Problems in mechanics

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Obsah

1 Problems

1.1 Statics

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1.2 Dynamics

1. Point mass m is repeled by force F, proportional to the third power of distance from the origin of space coordinate x. Solve the motion of the point mass, if it is released from rest from the point x_0.

2. Projectil of mass m is shooted perpendiculary up in the gravitational field of earth. Find all possible motions of the projectil depending of its initial energy. Neglect air resistance.

3. Boat of mass m is moving with the speed v_0 and stop the engine. Find time t_s, after which the boat stops, and the distance x_s it traveled, if it is decelerating by the force $F = -b\exp(av)$ where a and b are possitive constants.

4. Find the motion of rope, which is unraveling without friction at $x = 0$ and on which end at x is acting constant force F along possitive direction of axis x. Initial velocity and legnth of rope are $v_0 > 0$ and $x_0 > 0$.

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1.3 Oscillations and waves

1. A point mass m is attached to a string with spring constant k and rest legth x_0. The force in the string is $F = -k(x - x_0)$. Show that point mass undergoes harmonic oscillations.

2. Show that mathematical pendulum does not perform harmonic oscillations. Find it's period.

3. Point mass undergoes motion in a smooth potential $V(x)$. Find the frequuency of small oscillations about any equilibrium position. When are oscillations harmonic?

4. Solve the motion of small object falling through the diameter of earth. Consider denisity, radius R and mass M of earth to be constant.

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1.4 Lagrangian mechanics

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1.5 Hamiltonian mechanics

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2 Answers

2.1 Statics

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2.2 Dynamics

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$$x(t) = \sqrt{x_0^2 + \frac{kt^2}{mx_0^2}}$$

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$$\begin{aligned} \text{for} E < 0 \quad x &= \frac{GmM}{2|E|}(1 - \cos\tau) \\ t - t_0 &= \frac{GmM}{4|E|}\sqrt{\frac{2m}{|E|}}(\tau - \sin\tau) \\ \text{for} E > 0 \quad x &= \frac{GmM}{2E}(\cosh\tau - 1) \\ t - t_0 &= \frac{GmM}{4E}\sqrt{\frac{2m}{E}}(\sinh\tau - \tau) \end{aligned}$$

3.

$$\begin{aligned} t_s &= \frac{m}{ab}\left(1 - e^{-av_0}\right) \\ x_s &= \frac{m}{a^2b} = \frac{m}{a^2b}\left[1 - (1 + av_0)\, e^{-av_0}\right] \end{aligned}$$

4.

$$x(t) = \sqrt{\frac{F}{\rho}t^2 + 2x_0v_0t + x_0^2}$$

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2.3 Oscillations and waves

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$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\varphi_0}{2} + \frac{9}{64}\sin^4\frac{\varphi_0}{2} + \dots\right)$$

3.

$$\omega = \sqrt{\frac{1}{m}\frac{\mathrm{d}^2V}{\mathrm{d}x^2}\Big|_{x_0}}$$

All derivatives higher then second are equal to zero.

4.

$$x(t) = R\cos(\omega t) \quad \text{where } \omega = \sqrt{GM/R^3}$$

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2.4 Lagrangian mechanics

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2.5 Hamiltonian mechanics

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