An R Companion to James Hamilton's Times Series Analysis

with R

Robert Bell, Matthieu Stigler

Preliminary

Foreword

`RcompHam94` is an R package that implements many of the worked examples in `Time Series Analysis` (Hamilton, 1994) as well as providing access to the code and datasets used. In many cases `RcompHam94` provides both simplified implmentations "from scratch" to allow the reader to explore the underlying logic and calculations, and more realistic implementations that make use of the large body of contributed packages in the Comprehensive R Archive Network (CRAN). Thus readers who have cut their teeth on the textbook can use this package as a stepping stone to doing their own analysis and/or research. Readers looking for additional introductory treatment of facilities available in CRAN can explore other excellent introductions such as `http://cran.r-project.org/doc/contrib/Farnsworth-EconometricsInR.pdf` and `http://cran.r-project.org/web/packages/AER/AER.pdf` (Kleiber and Zeileis, 2008).

We assume the reader has downloaded the R language, and package "RcompHam94" from `http://www.r-project.org/` and has read "An Introduction to R" available here `http://cran.r-project.org/doc/manuals/R-intro.html` and also available as a PDF from the "Help" menu of the R package.

To load the package, just use:

```R
library("RcompHam94")
```

Code shown in this document (and some not shown for brevity) can be executed using the R "demo" function. For a list of available demos, use:

```R
demo(package = "RcompHam94")
```

To invoke a specific demo, say the demo called "p112", use:

```R
demo(topic = "p112", package = "RcompHam94")
```

In general the demos are written so that the results of individual calculations can be examined after the fact by examining variables containing the results of those calculations.

Page references in the body of this document refer to `Time Series Analysis`.

Contents

1 Difference Equations

1.1 Dynamic Multipliers for First Order Difference Equations

Page 3 describes calculations for dynamic multipliers for first order difference equations. An example of these calculations in action is given on page 4. A simple method to calculate dynamic multipliers is to simulate the difference equation calculating forward based on an initial shock at time t=1, assuming the value of y at time 0 is 0. R indexes arrays starting at 1 instead of 0, so subscripts are one more than the convention used in the text, meaning that the shock will be said to occur at time 2.

```
―――――――――――――――――――――――― R code ―――――――――――――――――――――――――
> T <- 20
> w <- 1 * (1:T == 2)
```

In the examples shown on page 4 there are actually four different equations being simulated, so we will use a matrix, rather than a vector, to store the results.

```
―――――――――――――――――――――――― R code ―――――――――――――――――――――――――
> phis <- c(0.8, -0.8, 1.1, -1.1)
> y <- array(dim = c(T, length(phis)))
> y[1, ] <- rep(0, length(phis))
> for (j in 2:T) y[j, ] <- phis * y[j - 1, ] + w[j]
```

We can check this calculation against the closed form expression on page 3.

```
―――――――――――――――――――――――― R code ―――――――――――――――――――――――――
> print(y[2:T, 1])
```

```
――――――――――――――――――――――――― output ―――――――――――――――――――――――――
 [1] 1.00000000 0.80000000 0.64000000 0.51200000 0.40960000 0.32768000
 [7] 0.26214400 0.20971520 0.16777216 0.13421773 0.10737418 0.08589935
[13] 0.06871948 0.05497558 0.04398047 0.03518437 0.02814750 0.02251800
[19] 0.01801440
```

```
―――――――――――――――――――――――― R code ―――――――――――――――――――――――――
> print(phis[[1]]^seq(0, T - 2))
```

```
――――――――――――――――――――――――― output ―――――――――――――――――――――――――
 [1] 1.00000000 0.80000000 0.64000000 0.51200000 0.40960000 0.32768000
 [7] 0.26214400 0.20971520 0.16777216 0.13421773 0.10737418 0.08589935
```

```
[13] 0.06871948 0.05497558 0.04398047 0.03518437 0.02814750 0.02251800
[19] 0.01801440
```

Finally we can plot the results using a histogram plot reproducing figure 1.1.
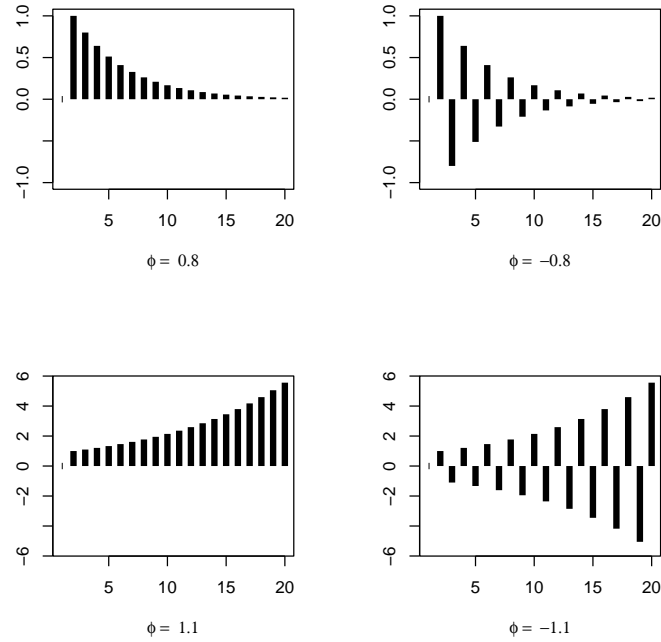
1.2 Comparing Transitory Versus Permanent Changes

The above example examined the effect changing ϕ on the dynamic multiplier. Pages 5 and 6 describe what happens when the permanence of the change is varied with a fixed multiplier, i.e. while leaving ϕ unchanged.

```
────────────────────────────── R code ──────────────────────────────
> phi <- 0.8
> T <- 20
> w <- 1 * cbind(1:T == 6, 1:T >= 6)
> y <- array(dim = c(T, 2))
> y[1:5, ] <- 0
> for (j in 6:T) y[j, ] <- phi * y[j - 1, ] + w[j, ]
```

The results can be plotted reproducing figures 1.2 and 1.3.
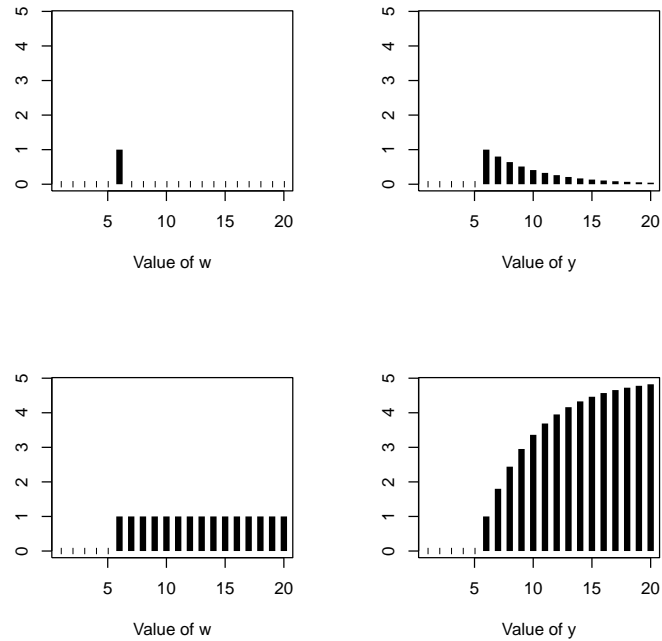
1.3 Dynamic Multipliers for Second Order Difference Equations

Finally we use similar techniques to calculate the effects of an impulse on a second order system. Here each column of phi represents the coefficients of a second order system.

```
―――――――――――― R code ――――――――――――
> T <- 20
> w <- 1 * (1:20 == 3)
> y <- array(dim = c(T, 2))
> y[1:2, ] <- 0
> phi <- array(c(0.6, 0.2, 0.5, -0.8), c(2, 2))
> for (j in 3:T) y[j, ] <- apply(X = phi * y[(j - 1):(j - 2), ],
+     MARGIN = 2, FUN = sum) + w[j]
```

The results can be plotted reproducing figure 1.4.

6
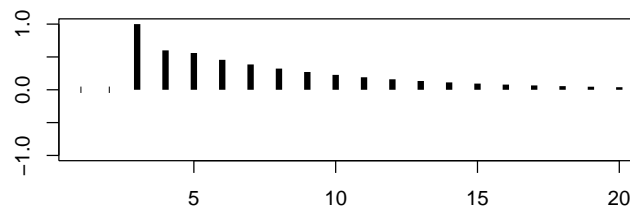
$\phi 1 = 0.6, \phi 2 = 0.5$
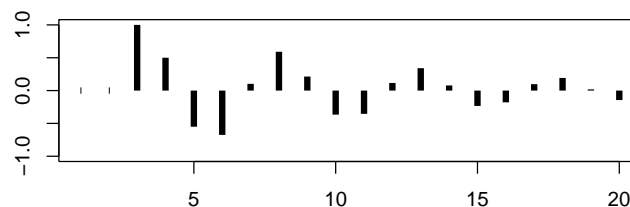
$\phi 1 = 0.2, \phi 2 = -0.8$

2 Stationary ARMA Processes

2.1 Autocorrelations for AR and MA Processes

Pages 50 to 59 describe the calculation of autocorrelation functions of AR and MA processes. Following the expressions in the text we can calculate results using separate formulae for white noise, moving average, and autoregressive processes.

```
―――――――――――――――――― R code ――――――――――――――――――
> T <- 20
> specifications <- list(list(label = "White Noise", MA = vector(mode = "numeric"),
+     AR = vector(mode = "numeric")), list(label = "MA(1)", MA = c(0.8),
+     AR = vector(mode = "numeric")), list(label = "MA(4)", MA = c(-0.6,
+     0.5, -0.5, 0.3), AR = vector(mode = "numeric")), list(label = "AR(1) with 0.8",
+     MA = vector(mode = "numeric"), AR = c(0.8)), list(label = "AR(1) with -0.8",
+     MA = vector(mode = "numeric"), AR = c(-0.8)))
> sigmasq <- 1
```

White noise calculations are described on bottom of page 47 and the top of page 48.

```
────────────────────── R code ──────────────────────
> specifications[[1]]$rho <- c(1, rep(0, T - 1))
```

Moving average calculations are described on page 51.

```
────────────────────── R code ──────────────────────
> for (i in 2:3) {
+     MA <- specifications[[i]]$MA
+     q <- length(MA)
+     gamma <- vector(mode = "numeric", length = T)
+     gamma[1] <- sigmasq * t(c(1, MA)) %*% c(1, MA)
+     for (j in 1:q) gamma[j + 1] <- sigmasq * t(MA[j:q]) %*% c(1,
+         MA)[1:(q - j + 1)]
+     gamma[(q + 2):T] <- 0
+     specifications[[i]]$rho <- gamma/gamma[1]
+ }
```

Autocorrelation calculations are described on page 59

```
────────────────────── R code ──────────────────────
> for (i in 4:5) {
+     AR <- specifications[[i]]$AR
+     p <- length(AR)
+     F <- rbind(AR, cbind(diag(p - 1), rep(0, p - 1)))
+     gamma <- vector(mode = "numeric", length = T)
+     gamma[1:p] <- sigmasq * solve(diag(p^2) - F %x% F)[1:p, 1]
+     for (j in (p + 1):T) gamma[[j]] <- t(gamma[(j - 1):(j - p)]) %*%
+         AR
+     specifications[[i]]$rho <- gamma/gamma[1]
+ }
```

White Noise

MA(1)

MA(4)

AR(1) with 0.8

AR(1) with −0.8
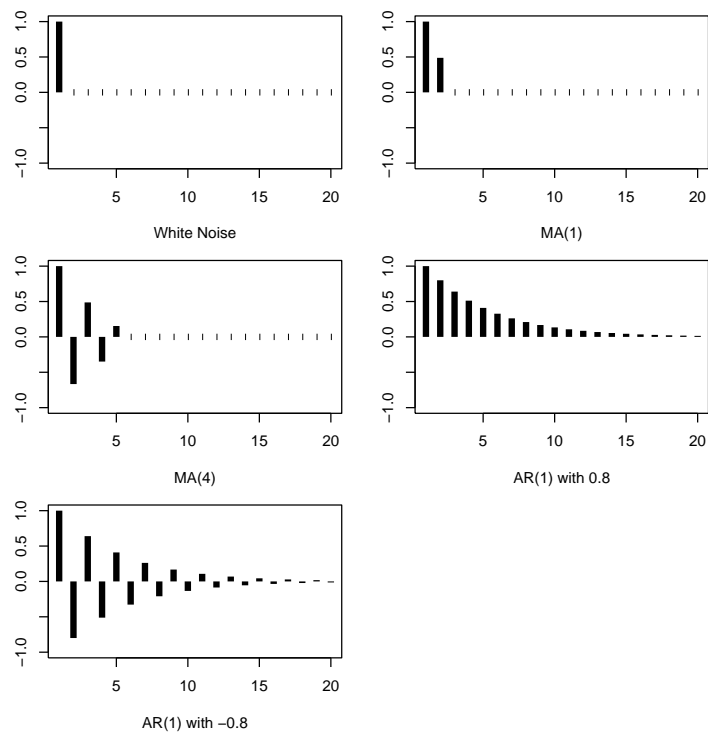
2.2   R Facilities for ARMA Autocorrelations

Function ARMAacf can be used to calculate autocorrelations for an arbitrary ARMA process.

```
──────────────────── R code ────────────────────
> g3 <- ARMAacf(ar = numeric(0), ma = specifications[[3]]$MA, lag.max = T,
+     pacf = FALSE)
> print(specifications[[3]]$rho)
```

```
──────────────────── output ────────────────────
 [1]   1.0000000  -0.6666667   0.4871795  -0.3487179   0.1538462   0.0000000
 [7]   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000
[13]   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000
[19]   0.0000000   0.0000000
```

```
──────────────────── R code ────────────────────
> print(g3)
```

```
──────────────────── output ────────────────────
         0           1           2           3           4           5           6
 1.0000000  -0.6666667   0.4871795  -0.3487179   0.1538462   0.0000000   0.0000000
```

9

```
         7           8           9          10          11          12          13
 0.0000000   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000
        14          15          16          17          18          19          20
 0.0000000   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000   0.0000000
```

```
──────────────────────────── R code ────────────────────────────
> g4 <- ARMAacf(ar = specifications[[4]]$AR, ma = numeric(0), lag.max = T -
+     1, pacf = FALSE)
> print(specifications[[4]]$rho)
```

```
──────────────────────────── output ────────────────────────────
 [1]  1.00000000  0.80000000  0.64000000  0.51200000  0.40960000  0.32768000
 [7]  0.26214400  0.20971520  0.16777216  0.13421773  0.10737418  0.08589935
[13]  0.06871948  0.05497558  0.04398047  0.03518437  0.02814750  0.02251800
[19]  0.01801440  0.01441152
```

```
──────────────────────────── R code ────────────────────────────
> print(g4)
```

```
──────────────────────────── output ────────────────────────────
         0           1           2           3           4           5           6
1.00000000  0.80000000  0.64000000  0.51200000  0.40960000  0.32768000  0.26214400
         7           8           9          10          11          12          13
0.20971520  0.16777216  0.13421773  0.10737418  0.08589935  0.06871948  0.05497558
        14          15          16          17          18          19
0.04398047  0.03518437  0.02814750  0.02251800  0.01801440  0.01441152
```

2.3 Autocorrelations as a Function of the Moving Average Parameter

Figure 3.2 is easily generated from the formula for autocorrelations of an MA(1) process.

```
──────────────────────────── R code ────────────────────────────
> theta <- (-300:300) * 0.01
> corrs <- theta/(1 + theta^2)
> plot(theta, corrs, type = "l")
> grid(nx = 2, ny = 2)
```

10
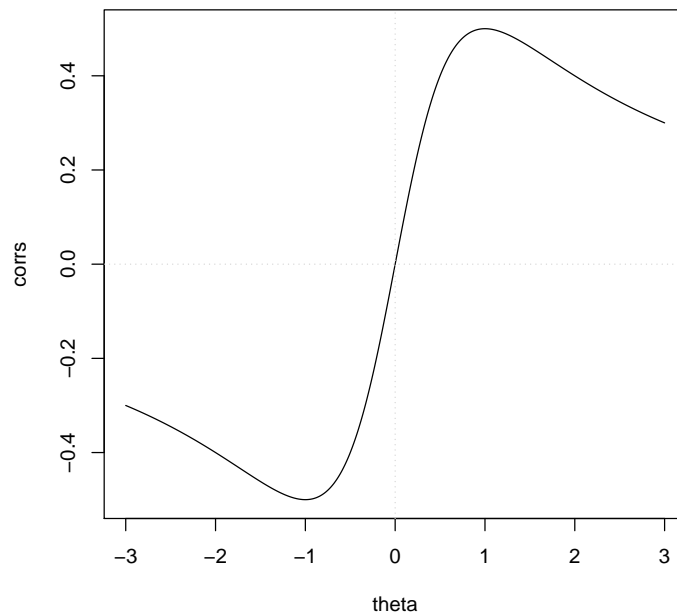
2.4   Realizations of ARMA Processes

Pages 55 shows some realizations of AR processes. We will assume the innovations are drawn from a standard normal distribution.

```
──────────────── R code ────────────────
> specifications <- list(list(label = "f = 0", MA = vector(mode = "numeric"),
+     AR = vector(mode = "numeric")), list(label = "f = .5", MA = vector(mode = "numeric"),
+     AR = c(0.5)), list(label = "f = .9", MA = vector(mode = "numeric"),
+     AR = c(0.9)))
> T <- 100
> set.seed(123)
> epsilon <- rnorm(T, 0, 1)
```

These can be calculated by iterating forward on the defining equations.

```
──────────────── R code ────────────────
> simulate.forward <- function(specification, epsilon) {
+     T <- length(epsilon)
+     AR <- specification$AR
+     MA <- specification$MA
+     presample <- rep(0, max(length(AR), length(MA)))
+     epsilon <- c(presample, epsilon)
```

11

```
+        Y <- vector(mode = "numeric", length = T + length(presample))
+        Y[1:length(presample)] <- 0
+        for (i in (length(presample) + 1):(T + length(presample))) Y[i] <- epsilon[[i]] +
+            ifelse(length(AR) > 0, t(AR) %*% Y[(i - 1):(i - length(AR))],
+                0) + ifelse(length(MA) > 0, t(MA) %*% epsilon[(i -
+            1):(i - length(MA))], 0)
+        Y[(length(presample) + 1):(T + length(presample))]
+ }
> for (i in 1:length(specifications)) specifications[[i]]$Y <- simulate.forward(specifications[[i]],
+        epsilon)
```


$\phi = 0$

$\phi = .5$

$\phi = .9$
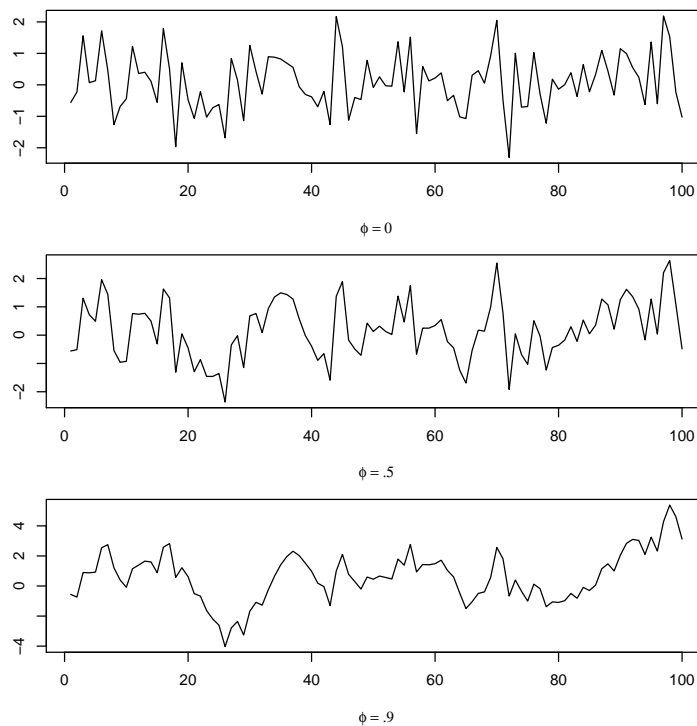
2.5   R Facilities for simulating ARMA process

Function "simulate.forward" is a special case of capabilities provided by the function arima.sim in package stats, as the following code verifies.

```
────────────────────────── R code ──────────────────────────
> for (specification in specifications) {
+        AR <- specification$AR
+        MA <- specification$MA
+        shift <- max(length(AR), length(MA))
+        Y <- arima.sim(model = list(order = c(length(AR), 0, length(MA)),
```

```
+          ar = AR, ma = MA), n = T, innov = epsilon[1:T], n.start = max(shift,
+          1), start.innov = rep(0, max(shift, 1)))
+      print(specification$Y[1:10])
+      print(Y[1:10])
+ }
```

```
──────────────────────── output ────────────────────────
 [1] -0.56047565 -0.23017749  1.55870831  0.07050839  0.12928774  1.71506499
 [7]  0.46091621 -1.26506123 -0.68685285 -0.44566197
 [1] -0.56047565 -0.23017749  1.55870831  0.07050839  0.12928774  1.71506499
 [7]  0.46091621 -1.26506123 -0.68685285 -0.44566197
 [1] -0.5604756 -0.5104153  1.3035007  0.7222587  0.4904171  1.9602735
 [7]  1.4410530 -0.5445347 -0.9591202 -0.9252221
 [1] -0.5604756 -0.5104153  1.3035007  0.7222587  0.4904171  1.9602735
 [7]  1.4410530 -0.5445347 -0.9591202 -0.9252221
 [1] -0.56047565 -0.73460557  0.89756330  0.87831536  0.91977156  2.54285939
 [7]  2.74948966  1.20947946  0.40167866 -0.08415118
 [1] -0.56047565 -0.73460557  0.89756330  0.87831536  0.91977156  2.54285939
 [7]  2.74948966  1.20947946  0.40167866 -0.08415118
```

4 Forecasting

4.1 A Box Jenkins Example

Example 4.1 from page 112 illustrates the Box-Jenkins approach based on autocorrelations. Here the data series is log changes of seasonally adjusted real US GNP from 1947 to 1988, available by simple transformations of the data in object "gnp1996". The data is prepared by selecting quarterly date from as shown, then computing the log of differences.

```
──────────────────────── R code ────────────────────────
> data(gnp1996, package = "RcompHam94")
> selection <- subset(gnp1996, Quarter >= "1947-01-01" & Quarter <=
+      "1988-10-01")
> y <- diff(log(selection$GNPH))
```

Page 110 shows how to compute sample autocorrelations - we will generate the first 20 to be used in plotting the results below.

```
──────────────────────── R code ────────────────────────
> max.lags <- 20
> T <- length(y)
```

```
> threshold <- 2/sqrt(T)
> gammas <- vector(mode = "numeric", length = max.lags + 1)
> gammas[[1]] <- 1/T * t(y - mean(y)) %*% (y - mean(y))
> for (j in 1:max.lags) gammas[j + 1] <- 1/T * t((y - mean(y))[(j +
+     1):T]) %*% (y - mean(y))[1:(T - j)]
> rhos <- gammas/gammas[[1]]
```

Page 111 shows how to compute sample partial autocorrelations.

```
──────────────────────────── R code ────────────────────────────
> subscripts <- outer(seq(1, max.lags), seq(1, max.lags), function(i,
+     j) {
+     abs(i - j)
+ })
> GAMMA <- array(gammas[as.vector(subscripts) + 1], c(max.lags,
+     max.lags))
> alphas <- vector(mode = "numeric", length = max.lags)
> for (m in 1:max.lags) alphas[m] <- solve(GAMMA[1:m, 1:m], gammas[2:(m +
+     1)])[[m]]
```

A plot of the outputs reproducing figure 4.2 is shown below. The source code
is provided in the demo.
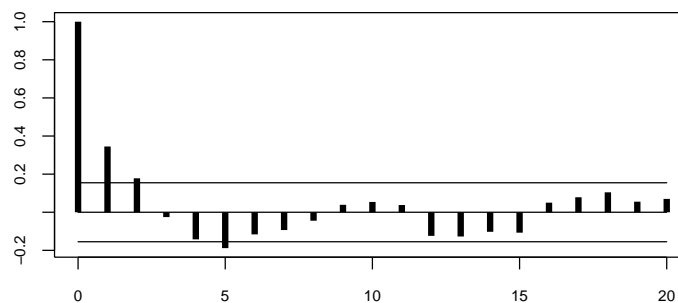
Figure 4.2(a) Sample autocorrelations
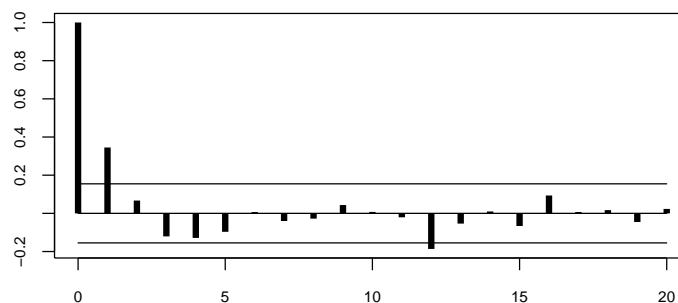
Figure 4.2(b) Sample partial autocorrelations

14

4.2 R Facilities for Sample Autocorrelations

Function acf from R package "stats" performs the same function as acf, as we can readily confirm.

```R
> acf.correlation <- acf(y, lag.max = max.lags, type = "correlation",
+     plot = FALSE, demean = TRUE)
> print(as.vector(acf.correlation$acf))
```

```
output
 [1]  1.00000000  0.34509475  0.17817758 -0.02537843 -0.14230681 -0.18827409
 [7] -0.11613672 -0.09335581 -0.04441490  0.03902657  0.05412612  0.03788102
[13] -0.12386994 -0.12725888 -0.10256196 -0.10719806  0.05022865  0.07874423
[19]  0.10451845  0.05540046  0.07001701
```

```R
> print(rhos)
```

```
output
 [1]  1.00000000  0.34509475  0.17817758 -0.02537843 -0.14230681 -0.18827409
 [7] -0.11613672 -0.09335581 -0.04441490  0.03902657  0.05412612  0.03788102
[13] -0.12386994 -0.12725888 -0.10256196 -0.10719806  0.05022865  0.07874423
[19]  0.10451845  0.05540046  0.07001701
```

```R
> acf.partial <- acf(y, lag.max = max.lags, type = "partial", plot = FALSE,
+     demean = TRUE)
> print(as.vector(acf.partial$acf))
```

```
output
 [1]  0.345094750  0.067075208 -0.120748043 -0.128609341 -0.096659383
 [6]  0.006935269 -0.040052970 -0.027544630  0.043507786  0.007543470
[11] -0.020592065 -0.186352407 -0.053599417  0.009939122 -0.066137883
[16]  0.093638650  0.007111983  0.016895000 -0.045185857  0.023227306
```

```R
> print(alphas)
```

```
output
 [1]  0.345094750  0.067075208 -0.120748043 -0.128609341 -0.096659383
 [6]  0.006935269 -0.040052970 -0.027544630  0.043507786  0.007543470
[11] -0.020592065 -0.186352407 -0.053599417  0.009939122 -0.066137883
[16]  0.093638650  0.007111983  0.016895000 -0.045185857  0.023227306
```

6 Spectral Analysis

Pages 167 to 170 give an example of the uses of spectral analysis, as applied to US Industrial Production from January 1947 to November 1989, available in data source "indprod". We will analyze the actual raw data, as well as one month and one year log changes.

```
─────────────────────────── R code ───────────────────────────
> data(indprod, package = "RcompHam94")
> selection <- subset(indprod, Month >= "1947-01-01" & Month <=
+      "1989-11-01")
> raw.data <- selection$IPMFG6
> logdiff.data <- 100 * diff(log(raw.data), lag = 1)
> yeardiff.data <- 100 * diff(log(raw.data), lag = 12)
```

For plotting purposes, generate frequencies at regular intervals as show on page 159. The first spectrum uses unsmoothed estimates, the last two use a Bartlett kernel.

We show this in two ways:

- Step by step function (page 16)

- Built-in function (page 17)

Step by step function

```
─────────────────────────── R code ───────────────────────────
> s.Y.omega <- function(omega, gammas, params) {
+      1/(2 * pi) * (gammas[[1]] + 2 * as.numeric(t(gammas[-1]) %*%
+          cos(1:(length(gammas) - 1) * omega)))
+ }
> s.Y.omega.Bartlett <- function(omega, gammas, params) {
+      1/(2 * pi) * (gammas[[1]] + 2 * as.numeric(t((1 - 1:params/(params +
+          1)) * gammas[2:(params + 1)]) %*% cos(1:params * omega)))
+ }
> generate.plot.data <- function(values, estimator, params) {
+      T <- length(values)
+      acf.covariance <- acf(values, lag.max = T - 1, type = "covariance",
+          plot = FALSE, demean = TRUE)
+      sapply(2 * pi/T * 1:((T - 1)/2), estimator, as.vector(acf.covariance$acf),
+          params)
+ }
> raw.s.Y.omega <- generate.plot.data(raw.data, s.Y.omega, NULL)
> logdiff.s.Y.omega <- generate.plot.data(logdiff.data, s.Y.omega.Bartlett,
```

```
+       12)
> yeardiff.s.Y.omega <- generate.plot.data(yeardiff.data, s.Y.omega.Bartlett,
+       12)
```

The resulting output is shown below.
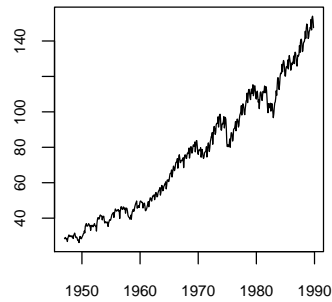
Figure 6.3 – FRB IP Index, NSA
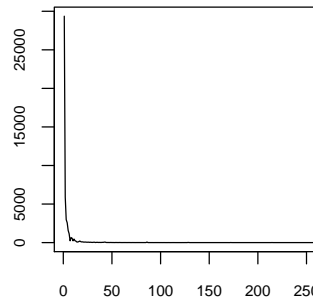
Figure 6.4 – Value of j
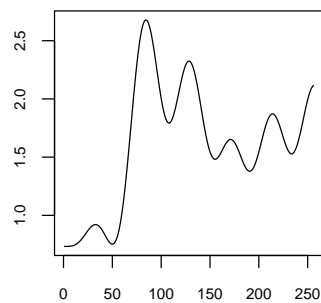
Figure 6.5 – Value of j
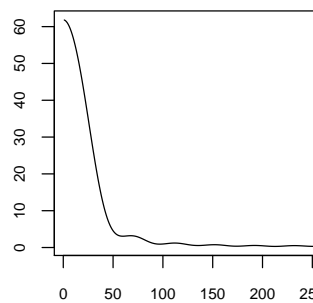
Figure 6.6 – Value of j

Built-in function   We use here the function spectrum:

─────────────────────────── *R code* ───────────────────────────
```
> args(spectrum)
```

─────────────────────────── output ───────────────────────────
```
function (x, ..., method = c("pgram", "ar"))
NULL
```

─────────────────────────── *R code* ───────────────────────────
```
> sp <- spectrum(raw.data, plot = FALSE, span = 10)
> x <- 100 * diff(log(raw.data))
> sp2 <- spectrum(x, span = 6, plot = FALSE)
> x12 <- 100 * diff(log(raw.data), lag = 12)
> sp3 <- spectrum(x12, span = 20, plot = FALSE)
```

17

The resulting output is shown below.

Figure 6.3 – FRB IP Index, NSA

Value of j
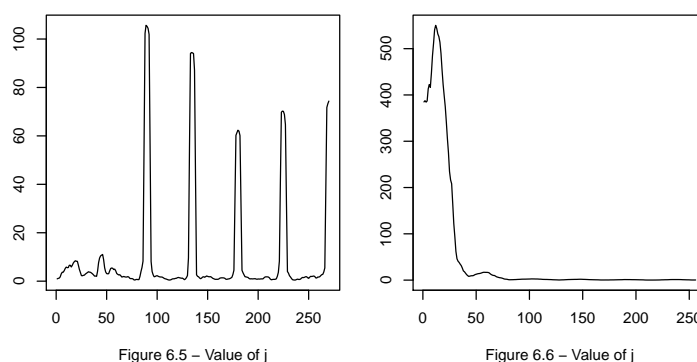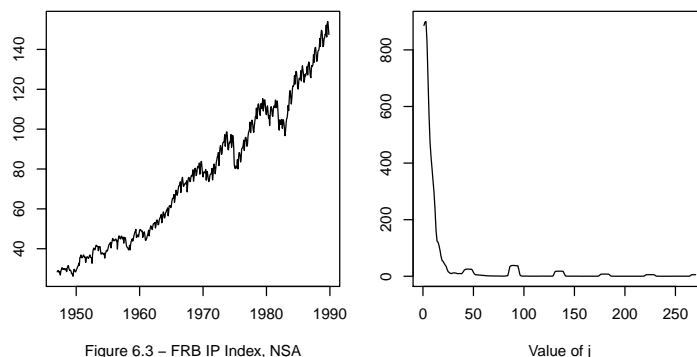
Figure 6.5 – Value of j

Figure 6.6 – Value of j

7   Asymptotic distribution theory

Further features in R

There exist a number of packages that illustrate interactively the central limit theorem or the law of large numbers, that can be useful to have an intuition on those theorems:

- Package `animation` has *clt.ani* and *lln.ani*

- Package `TeachingDemos` has *clt.examp*

- Package `ResearchMethods` has *cltDemo*

In addition, package `ConvergenceConcepts`, described in the R journal 1/2 2009 *enables one to investigate graphically the four classical modes of convergence of a sequence of random variables: convergence almost surely, convergence in probability, convergence in law and convergence in r-th mean.* (de Micheaux and Liquet (2009))

13 The Kalman Filter

13.1 Kalman Filtering Example Applied to Detecting Business Cycles

Page 376 describes an application of Kalman filtering to business cycles by James Stock and Mark Watson.

This can be implemented in two steps. The first is to implement the Kalman algorithm as described in the text. The following function generally follows the notation in Chapter 13, but several of the variables store the complete history of the iterations so that they include one more dimension. For example, rather than being a simple variable, the state vector is a vector of vectors (i.e. a two dimensionsal array) as shown.

```
─────────────────────── R code ───────────────────────
> xi.t.t_1 <- array(dim = c(length(xi.1.0), T + 1))
```

One minor difference is that the gain matrix as computed here does not include premultiplication by the F matrix as shown in [13.2.19] in the text.

```
─────────────────────── R code ───────────────────────
> K.t[, , tt] <- P.t.t_1[, , tt] %*% H %*% V
```

See also [13.2.16], [13.2.21] for updates of the second moments; [13.2.15], [13.2.17] for updates of the state vectors; [13.6.11], [13.6.16], [13.6.20] for computation of the smoothed inferences.

```
─────────────────────── R code ───────────────────────
> kalman <- function(H, R, F, x, A, y, Q, xi.1.0, P.1.0) {
+     T <- dim(x)[[2]]
+     P.t.t_1 <- array(dim = c(dim(P.1.0), T + 1))
+     P.t.t_1[, , 1] <- P.1.0
+     P.t.t <- array(dim = c(dim(P.1.0), T))
+     K.t <- array(dim = c(dim(H), T))
+     xi.t.t_1 <- array(dim = c(length(xi.1.0), T + 1))
+     xi.t.t_1[, 1] <- xi.1.0
+     xi.t.t <- array(dim = c(length(xi.1.0), T))
+     L <- 0
+     for (tt in 1:T) {
+         V <- solve(t(H) %*% P.t.t_1[, , tt] %*% H + R)
+         K.t[, , tt] <- P.t.t_1[, , tt] %*% H %*% V
+         P.t.t[, , tt] <- P.t.t_1[, , tt] - K.t[, , tt] %*% t(H) %*%
+             P.t.t_1[, , tt]
```

```
+          P.t.t_1[, , tt + 1] <- F %*% P.t.t[, , tt] %*% t(F) +
+              Q
+          w <- y[, tt] - t(A) %*% x[, tt] - t(H) %*% xi.t.t_1[,
+              tt]
+          xi.t.t[, tt] <- xi.t.t_1[, tt] + K.t[, , tt] %*% w
+          xi.t.t_1[, tt + 1] <- F %*% xi.t.t[, tt]
+          L <- L - 1/2 * dim(y)[[1]] * log(2 * pi) + 1/2 * log(det(V)) -
+              1/2 * t(w) %*% V %*% w
+      }
+      xi.t.T <- array(dim = c(length(xi.1.0), T))
+      xi.t.T[, T] <- xi.t.t[, T]
+      P.t.T <- array(dim = c(dim(P.1.0), T))
+      P.t.T[, , T] <- P.t.t[, , T]
+      for (tt in (T - 1):1) {
+          Jt <- P.t.t[, , tt] %*% t(F) %*% solve(P.t.t_1[, , tt +
+              1])
+          xi.t.T[, tt] <- xi.t.t[, tt] + Jt %*% (xi.t.T[, tt +
+              1] - xi.t.t_1[, tt + 1])
+          P.t.T[, , tt] <- P.t.t[, , tt] + Jt %*% (P.t.T[, , tt +
+              1] - P.t.t_1[, , tt + 1]) %*% t(Jt)
+      }
+      list(xi.t.t = xi.t.t, xi.t.t_1 = xi.t.t_1, P.t.t = P.t.t,
+          P.t.t_1 = P.t.t_1, K.t = K.t, log.likelihood = L, xi.t.T = xi.t.T,
+          P.t.T = P.t.T)
+ }
```

The second is to specify the state space model as described on pp376-377 and estimate the parameters via maximum likelihood. Data for this analysis is consumption and income data form dataset "coninc" in log differences.

```
─────────────────────── R code ───────────────────────
> data(coninc, package = "RcompHam94")
> YGR <- diff(log(as.vector(coninc[, "GYD82"])))
> CGR <- diff(log(as.vector(coninc[, "GC82"])))
> y <- t(cbind(YGR - mean(YGR), CGR - mean(CGR)))
```

The following helper function converts the parameters from a vector of labeled components into the correct inputs for the filter as shown in equations [13.1.28], [13.1.29], and [13.1.30].

```
─────────────────────── R code ───────────────────────
> THETA <- c(phic = 0.9, phi1 = 0.9, phi2 = 0.9, g1 = 0.5, g2 = 0.5,
+     sigc = 0.05^0.5, sig11 = 0.05^0.5, sig22 = 0.05^0.5, r11 = sd(YGR),
```

```
+      r22 = sd(CGR))
> theta.y.to.params <- function(THETA, y) {
+      params <- list(F = diag(THETA[c("phic", "phi1", "phi2")]),
+          Q = diag(THETA[c("sigc", "sig11", "sig22")]^2), H = rbind(THETA[c("g1",
+              "g2")], diag(2)), R = diag(THETA[c("r11", "r22")]^2),
+          A = diag(c(0, 0)), x = c(1, 1) %o% rep(1, dim(y)[[2]]),
+          xi.1.0 = c(0, 0, 0))
+      c(params, list(P.1.0 = array(solve(diag(length(params$xi.1.0)^2) -
+          params$F %x% params$F, as.vector(params$Q)), c(length(params$xi.1.0),
+          length(params$xi.1.0)))))
+ }
```

The objective function is the log.likelihood obtained from the Kalman iteration.

```
──────────────── R code ────────────────
> objective <- function(THETA, y) {
+      params <- theta.y.to.params(THETA, y)
+      kalman(params$H, params$R, params$F, params$x, params$A,
+          y, params$Q, params$xi.1.0, params$P.1.0)$log.likelihood
+ }
> optimizer.results <- optim(par = THETA, fn = objective, gr = NULL,
+      y = y, control = list(trace = 0))
```

Finally calculate the smoothed results based on the ML estimated parameters.

```
──────────────── R code ────────────────
> params <- theta.y.to.params(optimizer.results$par, y)
> smoothed.results <- kalman(params$H, params$R, params$F, params$x,
+      params$A, y, params$Q, params$xi.1.0, params$P.1.0)
> smoothed.data <- smoothed.results$xi.t.T[1, ]
```

The results of the smoothed inference are shown below.

CGR, YGR, and C

index(coninc)[−1]
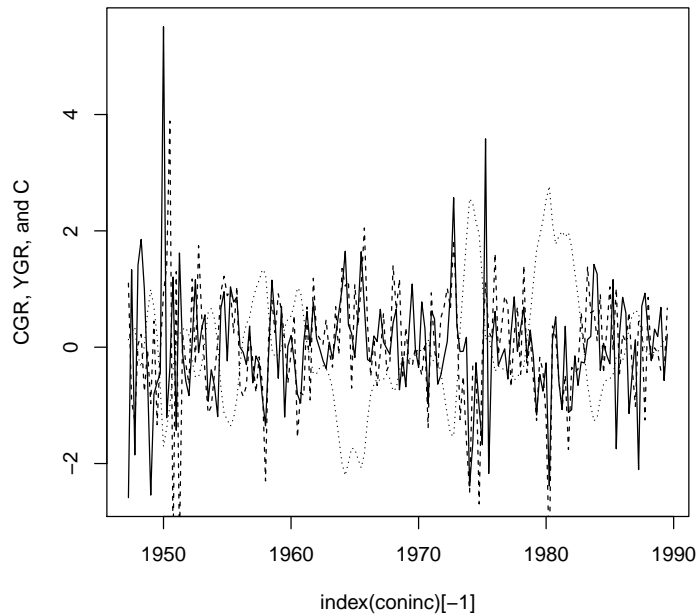
13.2 R facilities for Kalman Filtering

There are several different packages in R for Kalman filtering, some that provide univariate support, others multivariate support. For example, package `FKF` is a fast implementation, but there are others. One key aspect of using such packages is specifying an interface to allow for time varying inputs, and providing results under those conditions. Some packages use caller supplied functions, others check for dimensions of (up to three dimensional) arrays, etc.

For example, a simple implementation of the example on page 382 using function *kalman* above might look like:

```
―――――――――――――― R code ――――――――――――――
> sigmasq <- 2
> params <- list(F = array(c(0, 1, 0, 0), c(2, 2)), Q = diag(c(sigmasq,
+     0)), H = array(c(1, 0.8), c(2, 1)), R = array(0, c(1, 1)),
+     A = array(0.5, c(1, 1)), x = 1 %o% rep(1, 5), y = 1 %o% c(1,
+         seq(0.5, 4)), xi.1.0 = c(0, 0))
> params <- c(params, list(P.1.0 = array(solve(diag(length(params$xi.1.0)^2) -
+     params$F %x% params$F, as.vector(params$Q)), c(length(params$xi.1.0),
+     length(params$xi.1.0)))))
> myResults <- kalman(params$H, params$R, params$F, params$x, params$A,
+     params$y, params$Q, params$xi.1.0, params$P.1.0)
```

22

We can perform the some operations using package FKF with a slight alteration of the function arguments. In particular, many of the arguments using an outer product as a quick way to convert them into a structure of one additional dimension, with the length of the additional dimension being 1. This is a convenient calling convention to specifying a *non* time varying parameter. If the parameter *were* time varying then the full extra dimension would be used. For example, the F matrix can be time varying in FKF (called Tt). A call exploiting this would then have a vector of two dimensional F matrices, one for each time index, i.e. a three dimensional array. If F is not time varying, (as in the case of the simple example above) then a three dimensional array with the third dimension being of length 1 is used.

```R
> fkfResults <- FKF::fkf(a0 = params$xi.1.0, P0 = params$P.1.0,
+     dt = rep(0, length(params$xi.1.0)) %o% 1, Tt = params$F %o%
+         1, HHt = params$Q %o% 1, ct = t(params$A) %*% params$x,
+     Zt = t(params$H) %o% 1, GGt = params$R %o% 1, yt = params$y,
+     check.input = TRUE)
```

The results can be confirmed by examing the output:

```R
> print(myResults$xi.t.t)
```

```
          [,1]      [,2]        [,3]     [,4]     [,5]
[1,] 0.3048780 -0.19516  1.02502699 1.100137 2.031900
[2,] 0.2439024  0.24395 -0.03128374 1.124828 1.210125
```

```R
> print(fkfResults$att)
```

```
          [,1]      [,2]        [,3]     [,4]     [,5]
[1,] 0.3048780 -0.19516  1.02502699 1.100137 2.031900
[2,] 0.2439024  0.24395 -0.03128374 1.124828 1.210125
```

23

14   Generalized Method of Moments

14.1   Classical Method of Moments

Pages 409-410 gives a simple example of estimating the degrees of freedom of a
standard t distribution. To illustrate, first generate a sample of 500 observations
from a t distribution with 10 degrees of freedom.

```
────────────────────────────── R code ──────────────────────────────
> Y <- rt(500, 10)
```

Then maximize the sum of logs of a t density evaluated on the sample points.

```
────────────────────────────── R code ──────────────────────────────
> objective <- function(nu, Y) {
+     -sum(log(dt(Y, df = nu)))
+ }
> classical.results <- optimize(interval = c(1, 30), f = objective,
+     Y = Y)
> mu2 <- mean(Y^2)
> nu <- 2 * mu2/(mu2 - 1)
> print(classical.results)
```

```
────────────────────────────── output ──────────────────────────────
$minimum
[1] 12.30281

$objective
[1] 776.1968
```

```
────────────────────────────── R code ──────────────────────────────
> print(nu)
```

```
────────────────────────────── output ──────────────────────────────
[1] 8.861738
```

14.2   Generalized Method of Moments

Using the sample sample, we can estimate the degrees of freedom using GMM.
To this end define a function following the GMM recipe in the text.

```
────────────────────────────── R code ──────────────────────────────
> compute.estimates <- function(Y, h, interval) {
+     g <- function(Y, THETA) {
```

```
+            apply(X = apply(X = Y, MARGIN = 1, FUN = h, THETA = THETA),
+                MARGIN = 1, FUN = mean)
+        }
+        objective <- function(THETA, Y, W) {
+            g.value <- g(Y, THETA)
+            t(g.value) %*% W %*% g.value
+        }
+        r <- length(h(Y[1, ], interval[[1]]))
+        a <- length(interval[[1]])
+        T <- dim(Y)[[1]]
+        stage.1.results <- optimize(interval = interval, f = objective,
+            Y = Y, W = diag(r))
+        temp <- apply(X = Y, MARGIN = 1, FUN = h, THETA = stage.1.results$objective)
+        S <- 1/T * temp %*% t(temp)
+        stage.2.results <- optimize(interval = interval, f = objective,
+            Y = Y, W = solve(S))
+        J.test <- 1 - pchisq(T * stage.2.results$objective, r - a)
+        list(stage.1.results = stage.1.results, stage.2.results = stage.2.results,
+            overidentifying = J.test)
+ }
```

Using this function is then a matter of specifying an appropriate function h to
define an observation of the set of moments being targeted.

```
─────────────────────────── R code ───────────────────────────
> h <- function(Yt, THETA) {
+     nu <- THETA
+     c(Yt^2 - nu/(nu - 2), Yt^4 - 3 * nu^2/((nu - 2) * (nu - 4)))
+ }
> estimates <- compute.estimates(Y %o% 1, h, interval = c(5, 30))
> print(estimates)
```

```
─────────────────────────── output ───────────────────────────
$stage.1.results
$stage.1.results$minimum
[1] 13.04274


$stage.1.results$objective
           [,1]
[1,] 0.01223355
```

```
$stage.2.results
$stage.2.results$minimum
[1] 17.57763


$stage.2.results$objective
            [,1]
[1,] 0.007032297



$overidentifying
            [,1]
[1,] 0.06077352
```

A second example estimates the shape parameter of a two-sided gamma distribution.

```
────────────────────────── R code ──────────────────────────
> Yg <- rgamma(500, 10) * sign(runif(500, -1, 1))
> hg <- function(Yt, THETA) {
+     k <- THETA
+     nu <- k
+     mu <- k
+     sigma <- k
+     skew <- 2/sqrt(k)
+     kurt <- 6/k
+     c(Yt^2 - sigma - mu^2, Yt^4 - (kurt * (sigma^2) + 3) - 4 *
+         (skew * sigma^1.5) * mu - 6 * sigma * mu^2 - mu^4)
+ }
> gestimates <- compute.estimates(Yg %o% 1, hg, interval = c(5,
+     30))
> print(gestimates)
```

```
────────────────────────── output ──────────────────────────
$stage.1.results
$stage.1.results$minimum
[1] 9.968298


$stage.1.results$objective
         [,1]
[1,] 1.178662
```

```
$stage.2.results
$stage.2.results$minimum
[1] 9.932281


$stage.2.results$objective
             [,1]
[1,] 0.0004014549



$overidentifying
          [,1]
[1,] 0.6541343
```

14.3 R Facilities for Generalized Method of Moments

TBD

15 Models of Nonstationary Time Series

15.1 Fractional Integration

This example uses package package *fracdiff* to compute the exponent of fractional integration as described on pp 448-449. We use the function *fdGPH*:

────────────────────── R code ──────────────────────
```
> library(fracdiff)
> args(fdGPH)
```

────────────────────── output ──────────────────────
```
function (x, bandw.exp = 0.5)
NULL
```

Applied on US GDP and Treasury Yields data:

────────────────────── R code ──────────────────────
```
> data(gnptbill, package = "RcompHam94")
> print(fdGPH(log(gnptbill[, "GNP"])))
```

────────────────────── output ──────────────────────
```
$d
[1] 0.9832278
```

```
$sd.as
[1] 0.2427173


$sd.reg
[1] 0.04075541
```

```
> print(fdGPH(gnptbill[, "TBILL"]))
```

```
$d
[1] 0.9511594


$sd.as
[1] 0.2427173


$sd.reg
[1] 0.227921
```

17 Univariate Processes with Unit Roots

17.1 Preamble

This section uses a few utility functions that follow procedures in the text for testing hypotheses about unit roots. First is the Newey West estimator described by [10.5.10] and [10.5.15].

```
> print(Newey.West)
```

```
function (X, lags)
{
    S <- 0
    T <- dim(X)[[1]]
    for (lag in lags:1) S <- S + (lags + 1 - lag)/(lags + 1) *
        t(X[(lag + 1):T, ]) %*% X[1:(T - lag), ]
    1/T * (t(X) %*% X + S + t(S))
}
<environment: namespace:RcompHam94>
```

Next are the Dickey Fuller stats described in [17.4.7] and [17.4.9], with an optional correction for serial correlation defined in [17.7.35] and [17.7.38].

```R
──────────────────────────── R code ────────────────────────────
> print(Dickey.Fuller)
```

```
──────────────────────────── output ────────────────────────────
function (T, rho, sigma.rho, zeta = numeric(0))
{
    list(T = T, rho = rho, sigma.rho = sigma.rho, zeta = zeta,
        rho.stat = T * (rho - 1)/(1 - sum(zeta)), t.stat = (rho -
            1)/sigma.rho)
}
<environment: namespace:RcompHam94>
```

The Phillips Perron stats are defined by [17.6.8] and [17.6.12]

```R
──────────────────────────── R code ────────────────────────────
> print(Phillips.Perron)
```

```
──────────────────────────── output ────────────────────────────
function (T, rho, sigma.rho, s, lambda.hat.sq, gamma0)
{
    list(T = T, rho = rho, sigma.rho = sigma.rho, s.sq = s^2,
        lambda.hat.sq = lambda.hat.sq, gamma0 = gamma0, rho.stat = T *
            (rho - 1) - 1/2 * (T * sigma.rho/s)^2 * (lambda.hat.sq -
            gamma0), t.stat = (gamma0/lambda.hat.sq)^0.5 * (rho -
            1)/sigma.rho - 1/2 * (lambda.hat.sq - gamma0) * T *
            sigma.rho/s/(lambda.hat.sq^0.5))
}
<environment: namespace:RcompHam94>
```

Finally the Wald form of an F test as defined by [8.1.32].

```R
──────────────────────────── R code ────────────────────────────
> print(Wald.F.Test)
```

```
──────────────────────────── output ────────────────────────────
function (R, b, r, s2, XtX_1)
{
    v <- R %*% b - r
    as.numeric(t(v) %*% solve(s2 * R %*% XtX_1 %*% t(R)) %*%
        v/dim(R)[[1]])
```

29

```
}
<environment: namespace:RcompHam94>
```

For the following analyses we will use the R package dynlm which extends the formula language of the workhorse lm function of R to include constructs for expressing lags and differences. The raw data used is a series of treasury bill rates and real GNP. The GNP numbers are converted to logs and multiplied by 100 to get percentage growth rates, and we will use data from 1947:Q1 to 1989:Q1. Note that the text specifies a start date of 1947:Q2, but we include Q1 because it will be used in the lag calculation for the first "official" data point of Q2.

```R
————————————————————— R code —————————————————————
> data(gnptbill, package = "RcompHam94")
> dataset <- window(cbind(i = gnptbill[, "TBILL"], y = 100 * log(gnptbill[,
+       "GNP"]), tt = 1:dim(gnptbill)[[1]]), start = c(1947, 1),
+       end = c(1989, 1))
```

17.2   Dickey Fuller Tests for Unit Roots

Page 489 describes the analysis of nominal three month U.S. Treasury yield data from dataset gnptbill, shown below.
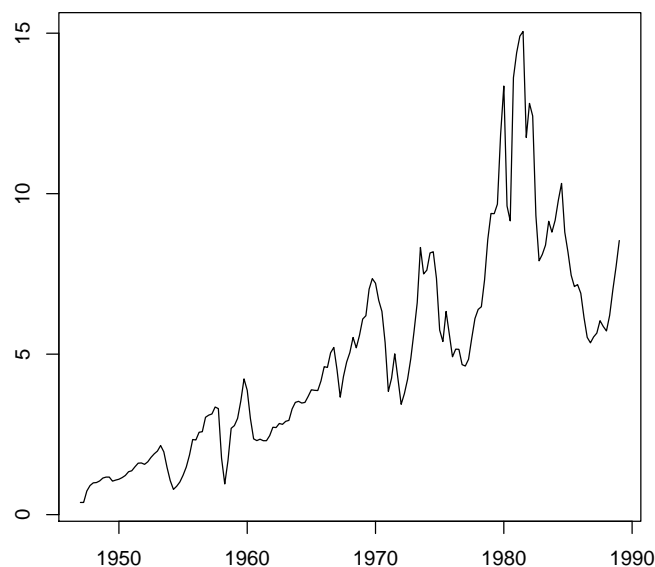
Figure 17.2 – Nominal Interest Rate

The regression model is shown in [17.4.13], and the results are shown below.

```R code
> case1.lms <- summary(dynlm(i ~ 0 + L(i), dataset))
> case1.DF <- Dickey.Fuller(T = length(case1.lms$residuals), rho = case1.lms$coefficients[["L(i)",
+     "Estimate"]], sigma.rho = case1.lms$coefficients[["L(i)",
+     "Std. Error"]])
> print(t(case1.lms$coefficients[, c("Estimate", "Std. Error"),
+     drop = FALSE]))
```

```output
                  L(i)
Estimate   0.99693575
Std. Error 0.01059183
```

```R code
> print(case1.DF)
```

```output
$T
[1] 168

$rho
[1] 0.9969357

$sigma.rho
[1] 0.01059183

$zeta
numeric(0)

$rho.stat
[1] -0.5147943

$t.stat
[1] -0.2893034
```

A similar analysis is described on page 494 , but a constant is included in the regression model [17.4.37].

```R code
> case2.lms <- summary(dynlm(i ~ 1 + L(i), dataset))
> case2.DF <- Dickey.Fuller(T = length(case2.lms$residuals), rho = case2.lms$coefficients[["L(i)",
```

```
+      "Estimate"]], sigma.rho = case2.lms$coefficients[["L(i)",
+      "Std. Error"]])
> print(t(case2.lms$coefficients[, c("Estimate", "Std. Error"),
+      drop = FALSE]))
```

────────────────────────── output ──────────────────────────
```
          (Intercept)       L(i)
Estimate    0.2105899 0.96691035
Std. Error  0.1121230 0.01913305
```

────────────────────────── R code ──────────────────────────
```
> print(case2.DF)
```

────────────────────────── output ──────────────────────────
```
$T
[1] 168

$rho
[1] 0.9669104

$sigma.rho
[1] 0.01913305

$zeta
numeric(0)

$rho.stat
[1] -5.559061

$t.stat
[1] -1.72945
```

Example 17.5 describes how to test the joint hypothesis that the trend coefficient is 0 and the autoregressive coefficient is 1.

────────────────────────── R code ──────────────────────────
```
> F <- Wald.F.Test(R = diag(2), b = case2.lms$coefficients[, "Estimate"],
+      r = c(0, 1), s2 = case2.lms$sigma^2, XtX_1 = case2.lms$cov.unscaled)
> print(F)
```

────────────────────────── output ──────────────────────────
```
[1] 1.806307
```

We can conduct a similar analysis for cases 1 and 2 using contributed package "urca" from CRAN.

```R code
> library(urca)
> args(ur.df)
```

```output
function (y, type = c("none", "drift", "trend"), lags = 1, selectlags = c("Fixed",
    "AIC", "BIC"))
NULL
```

```R code
> tbill.1.ur.df <- ur.df(dataset[, "i"], type = "none", lags = 0)
> print(summary(tbill.1.ur.df))
```

```output
#################################################
# Augmented Dickey-Fuller Test Unit Root Test #
#################################################


Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1)


Residuals:
    Min      1Q  Median      3Q     Max
-3.6957 -0.1223  0.0961  0.3987  4.4880


Coefficients:
         Estimate Std. Error t value Pr(>|t|)
z.lag.1 -0.003064   0.010592  -0.289    0.773


Residual standard error: 0.8045 on 167 degrees of freedom
Multiple R-squared: 0.0005009,       Adjusted R-squared: -0.005484
F-statistic: 0.0837 on 1 and 167 DF,  p-value: 0.7727


Value of test-statistic is: -0.2893
```

```
Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

```
> tbill.2.ur.df <- ur.df(dataset[, "i"], type = "drift", lags = 0)
> print(summary(tbill.2.ur.df))
```

```
################################################
# Augmented Dickey-Fuller Test Unit Root Test #
################################################


Test regression drift


Call:
lm(formula = z.diff ~ z.lag.1 + 1)

Residuals:
    Min      1Q  Median      3Q     Max
-3.5054 -0.1924 -0.0428  0.3515  4.5523

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.21059    0.11212   1.878   0.0621 .
z.lag.1     -0.03309    0.01913  -1.729   0.0856 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7985 on 166 degrees of freedom
Multiple R-squared: 0.0177,      Adjusted R-squared: 0.01178
F-statistic: 2.991 on 1 and 166 DF,  p-value: 0.08559


Value of test-statistic is: -1.7294 1.8063


Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.46 -2.88 -2.57
phi1  6.52  4.63  3.81
```

The test statistic quoted here is the t-statistic from text, and, conveniently the critical values from the appropriate tables in the book are printed as well. The ur.df function uses a slightly different form of the regression than the Hamilton text, using the first difference of the input variable as the left hand side, rather than its level. The resulting coefficient on the lagged value of the input variable will thus be 1 less than that obtained using the "manual" procedure above.

```R
R code
> print(case1.lms$coefficients["L(i)", "Estimate"])
```

```
output
[1] 0.9969357
```

```R
R code
> print(attr(tbill.1.ur.df, "testreg")$coefficients["z.lag.1",
+     "Estimate"] + 1)
```

```
output
[1] 0.9969357
```

```R
R code
> print(case2.lms$coefficients["L(i)", "Estimate"])
```

```
output
[1] 0.9669104
```

```R
R code
> print(attr(tbill.2.ur.df, "testreg")$coefficients["z.lag.1",
+     "Estimate"] + 1)
```

```
output
[1] 0.9669104
```

17.3 Analyzing GNP data

A similar analysis can be conducted on log real GNP data described beginning on
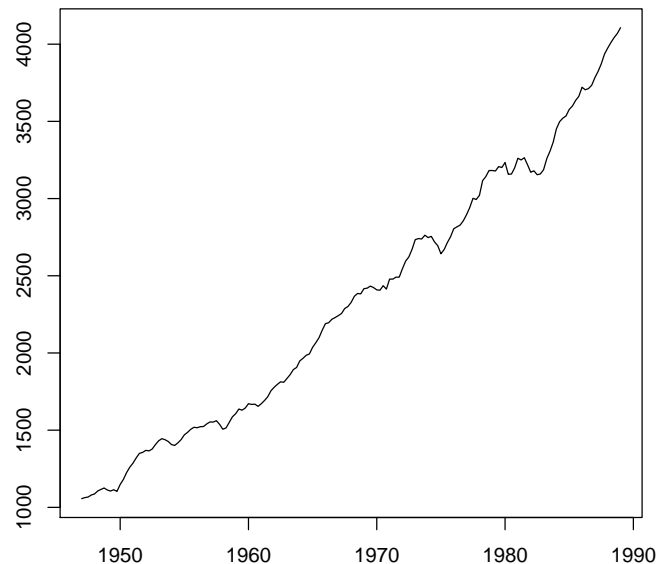
Figure 17.3 – Real GNP

page 501, shown below.

The regression model here incorporates a time trend, based on the shape of the GDP graph

```
                              R code
> case4.lms <- summary(dynlm(y ~ 1 + L(y) + tt, dataset))
> case4.DF <- Dickey.Fuller(T = length(case4.lms$residuals), rho = case4.lms$coefficients[["L(y)",
+      "Estimate"]], sigma.rho = case4.lms$coefficients[["L(y)",
+      "Std. Error"]])
> print(t(case4.lms$coefficients[, c("Estimate", "Std. Error"),
+      drop = FALSE]))
```

```
                              output
            (Intercept)       L(y)         tt
Estimate     27.23724 0.96252203 0.02753238
Std. Error   13.53483 0.01930452 0.01520877
```

```
                              R code
> print(case4.DF)
```

```
                              output
```

36

```
$T
[1] 168


$rho
[1] 0.962522


$sigma.rho
[1] 0.01930452


$zeta
numeric(0)


$rho.stat
[1] -6.296298


$t.stat
[1] -1.941409
```

```R
> F <- Wald.F.Test(R = cbind(rep(0, 2), diag(2)), b = case4.lms$coefficients[,
+     "Estimate"], r = c(1, 0), s2 = case4.lms$sigma^2, XtX_1 = case4.lms$cov.unscaled)
> print(F)
```

```
[1] 2.442251
```

Similarly we can use ur.df to get similar results, although ur.df gives a slightly different value for the intercept because of the way the trend dummy is calculated.

```R
> gnp.4.ur.df <- ur.df(dataset[, "y"], type = "trend", lags = 0)
> print(summary(gnp.4.ur.df))
```

```
#############################################
# Augmented Dickey-Fuller Test Unit Root Test #
#############################################


Test regression trend
```

```
Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt)


Residuals:
     Min       1Q   Median       3Q      Max
-3.03075 -0.61071  0.08077  0.62590  2.65110


Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 27.26477   13.54993   2.012   0.0458 *
z.lag.1     -0.03748    0.01930  -1.941   0.0539 .
tt           0.02753    0.01521   1.810   0.0721 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 1.075 on 165 degrees of freedom
Multiple R-squared: 0.02875,       Adjusted R-squared: 0.01698
F-statistic: 2.442 on 2 and 165 DF,  p-value: 0.0901



Value of test-statistic is: -1.9414 33.2587 2.4423


Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

17.4   Using Phillips Perron Tests

Examples 17.6 and 17.7 reanalyze the case 2 and case 4 regressions above using
the Phillips Perron tests as shown on pages 511-513.

```
────────────────────────── R code ──────────────────────────
> case2.PP <- Phillips.Perron(T = length(case2.lms$residuals),
+     rho = case2.lms$coefficients[["L(i)", "Estimate"]], sigma.rho = case2.lms$coefficients[["L(i)"
+         "Std. Error"]], s = case2.lms$sigma, lambda.hat.sq = as.numeric(Newey.West(case2.lms$resid
+         1, 4)), gamma0 = mean(case2.lms$residuals^2))
> print(t(case2.lms$coefficients[, c("Estimate", "Std. Error"),
+     drop = FALSE]))
─────────────────────────────────────────────────────────────
```

```
────────────────────────── output ──────────────────────────
```

```
          (Intercept)          L(i)
Estimate     0.2105899 0.96691035
Std. Error   0.1121230 0.01913305
```

```
────────────────────────── R code ──────────────────────────
> print(case2.PP)
```

```
────────────────────────── output ──────────────────────────
$T
[1] 168


$rho
[1] 0.9669104


$sigma.rho
[1] 0.01913305


$s.sq
[1] 0.6375998


$lambda.hat.sq
[1] 0.6880069


$gamma0
[1] 0.6300093


$rho.stat
[1] -6.028975


$t.stat
[1] -1.795686
```

```
────────────────────────── R code ──────────────────────────
> case4.PP <- Phillips.Perron(T = length(case4.lms$residuals),
+     rho = case4.lms$coefficients[["L(y)", "Estimate"]], sigma.rho = case4.lms$coefficients[["L(y)"
+         "Std. Error"]], s = case4.lms$sigma, lambda.hat.sq = as.numeric(Newey.West(case4.lms$resid
+         1, 4)), gamma0 = mean(case4.lms$residuals^2))
> print(t(case4.lms$coefficients[, c("Estimate", "Std. Error"),
+     drop = FALSE]))
```

```
────────────────────────── output ──────────────────────────
```

	(Intercept)	L(y)	tt
Estimate	27.23724	0.96252203	0.02753238
Std. Error	13.53483	0.01930452	0.01520877

```R code
> print(case4.PP)
```

```output
$T
[1] 168

$rho
[1] 0.962522

$sigma.rho
[1] 0.01930452

$s.sq
[1] 1.15627

$lambda.hat.sq
[1] 2.117173

$gamma0
[1] 1.135623

$rho.stat
[1] -10.76066

$t.stat
[1] -2.439143
```

We can get similar results for the PP t-stat by using the function ur.pp from the urca package. Note that the implied regression generates the trend centered at T/2, so that the intercept term shown is different than that calculated above, where the trend starts at 1.

```R code
> args(ur.pp)
```

```output
```

```
function (x, type = c("Z-alpha", "Z-tau"), model = c("constant",
    "trend"), lags = c("short", "long"), use.lag = NULL)
NULL
```

```
> case2.ur.pp <- ur.pp(dataset[, "i"], type = "Z-tau", model = "constant",
+     use.lag = 4)
> print(summary(case2.ur.pp))
```

────────────────────────── output ──────────────────────────
```
###################################
# Phillips-Perron Unit Root Test #
###################################


Test regression with intercept



Call:
lm(formula = y ~ y.l1)


Residuals:
    Min      1Q  Median      3Q     Max
-3.5054 -0.1924 -0.0428  0.3515  4.5523


Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.21059    0.11212   1.878   0.0621 .
y.l1         0.96691    0.01913  50.536   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7985 on 166 degrees of freedom
Multiple R-squared: 0.939,        Adjusted R-squared: 0.9386
F-statistic:  2554 on 1 and 166 DF,  p-value: < 2.2e-16



Value of test-statistic, type: Z-tau  is: -1.796


        aux. Z statistics
Z-tau-mu           1.9156


Critical values for Z statistics:
```

```
                    1pct      5pct      10pct
critical values -3.470245 -2.878694 -2.575818
```

```
────────────────────────── R code ──────────────────────────
> case4.ur.pp <- ur.pp(dataset[, "y"], type = "Z-tau", model = "trend",
+     use.lag = 4)
> print(summary(case4.ur.pp))
```

```
────────────────────────── output ──────────────────────────
##################################
# Phillips-Perron Unit Root Test #
##################################


Test regression with intercept and trend



Call:
lm(formula = y ~ y.l1 + trend)


Residuals:
    Min      1Q   Median      3Q      Max
-3.03075 -0.61071  0.08077  0.62590  2.65110


Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 29.57749   14.81856   1.996   0.0476 *
y.l1         0.96252    0.01930  49.860   <2e-16 ***
trend        0.02753    0.01521   1.810   0.0721 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.075 on 165 degrees of freedom
Multiple R-squared: 0.9992,       Adjusted R-squared: 0.9992
F-statistic: 1.055e+05 on 2 and 165 DF,  p-value: < 2.2e-16



Value of test-statistic, type: Z-tau  is: -2.4454


         aux. Z statistics
Z-tau-mu           0.4383
Z-tau-beta         2.5557
```

```
Critical values for Z statistics:
                   1pct      5pct     10pct
critical values -4.015201 -3.437273 -3.142561
```

17.5 Augmented Dickey Fuller Tests

Example 17.8 illustrates incorporates the use of lagged regressors to (putatively)
eliminate serial correlation in the residuals.

```
─────────────────────────────── R code ───────────────────────────────
> tbill.lms <- summary(dynlm(i ~ L(d(i), 1:4) + 1 + L(i), dataset))
> tbill.adf <- Dickey.Fuller(T = length(tbill.lms$residuals), rho = tbill.lms$coefficients[["L(i)",
+     "Estimate"]], sigma.rho = tbill.lms$coefficients[["L(i)",
+     "Std. Error"]], zeta = tbill.lms$coefficients[paste("L(d(i), 1:4)",
+     1:4, sep = ""), "Estimate"])
> print(t(tbill.lms$coefficients[, c("Estimate", "Std. Error"),
+     drop = FALSE]))
```

```
─────────────────────────────── output ───────────────────────────────
           (Intercept) L(d(i), 1:4)1 L(d(i), 1:4)2 L(d(i), 1:4)3 L(d(i), 1:4)4
Estimate     0.1954328     0.3346654    -0.38797356    0.27613320   -0.10670899
Std. Error   0.1086376     0.0788234     0.08082096    0.07998276    0.07944645
                 L(i)
Estimate    0.96904450
Std. Error  0.01860387
```

```
─────────────────────────────── R code ───────────────────────────────
> print(tbill.adf)
```

```
─────────────────────────────── output ───────────────────────────────
$T
[1] 164


$rho
[1] 0.9690445


$sigma.rho
[1] 0.01860387


$zeta
L(d(i), 1:4)1 L(d(i), 1:4)2 L(d(i), 1:4)3 L(d(i), 1:4)4
    0.3346654    -0.3879736     0.2761332    -0.1067090
```

```
$rho.stat
[1] -5.74363


$t.stat
[1] -1.663928
```

The next test checks whether or not the farthest lag is different from zero, i.e. whether or not the right number of lags are included in the equation.

```R code
> print(tbill.lms$coefficients[["L(d(i), 1:4)4", "t value"]])
```

```output
[1] -1.343156
```

Example 17.9 performs a similar analysis for the GNP data.

```R code
> gnp.lms <- summary(dynlm(y ~ L(d(y), 1:4) + 1 + L(y) + tt, dataset))
> gnp.adf <- Dickey.Fuller(T = length(gnp.lms$residuals), rho = gnp.lms$coefficients[["L(y)",
+     "Estimate"]], sigma.rho = gnp.lms$coefficients[["L(y)", "Std. Error"]],
+     zeta = gnp.lms$coefficients[paste("L(d(y), 1:4)", 1:4, sep = ""),
+         "Estimate"])
> F <- Wald.F.Test(R = cbind(rep(0, 2) %o% rep(0, 5), diag(2)),
+     b = gnp.lms$coefficients[, "Estimate"], r = c(1, 0), s2 = gnp.lms$sigma^2,
+     XtX_1 = gnp.lms$cov.unscaled)
> print(t(gnp.lms$coefficients[, c("Estimate", "Std. Error"), drop = FALSE]))
```

```output
           (Intercept) L(d(y), 1:4)1 L(d(y), 1:4)2 L(d(y), 1:4)3 L(d(y), 1:4)4
Estimate      35.91808    0.32908487    0.20856825   -0.08424648   -0.07453301
Std. Error    13.57200    0.07769385    0.08128118    0.08182895    0.07879621
                  L(y)            tt
Estimate    0.94969015    0.03783123
Std. Error  0.01938565    0.01521561
```

```R code
> print(gnp.adf)
```

```output
$T
[1] 164
```

```
$rho
[1] 0.9496901


$sigma.rho
[1] 0.01938565


$zeta
L(d(y), 1:4)1 L(d(y), 1:4)2 L(d(y), 1:4)3 L(d(y), 1:4)4
   0.32908487    0.20856825   -0.08424648   -0.07453301


$rho.stat
[1] -13.28363


$t.stat
[1] -2.595211
```

──────────────────────── R code ────────────────────────
```
> print(F)
```

──────────────────────── output ────────────────────────
```
[1] 3.743228
```

These calculations can be obtained using ur.df, noting that the coefficient on the lagged value of the variable will be 1 less than the method above.

──────────────────────── R code ────────────────────────
```
> tbill.ur.df <- ur.df(dataset[, "i"], type = "drift", lags = 4)
> print(summary(tbill.ur.df))
```

──────────────────────── output ────────────────────────
```
#################################################
# Augmented Dickey-Fuller Test Unit Root Test #
#################################################


Test regression drift


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)


Residuals:
```

```
   Min      1Q  Median      3Q     Max
-3.2648 -0.2153 -0.0315  0.2785  3.0678


Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.19543    0.10864   1.799 0.073936 .
z.lag.1     -0.03096    0.01860  -1.664 0.098109 .
z.diff.lag1  0.33467    0.07882   4.246 3.71e-05 ***
z.diff.lag2 -0.38797    0.08082  -4.800 3.64e-06 ***
z.diff.lag3  0.27613    0.07998   3.452 0.000713 ***
z.diff.lag4 -0.10671    0.07945  -1.343 0.181148
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.7396 on 158 degrees of freedom
Multiple R-squared: 0.1969,       Adjusted R-squared: 0.1715
F-statistic: 7.749 on 5 and 158 DF,  p-value: 1.522e-06



Value of test-statistic is: -1.6639 1.6539


Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.46 -2.88 -2.57
phi1  6.52  4.63  3.81
```

```R
> gnp.ur.df <- ur.df(dataset[, "y"], type = "trend", lags = 4)
> print(summary(gnp.ur.df))
```

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################


Test regression trend



Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)


Residuals:
```

```
       Min       1Q   Median       3Q      Max
-2.94175 -0.59496  0.02697  0.58862  2.85850


Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 35.95591   13.58711   2.646  0.00897 **
z.lag.1     -0.05031    0.01939  -2.595  0.01035 *
tt           0.03783    0.01522   2.486  0.01395 *
z.diff.lag1  0.32908    0.07769   4.236 3.87e-05 ***
z.diff.lag2  0.20857    0.08128   2.566  0.01122 *
z.diff.lag3 -0.08425    0.08183  -1.030  0.30481
z.diff.lag4 -0.07453    0.07880  -0.946  0.34566
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1


Residual standard error: 0.9883 on 157 degrees of freedom
Multiple R-squared: 0.2177,       Adjusted R-squared: 0.1878
F-statistic: 7.282 on 6 and 157 DF,  p-value: 7.069e-07



Value of test-statistic is: -2.5952 10.4162 3.7432


Critical values for test statistics:
     1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

17.6   Example 17.10 - Bayesian Test of Autoregressive Coefficient

Page 532 describes a test on the autoregressive coefficient that weights prior probabilities.

```
────────────────────────────── R code ──────────────────────────────
> t.value <- (1 - gnp.lms$coefficients[["L(y)", "Estimate"]])/gnp.lms$coefficients[["L(y)",
+     "Std. Error"]]
> print(t.value)
─────────────────────────────────────────────────────────────────────
```

```
────────────────────────────── output ──────────────────────────────
[1] 2.595211
─────────────────────────────────────────────────────────────────────
```

```R code
> print((1 - pt(t.value, length(gnp.lms$residuals)))/2)
```

```output
[1] 0.002577594
```

17.7 Determining Lag Length

Page 530 describes an iterative process to determine the correct lag length. This is easily expressed in terms of the structures used above.

```R code
> for (lag in 10:1) {
+     gnp.lm <- dynlm(formula = as.formula(paste("y ~ L(d(y), 1:",
+         lag, ") + 1 + L(y) + tt", sep = "")), data = dataset)
+     if (summary(gnp.lm)$coefficients[[paste("L(d(y), 1:", lag,
+         ")", lag, sep = ""), "Pr(>|t|)"]] < 0.05)
+         break
+ }
> print(lag)
```

```output
[1] 2
```

Annex: R Facilities

Further features in R

Since the tests of Dickey-Fuller and of Philips Perron, the issue of unit root testing has seen tremendous research, with hundreds of papers on the topic. For a survey of the literature, see the article of Phillips and Xiao (1998), or the book of Maddala and Kim (1998).

Concerning further developments, Elliott, Rothenberg, and Stock (1996) used a so-called GLS detrending method to test for the presence of drift and trends, and obtain tests with higher power. Concerning the lag length selection, Ng and Perron (2001) and Perron and Qu (2007) introduce a new information criterion which enables a better selection of the lag length. Finally, Kwiatkowski, Phillips, Schmidt, and Shin (1992) design a test where the null hypothesis is a stationary series (around a mean or a linear trend), while the alternative is the unit root. In an other direction, Hansen (1995) show that by adding other related variables in the testing regression, one can obtain tests with much higher power.

Package `urca`, well documented in the book of Pfaff (2008), contains a number of other tests:

- The DF-GLS test: *ur.ers*

- A LM test: *ur.sp*

- The KPSS test of stationarity: *ur.kpss*

- A test taking into account structural breaks: *ur.za*

Package `CADFtest`, described in Lupi (2009), implements the Hansen covariate test, nesting the ADF test when no covariate is given. It offers also the choice of the lag according to the Ng and Perron (2001) MAIC criterion.

19 Cointegration

19.1 Testing Cointegration when the Cointegrating Vector is Known

Section 19.2, beginning on page 582 describes cointegration testing of purchasing power parity between Italian lire and US dollars. The data used is 100 times log monthly price levels and spot nominal and real exchange rates, normalized to a value of zero at the start of the series.

```R
———————————————————————— R code ————————————————————————
> data(ppp, package = "RcompHam94")
> selection <- window(ppp, start = c(1973, 1), end = c(1989, 10))
> ppp.data <- cbind(pstar = 100 * log(selection[, "PC6IT"]/selection[[1,
+     "PC6IT"]]), p = 100 * log(selection[, "PZUNEW"]/selection[[1,
+     "PZUNEW"]]), ner = -100 * log(selection[, "EXRITL"]/selection[[1,
+     "EXRITL"]]))
> ppp.data <- cbind(ppp.data, rer = ppp.data[, "p"] - ppp.data[,
+     "ner"] - ppp.data[, "pstar"])
```
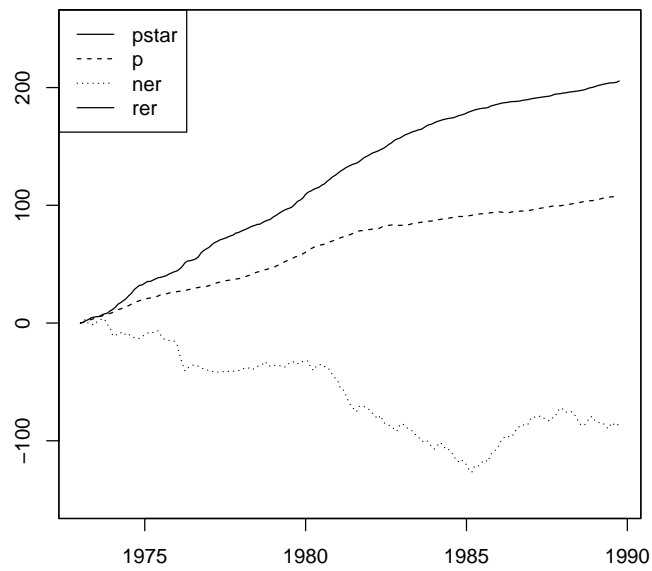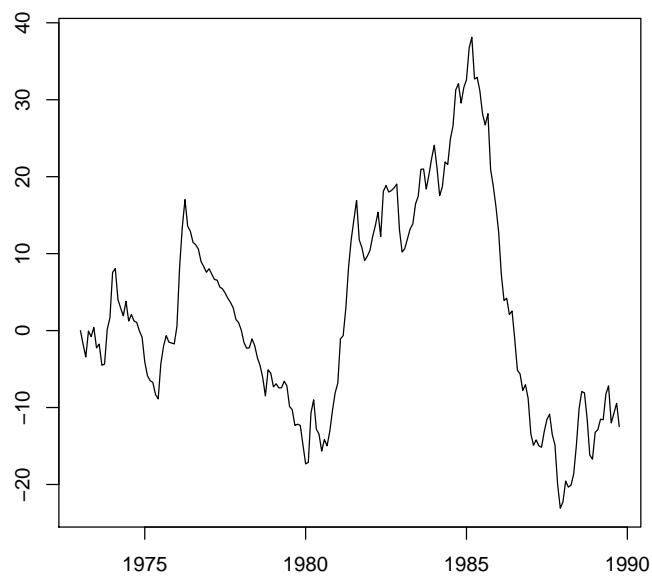
Figure 19.2

Figure 19.3

To save time define a simple utility function to perform augmented Dickey Fuller analysis according to the conventions in the text.

50

```R
──────────────────────────── R code ────────────────────────────
> do.DF <- function(series, lag) {
+     df.lms <- summary(dynlm(formula = as.formula(paste("y ~ L(y) + tt + L(d(y),1:",
+         lag, ") + 1", sep = "")), data = zooreg(cbind(y = series,
+         tt = 1:length(series)))))
+     df.results <- Dickey.Fuller(T = length(df.lms$residuals),
+         rho = df.lms$coefficients[["L(y)", "Estimate"]], sigma.rho = df.lms$coefficients[["L(y)",
+             "Std. Error"]], zeta = df.lms$coefficients[paste("L(d(y), 1:",
+             lag, ")", 1:lag, sep = ""), "Estimate"])
+     F <- Wald.F.Test(R = cbind(rep(0, 2), diag(2), rep(0, 2) %o%
+         rep(0, lag)), b = df.lms$coefficients[, "Estimate"],
+         r = c(1, 0), s2 = df.lms$sigma^2, XtX_1 = df.lms$cov.unscaled)
+     print(t(df.lms$coefficients[, c("Estimate", "Std. Error"),
+         drop = FALSE]))
+     print(df.results)
+     print(F)
+ }
```

Following the text, check each series with a Dickey Fuller test with a regression
estimated with twelve lags.

```R
──────────────────────────── R code ────────────────────────────
> for (series.name in c("p", "pstar", "ner", "rer")) do.DF(series = as.vector(ppp.data[,
+     series.name]), lag = 12)
```

```
──────────────────────────── output ────────────────────────────
            (Intercept)          L(y)           tt L(d(y), 1:12)1 L(d(y), 1:12)2
Estimate     0.13616093 0.994004087 0.002927051      0.55339784     -0.05690832
Std. Error   0.08577907 0.003067474 0.001766655      0.07521788      0.08544012
            L(d(y), 1:12)3 L(d(y), 1:12)4 L(d(y), 1:12)5 L(d(y), 1:12)6
Estimate        0.07012512     0.06038960    -0.07823250    -0.04837686
Std. Error      0.08490690     0.08196995     0.07848846     0.07072189
            L(d(y), 1:12)7 L(d(y), 1:12)8 L(d(y), 1:12)9 L(d(y), 1:12)10
Estimate        0.16584335    -0.07020745     0.24464455     -0.11004717
Std. Error      0.06891545     0.07001447     0.07016141      0.07257971
            L(d(y), 1:12)11 L(d(y), 1:12)12
Estimate         0.11758063      0.04670235
Std. Error       0.07293743      0.06865031
$T
[1] 189


$rho
[1] 0.9940041
```

51

```
$sigma.rho
[1] 0.003067474


$zeta
 L(d(y), 1:12)1  L(d(y), 1:12)2  L(d(y), 1:12)3  L(d(y), 1:12)4  L(d(y), 1:12)5
     0.55339784     -0.05690832      0.07012512      0.06038960     -0.07823250
 L(d(y), 1:12)6  L(d(y), 1:12)7  L(d(y), 1:12)8  L(d(y), 1:12)9 L(d(y), 1:12)10
    -0.04837686      0.16584335     -0.07020745      0.24464455     -0.11004717
L(d(y), 1:12)11 L(d(y), 1:12)12
     0.11758063      0.04670235


$rho.stat
[1] -10.78352


$t.stat
[1] -1.954675


[1] 2.412933
          (Intercept)           L(y)              tt L(d(y), 1:12)1 L(d(y), 1:12)2
Estimate       0.768008 0.999456707 -0.002406065       0.4207017     -0.01159213
Std. Error     0.253071 0.004116999  0.004989081       0.0761105      0.08152127
          L(d(y), 1:12)3 L(d(y), 1:12)4 L(d(y), 1:12)5 L(d(y), 1:12)6
Estimate      0.01343968     0.07720637    -0.03649430      0.1452822
Std. Error    0.08016238     0.08012553     0.08008714     0.0786705
          L(d(y), 1:12)7 L(d(y), 1:12)8 L(d(y), 1:12)9 L(d(y), 1:12)10
Estimate     -0.09911809     0.04671752    -0.04998236     -0.03463835
Std. Error    0.07883988     0.07859877     0.07811184     0.07816837
          L(d(y), 1:12)11 L(d(y), 1:12)12
Estimate       0.07555504     0.02186374
Std. Error     0.07799367     0.07334667
$T
[1] 189


$rho
[1] 0.9994567


$sigma.rho
[1] 0.004116999


$zeta
 L(d(y), 1:12)1  L(d(y), 1:12)2  L(d(y), 1:12)3  L(d(y), 1:12)4  L(d(y), 1:12)5
```

```
     0.42070173     -0.01159213      0.01343968      0.07720637     -0.03649430
 L(d(y), 1:12)6  L(d(y), 1:12)7  L(d(y), 1:12)8  L(d(y), 1:12)9 L(d(y), 1:12)10
     0.14528224     -0.09911809      0.04671752     -0.04998236     -0.03463835
L(d(y), 1:12)11 L(d(y), 1:12)12
     0.07555504      0.02186374


$rho.stat
[1] -0.2382095


$t.stat
[1] -0.1319633


[1] 4.249956
          (Intercept)         L(y)              tt L(d(y), 1:12)1 L(d(y), 1:12)2
Estimate   -0.3893374 0.98294130 -0.007384125      0.34882976     -0.02556740
Std. Error  0.4138009 0.01076644  0.006883901      0.07443904      0.07911076
          L(d(y), 1:12)3 L(d(y), 1:12)4 L(d(y), 1:12)5 L(d(y), 1:12)6
Estimate      0.002617322     0.01168946     0.09931411     0.001387289
Std. Error    0.078947706     0.08000793     0.07994826     0.080819939
          L(d(y), 1:12)7 L(d(y), 1:12)8 L(d(y), 1:12)9 L(d(y), 1:12)10
Estimate      0.06320540     0.11722338    -0.06112766      0.08173960
Std. Error    0.08061435     0.08056098     0.08078856      0.08069646
          L(d(y), 1:12)11 L(d(y), 1:12)12
Estimate       0.03726136     -0.03036347
Std. Error     0.08064652      0.07674078
$T
[1] 189


$rho
[1] 0.9829413


$sigma.rho
[1] 0.01076644


$zeta
 L(d(y), 1:12)1  L(d(y), 1:12)2  L(d(y), 1:12)3  L(d(y), 1:12)4  L(d(y), 1:12)5
    0.348829755     -0.025567401      0.002617322      0.011689457      0.099314112
 L(d(y), 1:12)6  L(d(y), 1:12)7  L(d(y), 1:12)8  L(d(y), 1:12)9 L(d(y), 1:12)10
    0.001387289      0.063205400      0.117223384     -0.061127657      0.081739596
L(d(y), 1:12)11 L(d(y), 1:12)12
    0.037261364     -0.030363466
```

```
$rho.stat
[1] -9.112996


$t.stat
[1] -1.584433


[1] 1.489674
          (Intercept)        L(y)             tt L(d(y), 1:12)1 L(d(y), 1:12)2
Estimate    0.05320142 0.97129326 -0.0004612496      0.31783702    -0.01491669
Std. Error  0.39055736 0.01414519  0.0032371854      0.07416327     0.07807885
          L(d(y), 1:12)3 L(d(y), 1:12)4 L(d(y), 1:12)5 L(d(y), 1:12)6
Estimate      0.01279732      0.0224258     0.08451558    -0.003065327
Std. Error    0.07772772      0.0786769     0.07833952     0.079071534
          L(d(y), 1:12)7 L(d(y), 1:12)8 L(d(y), 1:12)9 L(d(y), 1:12)10
Estimate      0.02991378     0.08241971    -0.04786150      0.07556671
Std. Error    0.07875080     0.07864164     0.07864791      0.07840588
          L(d(y), 1:12)11 L(d(y), 1:12)12
Estimate       0.05040823     -0.01247043
Std. Error     0.07827994      0.07599776
$T
[1] 189


$rho
[1] 0.9712933


$sigma.rho
[1] 0.01414519


$zeta
 L(d(y), 1:12)1  L(d(y), 1:12)2  L(d(y), 1:12)3  L(d(y), 1:12)4  L(d(y), 1:12)5
    0.317837019    -0.014916687     0.012797325     0.022425804     0.084515583
 L(d(y), 1:12)6  L(d(y), 1:12)7  L(d(y), 1:12)8  L(d(y), 1:12)9 L(d(y), 1:12)10
   -0.003065327     0.029913775     0.082419705    -0.047861504     0.075566713
L(d(y), 1:12)11 L(d(y), 1:12)12
    0.050408226    -0.012470431


$rho.stat
[1] -13.48204


$t.stat
[1] -2.029435
```

```
[1] 2.078078
```

Now check the real exchange rate with a Phillips Perron test

```R
> pp.lms <- summary(dynlm(z ~ L(z) + 1, zooreg(cbind(z = as.vector(ppp.data[,
+     "rer"])))))
> PP.results <- Phillips.Perron(T = length(pp.lms$residuals), rho = pp.lms$coefficients[["L(z)",
+     "Estimate"]], sigma.rho = pp.lms$coefficients[["L(z)", "Std. Error"]],
+     s = pp.lms$sigma, lambda.hat.sq = as.numeric(Newey.West(pp.lms$residuals %o%
+         1, 12)), gamma0 = mean(pp.lms$residuals^2))
> print(t(pp.lms$coefficients[, c("Estimate", "Std. Error"), drop = FALSE]))
```

```
                (Intercept)      L(z)
Estimate     -0.0297931 0.98654204
Std. Error    0.1783572 0.01275287
```

```R
> print(PP.results)
```

```
$T
[1] 201


$rho
[1] 0.986542


$sigma.rho
[1] 0.01275287


$s.sq
[1] 6.205887


$lambda.hat.sq
[1] 13.03064


$gamma0
[1] 6.144137


$rho.stat
[1] -6.35068
```

```
$t.stat
[1] -1.706128
```

Estimating the impulse response function gives a sense of the persistence of deviations from PPP. as shown in Figure 19.4, page 584. Proceed in three steps:

1. Estimate the AR(12)

2. Creta an innovation vector with only zeros and once a 1 value

3. Simulate an AR(12) process with parameters estimated in 1 and innovations defined in 2.

―――――――――――――――――――――― R code ――――――――――――――――――――――
```
> ar.results <- ar(ppp.data$rer, aic = FALSE, order.max = 13, method = "ols",
+     demean = TRUE)
> tt <- seq(1, 72)
> start.innov <- rep(0, 13)
> et <- c(start.innov, 1, rep(0, length(tt) - 14))
> arima.sim.output <- arima.sim(list(order = c(13, 0, 0), ar = ar.results$ar),
+     n = length(tt), innov = et, n.start = length(start.innov),
+     start.innov = start.innov)
> irf <- as.vector(arima.sim.output)
```
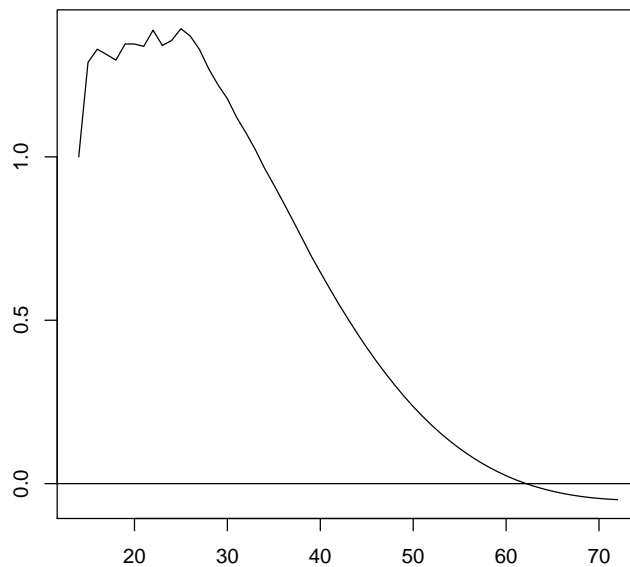
Figure 19.4

19.2   Estimating the Cointegrating Vector

Page 598 shows an example of the Phillips Ouliaris Hansen procedure for estimating a cointegrating vector.

```
──────────────────────────── R code ────────────────────────────
> poh.cointegration.lm <- lm(p ~ 1 + ner + pstar, ppp.data)
> poh.residual.lms <- summary(dynlm(u ~ 0 + L(u), zooreg(cbind(u = poh.cointegration.lm$residuals)))
> POH.results <- Phillips.Perron(T = length(poh.residual.lms$residuals),
+     rho = poh.residual.lms$coefficients[["L(u)", "Estimate"]],
+     sigma.rho = poh.residual.lms$coefficients[["L(u)", "Std. Error"]],
+     s = poh.residual.lms$sigma, lambda.hat.sq = as.numeric(Newey.West(poh.residual.lms$residuals %
+         1, 12)), gamma0 = mean(poh.residual.lms$residuals^2))
> print(t(summary(poh.cointegration.lm)$coefficients[, c("Estimate",
+     "Std. Error"), drop = FALSE]))
──────────────────────────────────────────────────────────────────
```

```
──────────────────────────── output ────────────────────────────
            (Intercept)         ner        pstar
Estimate      2.7123130  0.05134848  0.530040965
Std. Error    0.3676955  0.01204537  0.006708385
──────────────────────────────────────────────────────────────────
```

```
──────────────────────────── R code ────────────────────────────
```

```
> print(t(poh.residual.lms$coefficients[, c("Estimate", "Std. Error"),
+     drop = FALSE]))
```

────────────────────────── output ──────────────────────────
```
              L(u)
Estimate   0.98331085
Std. Error 0.01171956
```

────────────────────────── R code ──────────────────────────
```
> print(POH.results)
```

────────────────────────── output ──────────────────────────
```
$T
[1] 201

$rho
[1] 0.9833108

$sigma.rho
[1] 0.01171956

$s.sq
[1] 0.1630028

$lambda.hat.sq
[1] 0.4082242

$gamma0
[1] 0.1621919

$rho.stat
[1] -7.542281

$t.stat
[1] -2.020981
```

A second example performs a similar analysis on quarterly US consumption and income data from 1947Q1 to 1989Q3.

────────────────────────── R code ──────────────────────────
```
> data(coninc, package = "RcompHam94")
> coninc.data <- window(cbind(c = 100 * log(coninc[, "GC82"]),
```

```
+       y = 100 * log(coninc[, "GYD82"])), start = c(1947, 1), end = c(1989,
+       3))
> coninc.data <- cbind(coninc.data, tt = 1:dim(coninc.data)[[1]])
```
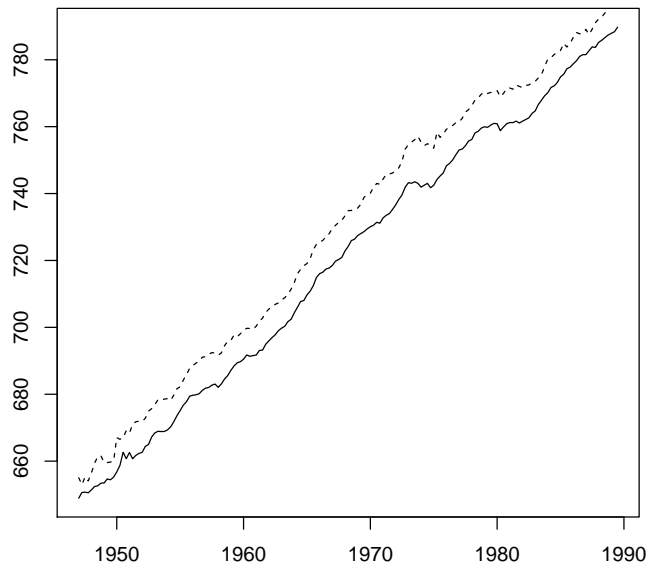

Figure 19.5

Test individual
series for unit root status using Dickey Fuller.

```
———————————————————— R code ————————————————————
> for (series.name in c("y", "c")) do.DF(series = as.vector(coninc.data[,
+       series.name]), lag = 6)
```

```
—————————————————————— output ——————————————————————
           (Intercept)       L(y)         tt L(d(y), 1:6)1 L(d(y), 1:6)2
Estimate      20.33673 0.97058490 0.02379684  -0.006528755    -0.03584632
Std. Error    15.04162 0.02306293 0.01985318   0.080928563     0.08025935
           L(d(y), 1:6)3 L(d(y), 1:6)4 L(d(y), 1:6)5 L(d(y), 1:6)6
Estimate      0.10212854   -0.18753634   -0.03718788    0.02785595
Std. Error    0.07758036    0.07699406    0.07813842    0.07662877
$T
[1] 164


$rho
[1] 0.9705849
```

```
$sigma.rho
[1] 0.02306293

$zeta
L(d(y), 1:6)1 L(d(y), 1:6)2 L(d(y), 1:6)3 L(d(y), 1:6)4 L(d(y), 1:6)5
 -0.006528755  -0.035846316   0.102128545  -0.187536343  -0.037187883
L(d(y), 1:6)6
  0.027855951


$rho.stat
[1] -4.242382


$t.stat
[1] -1.275428


[1] 1.132134
           (Intercept)         L(y)          tt L(d(y), 1:6)1 L(d(y), 1:6)2
Estimate      29.46860 0.95552168 0.03721088     0.03624864    0.25964745
Std. Error    15.19248 0.02360001 0.02006161     0.07979877    0.07935028
          L(d(y), 1:6)3 L(d(y), 1:6)4 L(d(y), 1:6)5 L(d(y), 1:6)6
Estimate      0.06273192   -0.05234112   -0.04791625   -0.06782142
Std. Error    0.08172798    0.08122252    0.07956524    0.07919698
$T
[1] 164


$rho
[1] 0.9555217


$sigma.rho
[1] 0.02360001


$zeta
L(d(y), 1:6)1 L(d(y), 1:6)2 L(d(y), 1:6)3 L(d(y), 1:6)4 L(d(y), 1:6)5
   0.03624864    0.25964745    0.06273192   -0.05234112   -0.04791625
L(d(y), 1:6)6
  -0.06782142


$rho.stat
[1] -9.011597


$t.stat
[1] -1.884673
```

```
[1] 1.85829
```

Estimate cointegration vector, then check for unit root status of the residual
using Phillips Perron.

```R
─────────────────────── R code ───────────────────────
> poh.cointegration.lm <- lm(c ~ 1 + y, coninc.data)
> poh.residual.lms <- summary(dynlm(u ~ 0 + L(u), zooreg(cbind(u = poh.cointegration.lm$residuals))))
> POH.results <- Phillips.Perron(T = length(poh.residual.lms$residuals),
+     rho = poh.residual.lms$coefficients[["L(u)", "Estimate"]],
+     sigma.rho = poh.residual.lms$coefficients[["L(u)", "Std. Error"]],
+     s = poh.residual.lms$sigma, lambda.hat.sq = as.numeric(Newey.West(poh.residual.lms$residuals %
+         1, 6)), gamma0 = mean(poh.residual.lms$residuals^2))
> print(t(summary(poh.cointegration.lm)$coefficients[, c("Estimate",
+     "Std. Error"), drop = FALSE]))
```

```
─────────────────────── output ───────────────────────
            (Intercept)           y
Estimate      0.6675807 0.986494296
Std. Error    2.3503489 0.003217444
```

```R
─────────────────────── R code ───────────────────────
> print(t(poh.residual.lms$coefficients[, c("Estimate", "Std. Error"),
+     drop = FALSE]))
```

```
─────────────────────── output ───────────────────────
                   L(u)
Estimate     0.78185415
Std. Error   0.04788553
```

```R
─────────────────────── R code ───────────────────────
> print(POH.results)
```

```
─────────────────────── output ───────────────────────
$T
[1] 170


$rho
[1] 0.7818542


$sigma.rho
```

```
[1] 0.04788553


$s.sq
[1] 1.22395


$lambda.hat.sq
[1] 1.030594


$gamma0
[1] 1.21675


$rho.stat
[1] -32.04525


$t.stat
[1] -4.27529
```

19.3 Testing Hypotheses About the Cointegrating Vector

Page 608-612 illustrate a technique that uses leads and lags to produce a stationary vector for hypothesis testing. The regression is estimated with both no trend and trend, and the corrected t-stat is calculated.

```
─────────────────────── R code ───────────────────────
> no.trend.lm <- dynlm(c ~ 1 + y + L(d(y), -4:4), coninc.data)
> trend.lm <- dynlm(c ~ 1 + y + tt + L(d(y), -4:4), coninc.data)
> for (model in list(no.trend.lm, trend.lm)) {
+     lags <- 2
+     cms <- summary(model)
+     T <- length(cms$residuals)
+     cfs <- cms$coefficients
+     t.rho <- (cfs[["y", "Estimate"]] - 1)/cfs[["y", "Std. Error"]]
+     rms <- summary(dynlm(as.formula(paste("u ~ 0 + L(u,1:", lags,
+         ")", sep = "")), zooreg(cbind(u = as.vector(cms$residuals)))))
+     sigma1.hat.sq <- mean(rms$residuals^2)
+     lambda.11 <- sigma1.hat.sq^0.5/(1 - sum(rms$coefficients[paste("L(u, 1:",
+         lags, ")", 1:lags, sep = ""), "Estimate"]))
+     t.a <- t.rho * cms$sigma/lambda.11
+     print(t(cfs[, c("Estimate", "Std. Error"), drop = FALSE]))
+     print(t(rms$coefficients[, c("Estimate", "Std. Error"), drop = FALSE]))
+     print(T)
+     print(cms$sigma)
```

```
+       print(t.rho)
+       print(sigma1.hat.sq)
+       print(lambda.11)
+       print(t.a)
+ }
```

```
────────────────────────────── output ──────────────────────────────
          (Intercept)         y L(d(y), -4:4)-4 L(d(y), -4:4)-3
Estimate    -4.519229 0.992158525       0.1453095       0.2861419
Std. Error   2.340225 0.003063317       0.1187996       0.1155945
          L(d(y), -4:4)-2 L(d(y), -4:4)-1 L(d(y), -4:4)0 L(d(y), -4:4)1
Estimate        0.2641186       0.4859239      -0.2403601      -0.01101143
Std. Error      0.1148920       0.1157048       0.1174159       0.11389942
          L(d(y), -4:4)2 L(d(y), -4:4)3 L(d(y), -4:4)4
Estimate      0.06969114     0.04055551      0.02150153
Std. Error    0.11150577     0.11115520      0.11008398
          L(u, 1:2)1 L(u, 1:2)2
Estimate   0.71796867 0.20574012
Std. Error 0.07722647 0.07684783
[1] 162
[1] 1.516006
[1] -2.559799
[1] 0.380918
[1] 8.089864
[1] -0.4796954
          (Intercept)          y         tt L(d(y), -4:4)-4 L(d(y), -4:4)-3
Estimate    198.87167 0.68117915 0.26895671       0.02782397       0.16559666
Std. Error   15.01478 0.02292367 0.01974617       0.08016237       0.07805023
          L(d(y), -4:4)-2 L(d(y), -4:4)-1 L(d(y), -4:4)0 L(d(y), -4:4)1
Estimate        0.15407283       0.40061828      -0.05124600      0.12737594
Std. Error      0.07749787       0.07787309       0.07998305      0.07708222
          L(d(y), -4:4)2 L(d(y), -4:4)3 L(d(y), -4:4)4
Estimate      0.23116996     0.20472613      0.18997478
Std. Error    0.07573754     0.07553655      0.07487875
          L(u, 1:2)1 L(u, 1:2)2
Estimate   0.68717133 0.12918203
Std. Error 0.07786238 0.07666487
[1] 162
[1] 1.017016
[1] -13.90793
[1] 0.3439489
[1] 3.193478
```

```
[1] -4.429212
```

20 Full-Information Maximum Likelihood Analysis of Cointegrated Systems

20.1 An Application of the Johansen Approach to the PPP data

Section 20.3 reanalyzes the data used in Chapter 19 using the FIML approach.

```
─────────────────────────────── R code ───────────────────────────────
> data(ppp, package = "RcompHam94")
> selection <- window(ppp, start = c(1973, 2), end = c(1989, 10))
> ppp.data <- cbind(p = 100 * log(selection[, "PZUNEW"]/selection[[1,
+     "PZUNEW"]]), ner = -100 * log(selection[, "EXRITL"]/selection[[1,
+     "EXRITL"]]), pstar = 100 * log(selection[, "PC6IT"]/selection[[1,
+     "PC6IT"]]))
> y <- as.matrix(ppp.data)
```

First conduct the auxiliary regressions. Given that the right hand sides consists of lagged values of the changes in y for both [20.2.4] and [20.2.5], construct a regression with both lagged y and lagged changes of y as left hand side.

```
─────────────────────────────── R code ───────────────────────────────
> lags <- 12
> n <- dim(y)[[2]]
> delta.y.lag <- embed(diff(y), lags)
> X <- delta.y.lag[, -(1:n)]
> T <- dim(X)[[1]]
> lhs <- cbind(delta.y.lag[, 1:n], y[2:(T + 1), ])
> aux.lm <- lm(lhs ~ 1 + X, list(lhs = lhs, X = X))
> uv <- sapply(summary(aux.lm), FUN = function(x) {
+     x$residuals
+ })
> u <- uv[, 1:n]
> v <- uv[, (n + 1):(2 * n)]
```

Now calculate the canonical correlations according to [20.2.6], [20.2.7], [20.2.8], and calculate eigenvalues according to [20.2.9], and log likelihood as in [20.2.10]. Note that u is T rows by n columns so that ut is the t-th row of matrix u, so only a single inner product, rather than sum of outer products, is needed.

```R
                                        R code
> SigmaUU <- 1/T * t(u) %*% u
> SigmaVV <- 1/T * t(v) %*% v
> SigmaUV <- 1/T * t(u) %*% v
> eigen.results <- eigen(solve(SigmaVV) %*% t(SigmaUV) %*% solve(SigmaUU) %*%
+    SigmaUV)
> lambda <- eigen.results$values
> LRT <- -T * sum(log(1 - lambda))
> print(SigmaUU)
```

```
                                        output
             Response Y1  Response Y2  Response Y3
Response Y1  0.04351139  -0.03162842   0.01542973
Response Y2 -0.03162842   4.68649773   0.03198756
Response Y3  0.01542973   0.03198756   0.17992709
```

```R
                                        R code
> print(SigmaVV)
```

```
                                        output
              Response p  Response ner  Response pstar
Response p      427.3661     -370.6990        805.8124
Response ner   -370.6990      424.0835       -709.0365
Response pstar  805.8124     -709.0365       1525.4480
```

```R
                                        R code
> print(SigmaUV)
```

```
                                        output
             Response p  Response ner  Response pstar
Response Y1  -0.484857     0.4987583       -0.8377012
Response Y2  -1.814008    -2.9592774       -2.4689545
Response Y3  -1.808357     1.4689736       -3.5899117
```

```R
                                        R code
> print(lambda)
```

```
                                        output
[1] 0.11046055 0.05603356 0.03039275
```

```R
                                        R code
> print(T * log(1 - lambda))
```

```
[1] -22.12272 -10.89862  -5.83333
```

```
> print(LRT)
```

```
[1] 38.85467
```

We can also use function ca.jo from package urca to calculate the lambda vector.

```
> ca.jo.results <- ca.jo(y, type = "eigen", ecdet = "none", K = 12,
+     spec = "transitory", season = NULL, dumvar = NULL)
> summary(ca.jo.results)
```

```
######################
# Johansen-Procedure #
######################


Test type: maximal eigenvalue statistic (lambda max) , with linear trend


Eigenvalues (lambda):
[1] 0.11046055 0.05603356 0.03039275


Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 2 |  5.83  6.50  8.18 11.65
r <= 1 | 10.90 12.91 14.90 19.19
r = 0  | 22.12 18.90 21.07 25.75


Eigenvectors, normalised to first column:
(These are the cointegration relations)


                p.l1     ner.l1    pstar.l1
p.l1      1.00000000  1.000000  1.0000000
ner.l1   -0.03695704  0.333539 -0.4217669
pstar.l1 -0.55679934 -0.276719 -0.6431898


Weights W:
```

(This is the loading matrix)

```
              p.l1       ner.l1     pstar.l1
p.d      -0.02117410 -0.002991059 -0.006182545
ner.d    -0.18953663 -0.073068358  0.040400775
pstar.d  0.07825214 -0.011212959 -0.004700730
```

Finally following page 648, calculate the first cointegrating vector normalized as in [20.3.9], and also normalized to have unity for the first coefficient.

```
──────────────────── R code ────────────────────
> ahat1 <- eigen.results$vectors[, 1]
> ahat1.tilde <- ahat1/sqrt(t(ahat1) %*% SigmaVV %*% ahat1)
> ahat1.normal <- ahat1/ahat1[[1]]
> print(ahat1)
```

```
──────────────────── output ────────────────────
[1]   0.87324045 -0.03227239 -0.48621971
```

```
──────────────────── R code ────────────────────
> print(ahat1.tilde)
```

```
──────────────────── output ────────────────────
[1]   0.75794168 -0.02801128 -0.42202143
```

```
──────────────────── R code ────────────────────
> print(ahat1.normal)
```

```
──────────────────── output ────────────────────
[1]   1.00000000 -0.03695704 -0.55679934
```

20.2   Likelihood Ratio Tests on the Cointegration Vector

Page 649 shows how to conduct hypothesis tests on the cointegration vector. The follow code implements [20.3.10] - [20.3.14] and subsequent calculations.

```
──────────────────── R code ────────────────────
> D = cbind(c(1, 0, 0), c(0, 0, 1))
> SigmaVV.tilde <- t(D) %*% SigmaVV %*% D
> SigmaUV.tilde <- SigmaUV %*% D
> eigen.results <- eigen(solve(SigmaVV.tilde) %*% t(SigmaUV.tilde) %*%
+     solve(SigmaUU) %*% SigmaUV.tilde)
```

```
> lambda.tilde <- eigen.results$values
> h <- 1
> LRT <- -T * sum(log(1 - lambda[1:h])) + T * sum(log(1 - lambda.tilde[1:h]))
> ahat1.normal.tilde <- eigen.results$vectors[, 1]/eigen.results$vectors[,
+      1][[1]]
> print(SigmaVV.tilde)
```

―――――――――――――― output ――――――――――――――
```
         [,1]      [,2]
[1,]  427.3661   805.8124
[2,]  805.8124  1525.4480
```

―――――――――――――― R code ――――――――――――――
```
> print(SigmaUV.tilde)
```

―――――――――――――― output ――――――――――――――
```
              [,1]        [,2]
Response Y1 -0.484857  -0.8377012
Response Y2 -1.814008  -2.4689545
Response Y3 -1.808357  -3.5899117
```

―――――――――――――― R code ――――――――――――――
```
> print(lambda.tilde)
```

―――――――――――――― output ――――――――――――――
```
[1] 0.10585119 0.04680752
```

―――――――――――――― R code ――――――――――――――
```
> print(T * log(1 - lambda.tilde))
```

―――――――――――――― output ――――――――――――――
```
[1] -21.145898  -9.060362
```

―――――――――――――― R code ――――――――――――――
```
> print(LRT)
```

―――――――――――――― output ――――――――――――――
```
[1] 0.9768206
```

―――――――――――――― R code ――――――――――――――
```
> print(ahat1.normal.tilde)
```

68

```
────────────────── output ──────────────────
[1]   1.0000000 -0.5386492
```

Page 650 shows a second example.

```
────────────────── R code ──────────────────
> h <- 1
> D = c(1, -1, -1) %o% 1
> SigmaVV.tilde <- t(D) %*% SigmaVV %*% D
> SigmaUV.tilde <- SigmaUV %*% D
> eigen.results <- eigen(solve(SigmaVV.tilde) %*% t(SigmaUV.tilde) %*%
+     solve(SigmaUU) %*% SigmaUV.tilde)
> lambda.tilde <- eigen.results$values
> LRT <- -T * sum(log(1 - lambda[1:h])) + T * sum(log(1 - lambda.tilde[1:h]))
> print(SigmaVV.tilde)
```

```
────────────────── output ──────────────────
        [,1]
[1,] 88.59774
```

```
────────────────── R code ──────────────────
> print(SigmaUV.tilde)
```

```
────────────────── output ──────────────────
                [,1]
Response Y1 -0.1459141
Response Y2  3.6142237
Response Y3  0.3125815
```

```
────────────────── R code ──────────────────
> print(lambda.tilde)
```

```
────────────────── output ──────────────────
[1] 0.04244982
```

```
────────────────── R code ──────────────────
> print(T * log(1 - lambda.tilde))
```

```
────────────────── output ──────────────────
[1] -8.198283
```

```
────────────────── R code ──────────────────
> print(LRT)
```

```
────────────────── output ──────────────────
[1] 13.92444
```

21 Time Series Models of Heteroskedasticity

21.1 Preamble

Page 658 and forward provide examples of ARCH models. Several utility functions are needed for these examples. The function "arch.fitted.values" calculates the value of ht given the conditional information set YT and a parameter vector THETA as described on page 660, [21.1.17] to [21.1.20].

```R
_____ R code _____
> arch.fitted.values <- function(THETA, YT) {
+     alpha <- THETA[grep("alpha.*", names(THETA))]
+     beta <- THETA[grep("beta.*", names(THETA))]
+     zeta <- THETA["zeta"]
+     m <- length(alpha)
+     h <- rep(as.vector(zeta), m)
+     u <- YT$y - YT$x %*% beta
+     indices <- (m + 1):length(YT$y)
+     z <- array(0, c(length(indices), 1 + m))
+     for (tt in indices) h[tt] <- t(c(zeta, alpha)) %*% c(1, u[(tt -
+         1):(tt - m)]^2)
+     list(u = u, h = h)
+ }
```

Function "arch.standard.errors" calculates values for standard errors according to the description on page 663, particularly equations [21.1.25], and also using [21.1.21] for the estimate of the outer product estimate of the information matrix.

```R
_____ R code _____
> arch.standard.errors <- function(THETA, YT) {
+     x <- YT$x
+     y <- YT$y
+     k <- dim(x)[[2]]
+     alpha <- THETA[grep("alpha.*", names(THETA))]
+     zeta <- THETA["zeta"]
+     m <- length(alpha)
+     T <- length(y) - m
+     a <- k + 1 + m
+     fv <- arch.fitted.values(THETA, YT)
+     h <- fv$h
+     u <- fv$u
+     u2 <- u^2
```

```
+       S <- array(0, c(a, a))
+       D <- array(0, c(a, a))
+       for (tt in (m + 1):length(y)) {
+           temp <- c(t(alpha) %*% ((u2[(tt - 1):(tt - m)] %o% rep(1,
+               k)) * x[(tt - 1):(tt - m), ]), c(1, u[(tt - 1):(tt -
+               m)]^2))
+           st <- (u2[tt] - h[tt])/(2 * h[tt]^2) * temp + c(u2[tt]/h[tt] *
+               x[tt, ], rep(0, a - k))
+           S <- S + 1/T * st %*% t(st)
+           D <- D + 1/T * (1/(2 * h[tt]^2) * temp %*% t(temp) +
+               rbind(cbind(1/h[tt] * x[tt, ] %*% t(x[tt, ]), array(0,
+                   c(k, a - k))), array(0, c(a - k, a))))
+       }
+       diag(1/T * solve(D) %*% S %*% solve(D))^0.5
+ }
```

The following two helper functions calculate the likelihood values under different distributional assumptions. The normal likelihood is calculated according to [21.1.20], the scaled t according to [21.1.24].

```
———————————————————— R code ————————————————————
> arch.normal <- function(THETA, YT) {
+     fv <- arch.fitted.values(THETA, YT)
+     m <- length(THETA[grep("alpha.*", names(THETA))])
+     h <- fv$h[-1:-m]
+     u <- fv$u[-1:-m]
+     -1/2 * (length(h) * log(2 * pi) - sum(log(h)) - sum(u^2/h))
+ }
> arch.scaled.t <- function(THETA, YT) {
+     fv <- arch.fitted.values(THETA, YT)
+     m <- length(THETA[grep("alpha.*", names(THETA))])
+     h <- fv$h[-1:-m]
+     u <- fv$u[-1:-m]
+     nu <- THETA[grep("nu", names(THETA))]
+     result <- length(h) * log(gamma((nu + 1)/2)/(sqrt(pi) * gamma(nu/2)) *
+         (nu - 2)^-0.5) - 1/2 * sum(log(h)) - (nu + 1)/2 * sum(log(1 +
+         u^2/(h * (nu - 2))))
+ }
```

GMM estimates are calculated according to the recipe in Chapter 14, notably equations [14.1.7] and [14.1.10]. Functions h and S are specified by the caller.

```
———————————————————— R code ————————————————————
```

```
> GMM.estimates <- function(YT, h, THETA, S) {
+     g <- function(YT, THETA) {
+         apply(X = apply(X = YT, MARGIN = 1, FUN = h, THETA = THETA),
+             MARGIN = 1, FUN = mean)
+     }
+     objective <- function(THETA, YT, W) {
+         g.value <- g(YT, THETA)
+         as.numeric(t(g.value) %*% W %*% g.value)
+     }
+     r <- length(h(YT[1, ], THETA))
+     a <- length(THETA)
+     stage.1.results <- optim(par = THETA, fn = objective, gr = NULL,
+         YT = YT, W = diag(r))
+     temp <- t(apply(X = YT, MARGIN = 1, FUN = h, THETA = stage.1.results$par))
+     ST <- S(temp)
+     stage.2.results <- optim(par = stage.1.results$par, fn = objective,
+         gr = NULL, YT = YT, W = solve(ST))
+     list(stage.1.results = stage.1.results, stage.2.results = stage.2.results)
+ }
```

21.2   Application of ARCH Models to US Fed Funds Data

The dataset for these examples is the US Fed Funds Rate, monthly between Jan 1955 and December 2000, shown below.

```
────────────────────────── R code ──────────────────────────
> data(fedfunds, package = "RcompHam94")
> selection <- window(fedfunds, start = c(1955, 1), end = c(2000,
+     12))
────────────────────────────────────────────────────────────
```
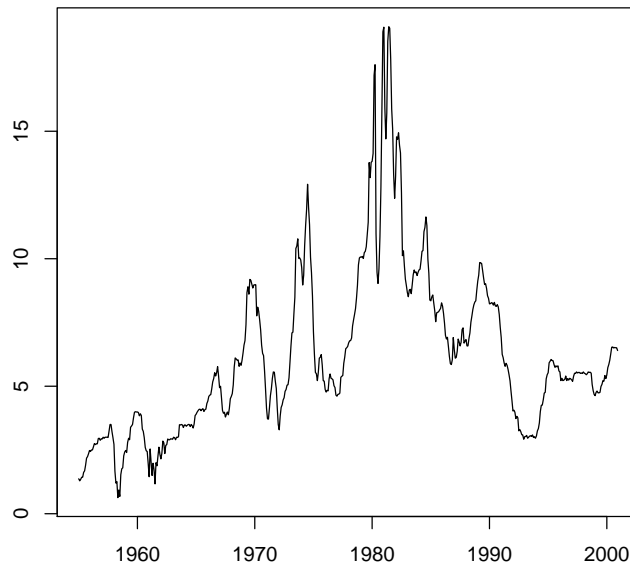
Figure 21.1 – US Fed Funds Rate

A first step is
to characterize the autocorrelation structure of the squared residuals. These
two regressions show that a second order AR process seems to fit the data
pretty well.

```
──────────────────────────── R code ────────────────────────────
> y.lm <- dynlm(y ~ 1 + L(y), data = zooreg(cbind(y = as.vector(selection[,
+     "FFED"])))))
> u2.lms <- summary(dynlm(u2 ~ 1 + L(u2, 1:4), zooreg(data.frame(u2 = y.lm$residuals^2))))
> F34 <- Wald.F.Test(R = cbind(rep(0, 2) %o% rep(0, 3), diag(2)),
+     b = u2.lms$coefficients[, "Estimate"], r = c(0, 0), s2 = u2.lms$sigma^2,
+     XtX_1 = u2.lms$cov.unscaled)
> F34.sig <- 1 - pf(F34, 2, u2.lms$df[[2]])
> F234 <- Wald.F.Test(R = cbind(rep(0, 3) %o% rep(0, 2), diag(3)),
+     b = u2.lms$coefficients[, "Estimate"], r = c(0, 0, 0), s2 = u2.lms$sigma^2,
+     XtX_1 = u2.lms$cov.unscaled)
> F234.sig <- 1 - pf(F234, 3, u2.lms$df[[2]])
> accept.arch <- pchisq(length(u2.lms$residuals) * u2.lms$r.squared,
+     4)
> print(F34)
```

```
──────────────────────────── output ────────────────────────────
[1] 0.8225742
```

73

```R code
> print(F34.sig)
```

```output
[1] 0.439847
```

```R code
> print(F234)
```

```output
[1] 11.88167
```

```R code
> print(F234.sig)
```

```output
[1] 1.513714e-07
```

```R code
> print(accept.arch)
```

```output
[1] 1
```

Next we use a maximum likelihood estimation to estimate the parameters for the second order equation assuming normal errors.

```R code
> y <- as.vector(selection[, "FFED"])
> YT <- list(y = y[-1], x = cbind(rep(1, length(y) - 1), y[-length(y)]))
> THETA <- c(beta = y.lm$coefficients, zeta = var(y.lm$residuals),
+     alpha = c(0.1, 0.1))
> optimizer.results <- optim(par = THETA, fn = arch.normal, gr = NULL,
+     YT = YT)
> print(optimizer.results$par)
```

```output
beta.(Intercept)       beta.L(y)            zeta          alpha1
      0.25226382      0.94858488      0.02734929      0.95530391
          alpha2
      0.29858866
```

```
―――――――――――――――――――――――――――――― R code ――――――――――――――――――――
> se <- arch.standard.errors(optimizer.results$par, YT)
> print(se)
```

```
―――――――――――――――――――――――――――――― output ――――――――――――――――――――
[1] 0.048149374 0.010283478 0.005256627 0.164308997 0.082197566
```

Now use GMM to estimate the same parameters following page 664. The initial values for the regression coefficients are derived from the (homoskedastic) regression above, as is the presample variance. The estimator for S assumes no correlation at leads and lags.

```
―――――――――――――――――――――――――――――― R code ――――――――――――――――――――
> h <- function(wt, THETA) {
+     beta <- THETA[grep("beta.*", names(THETA))]
+     zeta <- THETA["zeta"]
+     alpha <- THETA[grep("alpha.*", names(THETA))]
+     m <- length(alpha)
+     k <- length(beta)
+     yt <- wt[grep("yt.*", names(wt))]
+     xt <- wt[grep("xt.*", names(wt))]
+     ylagt <- wt[grep("ylagt.*", names(wt))]
+     xlagt <- t(array(wt[grep("xlagt.*", names(wt))], c(k, m)))
+     ut <- yt - t(xt) %*% beta
+     zt <- c(1, (ylagt - t(xlagt) %*% beta)^2)
+     c(ut * xt, (ut^2 - t(zt) %*% c(zeta, alpha)) * zt)
+ }
> S.estimator <- function(ht) {
+     1/dim(ht)[[1]] * t(ht) %*% ht
+ }
> THETA <- c(beta = y.lm$coefficients, zeta = var(y.lm$residuals),
+     alpha = c(0.1, 0.1))
> m <- length(THETA[grep("alpha.*", names(THETA))])
> T <- length(YT$y) - m
> w <- as.matrix(data.frame(yt = YT$y[-1:-m], xt = YT$x[-1:-m,
+     ], ylagt = embed(YT$y[-(T + m)], m), xlagt = embed(YT$x[-(T +
+     m), ], m)))
> estimates <- GMM.estimates(YT = w, h = h, THETA = THETA, S.estimator)
> print(estimates$stage.1.results$par)
```

```
―――――――――――――――――――――――――――――― output ――――――――――――――――――――
beta.(Intercept)          beta.L(y)              zeta             alpha1
      0.05788674         0.98955937        0.32491651         0.01073606
```

```
            alpha2
        0.02105476
```

```
> print(estimates$stage.2.results$par)
```

```
beta.(Intercept)        beta.L(y)            zeta            alpha1
      0.02579794       0.99791508      -0.17911928        0.01239927
            alpha2
        0.07770754
```

21.3   R Facilities For GARCH models

TBD

22   Modeling Time Series with Changes in Regime

22.1   Statistical Analysis of i.i.d. Mixture Distributions

Figure 22.2:

```
> curve(0.8 * dnorm(x, 0, 1), from = -2, to = 8, n = 100, col = 1,
+     ylab = "f(x)", main = "Density of mixture of 2 gaussians")
> curve(0.2 * dnorm(x, 4, 1), from = -2, to = 8, n = 100, add = TRUE,
+     col = 3)
> mixture <- function(x) 0.8 * dnorm(x, 0, 1) + 0.2 * dnorm(x,
+     4, 1)
> curve(mixture, from = -2, to = 8, n = 100, col = 2, add = TRUE)
```
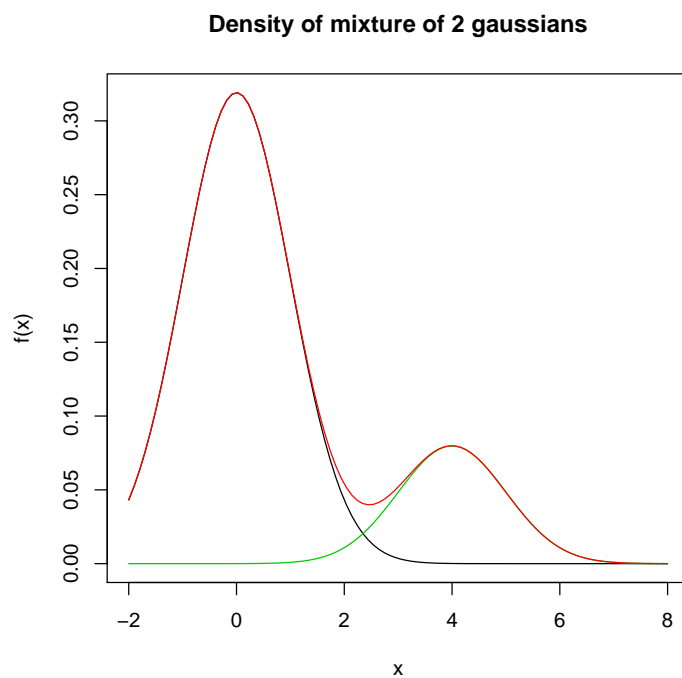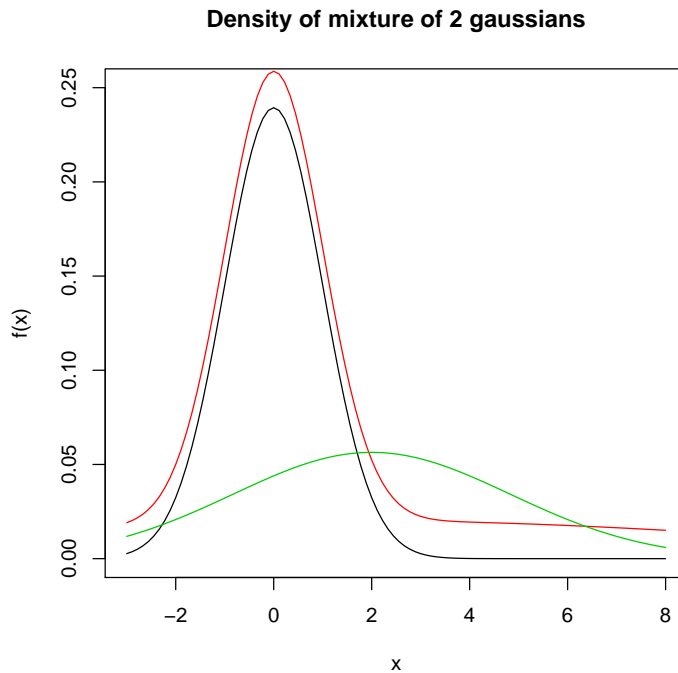
Density of mixture of 2 gaussians

Figure 22.3:

```
─────────────────────────── R code ───────────────────────────
> mixture2 <- function(x) 0.6 * dnorm(x, 0, 1) + 0.4 * dnorm(x,
+     2, 8)
> curve(mixture2, from = -3, to = 8, n = 100, col = 2, ylab = "f(x)",
+     main = "Density of mixture of 2 gaussians", ylim = c(0, 0.25))
> curve(0.6 * dnorm(x, 0, 1), from = -3, to = 8, n = 100, col = 1,
+     add = TRUE)
> curve(0.4 * dnorm(x, 2, sqrt(8)), from = -3, to = 8, n = 100,
+     add = TRUE, col = 3)
```

Density of mixture of 2 gaussians

22.2   Modeling Changes in Regime

Page 697 describes an example of the application of Markov switching models to US GNP from 1951Q1 to 1984Q4.

```
—————————————————— R code ——————————————————
> data(gnpdata, package = "RcompHam94")
> selection <- window(gnpdata, start = c(1951, 1), end = c(1984,
+     2))
> g <- diff(100 * log(as.vector(selection[, "GNP"])))
> d <- index(selection[-1])
```

The actual implementation uses the technique of collapsing multi-period states into a single state, p691, p698. During the maximum likelihood estimation process the state probabilities will change, but the layout of the matrix is still the same. The following code fragment precalculates the transition matrix structure with the five possible values, then uses a separate 5 element lookup vector to populate it.

```
—————————————————— R code ——————————————————
> nlags <- 4
> nstates <- 2^(nlags + 1)
> lagstate <- 1 + outer(1:nstates, 1:(nlags + 1), FUN = function(i,
```

```
+      j) {
+      trunc((i - 1)/2^(nlags + 1 - j))%%2
+ })
> head(lagstate)
```

<div style="text-align: center;">output</div>

```
      [,1] [,2] [,3] [,4] [,5]
[1,]    1    1    1    1    1
[2,]    1    1    1    1    2
[3,]    1    1    1    2    1
[4,]    1    1    1    2    2
[5,]    1    1    2    1    1
[6,]    1    1    2    1    2
```

<div style="text-align: center;">R code</div>

```
> transit <- outer(X = 1:nstates, Y = 1:nstates, FUN = function(i,
+      j) {
+      ((2 * lagstate[i, 1] + lagstate[j, 1] - 1) - 1) * (((i -
+          1)%%(2^nlags)) == trunc((j - 1)/2)) + 1
+ })
> head(transit)
```

<div style="text-align: center;">output</div>

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11] [,12] [,13] [,14]
[1,]    2    2    1    1    1    1    1    1    1     1     1     1     1     1
[2,]    1    1    2    2    1    1    1    1    1     1     1     1     1     1
[3,]    1    1    1    1    2    2    1    1    1     1     1     1     1     1
[4,]    1    1    1    1    1    1    2    2    1     1     1     1     1     1
[5,]    1    1    1    1    1    1    1    1    2     2     1     1     1     1
[6,]    1    1    1    1    1    1    1    1    1     1     2     2     1     1
      [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22] [,23] [,24] [,25] [,26]
[1,]     1     1     1     1     1     1     1     1     1     1     1     1
[2,]     1     1     1     1     1     1     1     1     1     1     1     1
[3,]     1     1     1     1     1     1     1     1     1     1     1     1
[4,]     1     1     1     1     1     1     1     1     1     1     1     1
[5,]     1     1     1     1     1     1     1     1     1     1     1     1
[6,]     1     1     1     1     1     1     1     1     1     1     1     1
      [,27] [,28] [,29] [,30] [,31] [,32]
[1,]     1     1     1     1     1     1
[2,]     1     1     1     1     1     1
[3,]     1     1     1     1     1     1
[4,]     1     1     1     1     1     1
```

```
[5,]     1     1     1     1     1     1
[6,]     1     1     1     1     1     1
```

The bulk of the work is done by the following function, based on the algorithm
in section 22.4. Ergodic probabilities are defined as on page 684, including
equation [22.2.26]. The loop uses equations [22.4.24], [22.4.2], [22.4.5], [22.4.8],
[22.4.7], [22.4.6] and [22.4.14].

```r
─────────────────────────────── R code ───────────────────────────────
> infer.regimes <- function(THETA, YT) {
+     phi <- THETA[grep("phi.*", names(THETA))]
+     mu <- THETA[grep("mu.*", names(THETA))]
+     sigma <- THETA["sigma"]
+     p11star <- THETA["p11star"]
+     p22star <- THETA["p22star"]
+     T <- length(YT)
+     tp <- c(0, p11star, 1 - p22star, 1 - p11star, p22star)
+     P <- array(tp[transit], c(nstates, nstates))
+     A <- rbind(diag(nstates) - P, rep(1, nstates))
+     ergodic.pi <- (solve(t(A) %*% A) %*% t(A))[, nstates + 1]
+     xi.t.t <- ergodic.pi %o% rep(1, nlags)
+     xi.t.t_1 <- xi.t.t
+     log.likelihood <- 0
+     for (tt in (nlags + 1):T) {
+         residuals <- as.vector(((rep(1, nstates) %o% YT[tt:(tt -
+             nlags)]) - array(mu[lagstate], c(nstates, nlags +
+             1))) %*% c(1, -phi))
+         eta.t <- dnorm(residuals, mean = 0, sd = sigma)
+         fp <- eta.t * xi.t.t_1[, tt - 1]
+         fpt <- sum(fp)
+         xi.t.t <- cbind(xi.t.t, fp/fpt)
+         log.likelihood <- log.likelihood + log(fpt)
+         xi.t.t_1 <- cbind(xi.t.t_1, P %*% xi.t.t[, tt])
+     }
+     xi.t.T <- xi.t.t[, T] %o% 1
+     for (tt in (T - 1):1) xi.t.T <- cbind(xi.t.t[, tt] * (t(P) %*%
+         (xi.t.T[, 1]/xi.t.t_1[, tt])), xi.t.T)
+     list(log.likelihood = log.likelihood, xi.t.t = xi.t.t, xi.t.T = xi.t.T)
+ }
──────────────────────────────────────────────────────────────────────
```

Initial values of the parameters for transition probabilities are set from historical
averages. The phi and sigma values are obtained from a (non-state) regression
of change in GDP on 4 of its own lags.

```R code
> g.lm <- dynlm(g ~ 1 + L(g, 1:4), data = zooreg(data.frame(g = g)))
> THETA <- c(p11star = 0.85, p22star = 0.7, mu = c(1, 0), phi = as.vector(g.lm$coefficients[1 +
+       (1:nlags)]), sigma = summary(g.lm)$sigma)
```

Now we are in a position to optimize, then calculate the smoothed probabilities from the optimal parameters.

```R code
> objective <- function(THETA, YT) {
+       -infer.regimes(THETA, YT)$log.likelihood
+ }
> optimizer.results <- optim(par = THETA, hessian = TRUE, fn = objective,
+       gr = NULL, YT = as.vector(g), method = "BFGS")
> se <- diag(solve(optimizer.results$hessian))^0.5
> print(optimizer.results$par)
```

```output
     p11star       p22star          mu1          mu2         phi1          phi2
  0.90030933    0.76062170   1.17515197  -0.31750266   0.02262260  -0.02950457
        phi3          phi4        sigma
 -0.22818176   -0.20243029   0.77954523
```

```R code
> print(se)
```

```output
     p11star       p22star          mu1          mu2          phi1         phi2          phi3
  0.04022558    0.09745502   0.08379353   0.27312797    0.12911244   0.14402459   0.11136972
        phi4         sigma
  0.11306913    0.06950831
```

```R code
> regimes <- infer.regimes(optimizer.results$par, as.vector(g))
> recession.probability <- as.vector((1:nstates > nstates/2) %*%
+       regimes$xi.t.t)
> smoothed.recession.probability <- as.vector((1:nstates > nstates/2) %*%
+       regimes$xi.t.T)
```
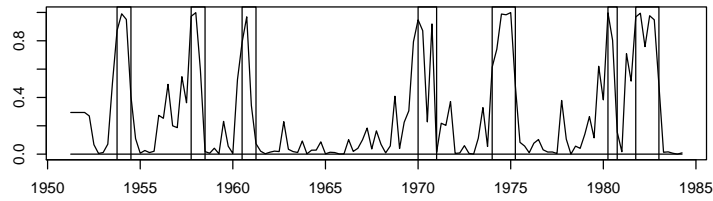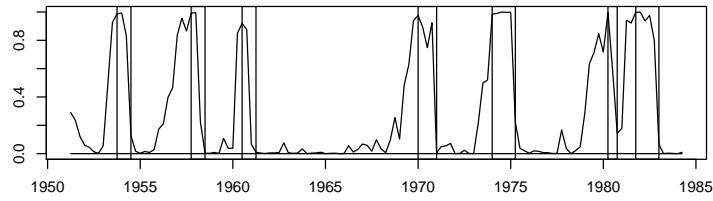
The results are shown below.

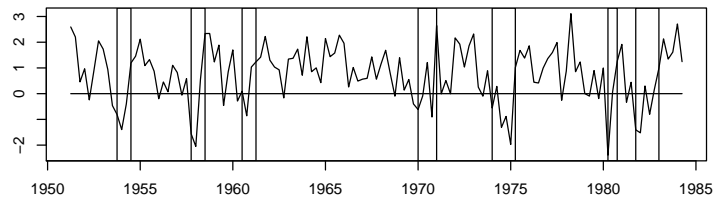Figure 22.4a

Smoothed recession probabilities

Figure 22.4b

References

DE MICHEAUX, P. L., AND B. LIQUET (2009): "ConvergenceConcepts : an R Package to Investigate Various Modes of Convergence," *The R Journal*, 1(2), 18–25.

ELLIOTT, G., T. ROTHENBERG, AND J. STOCK (1996): "Efficient Tests for an Autoregressive Unit Root," *Econometrica*, 64(4), 813–836.

HAMILTON, J. (1994): *Time series analysis*. Princeton University Press.

HANSEN, B. E. (1995): "Rethinking the Univariate Approach to Unit Root Testing: Using Covariates to Increase Power," *Econometric Theory*, 11(05), 1148–1171.

KLEIBER, C., AND A. ZEILEIS (2008): *Applied Econometrics with R*. Springer-Verlag, New York.

KWIATKOWSKI, D., P. PHILLIPS, P. SCHMIDT, AND Y. SHIN (1992): "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root?," *Journal of Econometrics*, 54, 159–178.

Lupi, C. (2009): "Unit Root CADF Testing with R," *Journal of Statistical Software*, 32(2)(i02).

Maddala, G. S., and I.-M. Kim (1998): *Unit roots, Cointegration and Structural Change.* Cambridge University Press.

Ng, S., and P. Perron (2001): "LAG Length Selection and the Construction of Unit Root Tests with Good Size and Power," *Econometrica*, 69(6), 1519–1554.

Perron, P., and Z. Qu (2007): "A simple modification to improve the finite sample properties of Ng and Perron's unit root tests," *Economics Letters*, 94(1), 12–19.

Pfaff, B. (2008): *Analysis of Integrated and Cointegrated Time Series with R.* Springer, New York, second edn., ISBN 0-387-27960-1.

Phillips, P. C. B., and Z. Xiao (1998): "A Primer on Unit Root Testing," *Journal of Economic Surveys*, 12(5), 423–69.