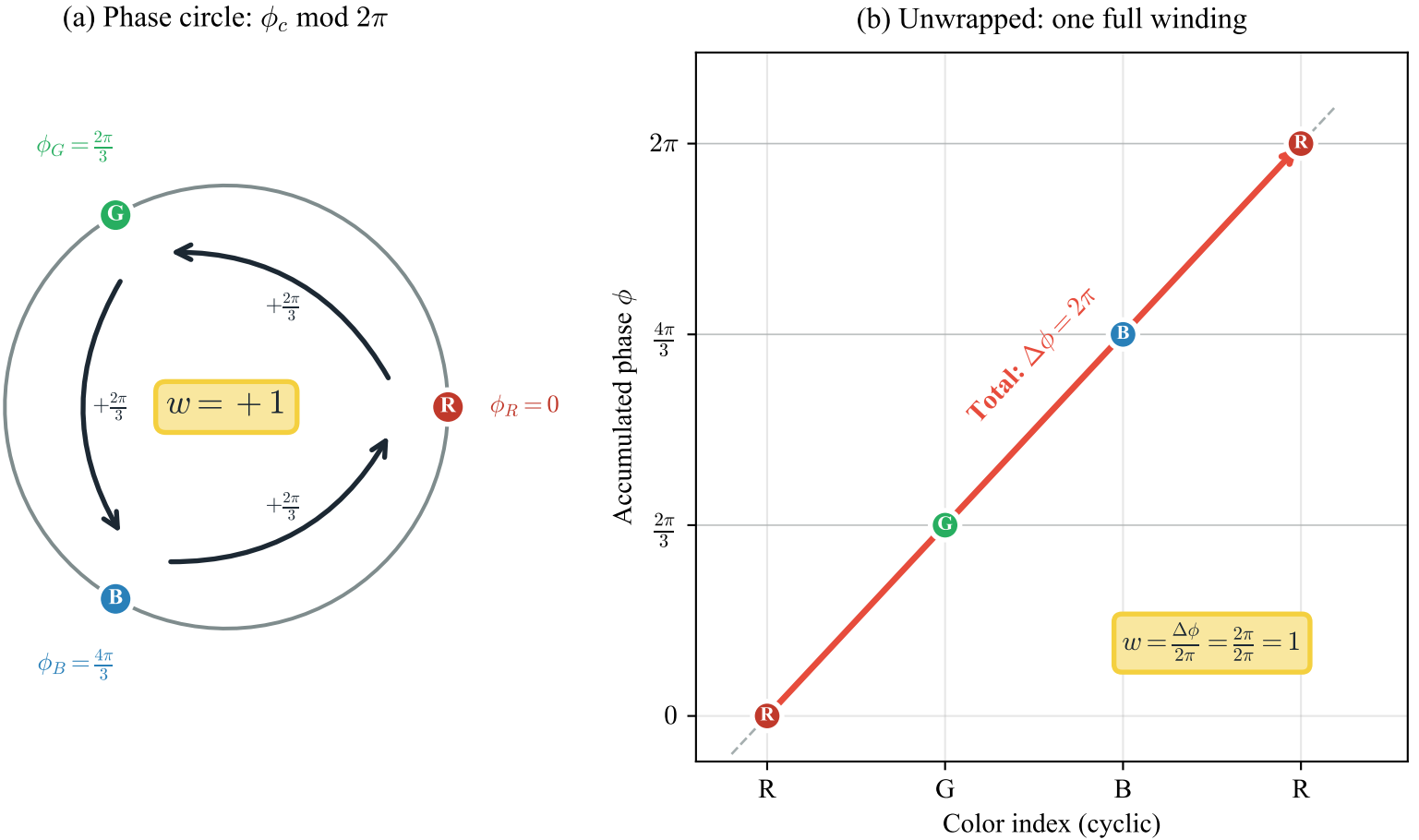

The $\mathbb{Z}_3$-symmetric phases complete one cycle: $w = +1$ is topologically protected.