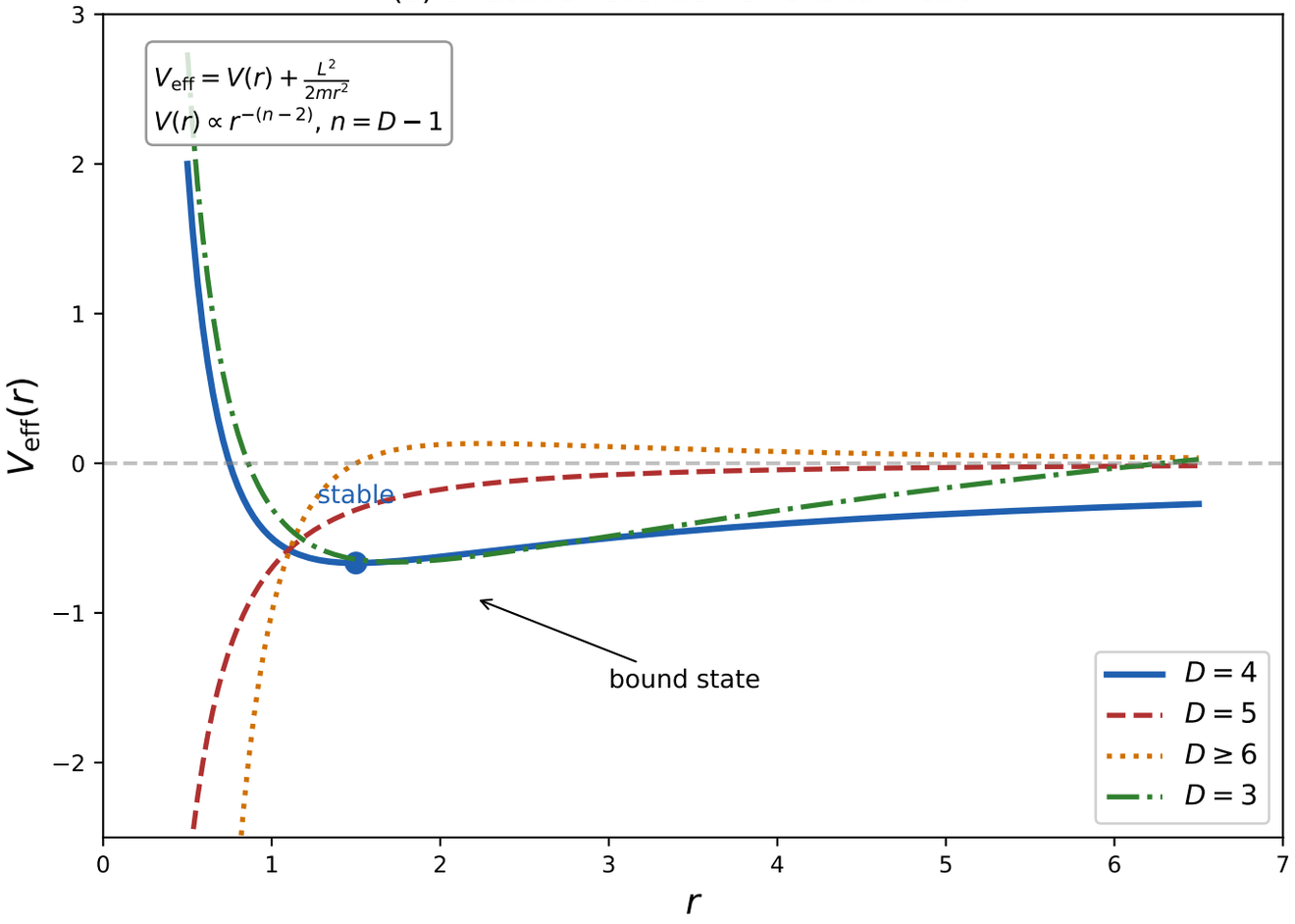
## (a) Effective Potential for Orbital Motion




## (b) Dimensional Selection: Four Constraints

| $D$ | P1: Gravity $n \leq 3$ | P2: Atoms $n = 3$ | P3: Huygens $n \geq 3$, odd | P4: Knots $n = 3$ | Result |
|---|---|---|---|---|---|
| $D = 2$ | ✓ | ✗ | ✗ | ✗ | — |
| $D = 3$ | ✓ | ✗ | ✗ | ✗ | — |
| $D = 4$ | ✓ | ✓ | ✓ | ✓ | **D = 4 UNIQUE** |
| $D = 5$ | ✗ | ✗ | ✗ | ✗ | — |
| $D = 6$ | ✗ | ✗ | ✓ | ✗ | — |
| $D = 7$ | ✗ | ✗ | ✗ | ✗ | — |

$$\{n \leq 3\} \cap \{n = 3\} \cap \{n \geq 3, \text{ odd}\} \cap \{n = 3\} = \{3\}$$
$$\Rightarrow D = n + 1 = 4$$

(P1) Gravitational stability: stable orbits require $D \leq 4$

(P2) Atomic stability: discrete spectrum requires $D = 4$

(P3) Huygens principle: sharp wave propagation for $D = 4, 6, 8, \ldots$ (odd $n \geq 3$)

(P4) Topological complexity: non-trivial knots require $D = 4$