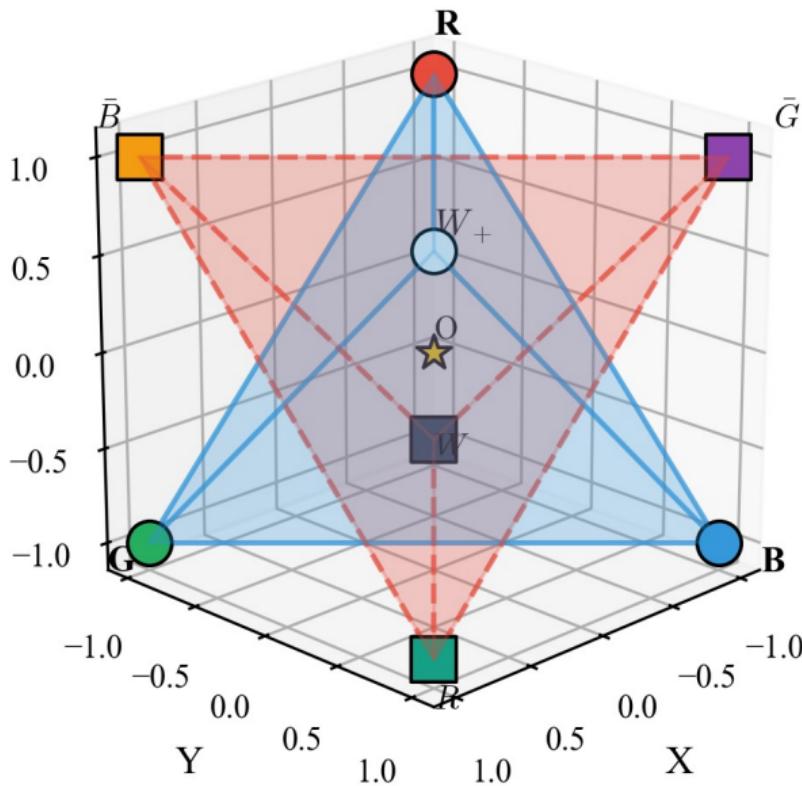


(a) Stella Octangula:  $T_+ \cup T_-$



(b) Uniqueness Proof Summary

Theorem 0.0.3: Stella Octangula Uniqueness

Vertex Structure:

- 6 color vertices  $\rightarrow 3 \oplus \bar{3}$
- 2 apex vertices  $\rightarrow$  singlet direction
- Total: 8 vertices (minimal)

Symmetry Group:

- Full symmetry:  $S_4 \times \mathbb{Z}_2$  (order 48)
- Contains:  $S_3 \cong \mathcal{W}(SU(3))$
- Charge conjugation:  $\mathbb{Z}_2$  swap

Geometric Conditions Satisfied:

- ✓ (GR1) Weight correspondence
- ✓ (GR2) Weyl group preservation
- ✓ (GR3) Conjugation compatibility
- ✓ (M1) Minimal vertex count
- ✓ (M2) Minimal embedding dimension

Alternatives Eliminated:

$\times$  Cube, Octahedron, 2D triangles...

Lean 4: `stella_is_unique_minimal_realization`