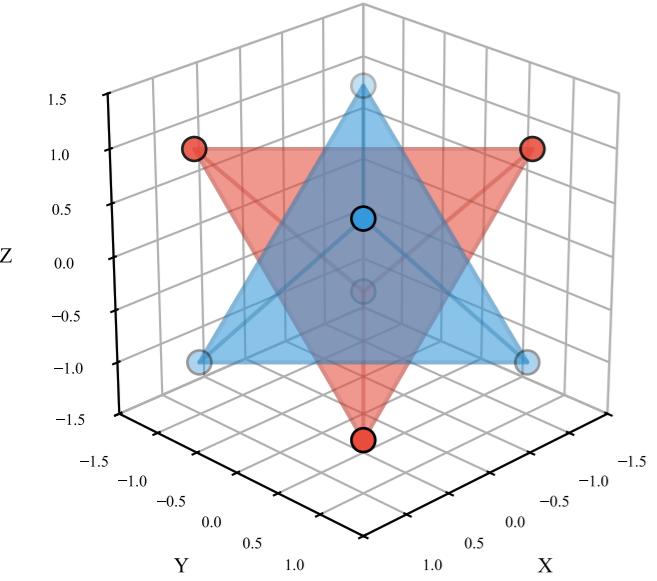
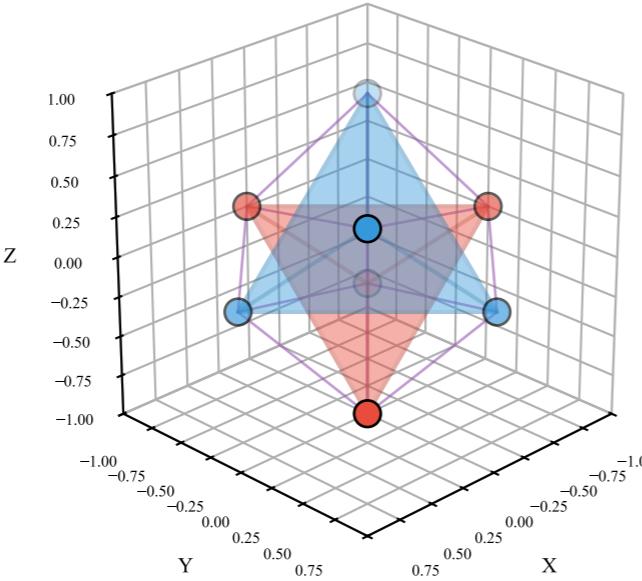


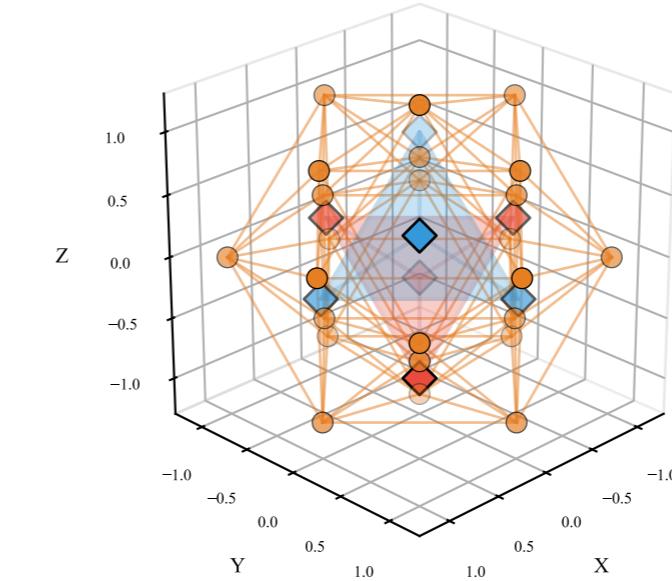
**(a) Stella Octangula**  
 $T_+$  (blue)  $\cup T_-$  (red)



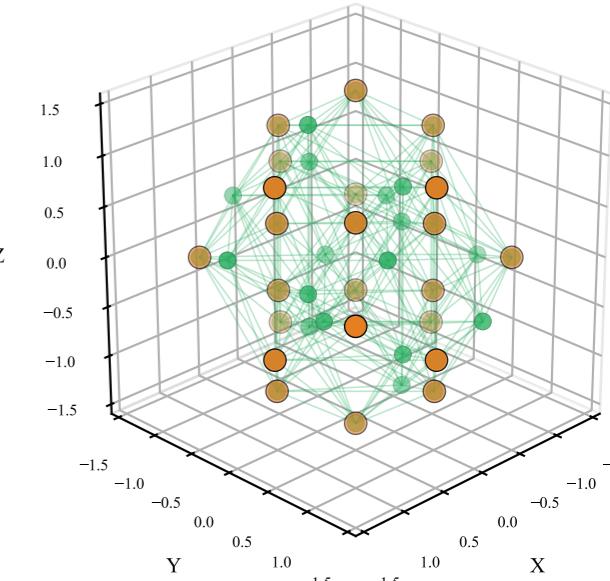
**(b) 16-cell ( $B_4$ )**  
 $\phi: T_{\pm} \rightarrow \pm e_i$



**(c) 24-cell ( $D_4$ )**  
16-cell  $\subset D_4$  roots



**(d)  $\mathfrak{so}(10)$  ( $D_5$ )**  
 $D_4 \subset D_5$  (40 roots)



**(e) GUT Embedding Chain**

$\subset$

$\subset$

$\subset$

$\supset$

break

**Stella**  
 $S_4 \times \mathbb{Z}_2$



**16-cell**  
 $B_4$



**24-cell**  
 $D_4$



**$\mathfrak{so}(10)$**   
 $D_5$



**$SU(5)$**



**$SU(3) \times SU(2) \times U(1)$**

$$\sin^2 \theta_W = 3/8 \text{ at GUT scale}$$