I. PRINCIPES OF SPECIAL RELATIVITY

1. The laws of physics are invariant in all inertial frames of reference.
2. The speed of light in vacuum is the same for all observers, regardless of the motion of the light source or observer.

II. LORENTZ TRANSFORMATION

1. Michelson-Morley experiment.
2. Lorentz factor $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$ and relatative velocity $\beta = v/c$.
3. Lorentz transformation between frames S' ad S with relative vel. v is $x'^\mu = \Lambda^\mu{}_\nu x^\nu$. Where Lorentz boost Λ takes form
$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$
4. $\Lambda^\alpha{}_\nu \Lambda_\alpha{}^\mu = \delta^\mu_\nu$ and $(\Lambda^\mu{}_\nu)^{-1} = \Lambda_\nu{}^\mu$
5. Lorentz boost in direction $\vec{n}$ of magnitude v:
$ct' = \gamma\,(ct - \beta\vec{n} \cdot \vec{r})$,
$\vec{r}' = [\vec{r} - (\vec{r} \cdot \vec{n})\vec{n}] + \gamma\,[(\vec{r} \cdot \vec{n})\vec{n} - \beta ct\vec{n}]$
where we used decomposition of $\vec{r}$,
$\vec{r}^{\,\parallel} = (\vec{r} \cdot \vec{n})\vec{n}$ and $\vec{r}^{\,\perp} = \vec{r} - (\vec{r} \cdot \vec{n})\vec{n}$.
6. $\beta = \tanh\xi$, $\gamma = \cosh\xi$ and $\beta\gamma = \sinh\xi$ where ξ is rapidity.
7. $$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cosh\xi & -\sinh\xi & 0 & 0 \\ -\sinh\xi & \cosh\xi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$
8. Time dilation $t = t_0\gamma$.
9. Length contraction $l = l_0/\gamma$.
10. Relativity of conplacemency $\Delta x = v\Delta t$.
11. Relativity of simultaneity $c\Delta t' = -\beta\gamma\Delta x$.
12. Transformation of 3-dim velocity: if v is along x axis: $w'_x = \frac{w_x - v}{1 - \frac{v}{c^2}w_x}$, $w'_y = \frac{1}{\gamma}\frac{w_y}{1 - \frac{v}{c^2}w_x}$.
13. Addition of vel. u and v: $w = (u+v)\left(1 + \frac{uv}{c^2}\right)$,
14. Hybrid velocity (distance in not-moving frame S but proper time) can be greater then c. If $\vec{w}$ is velocity in S, then hybrid vel. is $\frac{\mathrm{d}x}{\mathrm{d}t'} = \vec{w}\gamma$.
15. Space time interval $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$.
16. Space-time interval is invariant under Lorentz transformation.

III. SPACETIME

1. Vectors transform as $\mathrm{d}x'^\mu = \frac{\partial x'^\mu}{\partial x^\nu}\mathrm{d}x^\nu$ (choosen so bc. we want $\mathrm{d}s$ as a vector). Especially $\mathrm{d}x'^\mu = \Lambda^\mu{}_\nu \mathrm{d}x^\nu$.
2. Covectors transform as $\frac{\partial}{\partial x'^\mu} = \frac{\partial x^\nu}{\partial x'^\mu}\frac{\partial}{\partial x^\nu}$. Especially $\partial'_\mu = \Lambda_\mu{}^\nu \partial_\nu$.
3. $\partial_\nu \mathrm{d}x^\mu = \mathrm{d}x^\mu \partial_\nu = \delta^\mu_\nu = \mathrm{d}x'^\mu \partial'_\nu = \partial'_\nu \mathrm{d}x'^\mu$
4. Metric tensor $g_{\mu\nu}$ (a value of metric tensor field in the given point) is a billinear, symmetric, non-degenerate tensor.
5. Metric tensor defines invariant dot product $g'_{\mu\nu}V'^\mu V'^\nu = g_{\mu\nu}V^\mu V^\nu$.
6. $\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$
7. Metric tensor is invariant under Lorentz transformation $g_{\mu\nu} = g_{\alpha\beta}\Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu$, but gen. transforms as $g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}g_{\alpha\beta}$ (relation of ortogonality).
8. Sylvester's law of inertia: Signature (number of +, -, 0 eigenvalues) of matrix is invariant under change of basis.
9. Inverse metric tensor: $g^{\mu\nu} := \left(g^{-1}\right)^{\mu\nu}$, $g^{\mu\alpha}g_{\alpha\nu} = g_{\nu\alpha}g^{\alpha\mu} = \delta^\mu_\nu$
10. Lowering/rising inds. $g_{\mu\nu}A^\nu = A_\mu$, $g^{\mu\nu}A_\nu = A^\mu$
11. $g_{\mu\nu}V^\mu W^\nu = V^\mu g_{\mu\nu}W^\nu = W^\nu V^\mu g_{\mu\nu} = V_\nu W^\nu = W^\nu V_\nu$
12. Partial gradient transforms as a covector only under linear transformations: $\frac{\partial V'^\mu}{\partial x'^\nu} = \frac{\partial^2 x'^\mu}{\partial x^\alpha \partial x^\beta}\frac{\partial x^\beta}{\partial x'^\nu}V^\alpha + \frac{\partial x'^\mu}{\partial x^\alpha}\frac{\partial V^\alpha}{\partial x^\beta}\frac{\partial x^\beta}{\partial x'^\nu}$.
13. Dot product $g_{\mu\nu}V^\mu V^\nu$ is invariant $\Leftrightarrow$ trnasformation is linear and satisfy relation of orthogonality.
14. Metrix tensor of Minkowski spacetime
cartesian: $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$
cylindrical: $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \rho^2, 1)$
spherical: $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, r^2, r^2\sin^2\theta)$
15. Dot product of two timelike vectors oriented into the future is always negatve.
16. Levi-Civita tensor $\epsilon_{ijk\ldots}$ is 1 for even permutation of $\{1, 2, 3, \ldots\}$, -1 for odd permutation and zero otherwise. $\epsilon^{ijk\ldots}$ has opposite signs.
17. $\epsilon^{\alpha\beta\gamma\delta} = \eta^{\alpha i}\eta^{\beta j}\eta^{\gamma k}\eta^{\delta l}\epsilon_{ijkl}$

III. SPACETIME DIAGRAMS

1.

IV. RELATIVISTIC MECHANICS

1. Proper time τ along timelike wolrd line - time in a frame of the system.
2. Experimentaly is measured only the proper time.
3. $\mathrm{d}s = c\mathrm{d}\tau$, $\mathrm{d}t = \gamma\mathrm{d}\tau$ hence $\Delta\tau = \int \frac{\mathrm{d}s}{c} = \int \frac{\mathrm{d}t}{\gamma(t)}$

4. 4-velocity: $u^\mu = \frac{dx^\mu}{d\tau}$ and $u'^\mu = \Lambda^\mu{}_\nu u^\nu$
5. Minkowsi spacetime $\eta_{\mu\nu} u^\mu u^\nu = -c^2$.
6. $u^\mu = (c, \vec{v})^T$, where $\vec{v} = d\vec{x}/dt$ is classical 3-vel.
7. 4-acceleration: $a^\mu = \frac{du^\mu}{d\tau}$ and $a'^\mu = \Lambda^\mu{}_\nu a^\nu$
8. $a^\mu \perp u^\mu$ for every point of timelike world line, $\eta_{\mu\nu} a^\mu u^\nu = 0$
9. If $a^\mu = 0$ in one ref. frame, then it is in all ref. frames if transformation is linear.
10. Invariant mass m_0 is equal to the mass in the rest frame of the point mass.
11. Relativistic mass $m = m_0 \gamma$.
12. Conservation of **3-dim** lin. momentum works the same way as in classical mechanics, $\sum_{before} m_i v_i = \sum_{after} m_i v_i$, but with relativistic masses and with relat. addition of velocities.
13. Inavriant mass of resultant body after inealsitc collision is bigger then invariant masses of coliding bodies.
14. 4-momentum: $p^\mu = m_0 u^\mu = (c, \vec{p})^T$
15. 4-force: $F^\mu = \frac{dp^\mu}{d\tau} = \gamma \frac{dp^\mu}{dt} = \gamma \left(c\frac{dm}{dt}, \vec{f} \right)^T$, where $\vec{f} = \frac{d\vec{p}}{dt}$ is 3-force.
16. $\frac{dp^\mu}{d\tau} = \frac{dm_0}{d\tau} u^\mu + m_0 a^\mu$
17. 3-force: $\frac{d\vec{p}}{dt} = m\vec{a} + \left(m_0 \gamma^2 \frac{\vec{v}\cdot\vec{a}}{c^2} + \frac{dm_0}{dt} \right) \gamma \vec{v}$
18. $\frac{d\gamma}{dt} = \gamma^3 \frac{\vec{v}\cdot\vec{a}}{c^2}$
19. Mag. field (Lorenz force) $\vec{v} \cdot \vec{a} = 0$ and $\frac{dm_0}{dt} = 0$.
20. 4-forces for which $\frac{dm_0}{dt} = 0$ are spacelike.
21. If $dm_0/dt = 0$ then $\vec{f}_\perp = m\vec{a}_\perp$ and $\vec{f}_\parallel = m\gamma^2 \vec{a}_\parallel$.
22. $\eta_{\mu\nu} F^\mu u^\nu = -c^2 dm_0/d\tau$.
23. $c^2 dm = \frac{1}{\gamma} c^2 dm_0 + \vec{f} \cdot d\vec{r}$.
24. $E = mc^2 = m_0 c^2 + T = \sqrt{(m_0 c^2)^2 + (pc)^2}$, where $p = |\vec{p}|$.
25. Mass shell (3-dim hyperboloid): $m^2 c^2 - p^2 = m_0^2 c^2$
26. If $p^2 \ll m_0^2 c^2$ then $E = m_0 c^2 + p^2/2m_0$.
27. For masseles objects $|\vec{v}| = c$ and $E = T = pc$, or in wave-language $E = \hbar\omega$.
28. Covariant rel. for E
29. $v > c$, $v = c$, $v < c$ is invariant.

V. ELECTRODYNAMICS IN VACUUM

1. Charge density $\rho = \rho_0 \gamma$
2. 4-current: $J^\mu = \left(\rho c, \vec{J} \right)^T = \rho_0 u^\mu$, where $\vec{J} = \rho \vec{v}$
3. $\eta_{\mu\nu} J^\mu J^\nu = -c^2 \rho_0^2$.
4. Gradient: $\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla \right)^T$, $\partial^\mu = \frac{\partial}{\partial x_\mu}$, $\partial_\mu = \frac{\partial}{\partial x^\mu}$, $\partial_\mu \partial^\nu = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$.
5. 4-potential: $A^\mu = \left(\frac{\varphi}{c}, \vec{A} \right)^T$.
6. EM-field tensor: $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ or $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.
7. $F_{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & B_z & -B_y \\ E_y/c & -B_z & 0 & B_x \\ E_z/c & B_y & -B_x & 0 \end{pmatrix}$
Rising indicies just swaps the signs of zeroth row and column.
8. Continuity eq.: $\partial_\mu J^\mu = 0$.
9. 1st. series of Maxwell eq. $\partial_\nu F^{\mu\nu} = \mu J^\mu$.
10. 2st. series of Maxwell eq. $\partial_{[\alpha} F_{\beta\gamma]} = 0$.
11. Maxwell eqs. are invariant under any gauge transformation $\tilde{A}_\mu = A_\mu + \partial_\mu \chi$.
12. Lorenz gauge: $\partial_\mu \tilde{A}^\mu = 0$, hence $\partial_\mu \partial^\mu \chi = -\partial_\mu A^\mu$.
13. In Lorenz gauge 1st. s. of Maxwell eq. gives us wave equation $\partial_\mu \partial^\mu A^\nu = -\mu J^\nu$.
14. $\partial_\mu \partial^\mu F_{\alpha\beta} = \mu \left(\partial_\beta J_\alpha - \partial_\alpha J_\beta \right)$
15. Hodge dual elmag. tensor: $(*F)^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$.
16. $(*F)^{\mu\nu} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z/c & -E_y/c \\ B_y & -E_z/c & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{pmatrix}$
17. signs notation chcek $F_{\rho\sigma} F^{\rho\sigma} = 2B^2 - 2E^2/c^2$, $(*F)_{\rho\sigma} (*F)^{\rho\sigma} = -2B^2 + 2E^2/c^2$, $F_{\rho\sigma} (*F)^{\rho\sigma} = \frac{4}{c} \vec{E} \cdot \vec{B}$.
18. signs notation chcek Using notation where Levi-Civita symbol has indicies up ϵ^{ijkl}, $*F$ will have all signs opposite and $F_{\rho\sigma} (*F)^{\rho\sigma} = -\frac{4}{c} \vec{E} \cdot \vec{B}$.

OO. SOME OTHER SHIT

1. Thomas precession
2. Relativistic disk
3. Bell's spaceship paradox
4. Ehrenfest paradox