Combinatorial Optimization and Modern Heuristics: Assignment 1

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Chapter 1 Problems

Problem 1(d)

Find a cylinder with a given surface area A that has the largest volume V.

Solution:

Okay so this is just a basic b kinda maximization problem. Oh Also we could just make the cylinder out of a bubble and physics would do gradient descent on the surface area... aph ph oh wait I got it backwards. What am I doing? Microdosing might actually not be effectibe as tehy say.

So surface area is $2\pi r^2 + 2\pi rl$ where r is the radius and l is the length. Volume is $\pi r^2 l$. So given a fixed area, maximize the volume. Simply dimply.

$$2\pi r^2 + 2\pi rl = A$$
$$\text{maximize } \pi r^2 l$$

Which is the same as maximizing rl. r and l are greater than 0.

$$r^2 + rl = \frac{A}{2\pi}$$
$$\text{maximize } r^2 l$$

solve for l and plug into maximization

$$l = \frac{A}{2\pi r} - r$$
$$\text{maximize } \frac{Ar}{2\pi} - r^3$$

Taking the derivative of the cost function and finding where it is zero gives

$$\frac{A}{2\pi} - 3r^2 = 0$$
$$3r^2 = \frac{A}{2\pi}$$

TODO ask about what "instance" means at office hours
So uh at this point we should take a derivative or somethign. Dool.
Restrict r to be from 0 to inf,

Problem 3

Show that the neighborhood defined in Example 1.5 for the MST is exact.

Example 1.5

In the MST, an important neighborhood is defined by

$N(f) = \{g : g \in F$ and g can be obtained from f as follows: add an edge e to the tree f, producing a cycle; then delete any edge on the cycle$\}$ $\square$

Solution:

Oh god what. Exact? Fujk I sould have read the textbood.

Problem 6

Suppose we are given a set S containing $2n$ integers, and we wish to partition it into two sets S_1 and S_2 so that $|S_1| = |S_2| = n$ and so that the sum of the numbers in S_1 is as close as possible to the sum of those in S_2. Let the neighborhood N be determined by all possible interchanges of two integers between S_1 and S_2. Is N exact?

Solution:

Uhhhhh probably not, that algorithm sound insufficient to get a global solution, assuming that's the same thing as exact.

Problem 9

Let $f(x)$ be convex in R^n. Is $f(x+b)$, where b is constant, convex in R^n?

Solution:

So we're gonna have to use the actual definition of convexity to make this more formal but I would say yeah duh. Just translating the graph around is not going to change convexity.

Problem 10

Let $f(x_i)$ be a convex function of the single variable x_i. Then $g(x) = f(x_i)$ can also be considered a function of $x \in R^n$. Is $g(x)$ convex in R^n?

Solution:

I don't really understand, isnt g just a copy of f? fok

Chapter 2 Problems

Problem 8

Show that the set of optimal point of an instance of LP is a convex set.

Solution:

Oooh I know that that's true! Well each boundary is a hyperplane, and the region bounded by hyperplanes must be convex... that shouldn't be too hard to prove lol