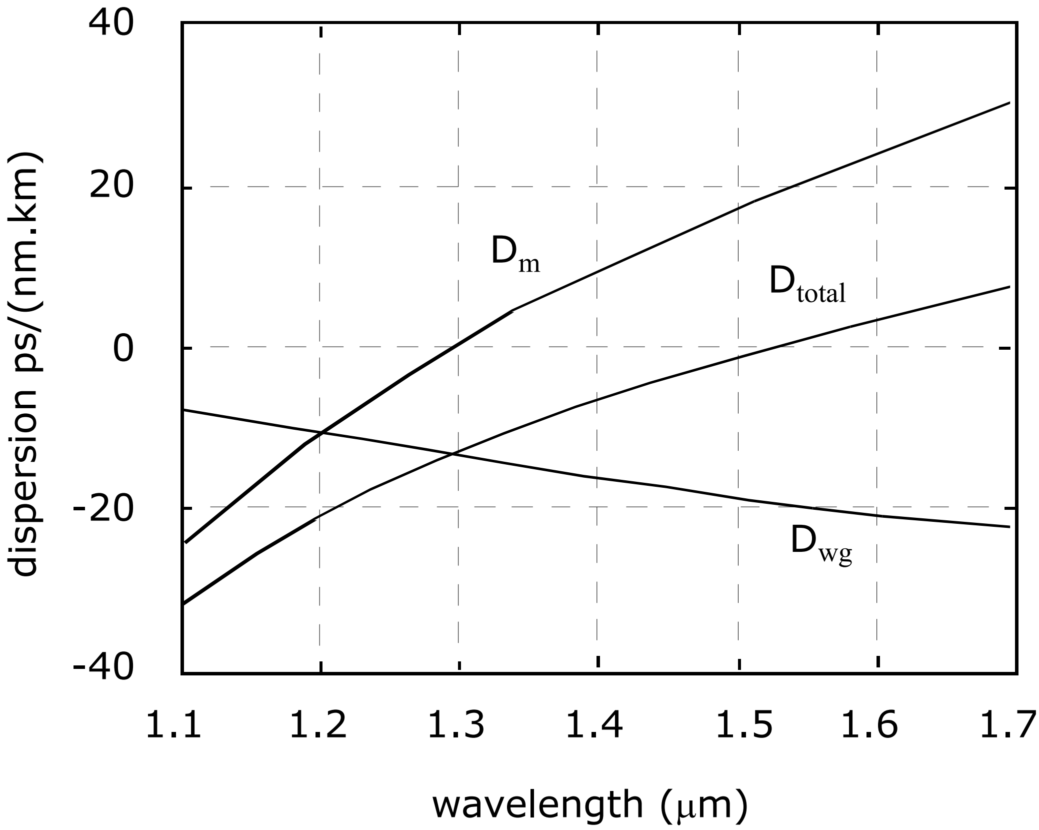
$$V \cdot \frac{d^2}{dV^2}(bV) = 0.92$$

$$D_{wg} = -\frac{NA^2}{2n_2\lambda c} \cdot V \cdot \frac{d^2}{dV^2}(bV) = -19.7 \text{ ps/(nm.km)}$$

$$D = D_m + D_{wg} = 0.3 \text{ ps/(nm.km)} \approx 0$$

Dispersion Shifted Fibre

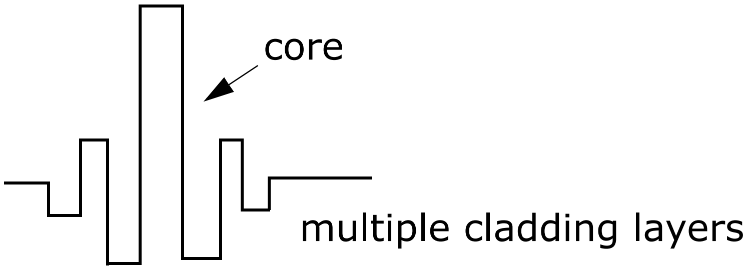
The previous example illustrates how the zero dispersion wavelength can be shifted from 1300nm to 1550nm by changing the fibre geometry and numerical aperture. Such fibres are called dispersion shifted fibres and their design exploits the opposite signs of material and waveguide dispersion at wavelengths above 1300nm, as shown below.



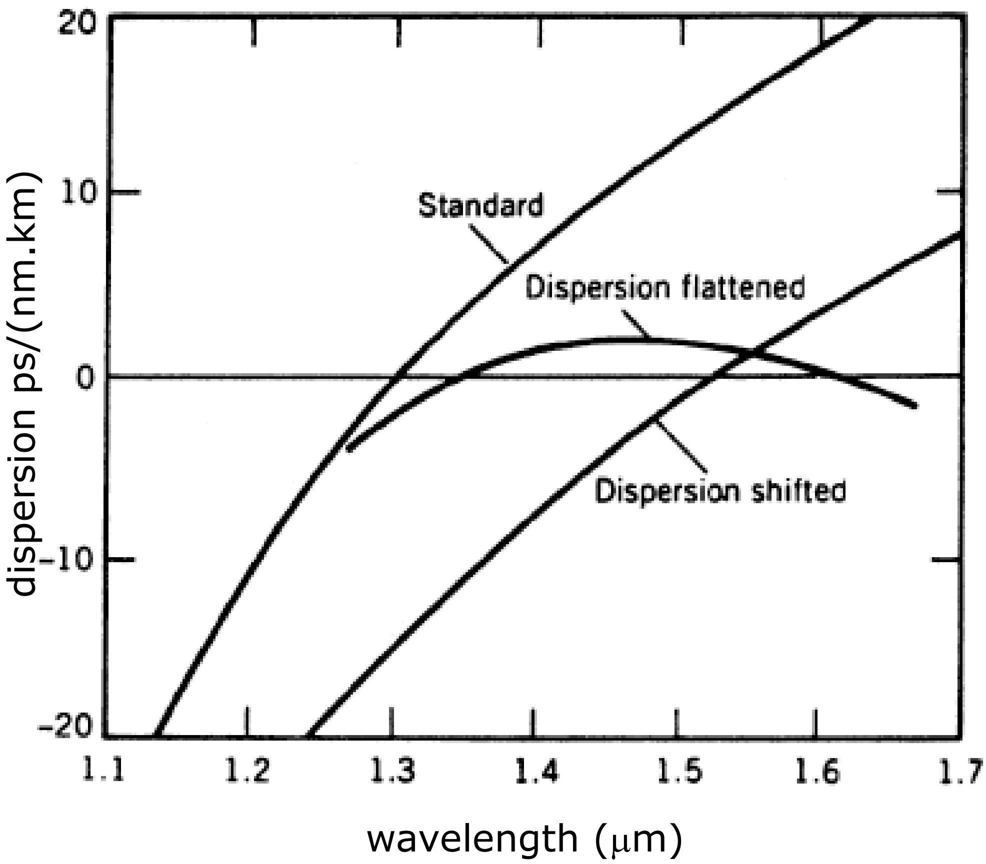
Material and waveguide dispersion for a fibre with NA = 0.17, a = 2.3 mm, n2 = 1.447.

Dispersion Flattened Fibre

By suitable design it is possible to make a fibre with small D over a wide spectral range, known as a dispersion flattened fibre. A typical refractive index profile is shown



Following graph shows typical dispersion

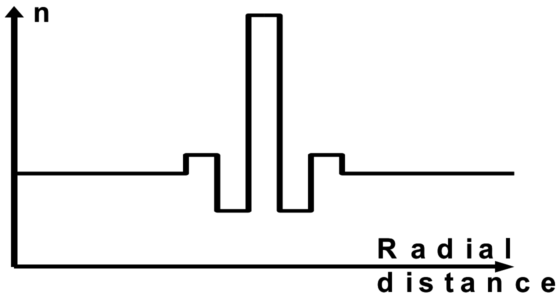


Dispersion compensating fibre (DCF)

- Fibre structure specially designed to have large amounts of negative dispersion, to balance the positive dispersion of NDSF (non-dispersion shifted fibre) at 1550nm.
- Very high negative dispersion, ~-100 ps/nm.km at 1550nm.
- Pay the price in terms of loss, which increases to between

0.5 and 1.0 dB/km.

- DCF is now being engineered to compensate for both the dispersion and dispersion slope - this is extremely important for multi-channel high bit-rate WDM systems.



Higher Order Dispersion Model

Can use the idea of group velocity to create a higher order model of propagation in fibres. Expand β as a Taylor expansion in ω so that $\omega = \omega_c + \omega_m$

$$\beta = \beta_c + \omega_m\left(\frac{\partial\beta}{\partial\omega}\right) + \frac{1}{2}\omega_m^2\left(\frac{\partial^2\beta}{\partial\omega^2}\right) + \ldots$$

$$\beta = \beta_c + \omega_m \cdot \frac{1}{v_g} + \frac{1}{2}\omega_m^2\beta_2$$

where the second term is the group delay and the third term β_2 is the group velocity dispersion.

Consider an envelope of frequencies propagating distance z along the fibre. Taking the Fourier transform to obtain the pulse shape

$$\psi(z, t) = \frac{1}{2\pi}\int\varphi(0, \omega)\exp j(\omega t - \beta z)d\omega$$

where $\varphi(0, \omega)$ is the input signal spectrum. Expand β in a Taylor series

$$\psi(z, t) = \frac{1}{2\pi}\int\varphi(0, \omega_m)\exp j\left((\omega_c + \omega_m)t - \left(\beta_c + \omega_m\beta_1 + \frac{1}{2}\omega_m^2\beta_2\right)z\right)d\omega_m$$

Rearranging to remove the carrier component

$$\psi(z,t) = \frac{\exp j(\omega_c t - \beta_c z)}{2\pi}\int \varphi(0,\omega_m)\exp j\left(\omega_m(t-\beta_1 z) - \frac{1}{2}\omega_m^2\beta_2 z\right)d\omega_m$$

The integral term describes the envelope of the wave which moves at the group velocity, the first term is the fast varying carrier.

Just considering the envelope, and moving to a time frame that moves at the group velocity with the wave, so that

$$t' = t - \beta_1 z$$

leads to

$$\psi(z, t' + \beta_1 z) = \frac{1}{2\pi}\int \varphi(0,\omega_m)\exp j\left(\omega_m t' - \frac{1}{2}\omega_m^2\beta_2 z\right)d\omega_m$$

This shows that the initial profile on entry to the fibre (z=0) is as would be expected, the Fourier Transform of the frequency spectrum. As the pulse propagates to a non-zero z the GVD β_2 causes evolution of the pulse shape.

Assuming a Gaussian pulse in time

$$\psi(0,t) = \exp\left(-\frac{t^2}{2T_0^2}\right)$$

The initial frequency response is the transform of the pulse

$$\varphi(0,\omega) = \frac{1}{2\pi}\int \exp\left(-\frac{t^2}{2T_0^2}\right)\exp(-j\omega t)d\omega = T_0\sqrt{2\pi}\exp\left(-\frac{\omega^2 T_0^2}{2}\right)$$

The broadened wave is

$$\psi(z, t' + \beta_1 z) = \frac{1}{2\pi}\int T_0\sqrt{2\pi}\exp\left(-\frac{\omega_m^2 T_0^2}{2}\right)\exp j\left(\omega_m t' - \frac{1}{2}\omega_m^2\beta_2 z\right)d\omega_m$$

This can be rewritten as

$$\psi(z, t' + \beta_1 z) = \frac{1}{2\pi}\int T_0\sqrt{2\pi}\exp\left(-\frac{\omega_m^2}{2}(T_0^2 + j\beta_2 z)\right)\exp(j\omega_m t')d\omega_m$$

From this we see that the output pulse is proportional to

$$\exp\left(-\frac{t'^2}{2T_0^2\left(1+j\frac{\beta_2 z}{T_0^2}\right)}\right) = \exp\left(-\frac{t'^2}{2T_0^2}\cdot\frac{\left(1-\frac{j\beta_2 z}{T_0^2}\right)}{\left(1+\left(\frac{\beta_2 z}{T_0^2}\right)^2\right)}\right)$$

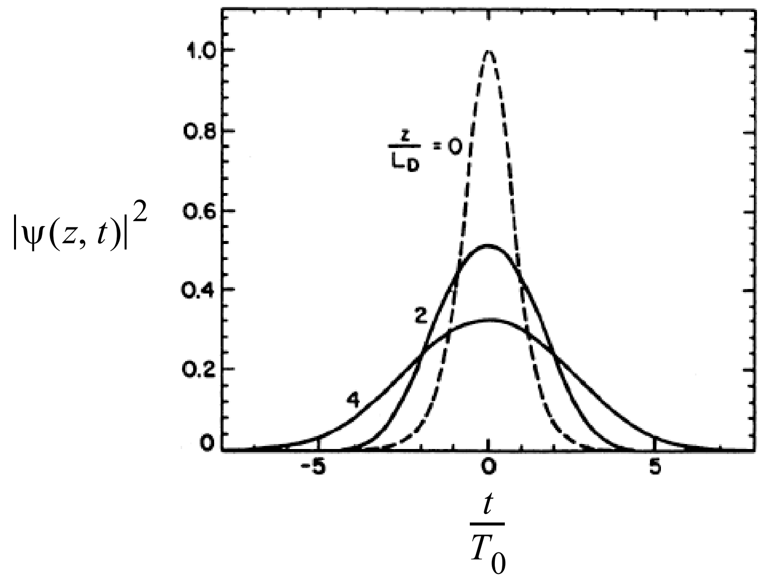
The output pulse has been broadened to

$$T = T_0\sqrt{1+\left(\frac{\beta_2 z}{T_0^2}\right)^2}$$

This analysis is transform limited- in that the source modulation creates the change in frequency. In practice many current systems are dominated by source linewidth, so that dispersion is a function of the source linewidth rather than the signal spectral width.

The dispersion length is defined as

$$L_D = \frac{T_0^2}{\beta_2}$$



Limits on Transmission

Communication channels are characterised by two fundamental parameters

Loss and dispersion

Loss Limited Transmission

In a loss limited situation the only factor limiting the length over which information can be transmitted is the loss of the channel. This can be expressed as

$$P_{out} = P_{in}\exp(-\alpha L)$$

It is common to express loss as dB/kM

$$\alpha_{dB} = \frac{10}{L}\log_{10}\left(\frac{P_{in}}{P_{out}}\right) \text{ dB/km}$$

It is straightforward to show

$$\alpha_{dB} = 10\alpha\log_{10}e$$

For a system to work the power in the output of the fibre should be the minimum power for which the receiver can work satisfactory, say , $\langle P_{smin}\rangle$. Then the maximum system length is given by solving the expression above for $\langle P_{smin}\rangle$. That is

$$\langle P_{smin}\rangle = 10^{-0.1\alpha_{dB}L}$$

The length now depends on the fibre loss and the launched power. The fibre loss varies between 0.14 – 0.2 dB depending on the details of its fabrication.

The launched power depends on the technology and for semiconductor lasers with external modulators can reach up to 20-25 mW (13 – 14 dBm).

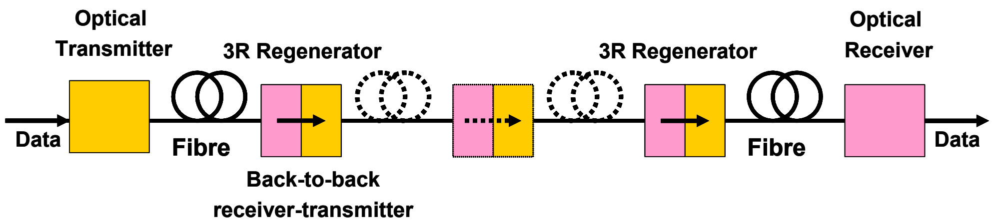
Worked Example

With a loss of 0.18 dB/km, minimum receiver power of 0.33 mW and launch power of 20 mW what is the maximum system length?

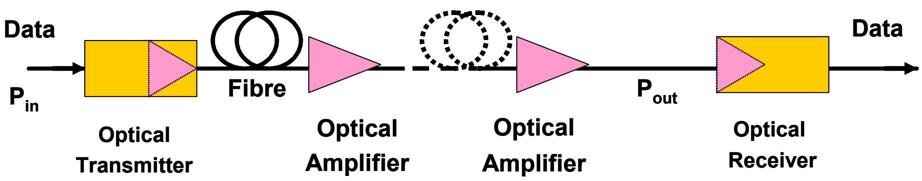
$$L_{max} = \frac{10}{\alpha_{dB}} \log_{10}\left(\frac{20 \times 10^{-3}}{0.33 \times 10^{-6}}\right) = 265 \text{ km}$$

If this system length is not sufficient then there are two alternatives.The first is to use a 3R digital regenerator and the second if to use optical amplifiers.

The architecture of a system with "3R" regenerators is shown below



The architecture of a system using optical amplifiers is shown below



Optical amplifiers have the significant advantage of simplicity, no electronic bottleneck and the ability to amplify a number of wavelengths simultaneously (wavelength division multiplexing) and as a result they are used in all modern optical fibre systems.

The key disadvantage of the 3R approach is that one needs one "3R" regenerator per wavelength which turns out to be very expensive.

Optical amplifiers do however have a number of disadvantages which include the aggregation of impairments over the length of the system which include noise and fibre non-linearities and difficulties in networking them in a dynamic network environment. These are the typical features of analogue systems but the cost advantage is such that they are used exclusively in modern systems.

Link Design

It is worth examining the choices the link designers themselves must deal with. These might include

Wavelength

•Fibre type

•Sources (Laser or LED)

•Detector/receiver choices

Crucial to the design are the power and dispersion budgets.

Power Budget

•Express Power in dBm (dB relative to 1mW).

•Losses in dB.

•Allows summation rather than multiplication.

Calculating Power Budgets

1. Source power dBm.

2. Coupling Loss (dB). What proportion of laser power gets into the fibre? Express coupling efficiency as a coupling loss (i.e. 50% efficiency=3dB loss).

3. Fibre loss = fibre loss in dB/km x length of fibre.

4. Splice losses= no of joints x joint loss in dB.

5. Outcoupling loss (How much light from out of the fibre gets into the receiver).

6. Safety margin- how much extra loss is allowed?.

7. Receiver sensitivity - quoted in dBm (i.e. -39dBm at 150Mbits/s at a BER of 10e-9.

•Link functions if Power at receiver>=Source power- sum of losses- safety margin

•If not system is said to be loss limited

Worked Example

You have a transmitter of power -10 dBm and a receiver that requires a signal of power -20 dBm (minimum) so that you have 10 dB of link budget.

You might allow:

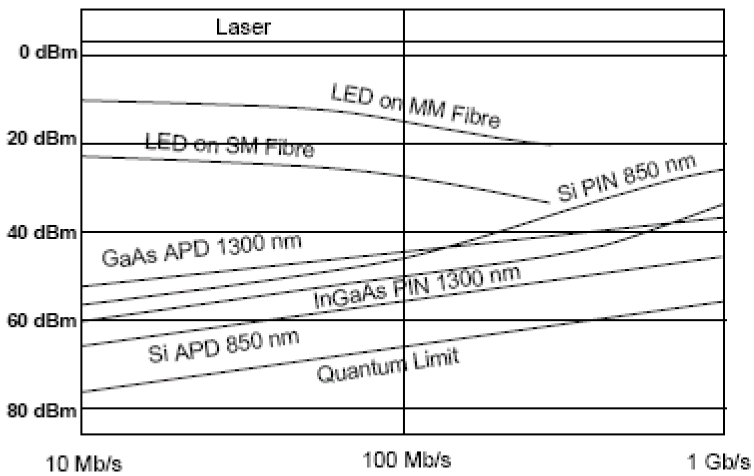
•10 connectors at .3 dB per connector = 3 dB

•2 km of cable at 2 dB per km (MM GI fibre at 1300 nm) = 4 dB

•Contingency of (say) 2 dB for deterioration due to ageing over the life of the system.

This leaves us with a total of 9 dB system loss. This is within our link budget and so we would expect such a system to have sufficient power.

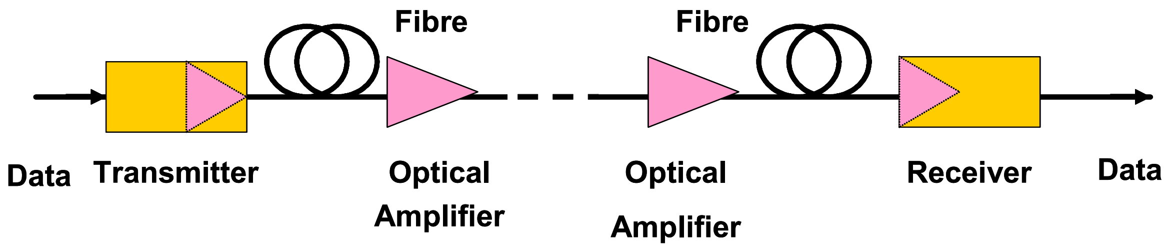
The amount of power that we have to use up on the link and in connectors is determined by the characteristics of the components we select as transmitters and receivers.



The above shows the characteristics of some typical devices versus the transmission speed (in bits per second).

Dispersion Limited Transmission

The architecture of a single channel optical communication system is as below

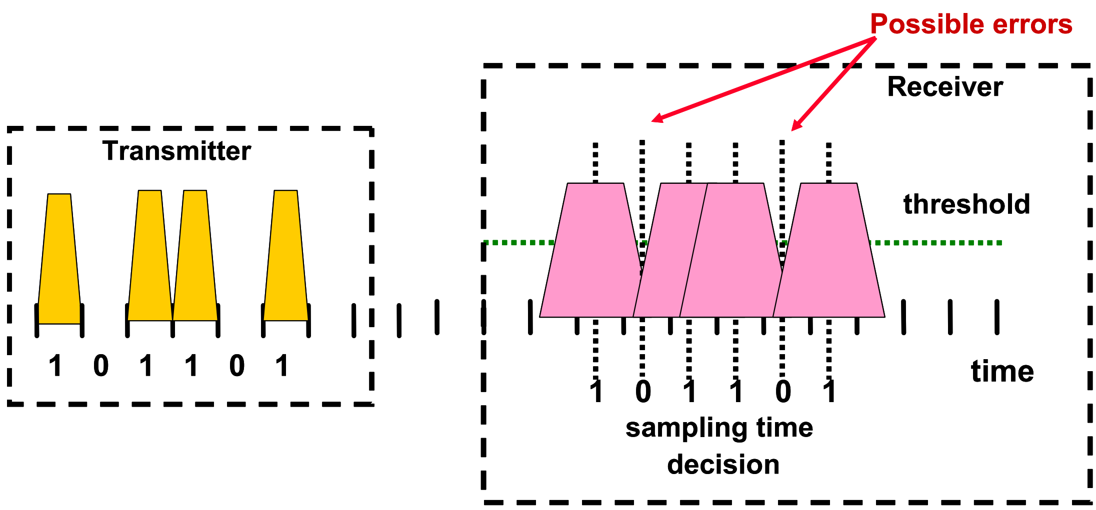


The fibre introduces attenuation (loss) and dispersion. The loss can be compensated by the optical amplifiers but the dispersion requires special treatment.

The receiver will detect the pulse and make decisions at each sampling point in time according the the decision rule:

Decide on "1" when signal above threshold

Decide on "0" when signal below threshold



The receiver will make errors because of the overlap of pulses belonging to different time slots (Inter Symbol Interference, ISI). In order to minimise the impact of ISI on the detection process the pulses arriving at the receiver have to be processed either before or after detection and "be squeezed " back to their original Slot. This process is known as dispersion compensation.

Because of the nature of the problem and the compensation techniques, dispersion cannot be compensated 100% so there will always be an element of residual dispersion which will give rise to ISI. This residual ISI will induce a penalty (errors) in the receiver which can be seen as equivalent to a reduction in receiver sensitivity. This penalty is known as dispersion penalty and dispersion compensation techniques aim at keeping it below a set level. The dispersion penalty by common consent is set to 1 dB. It can be shown theoretically that using Gaussian pulses the ISI for 1 dB penalty Is equivalent to a quarter of the bit period. This sets the following limit on pulse broadening

$$\boxed{\sigma \le \frac{T_{bit}}{4} \quad \text{for 1dB power penalty}}$$

or equivalently

$$\boxed{\sigma \le \frac{1}{4R_b} \quad \text{where } R_b = \text{bit rate}}$$

If return to zero (RTZ) pulses are used then the **bit rate ~ bandwidth**. If non-return to zero pulses are used then **bit-rate~ twice bandwidth**.

Dispersion from sources with large spectral width

Consider a source whose frequency spectral width σ_ω is broad, such that

$$\sigma_\omega \gg \frac{1}{\sigma_0}$$

where σ_0 is the initial pulse width. Then pulse broadening after distance L due to dispersion D is given by

$$\sigma = \sigma_\lambda \cdot L \cdot D$$

Worked Example

Estimate the maximum data rate that can be achieved at 1550nm over 10km of single mode fibre using a Fabry Perot laser with line width σ_λ *= 6nm and fibre dispersion D = 17ps/(nm.km)*

Since fibre is single mode there is no multimode dispersion, only waveguide and material dispersion.

Pulse broadening is

$$\sigma = \sigma_\lambda \cdot L \cdot D = 6 \times 10 \times 17 \text{ ps}$$

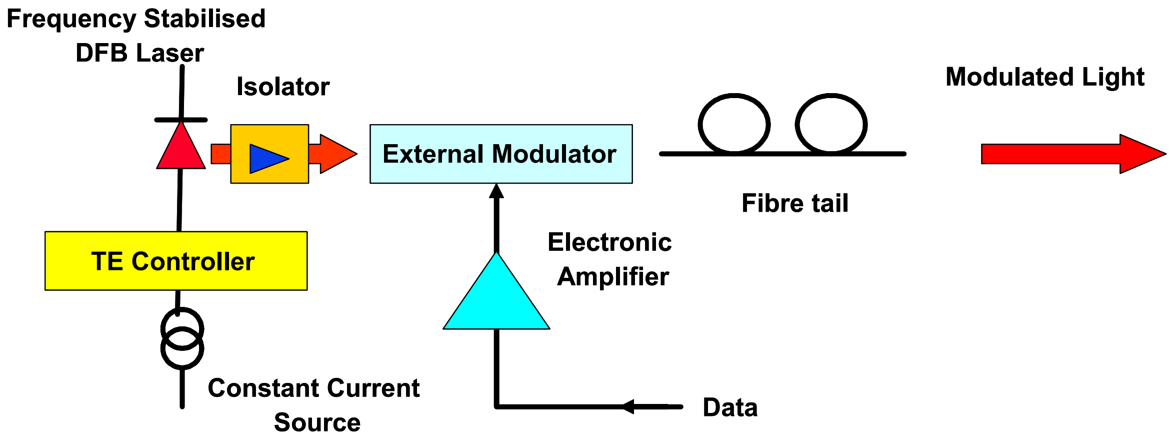
$$= 1.02 \text{ ns}$$

Maximum bit rate is then

$$R_b = \frac{1}{4\sigma} = 245 \text{ Mbs}^{-1}$$

Dispersion from sources with narrow spectral width

Previous example shows that using broad spectral sources severely limits bit rate. The solution is to use very narrow spectral lasers and to externally modulate. The architecture of a typical high speed optical transmitter is shown below:



Bit rate is given by

$$B \cdot L = \frac{1}{4D\sigma_\lambda}$$

The source rms spectral width will be that of the information. Frequency and wavelength are related by

$$\lambda = \frac{c}{f}$$

so the full width half magnitude (FWHM) of the source is given by

$$\sigma_\lambda = \frac{c}{f^2} \cdot \Delta f = \frac{\lambda^2}{c} \cdot \Delta f$$

Where Δf is the modulation rate. This is related to the bit rate by

$$B = \frac{1}{\Delta f}$$

Combining with previous equation we get

$$\boxed{B^2 L = \frac{c}{2D\lambda^2}}$$

Note that for very narrow spectral sources:

- B scales as $L^{-1/2}$
- B does not depend on source spectral width

Worked Example

Common non-dispersion shifted fibre has dispersion of D = 15 ps/km.nm at 1550nm. Calculate the maximum transmission length for 10 Gbit/s at 1550nm.

$$B^2 L = \frac{c}{2D\lambda^2}$$

$$= \frac{3 \times 10^8}{2 \times \frac{0.015 \times 10^{-12}}{10^{-9}} \times (1550 \times 10^{-9})^2}$$

$$= 4.162 \times 10^{24} \ (\text{bit/s})^2 . \text{m}$$

$$= 4162 \ (\text{Gbit/s})^2 \ \text{km}$$

For 10Gbit/s transmission rate the maximum length is:

$$\boxed{L = 41.6 \text{ km}}$$

Dispersion compensation

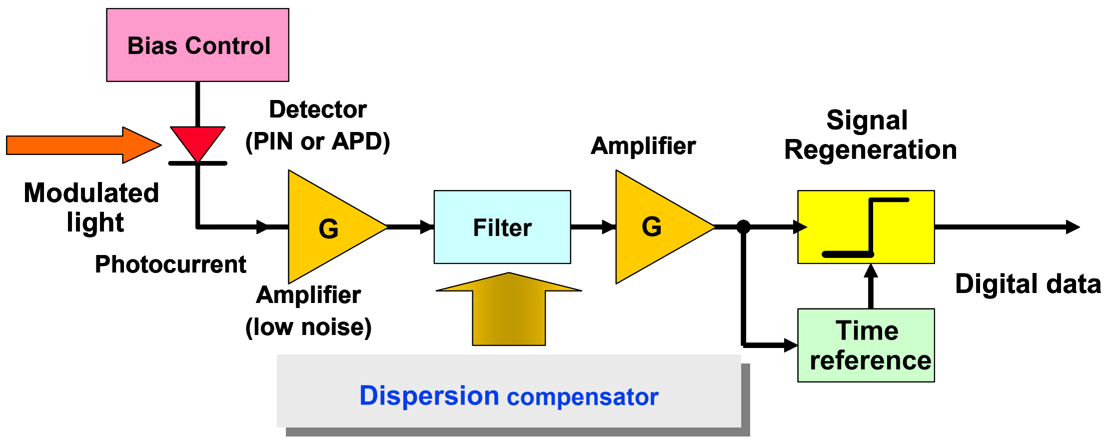
There are two approaches regarding dispersion compensation:

- post compensation techniques
- optical compensation techniques

In addition to these two approaches there is a third one whose main feature is the reduction of the bandwidth occupied by the information. This can be achieved through coding (multilevel, duobinary) or pre chirping. These techniques are also known as pre-compensation techniques.

Post compensation techniques

In this approach the compensation takes place after optical detection but before the threshold electronic detection:



The dispersion compensator is a high pass network so it tends to increase the noise power arriving at the threshold detector and also to turn the noise from white to "coloured".

In general equalisation invokes a loss penalty depending on the equaliser design. A simple rule for the penalty is:

where D_l is the extra channel loss arising from equalisation, $\Delta\tau$ is the dispersion induced pulse broadening, and B is the bit rate. This formula says that if you want to go fast you pay a large penalty.

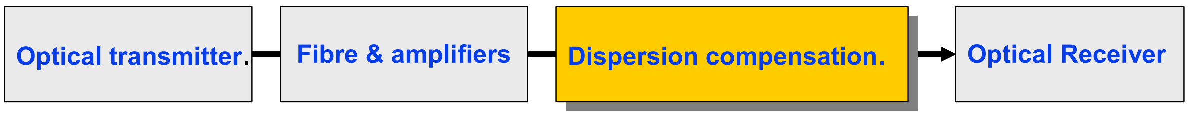
The technologies available for the physical implementation of the dispersion compensation are

- analogue; they can be used in principle at all bit rates but they present difficulties at high frequencies where microwave techniques are necessary and they also limit the sophistication of the compensator.

- digital; digital signal processing is now possible for bit rate up to 10 Gbit/s and since DSP allows the implementation of quite sophisticated compensators digital implementation will be a feature of the future.
- a fundamental feature of post compensation is that it is single channel.

Optical compensation techniques

In this approach the compensator is placed before the optical detector, that is it operates in the optical domain:



There are two key issues related to the use of optical compensation techniques

1 optimum position (where to place it)

2 compensation for its losses (amplifiers etc.)

Optimum position

Assuming a strictly linear propagation regime the compensator should be placed half in the transmit end and half in the receive end. However, if the losses of the devices are high then this approach will lead to high loss and the need to launch high powers which in their turn are difficult to generate.

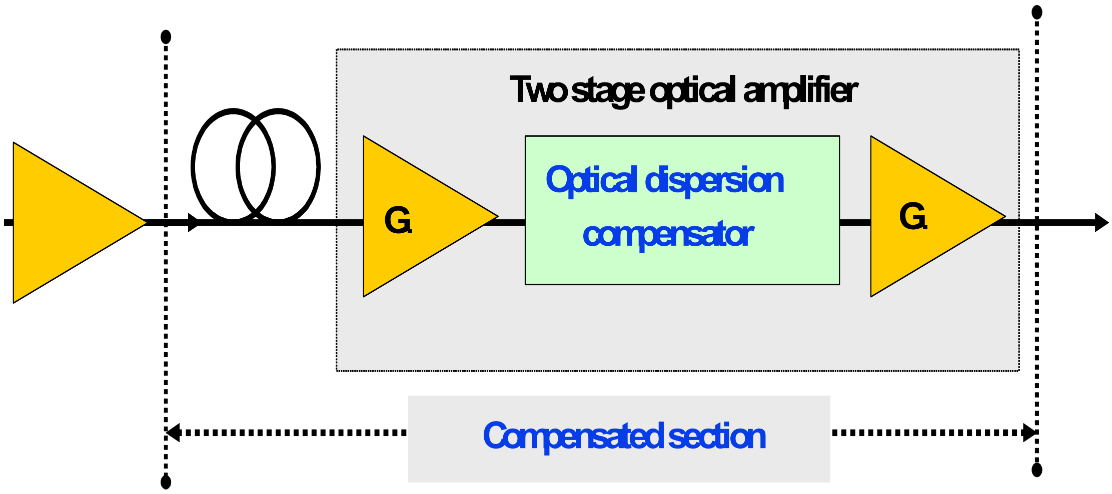
Compensation for the losses

Using the optimum placing of the compensators will necessitate the use of high gain / high power optical amplifiers. High gain / high power amplifiers with low noise figure are difficult to implement.

The solution to this dilemma is to compensate each section of the waveguide individually

In addition to this, the placing of the compensating device should not impair the performance of the system in terms of noise figure and high optical power. This requirement dictates that the compensator should be place in a position where its losses do not degrade the noise figure and the output power of the amplifiers.

The architecture of such an amplifier is shown below.



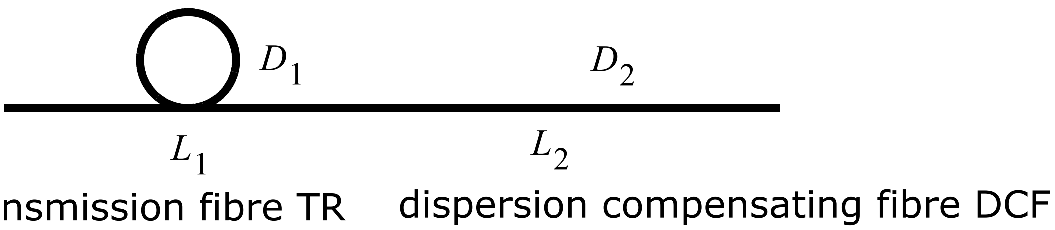
There are a number of technologies that can be used to implement optical dispersion compensation schemes. They include:

1 High dispersion fibre (dispersion compensating fibre)

2 Optical filters (Fabry – Perot and Mach – Zehnder interferometers)

3 Fibre Bragg gratings

Dispersion Compensating Fibre



Condition for dispersion compensation is

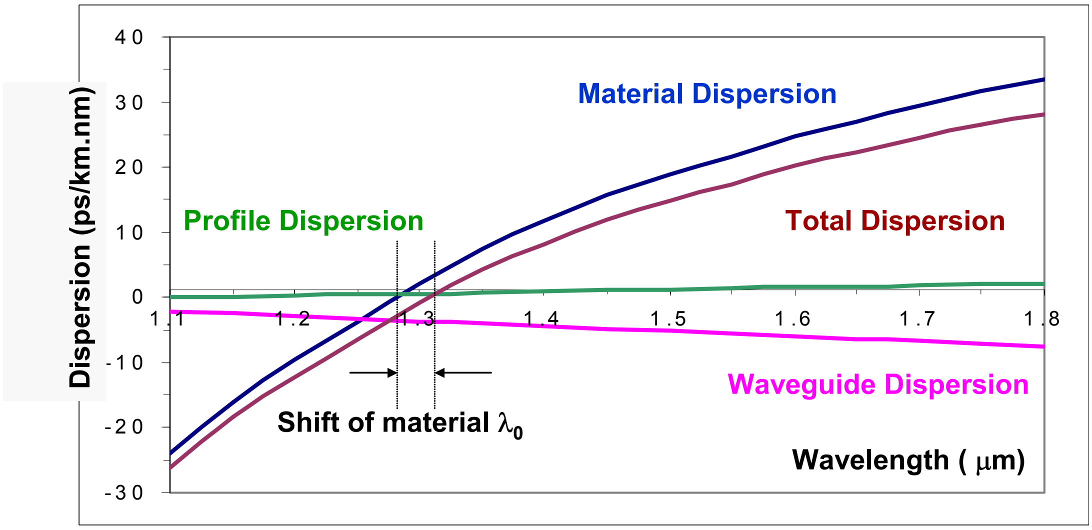
$$D_1 L_1 + D_2 L_2 = 0$$

This equation indicates that at 1550 nm and for standard telecommunication fibre with $D_1 > 0$ the dispersion compensating fibre (DCF) must have normal dispersion, that is, $D_2 < 0$. Its length must be chosen to satisfy

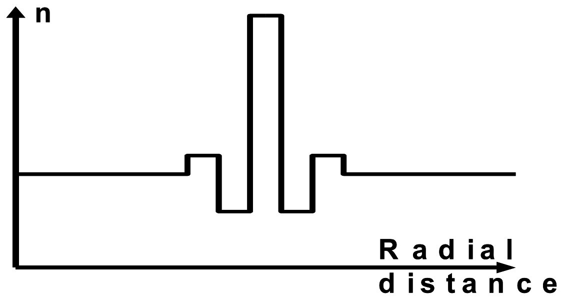
$$L_{DCF} = -\left(\frac{D_{TR}}{D_{DCF}}\right) L_{TR}$$

Where DCF and TR stand for dispersion compensating fibre and transmission fibre respectively.

The required dispersion D_{DCF} should be preferably achieved with the minimum length and the minimum loss. That in turn implies high dispersion compared to the dispersion of the transmission fibre (L_{TR}). In order to achieve the high dispersion and the negative sign the only design parameter is the waveguide dispersion which depends on the geometry of the fibre. Invariably in order to obtain high dispersion the core of the dispersion compensating fibre becomes small leading to relative high splicing loss and because of other design constrains to high bending losses.



The dispersion of a typical silica fibre. The total dispersion is the sum of material, profile and waveguide dispersion. The dispersion compensating fibre is obtained by increasing the waveguide dispersion and matching the slope to that of the material dispersion.



The key performance parameters, dispersion and loss are used to provide a figure of merit for DCF. The loss of the transmission fibre and the dispersion compensating fibre is given by

$$L_{total} = \alpha_{TR}L_{TR} + \alpha_{DCF}L_{DCF} = \left(\alpha_{TR} + \frac{\alpha_{DCF}}{L_{TR}}L_{DCF}\right)L_{TR}$$

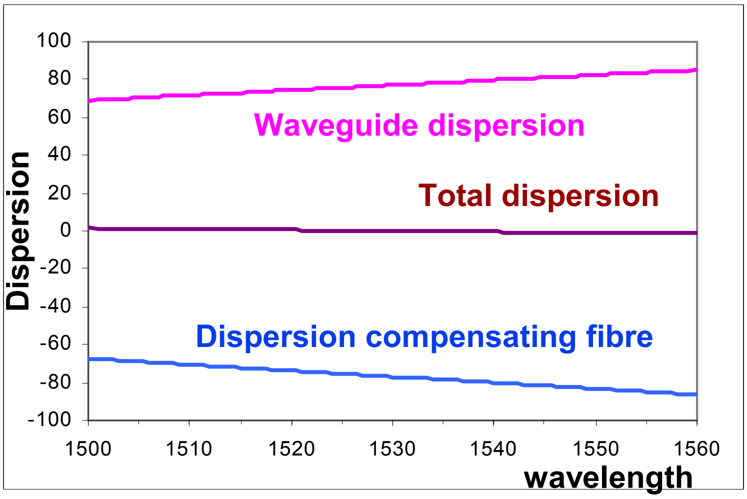
$$= \left(\alpha_{TR} + \frac{\alpha_{DCF}D_{TR}}{|D_{DCF}|}\right)L_{TR} = \left(\alpha_{TR} + \frac{D_{TR}}{\frac{|D_{DCF}|}{\alpha_{DCF}}}\right)L_{TR} = \left(\alpha_{TR} + \frac{D_{TR}}{FOM_{DCF}}\right)L_{TR}$$

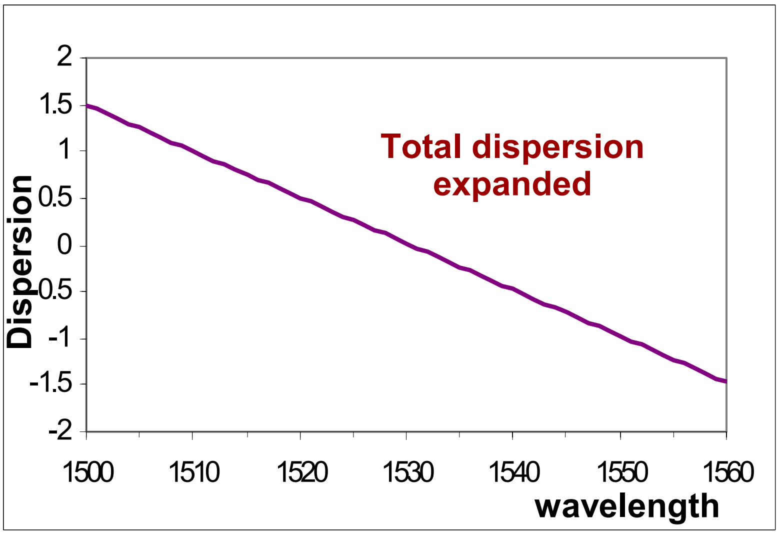
where the figure of merit for dispersion compensating fibres is given by

$$FOM_{DCF} = \frac{|D|}{\alpha}\ \text{ps/nm.dB}$$

The performance index for the DCF highlights the fact that the higher the index the lower the impact of the DCF on the loss of the system. Dispersion of up to 200 ps/km.nm with losses of the order of 0.5 dB have been achieved leading to FOM of 400 ps/nm.dB. Another key parameter of the DCM is its slope. Nominally the slope should match that of the transmission fibre over the operating optical bandwidth and in a single channel system that is not very difficult.

The performance of a typical DCF with standard optical fibre (λ_0 = 1300 nm) is shown below. The dispersion of the standard fibre corresponds to 5 km. That of DCF to one km.The graph of the total dispersion in shown expanded below.

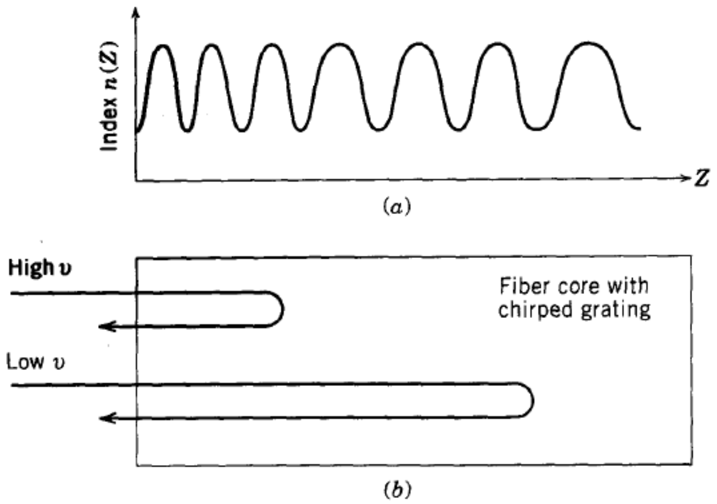


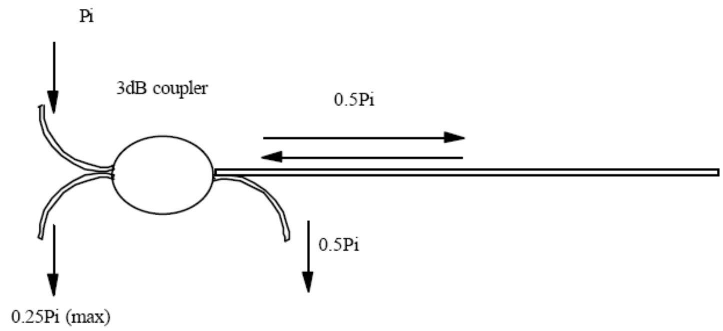


Fibre Bragg Gratings

Possible to engineer short (a few cm) of fibres with compensating dispersion characteristics (of opposite sign to the transmission fibre dispersion) to create a very low net dispersion.

The diagram shows the principle. A periodic variation in refractive index is written into the fibre using the UV sensitivity of the glass. The variation in index is approximately 0.01% of the mean value, and the gratings are written in from one side of the fibre using a UV laser and phase grating, or a holographic interferometry system. The fibre is used as a reflection filter, and therefore a coupler is required at the input. This has a normal 6dB loss.





The transfer function of the filter compensates for the GVD, so that the amplitude reflectivity should be as close to 1 as possible throughout the passband of the filter. The filter time (or group) delay needs to vary with wavelength ie the ps/nm number needs to be large and negative to compensate for the large accumulated positive number from the installed fibre. Instructive to recall results from fibre section. The pulse envelope at a distance z can be approximated by

$$\psi(z, t' + \beta_1 z) = \frac{1}{2\pi}\int \varphi(0, \omega_m)\exp j\left(\omega_m t' - \frac{1}{2}\omega_m^2 \beta_2 z\right) d\omega_m$$

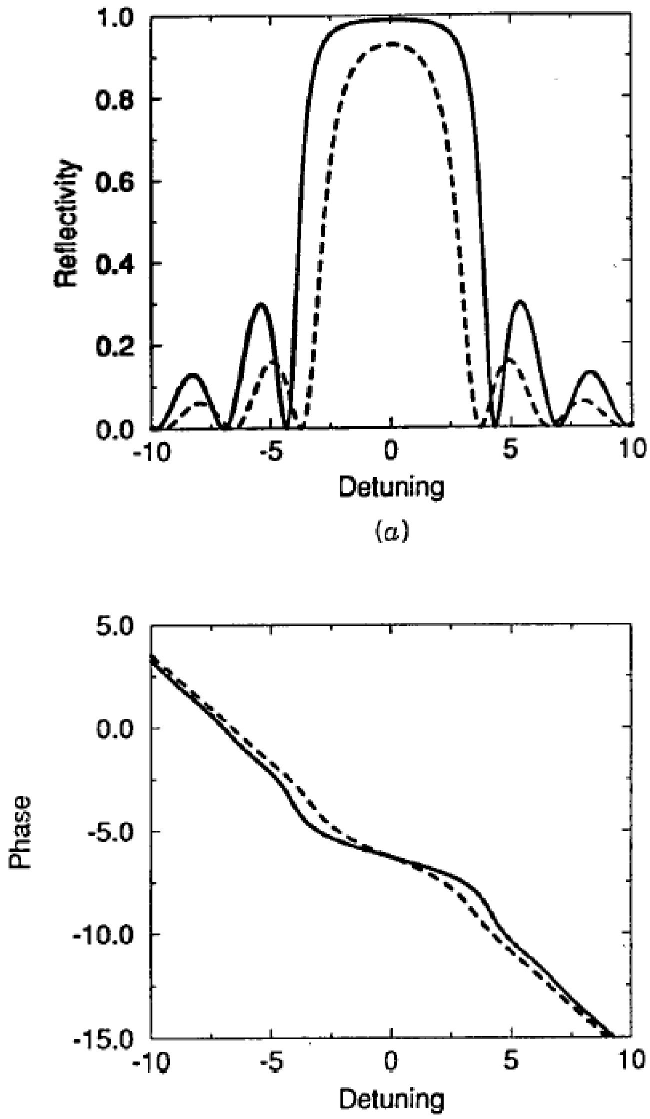
Recall that β_2 is what cause the broadening (Otherwise the expression would be a perfect Fourier Transform). Assuming that the grating response can be expanded using a Tayor series

$$H(j\omega_m) = \exp\left(j\left(\varphi + \omega_m \cdot \frac{d\varphi}{d\omega_m} + \frac{1}{2}\omega_m^2 \frac{d^2\varphi}{d\omega_m^2}\right)\right)$$

The first term is a constant and the second is a constant delay (as its proportional to frequency). The second derivative of phase can be used to compensate for the β_2 term, and careful design can be used for the higher order terms too.

Grating Theory

In general design is undertaken numerically, and only in a few cases is it possible to obtain analytical solutions. See Agrawal p 441 if you are interested in how solutions are obtained. The figure shows the phase and amplitude transfer function for a reflection fibre grating with a constant pitch and reflectivity. As can be seen the phase changes relatively linearly over the band with high reflectivity. This implies a low (close to zero) second derivative.



Chirped Fibre Bragg Grating

•The refractive index in a fibre is modified to vary periodically along the fibre - apodized grating

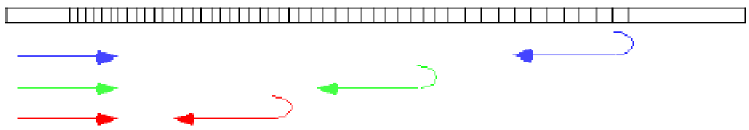
- fabricated using holographic UV exposure

• If the periodicity of the refractive index n(z) is changed along the fibre, we have a chirped fibre grating

– the Bragg wavelength, λ_B, at which the signal is reflected, will vary along the fibre

$$\lambda_B = 2n\Lambda$$

– different frequency components experience different time delays



•It can be shown that the grating dispersion is

$$D_g = \frac{2n}{c\Delta\lambda}$$

where $\Delta\lambda$ is the difference in λ_B at the two ends of the grating.

Worked Example

A chirped Fibre Bragg grating of length 10cm has $\Delta\lambda$ = 0.2nm. For what length of standard communications fibre will this cancel the dispersion?

Refractive index of silica fibre $n = 1.5$

$$D_g = \frac{2n}{c\Delta\lambda} = \frac{2 \times 1.5}{3 \times 10^8 \times 0.2 \times 10^{-9}} = 500 \text{ ps/(nm.cm)}$$

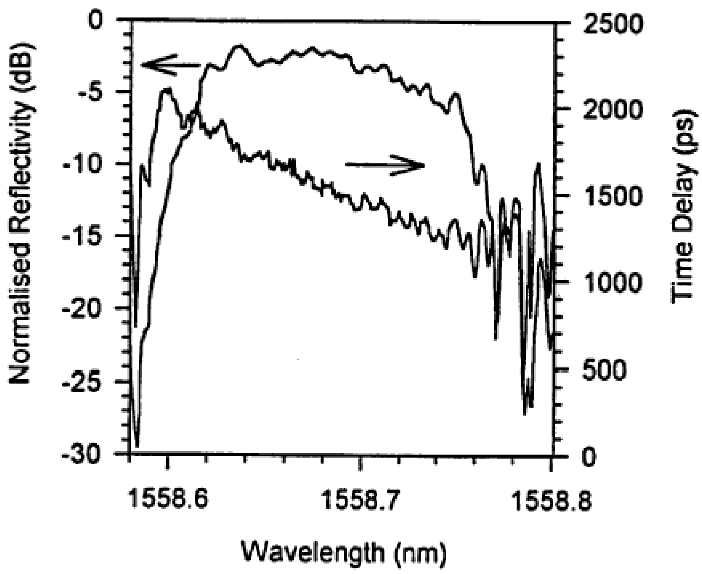
10cm grating has dispersion of 5000ps/nm

Dispersion of standard fibre is 17ps/(nm.km)

Length of standard fibre for which dispersion is cancelled is

$$L = \frac{5000}{17} = 294 \text{ km}$$

Finally, experimental results from an actual structure are shown

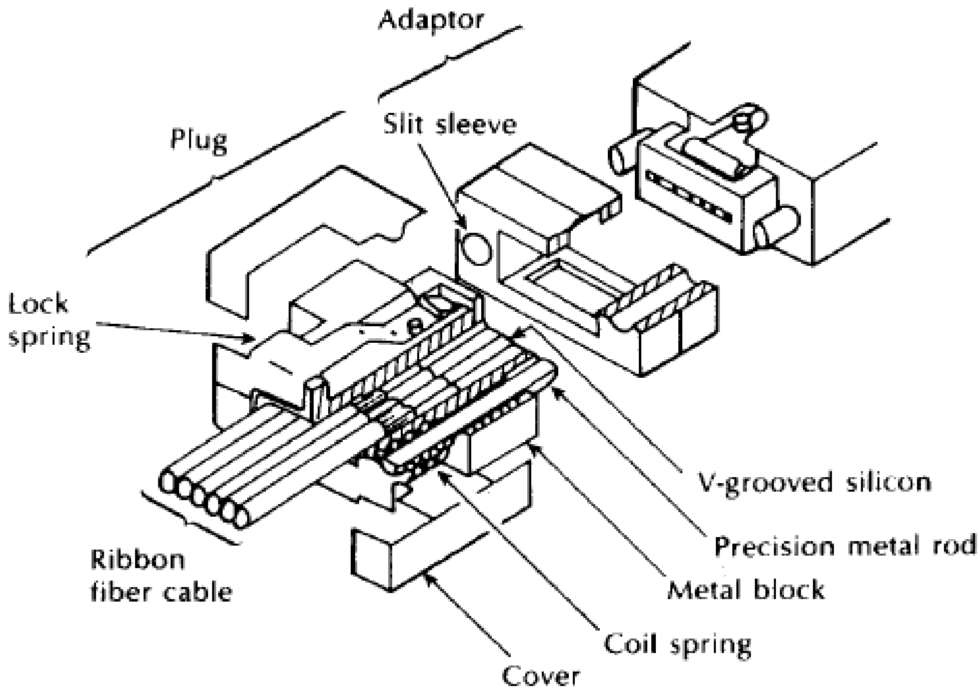


- •dispersion of 5000ps/nm
- •optical bandwidth 0.12nm - sufficient for 10 Gbit/s

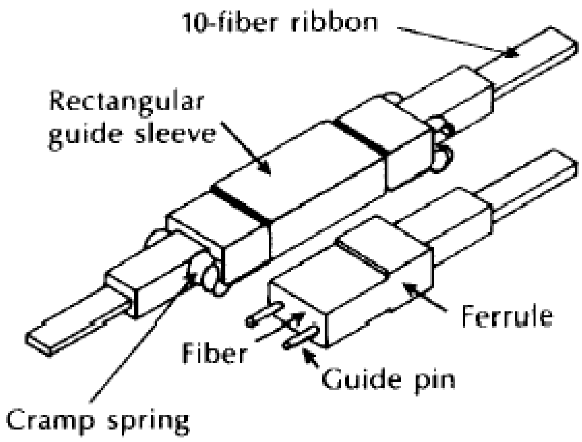
Connecting Fibres

•There are a large range of connector standards for single and multimode fibres that are manufactured to a very high tolerance. They are typically used at ends of systems and in LANs.

Figures below show some typical connector types. Note the use of lens beam expanders at the fibre ends to reduce the required alignment tolerance.

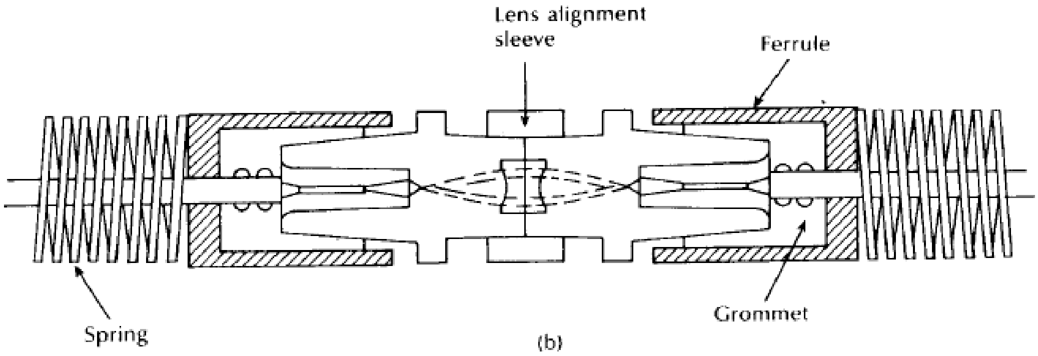


(a)



(b)

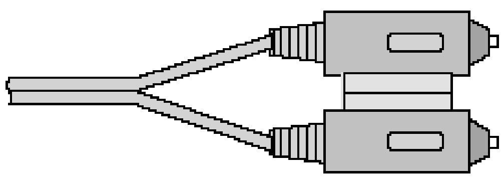
Multi-fibre connector



Single fibre connector

Following table gives typical losses for various fibre connectors. Most commonly used is the SC connector.

connector characteristics			
Connector Type	**Insertion Loss (MM) Typical**	**Insertion Loss (SM) Typical**	**Return Loss Typical**
ST	0.25 dB	0.2 dB	40 dB
SC	0.25 dB	0.2 dB	40 dB
SMA	1.5 dB		
FSD	0.6 dB		
FC	0.25 dB	0.2 dB	40 dB
D4	0.25 dB	0.2 dB	35 dB
DIN	0.25 dB	0.2 dB	40 dB
Biconic	0.6 dB	0.3 dB	30 dB



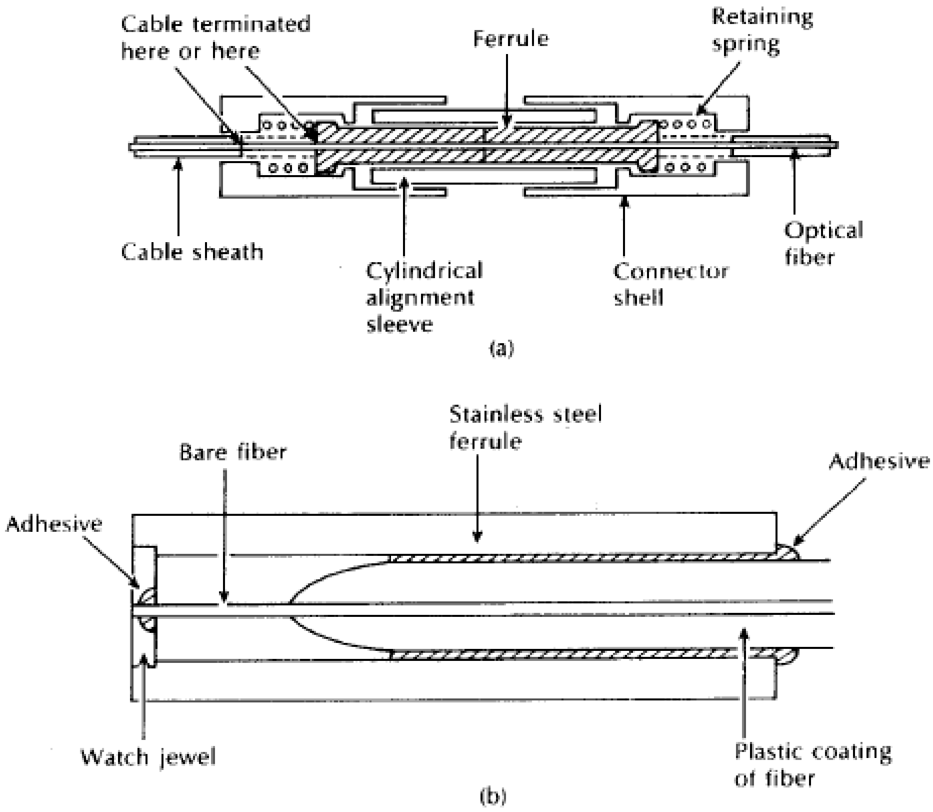
An SC duplex connector

Fibre splicing

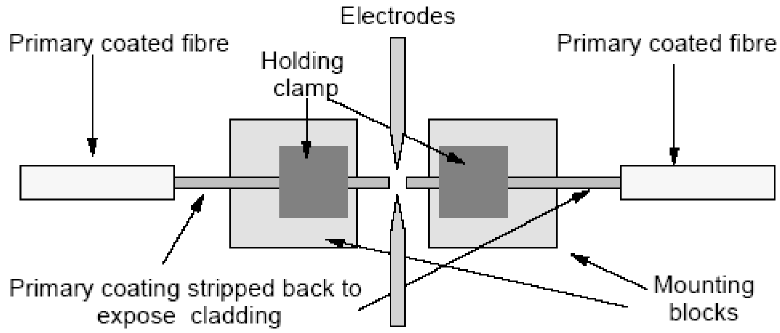
•Used for permanent fibre joins

•High precision mechanical and gluing techniques

•Fusion splicing, used in telecomms and other long haul applications. Lowest loss and high reliability.



Mechanical fibre splices



Fusion splicing - schematic

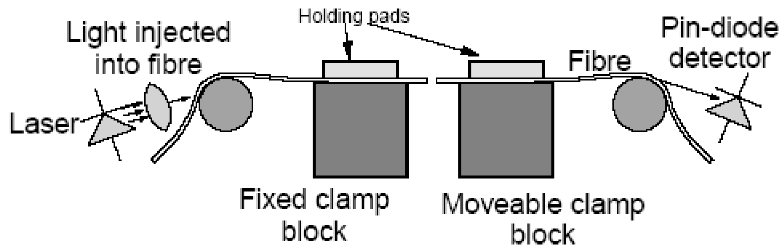
Following is sequence of operations for a fusion splicer:

1. Each fibre is stripped of its primary coating and the end cleaved such that it is square.

2. The fibre ends are positioned a few mm from one another and clamped to positioning blocks. There is often a groove provided in the mounting block to aid in correct alignment.

3. The fibre ends are then aligned with one another and brought closer together.

4. When alignment is satisfactory an electric arc is started between the two electrodes and the fibres brought into contact. Heat from the arc melts the glass and the join is made.



Fibre alignment using optical feedback

•A common method of fibre alignment is illustrated above. The primary coating is stripped from the fibre for several cm from the end.

•When it is mounted in the fusion splicer the fibre is bent tightly around two mandrels (one at each end).

•Light from a laser (or LED) is focused onto a spot on the fibre bend such that some of it enters the core in a guided mode.

•At the other side of the splicer there is an optical detector positioned to capture light radiated from the tight bend.

•One end of the fibre is moved (by moving the mounting block with a piezo-electric actuator) until the output of the detector is at a maximum.

Sources

Specifying a source

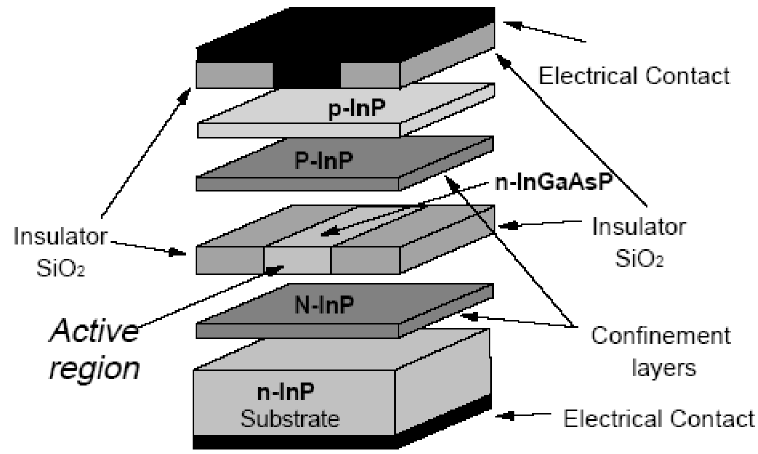
•Options are either Laser or LED. Structures and drive circuitry required are briefly described below.

Semiconductor Lasers

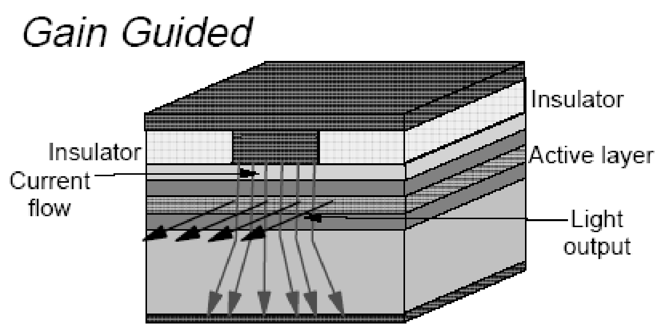
In-Plane Lasers - The Fabry-Perot (FP) Laser

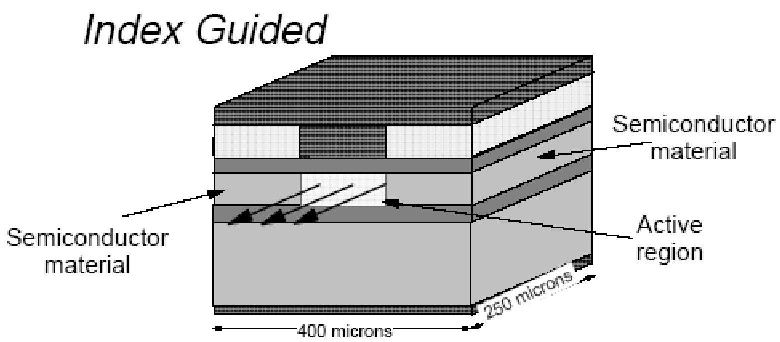
•based on the double heterojunction

•gain guided and index guided



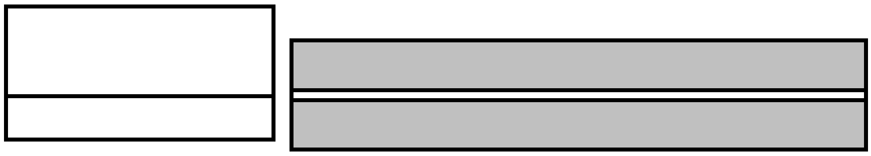
Exploded view of double heterojunction



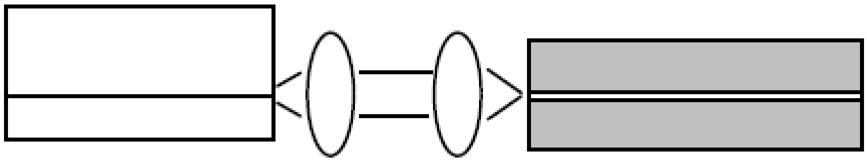


Coupling to Fibres

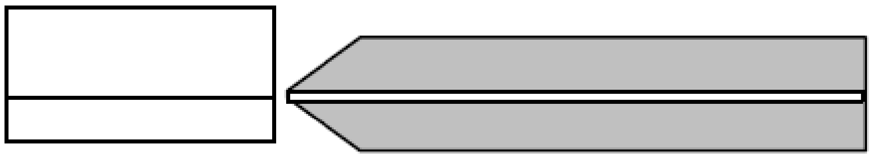
FP lasers can be coupled to fibres in several ways, illustrated in the following figures



Butt-coupling



Lensed coupling



Fibre taper

Typical Laser Characteristics

- Operate at 850nm, 1300nm, 1550nm
- Required for single mode systems
- Used to obtain high launch powers or linear modulation in multimode systems

Advantages

- High launch power / high speed modulation
- Narrow linewidth

Disadvantages

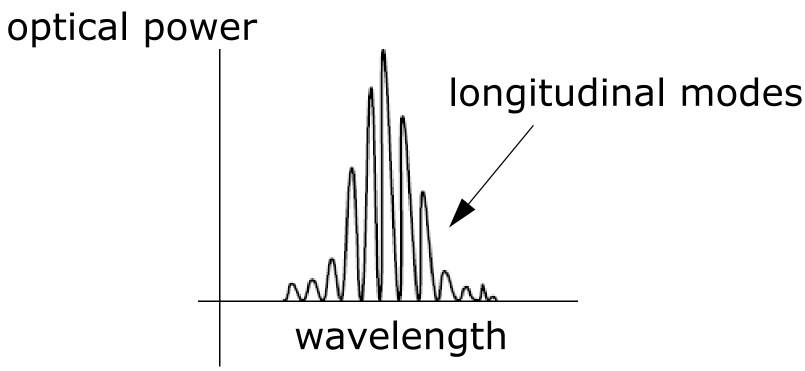
- Can be expensive
- Require temperature control/ power monitoring to control launch power and set bias and modulation

Applications

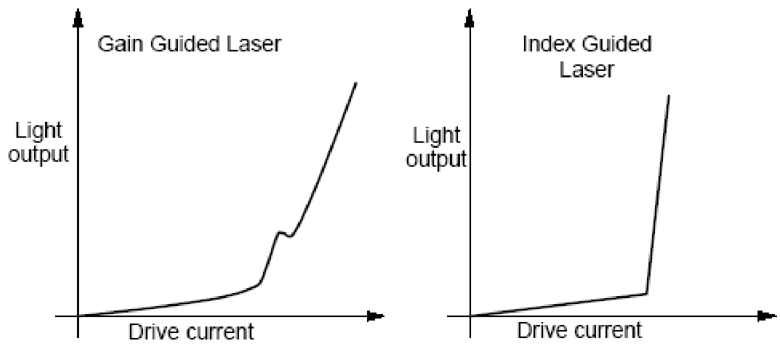
- Telecommunications (rather than data network application)

Output Characteristics of FP lasers

The following figure shows the typical output spectrum of a gain-guided Fabry-Perot laser. The asymmetry of the line structure is due to Raman scattering effects within the cavity.

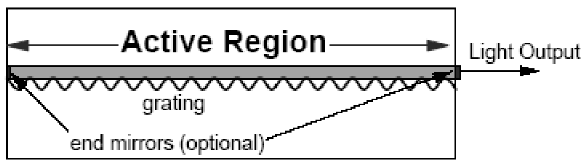


The light-current output characteristics of an index guided and gain guided laser are shown below.



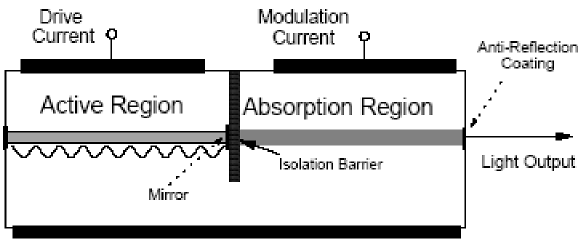
Note that the non-linear output of the gain guided laser makes it unsuitable for analogue systems, but is suitable for digital on-off keying systems (OOK).

Distributed Feedback Bragg (DFB) reflector laser



DFB laser with integrated electro-absorption modulator

External modulators are used for high speeds. The following figure shows a DFB laser with an integrated electro-absorption (EA) modulator.



DFB laser with integrated EA modulator

Vertical Cavity Surface Emitting Lasers (VCSEL)

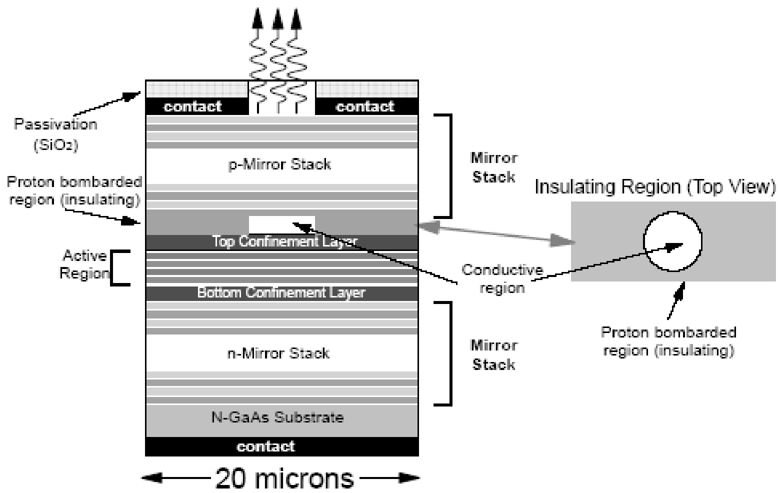
•Only one longitudinal mode-due to short cavity

•Low threshold- small active volume

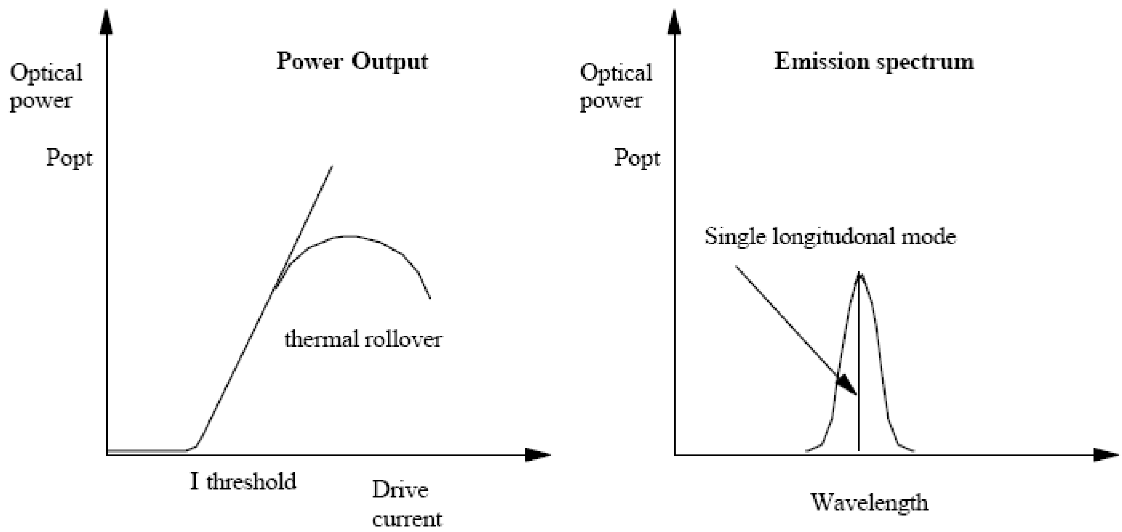
•Low cost- make a whole wafer at once and test before cleaving

•850nm,980nm possible 1300 in development ,1550 in development

•Data comms applications



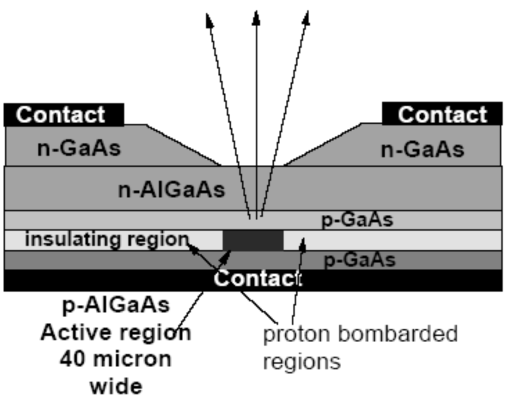
VCSEL structure

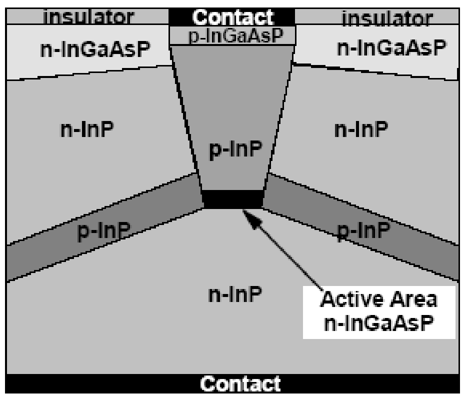


Output characteristics of VCSEL

Light Emitting Diodes (LEDs)

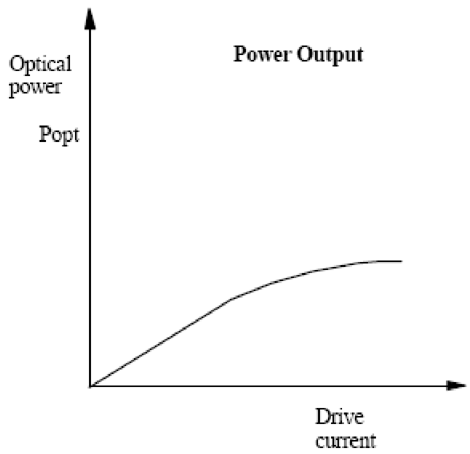
Simpler geometry than a laser, and easier to couple into multimode fibres. Two common types: Burrus diodes and edge emitting diodes.

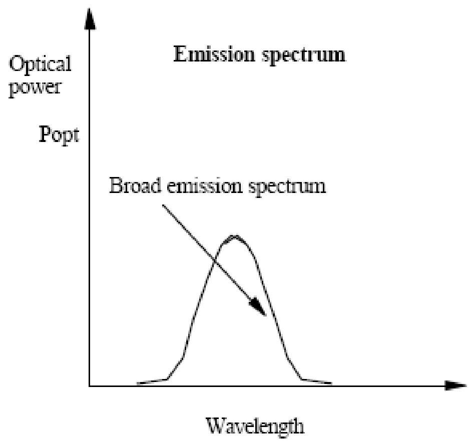




Output Characteristics

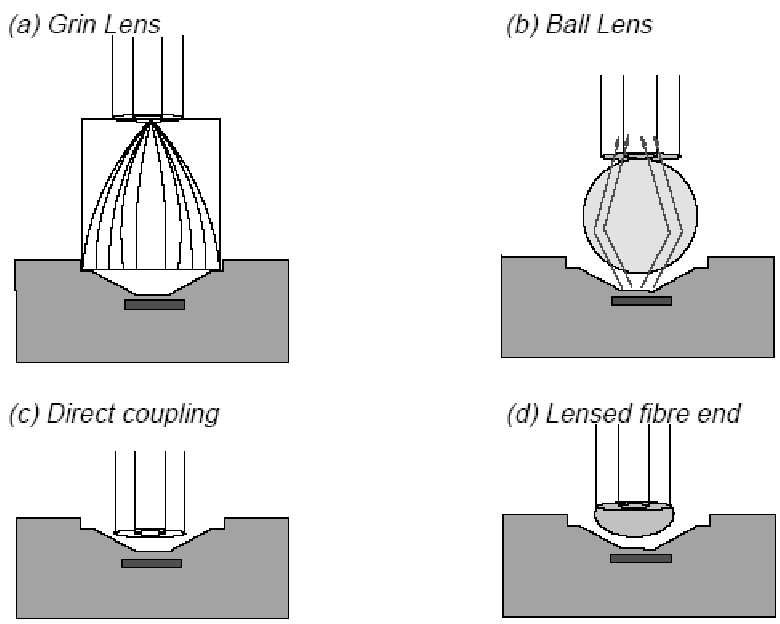
LEDs have a linear LI curve and a broad output spectrum, typically 50nm as shown on the figure below





Coupling LEDs to a fibre

Coupling light output to a fibre is the most difficult and costly part of manufacturing a real LED or laser device. Four common methods are illustrated below:



System Considerations

The table shows a comparison of the options for sources.

Sources	LED	LED/LASER	LASER
Detector	PIN detector	PIN/APD	APD/PIN
Wavelength of operation	Visible/0.85 µm	0.85/1.3 µm	1.3/1.55 µm
Uses	LANs/Computer interconnect	LANs/WANS (FDDI)	Telecommunications applications
Cost	Low speed/cost	Medium speed/cost	High speed/cost

Note the following points

•Single mode systems need semiconductor lasers that emit a single mode. The field shape from the laser must match that accepted by the fibre. All energy that doesn't 'fit' is wasted.

•Lasers are much more efficient than LEDs. 50% compared with about 1 or 2%. Therefore used in multimode systems to launch more power into the fibre.

•Lasers for high performance telecommunications systems are expensive, both due to laser and control circuitry required for high performance.

•LEDs are only suitable for multimode systems. They are cheap, and can be coupled (attached) to the fibre relatively easily.

Detectors

Material Systems

Material Systems

•500-1000nm band

Silicon PIN diodes operate over a range of 500 to 1120 nm as silicon has a bandgap energy of 1.11 eV. Since silicon technology is very low cost silicon is the material of choice in this band. However, silicon is an indirect bandgap material (at the wavelengths we are interested in) and this makes it relatively inefficient. (Silicon PIN diodes are not as sensitive as PIN diodes made from other materials in this band.) This is the same characteristic that prevents the use of silicon for practical lasers.

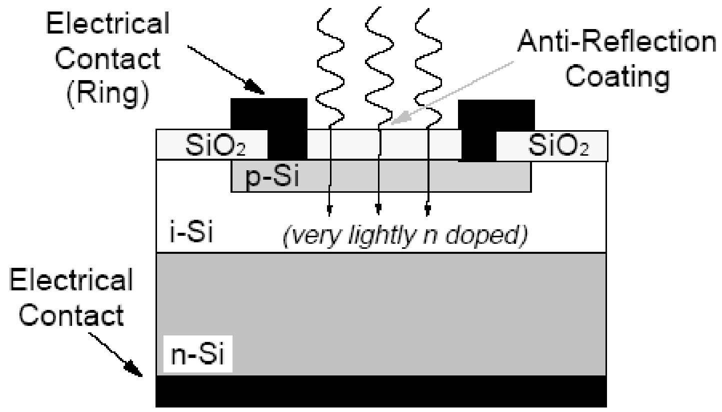
•1300 nm (1250 nm to 1400 nm) Band

In this band indium gallium arsenide phosphide (InGaAsP) and germanium can be used. Germanium has a lower bandgap energy (0.67 eV versus 0.89 eV for InGaAsP) and hence it can theoretically be used at longer wavelengths. However, other effects in Ge limit it to wavelengths below 1400 nm. InGaAsP is significantly more expensive than Ge but it is also significantly more efficient (devices are more sensitive).

•1550 nm Band (1500 nm to 1600 nm)

The material used here is usually InGaAs (indium gallium arsenide). InGaAs has a bandgap energy of 0.77 eV.

PIN photodetectors

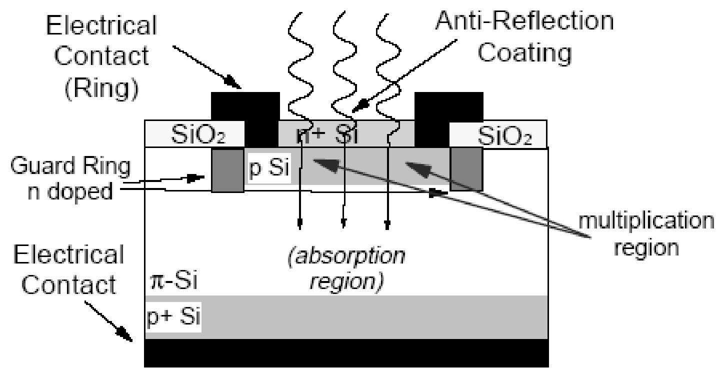


•Standard photodetector for low/medium performance systems. Have medium sensitivity - see figure above.

•Operates with reverse bias - 10V or so.

•Responsivities of 0.2-0.6 A/W

APD detectors

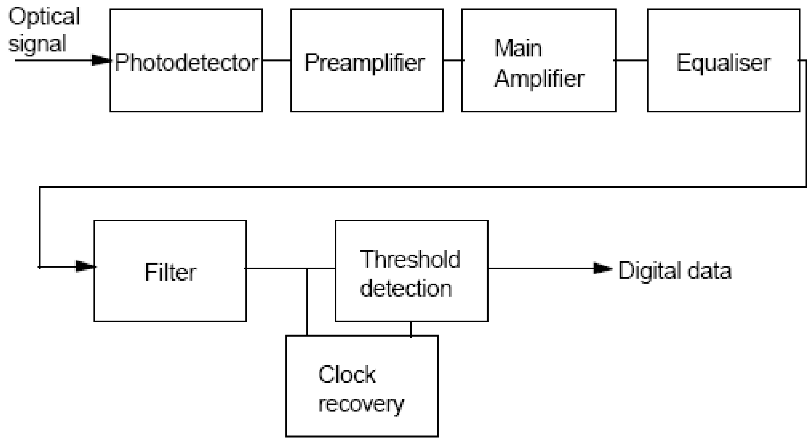


•Used in high performance telecommunications systems. Receiver structures are often 10dB or so more sensitive due to avalanche gain. Structures are more expensive to grow and need high DC bias (50-400V) and control to get avalanche effect.

•Avalanche process introduces excess noise.

•Figure shows typical structure. Guard ring is to stop breakdown of insulating depletion width.

Receivers



Normally don't buy a detector on its own. Detector and associated circuitry are combined to create a receiver that offers light in and a data out. Figure above shows a typical block diagram

- **Detector** converts photons into photocurrent
- **Preamplifier** converts photocurrent into a voltage signal, usually of the order of millivolts.
- **Main Amplifier** has a gain of 10-20dB and brings signal up to the order of volts. Often these amplifiers have Automatic Gain Control (AGC) to bring the input signal to a standard level that the decision circuits can then use to decide whether data is binary zero or one.
- **Equaliser** is used compensate for distortions introduced by the fibre dispersion, and the preamplifier and amplifier if necessary.
- **Filter** is used to optimise the signal spectrum, and reduce intersymbol interference.
- **Clock recovery** circuit recovers a clock signal from the data waveform, so that retimed digital data can be reproduced. Often special **Line Codes** are used on the optical fibre to preserve timing information. For example, it would be difficult to recover timing from a long train of binary '1' pulses.
- **Threshold detection** is where the received analogue signal is converted to digital data, using the timing information extracted by the clock recovery circuit.

Specifying a receiver

Error performance

•Bit Error Rate (BER) 1 error in each 10^9 on the link means 10^{-9} BER. In general telecommunications quoted at 9 10- (allowed because of error checking at higher levels in the system).

•Computers and data closer to 10^{-12}, as limited or no error checking. (a wrong bit on a computer bus can have big implications).

Quantum Limited Detection

Photons generated by the source have a Poisson distribution, so there is a small but finite chance that you meant to generate a pulse of light, but don't get one. The error caused by this always exists and is called the **quantum limit**. The probability of generating a number of photons z is given by

$$p(z) = \frac{z_m^{\ z} \exp - z_m}{z!}$$

where z_m is the average number of photons. This means that there is a small finite probability that no photons will be emitted (z=0) and this causes error. For a BER of 10^{-9} and with z=0 we have

$$10^{-9} = \exp - z_m$$

which gives

$$z_m \approx 21 \text{ photons}$$

So **21 photons per bit** is the **quantum limit**. This corresponds to -77dBm at 1000nm. (300 per bit is a more normal comms type number).

Real receivers

In practice the detection and amplification process introduces electrical noise, as well as the quantum fluctuations described above.

Noise sources

1. Shot noise. This is quantum noise from the detection.

2. Amplifier noise, expressed as Noise Figure. This is in excess of the thermal noise in the amplifier impedances.

3. Thermal noise (from the resistors in the circuit).

4. Avalanche gain excess noise. The avalanche multiplication in an APD introduces extra noise (not in PIN structures though).

Possible to analyse the noise performance of the photodetector and preamplifier process, and this determines the overall performance.

Preamplifiers

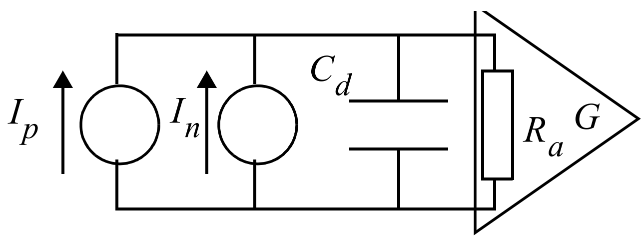
•The photodiode generates a photocurrent and this is converted into a voltage, which is amplified by a preamplifier then further amplifiers.

•First stage in the system, and as such the most important, as noise introduced here is amplified by the entire system.

•Most receivers use a transimpedance preamplifier. This converts the input photocurrent into a voltage. Offers high performance and a simple circuit. Most major semiconductor manufacturers offer a fibre optic chipset with a transimpedance preamp.

Noise calculation and analysis

Figure shows the equivalent circuit of a typical photodetector and preamplifier



where C_d = detector capacitance, R_a = amplifier resistance, I_n = noise current, I_p = peak photocurrent and G = amplifier gain.

Most circuits can be reduced to this form. All noise sources are 'moved' to the input stage of the amplifier, so the effects of the noise can be calculated and the rest of the system assumed noiseless (or at least included easily) in the analysis.

Bandwidth

The electrical bandwidth of the circuit is set by the 'RC' rise-time:

$$B = \frac{1}{2\pi R_a C_d}$$

where R_a is the equivalent resistance of the transimpedance amplifier, and is given by

$$R_a = \frac{R_f}{G}$$

where G is the open loop gain of the transimpedance amplifier, and R_f is the amplifier feedback resistance.

Noise

Shot noise is generated by the detection process, and is modelled as a noise current I_s^2

$$I_s^2 = 2eB(I_p + I_d)$$

where e is the electron charge, and I_d is the photodetector dark current. Note that it is proportional to the bandwidth and total photocurrent.

Thermal and amplifier noise

A resistor generates a thermal noise current , dependent on its temperature and resistance value

$$I_t^2 = \frac{4KTB}{R_f}$$

where K is Boltzmann's constant, and T is the temperature in Kelvin. An ideal amplifier has input and output impedances, so will always generate thermal noise. A real amplifier introduces extra noise, and this effect, along with the thermal noise can be modelled as one expression, so that

$$I_t^2 = \frac{4KTBF_N}{R_f}$$

where F_N is the amplifier noise figure.

Signal to noise ratio

The **peak** signal **power** to be amplified is simply

$$P_{sig} = I_P^2 R_a$$

The noise power to be amplified is

$$P_N = \left(2eB(I_p + I_d) + \frac{4KTBF_N}{R_f}\right)R_a$$

The electrical Signal to Noise Ratio (SNR) is then

$$SNR = \frac{I_P^2}{2e(I_p + I_d)B + \frac{4KTBF_N}{R_f}}$$

Modifications

Avalanche photodiodes multiply the photocurrent by a factor M thus increasing the signal. However, the multiplication generates extra shot noise, represented by a factor in the SNR expression. This becomes

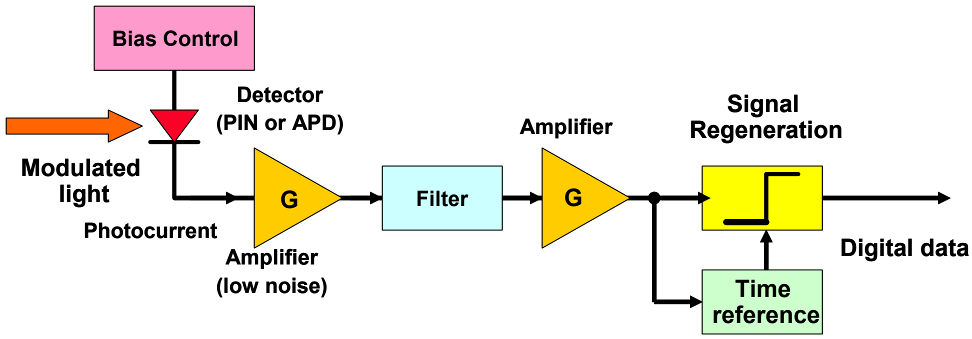
$$SNR = \frac{M^2 I_P^2}{2e(I_p + I_d)BM^{2+x} + \frac{4KTBF_N}{R_f}}$$

where x describes the extra shot noise (x=0.3-0.5 for Silicon APDs, 0.7-1.0 for Ge and III-V devices).

Note that for APD receivers there is an optimum value of M, the avalanche gain, that maximises the SNR for a particular input power level. M can be varied by altering photodiode bias, and sophisticated receivers might do this.

Relating SNR to BER

The architecture of an intensity modulation direct detection digital receiver (IM-DD) is as shown below.

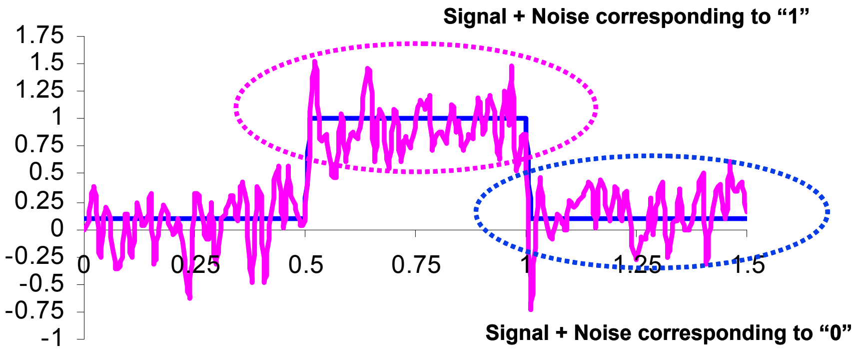


The theory of optical reception deals with the problem of detecting a signal delivered to the detector through an optical channel and identifying the optimum signal processing required to deliver the information with the highest possible quality.

There area number of approaches to defining the quality of a signal. In the context of an IM-DD receiver with binary format the typical performance index that defines the quality of the delivered signal is the Bit Error Rate (BER).

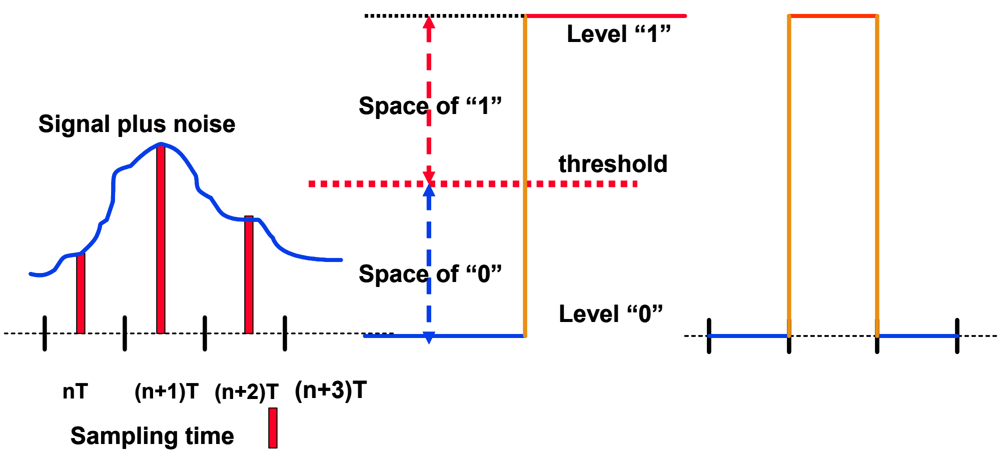
The BER is defines as the mean number of bits in error per second

The major cause of errors in detection is the presence of noise.The noise can be injected into the signal processing path from a large number of sources and one of the key issues in optical receiver theory is to identify these sources and quantify their impact on the detection process. For a binary format an IM - DD receiver must make a decision about the presence or absence of a digit (bit). To achieve this objective the receiver uses a ‟ threshold” level (voltage or current) against which the arriving information is judged.



The signal plus noise ensemble

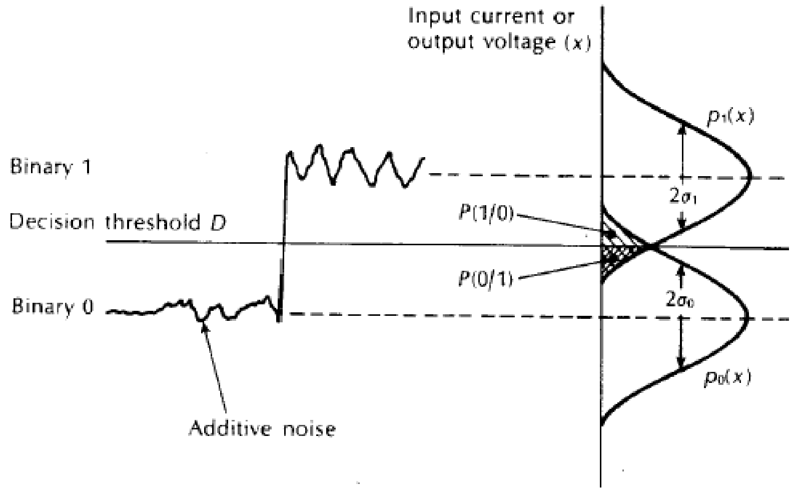
The figure below illustrates the concept of threshold detection for an IM-DD binary receiver



•It is possible to relate electrical SNR to Bit-error-Rate.

•Error when binary on level drops below the decision point (midway between the 1 and 0) level) or when zero level comes above the decision point.

•If the signal level variation follows a Gaussian distribution (i.e. noise statistics are Gaussian) then the error probability is equal to the shaded areas in the figure below.



The probability of an error is given by

$$p_{error} = p(1)p(0|1) + p(0)p(1|0)$$

where $p(1)$ is the probability that a one was transmitted, $p(0|1)$ is the probability that a zero was received, given it should have been a one , $p(0)$ is the prob. that a zero was transmitted, $p(1|0)$ is the prob. that a one was received given that it should have been a zero.

The noise probability distributions are

$$p_1(i) = \frac{1}{\sqrt{2\pi i_n^2}} \exp\left(-\frac{(i - i_{sig})^2}{2i_n^2}\right)$$

$$p_0(i) = \frac{1}{\sqrt{2\pi i_n^2}} \exp\left(-\frac{i^2}{2i_n^2}\right)$$

where we assume $\sigma_0^2 = \sigma_1^2 = i_n^2$

The shaded areas are given by

$$p(0|1) = \int_{-\infty}^{i_{dec}} p_1(i)di$$

and

$$p(1|0) = \int_{i_{dec}}^{\infty} p_0(i)di$$

These functions can be expressed in terms of the complementary error function defined as

$$erfc(z) = \frac{2}{\pi}\int_{-\infty}^{\infty} \exp(-x^2)dx$$

$$p(0|1) = \frac{1}{2}erfc\left(\frac{i_{sig} - i_{dec}}{\sqrt{2i_n^2}}\right)$$

and

$$p(1|0) = \frac{1}{2}erfc\left(\frac{i_{dec}}{\sqrt{2i_n^2}}\right)$$

The optimum point (for equiprobable ones and zeros) of decision is halfway between the one and zero points.

If $i_{dec} = \frac{1}{2}i_{sig}$ then

$$p_{error} = \frac{1}{2}erfc\left(\frac{i_{sig}}{2\sqrt{2i_n^2}}\right)$$

In terms of electrical signal to noise ratio this becomes

$$p_{error} = \frac{1}{2}erfc\left(\frac{1}{2} \cdot \sqrt{\frac{SNR}{2}}\right)$$

It is common to define an average signal power, rather than the peak value used here, so that an average SNR can be defined (as a half of the value used here).

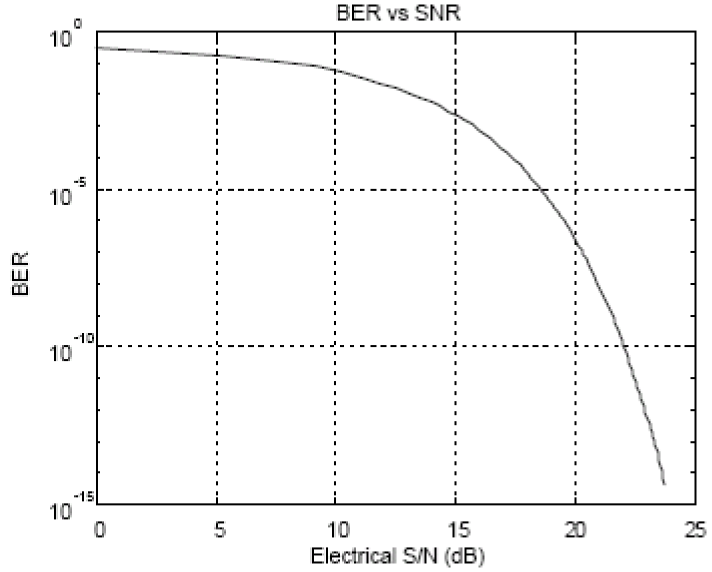
A parameter that is often used is the *Q* parameter, defined as

$$Q = \frac{\sqrt{SNR}}{2}$$

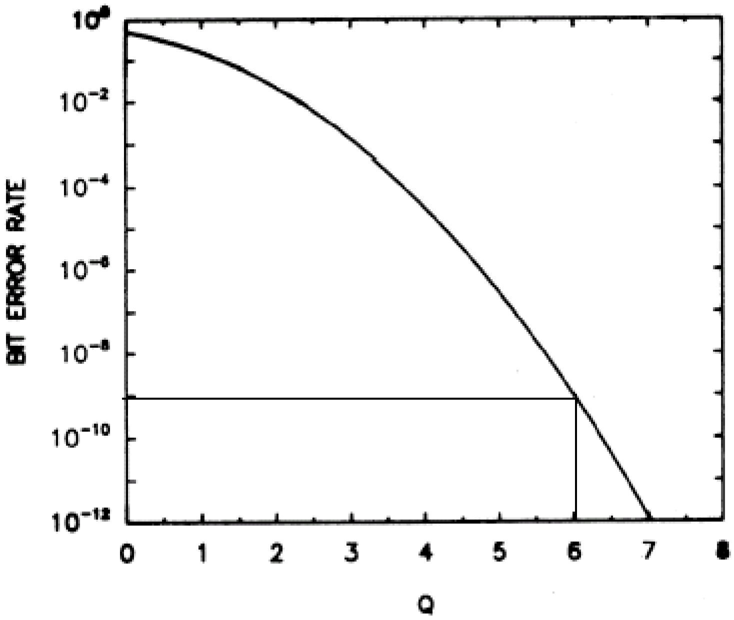
so that the BER is given by

$$p_{error} = erfc\left(\frac{Q}{\sqrt{2}}\right)$$

Following figure shows a graph of electrical SNR (in dB) vs BER for return to zero pulses.



The same information is shown below for BER vs Q .



The receiver sensitivity specifies the optical power received, from which the electrical SNR can be calculated, and hence the BER from the graph. Need 21.6dB SNR for BER of 10^{-9}. This corresponds to a Q value of $Q = 6$.

Receiver Specifications

•Sensitivity-specified in dBm at a certain Bit Rate at a certain Bit Error Rate.

•Dynamic range-specified in dB or dBm. This is the range of input optical power the receiver can accept and still function.

•Wavelength. This is a function of what detector material/ structure is used.

•Standard. Often receivers are optimised for a particular transmission standard (in terms of pulse shape and code used).

System Examples

FDDI (fibre distributed data interface)

•100Mb/s using 125Mbaud transmission (data is converted to a line code that makes timing recovery easier).

•100km of dual ring fibre (two fibres for redundancy).

•500 nodes, up to 2km between nodes.

•Token passing protocol.

•1300nm system InGaAsP LED sources.

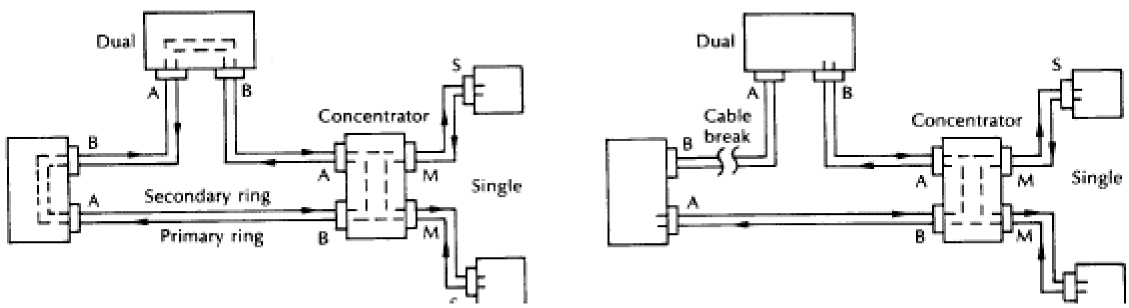
•62.5/125 μm and 85/125 μm graded index fibre, Distance bandwidth product >400MHz km.

•Germanium or InGaAs PIN detector, receiver sensitivity of -31dBm at BER of 10^{-12}.

•11dB link losses allowed, overall BER 10^{-9}.

•FDDI to the workstation becoming common, Integrated FDDI transceivers available for £80 or so (RS catalogue).

Figure shows how a network reconfigures on fault detection



FDDI fault reconfiguring

Gigabit Ethernet

•LAN standard for fibre based networks

•8B10B coding converts 8 bit symbol to 10 bit symbol for improved transmission, so 1.25GBaud line rate

•Must work with installed 62/12μm fibre (up to 550m)

•1000BASE-SX (780-860nm lasers) links up to 275m (worst case) on 62/125 fibre (too short)

•1000BASE-LX 1300nm (FP lasers) singlemode fibres up to 5km, 550m over 52/125 and 62/125

Computer interconnect

•Telecommunications market demands high reliability, 30 year lifetime, cost not critical as revenue so high. Fibre systems dominant.

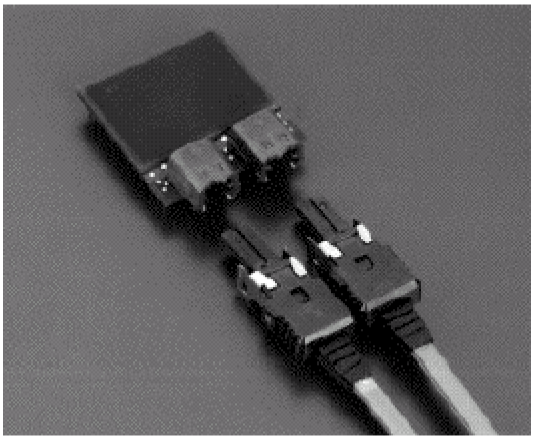
•Off-chip communications is one of bottlenecks where optical interconnect may have advantages over electrical.

•Computer market demands low cost, plug together, high performance and error free links. Projects such as POLO, and Optobus aim to make inroads.

•Possible due to development of arrays of Vertical Cavity Surface Emitting Lasers, that will be low cost, and compatible with fibre ribbon cable.

The figure below and the table show Motorola's offering 'Optobus'. Uses 850nm VCSELs and a multi-chip module transceiver. Asynchronous data transmission at 400Mb/s/channel x10 channels.

Target cost is $100 per module.



Optobus Transceiver Unit

Symbol	Parameter	Min	Typ	Max	Unit
V	Power Supply Voltage	4.75	5.0	5.25	V
PW	Power Consumption		1.35		W
Vout	Output Voltage Swing (Note 1)		0.25		V
Vin	Input Voltage Swing	0.25		1.2	V
CMR	Input Common Mode Range	VCC-2.25		VCC	V
Vp	Output Pull-Up Voltage (Note 2)	3.00		5.25	V
	Optical Wavelength	835	850	860	nm
PO	Optical Power (Peak)			800	uW/Chan
	Data Rate/Channel			400	Mbits/s
	Jitter (Note 3) (Total) Random/Channel Data Dependent Pulse Width Distortion		650 50 100 500		ps
	Channel to Channel Skew		200		ps
	Bit Error Rate		10^{-14}		
tr, tf	Rise/Fall Time (Receiver Output 20-80% Levels)		600		ps
	Optical Connects/Disconnects		250		
	Operating Temperature Range	0	35	70	°C
	Distance			300	Meters

Optobus specification

Telecommunications systems

•10Gb/s systems now available (single colour) (Nortel etc) with 40Gb/s advertised as available soon (lucent)

•Launch power of 0dBm or so into fibre.

•Singlemode fibre 8/125 μm, fusion spliced. Typical installed fibre loss 0.2dB/km, Dispersion 16ps/km

•Receiver using InGaAsP APD, sensitivity of -30 to -40dBm.

Table below gives line rate standards:

Synchronous Digital Hierachy (SDH)	Synchronous Optical Network (SONET)	'OC'
	STS-1 51.84 Mb/s	OC-1 51.84 Mb/s
STM-1 155.52Mb/s (2430bytes/ 125 us frame)	STS-3 155.52 Mb/s	
STM-n n*155.52Mb/s	STS-n n*51.84 Mb/s	OC-n n*51.84 Mb/s
		OC-48 2.4Gb/s
		OC-192 10 Gb/s

•STM (Synchronous Transport Mechanism) STS Synchronous Transport Structure.

•Single line capacity is increasing but Wavelength Division Multiplexing becoming the norm for new systems and network.

•Following Table shows growth in capacity for inland systems.

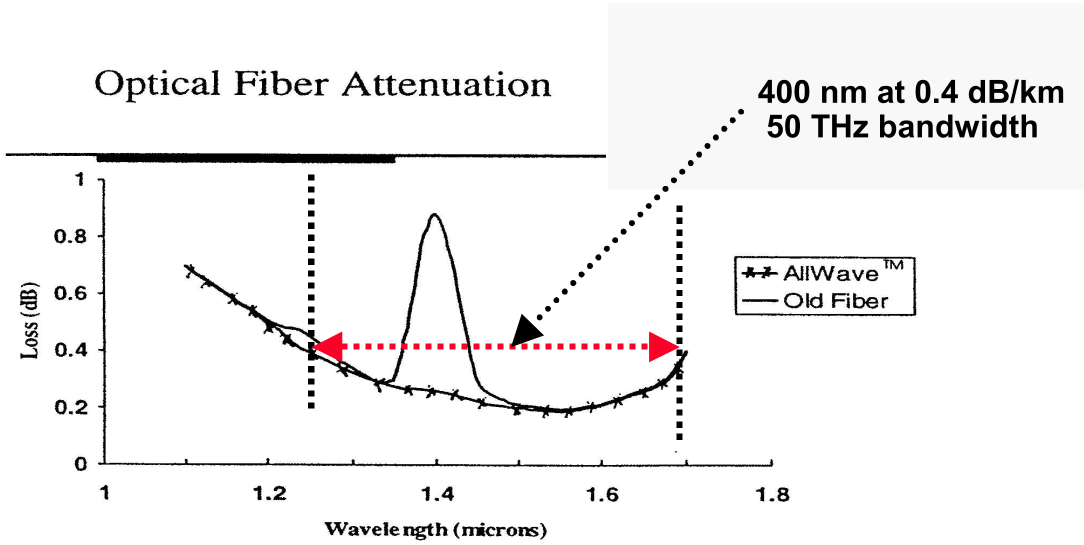
System	Year	λ (μm)	B (Mb/s)	L (km)	Voice Channels
FT–3	1980	0.825	45	< 10	672
FT–3C	1983	0.825	90	< 15	1,344
FT–3X	1984	1.30	180	< 25	2,688
FT–G	1985	1.30	417	< 40	6,048
FT–G-1.7	1987	1.30	1,668	< 46	24,192
STM–16	1991	1.55	2,488	< 85	32,256
STM–64	1996	1.55	9,953	< 90	129,024

Data rate increase for fibre optic systems

Wavelength Division Multiplexing (WDM)

The Information Capacity of Optical Fibre

The information carrying capabilities of optical fibre can be judged to first order analysis from the loss characteristics. From the figure below one can see that modern fibre has a very large bandwidth.



Improvements in fiber fabrication results in a much larger transmission window:	< 0.40 dB/km	400 nm
	<0.30 dB/km	300 nm
	<0.25 dB/km	200 nm

For example at a loss of 0.4 dB/km the bandwidth is around 50 THz. An immense capacity capability and the key question now is:

How can this capacity be used?

The straight forward approach is to use Time Division Multiplexing (TDM). From information theory we know that the capacity of a channel is given by

$$C = \Delta\nu \log_2\left(1 + \frac{S}{N}\right)$$

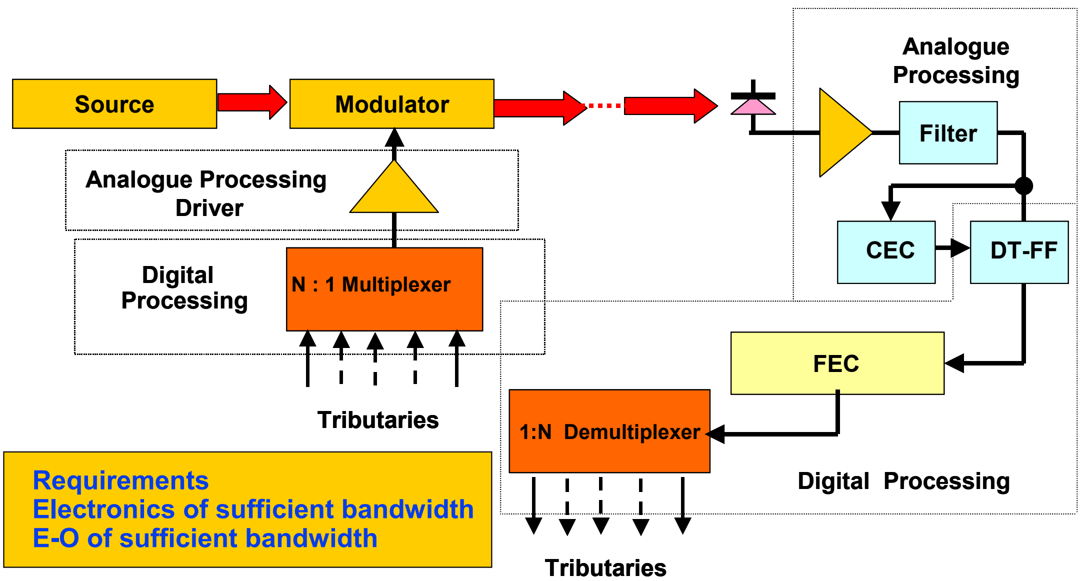
where $\Delta\nu$ is the bandwidth. For a S/N of 12 and for $\Delta\nu$ of 50 THz the expected information capacity is 185 Tbit/s.

The ability to implement a TDM system depends critically on three technologies:

electronics, optical modulators and optical detectors

•Electronics: electronics performs two basic processing functions. The first is the processing of information in digital form and the second is processing in analogue fashion. In order to satisfy these requirements the electronics must possess sufficient bandwidth (high speed) and the ability to drive the transducers, that is, modulators (speed and driving voltage) and detectors (sufficient bandwidth with low noise).

The typical architecture of a digital optical system (channel to be precise) is shown below



The limits of electronics

For digital processing the key components are the D-type Flip-flop and for analogue processing the analogue amplifier. At the present time and with the technologies in hand the following table illustrates the results that have been obtained

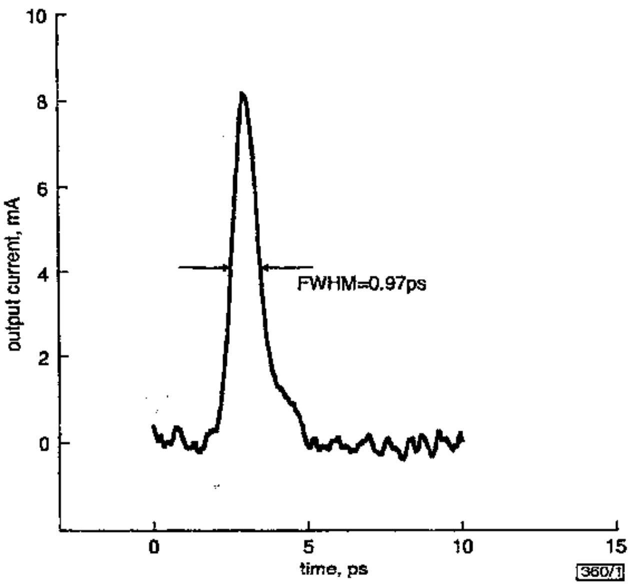
Function	FET /HEMT		HBT	
	GaAs	InP	GaAs	InP
Baseband amplifier GHz	56	90	85	57
D-type FF Gbit/s	40	50	40	
Multiplexer GBit/s	45	80	40	40
Demultiplexer GBit/s		40	30	
Modulator GBit/s	40		20	30

It is clear from this short table that mixed technologies will be required beyond 40 Gbit/s and there appear to be no electronics beyond 80 Gbit/s.

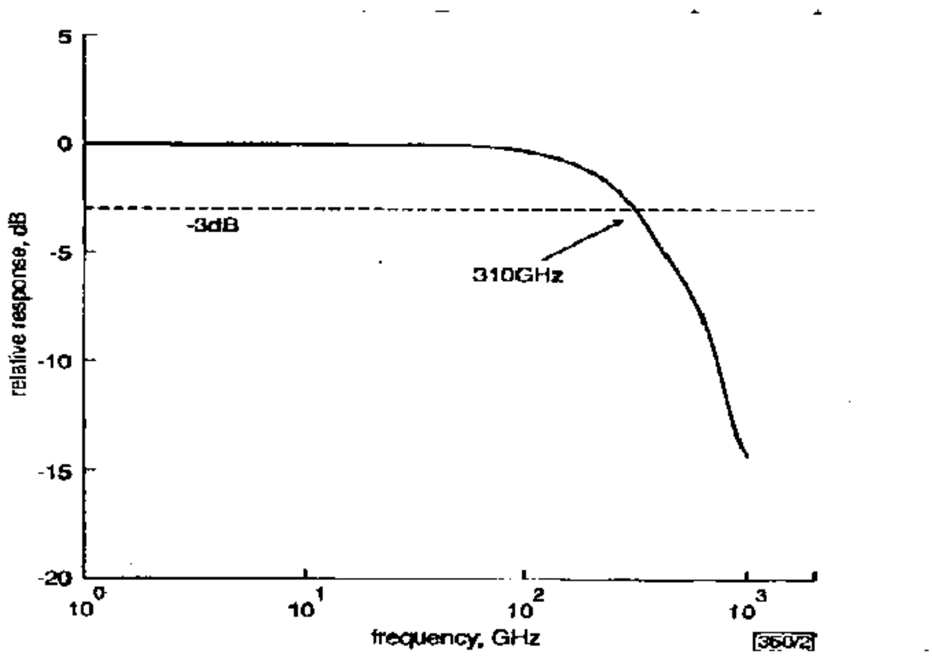
Limits of electro-optics

There are two components that limit the information bandwidth: modulators and detectors. The most successful material for external modulators is $LiNbO_3$ whose performance exceeds 100 GHz.

The detector performance is limited by capacitance and transit time effects of the device and the best result today using InP - InGaAs is shown below



Time response: FWHM 0.97ps



Frequency response: 310 GHz

The limits of technology:

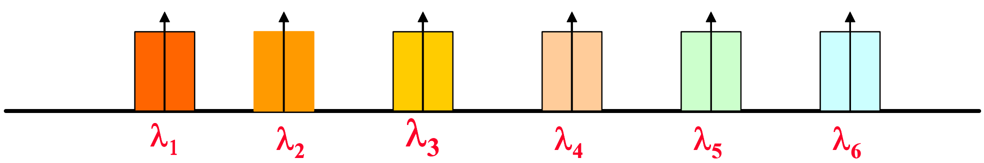
•Electronics: at this stage unlikely to work beyond 80 Gbit/s

•Electro-Optics: can work beyond 40 Gbit/s but not without electronics

Wavelength Division Multiplexing

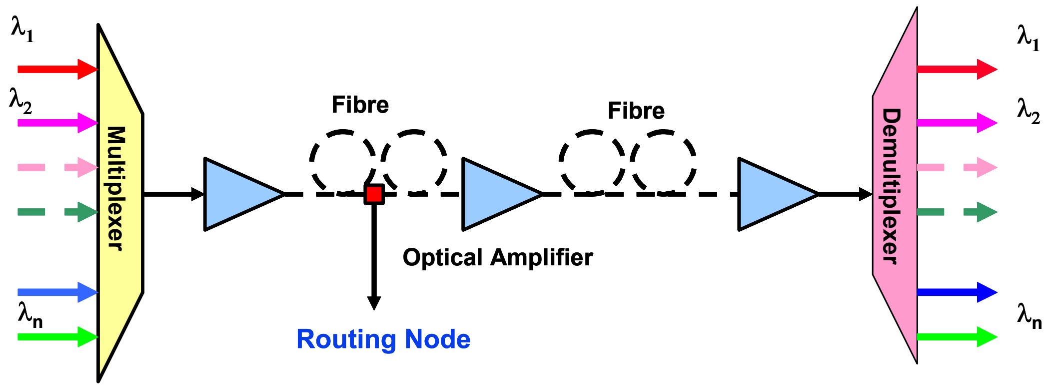
After this short investigation on the potential of electronics it is clear that unless we do something different the high information capacity of the fibre cannot be used. That something else is is " Wavelength Division Multiplexing " WDM for short.

The idea is not new and before the advent of fibre optics all communications over coaxial cable used a number of frequencies called, carriers, and the systems were knows as "Frequency Division Multiplexing" , FDM for short. In the WDM approach a number of wavelengths (carriers) are used simultaneously in a fibre each carrying X Gbit/s. The spectral decomposition of a WDM system looks like



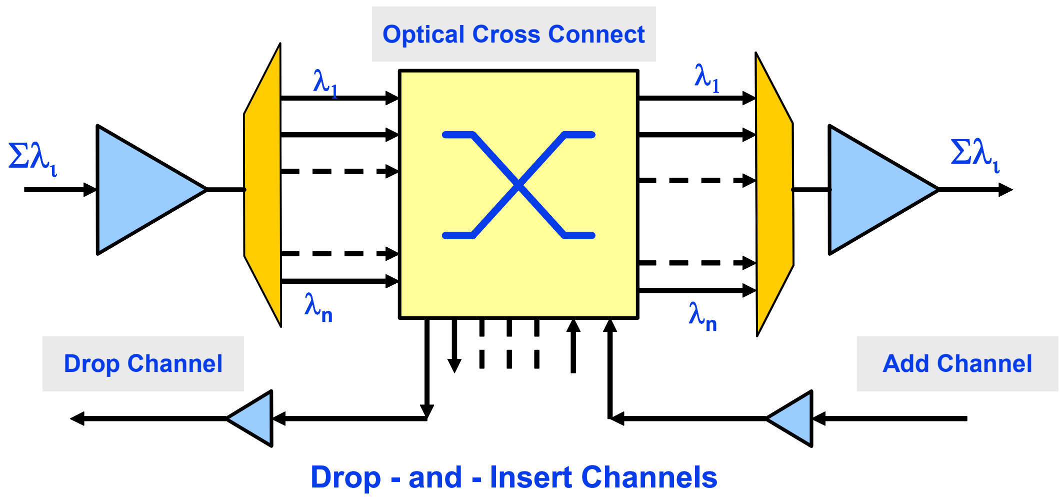
The use of WDM has a number of advantages over very high bit rate TDM. The advantages can be divided into two broad categories: technological and operational. The technological advantages are: optimum use of the fibre bandwidth and avoidance of the "electronic bottleneck".

The operational advantages are wavelength routing, dynamic routing and management and control in the optical layer. The generic description of a WDM systems is shown below



The key development behind the introduction and acceptance of WDM systems has been the optical amplifier based on erbium doped silica fibre. This class of amplifier provides the bandwidth and power capabilities that WDM systems require.

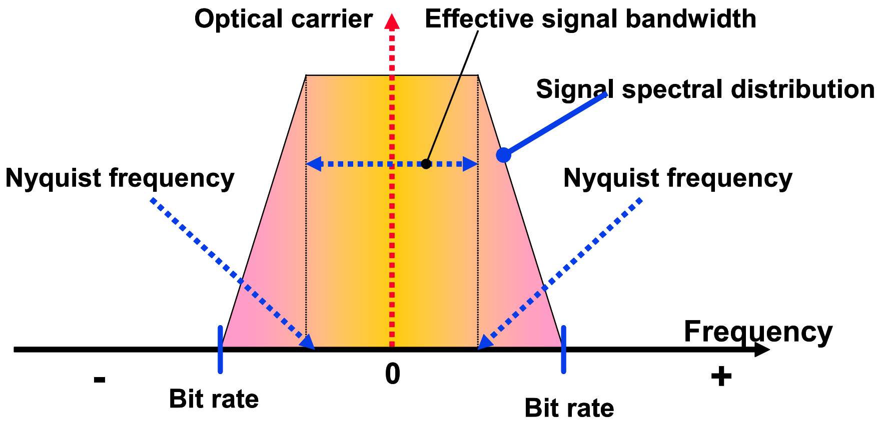
The operational feature of a WDM system for wavelength routing is summarised below



Fibre Capacity with WDM

The capacity of the fibre with WDM depends critically on the number of channels that can be packed together within a given optical bandwidth. In turn the optical bandwidth depends on the class of amplifier used. For erbium doped silica fibre amplifiers operating in the C - Band the optical bandwidth is around 30 nm (1530 – 1560 nm). If the L-Band is also used the bandwidth roughly speaking is another 30 nm. As a basis we will use a 30 nm optical bandwidth. The answer to the key question of how much bandwidth each channel of information is more complex because it depends on the details of modulation schemes and every case is different.

However, some appreciation of the information capacity of a fibre can be gain by using the concept of the Nyquist bandwidth

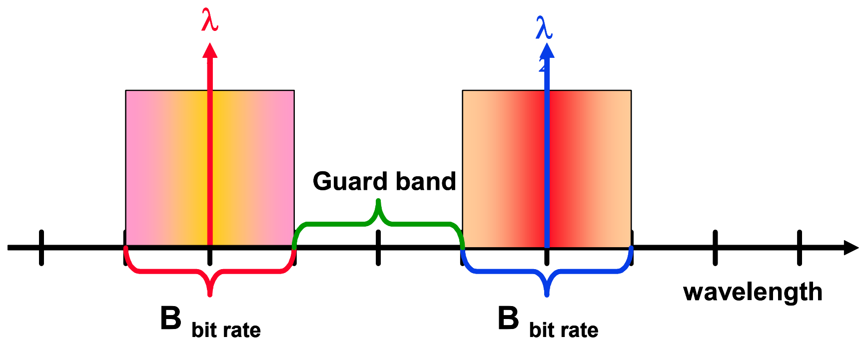


The Nyquist bandwidth is defined as being half the total bandwidth of the information. On that basis the total optical bandwidth occupied by a channel is twice the Nyquist bandwidth

$$\text{Channel Bandwidth } B_0 = 2B_N = B_{\text{bit rate}}$$

The next question is how close one can position the channels and still prevent. Interaction between the carriers. The separation depends on the capabilities of the technology employed and how much interference from other channel one can accept.

This is a complex question but let us assume that the separation between carriers is four time the Nyquist bandwidth, that is, $2B_N$ for information and $2B_N$ as guard band.



The optical bandwidth of 30 nm is converted to an equivalent frequency bandwidth through the relation

$$\Delta f_{\text{optical}} = \frac{c}{\lambda^2}\Delta\lambda$$

$\Delta\lambda = 30\ \text{nm}$ and $\lambda = 1550\ \text{nm}$

This gives $\Delta f_{\text{optical}} = 370\ \text{GHz}$

This gives number of channels:

$$\frac{\Delta f_{\text{optical}}}{\Delta f_{\text{channel}}} = \frac{\Delta f_{\text{optical}}}{2B_{\text{bit rate}}} = \frac{370}{2B_{\text{bit rate}}}$$

For 2.5 and 10 Gbit/s the number of channels are 740 and 185 respectively.

This approach leads to an appreciation of the capacity capabilities of the fibre but there is a subtle point underlining these numbers. In order to move from 2.5 to 10 Gbit/s channels the absolute frequencies of the carriers have to change. This in turn leads to a problem if one wants to interconnect two systems because the separation of the carriers is in fact arbitrary.

In order to prevent the confusion arising from an arbitrary assignment of the carrier wavelengths the International Telecommunication Union ,(ITU), has introduced a grid of frequencies with a minimum separation between carriers of 12.5 GHz.

According to the ITU recommendations the channel frequencies are assigned according to the following the rules

For a channel separation of 12.5 GHz the allowed channel frequencies in THz are defined by:

193.1+ n X 0.0125 where n is positive or negative including 0

For 25 GHz channel separation:

193.1 + n X 0.025 where n is positive or negative including 0

For 50 Ghz channel separation:

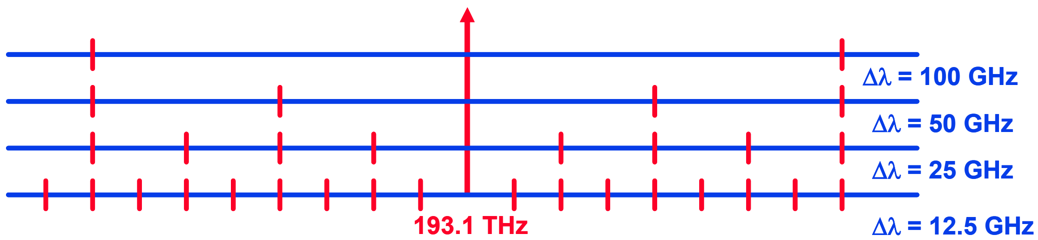
193.1 + n X0.05   where n is positive or negative including 0

For 100 Ghz channel separation:

193.1 + n X0.1   where n is positive or negative including 0

•Note that the reference frequency 193.1 THz corresponds to 1552.52 nm.

In order to convert frequencies to wavelengths the speed of light in vacuum is defined as 2.99792458x10^8 m/s. In their initial recommendations the ITU had specified frequencies within the C-band but in the latest recommendations there are no limitations on the frequency range. The defining frequency 193.1 THz corresponds to the absorption line of $H^{12}C^{14}N$ (1552.54 nm) making possible the frequency stabilisation of the whole grid if necessary.



The current ITU WDM frequency grid recommendations anchored at 193.1 THz (1555. 52 nm)

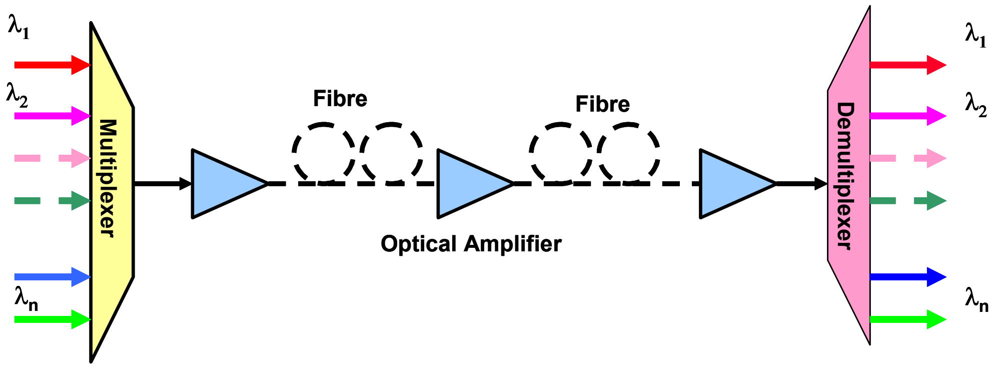
Clearly the ITU frequency grid can not accommodate the same capacity as an arbitrary channel carrier assignment but the grid satisfies two key objectives:

•accommodate the tolerance of optical components (active and passive) and their aging (makes manufacturing possible).

•accommodate the spectral broadening of the optical signal because of nonlinear propagation effects (ensures the operation of the system in spite the non linear fibre effects).

WDM Point to Point Links

The basic elements of a WDM point to point link are shown below



Required Components

•Multiplexers and Demultiplexers. A means of tapping off individual wavelengths, or all wavelengths. Can use gratings, waveguides, holographic filters.

•Stable sources of fixed wavelength separation

•Amplifiers to improve reach

Stable Wavelength Sources

Need narrow linewidth sources that stay on the ITU grid. High speed modulation can cause wavelength laser change (chirp) and design of electronics is difficult.

Options are:

•Externally modulated DFB lasers >10Gb/s. Laser is on all the time, thus stable, with external modulator controlling intensity.

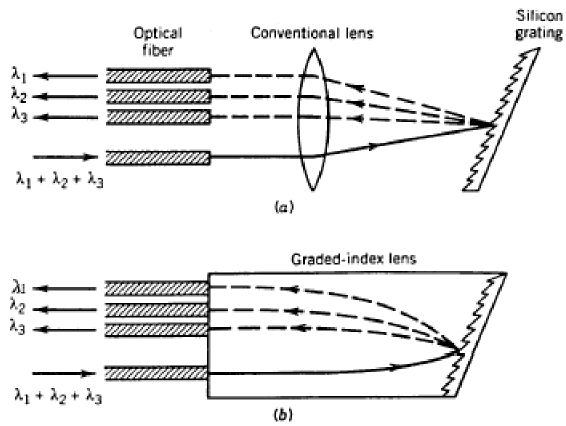
•Directly modulated DFB lasers <=10Gb/s,

•Directly modulated FP laser <10Gb/s (systems operating at 10Gb/s are becoming available).

Multiplexers

A Multiplexer combines the input signals from a number of sources into a single optical signal containing multiple wavelengths that can be launched into the fibre.

Grating based multiplexers and demultiplexers

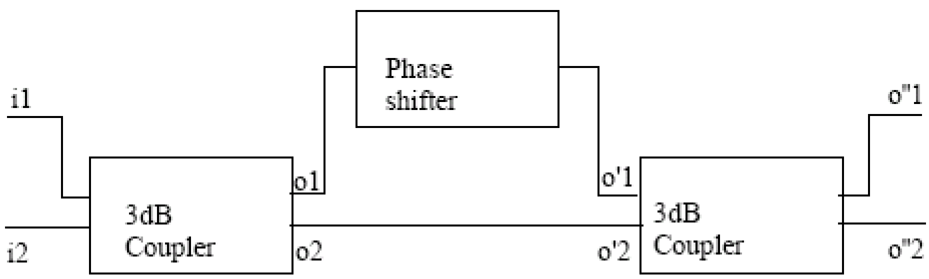


Free space demultiplexer

Grating dispersion is used to either combine or split wavelengths, either using Graded Index or bulk optics to image the fibres onto the gratings.

Mach-Zehnder Interferometer

The basic component that is used in many filters and multiplexers and demultiplexers is the Mach Zehnder interferometer, shown below



It consists of two 3dB couplers and two arms, one with a longer length than the other. The 3dB couplers have the amplitude transfer matrix

$$\begin{bmatrix} o_1 \\ o_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

The phase shifter is described by

$$\begin{bmatrix} o_1' \\ o_2' \end{bmatrix} = \begin{bmatrix} \exp(j2\pi f\tau) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} o_1 \\ o_2 \end{bmatrix}$$

where τ is the delay in the arm. The second coupler has a similar transfer matrix so the overall response can be written

$$\begin{bmatrix} o_1'' \\ o_2'' \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} \exp(j2\pi f\tau) & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

When either of the input signals i_1 or i_2 are zero the output powers take a very simple form:

When i_2=0 we have

$$|o_1''|^2 = \sin^2(\pi f\tau)|i_1|^2$$

$$|o_2''|^2 = \cos^2(\pi f\tau)|i_1|^2$$

and when i_1=0 we have

$$|o_1''|^2 = \cos^2(\pi f\tau)|i_2|^2$$

$$|o_2''|^2 = \sin^2(\pi f\tau)|i_2|^2$$

There are several ways to use this

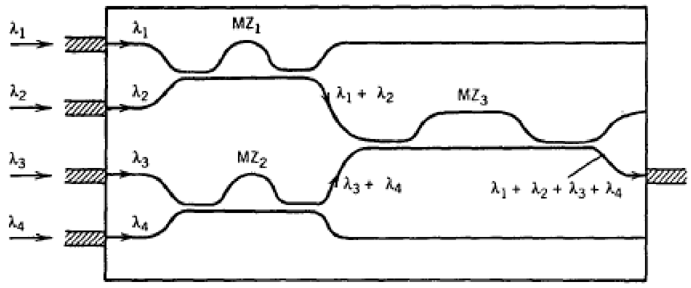
•As an optical filter, injecting a signal into port 1 and setting $\pi f\tau = \pi/2$ at the desired centre frequency, then taking the signal out from port 1.

•As a multiplexer. Arrange the input wavelengths and the delay τ so that the wavelength input to port 1 maximises $\sin^2(\pi f\tau)$ and the wavelength into port two maximises $\cos^2(\pi f\tau)$. The output port 1 then contains both wavelengths- a multiplexer. A similar process can be used to demultiplex.

•The cos and sin functions are periodic so with a grid of wavelengths a multiplexer can be used to combine 'odd and even' wavelengths on the grid. By changing the delay and chaining together multiplexers its possible to multiplex a whole grid of wavelengths (or demultiplex them). In this case the delay halves in each successive stage in order to double the frequency spacing between transfer maxima.

•In the case of a demultiplexer the first splits input into wavelengths 0 2 4 6 from one output and 1,3,5,7 from the other. The even stream is then further split by a multiplexer 0 4 8, and 2,6 etc until all are split. (How many stages for n wavelengths?)

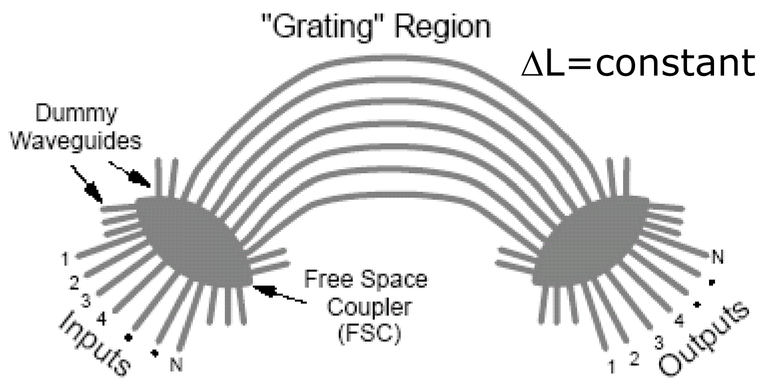
•Waveguide based multiplexers- chain together Mach-Zehnder (MZ) interferometers with adjustable phase shifts to combine multiple wavelengths. Figure below shows a particular arrangement



Multiplexer fabricated with MZ interferometers

Array Waveguide Grating Multiplexers/Demultiplexers

Array waveguide grating multiplexers and demultiplexers (known as AWGs) are important components in WDM systems.

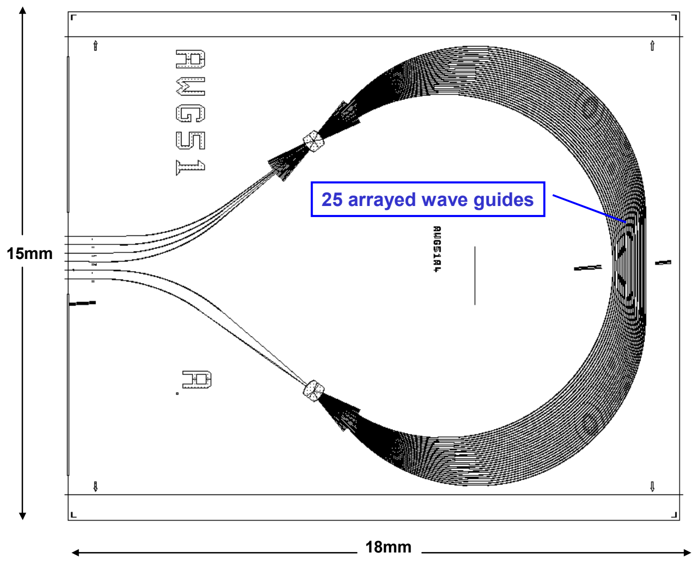


Array Waveguide Multiplexer

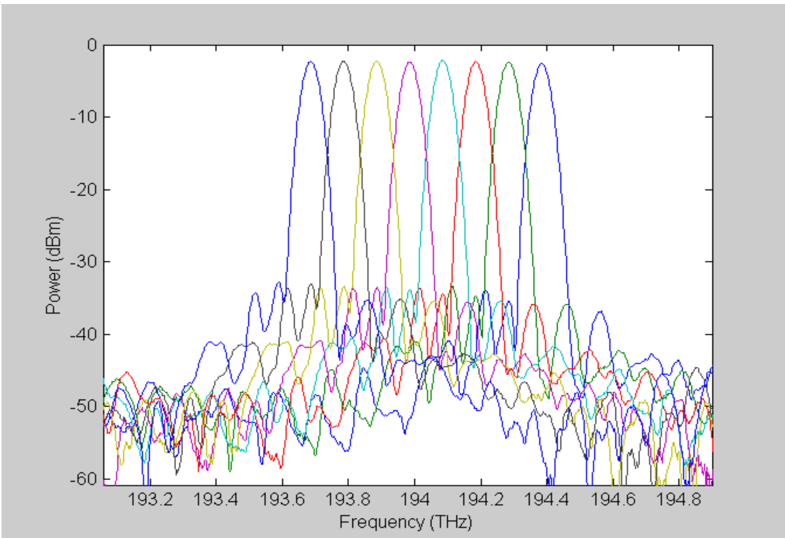
•The star coupler (free space coupler) is essentially a 'free propagation' section where light from each of the input waveguides expands and couples into each of the waveguide grating section equally. The star coupler is equivalent to a lens with the input fibres at the focus.

•Light then propagates along the grating sections to the second star coupler. The phase shift across the waveguides at the input to the second coupler is governed by the length of the arms. The phase shift varies linearly across the input to the star coupler, so the optical field is equivalent to a plane wave entering the section at an angle. The star coupler then 'focuses' this to a particular output waveguide. As the phase shift varies with wavelength each wavelength is focused to a different port.

•An example of the mask design for a 2x4 demultiplexer is shown below

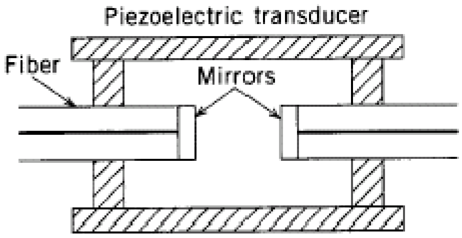


•The response of a 1x8 demultiplexer is shown below:



Filters

- Basic component required for WDM systems.
- Required to tap off a single wavelength.
- Based on Mach Zehnder inteferometers, two arms with a phase shift causing destructive/constructive interference at a particular wavelength.
- Also a Fabry Perot cavity that is tuneable (organ pipe mode approach).

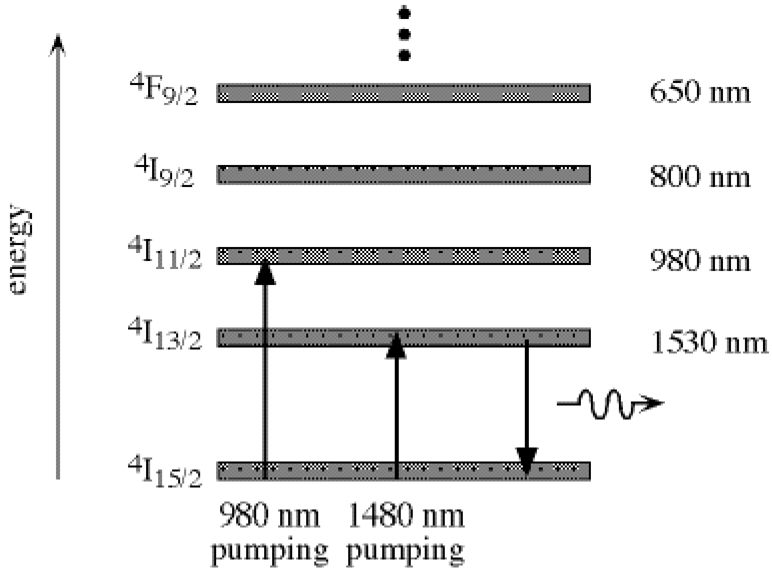


Filter structure using Fabry Perot Cavity

Optical Amplifier - Erbium Doped Fibre Amplifier (EDFA)

First demonstrated in 1987 at University of Southampton.

- In an EDFA, the silica fibre acts as a host for Erbium ions, which can be optically pumped to an excited state. An incoming signal then stimulates emission of new photons and gets amplified. Diagram below shows the erbium energy levels:



Energy levels of erbium

•The energy levels in Erbium are broadened into bands due to the non-crystalline glass host. The gain spectrum is thereby continuous.

•The two most important pump wavelengths (both of which are available with semiconductor lasers) are:

– 980 nm (4I15/2 - 4I11/2 transition). The population in the 4I11/2 state rapidly decays to the 4I13/2 state. This gives a lower noise figure.

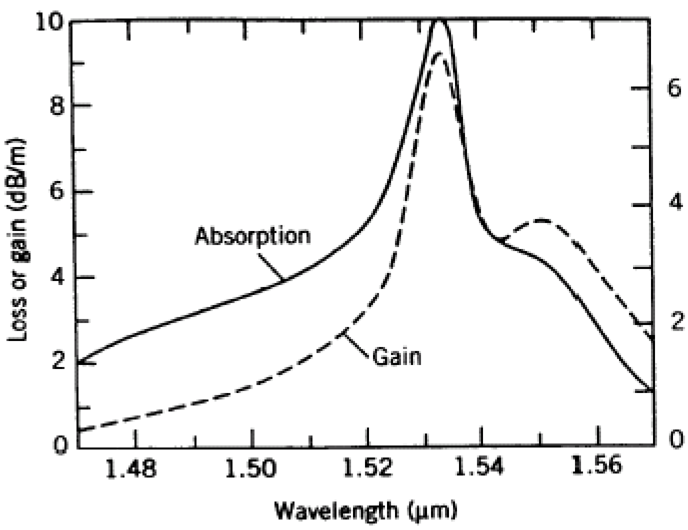
– 1480 nm (4I15/2 - 4I13/2 transition). This corresponds to pumping at the top of the first excited state. This enables higher output powers.

•The 4I13/2 state is called the metastable state and has a population lifetime of ~ 10 ms .

•When studying the gain characteristics, it is usually sufficient to consider only the ground state and the meta-stable state. The EDFA can be approximated as a two-level system at both pump wavelengths.

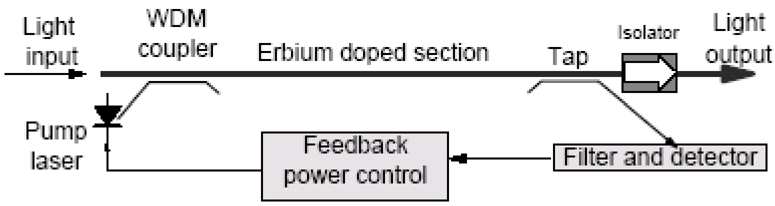
•The EDFA gain spectrum depends on: the co-dopants (usually Ge), the pump power, and the Erbium concentration.

•The figure below shows a typical gain spectrum at large pump power and the absorption spectrum (no pump).

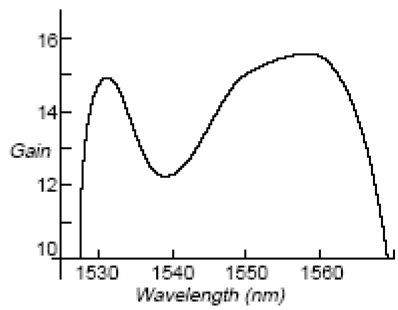


Amplifier Design

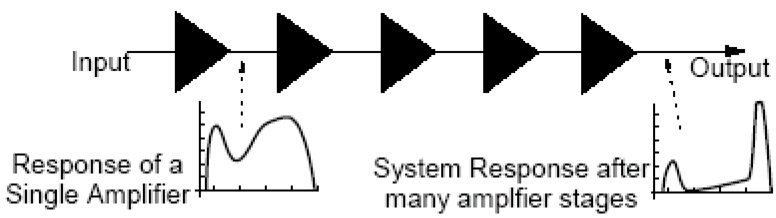
The basic amplifier design is shown below.



A typical gain curve for an EDFA is shown in the following figure



Gain at 1560 nm is some 3 dB higher than gain at 1540 nm (this is twice as much). In most applications (if you only have a single channel or if there are only a few amplifiers in the circuit) this is not too much of a limitation. However, WDM systems use many wavelengths within the amplified band. If we have a very long WDM link with many amplifiers (say two or three thousand km with 80 or so amplifiers) the difference in response in various channels adds up. This difference in signal levels between channels could become as great as perhaps 100 dB! Diagram below illustrates this

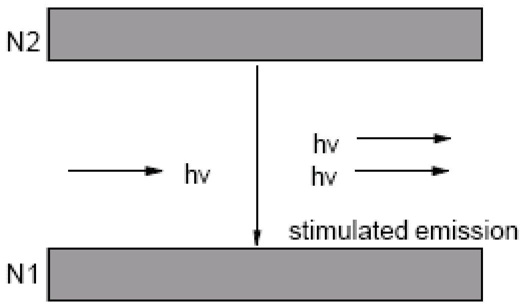


Response of cascaded EDFAs

In other words some channels will be so strong as to be dominant and others will be lost in the noise! This can be so severe that it will easily prevent successful system operation. In less severe cases it means that the SNR varies significantly from channel to channel. Of course the system is only as good as the worst channel. This means that for a WDM system we need each channel to have the same amount of gain. That is the amplifier's gain should be "flat" across the range of amplified wavelengths.

Amplifier Characteristics

The amplifier is modelled as a two level system:



It can be shown that for such a system the gain per unit length of fibre can be approximated by

$$g = \frac{g_0}{1 + \frac{P}{P_s}}$$

where P is the signal power in the fibre, and P_s is a saturation power level, and is the gain in the absence of any saturation effects. Note that this equation implies a decrease in the available gain as the signal power in the fibre reaches the saturation power level. (See Agrawal page 395 for a detailed analysis). Its then possible to describe the evolution of the signal

$$\frac{dP}{dz} = \frac{g_0 P}{1 + \frac{P}{P_s}}$$

This can be integrated

$$\int_0^z g_0 dz = \int_{P_i}^{P_0} \left(\frac{1}{P} + \frac{1}{P_s}\right) dP$$

This gives

$$g_0 z = \ln\frac{P_0}{P_i} + \frac{P_0 - P_i}{P_s}$$

So that

$$P_0 = P_i \exp\left(g_0 z - \frac{P_0 - P_i}{P_s}\right)$$

Note that this is not a linear amplifier near saturation. However, if $P \ll P_s$ then

$$P_0 = P_i \exp(g_0 z)$$

EDFA Amplifier Noise

•Gains of 30-40dB are possible using 10s of metres of fibre doped with ppm levels of Erbium.

•Noise added is due to spontaneous emission that is amplified along the length of the fibre. This Amplified Spontaneous Emission (ASE) is of major concern in high performance systems and networks.

Can show that (see Agrawal page 366) the noise figure of the EDFA

$$F_N = 2n_{sp}\left(\frac{G-1}{G}\right)$$

where n_{sp} is the spontaneous emission factor, which can be shown to be

$$n_{sp} = \frac{N_2}{N_2 - N_1}$$

If the amplifier is fully inverted $n_{sp} = 1$, leading to a minimum noise figure of 3dB. Practical amplifiers have figures of approximately 3.8dB.

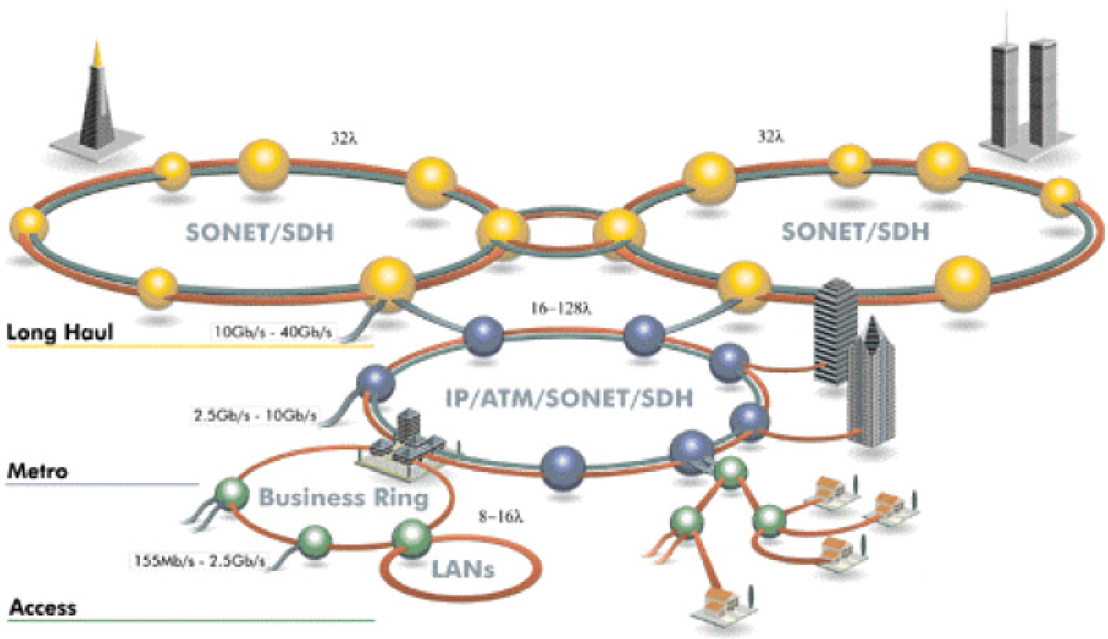
WDM Networks

The availability of amplifiers and WDM leads to the concept of WDM networking, rather than simple transmission.

In a network signals are routed according to wavelength, with the aim of all optical systems. These allow more wavelengths, and higher data rates to be added later. The idea is to minimise the electronic processing that must be upgraded later.

For this require we need extra components: optical switching nodes, the ability to tap-off (drop) and add single wavelength signals into the network, and the ability to convert signal wavelengths to another.

The following figure shows a Nortel diagram of how a WDM network might fit together.

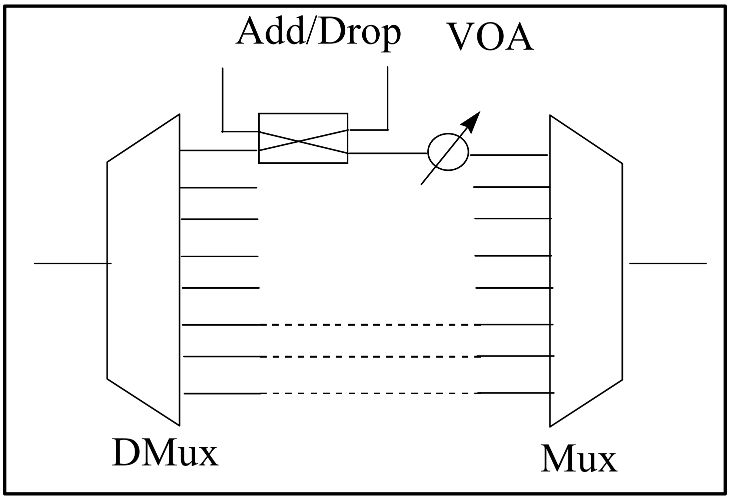


Add and drop multiplexers

•Possible to use the multiplexer structures shown earlier to separate off and add particular wavelengths to the network.

•Can be used for wavelength determined signal routing in some architectures.

•Typical add-drop filter is shown below.



Wavelength converters

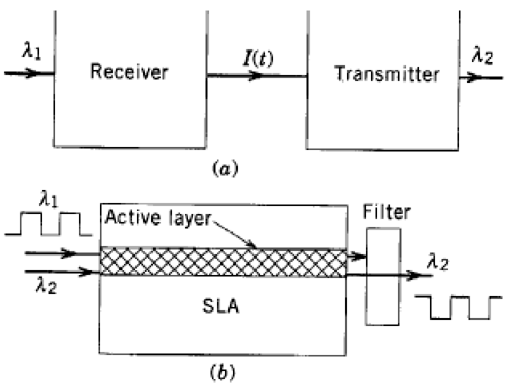
•The aggregate data rates on networks is enormous, and switching and controlling that data is difficult.

•EDFAs mean that electronic regeneration is very rarely used, or required.

•Possibility of routing wavelengths, in high level transit networks- one wavelength, one destination.

•Idea of a wavelength converter is attractive, as it allows forms of routing.

Diagram below shows how this might be done



Wavelength converter

Optical switching

•Want the equivalent of a set-of railway points - in theory the train isn't slowed down by the points. An optical equivalent is that the beams into the switch are just (somehow) diverted to the correct output fibre. This is called a transparent switch.

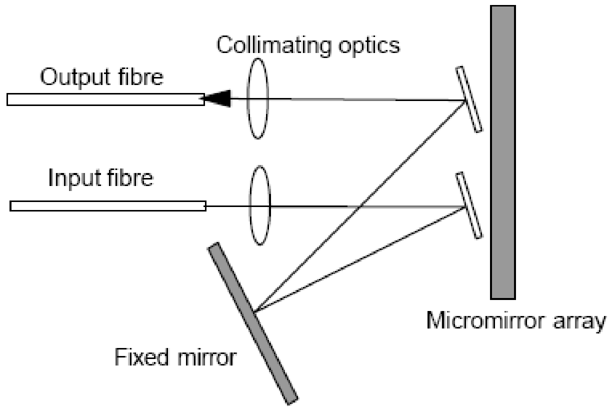
•Still a big issue as to how to do transparent switching (ie switch beams of light between fibre ends) with low (<40dB) of crosstalk. Crosstalk is a problem. If you use a switch with one input and two outputs you need many layers of switches in order to completely switch a large number of inputs and outputs- and 1000 ports is a target parameter. In this case the slightest amount of crosstalk in the simple element means the overall performance of the switch is insufficient. Such a switching element is called low fan-out, as there are a small number of outputs. (Nearly) All commercial approaches use high fanout.

Micromirror based switches

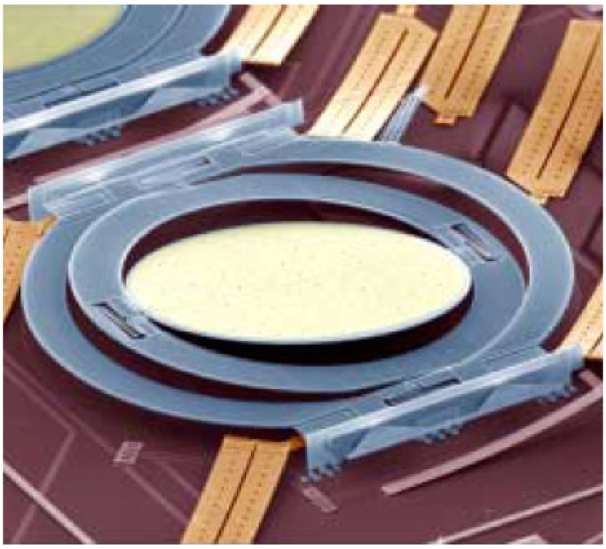
•Lucent and Nortel have both invested heavily in micromirror switching companies. Silicon IC processes can be used to make arrays of electrostatically controllable mirrors. Light from input fibres is collimated and these beams are steered using two mirrors, the first to direct the light to the correct output fibre and the second to align the beam with the output fibre, >50dB of crosstalk is possible, and the switches have 100s of nm of optical bandwidth.

•Switching speeds are in the ms range, so a return to the old circuit switching model (rather than the time space time switches). This switching does exist, but at much lower data rates. These types of cross connect sit high up in a WDM network where routing between cities is configured on a second by second basis.

•Commercial switches use arrays of micromirrors switched electrostatically, mechanical relay based switching or other 'brute force' techniques. The diagrams below illustrate the technique, and show a typical micromirror.



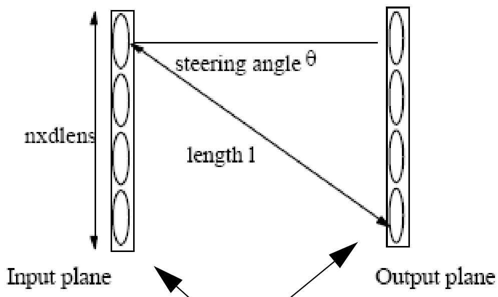
Micromirror based optical switching



Micromirror - Lucent Technology

Capacity of micromirror switches

The capacity of this is a function of how good the optics that collimates the light from the fibre is. This focuses the light at (approximately) the focal length of the collimating lens, and the light then expands again to strike the output lens. The total allowed path through the switch is therefore $2f_{coll}$ where f_{coll} is the collimating lens focal length. The diagram shows (conceptually) such a switch.



If the mirrors can steer through an angle θ then

$$\sin\theta = \frac{n \cdot d_{lens}}{2f_{coll}}$$

The capacity n is given by

$$n = \frac{2f_{coll}}{d_{lens}} \cdot \sin\theta = 2f\sin\theta$$

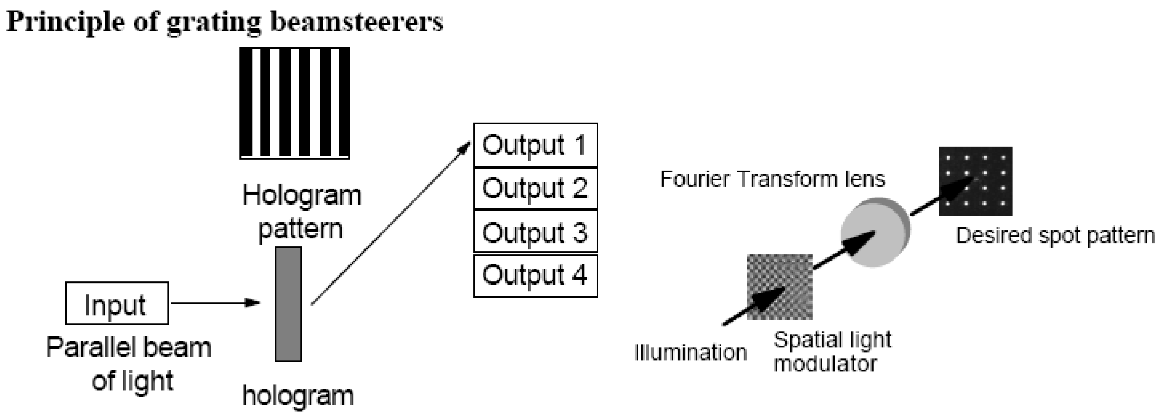
where the f-number of the lens is given by

$$f = \frac{f_{coll}}{d_{lens}}$$

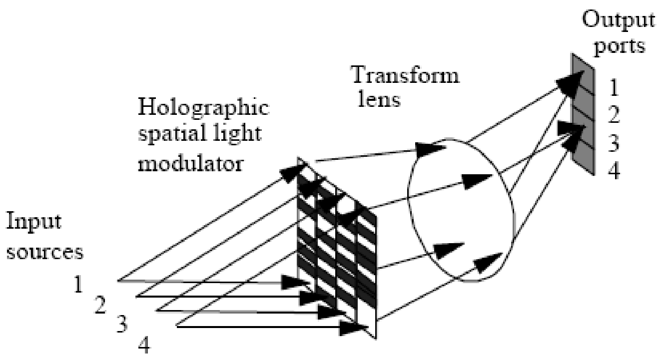
Typically a mirror might steer through an angle of 0.1 rads so for n=30 (corresponding to 1000 ports) an f-100 lens is required. In practice there are other limits: the size of the focused Gaussian beam must be less than the diameter of the collimating lens (and the mirror array) and this fixes the maximum allowed focal length as a function of the lens diameter. Loss is less of a problem given the EDFA, but dispersion management becomes the major problem for these networks, as well as crosstalk within the switches.

Liquid crystal based beamsteerers

The principle is based on a computer generated holographic diffraction grating. The beam of light is steered through an angle proportional to the grating pitch. The diagrams below show the principle and how this might be incorporated into a switch.



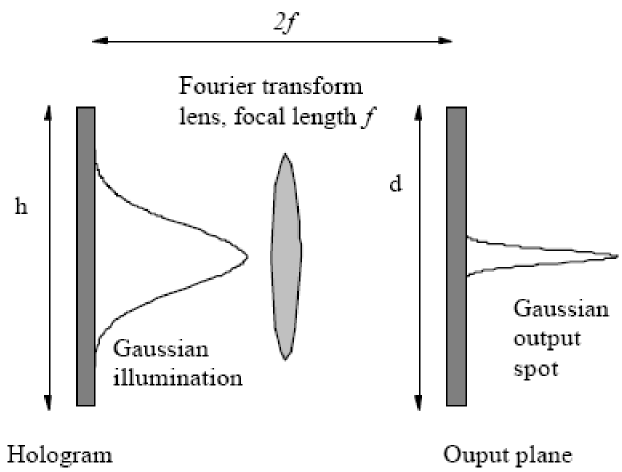
Computer generated hologram: principle



Grating based optical switch

Interconnection Capacity

This is linked to the space-bandwidth product (SBWP) of the hologram- the number of coding pixels. Assuming a perfect beamsteering hologram (all the light is steered to the desired position) the following analysis shows the relationship between capacity and the hologram SBWP for a simple 1-D interconnect:



The hologram is illuminated with a Gaussian field so that

$$a(x) = \exp\left(-\frac{x^2}{\omega^2}\right)$$

where ω is the Gaussian beam half width. The hologram dimension can be expressed in terms of this half width:

$$h = \alpha\omega$$

where α is the normalised hologram dimension. The Gaussian beam a(x) is transformed to a Gaussian a(x′) by the Fourier transform lens:

$$a(x') = \exp\left(-\frac{x'^2}{\omega'^2}\right)$$

where

$$\omega' = \frac{\lambda f}{\pi\omega}$$

The number of channels that are available in the output plane is set by the width of these Gaussians, as each channel must be sufficiently spaced so that the 'tails' of the Gaussian do not overlap. The dimension of the channel can be expressed, in normalised terms, as $\beta\omega'$. The hologram is effectively a diffraction grating that shifts this Gaussian spot to the desired point in the output plane. The maximum size of output field is set by the pitch of the grating, and is given by

$$d = \frac{2\lambda f}{\Delta}$$

where Δ is the grating pitch (or pixel size). This number is halved if the grating is binary as only half the order is independently addressable, as in the case considered here. The number of channels is simply this distance divided by the size of a channel

$$C = \frac{d}{\beta\omega'} = \frac{\lambda f}{\Delta\beta\omega'} = \frac{\pi\omega}{\Delta\beta}$$

Recall that the hologram size, h can be expressed in terms of α and ω. Using this result

$$C = \frac{\pi h}{\alpha\beta\Delta}$$

The hologram size, h divided by the pixel pitch gives the number of pixels, showing that the number of channels the idealised interconnect can route is a function of the SBWP (number of pixels) of the hologram only:

$$C = \frac{\pi \cdot \text{number of pixels}}{\alpha\beta}$$

The result shows the importance of the hologram SBWP in determining routing. The pixel pitch and lens focal length determine how compact (or not) the system may be.