Italian Judges

Authors

Affiliation

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Abstract

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1 Model

A judge has a docket of N cases. The n^{th} case comprises x_n hearings, a number drawn at random, in *i.i.d.* fashion. Completion rate $c(x) = \frac{\Pr(x_n = x)}{\Pr(x_n \geq x)}$ denotes the probability that a given case will finish in its x^{th} hearing, given that it has made it that far. Each hearing takes one period. The judge incurs cost ω for each open case in each period. The optimal scheduling policy π^* minimizes these expected discounted costs: $\pi^* = \arg\min_\pi \mathbb{E}_\pi \left(\sum_{n=1}^{N} \sum_{t=0}^{t_n - 1} \beta^t \omega \right)$, where $\beta \in \{0, 1\}$ is the discount rate and t_n is case n's completion time.

The judge considers two scheduling policies: The "see-it-through policy" sees each case through to completion; under this policy, the judge finishes one case's hearings before starting on another case's (the judge orders the cases randomly, since they are symmetric to him). And the "cycle-through policy" cycles through the open cases one hearing at a time; under this policy, the judge works on the case with the least number of completed hearings (breaking ties randomly).

Proposition 1. *The see-it-through policy is optimal when completion rate $c(x)$ is increasing and the cycle-through policy is optimal when it is decreasing.*

Proof. First, note that $\arg\min_\pi \mathbb{E}_\pi \left(\sum_{n=1}^{N} \sum_{t=0}^{t_n-1} \beta^t \omega\right) = \arg\min_\pi \mathbb{E}_\pi \left(\sum_{n=1}^{N} \frac{1-\beta^{t_n}}{1-\beta}\right) = \arg\max_\pi \mathbb{E}_\pi \left(\sum_{n=1}^{N} \beta^{t_n}\right)$. With this, we can reframe the problem from cost minimization to reward maximization, where the judge receives one unit of reward each time he closes a case. With this, the problem boils down to production scheduling with preemption (Pinedo, 2012, p. 275). Accordingly, in each period the judge chooses to work on the case with the largest Gittins index; the Gittins index of the n^{th} case is $G_n(x) = \max_{\tau \geq 1} \frac{\sum_{t=1}^{\tau} \beta^t \Pr(x_n = x+t)}{\sum_{t=1}^{\tau} \beta^t \Pr(x_n \geq x+t)}$, where state variable x denotes the number of completed hearings. If $c(x)$ increases in x, then $\frac{\sum_{t=1}^{\tau} \beta^t \Pr(x_n = x+t)}{\sum_{t=1}^{\tau} \beta^t \Pr(x_n \geq x+t)}$ increases in τ, and the optimal stopping time is $\tau = \infty$, which means the judge never preempts a case, working on it to completion. If $c(x)$ decreases in x, then $\frac{\sum_{t=1}^{\tau} \beta^t \Pr(x_n = x+t)}{\sum_{t=1}^{\tau} \beta^t \Pr(x_n \geq x+t)}$ decreases in τ, and the optimal stopping time is $\tau = 1$; in this case, $G_n(x) = c(x+1)$ also decreases in x, which means the judge works on the case with the least number of completed hearings. □

Proposition 1 orders the work by shortest expected processing time. When $c(x)$ is increasing—e.g., when half the hearing counts are geometric with rate $p_{fast} = .9$ and the other half were geometric with rate $p_{slow} = .1$—then the expected processing time increases with the number of completed cases, and the judge prioritizes the cases with the fewest number of completed hearings. However, when $c(x)$ is decreasing—e.g., when the hearing counts have a discrete uniform distribution—then the expected processing time decreases with the number of completed cases, and the judge prioritizes the case with the most number of completed cases.

Proposition 1 proves that the judges cycle-through policy *can* be optimal. Nevertheless, Exhibit *** demonstrates that it is not. The plots depict and increasing completion rate, which suggests the judges would be better off seeing a case through to completion.

References

Pinedo, Michael. 2012. *Scheduling: theory, algorithms, and systems*. Springer, New York, NY.