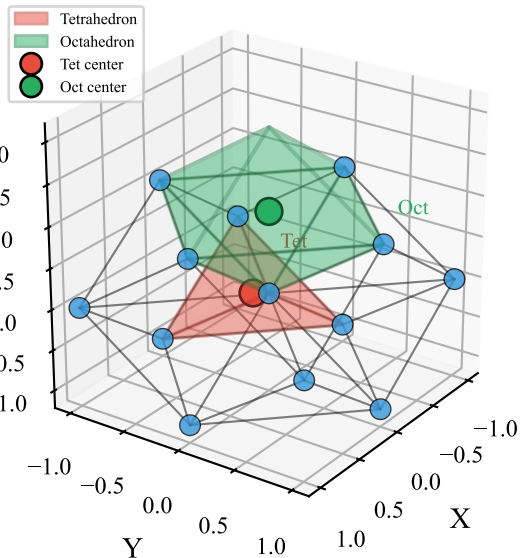


(a) Tet + Oct in FCC



(b) Properties

### Theorem 0.0.6: Spatial Extension

#### Honeycomb Structure:

- Tetrahedra + octahedra fill 3D space
- Also called "octet truss"
- Each vertex has 8 tetrahedra meeting

#### Stella Embedding:

- At each vertex: 8 tetrahedra partition into two groups of 4
- These form two interpenetrating tetrahedra
- = Stella octangula (Thm 0.0.3)

#### FCC Lattice Coordinates:

$$\Lambda_{FCC} = \{(n_1, n_2, n_3) \in \mathbb{Z}^3 : n_1 + n_2 + n_3 \equiv 0 \pmod{2}\}$$

#### Uniqueness: Dihedral angle argument

- Regular tetrahedron:  $\arccos(1/3) \approx 70.5^\circ$
- $5 \times 70.5^\circ = 352.7^\circ$  (gap),  $6 \times 70.5^\circ = 423.2^\circ$  (overlap)
- $\Rightarrow$  Must include octahedra: unique solution

#### Continuum Limit:

- $O_h$  symmetry  $\rightarrow SO(3)$  invariance
- Emergent metric assigns distances
- Bootstrap resolved: no prior metric needed

Lean 4: honeycomb\_uniqueness