Robert Heeter
BIOE 391 Numerical Methods – Due 11 March 2022

Problem Set 7 Solutions

1.

  a. **Output**
  PART A: Newton polynomial interpolation
  f(3.4) =
  4.700000 (1st order)
  5.260000 (2nd order)
  4.972000 (3rd order)

  b. **Output**
  PART B: Lagrange polynomial interpolation
  f(3.4) =
  4.700000 (1st order)
  5.260000 (2nd order)
  4.972000 (3rd order)

2. Output
PART A: Newton polynomial interpolation
Density (kg/m^3) at 330K:
1.035800 (1st order)
1.028720 (2nd order)
1.028888 (3rd order, best estimate)
1.027902 (4th order)
1.029021 (5th order)

PART B: Inverse interpolation with 3rd-order Newton interpolation
1.028888 kg/m^3 corresponds to a temperature of 330.000000 K

3.

  a. **Output**
  PART A: Newton polynomial interpolation (3rd order)
  T(x=4,y=3.2) = 43.368000 °C

  b. **Output**
  PART B: Interp2 2D interpolation
  T(x=4.3,y=2.7) = 46.006213 °C

4. Output
Depth of thermocline (m): 1.215110
Flux across interface (cal/(s*m^2)): 73.058930

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Figure (to show the cubic spline function with the data and the inflection area)

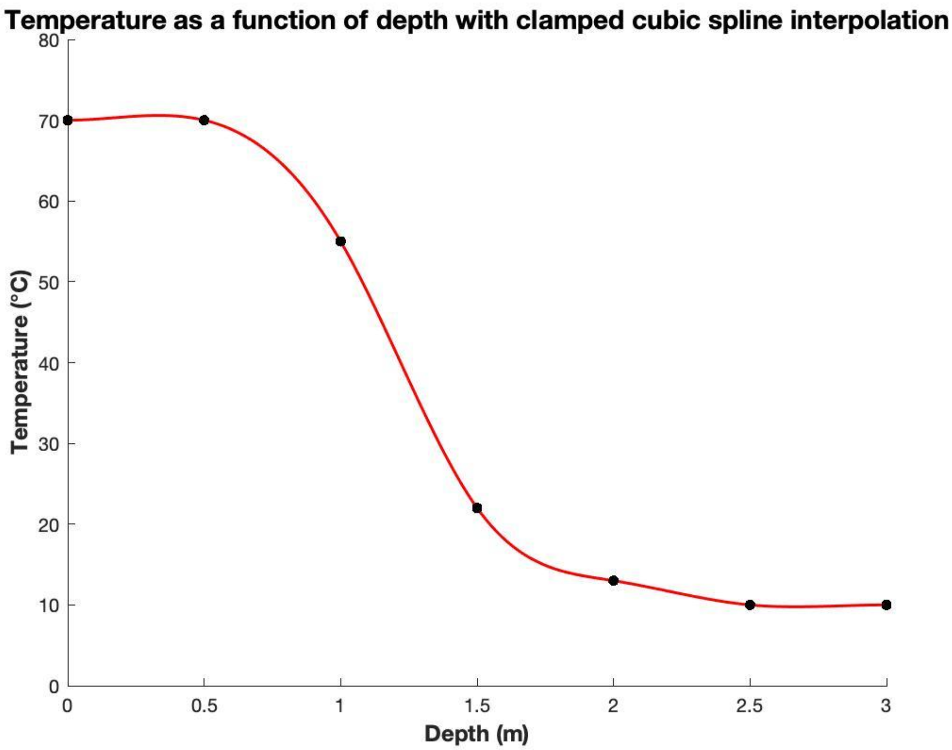
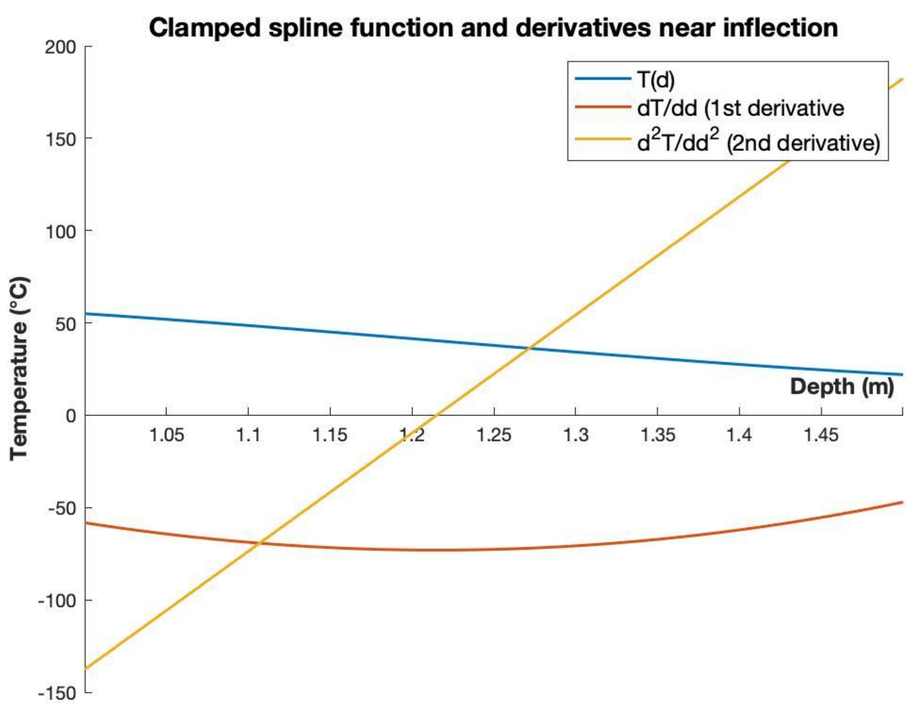


Figure (to show derivatives of the spline function near the inflection point to approximate inflection)



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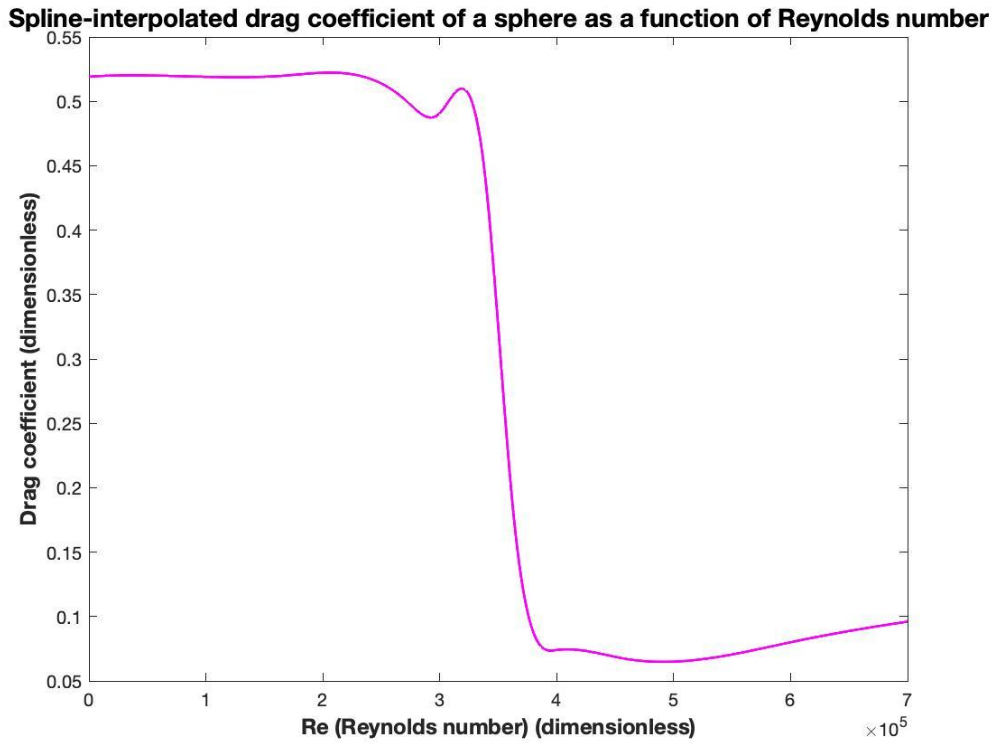
5.

a. See MATLAB code for function *drag*

b. Figure



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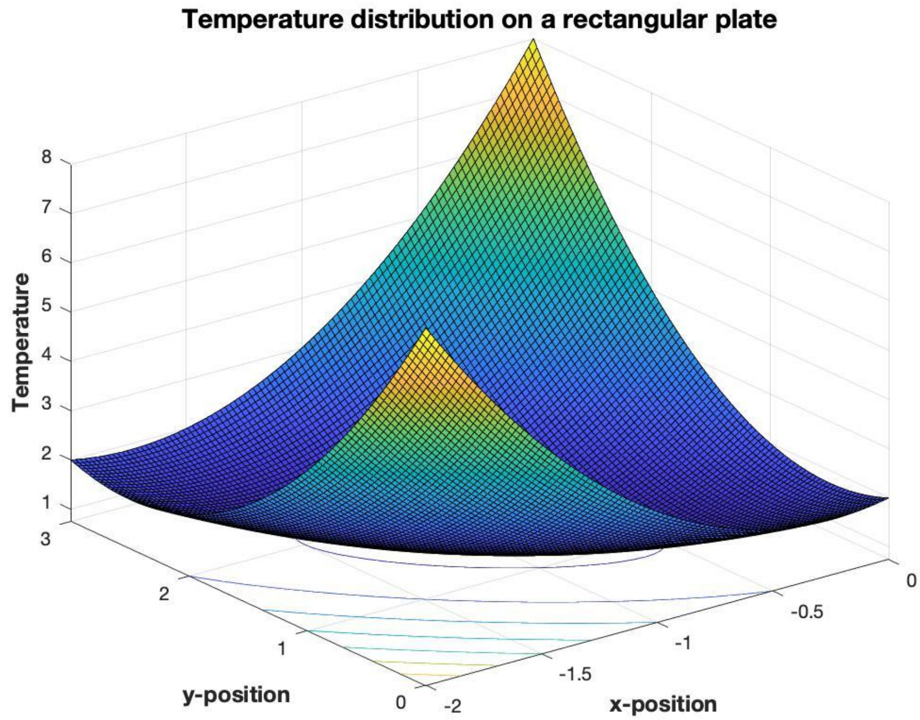
6.

a. Figure



b. Output
PART B: Interp2 2D linear interpolation
Interpolated T(x=-1.63,y=1.627) = 1.462605
Actual T = 1.399909
True relative error = 4.478577 percent

c. Output
PART C: Interp2 2D spline interpolation
Interpolated T(x=-1.63,y=1.627) = 1.399909
Actual T = 1.399909
True relative error = 0.000000 percent

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Complete MATLAB Code

```
% Robert Heeter
% BIOE 391 Numerical Methods
% HOMEWORK 7 MATLAB SCRIPT

clc, clf, clear, close all

%% P1. PROBLEM 17.4
disp('P1. PROBLEM 17.4');

x = [1 2 2.5 3 4 5]'; % x values
fx = [0 5 7 6.5 2 0]'; % f(x) values

% PART A: Newton interpolating polynomials
fx_newt_O1 = newtint(x(4:5),fx(4:5),3.4); % use newtint function (below) for interpolation
fx_newt_O2 = newtint(x(3:5),fx(3:5),3.4);
fx_newt_O3 = newtint(x(2:5),fx(2:5),3.4);

% PART B: Lagrange interpolating polynomials
fx_lagr_O1 = lagrint(x(4:5),fx(4:5),3.4); % use lagrint function (below) for interpolation
fx_lagr_O2 = lagrint(x(3:5),fx(3:5),3.4);
fx_lagr_O3 = lagrint(x(2:5),fx(2:5),3.4);

% Display results
fprintf('PART A: Newton polynomial interpolation\nf(3.4) =\n%f (1st order)\n%f (2nd order)\n%f (3rd
order)\n\n',fx_newt_O1,fx_newt_O2,fx_newt_O3);
fprintf('PART B: Lagrange polynomial interpolation\nf(3.4) =\n%f (1st order)\n%f (2nd order)\n%f (3rd
order)\n\n',fx_lagr_O1,fx_lagr_O2,fx_lagr_O3);


%% P2. PROBLEM 17.11
disp('P2. PROBLEM 17.11');

T = [200 250 300 350 400 450]'; % temperature data (K)
rho = [1.708 1.367 1.139 0.967 0.854 0.759]'; % density of nitrogen gas (kg/m^3)

% PART A: Newton polynomial interpolation (orders 1-5)
rho_newt_O1 = newtint(T(3:4),rho(3:4),330); % use newtint function (below) for interpolation
rho_newt_O2 = newtint(T(3:5),rho(3:5),330);
rho_newt_O3 = newtint(T(2:5),rho(2:5),330); % 3rd-order is best estimate
rho_newt_O4 = newtint(T(2:6),rho(2:6),330);
rho_newt_O5 = newtint(T(1:6),rho(1:6),330);

% PART B: Inverse interpolation to check 3rd-order estimate
p = polyfit(T(2:5),rho(2:5),3); % use polyfit to create 3rd-order polynomial through points
p(end) = p(end)-rho_newt_O3; % convert a = p(x) to 0 = p(x)-a, where a is the 3rd-order estimate
[roots] = roots(p); % find root of 0 = p(x)-a to find a = p(x)

% Display results
fprintf('PART A: Newton polynomial interpolation\nDensity (kg/m^3) at 330K:\n%f (1st order)\n%f (2nd
order)\n%f (3rd order, best estimate)\n%f (4th order)\n%f (5th
order)\n\n',rho_newt_O1,rho_newt_O2,rho_newt_O3,rho_newt_O4,rho_newt_O5);
fprintf('PART B: Inverse interpolation with 3rd-order Newton interpolation\n%f kg/m^3 corresponds to a
temperature of %f K\n\n',rho_newt_O3,roots(3));


%% P3. PROBLEM 17.20
disp('P3. PROBLEM 17.20');

T = [100.0 90.00 80.00 70.00 60.00;
     85.00 64.49 53.50 48.15 50.00;
     70.00 48.90 38.43 35.03 40.00;
     55.00 38.78 30.39 27.07 30.00;
     40.00 35.00 30.00 25.00 20.00]; % temperatures of heated plate (Â°C)
```

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```
x = [0 2 4 6 8]; % x-position
y = [0 2 4 6 8]'; % y-position

% PART A: Temperature estimate (x=4, y=3.2)
T_newt_O3 = newtint(y(1:4),T(1:4,3),3.2); % use newtint function (below) for interpolation

% PART B: Temperature estimate (x=4.3, y=2.7)
T_int_2D = interp2(x(2:5),y(1:4),T(1:4,2:5),4.3,2.7,'cubic'); % use in-built interp2 function for 2D
interpolation

% Display results
fprintf('PART A: Newton polynomial interpolation (3rd order)\nT(x=4,y=3.2) = %f Â°C\n\n',T_newt_O3);
fprintf('PART B: Interp2 2D interpolation\nT(x=4.3,y=2.7) = %f Â°C\n\n',T_int_2D);


%% P4. PROBLEM 18.2
disp('P4. PROBLEM 18.2');

% Data
d = [0 0.5 1 1.5 2 2.5 3]'; % depth (m)
T = [70 70 55 22 13 10 10]'; % temperature (Â°C)

h = diff(d); % differences between d values
csV = zeros(7,1); % preallocate vector for csM*csC=csV
csM = zeros(7,7); % preallocate matrix for csM*csC=csV

for i = 1:5
    % Fill csM with middle values along tridiagonal
    csM(i+1,i) = h(i);
    csM(i+1,i+1) = 2*(h(i)+h(i+1));
    csM(i+1,i+2) = h(i+1);
    % Fill csV with middle values
    csV(i+1) = 3*(((T(i+2)-T(i+1))/(d(i+2)-d(i+1)))-((T(i+1)-T(i))/(d(i+1)-d(i))));
end

% Fill csM and csV with endpoint values for clamped condition
csM(1,1:2) = [2*h(1) h(1)];
csV(1) = 3*(((T(2)-T(1))/(d(2)-d(1))-0));

% Fill csM and csV with endpoint values for clamped condition
csM(7,6:7) = [h(6) 2*h(6)];
csV(7) = 3*(0-((T(7)-T(6))/(d(7)-d(6))));

% Solve for vector of constants C
csC = csM\csV;

% Solve for vectors for constants A,B,D
csA = T(1:6);
csB = ((T(2:7)-T(1:6))./h)-((h./3).*((2.*csC(1:6))+csC(2:7)));
csD = (csC(2:7)-csC(1:6))./(3.*h);
csC = csC(1:6);

% Create plot of interpolated spline
figure
hold on
for i = 1:6
    Td = @(x) csA(i) + csB(i).*(x-d(i)) + csC(i).*(x-d(i)).^2 + csD(i).*(x-d(i)).^3;
    fplot(Td,[d(i) d(i+1)],'-r','LineWidth',1.5);
end
plot(d,T,'.k','MarkerSize',15);
xlabel('Depth (m)','FontSize',12,'FontWeight','bold');
ylabel('Temperature (Â°C)','FontSize',12,'FontWeight','bold');
title('Temperature as a function of depth with clamped cubic spline
interpolation','FontSize',14,'FontWeight','bold');
hold off
```

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```
% Create polynomial equation and differentiate to find inflection
syms Td(x)
Td(x) = csA(3) + csB(3)*(x-d(3)) + csC(3)*(x-d(3))^2 + csD(3)*(x-d(3))^3; % inflection occurs in third
segment from graph above

dTdd = diff(Td); % First derivative
d2Tdd2 = diff(Td,2); % Second derivative

% Create plot of spline function containing inflection and its derivatives
figure
hold on
fplot(Td,[d(3) d(4)],'-','LineWidth',1.5)
fplot(dTdd,[d(3) d(4)],'-','LineWidth',1.5)
fplot(d2Tdd2,[d(3) d(4)],'-','LineWidth',1.5)
legend('T(d)','dT/dd (1st derivative','d^2T/dd^2 (2nd derivative)','FontSize',12);
xlabel('Depth (m)','FontSize',12,'FontWeight','bold');
ylabel('Temperature (Â°C)','FontSize',12,'FontWeight','bold');
title('Clamped spline function and derivatives near inflection','FontSize',14,'FontWeight','bold');
ax = gca;
ax.XAxisLocation = 'origin';
hold off

% Find zero of second derivative for depth of thermocline
[depth,~] = fzero(d2Tdd2,[0,2]);

% Find heat flux across interface with Fourier's law
k = 1; % constant for Fourier's law (cal/(s*m*Â°C))
J = -1*k*dTdd(depth); % Fourier's law

% Display results
fprintf('Depth of thermocline (m): %f\nFlux across interface (cal/(s*m^2)): %f\n\n',depth,J);


%% P5. PROBLEM 18.13
disp('P5. PROBLEM 18.13');

% PART A: Function "Drag" (see below)

% PART B: Plot of C_D (drag coefficient) vs. Re (Reynolds number)
ReCD = [2 5.8 16.8 27.2 29.9 33.9 36.3 40 46 60 100 200 400;
    0.52 0.52 0.52 0.5 0.49 0.44 0.18 0.074 0.067 0.08 0.12 0.16 0.19]; % tabulated data; first row is
Re (dimensionless), second row is C_D (dimensionless)
ReCD(1,:) = 10^4.*ReCD(1,:);

Re_in = 10^4.*(0:0.01:70)'; % input vector for plot
CD_out = drag(ReCD,Re_in); % use drag function (below) for interpolated drag coefficient values

figure
plot(Re_in,CD_out,'-m','LineWidth',1.5);
xlabel('Re (Reynolds number) (dimensionless)','FontSize',12,'FontWeight','bold');
ylabel('Drag coefficient (dimensionless)','FontSize',12,'FontWeight','bold');
title('Spline-interpolated drag coefficient of a sphere as a function of Reynolds
number','FontSize',14,'FontWeight','bold');
disp(' ');


%% P6. PROBLEM 18.14
disp('P6. PROBLEM 18.14');

% PART A: Meshplot of temperature function
T = @(x,y) 2 + x - y +2.*x.^2 + 2.*x.*y + y.^2; % temperature function
x = linspace(-2,0); % vector of x-values
y = (linspace(0,3))'; % vector of y-values
z = T(x,y); % matrix of temperature values

figure
```

```
surfc(x,y,z);
xlabel('x-position','FontSize',12,'FontWeight','bold');
ylabel('y-position','FontSize',12,'FontWeight','bold');
zlabel('Temperature','FontSize',12,'FontWeight','bold');
title('Temperature distribution on a rectangular plate','FontSize',14,'FontWeight','bold');

% PART B: Linear interpolation with interp2
x_points = linspace(-2,0,9);
y_points = (linspace(0,3,9))';
z_points = T(x_points,y_points);
T_int_lin = interp2(x_points,y_points,z_points,-1.63,1.627,'linear'); % use in-built interp2 function
for 2D linear interpolation

% PART C: Spline interpolation with interp2
T_int_spl = interp2(x_points,y_points,z_points,-1.63,1.627,'spline'); % use in-built interp2 function
for 2D spline interpolation

% Display results
T_actual = T(-1.63,1.627);
error_lin = (abs(T_int_lin-T_actual)/T_actual)*100;
error_spl = (abs(T_int_spl-T_actual)/T_actual)*100;

fprintf('PART B: Interp2 2D linear interpolation\nInterpolated T(x=-1.63,y=1.627) = %f\nActual T =
%f\nTrue relative error = %f percent\n\n',T_int_lin,T_actual,error_lin);
fprintf('PART C: Interp2 2D spline interpolation\nInterpolated T(x=-1.63,y=1.627) = %f\nActual T =
%f\nTrue relative error = %f percent\n\n',T_int_spl,T_actual,error_spl);


%% Additional Functions

function CD_out = drag(ReCD,Re_in)
% ABOUT: Determines drag coefficient for a sphere using spline
% interpolation from the Reynolds number.
% INPUTS: ReCD = table of Reynolds numbers and corresponding drag
% coefficients for interpolation; Re_in = input Reynolds numbers to
% interpolate
% OUTPUTS: CD_out = output drag coefficient values from interpolation

    CD_out = spline(ReCD(1,:),ReCD(2,:),Re_in); %  % use in-built spline function for cubic
interpolation

end


function yint = newtint(x,y,xx)
% ABOUT: Newton polynomial interpolation, from textbook .m file.
% INPUTS: x = independent variable; y = dependent variable; xx = value of
% independent variable at which interpolation is calculated
% OUTPUTS: yint = interpolated value of dependent variable

% Compute the finite divided differences in the form of a % difference table
n = length(x);
if length(y)~=n
    error('x and y must be same length');
end

b = zeros(n,n);

% Assign dependent variables to the first column of b
b(:,1) = y(:); % the (:) ensures that y is a column vector
for j = 2:n
    for i = 1:n-j+1
        b(i,j) = (b(i+1,j-1)-b(i,j-1))/(x(i+j-1)-x(i));
    end
end
```

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```
% Use the finite divided differences to interpolate
xt = 1;
yint = b(1,1);
for j = 1:n-1
    xt = xt*(xx-x(j));
    yint = yint+b(1,j+1)*xt;
end

end


function yint = lagrint(x,y,xx)
% ABOUT: Lagrange polynomial interpolation, from textbook .m file.
% INPUTS: x = independent variable; y = dependent variable; xx = value of
% independent variable at which interpolation is calculated
% OUTPUTS: yint = interpolated value of dependent variable

n = length(x);
if length(y)~=n
    error('x and y must be same length');
end

s = 0;
for i = 1:n
    product = y(i);
    for j = 1:n
        if i ~= j
            product = product*(xx-x(j))/(x(i)-x(j));
        end
    end
    s = s+product;
end
yint = s;

end
```