

Task 2 – Regression using PyTorch (35%)

Part 1 (13 marks):

1. $y = 3(t^2 + 2)^2$, where $t = 2x + c$

```
In [ ]: import torch

# Initialize variables
x = torch.tensor(1.0, requires_grad=True)
c = torch.tensor(1.0)

# Define equation 1
t = 2*x + c
y = 3 * (t**2 + 2)**2

# Calculate gradient
y.backward()

# Print the gradient dy/dx
print("Gradient dy/dx for equation 1:", x.grad)
```

Gradient dy/dx for equation 1: tensor(792.)

2. $y = 3(s^3 + s) + 2c^4$, where $s = 2x$

```
In [ ]: # Reset gradient
x.grad = None

# Define equation 2
s = 2*x
y = 3 * (s**3 + s) + 2*c**4

# Calculate gradient
y.backward()

# Print the gradient dy/dx
print("Gradient dy/dx for equation 2:", x.grad)
```

Gradient dy/dx for equation 2: tensor(78.)

3. $y = 2t + c$, where $t = (p^2 + 2p + 3)^2$, $p = 2r^3 + 3r$, $r = 2q + 3$, $q = 2x + c$

```
In [ ]: # Reset gradient
x.grad = None


# Define equation 3
q = 2*x + c
r = 2*q + 3
```

```
p = 2*r**3 + 3*r
t = (p**2 + 2*p + 3)**2
y = 2*t + c

# Calculate gradient
y.backward()

# Print the gradient dy/dx
print("Gradient dy/dx for equation 3:", x.grad)
```

Gradient dy/dx for equation 3: tensor(5.1347e+13)

Q4) Draw (by-hand) two separate diagrams/ computational maps for the functions shown in Q1 & Q2 of this Task. That is, the diagram/ computational map should highlight the significant sub-components, demonstrating how inputs x & c are converted to the final function y 

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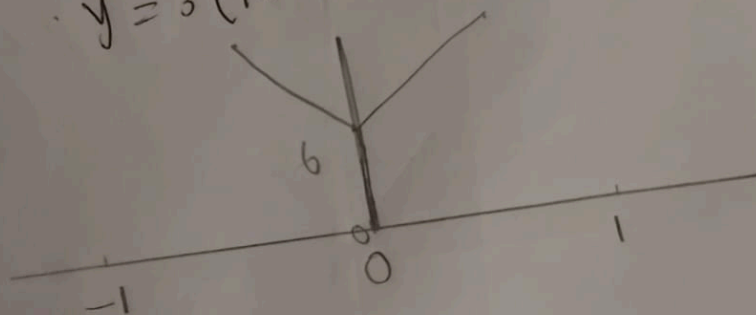
$$y = 3(t^2 + 2)^2 \quad t = 2u + c$$

$$y = 3(16c^4 + 8c^2u^2 + c^4 + 8u^2c + 4(u+2))$$

Assume

$$c = 0$$

$$y = 3(16u^4 + 8u^2 + 2)$$



$$y = 3(s^3 + 2s) + 2e^4, \text{ when } s = 2u$$

$$y = 3((2u)^3 + 2u) + 2e^4$$

$$y = 3(8u^3 + 2u) + 2e^4$$

$$y = 24u^3 + 6u + 2e^4$$

$$y = 24u^3 + 6u + 2e^4$$

Assume, $e = 0$

$$y = 24u^3 + 6u + 0$$

$$y = 24u^3 + 6u$$

when

u	-1	0	1
y	-30	0	30

Part 2 (22 marks):

Using PyTorch without helper functions (without using `torch.nn` or `torch.optim`), calculate the values of a & b that best fit the model $f(x)$ using the gradient descent algorithms with the following

$$f(x) = e^{-ax} + 2ax + b$$

where:

Certainly, here are the complete X and Y values:

$$X = [-2, -1.9, -1.8, -1.7, -1.6, -1.5, -1.4, -1.3, -1.2, -1.1, -1, -0.8, -0.7, -0.6, -1.5, 1.6, 1.7, 1.8, 1.9, 2.0]$$

$$Y = [6.255, 6.121, 6.005, 5.907, 5.825, 5.758, 5.704, 5.664, 5.636, 5.62, 5.614, 5.618, 5.63, 6.497, 6.593, 6.691, 6.793, 6.897, 7.003, 7.112, 7.223, 7.335, 7.45, 7.566, 7.684, 7.804, 7.9$$

```
In [ ]: import torch
import numpy as np

# Given data
X = torch.tensor([-2, -1.9, -1.8, -1.7, -1.6, -1.5, -1.4, -1.3, -1.2,
-1.1,
                    -1, -0.8, -0.7, -0.6, -0.5, -0.4, -0.3, -0.2, -0.1, 0,
                    0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1,
                    1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0],
dtype=torch.float32)
Y = torch.tensor([6.255, 6.121, 6.005, 5.907, 5.825, 5.758, 5.704, 5.664,
5.636, 5.62,
                    5.614, 5.618, 5.631, 5.652, 5.682, 5.719, 5.763, 5.814,
5.87, 5.933,
                    6.0, 6.072, 6.149, 6.231, 6.316, 6.405, 6.497, 6.593,
6.691, 6.793,
                    6.897, 7.003, 7.112, 7.223, 7.335, 7.45, 7.566, 7.684,
7.804, 7.924], dtype=torch.float32)

# Initialize parameters a and b with random values
a = torch.randn(1, requires_grad=True, dtype=torch.float32)
b = torch.randn(1, requires_grad=True, dtype=torch.float32)

# Learning rate
learning_rate = 0.01

# Number of epochs
epochs = 1000

# Gradient descent
for epoch in range(epochs):
    # Forward pass: compute predicted y
    y_pred = torch.exp(-a*X) + 2*a*X + b

    # Compute and print loss
    loss = torch.mean((y_pred - Y)**2)

    # Backpropagation to compute gradients of a and b with respect to loss
    loss.backward()

    # Update parameters using gradient descent
    with torch.no_grad():
        a -= learning_rate * a.grad
```

```
b -= learning_rate * b.grad

# Manually zero the gradients after updating
a.grad.zero_()
b.grad.zero_()

# Print the optimized values of a and b
print("Optimized value of a:", a.item())
print("Optimized value of b:", b.item())
```

```
Optimized value of a: 0.6887515783309937
Optimized value of b: 4.952696800231934
```