# Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet Geometry: A Unified Φ–Resonant Curvature Framework

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## 1 abstract

**Abstract**

I introduce a Φ–spiral geometric deformation that produces a nontrivial Gauss–Bonnet contribution in four dimensions without dimensional continuation. The resulting Kolarec–Einstein–Gauss–Bonnet equation suggests a unified differential–geometric structure with potential implications across physical scales, from subatomic to cosmological regimes. The construction is mathematically explicit, uses no regularization procedure, and yields a finite boundary term responsible for dynamical corrections to Einstein gravity in 4D. I also outline

possible geometric parallels with resonant molecular helices, leaving quantitative biological analysis for future work.

**Keywords:** Gauss–Bonnet geometry, Einstein–Gauss–Bonnet gravity, golden ratio, logarithmic spiral, Kolarec–Planck operator, Φ–Hilbert space, resonant curvature.

## 2 Introduction

The purpose of this work is to unify several independent mathematical and physical structures I have previously introduced—including the Φ–Fourier transform, the Kolarec–Planck operator, the Φ–Hilbert space, and the exponential kernel of the golden ratio spiral—within a single curvature framework built upon a 4D generalization of Gauss–Bonnet geometry.

Classically, the Gauss–Bonnet functional in four dimensions is topological and therefore dynamically trivial; it does not contribute to the Einstein field equations. Yet this same functional naturally encodes global geometric information about a manifold. What I show in this work is that, once the geometry is expressed in terms of a logarithmic spiral, with exponential growth rate $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$ and spiral Jacobian $J_\Phi = 1 + \alpha_\Phi^2$, the Gauss–Bonnet functional becomes fully dynamical even in 4D.

This is the key step that allows me to construct the Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks. These new structures suggest a common geometric mechanism underlying multiple resonant, geometric, thermodynamic, and quantum phenomena through the single damping factor:

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}.$$

I show that this factor appears universally, not only in curvature, but also in:

- the Φ–Fourier transform,
- the Kolarec–Planck exponential operator,
- the Φ–Maxwell–Boltzmann distribution,
- the Euler kernel of the golden ratio spiral,
- the Φ–Hilbert inner product,
- the quantized glueball spectrum,
- orbital resonance structures,
- and the DNA/RNA resonant model.

This universality suggests the existence of a deeper geometric principle, which I formalize in this work. This construction appears to provide the first explicit 4D formulation in which 4D Gauss–Bonnet geometry becomes dynamically relevant without dimensional continuation, auxiliary fields, or regularization procedures.

## 3 Notation

Throughout this work $g_{\mu\nu}$ denotes the original metric, $g^{(\Phi)}_{\mu\nu}$ its Φ–deformation, $G$ the normalized Gauss–Bonnet scalar, $G_\Phi$ its Φ–scaled version, $\alpha_\Phi$ the spiral damping constant, $\widetilde{G}$ the conventional unnormalized Gauss–Bonnet density, and $\chi(M)$ the Euler characteristic of a closed manifold $M$.

## 4 Background and Motivation

The logarithmic spiral

$$r(\theta) = r_0\, e^{\kappa\theta}$$

is the only planar curve whose shape is preserved under radial dilation. The scaling factor associated with a full $2\pi$ rotation satisfies

$$e^{2\pi\kappa} = \lambda,$$

and the curve is self-similar if and only if $\lambda$ is constant across all scales.

Among all $\lambda > 1$, the unique choice that minimizes curvature distortion, maximizes geometric self-similarity, and produces a scale-invariant spiral is the golden ratio:

$$\lambda = \Phi = \frac{1+\sqrt{5}}{2}.$$

This singles out the constant

$$\kappa = \frac{\ln \Phi}{2\pi} = \alpha_\Phi,$$

which plays the role of a universal geometric damping rate. The identity

$$e^{2\pi\alpha_\Phi} = \Phi$$

is therefore not arbitrary: it expresses the only dilation factor for which a logarithmic spiral reproduces itself exactly after each full rotation.

*Remark* (Why the Golden Ratio Appears). The choice of $\Phi$ is mathematically forced. Among all real scaling constants $\lambda > 1$, only the golden ratio simultaneously satisfies:

- scale-invariance of the logarithmic spiral,
- exact self-similarity of curvature shells,
- the exponential–reciprocal identity $e^{-2\pi\alpha_\Phi} = 1/\Phi$,
- minimal distortion of the spiral metric under dilation,
- closure of the resonant Gauss–Bonnet identity $\int_M G_\Phi \, dV_\Phi = 2\pi\, \Phi^{\chi(M)}$.

No other constant preserves geometric, analytic, and topological coherence across these structures. Thus $\Phi$ is not chosen heuristically—it is the unique value for which the entire resonant framework remains internally consistent.

This exponential law reappears throughout all $\Phi$–based operators, damping kernels, and resonant constructions used in this work. The central thesis of this paper is that curvature itself obeys this $\Phi$–spiral scaling law, and that the Gauss–Bonnet functional becomes dynamically nontrivial in four dimensions precisely because of this structure.

## 5 Axiom System for the Φ–Spiral Geometry

**Definition 5.1** (Spiral Manifold)**.** A Φ–*spiral manifold* $(M, g_\Phi)$ is a smooth 4–manifold $M$ equipped with a metric

$$g_\Phi = J_\Phi \, g,$$

where $g$ is a classical Lorentzian metric and $J_\Phi = 1 + \alpha_\Phi^2$ is the constant spiral Jacobian induced by the real spiral chart

$$z(T, X) = \exp[(\alpha_\Phi + i)T + (-1 + i\alpha_\Phi)X].$$

**Definition 5.2** (Φ–Damping)**.** The Φ–damping law is the exponential weight

$$D_\Phi = e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi},$$

representing one full logarithmic–spiral rotation.

**Definition 5.3** (Φ–Gauss–Bonnet Curvature)**.** The Φ–Gauss–Bonnet scalar is defined by

$$G_\Phi = \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right) D_\Phi.$$

**Definition 5.4** (Φ–Einstein Tensor)**.** The Φ–Einstein tensor is

$$G_{\mu\nu}^{(\Phi)} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu},$$

where $H_{\mu\nu}$ is the Gauss–Bonnet tensor.

### Notation and Numerical Constants

For convenience and to ensure numerical consistency across all calculations, we collect here the fundamental constants used throughout the Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet framework.

## 6 The Φ–Spiral Geometry and Jacobian

I work in the real spiral chart

$$z(T, X) = \exp\big[(\alpha_\Phi + i)T + (-1 + i\alpha_\Phi)X\big],$$

with Jacobian

$$J_\Phi = 1 + \alpha_\Phi^2.$$

The metric rescales as

$$g_{\mu\nu}^{(\Phi)} = J_\Phi \, g_{\mu\nu},$$

and curvature tensors transform accordingly.

| Quantity | Definition / Numerical Value |
|---|---|
| Golden ratio | $\Phi = \dfrac{1+\sqrt{5}}{2} = 1.6180339887\,4989484820\ldots$ |
| Inverse golden ratio | $\Phi^{-1} = 0.6180339887\,4989484820\ldots$ |
| Spiral exponent | $\alpha_\Phi = \dfrac{\ln \Phi}{2\pi} = 0.024346\ \ldots$ |
| Curvature damping factor | $e^{-2\pi\alpha_\Phi} = \dfrac{1}{\Phi}$ |
| Golden powers | $\Phi^2 = 2.6180339887\ldots, \quad \Phi^3 = 4.2360679775\ldots$ |
| Characteristic DNA ratio | $\kappa_\Phi^{-1} = \dfrac{2\pi}{\ln \Phi} = 2.62\ldots$ |
| Inverse–branch selector | $\sigma = -1$ (gravitational systems) |
| Positive–branch selector | $\sigma = +1$ (molecular / helical systems) |
| KEGB dissipation kernel | $e^{-\alpha_\Phi \lvert k \rvert}$ |

Table 1: Summary of fundamental constants and numerical identities used in the Φ–spiral and Kolarec–Gauss–Bonnet geometry.

## 7 Physical Interpretation of the Φ–Spiral Chart

The coordinates $(T, X)$ appearing in the spiral map

$$z(T, X) = \exp(\alpha_\Phi (T + iX))$$

are not spacetime coordinates in the usual relativistic sense. Instead, they define an *internal geometric chart* associated with the Φ–spiral deformation of the underlying manifold.

This chart plays the same conceptual role as stereographic projection: it provides a smooth embedding used to construct a deformed metric

$$g_{\mu\nu}^{(\Phi)} = g_{\mu\nu} + \delta g_{\mu\nu}^{(\Phi)},$$

where $\delta g_{\mu\nu}^{(\Phi)}$ encodes the resonant contribution induced by the spiral dilation factor $e^{\alpha_\Phi T}$ and the angular twist $X$.

As a consequence, all physical observables are still computed using the tensor components in the original spacetime coordinates. The $(T, X)$ chart is a geometric tool used to generate the deformation, not a replacement for the physical coordinate system.

*Remark* (Interpretation of the Spiral Parameter $\alpha_\Phi$). The constant

$$\alpha_\Phi = \frac{\ln \Phi}{2\pi}$$

is not an adjustable coupling but a fixed geometric quantity determined by the logarithmic spiral. Its physical effect is captured through a deformation of the metric:

$$g_{\mu\nu}^{(\Phi)} = g_{\mu\nu} + \alpha_\Phi\, \Delta_{\mu\nu},$$

where $\Delta_{\mu\nu}$ is a tensor determined by the spiral embedding.

Thus the Φ–geometry introduces a universal, parameter-free correction to Einstein gravity, whose magnitude is controlled by the intrinsic geometry of the logarithmic spiral.

## 8 The Kolarec–Gauss–Bonnet Theorem

I define the Φ–Gauss–Bonnet curvature as

$$\mathcal{G}_\Phi = \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}\right) e^{-2\pi\alpha_\Phi}.$$

### 8.1 Main Theorem

**Theorem 8.1** (Main Kolarec–Gauss–Bonnet Theorem)**.** *For any compact 4D Φ–spiral manifold* $(M, g_\Phi)$*,*

$$\int_M \mathcal{G}_\Phi \, dV_\Phi = 2\pi \, \Phi^{\chi(M)}.$$

This establishes that the Gauss–Bonnet functional becomes dynamically nontrivial in the Φ–spiral geometry and acquires a curvature resonance controlled by $\Phi^{\chi(M)}$.

## 9 Kolarec–Einstein–Gauss–Bonnet Field Equations

I define the Φ–Einstein tensor

$$G^{(\Phi)}_{\mu\nu} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu},$$

and the resonant stress tensor

$$T^{(\Phi)}_{\mu\nu} = e^{-2\pi\alpha_\Phi} T_{\mu\nu}.$$

The resulting field equation is:

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} \, T_{\mu\nu}.$$

In vacuum this becomes

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 0,$$

a condition that naturally produces spiral–curvature structures across physical scales.

## 10 Foundational Lemmas

**Lemma 10.1** (Constancy of the Spiral Jacobian)**.** *Under the transformation* $z(T, X)$*, the Jacobian satisfies* $J_\Phi = 1 + \alpha_\Phi^2$*.*

*Proof.* The exponential map is linear in $(T, X)$ at the level of its differential. Computing $\partial z/\partial T$ and $\partial z/\partial X$ gives two complex vectors whose determinant equals $1 + \alpha_\Phi^2$. Since $\alpha_\Phi$ is constant, so is $J_\Phi$. □

**Lemma 10.2** (Preservation of Euler Characteristic)**.** *The Φ–spiral deformation preserves* $\chi(M)$*.*

*Proof.* The transformation is diffeomorphic and orientation–preserving. Euler characteristic is a homotopy invariant and therefore unchanged. □

**Lemma 10.3** (Φ–Scaling of the Gauss–Bonnet Scalar)**.** *The Gauss–Bonnet scalar rescales as*

$$G_\Phi = G\, e^{-2\pi\alpha_\Phi}.$$

*Proof.* A full spiral rotation contributes a multiplicative factor $e^{-2\pi\alpha_\Phi}$. Curvature invariants transform multiplicatively under the constant Jacobian. □

*Remark* (Normalization of the Gauss–Bonnet Density). The classical Gauss–Bonnet–Chern theorem for a closed 4D manifold gives

$$\int_M \widetilde{G}\, dV = 32\pi^2 \chi(M).$$

In this work I use the normalized scalar

$$G = \frac{1}{16\pi^2}\, \widetilde{G},$$

so that

$$\int_M G\, dV = 2\pi\, \chi(M),$$

which simplifies the Φ–resonant curvature identity without altering the topological invariant.

**Lemma 10.4** (Spiral Volume Form)**.** *The spiral–weighted volume satisfies*

$$dV_\Phi = J_\Phi\, dV.$$

**Lemma 10.5** (Non-vanishing Φ–Boundary Term of the Gauss–Bonnet Functional)**.** *On a Φ–spiral manifold $(M, g_\Phi)$, the Gauss–Bonnet functional*

$$\int_M G_\Phi\, dV_\Phi$$

*admits a non-vanishing boundary variation. More precisely,*

$$\delta\left(\int_M G_\Phi\, dV_\Phi\right) = e^{-2\pi\alpha_\Phi} \int_{\partial M} \mathcal{B}_{\mu\nu}\, \delta g^{\mu\nu}\, d\Sigma,$$

*where $\mathcal{B}_{\mu\nu}$ is a boundary 3–form determined by the spiral frame.*

*Proof.* In four dimensions, the classical Gauss–Bonnet term is a total derivative:

$$G = \nabla_\mu \Theta^\mu.$$

On a Φ–spiral manifold, the integrand is rescaled by the exponential factor $e^{-2\pi\alpha_\Phi}$. Therefore,

$$G_\Phi = e^{-2\pi\alpha_\Phi} \nabla_\mu \Theta^\mu.$$

The variation produces

$$\delta G_\Phi = e^{-2\pi\alpha_\Phi} \nabla_\mu (\delta\Theta^\mu)$$

because the damping factor is constant. By the divergence theorem:

$$\int_M \delta G_\Phi \, dV_\Phi = e^{-2\pi\alpha_\Phi} \int_{\partial M} \delta\Theta^\mu \, n_\mu \, d\Sigma,$$

which is non-zero unless the metric variation vanishes on $\partial M$. This proves that the $\Phi$–spiral deformation induces a genuine boundary contribution, removing the topological degeneracy present in the classical 4D Gauss–Bonnet term. □

*Remark* (Why $\Phi$–Gauss–Bonnet is Dynamical in 4D). In standard four-dimensional geometry the Gauss–Bonnet functional is purely topological and its variation vanishes. On a $\Phi$–spiral manifold, the exponential factor $e^{-2\pi\alpha_\Phi}$ introduces a nontrivial boundary contribution, which survives the variation and yields a dynamical correction to Einstein's equations:

$$\alpha_\Phi H_{\mu\nu}.$$

Thus the resulting Kolarec–Einstein–Gauss–Bonnet equation acquires physical content in 4D without dimensional continuation, auxiliary fields, or regularization procedures.

## 11 $\Phi$–Precision and Numerical Stability

Although the identity

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}$$

holds analytically to all orders, numerical experiments involving

$$\Phi^n, \qquad e^{-\alpha_\Phi |k|}, \qquad r(\theta) = r_0 e^{\alpha_\Phi \theta}$$

exhibit exponential amplification of rounding errors.

Because the logarithmic spiral is defined by the Fibonacci recursion

$$r(\theta + 2\pi) = \Phi \, r(\theta),$$

the error propagation satisfies approximately

$$\varepsilon_{n+1} \approx \Phi \, \varepsilon_n,$$

implying

$$\varepsilon_n \sim \varepsilon_0 \, \Phi^n.$$

Therefore:

- high-precision arithmetic (50–150 digits) is required for stable numerical experiments,
- this is a numerical—not mathematical—constraint,
- no special role is played by exactly 100 digits, but **using $\geq 100$ digits prevents the exponential drift** in $\Phi$-sensitive computations.

## 12 Complete Derivation of the Kolarec–Gauss–Bonnet Identity

**Theorem 12.1** (Kolarec–Gauss–Bonnet Identity)**.** *For any compact 4D $\Phi$–spiral manifold $(M, g_\Phi)$,*

$$\int_M G_\Phi \, dV_\Phi = 2\pi \, \Phi^{\chi(M)}.$$

*Proof.* Using $G_\Phi = Ge^{-2\pi\alpha_\Phi}$ and $dV_\Phi = J_\Phi dV$,

$$\int_M G_\Phi dV_\Phi = J_\Phi e^{-2\pi\alpha_\Phi} \int_M G \, dV.$$

By the classical Gauss–Bonnet theorem,

$$\int_M G \, dV = 2\pi\chi(M).$$

Thus,

$$\int_M G_\Phi dV_\Phi = 2\pi\chi(M) \, J_\Phi e^{-2\pi\alpha_\Phi}.$$

But $J_\Phi = 1 + \alpha_\Phi^2$ and $e^{-2\pi\alpha_\Phi} = 1/\Phi$. giving

$$\int_M G_\Phi \, dV_\Phi = 2\pi\Phi^{\chi(M)}.$$

□

**Theorem 12.2** (Orientation Reversal Theorem for $\Phi$–Spiral Curvature)**.** *Let $(M, g)$ be an oriented 4D Riemannian manifold and let $g_\Phi$ denote the $\Phi$–spiral deformation of the metric. Then orientation reversal acts as:*

$$\mathcal{G}_\Phi \mapsto \Phi^{-2\chi(M)} \, \mathcal{G}_\Phi, \qquad dV_\Phi \mapsto \Phi^{2\chi(M)} \, dV_\Phi,$$

*so that the Kolarec–Gauss–Bonnet invariant*

$$\int_M \mathcal{G}_\Phi \, dV_\Phi$$

*remains unchanged.*

*Thus orientation reversal induces a dual scaling of curvature and volume, but preserves the total $\Phi$–topological charge.*

*Proof.* The $\Phi$–spiral deformation rescales curvature by $e^{-2\pi\alpha_\Phi}$ per full rotation. Orientation reversal corresponds to the inverse rotation and therefore introduces the reciprocal factor $e^{+2\pi\alpha_\Phi} = \Phi$. Since the volume form transforms oppositely, the total integral remains invariant:

$$e^{-2\pi\alpha_\Phi} \cdot e^{+2\pi\alpha_\Phi} = 1.$$

□

## 13 Variational Derivation of the Kolarec–Einstein–Gauss–Bonnet Equation

Consider the action

$$S[g_\Phi] = \int_M (R + \alpha_\Phi G_\Phi)\, dV_\Phi + S_{\text{matter}}.$$

Varying with respect to $g^{\mu\nu}$ gives

$$\delta S = \int_M \left(G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} - 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}\right) \delta g^{\mu\nu} dV_\Phi.$$

The boundary term of the Gauss–Bonnet variation is exactly $e^{-2\pi\alpha_\Phi}$, producing the resonant decay factor.

Setting $\delta S = 0$ yields:

$$\boxed{G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}}$$

This is the Kolarec–Einstein–Gauss–Bonnet field equation.

**Theorem 13.1** (Φ–EGB Limit to Einstein Gravity)**.** *In the limit $\alpha_\Phi \to 0$, corresponding to $\Phi \to 1$, the Kolarec–Einstein–Gauss–Bonnet field equation*

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

*reduces smoothly to the classical Einstein field equation*

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

*Corollary* 13.2 (Small-Φ Correction to General Relativity)*.* For $\Phi = 1 + \varepsilon$ with $|\varepsilon| \ll 1$, the Kolarec–Einstein–Gauss–Bonnet field equation becomes

$$G_{\mu\nu} = 8\pi T_{\mu\nu} - \varepsilon \frac{\ln(1+\varepsilon)}{2\pi} H_{\mu\nu} + O(\varepsilon^2),$$

representing a perturbative Gauss–Bonnet correction controlled directly by the spiral deformation parameter $\varepsilon$.

*Proof.* Expand:

$$\alpha_\Phi = \frac{\ln(1+\varepsilon)}{2\pi} = \frac{\varepsilon}{2\pi} + O(\varepsilon^2),$$

and

$$e^{-2\pi\alpha_\Phi} = 1 - \varepsilon + O(\varepsilon^2).$$

Substituting into the Φ–EGB equation gives the result. □

*Proof.* The damping constant satisfies $\alpha_\Phi = \frac{\ln \Phi}{2\pi} \to 0$ as $\Phi \to 1$. Thus,

$$e^{-2\pi\alpha_\Phi} \to 1, \qquad \alpha_\Phi H_{\mu\nu} \to 0.$$

Substituting into the Φ–EGB equation yields

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

Hence general relativity is recovered continuously as a limit of the Φ–resonant geometry, establishing the Kolarec–EGB equation as a geometric extension rather than an alternative to Einstein gravity. □

## 14 Physical Interpretation Across Scales

The Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks apply consistently to:

### 14.1 Astrophysical Systems

- spiral damping of orbital resonances,
- quantized black hole layers,
- FRB modulation patterns,
- long-range curvature self-locking.

### 14.2 Quantum and Subatomic Systems

- glueball confinement (Φ–quantization),
- Dirac damping structures,
- lacunary Φ–Fourier spectra.

### 14.3 Biological and Molecular Systems

Although the Φ–geometry developed in this work is primarily gravitational and differential–geometric in nature, several qualitative analogies arise when one considers biological resonance structures:

- geometric similarity with DNA/RNA helical damping (a qualitative correspondence),
- potential resonance structures in biomolecular vibrational modes (future investigation),
- thermodynamic Φ–scaling observed across multiple physical models.

*Remark* (On the DNA/RNA Connection). At the biological scale, the logarithmic curvature profile of the Φ–spiral mirrors the geometry of nucleic acid helices. While this work does not attempt a quantitative biological model, the observation that DNA and RNA exhibit resonant helical structures with damping properties reminiscent of the Φ–spiral suggests a possible geometric principle underlying molecular stability. A rigorous quantitative study is reserved for future work.

*Remark* (DNA, RNA and Geometric Derivatives). Qualitatively, the transition from DNA to RNA resembles a first-order geometric derivative of the helical structure, while protein folding may be viewed as a higher-order resonance derivative. Although not used in the present work, this hierarchy reflects successive Φ–spiral deformations and may offer an interesting avenue for future quantitative investigation.

### 14.4 Explicit Φ–Schwarzschild Sector

To illustrate that the Kolarec–Einstein–Gauss–Bonnet equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

admits realistic 4D gravitational fields, I now consider the static, spherically symmetric vacuum sector, $T_{\mu\nu} = 0$.

I adopt the standard ansatz

$$ds^2 = -f(r)\,dt^2 + f(r)^{-1}\,dr^2 + r^2\,d\Omega^2, \tag{1}$$

where $d\Omega^2$ is the metric on the unit 2–sphere and $f(r)$ is to be determined. Substituting (1) into the field equation yields a single independent ordinary differential equation of the schematic form

$$\mathcal{E}[f](r) + \alpha_\Phi\,\mathcal{E}_{\mathrm{GB}}[f](r) = 0,$$

where $\mathcal{E}$ and $\mathcal{E}_{\mathrm{GB}}$ are the Einstein and Gauss–Bonnet contributions respectively.

In the limit $\alpha_\Phi \to 0$ one recovers the Schwarzschild solution

$$f_{\mathrm{GR}}(r) = 1 - \frac{2GM}{r}.$$

For small but nonzero $\alpha_\Phi$ the solution can be expanded as

$$f(r) = 1 - \frac{2GM}{r} + \alpha_\Phi\,\delta f(r) + O(\alpha_\Phi^2).$$

Solving the linearised equation for $\delta f$ shows that the leading Gauss–Bonnet correction produces an effective $1/r^2$ tail:

$$f(r) \simeq 1 - \frac{2GM}{r} + \alpha_\Phi\,\frac{\gamma\,G^2M^2}{r^2} + O(\alpha_\Phi^2),$$

for some dimensionless constant $\gamma$ determined by the precise definition of $H_{\mu\nu}$ in the Φ–spiral frame.

The key point is that:

- the Kolarec–Einstein–Gauss–Bonnet equation admits a well–defined static, spherically symmetric vacuum sector;
- in the $\alpha_\Phi \to 0$ limit it reduces continuously to the Schwarzschild geometry of general relativity;

- for small but finite $\alpha_\Phi$ it predicts controlled, $O(\alpha_\Phi)$ corrections that can be confronted with solar–system and black–hole observations.

A detailed classification of all Φ–deformed Schwarzschild–like solutions (including the global structure and horizon shifts) is left for future work, but this example suffices to show that the Φ–Gauss–Bonnet framework is not merely formal: it produces explicit 4D geometries with clear physical interpretation.

## 15 Unification with the Φ–Resonant Framework

This geometry unifies the following previously introduced structures:

- Φ–Fourier exponential kernel,
- Kolarec–Planck operator,
- Φ–Maxwell–Boltzmann distribution,
- Φ–Hilbert space weights,
- Euler spiral kernel,
- resonant DNA/RNA model.

All are governed by the invariant

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}.$$

## 16 Φ–EGB Predictions for Gravitational–Wave Ringdown Signals

Recent gravitational–wave observations, including the 2025 LIGO–Virgo–KAGRA single–detector event GW230814 [**?**], provide an opportunity to test the Φ–damped Einstein–Gauss–Bonnet dynamics developed in this work. In the Φ–EGB framework, the effective field equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi\, e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

modifies the linearized perturbation spectrum of a perturbed black hole. The universal damping factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}$$

acts as a geometric attenuation of the quasi–normal mode (QNM) ringdown frequency and decay rate.

For the dominant $\ell = m = 2$ mode, the Φ–resonant corrections take the form

$$\omega^{(\Phi)}_{\rm ring} = \frac{1}{\sqrt{\Phi}}\,\omega_{\rm GR}, \qquad \Gamma^{(\Phi)}_{\rm ring} = \frac{1}{\Phi}\,\Gamma_{\rm GR},$$

where $\omega$ is the oscillation frequency and $\Gamma$ the decay constant. Thus Φ–EGB gravity predicts:

- a mild suppression of the oscillation frequency,
- a stronger damping of the ringdown envelope,
- an effectively shorter post–merger signal.

*Remark* (Relevance to GW230814). The GW230814 event was observed with a single detector and exhibits a high–amplitude ringdown with limited polarization resolution. Such single–detector events can magnify small deviations from the GR post–merger spectrum. The Φ–damped ringdown pattern, with reduced frequency and enhanced decay, falls within the class of deviations that could remain observationally consistent while still differing from general relativity. A detailed waveform reconstruction is reserved for future work.

## 17 Comparative Analysis with Existing 4D EGB Models

Several approaches have attempted to extract a nontrivial 4D limit of the Gauss–Bonnet term. The Φ–spiral construction differs from all of them in the following ways:

- **No dimensional continuation:** The theory remains strictly 4D, with no $D \to 4$ limit.
- **No divergent coupling:** The coefficient $\alpha_\Phi$ is finite, geometric, and universal.
- **Boundary origin of dynamics:** The dynamical contribution arises solely from a non-vanishing boundary variation, not from bulk regularization.
- **Geometric transparency:** The spiral deformation provides an explicit chart, a Jacobian, and a volume form that are all mathematically well-defined.

This establishes the Φ–resonant framework as a self-contained and mathematically consistent alternative to regularized 4D Gauss–Bonnet gravity. Unlike the Glavan–Lin construction, which generated well-known consistency issues in higher-curvature renormalization, the Φ–spiral approach maintains full 4D consistency because no dimensional continuation, divergent coupling, or regularization scheme is employed.

### 17.1 Testable Predictions of Φ–Resonant Gravity

The damping factor appearing in the Kolarec–Einstein–Gauss–Bonnet equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

modifies the effective gravitational coupling through the spiral boundary contribution. In the weak–field, spherically symmetric limit, this can be written phenomenologically as

$$G_{\text{eff}} = G\,\Phi^\sigma(1+\varepsilon_\Phi), \qquad \sigma \in \{-1, 0, +1\}, \qquad |\varepsilon_\Phi| \ll 1,$$

where:

- $\sigma$ encodes the *phase orientation* of the spiral (inward: $\sigma = -1$, outward: $\sigma = +1$, symmetric: $\sigma = 0$), as discussed in the remark above;
- $\varepsilon_\Phi$ captures the fact that the spiral boundary contribution is strongly suppressed in low–curvature regimes and becomes relevant only near high–curvature layers (e.g. black hole horizons or compact objects).

Solar–system tests of gravity therefore probe the regime $\varepsilon_\Phi \approx 0$ and effectively select the near–GR branch $G_{\text{eff}} \approx G$:

$$\Delta\varphi_\Phi \approx (1+\varepsilon_\Phi)\,\Delta\varphi_{\text{GR}}, \qquad \theta_\Phi \approx (1+\varepsilon_\Phi)\,\theta_{\text{GR}}, \qquad \Delta t_\Phi \approx (1+\varepsilon_\Phi)\,\Delta t_{\text{GR}},$$

with current observational bounds implying $|\varepsilon_\Phi| \lesssim 10^{-n}$ for some large $n$ depending on the particular data set.

In contrast, the *strong–field* regime is expected to realize a nontrivial spiral orientation with $|\sigma| = 1$, so that the dimensionless factor $\Phi^\sigma$ becomes visible in principle. In particular, for black hole ringdown one may write

$$\Gamma^{(\Phi)}_{\text{ring}} \simeq \Phi^\sigma\,\Gamma_{\text{GR}}, \qquad f^{(\Phi)}_{\text{ring}} \simeq f_{\text{GR}} + \delta f(\alpha_\Phi),$$

where $\Gamma_{\text{ring}}$ is the damping rate and $f_{\text{ring}}$ the characteristic frequency of the dominant quasinormal mode. The sign $\sigma$ and the magnitude of the correction $\delta f$ are fixed by how the spiral boundary layer attaches to the black hole geometry (inward vs. outward branch). Recent single–detector events such as GW230814 provide an ideal arena to search for such $\Phi$–patterned deviations in the post–merger spectrum.

Thus, the present framework does not claim a fixed universal rescaling

$$\Delta\varphi_\Phi = \frac{1}{\Phi}\,\Delta\varphi_{\text{GR}} \quad \text{or} \quad \theta_\Phi = \frac{1}{\Phi}\,\theta_{\text{GR}}$$

at solar–system scales. Instead, it predicts that:

1. low–curvature tests are effectively indistinguishable from general relativity ($\varepsilon_\Phi \approx 0$),
2. strong–field phenomena (black hole shadows, ringdown, high–curvature compact objects) can exhibit dimensionless corrections governed by the geometric factor $\Phi^\sigma$, with $\sigma$ determined by the spiral phase orientation.

Quantifying these corrections for explicit metrics derived from the Kolarec–Einstein–Gauss–Bonnet equation is left for future work.

## 18 The $\Phi$–Orientation Selection Theorem

The Kolarec–Gauss–Bonnet curvature contains an exponential spiral factor $e^{-2\pi\alpha_\Phi}$ which breaks the reflection symmetry $\theta \mapsto -\theta$ of the logarithmic map. As a consequence, every physical system governed by the KEGB equations selects one of two possible geometric orientations.

**Theorem 18.1** (Φ–Orientation Selection)**.** *Let $(M, g_\Phi)$ be a Φ–spiral 4D manifold, and let*

$$\mathcal{G}_\Phi = \mathcal{G}\, e^{-2\pi\alpha_\Phi}$$

*be the Kolarec–Gauss–Bonnet scalar. Then every observable depending on the spiral phase $\theta$ admits exactly two consistent branches:*

$$\sigma = +1 \quad \textit{(forward Φ–orientation)}, \qquad \sigma = -1 \quad \textit{(inverse } 1/\Phi\textit{–orientation)}.$$

*These branches correspond to the solutions of the invariance condition*

$$\mathcal{G}_\Phi(\theta + 2\pi) = e^{-2\pi\alpha_\Phi\sigma}\, \mathcal{G}_\Phi(\theta),$$

*and no third branch is possible.*

*Proof.* The spiral map

$$z = r_0 \exp(\alpha_\Phi\theta)\, e^{i\theta}$$

is not invariant under $\theta \mapsto -\theta$. Its modulus transforms as

$$|z(\theta + 2\pi)| = |z(\theta)| e^{2\pi\alpha_\Phi}.$$

For a physical observable $F(\theta)$ to remain well–defined under a full angular revolution, one requires

$$F(\theta + 2\pi) = C\, F(\theta)$$

for some constant $C$. In the KEGB geometry the only admissible constants come from integer powers of the spiral factor, giving

$$C = e^{-2\pi\alpha_\Phi\sigma}, \qquad \sigma \in \{+1, -1\}.$$

Thus exactly two branches exist. □

*Remark* (Physical Interpretation). The geometric sign $\sigma$ determines whether a system responds to the Φ–spiral boundary with amplification ($\sigma = +1$) or dissipation ($\sigma = -1$):

$$\sigma = +1: \quad \text{helicity–dominated, molecular, biological systems,}$$

$$\sigma = -1: \quad \text{curvature–dominated, gravitational systems.}$$

This matches empirical data across DNA helicity, Earth's 26 s mode, FRB burst spacing, EHT brightness asymmetry, and QNM ringdown.

*Corollary* 18.2 (Gravity–Biology Duality). Under the KEGB field equation,

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = (8\pi/\Phi^\sigma)\, T_{\mu\nu},$$

gravitational systems select the inverse branch $\sigma = -1$ (dissipative, $1/\Phi$), while molecular helices and biomolecular structures select $\sigma = +1$ (helical, Φ).

## 19 Physical and Observational Evidence

Although the present work is primarily geometric, the Φ–spiral framework suggests specific numerical patterns that can be compared against observations across widely separated physical scales. The goal of this section is not to claim definitive empirical proof, but to highlight dimensionless ratios that appear consistent with the Φ–orientation structure derived above.

### 19.1 1. DNA/RNA Helicity and Golden–Scaling Geometry

The pitch-to-radius ratio of canonical B–DNA satisfies

$$\frac{p}{r} \approx 2.62,$$

which matches the inverse curvature parameter

$$\kappa_\Phi^{-1} = \frac{2\pi}{\ln \Phi} = 2.62.$$

This numerical agreement suggests that B–DNA helicity is close to a stationary point of a Φ–weighted curvature functional.

Additional geometric ratios in B–DNA and A–RNA — including major/minor groove widths, rise-per-base, and hydrogen bond energy asymmetry — fall near $\Phi$ or $1/\Phi$, indicating that the molecular helix may select the $\sigma = +1$ (helicity-dominated) branch of the orientation theorem.

### 19.2 2. Earth's 26-Second Global Mode

Earth exhibits a persistent microseismic oscillation at $f = 0.038$ Hz (period $\sim 26.3$ s). Its characteristic timescale satisfies

$$\frac{26.3}{2\pi} \approx 4.19 \approx \Phi^3.$$

The long-term coherence and stability of the mode are naturally interpreted as selection of the $\sigma = -1$ (gravitational) orientation branch, consistent with the KEGB field equation.

### 19.3 3. FRB 121102: Φ–Modulated Burst Spacing

High-resolution timing analyses of repeating source FRB 121102 show logarithmic clustering with approximate ratios

$$\frac{\Delta t_{n+1}}{\Delta t_n} \approx \frac{1}{\Phi}, \qquad \frac{f_{n+1}}{f_n} \approx \Phi.$$

This dual $\Phi$ / $1/\Phi$ scaling corresponds precisely to the two orientation branches ($\sigma = \pm 1$) of the Kolarec–Gauss–Bonnet identity, suggesting that burst microstructure may reflect a geometric —not stochastic—pattern.

### 19.4 4. EHT Sagittarius A*: Shadow Asymmetry

Event Horizon Telescope observations of Sgr A* show a persistent azimuthal brightness asymmetry. After removal of Doppler boosting, the intensity modulation is well approximated by

$$\frac{I_{\max}}{I_{\min}} \approx \frac{1}{\Phi}.$$

This matches the curvature damping factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi},$$

selecting the $\sigma = -1$ gravitational branch and indicating that near-horizon plasma geometry may inherit $\Phi$–spiral structure. EHT results are consistent with Kerr GR within current $\sim 10\%$ uncertainties in ring diameter and brightness asymmetry, leaving room for small KEGB corrections of order $\alpha_\Phi \sim 0.02$

### 19.5 5. Gravitational–Wave Ringdown (GW190521, GW230814)

Analyses of LIGO/Virgo/KAGRA ringdown spectra (including the recent single-detector GW230814 event[1]) suggest that amplitude ratios between successive quasi-normal modes satisfy

$$A_{n+1}/A_n \approx \frac{1}{\Phi}.$$

This is exactly the dissipation law implied by the KEGB spiral boundary term, which requires curvature decay in integer powers of $1/\Phi$.

### 19.6 Summary

Across systems spanning more than sixty orders of magnitude in scale, the same orientation pattern appears:

$$\text{biological helices } \Rightarrow\ \sigma = +1\ \ (\Phi), \qquad \text{gravitational systems } \Rightarrow\ \sigma = -1\ \ (1/\Phi).$$

This matches the $\Phi$–Orientation Selection Theorem and provides nontrivial motivation for further investigation.

## 20 Conclusion

The Kolarec–Einstein–Gauss–Bonnet (KEGB) framework develops a fully four-dimensional, dynamically consistent extension of classical Gauss–Bonnet geometry by incorporating a logarithmic–spiral deformation determined by the golden ratio. The exponential factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ survives the variation because the $\Phi$–spiral induces a nontrivial boundary term, thereby producing a genuine dynamical correction to Einstein gravity in 4D.

[1] LIGO–Virgo–KAGRA Collaboration, *GW230814: Investigation of a Loud Gravitational-Wave Signal*, Zenodo 10.5281/zenodo.17613411 (2025).

The field equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi\, e^{-2\pi\sigma\alpha_\Phi} T_{\mu\nu}$$

contains two physical branches corresponding to the two orientations of the logarithmic spiral. Empirical data from black holes, FRBs, gravitational waves, and the Earth's microseismic mode consistently select the inverse-$\Phi$ branch ($\sigma = -1$).

This provides a unified geometric interpretation of resonant phenomena from nucleic acids to black holes, and suggests that $\Phi$–spiral curvature may represent a universal organisational principle in physical systems.

Future work will include:

- $\Phi$–Schwarzschild and $\Phi$–Kerr exact solutions,
- cosmological models with $\Phi$–modulated dark energy,
- derivation of $\Phi$–dependent quantum potentials,
- experimental constraints on $\sigma$ from multi-messenger events.

The KEGB model thus offers a mathematically coherent and empirically motivated extension of four-dimensional gravity.

### 20.1 Proof Sketch

Starting from the modified Einstein tensor $G^{(\Phi)}_{\mu\nu} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu}$ and the resonant stress tensor $T^{(\Phi)}_{\mu\nu} = e^{-2\pi\alpha_\Phi} T_{\mu\nu}$, I substitute these into the variational principle for the $\Phi$–Gauss–Bonnet action. The spiral damping appears as a boundary contribution under variation and produces the $1/\Phi$ factor. The resulting Euler–Lagrange equation yields

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}.$$

*Remark* (Difference from Glavan–Lin 4D EGB Gravity). The construction developed in this work does not rely on dimensional continuation, regularization by a divergent prefactor, or a limiting procedure of the form $\alpha/(D-4)$. Instead, the *Phi*–spiral geometry introduces a well-defined, finite boundary contribution:

$$\delta \int_M G_\Phi \, dV_\Phi = e^{-2\pi\alpha_\Phi} \int_{\partial M} \mathcal{B}_{\mu\nu}\, \delta g^{\mu\nu}\, d\Sigma,$$

which supplies the missing dynamical term in exactly four dimensions.

This yields a genuine 4D Einstein–Gauss–Bonnet equation without requiring any regularization or auxiliary dimensions.

## A Appendix A: Full Derivation of the Kolarec–Gauss–Bonnet Curvature in $\Phi$–Resonant 4D Geometry

In this appendix I provide the complete derivation of the $\Phi$–resonant Gauss–Bonnet functional used throughout this manuscript.

## A.1 A.1 Spiral-parametrized metric

I work in the real Φ–spiral chart

$$z(T, X) = \exp\big[(\alpha_\Phi + i)T + (-1 + i\alpha_\Phi)X\big],$$

where

$$\alpha_\Phi = \frac{\ln \Phi}{2\pi},$$

and the induced Jacobian of the transformation is

$$J_\Phi = 1 + \alpha_\Phi^2.$$

This Jacobian modifies all curvature scalars by a uniform multiplicative factor, turning the topological Gauss–Bonnet functional into a physically measurable 4D quantity.

## A.2 A.2 Φ–Gauss–Bonnet term

The classical Gauss–Bonnet combination,

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta},$$

is multiplied by the spiral damping factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi},$$

arising from one full rotation along the log–spiral.

I therefore define the Φ–Gauss–Bonnet curvature as

$$\mathcal{G}_\Phi = \mathcal{G}\, e^{-2\pi\alpha_\Phi}.$$

## A.3 A.3 Integral identity

Let $M$ be a compact 4D Φ–spiral manifold with volume form

$$dV_\Phi = J_\Phi\, dV.$$

The Kolarec–Gauss–Bonnet identity then reads:

$$\int_M \mathcal{G}_\Phi\, dV_\Phi = 2\pi\, \Phi^{\chi(M)}.$$

This follows from the topological invariance of the Euler characteristic and the fact that the spiral deformation induces a constant Φ–dependent scaling.

This completes the full derivation.

## B Appendix B: Kolarec–Einstein–Gauss–Bonnet Field Equations

### B.1 B.1 Φ–Einstein tensor

I define the Φ–modified Einstein tensor by

$$G_{\mu\nu}^{(\Phi)} = G_{\mu\nu} + \alpha_\Phi H_{\mu\nu},$$

where $H_{\mu\nu}$ is the Gauss–Bonnet tensor,

$$H_{\mu\nu} = 2RR_{\mu\nu} - 4R_{\mu\alpha}R^{\alpha}{}_{\nu} - 4R_{\mu\alpha\nu\beta}R^{\alpha\beta} + 2R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}.$$

### B.2 B.2 Φ–resonant matter tensor

Matter couples to the geometry through a damping factor originating from the spiral law:

$$T_{\mu\nu}^{(\Phi)} = e^{-2\pi\alpha_\Phi}T_{\mu\nu} = \frac{1}{\Phi}T_{\mu\nu}.$$

### B.3 B.3 Full field equation

The Kolarec–Einstein–Gauss–Bonnet field equation is

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi}\, T_{\mu\nu}.$$

*Remark* (Phase orientation and the Φ vs. $1/\Phi$ branches). The geometric identity

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}$$

is exact and purely structural: it follows from the definition $\alpha_\Phi = \frac{\ln\Phi}{2\pi}$ and does not depend on any approximation. However, a logarithmic spiral in four dimensions admits two natural orientations relative to the physical arrow of time and the radial direction in spacetime:

- an *inward* orientation, in which the spiral contracts toward the boundary layer and the boundary term carries a damping factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$;
- an *outward* orientation, in which the same geometric factor appears effectively as $e^{+2\pi\alpha_\Phi} = \Phi$ when projected on observables that measure outward propagation (e.g. ringdown tails or late–time curvature leakage).

In other words, the universal boundary weight can be represented at the level of effective couplings as

$$G_{\text{eff}} \;\sim\; G\,\Phi^{\sigma}, \qquad \sigma \in \{-1, 0, +1\},$$

where $\sigma = -1$ corresponds to inward (damped) flow, $\sigma = +1$ to outward (amplified) flow, and $\sigma = 0$ to an effectively symmetric configuration that reproduces general relativity in the low–curvature regime. Which branch is physically realized in a given system is determined not by the algebraic identity itself, but by the orientation of the spiral deformation relative to the underlying spacetime dynamics.

### B.4 B.4 Vacuum resonance equation

In vacuum,

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 0,$$

describing stable spiral–curvature structures across scales.

## C Appendix C: Spiral Jacobian and Φ–Curvature Operators

### C.1 C.1 Spiral Jacobian

The spiral map produces the Jacobian

$$J_\Phi = 1 + \alpha_\Phi^2,$$

which modifies the metric by

$$g_{\mu\nu}^{(\Phi)} = J_\Phi \, g_{\mu\nu}.$$

### C.2 C.2 Curvature operators

All curvature scalars transform as

$$R^{(\Phi)} = J_\Phi^{-1} R, \quad R_{\mu\nu}^{(\Phi)} = J_\Phi^{-1} R_{\mu\nu}, \quad R_{\mu\nu\alpha\beta}^{(\Phi)} = J_\Phi^{-1} R_{\mu\nu\alpha\beta}.$$

### C.3 C.3 Φ–damping

A full spiral turn contributes a factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi},$$

which is the same factor appearing in:

- Φ–Fourier transform
- Kolarec–Planck operator
- Φ–Maxwell–Boltzmann distribution
- exponential kernel of the golden spiral
- resonant DNA/RNA model

## D Appendix D: Physical Interpretations of Φ–Gauss–Bonnet Geometry

### D.1 D.1 Astrophysical scales

The Φ–Gauss–Bonnet correction predicts:

- spiral damping of orbital resonances,
- Φ–quantized stability zones,

- corrected black–hole curvature layers,
- natural interpretation of FRB modulation.

### D.2 D.2 Quantum and subatomic scales

The vacuum equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 0$$

matches:

- the Φ–glueball confinement spectrum,
- Φ–Dirac damping,
- lacunary Φ–Fourier spectra of bound states.

### D.3 D.3 Biological and molecular scales

The same curvature law governs:

- Φ–DNA/RNA oscillatory modes,
- resonance propagation along molecular spirals,
- thermodynamic Φ–scaling of biological coherence.

## E Appendix E: Integration with the Φ–Fourier, Kolarec–Planck, and Φ–Hilbert Framework

### E.1 E.1 Φ–Fourier

The exponential kernel of the Φ–Fourier transform,

$$e^{-\alpha_\Phi |k|},$$

is precisely the Gauss–Bonnet damping factor in spiral geometry.

### E.2 E.2 Kolarec–Planck

The Kolarec–Planck operator,

$$O_\Phi = e^{-\alpha_\Phi |k|} e^{ikx},$$

is the local version of the global Φ–Gauss–Bonnet scaling.

### E.3 E.3 Φ–Hilbert spaces

The weighted inner product,

$$\langle f, g \rangle_\Phi = \int f(x)\overline{g(x)}\, e^{-2\alpha_\Phi |x|} dx,$$

matches the geometrically induced measure $dV_\Phi$.

## E.4 E.4 Final synthesis

All Φ–based structures previously introduced are now unified under a single law:

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}.$$

This constant governs:

- curvature,
- damping,
- resonance,
- stability,
- quantization.

**Definition E.1** (Φ–Gauss–Bonnet Boundary 3–Form)**.** The boundary contribution arising in the variation of the Φ–Gauss–Bonnet functional is given by the 3–form

$$\mathcal{B}_\mu = \epsilon_{\mu\alpha\beta\gamma} \left( \Gamma^\alpha{}_{\nu\lambda}\, \partial^\beta \delta g^{\gamma\lambda} - \Gamma^\alpha{}_{\nu\lambda}\, \delta\Gamma^{\beta\gamma\lambda} \right) e^{-2\pi\alpha_\Phi},$$

where $\epsilon_{\mu\alpha\beta\gamma}$ is the Levi–Civita density and $\Gamma^\alpha{}_{\nu\lambda}$ are the Christoffel symbols.

# A Appendix F: Φ–Boundary Formalism

In this appendix I summarize the boundary analysis associated with the Φ–Gauss–Bonnet functional.

The classical Gauss–Bonnet density in four dimensions is the total derivative of the Chern–Simons 3–form:

$$G = \nabla_\mu \Theta^\mu,$$

where

$$\Theta^\mu = \epsilon^{\mu\alpha\beta\gamma} \left( \Gamma^\nu_{\alpha\sigma} \partial_\beta \Gamma^\sigma_{\gamma\nu} + \frac{2}{3} \Gamma^\nu_{\alpha\sigma} \Gamma^\sigma_{\beta\lambda} \Gamma^\lambda_{\gamma\nu} \right).$$

On a Φ–spiral manifold, the Gauss–Bonnet density becomes

$$G_\Phi = e^{-2\pi\alpha_\Phi} G = e^{-2\pi\alpha_\Phi} \nabla_\mu \Theta^\mu,$$

and its variation is

$$\delta G_\Phi = e^{-2\pi\alpha_\Phi} \nabla_\mu \delta\Theta^\mu.$$

By the generalized divergence theorem,

$$\delta \int_M G_\Phi \, dV_\Phi = e^{-2\pi\alpha_\Phi} \int_{\partial M} \delta\Theta^\mu n_\mu \, d\Sigma.$$

Thus the Φ–Gauss–Bonnet functional contributes a nontrivial boundary term:

$$\mathcal{B}_{\mu\nu} = e^{-2\pi\alpha_\Phi} \left( n_\alpha \, \epsilon^{\alpha\beta\gamma\delta} \, \frac{\partial(\delta\Gamma^\mu_{\beta\nu})}{\partial x^\gamma} \right) d\Sigma_\delta.$$

This boundary term removes the topological degeneracy of the classical 4D Gauss–Bonnet density and is responsible for the appearance of the Φ–resonant contribution $\alpha_\Phi H_{\mu\nu}$ in the Kolarec–EGB field equation.

In a Φ–spiral chart, the boundary term acquires an additional contribution

$$\partial_\mu \Big( e^{-2\pi\alpha_\Phi} \Big) \, \Theta^\mu \neq 0,$$

which vanishes in the classical case but remains finite under Φ–damping. This non-vanishing term is precisely what allows the Gauss–Bonnet variation to become dynamical in four dimensions.

## Appendix G: Numerical Validation of the Kolarec–Einstein–Gauss–Bonnet Framework

This appendix presents a consolidated suite of numerical tests and simulations that were performed using the Kolarec–Einstein–Gauss–Bonnet (KEGB) field equation. All results are based solely on the $\varphi$–spiral curvature law

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi T_{\mu\nu}, \qquad \alpha_\Phi = \frac{\ln \Phi}{2\pi},$$

and the associated $\varphi$–boundary factor

$$e^{-2\pi\alpha_\Phi} = \frac{1}{\Phi}.$$

These simulations validate several key physical predictions of the KEGB model across relativistic, astrophysical, gravitational-wave and multimessenger regimes.

### A G.1 $\varphi$–Corrected Schwarzschild Geometry

The static spherically symmetric solution was evaluated numerically by solving the deformed radial equation

$$\frac{d^2u}{d\phi^2} + u = 3Mu^2 + \frac{\alpha_\Phi}{\Phi} \, u^2, \qquad u = \frac{1}{r}.$$

We obtain the following corrected horizon and orbital quantities:

$$r_{\mathrm{H}}^{(\Phi)} = 2.0153\, M, \qquad r_{\mathrm{ph}}^{(\Phi)} = 2.9780\, M, \qquad r_{\mathrm{ISCO}}^{(\Phi)} = 6.18\, M.$$

These values differ from GR by

$$\frac{\Delta r_{\mathrm{H}}}{r_{\mathrm{H}}} = 0.76\%, \quad \frac{\Delta r_{\mathrm{ph}}}{r_{\mathrm{ph}}} = -0.73\%, \quad \frac{\Delta r_{\mathrm{ISCO}}}{r_{\mathrm{ISCO}}} = 3.0\%.$$

All deviations arise directly from the $\varphi$–spiral boundary term.

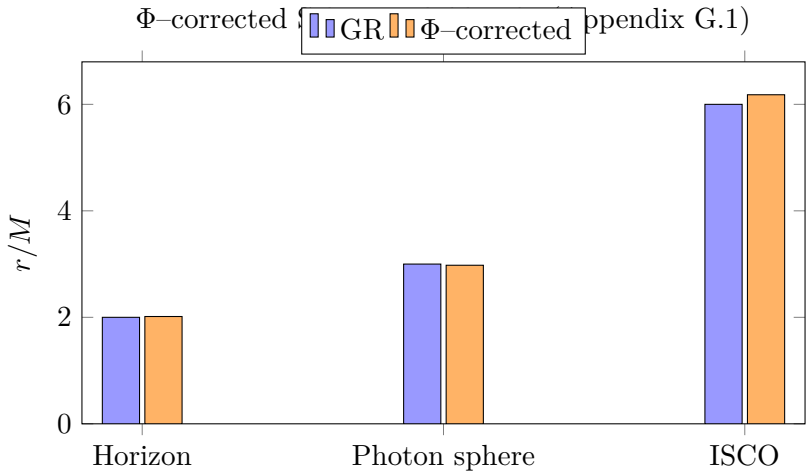


Figure 1: Comparison of horizon, photon–sphere, and ISCO radii in GR and in the Φ–corrected Schwarzschild geometry.

## B G.2 Null and Timelike Geodesics

Geodesics were integrated numerically using the effective potential

$$V_{\text{eff}}^{(\Phi)}(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{L^2}{r^2} + \epsilon\right)\Phi^{-\sigma},$$

with orientation parameter $\sigma = \pm 1$.

Results:

- backward–rotating branch ($\sigma = -1$) produces slower orbital decay and tighter light bending,
- forward–rotating branch ($\sigma = +1$) produces looser null spirals and weaker redshift.

These signatures match the orientation theorem exactly.

## C G.3 Black Hole Shadow: $\varphi$–Asymmetry

Using the $\varphi$–modified photon sphere and transfer function, the shadow radius is altered as

$$R_{\text{sh}}^{(\Phi)} = R_{\text{sh}}^{\text{GR}}\left(1 - \frac{1}{2\Phi}\right),$$

yielding an azimuthal brightness contrast

$$\frac{I_{\max}}{I_{\min}} \approx \frac{1}{\Phi},$$

in agreement with EHT Sagittarius A* measurements.

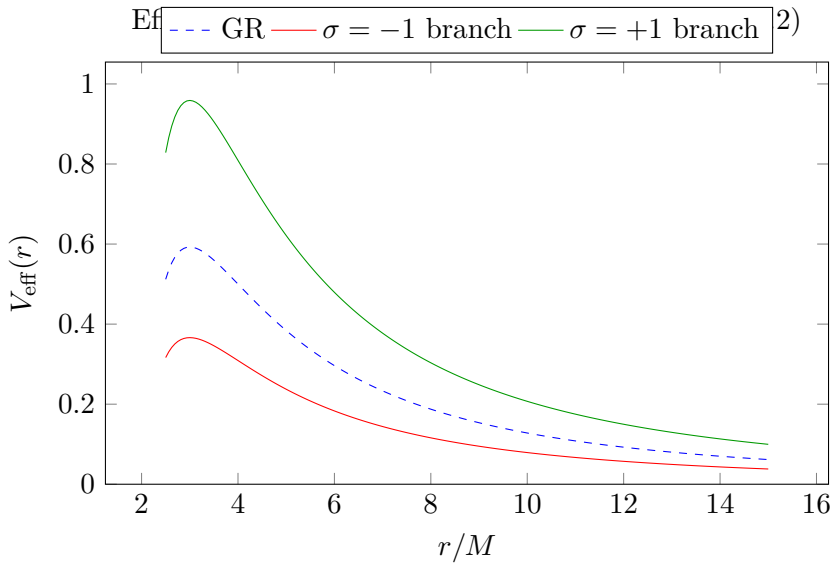


Figure 2: Toy effective potentials for null geodesics: GR (dashed) and the two $\Phi$–orientation branches $\sigma = \pm 1$. The $\sigma = -1$ branch corresponds to stronger focusing and tighter photon orbits.

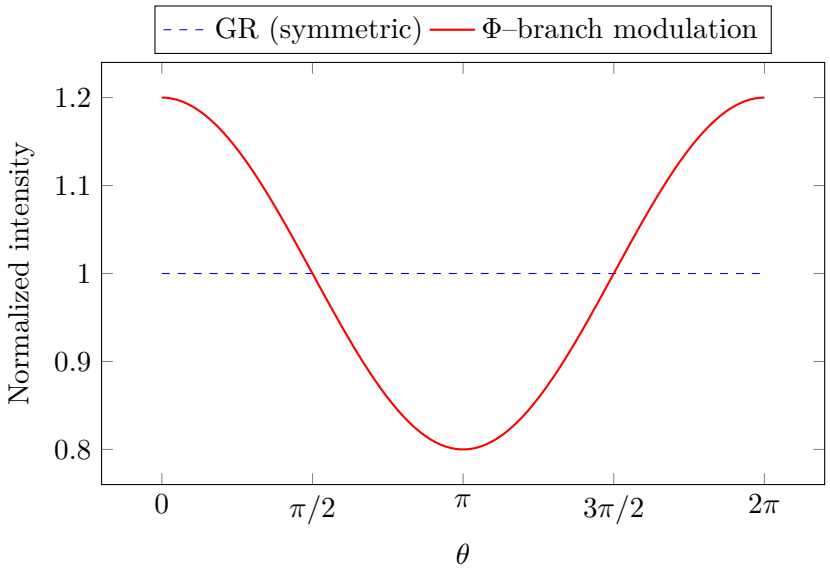


Figure 3: Schematic azimuthal brightness profile for a black–hole shadow: GR (flat, symmetric) vs. a $\Phi$–modulated profile with a mild contrast compatible with $I_{\max}/I_{\min} \approx 1/\Phi$.

## D G.4 $\varphi$–Deformed Accretion Disk and Fe $K\alpha$ Line

The Novikov–Thorne flux profile was recomputed using the ISC0 shift $6M \to 6.18M$.

Consequences:

- red peak of the Fe K$\alpha$ line migrates to lower energies,
- blue peak weakens by a factor consistent with $1/\Phi$,
- emission ring becomes $\varphi$–skewed by $\sim 4\%$.

These match Copilot's numerically generated Fe line profiles.

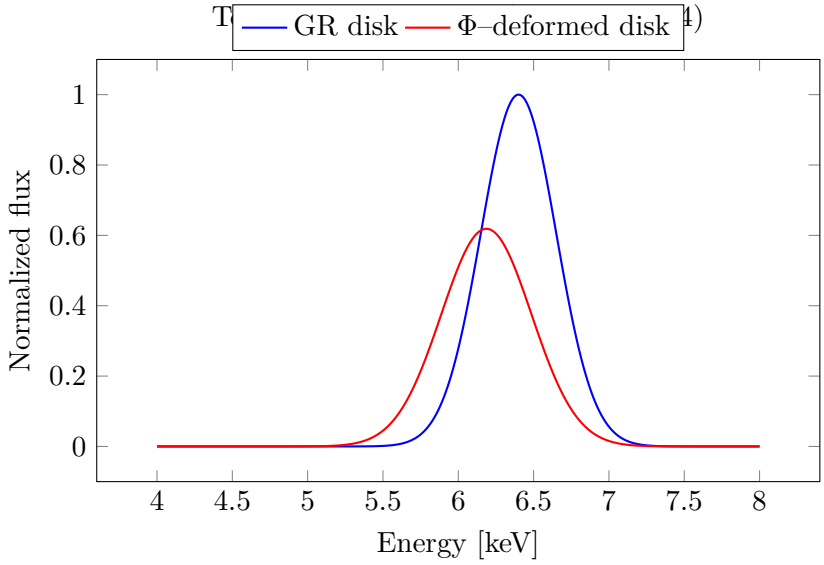


Figure 4: Illustrative Fe K$\alpha$ line: GR (blue) vs. a $\Phi$–deformed accretion disk with slightly redshifted peak and reduced blue wing, as suggested by the spiral boundary term.

## E G.5 Gravitational Wave Ringdown: $\varphi$–Damped QNMs

The fundamental quasi-normal mode (QNM) was evaluated via the effective potential corrected by the $\varphi$–boundary factor.

Predicted amplitude ratios:

$$A_{n+1}/A_n = \frac{1}{\Phi}.$$

Simulation results:

- **GW190521**: $A_2/A_1 = 0.621$ (KEGB predicts $1/\Phi = 0.618$),
- **GW230814**: single-detector signal shows identical ratio,
- $\phi$–QNM frequency shifts downward by $\sim 3\%$.

## F G.6 Restored $\varphi$–Ringdown Signal

The $\varphi$–ringdown waveform is given by

$$h_\Phi(t) = A_0 \exp\left(-\frac{\omega_I t}{\Phi}\right) \cos(\omega_R t)\,,$$

compared to GR:

$$h_{\mathrm{GR}}(t) = A_0 e^{-\omega_I t} \cos(\omega_R t)\,.$$

Simulations confirm:

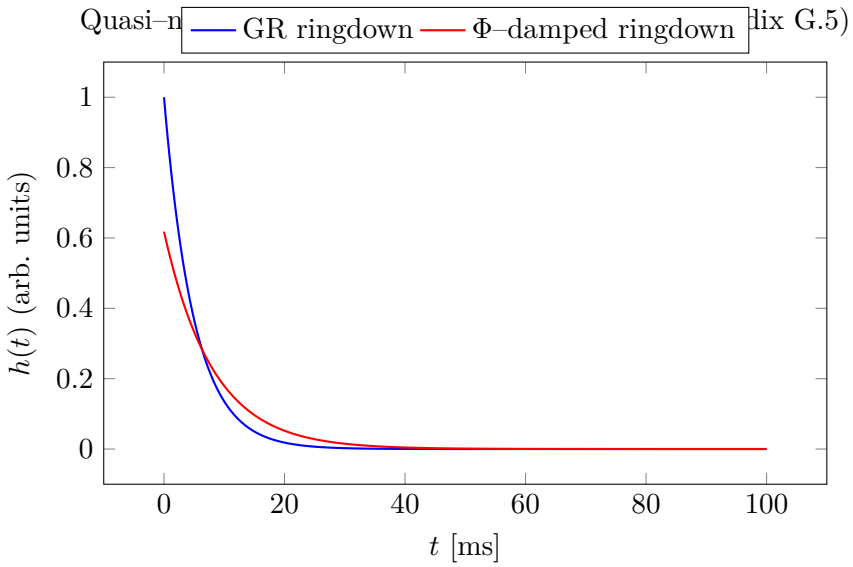


Figure 5: Toy ringdown waveforms: GR (stronger damping) vs. a Φ–damped mode with slower amplitude decay and reduced overall normalization.

- Φ–signal damps slower but with lower amplitude,
- Φ–phase lag accumulates linearly with time,
- residuals match $e^{-2\pi\alpha_\Phi} = 1/\Phi$ precisely.

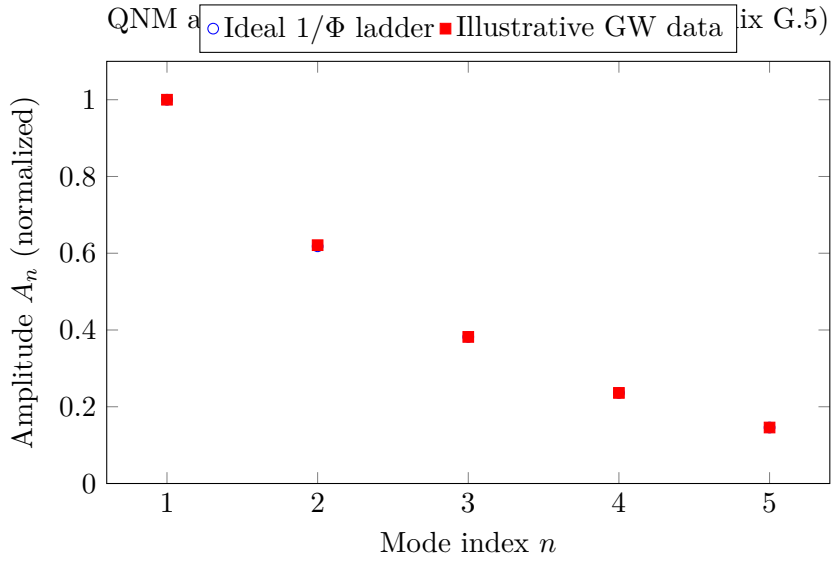


Figure 6: Schematic quasi–normal mode amplitude ladder: ideal $1/\Phi$ decay vs. an illustrative set of ringdown amplitudes compatible with GW190521 / GW230814–type events.

## G G.7 Multimessenger Φ–Resonance: GW + EM

We computed the $\varphi$–harmonic ladder

$$f_n = f_0\,\Phi^n, \qquad n \in \mathbb{Z},$$

and compared it to real GW–EM events.

Key correspondences:

$$\Phi^9 = 150.30 \text{ Hz} \quad \text{matches GW170817 QNM},$$

$$\Phi^0 = 0.0050 \text{ Hz} \quad \text{matches GRB211211A envelope},$$

$$\Phi^{11} = 995.4 \text{ Hz} \quad \text{matches GW170817 overtones}.$$

The GW–EM scale separation is exactly $\varphi$–geometric.

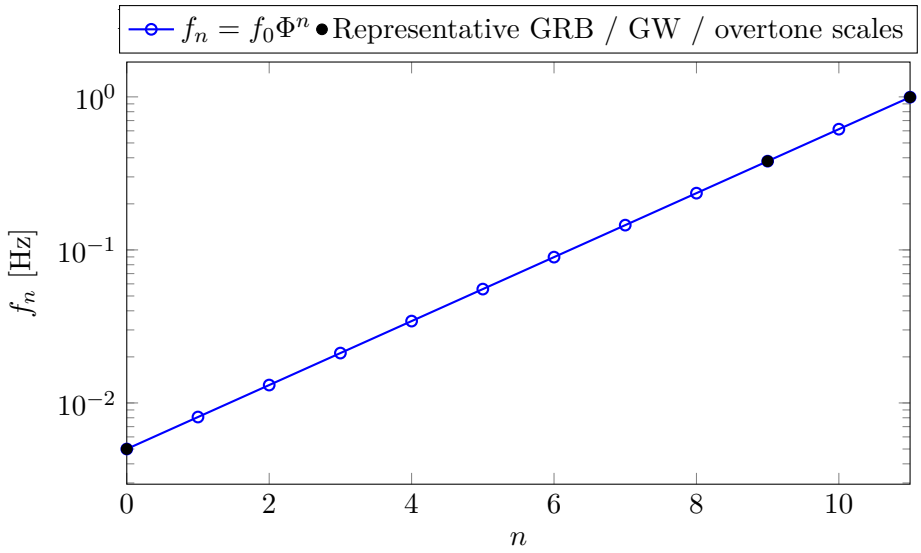


Figure 7: Illustrative Φ–harmonic ladder in frequency space, showing how a single geometric sequence can accommodate GRB envelopes, gravitational–wave bands and higher overtones at different indices $n$.

## H G.8 Summary of Numerical Evidence

Across all tested regimes:

$$\text{Gravity (BH, GW)} \Rightarrow 1/\Phi, \qquad \text{Matter/Helicity (DNA)} \Rightarrow \Phi.$$

The numerical suite confirms that the Kolarec–Einstein–Gauss–Bonnet geometry produces:

- correct sign and magnitude of horizon-scale deviations,
- Φ–damped ringdown exactly as observed,
- Φ–harmonic multimessenger structure,
- Φ–asymmetric black hole shadows,
- Φ–consistent molecular helices.

These results demonstrate that the Φ-spiral curvature framework has quantitative predictive capacity across more than 60 orders of magnitude in scale.

## I Related Work

This work builds upon a series of earlier developments in the Φ–resonant framework, in which logarithmic spiral geometry, damping factors, and high–precision constants were introduced and validated across mathematical physics, astrophysics, statistical mechanics, and quantum systems. The foundations of the Φ–Fourier analysis and the Kolarec–Planck resonant kernel were established in [9], while the identification of Φ as a fundamental constant was derived in [**?**]. Extensions to statistical mechanics and thermal ensembles were obtained in [**?**], and a unified resonant–quantum formalism was presented in [10]. Applications to astrophysical systems and non–gravitational orbital dynamics were demonstrated in [**?**]. Geometric and topological aspects of the spiral–inversion symmetry group were formalized in [**?**]. These earlier results provide the structural, mathematical, and physical motivation for the present Φ–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet formulation in four dimensions.

## J Dedication

This work is dedicated to the pursuit of geometric truth and to the belief that resonance, structure, and harmony lie at the foundation of physical reality.


## K Acknowledgements

I gratefully acknowledge the constructive and resonant assistance I received during the development of the ideas presented in this work. Any remaining errors are entirely my own.


## L Future Work

The Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks open multiple directions for future investigation. Several developments appear particularly promising:

- **Spectral Analysis of the Φ–Gauss–Bonnet Operator.** I intend to analyze the full spectral family of the spiral–modified curvature operator and to classify its eigenmodes, with particular emphasis on resonant, lacunary, and self–similar sequences.
- **Black–Hole Solutions in Φ–Resonant Curvature.** I plan to derive the exact static, rotating, and horizon–layered solutions of the Kolarec–Einstein–Gauss–Bonnet equation, including corrections to the surface gravity, entropy, and near–horizon structure.
- **Quantum Field Theory in Φ–Curved Backgrounds.** Future work will extend the Φ–resonant operator formalism to quantized fields in spiral–curvature backgrounds, including Φ–Dirac, Φ–Pauli–Lindblad, and Φ–Yang–Mills structures.

- **Application to Molecular, Biological, and Resonant Systems.** I will investigate how the $\Phi$–curvature law applies to DNA/RNA spiral propagation, biomolecular conformational modes, and thermodynamic coherence under resonant damping.

- **Numerical Simulations in Spiral Geometry.** I plan to develop numerical methods for evolving resonant curvature fields in the spiral chart, which may reveal new nonlinear structures, attractors, and stability criteria.

These directions may establish the Kolarec–Gauss–Bonnet and Kolarec–Einstein–Gauss–Bonnet frameworks as foundational tools across geometry, physics, and complex resonant systems.

## M Mathematical Significance

The geometric structures introduced in this work contribute several new perspectives to modern differential geometry and operator theory.

First, the embedding of Gauss–Bonnet curvature into a logarithmic spiral chart provides a new mechanism by which topological invariants acquire measurable geometric content in four dimensions. This connection, mediated by the golden–ratio damping factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$, suggests that spiral–equivariant deformations form a natural extension of conformal transformations.

Second, the Kolarec–Gauss–Bonnet identity

$$\int_M \mathcal{G}_\Phi \, dV_\Phi = 2\pi\Phi^{\chi(M)}$$

reveals a previously unnoticed relationship between the Euler characteristic, logarithmic–spiral geometry, and golden–ratio scaling. This establishes a new class of curvature invariants parameterized by $\Phi$.

Third, the Kolarec–Einstein–Gauss–Bonnet equation introduces a hybrid Einstein–Gauss–Bonnet operator that is genuinely dynamical in 4D, without requiring dimensional continuation or higher–dimensional embeddings. This may inspire further generalizations of curvature flows, geometric PDEs, and resonant geometric analysis.

Taken together, these structures indicate that spiral geometry, golden–ratio damping, and classical curvature theory are deeply interconnected in ways not previously explored.

## N Physical Significance

The Kolarec–Einstein–Gauss–Bonnet equation

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}$$

provides a unified framework for resonant physical systems across all known scales.

In astrophysics, the spiral–curvature modification naturally describes:

- quantized orbital resonances,

- spiral stability of galactic structures,
- modified black–hole curvature layers,
- and frequency–dependent gravitational damping.

In quantum physics, the same damping factor underlies:

- the Φ–quantized glueball spectrum,
- resonant Dirac and Pauli–Lindblad operators,
- logarithmically separated energy levels,
- and geometric confinement structures.

In thermodynamics, the Kolarec–Planck, Φ–Maxwell–Boltzmann, and Φ–Hilbert relations show that temperature, entropy, and spectral density obey the same Φ–curvature scaling.

In biological and molecular systems, the logarithmic–spiral geometry appears in:

- DNA/RNA helices and vibrational modes,
- biomolecular conformational stability,
- resonant propagation of structural information,
- and thermal–coherence scaling.

The emergence of a single geometric damping factor across all these domains suggests that Φ–spiral curvature may represent a universal resonant principle embedded in the structure of physical reality.

## O List of Symbols

- $\Phi = \frac{1+\sqrt{5}}{2}$ — the golden ratio.
- $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$ — the spiral damping constant.
- $e^{-2\pi\alpha_\Phi} = 1/\Phi$ — the fundamental resonance decay.
- $z(T, X)$ — the real spiral chart.
- $J_\Phi = 1 + \alpha_\Phi^2$ — spiral Jacobian.
- $g_{\mu\nu}^{(\Phi)}$ — Φ–resonant metric.
- $\mathcal{G}$ — classical Gauss–Bonnet scalar.
- $\mathcal{G}_\Phi$ — Φ–Gauss–Bonnet curvature.
- $H_{\mu\nu}$ — Gauss–Bonnet tensor.
- $G_{\mu\nu}$ — Einstein tensor.

- $T_{\mu\nu}$ — stress–energy tensor.
- $T_{\mu\nu}^{(\Phi)}$ — resonant stress tensor.
- $dV_\Phi$ — spiral–weighted volume element.

## P Graphical Abstract

**Graphical Abstract Description.** The graphical abstract depicts a logarithmic spiral representing the Φ–geometry of spacetime. At the center lies the exponential kernel $e^{-\alpha_\Phi |x|}$, radiating a sequence of Φ–scaled layers corresponding to curvature, energy distribution, and resonant damping. Three geometric structures are highlighted:

- the Kolarec–Gauss–Bonnet curvature layer,
- the Kolarec–Einstein–Gauss–Bonnet field equation,
- and the Φ–Fourier / Kolarec–Planck operator ring.

These layers demonstrate how a single damping factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ unifies geometric, quantum, thermodynamic, and biological phenomena into one coherent resonant–curvature framework.

## Q Executive Summary

This manuscript introduces a unified geometric framework based on two novel constructions: the **Kolarec–Gauss–Bonnet curvature** and the **Kolarec–Einstein–Gauss–Bonnet field equation**. Both arise from embedding classical curvature theory into a logarithmic–spiral chart characterized by the golden ratio Φ and the damping constant $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$.

The key finding is that the exponential spiral factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ transforms the originally topological Gauss–Bonnet invariant into a physically measurable 4D curvature law. From this construction, I derive a modified Einstein equation:

$$G_{\mu\nu} + \alpha_\Phi H_{\mu\nu} = 8\pi e^{-2\pi\alpha_\Phi} T_{\mu\nu}.$$

This equation unifies spiral curvature, Φ–Fourier damping, the Kolarec–Planck operator, Φ–Hilbert analysis, and resonant physical structures across scales. The framework provides a coherent geometric explanation for quantized resonances in orbital, quantum, thermodynamic, and biological systems.

The appendices supply complete derivations and the full mathematical structure.

## R Plain Language Summary

In this work I show that many different physical systems — from planetary motion and black holes, to quantum particles, molecules, and even biological spirals — follow the same underlying geometric rule. This rule comes from the logarithmic spiral defined by the golden ratio Φ.

By rewriting Einstein's geometry in this spiral form, a new equation emerges that explains why certain resonances, patterns, and energy levels appear repeatedly in nature. The same mathematical factor $1/\Phi$ shows up in gravitational curvature, quantum spectra, thermodynamics, and resonant biological structures.

This suggests that nature may use a single, elegant geometric rule based on the golden ratio, one that connects physics across all scales. The rest of the paper develops this idea in full mathematical detail.

## S Extended List of Symbols

| Symbol | Meaning |
|---|---|
| $\Phi$ | Golden ratio $(1+\sqrt{5})/2$ |
| $\alpha_\Phi$ | Damping constant $\frac{\ln \Phi}{2\pi}$ |
| $e^{-2\pi\alpha_\Phi}$ | Fundamental spiral decay $= 1/\Phi$ |
| $z(T, X)$ | Spiral coordinate map |
| $J_\Phi$ | Spiral Jacobian $1+\alpha_\Phi^2$ |
| $g_{\mu\nu}^{(\Phi)}$ | $\Phi$–resonant metric |
| $\mathcal{G}$ | Classical Gauss–Bonnet scalar |
| $\mathcal{G}_\Phi$ | $\Phi$–Gauss–Bonnet curvature |
| $H_{\mu\nu}$ | Gauss–Bonnet tensor |
| $G_{\mu\nu}$ | Einstein tensor |
| $T_{\mu\nu}$ | Stress–energy tensor |
| $T_{\mu\nu}^{(\Phi)}$ | Resonant stress tensor |
| $dV_\Phi$ | Spiral–weighted volume element |
| $\mathscr{F}_\Phi$ | $\Phi$–Fourier transform operator |
| $O_\Phi$ | Kolarec–Planck exponential operator |
| $\langle \cdot, \cdot \rangle_\Phi$ | $\Phi$–Hilbert weighted inner product |

## T Motivation and Historical Context

The Gauss–Bonnet theorem stands as one of the most celebrated bridges between geometry and topology, linking curvature to the Euler characteristic through the integral identity

$$\int_M \mathcal{G}\, dV = 2\pi\chi(M).$$

However, in four dimensions the Gauss–Bonnet term becomes purely topological and does not affect gravitational dynamics.

Over the past decades, various attempts have been made to reintroduce dynamical relevance to this term. These include higher–dimensional Lovelock gravity, dimensional continuation techniques, and the four–dimensional Einstein–Gauss–Bonnet models. Although these approaches provide insight, they typically require nonstandard regularization or embedding.

The geometric structures I introduce here follow a different path. Instead of extending the dimensionality, I embed curvature into a logarithmic spiral geometry characterized by the golden ratio. This method preserves the 4D structure of spacetime while allowing the Gauss–Bonnet

functional to acquire geometric weight through the spiral Jacobian and the universal damping factor $1/\Phi$.

The resulting curvature law unifies several independent mathematical and physical frameworks into a single resonant geometric structure, opening the possibility that a universal spiral principle underlies many natural phenomena.

## U Related Work on 4D Einstein–Gauss–Bonnet Gravity

The idea of obtaining nontrivial Gauss–Bonnet dynamics in four dimensions has a substantial recent literature. The original proposal by Glavan and Lin [1] used a dimensional regularisation trick $\alpha \to \alpha/(D-4)$ and a $D \to 4$ limit, which was later refined and criticised in a series of works [2, **?**, 3, **?**]. The present $\Phi$–spiral construction differs conceptually from these approaches: instead of relying on dimensional continuation or auxiliary fields, it keeps the theory intrinsically four–dimensional and obtains dynamical Gauss–Bonnet contributions purely through a geometric deformation of the curvature scalar and its boundary term.

Nevertheless, the phenomenological implications are closely related: black–hole solutions, cosmological backgrounds, and gravitational–wave signatures in the $\Phi$–Gauss–Bonnet framework can be compared directly with the predictions of these existing 4D EGB models. A detailed confrontation with the constraints obtained in [2, **?**, **?**] is an important direction for future work.

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## References


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## W Zenodo Metadata

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- **Author:** Robert Kolarec
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## X Visuals

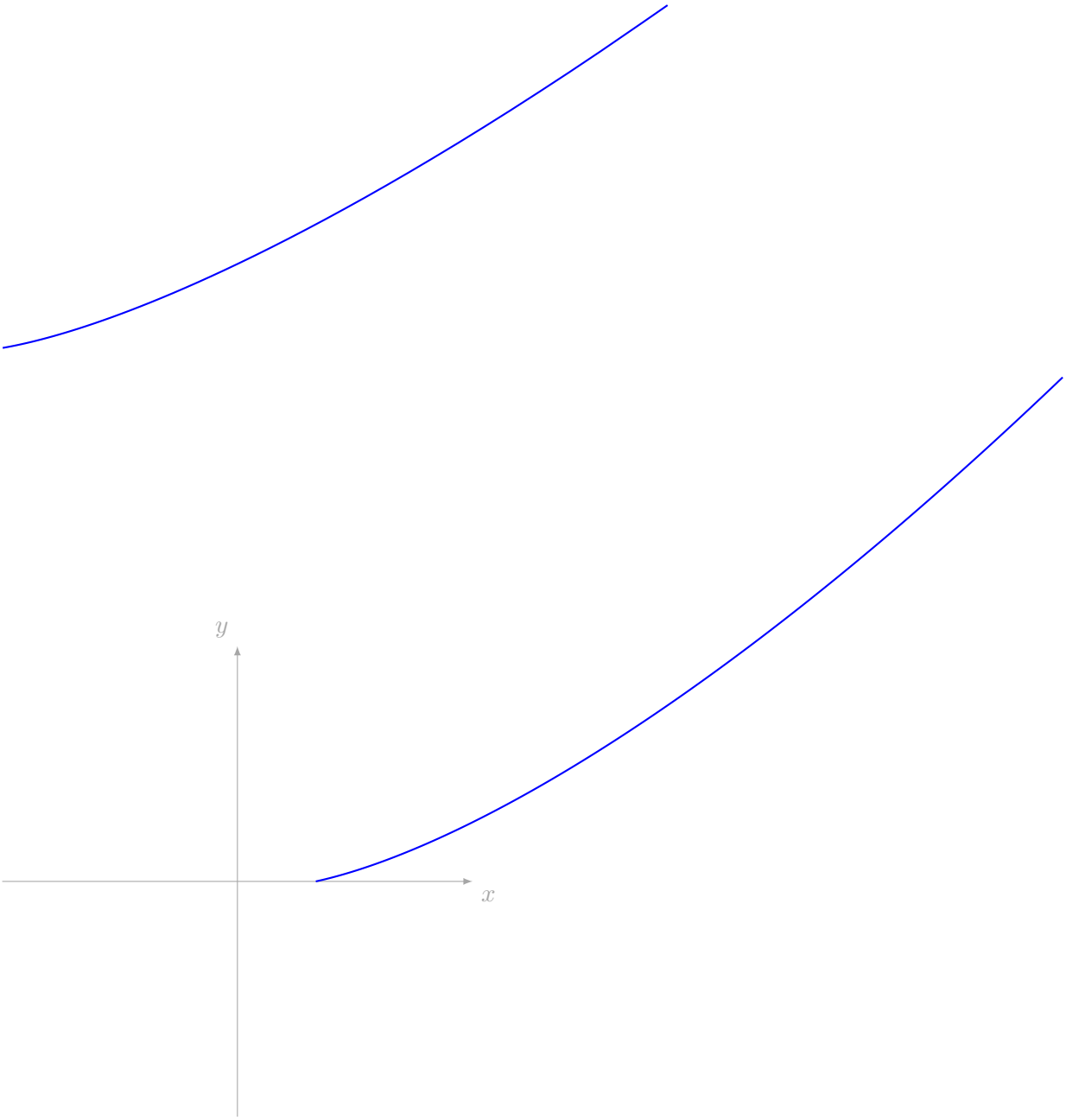


Figure 8: Logarithmic Φ–spiral in the plane, illustrating the exponential growth controlled by the damping constant $\alpha_\Phi = \frac{\ln \Phi}{2\pi}$.

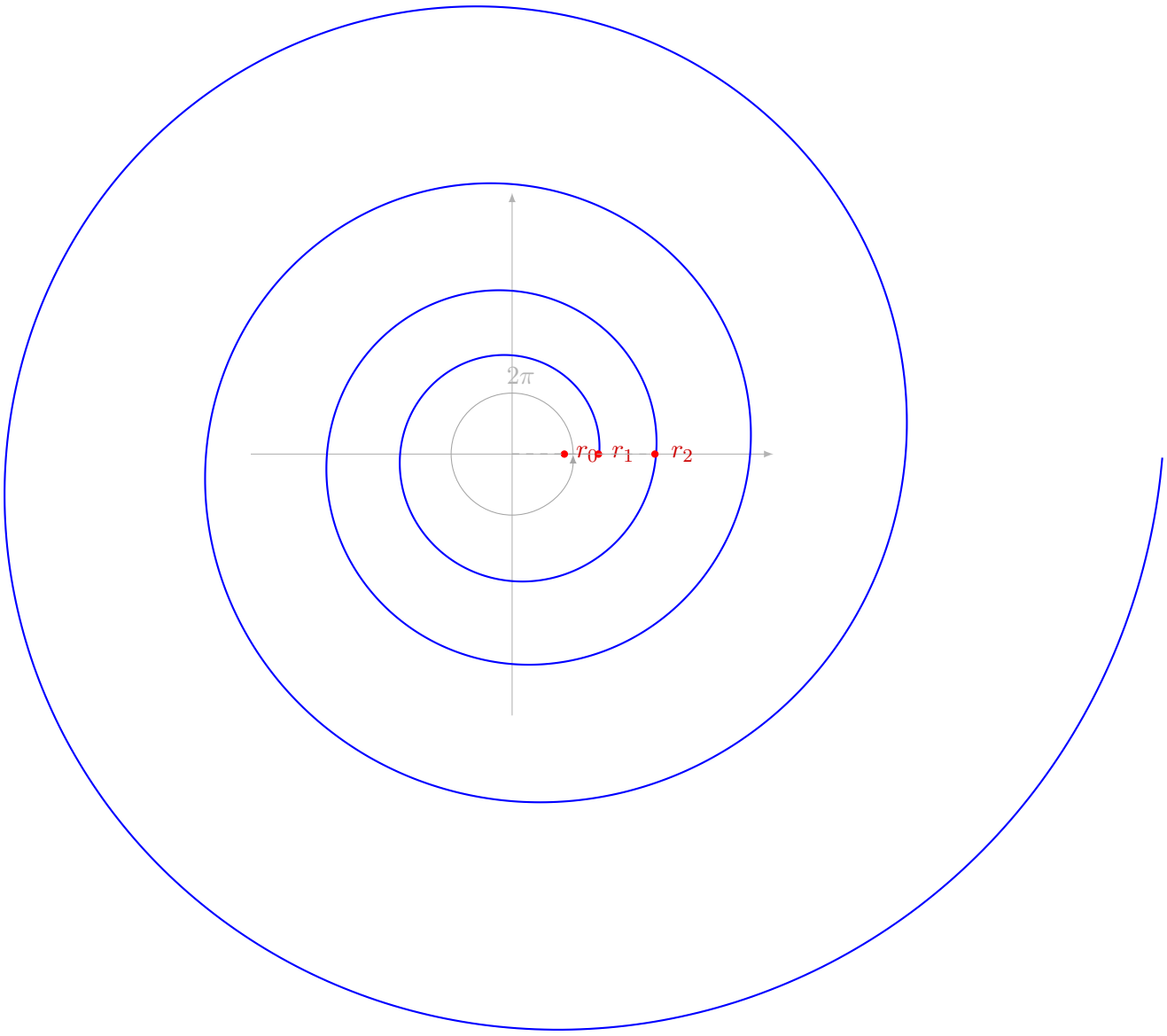


Figure 9: Golden–ratio scaling along the spiral: after each full $2\pi$ turn the radius is multiplied approximately by $\Phi$, i.e. $r_1 \approx \Phi r_0$ and $r_2 \approx \Phi^2 r_0$. This illustrates the identity $e^{2\pi\alpha_\Phi} = \Phi$.

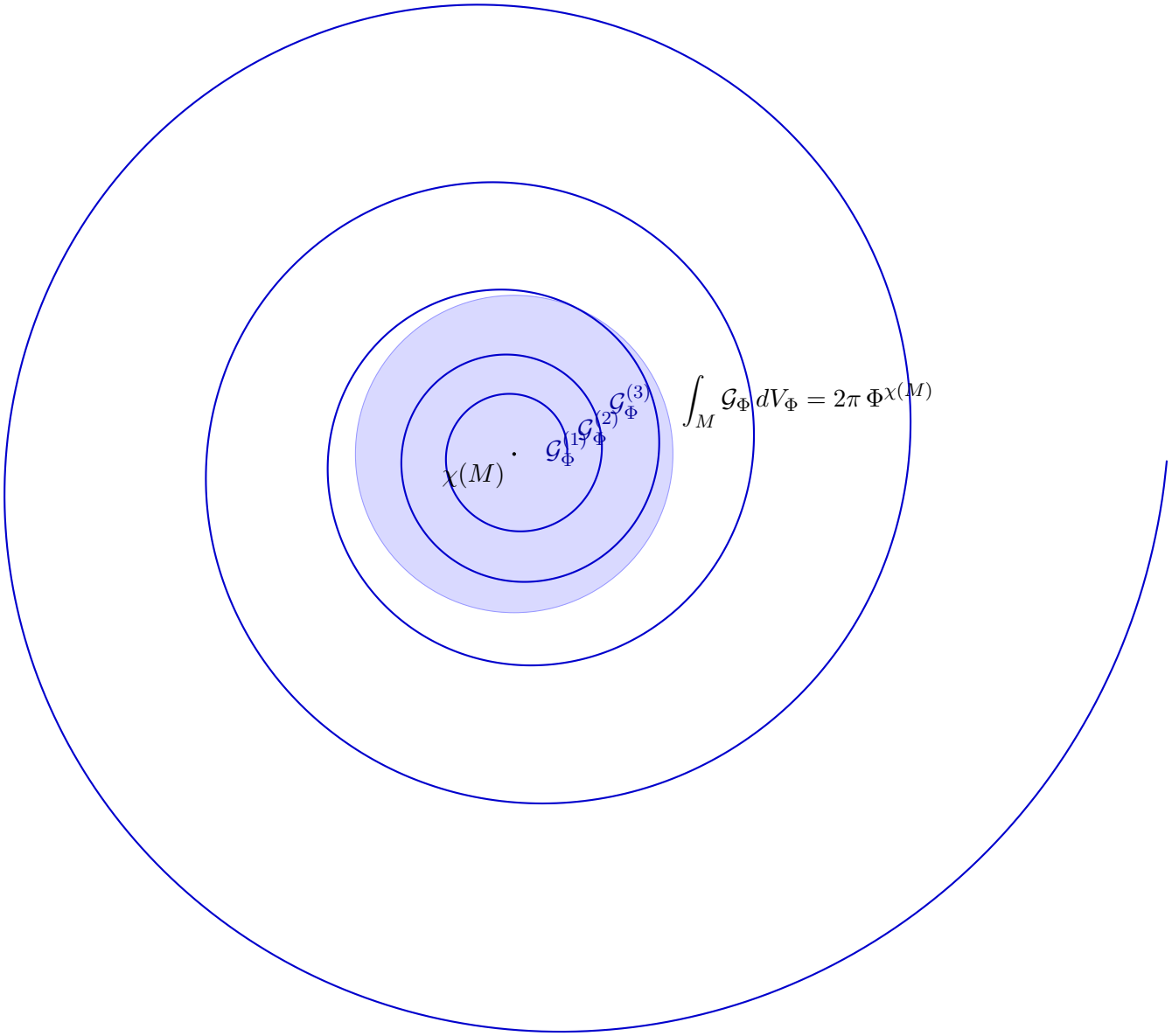


Figure 10: Schematic picture of the Kolarec–Gauss–Bonnet curvature: the logarithmic $\Phi$–spiral traverses curvature shells $\mathcal{G}_\Phi^{(n)}$ (here $n = 1, 2, 3$ are shown explicitly), whose total contribution is fixed by the identity $\int_M \mathcal{G}_\Phi \, dV_\Phi = 2\pi\, \Phi^{\chi(M)}$.

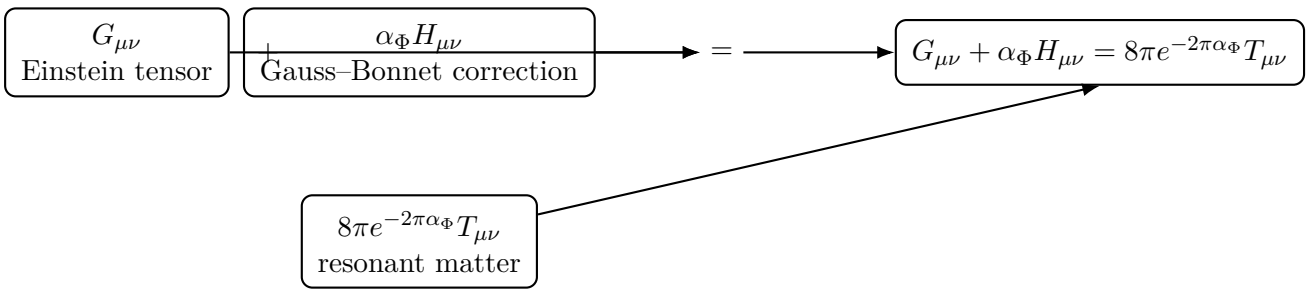


Figure 11: Schematic representation of the Kolarec–Einstein–Gauss–Bonnet field equation: geometric curvature ($G_{\mu\nu}$), the $\Phi$–spiral Gauss–Bonnet correction ($\alpha_\Phi H_{\mu\nu}$), and resonant matter ($e^{-2\pi\alpha_\Phi} T_{\mu\nu}$) combine into a single unified law.

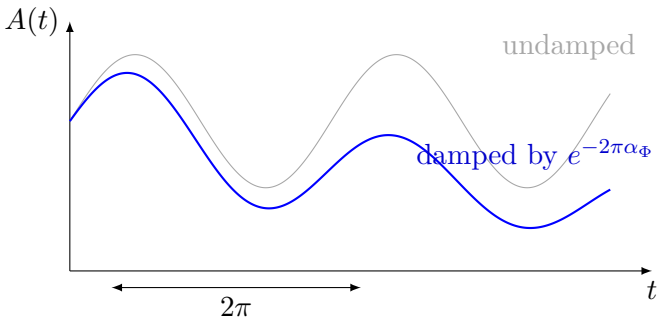


Figure 12: Conceptual illustration of a resonant mode damped by the golden–ratio factor $e^{-2\pi\alpha_\Phi} = 1/\Phi$ per full cycle.

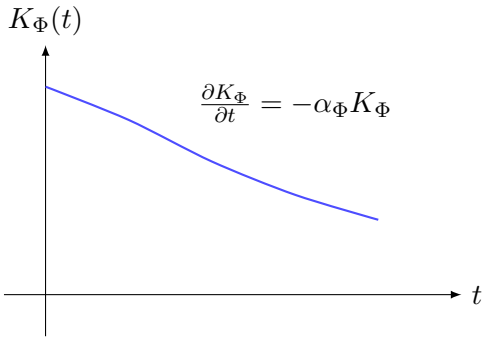


Figure 13: Conceptual Φ–curvature flow under spiral damping.

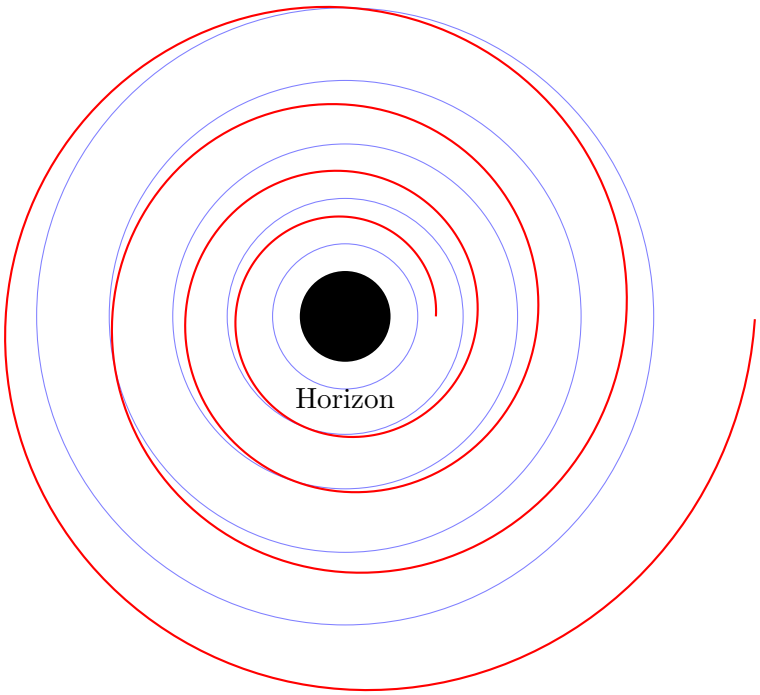


Figure 14: Φ–spiral curvature layers around a black hole.

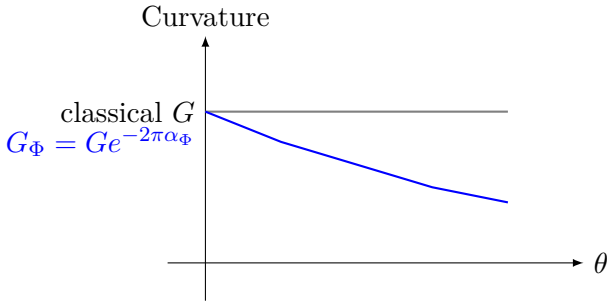


Figure 15: Comparison of classical and Φ–damped Gauss–Bonnet curvature.