

Transformations of Functions

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1  caseInsensitive = function(a,b) {
2      return a.toLowerCase() == b.toLowerCase();
3  };

```

Julia: Ugh!

Dylan: What's up Julia?

Julia: I have these functions I have to graph, and they're *so* close to functions I know really well, but they're a little bit different and it makes it so I have to calculate a bunch of points before I can confidently graph it!

James: Sounds like you could use some help Julia!

Julia and Dylan: James!

James: There are a ton of ways to transform functions, so let's get going and look at how we can modify our favorite functions!

Introduction

While you work with many different functions, there are a few basic types of functions. These include polynomials, rational functions, trigonometric functions, exponential functions, and logarithmic functions. In this lab we will explore different variations on these basic functions called **transformations**.

Guided Example

Consider the function $f(x) = x^2$.

Graph of $f(x) = x^2$

Learning outcomes:
Author(s):

Question 1 On the same axis graph $g(x) = f(x) + 2$. What change happened from $f(x)$ to $g(x)$?

The graph shifted units .

What can you infer about $f(x) - 2$?

The graph would shift units .

Consider the function $f(x + 2) = (x + 2)^2$. How do you think this graph will be different from the graph of $f(x)$?

Free Response:

Graph the function $f(x + 2)$ in the desmos window above, was your prediction correct? What can you infer about the function $f(x - 2)$? Graph this function to verify your prediction.

Free Response:

What rule can you write about a general function $f(x + c)$ where c is a positive constant? The function will shift units .

Why do you think the graph moves in the direction it does when using the rule you determined in the last question? Hint: Think about the x -intercept and how it changes when you add or subtract a constant from the x value

Free Response:

How do you think the graph of $f(x) = x^2$ be affected when you multiply the whole function by some constant c ?

Free Response:

Graph the function $c \cdot f(x)$ for the following values of $c = 2, \frac{1}{2}, -2, \frac{-1}{2}$

Graph of

Describe what is happening to the function based on the value of c , what can you generalize from this? It may be helpful to make a table with the x and y values to understand why this change happens.

Free Response:

On your own

Question 2 Using $f(x) = x^2$ as your base function create a new function that will shift the graph up 4 units, to the right 3 units, reflect it across the x-axis and stretch it vertically by a factor of 2 and graph it below

Graph of

Graph the function $f(2x)$

Graph of

What constant does this stretch or compress x^2 by?

Graph $f(2x + 6)$ on the same axis above, what transformation occurred?

Free Response:

Note the following expansion of the general function $f(x) = (ax + b)^2$:

$$f(x) = (ax + b)^2 = \left(a \left(x + \frac{b}{a} \right) \right)^2 = a^2 \left(x + \frac{b}{a} \right)^2$$

From this expansion, how is a function in the form $f(x) = (ax + b)^2$ being shifted and stretched/compressed in terms of a and b ?

Free Response:

In Summary

For the following questions, pick in which way the general graph $f(x)$ would change under certain transformations.

Question 3

$$c \cdot f(x)$$

When $c > 1$

Multiple Choice:

- (a) Shrink $f(x)$ vertically by c

- (b) Stretch $f(x)$ vertically by c ✓
- (c) Shrink $f(x)$ horizontally by c
- (d) Stretch $f(x)$ horizontally by c
- (e) Flip $f(x)$ over the x axis

When $c < -1$

Multiple Choice:

- (a) Flip $f(x)$ over the x axis
- (b) Shrink $f(x)$ horizontally by c
- (c) Flip $f(x)$ over the y axis and stretch horizontally by c
- (d) Flip $f(x)$ over the x axis and stretch vertically by c ✓
- (e) Flip $f(x)$ over the x axis and stretch horizontally by c

When $0 < c < 1$

Multiple Choice:

- (a) Stretch $f(x)$ horizontally by c
- (b) Shrink $f(x)$ vertically by c ✓
- (c) Shrink $f(x)$ horizontally by c
- (d) Stretch $f(x)$ horizontally by c
- (e) Flip $f(x)$ over the x axis

Question 4

$$f(x + c)$$

When $c > 0$

Multiple Choice:

- (a) Shift $f(x)$ left by $|c|$. ✓
- (b) Flip $f(x)$ over the x -axis.

- (c) Shift $f(x)$ right by $|c|$
- (d) Flip $f(x)$ over the x-axis and shift it up by $|c|$.
- (e) No change occurs to $f(x)$.

When $c < 0$

Multiple Choice:

- (a) Shift $f(x)$ left by $|c|$.
- (b) Flip $f(x)$ over the x-axis.
- (c) Shift $f(x)$ right by $|c|$ ✓
- (d) Flip $f(x)$ over the x-axis and shift it up by $|c|$.
- (e) No change occurs to $f(x)$.

When $c = 0$

Multiple Choice:

- (a) Shift $f(x)$ left by $|c|$.
- (b) Flip $f(x)$ over the x-axis.
- (c) Shift $f(x)$ right by $|c|$
- (d) Flip $f(x)$ over the x-axis and shift it up by $|c|$.
- (e) No change occurs to $f(x)$. ✓

Question 5

$$f(x) + c$$

When $c > 0$

Multiple Choice:

- (a) Shift $f(x)$ down by $|c|$.
- (b) Stretch $f(x)$ vertically by $|c|$.
- (c) Flip $f(x)$ over the x-axis.

- (d) Shift $f(x)$ up by $|c|$. ✓
- (e) No change will occur.

When $c = 0$

Multiple Choice:

- (a) Shift $f(x)$ down by $|c|$.
- (b) Stretch $f(x)$ vertically by $|c|$.
- (c) Flip $f(x)$ over the x -axis.
- (d) Shift $f(x)$ up by $|c|$.
- (e) No change will occur. ✓

When $c < 0$

Multiple Choice:

- (a) Shift $f(x)$ down by $|c|$. ✓
 - (b) Stretch $f(x)$ vertically by $|c|$.
 - (c) Flip $f(x)$ over the x -axis.
 - (d) Shift $f(x)$ up by $|c|$.
 - (e) No change will occur.
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