C3B1
MT2007

Modern Communications
Tutorial Sheet 1

(1) **Information**
Find the entropy, redundancy and information rate of a four symbol source (A,B,C,D) with a baud rate of 1024 symbols/s and symbol selection probabilities of 0.5, 0.2, 0.2 and 0.1.

(2) **Channel capacity**
A channel has a bandwidth B, and the channel information rate is R_b. The average energy received per bit of information is E_b and the noise power spectral density is N_o.
(a) Find the signal to noise ratio of the channel and use the Shannon expression of capacity to find a relationship between Spectral (or Bandwidth) Efficiency (defined as R_b/B (bits/s/Hz) vs (E_b/N_o). Sketch the curve.

(b) What signal to noise ratio (E_b/N_o) is required to achieve a capacity of 56kbits/s with a 3.4 kHz bandwidth channel working at the Shannon limit?

The measure E_b/N_o is used to commonly used to compare different modulation schemes, so that the spectral efficiency (R_b/B) can be compared at different values of channel noise.

3)A digital cellular telephone system is required to work at a bandwidth efficiency of 4 bits/sec/Hz in order to accommodate sufficient users to make it profitable.

(a) What is the minimum E_b/N_0 ratio that must be planned for in order to ensure that users on the edge of the coverage area receive error free communication?

(b) If the operator wishes to double the number of users on his existing network, how much more power must the base station and handsets radiate in order to maintain coverage and error free communication? (assuming the Shannon limit)

4) An Amplitude Shift Keying format is being used for transmitting data at a rate of 28.8kbps over a telephone channel with a bandwidth extending from 300Hz to 3400Hz.
(a) Make a sensible estimate of the symbol rate possible given the available bandwidth, justifying your answer, and hence work out how many symbol states (different levels) are required to achieve 28.8kbps.
(b) What would be the equivalent number of symbol states needed if the channel passband was from 0Hz to 3100Hz and baseband M-ary signalling was used?
(c) What is the theoretical capacity for the ASK system if the S/N ratio on the telephone link is 33dB?

There is no single correct answer to parts (a) and (b) of this question

5)PSD calculation
Manchester coding is often implemented by using an XOR function. The data clock, which consists of a logic 1 for half the data period then a zero for the rest, is XORed with the data, which is either 1 or 0 for the whole of the data period. The resulting Manchester data stream is then transmitted.
(a) Derive an expression for the PSD.
(b) Sketch the PSD and compare this with an NRZ PSD of the same bit rate. What are the (comparative) advantages and disadvantages might this scheme have?
(c) What is the bandwidth efficiency of this scheme?

6) Baseband Signalling
(a) What are the main advantages of using NRZ signalling?
(b) A transmission system is AC coupled to assist with the design of the transmitter and receiver electronics. (It can be thought to have a capacitor in series with the electrical signal transmission path.) What are the potential problems this might cause with NRZ signalling (either unipolar or bipolar)? How might you solve this?

(7) Error rate calculation
(a) A transmission system is required to communicate over 100km. The loss in the transmission medium (an optical fibre) is 0.2dB/km. The noise power at the decision point in the receiver is 10nW. A link margin of 5dBs (that is an extra 5dBs of 'headroom' to allow for variation in link components) is required.

(a) Show that the probability of error P_e for a unipolar NRZ data stream with an average signal power S and noise power N is given by

$$P_e = \frac{1}{2}\left\{1 - erf\left(\frac{1}{2}\sqrt{\frac{S}{N}}\right)\right\}$$

Where

$$erf(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}\,dy$$

(b) The specification requires the link to operate at BER (P_e) of 10^{-9}. What transmission power is required for the link to achieve this? Express your answer in dBm (i.e. dB's relative to 1mW)

This calculation requires you to evaluate the error function (or complementary error function). This is available in MATLAB. The complementary error function Erfc is plotted at the back of the sheet.

8)Orthogonality and Multi-level FSK

The signal waveforms in a symbol set are said to be orthogonal if they satisfy:

$$\int_0^{T_S} \varphi_n(t).\varphi_m(t)\,dt = \begin{cases} 0 & n \neq m \\ E_S & n = m \end{cases}$$

where T_S is the symbol period and E_S is the symbol energy.

(a) Consider a binary FSK modulation scheme. Show that the symbol states are orthogonal if:

$$\Delta f = \frac{1}{2T_S}$$

where Δf is the frequency spacing between the two signalling frequencies.

(b) This relationship can be extended to M-FSK (i.e. multi-level FSK) such that the minimum frequency separation between the signalling frequencies is $1/2T_S$.

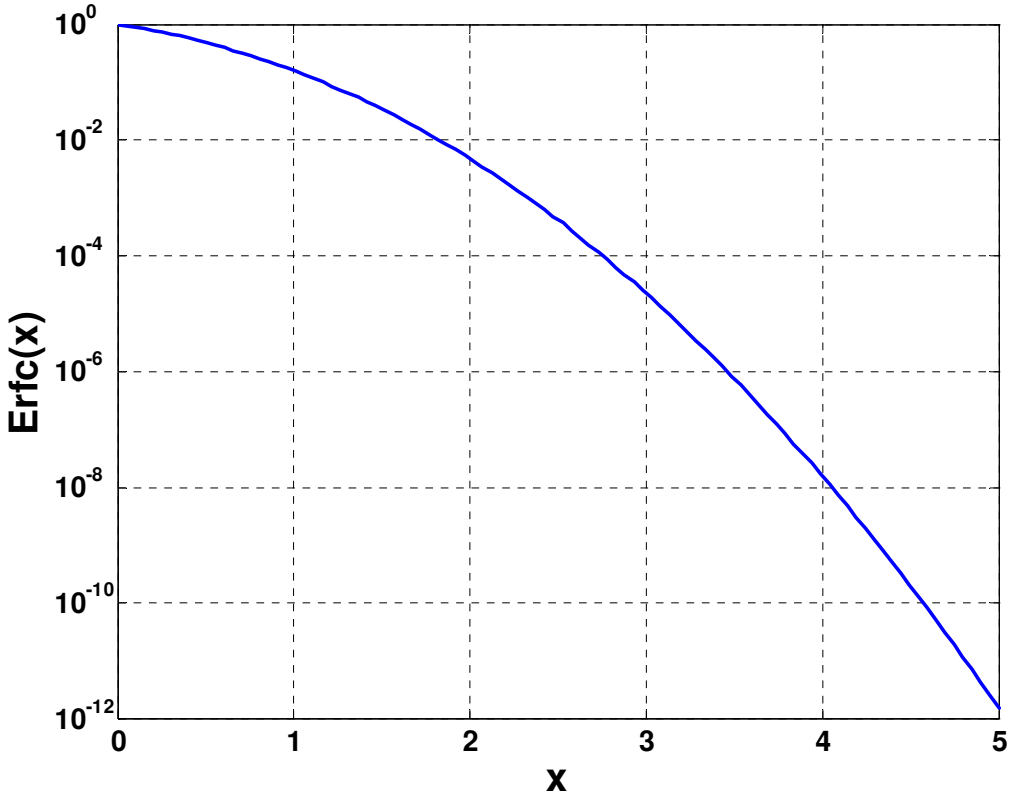
Given the result in (a) consider an FSK modem operating that is required to transmit a 1200 bits/s data stream. Assume the lowest signalling frequency is 2000 Hz. What would be the signalling frequencies for 2-FSK, 4-FSK and 8-FSK?

(c) For an m level FSK scheme show that the bandwidth efficiency R_b / B can be estimated to be

$$\frac{R_b}{B} = \frac{2\log_2 m}{(m+1)}$$

Sketch this curve for m=2,4,8, and comment on the bandwidth efficiency of FSK

Consider the spacing between frequency carriers, but also make an allowance for the bandwidth required either side of the upper and lower frequency values.



Plot of the complementary error function

Answers (some)

1)Entropy H=1.761 bits/symbol, Redundancy=0.239 bits/symbol, Information rate=1.803×10^3 bits/s.

2) $\frac{R_b}{B} = \log_2\left(1 + \frac{E}{N_o}\left(\frac{R_b}{B}\right)\right)$; E_b/No=37.4dB.

3) E_b/N_o=5.7dB; this value must be increased to 15.0dB to achieve double the capacity.

4)

(a) Assuming that for ASK the symbol rate is equal to the bandwidth need 9.3 bits/symbol to achieve the required data rate, rounds to 1024 levels or 10 bits/symbol

(b) Assuming that baseband scheme needs half the bandwidth compared with above this drops to 4.6 bits/symbol which rounds to 32 levels or 5 bits/symbols

(c) Capacity at the Shannon limit is 33.995 kbps.

8) 2-FSK:2000, 2600 Hz, 4-FSK:2000, 2300, 2600, 2900 Hz
8-FSK: 2000, 2200, 2400, 2600, 2800, 3000, 3200, 3400 Hz