Solution 4.4

```
function ep = macheps
% determines the machine epsilon
e = 1;
while (1)
  if e+1<=1, break, end
  e = e/2;
end
ep = 2*e;

>> macheps
ans =
  2.2204e-016

>> eps
ans =
  2.2204e-016
```

Solution 4.5

```
function s = small
% determines the smallest number
sm = 1;
while (1)
  s=sm/2;
  if s==0,break,end
  sm = s;
end
s = sm;
```

This function can be run to give

```
>> s=small
s =
  4.9407e-324
```

This result differs from the one obtained with the built-in `realmin` function,

```
>> s=realmin
s =
  2.2251e-308
```

Challenge question: We can take the base-2 logarithm of both results,

```
>> log2(small)
ans =
       -1074
>> log2(realmin)
ans =
       -1022
```

Thus, the result of our function is 2^{-1074} ($@4.9407\times10^{-324}$), whereas `realmin` gives 2^{-1022} ($\cong 2.2251\times10^{-308}$). Therefore, the function actually gives a smaller value that is equivalent to

`small` = 2^{-52} × `realmin`

Recall that machine epsilon is equal to 2^{-52}. Therefore,

`small` = `eps`×`realmin`

Such numbers, which are called *denormal* or *subnormal*, arise because the math coprocessor employs a different strategy for representing the significand and the exponent.

Solution 4.8

The true value can be computed as

$$f'(0.577) = \frac{6(0.577)}{(1-3\times 0.577^2)^2} = 2{,}352{,}911$$

Using 3-digits with chopping

$$6x = 6(0.577) = 3.462 \xrightarrow{\text{chopping}} 3.46$$
$$x = 0.577$$
$$x^2 = 0.332929 \xrightarrow{\text{chopping}} 0.332$$
$$3x^2 = 0.996$$
$$1-3x^2 = 0.004$$
$$f'(0.577) = \frac{3.46}{(1-0.996)^2} = \frac{3.46}{0.004^2} = 216{,}250$$

This represents a percent relative error of

$$\varepsilon_t = \left|\frac{2{,}352{,}911 - 216{,}250}{2{,}352{,}911}\right| \times 100\% = 90.8\%$$

Using 4-digits with chopping

$$6x = 6(0.577) = 3.462 \xrightarrow{\text{chopping}} 3.462$$
$$x = 0.577$$
$$x^2 = 0.332929 \xrightarrow{\text{chopping}} 0.3329$$
$$3x^2 = 0.9987$$
$$1-3x^2 = 0.0013$$
$$f'(0.577) = \frac{3.462}{(1-0.9987)^2} = \frac{3.462}{0.0013^2} = 2{,}048{,}521$$

This represents a percent relative error of

$$\varepsilon_t = \left|\frac{2{,}352{,}911 - 2{,}048{,}521}{2{,}352{,}911}\right| \times 100\% = 12.9\%$$

Solution continued on next page...

Although using more significant digits improves the estimate, the error is still considerable. The problem stems primarily from the fact that we are subtracting two nearly equal numbers in the denominator. Such subtractive cancellation is worsened by the fact that the denominator is squared.

Solution 4.23

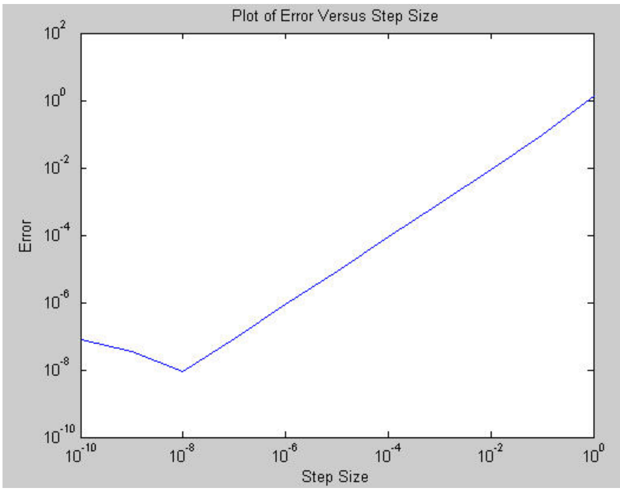
First, we must develop a function like the one in Example 4.5, but to evaluate a forward difference:

```
function prob0423(func,dfunc,x,n)
format long
dftrue=dfunc(x);
h=1;
H(1)=h;
D(1)=(func(x+h)-func(x))/h;
E(1)=abs(dftrue-D(1));
for i=2:n
  h=h/10;
  H(i)=h;
  D(i)=(func(x+h)-func(x))/h;
  E(i)=abs(dftrue-D(i));
end
L=[H' D' E']';
fprintf('   step size   finite difference    true error\n');
fprintf('%14.10f %16.14f %16.13f\n',L);
loglog(H,E),xlabel('Step Size'),ylabel('Error')
title('Plot of Error Versus Step Size')
format short
```

We can then use it to evaluate the same case as in Example 4.5:

```
>> ff=@(x) -0.1*x^4-0.15*x^3-0.5*x^2-0.25*x+1.2;
>> df=@(x) -0.4*x^3-0.45*x^2-x-0.25;
>> prob0423(ff,df,0.5,11)
   step size   finite difference    true error
  1.0000000000 -2.23750000000000  1.3250000000000
  0.1000000000 -1.00360000000000  0.0911000000000
  0.0100000000 -0.92128509999999  0.0087851000000
  0.0010000000 -0.91337535009994  0.0008753500999
  0.0001000000 -0.91258750349987  0.0000875034999
  0.0000100000 -0.91250875002835  0.0000087500284
  0.0000010000 -0.91250087497219  0.0000008749722
  0.0000001000 -0.91250008660282  0.0000000866028
  0.0000000100 -0.91250000888721  0.0000000088872
  0.0000000010 -0.91249996447829  0.0000000355217
  0.0000000001 -0.91250007550059  0.0000000755006
```

Solution continued on next page...


Plot of Error Versus Step Size
Error
Step Size
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^{0}

Solution 21.6

The true value

$$f'(0) = 8(0)^3 - 18(0)^2 - 12 = -12$$

Equation (21.21) can be used to compute the derivative as

$x_0 = -0.5$ $\quad f(x_0) = -1.125$
$x_1 = 1$ $\quad f(x_1) = -24$
$x_2 = 2$ $\quad f(x_2) = -48$

$$f'(0) = -1.125\frac{2(0)-1-2}{(-0.5-1)(-0.5-2)} + (-24)\frac{2(0)-(-0.5)-2}{(1-(-0.5))(1-2)} + (-48)\frac{2(0)-(-0.5)-1}{(2-(-0.5))(2-1)}$$

$$= 0.9 - 24 + 9.6 = -13.5$$

Centered difference:

$$f'(0) = \frac{-24-12}{1-(-1)} = -18$$

Solution 21.10

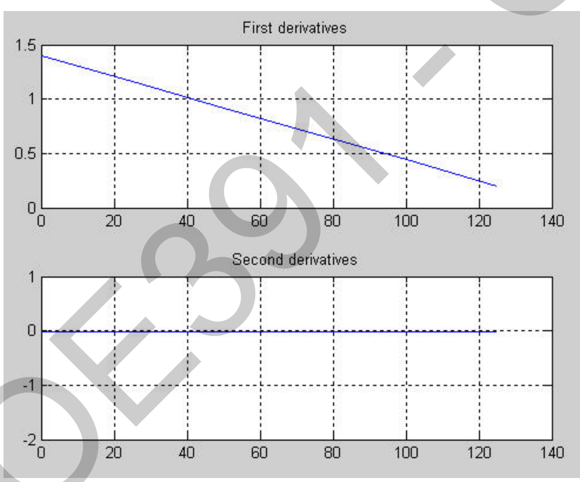
```
function [dydx, d2ydx2] = diffeq(x,y)
n = length(x);
if length(y)~=n, error('x and y must be same length'), end
if any(diff(diff(x))~=0), error('unequal spacing'), end
if length(x)<4, error('at least 4 values required'), end
dx=x(2)-x(1);
for i=1:n
  if i==1
    dydx(i)=(-y(i+2)+4*y(i+1)-3*y(i))/dx/2;
    d2ydx2(i)=(-y(i+3)+4*y(i+2)-5*y(i+1)+2*y(i))/dx^2;
  elseif i==n
    dydx(i)=(3*y(i)-4*y(i-1)+y(i-2))/dx/2;
    d2ydx2(i)=(2*y(i)-5*y(i-1)+4*y(i-2)-y(i-3))/dx^2;
  else
    dydx(i)=(y(i+1)-y(i-1))/dx/2;
    d2ydx2(i)=(y(i+1)-2*y(i)+y(i-1))/dx^2;
  end
end
subplot(2,1,1);plot(x,dydx);grid;title('First derivatives')
subplot(2,1,2);plot(x,d2ydx2);grid;title('Second derivatives')
```

The M-file can be run for the data from Prob. 21.11:

```
>> t=[0 25 50 75 100 125];
>> y=[0 32 58 78 92 100];
>> [dydx, d2ydx2] = diffeq(t,y)

dydx =
    1.4000    1.1600    0.9200    0.6800    0.4400    0.2000

d2ydx2 =
   -0.0096   -0.0096   -0.0096   -0.0096   -0.0096   -0.0096
```



Solution 21.28

Because the data is equispaced, we can use the second-order finite divided difference formulas from Figs. 21.3 through 21.5. For the first point, we can use

$$\frac{dT}{dt} = \frac{-30 + 4(44.5) - 3(80)}{10 - 0} = -9.2$$

For the intermediate points, we can use centered differences. For example, for the second point

$$\frac{dT}{dt} = \frac{30 - 80}{10 - 0} = -5$$

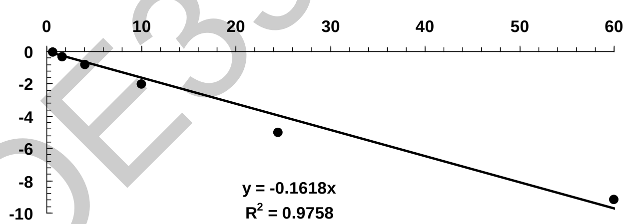
We can analyze the other interior points in a similar fashion. For the last point, we can use a backward difference

$$\frac{dT}{dt} = \frac{3(20.7) - 4(21.7) + 24.1}{25 - 15} = -0.06$$

All the values can be tabulated as

t	T	$T - T_a$	dT/dt
0	80	60	−9.2
5	44.5	24.5	−5
10	30	10	−2.04
15	24.1	4.1	−0.83
20	21.7	1.7	−0.34
25	20.7	0.7	−0.06

If Newton's law of cooling holds, we can plot dT/dt versus $T - T_a$ and the points can be fit with a linear regression with zero intercept to estimate the cooling rate. As in the following plot, the result is $k = 0.1618/\text{min}$.



Solution 21.33

Equation (21.21) can be used to estimate the derivatives at each temperature. This can be done using the M-file developed in Prob. 21.9 with the result

```
>> T=[750 800 900 1000];
>> h=[29629 32179 37405 42769];
>> cp=diffuneq(T,h)
cp =
   50.5800   51.4200   52.9500   54.3300
```

These results, which have units of kJ/(kmol K), can be converted to the desired units as in

$$c_p\left(\frac{\text{J}}{\text{kg K}}\right)=c_p\left(\frac{\text{kJ}}{\text{kmol K}}\right)\times\frac{\text{kmol}}{10^3\ \text{mol}}\times\frac{10^3\text{J}}{\text{kJ}}\times\frac{\text{mol}}{\left[12.011+2(15.9994)\right]\ \text{g}}\times\frac{10^3\text{g}}{\text{kg}}$$

```
>> cp=cp*1e3/(12.011+2*15.9994)

cp =
  1.0e+003 *
    1.1493    1.1684    1.2031    1.2345
```