

# Derivative

**Julia:** Ah, this sucks!

**Dylan:** What's up?

**Julia:** I'm supposed to find the slope of a parabola at a point, and I'm not sure how!

**Dylan:** Well if we had two points we could make a secant line to approximate it!

**Julia:** Secant line? What's that?

**Dylan:** A *secant line* is just a line which connects two points on a function!

**Julia:** But isn't the *tangent* line one that skims a curve at one point? So the slope of the tangent line is the slope at that point! See?

$$\text{Graph of } f(x) = x^2, g(x) = 2(x - 1) + 1$$

**Dylan:** Well do you know how to find the equation for a line with just one point?

**Julia:** ...

**James:** Come on guys we can approximate the tangent line using the secant line!

**Altogether:** Let's dive in!

## Guided Example

Consider the function  $f(x) = x^2$ . In green is  $g(x) = 4x - 4$  the tangent line at the point  $x = 2$ . Thus the slope of the tangent line at  $x = 2$  is 4.

$$\text{Graph of } f(x) = x^2, g(x) = 4x - 4$$

**Question 1** Find the slope of the secant line between  $x = 2$  and  $x = 7$ .

9

Does this seem to be a good approximation for the slope of the tangent line at  $x = 2$ ?

---

Learning outcomes:  
Author(s):

**Multiple Choice:**

- (a) Yes
- (b) No ✓

Dylan thinks we can solve the problem by just picking something closer than 7.  
Find the slope of the secant line between  $x = 2$  and  $x = 3$ .

5

Is this a good approximation for the slope of the tangent line at  $x = 2$ ?

**Multiple Choice:**

- (a) Yes
- (b) No ✓

Is it better than the last attempt?

**Multiple Choice:**

- (a) Yes ✓
- (b) No

**Julia:** Dylan, this still isn't a great approximation...

**Dylan:** Well, I think we need to get even closer. Like, infinitesimally close!  
But how would we do that....

**James:** You guys need some help?

**Julia and Dylan:** James! How do we find the slope of a line at a point?

**James:** It isn't too tough! Before, you were considering a certain point as your comparison. What if instead, you used the point you want to evaluate at plus something really small? Let's call it  $h$ .

**Question 2** How can you make the  $h$  in

$$\frac{f(2+h) - f(2)}{(2+h) - 2}$$

approach 0?

**Multiple Choice:**

- (a) Use  $\lim_{h \rightarrow 0}$ . ✓
- (b) Use  $\lim_{h \rightarrow \infty}$ .
- (c) Divide the fraction by  $h$ .
- (d) Pick a function  $f(x)$  so that  $f(2)$  is 0.

Using the method you determined, approximate the slope of the tangent line at the point  $x=2$ .

4

**James:** Want to know something really cool?

**Julia and Dylan:** What James?

**James:** The function we just discovered is how you determine a function's derivative! Using that process, you can find the instantaneous rate of change at any point on a function!

**Julia and Dylan:** Wow! So cool!

## On Your Own

Using what you've learned, find the derivative of the following functions at the given point.

**Question 1 Hint:** Remember you can enter  $\sqrt{x}$  as either

`sqrt(...)` or `(...)^{(1/2)}`

$$g(x) = x^2 + 1, x = 2$$

4

$$h(x) = \frac{1}{x}, x = 2$$

-0.25

$$f(x) = 3x^2 + 4x + 2, x = -1$$

-2

$$f(x) = \sqrt{x^2 + 1}, x = 3$$

$$\frac{3}{\sqrt{10}}$$

$$f(x) = x + x^{-1}, x = 4$$

$$15/16$$

By replacing the point in our formula for the derivative with  $x$ , we may determine the derivative at any point on the function. Determine the derivative for the following functions.

**Question 2**  $m(x) = x^3$

$$3x^2$$

$$n(x) = 3x + 2$$

$$3$$

$$f(x) = 4 - x^2$$

$$-2x$$

$$f(x) = 12 + 7x$$

$$7$$

$$f(x) = \frac{4}{x + 1}$$

$$\frac{-4}{(x + 1)^2}$$

## In Summary

**Julia:** So why is it called a secant line?

**James:** It comes from the Latin word *secare*, which means 'to cut'.

**Dylan:** Ohh, I get it now! Because a secant line is a line that 'cuts' a function!

In this lab we've (hopefully) learned the function for finding a derivative using the limit as  $h$  approaches 0. We also learned what secant and tangent lines are. For your convenience, the important definitions are listed below.

**Definition 1.** A *secant line* is any line that connects any two points on a curve.

**Definition 2.** A *tangent line* is a line which intersects a differentiable curve at a point where the slope of the curve equals the slope of the line.

**Definition 3.** The *derivative* of  $f(x)$  at  $a$  is defined by the following limit:

$$\left[ \frac{d}{dx} f(x) \right]_{x=a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}.$$