Untitled

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PORTFOLIO FINANCIAL ANALYSIS

In portfolio financial analysis, several models and techniques are commonly used to assess risk, return, and overall performance. Here are some key models:

1. Modern Portfolio Theory (MPT)

- Developed by: Harry Markowitz
- Concept: Investors can construct portfolios to maximize expected return based on a given level of market risk, emphasizing diversification.
- **Key Metric:** Efficient Frontier, which represents the set of optimal portfolios offering the highest expected return for a defined level of risk.

2. Capital Asset Pricing Model (CAPM)

- Concept: Describes the relationship between systematic risk and expected return for assets, particularly stocks.
- Formula:

$$E(R_i) = R_f + \beta_i (E(R_m) - R_f)$$

Where $E(R_i)$ is the expected return on the capital asset, R_f is the risk-free rate, β_i is the beta of the security, and $E(R_m)$ is the expected market return.

3. Arbitrage Pricing Theory (APT)

- Concept: An alternative to CAPM, APT considers multiple factors that might affect an asset's return, instead of just market risk.
- Formula:

$$E(R_i) = R_f + \beta_{i1}F_1 + \beta_{i2}F_2 + ... + \beta_{in}F_n$$

Where F represents various factors.

4. Fama-French Three-Factor Model

- Developed by: Eugene Fama and Kenneth French
- Concept: Expands on CAPM by adding size risk (SMB, Small Minus Big) and value risk (HML, High Minus Low) factors.
- Formula:

$$E(R_i) = R_f + \beta_{i1}(E(R_m) - R_f) + \beta_{i2}SMB + \beta_{i3}HML$$

5. Black-Litterman Model

- Developed by: Fischer Black and Robert Litterman
- Concept: Combines investor views with market equilibrium to estimate the expected returns of assets.
- Use: Often used in portfolio optimization to create a more stable and intuitive estimation of expected returns.

6. Value at Risk (VaR)

- Concept: Measures the potential loss in value of a portfolio over a defined period for a given confidence interval
- Usage: Helps in risk management to understand the maximum potential loss.

7. Monte Carlo Simulation

- Concept: Uses repeated random sampling to simulate the performance of a portfolio under various conditions.
- Usage: Useful for estimating the impact of risk and uncertainty in financial and investment decisions.

8. Sharpe Ratio

- Concept: Measures the performance of an investment compared to a risk-free asset, after adjusting for its risk.
- Formula:

Sharpe Ratio =
$$\frac{E(R_p) - R_f}{\sigma_p}$$

Where $E(R_p)$ is the expected portfolio return, R_f is the risk-free rate, and σ_p is the portfolio standard deviation.

These models and tools are essential for constructing, analyzing, and managing portfolios to achieve desired risk-return profiles.

getting the data

Installing and importing thelibrary

```
#install.packages("quantmod")
library(quantmod)

## Loading required package: xts

## Loading required package: zoo

## ## Attaching package: 'zoo'

## The following objects are masked from 'package:base':

## as.Date, as.Date.numeric

## Loading required package: TTR

## Registered S3 method overwritten by 'quantmod':

## method from

## as.zoo.data.frame zoo
```

importing data from yahoo

```
# Load the quantmod package
library(quantmod)
# Get S&P 500 (SPY) data
getSymbols("SPY", src = "yahoo", from = "2000-01-01", to = Sys.Date())
## [1] "SPY"
spy_data <- SPY
# Get Dow Jones Industrial Average (DJI) data
getSymbols("^DJI", src = "yahoo", from = "2000-01-01", to = Sys.Date())
## [1] "DJI"
dji_data <- DJI
colnames(spy_data)
## [1] "SPY.Open"
                      "SPY.High"
                                     "SPY.Low"
                                                     "SPY.Close"
                                                                    "SPY. Volume"
## [6] "SPY.Adjusted"
colnames(dji_data)
## [1] "DJI.Open"
                      "DJI.High"
                                     "DJI.Low"
                                                     "DJI.Close"
                                                                    "DJI.Volume"
## [6] "DJI.Adjusted"
#View(spy_data)
```

Calculating returns

```
# Calculate daily returns for SPY
spy_returns <- dailyReturn(spy_data[, "SPY.Adjusted"])
# Calculate daily returns for DJI
dji_returns <- dailyReturn(dji_data[, "DJI.Adjusted"])</pre>
```

Returns Summary

```
# Summary statistics for SPY returns
summary(spy_returns)
```

```
## Index daily.returns
## Min. :2000-01-03 Min. :-0.1094238
## 1st Qu.:2006-02-22 1st Qu.:-0.0046922
## Median :2012-04-07 Median : 0.0006730
## Mean :2012-04-07 Mean : 0.0003658
## 3rd Qu.:2018-05-23 3rd Qu.: 0.0060117
## Max. :2024-07-12 Max. : 0.1451976
```

```
# Summary statistics for DJI returns
summary(dji_returns)
```

```
## Index daily.returns

## Min. :2000-01-03 Min. :-0.1292655

## 1st Qu.:2006-02-22 1st Qu.:-0.0045844

## Median :2012-04-07 Median : 0.0004982

## Mean :2012-04-07 Mean : 0.0002723

## 3rd Qu.:2018-05-23 3rd Qu.: 0.0055937

## Max. :2024-07-12 Max. : 0.1136504
```

Vizualization

Closing prices

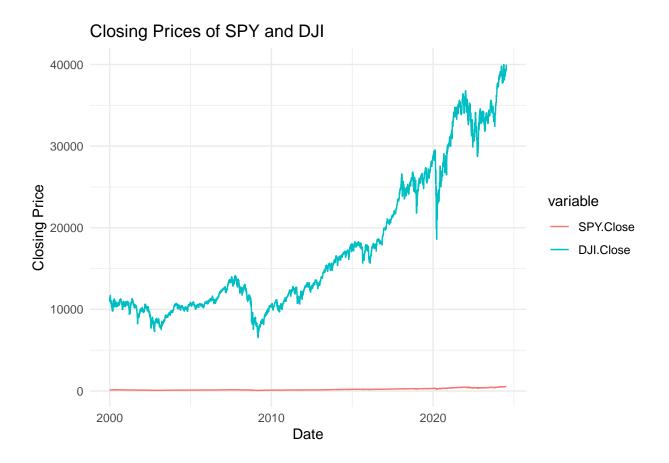
```
# Load necessary libraries
library(ggplot2)
library(reshape2)

# Combine data into one data frame for plotting
spy_close <- data.frame(Date = index(spy_data), SPY.Close = spy_data$SPY.Close)
dji_close <- data.frame(Date = index(dji_data), DJI.Close = dji_data$DJI.Close)

# Merge data frames by Date
combined_close <- merge(spy_close, dji_close, by = "Date", all = TRUE)

# Melt the data for ggplot2
combined_close_melt <- melt(combined_close, id.vars = "Date")

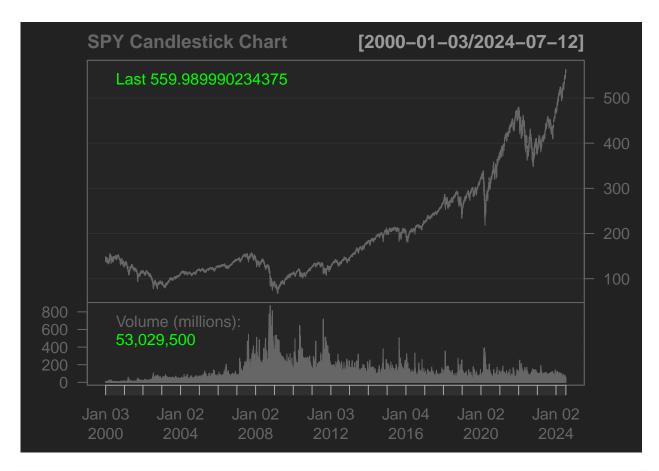
# Line plot
ggplot(combined_close_melt, aes(x = Date, y = value, color = variable)) +
    geom_line() +
    labs(title = "Closing Prices of SPY and DJI", x = "Date", y = "Closing Price") +
    theme_minimal()</pre>
```



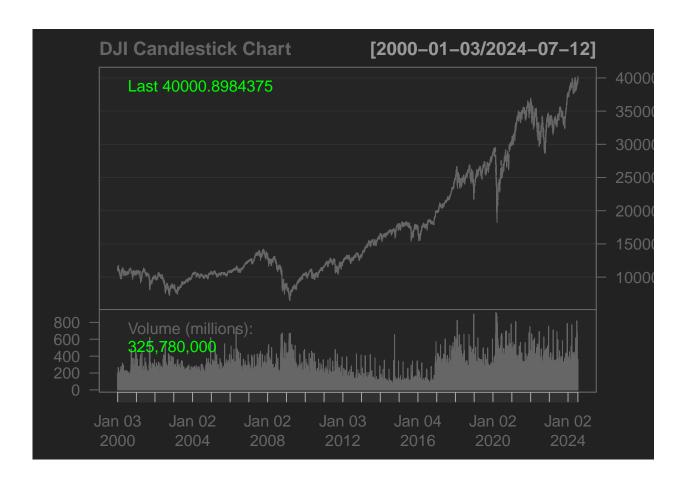
candlestick vizualization

```
# Load necessary library
library(quantmod)

# Candlestick chart for SPY
chartSeries(spy_data, type = "candlesticks", name = "SPY Candlestick Chart")
```



Candlestick chart for DJI
chartSeries(dji_data, type = "candlesticks", name = "DJI Candlestick Chart")

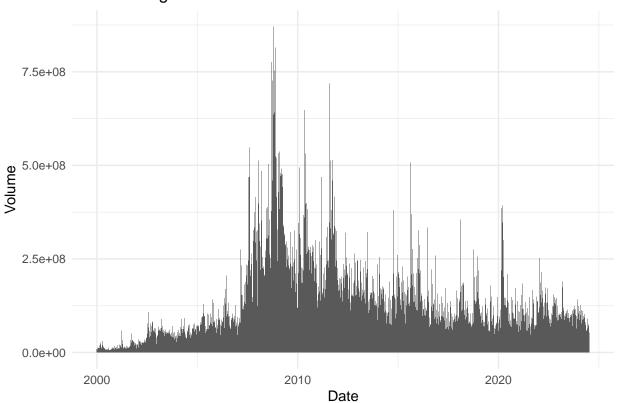


Volume plot

```
# SPY Volume Plot
ggplot(spy_close, aes(x = Date, y = spy_data$SPY.Volume)) +
   geom_bar(stat = "identity") +
   labs(title = "SPY Trading Volume", x = "Date", y = "Volume") +
   theme_minimal()
```

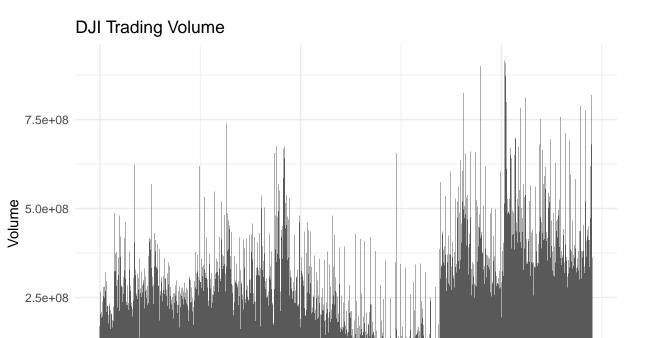
Don't know how to automatically pick scale for object of type <xts/zoo>.
Defaulting to continuous.

SPY Trading Volume



```
# DJI Volume Plot
ggplot(dji_close, aes(x = Date, y = dji_data$DJI.Volume)) +
  geom_bar(stat = "identity") +
  labs(title = "DJI Trading Volume", x = "Date", y = "Volume") +
  theme_minimal()
```

Don't know how to automatically pick scale for object of type <xts/zoo>.
Defaulting to continuous.



2010

Daily returns plot

2000

0.0e+00

```
# Calculate daily returns for SPY and DJI
spy_returns <- dailyReturn(spy_data[, "SPY.Adjusted"])
dji_returns <- dailyReturn(dji_data[, "DJI.Adjusted"])

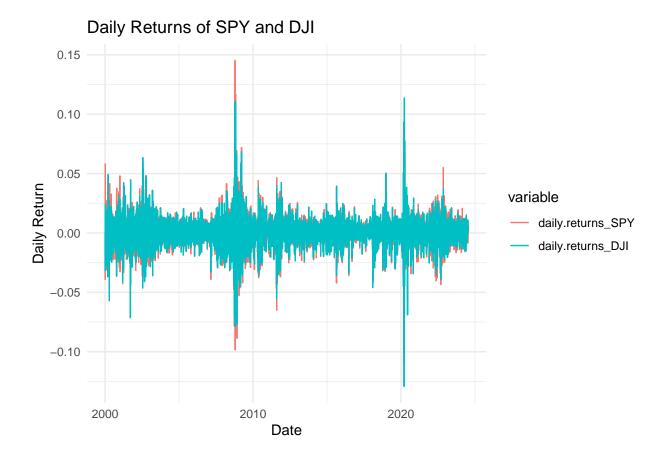
# Convert to data frames
spy_returns_df <- data.frame(Date = index(spy_returns), Return = coredata(spy_returns))
dji_returns_df <- data.frame(Date = index(dji_returns), Return = coredata(dji_returns))

# Combine returns for plotting
combined_returns <- merge(spy_returns_df, dji_returns_df, by = "Date", suffixes = c("_SPY", "_DJI"))
combined_returns_melt <- melt(combined_returns, id.vars = "Date")

# Line plot of daily returns
ggplot(combined_returns_melt, aes(x = Date, y = value, color = variable)) +
    geom_line() +
    labs(title = "Daily Returns of SPY and DJI", x = "Date", y = "Daily Return") +
    theme_minimal()</pre>
```

Date

2020



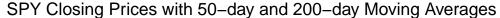
Moving averages plot

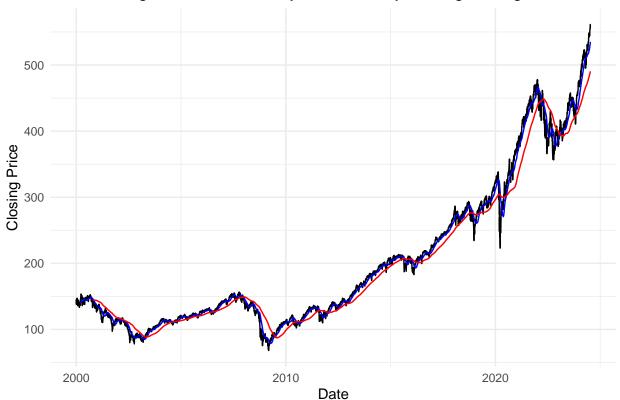
```
# Calculate 50-day and 200-day moving averages for SPY
spy_data$SPY.MA50 <- SMA(spy_data[, "SPY.Close"], n = 50)
spy_data$SPY.MA200 <- SMA(spy_data[, "SPY.Close"], n = 200)

# Plot SPY with moving averages
ggplot(spy_close, aes(x = Date, y = SPY.Close)) +
    geom_line() +
    geom_line(aes(y = spy_data$SPY.MA50), color = "blue") +
    geom_line(aes(y = spy_data$SPY.MA200), color = "red") +
    labs(title = "SPY Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices minimal()</pre>
```

Warning: Removed 49 rows containing missing values or values outside the scale range ## ('geom_line()').

Warning: Removed 199 rows containing missing values or values outside the scale range
('geom_line()').

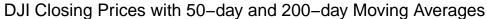




```
# Calculate 50-day and 200-day moving averages for DJI
dji_data$DJI.MA50 <- SMA(dji_data[, "DJI.Close"], n = 50)
dji_data$DJI.MA200 <- SMA(dji_data[, "DJI.Close"], n = 200)

# Plot DJI with moving averages
ggplot(dji_close, aes(x = Date, y = DJI.Close)) +
    geom_line() +
    geom_line(aes(y = dji_data$DJI.MA50), color = "blue") +
    geom_line(aes(y = dji_data$DJI.MA200), color = "red") +
    labs(title = "DJI Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages", x = "Date", y = "Closing Prices with 50-day and 200-day Moving Averages".</pre>
```

```
## Warning: Removed 49 rows containing missing values or values outside the scale range
## ('geom_line()').
## Removed 199 rows containing missing values or values outside the scale range
## ('geom_line()').
```

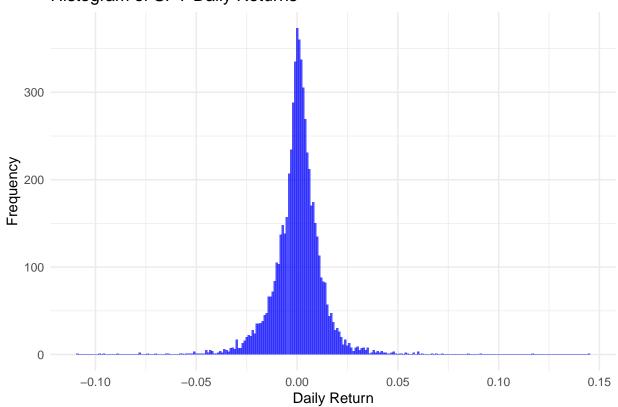




Returns Histogram

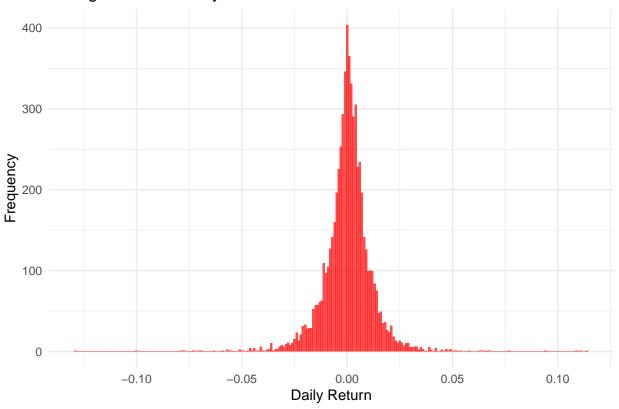
```
# Histogram of SPY returns
ggplot(spy_returns_df, aes(x = daily.returns)) +
  geom_histogram(binwidth = 0.001, fill = "blue", alpha = 0.7) +
  labs(title = "Histogram of SPY Daily Returns", x = "Daily Return", y = "Frequency") +
  theme_minimal()
```





```
# Histogram of DJI returns
ggplot(dji_returns_df, aes(x = daily.returns)) +
  geom_histogram(binwidth = 0.001, fill = "red", alpha = 0.7) +
  labs(title = "Histogram of DJI Daily Returns", x = "Daily Return", y = "Frequency") +
  theme_minimal()
```

Histogram of DJI Daily Returns



Summary Statistics

```
# Summary statistics for SPY data
print("Summary statistics for SPY data:")
```

[1] "Summary statistics for SPY data:"

summary(spy_data)

```
Index
                            SPY.Open
                                             SPY.High
                                                             SPY.Low
##
           :2000-01-03
                               : 67.95
                                                 : 70.0
                                                                : 67.1
   Min.
                         Min.
                                          Min.
                                                          Min.
                         1st Qu.:119.70
                                          1st Qu.:120.5
   1st Qu.:2006-02-22
                                                          1st Qu.:119.1
##
   Median :2012-04-07
                         Median :147.48
                                          Median :148.4
                                                          Median :146.4
           :2012-04-07
                                :204.11
                                                 :205.3
                                                                 :202.8
##
   Mean
                         Mean
                                          Mean
                                                          Mean
##
   3rd Qu.:2018-05-23
                         3rd Qu.:268.08
                                          3rd Qu.:270.2
                                                          3rd Qu.:266.8
##
   Max.
           :2024-07-12
                         Max.
                                :561.44
                                          Max.
                                                 :563.7
                                                                 :557.1
                                                          Max.
##
##
      SPY.Close
                       SPY. Volume
                                          SPY.Adjusted
                                                             SPY.MA50
##
  Min. : 68.11
                           : 1436600
                                         Min. : 50.86
                                                          Min.
                                                                : 79.1
                    Min.
                     1st Qu.: 51146925
##
   1st Qu.:119.84
                                         1st Qu.: 85.13
                                                          1st Qu.:119.6
  Median :147.42
                     Median : 80209450
                                         Median :110.01
                                                          Median :147.2
##
## Mean
         :204.13
                     Mean
                           :107186836
                                         Mean :175.60
                                                          Mean :203.0
   3rd Qu.:268.35
                     3rd Qu.:135968775
                                         3rd Qu.:243.14
                                                          3rd Qu.:268.8
```

```
## Max.
          :561.32
                   Max.
                         :871026300 Max.
                                              :561.32
                                                       Max.
                                                              :535.0
##
                                                       NA's
                                                              :49
##
     SPY.MA200
  Min. : 87.21
##
##
   1st Qu.:118.94
## Median:145.31
## Mean :200.00
## 3rd Qu.:269.90
## Max.
        :490.52
## NA's
        :199
# Summary statistics for DJI data
print("Summary statistics for DJI data:")
## [1] "Summary statistics for DJI data:"
summary(dji_data)
                          DJI.Open
                                          DJI.High
                                                         DJI.Low
##
       Index
          :2000-01-03
## Min.
                       Min. : 6547
                                       Min. : 6710
                                                      Min.
                                                            : 6470
  1st Qu.:2006-02-22
                       1st Qu.:10588
                                       1st Qu.:10641
                                                      1st Qu.:10522
## Median :2012-04-07
                       Median :13239
                                      Median :13303
                                                      Median :13168
## Mean
         :2012-04-07
                       Mean :17495
                                       Mean :17596
                                                      Mean
                                                            :17389
## 3rd Qu.:2018-05-23
                       3rd Qu.:24463
                                       3rd Qu.:24603
                                                      3rd Qu.:24247
## Max.
         :2024-07-12
                       Max. :39990
                                      Max.
                                             :40257
                                                      Max.
                                                            :39865
##
##
     DJI.Close
                    DJI.Volume
                                       DJI.Adjusted
                                                        DJI.MA50
                  Min. : 8410000
                                      Min. : 6547
                                                     Min. : 7522
## Min. : 6547
                 1st Qu.:163295000
                                                     1st Qu.:10570
   1st Qu.:10588
                                      1st Qu.:10588
## Median :13240 Median :236575000
                                      Median :13240
                                                     Median :13245
## Mean :17498
                 Mean :244573917
                                      Mean :17498
                                                     Mean :17437
   3rd Qu.:24414
                  3rd Qu.:307667500
                                      3rd Qu.:24414
                                                     3rd Qu.:24478
##
  Max. :40004
                  Max. :915990000
                                      Max. :40004
                                                     Max. :39106
##
##
                                                     NA's :49
     DJI.MA200
## Min. : 8280
## 1st Qu.:10529
## Median :13126
## Mean
         :17260
## 3rd Qu.:24625
## Max. :37424
## NA's
          :199
# Calculate daily returns for SPY
spy_returns <- dailyReturn(spy_data[, "SPY.Adjusted"])</pre>
# Convert to data frame
spy_returns_df <- data.frame(Date = index(spy_returns), Return = coredata(spy_returns))</pre>
# Summary statistics for SPY returns
print("Summary statistics for SPY returns:")
```

[1] "Summary statistics for SPY returns:"

summary(spy_returns_df)

```
##
         Date
                         daily.returns
##
   Min.
           :2000-01-03
                         Min.
                               :-0.1094238
   1st Qu.:2006-02-22
                         1st Qu.:-0.0046922
##
  Median :2012-04-07
                         Median: 0.0006730
##
  Mean
           :2012-04-07
                         Mean : 0.0003658
##
   3rd Qu.:2018-05-23
                         3rd Qu.: 0.0060117
##
  {\tt Max.}
           :2024-07-12
                         Max.
                              : 0.1451976
# Calculate daily returns for DJI
dji_returns <- dailyReturn(dji_data[, "DJI.Adjusted"])</pre>
# Convert to data frame
dji_returns_df <- data.frame(Date = index(dji_returns), Return = coredata(dji_returns))</pre>
# Summary statistics for DJI returns
print("Summary statistics for DJI returns:")
```

[1] "Summary statistics for DJI returns:"

```
summary(dji_returns_df)
```

```
##
        Date
                        daily.returns
##
   Min.
           :2000-01-03
                        Min.
                               :-0.1292655
##
   1st Qu.:2006-02-22
                        1st Qu.:-0.0045844
  Median :2012-04-07
                        Median: 0.0004982
##
  Mean
          :2012-04-07
                        Mean : 0.0002723
   3rd Qu.:2018-05-23
##
                        3rd Qu.: 0.0055937
   Max.
          :2024-07-12
                        Max. : 0.1136504
```

MODELs

Modern Portfolio Theory (MPT) Explanation

Modern Portfolio Theory (MPT) is a framework for constructing a portfolio of assets that maximizes expected return for a given level of risk. It was introduced by Harry Markowitz in his paper "Portfolio Selection" published in 1952. The key concept behind MPT is diversification, which helps reduce the overall risk of the portfolio.

Mathematical Explanation

1. Expected Return of a Portfolio $(E(R_n))$:

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

Where:

- $E(R_p)$ is the expected return of the portfolio.
- w_i is the weight of asset i in the portfolio.

- $E(R_i)$ is the expected return of asset i.
- 2. Variance of a Portfolio (σ_n^2) :

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$$

Where:

- σ_p^2 is the variance of the portfolio.
- σ_{ij} is the covariance between the returns of assets i and j.
- 3. **Efficient Frontier:** The efficient frontier is the set of portfolios that offers the highest expected return for a defined level of risk (or the lowest risk for a given level of expected return).

Steps to Perform MPT in R

To perform MPT on your data frames and visualize the efficient frontier, we can use the PortfolioAnalytics package. Below are the steps:

```
install.packages("quantmod")
1. Install and Load Necessary Packages
## Installing package into '/home/dragon/R/x86_64-pc-linux-gnu-library/4.4'
## (as 'lib' is unspecified)
install.packages("PortfolioAnalytics")
## Installing package into '/home/dragon/R/x86_64-pc-linux-gnu-library/4.4'
## (as 'lib' is unspecified)
install.packages("PerformanceAnalytics")
## Installing package into '/home/dragon/R/x86_64-pc-linux-gnu-library/4.4'
## (as 'lib' is unspecified)
install.packages("ROI")
## Installing package into '/home/dragon/R/x86_64-pc-linux-gnu-library/4.4'
## (as 'lib' is unspecified)
install.packages("ROI.plugin.quadprog")
## Installing package into '/home/dragon/R/x86_64-pc-linux-gnu-library/4.4'
## (as 'lib' is unspecified)
```

```
install.packages("ROI.plugin.glpk")
## Installing package into '/home/dragon/R/x86_64-pc-linux-gnu-library/4.4'
## (as 'lib' is unspecified)
library(quantmod)
library(PortfolioAnalytics)
## Loading required package: foreach
## Loading required package: PerformanceAnalytics
##
## Attaching package: 'PerformanceAnalytics'
## The following object is masked from 'package:graphics':
##
##
       legend
## Registered S3 method overwritten by 'PortfolioAnalytics':
    method
                      from
##
     print.constraint ROI
library(PerformanceAnalytics)
library(ROI)
## ROI: R Optimization Infrastructure
## Registered solver plugins: nlminb, symphony, glpk, quadprog.
## Default solver: auto.
##
## Attaching package: 'ROI'
## The following objects are masked from 'package:PortfolioAnalytics':
##
##
       is.constraint, objective
library(ROI.plugin.quadprog)
library(ROI.plugin.glpk)
# Load data for SPY and DJI
getSymbols("SPY", src = "yahoo", from = "2000-01-01", to = Sys.Date())
2. Get Data and Calculate Returns
```

[1] "SPY"

```
getSymbols("^DJI", src = "yahoo", from = "2000-01-01", to = Sys.Date())

## [1] "DJI"

# Calculate daily returns
spy_returns <- dailyReturn(Ad(SPY))
dji_returns <- dailyReturn(Ad(DJI))

# Combine returns into one data frame
returns_data <- na.omit(merge(spy_returns, dji_returns))
colnames(returns_data) <- c("SPY", "DJI")</pre>
```

```
# Define portfolio
portfolio <- portfolio.spec(assets = colnames(returns_data))

# Add constraints
portfolio <- add.constraint(portfolio, type = "full_investment")
portfolio <- add.constraint(portfolio, type = "box", min = 0.01, max = 0.99)

# Add objectives
portfolio <- add.objective(portfolio, type = "return", name = "mean")
portfolio <- add.objective(portfolio, type = "risk", name = "StdDev")</pre>
```

3. Define Portfolio and Constraints

```
# Optimize portfolio with tracing enabled
opt_portfolio <- optimize.portfolio(returns_data, portfolio, optimize_method = "ROI", trace = TRUE)</pre>
```

4. Optimize Portfolio

```
# Check optimization results
print(opt_portfolio)
```

5. Plot Efficient Frontier

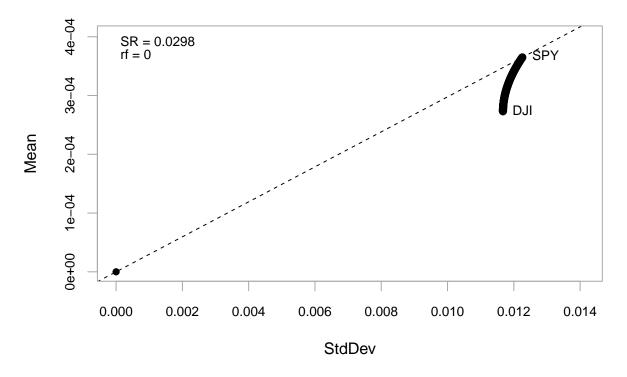
```
## *************************
## PortfolioAnalytics Optimization
## ***********************
##
## Call:
## optimize.portfolio(R = returns_data, portfolio = portfolio, optimize_method = "ROI",
## trace = TRUE)
##
```

```
## Optimal Weights:
##
   SPY DJI
## 0.99 0.01
##
## Objective Measure:
##
        mean
## 0.0003649
##
##
##
    StdDev
## 0.01225
# Efficient frontier
ef <- extractEfficientFrontier(opt_portfolio, match.col = "StdDev", n.portfolios = 100)</pre>
```

Warning: executing %dopar% sequentially: no parallel backend registered

```
# Plot efficient frontier
chart.EfficientFrontier(ef, match.col = "StdDev", main = "Efficient Frontier")
```

Efficient Frontier



Interpretation of Results

- 1. **Expected Return** ($E(R_p)$): This is the weighted average of the expected returns of the assets in the portfolio. It shows the return you can expect from the portfolio based on historical data.
- 2. Portfolio Variance (σ_p^2): This measures the risk of the portfolio. It considers not just the individual volatilities of the assets but also how they move together (covariance).

3.	Efficient Frontier: The efficient frontier plot shows the set of optimal portfolios. Portfolios on the efficient frontier offer the highest expected return for a given level of risk. Portfolios below the frontier are sub-optimal because they do not provide enough return for their level of risk.