

1. Modern Communications part II

1. Modern Communications part II	1
2. Multi-level modulation schemes	3
2.1. Quadrature Shift Keying (QPSK)	3
2.2. Generation	3
2.2.1 Power Efficiency of QPSK	6
2.2.2 System Example	8
2.3. Offset Quadrature Phase Shift Keying (OQPSK)	10
2.4. M-ary PSK	11
2.4.1 Waveform and spectrum	11
2.4.2 Differential M-PSK	13
2.5. Shannon Limit: How Efficient Are We ?	14
2.6. Continuous Phase Frequency Shift Keying (CPFSK)	15
2.7. Minimum Shift Keying (MSK)	16
2.8. Gaussian Minimum Shift Keying (GMSK)	18
3. Modern Communications: Error Detection, Correction and source coding	20
3.1. Introduction	20
3.2. Error Detection	20
Parity	20
3.3. Error detection and correction: Hamming Codes	22
3.3.1 Numerical Example	23
3.4. Single Bit Error Correction Performance	25
3.4.1 Assumptions	25
Model	25
3.5. Cyclic Redundancy Checking	27
Example (G&G p349)	27
3.6. Convolutional Codes	28

3.7. Reed-solomon codes	29
3.8. Concatentation of coding and interleaving	30
3.9. Source coding	30
4. Modern Communications:Orthogonal Frequency Division Multiplexing	32
4.1. Introduction	32
4.2. Spectrum.....	32
4.3. Advantages and disadvantages	33
4.4. Transmitter implementation.....	33
4.5. Receiver implementation.....	34
4.6. Typical systems	35
4.7. ADSL.....	35
4.8. Characteristics	36
5. Wireless LAN	38
5.1. Basics	38

2. Multi-level modulation schemes

2.1. Quadrature Shift Keying (QPSK)

- Quadrature shift keying (QPSK) techniques were established in the 1960's as a means of increasing bandwidth efficiency.
- Binary modulation schemes such as BPSK have a theoretical efficiency of 1b/s/Hz. Bandwidth is expensive, so increasing this number is a priority.

2.2. Generation

- A conceptual QPSK modulator is shown in Figure 1. The incoming NRZ data stream, has a bit rate $R_b = 1/T_b$ where T_b is the bit period.
- Data fed into a serial to parallel converter. This groups the bits together in pairs and assign a phase value to each of the pairs. The output of the converter then drives the quadrature branches at a symbol rate (baud rate) R_s , where $R_s = 1/T_s = R_b / 2$.
- Low pass filters are used to restrict the bandwidth and also perform the necessary pulse shaping. The data streams are then modulated in phase quadrature and combined. The band pass filter at the output is used to contain the spectral spill over and any other spurious mixer products that are produced during the modulation process.

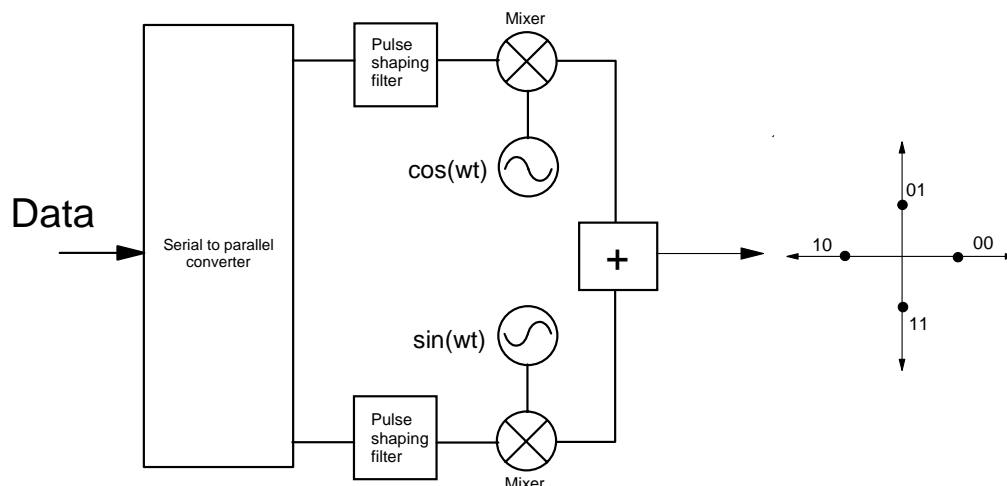


Figure 1. QPSK modulator

The information conveyed in the phase of the carrier (four states), so that the transmitted signal is of the form:

$$s_{QPSK}(t) = A \cos \left[2\pi f_o t + (i-1) \frac{\pi}{2} \right] \quad 0 \leq t \leq T_s \quad i = 1, 2, 3, 4$$

Waveform consists of a carrier with phase shifts of 0, 90, 180, and 270 degrees. This is shown in Figure 2.

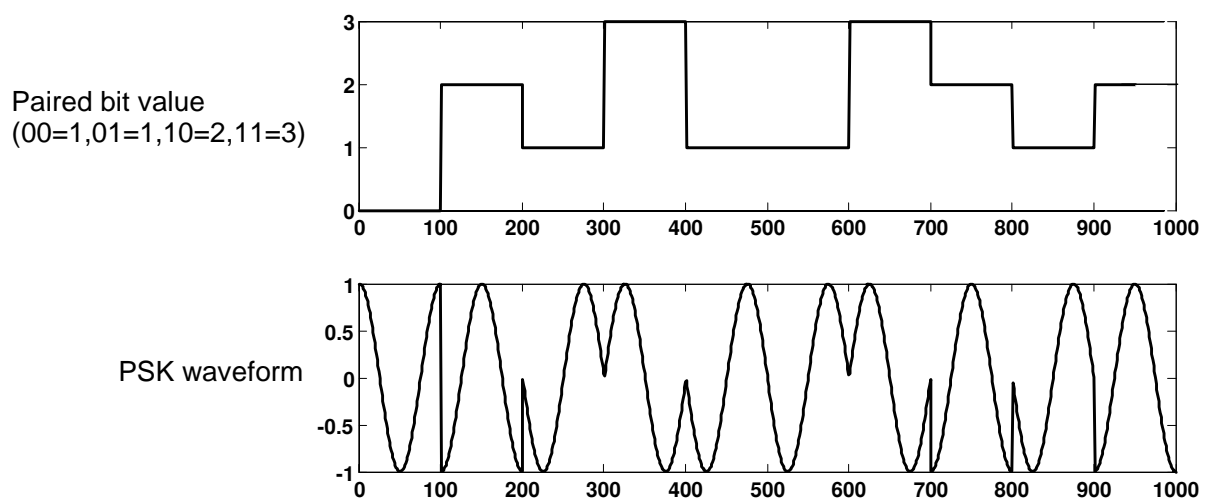


Figure 2. QPSK waveform

- QPSK waveform has a constant envelope with π and $\pi/2$ phase shifts at the data transmission points.

- Filtering causes constant envelope to be modified slightly
- QPSK signal can be considered as the sum of two phase reversal keyed – PRK (BPSK with $\{0,1\}$ signaling alphabet) signals in quadrature. The QPSK demodulator can be viewed as two PRK demodulators operating in quadrature with a parallel to serial converter at the output to combine the recovered 'I' and 'Q' data streams into a single binary data stream. Figure 3 shows the demodulator.

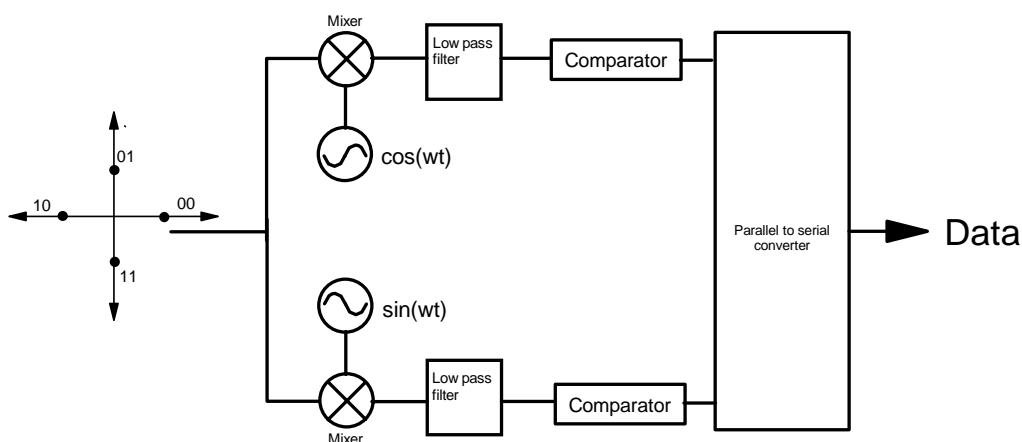


Figure 3. QPSK demodulator

Since the carrier information is conveyed in the phase of the carrier, a phase locked replica of the carrier is required at the receiver. In the case of QPSK, this could be achieved using a Costas loop (See Glover and Grant).

2.2.1 Power Efficiency of QPSK

QPSK can be considered as two independent PSK channels in quadrature. Both must be correct for received signal to be correct. Consider BER initially.

Signal energy is $E/2$ in each channel. Average probability of making a correct decision P_c is:

$$P_c = (1 - P^1)^2$$

Where P^1 is the probability of error of one of the PSK channels. Therefore

$$P_c = \left[1 - \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E}{2N_0}} \right) \right]^2$$

So that the average probability of error for QPSK is

$$P_e = 1 - P_c = \operatorname{erfc} \left(\sqrt{\frac{E}{2N_0}} \right) - \frac{1}{4} \operatorname{erf}^2 \left(\sqrt{\frac{E}{2N_0}} \right)$$

When $E/2N_0 \gg 1$

$$P_e \approx \operatorname{erfc} \left(\sqrt{\frac{E}{2N_0}} \right)$$

Also in QPSK we have 2 bits per symbol so $E = 2E_b$ leading to

$$P_e \approx \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0}} \right)$$

The error probability in each channel is

$$P^1 = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E}{2N_0}} \right)$$

So we have the Bit Error Rate is

$$BER \approx \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0}} \right)$$

The symbol error probability P_s is approximately:

$$P_s \approx \operatorname{erfc} \left(\sqrt{\frac{E_s}{2N_o}} \right)$$

where E_s denotes the energy per symbol. Figure 4 shows the BER vs SNR.

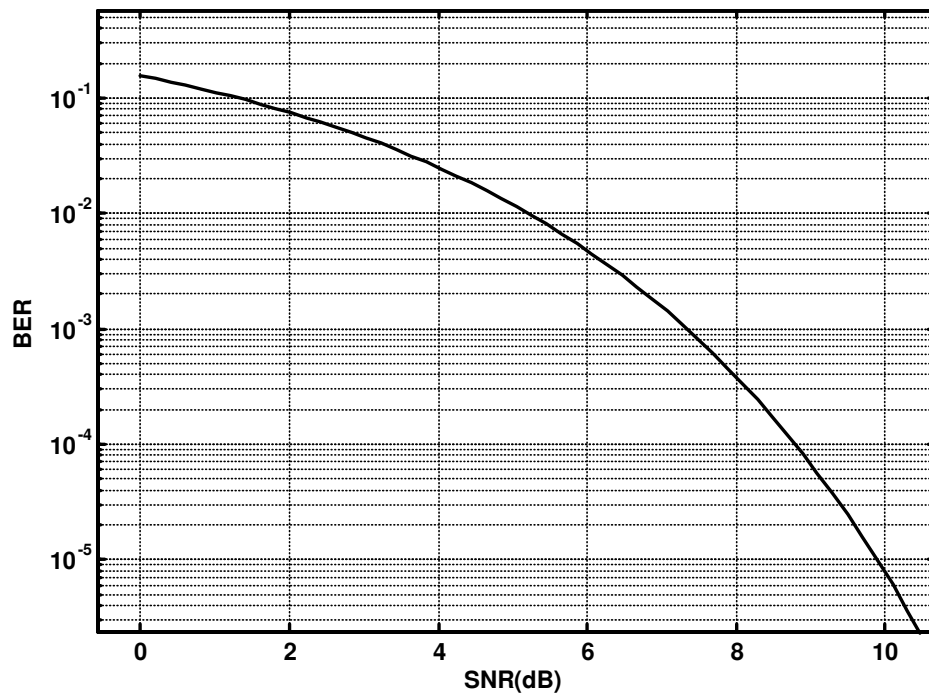


Figure 4. BER of QPSK vs. SNR

P_s is greater for a given signal to noise ratio when compared with PRK. However, twice as much information per symbol therefore performance same at low error rates. Due to the fact that symbol states of QPSK are orthogonal to each other.

Relationship between the probability of symbol error and the probability of bit error assuming Gray coding is:

$$P_e = \frac{P_s}{\log_2(M)} = \frac{P_s}{2}$$

For sufficiently low error rates. in terms of bit errors, the power efficiency of QPSK is equivalent to PRK.

2.2.2 System Example

Consider a QPSK system transmitting data at 44Mb/s. Assume that the average transmit power is 9 dBW, system losses are 120dB, noise power spectral density (N_o) = 1.67×10^{-20} W/Hz

The probability of symbol error, P_s is :

$$\begin{aligned} P_s &\approx \operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_o}}\right) = \operatorname{erfc}\left(\sqrt{\frac{ST_s}{2N_o}}\right) = \operatorname{erfc}\left(\sqrt{\frac{S}{2N_oR_s}}\right) \\ &= \operatorname{erfc}\left(\sqrt{\frac{S}{N_oR_b}}\right) = \operatorname{erfc}\left(\sqrt{\frac{10^{-11.1}}{(1.67 \times 10^{-20})(44 \times 10^6)}}\right) = 3.32 \times 10^{-6} \end{aligned}$$

Using Gray coding, the bit error rate (BER) or probability of error, P_e for QPSK is:

$$P_e = \frac{1}{2}P_s = \frac{1}{2} \times 3.32 \times 10^{-6} = 1.66 \times 10^{-6}$$

The bit error rate for an equivalent PRK system is:

$$\begin{aligned}P_e &= \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_o}} \right) = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{ST_b}{N_o}} \right) \\&= \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{S}{N_o R_b}} \right) = 1.66 \times 10^{-6}\end{aligned}$$

2.3. Offset Quadrature Phase Shift Keying (OQPSK)

- Problem of phase transitions create a signal of varying levels, so any amplifier must be able to cope with this. In practice this is hard. Offset QPSK is designed to solve this.

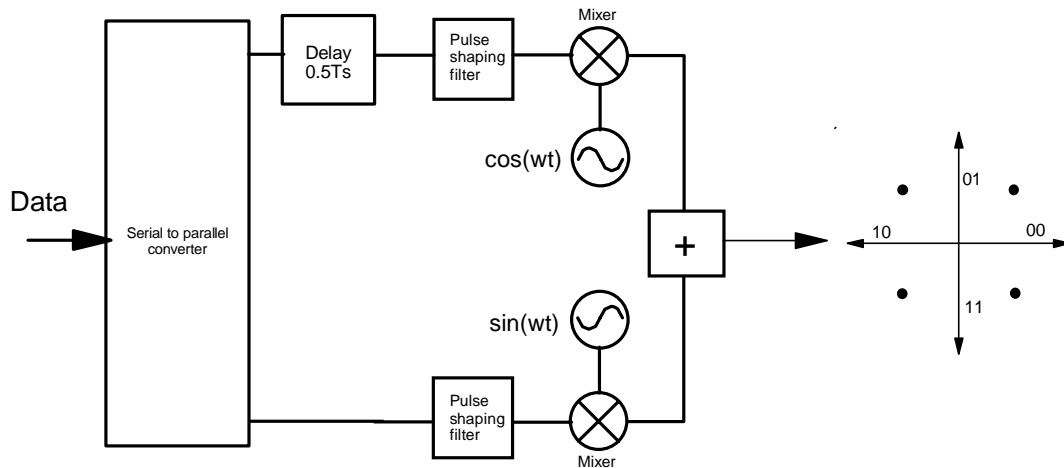


Figure 5. Offset QPSK modulator.

Conceptual OQPSK modulator is shown in Figure 5. This is the same as the QPSK except the in-phase and quadrature branches being offset by the bit duration, $T_b = T_s/2$. In this case, the phase transitions occur at a maximum rate of R_b (compared to R_s when QPSK is employed), but the maximum phase transition is restricted to $\pm \frac{\pi}{2}$. Figure 6 shows allowed phase transitions for QPSK and OPSK.

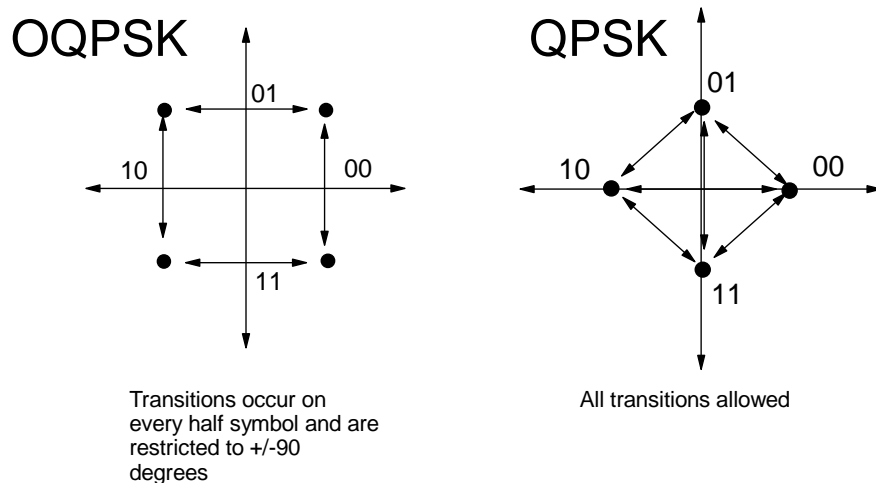


Figure 6. Allowed phase transitions for QPSK and OQPSK

2.4. M-ary PSK

2.4.1 Waveform and spectrum

- BPSK can be extended to M-ary PSK in order to cope with high data rates in bandwidth constrained channels.
- In M-ary PSK the binary PCM code is grouped into symbols consisting of $\log_2 M$ bits, which are then encoded into one of the M phases. Thus, the M-ary PSK waveform can be written as:

$$s(t) = A \cos(2\pi f_o t + \theta_i) \quad 0 < t \leq T_s$$

where T_s is the symbol duration and we have M possible phase angles, such that:

$$\theta_i = 0, \frac{2\pi}{M}, \frac{4\pi}{M}, \dots, \frac{2(M-1)\pi}{M}$$

The symbol duration is given by:

$$T_s = T_b \log_2 M$$

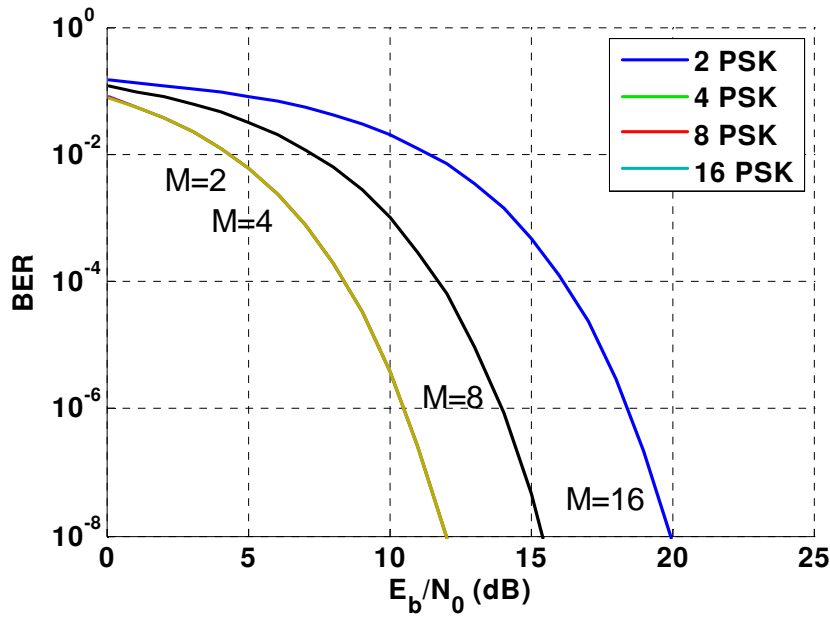


Figure 7. BER performance of MSK

The potential bandwidth efficiency for M-ary PSK is:

$$\frac{f_b}{f_s} = \log_2 M$$

It is difficult to derive an exact analytical expression for the probability of error. For a probability of symbol error $P_s < 10^{-3}$, the following provides a good approximation.

$$P_s \approx \operatorname{erfc} \left(\sqrt{\frac{E_s}{N_o} \sin \left(\frac{\pi}{M} \right)} \right) = \operatorname{erfc} \left(\sqrt{\log_2(M) \frac{E_b}{N_o} \sin \left(\frac{\pi}{M} \right)} \right)$$

Assuming Gray coding with the probability of symbol error small, the probability of bit errors can be obtained from:

$$P_e = \frac{P_s}{\log_2 M}$$

QPSK ($M=4$) offers a good trade off between power and bandwidth requirements. For high data rates in bandwidth limited channels, $M=8$ is typically used. Note moving from $M=4$ to $M=8$ incurs a power penalty of ~ 4 dBs (at $P_s \sim 10^{-5}$) whilst increasing the level from $M=8$ to $M=16$ incurs a ~ 5 dB penalty. Figure 7 shows the BER performance.

2.4.2 Differential M-PSK

Differential techniques (M-DPSK) can be used to simplify receiver complexity (no carrier recovery required). However, 4-DPSK incurs a penalty of ~ 2.3 dB, whilst for $M > 8$ the power penalty (relative to M-PSK) is of the order of 3dB.

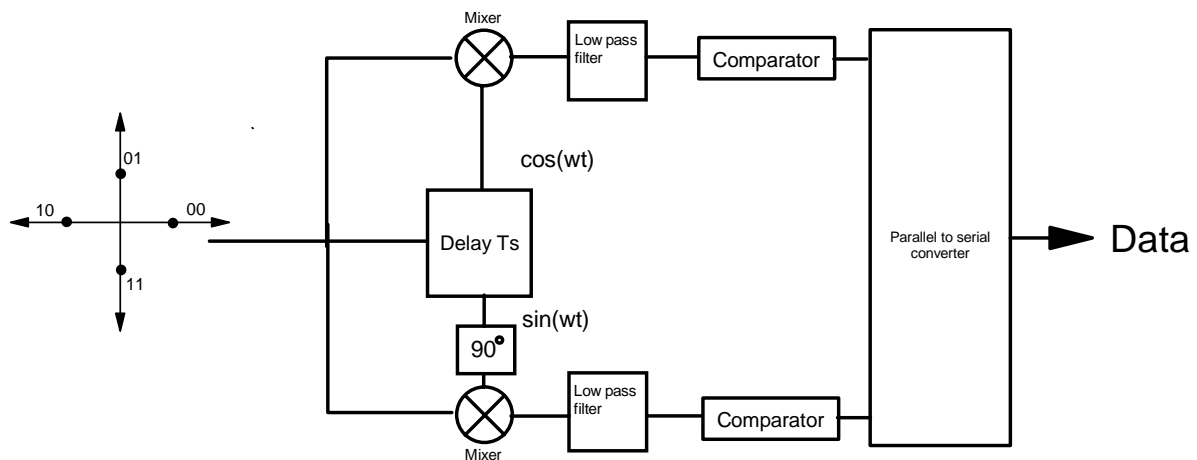


Figure 8. DQPSK demodulator

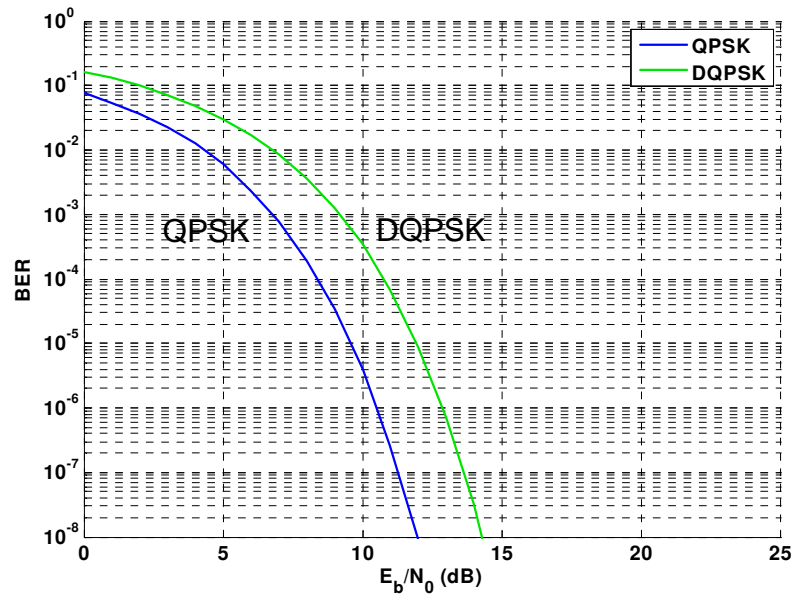


Figure 9. BER performance of DQPSK and QPSK.

2.5. Shannon Limit: How Efficient Are We ?

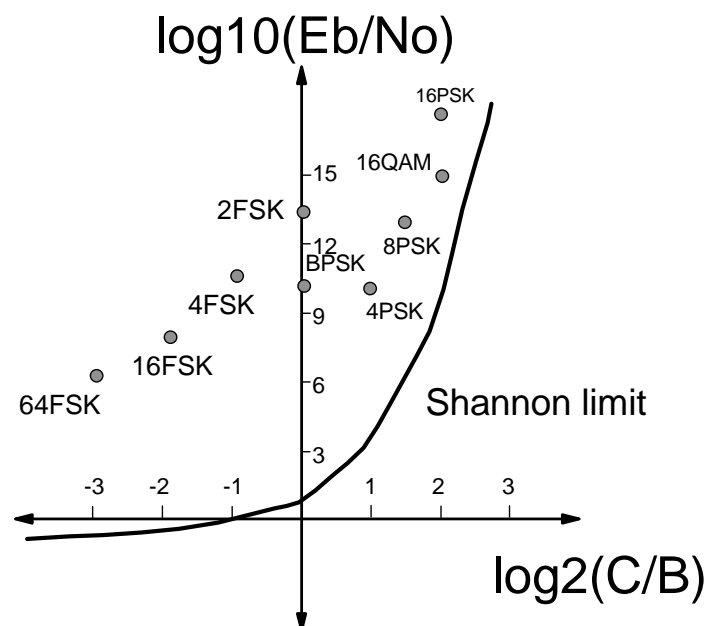


Figure 10. Performance of modulation schemes relative to the Shannon limit

- Figure 10 shows the performance of schemes. Can see not very efficient (graph is log scale) relative to Shannon limit. In practice

very difficult to reach this value, requiring information coding as well as complex modulation.

- Note that Shannon shows capacity grows as $\log(\text{SNR})$ so hard to improve capacity by improving quality of channel alone.

2.6. Continuous Phase Frequency Shift Keying (CPFSK)

- The aim of spectrally efficient modulation techniques such as QPSK/OQPSK is to maximise bandwidth efficiency.
- A further goal is to reduce the out of band response-phase jumps create high frequencies (frequency is rate of change of phase)

Continuous Phase Frequency Shift Keying (CPFSK) designed to reduce phase jumps. Phase controlled and information carried in the frequency of the carrier. The theoretical bandwidth efficiency is 2 bits per second per Hz and the spectral response rolls off at a rate of $\geq \omega^{-4}$.

Modulation	Bandwidths		
	-3dB	-50dB	99% Power Containment
QPSK, OQPSK	$0.44/T_b$	$100/T_b$	$10.3/T_b$
MSK (see later) Minimum Shift Keying	$0.59/T_b$	$8.18/T_b$	$1.17/T_b$

The generalised CPFSK waveform is written as :

$$s(t) = A \cos(2\pi f_o t + \gamma(t))$$

where $\gamma(t)$ the phase with respect to the carrier, is a continuous function of time. Over the bit interval, $0 < t < T_b$

$$s(t) = A \cos(2\pi(f_o \pm \Delta f)t + \gamma(0)) \quad 0 < t < T_b$$

with

$$f_o = \frac{f_1 + f_2}{2}, \Delta f = \frac{f_2 - f_1}{2}$$

where f_1 and f_2 are the frequencies that represent the zeros and ones respectively. $\gamma(t)$ can be expressed as:

$$\gamma(t) = \pm 2\pi\Delta f t + \gamma(0) \quad 0 \leq t \leq T_b$$

where the initial phase $\gamma(0)$ is chosen in such a way so as to avoid phase discontinuities. Since the aim is to **optimise** the spectral efficiency, Δf is chosen such that:

$$\Delta f T_b = 0.25$$

This case of CPSK is known as minimum shift keying or MSK (since it is the minimum frequency spacing between f_1 and f_2 that allows two FSK signals to be orthogonal to one another. Thus for MSK:

$$\gamma(t) = \pm \frac{\pi}{2T_b} t + \gamma(0) \quad 0 \leq t \leq T_b$$

2.7. Minimum Shift Keying (MSK)

Figure 11 shows a typical MSK waveform and phase. The minimum frequency deviation to retain **orthogonality** between frequencies is therefore denoted Minimum Shift Keying (for spectral efficiency). The MSK waveform can be expressed:

$$s(t) = A \cos \left[\omega t + \frac{\pi t}{2T_b} p_k + \gamma_k \right]$$

where γ_k is the excess phase at the start of the k^{th} bit and p_k is the binary switching function $\{-1, +1\}$ representing the base band data. From FM theory, the modulation index:

$$h \equiv \beta = \frac{\Delta f}{f_m} = 0.5|_{MSK}$$

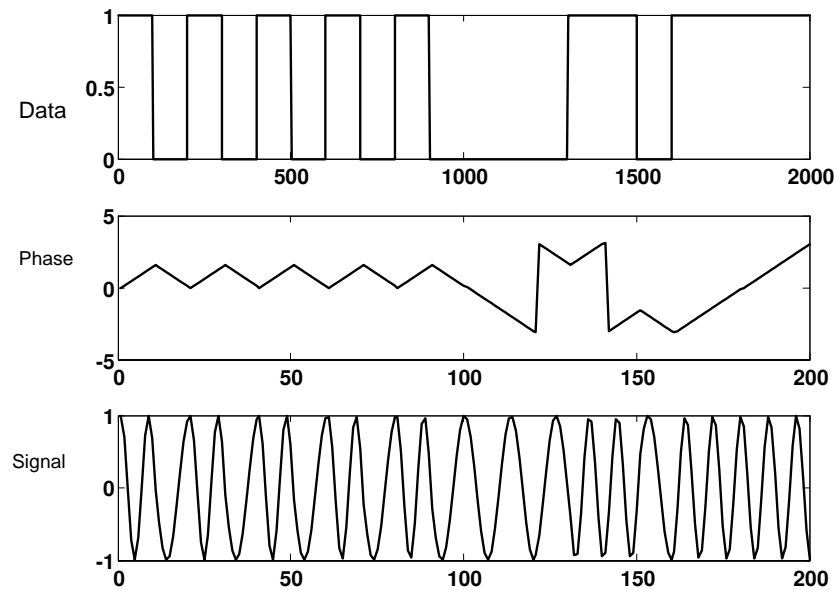


Figure 11. MSK phase and envelope

Recall that the MSK waveform is:

$$s(t) = A \cos \left[\omega_c t + \frac{\pi t}{2T_b} p_k + \gamma_k \right]$$

Expand the equation to obtain the quadrature form:

$$s(t) = A \left[a_I(t) \cos \left(\frac{\pi t}{2T_b} \right) \cos \omega_c t - a_Q(t) \sin \left(\frac{\pi t}{2T_b} \right) \sin \omega_c t \right]$$

Can consider MSK as a special case of OQPSK with sinusoidal weightings given to each pulse.

- Theoretical efficiency is the same as QPSK/PRK
- True constant envelope modulation scheme- hence it is relatively immune to spectral regrowth from non-linear amplification.
- Power Spectral Density has a rapid roll off in the sidelobes due to the continuous phase, a wider main lobe than QPSK and the slope of the phase (rate of change of phase + frequency) is not continuous.
- MSK is not sufficiently spectrally efficient for many band-limited applications (e.g. digital cellular telephony). To further increase spectral efficiency Gaussian Minimum Shift Keying was developed.

2.8. Gaussian Minimum Shift Keying (GMSK)

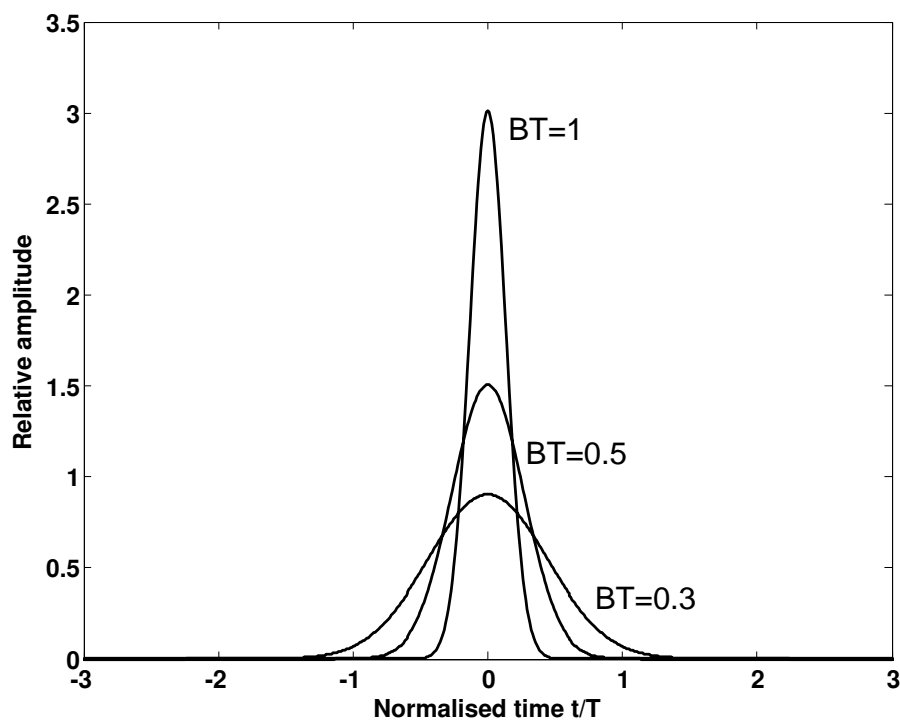


Figure 12. Gaussian filter impulse response

- prefilter the data prior to MSK modulation using a Gaussian Filter. Figure 12 shows the impulse response of a set of Gaussian filters (recall that a Gaussian Fourier Transforms to a Gaussian).

Gaussian filters

- A low bit overshoot in the time domain - this protects against excessive frequency deviation.
- They introduce a controlled amount of inter symbol interference (ISI) to reduce the occupied bandwidth.
- The normalised bandwidth $B_N = BT$ controls the interference tolerance and PSD characteristics: where B is the bandwidth of the Gaussian Filter and T is the bit period.

If the premodulation bandwidth B is decreased, the pulse duration increases

$B_N=0.5$ the symbol is spread over ~ 2 bit periods (DECT)

$B_N=0.3$ the symbol is spread over ~ 3 bit periods (GSM)

3. Modern Communications: Error Detection, Correction and source coding

3.1. *Introduction*

Two basic techniques.

- Error detection and retransmission of data. Information is added to transmitted data to allow detection of errors. A protocol is then used to ask for any retransmission
- Forward Error Correction Coding (FECC). If retransmission is not appropriate information can be coded to allow error correction.

Any coding operation will reduce the information transfer rate, as redundant information is added before transmission.

3.2. *Error Detection*

Parity

- Take a group of data bits and count the number of "ones" and insert an extra bit to indicate whether or not the total number of bits was even or odd (depending on convention)
- Receiver of this data stream can check to see if there is an error in the data.
- If errors are **equiprobable** throughout the data word then the receiver does not know if the error is in the original information word or the check bit itself, but there is an error (What about two errors?)
- Process can be implemented simply as an exclusive "OR" operation.

Compare coded and uncoded performance. Assume the probability of error in the uncoded case is P_e . The probability of an undetected error in the coded case will be the probability of an even number of errors. Parity check at the receiver will fail indicate an error, only if an odd number of errors occurs.

If we have n digits in each word, the probability of one digit being in error P_j is the joint probability of one incorrect and $(n-1)$ correct digits;

$$P_j = P_e(1 - P_e)^{n-1}$$

There are n chances of this occurring, so the probability of a single error in n digits P_{jn} is

$$P_{jn} = nP_e(1 - P_e)^{n-1}$$

For our bit error rate we want to know the probability of a certain number of errors occurring in a given information stream. So the probability that we get r errors in n digits is the joint probability that r digits will be received incorrectly and $(n-r)$ will be received correctly.

$$P_j = P_e^r (1 - P_e)^{n-r}$$

There is a total of nC_r ways of receiving r digits incorrectly in a total of n digits where:-

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

Thus the total probability of error is

$$P_j = {}^nC_r P_e^r (1 - P_e)^{n-r}$$

If there is an even number of errors then we cannot detect the error. The total probability of this occurring is the sum of all the probabilities of 2,4,6,8,... errors occurring in the word. (See the tutorial sheet problem for calculating this answer)

3.3. Error detection and correction: Hamming Codes

- Hamming codes can detect and correct single bit errors, and some combinations of multiple errors. Invented in the 1950s
- Choose groups of data and exclusive OR together to produce a check bit. Need enough check bits to span all the data. 16 bit data words then we need 5 check bits, since in order to be able to represent 16+ check bit positions (21 bits) we need at least a numerical range of 21 and the nearest is 32 (2^5). This allows us to recognise multiple bit errors (some of the time) though they cannot be corrected.
- At receiver perform an operation to generate more check bits- called a **syndrome**. Compare this with original check bits to determine error and location

Example

Consider single bit error detection and correction for an 8 bit data word. Applying 4 check bits we note that we have 16 discrete numerical values in our system which is adequate for our requirement.

To generate the code. The labels B0 –B7 represent the bit positions in the data word, the C0 – C3 denote the check bits and the S0 – S3 denote the syndrome operator.

For example C3 is generated by an exclusive 'OR' ing of B7, B6, B5, B4.

$$C3 = B7 \oplus B6 \oplus B5 \oplus B4$$

$$C2 = B7 \oplus B3 \oplus B2 \oplus B1$$

$$C1 = B6 \oplus B5 \oplus B3 \oplus B2 \oplus B0$$

$$C0 = B6 \oplus B4 \oplus B3 \oplus B1 \oplus B0$$

Data and check bits are transmitted in a message of the following format.

M12	M11	M10	M9	M8	M7	M6	M5	M4	M3	M2	M1
B7	B6	B5	B4	C3	B3	B2	B1	C2	B0	C1	C0

Then the data is received or read from a memory we generate a syndrome

$$S3 = B7 \oplus B6 \oplus B5 \oplus B4 \oplus C3$$

$$S2 = B7 \oplus B3 \oplus B2 \oplus B1 \oplus C2$$

$$S1 = B6 \oplus B5 \oplus B3 \oplus B2 \oplus B0 \oplus C1$$

$$S0 = B6 \oplus B4 \oplus B3 \oplus B1 \oplus B0 \oplus C0$$

This syndrome bit pattern then gives an indication (the position) of the bit in error and that bit is automatically inverted.

3.3.1 Numerical Example

Consider the data word:-

B7							B0
1	1	0	1	1	1	0	0

The check word becomes

$$\begin{aligned} C3 &= B7 \oplus B6 \oplus B5 \oplus B4 \\ &= 1 \oplus 1 \oplus 0 \oplus 1 = 1 \end{aligned}$$

$$\begin{aligned} C2 &= B7 \oplus B3 \oplus B2 \oplus B4 \\ &= 1 \oplus 1 \oplus 1 \oplus 0 = 1 \end{aligned}$$

$$\begin{aligned} C1 &= B6 \oplus B5 \oplus B3 \oplus B2 \oplus B0 \\ &= 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 = 1 \end{aligned}$$

$$\begin{aligned} C0 &= B6 \oplus B4 \oplus B3 \oplus B1 \oplus B0 \\ &= 1 \oplus 1 \oplus 1 \oplus 0 \oplus 0 = 1 \end{aligned}$$

And the whole word now becomes

M12	M11	M10	M9	M8	M7	M6	M5	M4	M3	M2	M1
1	1	0	1	1	1	1	0	1	0	1	1
B7	B6	B5	B4	C3	B3	B2	B1	C2	B0	C1	C0

Assuming no error the syndrome is calculated as above:

$$S3 = 0, S2 = 0, S1 = 0, S0 = 0$$

If we now assume data bit M7 is incorrect ($1 \rightarrow 0$). The syndrome now becomes:

$$S3 = 1$$

$$S2 = 1$$

$$S_1 = 0$$

$$S_0 = 0$$

The syndrome is 1100 which indicates M12 or B7 is in error.

3.4. Single Bit Error Correction Performance

You will readily see that this technique suffers from the problem of multiple bit errors mapping onto the space for correct data. If we have a 16 bit data word and a five bit check word then the error check word can resolve 32 bit patterns and hence detect and correct single bit errors in all 21 bit positions. So we have 21 valid error codes the others being multiple bit errors. However, it can be seen that a higher number of multiple bit errors must map into valid code space and therefore be incorrectly interpreted. (So manufacturers will claim something like all single bit errors are corrected and 11/32 of multiple bit errors will be detected but not corrected.) We now look at the performance of such codes with this in mind.

3.4.1 Assumptions

- Single bit corruption in each word
- Each bit has an equal probability of being corrupted.
- Single bit corruptions are independent.
- Single bit errors can be corrected.

Model

Data stream consisting of x words consisting of y bits

There are

- xy choices of placement of the single corruption.

- $(x-1)y$ choices for placement of the second corruption, out of a remaining $xy-1$ bits. Probability of two bits in error (in different words) is $\frac{(x-1)y}{xy-1}$
- $(x-2)y$ choices out of $xy-2$ remaining bits for the third corruption

To arrive at 3 single bit errors we have to have gone through the first two stages. So the probability of 3 random failures (errors) producing a correctable data stream is

$$\frac{xy}{xy} \cdot \frac{(x-1)y}{xy-1} \cdot \frac{(x-2)y}{xy-2}$$

In general, the probability of producing a working data stream with $n+1$ bits in error is

$$y^{n+1} \frac{x(x-1)(x-2)(x-3)\cdots(x-n)}{xy(x-1)(xy-2)(x-3)\cdots(xy-n)}$$

$$y^{n+1} \frac{x(x-1)!(xy-(n+1))!}{xy(xy-1)!(x-(n+1))!}$$

or

$$P = y^n \frac{(x-1)!(xy-(n+1))!}{(xy-1)!(x-(n+1))!}$$

Clearly our maximum error rate for any chance of working is one bit in error in each word, or $1/y$

The probability of an error occurring is

$$1 - \frac{y^n (x-1)! (xy - (n+1))!}{(xy-1)! (x - (n+1))!}$$

3.5. Cyclic Redundancy Checking

- Cyclic redundancy checking (CRC) generated by performing an operation on the whole of the data set and appending the result to the data.
- On reception the reciprocal operation is performed and if present an error is detected.
- Used in Ethernet and other transmission schemes
- CRC check bits are much more efficient than byte parity

Example (G&G p349)

Message $M(x)$ written as a polynomial:

$$M(x) = 1001 \text{ (equivalent to } M(x) = 1 + x^3 \text{)}$$

CRC uses a generator polynomial $P(x) = 1101$ (equivalent to $G(x) = 1 + x^2 + x^3$)

Transmitter

- Shift $M(x)$ by the order of $G(x)$ so $M'(x) = 1001000$ as highest coefficient in $G(x)$ is cubic.
- Divide $M'(x)$ by $G(x)$ and obtain a remainder. (011)
- Replace zeros in $M'(x)$ and transmit resulting sequence
 $M'(x) = 1001011$

Receiver

- Divide received message by the generator polynomial and if zero no errors were transmitted.

- If non-zero then further division and manipulation(G&G) can locate the bit positions of errors

Generally if the order of the generator polynomial is k a burst error of up to k consecutive bits can be detected.

CRCs are widely used as they are fast and low complexity. Correction of errors is normally achieved using a retransmission scheme.

A popular polynomial used in IEEE standards for LANS is

$$G(x) = x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x^1$$

3.6. Convolutional Codes

Codes that handle single bit errors not appropriate for burst errors (such as wireless channels). Want codes that can to an extent cope with multiple bits in error.

Convolutional codes process a continuous stream of data bits to generate a continuous stream of encoded digits each of which has a value dependant on the value of several previous information bits. Have memory

Coding is implemented by shift register of a given length and a number of modulo 2 adders (EXOR gates). In the Figure 13 the output data rate is double the input data rate- this is known as a half rate code. For every one input bit there are two outputs, one from each of the logic branch.

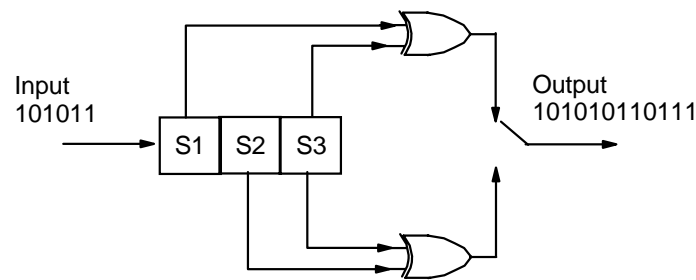


Figure 13. Convolutional encoder

These codes are usually decoded using a viterbi decoder, which is logically a ‘tree’ where the likely data path is mapped out. (See Glover and Grant)

3.7. Reed-Solomon codes

These codes take a block of k data bits and use a known polynomial to encode the data. The polynomial is evaluated at n points where $n > k$ and these values (ie $F(x), x = x_0 \dots x_{k-1}$) are then sent. As the polynomial is ‘over-determined’ ($k > n$) some points can be lost and the receiver can still reconstruct the data.

Theory says that a Reed-Solomon code can correct $\frac{n-k}{2}$ errored symbols in the code.

- Code invented in the 1960s, but only with modern digital electronics did it become feasible
- Used on CDs and media to correct ‘long’ bursts of errors rather than randomly distributed errors. (What would this be on a CD?)
- Also used in ADSL

3.8. Concatentation of coding and interleaving

- Modern signal processing allows codes to be added together. Figure 14 shows the principle.
- At the coding stage the RS coded stream is interleaved. This mixes up the data so that burst errors are redistributed in the interleaver at the receiver.
- At the decoding stage the convolutional code corrects some of the errors, but creates bursts of errors at the output. The de-interleaver distributes these bursts into smaller ones, which the RS coder can correct.
- Much complicated maths and analysis is expended on coding. Why?

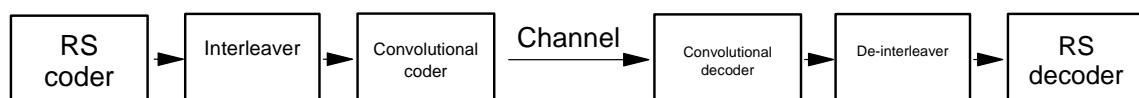


Figure 14. Code concatenation

3.9. Source coding

- Source coding looks at probability of symbols in message and tries to minimise the redundancy.
- Huffman coding is one such method. A modified method is used in Fax machines to remove whitespace. Figure 15 shows the coding technique.

Element	p(%)								Code
A	40							0	0
C	20					1		100	111
G	14				1	24		0	101
E	10				0			60	100
H	06					36		1	1100
B	05	—	1		16	0			11011
F	03	1		10	1				110101
D	02	0	5	0					110100

Figure 15. Huffman coding table

4. Modern Communications: Orthogonal Frequency Division Multiplexing

4.1. Introduction

- OFDM become extremely important for wireless communications, and for the efficient use of any bandwidth constrained and imperfect channel.

4.2. Spectrum

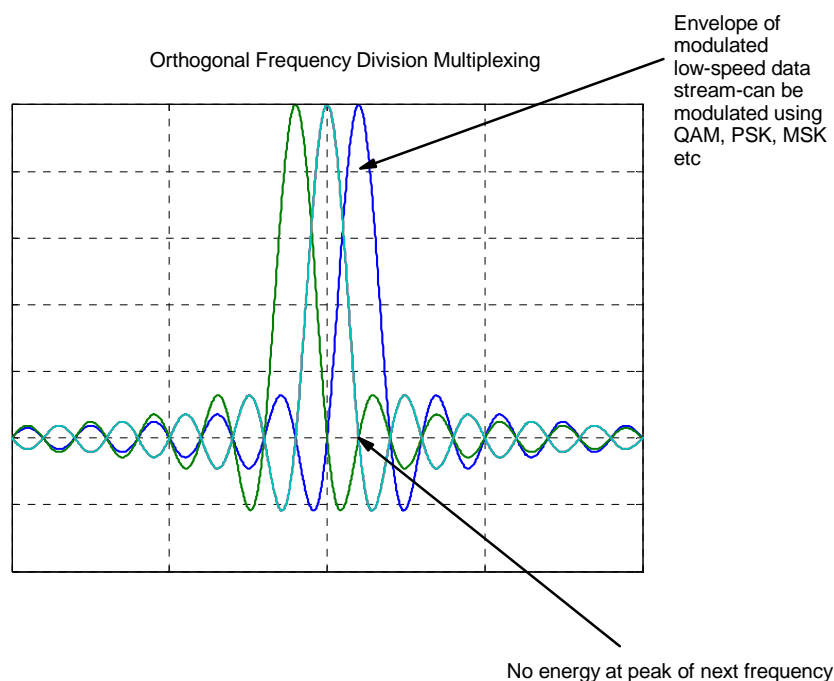


Figure 16. Frequency spectrum of OFDM.

Concept is to separate data into a large number of low-rate data streams. Each is coded using a particular modulation scheme (so each bit takes on a complex amplitude $(1, j, -1, -j)$ for QPSK as an example). These are then modulated onto one of a large number of orthogonal carriers.

4.3. Advantages and disadvantages

- Many wireless channels suffer from dispersion and fading. Rayleigh fading is narrowband (as it is coherent interference) so in OFDM only a small number of carriers will be affected.
- Dispersion affects wideband channels, but in OFDM the channel is too narrow for this to have a large effect.
- Similar for copper telephone lines or any channel where the frequency response of the channel is variable and or unpredictable.
- Disadvantage is the complexity of the modulator and demodulator scheme
- High peak-to-average-power ratio (PAPR)

4.4. Transmitter implementation

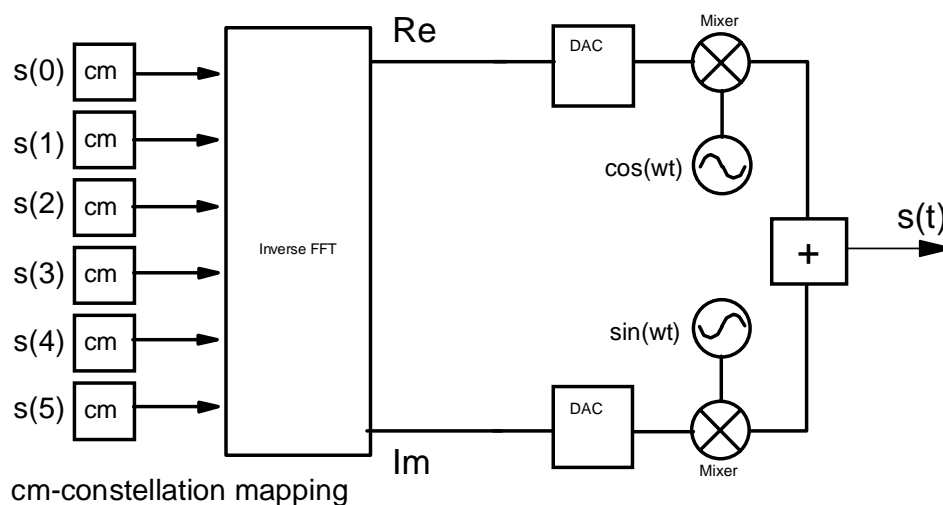


Figure 17. OFDM modulator.

- Based on IFFT and FFT
- Serial data stream is parallelised so $N \times m$ samples ($s(0) \dots s(5)$ in the figure) are inputs to N constellation mappers (ie the bit value is mapped onto an m level modulation scheme).

- Complex amplitude from this acts as input to the frequency side of an FFT block. Each input point represents one of the OFDM carriers. Task of the FFT is to change these into a time waveform.
- Output of FFT split into real and imaginary digital components.
- DACs convert these values, and these are then used to quadrature modulate the RF carriers

4.5. Receiver implementation

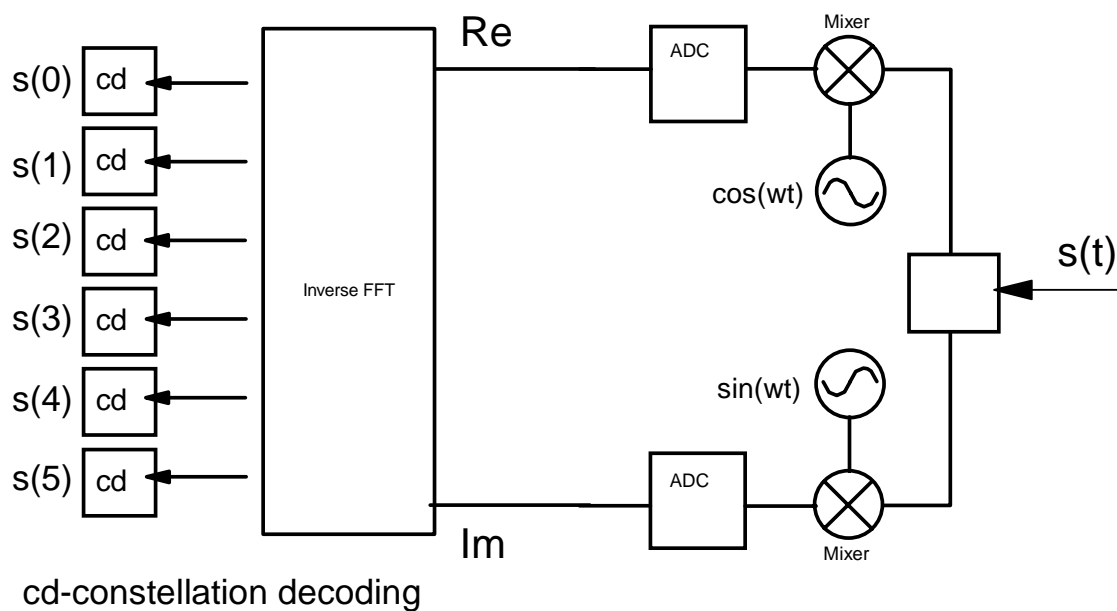


Figure 18. OFDM demodulator.

4.6. Typical systems

Standard name	DAB Eureka 147	DVB-T	
Ratified year	1995	1997	
Frequency range of today's equipment	174 - 240, 1452 - 1492	470 - 862	MHz
Channel spacing B	1.712	8	MHz
Number of subcarriers N	192, 384, 768 or 1536	2K mode: 1705 8K mode: 6817	
Subcarrier modulation scheme	DQPSK	QPSK, 16QAM or 64QAM	
Total symbol length T_S		2K mode: 224 + Guard Interval 8K mode: 896 + Guard Interval	μs
Guard interval T_G		1/4, 1/8, 1/16, 1/32	Fraction of T_S
Subcarrier spacing $\Delta f = 1/(T_S - T_G) \approx B/N$		2K mode: 4464 8K mode: 1116	Hz
Net bit rate R	0.576 - 1.152	4.98 - 31.67 (typically 24)	MHz
Link spectral efficiency R/B	0.34 - 0.67	0.62 - 4.0	bit/s/Hz
Inner FEC	Conv coding with code rates 1/4, 3/8 or 1/2	Conv coding with code rates 1/2, 2/3, 3/4, 5/6 or 7/8	
Outer FEC (if any)	None	RS(204,188,t=8)	
Maximum travelling speed	200 - 600	53 - 185	km/h
Time interleaving depth	385	0.6 - 3.5	ms
Adaptive transmission (if any)	None	None	
Multiple access method (if any)	None	None	
Typical source coding	192 kbit/s MPEG2 Audio layer 2	4 Mbit/s MPEG2	

Figure 19. OFDM standards.

4.7. ADSL

Figure 20 shows the spectral map for ADSL.

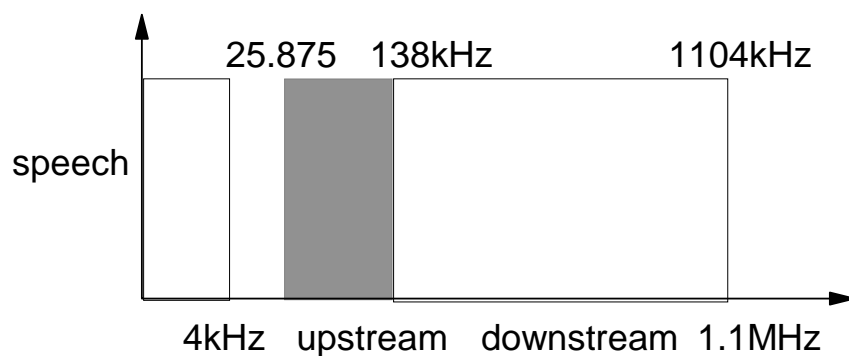


Figure 20. ADSL spectrum

As the copper pair is an unknown quantity the bit-rate available is variable. It is relatively easy to boost (or equalise) particular carriers that are subject to attenuation. Some ADSL standards are shown Figure 21.

Common name	Downstream rate	Upstream rate
ADSL	8 Mbit/s	1.0 Mbit/s
ADSL2	12 Mbit/s	1.0 Mbit/s
ADSL2+	24 Mbit/s	1.0 Mbit/s

Figure 21. ADSL standards

4.8. Characteristics

- 256 carrier ('bins') CODFM
- 0-4kHz phone line- 4-25kHz guard band-25-138kHz (25 bins on 4.3125kHz centres) and 138-1104kHz (224 bins)
- Within each bin (carrier) QPSK/QAM used to encode 2-15 bits/per symbol (This is adaptive depending on the SNR and attenuation of each bin)

Figure 22 shows the structure of an ADSL installation.

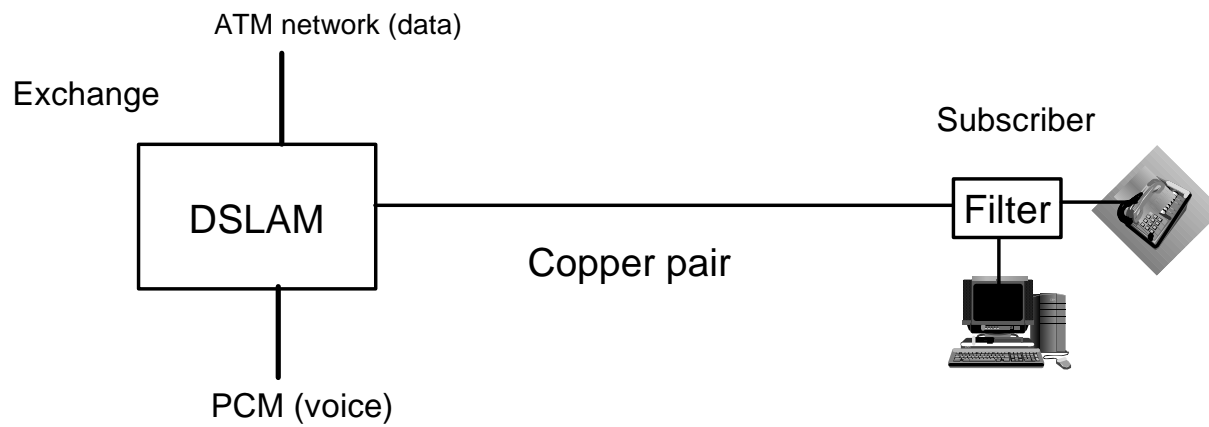


Figure 22. Structure of ADSL installation

5. Wireless LAN

5.1. *Basics*

- 2.4GHz Carrier frequency
- Bandwidth of 13 x5MHz channels
- Channel is shared between all stations in the network
- Stations (Computers etc) can talk or listen –not at once
- CSMA/CA Carrier Sense Multiple Access/Collision Avoidance. Stations listen to the channel before talking (CS). If channel is quiet station sends out signal that it wants to transmit, and then waits for a specified time before transmission (avoiding collisions). It then listens to hear for correct transmissions to be acknowledge.
- Problems. Cannot listen for collisions. Hidden terminal problem If two terminals want to transmit to the access point and they are out of range of each other Carrier Sense will not work. **Request to Send** (RTS) packet sent by the sender S, and a **Clear to Send** (CTS) sent by access point. Figure 23 shows a simple structure
- Coded Orthogonal Frequency Division Multiplexing (COFDM) used in 802.11(g). However, backward compatibility with 802.11(b) means complex control mechanisms required, as well as capability for simpler modulation schemes.

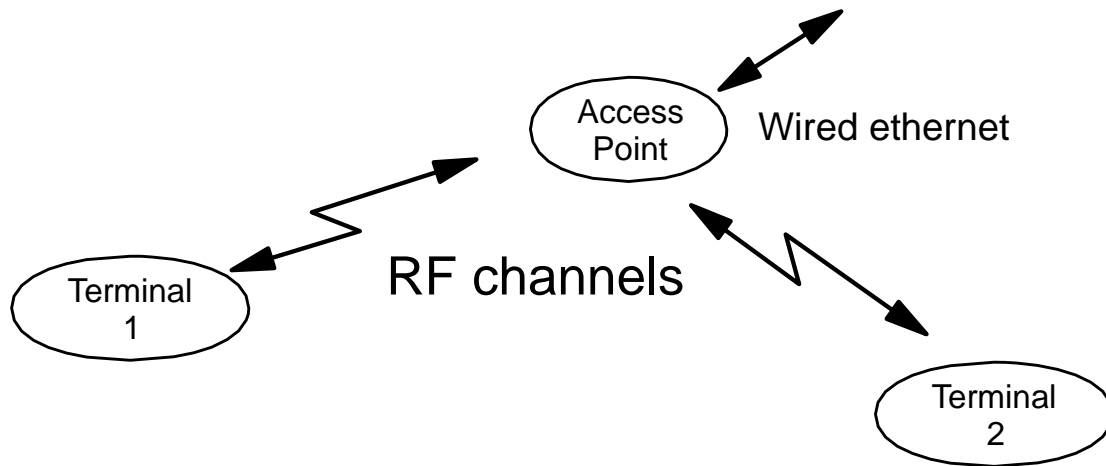


Figure 23. Simple wireless LAN

The frequency plan is show in Figure 24.

channel	frequency (MHz)
1	2412
2	2417
3	2422
4	2427
5	2432
6	2437
7	2442
8	2447
9	2452
10	2457
11	2462
12	2467
13	2472

Figure 24. Frequency plan