

2-Layer Lake Model (according to Chapra 1997)

(1) Epilimnion heat balance: $V_e \rho C_p \frac{dT_e}{dt} = Q \rho C_p T_{in} - Q \rho C_p T_e \pm J A_s + v_t A_t \rho C_p (T_h - T_e)$

(2) Hypolimnion heat balance: $V_h \rho C_p \frac{dT_h}{dt} = v_t A_t \rho C_p (T_e - T_h)$

Putting surface heat flux into equation (1)

(3)

$$\frac{dT_e}{dt} = \frac{Q}{V_e} T_{in} - \frac{Q}{V_e} T_e + v_t \frac{A_t}{V_e} (T_h - T_e) + \frac{J_{sw}}{\rho C_p H} + \frac{\sigma(T_{air}+273)^4(A+0.031\sqrt{e_{air}})(1-R_L)}{\rho C_p H} - \frac{\epsilon\sigma(T_e+273)^4}{\rho C_p H} - \frac{c_1 f(U_w)(T_e - T_{air})}{\rho C_p H} - \frac{f(U_w)(e_{sat} - e_{air})}{\rho C_p H}$$

and

$$(4) \frac{dT_h}{dt} = v_t \frac{A_t}{V_h} (T_e - T_h)$$

with $H = \frac{A_s}{V_e}$

Breaking down the units of the heat fluxes:

$$(5) \text{ incoming shortwave radiation: } \frac{J_{sw}}{\rho C_p H} = \frac{\text{cal} * \text{cm}^2 * \text{C}}{\text{cm}^2 * \text{d} * \text{cal}} = \frac{\text{C}}{\text{d}}$$

$$(6) \text{ incoming longwave radiation: } \frac{\sigma(T_{air}+273)^4(A+0.031\sqrt{e_{air}})(1-R_L)}{\rho C_p H} = \frac{\text{cm}^2 * \text{C} * \text{cal} * \text{K}^4}{\text{cal} * \text{cm}^2 * \text{d} * \text{K}^4} = \frac{\text{C}}{\text{d}}$$

$$(7) \text{ backscattering longwave radiation: } \frac{\epsilon\sigma(T_e+273)^4}{\rho C_p H} = \frac{\text{cm}^2 * \text{C} * \text{cal} * \text{K}^4}{\text{cal} * \text{cm}^2 * \text{d} * \text{K}^4} = \frac{\text{C}}{\text{d}}$$

$$(8) \text{ conduction: } \frac{c_1 f(U_w)(T_e - T_{air})}{\rho C_p H} = \frac{\text{cm}^2 * \text{C} * \text{C} * \text{m}^2}{\text{cal} * \text{C} * \text{s}^2} = \frac{\text{cm}^4 * \text{C}}{\text{cal} * \text{s}^2}$$

$$(9) \text{ evaporation: } \frac{f(U_w)(e_s - e_{air})}{\rho C_p H} = \frac{\text{cm}^2 * \text{C} * \text{m}^2}{\text{cal} * \text{s}^2} = \frac{\text{cm}^4 * \text{C}}{\text{cal} * \text{s}^2}$$

Here:

c_1 is Bowen's coefficient (=0.47 mmHg per deg C);

$f(U_w)$ is a wind function, here by Brady, Graves and Geyer (1969): $f(U_w) = 19.0 + 0.95 * U_w^2$; in which U_w must be in m/s

e_s and e_{air} are surface water vapor pressure and air vapor pressure in mmHG.