ITPC 104 – Discrete Structures

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Contents

4 CONTENTS

Preface

Course Syllabus

Placeholder

I. Degree Program: B. S. in Information Technology

II. Course Code: ITPC 104

III. Course Title: Discrete Structures

IV. Credits and duration

V. Course Description:

VI. Course Objectives

0.0.0.0.1 General :

0.0.0.0.2 Specific:

Chapter 1

Sets

Introduction

Objectives

Structure of the module

Lesson 1. What is a Set

Definition 1.1 (Set)**.** A **set** is an unordered collection of objects, called **elements** or **members** of the set. A set is said to **contain** its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

1.0.0.0.1 Examples

Examples of sets are:

1. the set of past presidents of the Philippines;
2. the set of State Universities in the Philippines;
3. the set of letters of the word "algebra"; and
4. the set of letters in the Filipino alphabet.

1.0.0.0.2 Remarks

Here are important concepts to remember about a set:

- A set may have a name for convenience purposes. An uppercase (capital) letter is used to name a set. For example,

$$A = the\ set\ of\ past\ presidents\ of\ the\ Philippines$$

.

- The objects in a set are called the **elements** or **members** of the set, and we say that a set **contain** its elements. The symbol $\in$ denotes membership in a set. Thus, "$a \in A$" (read *"a is an element of the set A"*) means that the object a belongs to set A while "$a \notin A$" (read *"a is not an element of the set A"*) means that a does not belong to the set A.
- The number of elements of a set A is called its **cardinality** or **size** and is denoted by $|A|$ or $n(A)$.
- The order, the number of occurrence of an element in a set, is not important and does not affect the size of the set.
- The **empty** or **null** set is the set with no elements and is denoted by { } or $\emptyset$.

Defining a Set

There are different ways of defining a set. One is to enumerate (or list) all the elements inside a pair of braces, {}, and separated by commas. This is called the **roster method** (or **list method** or **tabular method**).

Example 1.1. Suppose A is the set of odd numbers less than or equal to 10, then by the roster method, $A = \{1, 3, 5, 7, 9\}$. From this, we also have the following

- A is the name of the set.
- $7 \in A$ or $7 \in \{1, 3, 5, 7, 9\}$.
- $21 \notin A$ or $21 \notin \{1, 3, 5, 7, 9\}$
- $|A| = 5$ (read *"the cardinality of A is 5"*) which means that if you count the number of elements of the set A, it's 5.
- The same set is being referred to even if the set is written as $A = \{3, 1, 7, 9, 5\}$ since the ordering of the elements in the set does not matter.

Another method of defining a set is the **rule method**. This is much like the roster method but with a twist. Instead of listed elements inside a pair of braces, the criteria for membership in the set are given by using a symbol x to represent an element of the given set.

Example 1.2. In the illustration above, if we denote the set of odd numbers as $\mathbb{O}$, then set A may be defined as

$$A = \{x | x \leq 10, x \in \mathbb{O}\}.$$

This is read as *"A is the set of all x such that x is less than or equal to 10 and x is an element of the set of odd numbers"*.

Still another method of defining a set is by clear **description in plain language**. The set in the first illustration is defined this way (*"A is the set of odd counting numbers less than or equal to 10"*). Sometimes this is the easiest way to define a set but it may not be helpful when you want a definition of the set which can be computed or manipulated.

It is very important that you know how to define a set using these different methods and transform a set definition from one method to another.

Finite and Infinite Sets

Definition 1.2 (Finite and Infinite Set)**.** A set is said to be **finite** if its size corresponds to a counting number. Otherwise, it is **infinite**.

In other words, if it is possible to enumerate all the elements of a defined set however long it takes, then it is finite. If it is not, then it is said to be infinite.

Example 1.3. The set of months in a year and the set of past presidents of the Philippines are both finite while the set odd numbers greater than 3 is infinite.

Example 1.4. The set of all ant species in our planet is also finite as well as the set of hairs in your head. It may be difficult to enumerate the elements of a finite set as in the set of all ant species but the difficulty (which arises from a lot of factors) should not make you jump to conclusion that it is infinite.

Exercise 1.1. Answer the following:

1. Use both roster and the rule method to specify the following sets.

   (a) The children of your parents
   (b) The consonants in the English alphabet. (c)The digits in the binary number system.
   (c) Your classmates who are absent from Math 101 today.
   (d) The distinct letters of the phrase *"kakaba kaba ka ba?"*
   (e) The counting numbers greater than 3 but less than 11.
   (f) The colors of the rainbow.
   (g) The subjects (or courses) you are enrolled in this semester.

2. For each of the following sets, tell whether it is finite, infinite, or empty.

   (a) The set of counting numbers between 7 and 24.
   (b) The set of books in the MLUC libraries.
   (c) The set of points on the real number line between 0 and 1.
   (d) The set of distinct letters of the word *"controversial"*.
   (e) The set of even numbers.

   (f) The set of people living in the world today.
   (g) The set of real numbers which satisfy the equation $x^2 = -1$
   (h) The set of Philippine heroes.
   (i) The set of plate numbers available for Philippine automobiles.
   (j) The set of Eraserheads singles that reached Billboards Top 40 list.

3. Which of the following sets are different?

   (a) $\emptyset$

   (b) $\{0\}$

   (c) $\{\emptyset\}$

4. Which of the following sets is the null set?

   - $A = \{x | x \neq x\}$

   - $B = \{x | x + 5 = 5\}$

   - $C = \{x | x \in\}$

Equal and Equivalent Sets

Definition 1.3 (Equal Sets)**.** Two sets A and B are **equal**, denoted by $A = B$, if and only if they have exactly the same elements.

Example 1.5. If $A = \{2,3,5,6,9\}$, $B = \{3,5,2,9,6\}$, $C = \{2,5,3,6,9\}$, then the sets A, B, and C are equal. The order of the elements is not important.

Definition 1.4 (One-to-one Correspondence)**.** A **one-to-one correspondence** between two sets A and B is a pairing between the elements of A with the elements of B in such a way that each element of A is paired with one and only one element of B and each element of B is paired with one and only one element of A.

Definition 1.5 (Equivalent Sets)**.** Two sets A and B are **equivalent**, denoted by $A \sim B$ (or $A \leftrightarrow B$), if and only if there exists a one-to-one correspondence between A and B.

It should be noted that equal sets are always equivalent but equivalent sets may not be equal.

Example 1.6. Consider the sets $A = \{a, b, c\}$ and $B = \{sitaw, bataw, patani\}$. Since there is a one-to-one correspondence between the elements of A and B, we say that they are equivalent and we write, $A \sim B$. The easiest way to know whether two sets are equivalent is that they have the same cardinality. That is, if they have the same number of elements, then they are equivalent.

Definition 1.6 (Subset)**.** A set **A** is a subset of another set **B**, denoted by, $A \subseteq B$ (read *"A is a subset of B"*), if every element of A is in B. Otherwise, we write $A \nsubseteq B$.

It is very important to note that if A has at least one element that is not in B, then $A \nsubseteq B$.

Example 1.7. Let $A = \{2, 3, 7\}$, $B = \{1, 4, 5\}$ and $C = \{1, 2, 3, 4, 5, 6\}$. It is clear that A is not a subset of C and B is a subset of C. In symbols, we have $A \nsubseteq C$ and $B \subseteq C$.

1.0.0.0.3   Remarks

- Remember that the null set $ $ is a set containing no elements that is why we may write it this form $ \{ \}$. Since we can not find an element of $\emptyset$ that is not an element of any other set, say A, it follows that $\emptyset \subseteq A$. Thus, the null set $ $ is considered a subset of any set.
- Any set A is a subset of itself because all the elements A are found in the set itself.
- For any set A, the null set $ $ and the set A itself are called **trivial subsets** of A.
- A set A is a proper subset of a set B if $A \subseteq B$ and $A \neq B$. It means that while $A \subseteq B$, their cardinality (or their number of elements) are not the same. More specifically, A is a proper subset of B is denoted by $A \subset B$.
- *Two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.* This is the formal definition of equal sets. To prove that two sets are equal (that is, they have exactly the same element as in [Definition 3]), we must show that one is a subset of the other.

Venn Diagrams

The **Venn diagram** (or **Venn-Euler diagram**) is a pictorial representation of set relations and operations after the Cambridge logician John Venn (1834-1923). This set representation is very useful in helping you concretize abstract ideas on sets.

In the Venn diagram, the largest set in consideration is the universal set U and represented by a rectangle. The subsets of U are drawn as circles inside the rectangle as in the figure below. Also, in the figure, we can see the relationship that $A \subseteq B$.
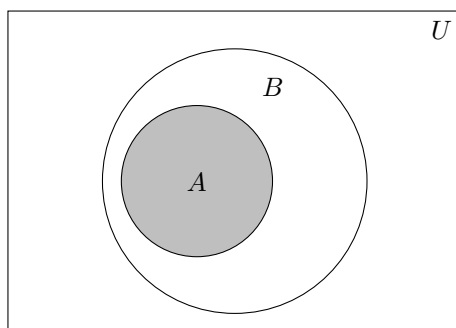
Figure 1.1: Venn diagram

Chapter 2

Propositional Logic

Placeholder

2.1 Introduction

2.2 Objectives

2.3 Structure of the module

Lesson 1. Proposition

2.3.0.0.1 Remarks:

Negation

2.3.0.0.2 Remarks:

2.3.0.0.3 Examples

2.3.0.0.4 Pahabol:

Lesson 2. Logical Operators (Connectives)

Conjunction (Logical And)

2.3.0.0.5 Remarks:

2.3.0.0.6 Examples

2.3.0.0.7 Try this.

Disjunction (Logical Or)

2.3.0.0.8 Remarks

2.3.0.0.9 Truth table

2.3.0.0.10 Examples

Logical Exclusive Or

2.3.0.0.11 Remarks

2.3.0.0.12 Examples

2.3.0.0.13 Truth table

Conditional Statement

2.3.0.0.14 Remarks

2.3.0.0.15 Truth table