Solution 1.4

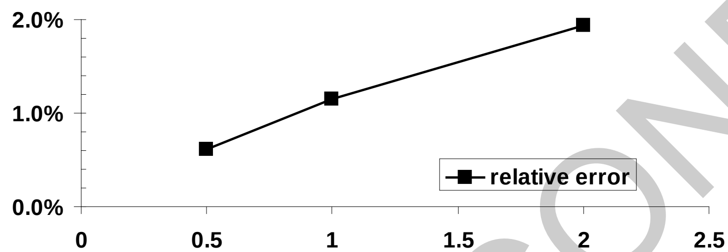
At t = 12 s, the analytical solution is 50.6175 (Example 1.1). The numerical results are:

step	$v(12)$	absolute relative error
2	51.6008	1.94%
1	51.2008	1.15%
0.5	50.9259	0.61%

where the relative error is calculated with

$$\text{absolute relative error} = \left|\frac{\text{analytical} - \text{numerical}}{\text{analytical}}\right| \times 100\%$$

The error versus step size can be plotted as



Thus, halving the step size approximately halves the error.

Solution 1.5

(a) The force balance is

$$\frac{dv}{dt} = g - \frac{c'}{m}v$$

Applying Laplace transforms,

$$sV - v(0) = \frac{g}{s} - \frac{c'}{m}V$$

Solve for

$$V = \frac{g}{s(s + c'/m)} + \frac{v(0)}{s + c'/m} \qquad (1)$$

The first term to the right of the equal sign can be evaluated by a partial fraction expansion,

$$\frac{g}{s(s + c'/m)} = \frac{A}{s} + \frac{B}{s + c'/m} \qquad (2)$$

$$\frac{g}{s(s + c'/m)} = \frac{A(s + c'/m) + Bs}{s(s + c'/m)}$$

Equating like terms in the numerators yields

$$A + B = 0$$

$$g = \frac{c'}{m}A$$

Therefore,

$$A = \frac{mg}{c'} \qquad B = -\frac{mg}{c'}$$

Solution continued on next page...

These results can be substituted into Eq. (2), and the result can be substituted back into Eq. (1) to give

$$V = \frac{mg/c'}{s} - \frac{mg/c'}{s + c'/m} + \frac{v(0)}{s + c'/m}$$

Applying inverse Laplace transforms yields

$$v = \frac{mg}{c'} - \frac{mg}{c'} e^{-(c'/m)t} + v(0) e^{-(c'/m)t}$$

or

$$v = v(0) e^{-(c'/m)t} + \frac{mg}{c'}\left[1 - e^{-(c'/m)t}\right]$$

where the first term to the right of the equal sign is the general solution and the second is the particular solution. For our case, $v(0) = 0$, so the final solution is

$$v = \frac{mg}{c'}\left[1 - e^{-(c'/m)t}\right]$$

Alternative solution: Another way to obtain solutions is to use separation of variables,

$$\int \frac{1}{g - \frac{c'}{m} v} dv = \int dt$$

The integrals can be evaluated as

$$-\frac{\ln\left(g - \frac{c'}{m} v\right)}{c'/m} = t + C$$

where C = a constant of integration, which can be evaluated by applying the initial condition

$$C = -\frac{\ln\left(g - \frac{c'}{m} v(0)\right)}{c'/m}$$

Solution continued on next page...

which can be substituted back into the solution

$$-\frac{\ln\left(g-\frac{c'}{m}v\right)}{c'/m}=t-\frac{\ln\left(g-\frac{c'}{m}v(0)\right)}{c'/m}$$

This result can be rearranged algebraically to solve for v,

$$v=v(0)e^{-(c'/m)t}+\frac{mg}{c'}\left[1-e^{-(c'/m)t}\right]$$

where the first term to the right of the equal sign is the general solution and the second is the particular solution. For our case, $v(0)=0$, so the final solution is

$$v=\frac{mg}{c'}\left[1-e^{-(c'/m)t}\right]$$

(b) The numerical solution can be implemented as

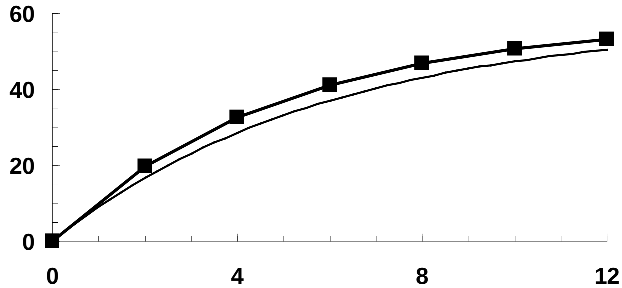
$$v(2)=0+\left[9.81-\frac{11.5}{68.1}(0)\right]2=19.62$$

$$v(4)=19.62+\left[9.81-\frac{11.5}{68.1}(19.62)\right]2=32.6136$$

The computation can be continued and the results summarized and plotted as:

t	v	dv/dt
0	0	9.81
2	19.6200	6.4968
4	32.6136	4.3026
6	41.2187	2.8494
8	46.9176	1.8871
10	50.6917	1.2497
12	53.1911	0.8276
∞	58.0923	

Solution continued on next page...



Note that the analytical solution is included on the plot for comparison.

Solution 1.9

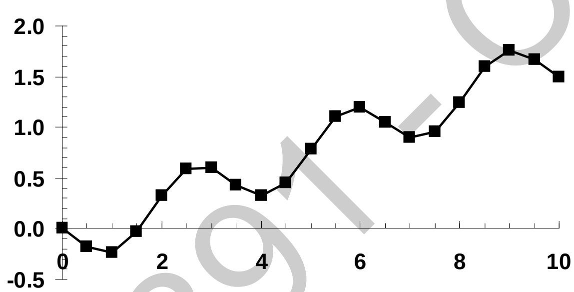
The first two steps yield

$$y(0.5) = 0 + \left[3\frac{450}{1250}\sin^2(0) - \frac{450}{1250} \right] 0.5 = \; 0 + (-0.36)\; 0.5 = -0.18$$

$$y(1) = -0.18 + \left[3\frac{450}{1250}\sin^2(0.5) - \frac{450}{1250} \right] 0.5 = \; -0.18 + (-0.11176)\; 0.5 = -0.23508$$

The process can be continued to give the following table and plot:

t	*y*	*dy/dt*	*t*	*y*	*dy/dt*
0	0.00000	-0.36000	5.5	1.10271	0.17761
0.5	-0.18000	-0.11176	6	1.19152	-0.27568
1	-0.23588	0.40472	6.5	1.05368	-0.31002
1.5	-0.03352	0.71460	7	0.89866	0.10616
2	0.32378	0.53297	7.5	0.95175	0.59023
2.5	0.59026	0.02682	8	1.24686	0.69714
3	0.60367	-0.33849	8.5	1.59543	0.32859
3.5	0.43443	-0.22711	9	1.75972	-0.17657
4	0.32087	0.25857	9.5	1.67144	-0.35390
4.5	0.45016	0.67201	10	1.49449	-0.04036
5	0.78616	0.63310			



Solution 1.13

$$\sum M_{in} - \sum M_{out} = 0$$

$$\text{Food} + \text{Drink} + \text{Air In} + \text{Metabolism} = \text{Urine} + \text{Skin} + \text{Feces} + \text{Air Out} + \text{Sweat}$$

$$\text{Drink} = \text{Urine} + \text{Skin} + \text{Feces} + \text{Air Out} + \text{Sweat} - \text{Food} - \text{Air In} - \text{Metabolism}$$

$$\text{Drink} = 1.4 + 0.35 + 0.2 + 0.4 + 0.3 - 1 - 0.05 - 0.3 = 1.3 \text{ L}$$

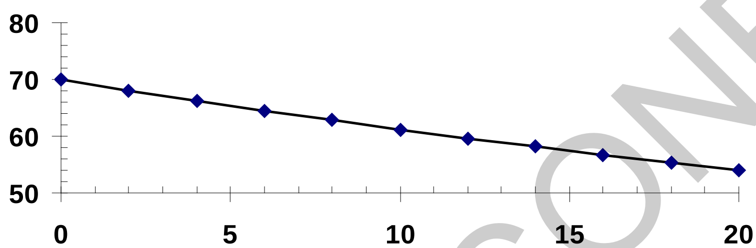
Solution 1.17

The first two steps can be computed as

$$T(1) = 70 + \left[-0.019(70-20)\right] 2 = 68 + (-0.95)2 = 68.1$$
$$T(2) = 68.1 + \left[-0.019(68.1-20)\right] 2 = 68.1 + (-0.9139)2 = 66.2722$$

The remaining results are displayed below along with a plot of the results.

t	T	dT/dt	t	T	dT/dt
0	70.00000	-0.95000	12.00000	59.62967	-0.75296
2	68.10000	-0.91390	14.00000	58.12374	-0.72435
4	66.27220	-0.87917	16.00000	56.67504	-0.69683
6	64.51386	-0.84576	18.00000	55.28139	-0.67035
8	62.82233	-0.81362	20.00000	53.94069	-0.64487
10	61.19508	-0.78271			



Solution 1.18

(a) For the constant temperature case, Newton's law of cooling is written as

$$\frac{dT}{dt} = -0.12(T - 10)$$

The first two steps of Euler's methods are

$$T(0.5) = T(0) - \frac{dT}{dt}(0) \times \Delta t = 37 + 0.12(10 - 37)(0.5) = 37 - 3.2400 \times 0.50 = 35.3800$$
$$T(1) = 35.3800 + 0.12(10 - 35.3800)(0.5) = 35.3800 - 3.0456 \times 0.50 = 33.8572$$

The remaining calculations are summarized in the following table:

t	T_a	T	dT/dt
0:00	10	37.0000	-3.2400
0:30	10	35.3800	-3.0456
1:00	10	33.8572	-2.8629
1:30	10	32.4258	-2.6911
2:00	10	31.0802	-2.5296
2:30	10	29.8154	-2.3778
3:00	10	28.6265	-2.2352
3:30	10	27.5089	-2.1011
4:00	10	26.4584	-1.9750
4:30	10	25.4709	-1.8565
5:00	10	24.5426	-1.7451

(b) For this case, the room temperature can be represented as

$$T_a = 20 - 2t$$

where t = time (hrs). Newton's law of cooling is written as

$$\frac{dT}{dt} = -0.12(T - 20 + 2t)$$

Solution continued on next page...

The first two steps of Euler's methods are

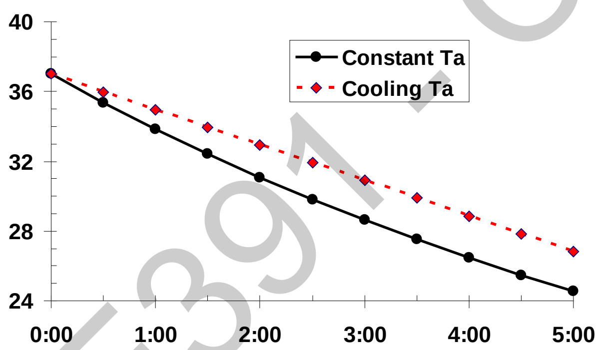
$$T(0.5) = 37 + 0.12(20 - 37)(0.5) = 37 - 2.040 \times 0.50 = 35.9800$$
$$T(1) = 35.9800 + 0.12(19 - 35.9800)(0.5) = 35.9800 - 2.0376 \times 0.50 = 34.9612$$

The remaining calculations are summarized in the following table:

t	T_a	T	dT/dt
0:00	20	37.0000	-2.0400
0:30	19	35.9800	-2.0376
1:00	18	34.9612	-2.0353
1:30	17	33.9435	-2.0332
2:00	16	32.9269	-2.0312
2:30	15	31.9113	-2.0294
3:00	14	30.8966	-2.0276
3:30	13	29.8828	-2.0259
4:00	12	28.8699	-2.0244
4:30	11	27.8577	-2.0229
5:00	10	26.8462	-2.0215

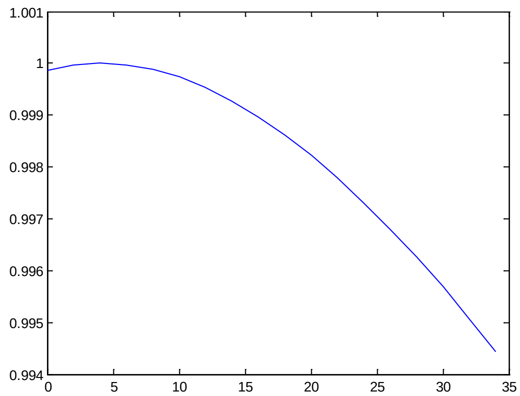
Comparison with (a) indicates that the effect of the room air temperature has a significant effect on the expected temperature at the end of the 5-hr period (difference = 26.8462 - 24.5426 = 2.3036°C).

(c) The solutions for (a) Constant T_a, and (b) Cooling T_a are plotted below:



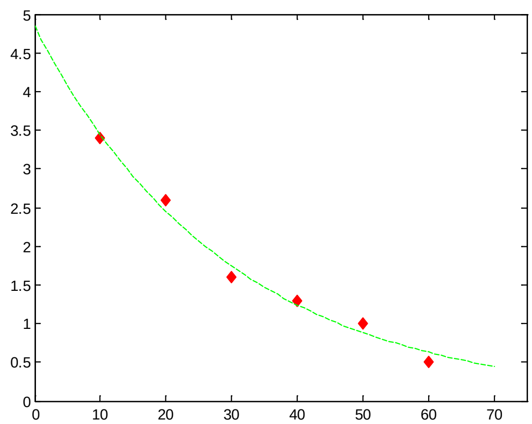
Solution 2.14

```
>> TF = 32:3.6:93.2;
>> TC = 5/9*(TF-32);
>> rho = 5.5289e-8*TC.^3-8.5016e-6*TC.^2+6.5622e-5*TC+0.99987;
>> plot(TC,rho)
```



Solution 2.16

```
clear, clc
t = 10:10:60;
c = [3.4 2.6 1.6 1.3 1.0 0.5];
tf = 0:70;
cf = 4.84*exp(-0.034*tf);
plot(t,c,'d','MarkerEdgeColor','r','MarkerFaceColor','r')
hold on
plot(tf,cf,'--g')
xlim([0 75])
hold off
```



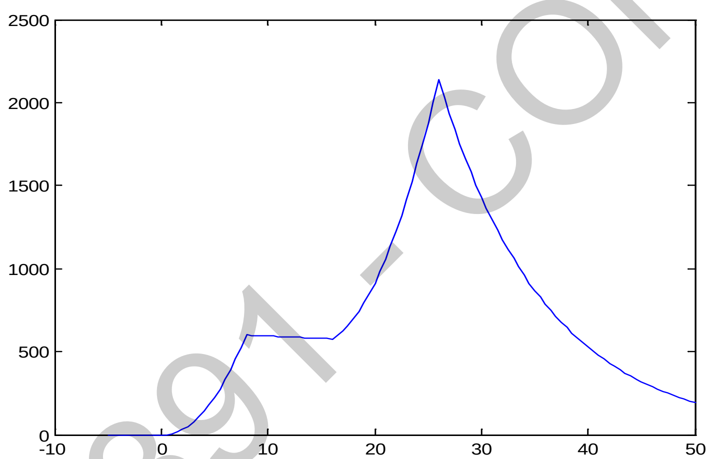
Solution 3.14

The following function implements the piecewise function:

```
function v = vpiece(t)
if t<0
  v = 0;
elseif t<8
  v = 10*t^2 - 5*t;
elseif t<16
  v = 624 - 3*t;
elseif t<26
  v = 36*t + 12*(t - 16)^2;
else
  v = 2136*exp(-0.1*(t-26));
end
```

Here is a script that uses `vpiece` to generate the plot

```
k=0;
for i = -5:.5:50
  k=k+1;
  t(k)=i;
  v(k)=vpiece(t(k));
end
plot(t,v)
```



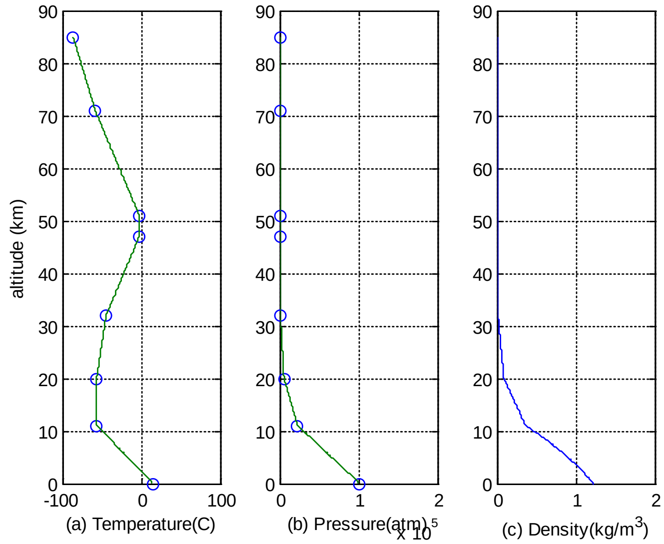
Solution 3.28

```
% Script to generate a plot of temperature, pressure and density
% for the International Standard Atmosphere
clc,clf
h=[0 11 20 32 47 51 71 84.852];
gamma=[-6.5 0 1 2.8 0 -2.8 -2];
T=[15 -56.5 -56.5 -44.5 -2.5 -2.5 -58.5 -86.28];
p=[101325 22632 5474.9 868.02 110.91 66.939 3.9564 0.3734];
hint=[0:0.1:84.852];
for i=1:length(hint)
  [Tint(i),pint(i),rint(i)]=StdAtm(h,T,p,gamma,hint(i));
end
subplot(1,3,1)
plot(T,h,'o',Tint,hint),grid,xlabel('(a) Temperature(C)')
ylabel('altitude (km)')
subplot(1,3,2)
plot(p,h,'o',pint,hint)
grid,xlabel('(b) Pressure(atm)')
subplot(1,3,3)
plot(rint,hint),grid,xlabel('(c) Density(kg/m^3)')
[Tint(i),pint(i),rint(i)]=StdAtm(h,T,p,gamma,85);

function [Tint,pint,rint] = StdAtm(h,T,p,gamma,hi)
R=8.3144621; M=0.0289644;
n=length(h);
if hi<h(1) || hi>h(n)
  error('Outside range')
else
  for i=1:n-1
    if hi<=h(i+1)
      Tint=T(i)+gamma(i)*(hi-h(i));
      pint=p(i)+(p(i+1)-p(i))/(h(i+1)-h(i))*(hi-h(i));
      rint=pint*M/(R*(Tint+273.15));
      break
    end
  end
end
end
```

<u>Results:</u>

```
Error using StdAtm (line 5)
Outside range

Error in Prob0328StdAtmScript (line 20)
[Tint(i),pint(i),rint(i)]=StdAtm(h,T,p,gamma,85);
```

Solution continued on next page...


altitude (km)
(a) Temperature(C)
(b) Pressure(atm)
x 10^5
(c) Density(kg/m^3)