(1)

$x$ $\boxed{8 \to 7}$

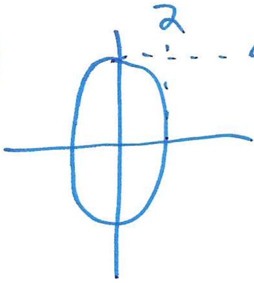
$y$ $\boxed{-3 \to 6}$

$t$ $\boxed{0 \to 1}$

$a + bt \to a + b\times 0 = 8 \to \boxed{a=8}$ $\quad 8 + b = 7 \to \boxed{b=-1}$

$c + dt \to \boxed{c=-3}$ $\quad -3 + d = 6 \to \boxed{d=9}$

(2)



2

4 $x: a \cos t$

$y: b \sin t$

$t = 0 \to y = 0 \quad x = a$

$t = \frac{\pi}{2} \to y = b \quad x = 0$

$$\boxed{\begin{array}{l} a = 2 \\ b = 4 \end{array}}$$

(3)

$x = e^{7t}$ find $y = f(x)$

$y = e^{3t}$

$\ln x = 7t \qquad \ln y = 3t$

$\frac{\ln x}{7} = t \qquad \frac{\ln y}{3} = t$

$\frac{\ln x}{7} = \frac{\ln y}{3} \qquad \frac{3}{7}\ln x = \ln y$

$e^{\frac{3}{7}\ln x} = y$

$(e^{\ln x})^{\frac{3}{7}} = y$

$$\boxed{x^{\frac{3}{7}} = y}$$

(4) $x = 5t^2 + 4t$ position find velocity x, y, total
$y = 4t^2 + 5$

velocity x: $5 \cdot 2 \cdot t + 4 \cdot 1 = 10t + 4$ at 3 $30 + 4 = \boxed{34 \text{ x velocity}}$

velocity y: $4 \cdot 2 \cdot t + 5 \cdot 0 = 8t$ $\boxed{24 \text{ y velocity}}$

speed: $\sqrt{vx^2 + vy^2}$ $\sqrt{34^2 + 24^2} = \boxed{\sqrt{1732} \approx 41.6 \text{ speed}}$

(5) $x = t^3 - 4t^2 + 1$ find $P_{x3}$ $V_{x3}$ $\frac{V_{y3}}{V_{x3}}$ $S_3$
$y = t^3 - 5t^2 + 8$ $P_{y3}$ $V_{y3}$

$P_{x3} = 3^3 - 4 \cdot 3^2 + 1 = 16$

$P_{y3} = 3^3 - 5 \cdot 3^2 + 8 = 20$

$V_{x3} \mid 3 \cdot t^2 - 4 \cdot 2 \cdot t + 1 \cdot 0 = 3t^2 - 8t$ at 3 $3 \cdot 3^2 - 8 \cdot 3 = 3$

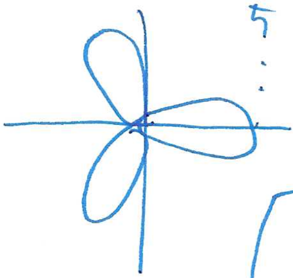
$V_{y3} \mid 3 \cdot t^2 - 5 \cdot 2 \cdot t + 8 \cdot 0 = 3t^2 - 10t$ $3 \cdot 3^2 - 10 \cdot 3 = -3$

$\frac{V_{y3}}{V_{x3}}$ $\frac{-3}{3} = -1$

$S_3 \mid \sqrt{V_{y3}^2 + V_{x3}^2}$ $\sqrt{3^2 + (-3)^2} = \sqrt{18} \approx 4.24$

$x(3) = 16$
$y(3) = 20$
$\frac{dx}{dt}(3) = 3$
$\frac{dy}{dt}(3) = -3$
$\frac{dy}{dx}(3) = -1$
$s(3) = \sqrt{18} \approx 4.24$

⑥



5 $r = 5\cos(3\theta)$

$0 = 5\cos(3\theta)$

$\frac{\cos^{-1}(0)}{3} = \theta$

$\cos(\theta) = 0$ at $\frac{\pi}{2}$ so $\frac{\pi}{2 \cdot 3}$

1 leaf $-\frac{\pi}{6} \to \frac{\pi}{6}$

2 leaf $\frac{\pi}{6}$ to $\frac{3\pi}{6}$

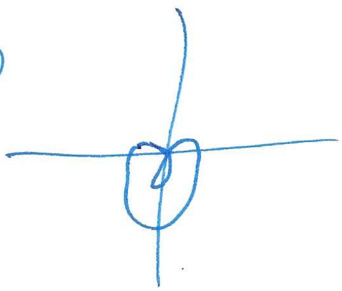
3 leaf $\frac{3\pi}{6}$ to $\frac{5\pi}{6}$ or $-\frac{\pi}{6}$

$\theta = 0, 2\pi$: symmetrical
$\theta = \frac{\pi}{2}$: not symmetrical

zeros: $\frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}$

rose

⑦

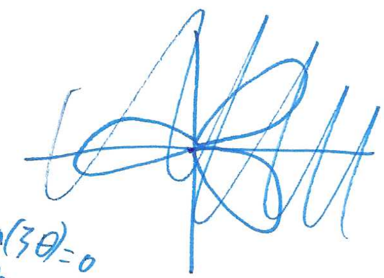


limacon

$\theta = 0, 2\pi$ not symmetrical
$\theta = \frac{\pi}{2}$ symmetrical

⑧

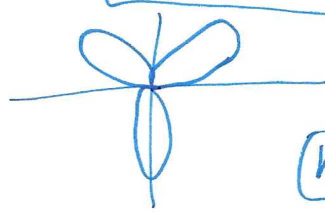
$6\sin(3\theta)$



$6\sin(3\theta)$ max is where
$3\theta = \frac{\pi}{2}$ | $\theta = \frac{\pi}{6}$
$6\sin(3 \cdot \frac{\pi}{6}) = 6$

$|r|$ max $= 6$

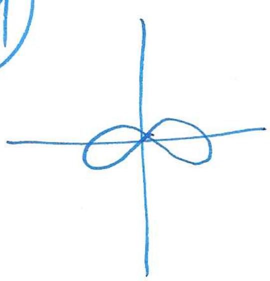
$r = 0$ when $\sin(3\theta) = 0$
$\sin(\phi) = 0$ when $\phi = \pi, 0, 2\pi$
$\phi = 3\theta$ so $\frac{\pi}{3}, 0, \frac{2\pi}{3}$ $r = 0$ at



symmetric on $\theta = \frac{\pi}{2}$

rose

⑨



rose

$|r|$ max is when $\cos 2\theta = 1$
$\cos\phi = 1$ when $\phi = 0, \pi, 2\pi$
$2\theta = \phi \to 0, \frac{\pi}{2}, \pi$
$|r|$ max at 0
$|r|\max = \sqrt{36\cos(0)} = \sqrt{1} \cdot \sqrt{36} = 6$

$|r|$ max $= 6$

symmetric $\theta = 0, 2\pi$
and
$\theta = \frac{\pi}{2}$

$r = 0$ when $\cos 2\theta = 0$
$\cos\phi = 0$ at $\frac{\pi}{2}, \frac{3\pi}{2}$
$\phi = 2\theta$ so

$r = 0$ at $\frac{\pi}{4}, \frac{3\pi}{4}$

(10) $r=8e^{2\theta}$ $\quad 0 \to 2\pi$ $\quad$ arc length $\quad r'=2\cdot 8e^{2\theta}$

$$\int_a^b \sqrt{f(x)^2+f'(x)^2}$$

$$\int_0^{2\pi}\sqrt{(8e^{2\theta})^2+(16e^{2\theta})^2} = \int_0^{2\pi}\sqrt{64e^{4\theta}+16^2e^{4\theta}}$$

$$= \int_0^{2\pi}\sqrt{(64+16^2)e^{4\theta}} = \int_0^{2\pi} e^{2\theta}\sqrt{64+16^2}$$

$$= \sqrt{64+16^2}\left[\frac{e^{2\theta}}{2}\right]_0^{2\pi} = \sqrt{64+16^2}\left(\frac{e^{4\pi}}{2}-1\right)$$

$$= \frac{(e^{4\pi}-2)\sqrt{64+16\cdot 16}}{2} = (e^{4\pi}-2)\frac{\sqrt{320}}{2} = (e^{4\pi}-2)\frac{\sqrt{320}}{\sqrt{4}}$$

$$\boxed{=(e^{4\pi}-2)\sqrt{80}}$$

(11) $r=4\sin(5\theta)$ $\quad$ area of 1 leaf

$0=4\sin(5\theta)$

$\frac{0}{4}=\sin(5\theta) \quad =0=\sin 5\theta$

$\arcsin(0)=5\theta \qquad \arcsin(0)= \quad 0,\pi$

$\frac{\arcsin(0)}{5}=\theta$

$\theta=\frac{0}{5},\ \frac{\pi}{5} = 0, \frac{\pi}{5}$

$\frac{1}{2}\int_a^b (4\sin(5\theta))^2\,d\theta$

$\frac{1}{2}\int_a^b 16\sin^2(5\theta)\,d\theta$

$8\int_a^b \frac{1-\cos(10\theta)}{2}\,d\theta$

$4\int_a^b 1-\cos(10\theta)\,d\theta$

$4\left[\theta-\frac{\sin(10\theta)}{10}\right]_a^b$

$4\left[\frac{\pi}{5}-\frac{\sin(2\pi)}{10}-\left(0-\frac{\sin(0)}{10}\right)\right]$

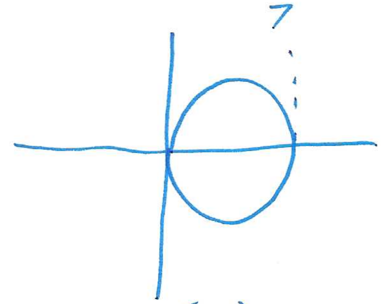
$4\left[\frac{\pi}{5}-0-(0-0)\right]$

$$\boxed{\frac{4\pi}{5}}$$

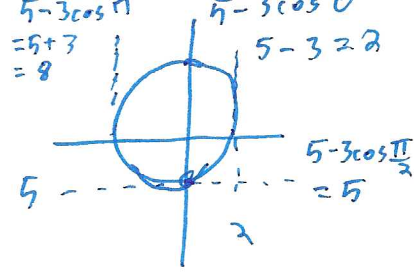
12

inside $7\cos(\theta)$

outside $5-3\cos(\theta)$




$7\cos(\theta)$




$5-3\cos\theta$

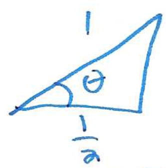
$$7\cos(\theta)=5-3\cos\theta$$

$$7\cos\theta+3\cos\theta=5$$

$$10\cos\theta=5$$

$$\cos\theta=\frac{1}{2}$$

$$\cos^{-1}\left(\tfrac{1}{2}\right)=\theta \rightarrow$$




$$\theta=\frac{\pi}{3},\ -\frac{\pi}{3}$$

$$\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(7\cos\theta)^2 d\theta-\frac{1}{2}\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}}(5-3\cos\theta)^2 d\theta$$

$$\frac{7^2}{2}\int_{\frac{\pi}{-3}}^{\frac{\pi}{3}}\cos^2\theta\, d\theta-\frac{1}{2}\int_{\frac{\pi}{-3}}^{\frac{\pi}{3}}5^2-30\cos\theta+9\cos^2\theta\, d\theta$$

$$\frac{7^2}{2}\int_{\frac{\pi}{-3}}^{\frac{\pi}{3}}\frac{1+\cos 2\theta}{2}d\theta-\frac{1}{2}\int_{\frac{\pi}{-3}}^{\frac{\pi}{3}}25-30\cos\theta+9\,\frac{1+\cos 2\theta}{2}$$

$$\frac{7^2}{4}\left[\theta+\frac{\sin 2\theta}{2}\right]_{\frac{\pi}{-3}}^{\frac{\pi}{3}}-\frac{1}{2}\left[25\theta-30\sin\theta+\frac{9}{2}\left(\theta+\frac{\sin 2\theta}{2}\right)\right]_{\frac{\pi}{-3}}^{\frac{\pi}{3}}$$

$$\frac{7^2}{4}\left[\frac{\pi}{3}+\frac{\sin\frac{2\pi}{3}}{2}--\frac{\pi}{3}-\frac{\sin\frac{-2\pi}{3}}{2}\right]-\frac{1}{2}\left[\frac{25\pi}{3}-30\sin\frac{\pi}{3}+\frac{9}{2}\left(\frac{\pi}{3}+\frac{\sin\frac{2\pi}{3}}{2}\right)-\frac{-25\pi}{3}--30\sin\frac{-\pi}{3}+\frac{9}{2}\left(\frac{-\pi}{3}+\frac{\sin\frac{-2\pi}{3}}{2}\right)\right]$$

$$\frac{7^2}{4}\left[\frac{2\pi}{3}+\frac{2\sqrt{3}}{4}\right]-\frac{1}{2}\left[\frac{50\pi}{3}-\frac{60\sqrt{3}}{2}+\frac{18\pi}{6}+\frac{9\sqrt{3}}{2}\right]=\frac{7^2\cdot 2\cdot\pi}{12}+\frac{7^2\sqrt{3}}{8}-\frac{1}{2}\left[\frac{82\pi}{6}-\frac{51\sqrt{3}}{2}\right]$$

$$=\frac{(7^2\cdot 2-82)\pi}{12}+\frac{(7^2+102)\sqrt{3}}{8}=\frac{16\pi}{12}+\frac{151\sqrt{3}}{8}$$

$$\boxed{=\frac{4}{3}\pi+18.875\sqrt{3}\approx 36.88}$$