BME2 – Biomedical Ultrasonics

Lecture 1: Introduction to Linear Acoustics

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1.1 Why study biomedical ultrasonics?

- When most people think of medical ultrasound, it is most often imaging applications that come to mind:

- However, since the 1950s, ultrasound has been found to induce a wide range of potentially beneficial *bioeffects.*

- All such effects became more pronounced
  - In the presence of gas or gas bubbles
  - At higher intensities and longer exposures

- Diagnostic ultrasound: 3.5-30 MHz, 0.1 W
- Therapeutic ultrasound: 0.1-5 MHz, 0.1-10 W

Therapeutic ultrasound bioeffects include:

- Rapid, highly localized heating:
  - Cessation of blood flow / internal bleeding (Acoustic Haemostasis)
  - Tissue ablation by High-Intensity Focused Ultrasound (HIFU) for cancer treatment

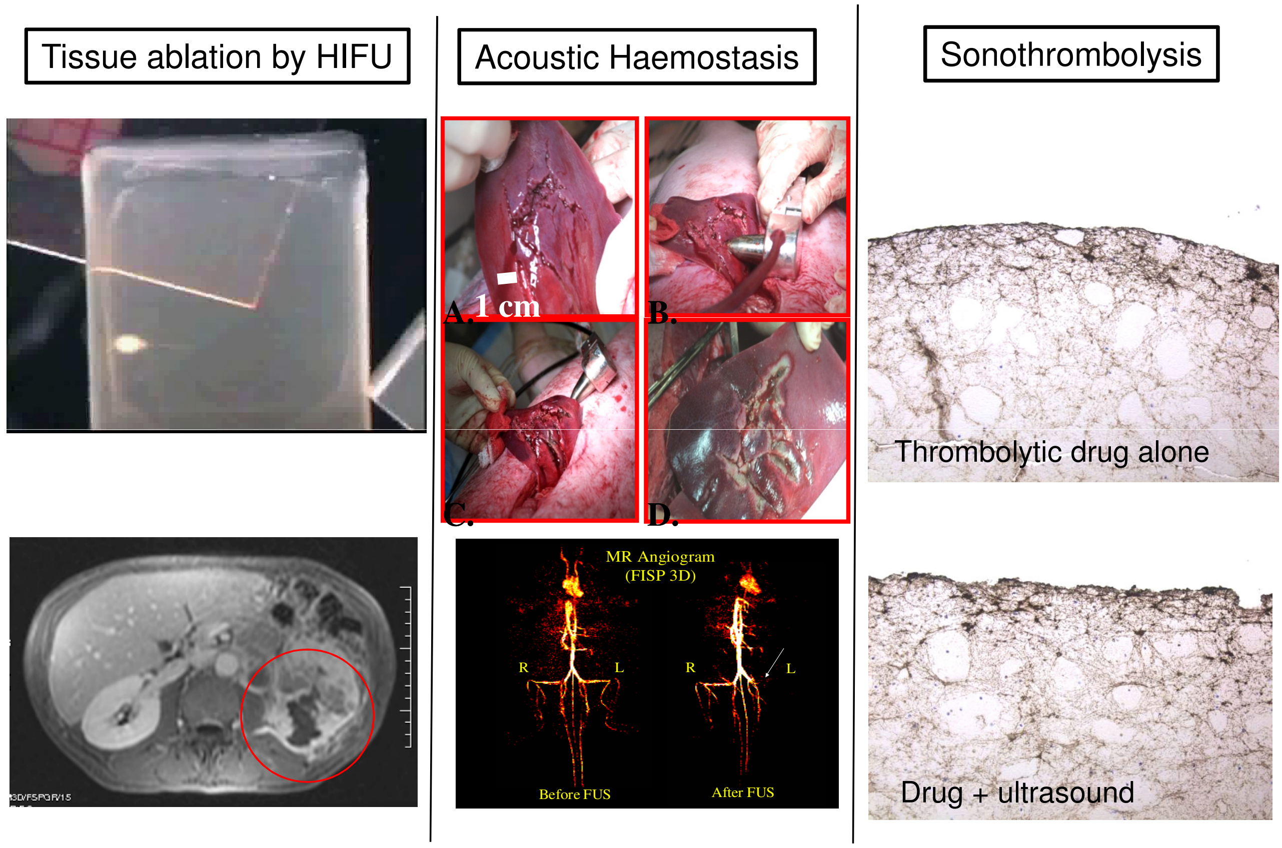
- Drug Activation/Enhancement of drug activity
  - Destruction of blood clots (SonoThrombolysis)
  - Targeted Chemotherapy/Drug Delivery

- Increase of the permeability of cell membranes to large molecules
  - Sonoporation

- Increased transport of molecules across interfaces, such as the skin
  - Sonophoresis

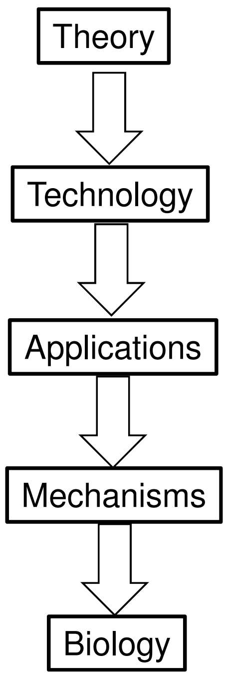
- Reversible, temporary opening of the blood-brain barrier

- Enhancement of musculoskeletal healing



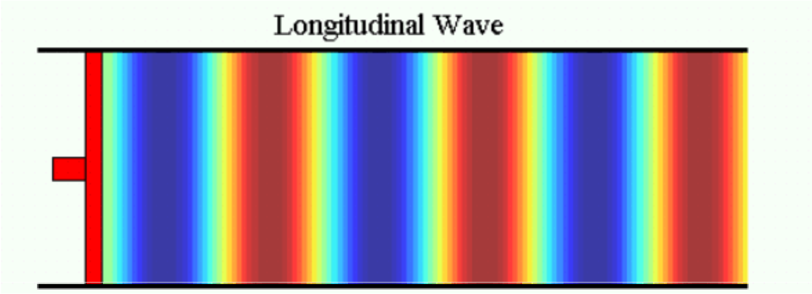
1.2 Course Structure (see detailed course outline)

- What is ultrasound?
- How does it propagate through tissue?
- How can it be generated for diagnosis & therapy?
- How are images formed?
- How can ultrasound be used for therapy?
  - Targeted drug delivery using contrast agents
  - Non-invasive cancer treatment
  - Haemostasis
  - Localized drug activation for stroke therapy



1.3 Definitions of sound and ultrasound

- Sound waves are *longitudinal* perturbations in pressure *p* / density *ρ*



- These perturbations are generated at a frequency *f* and propagate at a speed *c* (the speed of sound)

- Sound waves transport both information and energy:
  whisper ≈ 10^{-10} W   shout ≈ 10^{-5} W   jet engine ≈ 10^{5} W

- Some interesting facts:
  - The whole of Wembley Stadium shouting in unison barely produces enough energy to boil an egg
  - All of humankind shouting at once emits less sound power than a jet engine.

- Ultrasound is defined as sound of a frequency higher than the upper limit of the human hearing range (>20 kHz)

- Therefore, all ultrasound is sound and *the same physical principles that govern sound propagation are fully applicable to ultrasound*

- Compared to other modalities, such as light (laser) and other electromagnetic waves (microwaves), it is fortunate that ultrasound has the right attenuation characteristics to enable to penetrate the body to clinically relevant depths

- Most (but not all) sound consists of *small* perturbations in pressure and density.

- This creates an important distinction
  - LINEAR Acoustics

$$p(\mathbf{r},t) = p_o + p'(\mathbf{r},t) \quad p' << p_o \qquad [1.1]$$
$$\rho(\mathbf{r},t) = \rho_o + \rho'(\mathbf{r},t) \quad \rho' << \rho_o \qquad [1.2]$$

  - NON-LINEAR Acoustics

$$p(\mathbf{r},t) = p_o + p'(\mathbf{r},t) \quad p' \text{ not much smaller than } p_o$$
$$\rho(\mathbf{r},t) = \rho_o + \rho'(\mathbf{r},t) \quad \rho' \text{ not much smaller than } \rho_o$$

1.3 Governing equations of linear acoustics

- Equation of Continuity: *"The rate of change of mass within a control volume is equal to the mass flux in/out of that control volume"*

$$\boxed{\frac{\partial \rho(\mathbf{r},t)}{\partial t} = -\nabla \cdot \left(\rho(\mathbf{r},t)\mathbf{u}(\mathbf{r},t)\right)} \qquad [1.3]$$

where **u** represents the velocity of a fluid particle, which in a sound field is:

$$\boxed{\mathbf{u}(\mathbf{r},t) = 0 + \mathbf{u}'(\mathbf{r},t)} \qquad [1.4]$$

Substituting Equations [1.2] and [1.4] and ignoring products of small quantities gives:

$$\boxed{\frac{\partial \rho'(\mathbf{r},t)}{\partial t} = -\rho_0 \nabla \cdot \mathbf{u}'(\mathbf{r},t)} \qquad [1.5]$$

- Momentum Equation: *"The net change in momentum flux across a control volume is equal to the net force acting on that control volume"*

$$\boxed{\rho(\mathbf{r},t)\left(\frac{\partial \mathbf{u}(\mathbf{r},t)}{\partial t} + (\mathbf{u}(\mathbf{r},t)\cdot \nabla)\mathbf{u}(\mathbf{r},t)\right) = -\nabla p(\mathbf{r},t)} \qquad [1.6]$$

Substituting Equations [1.1], [1.2] & [1.4], and ignoring products of small quantities gives:

$$\boxed{\rho_0(\mathbf{r},t)\frac{\partial \mathbf{u}'(\mathbf{r},t)}{\partial t} = -\nabla p(\mathbf{r},t)} \qquad [1.7]$$

- Equation of state: *"Any thermodynamic quantity can be expressed as a function of any other two thermodynamic quantities"*

$$\boxed{p = p(\rho, s)} \qquad [1.8]$$

where *s* represents entropy.

Note also that sound propagation depends on two fundamental medium properties: the density (already defined), and the bulk modulus B, defined as:

$$\boxed{B = \rho_0 \left(\frac{\partial p}{\partial \rho}\right)_{\rho_0} = -V_0 \left(\frac{\partial p}{\partial V}\right)_{V_0} = \frac{1}{\kappa}} \qquad [1.9]$$

where κ is the compressibility.

Assuming that entropy remains constant (isentropic relationship), the pressure can be represented by a Taylor series expansion:

Giving the following relationship that relates pressure to density fluctuations:

$$\boxed{p'(\mathbf{r}, t) = \frac{1}{\kappa \rho_0} \rho'(\mathbf{r}, t)} \qquad [1.10]$$

Combining equations [1.5], [1.7] and [1.10] gives:

We thus obtain the linear wave equation, given by:

$$\boxed{\frac{\partial^2 p'(\mathbf{r},t)}{\partial t^2} = c^2 \nabla p'(\mathbf{r},t)} \qquad [1.11]$$

where *c* represents the speed of sound and is given by:

$$\boxed{c^2 = \frac{p'(\mathbf{r},t)}{\rho'(\mathbf{r},t)} = \left(\frac{\partial p}{\partial \rho}\right)_{\rho_0} = \frac{1}{\kappa \rho_0}} \qquad [1.12]$$

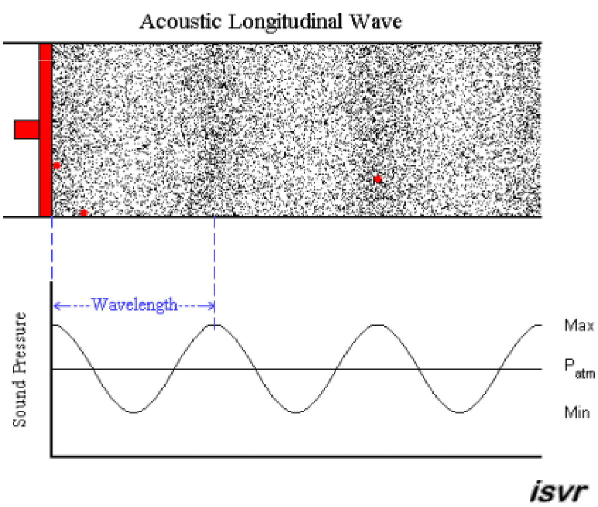
- From now on, we drop the superscript *p'* to indicate small fluctuations and we write:

$$\boxed{\frac{\partial^2 p(\mathbf{r},t)}{\partial t^2} = c^2 \nabla p(\mathbf{r},t)}$$

1.4 Speed of Sound, Frequency and Wavelength

- Sound Propagates as a wave characterized by it's frequency, wave speed, and wavelength
  - For air c = 340 m/sec
  - For water c = 1500 m/sec
  - For most materials, sound speed is independent of frequency

- Wavelength (many interpretations):
  - The distance through space that acoustic information (i.e. the pressure disturbance) travels in one acoustic period

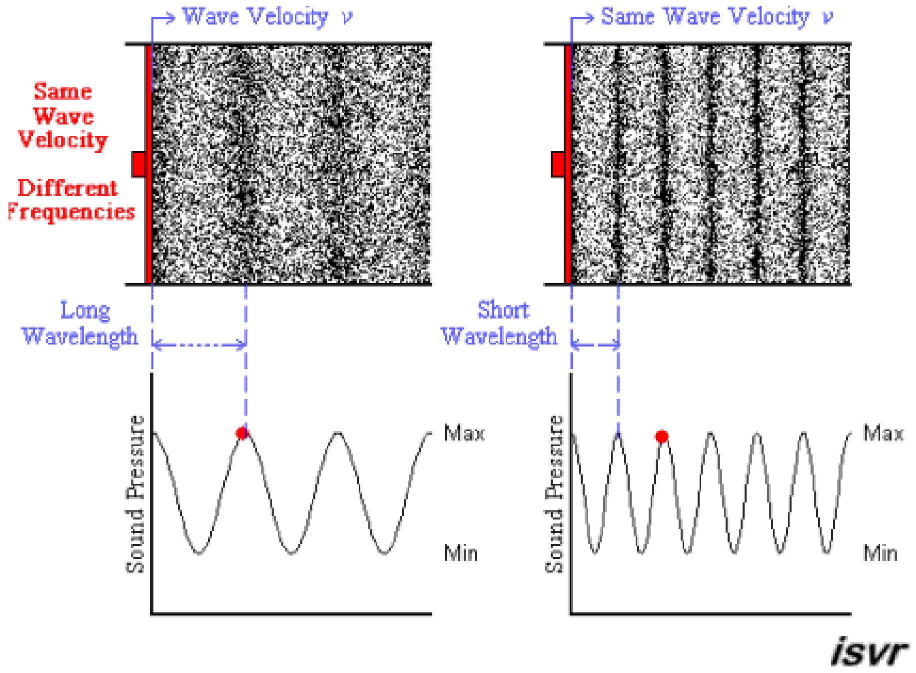


$$\lambda = cT = \frac{c}{f} \qquad [1.13]$$

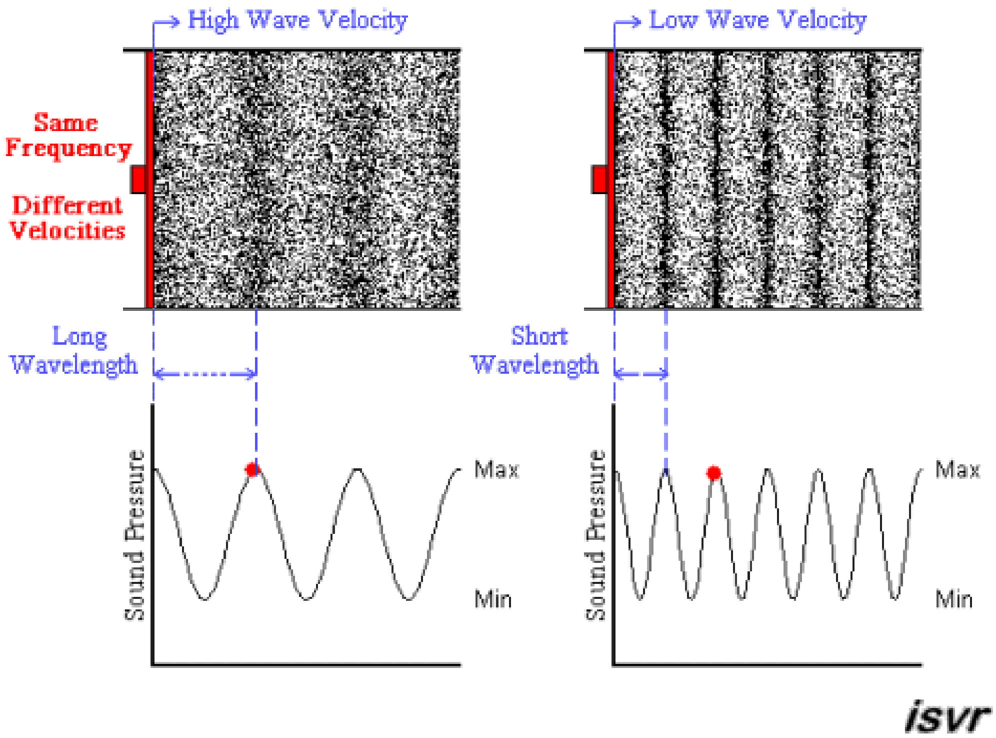
$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

$$\omega = 2\pi f$$

➢ Impact of wave frequency:



➢ Impact of wave velocity:



1.6 Energetics of Acoustic Waves

- Now define the following quantities:

$$\vec{I}_i \equiv p\vec{v} \quad ; \quad e_i \equiv \tfrac{1}{2}\rho_o v^2 + \tfrac{1}{2}\frac{p^2}{\rho_o c^2} \qquad [1.14]$$

- Substituting yields the following energy conservation equation:

$$\vec{\nabla}\cdot\vec{I}_i + \frac{\partial e_i}{\partial t} = 0 \quad \Longrightarrow \quad -\int_S \vec{I}_i \cdot \hat{n}\,ds = \frac{\partial E_i}{\partial t}$$

- e_i is the **instantaneous acoustic energy density** and *I* is an energy density flux vector referred to as the instantaneous intensity. The **acoustic intensity *I***, is the time average of :

$$\vec{I}(t) \equiv \left\langle \vec{I}_i \right\rangle = \frac{1}{\tau}\int_t^{t+\tau} p\vec{v}\,dt' \qquad [1.15]$$

$$e_{i,k} \equiv \tfrac{1}{2}\rho_o v^2 \quad ; \quad e_{i,p} \equiv \tfrac{1}{2}\frac{p^2}{\rho_o c^2}$$

Note that all energy and energy transport terms are 2nd order in acoustic variables