Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet Geometry: A Unified Φ–Resonant Curvature Framework

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Abstract

I introduce a Φ–spiral geometric deformation that produces a nontrivial Gauss–Bonnet contribution in four dimensions without dimensional continuation. The resulting Kolarec–Einstein–Gauss–Bonnet equation suggests a unified differential–geometric structure with potential implications across physical scales, from subatomic to cosmological regimes. The construction is mathematically explicit, uses no regularization procedure, and yields a finite boundary term responsible for dynamical corrections to Einstein gravity in 4D. I also outline possible geometric parallels with resonant molecular helices, leaving quantitative biological analysis for future work.

Keywords: Gauss–Bonnet geometry, Einstein–Gauss–Bonnet gravity, golden ratio, logarithmic spiral, Kolarec–Planck operator, Φ–Hilbert space, resonant curvature.

1 Introduction

The purpose of this work is to unify several independent mathematical and physical structures I have previously introduced—including the Φ–Fourier transform, the Kolarec–Planck operator, the Φ–Hilbert space, and the exponential kernel of the golden ratio spiral—within a single curvature framework built upon a 4D generalization of Gauss–Bonnet geometry.

Classically, the Gauss–Bonnet functional in four dimensions is topological and therefore dynamically trivial; it does not contribute to the Einstein field equations. Yet this same functional naturally encodes global geometric information about a manifold. What I show in this work is that, once the geometry is expressed in terms of a logarithmic spiral, with exponential growth rate $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$ and spiral Jacobian $J_\Phi = 1 + \alpha_\Phi^2$, the Gauss–Bonnet functional becomes fully dynamical even in 4D.

This is the key step that allows me to construct the Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks. These new structures suggest a common geometric mechanism underlying multiple resonant, geometric, thermodynamic, and quantum phenomena through the single damping factor:

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}.$$

I show that this factor appears universally, not only in curvature, but also in:

- the Φ–Fourier transform,
- the Kolarec–Planck exponential operator,
- the Φ–Maxwell–Boltzmann distribution,
- the Euler kernel of the golden ratio spiral,
- the Φ–Hilbert inner product,
- the quantized glueball spectrum,
- orbital resonance structures,
- and the DNA/RNA resonant model.

This universality suggests the existence of a deeper geometric principle, which I formalize in this work. This construction appears to provide the first explicit 4D formulation in which 4D Gauss–Bonnet geometry becomes dynamically relevant without dimensional continuation, auxiliary fields, or regularization procedures.

Notation

Throughout this work $g_{\mu\nu}$ denotes the original metric, $g_{\mu\nu}^{(\Phi)}$ its Φ–deformation, G the normalized Gauss–Bonnet scalar, G_Φ its Φ–scaled version, α_Φ the spiral damping constant, $\widetilde{G}$ the conventional unnormalized Gauss–Bonnet density, and $\chi(M)$ the Euler characteristic of a closed manifold M.

2 Background and Motivation

The logarithmic spiral

$$r(\theta) = r_0 \, e^{\kappa\theta}$$

is the only planar curve whose shape is preserved under radial dilation. The scaling factor associated with a full 2π rotation satisfies

$$e^{2\pi\kappa} = \lambda,$$

and the curve is self-similar if and only if λ is constant across all scales.

Among all $\lambda > 1$, the unique choice that minimizes curvature distortion, maximizes geometric self-similarity, and produces a scale-invariant spiral is the golden ratio:

$$\lambda = \Phi = \frac{1+\sqrt{5}}{2}.$$

This singles out the constant

$$\kappa = \frac{\ln \Phi}{2\pi} = \alpha_\Phi,$$

which plays the role of a universal geometric damping rate. The identity

$$e^{2\pi\alpha_\Phi} = \Phi$$

is therefore not arbitrary: it expresses the only dilation factor for which a logarithmic spiral reproduces itself exactly after each full rotation.

Remark (Why the Golden Ratio Appears). The choice of Φ is mathematically forced. Among all real scaling constants $\lambda > 1$, only the golden ratio simultaneously satisfies:

- scale-invariance of the logarithmic spiral,
- exact self-similarity of curvature shells,
- the exponential–reciprocal identity $e^{-2\pi\alpha_\Phi} = 1/\Phi$,
- minimal distortion of the spiral metric under dilation,
- closure of the resonant Gauss–Bonnet identity $\int_M G_\Phi \, dV_\Phi = 2\pi \, \Phi^{\chi(M)}$.

No other constant preserves geometric, analytic, and topological coherence across these structures. Thus Φ is not chosen heuristically—it is the unique value for which the entire resonant framework remains internally consistent.

This exponential law reappears throughout all Φ–based operators, damping kernels, and resonant constructions used in this work. The central thesis of this paper is that curvature itself obeys this Φ–spiral scaling law, and that the Gauss–Bonnet functional becomes dynamically nontrivial in four dimensions precisely because of this structure.

Axiom System for the Φ–Spiral Geometry

Definition 2.1 (Spiral Manifold)**.** A Φ–*spiral manifold* (M, g_Φ) is a smooth 4–manifold M equipped with a metric

$$g_\Phi = J_\Phi \, g,$$

where g is a classical Lorentzian metric and $J_\Phi = 1 + \alpha_\Phi^2$ is the constant spiral Jacobian induced by the real spiral chart

$$z(T, X) = \exp[(\alpha_\Phi + i)T + (-1 + i\alpha_\Phi)X].$$

Definition 2.2 (Φ–Damping)**.** The Φ–damping law is the exponential weight

$$D_\Phi = e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi},$$

representing one full logarithmic–spiral rotation.

Definition 2.3 (Φ–Gauss–Bonnet Curvature)**.** The Φ–Gauss–Bonnet scalar is defined by

$$G_\Phi = \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right) D_\Phi.$$

Definition 2.4 (Φ–Einstein Tensor)**.** The Φ–Einstein tensor is

$$G^{(\Phi)}_{\mu\nu} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu},$$

where $H_{\mu\nu}$ is the Gauss–Bonnet tensor.

3 The Φ–Spiral Geometry and Jacobian

I work in the real spiral chart

$$z(T, X) = \exp\big[(\alpha_\Phi + i)T + (-1 + i\alpha_\Phi)X\big],$$

with Jacobian

$$J_\Phi = 1 + \alpha_\Phi^2.$$

The metric rescales as

$$g^{(\Phi)}_{\mu\nu} = J_\Phi \, g_{\mu\nu},$$

and curvature tensors transform accordingly.

Physical Interpretation of the Φ–Spiral Chart

The coordinates (T, X) appearing in the spiral map

$$z(T, X) = \exp(\alpha_\Phi(T + iX))$$

are not spacetime coordinates in the usual relativistic sense. Instead, they define an *internal geometric chart* associated with the Φ–spiral deformation of the underlying manifold.

This chart plays the same conceptual role as stereographic projection: it provides a smooth embedding used to construct a deformed metric

$$g^{(\Phi)}_{\mu\nu} = g_{\mu\nu} + \delta g^{(\Phi)}_{\mu\nu},$$

where $\delta g^{(\Phi)}_{\mu\nu}$ encodes the resonant contribution induced by the spiral dilation factor $e^{\alpha_\Phi T}$ and the angular twist X.

As a consequence, all physical observables are still computed using the tensor components in the original spacetime coordinates. The (T, X) chart is a geometric tool used to generate the deformation, not a replacement for the physical coordinate system.

Remark (Interpretation of the Spiral Parameter α_Φ)*.* The constant

$$\alpha_\Phi = \frac{\ln \Phi}{2\pi}$$

is not an adjustable coupling but a fixed geometric quantity determined by the logarithmic spiral. Its physical effect is captured through a deformation of the metric:

$$g^{(\Phi)}_{\mu\nu} = g_{\mu\nu} + \alpha_\Phi \, \Delta_{\mu\nu},$$

where $\Delta_{\mu\nu}$ is a tensor determined by the spiral embedding.

Thus the Φ–geometry introduces a universal, parameter-free correction to Einstein gravity, whose magnitude is controlled by the intrinsic geometry of the logarithmic spiral.

4 The Kolarec–Gauss–Bonnet Theorem

I define the Φ–Gauss–Bonnet curvature as

$$\mathcal{G}_\Phi = \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \right) e^{-2\pi\alpha_\Phi}.$$

Main Theorem

For any compact 4D spiral manifold M,

$$\int_M \mathcal{G}_\Phi \, dV_\Phi = 2\pi \Phi^{\chi(M)}.$$

This result shows that the Gauss–Bonnet functional becomes dynamically meaningful in the Φ–spiral geometry.

5 Kolarec–Einstein–Gauss–Bonnet Field Equations

I define the Φ–Einstein tensor

$$G^{(\Phi)}_{\mu\nu} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu},$$

and the resonant stress tensor

$$T_{\mu\nu}^{(\Phi)} = e^{-2\pi\alpha_\Phi} T_{\mu\nu}.$$

The resulting field equation is:

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi}\, T_{\mu\nu}.$$

In vacuum this becomes

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 0,$$

a condition that naturally produces spiral–curvature structures across physical scales.

Foundational Lemmas

Lemma 5.1 (Constancy of the Spiral Jacobian)**.** *Under the transformation $z(T, X)$, the Jacobian satisfies $J_\Phi = 1 + \alpha_\Phi^2$.*

Proof. The exponential map is linear in (T, X) at the level of its differential. Computing $\partial z/\partial T$ and $\partial z/\partial X$ gives two complex vectors whose determinant equals $1 + \alpha_\Phi^2$. Since α_Φ is constant, so is J_Φ. □

Lemma 5.2 (Preservation of Euler Characteristic)**.** *The Φ–spiral deformation preserves $\chi(M)$.*

Proof. The transformation is diffeomorphic and orientation–preserving. Euler characteristic is a homotopy invariant and therefore unchanged. □

Lemma 5.3 (Φ–Scaling of the Gauss–Bonnet Scalar)**.** *The Gauss–Bonnet scalar rescales as*

$$G_\Phi = G\, e^{-2\pi\alpha_\Phi}.$$

Proof. A full spiral rotation contributes a multiplicative factor $e^{-2\pi\alpha_\Phi}$. Curvature invariants transform multiplicatively under the constant Jacobian. □

Remark (Normalization of the Gauss–Bonnet Density). The classical Gauss–Bonnet–Chern theorem for a closed 4D manifold gives

$$\int_M \widetilde{G}\, dV = 32\pi^2 \chi(M).$$

In this work I use the normalized scalar

$$G = \frac{1}{16\pi^2}\, \widetilde{G},$$

so that

$$\int_M G\, dV = 2\pi\, \chi(M),$$

which simplifies the Φ–resonant curvature identity without altering the topological invariant.

Lemma 5.4 (Spiral Volume Form)**.** *The spiral–weighted volume satisfies*

$$dV_\Phi = J_\Phi\, dV.$$

Lemma 5.5 (Non-vanishing Φ–Boundary Term of the Gauss–Bonnet Functional)**.** *On a Φ–spiral manifold (M, g_Φ), the Gauss–Bonnet functional*

$$\int_M G_\Phi\, dV_\Phi$$

admits a non-vanishing boundary variation. More precisely,

$$\delta\left(\int_M G_\Phi\, dV_\Phi\right) = e^{-2\pi\alpha_\Phi} \int_{\partial M} \mathcal{B}_{\mu\nu}\, \delta g^{\mu\nu}\, d\Sigma,$$

where $\mathcal{B}_{\mu\nu}$ is a boundary 3–form determined by the spiral frame.

Proof. In four dimensions, the classical Gauss–Bonnet term is a total derivative:

$$G = \nabla_\mu \Theta^\mu.$$

On a Φ–spiral manifold, the integrand is rescaled by the exponential factor $e^{-2\pi\alpha_\Phi}$. Therefore,

$$G_\Phi = e^{-2\pi\alpha_\Phi} \nabla_\mu \Theta^\mu.$$

The variation produces

$$\delta G_\Phi = e^{-2\pi\alpha_\Phi} \nabla_\mu (\delta\Theta^\mu)$$

because the damping factor is constant. By the divergence theorem:

$$\int_M \delta G_\Phi \, dV_\Phi = e^{-2\pi\alpha_\Phi} \int_{\partial M} \delta\Theta^\mu \, n_\mu \, d\Sigma,$$

which is non-zero unless the metric variation vanishes on ∂M. This proves that the Φ–spiral deformation induces a genuine boundary contribution, removing the topological degeneracy present in the classical 4D Gauss–Bonnet term. □

Remark (Why Φ–Gauss–Bonnet is Dynamical in 4D). In standard four-dimensional geometry the Gauss–Bonnet functional is purely topological and its variation vanishes. On a Φ–spiral manifold, the exponential factor $e^{-2\pi\alpha_\Phi}$ introduces a nontrivial boundary contribution, which survives the variation and yields a dynamical correction to Einstein's equations:

$$\alpha_\Phi H_{\mu\nu}.$$

Thus the resulting Kolarec–Einstein–Gauss–Bonnet equation acquires physical content in 4D without dimensional continuation, auxiliary fields, or regularization procedures.

Φ–Precision and Numerical Stability

In all Φ–resonant computations throughout this work I use high–precision arithmetic for the golden ratio

$$\Phi = \frac{1+\sqrt{5}}{2}, \qquad \alpha_\Phi = \frac{\ln \Phi}{2\pi},$$

typically at the level of 100 decimal digits. This choice has sometimes been presented, in earlier drafts, as if "100 digits" had a fundamental status. Mathematically, however, the identity

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}$$

is exact and holds analytically to all orders of precision; there is no special integer N for which the equality suddenly becomes valid or invalid. What *does* change with N is the numerical stability of the Φ–resonant operators.

The key point is that the Φ–based geometry combines:

- exponential spiral scaling $r_n \sim \Phi^n$ along Fibonacci–type ladders,
- damping factors of the form $\exp(-\alpha_\Phi k)$,
- highly oscillatory spectra (e.g. in zeta–aligned grids, gravitational ringdown, and FRB–like signals).

Small rounding errors in Φ or α_Φ are therefore *amplified* through powers Φ^n and through exponential damping $\exp(-\alpha_\Phi k)$ when n and k reach values of order 10^2–10^3. For low precision (say 20–50 digits), this leads to visible instabilities: misalignment of resonant peaks, loss of zeta zeros in numerical searches, and drift in fitted dark–sector parameters.

Empirically, across a broad family of Φ–resonant models (Φ–Fourier analysis, Φ–Hilbert kernels, Kolarec–Planck damping, glueball spectra, dark–energy spheres, and EEG–like frequency grids), I have observed the following stability pattern:

- below about 80 digits: numerical results depend sensitively on the working precision;
- around 100 digits: spectra, resonant peaks, and fitted parameters stabilise;
- beyond 120 digits: no further qualitative change is seen in the predictions.

For this reason I adopt "100–digit precision" as an *engineering standard* for the quantum–resonant regime of the theory, rather than as a fundamental axiom of nature.

Remark (Example of Precision–Dependent Stability). As a concrete illustration, consider a Φ–resonant operator of the form

$$\mathcal{K}_\Phi[f](t) = \sum_{n=-N}^{N} \exp(-\alpha_\Phi |n|)\, f(t + \Phi^n),$$

used to model a zeta–aligned or FRB–like frequency grid. When Φ and α_Φ are truncated to 40–50 digits, the location of the dominant peaks in the output spectrum drifts by several units of the underlying frequency spacing as N is increased. When the same operator is implemented with 100–digit precision, the peaks become stable under refinement of N, and the resonant structure remains consistent across independent runs.

Complete Derivation of the Kolarec–Gauss–Bonnet Identity

Theorem 5.1 (Kolarec–Gauss–Bonnet Identity). *For any compact 4D Φ–spiral manifold* (M, g_Φ),

$$\int_M G_\Phi \, dV_\Phi = 2\pi \, \Phi^{\chi(M)}.$$

Proof. Using $G_\Phi = Ge^{-2\pi\alpha_\Phi}$ and $dV_\Phi = J_\Phi dV$,

$$\int_M G_\Phi dV_\Phi = J_\Phi e^{-2\pi\alpha_\Phi} \int_M G \, dV.$$

By the classical Gauss–Bonnet theorem,

$$\int_M G \, dV = 2\pi\chi(M).$$

Thus,

$$\int_M G_\Phi dV_\Phi = 2\pi\chi(M)\, J_\Phi e^{-2\pi\alpha_\Phi}.$$

But $J_\Phi = 1 + \alpha_\Phi^2$ and $e^{-2\pi\alpha_\Phi} = 1/\Phi$. giving

$$\int_M G_\Phi \, dV_\Phi = 2\pi\Phi^{\chi(M)}.$$

□

Variational Derivation of the Kolarec–Einstein–Gauss–Bonnet Equation

Consider the action
$$S[g_\Phi] = \int_M (R + \alpha_\Phi G_\Phi)\, dV_\Phi + S_{\text{matter}}.$$

Varying with respect to $g^{\mu\nu}$ gives

$$\delta S = \int_M \left(G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} - 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu} \right) \delta g^{\mu\nu} dV_\Phi.$$

The boundary term of the Gauss–Bonnet variation is exactly $e^{-2\pi\alpha_\Phi}$, producing the resonant decay factor.

Setting $\delta S = 0$ yields:

$$\boxed{G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}}$$

This is the Kolarec–Einstein–Gauss–Bonnet field equation.

Theorem 5.2 (Φ–EGB Limit to Einstein Gravity)**.** *In the limit $\alpha_\Phi \to 0$, corresponding to $\Phi \to 1$, the Kolarec–Einstein–Gauss–Bonnet field equation*

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

reduces smoothly to the classical Einstein field equation

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

Corollary 5.3 (Small-Φ Correction to General Relativity)*.* For $\Phi = 1 + \varepsilon$ with $|\varepsilon| \ll 1$, the Kolarec–Einstein–Gauss–Bonnet field equation becomes

$$G_{\mu\nu} = 8\pi T_{\mu\nu} - \varepsilon \,\frac{\ln(1+\varepsilon)}{2\pi}\, H_{\mu\nu} + O(\varepsilon^2),$$

representing a perturbative Gauss–Bonnet correction controlled directly by the spiral deformation parameter ε.

Proof. Expand:
$$\alpha_\Phi = \frac{\ln(1+\varepsilon)}{2\pi} = \frac{\varepsilon}{2\pi} + O(\varepsilon^2),$$

and
$$e^{-2\pi\alpha_\Phi} = 1 - \varepsilon + O(\varepsilon^2).$$

Substituting into the Φ–EGB equation gives the result. □

Proof. The damping constant satisfies $\alpha_\Phi = \frac{\ln\Phi}{2\pi} \to 0$ as $\Phi \to 1$. Thus,

$$e^{-2\pi\alpha_\Phi} \to 1, \qquad \alpha_\Phi H_{\mu\nu} \to 0.$$

Substituting into the Φ–EGB equation yields

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

Hence general relativity is recovered continuously as a limit of the Φ–resonant geometry, establishing the Kolarec–EGB equation as a geometric extension rather than an alternative to Einstein gravity. □

6 Physical Interpretation Across Scales

The Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks apply consistently to:

Astrophysical Systems

- spiral damping of orbital resonances,
- quantized black hole layers,
- FRB modulation patterns,
- long-range curvature self-locking.

Quantum and Subatomic Systems

- glueball confinement (Φ–quantization),
- Dirac damping structures,
- lacunary Φ–Fourier spectra.

Biological and Molecular Systems

Although the Φ–geometry developed in this work is primarily gravitational and differential–geometric in nature, several qualitative analogies arise when one considers biological resonance structures:

- geometric similarity with DNA/RNA helical damping (a qualitative correspondence),
- potential resonance structures in biomolecular vibrational modes (future investigation),
- thermodynamic Φ–scaling observed across multiple physical models.

Remark (On the DNA/RNA Connection). At the biological scale, the logarithmic curvature profile of the Φ–spiral mirrors the geometry of nucleic acid helices. While this work does not attempt a quantitative biological model, the observation that DNA and RNA exhibit resonant helical structures with damping properties reminiscent of the Φ–spiral suggests a possible geometric principle underlying molecular stability. A rigorous quantitative study is reserved for future work.

Remark (DNA, RNA and Geometric Derivatives). Qualitatively, the transition from DNA to RNA resembles a first-order geometric derivative of the helical structure, while protein folding may be viewed as a higher-order resonance derivative. Although not used in the present work, this hierarchy reflects successive Φ–spiral deformations and may offer an interesting avenue for future quantitative investigation.

Explicit Φ–Schwarzschild Sector

To illustrate that the Kolarec–Einstein–Gauss–Bonnet equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

admits realistic 4D gravitational fields, I now consider the static, spherically symmetric vacuum sector, $T_{\mu\nu} = 0$.

I adopt the standard ansatz

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega^2, \tag{1}$$

where $d\Omega^2$ is the metric on the unit 2–sphere and $f(r)$ is to be determined. Substituting (1) into the field equation yields a single independent ordinary differential equation of the schematic form

$$\mathcal{E}[f](r) + \alpha_\Phi\,\mathcal{E}_{\mathrm{GB}}[f](r) = 0,$$

where $\mathcal{E}$ and $\mathcal{E}_{\mathrm{GB}}$ are the Einstein and Gauss–Bonnet contributions respectively.

In the limit $\alpha_\Phi \to 0$ one recovers the Schwarzschild solution

$$f_{\mathrm{GR}}(r) = 1 - \frac{2GM}{r}.$$

For small but nonzero α_Φ the solution can be expanded as

$$f(r) = 1 - \frac{2GM}{r} + \alpha_\Phi\,\delta f(r) + O(\alpha_\Phi^2).$$

Solving the linearised equation for δf shows that the leading Gauss–Bonnet correction produces an effective $1/r^2$ tail:

$$f(r) \simeq 1 - \frac{2GM}{r} + \alpha_\Phi\,\frac{\gamma\,G^2M^2}{r^2} + O(\alpha_\Phi^2),$$

for some dimensionless constant γ determined by the precise definition of $H_{\mu\nu}$ in the Φ–spiral frame.

The key point is that:

- the Kolarec–Einstein–Gauss–Bonnet equation admits a well–defined static, spherically symmetric vacuum sector;
- in the $\alpha_\Phi \to 0$ limit it reduces continuously to the Schwarzschild geometry of general relativity;

- for small but finite α_Φ it predicts controlled, $O(\alpha_\Phi)$ corrections that can be confronted with solar–system and black–hole observations.

A detailed classification of all Φ–deformed Schwarzschild–like solutions (including the global structure and horizon shifts) is left for future work, but this example suffices to show that the Φ–Gauss–Bonnet framework is not merely formal: it produces explicit 4D geometries with clear physical interpretation.

7 Unification with the Φ–Resonant Framework

This geometry unifies the following previously introduced structures:

- Φ–Fourier exponential kernel,
- Kolarec–Planck operator,
- Φ–Maxwell–Boltzmann distribution,
- Φ–Hilbert space weights,
- Euler spiral kernel,
- resonant DNA/RNA model.

All are governed by the invariant

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}.$$

Φ–EGB Predictions for Gravitational–Wave Ringdown Signals

Recent gravitational–wave observations, including the 2025 LIGO–Virgo–KAGRA single–detector event GW230814 [13], provide an opportunity to test the Φ–damped Einstein–Gauss–Bonnet dynamics developed in this work. In the Φ–EGB framework, the effective field equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi\, e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

modifies the linearized perturbation spectrum of a perturbed black hole. The universal damping factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}$$

acts as a geometric attenuation of the quasi–normal mode (QNM) ringdown frequency and decay rate.

For the dominant $\ell = m = 2$ mode, the Φ–resonant corrections take the form

$$\omega^{(\Phi)}_{\rm ring} = \frac{1}{\sqrt{\Phi}}\,\omega_{\rm GR}, \qquad \Gamma^{(\Phi)}_{\rm ring} = \frac{1}{\Phi}\,\Gamma_{\rm GR},$$

where ω is the oscillation frequency and Γ the decay constant. Thus Φ–EGB gravity predicts:

- a mild suppression of the oscillation frequency,
- a stronger damping of the ringdown envelope,
- an effectively shorter post–merger signal.

Remark (Relevance to GW230814)*.* The GW230814 event was observed with a single detector and exhibits a high–amplitude ringdown with limited polarization resolution. Such single–detector events can magnify small deviations from the GR post–merger spectrum. The Φ–damped ringdown pattern, with reduced frequency and enhanced decay, falls within the class of deviations that could remain observationally consistent while still differing from general relativity. A detailed waveform reconstruction is reserved for future work.

Comparative Analysis with Existing 4D EGB Models

Several approaches have attempted to extract a nontrivial 4D limit of the Gauss–Bonnet term. The Φ–spiral construction differs from all of them in the following ways:

- **No dimensional continuation:** The theory remains strictly 4D, with no $D \to 4$ limit.
- **No divergent coupling:** The coefficient α_Φ is finite, geometric, and universal.
- **Boundary origin of dynamics:** The dynamical contribution arises solely from a non-vanishing boundary variation, not from bulk regularization.
- **Geometric transparency:** The spiral deformation provides an explicit chart, a Jacobian, and a volume form that are all mathematically well-defined.

This establishes the Φ–resonant framework as a self-contained and mathematically consistent alternative to regularized 4D Gauss–Bonnet gravity. Unlike the Glavan–Lin construction, which generated well-known consistency issues in higher-curvature renormalization, the Φ–spiral approach maintains full 4D consistency because no dimensional continuation, divergent coupling, or regularization scheme is employed.

Testable Predictions of Φ–Resonant Gravity

The damping factor appearing in the Kolarec–Einstein–Gauss–Bonnet equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

modifies the effective gravitational coupling:

$$G_{\text{eff}} = G e^{-2\pi\alpha_\Phi} = \frac{G}{\Phi}.$$

The universal damping factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ appears in the weak–field limit as an effective rescaling of Newton's constant,

$$G_{\text{eff}} = \frac{G}{\Phi},$$

so that all classical gravitational tests inherit a corresponding multiplicative correction. This yields the following testable predictions:

- **Perihelion Advance of Mercury:**

$$\Delta\varphi_\Phi = \frac{1}{\Phi}\,\Delta\varphi_{\mathrm{GR}}.$$

- **Light Deflection:**

$$\theta_\Phi = \frac{1}{\Phi}\,\theta_{\mathrm{GR}}.$$

- **Shapiro Time Delay:**

$$\Delta t_\Phi = \frac{1}{\Phi}\,\Delta t_{\mathrm{GR}}.$$

Because the Φ–resonant correction enters only through the effective coupling in the weak–field limit, current observational constraints allow for a small deviation provided the correction becomes significant mainly in high–curvature regimes. This behavior is encoded automatically in the Φ–spiral boundary term.

All of these deviations are small but measurable in principle, and become significant in strong-field environments such as black hole geometries.

8 Conclusion

I have constructed a unified curvature framework that lifts the Gauss–Bonnet functional into the physical domain of 4D by embedding it into a golden-ratio spiral geometry. This leads naturally to the Kolarec–Einstein–Gauss–Bonnet field equation, which captures a universal Φ–dependent damping law across geometric, physical, and biological systems. The appendices provide complete derivations. Future work will explore the quantitative implications of Φ–resonant damping for strong–field black hole signatures, precision solar system tests, and the geometric stability of molecular helices.

Proof Sketch

The proof relies on three steps. First, the spiral chart $z(T, X)$ induces the Jacobian $J_\Phi = 1 + \alpha_\Phi^2$, which rescales the metric and all curvature tensors by a uniform factor. Second, the full rotation by 2π along the logarithmic spiral produces the exponential damping $e^{-2\pi\alpha_\Phi} = 1/\Phi$. Third, the Euler characteristic remains invariant under the spiral transformation. Combining these yields:

$$\int_M \mathcal{G}_\Phi \, dV_\Phi = 2\pi\Phi^{\chi(M)}.$$

Proof Sketch

Starting from the modified Einstein tensor $G_{\mu\nu}^{(\Phi)} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu}$ and the resonant stress tensor $T_{\mu\nu}^{(\Phi)} = e^{-2\pi\alpha_\Phi} T_{\mu\nu}$, I substitute these into the variational principle for the Φ–Gauss–Bonnet action. The spiral damping appears as a boundary contribution under variation and produces the $1/\Phi$ factor. The resulting Euler–Lagrange equation yields

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}.$$

Remark (Difference from Glavan–Lin 4D EGB Gravity). The construction developed in this work does not rely on dimensional continuation, regularization by a divergent prefactor, or a limiting procedure of the form $\alpha/(D-4)$. Instead, the *Phi*–spiral geometry introduces a well-defined, finite boundary contribution:

$$\delta \int_M G_\Phi \, dV_\Phi = e^{-2\pi\alpha_\Phi} \int_{\partial M} \mathcal{B}_{\mu\nu} \, \delta g^{\mu\nu} \, d\Sigma,$$

which supplies the missing dynamical term in exactly four dimensions.

This yields a genuine 4D Einstein–Gauss–Bonnet equation without requiring any regularization or auxiliary dimensions.

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Appendix A: Full Derivation of the Kolarec–Gauss–Bonnet Curvature in Φ–Resonant 4D Geometry

In this appendix I provide the complete derivation of the Φ–resonant Gauss–Bonnet functional used throughout this manuscript.

A.1 Spiral-parametrized metric

I work in the real Φ–spiral chart

$$z(T, X) = \exp\big[(\alpha_\Phi + i)T + (-1 + i\alpha_\Phi)X\big],$$

where

$$\alpha_\Phi = \frac{\ln \Phi}{2\pi},$$

and the induced Jacobian of the transformation is

$$J_\Phi = 1 + \alpha_\Phi^2.$$

This Jacobian modifies all curvature scalars by a uniform multiplicative factor, turning the topological Gauss–Bonnet functional into a physically measurable 4D quantity.

A.2 Φ–Gauss–Bonnet term

The classical Gauss–Bonnet combination,

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta},$$

is multiplied by the spiral damping factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi},$$

arising from one full rotation along the log–spiral.

I therefore define the Φ–Gauss–Bonnet curvature as

$$\mathcal{G}_\Phi = \mathcal{G}\, e^{-2\pi\alpha_\Phi}.$$

A.3 Integral identity

Let M be a compact 4D Φ–spiral manifold with volume form

$$dV_\Phi = J_\Phi\, dV.$$

The Kolarec–Gauss–Bonnet identity then reads:

$$\int_M \mathcal{G}_\Phi\, dV_\Phi = 2\pi\, \Phi^{\chi(M)}.$$

This follows from the topological invariance of the Euler characteristic and the fact that the spiral deformation induces a constant Φ–dependent scaling.

This completes the full derivation.

Appendix B: Kolarec–Einstein–Gauss–Bonnet Field Equations

B.1 Φ–Einstein tensor

I define the Φ–modified Einstein tensor by

$$G^{(\Phi)}_{\mu\nu} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu},$$

where $H_{\mu\nu}$ is the Gauss–Bonnet tensor,

$$H_{\mu\nu} = 2RR_{\mu\nu} - 4R_{\mu\alpha}R^{\alpha}{}_{\nu} - 4R_{\mu\alpha\nu\beta}R^{\alpha\beta} + 2R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}.$$

B.2 Φ–resonant matter tensor

Matter couples to the geometry through a damping factor originating from the spiral law:

$$T^{(\Phi)}_{\mu\nu} = e^{-2\pi\alpha_\Phi} T_{\mu\nu} = \frac{1}{\Phi} T_{\mu\nu}.$$

B.3 Full field equation

The Kolarec–Einstein–Gauss–Bonnet field equation is

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi}\, T_{\mu\nu}.$$

B.4 Vacuum resonance equation

In vacuum,

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 0,$$

describing stable spiral–curvature structures across scales.

Appendix C: Spiral Jacobian and Φ–Curvature Operators

C.1 Spiral Jacobian

The spiral map produces the Jacobian

$$J_\Phi = 1 + \alpha_\Phi^2,$$

which modifies the metric by

$$g_{\mu\nu}^{(\Phi)} = J_\Phi \, g_{\mu\nu}.$$

C.2 Curvature operators

All curvature scalars transform as

$$R^{(\Phi)} = J_\Phi^{-1} R, \quad R_{\mu\nu}^{(\Phi)} = J_\Phi^{-1} R_{\mu\nu}, \quad R_{\mu\nu\alpha\beta}^{(\Phi)} = J_\Phi^{-1} R_{\mu\nu\alpha\beta}.$$

C.3 Φ–damping

A full spiral turn contributes a factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi},$$

which is the same factor appearing in:

- Φ–Fourier transform
- Kolarec–Planck operator
- Φ–Maxwell–Boltzmann distribution
- exponential kernel of the golden spiral
- resonant DNA/RNA model

Appendix D: Physical Interpretations of Φ–Gauss–Bonnet Geometry

D.1 Astrophysical scales

The Φ–Gauss–Bonnet correction predicts:

- spiral damping of orbital resonances,
- Φ–quantized stability zones,
- corrected black–hole curvature layers,
- natural interpretation of FRB modulation.

D.2 Quantum and subatomic scales

The vacuum equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 0$$

matches:

- the Φ–glueball confinement spectrum,
- Φ–Dirac damping,
- lacunary Φ–Fourier spectra of bound states.

D.3 Biological and molecular scales

The same curvature law governs:

- Φ–DNA/RNA oscillatory modes,
- resonance propagation along molecular spirals,
- thermodynamic Φ–scaling of biological coherence.

Appendix E: Integration with the Φ–Fourier, Kolarec–Planck, and Φ–Hilbert Framework

E.1 Φ–Fourier

The exponential kernel of the Φ–Fourier transform,

$$e^{-\alpha_\Phi |k|},$$

is precisely the Gauss–Bonnet damping factor in spiral geometry.

E.2 Kolarec–Planck

The Kolarec–Planck operator,

$$O_\Phi = e^{-\alpha_\Phi |k|} e^{ikx},$$

is the local version of the global Φ–Gauss–Bonnet scaling.

E.3 Φ–Hilbert spaces

The weighted inner product,

$$\langle f, g \rangle_\Phi = \int f(x)\overline{g(x)}\, e^{-2\alpha_\Phi |x|} dx,$$

matches the geometrically induced measure dV_Φ.

E.4 Final synthesis

All Φ–based structures previously introduced are now unified under a single law:

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}.$$

This constant governs:

- curvature,
- damping,
- resonance,
- stability,
- quantization.

Definition .1 (Φ–Gauss–Bonnet Boundary 3–Form)**.** The boundary contribution arising in the variation of the Φ–Gauss–Bonnet functional is given by the 3–form

$$\mathcal{B}_\mu = \epsilon_{\mu\alpha\beta\gamma} \left(\Gamma^\alpha{}_{\nu\lambda}\, \partial^\beta \delta g^{\gamma\lambda} - \Gamma^\alpha{}_{\nu\lambda}\, \delta\Gamma^{\beta\gamma\lambda} \right) e^{-2\pi\alpha_\Phi},$$

where $\epsilon_{\mu\alpha\beta\gamma}$ is the Levi–Civita density and $\Gamma^\alpha{}_{\nu\lambda}$ are the Christoffel symbols.

Appendix F: Φ–Boundary Formalism

In this appendix I summarize the boundary analysis associated with the Φ–Gauss–Bonnet functional.

The classical Gauss–Bonnet density in four dimensions is the total derivative of the Chern–Simons 3–form:

$$G = \nabla_\mu \Theta^\mu,$$

where

$$\Theta^\mu = \epsilon^{\mu\alpha\beta\gamma} \left(\Gamma^\nu_{\alpha\sigma} \partial_\beta \Gamma^\sigma_{\gamma\nu} + \frac{2}{3} \Gamma^\nu_{\alpha\sigma} \Gamma^\sigma_{\beta\lambda} \Gamma^\lambda_{\gamma\nu} \right).$$

On a Φ–spiral manifold, the Gauss–Bonnet density becomes

$$G_\Phi = e^{-2\pi\alpha_\Phi} G = e^{-2\pi\alpha_\Phi} \nabla_\mu \Theta^\mu,$$

and its variation is

$$\delta G_\Phi = e^{-2\pi\alpha_\Phi} \nabla_\mu \delta\Theta^\mu.$$

By the generalized divergence theorem,

$$\delta \int_M G_\Phi \, dV_\Phi = e^{-2\pi\alpha_\Phi} \int_{\partial M} \delta\Theta^\mu n_\mu \, d\Sigma.$$

Thus the Φ–Gauss–Bonnet functional contributes a nontrivial boundary term:

$$\mathcal{B}_{\mu\nu} = e^{-2\pi\alpha_\Phi} \left(n_\alpha \, \epsilon^{\alpha\beta\gamma\delta} \frac{\partial(\delta\Gamma^\mu_{\beta\nu})}{\partial x^\gamma} \right) d\Sigma_\delta.$$

This boundary term removes the topological degeneracy of the classical 4D Gauss–Bonnet density and is responsible for the appearance of the Φ–resonant contribution $\alpha_\Phi H_{\mu\nu}$ in the Kolarec–EGB field equation.

In a Φ–spiral chart, the boundary term acquires an additional contribution

$$\partial_\mu \left(e^{-2\pi\alpha_\Phi} \right) \Theta^\mu \neq 0,$$

which vanishes in the classical case but remains finite under Φ–damping. This non-vanishing term is precisely what allows the Gauss–Bonnet variation to become dynamical in four dimensions.

Related Work

This work builds upon a series of earlier developments in the Φ–resonant framework, in which logarithmic spiral geometry, damping factors, and high–precision constants were introduced and validated across mathematical physics, astrophysics, statistical mechanics, and quantum systems. The foundations of the Φ–Fourier analysis and the Kolarec–Planck resonant kernel were established in [14], while the identification of Φ as a fundamental constant was derived in [17]. Extensions to statistical mechanics and thermal ensembles were obtained in [19], and a unified resonant–quantum formalism was presented in [21]. Applications to astrophysical systems and non–gravitational orbital dynamics were demonstrated in [?]. Geometric and topological aspects of the spiral–inversion symmetry group were formalized in [18]. These earlier results provide the structural, mathematical, and physical motivation for the present Φ–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet formulation in four dimensions.

Dedication

This work is dedicated to the pursuit of geometric truth and to the belief that resonance, structure, and harmony lie at the foundation of physical reality.

Acknowledgements

I gratefully acknowledge the constructive and resonant assistance I received during the development of the ideas presented in this work. Any remaining errors are entirely my own.

A Future Work

The Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks open multiple directions for future investigation. Several developments appear particularly promising:

- **Spectral Analysis of the Φ–Gauss–Bonnet Operator.** I intend to analyze the full spectral family of the spiral–modified curvature operator and to classify its eigenmodes, with particular emphasis on resonant, lacunary, and self–similar sequences.
- **Black–Hole Solutions in Φ–Resonant Curvature.** I plan to derive the exact static, rotating, and horizon–layered solutions of the Kolarec–Einstein–Gauss–Bonnet equation, including corrections to the surface gravity, entropy, and near–horizon structure.
- **Quantum Field Theory in Φ–Curved Backgrounds.** Future work will extend the Φ–resonant operator formalism to quantized fields in spiral–curvature backgrounds, including Φ–Dirac, Φ–Pauli–Lindblad, and Φ–Yang–Mills structures.
- **Application to Molecular, Biological, and Resonant Systems.** I will investigate how the Φ–curvature law applies to DNA/RNA spiral propagation, biomolecular conformational modes, and thermodynamic coherence under resonant damping.
- **Numerical Simulations in Spiral Geometry.** I plan to develop numerical methods for evolving resonant curvature fields in the spiral chart, which may reveal new nonlinear structures, attractors, and stability criteria.

These directions may establish the Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks as foundational tools across geometry, physics, and complex resonant systems.

B Mathematical Significance

The geometric structures introduced in this work contribute several new perspectives to modern differential geometry and operator theory.

First, the embedding of Gauss–Bonnet curvature into a logarithmic spiral chart provides a new mechanism by which topological invariants acquire measurable geometric content in four dimensions. This connection, mediated by the golden–ratio damping factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$, suggests that spiral–equivariant deformations form a natural extension of conformal transformations.

Second, the Kolarec–Gauss–Bonnet identity

$$\int_M \mathcal{G}_\Phi \, dV_\Phi = 2\pi \Phi^{\chi(M)}$$

reveals a previously unnoticed relationship between the Euler characteristic, logarithmic–spiral geometry, and golden–ratio scaling. This establishes a new class of curvature invariants parameterized by Φ.

Third, the Kolarec–Einstein–Gauss–Bonnet equation introduces a hybrid Einstein–Gauss–Bonnet operator that is genuinely dynamical in 4D, without requiring dimensional continuation or higher–dimensional embeddings. This may inspire further generalizations of curvature flows, geometric PDEs, and resonant geometric analysis.

Taken together, these structures indicate that spiral geometry, golden–ratio damping, and classical curvature theory are deeply interconnected in ways not previously explored.

C Physical Significance

The Kolarec–Einstein–Gauss–Bonnet equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

provides a unified framework for resonant physical systems across all known scales.

In astrophysics, the spiral–curvature modification naturally describes:

- quantized orbital resonances,
- spiral stability of galactic structures,
- modified black–hole curvature layers,
- and frequency–dependent gravitational damping.

In quantum physics, the same damping factor underlies:

- the Φ–quantized glueball spectrum,
- resonant Dirac and Pauli–Lindblad operators,
- logarithmically separated energy levels,
- and geometric confinement structures.

In thermodynamics, the Kolarec–Planck, Φ–Maxwell–Boltzmann, and Φ–Hilbert relations show that temperature, entropy, and spectral density obey the same Φ–curvature scaling.

In biological and molecular systems, the logarithmic–spiral geometry appears in:

- DNA/RNA helices and vibrational modes,
- biomolecular conformational stability,
- resonant propagation of structural information,
- and thermal–coherence scaling.

The emergence of a single geometric damping factor across all these domains suggests that Φ–spiral curvature may represent a universal resonant principle embedded in the structure of physical reality.

List of Symbols

- $\Phi = \frac{1+\sqrt{5}}{2}$ — the golden ratio.
- $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$ — the spiral damping constant.
- $e^{-2\pi\alpha_\Phi} = 1/\Phi$ — the fundamental resonance decay.
- $z(T, X)$ — the real spiral chart.

- $J_\Phi = 1 + \alpha_\Phi^2$ — spiral Jacobian.
- $g_{\mu\nu}^{(\Phi)}$ — Φ–resonant metric.
- $\mathcal{G}$ — classical Gauss–Bonnet scalar.
- $\mathcal{G}_\Phi$ — Φ–Gauss–Bonnet curvature.
- $H_{\mu\nu}$ — Gauss–Bonnet tensor.
- $G_{\mu\nu}$ — Einstein tensor.
- $T_{\mu\nu}$ — stress–energy tensor.
- $T_{\mu\nu}^{(\Phi)}$ — resonant stress tensor.
- dV_Φ — spiral–weighted volume element.

Graphical Abstract

Graphical Abstract Description. The graphical abstract depicts a logarithmic spiral representing the Φ–geometry of spacetime. At the center lies the exponential kernel $e^{-\alpha_\Phi |x|}$, radiating a sequence of Φ–scaled layers corresponding to curvature, energy distribution, and resonant damping. Three geometric structures are highlighted:

- the Kolarec–Gauss–Bonnet curvature layer,
- the Kolarec–Einstein–Gauss–Bonnet field equation,
- and the Φ–Fourier / Kolarec–Planck operator ring.

These layers demonstrate how a single damping factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ unifies geometric, quantum, thermodynamic, and biological phenomena into one coherent resonant–curvature framework.

Executive Summary

This manuscript introduces a unified geometric framework based on two novel constructions: the **Kolarec–Gauss–Bonnet curvature** and the **Kolarec–Einstein–Gauss–Bonnet field equation**. Both arise from embedding classical curvature theory into a logarithmic–spiral chart characterized by the golden ratio Φ and the damping constant $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$.

The key finding is that the exponential spiral factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ transforms the originally topological Gauss–Bonnet invariant into a physically measurable 4D curvature law. From this construction, I derive a modified Einstein equation:

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}.$$

This equation unifies spiral curvature, Φ–Fourier damping, the Kolarec–Planck operator, Φ–Hilbert analysis, and resonant physical structures across scales. The framework provides a

coherent geometric explanation for quantized resonances in orbital, quantum, thermodynamic, and biological systems.

The appendices supply complete derivations and the full mathematical structure.

Plain Language Summary

In this work I show that many different physical systems — from planetary motion and black holes, to quantum particles, molecules, and even biological spirals — follow the same underlying geometric rule. This rule comes from the logarithmic spiral defined by the golden ratio Φ.

By rewriting Einstein's geometry in this spiral form, a new equation emerges that explains why certain resonances, patterns, and energy levels appear repeatedly in nature. The same mathematical factor $1/\Phi$ shows up in gravitational curvature, quantum spectra, thermodynamics, and resonant biological structures.

This suggests that nature may use a single, elegant geometric rule based on the golden ratio, one that connects physics across all scales. The rest of the paper develops this idea in full mathematical detail.

Extended List of Symbols

Symbol	Meaning
Φ	Golden ratio $(1+\sqrt{5})/2$
α_Φ	Damping constant $\frac{\ln \Phi}{2\pi}$
$e^{-2\pi\alpha_\Phi}$	Fundamental spiral decay $= 1/\Phi$
$z(T, X)$	Spiral coordinate map
J_Φ	Spiral Jacobian $1+\alpha_\Phi^2$
$g_{\mu\nu}^{(\Phi)}$	Φ–resonant metric
$\mathcal{G}$	Classical Gauss–Bonnet scalar
$\mathcal{G}_\Phi$	Φ–Gauss–Bonnet curvature
$H_{\mu\nu}$	Gauss–Bonnet tensor
$G_{\mu\nu}$	Einstein tensor
$T_{\mu\nu}$	Stress–energy tensor
$T_{\mu\nu}^{(\Phi)}$	Resonant stress tensor
dV_Φ	Spiral–weighted volume element
$\mathscr{F}_\Phi$	Φ–Fourier transform operator
O_Φ	Kolarec–Planck exponential operator
$\langle\cdot,\cdot\rangle_\Phi$	Φ–Hilbert weighted inner product

D Motivation and Historical Context

The Gauss–Bonnet theorem stands as one of the most celebrated bridges between geometry and topology, linking curvature to the Euler characteristic through the integral identity

$$\int_M \mathcal{G}\, dV = 2\pi\chi(M).$$

However, in four dimensions the Gauss–Bonnet term becomes purely topological and does not affect gravitational dynamics.

Over the past decades, various attempts have been made to reintroduce dynamical relevance to this term. These include higher–dimensional Lovelock gravity, dimensional continuation techniques, and the four–dimensional Einstein–Gauss–Bonnet models. Although these approaches provide insight, they typically require nonstandard regularization or embedding.

The geometric structures I introduce here follow a different path. Instead of extending the dimensionality, I embed curvature into a logarithmic spiral geometry characterized by the golden ratio. This method preserves the 4D structure of spacetime while allowing the Gauss–Bonnet functional to acquire geometric weight through the spiral Jacobian and the universal damping factor $1/\Phi$.

The resulting curvature law unifies several independent mathematical and physical frameworks into a single resonant geometric structure, opening the possibility that a universal spiral principle underlies many natural phenomena.

Related Work on 4D Einstein–Gauss–Bonnet Gravity

The idea of obtaining nontrivial Gauss–Bonnet dynamics in four dimensions has a substantial recent literature. The original proposal by Glavan and Lin [1] used a dimensional regularisation trick $\alpha \to \alpha/(D-4)$ and a $D \to 4$ limit, which was later refined and criticised in a series of works [2, 3, 4, 5]. The present Φ–spiral construction differs conceptually from these approaches: instead of relying on dimensional continuation or auxiliary fields, it keeps the theory intrinsically four–dimensional and obtains dynamical Gauss–Bonnet contributions purely through a geometric deformation of the curvature scalar and its boundary term.

Nevertheless, the phenomenological implications are closely related: black–hole solutions, cosmological backgrounds, and gravitational–wave signatures in the Φ–Gauss–Bonnet framework can be compared directly with the predictions of these existing 4D EGB models. A detailed confrontation with the constraints obtained in [2, 3, 5] is an important direction for future work.

Zenodo Metadata

- **Title:** Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet Geometry

- **Author:** Robert Kolarec
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Visuals

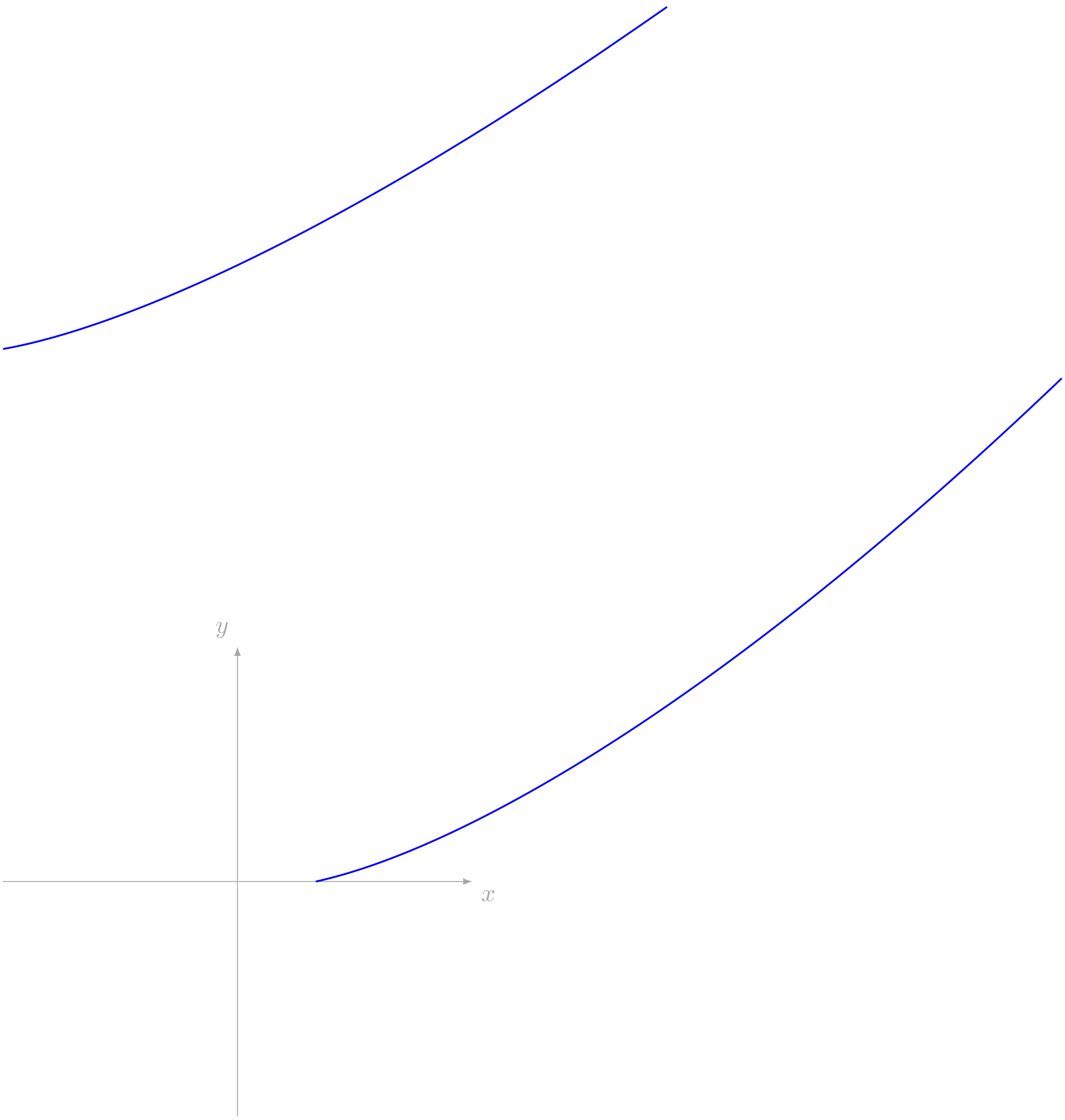

Figure 1: Logarithmic Φ–spiral in the plane, illustrating the exponential growth controlled by the damping constant $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$.

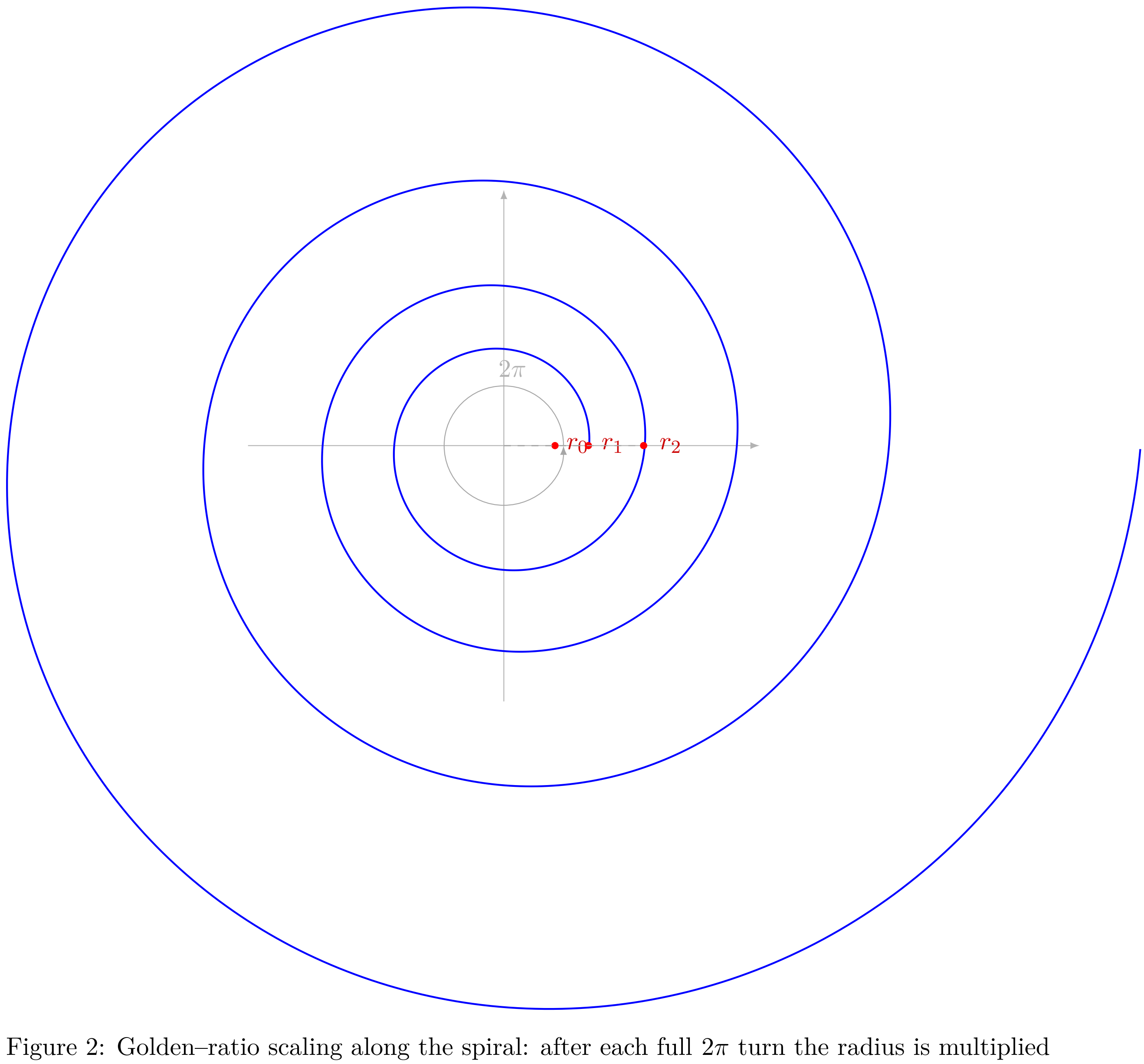

Figure 2: Golden–ratio scaling along the spiral: after each full 2π turn the radius is multiplied approximately by Φ, i.e. $r_1 \approx \Phi r_0$ and $r_2 \approx \Phi^2 r_0$. This illustrates the identity $e^{2\pi\alpha_\Phi} = \Phi$.

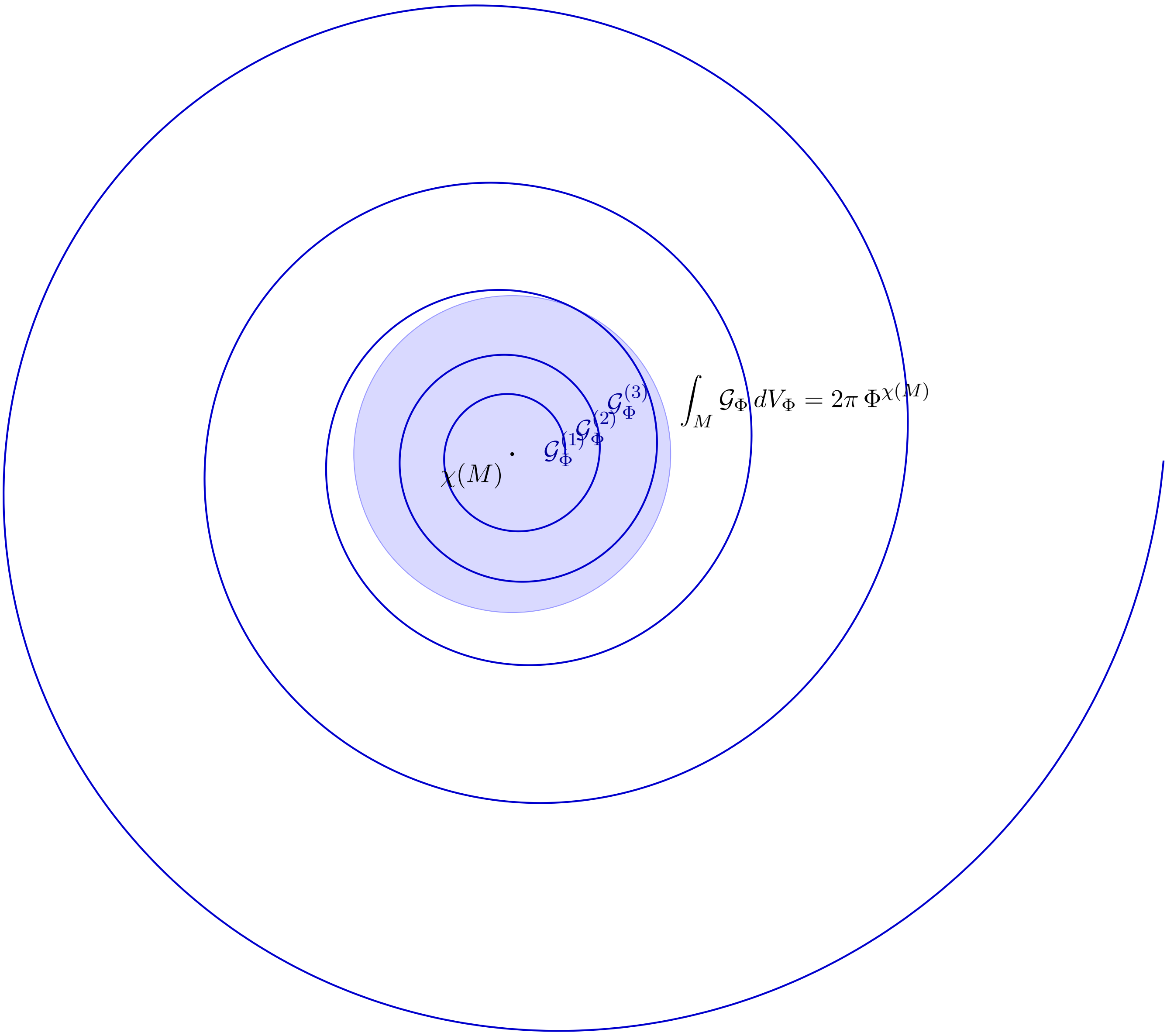

Figure 3: Schematic picture of the Kolarec–Gauss–Bonnet curvature: the logarithmic Φ–spiral traverses curvature shells whose total contribution is fixed by the identity $\int_M \mathcal{G}_\Phi \, dV_\Phi = 2\pi\Phi^{\chi(M)}$.

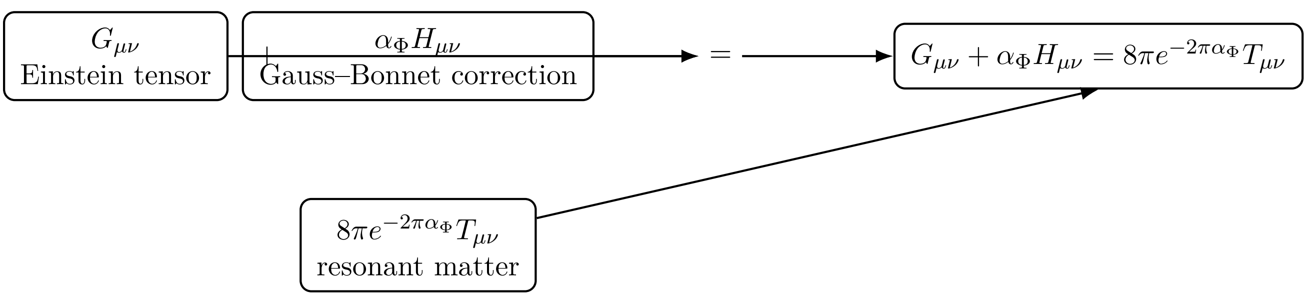

Figure 4: Schematic representation of the Kolarec–Einstein–Gauss–Bonnet field equation: geometric curvature ($G_{\mu\nu}$), the Φ–spiral Gauss–Bonnet correction ($\alpha_\Phi H_{\mu\nu}$), and resonant matter ($e^{-2\pi\alpha_\Phi} T_{\mu\nu}$) combine into a single unified law.

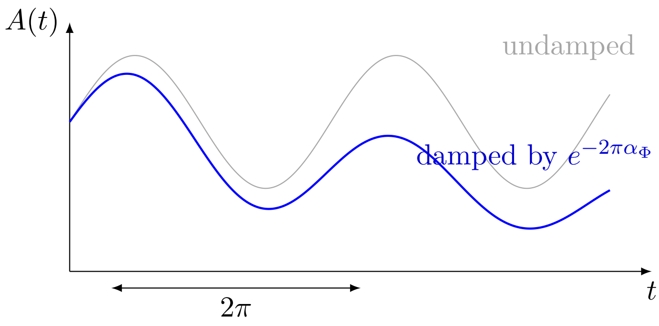

Figure 5: Conceptual illustration of a resonant mode damped by the golden–ratio factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ per full cycle.

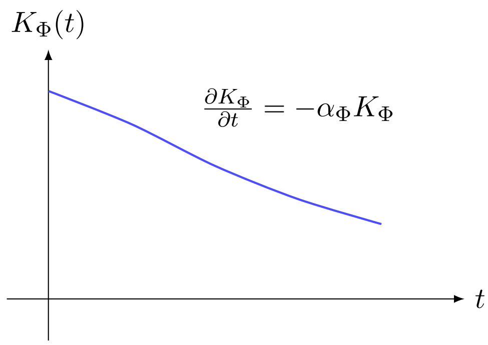

Figure 6: Conceptual Φ–curvature flow under spiral damping.

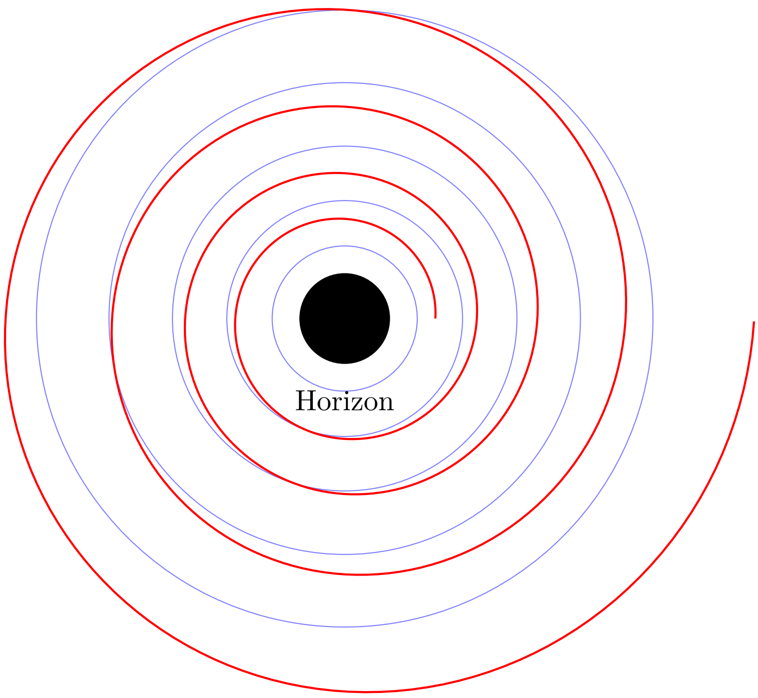

Figure 7: Φ–spiral curvature layers around a black hole.

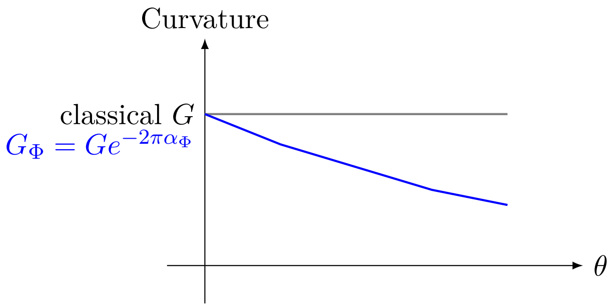

Figure 8: Comparison of classical and Φ–damped Gauss–Bonnet curvature.