Integrated Optics - Frank Payne HT 2008

Goals and applications of integrated optics

What is integrated optics? It is easier to say what its goals are, and then to enquire as to how they are achieved. Amongst the most important goals of integrated optics I would include:

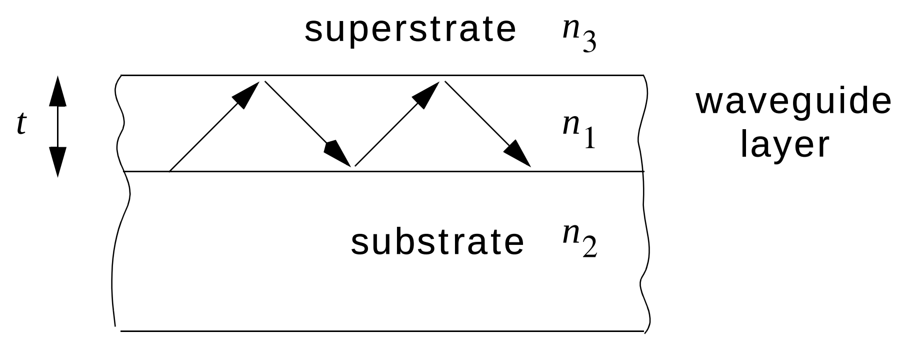
- ability to process optical signals directly
- compatibility with optical fibres
- compatibility with semiconductor lasers and detectors
- planar fabrication technology
- integration of components and devices
- small size and high speed

Having met these goals, what would be some of the applications of integrated optics? The list of applications is exhaustive and are dominated by the requirements of telecommunications, but amongst them I would include the following

- optical switching
- data encoding
- wavelength multiplexing
- polarisation control
- short pulse generation

The basic building block - the optical waveguide

The fundamental building block of integrated optics is the dielectric optical waveguide. An understanding of the operation of this component is essential for designing most optical circuits. The basic idea is that light is guided by total internal reflection in a longitudinally uniform layer whose refractive index is slightly higher than the material surrounding it. It usually consists of a substrate of refractive index n2 onto which is deposited a dielectric layer of higher refractive index n1. Further layers of lower index n3 may then be grown on top of the guiding layer, or it may simply be left with air as the top layer. Alternatively, the guiding layer may be formed by diffusing impurities into the substrate to increase the local refractive index. In either case, the waveguiding action is similar, and is illustrated below:



For efficient signal processing, the waveguide must support only one mode. Since the wavelengths used are in the range 0.85μm - 1.5μm, the guiding layer will be inconveniently thin to fabricate accurately unless the refractive indices n1 and n2 differ by only a few %. In the lectures we will show that the waveguide will be single-moded if the following condition is satisfied:

$$\frac{\pi t}{\lambda} \cdot \sqrt{n_1^2 - n_2^2} \approx 1$$

For an index difference of n_1-n_2~0.01 we require a thickness t~1-2 μm for λ ~1.3 μm and n1~3.5 . Such a thickness can be accurately controlled.

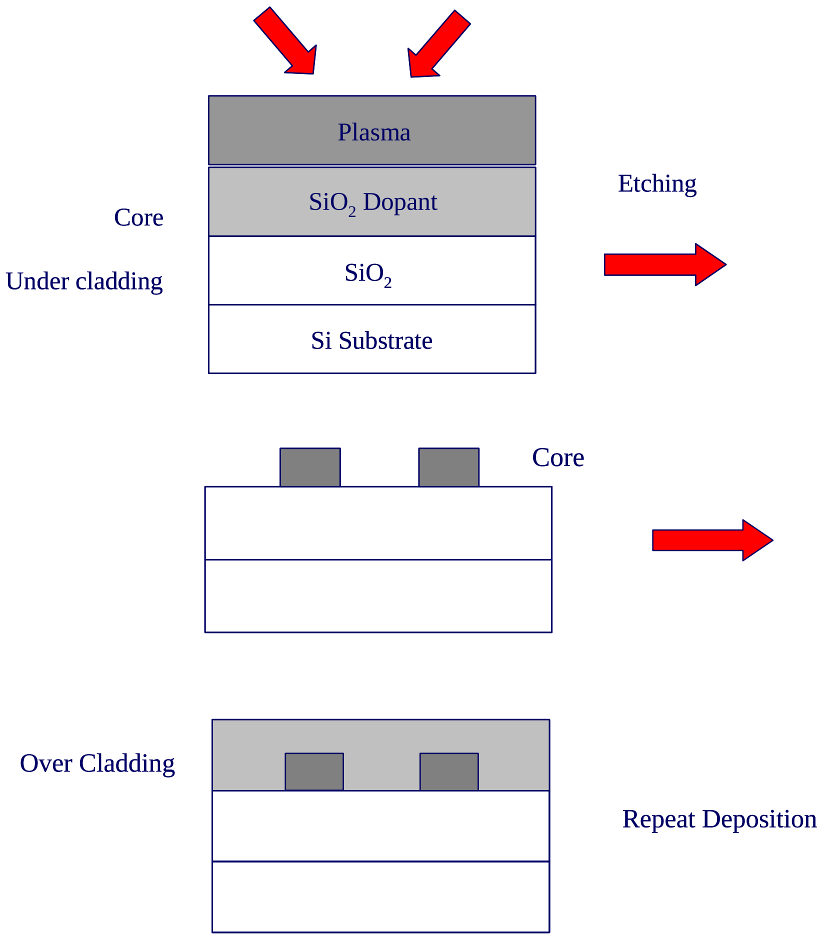
What do we make waveguides from?

We classify waveguides into passive and active. An active waveguide can be changed by the application of small bias voltages; it might generate light (as in a laser), or it might be able to amplify an optical signal - all this in addition to guiding light. A passive waveguide simply guides light; its waveguiding characteristics cannot be changed once it is made. The most commonly used substrate materials for these two types of waveguide are summarised below:

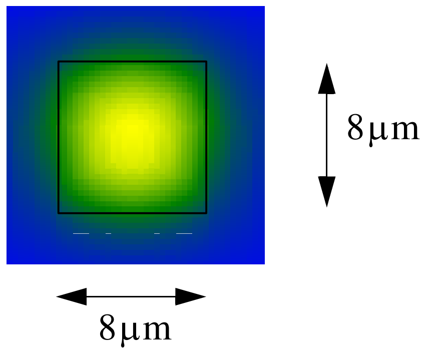
passive	active	active semiconductor
silica on silicon	lithium niobate $LiNbO_3$	gallium aresnide GaAs
silicon on insulator (SOI)		indium phosphide InP

Silica on silicon waveguides

- Most common material for passive integrated optical circuits
- Very low loss, typically ~ 0.01.dB/cm
- Waveguide core consists of SiO_2 doped with GeO_2
- Typical waveguide cross section 8x8 microns
- Plasma enhanced chemical vapour deposition PECVD:



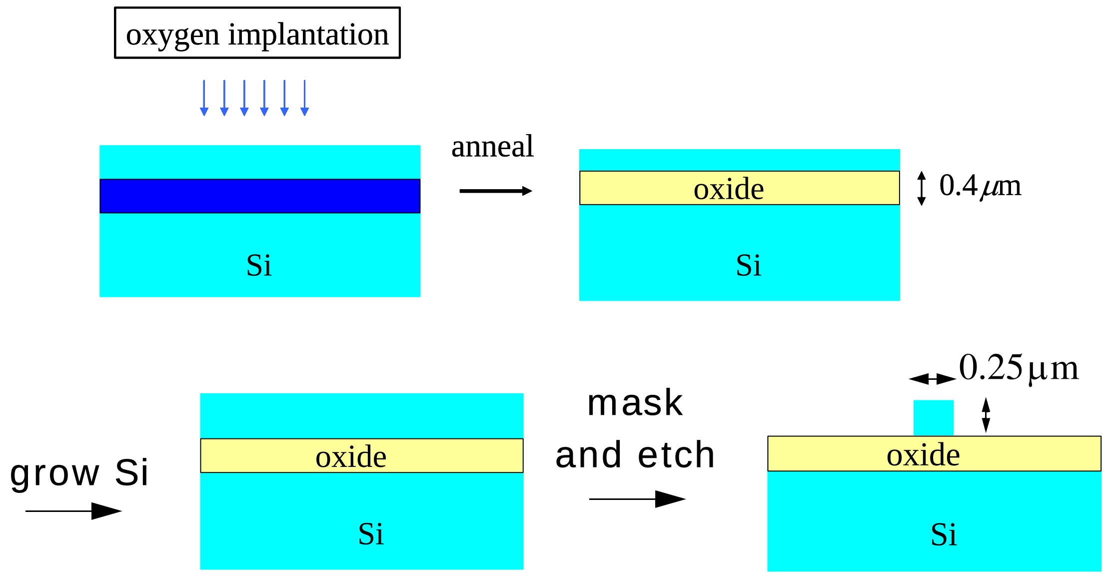
Light intensity distribution over cross section of 8x8 micron silica on silicon waveguide. The intensity distribution has a near Gaussian radial distribution close to that of a single mode fibre.



Silicon on insulator waveguides (SOI)

Sub-micron silicon waveguides

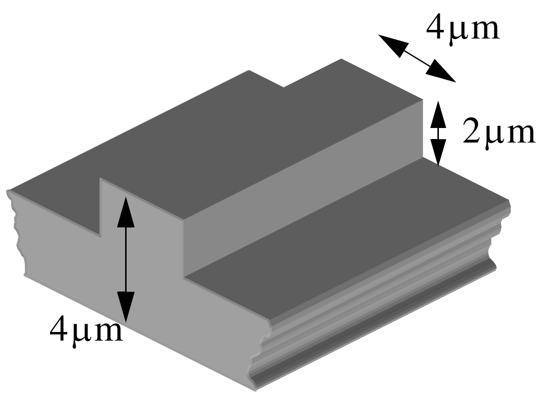
The starting point for the manufacture of all silicon on insulator waveguides is a SIMOX (separation by implantation of oxygen) wafer. Typically $2\times10^{18}/cm^2$ oxygen ions at 200 keV are implanted into a silicon wafer, followed by annealing for 6 hours at $1300°C$. This produces a buried oxide layer of about $0.4\mu m$ thickness, and a $0.2\mu m$ surface silicon layer. After annealing, the surface silicon layer is epitaxially grown to a thickness of upto 4 microns depending on the final waveguide design. The silicon waveguide pattern is then defined using simple photolithography, followed by plasma etching into the silicon.

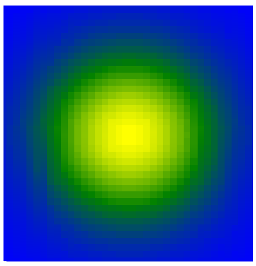


- very small waveguide cross section - 0.25 microns
- used to make nano-scale optical circuits and photonic crystal circuits
- compatible with CMOS processing
- high loss ~ 1dB/cm, but improving
- very difficult to couple light into from an optical fibre

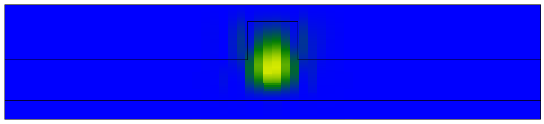
Silicon on insulator rib waveguides

- silicon rib waveguides can be made with dimensions of about 4x4 microns.
- much lower loss than sub-micron silicon waveguides, typically 0.1dB/cm.
- still very difficult to couple light into from fibres - US patent no 7035509 ‘semiconductor waveguide device’ solves this problem.
- used by companies such as Kotura (www.kotura.com) to make electrically controllable optical attenuators.





optical fibre
light intensity



silicon ridge waveguide
light intensity - same scale

Titanium diffused lithium niobate waveguides

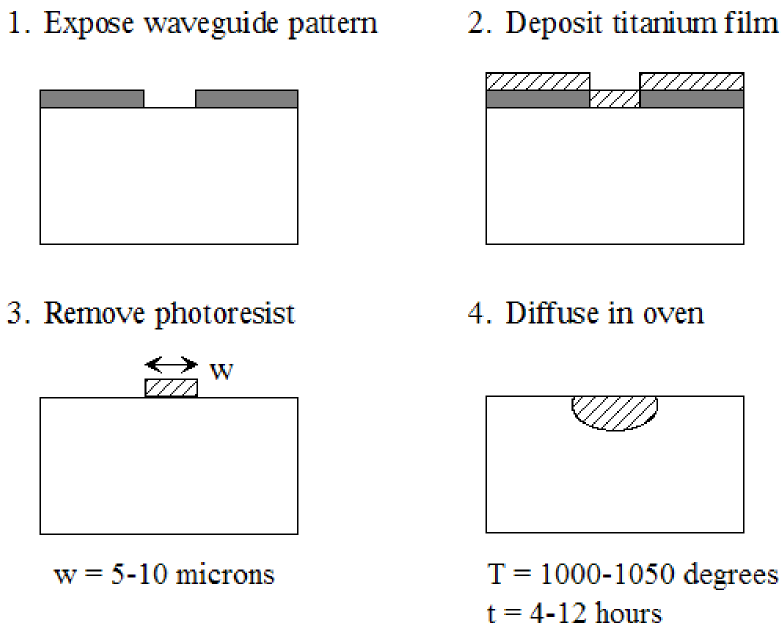
- electro-optic waveguides - can be tuned electrically
- low loss ~ 0.1dB/cm
- commonly used to make high speed external modulators for semiconductor lasers

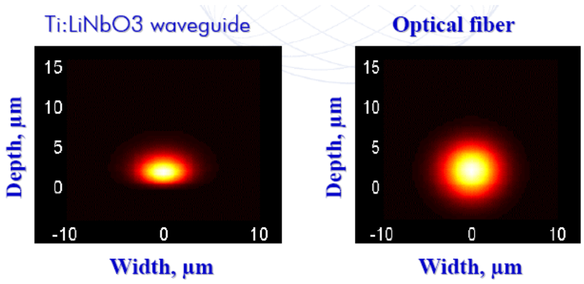
1. A layer of photoresist is deposited on a substrate of lithium niobate. The waveguide is defined on the surface of the photoresist by exposing a suitable mask with UV light. The photoresist is then developed to leave the waveguide pattern.

2. Titanium is now evaporated onto the surface to leave a thin film of metal covering the photoresist and waveguide pattern.

3. The remaining photoresist is dissolved, leaving the titanium layer defining the waveguide.

4. Finally, the $LiNbO_3$ is baked in an oven at about 1000° C for several hours. During this time, the Ti diffuses into the $LiNbO_3$ surface to form a channel waveguide





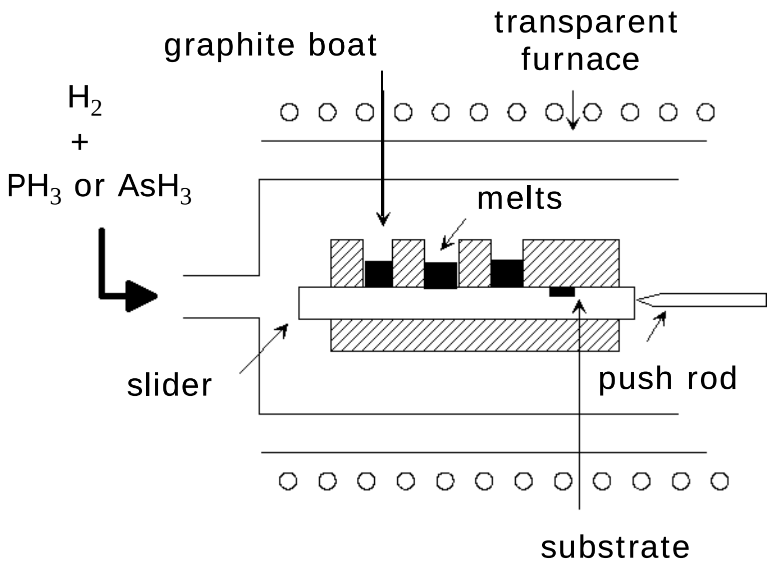
comparison between optical fields of $LiNbO_3$ waveguide and fibre

Semiconductor waveguides - GaAs and InP

Both GaAs and InP are used as waveguide materials. In the case of GaAs, the changes in refractive index needed to define waveguides are achieved by combining it with aluminium to form GaAlAs. By varying the amount of Al a large refractive index range can be achieved, always lower than that of pure GaAs. Refractive index changes can also be achieved by doping GaAs to form n+ or p+ material. In the case of InP it is much more difficult to grow material of different refractive index and which also has the same lattice spacing - necessary for the growth of good quality layers of semiconductor. However, the quaternary $Ga_{1-x}In_x As_{1-y}P_y$ provides a suitable material which can be lattice matched to InP for y=2.917x over the range InP to $Ga_{0.47}In_{0.53}As$. There are three commonly used methods for growing semiconductor waveguides: liquid phase epitaxy (LPE), metal organic chemical vapour deposition (MOCVD), and molecular beam epitaxy (MBE).

Liquid Phase Epitaxy

The LPE method is a simple and relatively cheap way of growing both InP and GaAs waveguides. It does not produce the lowest loss waveguides, or the most accurate. A schematic of the sort of equipment used is shown in the next diagram. The substrate is held on a slider which can be positioned under a graphite boat which has several bins containing saturated solutions of Ga, Al and As, or In, Ga, As and P. The substrate is slid under each bin in turn so that semiconductor crystallises on the substrate surface.

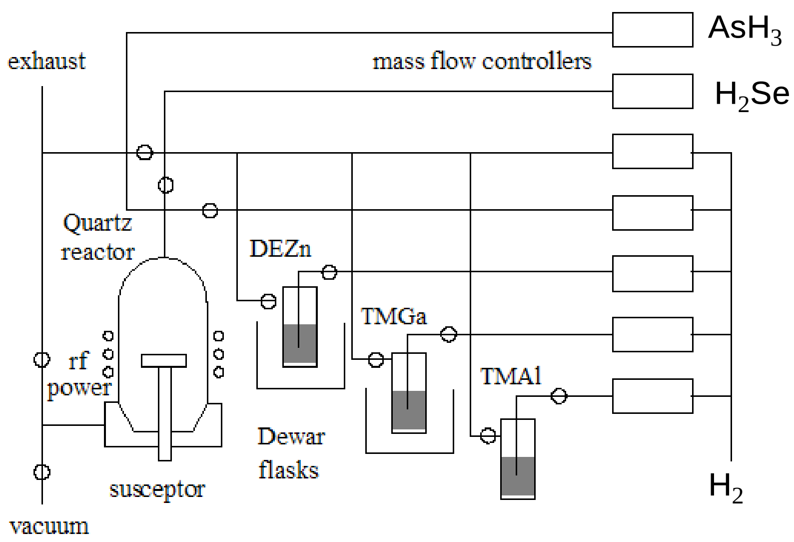


Metal Organic Chemical Vapour Deposition (MOCVD)

The MOCVD method has achieved some of the lowest loss waveguides reported. It also provides excellent control of the waveguide thickness. The semiconductors used must be in the form of metal alkyds, such as trimethyl gallium, $Ga(CH_3)_3$, trimethyl indium, $In(CH_3)_3$, triethyl indium, $In(C_2H_5)_3$ or hydrides, such AsH_3. The alkyds and hydrides are introduced into a cold walled quartz reaction tube, in which the substrate is supported on an RF heater. On coming in contact with the hot substrate the alkyds and halides react to form the semiconductor, eg:

$$Ga(CH_3)_3 + AsH_3 \rightarrow GaAs + 3CH_4$$

A schematic diagram of the MOCVD method is shown below.

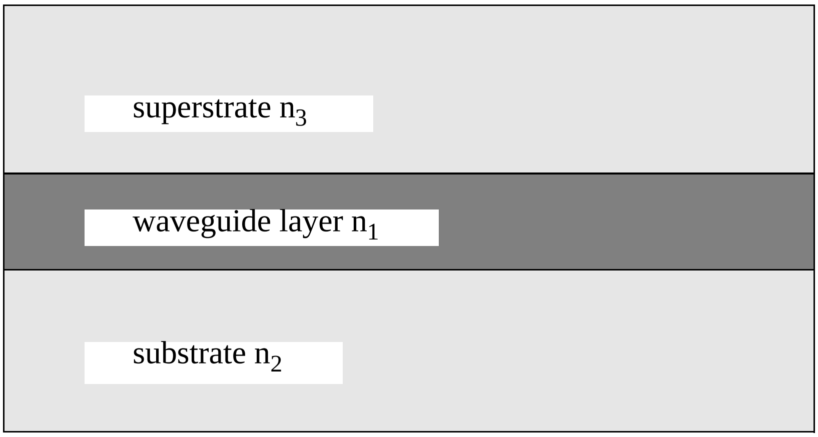


Molecular beam epitaxy

The MBE system is used for growing waveguiding layers where the highest accuracy is needed for the thickness of the layers. Losses are almost, but not quite, as good as those achieved with MOCVD. The growth rate is also low - less than 1 μm/hour, and so is not suitable for growing very thick layers. Essentially, the method is an ultra high vacuum technique in which the constituent elements are evaporated onto a heated substrate.

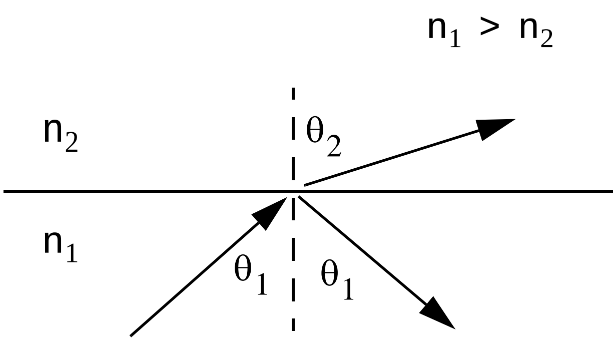
The Planar Optical Waveguide

A simple model for optical waveguides is the 2D planar (or slab) optical waveguide. It consists of a uniform layer of



refractive index n_1 surrounded by claddings of lower refractive indices n_2 and n_3, where we assume $n_1 > n_2 > n_3$. Light is guided along the waveguide by total internal reflection.

Total internal reflection



Snell's law tell's us

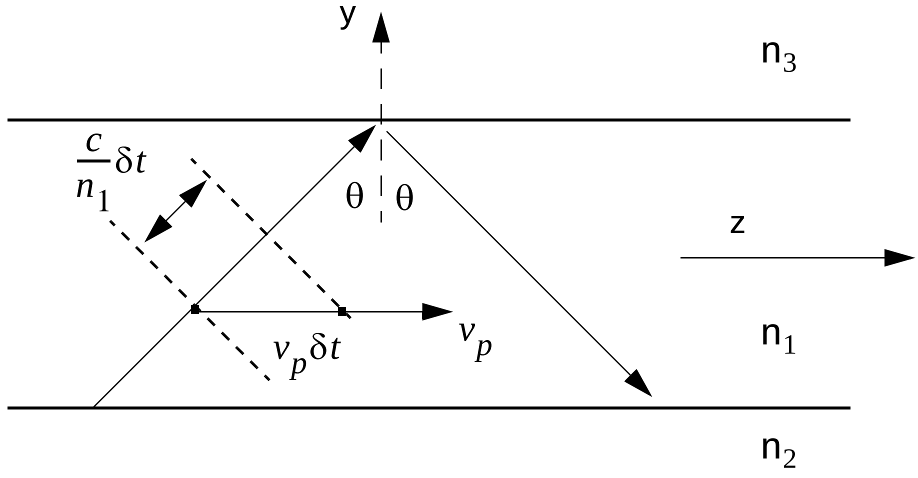
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Condition for total internal reflection:

$$\frac{n_1}{n_2} \sin\theta_1 \geq 1$$

A ray description of planar waveguides

As a light ray bounces along a the waveguide we are interested in the phase velocity v_p along the waveguide - that is the velocity along the waveguide at which the phase is seen to remain constant. In the diagram the dotted lines show the



planes of constant phase perpendicular to the ray direction. Consider movement of the phase front over a short time δt. Simple geometry gives

$$\frac{c/n_1\delta t}{v_p\delta t} = \sin\theta$$

which gives

$$v_p = \frac{c}{n_1\sin\theta}$$

A travelling wave in z direction is described by $\exp[j(\omega t - \beta z)]$ where β is called the propagation constant.

The phase velocity is

$$v_p = \frac{\omega}{\beta}$$

so that

$$\beta = \frac{\omega}{c}\cdot n_1\sin\theta$$

The wavenumber in free space is defined by

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c}$$

where λ_0 is the free space wavelength. Combining the above gives us an expression for β in terms of the ray direction along the waveguide

$$\beta = n_1 k_0 \sin\theta$$

In integrated optics we often define an effective refractive index n_{eff} which describes the average refractive index seen by the wave as it travels along the waveguide

$$n_{eff} = \frac{\beta}{k_0}$$

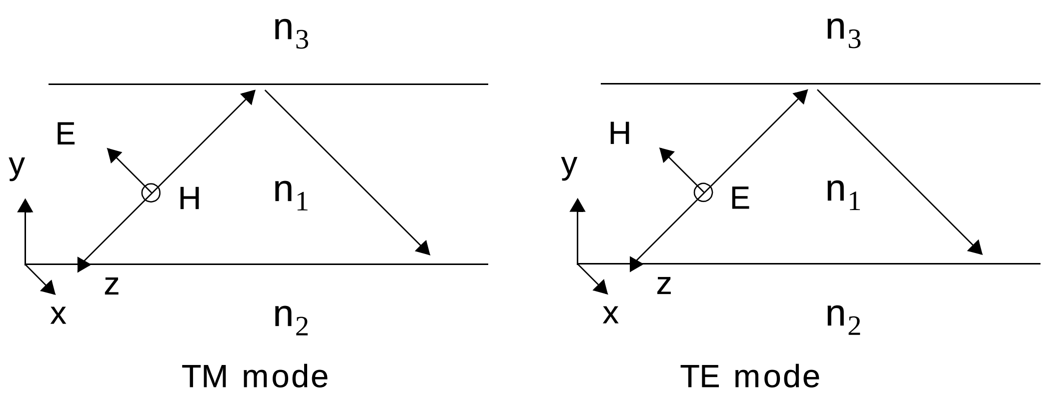
Remember $n_1 \sin\theta > n_2 > n_3$ so that

$$n_2 < n_{eff} < n_1$$

The last equation sets the absolute limits on n_{eff} and hence β. If β goes outside this range then the light is no longer guided by the waveguide and refracts away. We then say that the waveguide is cutoff.

Modal description of planar waveguide - TE and TM modes

A more precise description of the planar waveguide requires us to consider the electromagnetic fields. Two types of guided mode can exist on a planar waveguide: transverse electric (TE) and transverse magnetic (TM). We can visualise both types of mode in terms of a plane electromagnetic wave bouncing down the waveguide with either the E field (TM mode) or H field (TE mode) parallel to the walls of the waveguide, and perpendicular to the z direction along the waveguide



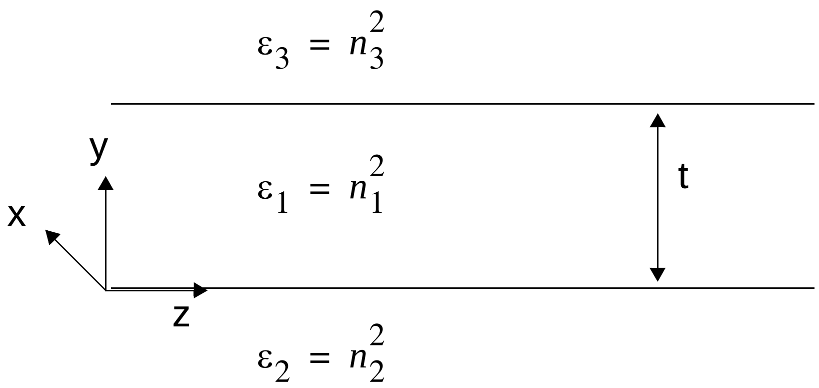
From the above diagrams we see that the field components associated with TE and TM modes will be:

TE mode	E_x , H_y , H_z
TM mode	H_x , E_y, E_z

For many waveguides $n_1 \approx n_2$ and then there is very little difference between the properties of TE and TM modes. For this reason we will only consider TE modes in detail.

Wave solution for TE modes on a planar waveguide

We will solve Maxwell's equations for a TE mode propagating in the z-direction on a planar optical waveguide defined by the following geometry



The TE mode has field components E_x , H_y , and H_z . The wave equation for E_x in the i^{th} layer of the waveguide is

$$\nabla^2 E_x - \frac{1}{c^2} n_i^2 \frac{\partial^2 E_x}{\partial t^2} = 0 \quad i = 1, 2, 3$$

Since the waveguide is infinite in the x-direction, we can assume the fields have no x-dependence, so that

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2} n_i^2 \frac{\partial^2 E_x}{\partial t^2} = 0$$

Now assume solutions for E_x take the form

$$E_x = \varphi(y) \exp[j(\omega t - \beta z)]$$

where the propagation constant β and modal field $\varphi(y)$ are to be determined. Substituting E_x into the wave equation gives us

$$\frac{\partial^2 \varphi}{\partial y^2} + (n_i^2 k_0^2 - \beta^2)\varphi = 0 \quad i = 1, 2, 3$$

We have used the fact that

$$\frac{\omega}{c} = k_0 = \frac{2\pi}{\lambda_0}$$

and λ_0 is the wavelength in free space. We must now consider the equation for φ in each of the three regions of the waveguide

$$\frac{\partial^2 \varphi}{\partial y^2} + (n_2^2 k_0^2 - \beta^2)\varphi = 0 \quad \text{where y<0}$$

$$\frac{\partial^2 \varphi}{\partial y^2} + (n_1^2 k_0^2 - \beta^2)\varphi = 0 \quad \text{where 0<y<t}$$

$$\frac{\partial^2 \varphi}{\partial y^2} + (n_3^2 k_0^2 - \beta^2)\varphi = 0 \quad \text{where y>t}$$

Now we define the following normalised variables:

$$V = k_0 t \sqrt{n_1^2 - n_2^2}$$

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}$$

$$b = \frac{\beta^2 - n_2^2 k_0^2}{(n_1^2 - n_2^2)k_0^2}$$

In terms of these, the equations for φ are

$$\frac{\partial^2 \varphi}{\partial y^2} - \frac{V^2 b}{t^2}\varphi = 0 \quad \text{where} \quad y < 0$$

$$\frac{\partial^2 \varphi}{\partial y^2} + \frac{V^2(1-b)}{t^2}\varphi = 0 \quad \text{where} \quad \text{0<y<t}$$

$$\frac{\partial^2 \varphi}{\partial y^2} - \frac{V^2(b+a)}{t^2}\varphi = 0 \quad \text{where} \quad y > t$$

These equations can be solved as follows

$$\varphi = A\exp\left(\frac{y}{t}V\sqrt{b}\right) \quad \text{where} \quad y < 0$$

$$\varphi = A\cos\left(\frac{y}{t}V\sqrt{1-b}\right) + B\sin\left(\frac{y}{t}V\sqrt{1-b}\right) \quad \text{where } 0<y<t$$

$$\varphi = (A\cos(V\sqrt{1-b}) + B\sin(V\sqrt{1-b}))\exp-\left(\frac{(y-t)}{t}V\sqrt{a+b}\right) \quad \text{where} \quad y > t$$

The amplitudes A and B have been arranged so the electric field is continuous across the waveguide boundaries.

The magnetic field component H_z is also continuous across the waveguide boundaries, and is given from Maxwell's equations by

$$H_z = \frac{1}{j\omega\mu_0}\frac{dE_x}{dy}$$

This means that the gradient $\frac{d\varphi}{dy}$ is also continuous. Applying this condition gives

$$A\sqrt{b} = B\sqrt{1-b} \quad \text{at y=0}$$

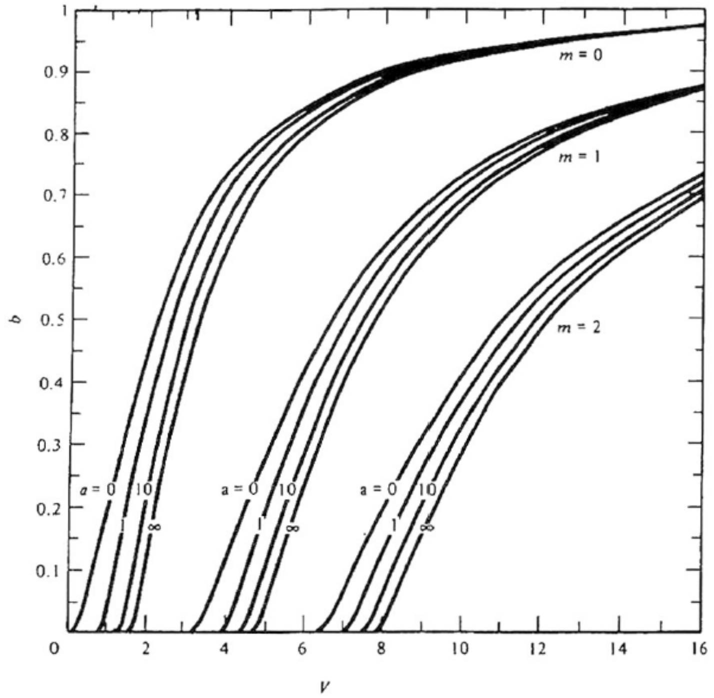
$$\sqrt{1-b}(-A\sin(V\sqrt{1-b}) + B\cos(V\sqrt{1-b}))$$
$$= -\sqrt{a+b}(A\cos(V\sqrt{1-b}) + B\sin(V\sqrt{1-b})) \quad \text{at y=t}$$

Eliminating A and B from these equations gives us an equation for the normalised propagation constant b

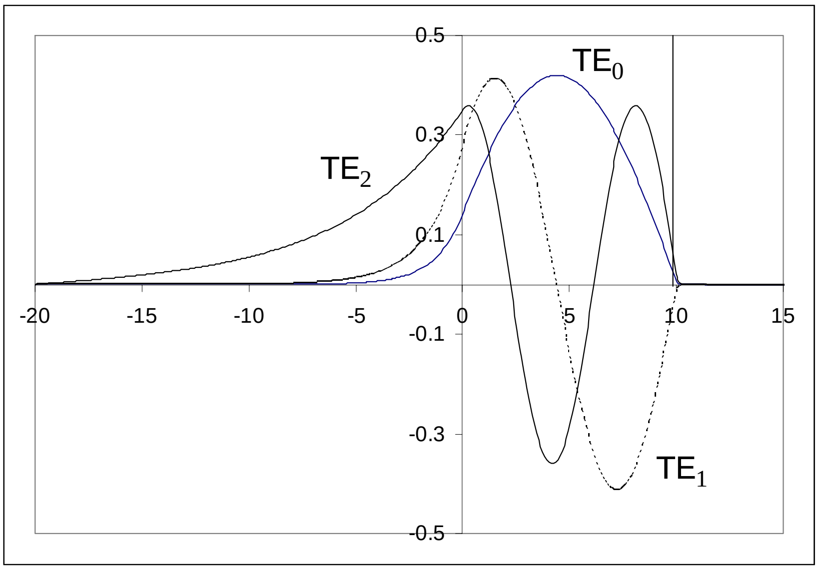
$$\tan(V\sqrt{1-b}) = \sqrt{1-b}\cdot\left(\frac{\sqrt{a+b}+\sqrt{b}}{(1-b)-\sqrt{b(a+b)}}\right)$$

This is the fundamental equation that determines many of the properties of planar optical waveguides. In general, it must be solved numerically for b given the known values for a and V. Once b is determined then β can be determined from the definition of b.

The following graph shows b against V for different values of a. The solutions group themselves into groups called modes labelled by the integers m = 0, 1, 2 ... and denoted by TE_0 , TE_1 , etc.



An example of the electric field functions $\varphi(y)$) are sketched below for m = 0, 1, and 2 for an asymmetric GaAs waveguide with asymmetry a = 262.



$n_1 = 3.44$

$n_2 = 3.434$

$n_3 = 1.0$

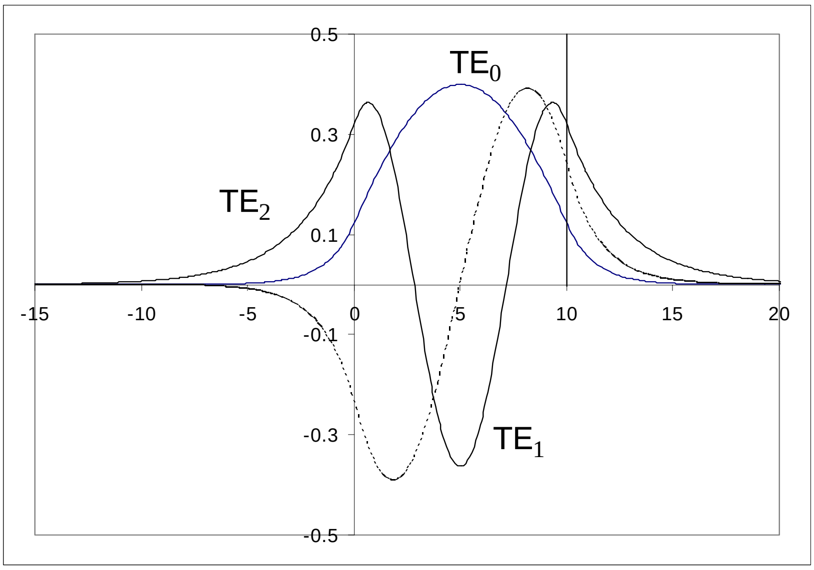
$\lambda = 1.55\ \mu\text{m}$

$t = 10\ \mu\text{m}$

- The field is oscillatory inside the waveguide and decays exponentially outside

- The exponential decay is faster in n_3 than in n_2 if $n_2 > n_3$.

The corresponding modal fields for a symmetric GaAs waveguide are shown



$$n_1 = 3.44$$
$$n_2 = 3.434$$
$$n_3 = 3.434$$
$$\lambda = 1.55\ \mu\text{m}$$
$$t = 10\ \mu\text{m}$$

- The exponential decay is the same in both n_2 and n_3 .
- The fields are either exactly symmetric or anti-symmetric about the middle of the waveguide.

Conditions for cutoff in the planar optical waveguide

For most application in integrated optics the waveguides must be single moded. We have already seen that β must be in the range

$$n_1 k_0 \geq \beta \geq n_2 k_0$$

or in terms of b,

$$0 < b < 1$$

When $\beta = n_2 k_0$ the condition for total internal reflection no longer holds, and light refracts out of the waveguide. We say the mode is cutoff. This condition is described by setting b = 0 in the equation of b above:

$$\tan(V) = \sqrt{a}$$

Solving for V the cutoff condition for the TE_m mode is

$$V = m\pi + \tan^{-1}(\sqrt{a})$$

For a waveguide to be single moded V must be in the range

$$\pi + \tan^{-1}(\sqrt{a}) > V > \tan^{-1}(\sqrt{a})$$

Recalling the definition of V and we have

$$\pi + \tan^{-1}(\sqrt{a}) > k_0 t\sqrt{n_1^2 - n_2^2} > \tan^{-1}(\sqrt{a})$$

So that the waveguide thickness must be in the range

$$\frac{\pi + \tan^{-1}(\sqrt{a})}{k_0\sqrt{n_1^2 - n_2^2}} > t > \frac{\tan^{-1}(\sqrt{a})}{k_0\sqrt{n_1^2 - n_2^2}}$$

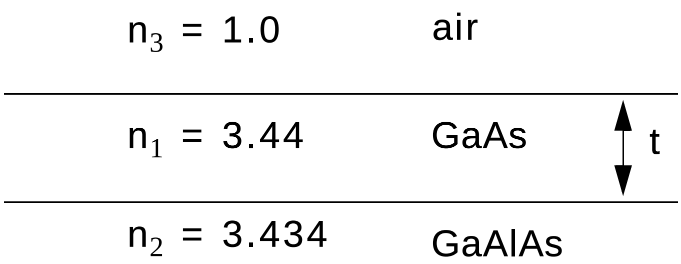
- For t to be reasonably large we need $n_1 \approx n_2$
- For a symmetric waveguide, a = 0 and t must satisfy

$$0 < t < \frac{\pi}{k_0\sqrt{n_1^2 - n_2^2}}$$

Worked Example

Determine the waveguide thickness for single mode operation at λ = 1.55μm for the following planar waveguides: (a) n_1 = 3.44, n_2 = 3.434, n_3 = 1 ; (b) n_1 = 1.4574, n_2 = n_3 = 1.4464; (c) n_1 = 3.4879, n_2 = 1.444, n_3 = 1.0;

(a) this is an air/GaAs/GaAlAs waveguide



Condition for single mode operation is

$$\frac{\pi + \tan^{-1}(\sqrt{a})}{k_0\sqrt{n_1^2 - n_2^2}} > t > \frac{\tan^{-1}(\sqrt{a})}{k_0\sqrt{n_1^2 - n_2^2}}$$

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} = 261.7$$

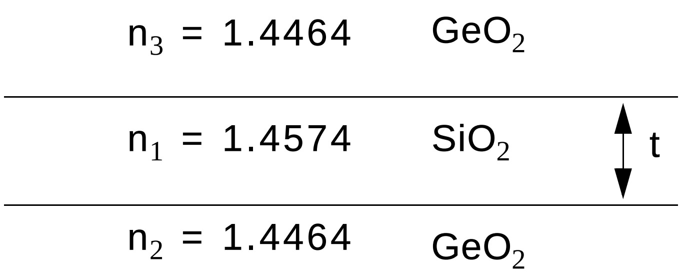
$$\sqrt{n_1^2 - n_2^2} = 0.2031$$

$$k_0 = \frac{2\pi}{\lambda} = 4.0537\ \mu\mathrm{m}^{-1}$$

This gives

$$1.83\mu\mathrm{m} < t < 5.65\mu\mathrm{m}$$

(b) this is a germania/silica waveguide



This waveguide is symmetric, so a = 0 and condition for single mode is

$$0 < t < \frac{\pi}{k_0\sqrt{n_1^2 - n_2^2}}$$

$$\sqrt{n_1^2 - n_2^2} = 0.1787$$

This gives

$$t < 4.33\mu\mathrm{m}$$

(c) this is a silicon on insulator (SOI) waveguide

$n_3 = 1.0$ air

$n_1 = 3.4879$ Si ↕ t

$n_2 = 1.444$ SiO_2

Condition for single mode operation is

$$\frac{\pi + \tan^{-1}(\sqrt{a})}{k_0\sqrt{n_1^2 - n_2^2}} > t > \frac{\tan^{-1}(\sqrt{a})}{k_0\sqrt{n_1^2 - n_2^2}}$$

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} = 0.1076$$

$$\sqrt{n_1^2 - n_2^2} = 3.175$$

This gives

$$0.02\mu m < t < 0.27\mu m$$

- note the very small size needed for silicon waveguides

Worked Example

Estimate the exponential decay of the electric field outside of the waveguide for waveguide (a) in the previous example (n_1 = 3.44, n_2 = 3.434, n_3 = 1) if t = 4μm.

The fields in the regions outside of the waveguide were derived previously:

$$\varphi = A\exp\left(\frac{y}{t}V\sqrt{b}\right) \text{ where } y < 0$$

$$\varphi = (A\cos(V\sqrt{1-b}) + B\sin(V\sqrt{1-b}))\exp-\left(\frac{(y-t)}{t}V\sqrt{a+b}\right) \text{ where } y > t$$

The exponential decay rates are then

$$\alpha_1 = \frac{V\sqrt{b}}{t}$$

$$\alpha_2 = \frac{V\sqrt{a+b}}{t}$$

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} = 261.7$$

$$V = k_0 t\sqrt{n_1^2 - n_2^2} = 3.3$$

We then find b = 0.5.

$$\alpha_1 = 0.58\mu\text{m}^{-1}$$

$$\alpha_2 = 13.3\mu\text{m}^{-1}$$

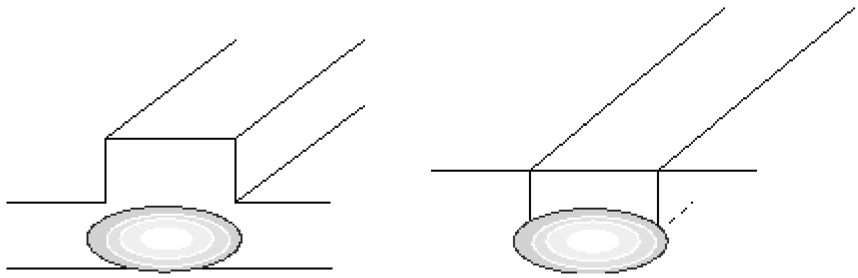
The associated penetration depths are

$$\frac{1}{\alpha_1} = 1.72\mu\text{m}$$

$$\frac{1}{\alpha_2} = 75.2\text{ nm}$$

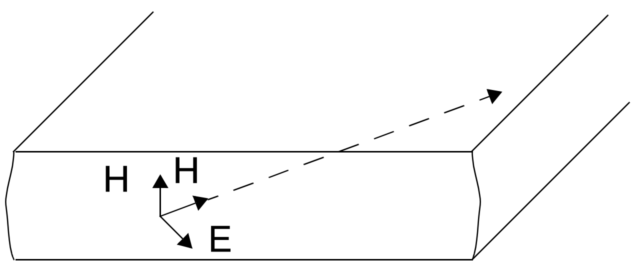
The effective index method

Although the planar optical waveguide is useful in some circumstances, it does suffer from the disadvantage of only confining the optical field in one direction. In many optical circuits it is necessary to confine the field both horizontally and vertically. There are several important types of waveguide used in integrated optics that meet this requirement: the rib guide, the rectangular guide and the channel guide. If you are making a waveguide in GaAs/GaAlAs, then it is most likely that you will be making a rib guide. On the other hand, a channel waveguide is usually made by Ti diffusion in lithium niobate and rectangular waveguides are usually made in silica. In all cases the optical intensity of the guided wave has a typically elliptical pattern, as illustrated below



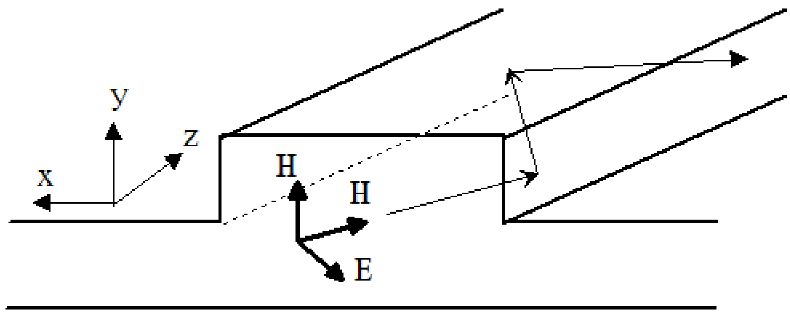
rib guide channel guide

To understand how these waveguides support a guided mode, let us start from a situation that we already understand - a TE mode propagating through a planar waveguide. Remember that the TE mode has an electric field component perpendicular to the direction of propagation, and magnetic field components both parallel and perpendicular to the mode direction. The following diagram shows a TE mode propagating in an arbitrary direction through a planar waveguide.



TE mode

Note that at this point we have not said anything about the position of the x-y-z coordinate axes. If the planar waveguide were infinite in the horizontal plane, then our TE mode would quite happily continue to propagate in the direction along which we launched it. But suppose that there is a rib somewhere on the surface of our planar waveguide, as illustrated below. What will this do to our TE mode? Well, nothing happens until the mode collides with the sides of the rib, but then it will be reflected and proceed towards the other side of the rib. We see then that the TE mode will zig-zag along the rib guide, with confinement in the horizontal plane being a result of reflection from the sides of the rib.

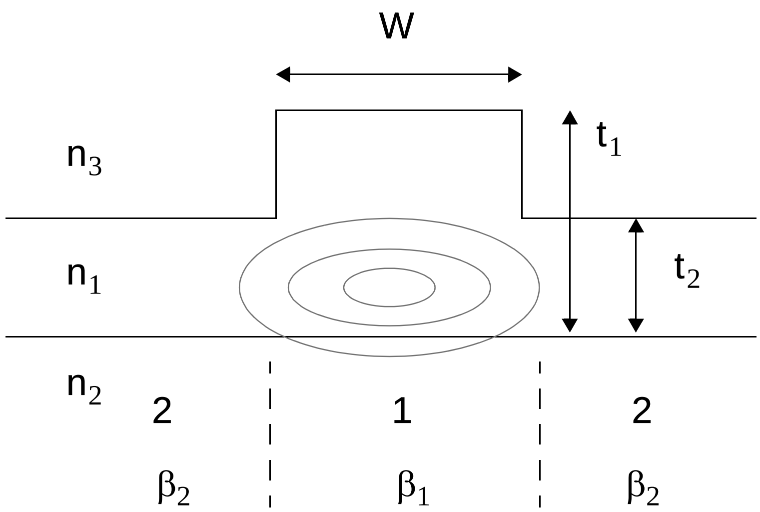


The effect of all this is to produce a modal structure which is confined in both the horizontal and vertical directions. The channel guide can be understood in the same way. We can now add coordinate axes to our waveguide, as shown in the previous diagram. The z direction is along the rib, the y direction is vertical, and the x direction is in the plane of the rib. With respect to these axes the zig-zagging mode has the following field components:

$$E_x, E_z, H_x, H_y, H_z$$

This is neither a TE or TM mode; because it is built up from a TE mode it is called a hybrid TE mode.

The following diagram shows a rib guide with the optical field confined under the rib. We have divided the waveguide into three parts, labelled 1 and 2, corresponding to the region under the rib, and the two regions either side.

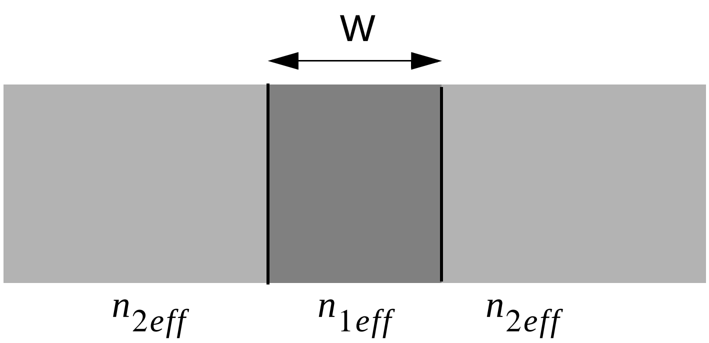


The Effective Index Method consists of a set of rules for constructing a planar guide whose propagation constant is close to the propagation constant of the original rib guide:

- For each of the regions 1 and 2 calculate β_1 and β_2 for a planar guide of thickness t1 and t2 , and indices n1 , n2 , and n3 .
- Calculate the effective indices:

$$n_{1eff} = \frac{\beta_1}{k_0} \text{ and } n_{2eff} = \frac{\beta_2}{k_0}$$

- Replace the rib guide by a planar optical waveguide with refractive indices n_{1eff} and n_{2eff} as shown below
- Calculate β for this equivalent planar waveguide. The result is a very good approximation to β of the original rib guide - usually to within 1% .



Having replaced the rib guide by an equivalent planar guide, the rest is easy. As with our earlier analysis, we introduce a normalised frequency V, and normalised propagation constant b:

$$V = \sqrt{n_{1eff}^2 - n_{2eff}^2} \cdot k_0 \cdot W$$

$$b = \frac{\beta^2 - n_{2eff}^2 k_0^2}{(n_{1eff}^2 - n_{2eff}^2) k_0^2}$$

The equivalent waveguide has zero asymmetry, so that b satisfies

$$\tan(V\sqrt{1-b}) = \frac{2\sqrt{b(1-b)}}{(1-2b)}$$

Recall the condition for single mode operation

$$V < \pi$$

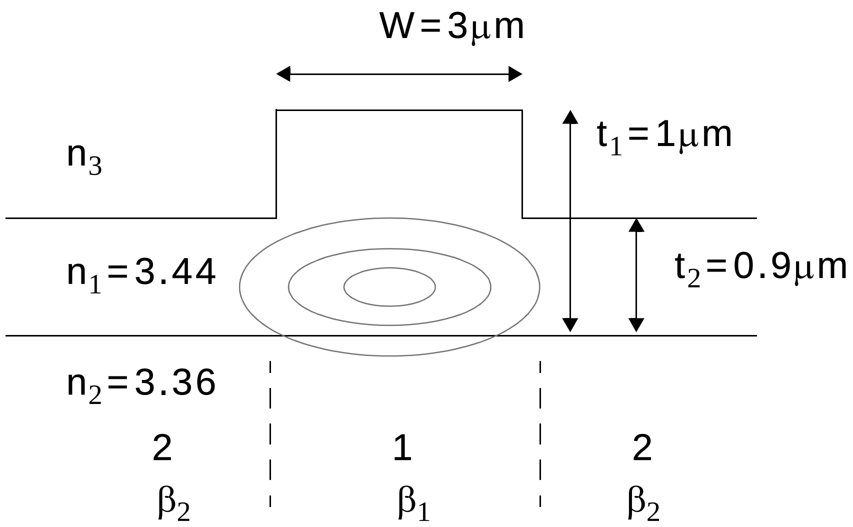
which gives

$$\sqrt{n_{1eff}^2 - n_{2eff}^2} \cdot k_0 \cdot W < \pi$$

$$W < \frac{\lambda}{2\sqrt{n_{1eff}^2 - n_{2eff}^2}}$$

Worked Example

Determine β for the following GaAs rib waveguide using the effective index method for λ = 1.55μm



Asymmetry a is

$$a = \frac{n_2^2 - n_3^2}{n_1^2 - n_2^2} = 19$$

$$V_1 = k_0 t_1 \sqrt{n_1^2 - n_2^2} = 3.0$$

From b-V graph we find

$$b_1 = 0.42 = \frac{\beta_1^2 - n_2^2 k_0^2}{(n_1^2 - n_2^2) k_0^2}$$

This gives the effective index as

$$n_{1eff} = \frac{\beta_1}{k_0} = 3.394$$

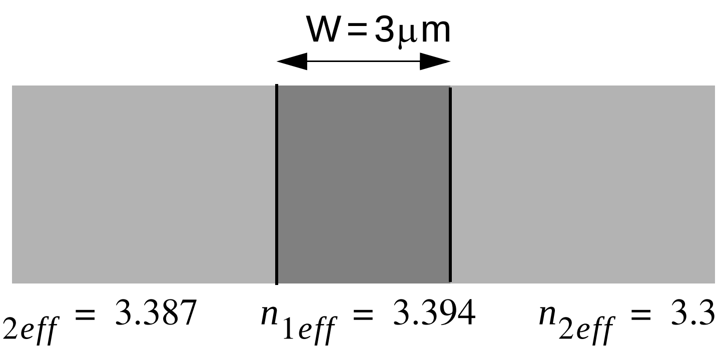
Similarly

$$V_2 = k_0 t_2 \sqrt{n_1^2 - n_2^2} = 2.7$$

$$b_2 = 0.34 = \frac{\beta_1^2 - n_2^2 k_0^2}{(n_1^2 - n_2^2) k_0^2}$$

$$n_{2eff} = \frac{\beta_2}{k_0} = 3.387$$

The equivalent planar waveguide is then



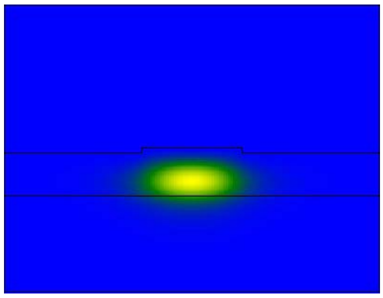
$$V = \sqrt{n_{1eff}^2 - n_{2eff}^2} \cdot k_0 \cdot W = 2.65$$

$$b = 0.56 = \frac{\beta^2 - n_{2eff}^2 k_0^2}{(n_{1eff}^2 - n_{2eff}^2) k_0^2}$$

From this we find β

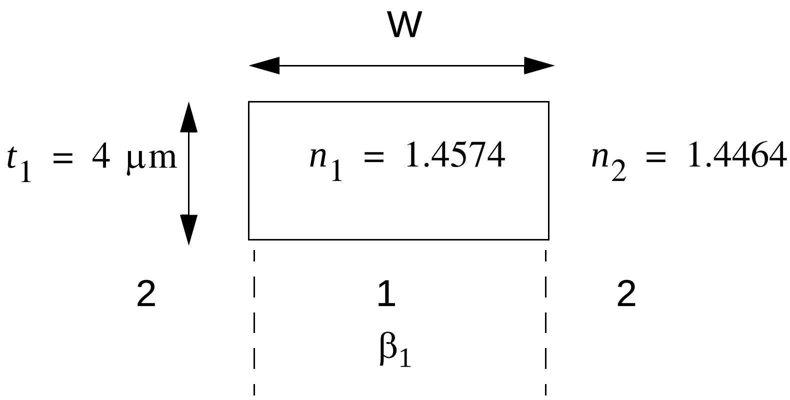
$$n_{eff} = \frac{\beta}{k_0} = 3.391$$

Numerical simulation of the optical intensity in the GaAs waveguide considered in this example. The numerical solution gives n_{eff} = 3.396, in good agreement with the effective index model.



Worked Example

Determine the maximum width W for single mode operation for the following silica waveguide at λ = 1.55μm



The asymmetry a = 0

$$V_1 = k_0 t_1 \sqrt{n_1^2 - n_2^2} = 2.89$$

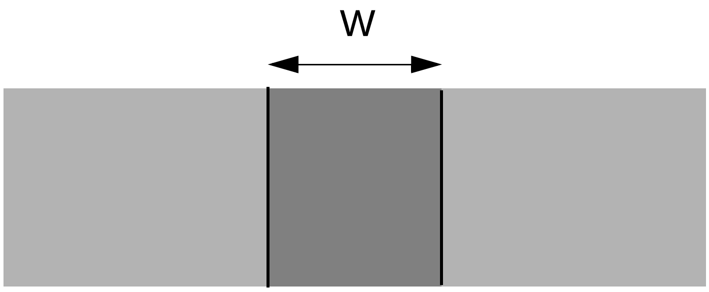
From b-V graph

$$b_1 = 0.61 = \frac{\beta_1^2 - n_2^2 k_0^2}{(n_1^2 - n_2^2)k_0^2}$$

This gives the effective index as

$$n_{1eff} = \frac{\beta_1}{k_0} = 1.4531$$

The equivalent planar waveguide is given by



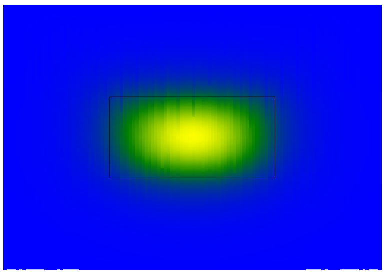
$n_{2eff} = n_2 = 1.4464$ $\quad n_{1eff} = 1.4531$ $\quad n_{2eff} = n_2 = 1.4464$

Single mode condition is

$$W < \frac{\lambda}{2\sqrt{n_{1eff}^2 - n_{2eff}^2}}$$

$$W < 5.6\ \mu\text{m}$$

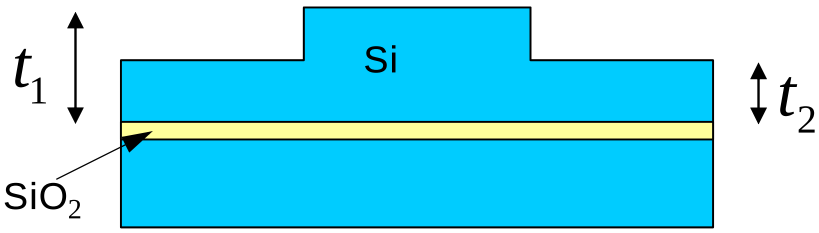
Numerical simulation of the optical intensity in the silica waveguide considered in this example.



A more exact analysis gives the cutoff width as W = 6μm, in good agreement with the simple effective index model.

Silicon Rib waveguides

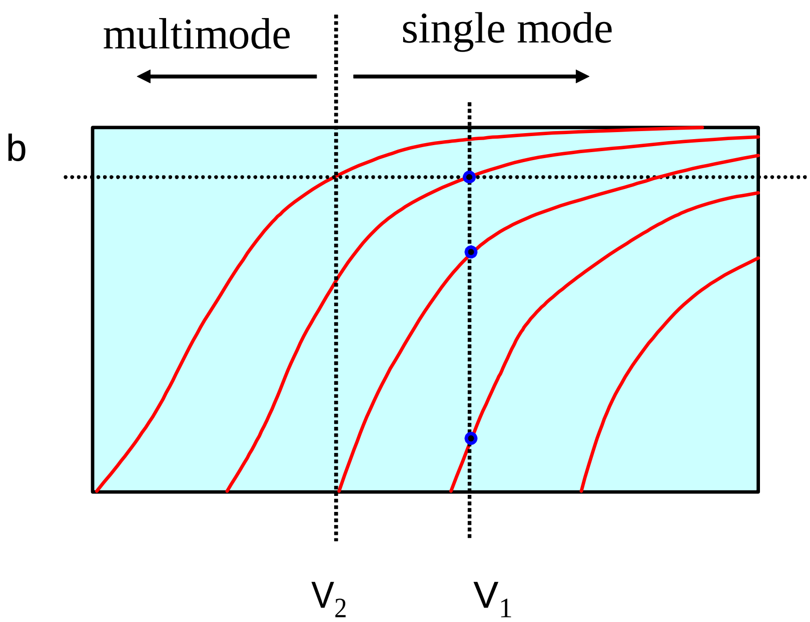
Silicon has a refractive index of about 3.5, and silicon dioxide about 1.45 . A rectangular waveguide made of silicon surrounded by SiO_2 as a cladding would have dimensions of less than about .24 micron square at 1.5μm in order to be single moded. It might be expected that, for a silicon rib guide on SiO_2 to be single moded, we would need similar dimensions. Surprisingly, this is not the case.



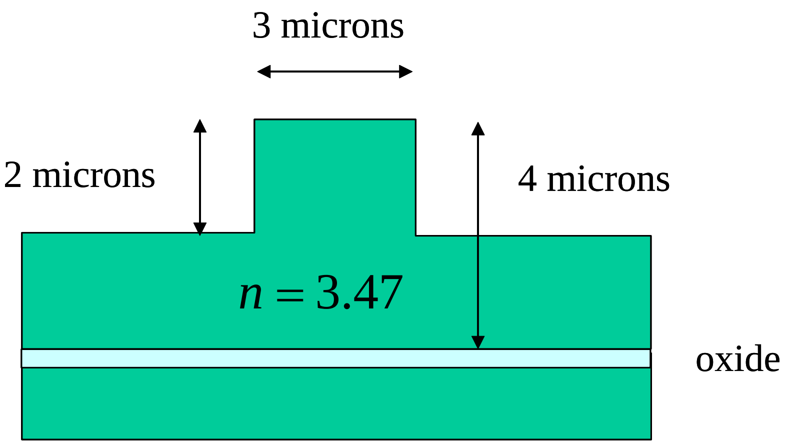
In the vertical direction the slab waveguides under the rib and either side of the rib are highly multimoded. However, if the effective index of the fundamental mode of the slab of thickness t_2 is higher than the effective indices of *all* the higher order modes above the fundamental of the slab under the rib, then according to the effective index model these higher order modes will be cutoff. The condition for this is given by

$$b_2 > b_1$$

This is illustrated in the following b-V plot



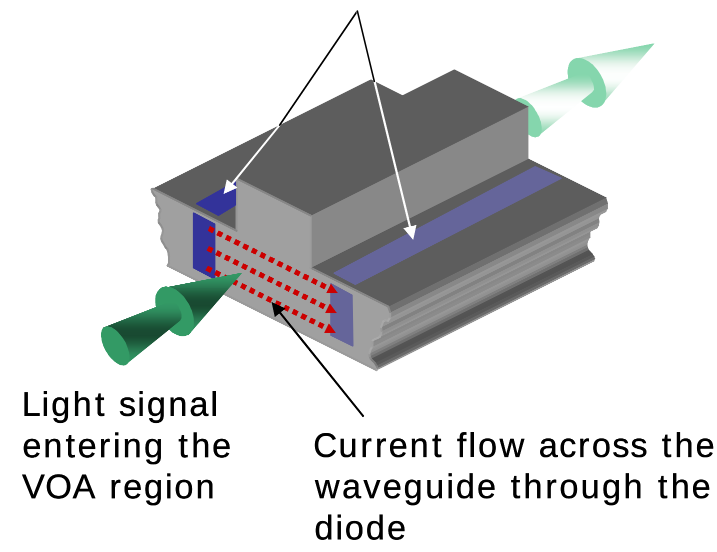
Typical dimensions of a single mode silicon rib waveguide based on this idea are shown below



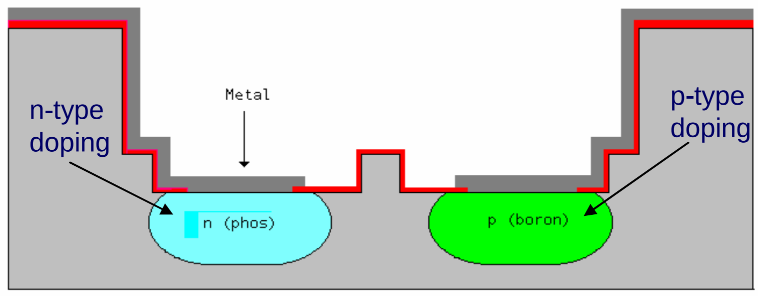
This type of waveguide is used in a number of commercial silicon optical circuits, most notably from Kotura (www.Kotura.com).

Silicon waveguide variable optical attenuator (VOA)

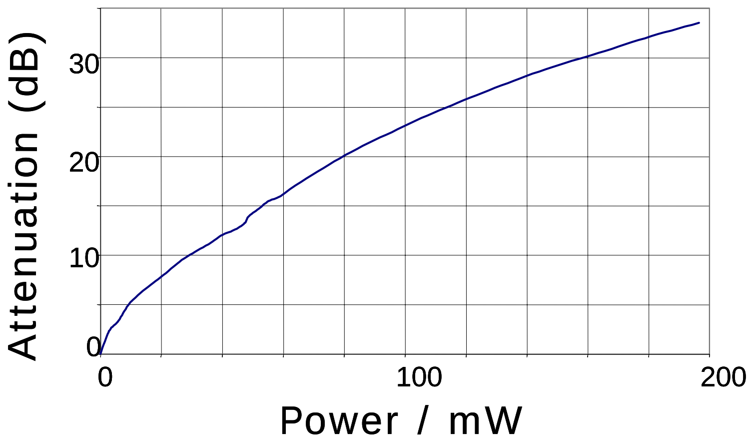
The VOA is an example of a silicon rib waveguide component, commercially available (www.kotura.com). It is a device which allows the electrical control of the optical power travelling through a waveguide.It operates on the principle of optical attenuation by electrons injected into the waveguide.



The basic design is shown below. When a voltage is applied to the p and n electrodes electrons are injected into the waveguide which attenuate the optical mode.



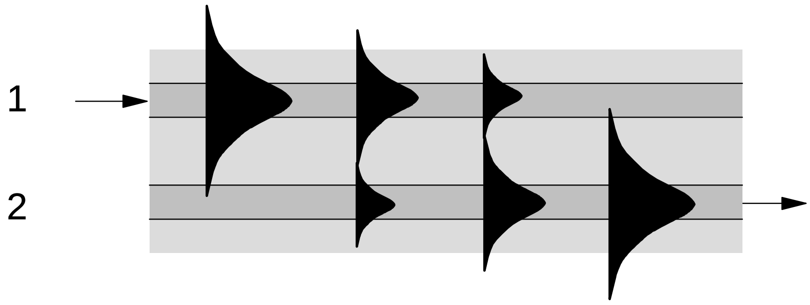
A typical response is shown below. Attenuation levels of over 30dB can be achieved with this type of attenuator.



The following picture shows a packaged multi channel device available commercially.

Coupled Waveguides

So far we have considered propagation through a single isolated waveguide. Now we examine what happens when two optical waveguides are placed near and parallel to each other, so that the optical field of one waveguide can couple to the field in the other. The diagram below illustrates what happens



with a pair of identical waveguides. We describe the electric field in each waveguide as follows

$$E_1 = a_1(z)\varphi(y)$$

$$E_2 = a_2(z)\varphi(y)$$

where $\varphi(y)$ is the modal field. Coupling between the waveguides is described by the following pair of coupled wave equations

$$\frac{da_1}{dz} = -j\beta a_1 + K_{12}a_2 \quad (1)$$

$$\frac{da_2}{dz} = -j\beta a_2 + K_{21}a_1 \quad (2)$$

where K_{12} and K_{21} describe the strength of coupling.

Power conservation

The total power in the waveguides must remain constant

$$P = P_1 + P_2 = |a_1(z)|^2 + |a_2(z)|^2 = \text{constant}$$

Then

$$\frac{dP}{dz} = 0$$

$$\frac{d}{dz}(|a_1(z)|^2 + |a_2(z)|^2) = 0$$

$$\frac{d}{dz}(a_1 a_1^* + a_2 a_2^*) = 0$$

Now take

[equation (1)] x a^*_1 + [equation (1)]* x a_1

$$\frac{d}{dz}|a_1(z)|^2 = K_{12} a_1^* a_2 + K_{12}^* a_1 a_2^*$$

Similarly using equation (2)

$$\frac{d}{dz}|a_2(z)|^2 = K_{21} a_2^* a_1 + K_{21}^* a_2 a_1^*$$

Adding these two equations

$$\frac{d}{dz}(|a_1(z)|^2 + |a_2(z)|^2) = a_1 a_2^*(K_{12}^* + K_{21}) + a_1^* a_2(K_{21}^* + K_{12}) = 0$$

The only way this can be true is if

$$K_{12}^* + K_{21} = K_{21}^* + K_{12} = 0$$

which implies

$$K_{12} = K_{21} = -jK$$

That is, the coupling coefficients are equal and pure imaginary.

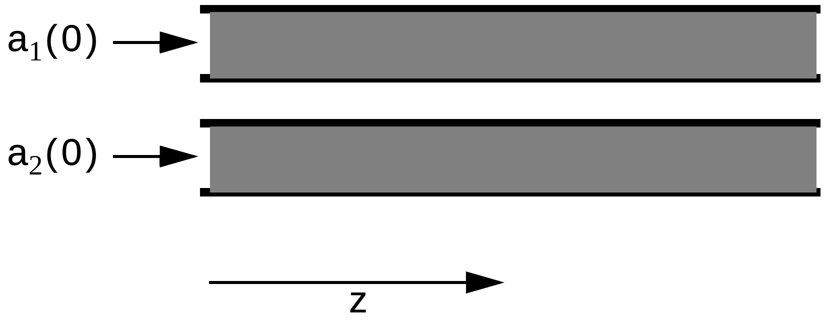
This gives the final form for the coupled wave equations:

$$\frac{da_1}{dz} = -j\beta a_1 - jK a_2$$

$$\frac{da_2}{dz} = -j\beta a_2 - jK a_1$$

Solving the coupled wave equations

Assume that at the inputs the amplitudes are $a_1(0)$ and $a_2(0)$



Easiest way to solve the coupled wave equations is to define new amplitudes

$$A_+ = a_1 + a_2$$

$$A_- = a_1 - a_2$$

Adding and subtracting the coupled wave equations gives us

$$\frac{dA_+}{dz} = -j(\beta + K)A_+$$

$$\frac{dA_-}{dz} = -j(\beta - K)A_-$$

these are easily solved

$$A_+ = A_+(0)\exp[-j(\beta + K)z] = (a_1(0) + a_2(0))\exp[-j(\beta + K)z]$$

$$A_- = A_-(0)\exp[-j(\beta - K)z] = (a_1(0) - a_2(0))\exp[-j(\beta - K)z]$$

$$a_1(z) = \frac{A_+ + A_-}{2}$$

$$a_2(z) = \frac{A_+ - A_-}{2}$$

From which we find

$$a_1(z) = \exp(-j\beta z)[a_1(0)\cos Kz - ja_2(0)\sin Kz] \quad (3)$$

$$a_2(z) = \exp(-j\beta z)[-ja_1(0)\sin Kz + a_2(0)\cos Kz] \quad (4)$$

The directional coupler

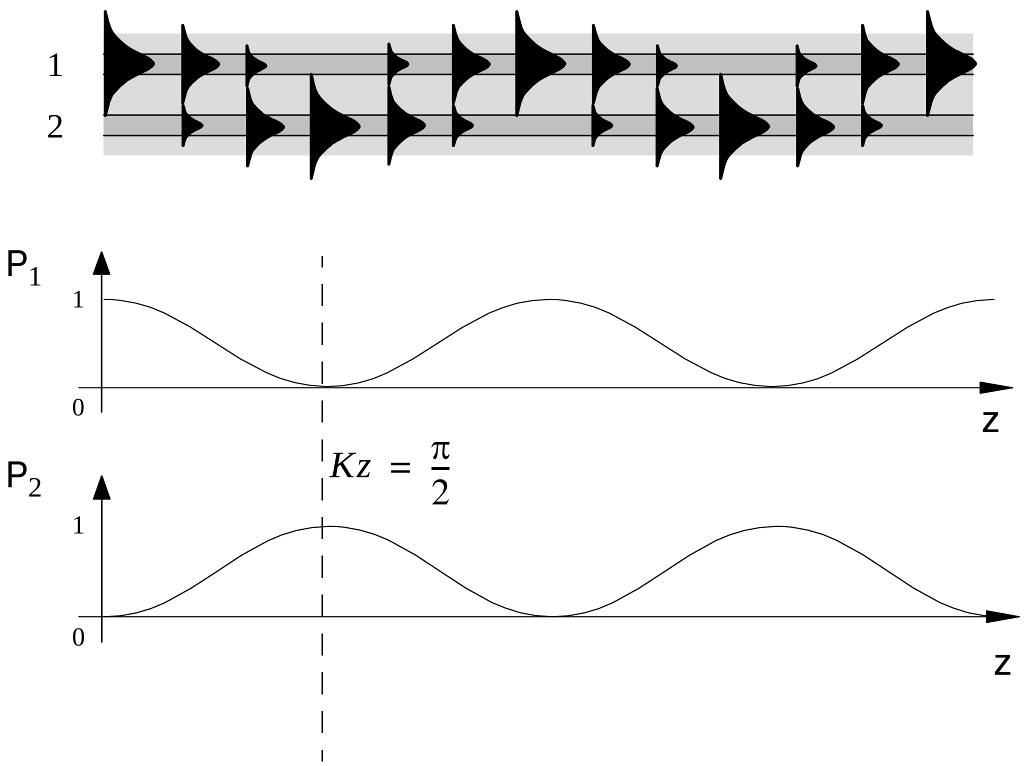
We have already met the directional coupler. It is a device whereby power input into one waveguide is divided between the two output waveguides. Assume we input a signal into waveguide 1, and nothing into waveguide 2

$$a_1(0) = 1 \quad \text{and} \quad a_2(0) = 0$$

The power in each waveguide varies with distance as follows

$$P_1(z) = |a_1(z)|^2 = \cos^2 Kz$$

$$P_2(z) = |a_2(z)|^2 = \sin^2 Kz$$



• when $Kz=\pi/2$ all power is transferred from waveguide 1 to waveguide 2.

• when $Kz=\pi/4$ the power is divided equally between the two waveguides - this is the 3DB coupler:

$$z = \frac{\pi}{4K}$$

Then equations (3) and (4) give

$$a_1(z) = \exp(-j\beta z)[a_1(0) - ja_2(0)] \cdot \frac{1}{\sqrt{2}}$$

$$a_2(z) = \exp(-j\beta z)[-ja_1(0) + a_2(0)] \cdot \frac{1}{\sqrt{2}}$$

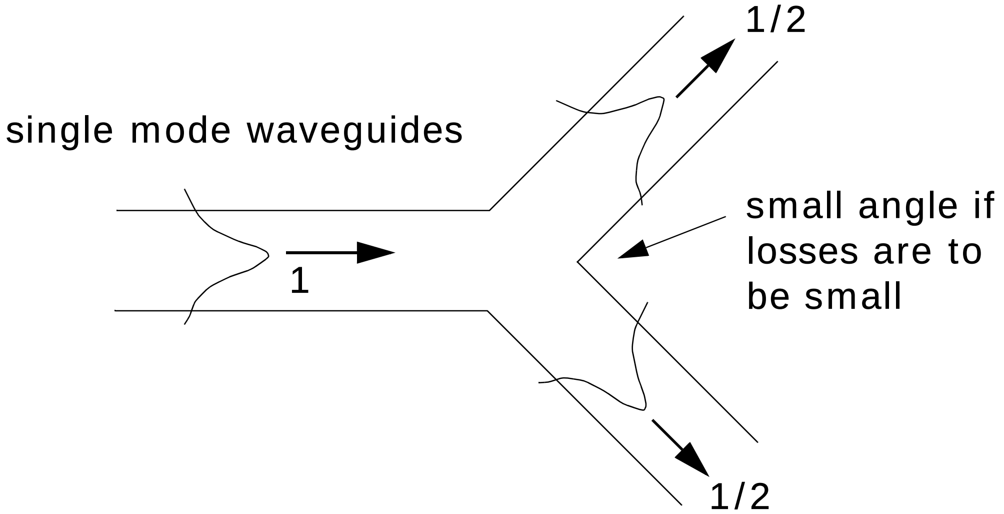
This can be written as

$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = \exp(-j\beta z) \cdot \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \cdot \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix}$$

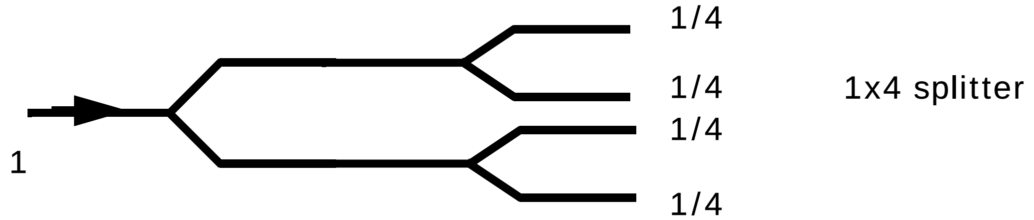
Apart from the phase factor, this is the same expression that we wrote down previously for the 3dB coupler.

The Y-junction

A simple way to divide light in a single mode waveguide into two equal signals is to use a y-junction



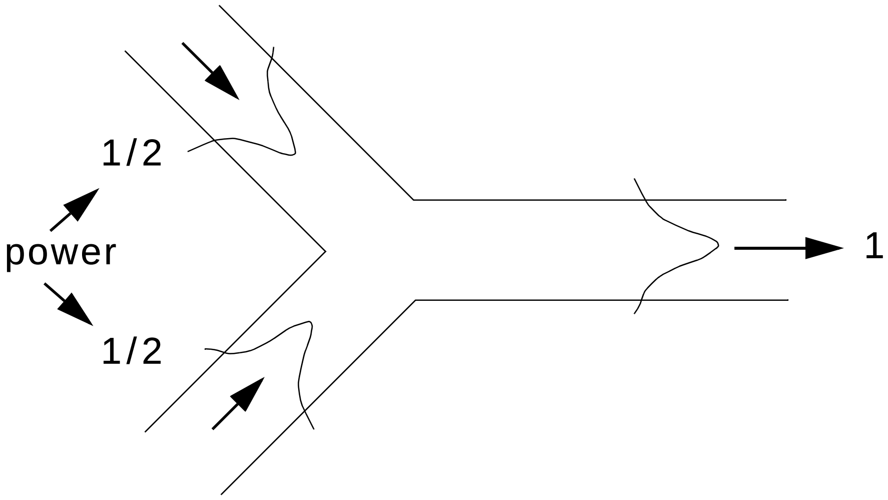
The y-junction forms the basis of 1xN splitting components



It is instructive to consider the y-junction operating in reverse. There are two cases:

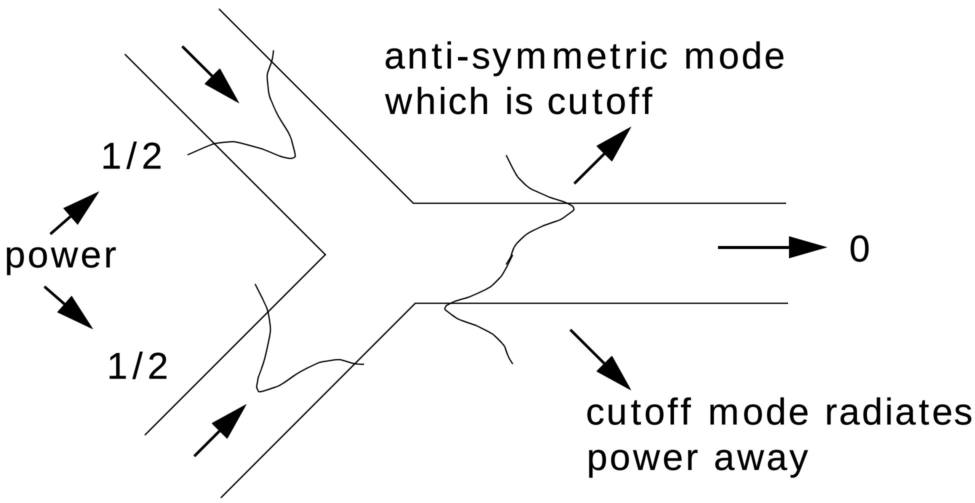
• equal symmetric excitation of the two waveguide arms.

These will combine to produce a symmetric field in the output waveguide, which, if the angle of the y-junction is small, and the waveguides are all single moded, will be the lowest order mode of the output guide. See next diagram.



In this way, the y-junction acts as a combiner for two in-phase signals.

• equal anti-symmetric excitation of the two input arms



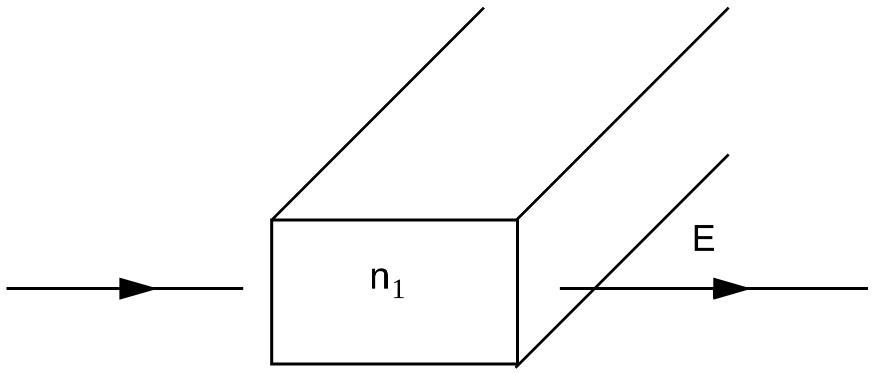
In this case the two inputs will combine to produce an anti-symmetric output, which will correspond to a second order mode. This will be cutoff if the waveguides are single mode, and will radiate away, leaving zero power in the output waveguide.

These properties are exploited in the design of electro-optic modulators, which we study next.

Electro-Optic Devices

The Electro-Optic Effect

Refractive index of a waveguide made from electro-optic materials can be changed by the application of an electric field.



Refractive index changes linearly with the electric field E

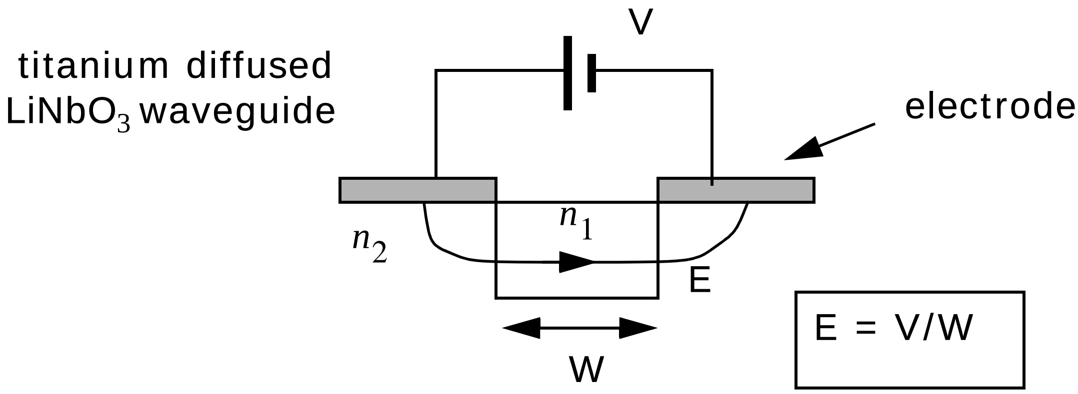
$$n = n_1 + \frac{1}{2} \cdot n_1^3 rE$$

where *r* is called the electro-optic coefficient.

material	r (10^{-12} m/V)	n_0
lithium niobate $LiNbO_3$	30	2.2
gallium arsenide GaAs	1.2	3.44

• note that the sign of the effect depends on the direction of the electric field

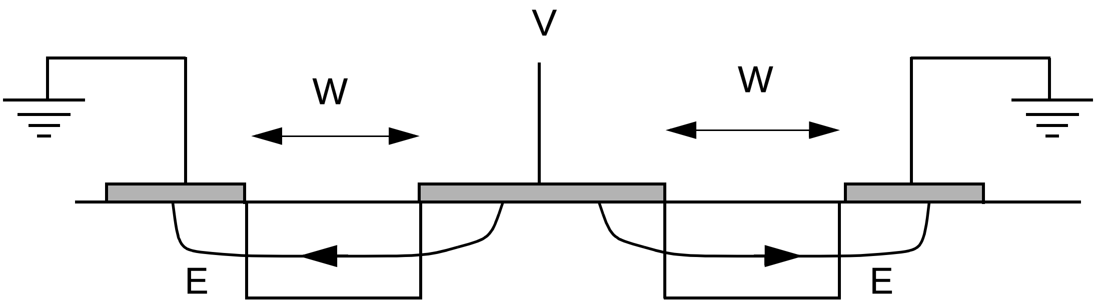
Application of electric field to optical waveguides



If n_1 and n_2 are nearly equal (which they will be in an optical waveguide) then the change in the waveguide propagation constant is given approximately by

$$\beta = \beta_0 \pm \frac{1}{2} n_1^3 r E k_0$$

where β_0 is the propagation constant of the unperturbed waveguide, and $k_0 = 2\pi/\lambda$. The sign depends on the direction of E.



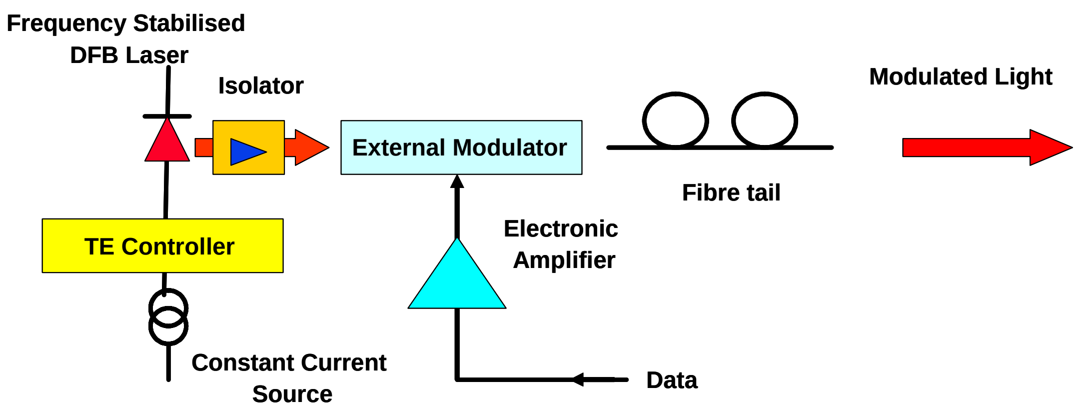
$$\beta = \beta_0 + \frac{1}{2} n_1^3 r E k_0 \qquad \beta = \beta_0 - \frac{1}{2} n_1^3 r E k_0$$

E = V/W

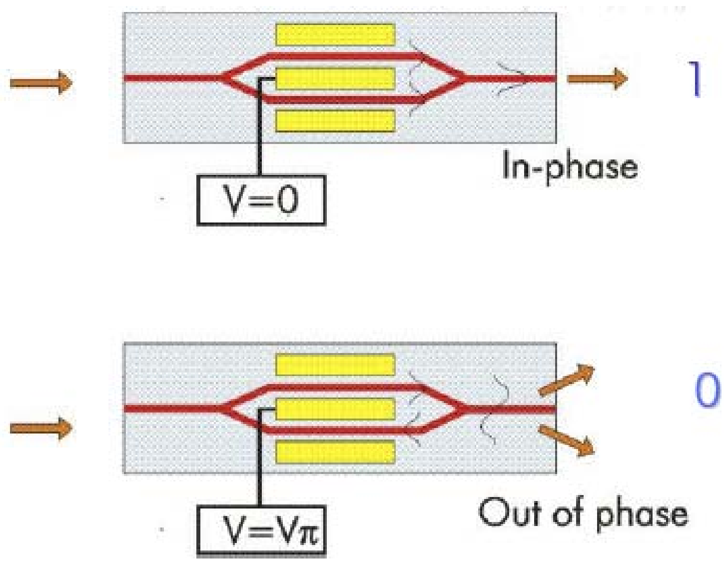
In this arrangement the electric fields are in opposite direction in each waveguide, so that β increases in one and decreases in the other.

The electro-optic modulator

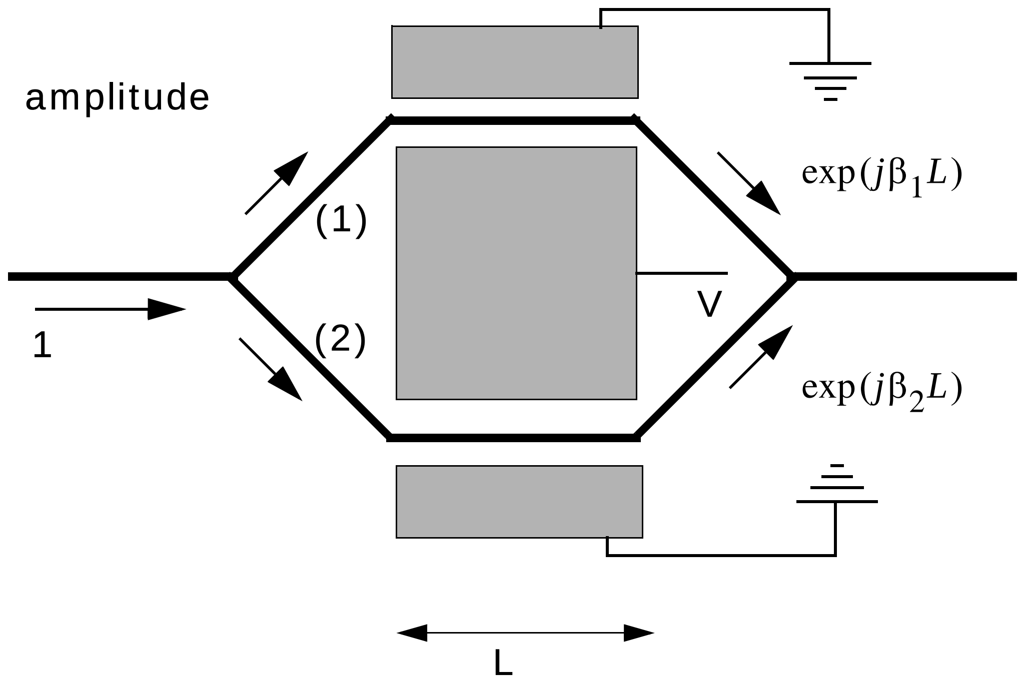
The electro-optic effect is exploited in titanium diffused lithium niobate waveguides to make high speed modulators for encoding data in fibre optic networks. Recall the basic design of the laser transmitter



The modulator is based on the Mach-Zehnder principle, but using y-junction waveguides rather than 3dB couplers:



The following represents a simple model for the modulator



The phases of the light in the output waveguide resulting from the signals passing through waveguides (1) and (2) are

$$\varphi_1 = \beta_1 L$$

$$\varphi_2 = \beta_2 L$$

where

$$\beta_1 = \beta + \frac{1}{2} \cdot n_1^3 r E k_0$$

$$\beta_2 = \beta - \frac{1}{2} \cdot n_1^3 r E k_0$$

The condition for a zero output is

$$\varphi_1 - \varphi_2 = \pi$$

Combining these equations, the condition for zero output is

$$n_1^3 r E k_0 L = \pi$$

Recall that for this geometry E = V/W, so that the voltage $V\pi$ needed for a zero is

$$V_\pi = \frac{\lambda W}{2 n_1^3 r L}$$

• typical $V\pi$ voltage is 5-6 volts.

• lithium niobate modulators are available for modulation rates upto 40Ghz (eg from JDSU: www.jdsu.com).